Tentative Designs for X- and W-Band Klystrons with Corkscrew-Modulated Hollow Electron Beam

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- This work was done wholly or mainly while in candidature for a research degree at this University.

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- Where I have consulted the published work of others, this is always clearly attributed.

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Abstract

Klystrons employing a hollow electron beam provide much higher RF power, compared to solid state amplifiers and compared to klystrons employing a pencil beam. This is the most important reason for the development of klystrons employing a hollow beam.

This paper presents tentative designs for X-band and W-band klystrons with corkscrew-modulated hollow electron beam. The X-band TM\textsubscript{110}-mode klystron operates in a frequency of 10 GHz and achieves 500 kW output power without great difficulties. The power gain of the klystron is 49 dB. In the X-band klystron employing the TM\textsubscript{310}-mode the hollow beam has an initial beam power of 5 MW, while the output power reaches 3.0 MW. The proposed 3rd harmonic W-band klystron design uses an input signal of 30 GHz to realize an output signal of 90 GHz. The output power reaches 1.12 kW with the help of a penultimate cavity. The total efficiency is 10.4% with a depressed collector. The linearity of the klystron is also investigated. For the electron gun design, an electron gun with annular beam as for the X-band klystron in TM\textsubscript{310}-mode is proposed. This electron gun uses a uncompressed beam. For applications which require beam compression, a possible beam compression strategy is presented.

In the investigation the hollow beam is present as continuous wave. But considering a long useful lifetime of the electron gun and better cooling of the klystron, a pulsed beam is recommended for practical applications.
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Physical Constants

<table>
<thead>
<tr>
<th>Physical Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of light in vacuum</td>
<td>( c = 2.99792458 \cdot 10^8 \text{ m/s} )</td>
</tr>
<tr>
<td>Mass of an electron</td>
<td>( m_e = 9.1093835 \cdot 10^{-31} \text{ kg} )</td>
</tr>
<tr>
<td>Electric charge of an electron</td>
<td>( e = -1.6021766208 \cdot 10^{-19} \text{ C} )</td>
</tr>
<tr>
<td>Vacuum permittivity</td>
<td>( \varepsilon_0 = 8.854187817620 \cdots \times 10^{-12} \text{ As/Vm} )</td>
</tr>
<tr>
<td>Vacuum permeability</td>
<td>( \mu_0 = 1.2566370614 \cdots \times 10^{-6} \text{ Vs/Am} )</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

The klystron is a very important type of vacuum tube operating with a linear electron beam, which was invented by Russell and Sigurd Varian in 1937, although the idea of generating electromagnetic waves by velocity modulation was published by A. Arsenyeva-Heil and Oskar Heil in 1935 already, [5]. It provides high power, high frequency signals which find uses in aerospace engineering (radar and communication), medical applications (oncology, cancer therapy), high-energy physics (particle accelerators etc.) and other industrial practices where such signals are needed. Despite the rapid development of solid state amplifiers in the last years, klystrons are still a reliable choice of high power amplifiers with long usable lifetime. The body of klystrons is usually made of copper and there are no electronic parts inside the body. The copper body provides excellent heat dissipation and is robust under harsh environmental conditions. Though the bandwidth of a klystron is narrow, the power gain is its great advantage compared to other types of electron tubes. A gain of 60 dB is not unusual for a multiple-cavity klystron.

![Figure 1.1: Layout of a klystron; the penultimate cavity is optional](image)

A basic schematic of a klystron is shown in Figure 1.1. The electron beam with certain velocity \( v_0 = \beta \cdot c_0 \) is generated in the Pierce-type electron gun. A small input signal modulates the electron beam velocity in the input cavity and the electron beam carrying the signal information moves through the intermediate section into the output cavity. The electrons give up kinetic energy in the output cavity due to shunt impedance. The idler cavities enhance the modulation of the electron beam and then the penultimate cavity improves the quality
Chapter 1. Introduction

of electron bunches or, in other words, it squeezes the bunches. These additional cavities are optional for a klystron design. In the first design of a klystron by the Varian Brothers only two cavities were required: the input cavity and the output cavity. Nowadays, multiple-cavity klystrons can be considered the standard. A multiple-cavity klystron has two advantages compared to conventional ones: the first is the broader bandwidth, the bandwidth of the klystron increases with the increased cavity number; another is the aforementioned high power gain.

The important parameters of a klystron’s quality are power gain $G_p$, which is the ratio of the output power $P_{out}$ to the input power $P_{in}$, and efficiency $\eta$, which is the ratio of output power to beam power $P_{beam}$. Klystrons have advantages in beam power and efficiency compared to other types of electron tubes.

$$G_p = \frac{P_{out}}{P_{in}} \text{dB}$$

$$\eta = \frac{P_{out}}{P_{beam}} \%$$

The beam power is determined from the beam current $I_0$ and the accelerating voltage $U_0$ in the electron gun:

$$P_{beam} = I_0 \cdot U_0$$

In the last two decades, klystron studies have been a continuous interest at the Fachgebiet Theoretische Elektrotechnik of Technische Universität Berlin. In the 1990s, a klystron with sheet electron beam was proposed and investigated at the institute [29] and later in Stanford [30]. The high power beam is what makes the sheet beam klystrons so attractive. The large cross-section of beam allows more electrons inside the beam without increasing electron density. However when using sheet beams, difficulties in beam focusing prove disadvantageous. Therefore a hollow beam klystron was proposed [12]. This hollow beam is velocity modulated with a rotating TM$_{m10}$-mode, where $m > 1$. A rotating TM mode is produced from two orthogonal modes, which have the same resonance frequency but with the phase of one mode $90^\circ$ shifted after the other’s. When these two modes oscillate in the cavity, the position of the electric field maximum rotates. This rotating electric field modulates the velocity of the hollow beam not only in the z-direction but also in the x and y-directions. But, compared to the velocity change of the electrons in the z-direction, the velocity growths in x and y directions are negligible. After a certain drift length this modulated hollow beam has a helical form. Klystrons with hollow beam also have a number of advantages over conventional beams like pencil beam, multiple beams or sheet beam.

---

A hollow beam is generated by a cathode of annular form. In this paper, annular beam is a general definition for beams, which are generated by an annular cathode.
1. Large cross-section of electron beam and easy beam focusing

This large cross-section of the electron beam means either that the beam power can be increased for the same current density or for the same beam power the current density is reduced. In addition to the propagation with velocity $\beta \cdot c_0$ in z-direction the hollow beam has extra spread tendency both in transversal and longitudinal directions due to space charge between electrons. Consider a beam has a continuous velocity $\beta c_0$ in the longitudinal direction $z$ and the beam is not bunched yet. The electron distribution in the longitudinal direction is homogeneous and the charge density $q$ has a radial dependence:

$$q(\rho, \varphi, z) = q(\rho)$$ (1.3)

The integral form of the Gauss’ law gives the electric field in radial direction

$$\int_S E_\rho dS = \int_V \frac{q(\rho)}{\varepsilon_0} dV$$

$$\Rightarrow E_\rho = \frac{1}{\varepsilon_0 q} \int q(\rho) \rho d\rho$$ (1.4)

The current density of the beam $J_0$ is

$$J_0 = q(\rho) v_z = q(\rho) \beta c_0$$ (1.5)

The magnetic field has a component in azimuthal direction only:

$$\int \vec{B}_\varphi \cdot d\vec{l} = \mu_0 \int \vec{J}_0 dS =$$

$$\Rightarrow B_\varphi = \frac{\mu_0 \beta c_0}{q} \int q(\rho) \rho d\rho$$ (1.6)

The space charge force exerting on the beam therefore has a radial component only:

$$F_\rho = q(E_\rho - v_z B_\varphi)$$

$$= q E_\rho (1 - \beta^2)$$ (1.7)

The force from the azimuthal magnetic field is actually focussing but it is negligible compared to the force from the electric field. The beam therefore diverges. This spread does not contribute to klystron’s operation and causes problems, which could be prevented easily with a solenoid. The electrons initially have a velocity component in axial direction $v_z$ only, but start rotating when they enter the magnetic focussing field and encounter the radial component of the magnetic flux $v_z \times B_\varphi$. The magnetic flux in hundreds of mT in longitudinal direction is stronger and it interacts with the rotating velocity component of electrons. The cross product produces focussing force for the beam. In an ideal
case this force compensates the defocussing force completely.

The space charge force in the longitudinal direction, on the contrary, if controlled well, makes contribution to the modulation of the hollow beam. The space charge forces between electrons in the longitudinal direction are responsible for the plasma oscillation, which plays the dominant role in the beam modulation at small signal level. More details about the bunching with space charge force are in Chapter 2.

2. Oversized cavities

The size of a higher order TM$_{m10}$ mode resonator ($m > 1$) is larger than a TM$_{010}$ mode cavity. A large cavity means larger cooling surface. Wall losses in a resonator are proportionally small. But a klystron with MW power level still causes considerable temperature rise in the resonator’s conducting wall. For possible cooling strategies the surface of cooling needs to be large enough.

The output cavities of this kind of klystrons with hollow beam are investigated in [15] and [13]. In simulations, an excellent extraction efficiency of 70\% could be reached. These investigations however are based on the assumption of a hollow beam with perfect helical form, Figure 1.2. This means 100\% of the electrons are in these bunches and 100\% participate in interaction with the modulation gap. As already mentioned, ideally the velocity of electrons in the $z$-direction is:

$$v_z = \beta \cdot c_0$$

(1.8)

Nowadays, the design of a klystron is typically done with the help of computer programs. Not only cold testing of klystron cavities is possible in numerical environments. Particle-in-cell (PIC) codes in 2-D or 3-D are also available to predict the electron performance in the klystron. This way, a klystron design is more convenient and less expensive, thanks to intensive developments of special codes for tube design. With the help of a Fortran program this helix beam, Figure 1.2, could easily be generated in simulation. In practice however it is nearly impossible to generate a helix beam like that. Even in case the beam was
generated, the desired form would get lost quite soon due to strong Lorentz-force between electrons. The arrangement of focusing magnets for this kind of beam is complicated. The B-field in the longitudinal direction only prevents the spread of electrons in the axial direction but not the spread in longitudinal direction. A more realistic modulated hollow beam has certain modulation depth $M$, Figure 1.3, and the velocity in $z$-direction is:

$$v_z = \beta c_0 \cdot (1 + M \cdot \cos(\omega t))$$

(1.9)

![Figure 1.3: Modulated hollow beam in vacuum](image)

The $z$-velocity of electrons depends on the phase of the mode inside the modulation cavity when electrons enter the cavity. After modulation, this electron beam needs a certain drift length to allow the slow and fast electrons to meet each other and to bunch. The detailed analysis of beam gap interaction follows in the next chapter.

This paper focuses on the investigation of klystrons with velocity modulated hollow beam in X-band and W-band and the simulations are carried out with the help of numerical programs. The beam itself is modulated into a helical pattern. The klystrons in X-band operate with a frequency of 10 GHz and with beam powers of 1 MW and 5 MW. Main purpose of a klystron in X-band is power amplification. The 3rd harmonic klystrons in W-band operate with an input signal of 30 GHz and an output signal of 90 GHz. The beam power is 50 kW. The W-band is used for satellite communications, because the signal with a frequency range of 94 GHz could penetrate the clouds with negligible attenuation. The purpose of the W-band klystron is to realise signal amplification and frequency tripling.

In this paper the analytical model of the klystron is introduced in Chapter 2. A short introduction of this klystron as numerical model and the problems in numerical solutions are given in Chapter 3. The software GdfidL, [28], is used for the eigenvalue and Particle-In-Cell simulations. Since each part of the klystron works discretely and individually, the investigation of the whole klystron is done for each part separately: Output cavity, idler and penultimate
cavity, and input cavity. The centre frequency, the quality factor and other features are studied thoroughly for each part. Afterwards all parts are put together to form a whole klystron. For the complete structure isolation among cavities is very important, since energy coupling could happen between gaps. This paper is concluded with an investigation of the electron gun for the hollow beam and of a depressed collector.

As results, the power gain for an X-band klystron reaches 49 dB while the W-band tube reaches 20 dB. The efficiency of the X-band klystron is over 50%, while only 2% for the W-band klystron. The W-band klystron shows good linearity and has great potential for signal amplification.

This new type of klystron raises a few challenges which are also mentioned and discussed in this paper:

1. Generation of hollow beam

Most electron guns in the market generate a pencil beam. This technology is mature, well tested and documented. Electron guns for hollow beam are seldomly seen and still in testing period, [20][19]. The emission of electrons must be uniform. This problem is solved by the fragmented annular cathode, [22]. But the electron gun for an annular beam in this paper is without compression and the beam radius and thickness are larger than the requirements in our investigation. To match the requirements the electron beam must be compressed without scalloping in the envelope.

2. Quality of modulated beam

Ideally all electrons of a modulated beam are in the bunches and all of them participate in the interaction with the cavities. But due to space charge force effects this ideal case does not exist. Almost one third of the electrons in the beam are outside of the bunches. In our investigation idler and penultimate cavities are introduced. In these cavities the electrons outside the bunches are pushed toward the bunches. The quality of bunches is therefore improved.

3. Difficulty of modelling cylindrical structures in the numerical program

This problem is common in numerical simulations. It was foreseeable that the cylindrical models would suffer accuracy loss when the models are rebuilt in a Cartesian coordinate system in the simulation program. As a prevention the mesh size of the grid was refined. But it caused more problems than expected, especially because the rotating mode is much more sensitive to the asymmetry in the axial direction. In some cases the rotating mode can not be excited successfully and the output efficiency is much lower than expected.
The analytical model of this cavity is built in a cylindrical coordinate system. The two orthogonal modes have exactly the same resonance frequency as in the analytical model. When the modulated beam enters the cavity, it induces a current and the current produces fields, and the fields, in return, accelerate the electrons.

The numerical program used in the investigation uses a Cartesian coordinate system. Therefore the numerical model has asymmetries in the geometry hence the two orthogonal modes show a slight frequency drift. Not only the frequencies are different but also the magnitudes of the fields. The average value of the two frequencies is chosen as the beam frequency. When this frequency and magnitude drifts get larger, it not only causes asymmetry in beam acceleration, but also impedes mode excitation in the first place. The difference between beam frequency and mode frequency could be beyond the bandwidth of the cavity, especially when the cavity in the klystron is normally narrow-banded. To minimize errors in the numerical program, small adjustments to the cylindrical form are made, mesh size is refined, and extra tuning blocks are added to adjust the modes independently. With these strategies the error can only be minimized, although not eliminated.
Chapter 2

Principles of the Klystron

In Chapter 1 the basic principles of a klystron were introduced. In this chapter it is explained in more detail how the electron beam and the cavity gap interact, how electrons gain and lose kinetic energy, how the cavities behave in the interaction and other phenomena in the klystron operation. Important parameters like shunt impedance $R_{sh}$ and quality factor $Q$ are introduced. The principles of other parts like electron beam generation and design of the electron gun are discussed in Chapter 6.

2.1 Beam Gap Interaction

Beam gap interaction could be divided into two general situations: a beam modulated by an excited gap and a modulated beam causes the excitation of a gap.

2.1.1 Beam modulation in an open pillbox resonator

![Pillbox cavity resonator with infinite long beam pipe](image)

**Figure 2.1:** Pillbox cavity resonator with infinite long beam pipe

The velocity in $z$-direction of an electron beam is supposed to be modulated by the excited input cavity. In the excited gap electric field and magnetic field
oscillate. But only the electric field participates in the interaction of electrons and gap and is therefore responsible for energy change of electrons. Only TM modes are candidates, because they have non-zero $E_z$. At first, an input signal excites a resonant mode in the input cavity. When electrons pass the input cavity, they gain or lose kinetic energy. The energy is present in the form of velocity change. For understanding the beam modulation the conditions of interaction between electron beam and beam gap are simplified as follows: the pencil beam has a negligibly small radius, the modulation gap is a pillbox cavity with an infinitely long beam pipe in the centre, Figure 2.1. The eigenmode for beam modulation is TM$_{010}$. The gap length is $g$, the radius of beam pipe is $a$ and the radius of the cavity is $b$.

It is supposed that the electric field at the position $\varrho = a$ inside the gap is constant $E_0$, outside the gap it is 0. An electron moves through this gap with an initial velocity of $v_0$ along the z-axis. The electric and magnetic fields are:

$$
\tilde{H} = \int_{-\infty}^{\infty} F(k_z) J_1(k_\varrho \varrho) \cdot e^{-jk_z z} \cdot \hat{e}_z dk_z,
$$

$$
k_\varrho = \sqrt{k^2 - k_z^2}
$$

$$
j \omega \varepsilon \tilde{E} = \nabla \times \tilde{H} = -\frac{\partial H_\varrho}{\partial z} \hat{e}_\varrho + \frac{1}{\varrho} \frac{\partial}{\partial \varrho} (\varrho H_\varrho) \hat{e}_z
$$

$$
= \int_{-\infty}^{\infty} j k_z F(k_z) J_1(k_\varrho \varrho) e^{-jk_z z} \hat{e}_z dk_z + \int_{-\infty}^{\infty} F(k_z) k_\varrho J_0(k_\varrho \varrho) e^{-jk_z z} \hat{e}_z dk_z
$$

$F(k_z)$ is an as-yet-undefined function. Applying the condition that $E_z(\varrho = a) = E_0$ for $\frac{g}{2} \leq \varrho \leq \frac{g}{2}$ and $E_z(\varrho = a) = 0$ for $|\varrho| > \frac{g}{2}$, the Inverse-Fourier-Transformation to Equation 2.1 leads to:

$$
\frac{1}{2\pi} \int_{-\frac{g}{2}}^{\frac{g}{2}} E_0 e^{jk_z z} dz = \frac{1}{j \omega \varepsilon} F(k_z) k_\varrho J_0(k_\varrho a)
$$

$$
\frac{1}{\pi} E_0 \frac{\sin(k_z \frac{g}{2})}{k_z} = \frac{1}{j \omega \varepsilon} F(k_z) k_\varrho J_0(k_\varrho a)
$$

The unknown function $F(k_z)$ is then:

$$
F(k_z) = \frac{j \omega \varepsilon \cdot \frac{g}{2}}{\pi} \frac{E_0}{k_\varrho J_0(k_\varrho a)} \cdot \frac{\sin(k_z \frac{g}{2})}{k_z \frac{g}{2}}
$$
and the electric field in z-direction, or propagation direction of electrons, is:

\[ E_z = \int_{-\infty}^{\infty} \frac{E_0 \cdot \frac{g}{2}}{\pi} \cdot \frac{J_0(\frac{g}{2})}{J_0(\frac{g}{2}a)} \cdot \frac{\sin(\frac{k_z \cdot g}{2})}{k_z \cdot g} \cdot e^{-jk_zz} dk_z \]  

(2.4)

The change in kinetic energy of a single electron moving along the axis in the cavity is:

\[ \Delta W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \]

\[ = q_0U(t) \]

\[ = q_0 \int_{-\infty}^{\infty} E_z e^{j\omega t} dz \]

(2.5)

Applying \( t = z/v_0 \), \( k_g = \sqrt{k^2 - k_z^2} \) the energy change \( \Delta W \) is:

\[ \Delta W = q_0 \frac{E_0 \cdot \frac{g}{2}}{\pi} \cdot \int_{-\infty}^{\infty} dk_z \frac{1}{J_0(\frac{g}{2}a)} \cdot \frac{\sin(\frac{k_z \cdot g}{2})}{k_z \cdot g} \cdot e^{j\omega t - k_zz} \]

\[ = 2\pi \delta \left( \frac{\omega}{v_0} - k_z \right) \]

(2.6)

\[ = q_0 E_0 g \cdot \frac{1}{J_0 \left( \sqrt{k^2 - \left( \frac{\omega}{v_0} \right)^2} \right)} \cdot \frac{\sin \left( \frac{\omega}{v_0} \cdot \frac{g}{2} \right)}{\frac{\omega}{v_0} \cdot \frac{g}{2}} \]

\[ M \]

\[ k^2 - \left( \frac{\omega}{v_0} \right)^2 < 0 \]

\[ \Rightarrow J_0 \left( a \sqrt{k^2 - \left( \frac{\omega}{v_0} \right)^2} \right) = J_0 \left( ja \sqrt{\left( \frac{\omega}{v_0} \right)^2 - k^2} \right) = I_0 \left( a \sqrt{\left( \frac{\omega}{v_0} \right)^2 - k^2} \right) \]

(2.7)

\( M \) is defined as the gap coupling coefficient. The maximum value of \( M \) is 1, when \( \frac{\omega}{v_0} \cdot \frac{g}{2} = 0 \). Thus, for a very narrow gap, the modulation is at its maximum and nearly constant.
2.1.2 Current induced in the gap by electrons

In the input cavity the input signal excites the resonant mode while in the output cavity the modulated electron beam excites the mode. When modulated electrons enter a resonant cavity, they induce current in the wall. The current produces electric and magnetic fields in the gap. The electric field in return affects the electron velocity.

Considering an electron sheet entering the gap with surface charge density $q_0$, the moving electron sheet induces current in the conducting walls at the left and right sides, Figure 2.2. The current density is $J_i$. The position of the electron is $z$. The induced charge densities are $q_L$ (left) and $q_R$ (right). The Maxwell’s equations in this situation yield:

$$
q_L = \varepsilon_0 E_L
$$
$$
q_R = \varepsilon_0 E_R
$$
$$
q_0 = -\varepsilon_0 (E_L + E_R)
$$
$$
\Rightarrow E_R = -\frac{q_0}{\varepsilon_0} - E_L \tag{2.8}
$$

The voltage across the gap (gap length $= g$) is:

$$
U = \int_{-g/2}^{z} E_L dz + \int_{z}^{g/2} (-E_R) dz
$$
$$
= E_L(z + g/2) + E_R(z - g/2) = 0 \tag{2.9}
$$
Combining Equation 2.8 and 2.9 gives:

\[ E_L = -\frac{q_0 \cdot \left( \frac{1}{2} - \frac{z}{g} \right)}{\varepsilon_0} \]
\[ E_R = -\frac{q_0 \cdot \left( \frac{1}{2} + \frac{z}{g} \right)}{\varepsilon_0} \]
\[ q_L = -q_0 \cdot \left( \frac{1}{2} - \frac{z}{g} \right) \]
\[ q_R = -q_0 \cdot \left( \frac{1}{2} + \frac{z}{g} \right) \]  

(2.10)

The change of current density is related to the change of position of the electron charge in the gap. The induced current in the gap is then:

\[ J_i = -\dot{q}_R = +\dot{q}_L = q_0 \cdot \frac{\partial z/\partial t}{g} = \frac{q_0}{g} v_0 \]  

(2.11)

This equation is an expression of Shockley-Ramo’s Theorem, [26]. A moving electric charge induces current the moment it enters the gap.

As the modulated beam enters the gap, the time-varying charge produces a current in the gap, the current leads to a magnetic field inside the gap. The time-varying magnetic field generates an electric field. The two fields oscillate inside the gap:

\[ \nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  

(2.12)

With shunt impedance the electric charge produces a voltage across the gap. The voltage, in return, affects the electrons themselves. The shunt impedance is the parameter measuring the interaction of electrons and the resonant cavity.

### 2.2 Coaxial Resonator

The beam in our investigation is of annular form, the modulation gap should be cylindrical or coaxial. Since the cylindrical gap has no inner conductor, it has less wall loss and is easier to be manufactured. It is the prime choice, Figure 2.3.

In klystron design this pillbox cavity is cut into two pieces by the coaxial beam pipe, Figure 2.4 (left). The inner part of the cavity is not supported since it is not attached to any other parts of the klystron. To solve the problem of the "floating" inner part of the cavity, an inner conductor inside the cavity is introduced to provide mechanical support for the inner part. This pillbox cavity is
Figure 2.3: Electric and magnetic fields of TM$_{110}$-mode in a cylindrical resonator

therefore altered into a coaxial cavity. Then the inner part of the beam pipe is connected to the inner conductor. This inner conductor part could be fixed at the very beginning and the end of the cavity structure, Figure 2.4 (right).

Figure 2.4: Left: cylindrical cavity with coaxial beam pipe opening; right: coaxial cavity with coaxial beam pipe opening

All the cavities in these hollow beam klystrons are altered into coaxial cavity resonators, Figure 2.5.

2.2.1 Eigenmodes of the coaxial resonator

The Maxwell’s law yields vector potential, electric- and magnetic fields of a coaxial structure for TM$_{mn0}$-mode$^1$:

$^1$The equations of electromagnetic fields of a cavity for an annual beam were introduced in [3].
$A = (C \cdot J_m(k_{mn}q) + D \cdot N_m(k_{mn}q)) \cos(m\varphi)\hat{e}_z$

$\vec{H} = \nabla \times \vec{A} = \frac{1}{\varrho} \cdot \frac{\partial (A_z)}{\partial \varphi} \hat{e}_\varphi - \frac{\partial (A_z)}{\partial \varrho} \hat{e}_\varrho$

$$= -\frac{m}{\varrho} \left( C \cdot J_m(k_{mn}q) + D \cdot N_m(k_{mn}q) \right) \sin(m\varphi) \cdot \hat{e}_\varphi$$

$$- \left( C \cdot J_m(k_{mn}q) + D \cdot N_m(k_{mn}q) \right)' k_{mn} \cos(m\varphi) \cdot \hat{e}_\varphi$$

$$= -\frac{m}{\varrho} G_m(k_{mn}q) \sin(m\varphi) \cdot \hat{e}_\varphi - G_m'(k_{mn}q) k_{mn} \cos(m\varphi) \cdot \hat{e}_\varphi$$

(2.13)

$j\omega E = \nabla \times \vec{H}$

$E_z = \frac{1}{j\omega} \nabla \times (\nabla \times \vec{A}) = \frac{1}{j\omega} (\nabla (\nabla \cdot A) - \nabla^2 A)$

$$= \frac{k_{mn}^2}{j\omega} \left( C \cdot J_m(k_{mn}q) + D \cdot N_m(k_{mn}q) \right) \cos(m\varphi)$$

$$= \frac{k_{mn}^2}{j\omega} G_m(k_{mn}q) \cos(m\varphi)$$

$E_\varphi = E_\varphi$
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$C$ and $D$ are unknown constants. Applying the boundary conditions $E_z|_{z=a} = 0$ and $E_z|_{z=b} = 0$, the equations of the electric field are written as:

\[ C \cdot J_m(k_{mn}a) + D \cdot N_m(k_{mn}a) = 0 \]

\[ C \cdot J_m(k_{mn}b) + D \cdot N_m(k_{mn}b) = 0 \]

\[ \Rightarrow \frac{J_m(k_{mn}a)}{N_m(k_{mn}a)} = \frac{J_m(k_{mn}b)}{N_m(k_{mn}b)} \] (2.14)

With these two boundary conditions alone, the equation above can not be solved. But since for a klystron design the desired operating frequency is known, the wave number $k_{mn}$ can be determined. As an example, the cavity for X-band klystron is used. Then we have the wave number:

\[ k_{mn} = \frac{2\pi \cdot 10 \text{ GHz}}{c_0} = 209 \text{ m}^{-1} \] (2.15)

The relation of inner- and outer radii to the mode number $m$ was also mentioned in [3]. The inner radius $b$ is chosen from 1 mm to 10 mm. The outer radius is determined in MATLAB with the known $b$, $k_{mn}$ and Equation 2.14. Figure 2.6 plots the inner and outer radii of TM$_{m10}$-mode, $m$ is 1 to 5.

