Optimization of Stationary Expansion Planning and Transient Network Control by Mixed-Integer Nonlinear Programming

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Abstract

Nowadays, transmission system operators of energy networks (TSO) have to enable secure energy supply even under challenging transport situations and increasing demand. Hence, they seek a stable network control and regularly expand the capacity of their networks. The challenging questions arise of how to obtain safe network operations for fixed network infrastructure and how to extend the network capacity at minimum cost. In this thesis, we formulate these two questions as mathematical optimization problems and present solution approaches that scale to real-world instances of size and complexity encountered in practice.

Model formulations of both problems are located in the field of Mixed-Integer Nonlinear Programming (MINLP). However, global state-of-the-art MINLP solvers are not mature enough to solve or even find primal solutions on large-scale real-world instances since the resulting degree of nonconvexity and nonlinearity pose principal difficulties. For this reason, we develop novel model formulations and optimization algorithms that significantly improve the performance of the solver SCIP[1].

The first part of the thesis focuses on the optimization of stationary network expansions by building new pipelines in parallel to existing ones ("looping"). Based upon a model reduction approach for multiple loops, we introduce new models for the discrete and split-pipe looping paradigm and compare them with existing models in the literature, both theoretically and empirically. It turns out that our novel split-pipe model performs best in several respects: running time, number of instances solved, and cost savings by up to 7000% over the discrete models. To further improve the performance, we analytically calculate the convex envelope of the nonconvex, nonlinear constraint function $f(x, y) = y\,\mathrm{sgn}(x)|x|^{\alpha}$, which models the physical effect of the expansion. In this way, we considerably tighten the convex relaxation of the network expansion problem. The resulting implementation significantly reduces the average solving time by up to 58% on difficult instances.

The second part of the thesis deals with the optimization of operations in transient large-scale networks. The problem consists of making day-ahead control decisions that enable feasible energy transport. To this end, we propose a specially-tailored solution approach. First, we aggregate network structures, which TSOs typically consider as of minor importance for the decision-making process. Then, we apply model reformulations that exploit the disjunctive nature of certain network structures and strengthen the linear relaxation. Finally, we present a primal heuristic based on time decomposition. The

[1] SCIP: Solving Constraint Integer Programs, see https://scip.zib.de

heuristic solves MINLP sub-models to acquire transport decisions for single time steps. We equip the heuristic with a mechanism that enables moving forward and backward on the time scale to compensate for possibly disadvantageous decisions taken at earlier stages. An extensive computational study shows that our approach reliably generates solutions for 96% to 100% of the instances for three different large-scale real-world networks.

Parts of the developed methods are already in use by our industrial cooperation partner Open Grid Europe GmbH[2] to facilitate the respective decision-making processes.

[2] https://oge.net, accessed in September 2020.

Acknowledgements

First of all, I would like to thank Thorsten Koch and Martin Grötschel for enabling me to work at Zuse Institute Berlin. This thesis would not have been possible without the inspiring environment at the optimization department. I would like to thank all my colleagues for the nice working atmosphere and all the beautiful work and non-work related time we had. I definitely made a couple of good friends here. I am very grateful for the financial support, which made it possible to attend optimization conferences in different places in the world and exchange with other optimizers. It also enabled me to get in touch with Andrea Lodi at the Operations Research Conference in Rotterdam in 2013. I thank Andrea Lodi for taking an interest in this thesis and agreeing to be a member of my Ph.D. committee. Also, I am thankful for the technical infrastructure and support I encountered at ZIB. In particular, I always appreciated the excellent administration of the computer cluster.

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[3] http://forschungscampus-modal.de

enriches my life and helped me recharge my empty batteries drained by working on the thesis.

Contents

Chapter 1

Introduction

Solving problems related to the production, storage and transportation of energy has widely stimulated the field of mathematical optimization and operations research. In practice, large energy-related optimization problems have to be solved since decades, including the design of networks (Rothfarb et al., 1970), but also the transportation of different energy types, such as natural gas (Ríos-Mercado and Borraz-Sánchez, 2015), water (D'Ambrosio et al., 2015), hydrogen (André et al., 2013) and electrical energy (van Ackooij et al., 2018).

In the past, markets of energy transportation systems have often been governed by monopolistic structures. These structures enabled transmission system operators (TSO) to conclude long-term contracts with clients of up to twenty years that simplified their network operations and expansion planning. For example, long-term contracts allowed TSOs to apportion high investment costs among their clients. Moreover, demand and transport situations were known long before being realized, which in turn enabled foresighted operation planning and safe gas transportations.

However, the markets' deregulations in the EU in 2009 led to incisive changes. Taking the example of natural gas, operations were decoupled from supply and production and open access was granted to all gas trading companies (Bundesministerium der Justiz, 2005; Gotzes et al., 2015). This introduced fundamental challenges concerning the network control for the TSOs. Most importantly, the specific amount of supplies and withdrawals were not under the direct control of the TSOs anymore. Instead, gas is now traded on the energy exchange, allowing traders to determine how much gas should be sent between destination and target points. While the deregulation of monopolistic structures makes the entire gas trading process more transparent, it has led to new challenges for the TSOs concerning the planning and control of their networks: First, TSOs face nowadays more volatile demand scenarios than ever. Hence, network operations will have to become more flexible requiring a robust and stable network control, while satisfying contractual, physical and technical restrictions. Second, their resources need to be employed as efficiently as possible in order to stay competitive in the market.

In a similar vein, the planned energy turnaround, decided in Germany in 2011, has intensified these challenges. Natural gas represents an important transit medium on the way to a more climate-friendly and sustainable energy system (DVGW, 2020). Until the phase-out of nuclear energy and fossil fuels, it can be used to mitigate shortcomings of intermittent renewable energy production and to balance fluctuations in electrical power supply. Under the assumption that the planned energy turnaround increases the share of renewable energy in the future, such as wind and solar power, whose production is subject to uncertainty, gas network operations will be additionally exposed to more volatile scenarios.

So far, dispatchers mostly control their networks manually by applying operational actions based on their personal experiences. Beyond the dispatching, TSOs apply further measures that aim at enabling safe operations. Certainly, the most prominent measure is to regularly expand the network capacities by building new network elements. Network expansions enlarge the possibilities of the network control and the transportation routing, however, they are extremely expensive and usually correspond to a massive encroachment on the environment. Other measures target the more efficient use of the existing network capacity. To this end, TSOs developed and launched tailor-made contracts in recent years that enable clients to buy and sell gas at certain network points, where the specific amount of gas depends, e.g., on the temperature. TSOs designed such contractual products with the intention to influence the resulting demand scenarios in a way that favors stable network control. Nevertheless, these measures might not necessarily be sufficient to guarantee secure gas supply. For example, the German gas network was allegedly at risk of a blackout in recent years (Die Welt, 2020), almost not being able to fulfill transport tasks and to satisfy crucial demands, even though feasible gas network operations are considered to be "systemrelevant" in Germany (Bundesnetzagentur, 2020), i.e., highly relevant for the basic infrastructure of society and economy.

These challenges motivate using computer-aided decision-making tools to tackle the strategical and operational tasks that dispatchers face in practice. In particular, mathematical programming enables automating the decision-making process and can effectively help to formalize and solve optimization problems on energy-driven networks. Mathematically speaking, transport systems of energy commodities are represented by meshed pipeline networks. Flows in pipelines arise from nonlinear potential differences between adjacent nodes, which can be represented by so-called "potential-driven" network models. Accounting also for the combinatorial complexity arising from network control elements, energy-driven problems are located in the field of Mixed-Integer Nonlinear Programming (MINLP). For this reason, we present an introduction to MINLP in Chapter 2 after this introduction.

In this thesis, we focus on the optimization of the *expansion planning* and the *daily control* of energy-driven networks. Expansion planning concerns a long time horizon. Hence, it is a reasonable assumption to apply stationary modeling, which assumes the network to be in steady-state. Provided that common stationary models lead to structurally equivalent equations for different types of energy, such as gas, hydrogen and water, the expansion planning applies to a broader setting of energy-driven networks. By contrast, the daily network control concerns short-term planning where the network state can change over time. Here, transient modeling is required instead, which restricts us to the application of energy types that feature similar gas physics, such as natural-gas and hydrogen.

Both problems canonically split the thesis in two parts: (i) *optimizing expansion costs in stationary energy networks*, and (ii) *optimizing the control of transient large-scale gas networks*. For both problems, we present model formulations and new algorithms that improve the model performance using a state-of-the-art MINLP solver. Figure 1.1 shows the organization of the thesis.

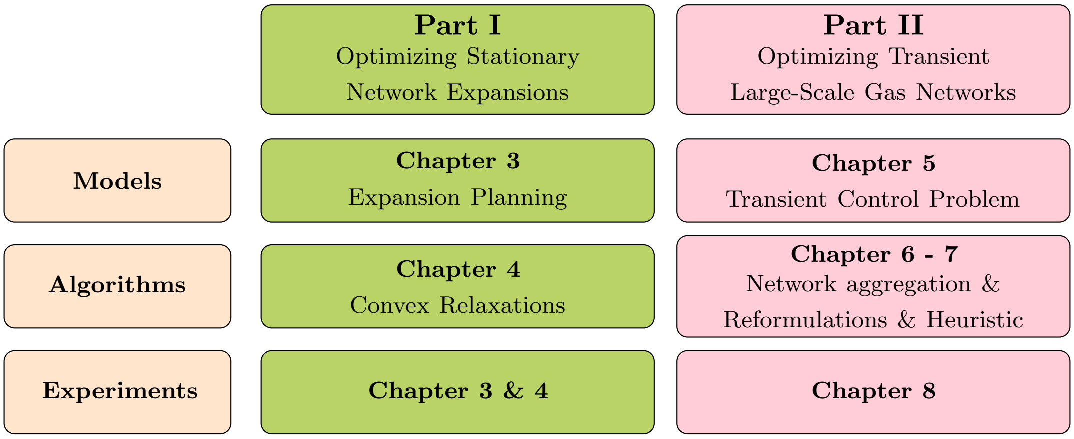


Figure 1.1: Organization of the thesis.

Part I - Optimizing Stationary Network Expansions. In the first part of the thesis, we investigate a type of network expansion that operators preferably apply in practice, namely to build new pipelines parallel to existing ones. This process is referred to as "looping" in the terminology of the industry. It has several advantages, compared to, for example, building completely new pipelines. Most importantly, it is significantly cheaper, since existing rights of way can be used simplifying the regulatory and the planning processes. Besides that, the encroachment on the environment is rather moderate and moreover, possible events are avoided that would potentially slow down the construction procedure, such as new archaeological findings[1].

In Chapter 3, we introduce new model formulations for the *stationary expansion planning problem of potential-driven networks* and compare these with existing models for the looping problem and related problems in the literature, both theoretically and experimentally. We conduct a computational study on various test sets comprising a large amount of differently structured instances. On this basis, we give recommendations about the circumstances under which a certain model should be used. In particular, it turns out that one of our new models outperforms the existing models with respect to computational time, the number of solutions found, the number of instances solved and cost savings. Moreover, to the best of our knowledge, this thesis is the first to include the practically relevant option that a particular pipeline may be looped several times, while keeping the model size reasonably small.

[1] See also the video about the construction of a North-West connection stream in Germany, called *NOWAL: Bau der Nord-West-Anbindungsleitung* (Gascade Gastransport GmbH, 2018).

A key constraint function in stationary expansion planning problems is the nonlinear and nonconvex potential loss function $f(x,y) = y\,\mathrm{sgn}(x)|x|^\alpha$ with $\alpha > 1$, where x represents the flow and y the physical impact of the expansion on the pressure loss reduction. It is well known that tight relaxations of nonlinear and nonconvex constraint functions significantly improve the performance of global MINLP solvers (Grossmann, 2002). For this reason, one of our main theoretical contributions in this thesis is to derive an algebraic description for the convex envelope of this function in Chapter 4. In this way, we obtain a tight convex relaxation of the expansion planning problem. Indeed, utilizing the closed-form expression for the convex envelope of f significantly improves the quality of relaxations that are constructed by state-of-the-art MINLP solvers for function f. Through a thorough computational study, we show that these tighter relaxations tremendously improve the performance of the MINLP solver SCIP on a large test set of practically relevant instances for the stationary expansion planning problem. The results show that our achievements improve the solver performance by up to 58% in shifted geometric mean runtime.

Part II - Optimizing the Control of Transient Large-Scale Gas Networks. In practice, the major task of dispatchers is to realize stable and safe gas network operations. To this end, they seek a network control that allows routing the gas such that demands, but also operational and physical restrictions are satisfied. In Chapter 5, we present an MINLP formulation for this problem, which we call the *Transient Control Problem* in this thesis. It generates decisions for the network control with respect to half day-ahead operations. In general, practically relevant gas networks frequently contain up to several thousands of elements (Carvalho et al., 2009) resulting in large sized MINLPs. It is not surprising that it is currently impossible to solve the *Transient Control Problem* on large-scale real-world networks. Motivated by the fact that it frequently suffices to generate high-quality solutions for the network control instead of solving the problem to global optimality, we develop a heuristic solution approach. It comprises several key ingredients, highlighted in Figure 1.2 (in yellow).

First, we reduce the model size. In fact, this is done implicitly by reducing the size of the network, given that the model is induced by the elements of the underlying network. To this end, we automatically generate a so-called backbone network representing the major network parts that TSOs consider as important for the network control. This step is referred to as *aggregation of transient gas networks* and described in Chapter 6. As input it takes the network topology, its current physical state and a demand forecast, see Figure 1.2. The aggregation outputs the topologically reduced network with corresponding state and demand. For the aggregation, we formalize the concept of *equivalent subnetwork replacements*, which allows to equivalently solve the *Transient Control Problem* on the original and aggregated networks. Based on this definition, we develop methods that equivalently replace sub-networks as well as additional heuristic procedures. Our evaluation of the heuristic aggregation methods based on fine-grained real-world data covering an entire year, shows that they reliably approximate the physical behavior of the original structures.

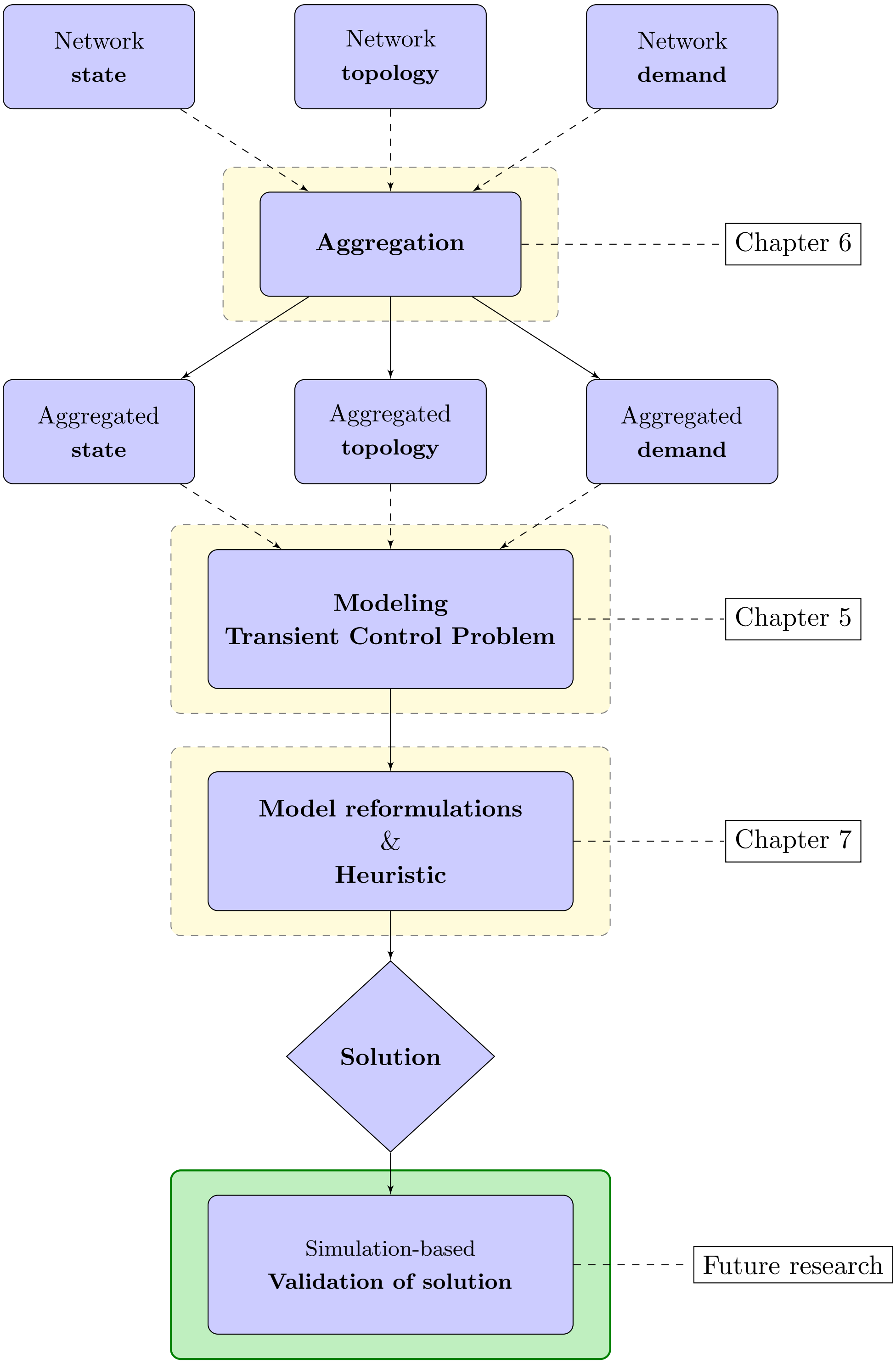


Figure 1.2: Solution approach to the optimization of operations in transient large-scale gas networks

Afterwards, we solve the *Transient Control Problem* on the aggregated network. In order to further reduce the computational burden, we develop model reformulations and a primal heuristic in Chapter 7. More precisely, we first exploit the disjunctive nature of specific network structures and apply Balas's convex-hull reformulation (Balas, 1985). Secondly, we develop a special-tailored moving horizon based heuristic. Most importantly, the resulting solutions yield a plan of how to control the network and maintain a stable network state, which can be interpreted as a recommendation for dispatchers in practice. Our extensive computational study in Chapter 8 shows the efficacy and reliability of our approach on large-scale real-world gas networks. For this reason, we understand our approach (Figure 1.2) as a step towards using MINLP models within a decision-support system. Such system could run in real-time and continuously generate recommendations for the next hours, while throughout updating the required input data for the network, its state and demand forecast. Let us mention that due to the more advanced solving technology of MIP solvers, so far most industrial driven optimization approaches primarily resort to complete linearizations and embed MIP models into online running decision-support systems, see for example, Hadjidimitriou et al. (2020); Hoppmann et al. (2019).

Finally, we remark that the entire procedure is only an approximation to solving the *Transient Control Problem* on the original (non-aggregated) network. To verify that the acquired decisions indeed yield feasible operations on the original network, future research concerns the development of a strategy that validates whether a solution for the aggregated network can be extended to a solution for the original network. This possible last step of the presented solution approach is highlighted in green in Figure 1.2.

Chapter 2

Preliminaries

This chapter presents introductory material that is relevant for the thesis. Both mathematical optimization problems investigated in this thesis, *the stationary expansion planning of potential-driven networks* and *the control of transient large-scale gas networks*, are located in the field of mixed-integer nonlinear programming (MINLP). For this reason, we give a rather extensive but basic introduction to MINLPs in Section 2.1, comprising a formal description and algorithmic solution concepts. Moreover, Section 2.2 discusses the experimental methodology deployed in the computational studies of this thesis, such as the used hardware, performance measures, and statistical significance testing.

2.1 Introduction to Mixed-Integer Nonlinear Programming

In this section, we give a brief introduction to Mixed-Integer Nonlinear Programming and present state-of-the-art algorithms to compute globally optimal solutions to convex and nonconvex MINLPs. Formally, a mixed-integer nonlinear program is of the following form

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & g_i(x) \le 0 && \forall i \in \mathcal{M}, \\ & x_j \in [\underline{x}_j, \overline{x}_j] && \forall j \in \mathcal{N}, \\ & x_j \in \mathbb{Z} && \forall j \in \mathcal{J}, \end{aligned} \tag{2.1}$$

where $\mathcal{N} := \{1, ..., n\}$ denotes the index set of all variables, $\mathcal{J} \subseteq \mathcal{N}$ the index set of integer variables and $\mathcal{M} := \{1, ..., m\}$ the index set of linear and nonlinear *constraint functions* $g_i \colon \mathbb{R}^n \to \mathbb{R}$ for $i \in \mathcal{M}$. The inequalities $g_i(x) \le 0$ are called *constraints* and $c^T x$ is called *objective function*. Here, we write the side constraints as inequalities, given that equalities can be equivalently expressed as a pair of opposed inequality constraints. The variables $x \in \mathbb{R}^n$ are bounded by $\underline{x}_j \in \mathbb{R} \cup \{-\infty\}$ and $\overline{x}_j \in \mathbb{R} \cup \{\infty\}$.

The feasible region of Problem (2.1) is given by the set

$$\mathcal{X} := \{x \in \mathbb{R}^n \mid g_i(x) \le 0 \text{ for all } i \in \mathcal{M}, x_j \in [\underline{x}_j, \overline{x}_j] \text{ for all } j \in \mathcal{N},\ x_j \in \mathbb{Z} \text{ for all } j \in \mathcal{J}\}.$$

If the value

$$c^* := \inf\{c^T x \mid x \in \mathcal{X}\}$$

is finite, we call it the *optimal solution value* of Problem (2.1). In the case of $c^* = \infty$, we call the problem *infeasible*, and *unbounded*, if $c^* = -\infty$. A point $x \in \mathcal{X}$ is called a *feasible solution* of Problem (2.1), and besides, all points $x \in \mathcal{X}$ that hold $c^T x = c^*$, are called *optimal solutions.* If $c = 0$, then the optimization problem reduces to a *feasibility problem.*

Note that without loss of generality, we can always assume a linear objective function. Otherwise, it is possible to equivalently model a nonlinear objective function $F\colon \mathbb{R}^n \to \mathbb{R}$ by introducing an auxiliary continuous variable z, adding the nonlinear inequality $F(x) - z \leq 0$ to the side constraints, and minimizing variable z. Moreover, maximization problems can equivalently be reformulated as minimization problems by virtue of the relationship $\max c^T \iff -\bigl(\min -c^T\bigr)$.

In the past twenty years, the development of solution techniques for MINLPs has gained growing attention, which gave rise to address a wide range of problems as MINLPs, including, among others, applications from chemical engineering, finance, transportation, engineering, process industry, and network design, see, for example, Tawarmalani and Sahinidis (2002b), Burer and Letchford (2012), and Grossmann and Kravanja (1997).

As MINLPs possibly contain linear and nonlinear constraint functions, as well as continuous and discrete variables, they constitute a general class of optimization problems that comprises different classes of mathematical programs. For example, by making restrictions on the nonlinearity of the functions or on the integrality of the variables, the following important subclasses arise (that are also relevant for this thesis):

- *linear program* (LP), where the objective function c^T and all constraint functions $g_i(x)$ are linear and $\mathcal{J} = \emptyset$;
- *mixed-integer linear program* (MIP), where the objective function c^T and all constraint functions $g_i(x)$ are linear;
- *nonlinear program* (NLP), where $\mathcal{J} = \emptyset$;
- *mixed-integer quadratically constraint program* (MIQCP), where c^T and all constraint functions g_i are linear or quadratic.

The drawback of this generality is that MINLPs typically turn out to be $\mathcal{NP}$-hard. This follows already from the fact that MINLP contains the $\mathcal{NP}$-hard subclass MIP. Even more sobering is that MINLPs are in general *undecidable*, which can be attributed to the decision whether a given polynomial has an integer root, see Matiyasevich (1970). However, undecidability only holds in the case that the variables are unbounded, otherwise the problem can be decided in finite time using enumeration. Fortunately in practical applications, all variables are typically bounded. For more information about $\mathcal{NP}$-hardness in general, we refer to Garey and Johnson (1990) and for complexity results related to MINLP to Köppe (2012).

As Vigerske (2012) states, "the solution of some of these subclasses can be considered to be 'cheap', thus LPs, MIPs, or convex NLPs are used as subroutines within different algorithms to solve a single MINLP." When it comes to solving practical applications as MINLPs, a key distinction concerning computational difficulty relies on whether the problems are convex or nonconvex. Note that the integrality requirement of the variables $x_j \in \mathbb{Z}$ for $j \in \mathcal{J}$ renders by default *nonconvexity* of the problem. However, the bounded integer variables can be expressed equivalently as nonconvex constraints using the relationship $x_j \in \{\underline{x}_j, \underline{x}_j + 1, ..., \overline{x}_j - 1, \overline{x}_j\} \iff \prod_{k=0}^{\overline{x}_j - \underline{x}_j} (x_j - \underline{x}_j - k) = 0$. This relationship enables one to transform an MINLP into a nonconvex NLP, whose solution techniques belong to the classical field of *global optimization*, see, e.g., Liberti and Maculan (2006), Horst and Tuy (1990), and Locatelli and Schoen (2013). We call Problem (2.1) a *nonconvex MINLP* if at least one constraint function g_i, $i \in \mathcal{M}$, is nonconvex. Otherwise, we call it *convex*. Convexity of all functions implies convexity of the feasible set $\mathcal{X}$, but not vice versa. Hence, Problem (2.1) can have a convex feasible region $\mathcal{X}$, even though some functions g_i are nonconvex, see, for example, Maugis (1977), Collins et al. (1978), and Humpola (2014), where this phenomenon occurs in energy-related problems.

We now state an example of a nonconvex MINLP.

Example 2.1.1. *Consider the following problem*

$$\begin{aligned}
\min \quad & -x - y, \\
s.t. \quad & \frac{1}{10} x|x| - y \leq 0, \\
& \frac{7}{10} \sin(x^2) + 1 - y \geq 0, \\
& -x + 2y - \frac{3}{2} \leq 0, \\
& (x, y) \in [-3, 3] \times [-1, 2], \\
& x, y \in \mathbb{Z}.
\end{aligned} \tag{2.2}$$

The problem contains two nonconvex constraints, one linear constraint, and two integer variables x and y. Altogether, the feasible region of Problem (2.2) *is given by the points* $\mathcal{X} = \{(-1, 0), (0, 0), (1, 1), (3, 1)\}$, *where the point* $(3, 1)$ *is the optimal solution, see Figure 2.1.*

2.1.1 Algorithmic concepts for solving MINLPs to global optimality

A key ingredient for solving general MINLPs is the concept of relaxations. Formally, we call a set $\mathcal{X}_{\text{rel}} \subseteq \mathbb{R}^n$ a relaxation of Problem (2.1), if $\mathcal{X} \subseteq \mathcal{X}_{\text{rel}}$ holds, and denote the optimal solution value of the relaxation as $c^*_{\text{rel}} := \min\{c^T \,|\, x \in \mathcal{X}_{\text{rel}}\}$. By construction, relaxations maintain feasible points of the original problem. Moreover, the optimal solution of the relaxation holds $c^*_{\text{rel}} \leq c^*$. In this way, the relaxation provides a lower bound c^*_{rel} of the optimal solution c^*, which is called a *dual bound* of the original problem. Together with an upper bound on the optimal solution, so-called *primal bound*, the dual bound can be used to determine a gap that measures their quality. Besides, it enables proving that an optimal solution has indeed been found. However, in some cases, a dual bound can even be gained from a relaxation that accounts for a different objective function, such as the *Lagrangian Relaxation*, where the objective function additionally penalizes constraints, see Shapiro (1979).

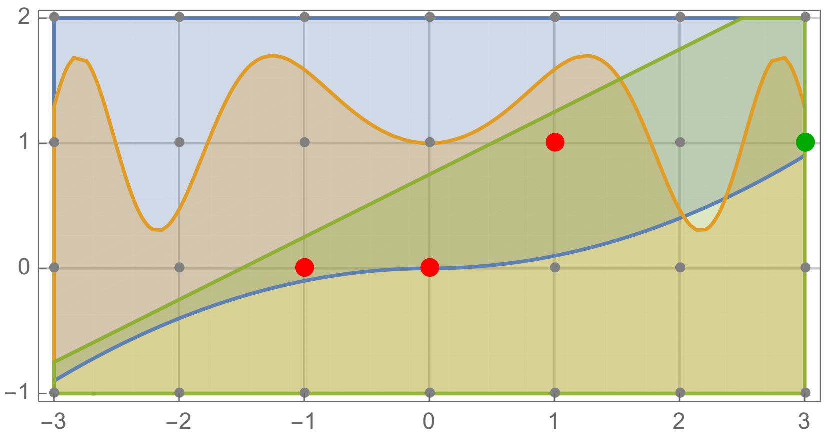


Figure 2.1: Illustration of the nonconvex MINLP from Example (2.2). The feasible region corresponds to all integer points that are contained in the intersection of the green, orange, and blue regions, which are determined by the three inequalities. The green point $(x, y) = (3, 1)$ is the optimal solution, whereas the remaining feasible solutions are highlighted in green.

Convex MINLPs. Nowadays, MIPs and convex NLPs are considered as optimization problems that can be solved efficiently, see Vigerske (2012). Thus, it is not surprising that different solution approaches decompose the original convex MINLP into MIP and NLP sub-problems. These sub-problems are constructed as relaxations, which can then be used to calculate dual bounds for the original MINLP. In particular, the following two relaxations play a significant role in convex MINLPs (i) the continuous relaxation (called NLP relaxation), and (ii) a linear relaxation obtained by substituting the nonlinearities of the problem for valid linear inequalities. Typically, the linear inequalities are derived from the calculation of *gradient cuts* to the nonlinear constraint functions.

In the first case, the NLP relaxation drops the integrality requirement and thus transforms Problem (2.1) to

$$\min \left\{ c^T \mid g_i(x) \le 0 \text{ for all } i \in \mathcal{M},\ x_j \in [\underline{x}_j, \overline{x}_j] \text{ for all } \mathcal{N} \right\}.$$

Given that the feasible region of this NLP relaxation is convex, a locally optimal solution c^*_{rel} is also globally optimal for this relaxation and hence corresponds to a valid dual bound of the original MINLP. Note that such NLP relaxation also applies to the case, when some integer variables are fixed to integral values. The resulting objective value underestimates the optimal objective value of the original problem. Nevertheless, the solution might violate some of the integrality conditions, and thus, require branching on a violated integer variable, as done in *NLP-based branch-and-bound*, see, Gupta and Ravindran (1985).

In the second case, a linear relaxation to the convex MINLP is generated. To this end, it can be exploited that for any point $x_0 \in [\underline{x}, \overline{x}]$, the following inequality holds

$$g_i(x_0) + \nabla g_i(x_0)(x - x_0)^T \le g_i(x),$$

for all $x \in [\underline{x}, \overline{x}]$, where $\nabla g_i(x_0)$ denotes the gradient of the differentiable function g_i at x_0. Note that for non-differentiable constraint functions g_i, subgradients can be used. This gives rise to generate the following *gradient cut*

$$g_i(x_0) + \nabla g_i(x_0)(x - x_0)^T \leq 0.$$

The gradient cut separates any point $x_0 \in [\underline{x}, \overline{x}]$ that violates the original constraint $g_i(x) \leq 0$, i.e., which holds $g_i(x_0) > 0$.

The notion of NLP and MIP relaxations has stimulated several separation algorithms that iteratively solve and refine the relaxations to obtain global optimally solutions. Numerous approaches exist that embed these relaxations into a branch-and-bound search, see, for example, *generalized Benders algorithm* by Geoffrion (1972), *outer approximation* by Duran and Grossmann (1986), *hybrid LP/NLP-based branch-and-bound* by Quesada and Grossmann (1992), Bonami et al. (2008), Abhishek et al. (2010), *extended cutting plane method* by Westerlund and Pettersson (1995) and *extended supporting hyperplane method* by Kronqvist et al. (2016). For a detailed overview of all these methods we refer to Vigerske (2012).

Nonconvex MINLPs. Here, some constraint functions g_i are nonconvex, and thus, the relaxations presented for convex MINLPs may not be helpful or may not even be valid relaxations when solving nonconvex MINLPs. For example, in the context of nonconvex MINLPs, the continuous relaxation is a nonconvex NLP, which is theoretically as difficult to solve as the original MINLP. Furthermore, applying the presented gradient cuts to nonconvex constraint functions is, in general, not possible as they may cut off valid points, see Figure 2.2. For this reason, solving MINLPs is more challenging, when constraint functions are nonconvex.

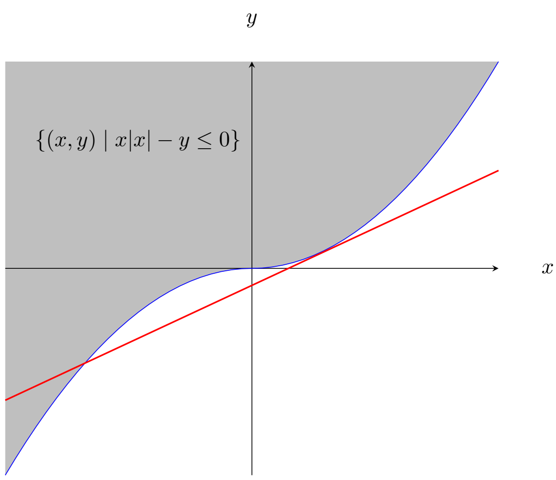


Figure 2.2: Example of an invalid gradient cut to a nonconvex constraint function.

In order to compute valid dual bounds for nonconvex MINLPs, convexification methods are needed yielding valid convex relaxations. Here, *convexification* refers to relaxing the probably nonconvex feasible region $\mathcal{X}$ to a convex set $\mathcal{X}_{\text{rel}}$, i.e., $\mathcal{X} \subseteq \mathcal{X}_{\text{rel}}$. Preferably, the convexification method yields the tightest possible convex set, which is given by the convex hull $\mathcal{X}_{\text{rel}} := \text{conv}\mathcal{X}$. However, it is rarely available. Hence, most solvers rely on a convex relaxation, which is obtained by underestimating each particular nonlinear constraint function g_i separately. In the case that nonlinear constraint functions appear as equality constraints, then the equalities are reformulated as pairs of inequalities, as explained above, for which finally convex underestimators are calculated. The following definition formalizes *convex underestimators*.

Definition 2.1.2. *Let $g_i\colon [\underline{x}, \overline{x}] \subseteq \mathbb{R} \to \mathbb{R}$ be a constraint function. We call $x \mapsto g_i^{conv}(x)$ a convex underestimator of g_i, if $g_i^{conv}(x) \leq g_i(x)$ holds for all $x \in [\underline{x}, \overline{x}]$. Accordingly, we call $x \mapsto g_i^{cave}(x)$ a concave overestimator of g_i, if $g_i(x) \leq g_i^{cave}(x)$ holds for all $x \in [\underline{x}, \overline{x}]$.*

These underestimators can be used to derive a convex relaxation of the nonconvex Problem (2.1). To this end, we need the following results about convex sets from Rockafellar (1970).

Lemma 2.1.3 (Rockafellar (1970), Corollary 4.6.1)**.** *Let $D \subseteq \mathbb{R}^n$ and $g\colon D \subseteq \mathbb{R}^n \to \mathbb{R}$ be convex. Then, the set*

$$\{x \in D \mid g(x) \leq \alpha\}$$

is convex for any $\alpha \in \mathbb{R}$.

Lemma 2.1.4 (Rockafellar (1970), Theorem 2.1)**.** *The intersection of an arbitrary collection of convex sets is convex.*

From Lemma 2.1.3 follows that the sets $\{x \in [\underline{x}, \overline{x}] \mid g_i^{conv}(x) \leq 0\}_{i \in \mathcal{M}}$ are convex, where g_i^{conv} are the convex underestimators of g_i. Since the intersection of convex sets is convex, see Lemma 2.1.4, a convex relaxation $\mathcal{X}_{rel}$ of Problem (2.1) is given by

$$\begin{aligned} \mathcal{X}_{rel} = \cap_{i \in \mathcal{M}} & \{x \in [\underline{x}, \overline{x}] \mid g_i^{conv}(x) \leq 0\} \\ = & \{x \in [\underline{x}, \overline{x}] \mid g_i^{conv}(x) \leq 0 \text{ for all } i \in \mathcal{M}\}. \end{aligned}$$

The best convex underestimator of a nonconvex function $g\colon D \subseteq \mathbb{R}^n \to \mathbb{R}$ over the domain $D = [\underline{x}, \overline{x}]$ is given by the *convex envelope* $x \mapsto \text{vex}_D[g](x)$. We refer to Chapter 4 for more information and how to utilize the convex envelope of a particular nonlinear constraint function in the solution process of an MINLP solver.

Even though convex envelopes are explicitly known for particular functions or classes of functions (Horst and Tuy, 1990; Tawarmalani and Sahinidis, 2002b), algebraic descriptions of the convex envelopes for general functions are typically not at hand. Hence, state-of-the-art MINLP solvers, in general, require factorable constraint functions g_i, which enable deriving a convex relaxation if convex underestimators are known for the "simpler" univariate functions, see e.g., Vigerske (2012). Factorable and separable functions allow for a reformulation by recursive sums and products of univariate functions, possibly introducing additional variables, see McCormick (1976). Exemplary, let us consider the nonlinear function $g(x, y) := \frac{1}{10}x|x| - y$ from Example 2.2. The constraint $g(x, y) \leq 0$ can equivalently be reformulated by using two auxiliary variables w_1 and w_2,

$$\begin{aligned} w_1 &= \frac{1}{10}x|x|, \\ w_2 &= w_1 - y, \\ w_2 &\leq 0, \end{aligned}$$

where the resulting constraint functions $h_1(x) := \frac{1}{10}x|x|$ and $h_2(y, w_1) = w_1 - y$ are assumed to be simple in the sense that the solver is equipped with tools that enable generating convex underestimators to those functions. Then, $(x, y) \mapsto h_2^{conv}(h_1^{conv}(x), y)$ is a convex underestimator of g when projected onto the (x, y)-space. Let us point out that the reformulation adds equality constraints in this example, which also requires to compute concave overestimators for the reformulated nonlinear functions, i.e., $(x, y) \mapsto h_2^{cave}(h_1^{cave}(x), y)$.

Apart from enabling the derivation of convex underestimators and concave overestimators automatically, the benefit of factorable functions is that they can efficiently be stored and evaluated in a so-called acyclic *expression graph* of the problem, where the sources represent the variables and constants of the constraint functions, and the sinks represent the functions themselves, see, e.g., Schichl and Neumaier (2005).

Using the concept of underestimators, the following optimization problem is a convex relaxation of Problem (2.1)

$$\begin{aligned} \min \quad & c^T x, \\ \text{s.t.} \quad & g_i^{conv}(x) \leq 0 && \forall i \in \mathcal{M}, \\ & x_j \in [\underline{x}_j, \overline{x}_j] && \forall j \in \mathcal{N}. \end{aligned} \tag{2.3}$$

In general, (2.3) is a nonlinear relaxation as long as some g_i^{conv} are nonlinear. In order to generate a linear relaxation (*LP-relaxation*), the nonlinear underestimators are replaced by gradient cuts. Nowadays, the LP-relaxation can be solved very efficiently, for example, by using warm starts and is mostly numerical stable. Most likely, the optimal solution of (2.3) is not feasible for the original Problem (2.1), where infeasibility comes in two shades: First, the solution might violate the integrality conditions, and second, the solution might violate some of the nonconvex nonlinear constraints. The latter case is due to the gap between the convex underestimator of the nonconvex nonlinear constraint function and the function itself. In both cases, a possible method to eliminate this infeasible solution is to resort to branching. In the first cases, let x^* be an optimal solution of Problem (2.3), and let x_i be an integer variable that has fractional solution value x_i^*. Then, the domain of x_i is partitioned into two subproblems with integer variable bounds $\underline{x}_i \leq x_i \leq \lfloor x_i^* \rfloor$ and $\lceil x_i^* \rceil \leq x_i \leq \overline{x}_i$, and a branch-and-bound scheme can be established, as first done by Land and Doig (1960). In the second case, the optimal solution x^* of Problem (2.3) violates some of the nonlinear and nonconvex constraints. Resolving such infeasibility might require to partition the domain of a variable x_j that occurs in a violated nonlinear constraint. For the relaxation value x_j^*, the partitioning then yields

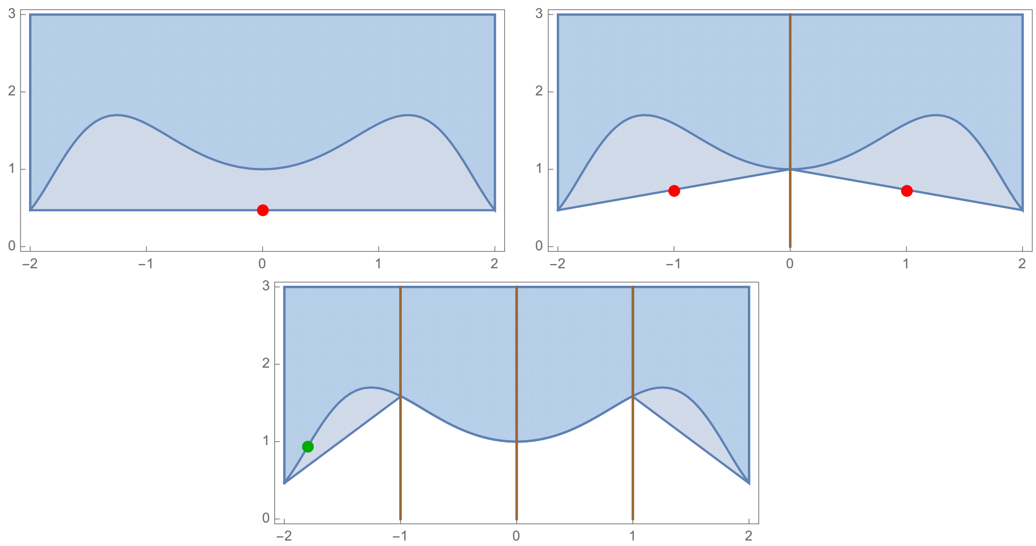


Figure 2.3: Illustration of spatial branch-and-bound, which intends to establish the inequality $\frac{7}{10}\sin(x^2) + 1 - y \geq 0$ on domain $[-2, 2] \times [0, 3]$ as part of some MINLP model. The feasible region is shown in blue, whereas light blue depicts the feasible region of the convexified regions, given by the convex envelope. Red points in all sub-figures represent the optimal solution of the relaxed subproblems. In the upper left figure, the optimal solution is not feasible for the original constraint and evokes branching on $x \leq 0 \vee x \geq 0$ yielding two subproblems in the upper right figure. The optimal solutions of the relaxations in both subproblems (red) are still not feasible, yielding four subproblems with smaller convexification gaps in the lower figure. Here, further spatial branching is required until the relaxations establish the optimal solution (green) of the original inequality.

$\underline{x} \leq x \leq x^*$ and $x^* \leq x \leq \overline{x}$. This process refers to as *spatial branching*. However, in contrast to branching on integer variables, spatial branching might not cut off the continuous solution x_j^*. However, due to the tighter variable bounds in the subproblems, tighter underestimator are obtained that refine the relaxations. Then, spatial branching is applied until it is possible to cut off x^*. Here, one of the key ingredients relies on the strong interdependency between the tightness of bounds and relaxations. More precisely, the tighter the bounds, the tighter the relaxations. Figure 2.3 shows an example of spatial branching.

Incorporating the concept of spatial branching together with the refinement of convex under- and concave overestimators into a branch-and-bound framework is called spatial branch-and-bound. The first versions of spatial branch-and-bound were introduced by Falk and Soland (1969) and Soland (1971) for *separable* nonconvex NLPs, followed by McCormick (1976), who developed the first global optimization algorithm for general *nonseparable* nonconvex NLPs. McCormick (1976) established an algorithm that first identifies those variables from the solution of a relaxation, which appear in violated constraints. Afterwards, branching on these variables enables recursively partitioning the feasible region. The underlying methodology of this algorithm is still the basis of how current state-of-the-art MINLP solvers handle nonconvex nonlinearities. The following global state-of-the-art solvers for nonconvex MINLPs use spatial branch-and-bound: For example, SCIP (Vigerske and Gleixner, 2018), BARON (Sahinidis, 1996), Couenne (Belotti et al., 2009) and Gurobi (Gurobi Optimization, LLC, 2020) apply spatial branch-and-bound in combination with an LP-relaxation, and ANTIGONE (Misener and Floudas, 2014) in combination with a MILP-relaxation.

Besides spatial branch-and-bound, further techniques exist to improve the performance of the solver. As for MIP solvers, essential ingredients are, for example, branching decisions on the selection of the subproblems. Moreover, domain propagation helps to tighten the bounds, which in turn strengthens the applied relaxations. The relaxations can further be tightened by generating valid (linear) inequalities that cut off the optimal solutions of the relaxations while retaining the feasible region of the original MINLP. Such constraints can, for example, be learned using conflict analysis when subproblems of the branch-and-bound tree are detected as infeasible. Likewise, the application of primal heuristics is crucial to improve the upper bound and to reduce the search space. For an overview of current (non)convex MINLP solvers and more detailed information about their major components, we refer to Vigerske (2012).

2.2 Experimental Methodology

In this thesis, we analyze the performance of our developed models and algorithms on large practically relevant test sets. In the following, we briefly describe the hardware, performance evaluation, and statistical testing used for our experiments.

2.2.1 Hardware

All experiments in this thesis were conducted on a cluster of 64-bit Intel Xeon CPU E5-2670 v2 CPUs at 2.5 GHz with 25 MB cache and 128 GB main memory. In order to safeguard against a potential mutual slowdown of parallel processes, we bind the processes to specific cores and run at most one job per node at a time. Only for the experiments in Chapter 3, where the test set contains a vast amount of instances, which is far beyond the size of typically used test sets, resulting in 57,000 single runs, we bind the processes to specific cores with at most four jobs per node. More details about the software and their versions used in the experiments can be found in the description of the computational studies in the particular chapters.

2.2.2 Performance measures

Besides measuring the number of solved instances, we evaluate the solver performance using the *running time* and *number of branch-and-bound nodes*. In all experiments, we test on benchmark sets that comprise a large number of instances. In order to obtain a single performance number across the individual instances of test sets, we average the performance results by using, among others, the classical *arithmetic mean* (amm)

$$\frac{1}{n}\sum_{i=1}^{n} t_i,$$

with values $t_1, ..., t_n \geq 0$. While the arithmetic mean is a suitable measure in some cases, for example, to indicate the running time required for applications in practice, it might be dominated by outliers with large absolute values. This effect can be reduced by using the *shifted geometric mean* (sgm), which provides a measure for the relative difference

$$\left(\prod_{i=1}^{n}(t_i + s)\right)^{1/n} - s, \tag{2.4}$$

with a shift value $s \geq 0$. While a shift value $s = 0$ (*geometric mean*) already limits the dominance of hard instances with large running times, a positive shift value avoids an over-representation of easy instances with very short running times. Hence, the *shifted geometric mean* averages outliers in both directions by reducing the impact of very easy and hard instances. In our experiments, we use a shift value of $s = 10$ seconds for running time and a shift value of $s = 100$ for branch-and-bound nodes. For further information, we refer to Achterberg and Wunderling (2013), Achterberg (2007), and Hendel (2014).

2.2.3 Statistical significance testing

In this thesis, we use statistical significance testing as an additional tool for performance measuring in order to analyze whether changes in the performance can be considered as consistent across a test set. Here, we apply statistical significance testing in the computational experiments in Chapter 3 when solving different, but equivalent model formulations, and in Chapter 4 for comparing the impact of different features on the solver performance.

In the following, we briefly describe this test. For more information about testing the statistical significance on benchmark results of MIP solvers, we refer to Hendel (2014). Let the results of two different approaches on a set of instances $\{1, ..., n\}$ be given by the sets of samples $X = (X_1, ..., X_n)$ and $Y = (Y_1, ..., Y_n)$. Both sets have the same size and are paired (X_i, Y_i) for $1 \leq i \leq n$, which means that X_i and Y_i describe the performance value of instance i for the first and second approach, respectively. For statistical tests employed in this thesis, we use the shifted geometric mean (2.4) as a performance measure. In fact, using only mean values as performance measures might lead to misinterpreted results. For example, a difference in mean values of both samples can have various reasons: First, it might arise from a huge improvement on a few instances, while the performance deteriorates only slightly on most of the other instances, or second, it might be due to a consistent improvement across the entire test set. Statistical significance testing helps to find out whether there is a significant difference between the averaged values over the results X and Y. Here, we apply a nonparametric *Wilcoxon signed-rank test*, because there is no reason to assume that the samples follow a normal probability distribution. Provided that the samples denote the total solving time in seconds, they can vary by orders of magnitude for individual instances across a test set. Therefore, we start by calculating a relative difference of sample pairs for each instance

$$Q_i = \log\left(\frac{X_i + s}{Y_i + s}\right),$$

with s being the shift value as defined in (2.4). Here, Q_i is well-defined, having non-negative values $X_i, Y_i, s \geq 0$. Negative values $Q_i < 0$ occur if $X_i < Y_i$ and indicate a speedup of X_i over Y_i, and accordingly, positive values $Q_i > 0$ indicate a slowdown of X_i. Let n_r be the remaining sample size when removing all occurrences of $Q_i = 0$. Subsequently, we sort the n_r-many instances in ascending order and rank them by their absolute value $|Q_1| \leq ... \leq |Q_{n_r}|$, where we resolve ties in the ranks, by assigning the average rank to each sample affected. Then, we compute the positive and negative rank sums

$$W^+ = \sum_{i:Q_i>0} i \qquad \text{and} \qquad W^- = \sum_{i:Q_i<0} i.$$

As n_r increases, the difference $W^+ - W^-$ converges to a normal distribution, having the variance $\sigma^2_{W^+-W^-} = n_r(n_r+1)(2n_r+1)/6$, see Wilcoxon (1992). This allows computing a z-score

$$z = \frac{|W^+ - W^-|}{\sigma_{W^+-W^-}}.$$

In this thesis, we test against the null hypothesis, which says that the underlying probability distributions of X and Y are equal and thus have the same mean values (one-sided test). In our context, the null hypothesis translates to the assumption that the approach, which yields better performance is actually not better. Then, the z-score can be used to derive the so-called p-value, which describes the (approximate) probability that the null hypothesis holds. Small p-values indicate that the hypothesis was wrong, and we may assert that the difference of the performance values in the shifted geometric mean is indeed significant. Here, we use a p-value of 1% for the significance testing. As an implementation, we use the *Wilcoxon signed-rank* test provided in the `SciPy` package (Jones et al., 2001; Virtanen et al., 2020) with the parameters `correction=False` and `zero_method='wilcox'`.

Part I

Optimizing Stationary Network Expansions

Chapter 3

Stationary Expansion Planning of Potential Driven Networks

3.1 Introduction

Operators of commodity transport networks such as natural gas, hydrogen and water networks regularly have to face both increasing demand and the need to handle more diverse transport situations. To deal with these challenges without having to resort to the expensive options of setting up new pipeline corridors or of demolishing pipelines and replacing them by larger ones, they must expand the capacity of the existing network. Without using compressor stations to increase the pressure in the pipelines, this can only be achieved by building pipelines parallel to existing ones, a process that is referred to as "looping" in the terminology of the industry. Due to the high costs involved, there has long been research about finding the cost minimal way of looping an existing network, i.e., of determining both the pipelines that are to be looped and the diameters of the pipelines to be built.

Commodity flows such as flows in gas and water networks arise from friction-induced potential differences between the nodes in the network, which leads to a type of nonlinear network model that is referred to as potential-driven network in the literature (Birkhoff and Diaz, 1956; Raghunathan, 2013; Robinius et al., 2019). Apart from the nonlinear character of potential-driven networks, a specific difficulty in finding optimal loops arises from the fact that the diameters of the pipelines typically have to be selected from a discrete set of commercially available diameters (André et al., 2009; Hansen et al., 1991; Fasold, 1999), which additionally imparts to the problem a combinatorial flavor. For these reasons, the problem of capacity expansion in potential-driven networks belongs to the family of mixed-integer nonlinear programming (MINLP) problems.

As a consequence, a large body of the literature on the topic is concerned with developing sophisticated special-purpose algorithms geared at finding locally optimal solutions or approximating a global optimum, partly based on novel ways of modeling the capacity expansion problem. While this research focus has certainly advanced our ability to solve this problem, it has also led to a variety of mathematical models for the looping problem that remain unconnected in the literature. In view of the recent advances in the development of general-purpose solvers for NLP and MINLP problems that allow for an efficient implementation of models that is significantly faster than implementing one of the special-purpose algorithms developed, it seems to be useful for practitioners involved in the design of networks to shift the research focus to the modeling stage. For this reason, the present chapter discusses various models for finding optimal loops in potential-driven networks.

In particular, it brings together, for the first time, the diversity of existing models in the literature and compares these, both theoretically and experimentally. For this comparison, we choose a comprehensive approach that includes two different, rather unconnected strands of research on the problem that are referred to as the discrete approach and the split-pipe approach (see below), and also takes into account literature that does not explicitly focus on the looping problem, but addresses closely related problems. Moreover, this chapter also introduces two novel models for the looping problem, one of which has turned out to outperform the existing models in the literature with respect to computational time, the number of solutions found, the number of instances solved and cost savings. Finally, in discussing these models, we go beyond the existing literature by including, the practically highly relevant case that a particular pipeline may be looped several times.

In line with our aim to bring together the diversity of existing approaches in the literature, the remainder of this section discusses the literature on the topic rather comprehensively before presenting an outline of the structure of this chapter.

During the past 40 years of research on capacity problems in potential-driven networks, three different modeling approaches have been considered for the looping problem (cf. Shiono and Suzuki, 2016): (i) a direct approach where the optimal diameters for the loops are chosen from the set of commercially available diameters (discrete approach), (ii) a continuous approach where continuous diameters are typically used to approximate the problem and (iii) an extended approach where the entire length of the pipeline to be looped is split into several segments of variable lengths each of which may have its own diameter from the discrete set of available diameters (split-pipe approach).

In the following discussion of the literature about these three approaches, we will not only look at papers addressing the looping problem (in fact, to the best of our knowledge, there are so far, apart from this thesis, only two papers, namely André et al. (2009) and Pietrasz et al. (2008) that solely focus on the looping problem), but will also consider papers about two closely related problems: the *network design problem*, which looks for the optimal diameters of pipelines between given unconnected nodes or determines both the location of nodes and the optimal pipelines between these, and the *network expansion problem*, which is about the optimal placement of new network elements of different types (such as pipelines, compressors and valves) at predefined (previously connected or unconnected) locations in the network.

(i) Discrete diameters. One of the first solution approaches using mathematical optimization techniques for this $\mathcal{NP}$-hard problem (Yates et al., 1984) has been proposed by Jacoby (1968). The author solves a nonlinear model using a gradient approximation method where the resulting continuous diameters are finally rounded to the nearest discrete-valued pipe diameters. The approach was tested on a small water network that contains seven pipelines and two cycles. Other early work where the discrete approach is used for networks with a simple structure include Liang (1971), who solved a gunbarrel system using dynamic programming, and Rothfarb et al. (1970), who used a serial and parallel merge algorithm to design tree shaped networks. Later, Gessler (1985) applied selective enumeration techniques to tackle a small sized network with two cycles. In the 1990s, a class of approaches was developed that relies on meta-heuristics, such as genetic algorithms, see, e.g., (Simpson et al., 1994; Savic and Walters, 1997) for water networks, and (Boyd et al., 1994; Castillo and González, 1998) for gas networks.

In the past decade, different papers applied an MINLP formulation to determine discrete pipe sizes, e.g., André et al. (2009), Bragalli et al. (2012) and Robinius et al. (2019). While André et al. (2009) solve the problem heuristically in two stages, where a first step identifies pipes to be looped by solving the continuous relaxation and a second step determines discrete-valued diameters for these selected pipes using branch-and-bound, Bragalli et al. (2012) solve the model directly with an MINLP solver using a continuous reformulation of the cost function. Robinius et al. (2019) determine discrete arc sizes in the context of tree shaped networks, which allows flows on arcs to be fixed and thus simplifying the MINLP model to an Mixed-Integer Programming model.

The nonlinear nature of the problem has led to a number of different MINLP models and approaches: Raghunathan (2013) presents a disjunctive program together with a convex relaxation, which is then solved to global optimality and Borraz-Sánchez et al. (2016) propose a new solution approach by presenting a new model together with a second-order cone relaxation. Humpola (2014) formulates a model that is solved to global optimality using convex reformulation techniques, special tailored cuts.

In this thesis, we investigate different existing MINLP approaches and propose a new model for the discrete looping problem.

(ii) Continuous diameters have been considered e.g., by Bhaskaran and Salzborn (1979b), Rowell and Barnes (1982), Bhave (1985), De Wolf and Smeers (1996), De Wolf and Bakhouya (2012), Babonneau et al. (2012) and André et al. (2013). Continuous diameters typically have been used in the literature to approximate the discrete diameters that are commercially available. Hansen et al. (1991), for example, use successive linear programming with a trust region strategy, where the algorithm adjusts the continuous diameters in each iteration to elements in the set of available discrete diameters. Osiadacz and Górecki (1995) apply sequential quadratic programming to the continuous relaxation of a gas network design problem and round the solution to the closest available diameter size. Shiono and Suzuki (2016) introduce an analytic approach that calculates the optimal diameter costs for the pipe-sizing problem of a tree-shaped gas network with continuous diameters, which are then heuristically converted to discrete pipe diameters. As we are concerned with models that lead to exact solutions, we will not further consider this line of research here.

(iii) Split-pipe approach. This approach, where the pipes can be split into several segments with different diameters each, combines features of both the discrete and the continuous approaches: while the diameters are chosen from the discrete set of available diameters, the option to split pipes at arbitrary points into sections of different diameters leads, as we will see in the next section, to a situation that is equivalent to choosing diameters from a continuous set. This concept was first used in designing networks with a tree structure, which allows the flows in the pipelines to be treated as constants and leads to a linear programming model (e.g., Karmeli et al. (1968) and Gupta et al. (1972) for water networks, and, independently, Bhaskaran and Salzborn (1979a) for gas networks). The first paper to attempt the split-pipe version of the capacity expansion problem as a nonlinear problem for general networks was Alperovits and Shamir (1977), who introduced the linear programming gradient (LPG) method, a two-stage heuristic that alternates between solving the linear program with fixed flows from Karmeli et al. (1968) for obtaining the pipeline diameters and modifying these flows on the basis of a sensitivity analysis of the solution of the first stage. This idea stimulated a number of subsequent papers (e.g., Quindry et al. (1981), Fujiwara and Khang (1990) and Kessler and Shamir (1989)) that improved on the method. Starting with Eiger et al. (1994), a strand of genuinely NLP-based global solution methods emerged, with further contributions by Zhang and Zhu (1996) and Sherali et al. (2001), for example. Surprisingly, despite 50 years of research on the split-pipe approach, all these contributions proceed from the very same basic model that goes back to the linear model by Karmeli et al. (1968). For this reason, we present a novel, alternative model for the split-pipe approach and compare it to the standard split-pipe model in the literature.

We have now sketched the history of research on the three different approaches to our looping problem and closely related problems. In the light of the variety of models and optimization methods for these problems, it is astonishing that there is nearly no research that brings together the discrete and the split-pipe approaches and compares the models proposed in the literature. In fact, to the best of our knowledge, the very early paper by Bhaskaran and Salzborn (1979a) with its linear model for a tree-shaped network is the only paper to compare the split-pipe approach with the discrete approach, albeit for very small trees with 9 and 14 nodes. Here we address this gap and present comparisons of the models discussed in the literature.

Another important aspect of the looping problem that has not been addressed in the literature is multiple looping, where each pipeline may be reinforced with several parallel pipelines of different diameters. This is a practically highly relevant problem as multiple looping can first replace large diameters that are commercially not available; may second lead to cost savings by substituting several parallel pipelines with smaller diameters for one pipeline with a large diameter; may third, as we will see in Section 3.2.2, allow for pipe characteristics that cannot be realized with single loops; and can finally provide a tool for strategic planning where several stages of successively looping a given network are to be considered. For these reasons, we take into account multiple looping throughout this chapter.

The remainder of the chapter is organized as follows: In the next section, we formalize the looping problem in potential-driven networks, show some of its basic properties and explain our approach of dealing with multiple loops. In Section 3.3, we present a new model for the discrete looping problem and contrast it with the existing models in the literature. The subsequent Section 3.4 turns to the split-pipe approach of the looping problem. Again we present a new model and address its relationship to the standard split-pipe model from the literature. Moreover, we discuss the way in which the feasible regions of all models presented so far are related to each other. In Section 3.5, we show theoretical expansion planning aspects, such as a convexity analysis, conditions that guarantee feasible loop expansions, Braess's paradoxon, and symmetries in split-pipe segments. In Section 3.6, we carry out extensive computational experiments on instances of both natural gas and water networks that allow us to compare the performance of all models and give recommendations regarding their use. The chapter ends with some concluding remarks in Section 3.7.

3.2 The Expansion Planning Problem

Let us begin by formally defining the planning problem.

3.2.1 Problem statement

Let $\mathcal{G} = (\mathcal{V}, \mathcal{A}, r)$ be a directed multigraph with node set $\mathcal{V}$, arc set $\mathcal{A}$ and a function $r\colon \mathcal{A} \to \mathcal{V} \times \mathcal{V}$ that maps each arc to its end points. The nodes can be partitioned into supply nodes (sources) $\mathcal{V}^+ \subseteq \mathcal{V}$, consumption nodes (sinks) $\mathcal{V}^- \subseteq \mathcal{V}$ and transshipment nodes $\mathcal{V}^* = \mathcal{V} \setminus (\mathcal{V}^+ \cup \mathcal{V}^-)$. We restrict to passive and connected networks, where the only arc type are pipes that are needed to transport the commodities. We are given a demand vector $b \in \mathbb{R}^{|\mathcal{V}|}$ where $b_v > 0$ denotes injection into the network at source $v \in \mathcal{V}$ and $b_v < 0$ withdrawal at sink $v \in \mathcal{V}$. Since we work with stationary and isothermal models, the demand vector is balanced, i.e., $\sum_{v\in\mathcal{V}} b_v = 0$. With each arc $a \in \mathcal{A}$, flow variables $x_a \in [\underline{x}_a, \overline{x}_a]$ are associated. Positive values of x_a indicate flow along the arc $a = (v, w)$ from v to w, whereas negative values indicate flow in the reversed direction. As in classical network flow problems, flow conservation is required at every node, i.e., $\sum_{a\in\delta^+(v)} x_a - \sum_{a\in\delta^-(v)} x_a = b_v$ with $\delta^+(v) := \{a \in \mathcal{A} \mid \exists\, w \in \mathcal{V} \text{ with } a = (v, w)\}$ and $\delta^-(v) := \{a \in \mathcal{A} \mid \exists\, w \in \mathcal{V} \text{ with } a = (w, v)\}$. In potential-based networks, the physical state is additionally described by nonnegative potential variables $\pi_v \in [\underline{\pi}_v, \overline{\pi}_v]$ at each node $v \in \mathcal{V}$.

In applications such as water transport problems, the nodal potentials π_v correspond to hydraulic heads, i.e., the sum of the elevation head, velocity head and pressure head (Walski et al., 2001), while they represent squared pressure values in gas transport problems (Koch et al., 2015).

The flow along a pipe $a = (v, w)$ depends on the potential difference at its adjacent nodes, the pipe length $L_a > 0$, the diameter $d_a > 0$, and a physical parameter $R_a > 0$ representing phenomena such as friction, gas temperature and density. The potential difference

$$\pi_v - \pi_w = \Phi(d_a, x_a) \qquad \forall a \in \mathcal{A} \text{ and } (v, w) = r(a), \tag{3.1}$$

is given by a function $\Phi(d_a, .) : \mathbb{R} \to \mathbb{R}$ that is strictly increasing, $\Phi(d_a, .) \in \mathcal{C}^1$ and antisymmetrical, i.e., $\Phi(d_a, -x_a) = -\Phi(d_a, x_a)$, and typically takes the form

$$\Phi(d_a, x_a) = \frac{L_a R_a}{d_a^\beta} \operatorname{sgn}(x_a)|x_a|^\alpha \tag{3.2}$$

with $\alpha, \beta > 0$. In line with most authors in the literature, we model R_a as a constant and assume the pipes to have zero slope, i.e., there is no influence of gravity on the potential drop $\pi_v - \pi_w$ (e.g., Zhang and Zhu, 1996; André et al., 2009; Babonneau et al., 2012). The value of α depends on the commodity and the type of approximation used for modeling the commodity flow. In water transport problems, the *potential loss function* (3.2) is typically given by the equations of Darcy-Weisbach with $\alpha = 2$ or Hazen-Williams with $\alpha = 1.852$ (Walski et al., 2001), whereas in gas transport problems Equation (3.2) takes the shape of the Weymouth equation with $\alpha = 2$ (Weymouth (1912)). In the approximations proposed by Darcy-Weisbach and Weymouth, the exponent of the diameter is $\beta = 5$, while it is $\beta = 4.87$ in the case of the approximation by Hazen-Williams.

The solution of the capacity expansion problem involves two decisions: (a) which pipelines $a \in \mathcal{A}$ should be looped? and (b) what pipeline diameters d_a should be used for the loops? As our goal is to solve the problem to global optimality and any pre-selection of certain pipes as looping candidates would be a heuristic procedure, we allow all pipes to be looped.

For the diameters d_a, we have in the discrete case $d_a \in \mathcal{D}_a := \{D_{a,0}, D_{a,1}, \dots, D_{a,k_a}\}$, where $D_{a,0}$ refers to the diameter of the already existing pipe and D_{a,k_a} denotes the maximal possible diameter when looping. In the continuous case the domain of the diameter is given by $d_a \in \mathcal{D}_a := [D_{a,0}, D_{a,k_a}]$. While we are not concerned with the continuous looping problem due to its approximative nature here, we will see in Section 3.2.2 that a continuous interval of diameters can also be interpreted as representing diameters in the split-pipe problem. Assuming a cost function $c\colon \mathcal{D}_a \to \mathbb{R}_{\geq 0}$ for the diameter candidates of the pipes to be given, the general looping problem then reads

$$
\begin{aligned}
&\min_{x,\, d,\, \pi} \sum_{a \in \mathcal{A}} c(d_a) L_a && && (3.3a)\\
\text{s.t.}\quad & \pi_v - \pi_w = \Phi(d_a, x_a) && \forall\, a \in \mathcal{A} \text{ and } (v,w) = r(a), && (3.3b)\\
& \sum_{a \in \delta^+(v)} x_a - \sum_{a \in \delta^-(v)} x_a = b_v && \forall\, v \in \mathcal{V}, && (3.3c)\\
& \underline{\pi}_v \leq \pi_v \leq \overline{\pi}_v && \forall\, v \in \mathcal{V}, && (3.3d)\\
& \underline{x}_a \leq x_a \leq \overline{x}_a && \forall\, a \in \mathcal{A}, && (3.3e)\\
& d_a \in \mathcal{D}_a && \forall\, a \in \mathcal{A}. && (3.3f)
\end{aligned}
$$

Note that even when no particular bounds $\underline{x}_a$ and $\overline{x}_a$ on the flow variables x_a are given, flow bounds are implied by the bounds on π_v and d_a by virtue of Equation (3.1). More precisely, with $D_{a,0}, D_{a,k_a}$ being the lower and upper bound of d_a, flow bounds on x_a are given by $\underline{x_a} := \operatorname{sgn}(m_a^1)\sqrt{|m_a^1|}$ and $\overline{x} := \operatorname{sgn}(m_a^2)\sqrt{|m_a^2|}$, where

$$
m_a^1 := \min \frac{1}{L_a R_a} \left\{ D_{a,0}^{\beta}(\underline{\pi}_v - \overline{\pi}_w), D_{a,0}^{\beta}(\overline{\pi}_v - \underline{\pi}_w), D_{a,k_a}^{\beta}(\underline{\pi}_v - \overline{\pi}_w), D_{a,k_a}^{\beta}(\overline{\pi}_v - \underline{\pi}_w) \right\},
$$
$$
m_a^2 := \max \frac{1}{L_a R_a} \left\{ D_{a,0}^{\beta}(\underline{\pi}_v - \overline{\pi}_w), D_{a,0}^{\beta}(\overline{\pi}_v - \underline{\pi}_w), D_{a,k_a}^{\beta}(\underline{\pi}_v - \overline{\pi}_w), D_{a,k_a}^{\beta}(\overline{\pi}_v - \underline{\pi}_w) \right\}.
$$

3.2.2 Loop diameters

After our general outline of the expansion planning problem, we now describe how the diameters used for looping pipelines are represented in the discrete and the split-pipe models to be discussed in this chapter. In doing so, we will go beyond the existing literature and allow that each pipeline $a \in \mathcal{A}$ may be looped several times. Our starting point is a finite set $\{d_1, \dots, d_n\}$ of commercially available diameters for looping. These diameters are associated with costs $c_1 < \dots < c_n$ per unit of pipeline length, see Figure 3.1a.

It is well known in the literature (see Katz (1959); André et al. (2009); André (2010) for the case of gas networks and Bragalli et al. (2012) for the case of water networks) that two parallel pipelines with diameters d_1 and d_2 can be replaced, without changing any physical properties of the network, by a single pipeline with diameter D_{d_1,d_2} (called *equivalent diameter*) by virtue of

$$D_{d_1,d_2}^{\beta/\alpha} = d_1^{\beta/\alpha} + d_2^{\beta/\alpha}. \tag{3.4}$$

For a derivation we refer to Proposition A.1.1 in the appendix.