![Figure 2.6](image)

**Figure 2.6:** Outer radii increase with the increased inner radii from 1 mm to 10 mm, while mode number $m$ of the TM$_{m10}$ resonant cavity is from 1 to 5

### 2.2.2 Power loss and quality factor

The inner conductor has to be strong enough to support the inner structure considering the klystron is mostly manufactured with copper and that copper has a certain weight. At the same time, the size of this conducting surface is related to the sizes of outer and inner radii. An unnecessarily large inner radius means unwanted wall losses. But with increased $m$ value, the near zero field area near the inner conducting wall is enlarged. For example, in Figure
2.6, the outer radii of TM\textsubscript{410} and TM\textsubscript{510} stay the same when the inner radii are under 10 mm. Consequently keeping the same operating frequency but choosing higher modes allows for a larger inner radius without an enlarged outer radius and without significantly increased wall losses. The quality factor $Q$ of a resonant cavity is related to these two parameters. The quality factor $Q$ is a characteristic parameter of oscillation systems and for a cavity resonator it can be written as, [16]:

$$Q = \frac{\omega W}{P_v}$$

(2.16)

$W$ is the total energy of electric and magnetic field of the eigenmode in the cavity, $P_v$ is the kinetic energy that the electron beam loses. $Q_0$ presents the quality factor without external loss:

$$Q_0 = \frac{\omega W}{P_{int}}$$

(2.17)

$$W = W_e + W_m = 2W_e = \frac{1}{4} \iiint_V E \cdot D^* dV + \frac{1}{4} \iiint_V B \cdot H^* dV$$

$$= \frac{\pi k_{mn} g}{2\pi^2 \delta_s} \int_b^a G_m^2(k_{mn} \varrho) \varrho d\varrho$$

(2.18)

Wall loss $P_{int}$ is caused by the current induced by the tangential magnetic field. The current flows through the surface of the resonator and causes heating in the conducting wall. The wall losses depend on the conducting surface area and the sheet resistance $R_w$, which is

$$R_w = \frac{\varrho}{\delta_s} = \frac{1}{\kappa \delta_s}$$

(2.19)

where $\varrho$ is the specific electrical resistance and $\kappa$ is the electrical conductivity of the resonator material, in our case copper, 58 MS/m. $\delta_s$ is the skin depth due to the RF frequency skin effect:

$$\delta_s = \sqrt{\frac{2}{\omega \mu \kappa}}$$

(2.20)

The sheet resistance $R_w$ is a measure of special resistance for the current that flows only in a very thin layer in the conducting material. This effect normally appears in the RF frequency range.

Wall losses occur on four surfaces of a closed coaxial resonator: Wall loss at top- and bottom surfaces of cavity $P_{v,\text{top}}$ and $P_{v,\text{bottom}}$, wall loss of side areas $P_{v,\text{coat}}$, e.g. inner conducting wall at $\varrho = b$ and of the outer conducting wall at
\( \varrho = a. \)

\[
\bar{P}_{\text{int}} = \int_S \frac{1}{2} R_\omega |J_\omega|^2 dS = \frac{1}{2} \frac{1}{\kappa_0} \int_S |H_{\text{tan}}|^2 dS
\]

\[
= \bar{P}_{v,\text{top}} + \bar{P}_{v,\text{bottom}} + \bar{P}_{v,\text{coat}} \tag{2.21}
\]

Top- and bottom areas are the same, the current in this area is induced by the tangential magnetic field components \( H_\varphi \) and \( H_\varphi' \):

\[
\bar{P}_{v,\text{top}} = \bar{P}_{v,\text{bottom}} = \frac{1}{2\kappa_0} \left( \int_b^a \int_0^{2\pi} |H_\varphi|^2 \varrho d\varphi d\varphi + \int_b^a \int_0^{2\pi} |H_\varphi'|^2 \varrho d\varphi d\varphi \right)
\]

\[
= -\frac{\pi}{2\kappa_0} \left( m^2 \int_b^a \frac{1}{\varrho} G_m^2(k_{mn}\varrho) d\varrho + k_{mn}^2 \int_b^a G_m^2(k_{mn}\varrho) d\varrho \right) \tag{2.22}
\]

The current at the inner and outer conducting walls is induced by the tangential component \( H_\varphi|_{\varrho=a,b} \):

\[
\bar{P}_{v,\text{coat}} = \frac{1}{2\kappa_0} \int_0^g \int_0^{2\pi} |H_\varphi|^2 \varrho d\varphi d\varphi|_{\varrho=a,b}
\]

\[
= \frac{g\pi k_{mn}^2}{2\kappa_0} \left( aG_m^2(k_{mn}a)^2 + bG_m^2(k_{mn}b)^2 \right) \tag{2.23}
\]

Field \( E_z \) and quality factor \( Q_0 \) are both parameters characterizing a resonant cavity without interaction with electrons. For the interaction between electron and modulation strength the parameter shunt impedance \( R_{sh} \) is introduced. To add longitudinal force to electrons the resonant cavity is excited. The longitudinal voltage inside the cavity is \( U \):

\[
U = \left| \int_z E_z \cdot e^{j\omega t} dz \right| = \left| \int_z E_z \cdot e^{j\omega t/\omega_c} dz \right|
\]

\[
= k_{mn} \frac{g}{\omega_c} \cos(m\varphi)|G_m(k_{mn}\varrho)| \cdot \sin \left( \frac{\omega g}{2\beta C_0} \right) \tag{2.24}
\]

with

\[
\sin \left( \frac{\omega g}{2\beta C_0} \right) = \frac{\sin \left( \frac{\omega g}{2\beta C_0} \right)}{\frac{\omega g}{2\beta C_0}} \tag{2.25}
\]

When the resonator has no external losses, the shunt impedance \( R_{sh} \) is only related to the voltage \( U \) and the wall losses \( \bar{P}_{\text{int}} \):

\[
R_{sh} = \frac{U^2}{\bar{P}_{\text{int}}} \tag{2.26}
\]
Combining Equations 2.21, 2.22, 2.23, 2.24 and 2.26 gives:

\[
R_{sh} = \frac{k_{mn}^2 g^2 \int_0^\pi \cos(m\varphi)^2 |G_m(k_{mn}q)|^2 \cdot \sin(\frac{\omega_0}{2c_0})^2}{m^2 \int_b^a G''_m(k_{mn}q) dq + k_{mn}^2 \int_b^a G'_m(k_{mn}q)^2 dq + \frac{g^2}{2}k_{mn}^2 (aG'_m(k_{mn}a)^2 + bG'_m(k_{mn}b)^2)}
\]  

(2.27)

\(R_{sh}/Q\) is an important ratio measuring the acceleration force of resonant cavities. The value of the shunt impedance \(R_{sh}\) is related to several parameters, Equation 2.24 and Equation 2.27. The first is the mode number \(m\). Besides, the shunt impedance is also related to the gap length \(g\), [36]. Figure 2.7 plots the relation of gap length and shunt impedance maxima. The shunt impedance reaches a maximum, where the gap length is a little less than half the wavelength of the electron beam. One wavelength of the electron beam is \(c_0 f\). For resonators with more than one cavity the thickness of the iris, which is between the gaps is also taken into consideration. The total length of the gap and the subsequent iris must be less than \(\frac{c_0}{2f}\).

![Figure 2.7: Normalized shunt impedance R_{sh} inside the modulation gap with respect to gap length; \lambda = \frac{\beta_c}{f}](image)

### 2.2.3 Coupling factor \(\beta_k\)

With a proper coupling method the external load is integrated into this cavity resonator. A cavity with external losses is called a "loaded" cavity. Its shunt impedance grows smaller, bandwidth grows broader, and the quality factor grows smaller. They all drift from the initial values with respect to the external loss, and the resonance frequency drifts too. In the "loaded" output cavity the electron beam gives up kinetic energy \(P_v\) and this energy is converted to wall losses \(P_{int}\) and output power / external losses \(P_{out}\). The coupling factor \(\beta_k\) is introduced as a measure of the ratio of external power \(P_{out}\) to internal energy loss \(P_{int}\):
\[ \beta_k = \frac{P_{\text{out}}}{P_{\text{int}}} \]  
(2.28)

The total kinetic energy loss \( P_v \) is the sum of \( P_{\text{int}} \) and \( P_{\text{out}} \):
\[
P_v = P_{\text{out}} + P_{\text{int}} = (1 + \beta_k)P_{\text{int}}
\]
(2.29)

\( I \) is dc beam current. A beam current of a modulated beam has both dc and ac components. The detailed analysis of the beam current follows in Section 2.4.2. \( U \) is from Equation 2.24:
\[
U = \sqrt{P_{\text{int}}R_{sh}}
\]
(2.30)

\[
\Rightarrow I \cdot \sqrt{P_{\text{int}}R_{sh}} = (1 + \beta_k)P_{\text{int}}
\]
(2.31)

\( P_{\text{out}} \) and \( P_v \) are then as follows:
\[
P_{\text{out}} = \beta_k \cdot P_{\text{int}} = I^2 R_{sh} \cdot \frac{\beta_k}{(1 + \beta_k)^2}
\]
(2.32)

\[
P_v = (1 + \beta_k)P_{\text{int}} = I^2 R_{sh} \cdot \frac{1}{1 + \beta_k}
\]

The relations of \( P_v, P_{\text{int}} \) and \( P_{\text{out}} \) are plotted in Figure 2.8. If the coupling factor \( \beta_k < 1 \), the cavity is under coupled and the output cavity loses more power in the form of wall losses than power is transported to the output wave guides. The output cavity only fulfils its purpose in the case of \( \beta_k \geq 1 \), where the cavity is over coupled. For an output cavity a proper coupling factor \( \beta_k \) delivers good output power while wall losses are kept proportionally small. \( \beta_k \) between 2 to 5 is the regular choice, because the ratio of \( P_{\text{out}} \) to \( P_{\text{int}} \) improves with increased \( \beta_k \). But the total converted power \( P_v \) decreases. When \( \beta_k = 1 \), then the cavity is critical coupled.

The "loaded" shunt impedance \( R_{sh,L} \), "loaded" quality factor \( Q_L \) and "loaded" bandwidth \( \omega_B \) are written as:
\[
R_{sh,L} = \frac{U^2}{P_{\text{int}} + P_{\text{out}}} = \frac{U^2}{P_{\text{int}}(1 + \beta_k)}
\]
(2.33)

\[
\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{\text{out}}}
\]

\[
Q_L = \frac{\omega_0 W}{P_{\text{int}} + P_{\text{out}}} = \frac{1}{1 + \beta_k}Q_0
\]

\[
\omega_B = \frac{\omega_0}{Q_L}
\]
The quality factor $Q$ is not only related to bandwidth but also to the exponential time constant $\tau$

$$Q = \frac{\omega_0 \tau}{2}$$

(2.34)

where $\tau$ is the mean lifetime of the voltage of a decaying system:

$$U(t) = U \cdot (e^{-t/\tau})$$

(2.35)

Thus, keeping $Q_L$ small ensures a broader bandwidth and shorter charging time of a cavity. On the other hand, a small $Q_L$ leads to low loaded shunt impedance and less power extraction from the electron beam.

$Q_0$, $Q_L$, $P_v$, $P_{\text{int}}$ and $P_{\text{out}}$ are related to each other.

$$Q_0 = \frac{\omega W}{P_{\text{int}}}$$

$$Q_L = \frac{\omega W}{P_v} = \frac{\omega W}{P_{\text{int}} + P_{\text{out}}}$$

(2.36)

$$\beta_\kappa = \frac{P_{\text{out}}}{P_{\text{int}}} = \frac{Q_0}{Q_L} - 1$$

### 2.3 Coupled Cavity Chain

A single cavity is a standing wave structure. But a single gap provides limited shunt impedance and more gaps may be needed for extraction of beam power. Of course each gap could be isolated and operate just individually. It then requires waveguides for power feeding or for power extraction installed in each gap. An alternative is the gaps are connected to each other through coupling structures. If coupling takes place, the extracted power flows through this cavity chain. In this case output waveguides only have to be installed in the first
or the last gap depending on the coupling method. The mode in this coupled cavity chain could be 0 or $\pi$ or other values in between.

Here mode describes the electric field pattern due to the coupling and the interaction between gaps in this chain, and not the resonant mode of one single cavity. For the output section, a $\pi$ mode is chosen. It means that the phase in one gap is $180^\circ$ ahead of the phase in the next gap. This way electrons always experience the maximum electric field when entering each gap. The effect of impedance in coupled cavities on electrons is cumulative. For this $\pi$ mode cavity chain the distance from the centre of one gap to the centre of the next gap is $\beta\lambda/2$.

The gaps could be capacitively or inductively coupled. In the case of inductive coupling, extra small coupling slots near the outer conducting wall are required for the coupling, Figure 2.9. In this area magnetic coupling happens. In the 1960s D. E. Nagle and et al., [21], introduced a mathematical model of coupled cavity chain using coupled series LC circuits, Figure 2.10. Each gap is
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Suppose a cavity chain has \( N + 1 \) cavities and all the cavities are identical in capacitance \( C \) and inductance \( L \). The coupling between two neighbouring gaps is also the same and is \( \kappa \). For a finite cavity chain the 0\(^{th} \) gap, which is the first gap in the chain, and the \( N^{th} \) gap, which is the last gap in the chain, are half gaps. The \( n^{th} \) gap is in between and \( n \) is greater than 0 and less than \( N \). The voltage in the cavity is zero according to Kirchhoff’s law:

\[
\begin{align*}
&j\omega Li_0 - \frac{j}{2\omega C}i_0 - \kappa j\omega Li_1 = U_0 = 0 \\
2j\omega Li_n - \frac{j}{\omega C}i_n - \kappa j\omega L(i_{n-1} + i_{n+1}) = U_n = 0 \\
j\omega Li_N - \frac{j}{2\omega C}i_N - \kappa j\omega Li_{N-1} = U_N = 0
\end{align*}
\] (2.37)

Since \( \omega_0^2 = \frac{1}{2LC} \), the equations can be rewritten as:

\[
\begin{align*}
i_0 - \kappa i_1 &= \omega_0^2 i_1 \\
-\frac{\kappa}{2}i_{n-1} + i_n - \frac{\kappa}{2}i_{n+1} &= \omega_0^2 i_n \\
i_N - \kappa i_{N-1} &= \omega_0^2 i_N
\end{align*}
\] (2.38)

The problems are transformed to matrix relations as follows:

\[
\begin{pmatrix}
1 & -\kappa & 0 & \ldots & 0 \\
-\frac{\kappa}{2} & 1 & -\frac{\kappa}{2} & \ddots & 0 \\
0 & \ddots & \ddots & \ddots & \vdots \\
& \ddots & -\frac{\kappa}{2} & 1 & -\frac{\kappa}{2} \\
0 & \ldots & 0 & -\kappa & 1
\end{pmatrix}
\begin{pmatrix}
i_0 \\
i_1 \\
i_n \\
i_N
\end{pmatrix}
= W
\begin{pmatrix}
i_0 \\
i_1 \\
i_n \\
i_N
\end{pmatrix},
\]

The \( N + 1 \) solutions of eigenvalue \( W \) correspond to frequencies of modes of this passband and solutions of eigenvector \( i_n \) correspond to magnitudes of these modes. If the coupling \( \kappa \) and resonance frequency \( \omega_0 \) are known, eigenvalue \( W \) and eigenvector \( u_n \) can be solved with MATLAB. Mode frequencies of 0, to \( \pi \)-mode and their magnitude are to be determined.

As an example we chose:

\[
\kappa = 0.01, 0.02, 0.03, \\
\omega_0 = 2\pi \cdot 10 \cdot 10^9 \text{ Hz}, N = 5
\] (2.39)

The electric field amplitudes of coupling modes are shown in Figure 2.11. The dispersion curves of this cavity chain are shown in Figure 2.12. The slope of operating points on the dispersion curve corresponds to group velocity \( v_g \) and
$v_g$ is equal to the energy velocity $v_e$. In the inductive chain the gradient of $v_g$ is negative, the gradient of $v_e$ is negative. The energy flows in the opposite direction of wave propagation. If in contrary the gradient of $v_g$ of the capacitive chain is positive, the energy flows in the same direction of wave propagation.

$$v_g = \frac{d\omega}{d\beta} = v_e \quad (2.40)$$

**Figure 2.11:** Amplitudes of electric fields of different modes in the inductively coupled cavity chain with 5 + 1 gaps

The passband refers to the frequency band between 0-mode and $\pi$-mode and the bandwidth is $\omega_B$. When the coupling $\kappa$ between the gaps increases, the frequencies of 0-mode and $\pi$-mode drift far away from the initial resonance frequency of the resonator and the passing band grows broader. This relation can be simplified as:

$$\omega_B \approx 2\kappa\omega_0 \quad (2.41)$$
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2.4 Electron Beam: Realization of a Helical Beam and its Drift Length

One basic method of beam modulation is velocity modulation. The electron beam has a homogeneous electron distribution in the longitudinal direction. When the electrons travel through the modulation gap, they meet the longitudinal electric field inside the gap and are either accelerated or decelerated, while some retain a constant velocity. The accelerated electrons and slowed electrons continue the travelling with changed velocity and they may meet each other after a certain travelling time and increase the local electron density. This process is called bunching. This density change has a certain periodical pattern. In the following the way to helix beam is explained.

2.4.1 Rotating TM-mode

The electron beam is modulated in the input cavity with a rotating mode and therefore obtains a helical pattern. The peak of electron bunches seems to rotate in the radial direction, but actually all the electrons move in z-direction only. To extract the maximum energy from the beam the output cavity has to obtain a rotating $\text{TM}_{m10}$-mode too. In this cavity the maximum of electric field synchronizes with the electron bunches. It is also required to realise this helical modulation rotating mode in the input cavity. As mentioned before a rotating mode comes from superposition of two orthogonal modes of $\text{TM}_{m10}$, whose...
phases are $90^\circ$ apart.

As an example Figure 2.13 shows two orthogonal modes TM$_{110}$ in a time series. At an initial time point $t = 0$ mode 1 is present in the form of electric field and the orthogonal mode 2, which is one quarter period delayed, is present in the form of magnetic field. This magnetic field induces an electric field and is present in the form of an electric field at $t = T/4$ and mode 1 is now in form of a magnetic field. At $t = T/2$ the electric field from mode 1 is present again but with $E_z$ in the opposite direction and electric field of mode 2 is dominant at $t = 3T/4$ and so on.

The real part of a rotating mode is then the sum of two TM$_{m10}$ modes, [3]:

$$\Re\{E_{\text{rotate}1}\} = \Re\{E_0 \sin(m\varphi) \cdot e^{j\omega t} + jE_0 \cos(m\varphi) \cdot e^{j\omega t}\}$$
$$= E_0 (\sin(m\varphi) \cos(\omega t) - \cos(m\varphi) \sin(\omega t))$$
$$= E_0 \sin(m\varphi - \omega t) \quad (2.42)$$

The maximum of the electric field therefore rotates with an angular velocity $\omega/m$. The field combination above rotates clockwise. That is the cause of helical modulation. The modulation happens only in $z$-direction. The direction of rotation depends on the choice of the two modes. Another combination is:

$$\Re\{E_{\text{rotate}2}\} = \Re\{E_0 \sin(m\varphi) \cdot e^{j\omega t} - jE_0 \cos(m\varphi) \cdot e^{j\omega t}\}$$
$$= E_0 (\sin(m\varphi) \cos(\omega t) + \cos(m\varphi) \sin(\omega t))$$
$$= E_0 \sin(m\varphi + \omega t) \quad (2.43)$$

In this case the field maximum rotates counter-clockwise.

2.4.2 Drift length

After leaving the modulation gap, the electrons still need a drift length to gain the helical pattern. The fast electrons from a period later meet slow electrons
from a period earlier during the travelling. The electrons fly into the bunches after a certain time interval and a certain drifting space. The electron current develops into a function of axial position. The bunching development is either space charge force dominant or "ballistic" bunching dominant, which is related to the current and the modulation voltage.

"ballistic" bunching

"Ballistic" bunching describes the bunching when the electrons leave the modulation gap and the space charge force is neglected. The velocity of electrons after the modulation gap is:

\[ v = v_0(1 + M \sin(\omega t)) \]  

(2.44)

where \( M \) is modulation depth. The time point when the electrons reach position \( z \) is:

\[ t_2(z) = t_1 + \frac{z}{v} = t_1 + \frac{z}{v_0(1 + M \cdot \sin(\omega t_1))} \]  

(2.45)

\( t_1 \) is the time point when electrons leave the modulation cavity and \( i_1 \) is the current at this moment. The law of charge conservation requires:

\[ i_2(z) dt_2(z) = i_1 dt_1 \]  

(2.46)

The current \( i_2(z) \) at the position \( z \) is written as the following expression, [34]:

\[ i_2(z) = i_1 \cdot \frac{1}{1 - \frac{\omega z}{v_0} M \cos(\omega t_1)} \]  

(2.47)

When the modulation is small \( M \ll 1 \), then \( i_1 \approx I_0 \). \( I_0 \) is the dc beam current. The travelling electron bunches constitute an RF current in the electron beam, [1] and the current wave is rich in high harmonics. The current \( i_2(z) \) is expressed as a Fourier series and in terms of Bessel functions of the first kind:

\[ i_2(z) = I_0(1 + 2\{J_1(X) \cos(x) + J_2(2X) \cos(2x) + \cdots + J_n(nX) \cos(nx)\}\} \]  

(2.48)

\[ = I_0(1 + 2 \sum_{n=1}^{\infty} J_n(nX) \cos(nx)) \]

where

\[ x = \omega t_2(z) - \frac{\omega z}{v_0} \]

\[ X = \frac{\omega z}{v_0} M \]  

(2.49)

The fundamental part of the RF current of \( i_2 \) is

\[ 2J_1(X) \]  

(2.50)
In this equation the maximum is reached when $X = 1.84$ and the axial position $z$ at which the value occurs is:

$$z = 1.84 \frac{v_0}{M \omega} \quad (2.51)$$

**space charge bunching**

For small signal levels, the velocity change of electrons after the modulation gap is very small. Assume an electron clouds is moving and the electrons have a uniform velocity. When electrons are pushed away from their original positions, for example by the electric field in a modulation gap, the velocities of electrons are changed. They form bunches. In the high electron density area, space charge force stops the movement of electrons and pushes electrons in the reversed direction. When electrons fly toward reversed direction, new bunches are formed and the space charge force acts in the new bunches. Electrons oscillate thus back and forth. This is called "plasma oscillation" and the frequency with which the electrons oscillate is called "electron plasma frequency" $\omega_p$:

$$\omega_p = \sqrt{\frac{e^2 n_0}{\varepsilon_0 m_e}} \quad (2.52)$$

e is the charge of one single electron, $n_0$ is the electron density, and $m_e$ is the mass of an electron. The only variable in the definition of the plasma frequency is the electron density of the beam. $\lambda_p$ is referred to as plasma frequency wavelength:

$$\lambda_p = \frac{\omega_p}{2\pi} = \frac{\beta c_0}{f_p} \quad (2.53)$$

When electrons are in an infinite beam only the variations in axial direction are considered. This means that all quantities are only related to $z$ and $t^2$:

$$v(z, t) = v_0 + v_1 e^{i(\omega t - \beta z)}$$
$$q(z, t) = q_0 + q_1 e^{i(\omega t - \beta z)}$$
$$E(z, t) = E_0 + E_1 e^{i(\omega t - \beta z)}$$
$$H(z, t) = H_0 + H_1 e^{i(\omega t - \beta z)} \quad (2.54)$$

Maxwell’s equations yield the following relations:

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$
$$\nabla \times \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (2.55)$$
$$\varepsilon_0 \nabla \cdot \vec{E} = q$$

\textsuperscript{2}index 0: large quantities; index 1: small quantities
The continuity equation $\nabla \cdot J = -\frac{\partial q}{\partial t}$ yields

$$q_1 = \frac{\beta}{\omega} J_1$$

(2.56)

And from Equation 2.55:

$$J_1 = -\frac{j k^2}{\omega \mu_0} E_1$$

(2.57)

The current density $\overline{J}$ could be expressed with $q \cdot \overline{v}$. With Equation 2.54 $J(z, t)$ is written as:

$$J(z, t) = q_0 v_0 + q_1 v_0 e^{j(\omega t - \beta z)} + q_0 v_1 e^{j(\omega t - \beta z)} + q_1 v_1 e^{2j(\omega t - \beta z)}$$

(2.58)

$q_1 v_1$ is the product of small signal level and is neglected in this analysis. Therefore the current density $J_1$ is also:

$$J_1 = q_0 v_1 + q_1 v_0$$

(2.59)

Combining Equation 2.56 and Equation 2.59 yields

$$J_1 = \frac{q_0}{1 - \frac{\beta v_0}{\omega}} v_1$$

(2.60)

From absolute derivation

$$\frac{d(v(z, t))}{dt} m_e = \left( \frac{\partial}{\partial t} (v_1 e^{j(\omega t - \beta z)}) + v_1 \frac{\partial}{\partial z} (v_1 e^{j(\omega t - \beta z)}) \right) m_e$$

$$= v_1 (j \omega e^{j\omega t} e^{-j\beta z} - j \beta \frac{dz}{dt} e^{j\omega t} e^{-j\beta z}) m_e$$

$$= -e E_1$$

$$\Rightarrow v_1 = -\frac{e}{j m_e (\omega - \beta v_0)} E_1$$

(2.61)

The current density $J_1$ can be expressed with the help of Equation 2.59 and 2.61:

$$J_1 = j \frac{\omega q_0 e}{m_e (\omega - \beta v_0)^2} E_1, q_0 = -n_0 e$$

(2.62)

Plasma frequency $\omega_p = \sqrt{\frac{e^2 n_e}{\varepsilon_0 m_e}}$ is integrated in to the equation above and

$$J_1 = -j \omega_0 \frac{\omega_p^2}{(\omega - \beta v_0)^2} E_1$$

(2.63)

Combining Equations 2.57 and 2.63 gives then

$$\beta = \frac{\omega \pm \omega_p}{v_0}$$

(2.64)

This equation gives two space charge waves, a slow wave with phase velocity
Chapter 2. Principles of the Klystron

\( \omega - \omega_p \) and a fast wave with phase velocity \( \omega + \omega_p \). The velocity of the beam with both slow- and fast waves \( v_{f,s}(z, t) \) can be written as:

\[
v_{f,s}(z, t) = v_0 + v_1 \left( e^{j(\omega t - (\frac{\omega}{v_0} + \beta_p)z)} + e^{j(\omega t - (\frac{\omega}{v_0} - \beta_p)z)} \right)
\]

(2.65)

\[
v_{f,s}(z, t) = v_0 + v_1 2 \cos(\beta_p z) e^{j(\omega t - (\frac{\omega}{v_0})z)}
\]

The ac velocity quantity \( v_1 2 \cos(\beta_p z) e^{j(\omega t - (\frac{\omega}{v_0})z)} \) is 0 at positions where \( \cos(\beta_p z) = 0 \). Here \( z = \frac{\pi}{2} / \beta_p = \lambda_p / 4 \). The slow or fast electrons all come to dc velocity at this position and there a bunch forms. The current density \( J_1 \) of the beam at this position reaches maximum. Substituting Equation 2.65 into Equation 2.60 yields:

\[
J_1(\lambda_p / 4) = \frac{q_0 \omega_p}{\omega} v_1 \sin(\beta_p \frac{\lambda_p}{4}) \sin(\omega t - \frac{\omega \lambda_p}{v_0 / 4})
\]

(2.66)

\[
J_1(\lambda_p / 4) = \frac{q_0 \omega_p}{\omega} v_1 \sin(\omega t - \frac{\pi \omega}{2 \omega_p})
\]

Passing this \( \lambda_p / 4 \) position the electrons are pushed faster or slower again and the next bunch occurs at position \( \frac{3}{4} \cdot \lambda_p \) and so. The "ballistic" bunching and the space charge force bunching both have influence on the drift length of the electron beam. At small signal level \( v_1 \ll v_0 \), the modulated electron beam reaches the maximum beam current at the quarter wavelength of the plasma oscillation. With increased energy level, the "ballistic" bunching dominates and the maximum beam current position drifts away from this quarter wavelength of the plasma oscillation. The small velocity gain \( \Delta v_z \) due to space charge is neglected. For all modulated beams, the maximum current position is always the result of the both bunching forces. The proper drift length of a modulated beam is to be found with the help of simulations and experiments.

2.5 Principles of Idler and Penultimate Cavities

So far the movement changes of electrons in input and output cavities are explained and also the influence of these changes. The resonance frequencies of input and output cavities are identical. The electron beam is modulated in the input cavity and the modulated beam then excites the output cavity and gives up its kinetic energy. Modern klystrons have evolved from two-cavity klystrons invented by the Varian Brothers to multiple-cavity klystrons, which also have extra cavities inbetween, i.e. idler and penultimate cavities, to increase the bandwidth and the power gain of klystrons. The resonance frequency of these cavities is tuned slightly higher than their beam modulation frequency. For a resonator, the excitation only happens with a signal within the bandwidth \( \omega_B \). \( \omega_B \) is defined as:

\[
\omega_B = \frac{\omega_0}{Q}
\]

(2.67)
When the beam frequency is higher than the resonance frequency, the resonator is capacitively excited. When the beam frequency is lower, the resonator is inductively excited. In both cases the voltage and current inside the resonator show a phase shift. To determine this phase difference, a cavity resonator is treated as a parallel LC circuit with a resistance $R_p$, Figure 2.14.

![Figure 2.14: Equivalent circuit of a cavity resonator as a parallel LC circuit](image)

The admittance of an LC circuit of a resonant cavity is written as:

$$Y(\omega) = \frac{1}{R_p} + j\omega C_p + \frac{1}{j\omega L_p}$$

(2.68)

$\omega$ is the frequency of the input signal $I_e$. This equation can be rewritten in the impedance form:

$$Z(\omega) = \frac{1}{Y(\omega)} = \frac{j\omega L_p R_p}{R_p - \omega^2 L_p C_p R_p + j\omega L_p}$$

(2.69)

$\omega_0$ is the resonance frequency of the cavity. A cavity with internal losses only could be seen as an ideal LC parallel circuit with resonance frequency $\omega_0$, [8]

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{L_p C_p}}$$

(2.70)

When $\omega L_p / R_p = 1/Q$, $\omega_B = \omega_0 / Q$, the impedance equation Equation 2.69 is written as:

$$Z(\omega) = R_p \frac{j\omega \omega_B}{\omega_0^2 - \omega^2 + j\omega \omega_B}$$

(2.71)

The phase $\phi$ of the impedance is then:

$$\phi = \tan^{-1} \frac{\Im\{Z(\omega)\}}{\Re\{Z(\omega)\}}$$

(2.72)

$$= \tan^{-1} \frac{\omega_0^2 - \omega^2}{\omega_0 \omega_B} \text{[rad]}$$
Here $\omega$ is the beam modulation frequency and $\omega_0$ is the resonance frequency of the cavity. The maximum phase difference of current and voltage is $90^\circ$. The phase shift in the idler cavity is tuned to between $60^\circ$ and $90^\circ$, Figure 2.15. The phase shift in the penultimate cavity is near $90^\circ$. If there is a phase difference between the electron current and voltage, the electric field/voltage does not have an acceleration or deceleration effect on the major population of electrons. The induced voltage in the penultimate cavity has its maximum value when the electron current is $0$. This accelerates the electrons behind a bunch and decelerates the electrons ahead of a bunch, [1]. The resulting effect on the bunches is that the margin skirt of a bunch is squeezed and the bunch quality is improved. Ideally the beam barely loses its kinetic energy during the passage in the cavity. For the induced voltage in the idler cavity, the phase shift is smaller than in the penultimate cavity. Not only the electrons at the bunch margin but also the electrons in bunches experience more force. The enhancement of modulation depth is greater than in the penultimate cavity but the kinetic energy loss is also greater. When a cavity is capacitively excited the electrons are pushed away from the bunches and the bunch quality is impaired.

![Figure 2.15](image)

*Figure 2.15: Left: induced voltage lags electron current by $90^\circ$ in the penultimate cavity; right: induced voltage lags electron current by $60^\circ$ in the idler cavity; induced voltage in solid red lines; electron current in solid blue lines.*

### 2.6 Input and Output of the RF Signal

Certain methods are required to insert an input signal into a cavity or to transport the output power from the output cavity to the external network. An output waveguide could be installed in the modulation gap. But in this case the coupling factor $\beta_n$ is not adjustable. So that the resonator could be overcritically coupled, which means $\beta_n > 1$. The resonator may lose the function as resonator and work only as a piece of waveguide with bad scattering parameters. A small metal loop is used for inductive coupling, a metal stub for capacitive coupling, or just small apertures in the conducting wall of the cavity, which could result in capacitive, inductive or mixed coupling depending on the location of the coupling aperture, [17][25]. Here, the coupling method is to control the integration of the external load to the inner circuit, i.e. resonator circuit. Aperture coupling
is our choice because it is easy to be constructed. In our case, the location of the aperture is chosen to be in the middle area of the outer conducting wall, where the magnetic field is dominant. Therefore the magnetic coupling happens. The aperture is rectangular and connects a standard waveguide to the cavity. When the size of the aperture changes the coupling factor \( \beta \) between the external network and the cavity also changes. The relation of coupling factor and beam power extraction, Figure 2.16 (left), and the choice of coupling factor were discussed earlier.

For a rotating mode, two orthogonal modes are excited in the cavity and, therefore, at least two input coupling apertures are needed for rotating mode excitation. One is responsible for one orthogonal mode. But input apertures change the boundary condition, which means that electric and magnetic fields change in the local area. The field change has impact on the symmetry of the modes. \( \text{TM}_{110} \) has at least \( 2m \) field maxima. With only one aperture for one mode, one field maximum is weakened and the other \( (m - 1) \) field maxima are left intact. The mode is no longer uniform. Thus, \( 2m \) coupling apertures are needed to ensure the symmetry of the rotating mode \( \text{TM}_{m10} \). The coupling apertures are allocated with a \( \frac{180^\circ}{m} \) azimuth.

As for the output waveguide, a rectangular waveguide with standard size is envisaged. The waveguides are designed in a way that only the \( \text{TE}_{10} \) mode propagates. For the 10 GHz signal is WR90 and for 90 GHz it is WR10. A ring coupler is optional for combining these output waveguides. Figure 2.16 (right) is a design proposal of this ring coupler. This proposal is also investigated in Section 4.1.1, 4.2.1. The advantages and disadvantages of this coupler are discussed.
Chapter 3

Klystron Simulation

An analysis based on the mathematical model gives the first impression of the working principles of klystrons with modulated hollow electron beam. It serves well as a starting point of klystron design, but with many constraints neglected. A more realistic resonant cavity also includes an opening of the beam pipe, coupling slots and feeding apertures. The geometry changes in local areas change the electric field in the local area and influence the electric field of the cavity as a whole. All these together make the cavity complex and analytical solutions become excessively complicated. But with help of numerical simulations complicated structures can be investigated thoroughly. In numerical simulation programs, the investigation of static electric and magnetic fields, the interaction of beam and gap are carried out. Simulations are carried out mostly employing the program GdfidL, in association with Fortran scripts, [7]. MATLAB, [23], is used for data evaluation.

3.1 Routine of Modelling and Development

The numerical investigation needs a proper routine to solve the problem effectively. The basic flow diagram of simulation is shown in Figure 3.1. This working flow diagram is valid not only for a single cavity but also for the whole klystron. The input, idler and output cavities are characterized individually with eigenmode, decay and PIC (Particle-in-cell) simulations. It ensures that the resonance frequency, the coupling factor, the quality factor and so on in each cavity, meet the requirements of the whole klystron design. The beam and gap interaction is then investigated with PIC simulation. In the next Chapter 4 and Chapter 5 the analysis, problems, considerations and results are documented in detail.

The first step in the flow diagram is to determine the basic parameters of a klystron analytically. The radii and the length of a modulation gap are calculated with Equation 2.14. But a numerical model uses a discrete mesh and the resonance frequency found in the computational volume is thus different from the analytical value. The opening of the beam pipe adds more drifting to the resonance frequency. The right eigenmode with the right eigenfrequency is to be found in the eigenmode simulation. In the standing wave structure, i.e. cavity resonator, Maxwell’s equations for the resonant structure modes and
Chapter 3. Klystron Simulation

Evaluation

Analytical Calculation
- length, radii of cavity

Eigenvalue Simulation
- new radii of cavity
- beam pipe opening
- \( f_0, Q_0 \)
- shunt impedance in the cavity
- flat electric field in z-direction

Decay Simulation
- \( f_L, Q_L, \beta_k \)

PIC Simulation
- 3 D plot of electric fields and electrons
- output power
- electron velocity change
- voltage across the gap

FIGURE 3.1: Flow diagram of simulation of one cavity
resonance frequencies are:

\[ \nabla \times \vec{E} = -j\omega \mu \vec{H} \]
\[ \nabla \times \vec{H} = j\omega \varepsilon \vec{E} + \sigma \vec{E} = j\omega (\varepsilon + \frac{\sigma}{j\omega}) \vec{E} = j\omega \varepsilon \vec{E} \]
\[ \Rightarrow \nabla \times \frac{1}{\mu} \nabla \times \vec{E} = \omega^2 \varepsilon \vec{E} \]

\( (3.1) \)

The found eigenvalue is \( \omega \) and the found eigenvector is \( \vec{E} \). The eigenmode simulation in GdfidL calculates all the modes from the one with the lowest resonance frequency to the one with the estimated frequency. The mode with the desired fields and resonance frequency is chosen. The decay simulation follows. The frequency found in eigenmode simulation is \( f_0 \), but the external load has not been taken into consideration. The decay simulation applies the finite-difference time-domain method (FDTD-method), which was proposed by Kane Yee in 1966, [37]. The FDTD algorithm discretizes both time and space. The electric fields and magnetic fields are evaluated in the discrete time intervals, [27]. In the given time span the electric field and magnetic field of the chosen mode evolve to steady-state. The loaded quality factor could be found in this process by observing the damping of electric voltage across the gap. In this process the quality factor \( Q_L \) with respect to external load, and the resonance frequency \( f_L \) with respect to the external load are found. The detailed method is described in Section 3.2.

With \( f_L \) and \( Q_L \) determined in the decay simulation the Particle-in-Cell (PIC) simulation is carried out. The Particle-in-Cell method was introduced in 1955 by Francis H. Harlow of the Los Alamos National Laboratory for plasma simulation, [14]. The particles are tracked in continuous phase space. Electrons in the computational volume are generated with a Fortran program. The values of the initial velocity, the mass, and the position and all the other parameters are defined in the Fortran file. During the passage of electrons inside the computational volume all these values could change due to interaction with electric and magnetic fields. The values of electrons are well tracked in defined time steps as well as electric and magnetic field in the computational volume in time. In post-processing the function and efficiency of cavity also are evaluated thoroughly.

3.2 Numerical Model of the Klystron

The numerical program requires that a subject is discretized into meshes (grids) in the program. There are different grid forms, e.g. hexagon, tetragon. Also the grid size can be defined. But a grid has a finite size no matter what and a numerical model has a slightly different size than the analytical model. This is caused by discretization errors. There is a limited number of software products specialized in microwave tubes in this niche market. MAFIA, Microwave
Studio, GdfidL they all perform the eigenmode and PIC simulations in Cartesian coordinates. Poisson Superfish calculates the eigenmodes not only in static electric and magnetic fields but also in radio-frequency fields in 2 dimensions in both Cartesian coordinates and axially symmetric cylindrical coordinates. But PIC simulation can not be done in this program. The geometry found through Eigenvalue simulation must be imported to other software products for further analysis. The advantages of cylindrical coordinates of Poisson Superfish are lost in this process.

In this paper the simulation program GdfidL is used. GdfidL also uses Cartesian coordinates. The geometry parameters $\theta$ and $\phi$ of the cylindrical system of the klystron are transformed to $x,y$-planes. The two orthogonal modes found inside the gap have slightly different resonance frequencies $\omega_1$ and $\omega_2$ but the same magnitudes. The beam frequency is $\omega$. When the rotating mode inside the resonator is excited, the electric field of rotating mode is the superposition of these two modes and written as, \[35\]:

$$E = E_0 \sin(m\phi) \cos(\omega_1 t) + E_0 \cos(m\phi) \sin(\omega_2 t)$$  \hspace{1cm} (3.2)

The differences of two frequencies to beam frequency are $\Delta \omega = \omega - \omega_1 = -(\omega - \omega_2)$. The equation for the electric field is rewritten as:

$$E = E_0 \sin(m\phi) \cos((\omega - \Delta \omega)t) + E_0 \cos(m\phi) \sin((\omega + \Delta \omega)t)$$  \hspace{1cm} (3.3)
The energy stored inside the cavity is:

\[
W = \frac{1}{2} \varepsilon_0 \int \int \int_V |E|^2 dV
\]

\[
= A \frac{2\pi}{\pi} \int_0^{2\pi} \left| \sin(m\varphi) \cos((\omega - \Delta\omega)t) + \cos(m\varphi) \sin((\omega + \Delta\omega)t) \right|^2 d\varphi
\]

\[
= A \left( \frac{1}{2} \cos(2(\omega - \Delta\omega)t) - \frac{1}{2} \cos(2(\omega + \Delta\omega)t) + 1 \right)
\]

\[
= A(1 + \sin(2\omega t) \sin(2\Delta\omega t))
\]  

(3.4)

The term \(\sin(2\Delta\omega t)\) in Equation 3.4 can be written as \(\sin(2\Delta\phi)\). \(\Delta\phi\) is the phase drift of one orthogonal mode to the electron beam. From Equation 2.72 is \(\Delta\phi\) rewritten as:

\[
\Delta\phi \approx \frac{\omega^2 - \omega_B^2}{\omega_2\omega_B}, \omega_2 = \omega + \Delta\omega
\] 

(3.5)

for the case \(\frac{\omega^2 - \omega_B^2}{\omega_2\omega_B} \ll 1\).

\[
\Delta\phi = \frac{(\omega^2 - (\omega + \Delta\omega)^2)}{(\omega + \Delta\omega)\omega_B} = \frac{-\Delta\omega^2 + 2\omega\Delta\omega}{(\omega + \Delta\omega)\omega_B}
\]

\[
\approx -2\frac{\Delta\omega}{\omega_B}
\]

(3.6)

\[
\Rightarrow \Delta\omega t = \Delta\phi \approx -2\frac{\Delta\omega}{\omega_B}
\]

\[
\Rightarrow \sin(2\Delta\omega t) \approx -\sin\left(2\frac{\Delta\omega}{\omega_B}\right)
\]

The expression \(\sin(2\Delta\omega t)\) defines the amplitude of the oscillation of \(W\) and can be replaced with \(a\), then Equation 3.4 is rewritten as:

\[
W = W_0(1 + a \cdot \sin(2\omega t))
\] 

(3.7)

Oscillation of \(W\) leads to the output power oscillation. The oscillation amplitude of the output power should be kept under 10%. It means

\[
\sin(2\Delta\omega t) \leq 0.1
\] 

(3.8)

Supposing the cavity has a "loaded" \(Q\) of 500 and beam frequency is 10 GHz and the acceptable frequency difference \(\Delta f\) is under 1 MHz which is 50% of bandwidth. The asymmetry of two orthogonal modes is caused by the incompatibility of cylindrical structure and Cartesian coordinates. Given different values to \(\mu_r\) and \(\kappa_m\) of tuning rings can minimize the difference.

The tuning ring is a tool that was introduced to simulate all these losses in the numerical environment. The tuning ring is made of magnetic lossy material with tunable \(\mu_r\) and \(\kappa_m\). \(\mu_r\) is the relative permeability of the material and
may be given a value from 0 to 10 to adjust the field amplitude in the cavity, which is related to the resonance frequency. $\kappa_m$ is the magnetic conductivity of the material. The tuning ring is placed at the location with high magnetic field. Tunable $\kappa_m$ leads to tunable power loss, which can add additional power loss for different situations. A change in $\kappa_m$ also causes change in the resonance frequency of the cavity, Equation 3.9, but this change is smaller than the one through $\mu_r$ tuning.

With the output apertures the asymmetry of these two modes is worsened. The openings of the output apertures are rectangular and are given identical sizes. But in the numerical environment, the different apertures could contain different mesh cells. It means that the sizes of the output apertures are changed unwillingly. The field changes at local areas are different due to this mismatch in the aperture sizes. This leads to an enhanced asymmetry of the two modes. In GdfidL it is possible to define the fixed perpendicular lines (function "fixlines") in the x,y or z planes at certain positions of the geometry. It forces the boundaries of the output apertures to allocate at the intended positions. Thereby the asymmetry caused by the output apertures is reduced.

**Decay simulation**

Eigenmode simulation can determine the eigenfrequency $f_0$ and the unloaded quality factor $Q_0$, but in the eigenmode simulation the influence of skin effect and possible external losses are not considered. The simulation treated the cavity as an ideal LC circuit without external losses. The unloaded quality factor $Q_0$ in the simulation is determined from the power loss in the wall only. The loaded quality factor $Q_L$ and the corresponding resonance frequency $f_L$ with respect to the external loss could only be found with finite-difference time-domain (FDTD) method. The surface roughness during the fabrication is not as smooth as in the ideal numerical model. The surface roughness is also a factor that affects the quality of a resonator. Due to the rough surface the power loss inside the cavity $P_{int}$ is increased by 15% to 30%. This additional loss can be simulated with increased $\kappa_m$.

In some cases the existence of input/output apertures increases the asymmetry of the two orthogonal modes dramatically, in some other cases the work to find out the proper input/output coupling factor is challenging. In all of these situations a tuning ring is used instead of the input/output apertures. For the unloaded cavity $\kappa_m$ is chosen at a value, at which the magnetic loss equals the electric loss. Then for the loaded cavity $\kappa_m$ is chosen $(1 + \beta_n)$ times higher than the $\kappa_m$ in the unloaded cavity. Thus, $Q_0/Q_L - 1 = P_{out}/P_{int}$.

A resonant cavity contains unlimited modes, if the frequency range is unlimited. But the right mode can only be excited properly with a signal of the right frequency. In the ideal case a resonant cavity is without losses. But a real cavity is always with losses. These losses can be wall losses only or also additional external losses. When the cavity is no longer lossless, the resonance
frequency $f_r$ is no longer $\frac{1}{2\pi\sqrt{LC}}$ and it drifts under the influence of this loss. A parallel LC circuit with loss is usually transformed to an LC circuit with series resistance $R_L$ for the resonance frequency analysis, Figure 3.3. The resonance frequency $f_r$ is then:

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} = f_0 \sqrt{1 - \frac{R_L^2 C}{L}}$$

(3.9)

When the cavity is without external loss, then

$$\frac{R_L^2 C}{L} = \frac{1}{Q_0^2}$$

(3.10)

For a good cavity the internal loss is small and the quality factor is high. The impact of the internal loss on the resonance frequency is minimum. In this situation the $f_r$ from Equation 3.9 is still called the "unloaded" resonance frequency $f_0$.

The external losses are introduced to the circuit through the input/output apertures. When the cavity is with external losses, $Q_L = \frac{Q_0}{1 + \beta_n}$. Similar to Equation 3.10, the series impedance $R_L$ of this "loaded" cavity is

$$\frac{R_L^2 C}{L} = \frac{(1 + \beta_n)^2}{Q_0^2}$$

(3.11)

The resonance frequency with external losses, in other words, the "loaded" frequency $f_L$, is calculated from Equation 3.9. The "loaded" resonance frequency drifts further away from the $f_0$ and is smaller than the "unloaded". With growing $\beta_n$ the $f_L$ gets lower. Through the decay simulation the right resonance frequency $f_L$ with external losses is determined. $f_L$ of the input cavity and of the output cavity shall be identical. $f_L$ of the idler cavity is tuned to a higher value. When the decay simulation is done the PIC simulation can take place.
Chapter 3. Klystron Simulation

But in the numerical simulations a special method is applied for the determination of $f_L$. In the previous chapter it is introduced that loaded quality factor $Q_L$, voltage across the gap $U$ and charging time $\tau$ are related, Equation 2.35. Eigenmode and decay simulations are done without the presence of electrons. The important parameters of the cavity are determined. Figure 3.4 plots voltage decaying inside the cavity as a function of time. Loaded quality factor $Q_L$ is calculated from:

$$\Delta U = U e^{-\Delta t/\tau}$$
$$\tau = \Delta t / \ln(U/\Delta U)$$
$$Q_L = \frac{\tau \omega}{2}$$

(3.12)

$\Delta t$ is the time interval of two neighbouring maxima of $U$. The loaded resonance frequency $f_L$ is determined from the following equation, Figure 3.5:

$$f_L = 1/\Delta t$$

(3.13)

Figure 3.5 indicates that the cavity enters a steady state after the oscillation at the beginning. This evaluation method is introduced in [10].

![Figure 3.4: Blue line plots the voltage $U$ across the gap over time (resonant periods) and red line plots exponential decay of voltage as a function of time](image)

**PIC simulation**

With the found $f_L$, Particle-in-Cell (PIC) simulations are carried out. The mesh size is defined as fine as possible to resemble a cylindrical cavity. When the mesh size is chosen as 200 $\mu$m, the computational volume of a 10 GHz cavity contains nearly one millions cells. As for the numerical model of the W-band
klystron the computational volume contains more than four million cells. The enormous number of cells is not the only problem. In the first simulations, eigenmode and decay simulation, are done without the presence of an electron beam, in other words, the klystron is non-driven. The participation of an electron beam is necessary for PIC simulation: Particles are taking part in the interaction. The program has to determine the electric and magnetic field in each cell and also other parameters, like position and velocity of each electron that is emitted in the computational volume. It requires a lot of computational capacity. In order to save computational capacity, the common solution is placing a PEC or PMC wall in a symmetry plane $x = 0$, or $y = 0$ to save half of the computational volume. But the rotating mode is very sensitive to mesh changes of the numerical model and the parameters. That means $f_L$, $Q_L$, $\beta_k$, which are found in this half structure could not be used in the PIC simulation. PIC simulation for a helical beam needs a complete cavity because the fields and electron movement are not mirrored in the $x$ or $y$ plane. For PIC simulation the electron beam is not generated by an electron gun but by a Fortran program which defines the inner and outer radii of the beam, the start position and the start velocity of electrons. This beam is a nearly perfectly modulated beam with the required modulation depth. The number of emitted electrons is chosen higher than that of the mesh cells of the emitting surface. Figure 3.6 shows that the electron beam loses its initial kinetic energy during its passage through the output gap. The extracted beam power could be either directly monitored at the output wave guides (if available) or calculated from the change of the electron velocity. Supposing an electron moves with relativistic velocity. The whole energy $E$ of this electron is the sum of electron energy at rest $E_0$, and the kinetic
energy $E_{\text{kin}}$:

$$E = E_0 + E_{\text{kin}} = \gamma m_0 c^2$$

$$E_0 = m_0 c^2$$

$$E_{\text{kin}} = q U_0$$

$U_0$ is the dc voltage for electron acceleration. The electrons gain kinetic energy in the electron gun. The Lorentz factor $\gamma$ is the ratio of relativistic electron mass $m_{\text{rel}}$ to its mass at rest $m_0$:

$$\gamma = \frac{m_{\text{rel}}}{m_0} = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

FIGURE 3.6: Electrons gradually lose energy during their passage in the output gap; the distribution of electron velocity in solid blue line, the average velocity of electrons in green lines, the position of gap marked with dashed blues lines.