It is easy to see that for looping an arc a of length L with existing diameter d_a multiple (k) times with (not necessarily distinct) diameters $d_{i_1}, d_{i_2}, \dots, d_{i_k} \in D$, this relationship can be extended to

$$D_{d_a,d_{i_1},\dots,d_{i_k}}^{\beta/\alpha} = d_a^{\beta/\alpha} + d_{i_1}^{\beta/\alpha} + \dots + d_{i_k}^{\beta/\alpha}, \tag{3.5}$$

where this multiple loop is associated with costs

$$\sum\nolimits_{j=1}^{k} c_{i_j} L. \tag{3.6}$$

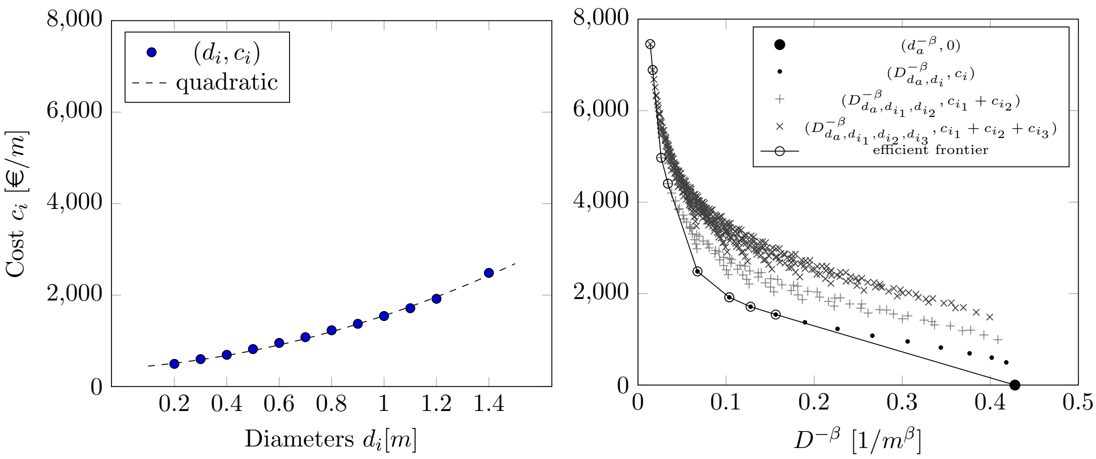
Equation (3.5) directly follows by successively applying Equation (3.4) to the different pairs of diameters. Note that $D_{d_a,d_{i_1},\dots,d_{i_k}}^{\beta/\alpha}$ is independent of the order of the different pairs of diameters, since $D_{d_a,d_{i_1},\dots,d_{i_k}}$ is commutative.

Equation (3.5) implies that when allowing for multiple loops, we cannot only choose among the discrete set of commercially available diameters, but we have at our disposal the much larger discrete set of equivalent diameters that result from all possible combinations of available loops.

A type of equivalent diameter also exists for the case of two *serial* pipelines with no source or sink in between. Assume we have two pipelines, one from node v to node w and the other one from w to z, with diameters d_1 and d_2, the same physical parameter $R := R_1 = R_2$, a total length of L from v to z, and with λL being the length of the first pipeline from v to w, where $0 < \lambda < 1$. Then, due to (3.1) and (3.2), the total potential loss along the flow x from v to z is given by

$$(\pi_v - \pi_w) + (\pi_w - \pi_z) = \frac{\lambda L R_1}{d_1^\beta} \operatorname{sgn}(x)|x|^\alpha + \frac{(1-\lambda) L R_2}{d_2^\beta} \operatorname{sgn}(x)|x|^\alpha. \tag{3.7}$$

The same potential loss can be achieved with a single pipeline of length L when its diameter D is given by $\pi_v - \pi_z = LR/D^\beta \operatorname{sgn}(x)|x|^\alpha$. Equations (3.6) and (3.7) imply that the



(a) Data points (d_i, c_i) (b) Efficient frontier

Figure 3.1: Available diameters $\{d_1, \ldots, d_n\}$ with associated cost factors (left). On the right: construction of the efficient frontier of dominating equivalent diameters. Note that the cost components of the points (d_i, c_i) on the left side correspond with those of the points $(D_{d_a,d_i}^{-\beta}, c_i)$ on the right side.

equivalent diameter satisfies $D^{-\beta} = \lambda d_1^{-\beta} + (1-\lambda) d_2^{-\beta}$. Analogously, we have for the case of k serial pipeline segments with diameters $d_1, d_2, \ldots, d_k$ and lengths $\lambda_1 L, \lambda_2 L, \ldots, \lambda_k L$, where $\sum_{j=1}^k \lambda_j = 1$ with $\lambda_j \geq 0$:

$$D^{-\beta} = \sum\nolimits_{j=1}^{k} \lambda_j d_j^{-\beta} \qquad \text{with associated costs} \qquad L \sum\nolimits_{j=1}^{k} \lambda_j c_j. \tag{3.8}$$

For practical purposes, relationship (3.8) implies that, when looping pipelines, we are not restricted to the discrete set of equivalent diameters that results from multiple loops with the commercially available diameters according to (3.5). Instead, we can decide to split a pipeline into several segments of lengths $\lambda_i L$ and have different, possibly multiple loops, i.e., loops with different equivalent diameters, on all segments. In this way, (3.8) allows us to realize in the network all diameters in the (continuous) interval between the smallest and the largest possible equivalent diameters. This is the approach that the *split-pipe models* use and it enables us to benefit, despite the limited number of commercially available pipeline diameters, from having the flexibility of a continuous set of equivalent diameters at our disposal.

We will allow a single pipe a to be looped up to j times, i.e., in the capacity expansion problem we have to pick up to j diameters for each existing pipeline of the network. With $\binom{n+k-1}{k}$ being the number of k-combinations with repetitions from the diameter set $\mathcal{D}$, we obtain $\sum_{k=0}^{j} \binom{n+k-1}{k}$ equivalent diameters for each pipe. To avoid a factorial growth of the number of variables in the models, we apply a model reduction technique to the split-pipe models that has been introduced in the literature for the case of single looping several times (e.g., Bhaskaran and Salzborn, 1979a; Fujiwara and Dey, 1987; Zhang and Zhu, 1996). We will briefly outline this method for the general case of multiple loops.

Our aim is to compare all potential equivalent diameters resulting from both parallel pipelines (multiple loops) and serial pipelines (split-pipe setting) with respect to their costs and to incorporate into the models only those equivalent diameters that are not dominated by equivalent diameters with lower costs. To this end, we place all pairs of equivalent diameters D generated according to (3.5) and their corresponding costs per unit of pipeline length c_D according to (3.6) into a coordinate system where the horizontal axis depicts the *exponentiated* equivalent diameter $D^{-\beta}$ and the vertical axis the cost c_D, i.e., we consider the points $(D^{-\beta}, c_D)$. In this coordinate system, the $D^{-\beta}$-coordinate of each convex combination of points $(D_i^{-\beta}, c_{D_i})$ represents, due to (3.8), an equivalent diameter that results from splitting a looped pipe into several sections with equivalent diameters D_i each, while the c_D-coordinate of such a convex combination indicates, also due to (3.8), the corresponding cost of such a split-pipe arrangement. This is illustrated in Figure 3.1b. It shows for an arbitrary pipeline the original diameter $d_a^{-\beta}$ of the pipeline, which has zero cost (solid circle); all equivalent diameters $D_{d_a,d_i}^{-\beta}$ that result from looping the existing pipeline once at cost c_i (dots in the diagram); all equivalent diameters $D_{d_a,d_{i_1},d_{i_2}}^{-\beta}$ that result from looping the existing pipeline twice at cost $c_{i_1} + c_{i_2}$ (plus signs in the diagram); and finally, all equivalent diameters $D_{d_a,d_{i_1},d_{i_2},d_{i_3}}^{-\beta}$ that result from triple looping at cost $c_{i_1} + c_{i_2} + c_{i_3}$, which are represented by crosses in the diagram.

This setting allows us to identify those among the equivalent diameters generated by (3.5) that are crucial for the models, namely those equivalent diameters that are represented by the extreme points of the *efficient frontier* in Figure 3.1b (circled in the diagram). All points on the line that represent the efficient frontier in the figure correspond to convex combinations of two equivalent diameters, i.e., a split-pipe setting where the pipeline is split into two segments, and these points represent the cost minimal ways of equipping a particular arc of the network with a certain equivalent diameter. As a consequence, for finding an optimal solution to the capacity extension problem, it suffices to incorporate into the split-pipe models only the equivalent diameters that correspond to the extreme points of the efficient frontier and to discard all others. As we will see when introducing the models, this will greatly reduce the number of variables.

In the case of the discrete capacity extension problem, we cannot use convex combinations of equivalent diameters and have to get along with the equivalent diameters given by (3.5). Unfortunately, however, we do not use all possible equivalent diameters depicted on the right of Figure 3.1b, because depending on the number of loops and diameter candidates, this would significantly increase the combinatorial complexity. For this reason, in our computational experiments in Section 3.6, we will restrict the discrete models to the equivalent diameters that result from looping an existing pipeline once and, additionally, to those equivalent diameters that result from looping an existing pipeline several times and are extreme points of the efficient frontier described above. As a consequence, we neglect all equivalent diameters resulting from multiple loops that are not extreme points of the efficient frontier. In this way, we may have to do without some potentially cost-minimal diameters, but will keep the models at a reasonable size and will, due to having incorporated the option of multiple looping, still achieve results that are as least as good as the approaches presented in the literature.

In the following, we denote the set of equivalent extension diameters of pipe a that we use in the models by $\{D_{a,0}, \dots, D_{a,k_a}\}$ (sorted in ascending order), where $D_{a,0}$ represents the (unlooped) original diameter d_a of the existing pipe, and the corresponding cost factors by $0 = c_{a,0} < \dots < c_{a,k_a}$. Moreover, for the sake of convenience, we write $[k_a] = \{0, 1, \dots, k_a\}$.

3.3 Discrete Loop Expansions

In this section, we present different approaches to model discrete loop expansions. We will begin with the model that is closest to the generic formulation of the capacity expansion Problem (3.3).

3.3.1 Discrete looping with potential function constraints (Model A)

To formulate an MINLP based on the generic Formulation (3.3) of the looping problem, which can be found in its discrete version in several recent papers (e.g., André et al., 2009; Bragalli et al., 2012; Robinius et al., 2019), we have to specify the way in which the discrete values of the diameter variables are selected. Here this happens in a straightforward way by means of binary variables $\lambda_{a,i}$ each of which represents one diameter $D_{a,i} \in \{D_{a,0}, \dots, D_{a,k_a}\}$. Constraint (3.9c) ensures that exactly one diameter is chosen for each arc $a \in \mathcal{A}$, and in constraint (3.9b) the potential function has been rewritten in a way such that the chosen diameter is used for modeling the potential loss. With these variables $\lambda_{a,i}$, the objective function is a direct consequence of (3.6) in conjunction with the generic objective function (3.3a), while all other constraints are identical with the generic model:

$$\min_{\lambda,x,\pi} \quad \sum_{a\in\mathcal{A}} L_a \sum_{i=0}^{k_a} \lambda_{a,i} c_{a,i} \tag{3.9a}$$

$$\text{subject to} \quad \pi_v - \pi_w = L_a R_a \left(\sum_{i=0}^{k_a} \frac{\lambda_{a,i}}{D_{a,i}^{\beta}} \right) \operatorname{sgn}(x_a)|x_a|^{\alpha} \quad \forall a \in \mathcal{A},\ (v,w) = r(a), \tag{3.9b}$$

$$\sum_{i=0}^{k_a} \lambda_{a,i} = 1 \qquad \forall\, a \in \mathcal{A}, \tag{3.9c}$$

$$\sum_{a\in\delta^+(v)} x_a - \sum_{a\in\delta^-(v)} x_a = b_v \qquad \forall\, v \in \mathcal{V}, \tag{3.9d}$$

$$\underline{\pi}_v \le \pi_v \le \overline{\pi}_v \qquad \forall\, v \in \mathcal{V}, \tag{3.9e}$$

$$\underline{x}_a \le x_a \le \overline{x}_a \qquad \forall\, a \in \mathcal{A}, \tag{3.9f}$$

$$\lambda_{a,i} \in \{0,1\} \qquad \forall\, a \in \mathcal{A},\ \forall i \in [k_a]. \tag{3.9g}$$

3.3.2 Discrete looping with flow direction variables (Model B)

This model is based on an approach by Raghunathan (2013) to tackle the network design problem. The main difference to the previous model is that we split the flow variable x_a into two sets of variables $x^+_{a,i}$ and $x^-_{a,i}$, which represent the flow on arc a with diameter $D_{a,i}$ in forward and backward direction, respectively (see constraints (3.10d), (3.10f)), and correspondingly, variables Δ^+_a, Δ^-_a that describe the potential drop along the arcs according to the flow direction (see constraints (3.10b) and (3.10e)). The idea behind this formulation is that constraint (3.9b) uses two nonlinear functions, namely the power function and the sign function, and by splitting the variable x_a into a part for a forward flow and a part for a backward flow, we are able to do without the latter function. This comes at a price, however, as we have to introduce additional binary variables z_a to choose between the two flow directions, see (3.10g) and (3.10h).

In his paper, Raghunathan (2013) develops his own solution algorithm to the network design problem, where he uses a convex relaxation of the problem by relaxing the potential constraint function (3.1) to $\Delta^+_a \leq \Phi(D_{a,i}, x^+_{a,i})$ and $\Delta^-_a \leq \Phi(D_{a,i}, x^-_{a,i})$ for all $a \in \mathcal{A}$ and $i \in [k_a]$. In our context, to model the expansion problem in an exact way, our aim is to enforce these inequalities with equality. In general, this can be done in different ways, such as using big-M constraints for modeling the disjunctions $\vee_{i=0}^{k_a} \Delta^+_a = \Phi(D_{a,i}, x^+_{a,i})$ and $\vee_{i=0}^{k_a} \Delta^-_a = \Phi(D_{a,i}, x^-_{a,i})$ or introducing the following constraints: $\Delta^+_a = \sum_{i=0}^{k_a} \Phi(D_{a,i}, x^+_{a,i})$ and $\Delta^-_a = \sum_{i=0}^{k_a} \Phi(D_{a,i}, x^-_{a,i})$ for all $a \in \mathcal{A}$. Preliminary computational tests revealed that the best performance is achieved with Balas's convex-hull formulation (Balas, 1985), i.e., when resolving these disjunctions by explicitly modeling the potential drop along arc a for diameter $D_{a,i}$ with the variables $\Delta^+_{a,i}, \Delta^-_{a,i}$. The model then reads as follows:

$$\min_{\substack{\lambda, x^+, x^-, \pi,\\ \Delta^+, \Delta^-, z}} \quad \sum\nolimits_{a \in \mathcal{A}} L_a \sum\nolimits_{i=0}^{k_a} \lambda_{a,i} c_{a,i} \tag{3.10a}$$

$$\text{subject to} \quad \pi_v - \pi_w = \sum\nolimits_{i \in [k_a]} \left(\Delta^+_{a,i} - \Delta^-_{a,i} \right) \qquad \forall a \in \mathcal{A},\ (v,w) = r(a), \tag{3.10b}$$

$$\sum\nolimits_{i=0}^{k_a} \lambda_{a,i} = 1 \qquad \forall\, a \in \mathcal{A},\ \forall i \in [k_a], \tag{3.10c}$$

$$\sum_{a \in \delta^+(v)} \sum_{i \in [k_a]} (x^+_{a,i} - x^-_{a,i}) - \sum_{a \in \delta^-(v)} \sum_{i \in [k_a]} (x^+_{a,i} - x^-_{a,i}) = b_v \quad \forall\, v \in \mathcal{V}, \tag{3.10d}$$

$$\Delta^+_{a,i} = \frac{L_a R_a}{D^\beta_{a,i}} (x^+_{a,i})^\alpha, \qquad \Delta^-_{a,i} = \frac{L_a R_a}{D^\beta_{a,i}} (x^-_{a,i})^\alpha \qquad \forall\, a \in \mathcal{A},\ \forall i \in [k_a], \tag{3.10e}$$

$$0 \leq x^+_{a,i} \leq \overline{x}_{a,i} \lambda_{a,i}, \qquad 0 \leq x^-_{a,i} \leq \overline{x}_{a,i} \lambda_{a,i} \qquad \forall\, a \in \mathcal{A},\ \forall i \in [k_a], \tag{3.10f}$$

$$x^+_{a,i} \leq \overline{x}_{a,i} z_a, \qquad x^-_{a,i} \leq \overline{x}_{a,i} (1 - z_a) \qquad \forall\, a \in \mathcal{A},\ \forall i \in [k_a], \tag{3.10g}$$

$$0 \leq \Delta^+_{a,i} \leq \overline{\Delta}^+_a z_a, \qquad 0 \leq \Delta^-_{a,i} \leq \overline{\Delta}^-_a (1 - z_a) \qquad \forall\, a \in \mathcal{A},\ \forall i \in [k_a], \tag{3.10h}$$

$$\underline{\pi}_v \leq \pi_v \leq \overline{\pi}_v \qquad \forall\, v \in \mathcal{V}, \tag{3.10i}$$

$$\lambda_{a,i} \in \{0,1\} \qquad \forall\, a \in \mathcal{A},\ \forall i \in [k_a], \tag{3.10j}$$

$$z_a \in \{0,1\} \qquad \forall\, a \in \mathcal{A}, \tag{3.10k}$$

with $\overline{\Delta}^+_a := \overline{\pi}_v - \underline{\pi}_w$ and $\overline{\Delta}^-_a := \overline{\pi}_w - \underline{\pi}_v$. Inequalities (3.10f) – (3.10h) model that a non-negative flow of arc a takes only place along a given direction for the selected diameter $D_{a,i}$

inducing the corresponding potential drop. To provide another upper bound on the potential loss on the arcs, Raghunathan (2013) introduces further constraints, which in our case read $\Delta^-_{a,i} \leq (L_a R_a / D^\beta_{a,i})(\overline{x}_{a,i})^{\alpha-1} x^-_{a,i}$ and $\Delta^+_{a,i} \leq (L_a R_a / D^\beta_{a,i})(\overline{x}_{a,i})^{\alpha-1} x^+_{a,i}$.

3.3.3 Discrete looping with potential difference variables (Model C)

The model presented in this section is a novel approach to tackle the nonlinearities of the problem. Instead of introducing variables for the flow direction as in the previous model, the idea here is to reduce the two-dimensional function $\Phi(d_a, x_a)$ to a one-dimensional function $\phi(x_a)$. To this end, we shift the loop diameters from the flow-related term towards the potential loss term, see (3.11b), similarly as done in (De Wolf and Smeers, 1996). Then, we introduce continuous variables $\Delta_{a,i}$ to model the potential loss that corresponds to the loop diameter chosen (3.11c) and use binary variables $\lambda_{a,i}$ not to choose the diameters, but to select the potential loss. As a trade-off we have to introduce big-M constraints to determine the potential loss variables $\Delta_{a,i}$ in (3.11d). In fact, the idea behind this model formulation is to see whether global MINLP solvers perform better on models that introduce big-M constraints, but in return reduce the degree of nonlinear and nonconvex constraint functions.

$$\min_{\lambda,x,\pi,\Delta} \quad \sum\nolimits_{a\in\mathcal{A}} L_a \sum\nolimits_{i=0}^{k_a} \lambda_{a,i} c_{a,i} \tag{3.11a}$$

$$\text{subject to} \quad \operatorname{sgn}(x_a)|x_a|^\alpha - \sum\nolimits_{i=0}^{k_a} \frac{D^\beta_{a,i}}{L_a R_a} \Delta_{a,i} = 0 \qquad \forall\, a \in \mathcal{A}, \tag{3.11b}$$

$$\sum\nolimits_{i=0}^{k_a} \Delta_{a,i} = \pi_v - \pi_w \qquad \forall a \in \mathcal{A},\ (v,w) = r(a), \tag{3.11c}$$

$$M^{(1)}_{a,i}\, \lambda_{a,i} \leq \Delta_{a,i} \leq M^{(2)}_{a,i}\, \lambda_{a,i} \qquad \forall a \in \mathcal{A},\ \forall i \in [k_a], \tag{3.11d}$$

$$\sum\nolimits_{i=0}^{k_a} \lambda_{a,i} = 1 \qquad \forall\, a \in \mathcal{A}, \tag{3.11e}$$

$$\sum_{a\in\delta^+(v)} x_a - \sum_{a\in\delta^-(v)} x_a = b_v \qquad \forall\, v \in \mathcal{V}, \tag{3.11f}$$

$$\underline{\pi}_v \leq \pi_v \leq \overline{\pi}_v \qquad \forall\, v \in \mathcal{V}, \tag{3.11g}$$

$$\underline{\pi}_v - \overline{\pi}_w \leq \Delta_{a,i} \leq \overline{\pi}_v - \underline{\pi}_w \qquad \forall a \in \mathcal{A},\ \forall i \in [k_a], \tag{3.11h}$$

$$\underline{x}_a \leq x_a \leq \overline{x}_a \qquad \forall\, a \in \mathcal{A}, \tag{3.11i}$$

$$\lambda_{a,i} \in \{0,1\} \qquad \forall a \in \mathcal{A},\ \forall i \in [k_a], \tag{3.11j}$$

where $M^{(1)}_{a,i} := \Phi(D_{a,i}, \underline{x}_a)$, $M^{(2)}_{a,i} := \Phi(D_{a,i}, \overline{x}_a)$.

3.3.4 Further approaches for discrete looping in the literature

There are two further models in the literature for the network design problem, both of which are not intended as “stand-alone models”, but as starting points for solution algorithms. While we will not go into the details of the models here because preliminary computational experiments have demonstrated that they do not perform particularly well, see Section 3.6.3, the approaches still deserve mentioning.

Borraz-Sánchez et al. (2016) present an MINLP formulation that is similar to the model developed by Raghunathan (2013) (Section 3.3.2) in the sense that it distinguishes between forward and backward flow directions. Here, however, this is achieved through binary variables z_a^+, z_a^- that are then used to model the potential loss function in product form $\lambda_{a,i}\,(z_a^+ - z_a^-)(\pi_v - \pi_w) = L_a R_a / D_{a,i}^{\beta} x^{\alpha}$.

Humpola (2014) uses indicator constraints to select arcs a in the network design problem (or diameters $D_{a,i}$ in the context of our expansion problem), i.e., constraints of the form $\lambda_{a,i} = 1 \Rightarrow \pi_v - \pi_w = \Phi(d_{a,i}, x_a)$ and $\lambda_{a,i} = 0 \Rightarrow x_{a,i} = 0$, which may be represented by big-M constraints in an MINLP model.

3.3.5 Equivalence of the discrete Models A, B and C

To finish the section on discrete models for the capacity expansion problem, we show that the three models presented so far are equivalent. More precisely, we show that for each solution to one of the models, there exist solutions to the other two models such that all of them represent the (i) same physical network state in terms of flow and potential, (ii) same decisions on loop extensions, and (iii) same objective function values. To this end, we use the standard projection to map the feasible regions of the discrete models onto the space of their common variables, i.e. onto the $\big((\pi_v, \lambda_{a,i})_{v\in\mathcal{V}, a\in\mathcal{A}, i\in[k_a]}\big)$-space: In this context we denote the feasible regions of the discrete models as: X_A for Model A, X_B for Model B, and X_C for Model C:

$$X_A = \operatorname{Proj}_{\pi_v,\lambda_{a,i}} \Big\{ \big(\pi_v, \lambda_{a,i}, x_a\big) \mid \big(\pi_v, x_a, \lambda_{a,i}\big) \text{ satisfy Eqs. (3.9b) – (3.9g) } \Big\},$$

$$X_B = \operatorname{Proj}_{\pi_v,\lambda_{a,i}} \Big\{ \big(\pi_v, \lambda_{a,i}, x_{a,i}^+, x_{a,i}^-, \Delta_{a,i}^+, \Delta_{a,i}^-, z_a\big) \mid \big(\pi_v, \lambda_{a,i}, x_{a,i}^+, x_{a,i}^-, \Delta_{a,i}^+, \Delta_{a,i}^-, z_a\big) \text{ satisfy Eqs. (3.10b) – (3.10k) } \Big\},$$

$$X_C = \operatorname{Proj}_{\pi_v,\lambda_{a,i}} \Big\{ \big(\pi_v, \lambda_{a,i}, x_a, \Delta_{a,i}\big) \mid \big(\pi_v, \lambda_{a,i}, x_a, \Delta_{a,i}\big) \text{ satisfy Eqs. (3.11b) – (3.11j) } \Big\}.$$

Proposition 3.3.1. *The discrete Models A, B and C are equivalent, i.e.*

$$X_A = X_C = X_B.$$

Proof. "$X_A \subseteq X_C$":
Let $(x_a, \pi_v, \lambda_{a,i})_{v\in\mathcal{V}, a\in\mathcal{A}, i\in[k_a]}$ be a solution of Model A, i.e. $(\pi_v, \lambda_{a,i})_{v\in\mathcal{V}, a\in\mathcal{A}, i\in[k_a]} \in X_A$. Then (3.11e) – (3.11g) and (3.11i), (3.11j) follow right away with $(x_a, \pi_v, \lambda_{a,i})_{v\in\mathcal{V}, a\in\mathcal{A}, i\in[k_a]}$. For pipe a, let $\check{i}_a \in [k_a]$ be such that $\lambda_{a,\check{i}_a} = 1$ and define

$$\Delta_{a,i} := \begin{cases} 0 & \forall i \in [k_a] \setminus \{\check{i}_a\} \\ \frac{L_a R_a}{D_{a,i}^{\beta}} \operatorname{sgn}(x_a)|x_a|^{\alpha} & i = \check{i}_a. \end{cases}$$

Then (3.11b) follows by construction, (3.11c) equals (3.9b), (3.11d) holds by virtue of $M_{a,i}^{(1)} = \Phi(D_{a,\check{i}_a}, \underline{x}_a)$, $M_{a,i}^{(2)} = \Phi(D_{a,\check{i}_a}, \overline{x}_a)$ and (3.11h) follows from (3.11c), (3.11g). Thus, $(x_a, \pi_v, \lambda_{a,i}, \Delta_{a,i})_{v\in\mathcal{V}, a\in\mathcal{A}, i\in[k_a]}$ is a solution of Model C, i.e., $(\pi_v, \lambda_{a,i})_{v\in\mathcal{V}, a\in\mathcal{A}, i\in[k_a]} \in X_C$.

"$X_C \subseteq X_A$":
Let $(x_a, \pi_v, \lambda_{a,i}, \Delta_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$ be a solution of Model C, i.e., $(\pi_v, \lambda_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_C$. With $(x_a, \pi_v, \lambda_{a,i}, \Delta_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$, (3.9b) follows from (3.11b) – (3.11d) and (3.9c) – (3.9g) equals (3.11e) – (3.11g), (3.11i) – (3.11j). Hence, $(\pi_v, \lambda_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_A$.

"$X_A \subseteq X_B$":
Let $(\pi_v, \lambda_{a,i}, x_a)_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$ be a solution of Model A, i.e.,$(\pi_v, \lambda_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_A$. For pipe a let $\check{i}_a \in [k_a]$ be such that $\lambda_{a,\check{i}_a} = 1$. We set $x^+_{a,i}, x^-_{a,i}, \Delta^+_{a,i}, \Delta^-_{a,i} := 0$ for all $i \in [k_a] \setminus \{\check{i}_a\}$.

Case 1: if $x_a \geq 0$, define $x^+_{a,\check{i}_a} := x_a$, $x^-_{a,\check{i}_a} = 0$ and $\Delta^+_{a,\check{i}_a} := \pi_v - \pi_w$ and $z_a := 1$.

Case 2: if $x_a < 0$, define $x^+_{a,\check{i}_a} := 0$, $x^-_{a,\check{i}_a} := -x_a$ and $\Delta^-_{a,\check{i}_a} := \pi_w - \pi_v$ and $z_a := 0$.

Then in both cases (3.10b) – (3.10k) hold. Hence, $(\pi_v, \lambda_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_B$.

"$X_B \subseteq X_A$":
Let $\left(x^+_{a,i}, x^-_{a,i}, z_a, \pi_v, \lambda_{a,i}, \Delta^+_{a,i}, \Delta^-_{a,i}\right)_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$ be a solution of Model B, that means, $(\pi_v, \lambda_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_B$. Set $x_a := \sum_{i\in k_a}(x^+_{a,i} - x^-_{a,i})$ for all $a \in \mathcal{A}$, then the equations of Model A are satisfied. Hence, $(\pi_v, \lambda_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_A$. □

3.4 Split-Pipe Loop Expansions

In this section, we present two equivalent modeling approaches for continuous loop expansions. The first one is a model common in the literature (e.g., Alperovits and Shamir, 1977; Zhang and Zhu, 1996), while the second one is a novel approach.

3.4.1 Split-pipe looping with length variables (Model D)

This split-pipe model is identical to the first discrete Model A, except that the binary variables $\lambda_{a,i} \in \{0,1\}$ have been relaxed to continuous length variables $\lambda_{a,i} \in [0,1]$ here, turning the MINLP of Section 3.3.1 into an NLP. As explained in Section 3.2.2, these length variables denote the proportion of pipe a that is looped with diameter $D_{a,i}$. This means that for each arc a, the pipeline consists of segments of those equivalent diameters $D_{a,i}$ for which $\lambda_{a,i} > 0$.

Since the objective is to minimize costs, any local optimal solution has the property that each arc consists of at most two pipe segments (Fujiwara and Dey, 1987), where the corresponding equivalent diameters correspond to adjacent extreme points of the efficient frontier. Hence, the variables $\lambda_{a,i}$ inherently form a special ordered set of type 2 (SOS 2).

$$\min_{\lambda,x,\pi} \quad \sum\nolimits_{a\in\mathcal{A}} L_a \sum\nolimits_{i=0}^{k_a} \lambda_{a,i} c_{a,i} \tag{3.12a}$$

$$\text{subject to} \quad \pi_v - \pi_w = L_a R_a \left(\sum_{i=0}^{k_a} \frac{\lambda_{a,i}}{D^\beta_{a,i}}\right) \operatorname{sgn}(x_a)|x_a|^\alpha \quad \forall a \in \mathcal{A},\ (v,w) = r(a), \tag{3.12b}$$

$$\sum_{i=0}^{k_a} \lambda_{a,i} = 1 \qquad \forall\, a \in \mathcal{A}, \quad (3.12c)$$

$$\sum_{a\in\delta^+(v)} x_a - \sum_{a\in\delta^-(v)} x_a = b_v \qquad \forall\, v \in \mathcal{V}, \quad (3.12d)$$

$$\underline{\pi}_v \le \pi_v \le \overline{\pi}_v \qquad \forall\, v \in \mathcal{V}, \quad (3.12e)$$

$$\underline{x}_a \le x_a \le \overline{x}_a \qquad \forall\, a \in \mathcal{A}, \quad (3.12f)$$

$$\lambda_{a,i} \in [0,1] \qquad \forall\, a \in \mathcal{A},\ \forall i \in [k_a]. \quad (3.12g)$$

3.4.2 Split-pipe looping with efficient frontier constraints (Model E)

In the previous section, we modeled the efficient frontier of equivalent diameters using length variables $\lambda_{a,i}$ to express convex combinations of the equivalent diameters $D_{a,i}$ that are the extreme points of the frontier. In the present section, the efficient frontier is modeled explicitly by using linear constraints.

As the efficient frontier consists of points $(D^{-\beta}, c)$ (cf. Section 3.2.2), we introduce new continuous variables y_a to model the exponentiated diameter, see constraint (3.13b), and variables c_a that represent the costs per unit length of pipe a for the equivalent diameters. On this basis, the efficient frontier can be represented by linear constraints (3.13d), each of which models the frontier between a pair of adjacent extreme points (cf. Figure 3.1b). The parameters $s_{a,i}$ and $t_{a,i}$ can be calculated in advance as $s_{a,i} = (c_{a,i} - c_{a,i+1})/(D_{a,i}^{-\beta} - D_{a,i+1}^{-\beta})$ and $t_{a,i} = -s_{a,i} D_{a,i+1}^{-\beta} + c_{a,i+1}$.

$$\min_{y,c,x,\pi} \quad \sum\nolimits_{a\in\mathcal{A}} L_a c_a \qquad (3.13a)$$

$$\text{subject to} \quad \pi_v - \pi_w = L_a R_a\, y_a \,\mathrm{sgn}(x_a)|x_a|^\alpha \qquad \forall a \in \mathcal{A},\ (v,w) = r(a), \qquad (3.13b)$$

$$\sum_{a\in\delta^+(v)} x_a - \sum_{a\in\delta^-(v)} x_a = b_v \qquad \forall\, v \in \mathcal{V}, \qquad (3.13c)$$

$$c_a \ge s_{a,i} y_a + t_{a,i} \qquad \forall a \in \mathcal{A},\ \forall i \in [k_a - 1], \qquad (3.13d)$$

$$\underline{\pi}_v \le \pi_v \le \overline{\pi}_v \qquad \forall\, v \in \mathcal{V}, \qquad (3.13e)$$

$$\underline{x}_a \le x_a \le \overline{x}_a \qquad \forall\, a \in \mathcal{A}, \qquad (3.13f)$$

$$\underline{y}_a \le y_a \le \overline{y}_a \qquad \forall\, a \in \mathcal{A}, \qquad (3.13g)$$

$$c_a \ge 0 \qquad \forall\, a \in \mathcal{A}. \qquad (3.13h)$$

Note that the bounds in (3.13g) can be calculated as $\underline{y}_a = D_{a,k_a}^{-\beta}$ and $\overline{y}_a = D_{a,0}^{-\beta}$.

3.4.3 Equivalence of the split-pipe Models D and E

To compare the feasible regions of the split-pipe Models D and E, we use the standard projection to map their feasible regions onto the space of their common variables, i.e. onto the $((\pi_v, x_a)_{v\in\mathcal{V},a\in\mathcal{A}})$-space, as done in Section 3.3.5 for the discrete models:

$$X_D = \mathrm{Proj}_{\pi_v,x_a} \Big\{ \big(\pi_v, x_a, \lambda_{a,i}\big) \mid \big(\pi_v, x_a, \lambda_{a,i}\big) \text{ satisfy Eqs. (3.12b) – (3.12g)} \Big\},$$

$$X_E = \operatorname{Proj}_{\pi_v,x_a} \Big\{ \big(\pi_v, x_a, y_a, c_a\big) \mid \big(\pi_v, x_a, y_a, c_a\big) \text{ satisfy Eqs. (3.13b) – (3.13h)} \Big\}.$$

The feasible region of Model D is a subset of the feasible region of Model E due to the inequality constraints (3.13d), i.e. $X_D \subseteq X_E$. But since the objective is to minimize the cost for building loops, solutions of both models are forced to be on the efficient frontier. The proof of the following proposition formalizes this argument.