The dc voltage $U_0$ is related to the electron’s initial relativistic velocity $v_0$ with the following equation:

$$v_0 = c \cdot \sqrt{1 - \frac{1}{\left(1 + \frac{U_0 e}{m_e c^2}\right)^2}}$$

The velocity of electrons after the output gap is $v_{\text{new}}$ and dc voltage $U_{\text{new}}$ is:

$$U_{\text{new}} = \left(\sqrt{1 - \left(\frac{v_{\text{new}}}{c}\right)^2} - 1\right) \cdot \frac{m_e c^2}{e}$$
The lost kinetic energy of the electron beam, which has a current of $I_0$, in the gap is:

$$\overline{P}_v = U_0 \cdot I_0 - U_{\text{new}} \cdot I_0$$

$$= \overline{P}_{\text{int}} + \overline{P}_{\text{out}} = (1 + \beta_n) \cdot \overline{P}_{\text{int}}$$

(3.18)

### 3.3 From Design to Manufacture

The goal of the simulations is to test an idea before investing more time, personnel and money in production. There are tricks and methods used in simulations to resemble the problems that may occur in real life. But still there are constraints in real life which are ignored in the computational environment. The arcing and breakdown protection is one of them. The cavity is evacuated. The breakdown field of vacuum normally is $0.5 - 3 \times 10^5 \, \text{V/cm}$. In the simulations for the X-band klystron a continuous beam with beam power up to 5 MW is used. The voltage across the gap is over $3 \times 10^5 \, \text{V/cm}$. At the cavity edges the field strengths are even higher. A real life material could hardly withstand this voltage. Therefore, in real life applications, a pulsed electron beam is the recommended choice. The length of the pulse is variable depending on application requirements.

The electron beam in the simulation is also idealized. In simulation, the electrons of the hollow beam are casted homogeneously over the emission area which is difficult for an electron gun in real life. Calabazas Creek Research, Inc and Karlsruhe Institute of Technology developed a segmented Pierce cathode with increased homogeneity of the electron emission for an electron gun with annular beam, [22].

Another problem is that the stability of the whole klystron structure is taken for granted in the simulation. But the truth is, the inner part and outer part of the klystron stay in place only by the supporting points at the beginning and the end of the whole structure. The cavity’s self-weight may cause stretching of the metal in the middle of the structure after long time horizontal operation. The W-band structure is in $\mu m$ scale. Small changes of the beam pipe position may lead to collision of electrons with the conducting wall.

At last the beam focussing is also a matter of concern. For the simulation a homogeneous magnetic field in 400 mT is applied in beam transmission direction for the whole cavities. As in real life a solenoid is used.
Chapter 4

X-band Klystron

In this chapter the potential of a 10 GHz hollow beam klystron is investigated, focusing on the part from the input cavity, the intermediate cavity to the output cavity, Figure 1.1. The electron gun and the depressed collector are investigated in the later chapters. The beam current is chosen as high as possible with respect to the physical limitation. For the simulation, a CW signal is used but in the laboratory environment it can be replaced with a pulsed signal. For beam focussing a magnetic field of 400 mT in z-direction is applied. The cylindrical structure is very sensitive to mesh changes in the simulation and the klystron is a narrow band device. Small adjustments in geometry could cause unexpected frequency drifts in the cavity. Therefore the exact values of frequencies mentioned in the following investigation can undergo minor changes over the course of the investigation. The differences of frequencies are much more important. The beam modulation frequency equals the output cavity resonance frequency; the input cavity and the output cavity have the same resonance frequency, or the frequency difference of the two orthogonal modes is less than a few percent.

4.1 TM\textsubscript{110}-Mode X-band Klystron

The design of the X-band klystron starts with a resonator operating in the TM\textsubscript{110}-mode at a beam power of 1 MW. The dc acceleration voltage of the electron beam is 100 kV and the velocity of electrons is 0.55c\textsubscript{0}. The dc current of the electron beam is 10 A. The modulation depth of electron beam is chosen as 5% for output cavity investigations. Important parameters of this output cavity are in Table 4.1. These parameters define the character of a klystron and their values need to be chosen at the beginning of the design.

4.1.1 Output section

The design begins with the output cavity. For a single gap cavity, various prospects of the cavity are examined. This is the first part of the klystron which is investigated.

The resonance frequency of the cavity equals the beam modulation frequency. The beam frequency is given and the initial radii and the length of the output
centre frequency of the input signal $f_0$ & 10 GHz  \\
dc current of electron beam $I_0$ & 10 A  \\
accelerating voltage of electron beam $V_0$ & 100 kV  \\
speed of electron beam relative to the speed of light $\beta_0$ & 0.55  \\
mode in the input section & $\text{TM}_{110}$  \\
mode in the output section & $\text{TM}_{110}$  \\
mode in the intermediate section & $\text{TM}_{110}$

**Table 4.1:** Parameters of X-band klystron operating in the $\text{TM}_{110}$-mode

---

**Figure 4.1:** Normalized shunt impedance (left) and $E_\rho$ (right) of one cavity without beam pipe opening (solid blue lines), with beam pipe opening of 1.2 mm (solid green lines), and beam pipe opening of 2.8 mm (solid red lines)
gap are determined with Equation 2.14 and are adjusted to the correct values in the simulation. The inner radius should be large enough to support the whole inner conductor but not more in order to avoid unnecessary power loss caused by large conductor surface. Here for the one gap cavity the inner and outer radii are 4 mm and 20 mm and the resonance frequencies for the TM_{110}-mode are found as 10.044 GHz and 10.050 GHz in eigenmode simulations.

When designing the beam pipe opening, the following effects have to taken into consideration. The opening of the beam pipe at the maximum of the electric field changes the electric field in this local area and \( E_\theta \) increases, Figure 4.1. The electric field \( E_z \) near the beam pipe increases slightly compared to the field in an closed structure. \( E_z \) decreases in the beam pipe area and at the centre of opening it reaches its minimum. The shunt impedance of the cavity is proportional to \( E_z^2 \). With increased beam pipe opening the shunt impedance drops. And, furthermore, the increased opening results in an increased \( E_\theta \). This quantity accelerates electrons in radial direction and means greater chance of collision with the conducting wall. Only at the opening centre \( E_\theta \) ceases. These are the arguments for a narrow opening. But the electron beam has to maintain certain distance to the conducting wall and the beam itself should not be compressed to a thin sheet to avoid excessive space charge force effect. The size of the beam pipe has to be constrained to avoid unnecessary shunt impedance drop but at the same time providing enough space for beam passage.

The opening of the beam pipe changes not only the field magnitudes but also the resonance frequency. The resonance frequency drifts away from the initial value due to change of impedance, Equation 3.9. The two orthogonal modes in the cavity with beam pipe opening 1.2 mm have the frequencies of 10.091 GHz and 10.097 GHz. The resonance frequencies are 10.283 GHz and 10.289 GHz, when the opening is 2.8 mm wide.
Four waveguides are installed through the output apertures in the output cavity. The location of output apertures is chosen at the outer conducting wall. As for the output waveguides, standard waveguides WR-90 are used. With these output apertures in the cavity the frequency of cavity drifts again. The opening cuts through the surface of the cavity and increases the frequency. But through the output apertures the external load is introduced in the cavity. With this extra loss the resonance frequency drops slightly. The output waveguides cross the computational boundaries. At the boundaries the locations of output waveguides are defined as “ports”. In GdfidL ports location can be specified only on x,y or z planes. A "port" is a part of the border of the computational volume that shall be treated as an infinitely long waveguide, [4]. Frequency and amplitudes of the chosen modes are monitored at these ports. In our design this provides a direct method to monitor the extracted output ac power in time sequence.

\[ R/Q \quad [\Omega/m] \]

![Figure 4.3: R upon Q inside the coaxial cavity with beam pipe (left) and without beam pipe (right); $\beta = 0.55$](image)

The basic schematic of the output cavity is plotted in Figure 4.2. The inner and outer radii of the cavity are found as 4 mm and 20.5 mm respectively. The inner radius of the beam pipe is 10 mm and the outer radius 12 mm. Shunt impedances in this cavity with and without beam pipe are shown in Figure 4.3. The shunt impedance remains high in the beam pipe area. The beam has an inner radius of 10.65 mm and an outer radius of 11.3 mm. The centre of the beam matches the centre of the beam pipe. It ensures the maximum interaction of electrons with $E_z$ and minimum interaction with $E_x$. The current density of the beam is 20 A/cm$^2$. The gap length is 5 mm, which is 3 mm shorter than the value from $\frac{\delta e}{\delta t}$. This 3 mm length is reserved for the iris thickness between gaps. The mesh cell size is chosen as 400 $\mu$m. It is a compromise between simulation time and an accurate model of the cavity.

Klystron operation produces heat in the cavity. With a beam power of 1 MW an efficient heat dissipation strategy is a must to ensure the lifespan of the device, [24]. The specific cooling method is not discussed in this paper. But an
iris thickness of 3 mm provides sufficient surface for different possibilities. Microchannel cooling is one of them.

\[ \eta = \frac{P_{\text{out}}}{P_{\text{beam}}} \]  \hspace{1cm} (4.1)

The size of output apertures needs to be modified in order to achieve the desired coupling factor. The modulated beam is generated with a FORTRAN program with an ideal sine signal. An electron beam modulation depth of 5% is chosen. The electron beam has travelled a certain drift space before entering the computational volume, i.e. cavity. Bunches are formed. An output cavity which is excited with modulated beam is shown in Figure 4.4 and Figure 4.5. The output power $P_{\text{out}}$ is read directly at the ports and the extraction efficiency is calculated from

A thin layer ring with lossy magnetic material is placed next to the inner conductor. It works as tuning ring to adjust the resonance frequency to the desired value. Another function of it will be discussed later. The quality factor $Q_0$ of this cavity is 5100. The size of output apertures can be adjusted to adjust
the loaded quality factor $Q_L$. The coupling factor $\beta_k$ is

$$\beta_k = \frac{Q_0}{Q_L} - 1$$

(4.2)

In Chapter 2 it was lined out that $\beta_k$ between 3 to 6 ensures balance of output power and wall losses. A proper $\beta_k$ has to be chosen with respect to the loaded shunt impedance, the output power and other factors. Table 4.2 gives information about loaded Q factors, centre frequencies of the two orthogonal modes of a one gap cavity under different widths of output apertures when the height of the output apertures is 4.8 mm. The difference of $Q_L$ values of the two modes is not to be ignored. The asymmetry of two modes caused by discretization errors through Cartesian coordinates has been noted in Chapter 3. Not only the radially symmetrical cavity but also the output apertures are affected. When the output apertures have slightly different sizes, the disruptions to the field are different. But the asymmetry in Q factors and in centre frequencies of the two modes decline with increased width of output apertures, or more precisely, with increased coupling factor $\beta_k$, because the mesh discretization errors decrease with increased aperture size. The extracted RF power and the average velocity of the electrons through the output gap are plotted in Figure 4.6 and
Figure 4.7. The electron beam here is given a drift length of 70 mm and is not optimized.

<table>
<thead>
<tr>
<th>width of output apertures [mm]</th>
<th>$Q_{L1}$</th>
<th>$Q_{L2}$</th>
<th>$f_{L1}$ [GHz]</th>
<th>$f_{L2}$ [GHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.4</td>
<td>800</td>
<td>2300</td>
<td>10.32483</td>
<td>10.32476</td>
</tr>
<tr>
<td>7.2</td>
<td>510</td>
<td>920</td>
<td>10.27025</td>
<td>10.27016</td>
</tr>
<tr>
<td>7.6</td>
<td>420</td>
<td>640</td>
<td>10.23965</td>
<td>10.23950</td>
</tr>
<tr>
<td>8.2</td>
<td>300</td>
<td>400</td>
<td>10.19200</td>
<td>10.19201</td>
</tr>
<tr>
<td>9.0</td>
<td>190</td>
<td>220</td>
<td>10.11865</td>
<td>10.11866</td>
</tr>
<tr>
<td>9.8</td>
<td>120</td>
<td>130</td>
<td>10.01297</td>
<td>10.01304</td>
</tr>
</tbody>
</table>

**Table 4.2:** Results of one gap TM$_{110}$-mode output cavity in the X-band under different $Q_L$

In some cases the rotating mode could not be excited evenly. For example in the output cavity with output aperture width of 6.4 mm, one mode has a 50% higher amplitude than the other mode, Figure 4.8 and Figure 4.9. The electrons experience an uneven deceleration in the radial direction. The beam is deformed, Figure 4.10. In the worst case a deformed beam could not excite a rotating mode any more and the charged cavity loses power gradually. The extracted power drops to zero. Not only asymmetry of the modes but also strong fields in the low $\beta_e$ area are responsible for the deformation. The strong electric field decelerates electrons dramatically and the trajectory of the beam is sharply interrupted. The gradual deceleration does not happen in this case.

The problem of excessively strong fields can be solved by increasing the coupling factor $\beta_e$. But the asymmetry of two modes caused by the mesh errors,
that occurs here, can not be solved. Reducing the mesh size can reduce the
difference but with great sacrifice on simulation time. There is a better option
to limit the mesh discretization error’s impact. Using the $-x_{\text{fixed}},-y_{\text{fixed}}$ or
$-z_{\text{fixed}}$ functions in GdfiD to enforce the mesh-planes at the boundaries of
output apertures could at least make sure the output apertures have the same
size in the simulation and reduce the negative influences of the openings on the
symmetry. These enforced mesh-planes are to be placed at the outer radius of
the cavity too. But for the cavities with relatively high $\beta_n$ values, e.g. cavities
with output apertures of 9.0 mm and 9.8 mm, the symmetry is improved but the
output power is also significantly lower compared to the cavities with smaller
apertures. Thus the choice falls on cavities with $\beta_n$ around 8 to 10. The chosen
output cavity has an aperture width of 7.6 mm.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.7}
\caption{Average normalized velocity $\beta$ of electrons in one gap output cavity with different output aperture widths}
\end{figure}
Figure 4.9: A zoomed out plot of voltages of these two modes, the 90° phase shift between them is easily recognized.

Figure 4.10: Electron beam excites the TM$_{110}$-mode and the beam deforms through deceleration.

Figure 4.11 shows the velocities of electrons travelling through an output gap with an aperture width of 7.6 mm. The average normalized velocity $\beta$ is reduced from 0.55 to 0.49. Using Equation 3.17 and Equation 3.18 the extracted beam power is 250 kW. The coupling factor $\beta_n$ of this cavity is about 9 and the power transported to the output waveguides is 220 kW. The new value of $\beta = 0.49$ is only estimated, since with error margin the beam power calculated here equals the power monitored at the ports. The results of these two power observing methods are consistent. The velocity components in x and y directions show no noticeable changes. It means the slight increase of $E_y$ in the gap has no impact on the electrons. The normalized velocity $\beta$ in the z-direction distributes over a broader spectrum. While the maximum value reaches 0.64, the minimum touches down to 0.37. Since there is still significant power left in the beam after deceleration in the one gap cavity, another gap is introduced.
into the structure.

![Figure 4.11: Velocity of electrons in the output cavity with an aperture width of 7.6 mm; left: \( \beta \) of electrons in z-direction in solid green line and average \( \beta \) in solid blue line; right: \( \beta \) of electrons in x-direction (blue dots) and in y-direction (red dots) before and after the travelling in output cavity.](image)

The first step in the two-gap cavity design is to find the proper coupling mode between the two gaps. The easiest way to transport the extracted RF power to the external network is of course using output apertures, which are connected to the output waveguides. But in the two gap cavity, it means four output waveguides installed in the first gap, another four installed in the second. The space outside the output cavity becomes crowded. When two gaps are coupled, the extracted power from the gap without output apertures could be transported to the gap with output apertures. Then the whole portion of RF power is transported to the external network. As discussed in Section 2.3 two coupling methods are possible. Electric coupling uses the existing beam pipe and it may be necessary to broaden the pipe for a better coupling efficiency. A broader opening leads to a further drop of shunt impedance in the beam pipe area. Magnetic coupling needs extra slots, but it avoids the shunt impedance drop. Therefore, four coupling slots are placed in the high magnetic field area, which in this case is near the outer conductor of the coaxial resonator.

Four output waveguides are installed in the first gap of the cavity, Figure 4.12. Four coupling slots connect two gaps. The coupling mode is \( \pi \)-mode. The phase shift from one gap to the next gap is \( 180^\circ \). It makes sure that when electrons enter each gap, they always experience maximum deceleration. As for the coupling, there are several defining quantities. One important quantity is the passband of the coupled chain. This is defined as the frequency range between 0-mode and \( \pi \)-mode. The size of coupling slots is tuned in a way that
good coupling happens between gaps. It means the passband is over 2% of the resonance frequency. The resonance frequency of \( \pi \)-modes drifts away from the resonance frequency of a single cavity and is higher than the cavity resonance frequency. Meanwhile the resonance frequency of 0-mode is below the cavity resonance frequency.

![Figure 4.12: Side (left) and top view of two-gap X-band output cavity in 3d](image)

Another feature in the two-gap output cavity is that the electric field inside two gaps is tuned to the same magnitude through the tuning ring inside the cavity, Figure 4.13. It is important for an even deceleration of the electron beam.

![Figure 4.13: Magnitude of electric field in two-gap structure with (right) and without (left) tuning; blue dashed lines mark the position of gaps](image)
When these two points above are set, it is time to consider the proper length for the second gap. The minimum normalized velocity $\beta_z$ of electrons is about 0.37 after the one-gap structure. The corresponding gap length is 5 mm. The electrons should only be decelerated in the gap and escape the gap before being accelerated again. To ensure this the gap length is shortened to 4 mm. The width of the output apertures is carefully chosen as 8 mm. $Q_0$ of this cavity is 4700 and the loaded Q factor $Q_L$ is 940. The velocity of electrons has a wide range as plotted in Figure 4.14 but the average velocity decreases gradually through the two gaps. The output power is then 350 kW. At last, the important parameters and results of the two-gap output cavity are shown in Table 4.3.

![Figure 4.14: The green line plots the velocity distribution of electrons through out the two-gap output cavity and the solid blue line plots the average velocity of electrons; gaps are marked with dashed blues lines](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>first gap length</td>
<td>5 mm</td>
</tr>
<tr>
<td>second gap length</td>
<td>4 mm</td>
</tr>
<tr>
<td>iris thickness</td>
<td>3 mm</td>
</tr>
<tr>
<td>output power</td>
<td>350 kW</td>
</tr>
</tbody>
</table>

**Table 4.3: Results of two-gap output TM$_{110}$-mode cavities in X-band**

The electron beam travels through the two-gap output cavity and leaves the cavity with a certain amount of kinetic energy left in the beam. That is the reason for introduction of the third gap to the structure. The length of the second gap is reduced to 2.4 mm in order to make room for the iris between the second gap and the additional gap. The iris is 3 mm thick. The third gap has a length of 2 mm. The coupling between gaps reduces the resonance frequency further. To obtain the centre frequency of 10 GHz, the outer radius of the cavity is reduced
Four coupling slots connect each two gaps together. Staggered slots, Figure 4.15, and aligned slots, Figure 4.16, are two arrangements for a coupled cavity chain, which contains more than two gaps. They are mentioned and investigated by J. F. Gittins, [9]. These two arrangements lead to different characters of coupling chains. The primary difference is that aligned-slot structure has a smaller bandwidth. Thus a staggered arrangement is chosen for our application, Figure 4.17. The coupling slots connecting the first and the second gap are in red and the slots connecting the second and the third gap are in blue. The three-gap cavity with coupling slots is plotted in Figure 4.18. It is not easy to recognise that the coupling slots are not placed symmetrically in the numerical model. The angle of coupling slots connecting the first and the second gap have an offset of $30^{\circ}$ to the x axis instead. But the offset between the slots connecting

\footnote{The $30^{\circ}$ offset of the coupling slots to the x axis were originally a mistake in script-writing but in simulation the coupling slots in this arrangement turned out to achieve better symmetry than the perfect symmetrically arranged structure. The probable reason is that the numerical error of the cavity model is minimized by the asymmetric arrangement of coupling slots.}
Chapter 4. X-band Klystron

FIGURE 4.18: Three-gap structure excited in $\pi$ mode

the first and the second gap and the slots connecting the second and third gap is still 45°.

FIGURE 4.19: Dispersion curve of three-gap cavity; solid red line plots a fitting curve

The dispersion curve of this inductively coupled cavity chain is plotted in Figure 4.19. The passband of this chain cavities is about 3% of $\omega_0$. The slope

Since discretization errors will not occur in the laboratory environment, the asymmetry is to be corrected in the manufacture but not in the simulation.
of this curve is negative and because of that the group velocity $v_g$ is also negative. The extracted beam power travels backwards in the gaps. Four output waveguides are then installed in the first gap, Figure 4.18. $E_z$ along the beam pipe through out the three-gap structure is plotted in Figure 4.20. The width of the output apertures is chosen as 8.8 mm. $Q_0$ of this cavity is 5000 and $Q_L$ is 800. The average output power reaches 435 kW.

![Figure 4.20: Tuned $E_z$ along the centre of beam pipe in the output cavity; dashed blue lines mark the position of three gaps](image)

The average velocity of the beam is gradually decreased in three gaps, Figure 4.21. The minimum normalized velocity $\beta$ is under 0.1. It is not necessary to extend the output gap any further.

![Figure 4.21: Average normalized velocity $\beta$ of electrons in three-gap output cavity](image)
Electron bunching in the modulated beam needs a certain drift length to reach the maximum of the electron density. The input power level and the form of the beam both affect the drift length. The exact length cannot be predicted from space charge- and ballistic analyses alone. For a start, a drift length of 70 mm is chosen. Since the geometry parameters of the three-gap output cavity are defined, it is time to find the right drift length for the modulated beam. Our interest is to find out the drift length for which the output power is maximum. Simulations are undertaken. The results are plotted in Figure 4.22. The maximum output power is 460 kW with a drift length of 100 mm. The important parameters and results of the three-gap output cavity are summarized in Table 4.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>first gap length</td>
<td>5 mm</td>
</tr>
<tr>
<td>second gap length</td>
<td>2.4 mm</td>
</tr>
<tr>
<td>third gap length</td>
<td>2 mm</td>
</tr>
<tr>
<td>iris thickness</td>
<td>3 mm</td>
</tr>
<tr>
<td>output power</td>
<td>460 kW</td>
</tr>
</tbody>
</table>

Table 4.4: Results of three-gap output cavity in X-band with TM_{110}-mode

ring coupler

A ring coupler is employed to connect all the output waveguides together, [18], [11]. Figure 4.23 (left) shows a tentative design of a ring coupler connecting to a one gap cavity resonator. The ring coupler has the width of a WR-90 standard waveguide. The first version is a simple ring, Figure 4.23. The output apertures connect directly to the waveguide. The S-parameter investigation is not possible for this structure while the input signal could only be placed at a "Port", 

![Figure 4.22: Output power of the three-gap cavity with different drift lengths for the electron beam](image-url)
which can only be defined at boundaries of the computational volume. Therefore the quality of this waveguide is to be verified in the PIC simulation. The design of the ring coupler is similar to the investigation of the output cavity. The eigenvalue simulation is to identify two orthogonal modes in this cavity. Then decay simulation is done to determine $f_L$, $Q_L$ and $\beta_r$ of this cavity.

In PIC simulation, the output power travels clockwise along the output waveguide to the external network. The eigenvalue simulation and decaying simulation seem fine. Two modes are found in the eigenvalue simulation. The voltage decaying processes of the two modes are different, but this is no surprise for our cylindrical cavity because of the already discussed discretization errors. In the PIC Simulation an unexpected problem occurs. Only one orthogonal mode is excited, Figure 4.23 (right). The electric field maximum doesn’t rotate. The extracted beam power is only half compared to the cavity with a similar coupling factor in Section 4.1.1 but in rotating mode. Later in the TM_{310}-mode design, more proposals of ring couplers will be brought up and the results will be discussed.

**Figure 4.23:** Left: tentative design of a ring coupler connecting to one gap output cavity; right: output cavity is excited with only one mode

### 4.1.2 Intermediate section

In the intermediate section between the input cavity and the output cavity a so called "idler" cavity is introduced. There are two advantages of “idler” cavities. First, the bandwidth of the whole klystron is increased, and second, the power gain of the klystron is improved. The idler cavity operates in the TM_{110}-mode with a resonance frequency of around 10 GHz. But its resonance frequency is slightly higher than the modulation frequency of the electron beam. This makes sure that the "idler" cavity is inductively excited. In Chapter 3 the excitation of a resonant cavity by electrons is introduced. When a cavity is inductively excited, the voltage lags the current by 90° in the cavity. The electrons left in the area between two electron bunches could be either accelerated or decelerated, in both cases they get pushed toward the bunches. The bunches are squeezed and
the concentration of electrons in the bunches is improved. An "idler" cavity increases the modulation depth with little loss of beam power. The detailed equations and plots are presented in Section 2.5. From the same section it is known that the phase $\phi$ of the excited "idler" cavity is:

\[
\phi = \tan^{-1} \frac{\omega_0^2 - \omega^2}{\omega \omega_0 / Q_L}[\text{rad}]
\]  

(4.3)

Here $\omega$ is the angular beam modulation frequency and so $\phi$ is related to $\omega_0$ and $Q_L$ of the idler cavity. There is also a tuning ring placed inside the cavity. Its material parameters $\mu_k$ and $\kappa_m$ are adjustable to adjust the quality factor and the power loss of the cavity. With the help of a tuning ring, extra 20% power loss is added to the cavity to simulate the internal loss through the surface roughness. Normally the phase shift is chosen between 50° and 90°. If the phase shift is too small the “idler” cavity works more like an output cavity and extracts energy from the beam. When the phase shift gets too large (near 90°), the cavity works more like a penultimate cavity, which improves the bunching quality instead of enhancing the modulation degree. Thus choosing phase shift carefully is vital for an "idler" cavity’s function.

\[
Q_L = \begin{cases} 
6000 & \text{for } m = 15 \text{ and } f_0 = \omega_0/4 \\
3000 & \text{for } m = 1 \text{ and } f_0 = \omega_0/2 \\
1500 & \text{for } m = 0.5 \text{ and } f_0 = \omega_0 \\
750 & \text{for } m = 0 \text{ and } f_0 = 3\omega_0/4 
\end{cases}
\]

The outer radius is 19.5 mm and the inner radius is 4 mm. The length of the cavity gap is 8 mm. The quality factor $Q_0$ of this cavity is 7500. A tuning ring is placed inside the cavity next to outer conductor. To reduce the electric field magnitude across the gap, the length of the gap is reduced to 6.8 mm. In Figure 4.24 the relations of phase and $Q_L$ and frequency differences $\Delta f = (\omega - \omega_0)/2\pi$ are plotted in solid lines. A larger frequency difference is allowed with reduced $Q_L$. At this working point the quality factor $Q_L$ is tuned to 6250 with respect to loss of surface roughness. The surface roughness loss is about 20%. The quality factor should be tuned down to a smaller value to ensure the width of the pass-band. But at the moment of design it did not seem to be a bad idea to maximize
Chapter 4. X-band Klystron

the power gain through the idler cavity. It means enhancing the modulation depth as much as possible. Furthermore, the rotating mode is very sensitive to the frequency change. The exciting signal with frequency within the bandwidth does not secure the successful excitation of a rotating mode. It happens often that only one mode of the two modes is excited successfully. The two orthogonal modes have to be excited homogeneously. The small bandwidth due to high $Q$ is acceptable. The resonance frequency of the idler cavity is $2.7\,\text{MHz}$ higher than the beam frequency. The phase shift in the cavity is $73^\circ$.

<table>
<thead>
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<th>Value</th>
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</thead>
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<td>Q factor of cavity</td>
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</tr>
<tr>
<td>Q factor with surface roughness loss</td>
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</tr>
<tr>
<td>inner radius of cavity</td>
<td>4 mm</td>
</tr>
<tr>
<td>outer radius of cavity</td>
<td>19.5 mm</td>
</tr>
<tr>
<td>length of cavity</td>
<td>6.8 mm</td>
</tr>
</tbody>
</table>

**Table 4.5:** Parameters of an "idler" cavity for TM$_{110}$ mode X-band klystron

PIC simulation was done. The excited idler calculated by the GdfidL program is shown in Figure 4.25. The electron cloud is NOT plotted in the figure. Chokes and fins are going to be introduced in Section 4.1.4. A beam with an initial modulation depth of 0.7% enters the "idler" cavity and the modulation depth reaches 5% when the beam leaves the cavity, Figure 4.26. The drift space for this beam is found as 70 mm long which matches $\lambda_p/4$ of this hollow beam. The average normalized velocity $\beta$ of electrons drops to 0.548 and this means that the electron beam loses 1200 W of kinetic energy inside the cavity. It is only 1% of the beam power. Overall the "idler" cavity enhances the modulation depth enormously with little beam power loss.

4.1.3 Input section

Like the output cavity and the idler cavity, the input cavity design is investigated at first as a stand alone device. For the input section, one simple circular cavity is needed. The beam pipe size constrains the size of the input cavity. Four power feeding waveguides WR-90 are connected to the input cavity through rectangular input apertures, Figure 4.27 (left). Similar to the output cavity, a tuning ring adjusts the resonance frequency and the quality factor $Q_0$. The input aperture defines the coupling factor $\beta_k$. The unloaded $Q_0$ is 9100 in this cavity. The quality factor $Q_0$ is reduced to about 7500 with respect to extra loss due to surface roughness. If the loaded quality factor $Q_L$ is 3200, then the coupling factor $\beta_k$ is about 1 and the input cavity is critically coupled. The reflection of the input signal is at minimum. But $Q_L$ at 3200 means that the input cavity has a bandwidth of 3 MHz, which is only one quarter of the bandwidth of the output cavity. Furthermore, the small bandwidth of the input cavity means
Figure 4.25: Excited idler cavity with chokes and fins
Figure 4.26: Normalized velocity $\beta$ of electrons through idler cavity; green dashed lines mark the position of the cavity.

Figure 4.27: Left: drawing of the excited input cavity with four input waveguides in the simulation program; right: normalized velocity $\beta$ in the z-direction of the electrons through the input cavity.
the excitation of a rotation mode is extremely difficult. Thus the loaded quality factor $Q_L$ is chosen as 700 and the coupling factor $\beta_k$ is 10. The modulation depth of the beam after the input cavity should be 0.7%, Figure 4.27 (right), and the required average input power is 7 W. The input cavity is excited at a low energy level. The electron beam is modulated by a small signal for the first time inside the cavity. Small signal means that the fields in the input cavity are small compared to the idler cavity and the fields are sensitive to disruption. The input signals are fed into the input cavity one quarter period after each other clockwise or counter-clockwise. Thus a rotating mode can then be excited. This step decides the rotating direction of excited mode and beam. Table 4.6 lists the important information of this input cavity.

<table>
<thead>
<tr>
<th>Q factor of cavity</th>
<th>9100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q factor with surface roughness loss</td>
<td>7500</td>
</tr>
<tr>
<td>loaded Q-factor of cavity</td>
<td>400</td>
</tr>
<tr>
<td>inner radius of cavity</td>
<td>4 mm</td>
</tr>
<tr>
<td>outer radius of cavity</td>
<td>19.5 mm</td>
</tr>
<tr>
<td>length of cavity</td>
<td>8 mm</td>
</tr>
</tbody>
</table>

**Table 4.6:** Parameters of the input cavity for a TM$_{110}$ mode X-band klystron

### 4.1.4 De-coupling of cavities and whole klystron investigation

The coaxial structure of the beam pipe is an ideal waveguide for coupling between input and idler cavities, and all the possible modes can propagate. From the earlier results, it is known that an input power of 7 W means that the output is at 400 kW. The power gain in this klystron is over 47 dB. The isolation is very critical for this power level. The fields inside the input cavity are much weaker than inside the idler. When a minor portion of power propagates from the idler cavity to the input cavity, the fields in the input cavity are disrupted. The disrupted fields in the input cavity result in a deformed beam. The deformed electron beam means no more excitation of the right mode in the input cavity or in the idler cavity. At first the whole klystron is investigated without isolation. The helical pattern of the beam disappears after about 500 periods.

To prevent this chain effect, there are two approaches to de-couple the cavities from each other. One is to install fins between inner and outer conductors of beam pipe (Figure 4.29 (left)). $a$ is the length of the curve along the centre of the beam pipe. The four fins connect the inner and the outer conductor and turn a coaxial waveguide into four nearly rectangular waveguides. The hollow beam itself has to be modulated into a hollow beam with four slits avoiding collision of the electrons with the waveguide. Only the frequency above cut-off frequency of this rectangular waveguide can propagate through. The cut-off
frequency is approximated from a rectangular waveguide:

\[ f_{c, mn} = \frac{c}{2\pi} \sqrt{\left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2} \]  (4.4)

The dominant mode of a rectangular waveguide is \( TE_{10} \) and fins have a thickness of \( 1 \text{ mm} \). \( a \) is then \( 16.3 \text{ mm} \). The cut-off frequency for the \( TE_{10} \) mode of one beam pipe segment is \( 9.2 \text{ GHz} \), which is under the beam frequency. The thickness of the fins can be increased to increase the cut-off frequency. A small increment seems enough, because the signal only propagates properly when it is about 15% higher than the cut-off frequency. The new thickness of the fin is chosen as \( 2 \text{ mm} \). The cut-off frequency of the quasi rectangular waveguide is \( 9.8 \text{ GHz} \), which is still under the beam frequency of \( 10 \text{ GHz} \). But still the thickness of the fins are not increased further. Thicker fins also mean larger slits of the beam, which leads to a higher electron density in the beam. Higher electron density in the beam means that the effects of space force between electrons is stronger. Another disadvantage of fins is that the electrons may touch the surface of fins when they slightly rotate in radiation direction during travelling. Because of the focusing force provided by magnetic field in the longitudinal direction, the electrons also gain rotating velocity. Without investigation it is hard to tell, how likely collisions are. Collisions of the electrons and the conducting wall should be avoided, because it causes secondary emissions. The PIC simulation also shows that only four fins in the beam pipe can not prevent the vanish of the rotating mode in the cavities. The cavities were charged in the first place and then discharged eventually.

Another option is to use a choke flange (Figure 4.29 (right)): a choke is a radial transmission line with a radius \( r_{\text{choke}} \) approximately one quarter wavelength. The impedance that a signal "sees", when it enters this one quarter wavelength transmission line, is:

\[ Z_{\text{in}} = \frac{Z_0^2}{Z_L} \]  (4.5)

In this case the \( \lambda/4 \) transmission line is short-circuited. It means this choke has nearly an infinite impedance.

\[ Z_{\text{in}} = \infty \]  (4.6)

To determine the proper \( r_{\text{choke}} \) this radial transmission line is treated as a regular rectangular waveguide. The inner circular length is treated as the width of waveguide. The radius of this choke, \( r_{\text{choke}} \), is then:

\[
\begin{align*}
  r_{\text{choke}} &= \frac{1}{4\lambda_g} = 1/4 \cdot \frac{2\pi}{\sqrt{k^2 - k_c^2}} \\
  &= 1/4 \cdot \frac{2\pi}{\sqrt{(\frac{\pi}{c})^2 - (\frac{\pi}{a})^2}}
\end{align*}
\]  (4.7)
Here \( a = 75.4 \text{ mm} \) is considered as outer radius of the beam pipe and the cut-off frequency is 10 GHz. The radius of the choke \( r_{\text{choke}} \) is 7.5 mm. The height is 2 mm. Neither chokes nor fins should have influence on the fields inside the gaps. Thus, they are placed at least at the location with a distance of 2 mm to the gaps.

The transmission factors of different isolation variations are plotted in Figure 4.28. There are four fins with thickness of 1 mm, and with a fin thickness of 2 mm. There are also isolations with two chokes and the combination of two chokes and fins, the latter has a fin thickness of 2 mm. The isolation with four 1 mm thick fins has a transmission factor over 0.8 at 10 GHz, while the fins with a thickness of 2 mm reduce the transmission to 0.3 at 10 GHz. The isolations with chokes have much better performance than with fins. The transmission factor of the two choke variation is 0.15 at 10 GHz, and transmission scattering parameter \( S_{21} \) of the isolation with chokes and fins is around 0.25. No matter which isolation configuration, the lowest value of \( S_{21} \) is not at 10 GHz exactly. Because the mesh size in the computational volume can not be defined infinite small, the lowest point of \( S_{21} \) could not be tuned to the exact value we wish.

![Diagram](image)

**Figure 4.28:** Transmission factors \( S_{21} \) of the beam pipe with different isolation strategies;

This isolation problem reminds us that the idler cavity should not work with an electric field which has large amplitude difference compared to the field in the input cavity. It is useful to reduce \( Q_L \) in the idler cavity, not only for the bandwidth of the whole klystron but also for the possible isolation from input section to output section.

A klystron with input, idler and output sections is plotted in Figure 4.30. In this figure, the chokes and fins serve as coupling suppression. Another variation of the coupling suppression is using chokes alone, which are placed before,
and after the idler cavity and before the output cavity, Figure 4.31. The complete PIC simulations are done for these two variations with the whole klystrons from the input cavity, over the idler cavity to the output cavity. The investigations show that the chokes alone are sufficient to isolate cavities from each other, which is consistent with the investigation results of the transmission factor, Figure 4.28. Without fins means that the hollow beam does not have to be divided by slits. This avoids manufacturing difficulties because of the fins in the beam pipe. For the klystron variation with chokes alone the input signal has an average power of 7 W and the average output power is then 500 kW at an extraction efficiency $P_{out}/P_{beam}$ of 50%, [11]. The output power is increased by 100 kW compared to the output cavity investigation. The beam’s improved bunching quality is the main reason for this output efficiency improvement. That is a gain of 49 dB. The charging processes of the idler cavity and the output cavity are plotted in Figure 4.32. The voltage in the idler cavity has a slight overshoot and it is consistent with the fact that the cavity is inductively excited. The average normalized velocity of the electrons is plotted in Figure 4.33, [11].

So far the electron beam is working in continuous wave (CW) in simulation. The possible breakdown and the protection of the breakdown of the RF-structure have not been considered. The X-band klystron works with a power level of MW and the power gain is 49 dB. Breakdown may occur depending on two factors: 1. The applied field level and local field level enhancement effects; 2. The breakdown field of the medium. With increased signal level the mechanical structure could suffer mechanical failure. The breakdown could happen when the electric field reaches $0.5 - 3 \times 10^5$ V/cm. To protect the RF structure, the electron beam should be switched to pulsed with a pulse length of $10^{-3}$ s in the laboratory environment with a tunable pulse length to fulfil the experimental requirement. At this stage of the investigation, the determination
Chapter 4. X-band Klystron

Figure 4.30: TM$_{110}$ X-band klystron with fins and chokes

Figure 4.31: TM$_{110}$ X-band klystron with chokes
Chapter 4. X-band Klystron

Figure 4.32: Voltage inside idler (upper) and output (lower) cavities in klystron

Figure 4.33: Normalized velocity $\beta$ of electrons in TM$_{110}$ X-band klystron with chokes from input cavity (marked with dashed magenta lines), through idler cavity (marked with dashed green lines) till three-gap output cavity (marked with dashed blue lines; velocity stays at original value in the input and intermediate sections and then decreases gradually in the output section
4.2  **TM$_{310}$-Mode X-band Klystron**

The TM$_{410}$ mode X-band klystron has shown great potential in power amplification. The desire to achieve even higher power leads to the investigation of a klystron operating in the TM$_{310}$ mode. The centre modulation frequency of the electron beam is still in X-band, i.e. 10 GHz. In Chapter 3 the relationship between shunt impedance and modes is explained. It is known that the maximum shunt impedance decreases significantly with increasing mode number. But the rise of the Bessel Function $J_m$ gets slower with increased mode number $m$. Thus a higher mode number allows a larger inner radius of the cavity without much sacrifice on power. The inner radius of the cavity $R_i$ is enlarged to 25 mm. The outer radius $R_a$ is 41 mm. Due to the large inner conductor the mechanical stability of this structure is expected to be excellent. That’s not the only advantage of klystrons operating in the TM$_{310}$ mode. Enlarged radii lead to enlarged conducting surface, which is essential for the cooling strategy.

The calculated resonance frequency is 10.36099 GHz and the two resonance frequencies found in simulation are 10.251913 GHz and 10.252310 GHz. The frequency difference of the two modes is 400 kHz. The value seems proportionally small. But the asymmetry of the two orthogonal modes was a great problem in the TM$_{410}$ mode klystron design. In order to reduce the asymmetry of the two orthogonal modes in simulation small modifications in resonator geometry are undertaken. One TM$_{310}$ mode has six electric field maxima in the rotation direction and two modes have twelve. The cavity is changed into a dodecagon cavity with a dodecagon inner conductor, Figure 4.34.

The inner radius $R_i$ is 25 mm and the outer radius $R_a$ is 43 mm. The gap length is 5 mm. The quality factor $Q_0$ is then 5100. The frequencies of the two orthogonal modes are 10.36656727 GHz and 10.36656727 GHz respectively. The frequency difference is invisible in the eigenvalue simulation. The radius of the electron beam is also increased. The centre radius of the beam is 34 mm with a thickness of 1.5 mm, while the outer radius of the beam pipe opening is 35.5 mm and the inner radius is 32.5 mm. The enlarged cross section of the electron beam
centre frequency of input signal | 10 GHz
resonance frequency of input cavity | 10 GHz
resonance frequency of intermediate cavity | 10 GHz
resonance frequency of output cavity | 10 GHz
current of electron beam | 50 A
accelerating velocity of electron beam | 100 kV
speed of electron beam relative to the speed of light $\beta_0$ | 0.55
mode in the input section | TM$_{310}$
mode in the output section | TM$_{310}$
mode in the intermediate section | TM$_{310}$

Table 4.7: Parameters in category a of X-band klystron operating in the TM$_{310}$-mode

means that an increased beam current is allowed, here 50 A. The current density of the electron beam is now 17 A/cm$^2$. It is even lower than the current density of the hollow beam in the TM$_{110}$ klystron. Not only the output cavity but also the input and intermediate sections all operate in the TM$_{310}$-mode. The electric field inside the output cavity has three positive magnitude maxima and three negative magnitude maxima in z-direction. Three helical beams are required to benefit fully from the deceleration force of the electric field. These three helical beams are generated from a hollow beam in the input cavity, which also operates in the TM$_{310}$-mode. Important parameters for a TM$_{310}$-mode X-band klystron in are shown in Table 4.7.

4.2.1 Output section

The design starts with the investigation of the output cavity. In this section the output cavity is investigated in the variant with output couplers and in the variant without output couplers.

output cavity with 12 couplers

Each TM$_{310}$-mode has six magnitude maxima, so two orthogonal modes have twelve. Twelve output couplers are needed for the power extraction without negative influence of the waveguide openings on the fields inside the cavity. The easiest and safest way is to directly connect twelve linear waveguides to the output cavity through the coupling apertures. At the time that this numerical model was built (2014), it was impossible to define "ports" in polar coordinates in GdfidL. The output waveguides are extended with bent waveguides to the z-plane, where the output "ports" are defined, Figure 4.35. At the output "ports" the frequency, the amplitude, and other information of the output signals are read directly.

The bent waveguide only has its meaning in the numeric environment. But its scattering parameters are related to the quality of the output cavity. The
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Figure 4.35: Side view (left) of the TM$^{310}$-mode X-band output cavity with twelve output waveguides in CAD program; model of output cavity in simulation program.

The radius of curvature of this extended waveguide is 10.2 mm and the angle is 90°, Figure 4.36 (left). The extended waveguide is examined in the simulation program Microwave Studio and the transmission parameter $S_{21}$ is near 0 dB, Figure 4.36 (right). This curved waveguide is not expected to be an obstacle for the output signal propagation. The choices of coupling factor $\beta_\kappa$, gap length and some other basic parameters which were mentioned in the investigation of the TM$^{110}$ klystron are not repeated here. The radii of the cavities are 43 mm and 27 mm respectively and the length of the first gap is 5 mm. The iris between gaps is assumed as 3 mm thick. The results of the one gap cavity with different coupling factors $\beta_\kappa$ are in Table 4.8.

Figure 4.36: Left: curved waveguide in simulation program GdfidL; right: transmission parameter $S_{21}$ of this curved waveguide.

$\beta_\kappa \approx 9$ is chosen as the proper operation point. The two excited modes have a difference in field amplitude of only around 3%. The two modes have almost
the same contribution to beam power extraction. This means the electron beam is decelerated homogeneously and no deformation of the helical pattern occurs. The minimum normalized velocity $\beta$ of electrons after the output cavity in this case is 0.35. The output cavity is extended to a two-gap structure. The length of the second gap is chosen as 2.4 mm. The iris between gaps is 3 mm thick. For structures with more than one gap, coupling is necessary between gaps for power transport.

In the TM$_{310}$-mode output cavity magnetic coupling was used. Here, twelve coupling slots are placed near the outer wall of the cavities, Figure 4.37. Figure 4.38 shows an excited two-gap cavity with extended waveguides. But for the TM$_{310}$-mode, the magnetic coupling is problematic. The asymmetry of two orthogonal modes already exists in a single gap structure. The coupling slots are placed with an azimuth of 30°. Four are on the x and y axes and eight of them are not. They have thus different sizes due to the numerical error. Different coupling slot sizes enhance the asymmetry of the two modes. The coupling factor $\beta_k$ of this two-gap is chosen as 9. The output power reaches 1.8 MW. One mode contributes 0.98 MW output power and the other 0.79 MW. The power difference increases from 3% to over 10%.

Despite the worsened symmetry in the two-gap structure, the cavity is extended to the three-gap structure. Because the minimum velocity $\beta$ of electrons

<table>
<thead>
<tr>
<th>$\beta_k$</th>
<th>$P_{\text{out}}$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.9</td>
<td>1.54 MW</td>
<td>31%</td>
</tr>
<tr>
<td>7.0</td>
<td>1.39 MW</td>
<td>28%</td>
</tr>
<tr>
<td>9.2</td>
<td>1.21 MW</td>
<td>24%</td>
</tr>
<tr>
<td>23.3</td>
<td>0.64 MW</td>
<td>13%</td>
</tr>
</tbody>
</table>

Table 4.8: Results of one gap TM$_{310}$-mode X-band output cavity with output waveguides under different coupling factor $\beta_k$
Chapter 4. X-band Klystron

<table>
<thead>
<tr>
<th>cavity</th>
<th>$P_{out}$</th>
<th>$\eta$</th>
<th>$P_{out}/P_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>one gap</td>
<td>1.20 MW</td>
<td>24%</td>
<td>9</td>
</tr>
<tr>
<td>two gaps</td>
<td>1.80 MW</td>
<td>36%</td>
<td>9</td>
</tr>
</tbody>
</table>

**Table 4.9:** Results of TM$_{310}$ mode X-band output cavity with 12 output waveguides

is about 0.2 after the two-gap cavity, the third gap is chosen as 2 mm long. In the three-gap cavity, the coupling slots that connect the second and the third gaps are placed at the positions with the same $x$, $y$ coordinates like the coupling slots connecting the first gap and the second. The coupling slots are aligned, Figure 4.16. The coupling factor $\beta_e$ is still chosen as 9. But the efforts to excite this three-gap cavity deliver unsatisfying results. The output power is plotted in Figure 4.40. It is easy to recognize that the two orthogonal modes are not excited equally at the same time. One mode (mode 2 in Figure 4.40) is not actually excited. The output power only reads 1 MW and it is lower than the output power from the one gap cavity.

Some electrons suffer severe deceleration in the interaction with output gaps and lose all of their kinetic energy before escaping the cavity, Figure 4.39. Their velocities reach the value zero, when they are still inside the cavity. The electrons may gain kinetic energy again and be accelerated in the opposite direction. This phenomenon has been discussed in [29]. The electron beam is “saturated” when it gives up its whole kinetic energy inside the cavity. When beam saturation happens in the beam gap interaction, the efficiency of the klystron drifts away from ideal settings. That is exactly what happens here in the output cavity with three gaps. The output efficiency is lower in this situation. The possible reason for this asymmetry problem of the three-gap cavity is still the incompatibility of the analytical model in cylindrical coordinates and the numeric model in Cartesian coordinates. The coupling slots between gaps and the output apertures are placed with an azimuth of 30°. But in Cartesian coordinates the angle has a small variation and also the sizes of the openings are slightly different. They are not placed symmetrically in the $x$ or $y$ plane. $-xfixed$, $-yfixed$ these functions can be applied to the openings, but they cannot fix the boundaries of the openings which are not in the $x$ or $y$ plane. Therefore the lengths and widths of the openings still differ from each other. The important results for the output cavity with 12 output waveguides are in Table 4.9.

**output cavity with fewer than 12 output waveguides**

We discussed the possibility of a ring coupler in the klystron design with the TM$_{110}$ mode. The results were not satisfying. A TM$_{310}$-mode cavity has twelve output waveguides and if it is possible to have a few less, this should be investigated. Here a one gap output cavity is under investigation, because with increased gap number, much more simulation time and much more memory...
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Figure 4.39: Velocity (green) and average velocity (blue) of electrons throughout the output cavity; gaps are marked with dashed blue lines.

Figure 4.40: Output power extracted from the three-gap output cavity; the power from mode 1 reaches the maximum and the power from mode 2 not.

are needed. Changes in the outer conducting wall of the cavity also change the electric and magnetic fields inside the cavity. The design of an output cavity has to respect this and avoid minimizing the negative influences on the fields. Twelve output apertures are kept to avoid the direct change in the outer conductor. The adjustments are only applied on the output waveguides connecting to the apertures.

The design starts with a ring coupler similar to the one in the TM_{110}-mode klystron, but with small adjustments, Figure 4.41. Only a very slim slice instead of a wedge is cut out of the ring. This slim slice makes the waveguide a one-direction waveguide. The purpose of this change is to minimize the difference, when the fields look towards the output apertures from the inside. The two orthogonal modes of the output cavity with this ring coupler have a frequency difference of 80 MHz. This small adjustment in the ring coupler brought no improvement in the cavity performance. The PIC simulation result shows, only one mode is excited and the output power is only half of the value compared to the output cavity with similar \( \beta_e \) but excited in the rotating mode.

The second version of the ring coupler is with two output waveguides, Figure 4.42. There is no separating wall in the ring coupler any more. The two modes should see the same impedance, when they look towards the output apertures. The frequency difference of the two modes is, as expected, reduced to 60 MHz. Frequency difference is not the only problem in this configuration. More severe is that the TM_{310} mode found here can barely be called a TM_{310} mode. The fields are disrupted and the six maxima of the electric field obviously have different amplitudes. The performance of this ring coupler arrangement is as poor as expected. Only one mode is excited and the deceleration
of electrons in the rotation direction is not homogeneous. All these lead to a poor output efficiency. 5% kinetic energy of the beam is converted to the output power in this cavity. The results of these two coupler arrangements are not ideal. The main reason is, as mentioned before, the asymmetry in the ring coupler leads to the asymmetry of the two modes.

Therefore, blinds/chokes are connected to the output apertures. Blinds/chokes are basically λ/4 transformers in short circuit. They make sure that the fields inside the cavity should see the same impedance, when they look towards the apertures.

One arrangement is with two output waveguides, Figure 4.43. To reduce the frequency difference between the two modes, two groups of small tuning blocks are placed inside the cavity and near the inner conductor. Each six tuning blocks form one group and have the same μ_r values. They are placed next to the inner conductor in the cavity and 60° apart. The azimuth of the two groups is 90°, Figure 4.44. The values of μ_r of these two groups are tuned individually to minimize the frequency difference. The difference is reduced to be invisible in the simulation program.

At this point another problem arises. Two orthogonal modes are found in the eigenvalue simulation. The amplitudes of these two modes are slightly different, which is also recognizable in Figure 4.43. This means that the deceleration force is not homogeneous in rotation direction. Further investigations are done with the help of decaying and PIC simulations. The performance of this arrangement, Figure 4.43, is much better than the ring couplers in Arrangement 1 and Arrangement 2. The two orthogonal modes are excited with slight difference. The average output power from one mode is 20% higher than from the other. This is clearly higher than 3% power difference of an output cavity with twelve output waveguides, but this output waveguide arrangement does not
bring obvious disadvantages in the output cavity function, i.e. in mode excitation or power extraction. With coupling factor $\beta_n = 11$, an average output power of 1.15 MW is comparable with that of an output cavity with 12 output waveguides. Arrangement 3 has shown an output coupler that can fulfil its assignment.

**Figure 4.43:** Output coupler arrangement 3

**Figure 4.44:** Two groups of tuning blocks in green and grey near the inner conductor
output cavities without couplers

After struggling with output couplers, a design is considered without output couplers. The purpose of this investigation is to explore the power amplification potential of the klystron operating in the TM$_{310}$-mode without the disruption of waveguides. A tuning ring is used to induce magnetic loss, which simulates the power extraction by the output waveguides. Loaded quality factor $Q_L$ and shunt impedance $R_{sh}$ of the cavity are also adjustable by the tuning ring. The resonance frequency is also tunable with the tuning ring. Another change is that electric coupling is used instead of magnetic coupling between gaps. Thus there are no more coupling apertures near the outer conducting wall in the gap. Electric coupling happens through the beam pipe. The opening of the beam pipe is slightly widened and the relative passband width of this coupled chain is 2%. Then the structure is transformed into a forward wave structure.
The extracted energy travels forward to the last gap. With a broader beam pipe the shunt impedance in the opening area is reduced. The coupling factor $\beta_c$ of this cavity is chosen as 6, smaller than for the cavity with magnetic coupling, to achieve a similar deceleration. The electric field through the cavity is tuned flat and the electrons experience gradual deceleration in each gap. The output cavity is easily extended to a three-gap structure, Figure 4.45. Without the output couplers in the radial direction two orthogonal modes in the three-gap cavity are excited without magnitude difference in the electric fields, Figure 4.46. A four-gap structure is also realised in simulation. The average normalized velocities of electrons in three-gap and four-gap cavities are plotted in Figure 4.47.