Proposition 3.4.1. *Each optimal solution to Model D is an optimal solution to Model E, and vice versa.*

Proof. (i) Let $\big(x_a, \pi_v, \lambda_{a,i}\big)_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$ be an optimal solution of Model D, that is, $(x_a, \pi_v)_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_D$. According to Fujiwara and Dey (1987), an optimal solution has the property that each arc consists of at most two pipe segments, where the corresponding equivalent diameters correspond to adjacent extreme points of the efficient frontier $\{(D_{a,0}^{-\beta}, c_{a,0}), \dots, (D_{a,k_a}^{-\beta}, c_{a,k_a})\}$. Hence, for all $a \in \mathcal{A}$ there exists a unique $i_0 \in [k_a - 1]$ such that $0 \le \lambda_{a,i_0}, \lambda_{a,i_0+1} \le 1$ with $\lambda_{a,i_0} + \lambda_{a,i_0+1} = 1$ and $\lambda_{a,i} = 0\ \forall i \in [k_a] \setminus \{i_0, i_0+1\}$. Define

$$y_a := \lambda_{a,i_0} D_{a,i_0}^{-\beta} + \lambda_{a,i_0+1} D_{a,i_0+1}^{-\beta} \text{ and } c_a := \lambda_{a,i_0} c_{a,i_0} + \lambda_{a,i_0+1} c_{a,i_0+1}.$$

Then, $(x_a, \pi_v, y_a, c_a)_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$ obviously fulfills the variable bounds (3.13e) – (3.13g) and Equation (3.13c). From (3.12b) and the definition of y follows (3.13b). We now show that (3.13d) holds: From our definition of y_a, we obtain

$$y_a = \lambda_{a,i_0}(D_{a,i_0}^{-\beta} - D_{a,i_0+1}^{-\beta}) + D_{a,i_0+1}^{-\beta} \tag{3.14}$$

$$\Leftrightarrow \quad \lambda_{a,i_0}(c_{a,i_0} - c_{a,i_0+1}) = \frac{c_{a,i_0} - c_{a,i_0+1}}{D_{a,i_0}^{-\beta} - D_{a,i_0+1}^{-\beta}}(y_a - D_{a,i_0+1}^{-\beta}) \tag{3.15}$$

$$\Leftrightarrow \quad \lambda_{a,i_0} c_{a,i_0} + \lambda_{a,i_0+1} c_{a,i_0+1} = \frac{c_{a,i_0} - c_{a,i_0+1}}{D_{a,i_0}^{-\beta} - D_{a,i_0+1}^{-\beta}}(y_a - D_{a,i_0+1}^{-\beta}) + c_{a,i_0+1} \tag{3.16}$$

$$\Leftrightarrow \quad c_a = s_{a,i_0} y_a + t_{a,i_0}. \tag{3.17}$$

Since the objective is to minimize and the slopes $s_{a,i}$ in (3.13d) are decreasing for increasing i, we have $c_a = s_{a,i_0}\, y_a + t_{a,i_0} = \max_{i\in[k_a-1]} s_{a,i}\, y_a + t_{a,i}$ which implies $c_a \ge s_{a,i} y_a + t_{a,i}$ for all $a \in \mathcal{A}$ and for all $i \in [k_a - 1]$, and hence, the solution of Model D satisfies (3.13d). Therefore, all optimal solutions of Model D are feasible for Model E, i.e., $(x_a, \pi_v)_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_E$. The optimal objective function value of Model D is $\sum_{a\in\mathcal{A}} L_a(\lambda_{a,i_0} c_{a,i_0} + \lambda_{a,i_0+1} c_{a,i_0+1})$ and equals $\sum_{a\in\mathcal{A}} L_a c_a$ by construction.

(ii) Let $(x_a, \pi_v, y_a, c_a)_{v\in\mathcal{V},a\in\mathcal{A}}$ be an optimal solution of Model E, i.e., $(x_a, \pi_v)_{v\in\mathcal{V},a\in\mathcal{A}} \in X_E$. Consider $i_0 \in [k_a - 1]$ such that $D_{a,i_0+1}^{-\beta} \le y_a \le D_{a,i_0}^{-\beta}$. Then, there exist unique $0 \le \lambda_{a,i_0}, \lambda_{a,i_0+1} \le 1$ with $\lambda_{a,i_0} + \lambda_{a,i_0+1} = 1$ such that $y_a = \lambda_{a,i_0} D_{a,i_0}^{-\beta} + \lambda_{a,i_0+1} D_{a,i_0+1}^{-\beta}$ for all $a \in \mathcal{A}$. We set $\lambda_{a,i} := 0$ for all $i \in [k_a] \setminus \{i_0, i_0+1\}$, then $(x_a, \pi_v, \lambda_a)_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$ fulfills (3.12b) – (3.12g). Hence, $(x_a, \pi_v)_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_D$. Therefore, all optimal solutions of Model E are feasible for Model D, i.e. $(x_a, \pi_v)_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_D$.

From (3.13d), we have $c_a = s_{a,i_0} y_a + t_{a,i_0}$. However, rearranging $y_a = \lambda_{a,i_0} D_{a,i_0}^{-\beta} + \lambda_{a,i_0+1} D_{a,i_0+1}^{-\beta}$ as in (3.14) to (3.16) yields $\lambda_{a,i_0} c_{a,i_0} + \lambda_{a,i_0+1} c_{a,i_0+1} = s_{a,i_0} y_a + t_{a,i_0}$, hence $\sum_{a\in\mathcal{A}} L_a\ (\lambda_{a,i_0} c_{a,i_0} + \lambda_{a,i_0+1} c_{a,i_0+1}) = \sum_{a\in\mathcal{A}} L_a c_a$. □

3.4.4 Comparison of relaxations

In this section, we consider the continuous relaxations of the discrete models. As solvers relax the combinatorial part during the solution procedure, the tightness of the continuous relaxation plays a major role for the performance of the models. To this end, we use the standard projection to map their feasible regions onto the space of their common variables, i.e. onto the $((\pi_v, x_a)_{v\in\mathcal{V},a\in\mathcal{A}})$-space: In the following, we denote the feasible regions of the continuous relaxations of the discrete Models A, B and C by $X_{\text{A, rel}}$, $X_{\text{B, rel}}$ and $X_{\text{C, rel}}$, respectively:

$$\begin{aligned}
X_{A,rel} = \text{Proj}_{\pi_v,\lambda_{a,i}} \Big\{ \big(\pi_v, \lambda_{a,i}, x_a\big) \mid \big(\pi_v, x_a, \lambda_{a,i}\big) &\text{ satisfy Eqs. (3.9b) -- (3.9f)} \\
&\text{and } \lambda_{a,i} \in [0,1]\ \forall a \in \mathcal{A}, \forall i \in [k_a] \Big\}, \\
X_{B,rel} = \text{Proj}_{\pi_v,\lambda_{a,i}} \Big\{ \big(\pi_v, \lambda_{a,i}, x^+_{a,i}, x^-_{a,i}, \Delta^+_{a,i}, \Delta^-_{a,i}, z_a\big) &\mid \big(\pi_v, \lambda_{a,i}, x^+_{a,i}, x^-_{a,i}, \Delta^+_{a,i}, \Delta^-_{a,i}, z_a\big) \\
&\text{satisfy Eqs. (3.10b) -- (3.10i) and } \lambda_{a,i}, z_a \in [0,1]\ \forall a \in \mathcal{A}, \forall i \in [k_a] \Big\}, \\
X_{C,rel} = \text{Proj}_{\pi_v,\lambda_{a,i}} \Big\{ \big(\pi_v, \lambda_{a,i}, x_a, \Delta_{a,i}\big) \mid \big(\pi_v, \lambda_{a,i}, x_a, \Delta_{a,i}\big) &\text{ satisfy Eqs. (3.11b) -- (3.11i)} \\
&\text{and } \lambda_{a,i} \in [0,1]\ \forall a \in \mathcal{A}, \forall i \in [k_a] \Big\}.
\end{aligned}$$

Proposition 3.4.2 shows relationships between the feasible regions of the split-pipe models and Figure 3.2 compares the feasible regions of both discrete and split-pipe models.

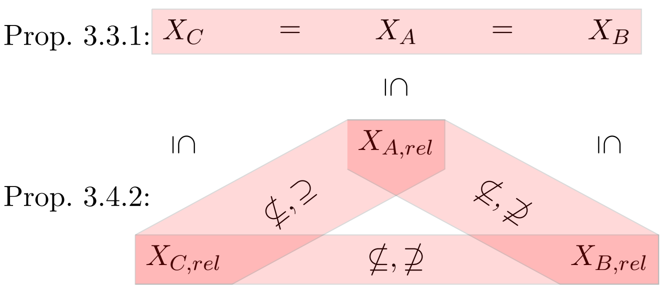


Figure 3.2: Relation of feasible regions. The highlighted relations in red are proven as propositions. The other relations $X_A \subseteq X_{A,rel}$, $X_C \subseteq X_{\text{C, rel}}$ and $X_B \subseteq X_{\text{B, rel}}$ are the canonical continuous relaxations of the corresponding discrete models. Proposition 3.3.1 also implies $X_C, X_B \subseteq X_{A,rel}$. Since $X_D = X_{A,rel}$, the split-pipe model can be seen as a relaxation of Models A, B and C.

Proposition 3.4.2. *Let $\alpha \neq 1$. Then, the following relationships hold for the continuous relaxation of Models A, B and C:*

i) $X_{A,rel} \subseteq X_{C,\ rel}$ *but* $X_{C,\ rel} \neq X_{A,rel}$,

ii) $X_{A,rel} \not\subseteq X_{B,\ rel}$ *and* $X_{B,\ rel} \not\subseteq X_{A,rel}$,

iii) $X_{C,rel} \not\subseteq X_{B,\ rel}$ *and* $X_{B,\ rel} \not\subseteq X_{C,rel}$.

Proof. "$X_{A,rel} \subseteq X_{\text{C, rel}}$":

Let $(x_a, \pi_v, \lambda_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$ be a solution to the continuous relaxation of Model A, i.e., $(\pi_v, \lambda_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]} \in X_{A,rel}$. We show that it can be transformed to a point in $X_{\text{C, rel}}$. First, $(x_a, \pi_v, \lambda_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$ satisfy Equations (3.11e) – (3.11g) and (3.11i). By defining

$$\Delta_{a,i} := \lambda_{a,i}\,\text{sgn}(x_a)|x_a|^\alpha L_a R_a D_{a,i}^{-\beta} \quad \forall a \in \mathcal{A},\ \forall i \in [k_a] \tag{3.18}$$

$$\Rightarrow \sum\nolimits_{i=0}^{k_a} \Delta_{a,i} \overset{(3.18)}{=} \text{sgn}(x_a)|x_a|^\alpha \sum\nolimits_{i=0}^{k_a} \lambda_{a,i} L_a R_a D_{a,i}^{-\beta} \overset{(3.12b)}{=} \pi_v - \pi_w \quad \forall a \in \mathcal{A},$$

the potential loss constraint (3.11c) and the bounds of $\Delta_{a,i}$ (3.11h) are satisfied. With $M_{a,i}^{(1)} = \Phi(D_{a,i}, \underline{x}_a)$, $M_{a,i}^{(2)} = \Phi(D_{a,i}, \overline{x}_a)$, the big M-formulation (3.11d) holds by construction. Equation (3.11b) also holds, since

$$\overset{(3.18)}{\Rightarrow} \lambda_{a,i}\,\text{sgn}(x_a)|x_a|^\alpha = \frac{D_{a,i}^\beta}{L_a R_a}\Delta_{a,i} \qquad \forall a \in \mathcal{A},\ \forall i \in [k_a],$$

$$\Rightarrow \sum_{i=0}^{k_a} \lambda_{a,i}\,\text{sgn}(x_a)|x_a|^\alpha = \sum_{i=0}^{k_a} \frac{D_{a,i}^\beta}{L_a R_a}\Delta_{a,i} \overset{(3.11e)}{\Rightarrow} \text{sgn}(x_a)|x_a|^\alpha = \sum_{i=0}^{k_a} \frac{D_{a,i}^\beta}{L_a R_a}\Delta_{a,i} \ \ \forall a \in \mathcal{A},$$

hence $(x_a, \pi_v, \lambda_{a,i}, \Delta_{a,i})_{v\in\mathcal{V},a\in\mathcal{A},i\in[k_a]}$ is a solution of the continuous relaxation of Model C, i.e. $X_{A,rel} \subseteq X_{\text{C, rel}}$.

The remaining relations between the continuous relaxations of the discrete models are shown by counterexamples. Throughout we consider a single pipe $a = (v, w)$ with $L_a R_a = 1, D_{a,0}^\beta = 1, D_{a,1}^\beta = 2$ for given $\alpha > 0$ and $\beta > 0$.

"$X_{\text{C, rel}} \not\subseteq X_{A,rel}$ and $X_{\text{B, rel}} \not\subseteq X_{A,rel}$":

Let $b_v = \sqrt[\alpha]{100}$, $b_w = -\sqrt[\alpha]{100}$ and $\pi_v, \pi_w \in [0, 3600]$ be given. Then $(\lambda_{a,0}, \lambda_{a,1}, \pi_v, \pi_w) := (0.5, 0.5, 3600, 3500) \in X_{B,rel}, X_{C,rel}$, where the remaining variables of Model B are given by e.g. $x_{a,0}^+ = \sqrt[\alpha]{100}, x_{a,1}^+ = x_{a,0}^- = x_{a,1}^- = \Delta_{a,1}^+ = \Delta_{a,0}^- = \Delta_{a,1}^- = 0, z_a = 1$, $\Delta_{a,0}^+ = 100$, and of Model C by $x_a = \sqrt[\alpha]{100}, \Delta_{a,0} = 100$, and $\Delta_{a,1} = 0$. However, $(\lambda_{a,0}, \lambda_{a,1}, \pi_v, \pi_w) = (0.5, 0.5, 3600, 3500) \notin X_D$, where the remaining variable $x_a = \sqrt[\alpha]{100}$ is uniquely determined by Equation (3.9d), which contradicts Equation (3.9b), i.e., $\pi_v - \pi_w = 100 \neq 0.75(\sqrt[\alpha]{100})^\alpha = \sum_{i=0}^{k_a}(\lambda_{a,i}/D_{a,i}^\beta)\,\text{sgn}(x_a)|x_a|^\alpha$.

"$X_{A,rel} \not\subseteq X_{\text{B, rel}}$ and $X_{\text{C, rel}} \not\subseteq X_{\text{B, rel}}$":

Now slightly change the above example to $b_v = 1000 = -b_w$. Moreover, we set $\overline{x}_a = 1000$. Then $(\lambda_{a,0}, \lambda_{a,1}, \pi_v, \pi_w) := (0.5, 0.5, 0.75 \times 1000^\alpha, 0) \in X_{A,rel}, X_{C,rel}$, where the remaining variables of the continuous relaxation of Model A are given by $x_a = 1000$ and of the continuous relaxation of Model C by $\Delta_{a,0} = 0.5 \times 1000^\alpha$, $\Delta_{a,1} = 0.25 \times 1000^\alpha$ and $x_a = 1000$. However, $(\lambda_{a,0}, \lambda_{a,1}, \pi_v, \pi_w) = (0.5, 0.5, 0.75 \times 1000^\alpha, 0) \notin X_{\text{B, rel}}$, because it follows from Equations (3.10d) and (3.10f) that $x_{a,0}^+ = x_{a,1}^+ = 500 = 0.5\,\overline{x}_a$ and $x_{a,0}^- = x_{a,1}^- = 0$, and from (3.10e) and (3.10g), it follows that $z_a = 0.5$ and $\Delta_{a,0}^+ = 500^\alpha$, $\Delta_{a,1}^+ = 0.5 \times 500^\alpha$ and $\Delta_{a,0}^-, \Delta_{a,1}^- = 0$. With these values, (3.10b) implies for $\alpha \neq 1$

$$\sum\nolimits_{i\in[k_a]} \left(\Delta_{a,i}^+ - \Delta_{a,i}^-\right) = 1.5 \times 500^\alpha \neq 0.75 \times 1000^\alpha = \pi_v - \pi_w.$$

"$X_{\text{B, rel}} \not\subseteq X_{\text{C, rel}}$":

We now choose $b_v = \sqrt[\alpha]{75} = -b_w$ and set $\underline{x}_a > 0$. Then $(\lambda_{a,0}, \lambda_{a,1}, \pi_v, \pi_w, x^+_{a,0}, x^+_{a,1}, x^-_{a,0}, x^-_{a,1}, z_a, \Delta^+_{a,0}, \Delta^+_{a,1}, \Delta^-_{a,0}, \Delta^-_{a,1}) := (0.75, 0.25, 3600, 3525, \sqrt[\alpha]{75}, 0, 0, 0, 1, 75, 0, 0, 0) \in X_{\text{B, rel}}$. However, $(\lambda_{a,0}, \lambda_{a,1}, \pi_v, \pi_w) := (0.75, 0.25, 3600, 3525) \notin X_{\text{C, rel}}$, since $x_a = \sqrt[\alpha]{75}$ follows uniquely from (3.11f), which together with Equation (3.11b) leads to $\Delta_{a,0} + 2\Delta_{a,1} = 75$. Moreover, from (3.11d) it follows that $\Delta_{a,0}, \Delta_{a,1} > 0$, since $\lambda_{a,0}, \lambda_{a,1} > 0$ and $M^{(1)}_{a,i} = \Phi(D_{a,i}, \underline{x}_a) > 0$ holds by virtue of $\underline{x}_a > 0$. But then, Equations (3.11b) and (3.11c) cannot be satisfied simultaneously, because (3.11c) yields $\Delta_{a,0} + \Delta_{a,1} = 75$. □

3.5 Theoretical Expansion Planning Aspects

After having presented models for the discrete and split-pipe expansion planning problems in the previous sections, we provide the reader with further insights into the expansion planning problem by showing theoretical aspects in this section. We start with a convexity analysis in Section 3.5.1. It turns out that the problem is nonconvex, which is important for the choice of an appropriate solver, in particular when solving the problem to global optimality. Motivated by the fact that feasible network operations are crucial in practice, we state conditions in Section 3.5.2 that always enable feasible loop expansions. However, expansions, in general, might worsen the flow situation and allow for less throughput as before. This phenomenon is known as Braess's paradox. In Section 3.5.3, we show that it might also occur in the context of looping. In the extreme, it might turn a feasible state infeasible. Therefore, we present a simple strategy to overcome such shortcomings that utilizes the conditions on feasibility from Section 3.5.2. We conclude this theoretical part in Section 3.5.4 by illustrating that split-pipe loop expansions generally introduce symmetries, which depend on the order of both split-pipe segments. However, the presented split-pipe model formulations avoid these symmetries.

Let us recall the general loop expansion Problem 3.3 from Section 3.2,

$$\min_{x,\, d,\, \pi} \sum_{a \in \mathcal{A}} c(d_a) L_a \tag{3.3a}$$

$$\text{s.t.} \quad \pi_v - \pi_w = \Phi(d_a, x_a) \quad \forall\, a \in \mathcal{A} \text{ and } (v, w) = r(a), \tag{3.3b}$$

$$\sum\nolimits_{a \in \delta^+(v)} x_a - \sum\nolimits_{a \in \delta^-(v)} x_a = b_v \quad \forall\, v \in \mathcal{V}, \tag{3.3c}$$

$$\underline{\pi}_v \le \pi_v \le \overline{\pi}_v \quad \forall\, v \in \mathcal{V}, \tag{3.3d}$$

$$\underline{x}_a \le x_a \le \overline{x}_a \quad \forall\, a \in \mathcal{A}, \tag{3.3e}$$

$$d_a \in \mathcal{D}_a \quad \forall\, a \in \mathcal{A}. \tag{3.3f}$$

3.5.1 Convexity analysis

In the previous sections, we have modeled the expansion planning problem as MINLP for discrete looping decisions and as NLP for the split-pipe approach. We now show that the feasible regions of the discrete and continuous (or split-pipe) capacity expansion planning problems are non-convex.

When neglecting loop expansions, i.e., $d_a = D_{a,0}$ for all pipes $a \in \mathcal{A}$, the continuous and the discrete Problems (3.3) reduce to the same existence problem of validating a given demand scenario for feasibility. The feasible region of the resulting problem is convex (Maugis (1977) and Collins et al. (1978)), even though it comprises nonlinear nonconvex constraints of type (3.3b). However, the following proposition shows that this property does not hold for the feasible region of the expansion planning problem.

Proposition 3.5.1. *The feasible regions of the continuous and discrete expansion planning problems are nonconvex.*

Proof. The discrete expansion planning problem is trivially nonconvex. Regarding the continuous (and split-pipe) problem, we consider a network of two pipes a_1, a_2 in parallel with adjacent nodes v and w. For given $\alpha = 2$ and $\beta = 5$, let a demand situation $b_v = 10 = -b_w$, pipe properties $L_a, R_a = 1$ and diameters $d_a \in [1, 2]$ for both pipes $a \in \{a_1, a_2\}$ be given. Then, the flow distribution among the parallel pipes is unique, where both pipes have the same flow direction, i.e., $\operatorname{sgn}(x_{a_1}) = \operatorname{sgn}(x_{a_2})$, and thus Equation (3.1) reduces to $\pi_v - \pi_w = L_a R_a / d_{a_1}^5 x_{a_1}^2 = L_a R_a / d_{a_2}^5 x_{a_2}^2$. Then,

$$\hat{s} := (\hat{x}_{a_1}, \hat{x}_{a_2}, \hat{d}_{a_1}, \hat{d}_{a_2}, \hat{\pi}_v, \hat{\pi}_w) \approx \Big(\frac{10}{31}(4\sqrt{2} - 1), -\frac{40}{31}(\sqrt{2} - 8), 1, 2, 100, 97.743362\Big)$$

and by symmetry

$$\tilde{s} := (\tilde{x}_{a_1}, \tilde{x}_{a_2}, \tilde{d}_{a_1}, \tilde{d}_{a_2}, \tilde{\pi}_v, \tilde{\pi}_w) \approx \Big(-\frac{40}{31}(\sqrt{2} - 8), \frac{10}{31}(4\sqrt{2} - 1), 2, 1, 100, 97.743362\Big)$$

are two different solutions of Problem (3.3), where the diameters and the corresponding flow values are swapped. But there exists an infeasible convex combination of both solutions, namely $s^* := 0.5\,(\hat{s}+\tilde{s}) = (x_{a_1}^*, x_{a_2}^*, d_{a_1}^*, d_{a_2}^*, \pi_v^*, \pi_w^*) \approx (5, 5, 1.5, 1.5, 100, 97.743362)$, because s^* violates Equation (3.3b), i.e.,

$$\pi_v^* - \pi_w^* \neq L_a R_a / (d_{a_1}^*)^5 (x_{a_1}^*)^2 = L_a R_a / (d_{a_2}^*)^5 (x_{a_2}^*)^2,$$

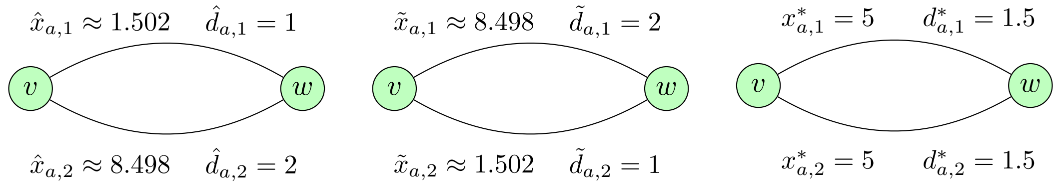
$$\text{since} \quad \pi_v^* - \pi_w^* \approx 2.256638 \quad \text{and} \quad L_a R_a / (d_{a_1}^*)^5 (x_{a_1}^*)^2 = 1/1.5^5 \cdot 5^2 \approx 3.292181.$$

□

An illustration of the proof is depicted in Figure 3.3. As a consequence of Proposition 3.5.1, the discrete Models A, B, C and the split-pipe Models D and E are nonconvex. For this reason, we use a general-purpose solver for nonconvex MINLPs in our computational study in the next section.

3.5.2 Ensuring feasibility by loop expansions

In this section, we demonstrate that it is always possible to find a solution to the expansion planning problem under the assumptions that no bounds on the arc flow exist, the intersection of all potential variable bounds is a non-empty interval, and the diameter candidates can be chosen sufficiently large, as formalized by Theorem 3.5.3. For the proof, we first show that no flow along any cycle occurs in passive pipe networks.



(a) feasible
$\hat{\pi}_v - \hat{\pi}_w \approx 2.257$

(b) feasible
$\tilde{\pi}_v - \tilde{\pi}_w \approx 2.257$

(c) infeasible
$\pi^*_v - \pi^*_w \neq 3.292$

Figure 3.3: An example that illustrates the nonconvexity of the split-pipe problem.

Lemma 3.5.2 (Cycle flow). *Given a passive pipe network $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ and a demand vector $(b_v)_{v \in \mathcal{V}}$ that satisfies $\sum_{v \in \mathcal{V}} b_v = 0$. Assume that $\mathcal{G}$ contains a cycle $C = (v_1, a_1, \ldots, a_{n-1}, v_n)$ with nodes $v_1, \ldots, v_n$ and possibly undirected pipes $a_1, \ldots, a_{n-1}$. Then, a vector $(\pi_v, x_a, d_a)_{v \in \mathcal{V}, a \in \mathcal{A}}$, which satisfies the flow conservation* (3.3b) *and the potential loss constraint* (3.3c) *holds that there is no flow along cycle C except for zero flow.*

Proof. Let $(\pi_v, x_a, d_a)_{v \in \mathcal{V}, a \in \mathcal{A}}$ be given and satisfy (3.3c) and (3.3b). Assume that there is non-zero flow along a cycle $C = (v_1, a_1, \ldots, a_{n-1}, v_n)$, i.e., $x_{a_i} \neq 0$ holds for all $a_i = (v_i, v_{i+1})$ with $i \in \{1, \ldots, n-1\}$. Then,

$$0 = \pi_{v_1} - \pi_{v_1} = \sum_{i: a_i = (v_i, v_{i+1}) \in C} \pi_{v_i} - \pi_{v_{i+1}} = \sum_{i: a_i = (v_i, v_{i+1}) \in C} \frac{L_{a_i} R_{a_i}}{d_{a_i}^{\beta}} \operatorname{sgn}(x_{a_i}) |x_{a_i}|^{\alpha} \neq 0.$$

Hence, there is no flow along a cycle (only zero flow). □

Theorem 3.5.3. *Given a passive pipe network $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ and a demand vector $(b_v)_{v \in \mathcal{V}}$ that satisfies $\sum_{v \in \mathcal{V}} b_v = 0$. Assume that $int\big(\cap_{v \in \mathcal{V}} [\underline{\pi}_v, \overline{\pi}_v]\big) \neq \emptyset$ and $x_a \in \mathbb{R}$ for all $a \in \mathcal{A}$. If the pipeline diameters can be chosen sufficiently large, then Problem* (3.3a) – (3.3d) *is feasible.*

Proof. Let $[\pi_{\min}, \pi_{\max}] = int\big(\cap_{v \in \mathcal{V}} [\underline{\pi}_v, \overline{\pi}_v]\big)$ with $\pi_{\min} < \pi_{\max}$ and let $\overline{b}$ be the overall network inflow, i.e., the sum of all flows that enter the network at the entry nodes. According to Lemma 3.5.2, no flow occurs along cycles in the network, hence it follows that $\overline{b}$ is an upper bound on the flow along any arc in the network. We now select a diameter d_a for each arc $a \in \mathcal{A}$ such that

$$\frac{L_a R_a}{{d_a}^{\beta}} \operatorname{sgn}(\overline{b}) |\overline{b}|^{\alpha} \leq \frac{\pi_{\max} - \pi_{\min}}{|\mathcal{A}|}.$$

For fixed values $(d_a)_{a \in \mathcal{A}}$, let $(\pi_v, x_a)_{v \in \mathcal{V}, a \in \mathcal{A}}$ be a solution of Problem (3.3a) – (3.3c). According to Collins et al. (1978) and Humpola (2014), the solution of this relaxed problem exists. Furthermore it has unique flow values and the potential values are uniquely determined up to a constant shift, which obviously satisfy

$$\pi_v - \pi_w = \frac{L_a R_a}{{d_a}^{\beta}} \operatorname{sgn}(x_a) |x_a|^{\alpha} \leq \frac{L_a R_a}{{d_a}^{\beta}} \, \overline{b}^{\alpha} \leq \frac{\pi_{\max} - \pi_{\min}}{|\mathcal{A}|} \quad \forall a \in \mathcal{A} \text{ and } (v, w) = r(a).$$

In the following, we show that the potential variables lie within $[\pi_{\min}, \pi_{\max}]$. Summing up these inequalities along any cycle-free path $\overrightarrow{v_0 v_k}$ between two connected nodes v_0 and v_k yields:

$$|\pi_{v_0} - \pi_{v_k}| = \left|\sum_{i=0}^{k-1} \pi_{v_i} - \pi_{v_{i+1}}\right| = \left|\sum_{i=0}^{k-1} \frac{L_{(v_i,v_{i+1})} R_{v_i,v_{i+1}}}{{d_{(v_i,v_{i+1})}}^{\beta}} \operatorname{sgn}(x_{(v_i,v_{i+1})}) \left|x_{(v_i,v_{i+1})}\right|^{\alpha}\right|$$
$$\leq \sum_{i=0}^{k-1} \frac{L_{(v_i,v_{i+1})} R_{(v_i,v_{i+1})}}{{d_{(v_i,v_{i+1})}}^{\beta}} \,\overline{b}^{\alpha} \leq \sum_{i=0}^{k-1} \frac{\pi_{\max} - \pi_{\min}}{|\mathcal{A}|} \overset{k \leq |A|}{\leq} \pi_{\max} - \pi_{\min}.$$

We now choose a node v_0 with the highest potential in the solution, i.e., $\pi_{v_0} = \max_{v \in \mathcal{V}}\{\pi_v\}$, and shift this potential value by a constant $\Delta_{v_0} \in \mathbb{R}$ such that it is equal to the upper bound $\pi_{\max}$, i.e., we have for node v_0 a new potential value: $\tilde{\pi}_{v_0} := \pi_{v_0} + \Delta_{v_0} = \pi_{\max}$. If we shift the potentials π_v of the other nodes by the same constant Δ_{v_0}, all potentials are guaranteed to satisfy (3.3d). Besides, the flow distribution $(x_a)_{a \in \mathcal{A}}$ is invariant to the shift value Δ_{v_0}, since $\tilde{\pi}_v := \pi_v + \Delta_{v_0}$ implies

$$\begin{aligned} \pi_v - \pi_w &= \frac{L_a R_a}{{d_a}^{\beta}} \operatorname{sgn}(x_a)|x_a|^{\alpha} \\ \Rightarrow \quad \tilde{\pi}_v - \tilde{\pi}_w &= \frac{L_a R_a}{{d_a}^{\beta}} \operatorname{sgn}(x_a)|x_a|^{\alpha}. \end{aligned}$$

Then, $(\tilde{\pi}_v, x_a, d_a)_{v \in \mathcal{V}, a \in \mathcal{A}}$ is a solution of Problem (3.3a) – (3.3d). □

As a result of Theorem 3.5.3, a solution exists for all presented Models A – E for any instance of a passive pipe network that satisfies the assumptions of the theorem.

3.5.3 Braess's paradox

In the context of road networks, Braess discovered that potential improvements of traffic flows, e.g., opening new streets, could lead to a deterioration of the travel time (Braess, 1968). The paradox is a counterexample to the monotonicity assumption that more road network capacity leads to more throughput. This effect is called Braess's paradox and has also been discovered to appear in nonlinear potential-driven networks, see Calvert and Keady (1993). It is also known in gas network expansion planning, as "more pipeline - less throughput phenomenon" and occurs when adding a new element to the gas network might lead to infeasibility (Szabó, 2012).

Figure 3.4 illustrates that this phenomenon may also occur in the context of looping even though practitioners report that the Braess's paradox is unlikely to happen in gas network operations[1]. The maximal potential difference in the network of a feasible state (left Figure 3.4) might be increased when doubling pipe (z, w) (middle Figure 3.4). While e.g., André (2010) states conditions on the network topology that avoid such paradox, we provide a way to resolve an infeasible network state (where the infeasibility is not necessarily due to Braess's paradox).

[1]Communication with engineers from Open Grid Europe GmbH.

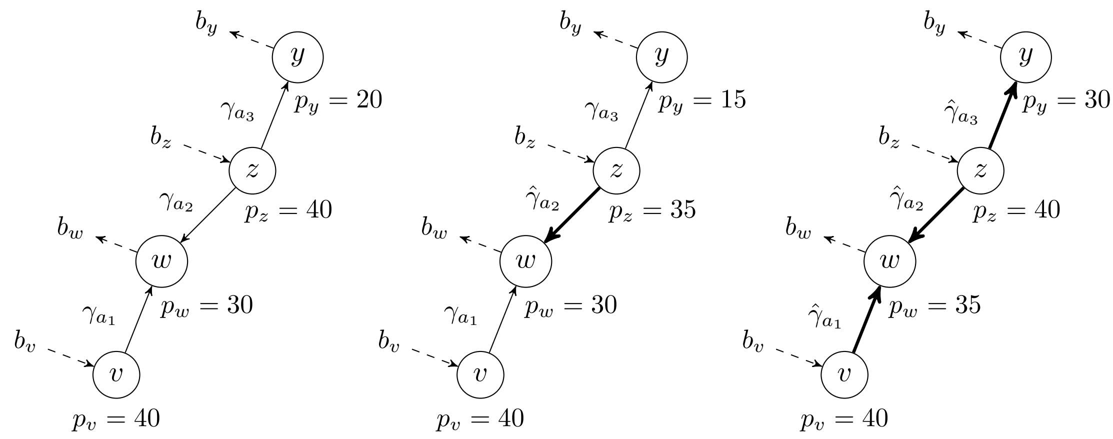


Figure 3.4: Feasible network state (left) with source nodes v and z and exit nodes w and y for given potential bounds $p_v, p_z, p_w, p_y \in [20, 40]$. Looping pipe $a_2 = (z, w)$ (thick, middle) with factor s decreases the potential loss along pipe $a_2 = (z, w)$, but increases the maximal potential difference in the network (Braess's Paradox) between the nodes v and y and leads to an infeasible state, depending on the potential bounds, here $p_y \notin [20, 40]$. Instead, fixing the highest occurring potential value ($p_v = 40$) and multiplying the pipes constants $\gamma_{a_1} := L_{a_1} R_{a_1} / d_{a_1}^{\beta}$, $\gamma_{a_2} := L_{a_2} R_{a_2} / d_{a_2}^{\beta}$, and $\gamma_{a_3} := L_{a_3} R_{a_3} / d_{a_3}^{\beta}$ by the same factor $0 < s < 1$, i.e., $\hat{\gamma}_{a_1} = s\,\gamma_{a_1}$, $\hat{\gamma}_{a_2} = s\,\gamma_{a_2}$ and $\hat{\gamma}_{a_3} = s\,\gamma_{a_3}$ resolves the infeasibility by shrinking the overall potential difference in the network (right).

Conversion of infeasible instances into feasible ones Motivated by the fact that a feasible network state exists under the assumptions of Theorem 3.5.3, Proposition 3.5.4 explicitly shows how to turn an infeasible network state feasible: First, we solve the relaxed Problem (3.3a) – (3.3c), for which a feasible solution exists (Collins et al., 1978; Humpola, 2014), and then tighten the resulting potential differences along the pipes of the network such that Equation (3.3d) is satisfied. To this end, we multiply the term $d_a^{-\beta}$ of all pipes with a common factor $0 < s < 1$ that is chosen appropriately small. This multiplication shifts $d_a^{-\beta} > 0$ towards zero, and corresponds to enlarging all diameters by the same factor, cf. right Figure 3.4.

Proposition 3.5.4. *Given a passive pipe network and a demand vector* $(b_v)_{v \in \mathcal{V}}$ *that satisfies* $\sum_{v \in \mathcal{V}} b_v = 0$. *Assume* $int\big(\cap_{v \in \mathcal{V}} [\underline{\pi}_v, \overline{\pi}_v]\big) \neq \emptyset$ *and* $x_a \in \mathbb{R}$ *for all* $a \in \mathcal{A}$. *If the pipeline diameters can be chosen sufficiently large, i.e.,* $\tilde{d}_a \in [D_{a,0}, \infty)$*, then a solution* $(\tilde{\pi}_v, \tilde{x}_a, \tilde{d}_a)_{v \in \mathcal{V}, a \in \mathcal{A}}$ *of Problem* (3.3a) – (3.3d) *exists with* $\tilde{\pi}_v := s\,\pi_v^* + \pi_{\min}$, $0 < s < 1$, *and* $\tilde{x}_a = x_a^*$ *and* $\tilde{d}_a = d_a^* / \sqrt[\beta]{s}$, *where* $(\pi_v^*, x_a^*, d_a^*)_{v \in \mathcal{V}, a \in \mathcal{A}}$ *is a solution of the relaxed Problem* (3.3a) – (3.3c).

Proof. Let $[\pi_{\min}, \pi_{\max}] = int\big(\cap_{v \in \mathcal{V}} [\underline{\pi}_v, \overline{\pi}_v]\big)$ with $\pi_{\min} < \pi_{\max}$ and let $(\pi_v^*, x_a^*, d_a^*)_{v \in \mathcal{V}, a \in \mathcal{A}}$ be a solution of the relaxed Problem (3.3a) – (3.3c). At first, we select $0 < s < 1$ such that $s\,(\max_{v \in \mathcal{V}} \pi_v^*) + \pi_{\min} \leq \pi_{\max}$. Then, it follows that $s\,\pi_v^* \in [0, \pi_{\max} - \pi_{\min}]$ for all

$v \in \mathcal{V}$. By defining $\tilde{\pi}_v := s\pi_v^* + \pi_{\min}$, the common potential bounds are satisfied, i.e.,

$$\tilde{\pi}_v := (s\pi_v^* + \pi_{\min}) \in [\pi_{\min}, \pi_{\max}]$$

for all $v \in \mathcal{V}$, and hence the original bounds (3.3d) are satisfied. Let $\tilde{x}_a := x_a^*$ and $\tilde{d}_a := d_a^* \,/\, \sqrt[\beta]{s}$, then we can deduce from the definition of $\tilde{\pi}_v, \tilde{x}_a, \tilde{d}_a$ and the solution $(\pi_v^*, x_a^*, d_a^*)_{v\in\mathcal{V}, a\in\mathcal{A}}$ of the relaxed problem:

$$\pi_v^* - \pi_w^* = \frac{L_a R_a}{{d_a^*}^\beta} \operatorname{sgn}(x_a^*)|x_a^*|^\alpha \tag{3.19}$$

$$\Rightarrow \quad \tilde{\pi}_v - \tilde{\pi}_w = s\,(\pi_v^* - \pi_w^*) = s\frac{L_a R_a}{{d_a^*}^\beta} \operatorname{sgn}(x_a^*)|x_a^*|^\alpha$$

$$\Rightarrow \quad \tilde{\pi}_v - \tilde{\pi}_w = \frac{L_a R_a}{\tilde{d}_a^\beta} \operatorname{s\tilde{g}n}(x_a)|\tilde{x}_a|^\alpha \tag{3.20}$$

for all $a = (v, w) \in \mathcal{A}$, with $(v, w) = r(a)$. Thus, constraint (3.3b) is satisfied. Furthermore, the flow conservation (3.3c) holds, since the flow distribution $(x_a^*)_{a\in\mathcal{A}}$ is invariant to scaling the term $L_a R_a / {d_a^*}^\beta$ in (3.3b) for all arcs with a common arc-independent factor $0 < s < 1$, and hence both Equations (3.19) and (3.20) can be resolved to

$$x_a^* = \operatorname{sgn}(\pi_v^* - \pi_w^*) \sqrt[\alpha]{|(\pi_v^* - \pi_w^*)| \,/\, (L_a R_a / {d_a^*}^\beta)}$$

$$\iff \quad x_a^* = \operatorname{sgn}(s\,(\pi_v^* - \pi_w^*)) \sqrt[\alpha]{|s\,(\pi_v^* - \pi_w^*)| \,/\, (s\, L_a R_a / {d_a^*}^\beta)}$$

$$\iff \quad \tilde{x}_a = \operatorname{sgn}(\tilde{\pi}_v - \tilde{\pi}_w) \sqrt[\alpha]{|(\tilde{\pi}_v - \tilde{\pi}_w)| \,/\, (L_a R_a / \tilde{d}_a^\beta)}$$

for all $a = (v, w) \in \mathcal{A}$, with $(v, w) = r(a)$. □

Realizing the solution $(\tilde{\pi}_v, \tilde{x}_a, \tilde{d}_a)_{v\in\mathcal{V}, a\in\mathcal{A}}$ from Proposition 3.5.4 might only be possible within the split-pipe and not within the discrete approach because $\tilde{d}_a$ is (most probably) continuous. Even rounding up $\tilde{d}_a$ to discrete values might not be feasible due to Braess's paradox. For sure, from Theorem 3.5.3 we already know that a solution also exists to the discrete approach when selecting sufficiently large discrete diameters. But such solution results in different flow distributions than the one of the relaxed Problem. In general, enabling solutions that are invariant to certain flow patterns is a significant advantage of the split-pipe approach over the discrete approach.

For split-pipes, the extension diameters $(\tilde{d}_a)_{a\in\mathcal{A}}$ can be achieved by selecting $i_0 \in [k_a - 1]$ such that $\lambda_{a,i_0}^* + \lambda_{a,i_0+1}^* = 1$ and $0 \le \lambda_{a,i_0}^*, \lambda_{a,i_0+1}^* \le 1$ yielding

$$\tilde{d}_a^{-\beta} = \lambda_{a,i_0+1}^*\, D_{a,i_0+1}^{-\beta} + \lambda_{a,i_0}^* D_{a,i_0}^{-\beta}. \tag{3.21}$$

In our computational study, we will utilize Proposition (3.5.4) to resolve infeasible split-pipe instances, as its proof is constructive. For an infeasible instance, we first solve the relaxed Problem (3.3a) – (3.3c), where the pipe diameters are fixed to $D_{a,0}$. Then, from its solution, we calculate the shifting factor $s = (\pi_{\max} - \pi_{\min})/(\max_{v\in\mathcal{V}} \pi_v^*)$ and the extension diameters $\tilde{d}_a = D_{a,0} \,/\, \sqrt[\beta]{s}$.

3.5.4 Symmetry in looping split-pipe segments

In general, optimization problems are called symmetric, if their structure is invariant to a permutation of the variables. Symmetric optimization problems arise in several applications, for example, in graph coloring problems when permuting the colors, in cutting stock problems, where rolls have identical widths, in scheduling and assignment problems with parallel identical machines, in packing, and covering problems. It is well known that symmetric problems are difficult to solve using branch-and-bound solvers, since branching creates several isomorphic subproblems in the tree, whose exploration might cause a significant slowdown. To avoid symmetries and to save computational efforts, different strategies have been developed, such as applying symmetry-breaking constraints, perturbing the problem, fixing variables, just to mention a few, for more information we refer to Margot (2010). Another possibility are model reformulations that typically introduce extended formulations (Vanderbeck and Wolsey, 2010) where the symmetry disappears. Examples of symmetry avoiding reformulations for cutting stock problems can be found in Gilmore and Gomory (1961), and for graph coloring problems in Mehrotra and Trick (1996).

In the context of split-pipe loop expansions, symmetries, in general, arise through the twofold possibility of arranging two split-pipe segments along a pipeline, see Figure 3.5.

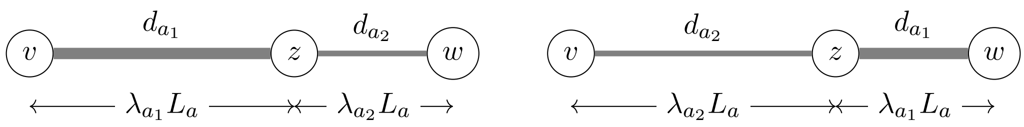


Figure 3.5: Symmetry in split-pipe segments. Proposition 3.5.5 states that the potential loss along pipe $a = (v, z)$ is independent of the order of the pipe segments (v, z) and (z, w).

The following proposition shows that looping split-pipe segments with no source or sink in between is independent of their order.

Proposition 3.5.5 (Symmetry of looping split-pipe segments)**.** *Let a pipe $a = (v, w)$ be given that is split into serial segments $a_1 = (v, z)$ and $a_2 = (z, w)$, where the intermediate node z has zero demand $b_z = 0$ and let $R_a = R_{a_1} = R_{a_2}$. Then, looping both segments with different diameters and lengths is independent of their order.*

Proof. Let the pipe characteristics of both segments be given by $\gamma_{a_1} := \lambda_{a_1} L_a R_a / d_{a_1}^{\beta}$ and $\gamma_{a_2} := \lambda_{a_2} L_a R_a / d_{a_2}^{\beta}$, where d_{a_1}, d_{a_2} denote the diameters and $\lambda_{a_1} L_a$, $\lambda_{a_2} L_a$ the length of both segments, with $0 \le \lambda_{a_1}, \lambda_{a_2} \le 1$ and $\lambda_{a_1} + \lambda_{a_2} = 1$, while L_a denotes the length of pipe $a = (v, w)$. Since flow along the segments holds $x_a = x_{a_1} = x_{a_2}$, it follows

$$\begin{aligned}\pi_v - \pi_w = \pi_v - \pi_z + \pi_z - \pi_w &= \gamma_{a_1} x_{a_1} \, |x_{a_1}| + \gamma_{a_2} \, x_{a_1} |x_{a_1}| \\ &= (\gamma_{a_1} + \gamma_{a_2}) \, x_{a_1} |x_{a_1}| = (\gamma_{a_2} + \gamma_{a_1}) \, x_{a_1} |x_{a_1}|.\end{aligned}$$

□

However, let us mention that the model formulations D and E of the split-pipe approach neglect the decisions on the order of the segments and thereby avoid the resulting symmetries. As a consequence, the choice of how to arrange both segments in practice belongs to the network planner.

3.6 Computational Study

In this computational study, we first investigate the performance of the discrete Models A, B and C, and then compare the performance of the split-pipe Models D and E. In the case that infeasible split-pipe instances allow to apply Proposition 3.5.4, we convert them into feasible ones. Afterwards, we analyze the difference in the performance of the discrete versus the split-pipe models. Especially from the point of view of practical applications, it is important to see whether the split-pipe approach can lead to significant cost-savings when compared to the discrete looping approach.

3.6.1 Test sets

Gas and water networks. The computational study was carried out on different data sets that vary in size and structure and are based on two major types of potential-driven networks: gas networks and water networks. As the starting point, we used the *Belgian* gas network from the GAMS model library[2], which is of rather smaller size and has a simple, almost tree-shaped structure with the only exception being 5 arcs that have 2 parallel pipelines. We continued our computational experiments on the *GasLib-40* and *GasLib-135* networks (Schmidt et al., 2017a) to test the models on networks that are considerably larger and more complex. As the basis for our water instances, the well-known *New York* network[3] and an extended version with four additional arcs, simply denoted as *New York+*, was employed. For the new pipelines of the extended network, we use an original diameter $D_{a,0}$ that is equal to the average diameter of the existing pipelines in the network.

Finally, to focus not only on computational difficulties that arise from the size of the network, we systematically investigate the impact of cycles on the model performance. It is well known that optimization problems on potential-driven networks get more challenging to solve the more cycles they contain because the existence of cycles leads to more complex patterns of flow directions, cf. the literature review in Shiono and Suzuki (2016). As measure for the number of cycles, we use the so-called *circuit rank*, see the following definition.

Definition 3.6.1 (Circuit rank)**.** *Let $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ be given. The circuit rank of $\mathcal{G}$ is given by $|\mathcal{A}| - |\mathcal{V}| + 1$.*

To systematically test the models also with respect to this type of difficulty, we successively increase the circuit rank by adding up to ten new pipelines to the *Belgian* network, which results in five new networks *Belgium* + (2, 4, 6, 8, 10) arcs, see Figure 3.6a.

[2] General Algebraic Modeling System (GAMS) Model Library
`https://www.gams.com/modlib/modlib.htm`

[3] Library of Codes and Instances Page – Operations Research Group Bologna
`http://or.dei.unibo.it/library`

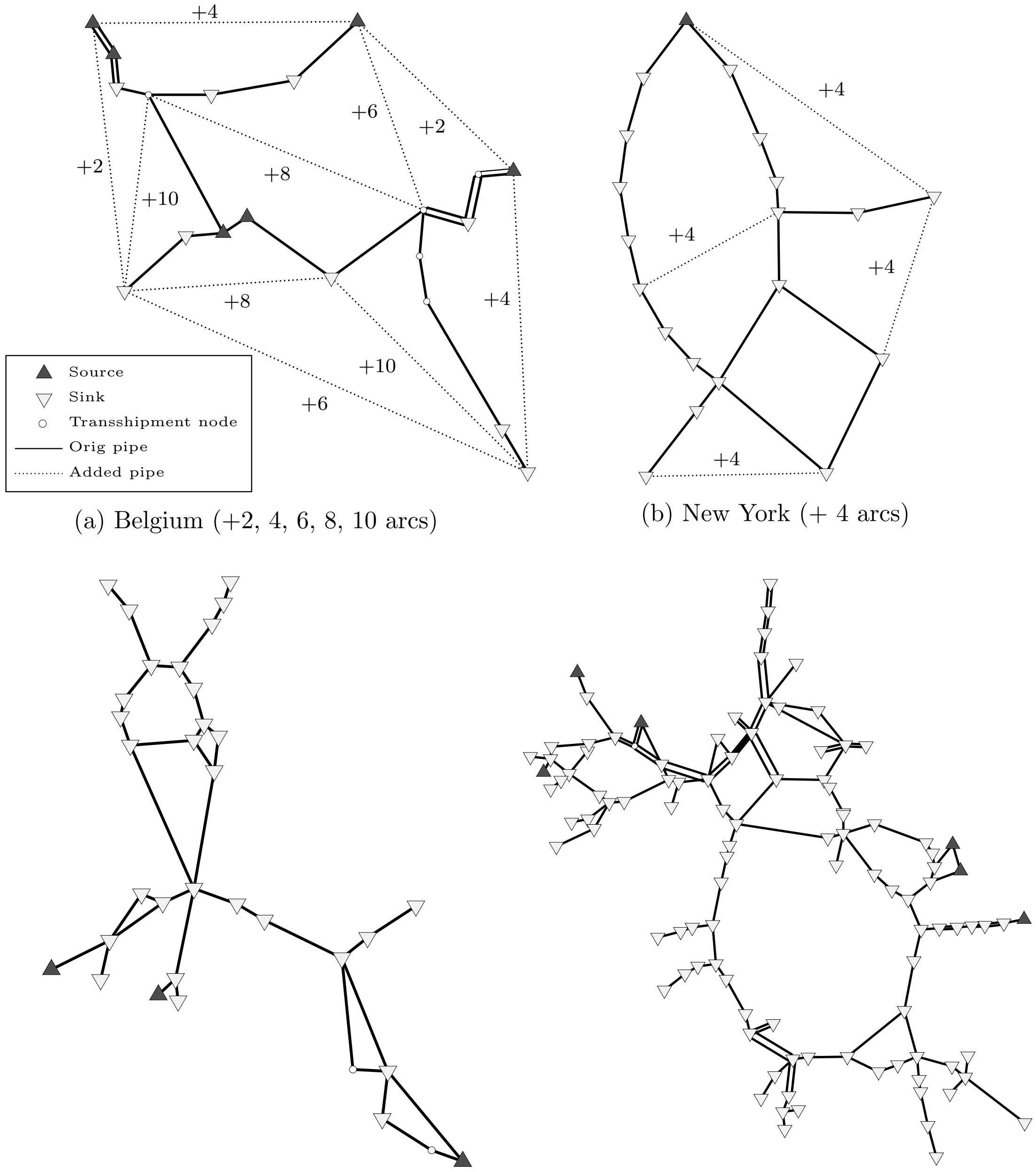
Again, we assign a diameter $D_{a,0}$ to the new pipelines that is equal to the average diameter of the existing pipelines in the *Belgian* network. Here, but also for the extended version of the *New York* network, the intention is to add arcs that connect different regions of the network and have a high impact on the network topology. The details about the different networks are found in Table 3.1 and a visualization in Figure 3.6.

	# Nodes	# Sources	# Sinks	# Arcs	Circuit rank	# Diameters Models	
	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{V}^+\rvert$	$\lvert\mathcal{V}^-\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{A}\rvert - \lvert\mathcal{V}\rvert + 1$	Split-Pipe	Discrete
Belgium	20	5	9	24	5	308	418
GasLib-40	34	3	29	39	6	488	678
GasLib-135	105	6	99	141	37	2,093	2,488
New York	20	1	20	21	2	191	327
New York + 4 arcs	20	1	20	25	6	223	387
Belgium + 2 arcs	20	5	9	26	7	332	452
Belgium + 4 arcs	20	5	9	28	9	356	486
Belgium + 6 arcs	20	5	9	30	11	380	520
Belgium + 8 arcs	20	5	9	32	13	404	554
Belgium + 10 arcs	20	5	9	34	15	428	588

Table 3.1: Used networks in the computational study. The number of diameters corresponds to an aggregated number of the extension candidates for all pipes in the network.

We note that originally the *Belgian*, *GasLib-40* and the *GasLib-135* networks contain compressor stations the treatment of which is outside the scope of this chapter. In the case of the *Belgian* network, we used data available in the GAMS model library to model these as normal pipelines, while arcs representing compressor stations had to be contracted in the case of the *GasLib-40* and *GasLib-135* networks due to lack of available data.

Demands. For each network, we generated instances that cover a wide range of possible network demands. It is known in practice that increasing the overall network demand typically results in more complex transport situations that stress the networks. Practitioners therefore use the so-called transport moment to detect severe demand situations by approximating the transport load in the network (cf. Maring, 2017; Hiller et al., 2018). It can even be observed in many computational experiments that higher network demands tend to slow down the model performance, provided that the transport situations are not trivially infeasible. Hence, to test the performance of the models in diverse and severe situations, we consider different demand loads that represent the whole spectrum from "easier" instances with lower demands up to "harder" instances with higher demands for each network, whereas for the *Circuit rank* experiment, we use demand scenarios with a constant overall network demand. An overview of the different demand loads $\mathcal{B}$ (in $[10^6\ \text{m}^3/\text{day}]$) used for the particular networks is given in Table 3.2.



(a) Belgium (+2, 4, 6, 8, 10 arcs)

(b) New York (+ 4 arcs)

(c) GasLib-40

(d) GasLib-135

Figure 3.6: *Belgian* (a) and *New York* (b) and *GasLib-40* (c) and *GasLib-135* networks (d) in schematic view, being illustrated by the bold lines. Dotted lines indicate the *Belgian* network with $2, 4, 6, 8, 10$ additional arcs and the *New York* network with 4 additional arcs.

Test set	Network	Total demand $\mathcal{B}$					
		50	100	150	200	500	1,000
Belgium	*Belgium*		✓		✓	✓	✓
GasLib-40	*GasLib-40*	✓	✓		✓	✓	
GasLib-135	*GasLib-135*	✓	✓	✓		✓	
Circuit rank	*Belgium + 2, 4, 6, 8, 10 arcs*				✓		
New York	*New York*		✓		✓	✓	
New York+	*New York + 4 arcs*		✓		✓	✓	

Table 3.2: Test sets with network and demand loads used in the computational study. The total network demand value $\mathcal{B}$ is denoted in $[10^6\ \mathrm{m}^3/\mathrm{day}]$.

Throughout, we randomly generated scenarios for each given total network demand $\mathcal{B}$ according to a random uniform distribution of the demand at sources and sinks. More precisely speaking, for all sources $\mathcal{V}^+$ and for all sinks $\mathcal{V}^-$, we generated uniformly distributed independent random numbers $r_v \in [0, 1)$ and then normalized the source and sink flows to calculate the demand as $b_v = \mathcal{B}\ r_v / \sum_{v \in \mathcal{V}^+} r_v$ for all $v \in \mathcal{V}^+$ and $b_v = \mathcal{B}\ r_v / \sum_{v \in \mathcal{V}^-} r_v$ for all $v \in \mathcal{V}^-$. For more sophisticated methods about the generation of balanced scenarios in gas networks, we refer to Hiller et al. (2015).

For each given total demand $\mathcal{B}$ and for each network, we generated 500 scenarios because preliminary tests indicated that this sample size leads to sufficiently stable results for the given distribution with respect to model performances. Including preliminary tests, we calculated about 300,000 instances. Due to this large number and variety of instances, we can assume that the results provide us with a representative picture for model comparisons. Here, we present a selection of 7,500 different demand scenarios and a total of 57,500 instances that represent the major effects observed.

Expansion capacities. Throughout the test sets, the expansion capacities of all instances are given by the diameter candidates $\{D_{a,0}, \ldots, D_{a,k_a}\}$ resulting from at most triple loops.

In our experiments, we use a quadratic cost function as this is the most common form in practical applications, (see e.g., De Wolf and Smeers, 1996; Parker, 2005). The data points for the cost function used for the gas related instances are shown in Figure 3.1a and were provided to us by practitioners, and for the water instances it was part of the *New York* network data. We remark that cost functions from linear to cubic can also be found in the literature, see Osiadacz and Górecki (1995) and Babonneau et al. (2012), for example, and were used as part of our preliminary tests.

Summarizing, with the maximum number of parallel loops fixed to three and the quadratic cost function being given, one instance is characterized by the choice of network and demand scenario.

3.6.2 Experimental setup

The computational experiments were conducted on a cluster of 64-bit Intel Xeon CPU E5-2670 v2 CPUs at 2.5 GHz with 25 MB cache and 128 GB main memory. In order to safeguard against a potential mutual slowdown of parallel processes, we bind the processes to specific cores and run at most 4 jobs per node. We solve the split-pipe and discrete expansion planning problems to global optimality using the nonconvex MINLP solver SCIP version 5.0.1 (Gleixner et al., 2017) with CPLEX 12.7.1 (Cplex, IBM ILOG, 2019) as LP solver and Ipopt 3.12.6 (Wächter and Biegler, 2006) as NLP solver. As modeling languages we use GAMS 25.0.2 (GAMS, 2019) for the water instances and ZIMPL 3.3.5 (Koch, 2004) for the gas network instances. In all experiments, we ran SCIP with default settings and set a time limit of four hours. This time limit seems reasonable for practical purposes since expansion planning is carried out with a long-term perspective and decisions on short notice are not required as in daily operations. We solve the instances to ϵ-global optimality[4] with a gap limit of $\epsilon = 10^{-4}$. As performance measure, we use the number of solved instances, the runtime and the number of branch-and-bound nodes. In order to reduce the impact of very easy or hard instances in the mean values, we report the shifted geometric mean ($\sqrt[n]{\prod(t_i + s)} - s$) for values $t_1, \ldots, t_n$ with a shift of $s = 10$ for the runtime in seconds and $s = 100$ for the number of branch-and-bound nodes, as explained in Section 2.2.

Presentation of results. The computational results for the comparison of the discrete Models A, B and C are found in Figures 3.7 – 3.12 and Tables 3.3a – 3.3f, and for the comparison of the split-pipe Models D and E in Figures 3.13 – 3.18 and Tables 3.4a – 3.4f. The figures illustrate for each data set and for each model under consideration the number of instances solved to optimality (green), the number of instances for which a solution with non-zero optimality gap has been found (orange), the number of infeasible instances (red), and the remaining instances (gray). We visualize this information in stacked bar charts, where each bar summarizes the results of all 500 instances of a particular data set. Consequently, bars with longer amounts in green and red for the optimal and infeasible instances, and in orange for the additional feasible instances are preferred over gray amounts. Information on the runtime can be found in the tables. More precisely, the tables present for each data set the model with the best performance (row *best model*), its runtime as shifted geometric mean in seconds (row *sgm of best model*), and the relative time factor of the remaining models to the best performing model (row *factor*).

Further aggregated information on the computational results can be found in Tables A.47 – A.57 in the appendix. There, we additionally report the arithmetic mean of the computation time to indicate the time required for applications in practice. Moreover, these tables report the number of branch-and-bound nodes for those instances that are solved by all models under comparison (row *All opt*) and the performance of the models with respect to the instances that they alone were able to solve to global optimality (row *Only opt by*). Finally, detailed instance-wise results are reported in Tables A.1 – A.46.

[4]For best known upper and lower bounds U, L, the solver SCIP terminates if either $U - L \leq \epsilon$ or the relative gap limit is reached, i.e., U, L have the same sign, $|U|, |L| > 10^{-9}$, and $|U - L|/\min\{|U|, |L|\} \leq \epsilon$.

3.6.3 Comparison of discrete models (Models A, B and C)

***Belgian* gas network.** We begin the computational experiments with the *Belgian* gas network, which has frequently been used as a tool in research about network optimization (see e.g., De Wolf and Smeers (1996), De Wolf and Smeers (2000) and Babonneau et al. (2012)). The computational results for the network are found in Figure 3.7 and Tables 3.3a and A.47.

We first consider the performance of the models as depending on the demand. For the lowest demand $\mathcal{B} = 100$ about all instances are solved to optimality within a couple of minutes. With increasing demand, computational time increases and the number of instances solved to optimality decreases. However, at a demand of $\mathcal{B} = 1{,}000$, we have reached a point where an optimum has been found for less than a quarter of the instances and more than 60 % of the instances are detected as infeasible (Figure 3.7d). As a consequence of the high number of infeasibilities, the number of solved instances has increased and computational time decreased. This pattern suggests that the range of demand used here is sufficient to obtain an overview of the general behavior of the models and that it is not necessary to test the models for a demand of less than $\mathcal{B} = 100$ and greater than $\mathcal{B} = 1{,}000$. Due to this typical pattern, which was frequently observed also during our preliminary computational experiments, we also use a test range from a low demand to a high demand with a large number of infeasibilities (or a large number of unsolved instances) in the other test sets.

We observe that Model A is the fastest model throughout, up to a factor 4.3 faster than Model B for a demand of $\mathcal{B} = 100$, and up to a factor 2.6 faster than Model C for $\mathcal{B} = 200$ and $\mathcal{B} = 500$. On the other hand, Model B solves the largest number of optimal and infeasible instances, in sum 53 instances more than Model A. Model B also has a high number of instances that were solved fast, for this reason B is the fastest with respect to the arithmetic mean, see Table A.47 in the appendix.

Let us remark in passing that during our preliminary tests of the two models mentioned in Section 3.3.4, which were carried out on the rather simple *Belgian* network, the model in Humpola (2014) ran into the time limit at a demand of $\mathcal{B} = 200$ with about 2/3 of all instances, while the model in Borraz-Sánchez et al. (2016) exceeded the time limit in all cases for the same demand. In contrast with this, the three models considered here ran into the time limit only with 0.8% to 6.6% of the same instances.

***GasLib-40* and *GasLib-135* gas networks.** We now turn our attention to *GasLib-40*, see Figure 3.8 and Tables 3.3b and A.48. Again we can see that Model A is the fastest model. This time, Models A and B solve a similar amount of instances. While Model A solves 19 more instances for $\mathcal{B} = 500$, Model B solves 16 more instances for $\mathcal{B} = 1{,}000$ including the infeasible ones. Finally note that a demand of $\mathcal{B} = 500$ is too difficult for all three models, i.e., for unfavorable demand situations we have reached the limits of what is computational tractable. (But obviously, this does not imply that expansion problems on more complex networks cannot be solved for favorable demand situations.)

On *GasLib-135*, Models A and C solve nearly all instances for a demand of $\mathcal{B} = 50$, while their limit is reached at a demand of $\mathcal{B} = 150$, see Figure 3.9c. In contrast, Model B practically does not solve any instance, not even for the lowest demand $\mathcal{B} = 50$. Even though Model A is the fastest again (Table 3.3c), it is worth mentioning that Model C solves the most instances (at all 11 more optimal + infeasible instances than Model A). Moreover, Model C throughout performs best on *GasLib-135* with respect to the runtime of all instances in the arithmetic mean, see Table A.49.

Circuit rank on *Belgian* gas network. To focus not only on computational difficulties that arise from the size of the network, we systematically investigate the impact of cycles on the model performance. Recall that we increase the circuit rank by adding up to ten new pipelines to the *Belgian* network, which results in five new networks *Belgium* + (2, 4, 6, 8, 10) arcs. Here, we throughout use the same demand scenarios with $\mathcal{B} = 200$ as for the *Belgian* network.

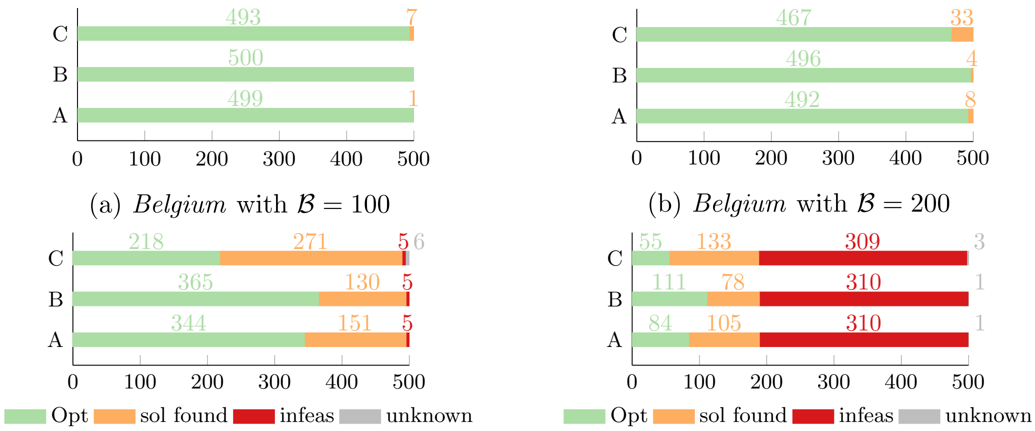
We observe in Figure 3.10 that for increasing circuit rank, the number of instances solved to optimality at first decreases and then increases again for Models A and C, while it continues to decrease for Model B. A similar pattern applies with respect to computational time, where the runtime at first increases for Models A and C and then decreases for the instances with 6 or more additional arcs, while the runtime of Model B increases nearly throughout, see Tables 3.3d and A.50. This leads to a situation where Model A turns out to have the best runtime for all circuit ranks, B solves the most instances for smaller circuit ranks, and A solves the most instances for larger circuit ranks. Across all circuit ranks, Model A solves 61 more instances than B. Finally, while Model C shows poor results for small circuit ranks, it gains ground with increasing circuit ranks and performs second best for the instances with *Belgium* + 10 arcs. Besides, it is worth mentioning that Model C is the only Model which finds a feasible solution to all instances of the entire test set.

***New York* water networks.** The famous *New York* network is of particular interest for the capacity expansion problem, as it is the only network in the water library where the arcs are already laid out with given diameters. Introduced by Schaake and Lai (1969), these instances have been used extensively, (see e.g., Quindry et al., 1981; Bhave, 1985; Bragalli et al., 2012). On this network, Model B clearly performs best, both regarding the number of instances solved and the runtime, see Figure 3.11 and Tables 3.3e, A.51. We note that we have instances here, where Model C performs better than Model A in terms of both solved instances and runtime ($\mathcal{B} = 200$).

To further test the models' performance on a second water network, we add four new arcs to the *New York* network akin to the requirements of the previous circuit rank experiment. Here, we take the same demand scenarios as generated for *New York*. The impact of the four additional arcs can directly be seen by comparing the results of the original and the extended networks *New York* and *New York+* for $\mathcal{B} = 200$ in Figures 3.11b and 3.12b: The arc extensions turn at least 55 (= 79 - 24) instances feasible. This can be derived by the fact that 79 instances are infeasible for *New York* (cf. Model B in Figure 3.11b), whereas for *New York+* no instance is detected infeasible by any of the three models, and all but 24 instances are feasible (= 500 - 12 optimal - 464 feasible, cf. Model B). The solving status of these 24 instances is unknown.

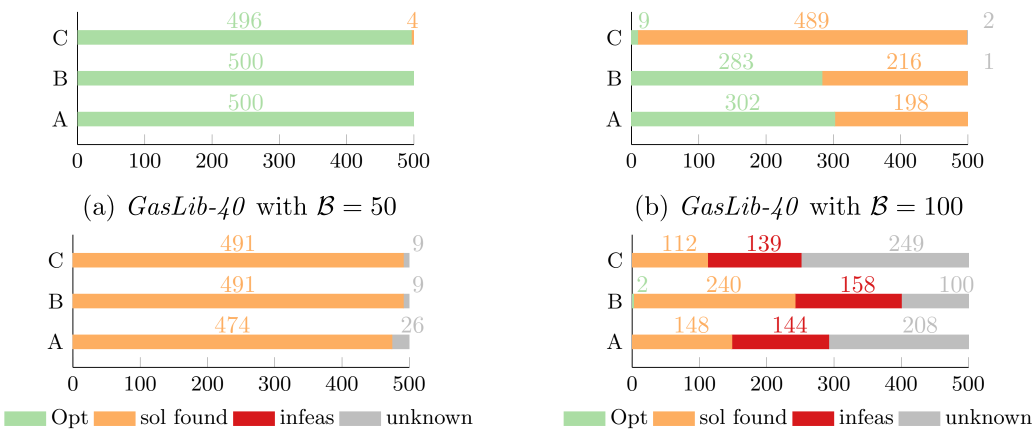
On the *New York+* test set, the superior performance of Model B is confirmed with respect to the number of instances that are solved to optimality and detected as infeasible, and for which a solution has been found. On the other hand, Model A exhibits a better runtime for instances with a demand of $\mathcal{B} = 100$ and 500. While for $\mathcal{B} = 100$ this improvement is due to the fact that Model A solves 9 more instances than Model B, Model A performs significantly better for $\mathcal{B} = 500$ (up to a factor 7.73 better than B), even though Model B solves 63 more instances, see Table 3.3f. That is, Model A has a higher number of instances that were solved considerably faster and a higher number of outliers, among others those instances that ran into the time limit.