The average output power $\overline{P}_{out}$ is calculated with the help of Equation 3.17 and Equation 3.18. For example the average new velocity $v_{new}$ of electrons after three-gap output cavity is $0.38 \cdot c_0$ and we have

$$U_{new} = \frac{m_e e^2}{e} \left( \sqrt{\frac{1}{1 - \left( \frac{0.38 \cdot c_0}{c_0} \right)^2}} - 1 \right)$$

$$= 41.5 \text{kV} \quad (4.8)$$

The output power $\overline{P}_{out}$ is

$$\overline{P}_{out} = (U - U_{new}) I \cdot \frac{\beta_c}{1 + \beta_c}$$

$$= 2.51 \text{MW} \quad (4.9)$$

The results of the three-gap structure and the four-gap structure are shown in Table 4.10. Up to this point the TM$_{310}$-mode output cavity shows great potential in power amplification.
FIGURE 4.47: Normalized velocities of electrons in the beam throughout three-gap (left) and four-gap (right) output cavities; green lines plot the velocity distributions and blue lines plot the average velocities.

<table>
<thead>
<tr>
<th>cavity</th>
<th>average $\beta$</th>
<th>$P_{out}$</th>
<th>$\eta$</th>
<th>$P_{out}/P_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>three gaps</td>
<td>0.40</td>
<td>2.29 MW</td>
<td>45.8%</td>
<td>6</td>
</tr>
<tr>
<td>four gaps</td>
<td>0.38</td>
<td>2.51 MW</td>
<td>50.2%</td>
<td>6</td>
</tr>
</tbody>
</table>

TABLE 4.10: Results of output TM$_{310}$-mode X-band cavity without output waveguides.
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4.2.2 Intermediate and input sections

The idler and input sections of the klystron both operate the in TM$_{310}$ mode with a centre frequency around 10 GHz. The cavities have a polygon form with 12 edges and have the same radii of 43 cm and 27 mm respectively, and a length of 5 mm, Figure 4.48. There are no input waveguides connected to the input cavity, while twelve input apertures would disturb the symmetry of the two orthogonal modes, as happened in the output section. Instead, small voltage sources are placed near the outer conductor in the cavity. These voltages excite the rotating mode without compromise on the symmetry. In the laboratory environment, the voltage sources are replaced with twelve regular input apertures. The input power $P_{\text{in}}$ is the sum of the power dissipated in the input cavity $P_{\text{cav}}$ and the power dissipated in the source load $P_{\text{ext}}$. The coupling factor $\beta_{\kappa}$ is:

$$\beta_{\kappa} = \frac{P_{\text{ext}}}{P_{\text{cav}}} \quad (4.10)$$

$P_{\text{cav}}$ is related to the voltage across the gap according to:

$$P_{\text{cav}} = \frac{U^2}{R_{\text{sh}}} \quad (4.11)$$

The voltage $U$ across the gap is monitored directly. The geometries of the input and idler cavities are identical in the simulation. Only the resonance frequencies and loaded quality factors $Q_L$ are tuned to different values to fulfil their different functions. An input cavity is responsible for the beam modulation. The resonance frequency of the input cavity decides the modulation frequency for the beam. The function of the idler cavity is to enhance the modulation depth. The resonance frequency of the idler cavity is slightly higher than the beam modulation frequency. To achieve better power gain, the input cavity
is excited with a small signal and the main modulation is realised in the idler cavity. Therefore the electric field strength in the idler cavity is much stronger than in the input cavity. The beam pipe as coaxial waveguide allows coupling between these two gaps. Due to the experience with the TM$_{110}$-mode klystron especially the input cavity needs isolation from the idler cavity for a successful beam modulation. Suppressing the coupling between input and intermediate cavities plays an important role.

The isolation for the TM$_{310}$-mode klystron is more difficult than for the klystron operating in the TM$_{110}$ mode. The first reason for this is the fivefold beam power. More power is coupled into the input gap, when the transmission factor $S_{21}$ of the isolation is the same. Another reason is, the frequencies of different modes are close to each other, Table 4.11. The frequency found in the simulation is a little lower. Figure 4.49 shows the unsuccessful charging process inside the input cavity and the idler cavity. The voltage inside the input cavity is much smaller than the voltage inside the idler cavity. The excited mode has a centre frequency of 10 GHz. But after about 300 periods, the excited 10 GHz-mode vanishes and a mode at 15 GHz is excited. This mode originates from the deformed modulated beam. The disturbed fields in the input cavity result in a deformed beam and this deformed beam excites an unidentified mode in the idler cavity. The difference between the two voltages declines and they reach an equal value. The input and idler cavities lose their function at this point. There are basically two suppressing methods used in our investigation: chokes and fins. A choke is a $\lambda/4$ transformer in short circuit and works as band-stop filter. The bandwidth of the chokes is relatively narrow. Signals with frequencies outside the band can pass the choke easily. The radius $r_{\text{choke}}$ and height $h$ of the choke, Figure 4.29, in the investigation are:

$$r_{\text{choke}} = 7.5 \text{ mm}$$
$$h = 3 \text{ mm}$$  \hspace{1cm} (4.12)

<table>
<thead>
<tr>
<th>mode</th>
<th>frequency found in analytical model</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM$_{310}$</td>
<td>10.22 GHz</td>
</tr>
<tr>
<td>TM$_{610}$</td>
<td>12.46 GHz</td>
</tr>
<tr>
<td>TM$_{910}$</td>
<td>15.52 GHz</td>
</tr>
</tbody>
</table>

Table 4.11: Resonance frequencies of TM-mode in intermediate cavity

Fins are high-pass filters and only stop signals below the cut-off frequency. Two aspects should be considered to avoid the impacts of fins on the input- and idler section. First: fins and chokes should be located with certain distance from both cavities. Otherwise the fields could be slightly modified. Second: fins should come in quantities of 12, 24 or even 36. Only the frequency which is higher than the cut-off frequency can propagate. The centre radius of the beam pipe is 34 cm. Supposing the thickness of fins is 1 mm, the cut-off frequencies of
12 fins, 24 fins, 36 fins and 48 fins are as following:

\[
\begin{align*}
    f_{c,12} &= 4.1 \text{ GHz} \\
    f_{c,24} &= 6.0 \text{ GHz} \\
    f_{c,36} &= 7.6 \text{ GHz} \\
    f_{c,48} &= 9.0 \text{ GHz}
\end{align*}
\]  

(4.13)

Only when the number of fins is increased to 60, the cut-off frequency is over 10 GHz, i.e. at 12 GHz. A beam with an infinite number of slits is not desired in the design. It causes an increased current density by reducing the cross section of the beam. The current density is increased from 17 A/cm\(^2\) to 20 A/cm\(^2\) for a 12 fins beam pipe, and 24 A/cm\(^2\) for a 24 fins beam pipe, which increases the space charge effect between electrons too. It is difficult keeping the electron trajectory straight forward, which is but necessary to avoid possible collisions with the conducting wall. Fins or chokes alone can not fulfil the function as isolator. Hence the single use of chokes and fins, or possible combinations are tried out. Figure 4.50 shows a variation with 2 chokes and 24 fins as isolation solution between the input cavity and the idler cavity. S-parameters of this part of the beam pipe with different isolation strategies are illustrated in Figure 4.51.

\[
\text{Figure 4.50: 2 chokes and 24 fins placed between input cavity and idler cavity}
\]

It is not easy to reach a clear judgement of the quality and performance of one isolation method from only the plots of the transmission factor. But there are two forerunners in the plots. One is with 24 fins and another is with 24 fins and two chokes. The transmission factors of both are still under 0.1 even for 14 GHz, while the two choke variant only stops 10 GHz signals and 12 fins variation has \(S_{21} = 0.7\) for 10 GHz.

PIC simulations examine the quality of the isolation strategies. The three-gap cavity without output waveguides is chosen for the output section. The
Chapter 4. X-band Klystron

Figure 4.51: $S_{21}$ of beam pipe with different filters

Figure 4.52: Configuration of the whole klystron with the isolation variant of 24 fins and 2 chokes
coupling factor \( \beta_c \) is chosen as 6. The isolation strategy with 24 fins and 2 chokes is the only one that works. The S-parameters of all the isolation strategies are plotted in Figure 4.51. Therefore this configuration decouples the input cavity and the idler cavity, and the idler cavity and the output, Figure 4.52. The input cavity has a loaded quality factor \( Q_L \) of 3000. The resonance frequency of the idler cavity is slightly higher than the beam modulation frequency, here 100 MHz higher. The quality factor was initially 5000 and is tuned to 500 with the help of a tuning ring. The phase difference of voltage and current in the gap is 85°. The phase shift in the idler cavity is tuned larger than the normal value, and, the input signal is given a relatively high power of 140 W. These are to avoid the huge difference of fields between two cavities, which is essential for cavity isolation. The electric fields inside the input and the idler have similar values. In this case, even when a small portion coupling takes place, the fields in the input cavity are still strong enough to survive the disruption and the input cavity can fulfil its function.

![Normalized velocity of electrons in the klystron](image)

**Figure 4.53:** Normalized velocity \( \beta \) of electrons while the beam is moving through the klystron operating in the TM\(_{310}\)-mode; dashed magenta lines mark the position of the input cavity, dashed green lines mark the position of the idler cavity, dashed blue lines mark the position of the output cavity.

A modulation depth of 2.7% of the electron beam is achieved in the input cavity. The value of modulation depth is enhanced to 5% in the idler cavity. The drift length for the modulated beam between the input cavity and the idler cavity is 110 mm. The distance from the idler cavity to the output is 55 mm. The average normalized velocity of electrons through this klystron is plotted in Figure 4.53. The average \( \beta \) of electrons stays stable at 0.55 from the starting point, through the input cavity and idler cavity. The velocity change of electrons is merely visible in the plot and only a very small amount of the kinetic energy of the electrons is lost in the input and in the idler cavities. After escaping the output cavity, the average \( \beta \) of electrons drops to 0.33, while the minimum \( \beta \) touches down to 0.2. The output cavity extracts most of the kinetic energy from
the beam, i.e. 3.0 MW. The total power gain of this klystron configuration is 29 dB. It is possible to improve the power gain of the klystron with more simulations. The key is to find the limit of the isolation strategy. But due to limited time and computational capacity, no more simulations were done in the investigation.

One of the concerns when using an electron beam with slits is that electrons with increased velocity in $\varphi$ direction may collide with the fins in the beam pipe. A few electrons are tracked from the beginning of their trip in the beam pipe to the end, Figure 4.54. The electrons are chosen randomly around the circle, at the time that all the cavities in the klystron reach saturation. Their positions in time sequence are read and plotted. A few electrons have only part of their trajectories recorded due to the tracking method used. But for the most selected electrons their whole journey is presented in the figure. Electrons may rotate with small radius, but this is not visible in the figure. And the distance between electrons and fins is given as 0.75 mm, the same distance as between electron beam and the conducting walls of the beam pipe. This is enough space for small variations of beam trajectory and should avoid collisions of the electrons with the conducting wall.

![Figure 4.54: Trajectories of electrons moving through the whole klystron; the starting point of output cavity is at z = 0 mm](image-url)
The phases of electrons are plotted in Figure 4.55. The electrons enter the klystron with different phase. Electrons in the same periods come closer to the other electrons while travelling and their phases also come closer, but not too close, due to space charge force. In the output cavity the electrons experience strong deceleration and are all slowed down. The phases of electrons in the output cavity show the same tendency.

**Figure 4.55:** Phase of electrons while the beam moving through the klystron operating in the TM$_{310}$-mode; dashed magenta lines mark the position of the input cavity, dashed green lines mark the position of the idler cavity, dashed blue lines mark the position of the output cavity.
Chapter 5

W-band Klystron

The purpose of a W-band klystron besides power amplification, is to realize frequency tripling. This means that the input, the idler and the optional penultimate cavities all operate in the TM_{110}-mode at 30 GHz, while the output cavity operates in the TM_{310}-mode at 90 GHz. Important parameters for this klystron are carefully chosen with respect to physical restrictions and experience from the X-band klystron investigation. In the simulation, the cavities are 12 edge prisms, like the cavities in the X-band klystron, Figure 4.34, in order to reduce the frequency difference between the two modes. The beam current is chosen as 1 A, which is much smaller than the beam current for the X-band klystron. Because the cavity size is reduced, the cooling surface is reduced. But wall losses are higher at higher frequencies. 0.4 T homogeneous magnetic flux $B$ in propagation direction (here z-direction) is applied for beam focusing.

The radii of the cavities also have a large difference from each other because of the different resonance frequencies and resonant modes. In the analytical model, if all the cavities have the same inner radius of 2.0 mm, the outer radius of the input cavity is 7.0 mm and the outer radius of the output cavity is 4 mm, Equation 2.14. The beam pipe should be placed at the position where the longitudinal electric field component has its maximum. The centre radius of the beam pipe at the input cavity is then 4.5 mm and the beam pipe has a centre radius of 3 mm at the output cavity. This means that the radii of the beam pipe and the radius of the electron beam are changing along the klystron. But a smooth transition of the radius of the beam is a great challenge. Therefore a much easier solution is that the beam pipe size stays the same and the radii of the cavities adapt to the beam pipe size. The beam pipe size is determined with respect to the input cavity size.

The inner radius and the outer radius of a 30 GHz input cavity operating in the TM_{110} mode are found as 2 mm and 7 mm respectively with the help of numerical simulations. The electron beam therefore has a centre radius of 4.0 mm. The matching inner and outer radius of the output cavity are then 3.15 mm and 5.1 mm. The opening of the beam pipe should be as small as possible, because the shunt impedance drops in the beam pipe area with the enlarged opening. But, at the same time, the beam pipe opening should be large enough to allow a beam with a reasonable thickness and a reasonable current density passing through. There should also be enough free space between the beam and the
conducting walls of the beam pipe. Due to these considerations, the outer radius and inner radius of the beam pipe are chosen as 4.2 mm and 3.8 mm, respectively. The beam has a thickness of 300 μm. The beam current density is 13.3 A/cm². The beam surface has a 50 μm distance to the surface of the beam pipe.

At last the dc voltage of the electron is to be be chosen. The favourites are 16 kV, 20 kV and 50 kV, and the normalized velocity β of the electron beam is 0.244, 0.272 and 0.412, respectively. With the help of PIC simulations the output cavities for these acceleration voltages are investigated. There are no waveguides at the output cavity. The power extraction is simulated with a tuning ring, which is made of magnetic lossy material, inside the cavity. The coupling factor βκ is calculated with QL/Q₀ – 1. βκ is chosen as 2 to ensure a sufficient deceleration field for the beam power extraction. A helix beam with 5% modulation depth excites the output cavity. The cavity lengths match the different beam dc voltages.

The output cavity for the electron beam with 50 kV voltage has the highest extraction efficiency, Table 5.1. The cavities for 16 kV and 20 kV have reversed efficiency values. A reasonable explanation is that these two cavities have similar lengths, 0.38 mm and 0.40 mm. The mesh size of the grid is chosen as 0.02 mm, which is the finest mesh size that the computational capacity could afford. The gap lengths of the cavities may have changed due to numerical error. For the same gap length one gap may be discretized into 4 mesh cells and the other one into 5 mesh cells. The two gaps with the same length in the analytical environment have suddenly different lengths in the numerical environment. Figure 5.1 shows an excitation of the TM₃₁₀ mode at 90 GHz in an output cavity with one helical beam with 30 GHz modulation frequency.

<table>
<thead>
<tr>
<th>dc voltage of the beam</th>
<th>gap length</th>
<th>energy loss in the gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 kV</td>
<td>0.38 mm</td>
<td>2.5%</td>
</tr>
<tr>
<td>20 kV</td>
<td>0.40 mm</td>
<td>2.4%</td>
</tr>
<tr>
<td>50 kV</td>
<td>0.68 mm</td>
<td>3.9%</td>
</tr>
</tbody>
</table>

Table 5.1: Results of PIC simulations of one gap output cavity in W-band with TM₃₁₀ mode with different acceleration voltages

The results from the PIC simulation with different dc voltages seem promising. Frequency tripling seems realistic with a good output efficiency. Although nearly 4% extraction efficiency is not much for a klystron, it is more than enough for a one gap cavity. When the cavity is extended to the multiple gap structure, the efficiency should also increase. These conclusions are made before considering one important fact: the simulation results above are achieved only with a helical beam, which is supposed to have a better extraction efficiency compared to the modulated hollow beam. But the goal in our investigation is a
design with modulated hollow beam, which is already mentioned at the beginning of this thesis in Chapter 1. Figure 5.2 plots the average normalized velocity $\beta$ of electrons of the modulated hollow beam, Figure 1.3, and of the helical beam, Figure 1.2, through a one gap output cavity. The modulation depths of these two beams are both 5%. The average velocity in the hollow beam changes barely, although the beam enters the output cavity after a drift length, which is long enough to form quality bunches. The average $\beta$ of electrons in the helical beam drops from 0.412 to 0.407 when passing the output cavity.

Comparing to the X-band TM$_{110}$-mode klystron, the output cavity here works not only in a higher mode, but also with higher harmonic, e.g. 3rd harmonic. But a helical beam is the ideal form, Figure 1.3. The electrons are 100% in the bunches and 100% participate in the interaction with the gap. Figure 5.3 shows electrons in this helical beam and its electron density. The electrons in this beam
distribution of electrons with azimuth 0° to 5°

distribution of electrons with azimuth 0° to 5°

distribution of electrons with azimuth 0° to 5°

distribution of electrons with azimuth 0° to 5°

**Figure 5.3:** Electron distribution and electron density in the helical beam

**Figure 5.4:** Electron distribution and electron density in the hollow beam
participate 100% in the beam gap interaction. But the electron concentration reduces during the travelling and the distribution width of the beam in the longitudinal direction increases. To constrain this kind of trajectory spread a very sophisticated magnet arrangement is required. Only 60% of the electrons in the modulated hollow beam (modulation depth of 5% and the drift length is about 16 mm, Figure 5.4) participate in the interaction. This may be the only disadvantage compared to the helical beam. But this lower electron density in the hollow beam leads to less extracted beam power in the output cavity.

The marked areas in Figure 5.3 and Figure 5.4 highlight the signals which enter the gap and interact with the output cavity. For the output cavity, they only "see" these signals periodically. Therefore these signals are converted into signals in time domain, Figure 5.5\(^1\). The frequency spectra of these periodic signals are produced by discrete Fourier transformation (DFT), Figure 5.5. The helical beam has a near perfect impulse comb and therefore the harmonics in the frequency domain have almost the same amplitude. But the situation in the frequency domain of a modulated hollow beam is much worse. The amplitude of the third harmonic in the frequency domain is only 40% of the amplitude of the fundamental signal. This means with the modulated beam the excited field

\(^1\)The electron beams in dashed boxes in Figure 5.3 and Figure 5.4 are functions of distance. They can be transformed into the functions of time with just \(t = \frac{Ax}{v} \)
across the gap is much lower than the field excited by the helical beam.

The difficulties with the modulated hollow beam are presented here and the design of the W-band klystron is carried out with these in mind. Table 5.2 gives the important parameters of the W-band klystron.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre frequency of input signal</td>
<td>30 GHz</td>
</tr>
<tr>
<td>Resonance frequency of input cavity</td>
<td>30 GHz</td>
</tr>
<tr>
<td>Resonance frequency of output cavity</td>
<td>90 GHz</td>
</tr>
<tr>
<td>Resonance frequency of intermediate cavity</td>
<td>30 GHz</td>
</tr>
<tr>
<td>Current of electron beam</td>
<td>1 A</td>
</tr>
<tr>
<td>Accelerating velocity of electron beam</td>
<td>50 kV</td>
</tr>
<tr>
<td>Speed of electron beam relative to the speed of light $\beta_0$</td>
<td>0.412</td>
</tr>
<tr>
<td>Mode in the input section</td>
<td>TM$_{110}$</td>
</tr>
<tr>
<td>Mode in the output section</td>
<td>TM$_{310}$</td>
</tr>
<tr>
<td>Mode in the intermediate section</td>
<td>TM$_{110}$</td>
</tr>
</tbody>
</table>

**Table 5.2: Parameters of W-band klystron**

### 5.1 Output Cavity

To find the proper drift length of the electron beam, knowledge of the plasma wavelength is important. The beam in W-band klystron has an outer radius of 4.15 mm and an inner radius of 3.85 mm, and the current is 1 A. One quarter wavelength of plasma frequency is then:

$$\omega_p = \sqrt{\frac{e^2 n_0}{\varepsilon_0 m_e}} = \sqrt{\frac{e}{\varepsilon_0 m_e} \frac{I_0}{\beta_0 c_0 \pi (R^2 - r^2)}} = 4.62 \cdot 10^9 \text{ rad/s}$$  

$$\frac{1}{4} \lambda_p = \frac{1}{4} \cdot \frac{\beta c_0}{f_p} = \frac{1}{4} \cdot \frac{2\pi \beta c_0}{\omega_p} = 42 \text{ mm}$$  

42 mm is one quarter wavelength corresponding to the plasma frequency of this beam and is also a candidate for the proper drift length of this beam. But this value is based on space charge theory and needs adjustment for large signals. In our case, when the modulation depth is 5\%, the quality electron bunch occurs at a drift length of around 16 mm, Figure 5.4. The quality electron bunch means that the bunch contains the most electrons of this period and the bunch has a very small longitudinal spread. The drift length for the output cavity is therefore chosen as 16 mm. With increased gap number, the drift length will be shortened slightly.

The opening of the beam pipe is large compared to the radii of the cavity. The shunt impedance drops dramatically in the beam pipe, Figure 5.6. It becomes much more difficult to extract the RF power in higher modes from the
beam. The investigation starts directly with the two-gap structure, Figure 5.7. The output cavity here has no output waveguides and is tuned to be critically coupled, which means the coupling factor is 1. The ratio of the extracted output power and ohmic loss in the cavity is 1:1. A low coupling factor value could help to obtain a certain acceleration field. The investigation begins with a two-gap structure. As mentioned before, the inner radius of dodecagon is 3.15 mm and the outer radius of 5.1 mm. The length of gap $\lambda_{gap}$ is defined with respect to the beam velocity.

$$\lambda_{gap} \leq \frac{1}{2} \frac{\beta c_0}{f_0} = 0.687 \text{ mm} \quad (5.2)$$

But with respect to the iris thickness, which is between gaps, the lengths of two gaps are both chosen as 0.38 mm and the iris between gaps is 0.3 mm thick. No extra coupling slots are placed and the gap coupling happens through the beam pipe opening. With shorter iris, the cooling surface is smaller, the mechanical stability would be weaker. The quality factor of this two-gap structure is 1300 and $f_0$ is 91.5385233 GHz. $\beta_k$ is tuned to 1 and the loaded Q-factor is then 650.

For the output cavity structure with more than one gap, the gap length is supposed to be reduced to match the reduced electron velocity. But Figure 5.2 shows that the velocity of the electrons merely changes after the one gap cavity. The electric field is tuned flat with the help of a tuning ring to ensure the gradual deceleration in each gap, Figure 5.8. The drift length of the electron beam before entering the output section is 15 mm. Figure 5.9 shows the distribution of the normalized electron velocity through a two-gap structure. The beam drops its kinetic power gradually and slightly. The output power is only 0.3 kW, which is calculated from the average $\beta$ at 0.410 of the electrons after the output cavity. It is only 0.6% of beam power. The same amount of power is lost
Chapter 5. W-band Klystron

Figure 5.7: Two-gap output cavity excited with modulated hollow beam; left is side view and right is top view. The arrows in the plots show the electric field vector.

as ohmic loss inside the cavity. More details about the two-gap structure are summarized in Table A.1.

Figure 5.8: Magnitude of electric field in propagation direction inside two-gap W-band output cavity; two gaps are marked with dashed blue lines.

Due to the poor performance of the two-gap cavity, the output cavity is extended to a four-gap structure. Because the average velocity of the electrons still has no visible change after the two-gap output cavity, all four gaps are given a length of 0.38 mm. As with the output cavity with two gaps, the electric field inside the four-gap structure has also been tuned flat, Figure 5.11. The unloaded $Q_0$ is about 1300 and the loaded $Q$ at 650. With this four-gap output cavity, the average $\beta$ of electrons drops to 0.4078, the output power is 0.65 KW, and an extraction efficiency of 1.3% is achieved. More details about this four-gap structures are given in Table A.2. The excited output cavity is shown in Figure 5.10.
The investigation of a six-gap output cavity is treated with the same procedure, Figure 5.12, Figure 5.13. The first five of the six gaps are 0.38 mm long and the sixth is 0.32 mm. The average normalized electron velocity $\beta$ decreases
to 0.4052 after passing the output cavity. The output power is 0.99 kW and the extraction efficiency of the six-gap structure is 1.98%. The parameters of the cavities are shown in Table A.1, A.2 and A.3. The performances of these cavities are not satisfying.

**Figure 5.12:** Six-gap output cavity without the "penultimate" cavity

**Figure 5.13:** Magnitude of electric field in propagation direction inside six-gap W-band output cavity; six gaps are marked with dashed blue lines

**penultimate cavity**

The output cavity investigation has less satisfying results than expected. One approach to improve the extraction efficiency of the output cavity is to improve the quality of the electron bunches. A "penultimate" cavity is placed ahead of the output cavity and provides bunching improvement. The principle of a "penultimate" cavity is similar to an "idler" cavity. They are both inductively excited. The difference is that a "penultimate" cavity works usually with a phase shift of near 90°, while the resonance frequency of "idler" is normally chosen between 60° and 90°. This means that the fields inside the penultimate cavity will have most of the impact on the electrons which are left between two bunches. The electrons in the transition area are pushed toward the bunches. At the end the number of electrons inside the bunches is increased. The cavity has very little effect on the ones in the bunches. Depending on the phase of
electrons, they will either be pushed forward or backward toward the centres of the bunches. But the modulation degree of the electron beam won’t be significantly enhanced.

![Figure 5.14: Four-gap output cavity with the "penultimate" cavity and a choke](image)

In our investigation, the penultimate cavity is also a polygon with twelve sides. The outer radius of the penultimate cavity is 7 mm and the inner radius is 2 mm. The cavity has a gap length of 1.8 mm and a resonance frequency which is 69 MHz higher than the beam modulation frequency and the resonance frequency of the output cavity. It is specialized to suppress the fundamental frequency 30 GHz. The quality factor of this cavity is 2400, which includes 20% more power loss with respect to surface roughness of the cavity. The phase difference of the current and the voltage in this cavity is $85^\circ$, which fulfils the requirements. The "penultimate" cavity is to improve the bunching quality, but not to disturb the fields of the output cavity. It is placed at a distance of 3.2 mm to the output cavity. This distance shall ensure the correct fields of the output cavity despite the structure modification of the beam pipe. To suppress the possible coupling between penultimate cavity and output cavity, a choke is used. The gap length of the choke is 1.2 mm. The choke is determined for the frequency 30 GHz and has a radius $r_{\text{choke}}$ of 2.5 mm, Figure 4.29 and Equation 5.3:

$$
\frac{1}{4} \lambda_g = 0.25 \cdot \frac{2\pi}{\sqrt{\left(\frac{c}{\omega_0}\right)^2 - \left(\frac{n}{\pi}\right)^2}} = 2.5 \text{ mm}
$$

This "penultimate" cavity is applied to the output cavities with four gaps and six gaps, Figure 5.14 and Figure 5.15. The output efficiencies are improved as expected, Figure 5.16 and Figure 5.17. The output efficiency of a four-gap cavity is increased from 1.3% to 1.5%, while the output efficiency of a six-gap cavity is increased from 1.98% to 2.24%. For the investigation of the six-gap output cavity a beam drift length of 16 mm gave the best results.
5.2 Idler Cavities and Input Cavity

5.2.1 Idler types

In the region between the input cavity and the output cavity, several idler cavities are added along the beam pipe to increase the gain of the whole klystron. These cavities operate in the fundamental $TM_{110}$-mode at $30\,\text{GHz}$ to make sure the enhancement is not weakened. The cavities themselves are coaxial. They are tuned to a resonance frequency which is slightly higher than the modulation frequency of the electron beam. As a result, the intermediate cavities will be inductively excited. The electron current in the cavities has a phase shift between $60^\circ$ and $90^\circ$ to the voltage. The modulation of the beam will be enhanced in the intermediate idler cavities. The multiple gap output structure...
defines the maximum bandwidth of the whole klystron. In addition, the Q-factor of these intermediate cavities should be chosen carefully to avoid further narrowing down of the bandwidth of the klystrons. In the following several configurations of idler cavity are introduced.

**single idler cavity**

A simple configuration is one single idler cavity placed in the space between input and output sections, Figure 5.18. The resonance frequency is tuned slightly higher than the modulation frequency of the beam and it is inductively excited and works as an enhancement of the modulation.

<table>
<thead>
<tr>
<th>phase shift $\Delta \phi$</th>
<th>modulation degree in %</th>
<th>energy loss in cavity %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$65^\circ$</td>
<td>2.35</td>
<td>0.025</td>
</tr>
<tr>
<td>$70^\circ$</td>
<td>2.14</td>
<td>0.016</td>
</tr>
<tr>
<td>$75^\circ$</td>
<td>1.87</td>
<td>0.009</td>
</tr>
<tr>
<td>$80^\circ$</td>
<td>1.65</td>
<td>0.006</td>
</tr>
<tr>
<td>$85^\circ$</td>
<td>1.36</td>
<td>0.002</td>
</tr>
</tbody>
</table>

*Table 5.3: Modulation degree of electron beam after one single idler cavity*

The idler resonator is a polygon cavity with dodecagonal top and bottom surfaces. To save the exhausting frequency tuning in simulation the single idler cavity is tuned to about 30 GHz in the simulation. The gap length is 1.8 mm,
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Figure 5.18: Single idler cavity excited by an electron beam with 1% modulation degree

which is a little less than half the wavelength of 30 GHz. The outer radius of the cavity is 6.9 mm and the inner radius is 2.1 mm. To suppress the coupling between the input and the idler cavity, a choke for 30 GHz is placed ahead of the idler cavity. The $Q_0$ of this cavity is 3100, but is tuned down to 1100 with respect to the bandwidth defined by the output cavity and due to extra loss of surface roughness of this structure. The resonance frequency is 30.3224 GHz. The operating frequencies of the beam are chosen as 30.293 GHz, 30.285 GHz, 30.270 GHz, 30.245 GHz and 30.165 GHz. The phase differences in the idler cavity are then 65$^\circ$, 70$^\circ$, 75$^\circ$, 80$^\circ$, and 85$^\circ$ respectively. The drift length of the electron beam with 1% modulation is chosen as 42 mm, which is one quarter wavelength of plasma frequency, Equation 5.1. This drift length is also chosen for the following idler cavity investigation. The modulation degree of the electron beam is 1%. The increased modulation degrees and the average power left in the beam after one single idler cavity are shown in Table 5.3. Figure 5.19 shows the normalized electron velocity through the single idler cavity.

The final choice of the single idler cavity is with a phase difference of 80$^\circ$ between the current and the voltage. This cavity provides a balance of the achieved modulation depth and the power loss. This may cause confusion, because the cavity with a smaller phase difference has actually better enhancement performance but still a relatively small wall loss. One important conclusion from the experience of the X-band idler cavity design with 5 MW beam power is that the field difference between the idler cavity and the input cavity should be kept small. Otherwise the coupling between these two cavities brings destructive influence on the function of the whole klystron. For the following idler cavities, the phase difference is always chosen as 80$^\circ$.

coupled idler cavities

A single idler cavity can be replaced by two cavities, which are near each other and coupled with each other, Figure 5.20. The coupled cavities have been given
Figure 5.19: Normalized velocity of electrons through single idler cavity with an 80° phase difference in the cavity; green line plots the velocity distribution and the blue line plots the average value of β; the gap is marked with dashed lines.

Figure 5.20: Coupled idler cavities excited by an electron beam with 1% modulation degree.
Figure 5.21: Normalized velocity of electrons along the coupled idler cavities which have a 80° phase difference inside the gap; gaps are marked with green dashed lines

...
gaps here have the same cavity size. These two clustered idler cavities are also called "clustered-cavity pair" in this paper.

The inner radius is 2 mm and the outer radius 7 mm. The gap length is 1.8 mm. Here the clustered cavities are 2 mm away from each other, Figure 5.22. The coupling between the two cavities is less than 0.02%. The two gaps could be considered isolated from each other. The tuned quality factor is about 1100 and the resonance frequency is 30.7962 GHz. The clustered cavities are excited with a beam in an operating frequency of 30.667 GHz. The phase difference in the gap is 80°. The beam has 1% modulation and a drift length of 42 mm.

Two variants of these clustered cavities are tested: one is with half Q value of the single idler cavity; one with Q value of the single idler cavity. The former achieved a modulation depth of 1.6%, and the latter of 2.7%, Figure 5.23. The modulation depth increases continuously in the two gaps. As for the latter
variant, the average $\beta$ of electrons is 0.4115 when they leave the cavities and the power loss in the cavities is 139 W, which is only 0.27% of the beam power.

The achieved modulation depths of the electron beam after different idlers are listed in Table 5.4. The chosen idler cavity for the W-band klystron design is the clustered-cavity with full value of quality factor. It has smaller bandwidth than the coupled-cavity, but it achieves better modulation depth. Fewer cavity pairs are needed to realise the same modulation depth for one beam compared to the alternatives. Fewer cavity pairs also mean less drifting space and a shorter beam pipe.

<table>
<thead>
<tr>
<th>idler type</th>
<th>modulation degree [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>one single cavity with Q</td>
<td>1.65</td>
</tr>
<tr>
<td>coupled cavities with half Q</td>
<td>1.38</td>
</tr>
<tr>
<td>clustered cavities with half Q</td>
<td>1.58</td>
</tr>
<tr>
<td>clustered cavities with Q</td>
<td>2.67</td>
</tr>
</tbody>
</table>

**Table 5.4**: Modulation degree of electron beam after one single idler cavity, coupled cavities and clustered cavities; Q is about 1200 and half Q is about 600

### 5.2.2 Length of drift zone

In the investigation of the idler cavities, the drift length of the beam before it enters the idler cavity is chosen as 42 mm, which corresponds to one quarter wavelength at the plasma frequency of the beam. But it might not be the optimal length. The optimal drift length helps to achieve a better modulation depth of the beam without increasing the idler numbers.

Electron bunching happens under the influence of the modulation they gained in the modulation gaps, i.e. ballistic bunching, and, at the same time, under the influence of the space charge force between the electrons, i.e. space charge bunching. Depending on the power level, one bunching is more dominant than the other. Therefore, for the beam with different modulation depth, the proper drift length varies. As for our investigation, the desired modulation depth is 5%, before the electron beam enters the output cavity. Using as few idler pairs as possible is another requirement for this investigation. The third constraint is that the achieved modulation depth in one gap should be kept moderate, avoiding large differences of field strengths in the successive cavities.

The investigation is done with three beams, whose initial modulation depths are 1%, 2.5% and 4%, respectively. They enter the clustered-cavity after a certain drift length and reach different modulation depths after the cavity, Figure 5.24. The drift length is chosen between 10 mm and 85 mm.
Figure 5.24: Achieved modulation degree of electron beam with different starting modulation degree in the clustered idler pair

As the results show, the beam with an initial modulation depth of 4% reaches the maximum modulation depth after just 12 mm drift length, while the beam with 2.5% needs 42 mm, and the beam with 1% needs more than 85 mm. This verifies the analysis in Chapter 2 that the space charge takes over the dominant role in the bunching with increased power level. The shortened drift length matches the drift length that has been determined in the "ballistic analysis". At the same time, there are peaks of the achieved modulation depths for the three beams. They all occur at the drift length of 42 mm, which matches the quarter wavelength of the plasma frequency of this hollow beam. In the space charge analysis, the beam reaches its current peak at the quarter wavelength position.

The investigation results help to decide on the number of idler cavities needed in the klystron design, supposing the two clustered cavities are treated as one clustered-cavity pair. Supposing the beam gains 1% modulation depth in the input cavity, the modulation depth could reach 2.5% after the first clustered-cavity with drift length of around 40 mm. The modulation depth of this beam reaches 5% after the second idler cavity with a drift length of 42 mm. Thus, two clustered-cavity pairs are required to fulfil the beam modulation function.

The idler cavity enhances the modulation depth of the beam and improves the bunch quality. The output cavity with this improved beam is expected to achieve a better output efficiency. But due to the limited computational capacity and constrains in the mesh cells the PIC simulation with idler cavity and output cavity is not done. With 20 μm mesh size for the output cavity, one single PIC simulation iteration took over five days and more than 24 GB RAM. To save simulation time and reduce the used RAM, the mesh size is defined as 80 μm for the idler cavity and the input cavity investigations. For a PIC simulation the mesh of the whole computational volume has to be uniformly. This means
the whole klystron has be discretized with the same mesh size of $20\,\mu m$. This situation requires enormous simulation time and enormous RAM size, which is not available in this investigation. The limited computational capacity is also the reason that the modulated electrons from the idler cavity PIC simulation can not be imported directly into the PIC simulation of the output cavity.

5.2.3 Input cavity

The design of the input cavity is straightforward. The input cavity is also in the form of a twelve edged polygon and it operates in TM$_{110}$ with a resonance frequency of 30 GHz like the idler cavities. The outer and inner radii of this cavity are found in the simulation as $2.1\,mm$ and $7\,mm$ respectively. The length is $1.8\,mm$. Four power feeding waveguides WR-10 are installed through the rectangular input apertures, Figure 5.25. The width and height of feeding waveguides are $2.54\,mm$ and $1.27\,mm$, respectively. There is also a tuning ring inside the cavity which is responsible for the quality factor and resonance frequency tuning. $Q_0$ of this input cavity is 3000 and is tuned to 2000 using a tuning ring. $Q_L$ is scaled down to 1000 with respect to the bandwidth of input cavity and the bandwidth of the whole klystron. This $Q_L$ is achieved by adjustment of the input aperture size. The coupling factor $\beta_k$ is then 1 which ensures the best transmission of the input signal.

![Figure 5.25: Input cavity of the W-band klystron with four power feeding waveguides](image)

The input signal is fed through four input waveguides. The performance of this input cavity and the modulation depths achieved with different input power are discussed in the next section.
5.3 Character of the Klystron

5.3.1 Linearity

The choice of number of idler cavity is made under the assumption that the beam reaches a modulation depth of 1% in the input cavity. This means, to reach the best amplification performance of the klystron, the input signal has to maintain the same power. This is not realistic. When used as a signal amplifier, the linear range of the W-band klystron shall be found. In the linear region, the increment of the modulation depth of the electron beam, as also the output power are proportional to the increment of the input power, Figure 5.26. From a certain point on, about 50 W input power, increasing the input power no longer leads to proportional increases in the modulation depth and in the output power. The output values converge to a certain limit, Figure 5.27. This region is called the saturation region or the non-linear region.

The blue line in Figure 5.27 flattens out when the input reaches 50 W and starts to decrease from an input power of 100 W. The distances between the input cavity and idler cavity and between the two idler cavities are both 42 mm, chosen under the assumption that the modulation depth of the beam reaches 1% after passing through the input cavity. Under this assumption the input power is 70 W and the output power reaches 1.12 kW. The power gain of the whole klystron is 15 dB. When the input power is higher than 70 W, a shorter drift length is required, i.e. shorter distances between cavities. Otherwise the bunches in these beams reach their best form and lose the form again before
Figure 5.27: Klystron in the non-linear region; the magenta lines describe the modulation depths the beam achieved with only the input cavity, the blue lines show the modulation depth in the klystron with the input cavity and only one intermediate clustered-cavity, the green lines are the modulation depth of a klystron with the input cavity and two clustered-cavities entering the cavities, when the drift lengths are too long. This is the cause of the falling blue line.

In 2015, A. Yu Baikov etc. proposed another method of searching for the proper drift length in hope of achieving the maximum RF power conversion efficiency, over 90%, [31]. The conventional method, including the method applied in this thesis, locates the bunching cavity at the position where the RF current has its maximum. This method proposed here implies that peripheral electrons in the bunches should receive much stronger relative phase shift than the core electrons and this could happen, only if the core of the bunch experiences oscillations due to the space charge forces, whilst the peripherals approach the bunch centre monotonously, [32]. But this method requires much longer drift length and many more simulation rounds.

However due to the limited time and the limited computational capacity, this method is not applied in our investigation. Due to the same reasons the simulation of the klystron as a whole is not pursued. As mentioned in Chapter 5.2.2, with the given computational capacity it is not possible to simulate the W-band output cavity together with any other cavity in the same computational volume. Therefore the simulation of the assembly of all the cavities in the X-band klystron is not presented.

5.3.2 Power recovery

So far we have discussed the input cavity, the idler cavity and the output cavity of a klystron. There are still two important parts remaining: electron gun and the collector. The design of an electron gun for a hollow beam is introduced in the next chapter. The collector design is not included in our investigation,
but the purpose of a depressed collector is to increase the total efficiency of the klystron through power recovery.

This total efficiency has not been discussed in the X-band klystron design, because the klystron already reaches over 50% efficiency without power recovery. The electrons that escape the output cavity also have a wide velocity spread. The lowest normalized velocity $\beta$ of the electrons is near zero. When a collector for these klystrons is designed, a multi-stage depressed collector is the basic requirement.

But a depressed collector for the beam in the W-band klystron has greater significance. The normalized velocities $\beta$ of the electrons are between 0.45 and 0.37. Supposing the depressed collector has only one stage, the voltage has to be equal to or lower than the voltage of the electrons, which have a $\beta$ of 0.37. Therefore the voltage of the depressed collector supply $V_{coll}$ is:

$$V_{coll} = \left( \sqrt{ \frac{1}{1 - (0.37)^2} - 1 } \right) \cdot \frac{m_e c^2}{e} = 39.14 \text{kV}$$

Supposing the recovered power $P_{rec}$ from this depressed collector is

$$P_{rec} = V_{coll} \cdot I_0 = 39.14 \text{kW}$$

the output power $P_{out}$ from the six-gap output cavity with penultimate cavity is 1.12 kW and the overall efficiency of this klystron $\eta_{ov}$ is then increased to:

$$\eta_{ov} = \frac{P_{out}}{P_{beam} - P_{rec}} = \frac{1.12 \text{kW}}{50 \text{kW} - 39.14 \text{kW}} = 10.4\%$$

This is a significant improvement compared to the klystron without power recovery strategy.
Chapter 6  
Electron Gun

The electron gun is a primary component of a klystron. It supplies the beam power, which is later converted into RF power. The hollow beam used in the present investigation is produced in the Fortran program with the optimal radius, optimal electron density, and the required velocity $\beta c_0$. Whether it’s possible to generate this kind of hollow beam with the desired quality was not in the consideration in the phase of this investigation which is documented in Chapter 4 and in Chapter 5. The constraints of the usable lifetime and the capacity of emission of the cathode were also ignored. In this chapter the tentative designs of the electron gun for hollow beam are introduced. The numerical model of the electron gun is built and tested in GdfidL. The emitted current value is defined in the Fortran program.

6.1 Electron Gun Design for TM$_{310}$-mode X band Klystron

A specialized design of an electron gun always starts from a similar design. Researchers from Calabazas Creek Research, Inc have demonstrated a cathode with emission current density of $30 \text{ A/cm}^2$ could have an estimated
lifetime over 30,000 hours and a lifetime over 100,000 hours, when the beam current density is $17 \text{ A/cm}^2$, [20], [19]. A cathode with diameter of 0.7 mm is mentioned, [20], [19]. When the diameter of 0.7 mm is realisable in fabrication, there is no need for the beam to be compressed for the use of this electron gun in the TM$_{310}$-mode X band klystron. The electron beam in this klystron has a current density of $17 \text{ A/cm}^2$ and a thickness of 1.5 mm. The inner radius of this beam is 32.5 mm. For our investigation the drawing of the electron gun is shown in Figure 6.1.

A voltage of 100 kV is supplied to the anode, which means the DC velocity that electrons gain in the gun is $0.55c_0$. The emitted electron current is defined as 50 A. The control electrodes are grounded. The inner conductor of the beam pipe is introduced here. The magnetic flux for the beam focusing is introduced already in the electron gun with a value of 800 mT.

When the cathode is heated, the electrons overcome the work function of the material and escape from the cathode. When the emitted electrons gather in front of the cathode after escaping the emitting surface, they cause a reduction of the potential near the emitting surface. When the potential is reduced to a negative value, the electrons are thrown back to the cathode surface and the emitted electron current reaches the space charge limitation. The allowed electron quantity under space charge limitation is related to the anode voltage $V$ and the distance $d$ of the cathode and the anode. Equation 6.1 gives out the current density regarding the space charge limitation of the cathode due to the Child-Langmuir Law for a planar diode, [6]:

$$J = \frac{4}{9} \varepsilon_0 \sqrt{2 \eta_e} \frac{V^{3/2}}{d^2}$$  \hspace{1cm} (6.1)

$J$ is the current density of the electron flow; $\eta_e$ is the electron charge-mass ratio $\frac{e}{m_e}$; $\varepsilon_0$ is the permittivity of free space. In other words, the emitted electron current is subject to certain limitations, and the current limitation is related inversely to the square of the distance $d$. In our case $J$ is $17 \text{ A/cm}^2$, the maximum distance between the cathode and the anode $d_{\text{max}}$ is

$$d_{\text{max}} = \sqrt{\frac{2.33 \times 10^{-6} V^{3/2}}{J}} \hspace{1cm} (6.2)$$

Only when $d$ is less than $d_{\text{max}}$, the emitted current density will reach $17 \text{ A/cm}^2$ without trouble. The distance from the emitting surface of the cathode to the tip of the anode is 20 mm, which is sufficient to overcome the space charge. The 3-D model of this electron gun is shown in Figure 6.2.
The velocity of electrons increases from near zero to $0.55 \cdot c_0$ during the passage through the electron gun, Figure 6.3. The overshoot of the velocity increment vanishes after 500 nanoseconds operating time. Figure 6.4 plots the trajectory of electrons. The green dashed lines in Figure 6.3 and Figure 6.4 mark the position of the emitting surface. The blue lines mark the entrance of the beam pipe. The electron flow travels without spread tendency and has the same thickness of 1.5 mm through the electron gun. The trajectory of electrons also suggests that the beam may maintain the thickness in the further trip.

### 6.2 Tentative Design of an Electron Gun for Hollow Beam with Compression

Having considered an electron gun for the hollow beam without beam compression, this section explores the possibility of an electron gun with beam compression. There are several motivations for the investigation of an electron gun with a compressed beam. One is that the emitting surface of the cathode has a very limited lifespan if the electron flow from the emitting surface is excessive. But in many tubes a beam with high current density is desired. Compression of the beam is the typical solution to increase the electron density of the gun without reducing the lifespan of the cathode. Another reason is that, despite the development of the material research for the emitter, there are still certain limitations in the electron gun design. The desired beam thickness is 0.3 mm, but it is not realistic to fabricate a cathode with a diameter in this dimension at present. Therefore the emission surface of the cathode is chosen as 3 mm thick.

The pursuing compression ratio of the beam is 10 : 1. It means, when the beam leaves the electron gun, the thickness of the electron beam has to be reduced to 1/10 of the emitted electron flow thickness.

One commonly used favourite design is Pierce Gun. A Pierce gun has a cathode disc, which is spherical, [2]. The anode is also spherical. The electron emission has a convergent flow. In our investigation the cathode disc is altered into an annular form. The control electrodes are placed in the centre of the cathode disc and around the disc.

A voltage of 16 kV is applied to the anode. The electron current emitted from the cathode surface is 0.01 A. Here the electrons gain their initial velocity of $0.244 \cdot c_0$ for the later usage. The hole of the anode has a tapered opening to avoid defocussing of the electron beam. The velocity is defined by the potential difference between the cathode and the anode. The maximum distance $d_{\text{max}}$ of the cathode to the anode due to the space charge limitation is, Equation 6.1:

$$d_{\text{max}} = 265 \text{ mm}$$  \hspace{1cm} (6.3)

The distance $d$ chosen in the investigation is 13.75 mm. For the beam compression only electrodes next to the cathode are not enough. Extra solenoid pairs and an electrode (for beam compression) are placed at the entrance of the
beam pipe. The drawing of this electron gun is shown in Figure 6.5. The potential difference from the electrode for beam compression to the anode is 2.4 kV. The solenoid produces a 100 mT magnetic flux at the centre of the beam pipe, Figure 6.7. The numerical model and the electron flow travelling through the gun are shown in Figure 6.6.

The electron trajectory is plotted in Figure 6.9. The thickness of the beam at the entrance of the beam pipe is reduced to around 0.4 mm, while the velocity in the longitudinal direction is increased to $0.244 \cdot c_0$, Figure 6.8. The green dashed lines in Figure 6.7 and Figure 6.8 mark the position of the emitting surface, while the dashed blue lines mark the tip of the anode. But whether the thickness can be maintained is uncertain. The variation of the thickness of the beam in the beam pipe area raises the same question. Further investigations to explore this beam behavior could be very interesting.
Figure 6.2: Top: cross-section of the electron gun for the hollow beam without compression; bottom: electrons travelling from the emitting surface to the beam pipe
Chapter 6. Electron Gun

**Figure 6.3:** Normalized electron velocity as a function of longitudinal distance $z$ for different calculation times.

**Figure 6.4:** Electron distribution in the electron gun.

**Figure 6.5:** Drawing of the electron gun for a compressed hollow beam.
Figure 6.6: Left: Cross-section of the electron gun for the hollow beam with compression; right: electrons travelling from the emitting surface to the beam pipe.

Figure 6.7: Magnetic flux in z direction at radius of 2.85 mm.

Figure 6.8: Velocity of electrons in longitudinal direction continuously increases in the electron gun.
Figure 6.9: Cross-section of electron beam in the electron gun with convergent trajectory; the beam pipe is marked with dashed blue lines.
Chapter 7

Conclusions

In this paper the design proposals and simulation results of the investigation of X-band and W-band klystrons with hollow electron beam are presented. The purpose of the hollow beam application is to increase the power level of the klystron and to emphasize the advantage of an electron tube in comparison to solid states amplifiers. At first the fundamental principles of klystrons were introduced and were followed by the preparation from the analysis to numerical models. The designs of concrete klystrons were investigated with the help of numerical software, in our case, GdfidL and Matlab.

The X-band klystron design focused on power amplification where the 1 MW hollow beam has shown its advantages too. The beam is supplied for the TM$_{110}$-mode klystron and 5 MW for the TM$_{310}$-mode klystron. The mode excitation difficulties in the cylindrical klystron were mostly caused by the incompatibility of the cylindrical structure of the cavities and the Cartesian coordinate system in the numerical program. But in the investigation it was found that the asymmetry of the modes caused by the discretization error in the mesh can be compensated by clever choice of the coupling factor of the output network, e.g. the choice of the field strength inside the cavity. In general a coupling factor $\beta_c$ under 5 is better, compared to a higher coupling factor, for efficient beam power extraction. But in the TM$_{110}$-mode section, a $\beta_c$ at the value of 7 to 9 showed better results. The key is to decelerate the electrons gradually and evenly while avoiding excessive acceleration force on the electrons. The achieved output efficiencies in the TM$_{110}$-mode structure and TM$_{310}$-mode structure are both over 50%. A big problem is the isolation between different parts. Most klystrons operate with a single beam, that means the beam pipe is a very small tube. But in the hollow beam klystron the beam pipe is a coaxial waveguide. It was very difficult to suppress coupling between the cavities due to the character of the coaxial waveguide, especially in the TM$_{310}$-mode klystron, and especially between input cavity and idler cavity. The two cavities have a large field strength difference. At last with 24 fins and 2 chokes the coupling was suppressed. The possibility to use a ring coupler in the output network was also investigated. With the help of blinds the ring coupler was installed in the TM$_{310}$-mode output cavity. Of course, whether this output network could withstand the very large thermal stress is not certain, while the beam power is 5 MW and the output efficiency is 50%.
Chapter 7. Conclusions

The goal of the W-band klystron investigation seemed more simple. The W-band klystron is designed to use frequency tripling from 30 GHz to 90 GHz, while the beam has a moderate power level, i.e. in the kW range. But the output cavity operates in a higher mode with a higher harmonic. We confronted the difficulties of operating the cavity with harmonics already at the very beginning. The higher mode requires a larger cavity, the higher harmonic requires finer mesh size. Together it led to an extreme number of mesh cells in the computational volume, which led to higher demand on the computer RAM memory and the much longer simulation time. One single eigenvalue simulation took several hours at the time of the research in 2015 (with personal computer with Intel® Xeon® Processor E5-1650 and 32 GB RAM). The extreme demand on the computational capability was not the only problem. It was much more difficult to achieve a reasonable efficiency in the output cavity. The higher mode didn’t supply enough deceleration field. The output efficiency with a six gap structure was still under 2%. Therefore the application of the penultimate cavity and different types of the idler cavities were investigated. The efficiency was only slightly improved. If a depressed collector is used, the total efficiency could be increased to 10.4%. With an input signal at 1 W to 8 W the whole klystron operates in the linear area.