Summary. In summary, we can state that Model A has the smallest runtime nearly on all data sets, with a few exceptions, especially on the *New York* water networks. Overall Models A and B perform better than Model C, with Model A clearly outperforming Model B on problems with a higher circuit rank, i.e., Model A solves more instances on *Belgium* + 6, 8 and 10 arcs, *GasLib-40* and *GasLib-135*, whereas Model B solves more instances on the water networks and *Belgium* + 0, 2 and 4 arcs. The performance of Model C may partly be a consequence of the fact that Model A, as we have shown in Section 3.4.4, has a tighter relaxation than Model C, even though there is a smaller number of entire data sets and a larger number of individual instances where Model C outperforms both Model A and Model B, in particular instances with a higher circuit rank. Model C is therefore a valuable alternative in cases where Models A and B require too much computational time to prove infeasibility or solve an instance to optimality.



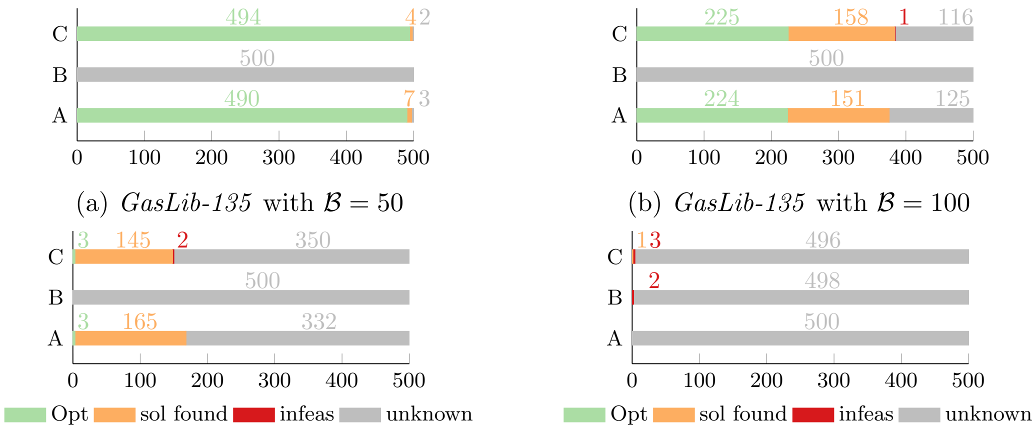
(c) *Belgium* with $\mathcal{B} = 500$ (d) *Belgium* with $\mathcal{B} = 1{,}000$

Figure 3.7: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the discrete Models A, B and C on *Belgium* for different demand loads $\mathcal{B}$.



(c) *GasLib-40* with $\mathcal{B} = 500$ (d) *GasLib-40* with $\mathcal{B} = 1{,}000$

Figure 3.8: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the discrete Models A, B and C on *GasLib-40* for different demand loads $\mathcal{B}$.



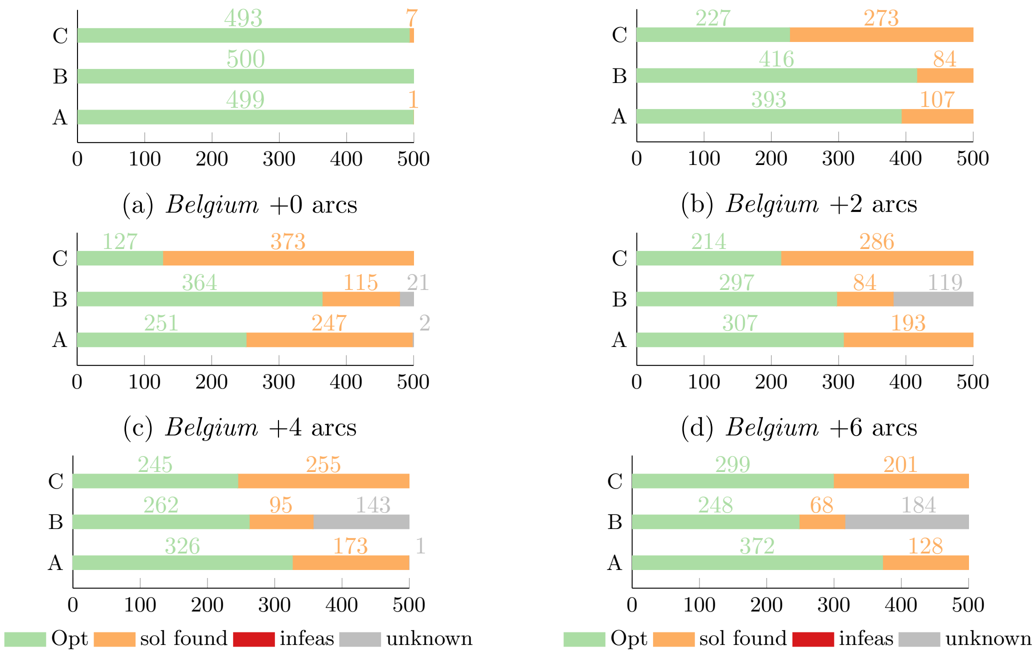
(a) *GasLib-135* with $\mathcal{B} = 50$

(b) *GasLib-135* with $\mathcal{B} = 100$

(c) *GasLib-135* with $\mathcal{B} = 150$

(d) *GasLib-135* with $\mathcal{B} = 500$

Figure 3.9: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the discrete Models A, B and C on *GasLib-135* for different demand loads $\mathcal{B}$.



(a) *Belgium* +0 arcs

(b) *Belgium* +2 arcs

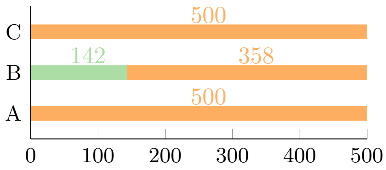
(c) *Belgium* +4 arcs

(d) *Belgium* +6 arcs

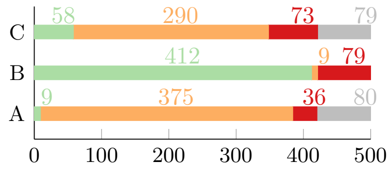
(e) *Belgium* +8 arcs

(f) *Belgium* +10 arcs

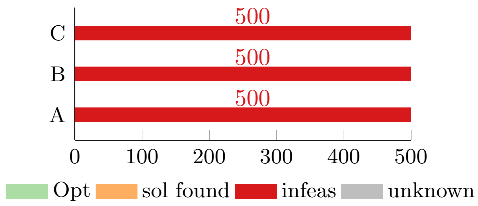
Figure 3.10: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the discrete Models A, B and C on *Circuit rank*.



(a) *New York* with $\mathcal{B} = 100$

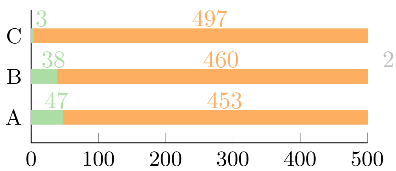


(b) *New York* with $\mathcal{B} = 200$

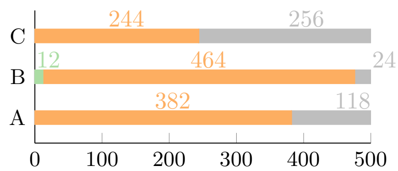


(c) *New York* with $\mathcal{B} = 500$

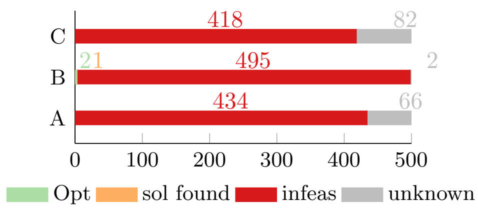
Figure 3.11: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the discrete Models A, B and C on *New York* for different demand loads $\mathcal{B}$.



(a) *New York+* with $\mathcal{B} = 100$



(b) *New York+* with $\mathcal{B} = 200$



(c) *New York+* with $\mathcal{B} = 500$

Figure 3.12: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the discrete Models A, B and C on *New York+* for different demand loads $\mathcal{B}$.

	$\mathcal{B} = 100$		$\mathcal{B} = 200$		$\mathcal{B} = 500$		$\mathcal{B} = 1{,}000$	
best model	A		A		A		A	
sgm of best model	9.1		41.4		1,424.6		88.4	
relative time to	4.3 (B)	2.2 (C)	3.9 (B)	2.6 (C)	1.7 (B)	2.6 (C)	1.7 (B)	1.3 (C)

(a) Performance of discrete Models A, B and C on *Belgium*

	$\mathcal{B} = 50$		$\mathcal{B} = 100$		$\mathcal{B} = 500$	$\mathcal{B} = 1{,}000$	
best model	A		A		A, B, C	A	
sgm of best model	4.0		3,923.0		14,400.0	1,794.8	
factor	4.3 (B)	3.2 (C)	2.0 (B)	3.6 (C)	-	1.8 (B)	1.2 (C)

(b) Performance of discrete Models A, B and C on *GasLib-40*

	$\mathcal{B} = 50$		$\mathcal{B} = 100$		$\mathcal{B} = 150$		$\mathcal{B} = 500$	
best model	A		A		A		C	
sgm of best model	3.4		677.8		13,846.9		14,336.1	
factor	4,235.3 (B)	2.8 (C)	21.3 (B)	1.3 (C)	1.04 (B)	1.00 (C)	1.01 (B)	1.01 (A)

(c) Performance of discrete Models A, B and C on *GasLib-135*

	+0 arcs		+2 arcs		+4 arcs		+6 arcs		+8 arcs		+10 arcs	
best model	A		A		A		A		A		A	
sgm of best model	9.1		1,368.6		3,492.2		2,327.2		2,489.2		1,443.4	
factor	4.3 (B)	2.2 (C)	1.5 (B)	3.5 (C)	1.2 (B)	1.8 (C)	2.5 (B)	1.9 (C)	3.0 (B)	1.5 (C)	4.7 (B)	1.7 (C)

(d) Performance of discrete Models A, B and C on *Circuit rank*

	$\mathcal{B} = 100$		$\mathcal{B} = 200$		$\mathcal{B} = 500$
best model	B		B		A, B, C
sgm of best model	12,183.0		1,000.0		< 1.0
factor	1.2 (A)	1.2 (C)	8.8 (A)	7.5 (C)	-

(e) Performance of discrete Models A, B and C on *New York*

	$\mathcal{B} = 100$		$\mathcal{B} = 200$		$\mathcal{B} = 500$	
best model	A		B		A	
sgm of best model	13,015.3		14,323.7		46.9	
factor	1.07 (B)	1.10 (C)	1.01 (A)	1.01 (C)	7.73 (B)	3.45 (C)

(f) Performance of discrete Models A, B and C on *New York+*

Table 3.3: Performance of discrete Models A, B and C, where the shifted geometric mean (sgm) for the best performing model is denoted in seconds, and the change of solving time relative to the best performing model for the other models.

3.6.4 Comparison of split-pipe models (Models D and E)

***Belgian* gas network.** Again we start with the *Belgian* network (Figure 3.13 and Table 3.4a) and proceed from easily solved instances to scenarios with a large number of infeasible instances. It turns out that both models solve a comparable number of instances, with Model E being throughout faster, around 5% for $\mathcal{B} = 500$, while $\mathcal{B} = 100, 200$ can be considered as easy instances for both models. It is worth noting that for a demand of $\mathcal{B} = 500$, i.e., for more difficult instances, 10% of the instances are solved to optimality by only one of the two models, i.e., in this case both models complement each other well, see row *Only opt by* in Table A.53 in the appendix.

***GasLib-40* and *GasLib-135* gas networks.** To compare the performance on gas networks of larger sizes with a more complex cyclic structure, we first turn to the *GasLib-40* network. Here, Model E performs clearly better: it solves more instances on all four data sets, i.e., 1, 42, 7 and 37 more instances for a demand $\mathcal{B} = 50, 100, 500$ and 1,000 (Figure 3.14). Besides, the runtime of Model E is remarkably better, up to a factor of 2.5 for $\mathcal{B} = 50$, see Table 3.4b.

These factors even increase significantly on the *GasLib-135* network, where Model E solves significantly more instances up to 165 times faster than Model D for $\mathcal{B} = 50$, see Table 3.4c. Concerning the number of instances, Model E solves 96 and 74 more instances for the demands $\mathcal{B} = 50, 100$ than Model D, and finds 236 and 388 more solutions for $\mathcal{B} = 150, 500$ (Figure 3.15). For more detailed information about the results on *GasLib-40* and *GasLib-135*, we also refer to the Tables A.54 and A.55 in the appendix.

Circuit rank on *Belgian* gas network. To further test the split-pipe Models D and E, we consider the performance for increasing circuit rank. We observe that Model E consistently outperforms Model D, with the ratio of the solving times of the model becoming the more favorable for E the higher the circuit rank, up to a factor of 13.3 for 10 additional arcs (Table 3.4d).

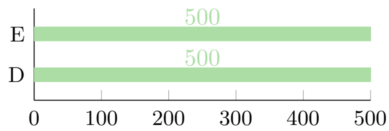
Similarly, Model E performs significantly better with respect to the number of solved instances, as it solves 80, 78, 144, 182 and 278 more instances to optimality for +2, 4, 6, 8 and 10 additional arcs, see green bars in Figure 3.16.

***New York* water networks.** Finally, we investigate the performance of the Models D and E on the *New York* networks. On the original *New York* network, both models solve all instances for $\mathcal{B} = 100, 200$ and 500 in at most 3 seconds, see Figure 3.17 and Table 3.4e. On the extended network with four additional arcs *New York+*, Model E again solves 36 and 9 more instances for a demand $\mathcal{B} = 100$ and 200 (Figure 3.18) in significantly less computation time, being up to 72% faster for instances with a demand $\mathcal{B} = 100$, see Table 3.4f.

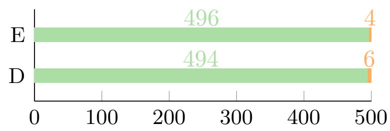
To analyze how consistently the improvements in the performance with respect to the shifted geometric mean are distributed across all test sets *Belgium*, *GasLib-40*, *GasLib-135*, *Circuit rank*, *New York* and *New York+*, we apply the Wilcoxon signed-rank test from Chapter 2. The test judges all improvements of Model E over Model D as significant, with the only exceptions on the *New York* test set for $\mathcal{B} = 200$, and the *New York+* test set for $\mathcal{B} = 500$. However, both models D and E solve the instances of these two sets in less than five seconds, and thus, the statistical significance can be considered as negligible on these two test sets.

Conversion of infeasible instances into feasible ones. Altogether, both split-pipe Models D and E detect the following number of infeasible instances across all test sets: 310 infeasible instances of *Belgium* with $\mathcal{B} = 1{,}000$ (Figure 3.13), 160 infeasible instances of *GasLib-40* with $\mathcal{B} = 1{,}000$ (Figure 3.14), 579 infeasible instances of *New York* with $\mathcal{B} = 200$ and $\mathcal{B} = 500$ (Figure 3.17), and 496 infeasible instances of *New York+* with $\mathcal{B} = 500$ (Figure 3.18c). In order to also provide solutions to these instances, we apply Proposition 3.5.4. The proposition requires that (i) the intersection of all nodal potential bounds is a non-empty interval, and (ii) no flow bounds exist, and (iii) the loop diameters can be chosen sufficiently large. While the instances of *Belgium* and *GasLib-40* satisfy the first requirement, the potential bounds at the only source node of all *New York* and *New York+* instances are fixed to the same value, which contradicts the requirement on a non-empty interval and hence prevents us to apply the proposition. For this reason, we restrict our investigation to the infeasible instances of the test sets *Belgium* and *GasLib-40*. First, we solve the relaxed problems of theses instances and based on their solutions, we determine the shifting factor s and the final diameters $(\tilde{d}_a)_{a\in\mathcal{A}}$, as explained in Section 3.5.3. In this way, a solution can be provided to all of the 310 infeasible *Belgium*-instances and 160 infeasible *GasLib-40*-instances.

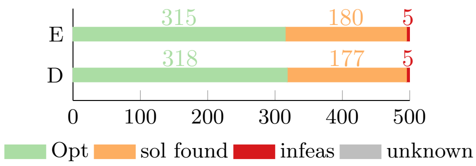
Summary. On the basis of our computational experiments for the split-pipe case, we can clearly recommend the use of our novel Model E instead of the Model D from the literature, particularly for networks with a more complex cyclic structure, even though one will, of course, encounter instances where Model D is more successful.



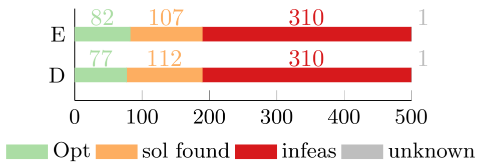
(a) *Belgium* with $\mathcal{B} = 100$



(b) *Belgium* with $\mathcal{B} = 200$

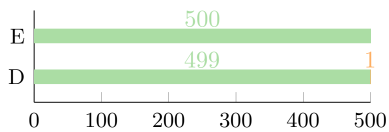


(c) *Belgium* with $\mathcal{B} = 500$

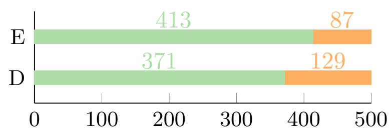


(d) *Belgium* with $\mathcal{B} = 1{,}000$

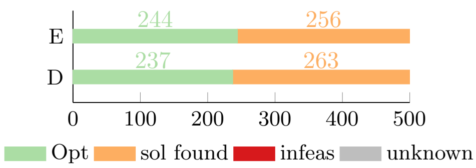
Figure 3.13: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the split-pipe Models D and E on *Belgium* for different demand loads $\mathcal{B}$.



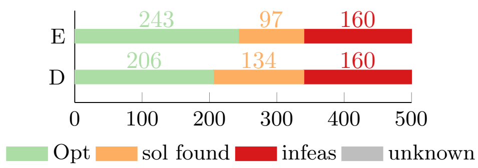
(a) *GasLib-40* with $\mathcal{B} = 50$



(b) *GasLib-40* with $\mathcal{B} = 100$

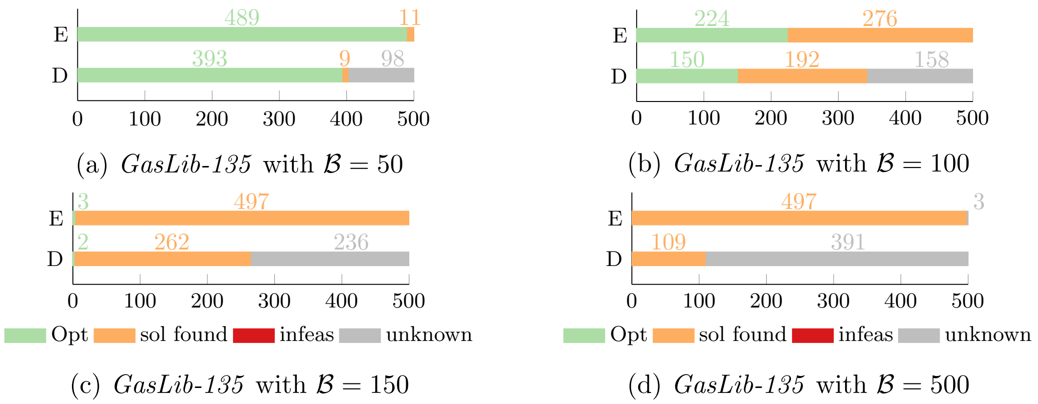


(c) *GasLib-40* with $\mathcal{B} = 500$



(d) *GasLib-40* with $\mathcal{B} = 1{,}000$

Figure 3.14: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the split-pipe Models D and E on *GasLib-40*.



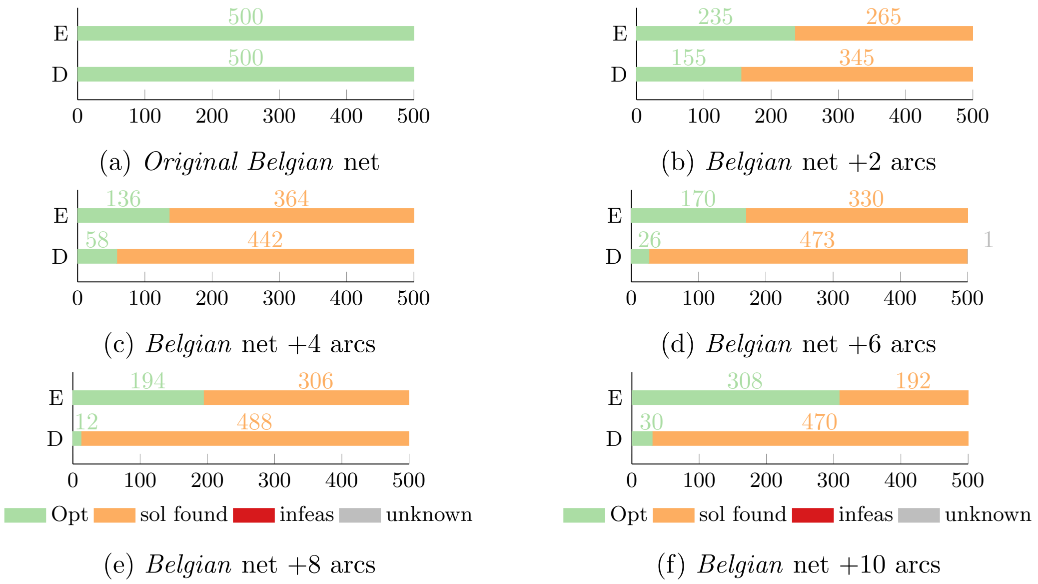
(a) *GasLib-135* with $\mathcal{B} = 50$

(b) *GasLib-135* with $\mathcal{B} = 100$

(c) *GasLib-135* with $\mathcal{B} = 150$

(d) *GasLib-135* with $\mathcal{B} = 500$

Figure 3.15: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the split-pipe Models D and E on *GasLib-135*.



(a) *Original Belgian* net

(b) *Belgian* net +2 arcs

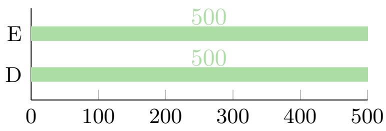
(c) *Belgian* net +4 arcs

(d) *Belgian* net +6 arcs

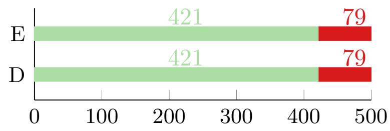
(e) *Belgian* net +8 arcs

(f) *Belgian* net +10 arcs

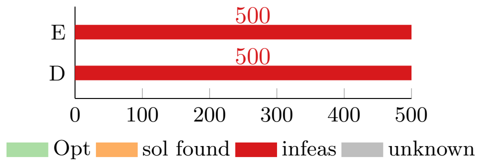
Figure 3.16: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the split-pipe Models D and E on *Circuit rank*.



(a) *New York* with $\mathcal{B} = 100$

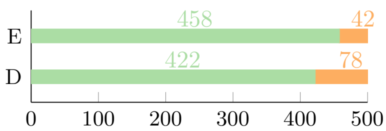


(b) *New York* with $\mathcal{B} = 200$

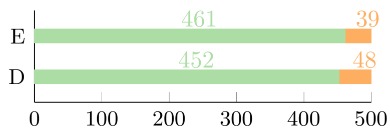


(c) *New York* with $\mathcal{B} = 500$

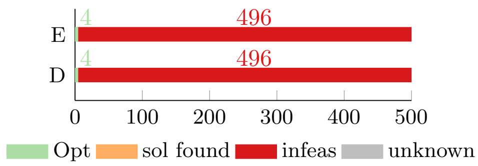
Figure 3.17: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the split-pipe Models D and E on *New York*.



(a) *New York+* with $\mathcal{B} = 100$



(b) *New York+* with $\mathcal{B} = 200$



(c) *New York+* with $\mathcal{B} = 500$

Figure 3.18: Number of optimal, infeasible and feasible instances with non-zero optimality gap, for the split-pipe Models D and E on *New York+*.

	$\mathcal{B} = 100$	$\mathcal{B} = 200$	$\mathcal{B} = 500$	$\mathcal{B} = 1000$
best model	E	E	E	E
sgm of best model	0.6	4.8	640.9	79.1
relative time to	1.67 (D)	1.46 (D)	1.06 (D)	1.02 (D)

(a) Performance of split-pipe Models D and E on *Belgium*

	$\mathcal{B} = 50$	$\mathcal{B} = 100$	$\mathcal{B} = 500$	$\mathcal{B} = 1{,}000$
best model	E	E	E	E
sgm of best model	1.4	164.3	4,452.5	687.8
relative time to	2.50 (D)	1.31 (D)	1.09 (D)	1.10 (D)

(b) Performance of split-pipe Models D, and E on *GasLib-40*

	$\mathcal{B} = 50$	$\mathcal{B} = 100$	$\mathcal{B} = 150$	$\mathcal{B} = 500$
best model	E	E	E	D,E
sgm of best model	13.8	841.6	13,879.2	14,400.0
relative time to	163.58 (D)	7.68 (D)	1.02 (D)	-

(c) Performance of split-pipe Models D, and E on *GasLib-135*

	+0 arcs	+2 arcs	+4 arcs	+6 arcs	+8 arcs	+10 arcs
best	E	E	E	E	E	E
sgm of best model	0.6	1,173.1	3,952.4	2,790.4	2,675.5	844.2
factor	1.67 (D)	2.72 (D)	2.43 (D)	4.24 (D)	4.84 (D)	13.32 (D)

(d) Performance of split-pipe Models A, B and C on *Circuit rank*

	$\mathcal{B} = 100$	$\mathcal{B} = 200$	$\mathcal{B} = 500$
best	E	D	D & E
sgm of best model	2.8	1.8	< 1.0
factor	1.07 (D)	1.56 (E)	-

(e) Performance of split-pipe Models D and E on *New York*

	$\mathcal{B} = 100$	$\mathcal{B} = 200$	$\mathcal{B} = 500$
best	E	E	D
sgm of best model	186.9	275.2	3.9
factor	1.72 (D)	1.42 (D)	1.26 (E)

(f) Performance of split-pipe Models D and E on *New York+*

Table 3.4: Performance of split-pipe Models D and E, where the shifted geometric mean (sgm) for the best performing model is denoted in seconds, and the factor for the change of solving time relative to the best performing model for the other models.

3.6.5 Comparison of discrete and split-pipe models

To conclude the computational experiments, we compare the discrete with the split-pipe approach. As mentioned in the introduction, this is the first time that such a comparison is carried out for networks of practically relevant size and complexity. Of course, one may expect that the split-pipe Models D and E will perform better on average, but as the computational experiments show, there are data sets where the discrete approach yields more optimal solutions within the time limit of 4 hours than the split-pipe approach, cf. for example the results for *Belgium* +2, +4, +6, and +8 arcs in Figures 3.10 and 3.16. Hence, it is unclear what size the performance gap may actually take. In the following, we will look at two criteria: (a) potential cost savings and (b) computational time.

Concerning cost savings, Table 3.5 shows the average gain of the best known solutions of the split-pipe Models D and E over the discrete Models A, B and C. Here, we consider only those instances where an expansion actually takes place and where either at least one split-pipe and at least one discrete model have been solved to optimality (column *Optimal*) or where both approaches provide at least one solution (column *Feasible*). For these instances, we then consider the average gain of the best known split-pipe solution over the best-known discrete one. The column *Feasible* additionally shows the percentage of instances where the best-known solution is provided by one of the split-pipe Models D and E.

As Table 3.5 demonstrates, the split-pipe approach yields cost savings on all data sets. While the results for the comparably simple networks of *Belgium* and *New York* are rather low, the benefit of realizing a split-pipe solution can be considerable for networks with a complex cyclical structure. Moreover, Figure 3.19 shows that for all feasible instances with non-zero objective function value, the best solutions were practically always found by the split-pipe models, with Model E delivering the best results. Even more, further analysis of the data presented in Figures 3.7 – 3.18 reveals that the split-pipe models optimally solve or detect (in)feasibility in all cases except 4 (Model E) or 987 (Model D) instances, whereas the number of instances with unknown status is much higher for the discrete models, namely 1,462 (Model A), 2,604 (Model B) and 1,650 (Model C) out of the 11,500 instances we have calculated per model. Therefore, in view of the much larger number of feasible solutions by the split-pipe Models D and E, the economic benefit of these models goes well beyond the cost savings and the percentage of instances depicted in Table 3.5 and Figure 3.19.

To compare the overall runtime performance of all discrete and split-pipe models, we use a performance profile (Dolan and Moré, 2002). It is based on the performance ratio, i.e., the runtime of a particular data set with the model under consideration divided by the best runtime for that data set with any of the five models. The performance profile describes on the y-axis the fraction of instances among all solved (i.e., optimal or infeasible) instances that the model could solve with a performance ratio of up to the corresponding number on the x-axis. Clearly, models are to be preferred when their profile shows higher y-values for fixed x-values and lower x-values for fixed y-values. To exclude trivial cases, we disregard instances that were not solved by any model and those that were solved by all models in less than 1 second CPU time.

		Optimal		Feasible		
Network	Data set	Gain [%]	# instances	Gain [%]	# instances	SP better on # instances [%]
Belgium	$\mathcal{B} = 100$	2.2	500	2.2	500	100.0
Belgium	$\mathcal{B} = 200$	1.6	498	1.6	500	100.0
Belgium	$\mathcal{B} = 500$	1.1	332	1.2	495	100.0
Belgium	$\mathcal{B} = 1000$	1.3	74	1.4	189	100.0
GasLib-40	$\mathcal{B} = 50$	321.5	147	321.5	147	100.0
GasLib-40	$\mathcal{B} = 100$	2.1	326	3.7	500	100.0
GasLib-40	$\mathcal{B} = 500$	-	0	10.7	498	100.0
GasLib-40	$\mathcal{B} = 1{,}000$	0.7	2	8.0	175	100.0
GasLib-135	$\mathcal{B} = 50$	-	0	7,844.3	9	100.0
GasLib-135	$\mathcal{B} = 100$	-	0	435.0	229	100.0
GasLib-135	$\mathcal{B} = 150$	-	0	613.0	238	100.0
GasLib-135	$\mathcal{B} = 500$	-	0	550.0	1	100.0
New York	$\mathcal{B} = 100$	1.0	142	1.1	500	100.0
New York	$\mathcal{B} = 200$	0.8	413	0.8	415	100.0
New York	$\mathcal{B} = 500$	-	0	-	0	-
New York+4 arcs	$\mathcal{B} = 100$	2.8	82	3.4	499	99.8
New York+4 arcs	$\mathcal{B} = 200$	1.0	12	7.3	434	100.0
New York+4 arcs	$\mathcal{B} = 500$	2.6	2	1.9	3	100.0
Belgium+2 arcs	$\mathcal{B} = 200$	1.6	268	1.5	500	99.8
Belgium+4 arcs	$\mathcal{B} = 200$	8.2	151	5.3	499	96.4
Belgium+6 arcs	$\mathcal{B} = 200$	14.1	169	12.1	498	98.8
Belgium+8 arcs	$\mathcal{B} = 200$	17.4	185	14.5	498	99.6
Belgium+10 arcs	$\mathcal{B} = 200$	71.0	284	50.7	490	100.0

Table 3.5: Gain of split-pipe over discrete problem.

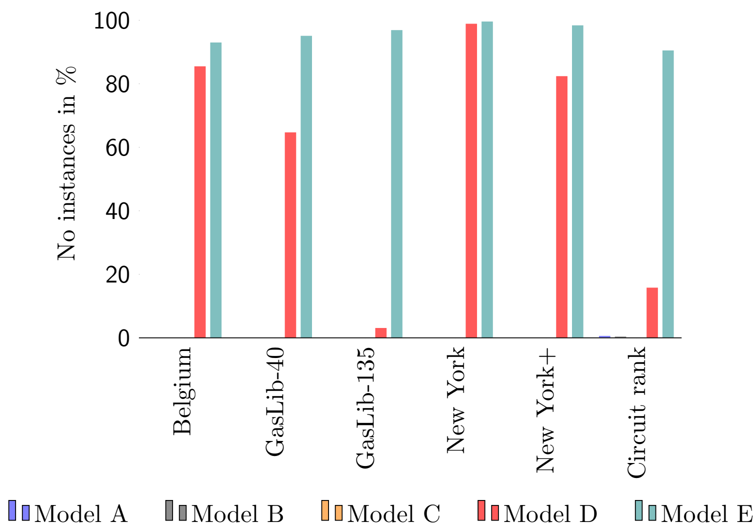


Figure 3.19: Percentage of instances, for which the best solution was found by the respective expansion planning models.

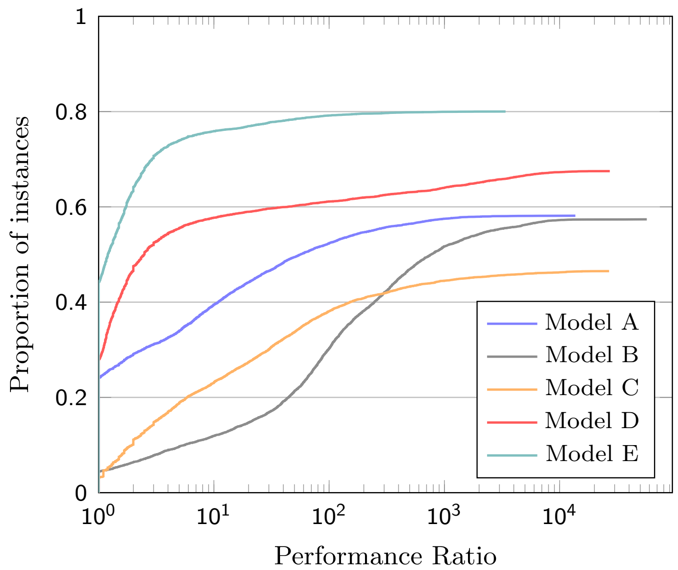


Figure 3.20: An illustration of the performance profile, where the performance ratio is logarithmically scaled on the x-axis.

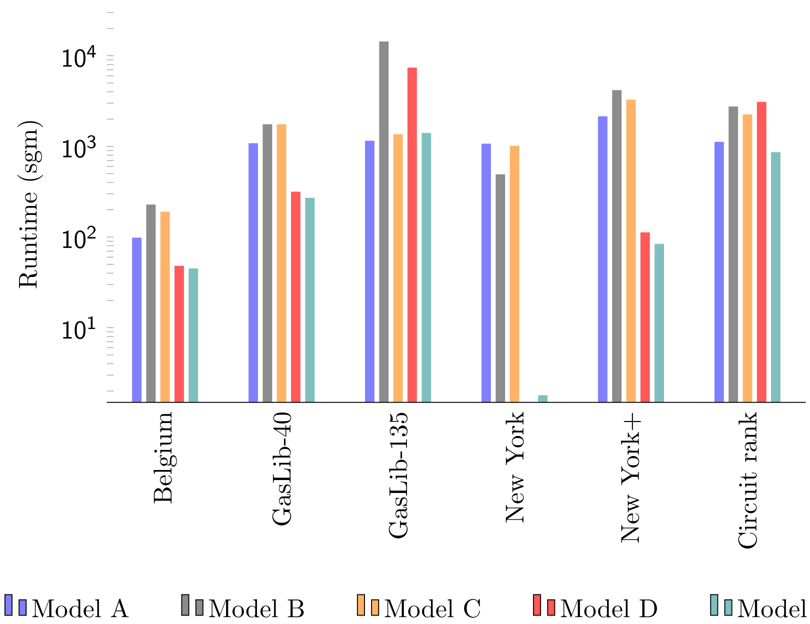


Figure 3.21: Average runtime (logarithmically scaled) of discrete and split-pipe models on different test sets.

As we can see in Figure 3.20, the runtime of Model E dominates the runtime of all other models across the spectrum of performance ratios. Further insights about the model performances for different data sets are gathered from Figure 3.21, which depicts the shifted geometric mean of the runtime of all instances for different network types and confirms the dominance of the split-pipe models. Again Model E turns out to perform best.

Recommendations. To conclude this section, our computational experiments suggest the following recommendations for practitioners:

i) In view of the advantages of the split-pipe approach discussed in the previous subsection, practitioners should use the split-pipe approach whenever possible.

ii) The novel Model E should be the model of choice as it leads to more instances that are solved, to more instances for which feasible solutions are found, to a higher percentage of instances with a better objective function value and cost savings, and performs better with respect to computational time.

iii) When the split-pipe approach is not possible, Models A or B are likely to be the most useful. Among these two models, Model A should be tried out first on networks with a more complex cyclical structure.

iv) Finally, Models C and D may be useful complements when dealing with instances that happen to be particularly hard for the other models.

3.7 Conclusion

In this chapter, we studied the problem of capacity expansion of potential-driven networks using loops. We showed properties of the looping problem, such as its non-convexity, and, building on an existing method of selecting cost-minimal loop diameters a priori, we presented a model reduction approach for multiple loops. On this basis, we introduced new models for both discrete and split-pipe looping and contrasted these with existing models for the looping problem and related problems in the literature, both theoretically and empirically. This was also the first time that discrete and split-pipe approaches were compared for networks of a practically relevant size and complexity. The performance of the models was analyzed in an extensive computational study with a large set of demand vectors and diverse networks of different sizes and topologies, including network variations for different circuit ranks, and led to recommendations regarding the use of different models. In particular, our experiments showed that overall our novel Model E outperforms the existing models with respect to computational time, the number of solutions found, the number of instances solved and cost savings.

An interesting avenue for further research would be to improve our split-pipe Model E by generating the efficient frontier dynamically. Instead of adding all constraints for the frontier globally to the model, they could be generated dynamically during the solving process whenever the frontier is violated by the current LP solution. This could drastically reduce the model size, depending on the number of arcs and extreme points of the efficient frontier.

Moreover, while we allowed all arcs to be looped, we could reduce the model size by a heuristic pre-selection of the arcs that are most likely to be looped. Such an approach could gather the pipes that have a higher expected potential loss along the pipe, which could be determined by simple auxiliary models. Finally, further work could also include loop expansion planning in the context of time-dynamic commodity transport.

Chapter 4

Tight Convex Relaxations for the Expansion Planning Problem

Tight relaxations are one of the most important ingredients for solving MINLPs with spatial branch-and-bound to global optimality. For a nonconvex constraint $f(x) \leq 0$ with $x \in D \subseteq \mathbb{R}^n$, a convex relaxation is obtained by computing a convex function $g\colon D \to \mathbb{R}$ that satisfies $g(x) \leq f(x)$ for all $x \in D$. Then, $\{x \in D \mid f(x) \leq 0\} \subseteq \{x \in D \mid g(x) \leq 0\}$ yields a convex relaxation of the constraint $f(x) \leq 0$. The best possible convex underestimator of a nonconvex function f over domain D is given by the convex envelope $\operatorname{vex}_D(f)$. For general functions, computing the convex envelope is a difficult task, but for specific functions it might be tractable. Note that constraints can also be of the form $f(x) \geq 0$. Then, concave overestimators are required to build a convex relaxation. In particular, for equality constraints, both the convex under and concave overestimators are needed. Our goal is to improve the algorithmic performance of the MINLP solver for the stationary expansion planning problem. In this chapter, we will analytically derive the convex envelope of the potential loss function $f(x, y) = y \operatorname{sgn}(x)|x|^\alpha$ with $\alpha > 1$ from the previous chapter. As we will see in Section 4.2.1, the concave envelope of f can be deduced from the convex envelope of f, thus we concentrate on the convex envelope in the following.

To the best of our knowledge, the convex envelope of this function is unknown in the literature. One of our main contributions is to derive an explicit algebraic description for the convex envelope of f over a rectangular domain. This allows us to significantly improve the quality of relaxations that are constructed by state-of-the-art MINLP solvers for function f. Through an extensive computational study, we show that these tighter relaxations tremendously improve the performance of the MINLP solver SCIP on a large test set of practically relevant instances that stem from real-world applications. Our results show that the performance improves by up to 58% for hard instances.

This chapter is structured as follows: In Section 4.1, we state relevant literature on convex envelopes and introduce preliminary theoretical results from convex analysis that we need for the derivation of the convex envelope of function $f(x, y) = y \operatorname{sgn}(x)|x|^\alpha$. In Sections 4.2 – 4.4, we present our main contribution, which is the derivation of a closed-form expression for the convex envelope of f. There, we present two different solution approaches, where the second approach in Section 4.4 can be considered as additional material and intends to strengthen the geometrical intuition of the solution. In Section 4.5, we compare the convex envelope with the standard factorable relaxation for the constraint function f followed by an extensive computational study in Section 4.6. This study illustrates the impact of the convex envelope over the standard relaxation generated by MINLP solvers using stationary expansion planning problem instances from the previous chapter. To this end, we first study the impact on the dual bound in the root node, and second, the overall performance within the spatial branch-and-bound search. Finally, Section 4.7 provides conclusions.

4.1 Introduction to Convex Envelopes

In general, the derivation of convex and concave envelopes is a challenging task depending on the nonlinear constraint function and on the domain of the variables involved, see, for example, Horst and Tuy (1990). It can be shown that for a l.s.c. function f over a compact and convex domain D, $\min_{x \in D} f(x) = \min_{x \in D} \text{vex}_D[f](x)$ and hence computing the convex envelope is in general $\mathcal{NP}$-hard. Unless $\mathcal{P} = \mathcal{NP}$, no efficient method exists to compute convex envelopes of general nonlinear functions.

In fact, the standard representation corresponds to a nonconvex optimization problem that is, in general intractable, as we highlight below. Due to the necessity to have envelopes at hand, they have been widely studied for particular functions or classes of functions, among those bivariate functions. For example, McCormick (1976) constructs a linear relaxation over a box domain for the bilinear term $x\,y$, which is proven to be the convex envelope by Al-Khayyal and Falk (1983). In the last decade, several approaches arose to convexify the bilinear term $x\,y$ over special domains, see, e.g., Sherali and Alameddine (1990), Linderoth (2005), Locatelli (2018), Hijazi (2019), and Müller et al. (2019). The convex envelope of the fractional term y/x has been studied, e.g., in Zamora and Grossmann (1999), and Tawarmalani and Sahinidis (2001).

Moreover, Tawarmalani and Sahinidis (2001) calculate the convex envelope of the function $f(x, y) = g(y)\, x^2$ over a rectangular domain $D \subseteq \mathbb{R}^2$, where $g\colon D \to \mathbb{R}$ is convex in x and concave in y. Even though this function is close to our constraint function $f(x, y) = y\, \text{sgn}(x)|x|^\alpha$, this approach is not applicable in our setting, given that our function involves the nonconvex term $\text{sgn}(x)|x|^\alpha$ instead of x^2.

Note that $f(x, y) = y\, \text{sgn}(x)|x|^\alpha$ is twice differentiable only for $\alpha > 2$ and thus it is not possible to compute the convex envelope using techniques based on the Hessian, as done, for example, for $(n-1)$-convex functions in Jach et al. (2008). In principle, when $f \in C^2$ (i.e., $\alpha > 2$), we could generate valid tangential hyperplanes of function f to separate violated LP solutions without explicitly calculating the convex envelope, as done in Ballerstein et al. (2013) for twice continuously differentiable functions with fixed convexity behavior over a box. However, our analysis is valid for any $\alpha > 1$. Furthermore, we recall from Section 3.2 in the previous chapter that for relevant applications the values of α are $\alpha = 2$ for gas network operations and $\alpha = 1.85$ for water network operations.

In the remaining of this section, we introduce some mathematical notation and necessary theoretical results that are required to derive $\text{vex}_D(f)$. Even though our function is continuous, many of the following results hold for lower semicontinuous functions, which we define as follows:

Definition 4.1.1. *Let* $f\colon D \to \mathbb{R}$ *with* $D \subseteq \mathbb{R}^n$*. The* epigraph *of* f *is*

$$epi_D f = \{(x,\mu)|\ x \in D, \mu \in \mathbb{R}, \mu \geq f(x)\}.$$

Definition 4.1.2. *A function* $f\colon D \to \mathbb{R}$ *is* lower semicontinuous *(l.s.c.) if its epigraph is closed.*

We continue by formally defining the convex envelope of a function.

Definition 4.1.3. *The convex envelope of a function* $f\colon D \to \mathbb{R}$ *over a convex set* $D \subseteq \mathbb{R}^n$ *is given by the tightest convex underestimator* $\eta\colon D \to \mathbb{R}$ *of* f*, defined pointwise as*

$$vex_D[f](x) = \sup\{\ \eta(x) \mid \eta(y) \leq f(y) \text{ for all } y \in D, \eta \text{ convex}\ \}.$$

The concave envelope of a nonconvex function $f\colon D \to \mathbb{R}$ *over* $D \subseteq \mathbb{R}^n$ *is given by the tightest concave overestimator* $\eta\colon D \to \mathbb{R}$ *of* f*, defined pointwise as*

$$cave_D[f](x) = \inf\{\ \eta(x) \mid \eta(y) \geq f(y) \text{ for all } y \in D, \eta \text{ concave}\ \}.$$

Next, we characterize the convex envelope geometrically. For this, we need the following definition and theorem.

Definition 4.1.4. *Let* $f\colon D \to \mathbb{R}$ *with* $D \subseteq \mathbb{R}^n$*. The epigraph of* f *is*

$$epi_D f = \{(x,\mu)|\ x \in D, \mu \in \mathbb{R}, \mu \geq f(x)\}.$$

Theorem 4.1.5 (Rockafellar (1970), Theorem 5.3)**.** *Let* E *be any convex set in* $\mathbb{R}^{n+1}$ *and let*

$$g(x) = \inf\{\mu|\ (x,\mu) \in E\}.$$

Then $g\colon \mathbb{R}^n \to \mathbb{R}$ *is a convex function.*

The convex envelope of a l.s.c. function f corresponds to the function g, obtained by applying Theorem 4.1.5 to the convex set $\text{conv}(\text{epi}_D f)$. Therefore, it follows

$$\text{vex}_D[f](x) = \min\{\mu|\ (x,\mu) \in \text{conv}(\text{epi}_D f)\}, \tag{4.1}$$

see also Rockafellar (1970). In order to determine the convex envelope of a l.s.c. function f at point $x \in D$, its representation in (4.1) leads to the following minimization problem

$$\begin{aligned} \text{vex}_D[f](x) = \min_{x_k, \lambda_k} & \sum_{k=1}^{n+1} \lambda_k \, f(x_k) \\ \text{s.t.} & \sum_{k=1}^{n+1} \lambda_k \, x_k = x, \\ & \sum_{k=1}^{n+1} \lambda_k = 1, \\ & \lambda_k \geq 0, x_k \in D \;\text{ for }\; 1 \leq k \leq n+1. \end{aligned} \tag{4.2}$$

Note that taking the convex combination of at most $n+1$ points in D is justified by *Caratheodory's theorem* (cf. Theorem 17.1 in Rockafellar, 1970). Problem (4.2) is nonconvex and nonlinear as the multipliers λ_k and the variables x_k are unknown. However, it is possible to reduce the degree of nonlinearity and nonconvexity of Problem (4.2) by studying the convex set $\text{conv}(\text{epi}_D f)$ in more detail. In particular, we reduce the domain D by determining a subset $A \subseteq D$, such that $\text{conv}(\text{epi}_D f) = \text{conv}(\text{epi}_A f)$ holds.

Definition 4.1.6. *Let D be a compact and convex subset of $\mathbb{R}^n$ and let $f \colon D \to \mathbb{R}$ be a l.s.c. function. The* generating set *of the convex envelope of f is defined by*

$$\mathcal{G}_D(f) := \{x \in D \mid (x, \mathit{vex}_D[f](x)) \text{ is an extreme point of conv } (\mathit{epi}_D f)\}.$$

For more information about the generating set, we refer to Tawarmalani and Sahinidis (2002b).

Theorem 4.1.7. *Let D be a compact and convex subset of $\mathbb{R}^n$, $f \colon D \to \mathbb{R}$ be a l.s.c. function, and $A \subseteq D$. Then $\mathit{conv}(\mathit{epi}_D f) = \mathit{conv}(\mathit{epi}_A f)$ if and only if $\mathcal{G}_D(f) \subseteq A$.*

Proof. To prove the statement we need two claims. First, the convex hull of a closed convex set that contains no lines is characterized by the set of its extreme points and extreme directions, cf. Theorem 18.5 in Rockafellar (1970):

$$\text{conv}(\text{epi}_D f) = \text{conv}\left\{(x, f(x)) \,|\, x \in \mathcal{G}_D(f)\right\} + \text{cone}\left\{(0,1)\right\}. \tag{4.3}$$

Note that $\text{conv}(\text{epi}_D f)$ is closed, since f is lower semi-continuous on a compact domain D. Second, the convex envelope and the function itself coincide in all points of the generating set, i.e.,

$$\text{vex}_D[f](x) = f(x) \quad \text{for all } x \in \mathcal{G}_D(f). \tag{4.4}$$

"$\Rightarrow$" Proof by contrapositive. Assume that $\mathcal{G}_D(f) \nsubseteq A$ holds. Then there exists $x \in \mathcal{G}_D(f) \setminus A$, which implies $(x, f(x)) \notin \text{epi}_A f$. Furthermore, as $(x, f(x))$ is an extreme point of $\text{conv}(\text{epi}_D f)$, it cannot be expressed as a convex combination of points in $\text{epi}_A f$, which implies $\text{conv}(\text{epi}_D f) \neq \text{conv}(\text{epi}_A f)$.

"$\Leftarrow$" We assume that $\mathcal{G}_D(f) \subseteq A$ holds. We need to show that $\text{conv}(\text{epi}_D f) \subseteq \text{conv}(\text{epi}_A f)$, since $\text{conv}(\text{epi}_A f) \subseteq \text{conv}(\text{epi}_D f)$ follows from $A \subseteq D$.

Let $(x, \mu) \in \text{conv}(\text{epi}_D f)$. Then there exist points $x_i \in \mathcal{G}_D(f)$ and $\nu \geq 0$, such that

$$\begin{aligned}(x, \mu) &\overset{(4.3)}{=} \sum_i \lambda_i \left(x_i, \text{vex}_D[f](x_i)\right) + \nu(0, 1) \\ &\overset{(4.4)}{=} \sum_i \lambda_i \left(x_i, f(x_i)\right) + \nu(0, 1).\end{aligned}$$

Since $\mathcal{G}_D(f) \subseteq A$, it follows $(x_i, f(x_i)) \in \text{epi}_A f$. Thus, $(x, \mu) \in \text{conv}(\text{epi}_A f)$ and we conclude that $\text{conv}(\text{epi}_D f) \subseteq \text{conv}(\text{epi}_A f)$. □

Even though there exists no general formula to compute $\mathcal{G}_D(f)$ of a nonconvex function f, Tawarmalani and Sahinidis (2002a) provide a condition to determine when a point $x \in D$ is not in $\mathcal{G}_D(f)$. For this we need the relative interior of a convex set D, $\text{ri}(D) := \{x \in D : \forall y \in D\; \exists\, \lambda < 1 : \lambda\, x + (1 - \lambda)\, y \in D\}$.

Corollary 4.1.8 (Tawarmalani and Sahinidis (2002a), Corollary 5)**.** *Let $f\colon D \to \mathbb{R}$ be l.s.c. and $D \subseteq \mathbb{R}^n$ compact and convex. If for $x \in D$ there exists a line segment $l \subseteq D$ such that $x \in \text{ri}(l)$ and f restricted to l is concave, then $x \notin \mathcal{G}_D(f)$.*

4.2 Computing the Convex Envelope of $f(x, y) = y\, sgn(x)|x|^\alpha$

In this section, we show how to compute the convex and concave envelope of $f\colon D \subseteq \mathbb{R}^2 \to \mathbb{R}$, with $f\colon (x, y) \mapsto y\, \text{sgn}(x)|x|^\alpha$, where $\alpha > 1$, over $D = [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}] \subseteq \mathbb{R}^2$. In the remainder of this chapter, we assume $0 < \underline{y} < \overline{y}$ and $\underline{x} < 0 < \overline{x}$, as it typically occurs in practical applications, see Chapter 3. For a visualization of the nonconvex function f with $\alpha = 2$, we refer to Figure 4.6a.

4.2.1 Preliminaries about the convex envelope of $f(x, y) = y\, sgn(x)|x|^\alpha$.

At first, we remark that in fact, we would need to determine the convex envelope of the constraint function $f(x, y, \pi_v, \pi_w) = y\, \text{sgn}(x)|x|^\alpha - \pi_v + \pi_w$, as this is the form of how it appears in the stationary expansion planning problem from Chapter 3, see, e.g., Model (3.13). Nevertheless, given that π_v, π_w only appear linearly in the constraint $f(x, y, \pi_v, \pi_w) = 0$, the term $-\pi_v + \pi_w$ is already convex and can be neglected. We thus derive the convex envelope of the function $f(x, y) = y\, \text{sgn}(x)|x|^\alpha$ over the domain $D = [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}] \subseteq \mathbb{R}^2$.

For $f(x, y) = y\, \text{sgn}(x)|x|^\alpha$ over $D = [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}] \subseteq \mathbb{R}^2$, Problem (4.2) reads as

$$\begin{aligned}\text{vex}_D[f](x, y) = \min_{x_k, y_k, \lambda_k}\; & \sum_{k=1}^{3} \lambda_k\, y_k\, \text{sgn}(x_k)|x_k|^\alpha \\ \text{s.t.}\; & \sum_{k=1}^{3} \lambda_k\, x_k = x,\end{aligned}$$

$$\sum_{k=1}^{3} \lambda_k y_k = y,$$
$$\sum_{k=1}^{3} \lambda_k = 1,$$
$$\lambda_k \geq 0, (x_k, y_k) \in D \quad \forall 1 \leq k \leq 3.$$

Note that the above problem has nine variables (x_k, y_k, λ_k for $1 \leq k \leq 3$). In the next section, we show how to reduce this problem to a one-dimensional problem.

Here, the structure of the function $f(x, y) = y \operatorname{sgn}(x)|x|^\alpha$ allows us to deduce both envelopes from each other. The function f is odd in x when y is fixed. This enables us to retrieve the concave envelope from the convex envelope, as formalized in the following proposition, see also Figure 4.1. Thus, from now on, we restrict ourselves to the derivation of the convex envelope.

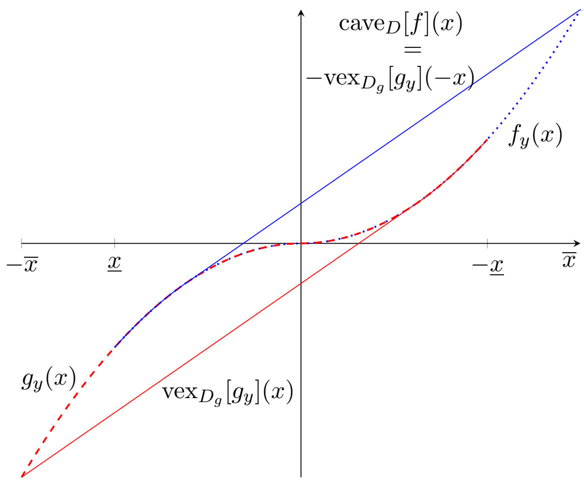


Figure 4.1: Illustration of Proposition 4.2.1 for the one-dimensional functions $f_y(x) = y \operatorname{sgn}(x)x^2$ (blue, dotted) and $g_y(x) := -f_y(-x) = -y \operatorname{sgn}(-x)(-x)^2$ (red, dashed) with $f_y : D := [\underline{x}, \overline{x}] \to \mathbb{R}$ and $g_y : D_g := [-\overline{x}, -\underline{x}] \to \mathbb{R}$ for fixed y, where $\text{vex}_D[g_y](x)$ (red, solid) is the convex envelope of g_y and $-\text{vex}_{D_g}[g_y](-x) = \text{cave}_D[f](x)$ (blue, solid) the concave envelope of f_y.

Proposition 4.2.1. *Let $f : \mathbb{R} \times [\underline{y}, \overline{y}] \to \mathbb{R}$ be an odd function in x, i.e., $f(x, y) = -f(-x, y)$ and let $D := [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$. Let $D_g := [-\overline{x}, -\underline{x}] \times [\underline{y}, \overline{y}]$ and $g : D_g \to \mathbb{R}$ be given by $g(x, y) := -f(-x, y)$. Then the concave envelope of f over D is given by $\text{cave}_D[f](x, y) = -\text{vex}_{D_g}[g](-x, y)$.*

Proof. Following Rockafellar (1970), Theorem 12.1, the description of the convex (concave) envelope of f in Definition 4.1.3 can be restricted to the pointwise maximum (minimum) of all linear under (over)estimators of f, i.e.,

$$\text{vex}_D[f](x) = \max_{a_0, a_1, a_2 \in \mathbb{R}} \{a_0^T x + a_1^T y + a_2 \mid a_0^T x + a_1^T y + a_2 \leq f(x, y) \text{ for all } (x, y) \in D\},$$

$$\text{cave}_D[f](x) = \min_{a_0,a_1,a_2\in\mathbb{R}} \{a_0^T x + a_1^T y + a_2 \mid a_0^T x + a_1^T y + a_2 \geq f(x,y) \text{ for all } (x,y) \in D\}.$$

Then, it follows

$$\begin{aligned}
& -\text{vex}_{D_g}[g](-x_0, y_0) \\
\overset{Def\ 4.1.3}{=} & -\max_{a_0,a_1,a_2} \{-a_0^T x_0 + a_1^T y_0 + a_2 \mid a_0^T x + a_1^T y + a_2 \leq g(x,y)\ \forall (x,y) \in D_g\} \\
\overset{z=-x}{=} & -\max_{a_0,a_1,a_2} \{-a_0^T x_0 + a_1^T y_0 + a_2 | - a_0^T z + a_1^T y + a_2 \leq g(-z,y)\ \forall (z,y) \in D\} \\
\overset{g(-z,y)=-f(z,y)}{=} & -\max_{a_0,a_1,a_2} \{-a_0^T x_0 + a_1^T y_0 + a_2 | - a_0^T z + a_1^T y + a_2 \leq -f(z,y)\ \forall (z,y) \in D\} \\
= & -\max_{a_0,a_1,a_2} \{-a_0^T x_0 + a_1^T y_0 + a_2 \mid a_0^T z - a_1^T y - a_2 \geq f(z,y) \quad \forall (z,y) \in D\} \\
= & \min_{a_0,a_1,a_2} \{a_0^T x_0 - a_1^T y_0 - a_2 \mid a_0^T z - a_1^T y - a_2 \geq f(z,y) \quad \forall (z,y) \in D\} \\
\overset{\substack{b_0=a_0 \\ b_1=-a_1 \\ b_2=-a_2}}{=} & \min_{b_0,b_1,b_2} \{b_0^T x_0 + b_1^T y_0 + b_2 \mid b_0^T z + b_1^T y + b_2 \geq f(z,y) \quad \forall (z,y) \in D\} \\
\overset{Def\ 4.1.3}{=} & \text{cave}_D[f](x_0, y_0).
\end{aligned}$$

□

4.2.2 Problem reduction by using the generating set $\mathcal{G}_D(f)$

To simplify Problem (4.2), we use Corollary 4.1.8 to find a set $A \subseteq D$ such that $\mathcal{G}_D(f) \subseteq A$ holds for $f(x,y) = y\,\text{sgn}(x)|x|^\alpha$.

First, notice that after fixing x to a value in $[\underline{x}, \overline{x}]$, the function f is linear and thus concave over the line segment $l = \{(x,y) \,|\, y \in [\underline{y}, \overline{y}]\}$. Then, Corollary 4.1.8 implies that all points in $\text{ri}(l) = \{(x,y) \,|\, y \in (\underline{y}, \overline{y})\}$ are not in $\mathcal{G}_D(f)$. Thus,

$$\mathcal{G}_D(f) \subseteq \big\{(x,\underline{y}),\ \underline{x} \leq x \leq \overline{x}\big\} \cup \big\{(x,\overline{y}),\ \underline{x} \leq x \leq \overline{x}\big\}.$$

Let $D_{\underline{y}} := \{(x,\underline{y}) \,|\, \underline{x} \leq x \leq \overline{x}\}$ and $D_{\overline{y}} := \{(x,\overline{y}) \,|\, \underline{x} \leq x \leq \overline{x}\}$. Without proving, we mention that the actual generating set is given by

$$\mathcal{G}_D(f) = \big\{(x,y) \,|\, x \in [\beta\underline{x}, \overline{x}],\ y \in \{\underline{y}, \overline{y}\}\big\} \cup (\underline{x}, \underline{y}) \cup (\underline{x}, \overline{y}),$$

where β is specified in Proposition 4.2.2.

Given that $\mathcal{G}_D(f) \subset D_{\underline{y}} \cup D_{\overline{y}}$, Theorem 4.1.7 allows us to express the convex hull of $\text{epi}_D f$ as

$$\begin{aligned}
\text{conv}(\text{epi}_D f) &= \text{conv}(\text{epi}_{D_{\underline{y}} \cup D_{\overline{y}}} f) \\
&= \text{conv}(\text{epi}_{D_{\underline{y}}} f \cup \text{epi}_{D_{\overline{y}}} f) \\
&= \text{conv}(\text{conv}(\text{epi}_{D_{\underline{y}}} f) \cup \text{conv}(\text{epi}_{D_{\overline{y}}} f)). \qquad (4.5)
\end{aligned}$$

The convex hulls $\text{conv}(\text{epi}_{D_{\underline{y}}} f)$ and $\text{conv}(\text{epi}_{D_{\overline{y}}} f)$ correspond to the epigraphs of the convex envelopes of the one-dimensional functions $f_{\underline{y}}(x) := f(x,\underline{y})$ and $f_{\overline{y}}(x) := f(x,\overline{y})$. The following proposition introduces these one-dimensional convex envelopes, see Figure 4.2 for an illustration.

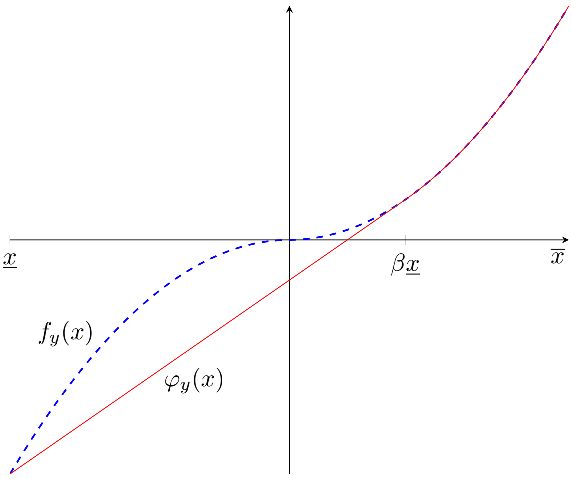


Figure 4.2: The convex envelope φ_y (red) of the one-dimensional function f_y (blue) depends on the relation of the values $\beta\underline{x}$ and $\overline{x}$. In the case of $\beta\underline{x} < \overline{x}$, the convex envelope is given by the secant between the points $(\underline{x}, f_y(\underline{x}))$ and $(\beta\underline{x}, f_y(\beta\underline{x}))$ for $x \in [\underline{x}, \beta\underline{x}]$ and the function $x \mapsto f_y(x)$ itself for $x \in [\beta\underline{x}, \overline{x}]$. Otherwise, if $\beta\underline{x} > \overline{x}$, then φ_y is given by the secant between $(\underline{x}, f_y(\underline{x}))$ and $(\overline{x}, f_y(\overline{x}))$.

Proposition 4.2.2 (Vigerske (2012), Section 7.5.2)**.** *The convex envelope of the function $f_y\colon [\underline{x}, \overline{x}] \to \mathbb{R}$ given by $f_y(x) = y\, sgn(x)|x|^\alpha$, where $\underline{x} < 0 < \overline{x}$, $\alpha > 1$, and $y > 0$ is given by $\varphi_y\colon [\underline{x}, \overline{x}] \to \mathbb{R}$, with*

$$\varphi_y(x) = \begin{cases} yx^\alpha & x \geq \beta\underline{x} \\ \alpha\gamma^{\alpha-1}xy + (1-\alpha)\gamma^\alpha y & x < \beta\underline{x}, \end{cases} \tag{4.6}$$

where $\gamma = \min\{\overline{x}, \beta\underline{x}\}$ and β is the unique negative root of $(\alpha-1)(-\beta)^\alpha + \alpha(-\beta)^{\alpha-1} - 1$.

Proof. We restrict the proof to the interesting case, when $\underline{x} < 0 < \overline{x}$. Then, the convex envelope $x \mapsto \varphi_y(x)$ is given by a linear part that corresponds to the secant between the points $(\underline{x}, f_y(\underline{x}))$ and $(z, f_y(z))$ for $x \in [\underline{x}, z]$, where the secant touches f_y tangentially in $z \in [\underline{x}, \overline{x}]$, and by the function itself for $x \in [z, \overline{x}]$, see Figure 4.2. Note that in the case of $\overline{x} < z$, the convex envelope is given by the secant between the points $(\underline{x}, f_y(\underline{x}))$ and $(\overline{x}, f_y(\overline{x}))$. From the curvature of f_y, we know that $z > 0$. Then,

$$\begin{aligned}
& f'(z) &&= \frac{f(z) - f(\underline{x})}{z - \underline{x}} \\
\Rightarrow\quad & \alpha y\,|z|^{\alpha-1} &&= \frac{y\,|z|^\alpha \mathrm{sgn}(z) - y\,|\underline{x}|^\alpha \mathrm{sgn}(\underline{x})}{z - \underline{x}} \\
\Rightarrow\quad & \alpha z^{\alpha-1} &&= \frac{z^\alpha + (-\underline{x})^\alpha}{z - \underline{x}} \\
\Rightarrow\quad & 0 &&= \alpha z^\alpha - \alpha\underline{x}z^{\alpha-1} - z^\alpha - (-\underline{x})^\alpha \\
\Rightarrow\quad & 0 &&= (\alpha-1)z^\alpha - \alpha\underline{x}z^{\alpha-1} - (-\underline{x})^\alpha
\end{aligned}$$

$$\Rightarrow \qquad 0 = (\alpha-1)\Big(\frac{z}{-\underline{x}}\Big)^{\alpha} + \alpha\Big(\frac{z}{-\underline{x}}\Big)^{\alpha-1} - 1$$

$$\beta := \frac{z}{\underline{x}} \quad \Rightarrow \qquad 0 = (\alpha-1)(-\beta)^{\alpha} + \alpha(-\beta)^{\alpha-1} - 1. \tag{4.7}$$

For a given value $\alpha > 1$, Equation (4.7) has a unique solution value β, which lies in the interval $[-1,0]$. This follows by considering the function $g\colon \mathbb{R} \to \mathbb{R}$ with $g\colon \beta \mapsto (\alpha-1)(-\beta)^{\alpha} + \alpha(-\beta)^{\alpha-1} - 1$. Similar as done in Liberti and Pantelides (2003) for monomials of odd degree, the function g has exactly one root, which lies in $\beta \in [-1,0]$ given that g is strictly decreasing in $[-1,0]$ and takes values in $[-1, 2\alpha-2]$. Using numerical methods to determine the root β yields $z = \beta\,\underline{x}$.

Finally, calculating the secant through $(\underline{x}, f_y(\underline{x}))$ and $(z, f_y(z))$ yields the convex envelope φ_y of function f_y as given in (4.6). ☐

We do not state β explicitly here, but mention that it reads $\beta = (1-\sqrt{2})$ for $\alpha = 2$. Further values for β are given in Vigerske (2012).

Remark 4.2.3. *The function $(x,y) \mapsto \varphi_y(x)$ is not the convex envelope of $f(x,y)$ as can be seen from Figure 4.6b. This function is not even convex!*

Given that $\operatorname{conv}(\operatorname{epi}_{D_{\underline{y}}} f) = \operatorname{epi}_{D_{\underline{y}}} \varphi_{\underline{y}}$ and $\operatorname{conv}(\operatorname{epi}_{D_{\overline{y}}} f) = \operatorname{epi}_{D_{\overline{y}}} \varphi_{\overline{y}}$, we can further rewrite (4.5) to

$$\operatorname{conv}(\operatorname{epi}_D f) = \operatorname{conv}\Big(\operatorname{epi}_{D_{\underline{y}}} \varphi_{\underline{y}} \cup \operatorname{epi}_{D_{\overline{y}}} \varphi_{\overline{y}}\Big).$$

Therefore, we have

$$\operatorname{vex}_D[f](x,y) = \min\Big\{\mu \,\big|\, (x,y,\mu) \in \operatorname{conv}\Big(\operatorname{epi}_{D_{\underline{y}}} \varphi_{\underline{y}} \cup \operatorname{epi}_{D_{\overline{y}}} \varphi_{\overline{y}}\Big)\Big\}$$

and equivalently,

$$\operatorname{vex}_D[f](x,y) = \min_{x_1,x_2,\lambda} \; (1-\lambda)\varphi_{\underline{y}}(x_1) + \lambda\varphi_{\overline{y}}(x_2) \tag{4.8a}$$

$$\text{s.t. } (1-\lambda)x_1 + \lambda x_2 = x \tag{4.8b}$$

$$(1-\lambda)\underline{y} + \lambda\overline{y} = y \tag{4.8c}$$

$$\lambda \in [0,1] \tag{4.8d}$$

$$(x_1,x_2) \in [\underline{x},\overline{x}]^2. \tag{4.8e}$$

Notice that $\operatorname{vex}_D[f](x,\underline{y}) = \varphi_{\underline{y}}(x)$ and $\operatorname{vex}_D[f](x,\overline{y}) = \varphi_{\overline{y}}(x)$. Hence, we just need to determine the convex envelope of $f(x,y)$ for $x \in [\underline{x},\overline{x}]$ and $y \in (\underline{y},\overline{y})$. In the following, we assume that $y \in (\underline{y},\overline{y})$.

4.2.3 Reduction to a one-dimensional optimization problem

We can further reduce optimization Problem (4.8) to a one-dimensional problem.