At the very end of this paper the investigation of electron guns for the hollow beam are described. The simulation of the electron gun without beam compression was straightforward and without obvious difficulties. The results were satisfying too. But the design of the electron gun with beam compression is not specialized for a certain klystron in this paper. Instead the potentials of hollow beam generation and beam compression for a beam with a thickness of 0.3 mm were investigated. The final result didn’t shown the copious variables in the design. Nevertheless, the investigation results showed the direction of an electron gun design in this size.

Overall this investigation of X-band and W-band klystrons achieved the initial purpose, not only the power amplification function of the X-band klystron, but also the signal amplification of the W-band klystron with. The investigation provides many details for manufacture and realisation of the structure.
Appendix A

Tables of Output Cavity in W-Band Klystron

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>unloaded Q-factor of the cavity $Q_0$</td>
<td>1300</td>
</tr>
<tr>
<td>loaded Q-factor of the cavity $Q_L$</td>
<td>650</td>
</tr>
<tr>
<td>length of the first gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>length of the second gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>$\kappa_{m,1}$ in the first gap</td>
<td>1500</td>
</tr>
<tr>
<td>$\kappa_{m,2}$ in the second gap</td>
<td>1500</td>
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<tr>
<td>the average gamma $\gamma$ of electrons after the cavity</td>
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<tr>
<td>extraction efficiency of the cavity</td>
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</tr>
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Table A.1: Parameters and results of output cavity with two gaps

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>unloaded Q-factor of the cavity $Q_0$</td>
<td>1300</td>
</tr>
<tr>
<td>loaded Q-factor of the cavity $Q_L$</td>
<td>650</td>
</tr>
<tr>
<td>length of the first gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>length of the second gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>length of the third gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>length of the fourth gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>$\kappa_{m,1}$ in the first gap</td>
<td>1500</td>
</tr>
<tr>
<td>$\kappa_{m,2}$ in the second gap</td>
<td>1500</td>
</tr>
<tr>
<td>$\kappa_{m,3}$ in the third gap</td>
<td>1500</td>
</tr>
<tr>
<td>$\kappa_{m,4}$ in the fourth gap</td>
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<tr>
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Table A.2: Parameters and results of output cavity with four gaps
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<td>unloaded Q-factor of the cavity $Q_0$</td>
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</tr>
<tr>
<td>loaded Q-factor of the cavity $Q_L$</td>
<td>650</td>
</tr>
<tr>
<td>length of the first gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>length of the second gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>length of the third gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>length of the fourth gap</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>length of the fifth gap</td>
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<tr>
<td>length of the sixth gap</td>
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<tr>
<td>$\kappa_{m,1}$ in the first gap</td>
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<tr>
<td>$\kappa_{m,2}$ in the second gap</td>
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</tr>
<tr>
<td>$\kappa_{m,3}$ in the third gap</td>
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</tr>
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<td>$\kappa_{m,4}$ in the fourth gap</td>
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</tr>
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<td>$\kappa_{m,5}$ in the fifth gap</td>
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<td>$\kappa_{m,6}$ in the sixth gap</td>
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</tr>
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<td>1.986%</td>
</tr>
</tbody>
</table>

**Table A.3:** Parameters and results of output cavity with six gaps
Appendix B

Important Source Codes for Numerical Simulation

B.1 GdfidL Source Codes

As an example the source codes for the output cavity with four gaps in W-band, including the Fortran program for hollow beam, are as following. The geometry parameters of this output cavity are in Table A.2.

B.1.1 Source Code for Eigenvalue Simulation

```plaintext
define (RA, 5.1e-3)#Outer radius of the resonator
define (RA1, RA*0.5)
define (RA2, RA*0.866025403)
define (RI, 3.15e-3)#Inner radius of the resonator
define (RI1, RI*0.5)
define (RI2, RI*0.866025403)
define (RH, 0.38e-3)#Gap length for the first three gaps
define (RH2, 0.3e-3)#Gap length for the fourth gap
define (BRI, 3.8e-3)#Inner Radius of beam pipe
define (BRA, 4.2e-3)#Outer Radius of beam pipe
define (EL, 5)

define (AB, 0.3e-3)#Distance between 2 Resonators
define (STPSZE, 0.04e-3)#Meshing size in Simulation

- general
  outfile= /scratch/ausgangsresonator-w-band-4-zellen
  scratch= /tmp/scratch1

- material
  material= 6, type= normal, epsr= 1, muer= 1.05, mkappa= 11.3e3*3.7

- material
  material= 7, type= normal, epsr= 1, muer= 1.2, mkappa= 11.3e3*3.7
```
Appendix B. Important Source Codes for Numerical Simulation

- material
  material = 8, type = normal, epsr = 1, muer = 1.18, mkappa = 11.3e3*3.7

- material
  material = 9, type = normal, epsr = 1, muer = 1.05, mkappa = 11.3e3*3.7

- material
  material = EL, type = normal, epsr = 1, muer = 1, kappa = 58e6

-mesh
  
  # spacing: The default grid spacing to use.
  # Additionally, one may enforce grid planes at selected coordinates
  # spacing = STPSZE

  zgraded = no

  
  # pxlow: (P)lane XLOW: The borders of the computational volume.
  # only what is inside of the box
  # (pxlo, oylow, pzlow), (pxhigh, pyhigh, pzhight)
  # is really discretised and used for the computation.
  # pxlow = -1.5*RA, pxhigh = 1.5*RA

  pylow = -1.5*RA, pyhigh = 1.5*RA
  pzlow = -1.5e-3, pzhight = 4*RH+3*AB+1.5e-3

  if (0) then
    xfixed (2, -1*RA, 1*RA)
    yfixed (2, -1*RA, 1*RA)
    zfixed (2, 0+ZOFF, RH+ZOFF)
  end if

  # pzhight = 0 # WB

  # cxlow: Condition at XLOW
  # The boundary conditions at the outermost borders of the computational volume.
  # cxlow = electric, cxhigh = electric
  # cylow = electric, cyhigh = electric
  # czlow = electric, czhigh = electric
# czlow= magnetic, czhigh=mag

#zperiodic= yes, zphase= PHASE

# perfectmesh= yes

######
######
######
#
# We define the geometry..#
#

# Fill the 'Universe' with metal:
brick
material= 1
xlow= -INF, xhigh= INF
ylow= -INF, yhigh= INF
zlow= -INF, zhigh= INF
doit

d er 1. Resonator
# Cavity
-ggcylinder
material= 0
originprime= ( 0, 0, 0 )
xprimedirection= ( 1, 0, 0 )
yprimedirection= ( 0, 1, 0 )
range= ( 0, RH )
clear  # Clear the previous Polygon-List, if any.
point= ( RA, 0)
point= ( RA2, RA1)
point= ( RA1, RA2)
point= ( 0, RA)
point= ( -RA1, RA2)
point= ( -RA2, RA1)
point= ( -RA,0)
point= ( -RA2, -RA1)
point= ( -RA1, -RA2)
point= ( 0, -RA)
point= ( RA1, -RA2)
point= ( RA2, -RA1)
point= ( RA, 0)
doit
### tuning ring
Appendix B. Important Source Codes for Numerical Simulation

```plaintext
# gbor clear
point= (0+STPSZE, RI+STPSZE ),
point= (0+STPSZE, RI+2*STPSZE),
# arc, radius=Beampipe_Radius, type= clockwise
point= (RH-STPSZE, RI+2*STPSZE),
point= (RH-STPSZE, RI+STPSZE),
    # arc, radius=g+t/2, type= clockwise
material=6,
origin= ( 0, 0, 0),
zprimedir= ( 0, 0, 1),
rprimedir= ( 1, 1, 0 ),
range= ( 0, 360 ),
#show= now
doit

der 2. Resonator
# Cavity
-ggcylinder
material= 0
    originprime= ( 0, 0, RH+AB )
xprimedirection= ( 1, 0, 0 )
yprimedirection= ( 0, 1, 0 )
range= ( 0, RH )

# Clear the previous Polygon–List, if any.
point= ( RA, 0)
point= ( RA2, RA1)
point= ( RA1, RA2)
point= ( 0, RA)
point= ( –RA1, RA2)
point= ( –RA2, RA1)
point= (–RA, 0)
point= ( –RA2, –RA1)
point= ( –RA1, –RA2)
point= ( 0, –RA)
point= ( RA1, –RA2)
point= ( RA2, –RA1)
point= ( RA, 0)
doit
### tuning ring

# gbor clear
```
Appendix B. Important Source Codes for Numerical Simulation

point= (0+STPSZE, RI+STPSZE ),
point= (0+STPSZE, RI+2*STPSZE ),
# arc, radius=Beampipe_Radius, type= clockwise
point= (RH-STPSZE, RI+2*STPSZE ),
point= (RH-STPSZE, RI+STPSZE ),
# arc, radius=g+t/2, type= clockwise

material=7,
origin= ( 0, 0, RH+AB),
zprimedir= ( 0, 0, 1),
rprimedir= ( 1, 1, 0 ),
rang= ( 0, 360 ),
#show= now
doit

der 3. Resonator
# Cavity
ggcylinder
material= 0
originprime= ( 0, 0, 2*(RH+AB) )
xprimedirection= ( 1, 0, 0)
yprimedirection= ( 0, 1, 0)
rang= ( 0, RH )
clear # Clear the previous Polygon–List, if any.
point= ( RA, 0)
point= ( RA2, RA1)
point= ( RA1, RA2)
point= ( 0, RA)
point= ( –RA1, RA2)
point= ( –RA2, RA1)
point= (–RA,0)
point= (–RA2, –RA1)
point= (–RA1, –RA2)
point= ( 0, –RA)
point= ( RA1, –RA2)
point= ( RA2, –RA1)
point= ( RA, 0)
doit
###tuning ring

gbor
clear
point= (0+STPSZE, RI+STPSZE ),
point= (0+STPSZE, RI+2*STPSZE ),
# arc, radius=Beampipe_Radius, type= clockwise
Appendix B. Important Source Codes for Numerical Simulation

# der 4. Resonator
# Cavity
-ggcylinder
material = 0
originprime = (0, 0, 3*(RH+AB))
xprimedirection = (1, 0, 0)
yprimedirection = (0, 1, 0)
range = (0, RH)
clear # Clear the previous Polygon–List, if any.
point = (RA, 0)
point = (RA2, RA1)
point = (RA1, RA2)
point = (0, RA)
point = (-RA1, RA2)
point = (-RA2, RA1)
point = (-RA, 0)
point = (-RA2, -RA1)
point = (-RA1, -RA2)
point = (0, -RA)
point = (RA1, -RA2)
point = (RA2, -RA1)
point = (RA, 0)
doit
### tuning ring
-gbor
clear
point = (0+STPSZE, RI+STPSZE),
point = (0+STPSZE, RI+2*STPSZE),
# arc, radius=Beampipe_Radius, type= clockwise
point = (RH-STPSZE, RI+2*STPSZE),
point = (RH-STPSZE, RI+STPSZE),
# arc, radius=g+t/2, type= clockwise

material = 9,
origin = (0, 0, 3*(RH+AB)),
zprimedir = (0, 0, 1),
rprimedir = (1,1, 0),
rangep = (0, 360 ),
#show = now
doit
#Innenleiter
-ggcylinder
material = 1
originprime = (0, 0,-STPSZE)
xprimedirection = (1, 0, 0)
yprimedirection = (0, 1, 0)
rangep = (0, 4*RH+3*AB+2*STPSZE)
clear # Clear the previous Polygon–List, if any.
point = (RI, 0)
point = (RI2, RI1)
point = (RI1, RI2)
point = (0, RI)
point = (-RI1, RI2)
point = (-RI2, RI1)
point = (-RI, 0)
point = (-RI2, -RI1)
point = (-RI1, -RI2)
point = (0, -RI)
point = (RI1, -RI2)
point = (RI2, -RI1)
point = (RI, 0)
doit

#########
#########
#########
#########
macro Bend
-gbor
origin = (WA+10e3, 0, -WH+RH/2)
zprimedirection = (0, -1, 0)
rprimedirection = (0, 0, -1)
rangep = (@arg1, @arg2)
material = 0
clear  # clear any previous polygon-description.
point = ( -WB/2, BendRadius-WH/2 )  # (Z,R)
point = ( -WB/2, BendRadius+WH/2 )  # (Z,R)
point = ( WB/2, BendRadius-WH/2 )  # (Z,R)
point = ( WB/2, BendRadius+WH/2 )  # (Z,R)

# show= all
doit
# show= off

endmacro  # Bend

# Beampipe

-gbor
clear

point = (- 5e-3, BRI ),
point = (- 5e-3, BRA),
# arc, radius=Beampipe_Radius, type= clockwise
point = ( RH+1, BRA),
point = ( RH+1, BRI),
# arc, radius=g+t/2, type= clockwise

material = 0,
origin = ( 0, 0, -0.5),
zprimedir = ( 0, 0, 1),
rprimedir = ( 1, 1, 0 ),
range = ( 0, 360 ),
# show= now
doit
-volumeplot
bbyl 0
doit

# Done with modeling the geometry..
# How does this look like?

# We want to compute the eigenvalues in that geometry.
#
# \texttt{eigenvalues estimation=151.10554361e+9} 
# \texttt{solutions= 160} 
# \texttt{passes=2} 
# \texttt{doit}

## B.1.2 Source Code for PIC Simulation

The PIC simulation is done together with the FORTRAN program of the hollow beam.

```plaintext
define(RA,5.1e-3)#Outer radius of the resonator
define(RA1,RA*0.5)
define(RA2,RA*0.866025403)
define(RI,3.15e-3)#Inner radius of the resonator
define(RI1,RI*0.5)
define(RI2,RI*0.866025403)
define(RH,0.38e-3)#Gap length for the first three gaps
define(RH2,0.3e-3)#Gap length for the fourth gap
define(BRI,3.8e-3)#Inner Radius of beam pipe
define(BRA,4.2e-3)#Outer Radius of beam pipe
define(EL,5)
define(AB,0.3e-3)#Distance between 2 Resonators

-genera

-nrofthreads=8

-genera


-scratch= /tmp/stratch

-geom

-infile= /scratch/ausgangsresonator–w–band–4–zellen

-doit

-fdtd

-fdtd
```
Appendix B. Important Source Codes for Numerical Simulation

```
# initial fields
abstatic  =0.6

# nbstatic = ( 0.0, 0.0, 1.0 )

# We compile the executable file with the Fortran Compiler,
# (which is called 'fortran' on this system)

system( f95 hollow−beam−4−zellen−mod006−16mm−drift.f90 −o ./hollow−beam−4−zellen−mod006−16mm−drift.out )

# We specify that the properties of free moving charges
# shall be described by the executable file 'Eject −a.out'

ejectioncommand=./hollow−beam−4−zellen−mod006−16mm−drift.out

define (TEND, TMAX/FREQ)
tmin=TEND
tmax=TEND

# We want to compute the eigenvalues in that geometry..

−time

−fdtd
    −storefieldsat
        name= a, what= e
        firstsaved= 10/FREQ
        lastsaved= TMAX/FREQ
        distance= 50/FREQ
        doit

−storefieldsat
    name= b, what= e
    firstsaved= 500/FREQ
    lastsaved= 502/FREQ
    distance= 0.1/FREQ
```
doit

-voltages
  startpoint=(0, BRA+2*STPSZE, + STPSZE)
  endpoint  =(0, BRA+2*STPSZE, + RH-STPSZE)
  amplitude =1e-10
  resistance=1e10
  frequency =0
  risetime  =10/FREQ
  logcurrent=yes
  name=Spannung-1

doit

-voltages
  startpoint=(BRA+2*STPSZE, 0, + STPSZE)
  endpoint  =(BRA+2*STPSZE, 0, + RH-STPSZE)
  amplitude =1e-10
  resistance=1e10
  frequency =0
  risetime  =10/FREQ
  logcurrent=yes
  name=Spannung-2

doit

-voltages
  startpoint=(0, BRA+2*STPSZE,RH+AB+STPSZE)
  endpoint  =(0, BRA+2*STPSZE,RH+AB+RH-STPSZE)
  amplitude =1e-10
  resistance=1e10
  frequency =0
  risetime  =10/FREQ
  logcurrent=yes
  name=Spannung-3

doit

-voltages
  startpoint=(BRA+2*STPSZE, 0,RH+AB+STPSZE)
  endpoint  =(BRA+2*STPSZE, 0,RH+AB+RH-STPSZE)
  amplitude =1e-10
  resistance=1e10
  frequency =0
  risetime  =10/FREQ
  logcurrent=yes
  name=Spannung-4

doit
B.1.3 Fortran Program for Hollow Beam

The electrons generated by this code have different colours depending on the time, when they come to existence. This colouring is later used for electron trajectory plotting:

```fortran
program Strahl

implicit none

! Setzen nicht veränderbarer Großen

double precision, parameter :: 
  c_licht = 299792458.0D0 & ! Lichtgeschwindigkeit
  ladung_elektron = -1.602176462D-19 & ! Elektronenladung
  masse_elektron = 0.910938188D-30 & ! Elektronenmasse
  strom = 50D0 & ! Strahlstrom
  beta = 0.55D0 & ! relative Strahlgeschwindigkeit
  radius = 33.25D-3 & ! Radius des Strahls
  modu = 0.024 & ! Geschwindigkeitsmodulation in z-Richtung
  frequenz = 10497024505.728018 &
  dicke = 1.5D-3

integer, parameter :: teilchenanzahl = 60

! Variablen

integer :: &
  nr & ! bis jetzt emittierte
    + neu emittierte Partikel
  , bis_jetzt & ! bis jetzt emittierte
    Teilchen (aus GdfidL einlesen)
  , jzeit & ! Zeitpunkt der
    Simulation bei Programmaufruf (a. GdfidL e.)
  , partikel_i & ! Index für das
    Einschieben der Partikel pro Zeitschritt
```
Programmaufruf (a. GdfidL e.)
mache_schritte = 500 & ! Anzahl der im voraus zu berechnenden Zeitschritte
farbe = 32 & ! Farbe der Teilchen

double Precision :: &
zeit & ! Momentane Zeit (= aktueller_zeitschritt * zeitschrittweite)
charge & ! Ladung
Charge_Over_Mass & ! Ladung ueber Masse
x_start & ! x-Startkoordinate des Strahls
y_start & ! y-Startkoordinate des Strahls
z_start & ! z-Startkoordinate des Strahls
zeitschrittweite & ! Zeitschrittweite
delta_z & ! Driftstrecke pro Zeitschritt
r_zufallszahl & ! =0...1, um den Strahl zu "mischen"
phi_zufallszahl & ! =0...1, um den Strahl zu "mischen"
z_zufallszahl & ! =0...1, um den Strahl zu "mischen"
wolke & ! geometrische Groe e der Wolke in GdfidL
random_z

real :: &
x_position = 0 & ! X-Position
y_position = 0 & ! Y-Position
z_position = 0 ! Z-Position

real, dimension(3) :: &
Ort & ! 3D-Array Position eines Teilchens
Geschw & ! 3D-Array Geschwindigkeit eines Teilchens
Appendix B. Important Source Codes for Numerical Simulation

\begin{verbatim}
x_start = 0.0D0
y_start = 0.0D0
z_start = -1e-6

! Einlesen der Daten aus GdfidL über die Standardeingabe

read (*) aktueller_zeitschritt ! aktueller Zeitschritt
    von GdfidL (integer)
read (*) zeitschrittweite ! Zeitschrittweite (double)
read (*) wolke ! Größe der Wolke
read (*) bis_jetzt ! bis jetzt in GdfidL
    eingespeiste Anzahl von Partikel

! Wie weit driften die Teilchen ungefähr in einem Zeitschritt
nr = bis_jetzt ! Nr. des aktuellen Partikels

write (*) mache_schritte ! GdfidL wird mitgeteilt wie viele Zeitschritte im voraus berechnet werden

Do-Schleifen zur Berechnung und Ausgabe der Partikelpositionen
! Die äußere Do-Schleife legt fest, für welchen Zeitraum (wie viele Zeitschritte)
! die Daten berechnet werden sollen
! Dazu wird der aktuelle, aus GdfidL eingelesene Zeitschritt, mit der Anzahl der im voraus zu
! berechnenden Zeitschritte addiert
! Danach wird GdfidL die Partikelanzahl pro Zeitschritt mitgeteilt

do jzeit = aktueller_zeitschritt , aktueller_zeitschritt +
    mache_schritte -1, 1

    zeit = jzeit * zeitschrittweite

write (*) teilchenanzahl

x_position = x_start
y_position = y_start
z_position = z_start

do partikel_i=1, teilchenanzahl , 1
\end{verbatim}
CALL RANDOM_NUMBER(r_zufallszahl)
CALL RANDOM_NUMBER(phi_zufallszahl)
CALL RANDOM_NUMBER(z_zufallszahl)

Geschw(1) = 0.0D0
Geschw(2) = 0.0D0
Geschw(3) = beta * c_licht * (1.0D0 + modu * cos(3 * phi_zufallszahl * 2 * 3.1415D0 - frequenz * zeit * 2 * 3.1415D0))

delta_z = Geschw(3) * zeitschrittweite

Ort(1) = x_position + (radius + dicke * r_zufallszahl) * cos(phi_zufallszahl * 2 * 3.1415D0)
Ort(2) = y_position + (radius + dicke * r_zufallszahl) * sin(phi_zufallszahl * 2 * 3.1415D0)
Ort(3) = z_position + z_zufallszahl * delta_z

nr=nr+1

 Charge_Over_Mass = ladung_elektron / masse_elektron
 Charge = SIGN(strom * zeitschrittweite / teilchenanzahl, (ladung_elektron))
 phase = mod(zeit, 1 / frequenz) * frequenz * 20
 IF (phase.GE. 0 .AND. phase.LT. 1) farbe = 21
 IF (phase.GE. 1 .AND. phase.LT. 2) farbe = 22
 IF (phase.GE. 2 .AND. phase.LT. 3) farbe = 23
 IF (phase.GE. 3 .AND. phase.LT. 4) farbe = 24
 IF (phase.GE. 4 .AND. phase.LT. 5) farbe = 25
 IF (phase.GE. 5 .AND. phase.LT. 6) farbe = 26
 IF (phase.GE. 6 .AND. phase.LT. 7) farbe = 27
 IF (phase.GE. 7 .AND. phase.LT. 8) farbe = 28
 IF (phase.GE. 8 .AND. phase.LT. 9) farbe = 29
 IF (phase.GE. 9 .AND. phase.LT. 10) farbe = 30
 IF (phase.GE. 10 .AND. phase.LT. 11) farbe = 31
 IF (phase.GE. 11 .AND. phase.LT. 12) farbe = 32
 IF (phase.GE. 12 .AND. phase.LT. 13) farbe = 33
 IF (phase.GE. 13 .AND. phase.LT. 14) farbe = 34
 IF (phase.GE. 14 .AND. phase.LT. 15) farbe = 35
 IF (phase.GE. 15 .AND. phase.LT. 16) farbe = 36
 IF (phase.GE. 16 .AND. phase.LT. 17) farbe = 37
 IF (phase.GE. 17 .AND. phase.LT. 18) farbe = 38
B.2 MATLAB codes for Electron Cloud Tracking

This two Matlab files as following are used to plot the electron trajectory through the whole klystron. The phase is also plotted.

c1c
clear
start_nr=40;
end_nr=115;
for kk = start_nr:end_nr
    % Create a text file name, and read the file.
    textFileName = ['xband-tm310color-clouds-' num2str(kk)]
    names=[['test ' num2str(kk)]]
    if exist(textFileName, 'file')
        for ind = 1:length(names)
            wolken.(names{ind})=importdata(textFileName);
        end
    else
        fprintf('File %s does not exist.\n', textFileName);
    end
end
color=wolken.(names{1}).data(:,2);
for ii=40:1:21
    names2={'position ' num2str(kk)}
    if kk==start_nr
        color_nr=find(color==ii);
        particle_nr(ii*(-1)+41)=wolken.(names{1}).data(color_nr(10),1);
        no=find(wolken.(names{1}).data(:,1)==
            particle_nr(ii*(-1)+41));
        nr(ii*(-1)+41)=no(1);
        color_.(ii*(-1)+41)=wolken.(names{1}).data(nr(ii*(-1)+41),2)
        position.(names2{ind})(ii*(-1)+41,:)=
            wolken.(names{1}).data(nr(ii*(-1)+41)+1,1:3)
    else
        no = find((wolken.(names{ind}).data(:,1))==
            particle_nr(ii*(-1)+41));
        nr(ii*(-1)+41,:)=no(1);
        color_.(ii*(-1)+41)=wolken.(names{ind}).data(nr(ii*(-1)+41),2)
        position.(names2{ind})(ii*(-1)+41,:)=
            wolken.(names{1}).data(nr(ii*(-1)+41)+1,1:3)
    end
end
end

start_nr=40;
end_nr=90;
freq=10e9;
clight=3e8;
v0=0.55*clight;
figure;
for kk=start_nr:1:end_nr
    names2={'position ' num2str(kk)}
    for ind = 1:length(names2)
        for ii=1:20
            z(kk,ii)=position.(names2{ind})(ii,3);
            time(kk,ii)=z(kk,ii)/v0;
            phi_z(kk,ii)=2*pi*freq*time(kk,ii);
            phi_soll(kk,ii)=2*pi*freq*time(kk,ii);
            phi_delta(kk,ii)=2*pi*freq*time(kk)-phi_z(kk,ii);
        end
    end
Appendix B. Important Source Codes for Numerical Simulation

```matlab
scatter3(position.(names2{ind})(ii,1),position.
    (names2{ind})(ii,2),position.(names2{ind})(
    ii,3),’filled’);
hold on;
end
end
figure;
for ii =1:20
    plot(phi_z(start_nr:end_nr,1 ’,phi_delta(start_nr:end_nr,
        ii))
    hold on
end
```
Bibliography