At first, (4.8c) enables us to fix the multiplier λ

$$\lambda_y := \lambda = \frac{y - \underline{y}}{\overline{y} - \underline{y}} \in (0, 1).$$

Given that λ is fixed, we can use (4.8b) to define x_2 in terms of x_1. For this we introduce the functions $t_{x,y}, T_{x,y} \colon [\underline{x}, \overline{x}] \to \mathbb{R}$ given by

$$t_{x,y}(z) = \frac{x - (1 - \lambda_y)\, z}{\lambda_y}, \quad \text{and} \tag{4.9}$$

$$T_{x,y}(z) = \frac{x - \lambda_y z}{1 - \lambda_y}. \tag{4.10}$$

These functions satisfy $x_2 = t_{x,y}(x_1)$ and $x_1 = T_{x,y}(x_2)$ for every feasible solution x_1, x_2 of Problem (4.8). Both functions are affine linear, strictly decreasing, and inverse to each other. For an interpretation of $T_{x,y}$, see Figure 4.3. Note that $t_{x,y}$ and $T_{x,y}$ are well-defined, since we assume $\lambda_y \in (0, 1)$, as described above.

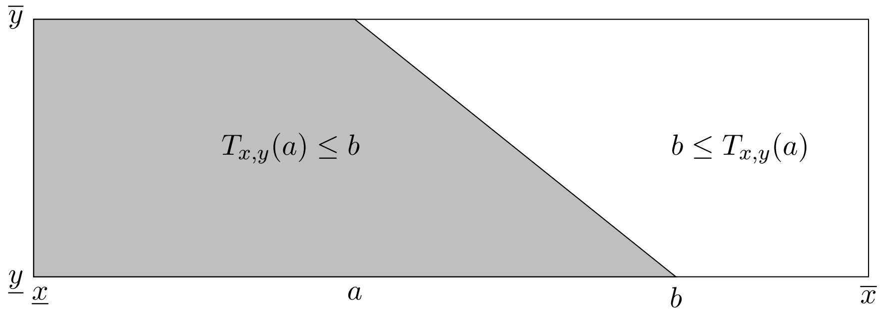


Figure 4.3: An interpretation of function $T_{x,y}$ on $D = [\underline{x}, \overline{x}] \times (\underline{y}, \overline{y})$. The set $\{(x, y) \colon T_{x,y}(a) \leq b\}$ with $\underline{x} \leq a \leq b \leq \overline{x}$ represents the gray shaded region in D, and $\{(x, y) \colon b \leq T_{x,y}(a)\}$ corresponds to the region in white in D. Both regions are separated by the line segment that connects the points $(a, \overline{y})$ and $(b, \underline{y})$. As we will see, the value of the convex envelope will depend on conditions of the form $T_{x,y}(a) \leq b$ for appropriate values of a and b. Thus, the reader can use this figure to understand how the position of the point $(x, y) \in D$ relates to the expression of the convex envelope.

Projecting out x_2 in Problem (4.8) yields a one-dimensional problem, which we can express with the help of $t_{x,y}$ and $T_{x,y}$ as

$$\begin{aligned} \min\ & (1 - \lambda_y)\varphi_{\underline{y}}(x_1) + \lambda_y \varphi_{\overline{y}}(t_{x,y}(x_1)) \\ \text{s.t. } & \max\{\underline{x}, T_{x,y}(\overline{x})\} \leq x_1 \leq \min\{\overline{x}, T_{x,y}(\underline{x})\}. \end{aligned} \tag{4.11}$$

The expression for the bounds comes from combining both bounds of x_1, namely, $\underline{x} \leq x_1 \leq \overline{x}$ and $T_{x,y}(\overline{x}) \leq x_1 \leq T_{x,y}(\underline{x})$. The latter bounds arise from $\underline{x} \leq x_2 \leq \overline{x}$, which after projecting x_2 becomes $\underline{x} \leq t_{x,y}(x_1) \leq \overline{x}$. Applying the function $T_{x,y}$ to the inequality yields $T_{x,y}(\overline{x}) \leq x_1 \leq T_{x,y}(\underline{x})$.

Let $\underline{x}_1 := \max\{\underline{x}, T_{x,y}(\overline{x})\}$, $\overline{x}_1 := \min\{\overline{x}, T_{x,y}(\underline{x})\}$, and let $F_{x,y} \colon [\underline{x}_1, \overline{x}_1] \to \mathbb{R}$ represent the objective function, i.e.,

$$F_{x,y}(x_1) := (1 - \lambda_y)\varphi_{\underline{y}}(x_1) + \lambda_y \varphi_{\overline{y}}(t_{x,y}(x_1)). \tag{4.12}$$

Then, Problem (4.11) is equivalent to

$$\begin{aligned} \min \ & F_{x,y}(x_1) \\ \text{s.t.} \ & \underline{x}_1 \leq x_1 \leq \overline{x}_1. \end{aligned} \tag{4.13}$$

Remark 4.2.4. *Problem* (4.13) *allows for a geometrical interpretation, as illustrated in Figure 4.4. For a given point* $(x, y) \in D$*, Problem* (4.13) *corresponds to finding the points* $(x_1, \underline{y})$ *and* $(t_{x,y}(x_1), \overline{y})$ *such that the line segment between these points contains* (x, y)*, and the line segment with endpoints* $(x_1, \underline{y}, \varphi_{\underline{y}}(x_1))$ *and* $(t_{x,y}(x_1), \overline{y}, \varphi_{\overline{y}}(t_{x,y}(x_1)))$ *has the lowest height at* (x, y)*. Note that the compact formulation of Problem* (4.13) *only contains* x_1 *as a variable, where constraint* (4.9) *allows us to recover* $x_2 = t_{x,y}(x_1)$*.*

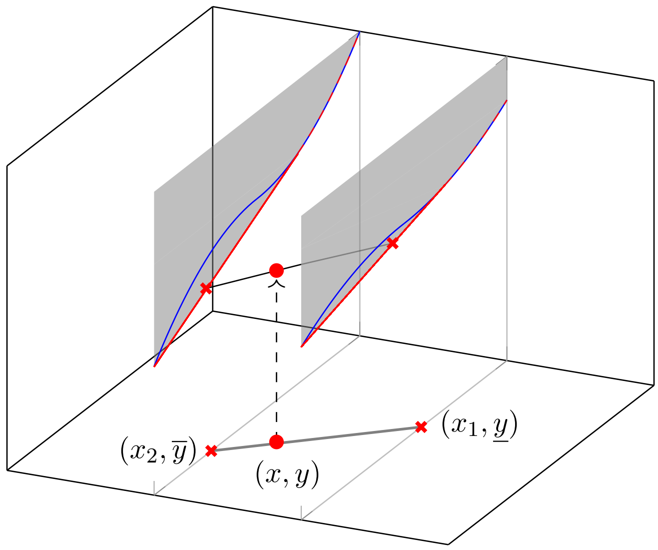


Figure 4.4: A geometrical interpretation of Problem (4.13) to determine $\text{vex}_D[f](x, y)$, as described in Remark 4.2.4. The one-dimensional functions $f_{\underline{y}}(x) = \underline{y}\,\text{sgn}(x)|x|^\alpha$ and $f_{\overline{y}}(x) = \overline{y}\,\text{sgn}(x)|x|^\alpha$ in blue are obtained after fixing y to $\underline{y}$ and $\overline{y}$ in $f(x, y) = y\,\text{sgn}(x)|x|^\alpha$. Their convex envelopes $\varphi_{\underline{y}}(x)$ and $\varphi_{\overline{y}}(x)$ in red are given by (4.6) and their epigraphs $\text{epi}_{D_{\underline{y}}}\varphi_{\underline{y}}$ and $\text{epi}_{D_{\overline{y}}}\varphi_{\overline{y}}$ in gray. Red crosses in the (x, y)–space correspond to the points $(x_1, \underline{y})$ and $(t_{x,y}(x_1), \overline{y})$ with $x_2 = t_{x,y}(x_1)$, and red crosses in the $(x, y, \varphi_y(x_1))$–space correspond to the points $(x_1, \underline{y}, \varphi_{\underline{y}}(x_1))$ and $(t_{x,y}(x_1), \overline{y}, \varphi_{\overline{y}}(t_{x,y}(x_1)))$.

At this point we can already compute the convex envelope for a simple case.

Proposition 4.2.5. *If* $\overline{x} < \beta\underline{x}$*, then* $\text{vex}_D[f](x, y) = F_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x})\})$*.*

Proof. When $\overline{x} < \beta\underline{x}$ holds, then $\varphi_y(x) = \alpha\overline{x}^{\alpha-1}xy + (1-\alpha)\overline{x}^\alpha y$. Therefore, $F_{x,y}$ is a linear function and (4.13) reduces to minimizing a linear function over an interval. A simple computation shows that the coefficient of x_1 in $F_{x,y}(x_1)$ is negative, and so the minimum is achieved at the upper bound, i.e., $\text{vex}_D[f](x, y) = F_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x})\})$. □

In light of the previous proposition, we assume that $\overline{x} \geq \beta\underline{x}$ in the remaining of this chapter. In particular, $\gamma = \beta\underline{x}$ (see Proposition 4.2.2).

4.2.4 Properties of the objective function

The objective function of Problem (4.13), $F_{x,y}$, is convex as it is a convex combination of convex functions. Note that the function $\varphi_{\overline{y}}(t_{x,y}(\cdot))$ is convex, since it is the composition of the convex function $\varphi_{\overline{y}}$ with the affine linear function $t_{x,y}$.

The following proposition characterizes the optimal solution for a large class of problems of a form similar to Problem (4.13).

Proposition 4.2.6. *Suppose that a function $F\colon \mathbb{R} \to \mathbb{R}$ is convex and has a global minimum $z^* \in \mathbb{R}$. Then the solution of $\min\{F(z) : a \leq z \leq b\}$ is given by*

$$F\big(\operatorname{mid}(a, b, z^*)\big),$$

where $\operatorname{mid}(a, b, z^*)$ selects the middle value between a, b and z^*.

Proof. The claim follows directly by comparing $F(a), F(b)$ and $F(z^*)$. If z^* is within the bounds, i.e., $a \leq z^* \leq b$, then $F(\operatorname{mid}(a, b, z^*)) = F(z^*)$. If $z^* \leq a$, then the convexity of F implies that F is increasing in $[z^*, \infty)$ and so $F(a) = F(\operatorname{mid}(a, b, z^*))$ is the minimum. Under the same logic follows the case $z^* \geq b$. □

In order to apply Proposition 4.2.6, we need to extend $F_{x,y}$ from $\underline{x}_1 \leq x_1 \leq \overline{x}_1$ to $x_1 \in \mathbb{R}$ and show that $F_{x,y}$ has a global minimum. The extension is immediate, given that all functions involved can be evaluated in $\mathbb{R}$. To deduce that $F_{x,y}$ has a global minimum over $\mathbb{R}$, we show that the sublevel sets of $F_{x,y}$ are bounded. This follows from the following proposition.

Proposition 4.2.7. *The function $F_{x,y}\colon \mathbb{R} \to \mathbb{R}$, given by Equation (4.12) with $\alpha > 1$, is coercive, i.e., it satisfies*

$$\lim_{x_1 \to \infty} F_{x,y}(x_1) = \infty \quad \textit{and} \quad \lim_{x_1 \to -\infty} F_{x,y}(x_1) = \infty.$$

Proof. We compute the limit for $x_1 \to +\infty$. The case $x_1 \to -\infty$ follows by the same argumentation. By definition (4.12),

$$F_{x,y}(x_1) = (1 - \lambda_y)\varphi_{\underline{y}}(x_1) + \lambda_y \varphi_{\overline{y}}(t_{x,y}(x_1)).$$

For every x_1 large enough, we have $x_1 \geq \beta\underline{x}$ and $t_{x,y}(x_1) < \beta\underline{x}$. Thus,

$$F_{x,y}(x_1) = (1 - \lambda_y)\,\underline{y}x_1^{\alpha} + \lambda_y\Big(\alpha(\beta\underline{x})^{\alpha-1}t_{x,y}(x_1)\overline{y} + (1-\alpha)(\beta\underline{x})^{\alpha}\overline{y}\Big).$$

This can be rewritten as $ax_1^{\alpha} + bx_1 + c$, where $a > 0$, $b < 0$ (since $t_{x,y}$ is strictly decreasing and linear), and $c \in \mathbb{R}$. Given that $\alpha > 1$, the sign of a determines the behavior of $F_{x,y}$ at infinity, hence, $\lim_{x_1 \to \infty} F_{x,y}(x_1) = \infty$. □

Figure 4.5 visualizes function $F_{x,y}$ for different pairs of points (x, y).

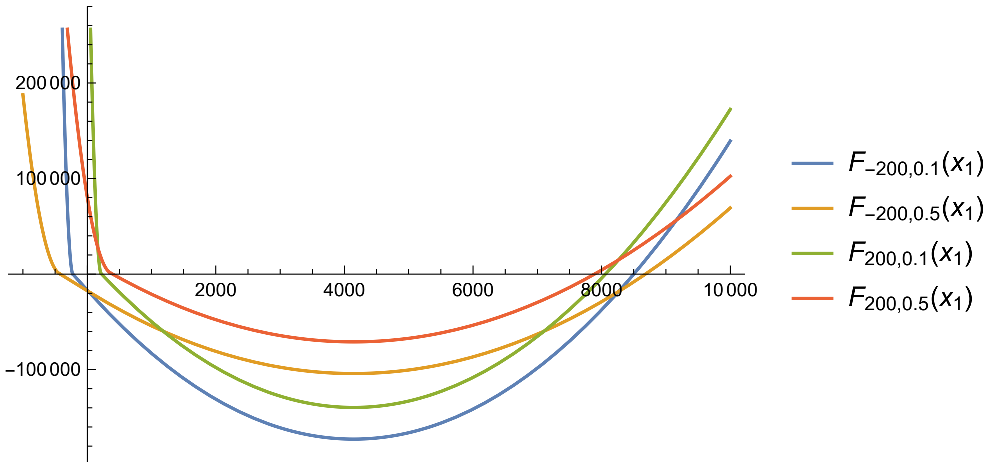


Figure 4.5: An illustration of $F_{x,y}$ for different pairs of values $(x, y) \in \{-200, 200\} \times \{0.1, 0.5\}$ and $\underline{y} = 0.01$, $\overline{y} = 1$. The function $F_{x,y}$ is continuous, convex and coercive.

4.3 First Solution Approach

With these properties of $F_{x,y}$, we are ready to compute its global minimum over $\mathbb{R}$. To this end, we use that $F_{x,y}$ is differentiable, since $\varphi_y \in C^1$ for $\alpha > 1$, and solve $F'_{x,y}(x_1) = 0$. We have,

$$\begin{aligned} F'_{x,y}(x_1) &= (1-\lambda_y)\varphi'_{\underline{y}}(x_1) + \lambda_y \varphi'_{\overline{y}}\left(t_{x,y}(x_1)\right) t'_{x,y}(x_1) \\ &\overset{(4.9)}{=} (1-\lambda_y)\Big(\varphi'_{\underline{y}}(x_1) - \varphi'_{\overline{y}}(t_{x,y}(x_1))\Big). \end{aligned}$$

Then, given the piecewise nature of φ_y, see (4.6), we get

$$F'_{x,y}(x_1) = (1-\lambda_y)\begin{cases} \alpha(\beta\underline{x})^{\alpha-1}\underline{y} - \alpha(\beta\underline{x})^{\alpha-1}\overline{y} & \text{if } x_1 \in LL \\ \alpha(\beta\underline{x})^{\alpha-1}\underline{y} - \alpha t_{x,y}(x_1)^{\alpha-1}\overline{y} & \text{if } x_1 \in LR \\ \alpha x_1^{\alpha-1}\underline{y} - \alpha(\beta\underline{x})^{\alpha-1}\overline{y} & \text{if } x_1 \in RL \\ \alpha x_1^{\alpha-1}\underline{y} - \alpha t_{x,y}(x_1)^{\alpha-1}\overline{y} & \text{if } x_1 \in RR, \end{cases} \tag{4.14}$$

where

$$\begin{aligned} LL &= \{x_1 \in \mathbb{R} \colon x_1 \le \beta\underline{x},\ t_{x,y}(x_1) \le \beta\underline{x}\}, \\ LR &= \{x_1 \in \mathbb{R} \colon x_1 \le \beta\underline{x},\ t_{x,y}(x_1) \ge \beta\underline{x}\}, \\ RL &= \{x_1 \in \mathbb{R} \colon x_1 \ge \beta\underline{x},\ t_{x,y}(x_1) \le \beta\underline{x}\}, \\ RR &= \{x_1 \in \mathbb{R} \colon x_1 \ge \beta\underline{x},\ t_{x,y}(x_1) \ge \beta\underline{x}\}. \end{aligned}$$

To solve $F'_{x,y}(x_1) = 0$, we restrict x_1 to be in LL, LR, RL, or RR.

Case $x_1 \in LL$: From (4.14) follows that $F'_{x,y}(x_1) = 0$ if and only if

$$\alpha(\beta\underline{x})^{\alpha-1}\underline{y} = \alpha(\beta\underline{x})^{\alpha-1}\overline{y}.$$

Given that $\underline{y} < \overline{y}$, it follows $F'_{x,y}(x_1) \neq 0$ for all $x_1 \in LL$. Thus, the global minimum is not in LL.

Case $x_1 \in LR$**:** The solution of $F'_{x,y}(x_1) = 0$ is given by

$$x_{lr} := T_{x,y}(\beta \underline{x} \, (\underline{y}/\overline{y})^{\frac{1}{\alpha-1}}). \tag{4.15}$$

Note, however, that $x_{lr} \notin LR$, as $t_{x,y}(x_{lr}) < \beta\underline{x}$. Thus, the global minimum is not in LR.

Case $x_1 \in RL$**:** The solution of $F'_{x,y}(x_1) = 0$ is given by

$$x_{rl} := \beta \underline{x} \, (\overline{y}/\underline{y})^{\frac{1}{\alpha-1}} . \tag{4.16}$$

The point x_{rl} is in RL if and only if $x_{rl} \geq \beta\underline{x}$ and $t_{x,y}(x_{rl}) \leq \beta\underline{x}$. Given that $\beta, \underline{x} < 0$, we have that $x_{rl} > \beta\underline{x}$. Thus, the global minimum is x_{rl} if and only if $t_{x,y}(x_{rl}) \leq \beta\underline{x}$. Otherwise, the global minimum must be in the next case.

Case $x_1 \in RR$**:** The solution of $F'_{x,y}(x_1) = 0$ is given by

$$x_{rr} := x \frac{\overline{y}^{\frac{1}{\alpha-1}}}{\lambda_y \underline{y}^{\frac{1}{\alpha-1}} + (1-\lambda_y)\overline{y}^{\frac{1}{\alpha-1}}}. \tag{4.17}$$

From the above case distinction follows that in the case $t_{x,y}(x_{rl}) \leq \beta\underline{x}$, x_{rl} is the optimal solution, otherwise it is x_{rr}. Therefore, from Proposition 4.2.6, we conclude that

$$\mathrm{vex}_D[f](x,y) = \begin{cases} F_{x,y}(\mathrm{mid}(\underline{x}_1, \overline{x}_1, x_{rl})) & \text{if } t_{x,y}(x_{rl}) \leq \beta\underline{x} \\ F_{x,y}(\mathrm{mid}(\underline{x}_1, \overline{x}_1, x_{rr})) & \text{else.} \end{cases} \tag{4.18}$$

Figure 4.6 visualizes function $f(x,y) = y\,\mathrm{sgn}(x)|x|^\alpha$ and its convex envelope $\mathrm{vex}_D[f](x,y)$.

Simplifying the convex envelope. We now derive a representation of $\mathrm{vex}_D[f](x,y)$ that is more compact than (4.18).

Theorem 4.3.1. *The convex envelope of* $f(x,y) = y\,sgn(x)|x|^\alpha$ *over* $D = [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$ *is given by*

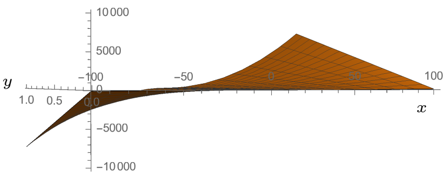
$$vex_D[f](x,y) = F_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x}), \max\{x_{rl}, x_{rr}\}\}). \tag{4.19}$$

Proof. To prove the claim we rewrite (4.18). We first show that

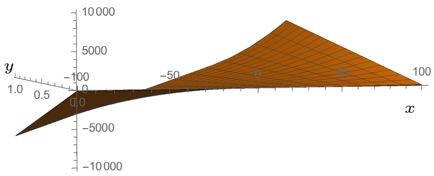
$$t_{x,y}(x_{rl}) \leq \beta\underline{x} \iff x_{rl} \geq x_{rr}, \tag{4.20}$$

where x_{rl} and x_{rr} are given in (4.16) and (4.17). It enables us to rewrite (4.18) as

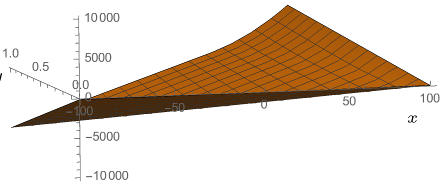
$$\mathrm{vex}_D[f](x,y) = F_{x,y}\big(\mathrm{mid}(\underline{x}_1, \overline{x}_1, \max\{x_{rl}, x_{rr}\})\big).$$



(a) $f(x,y) = y\,\mathrm{sgn}(x)|x|^\alpha$



(b) $(x,y) \mapsto \varphi_y(x)$



(c) $\mathrm{vex}_D[f](x,y)$

Figure 4.6: An illustration of the functions $f(x,y) = y\,\mathrm{sgn}(x)|x|^\alpha$, $(x,y) \mapsto \varphi_y$ given in (4.6), and $\mathrm{vex}_D[f](x,y)$ over the domain $(x,y) \in [-100,100] \times [0.01,1]$ for $\alpha = 2$.

Then, we show that

$$\underline{x}_1 \leq \max\{x_{rl}, x_{rr}\}. \tag{4.21}$$

This implies that $\mathrm{mid}(\underline{x}_1, \overline{x}_1, \max\{x_{rl}, x_{rr}\}) = \min\{\overline{x}_1, \max\{x_{rl}, x_{rr}\}\} = \min\{\overline{x}, T_{x,y}(\underline{x}), \max\{x_{rl}, x_{rr}\}\}$, which proves the result.

To prove (4.20) and (4.21), define $G_{x,y} \colon \mathbb{R} \to \mathbb{R}$ as

$$G_{x,y}(z) := z - t_{x,y}(z)\left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}}, \tag{4.22}$$

and notice that $G_{x,y}(x_{rr}) = 0$ and $G_{x,y}$ is strictly increasing.

The equivalence (4.20) follows from the following computation,

$$\begin{aligned}
x_{rl} \leq x_{rr} &\iff G_{x,y}(x_{rl}) \leq G_{x,y}(x_{rr}) \\
&\iff G_{x,y}(x_{rl}) \leq 0 \\
&\iff x_{rl} - t_{x,y}(x_{rl})\left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}} \leq 0 \\
&\iff x_{rl} \leq t_{x,y}(x_{rl})\left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}} \\
&\iff \beta\underline{x} \leq t_{x,y}(x_{rl}).
\end{aligned}$$

To prove (4.21), it suffices to show $\underline{x}_1 \leq x_{rr}$. Recall that $\underline{x}_1 = \max\{\underline{x}, T_{x,y}(\overline{x})\}$. The inequality $\underline{x} \leq x_{rr}$ follows from the definition of x_{rr} and $\lambda_y, \underline{y}, \overline{y} > 0$.

Finally, let us show

$$T_{x,y}(\overline{x}) \leq x_{rr}. \tag{4.23}$$

This follows from the following computation,

$$T_{x,y}(\overline{x}) \leq x_{rr} \iff G_{x,y}(T_{x,y}(\overline{x})) \leq 0$$

$$\iff T_{x,y}(\overline{x}) \leq \overline{x}\left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}}$$
$$\iff x \leq (1-\lambda_y)\overline{x}\left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}} + \lambda_y \overline{x},$$

and the fact that $x \leq \overline{x} \leq (1-\lambda_y)\overline{x}\left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}} + \lambda_y\overline{x}$. □

Remark 4.3.2. *From* (4.19) *we see that if* $x_{rl} \geq \overline{x}$*, then*

$$\mathit{vex}_D[f](x,y) = F_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x})\}) = F_{x,y}(\overline{x}_1).$$

The condition $x_{rl} \geq \overline{x}$ *depends only on the bounds, and not on the given point* (x, y)*. Therefore, if the bounds satisfy* $x_{rl} \geq \overline{x}$*, then we can further reduce the description of the convex envelope as above.*

In summary, we derived an analytic solution for the convex envelope of the function $f(x,y) = y\,\mathrm{sgn}(x)|x|^\alpha$, with $\alpha > 1$, which is stated in (4.19). To evaluate the convex envelope only requires the computation of the function $F_{x,y}$ at the minimum between $\overline{x}_1$, x_{rl}, and x_{rr}.

4.4 Second Solution Approach

We now provide an alternative approach for the derivation of $\mathrm{vex}_D[f]$. Our motivation to present this second approach is to provide additional insights into the structure of the closed-form solution to $\mathrm{vex}_D[f]$ and to strengthen the geometrical intuition of Problem (4.11), which enables us to better understand when the solution is determined by the variable bounds and when by the values x_{rl} and x_{rr}. To this end, we split again the domain according to the piecewise nature of the objective function $F_{x,y}$. Then, we solve Problem (4.11) on each of the four resulting regions and finally conduct a detailed analysis of the corresponding solutions to deduce the optimal objective value of the overall problem.

In the previous section we have seen that the main difficulty of solving Problem (4.11) results from the piecewise nature of the objective function $F_{x,y}$. While we considered $x_1 \in \mathbb{R}$ there, we will partition the domain $[\underline{x}, \overline{x}]^2$ of x_1 and $t_{x,y}(x_1)$ in this section. That is, we introduce $L := [\underline{x}, \beta\underline{x}]$ and $R := [\beta\underline{x}, \overline{x}]$ and set $LR := L \times R$, $RR := R \times R$, $RL := R \times L$, and $LL := L \times L$, where

$$LR = \left\{\left(x_1, t_{x,y}(x_1)\right) \in \mathbb{R}^2 \mid x_1 \in [\underline{x}, \beta\underline{x}],\ t_{x,y}(x_1) \in [\beta\underline{x}, \overline{x}]\right\}$$
$$RR = \left\{\left(x_1, t_{x,y}(x_1)\right) \in \mathbb{R}^2 \mid x_1 \in [\beta\underline{x}, \overline{x}],\ t_{x,y}(x_1) \in [\beta\underline{x}, \overline{x}]\right\}$$
$$RL = \left\{\left(x_1, t_{x,y}(x_1)\right) \in \mathbb{R}^2 \mid x_1 \in [\beta\underline{x}, \overline{x}],\ t_{x,y}(x_1) \in [\underline{x}, \beta\underline{x}]\right\}$$
$$LL = \left\{\left(x_1, t_{x,y}(x_1)\right) \in \mathbb{R}^2 \mid x_1 \in [\underline{x}, \beta\underline{x}],\ t_{x,y}(x_1) \in [\underline{x}, \beta\underline{x}]\right\}.$$

However, by doing this, we shift the difficulty of solving Problem (4.11), since the domain partitioning leads to

$$\begin{aligned} \text{vex}_D[f](x,y) = \min_{LL,LR,RL,RR} \min_{x_1} \; & (1-\lambda_y)\varphi_{\underline{y}}(x_1) + \lambda_y \varphi_{\overline{y}}(t_{x,y}(x_1)) \\ \text{s.t.} \; & \max\{\underline{x}, T_{x,y}(\overline{x})\} \le x_1 \le \min\{\overline{x}, T_{x,y}(\underline{x})\} \\ & (x_1, t_{x,y}(x_1)) \in LL, LR, RL, RR. \end{aligned} \tag{4.24}$$

Later on, we see how to handle Problem (4.24) when comparing the optimal objective values of the different cases LL, LR, RL and RR. Now, we concentrate on solving the inner minimization problem for each case separately. The geometrical interpretation of the inner minimization problem restricted to a particular case is as follows: For a given element $(x,y) \in D$, the inner minimization Problem of (4.24) corresponds to finding the points $(x_1, \underline{y})$ and $(t_{x,y}(x_1), \overline{y})$ in the respective domain such that (i) the line segment between these points contains (x,y), and (ii) the line segment with endpoints $(x_1, \underline{y}, \varphi_{\underline{y}}(x_1))$ and $(x_2, \overline{y}, \varphi_{\overline{y}}(x_2))$ has the lowest height at (x,y), where $x_2 = t_{x,y}(x_1)$.

The case partitioning is well-defined because $[\underline{x}, \overline{x}]^2 = LR \cup RR \cup RL \cup LL$. Hence, for each point $(x,y) \in [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$ there exists a line segment that contains (x,y), has end points $(x_1, \underline{y})$ and $(t_{x,y}(x_1), \overline{y})$, and belongs to at least one of the cases LL, LR, RL and RR. By turning our investigations to each particular case separately, one can think of considering all possible line segments that can be realized within the corresponding case.

4.4.1 Solving the inner minimization problems

Now, we determine the solution of the inner minimization problem of Problem (4.24) for each case separately. To this end, we throughout consider x_1^* that solves $F'_{x,y}(x_1^*) = 0$ and then derive the mid-value between the lower bound, upper bound, and x_1^* following Proposition 4.2.6. We will illustrate the solutions in Figures 4.7 – 4.12, where we throughout visualize case LR in gray, RR in green, RL in orange, and LL in hatched gray.

Case LR. In this case, the inner minimization problem reads

$$\begin{aligned} \min_{x_1} \; & (1-\lambda_y)\varphi_{\underline{y}}(x_1) + \lambda_y \varphi_{\overline{y}}(t_{x,y}(x_1)) \\ \text{s.t.} \; & \max\{\underline{x}, T_{x,y}(\overline{x})\} \le x_1 \le \min\{\beta\underline{x}, T_{x,y}(\beta\underline{x})\}. \end{aligned} \tag{4.25}$$

Solving $F'_{x,y}(x_1) = 0$ with $x_1 \in LR$, where $F'_{x,y}$ is given in (4.14) yields

$$x_{lr} = T_{x,y}\left(\beta\underline{x}\,(\underline{y}/\overline{y})^{\frac{1}{\alpha-1}}\right), \tag{4.26}$$

cf. also Equation (4.15). The following proposition states that the mid-value between the lower and upper bound of x_1 and x_{rl} is obtained when x_1 is at its upper bound, which is given by $\min\{\beta\underline{x}, T_{x,y}(\beta\underline{x})\}$.

Proposition 4.4.1. *It holds*

$$\operatorname{mid}\Big\{ \max\{\beta\underline{x}, T_{x,y}(\overline{x})\}, x_{rl}, \min\{\beta\underline{x}, T_{x,y}(\beta\underline{x})\} \Big\} = \min\{\beta\underline{x}, T_{x,y}(\beta\underline{x})\},$$

with x_{lr} given in (4.26).

Proof. It suffices to show $\min\{\beta\underline{x}, T_{x,y}(\beta\underline{x})\} \leq x_{lr}$, since the lower and upper bounds naturally satisfy $\max\{\beta\underline{x}, T_{x,y}(\overline{x})\} \leq \min\{\beta\underline{x}, T_{x,y}(\beta\underline{x})\}$. Given that $\beta\underline{x}\,(\underline{y}/\overline{y})^{\frac{1}{\alpha-1}} < \beta\underline{x}$ (since $\underline{y} < \overline{y}$) and $T_{x,y}$ is decreasing, it follows $T_{x,y}(\beta\underline{x}) \leq T_{x,y}\left(\beta\underline{x}(\underline{y}/\overline{y})^{\frac{1}{\alpha-1}}\right) = x_{lr}$. Therefore, $\min\{\beta\underline{x}, T_{x,y}(\beta\underline{x})\} \leq T_{x,y}(\beta\underline{x}) \leq x_{lr}$. □

From Proposition 4.4.1, we conclude that the minimum of $F_{x,y}$ in case LR is

$$\min_{x_1 \in LR} F_{x,y} = \begin{cases} F_{x,y}(T_{x,y}(\beta\underline{x})) & \text{if } T_{x,y}(\beta\underline{x}) \leq \beta\underline{x} \\ F_{x,y}(\beta\underline{x}) & \text{if } \beta\underline{x} \leq T_{x,y}(\beta\underline{x}), \end{cases} \tag{4.27}$$

illustrated in Figure 4.7.

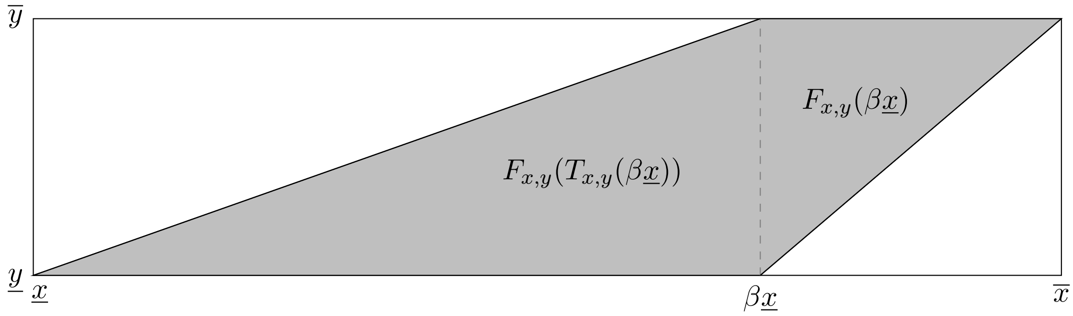


Figure 4.7: Optimal objective value of Problem (4.25) in case LR. For any point (x, y) in the gray area, the endpoints $(x_1, \underline{y})$ and $(t_{x,y}(x_1), \overline{y})$ of the line segment that contains (x, y) and minimizes (4.25) satisfy $x_1 \in [\underline{x}, \beta\underline{x}]$ and $t_{x,y}(x_1) \in [\beta\underline{x}, \overline{x}]$.

Remark 4.4.2. *It is possible to neglect case LR without losing the optimal solution of the overall Problem* (4.24)*, since every optimal solution* (4.27) *of case LR is feasible in some other case. In fact, when $T_{x,y}(\beta\underline{x}) \leq \beta\underline{x}$, then the optimal solution of* (4.27) *is $x_1 = T_{x,y}(\beta\underline{x})$, and with $t_{x,y} = T_{x,y}^{-1}$ it follows $(x_1, t_{x,y}(x_1)) = (T_{x,y}(\beta\underline{x}), \beta\underline{x}) \in LL$. Analogously, for $\beta\underline{x} \leq T_{x,y}(\beta\underline{x})$, the optimal solution of* (4.27) *is $x_1 = \beta\underline{x}$. Then, it follows $(x_1, t_{x,y}(x_1)) = (\beta\underline{x}, t_{x,y}(\beta\underline{x}) \in RR$.*

Case RR. In this case, the inner minimization problem reads

$$\begin{aligned} &\min_{x_1}\ (1-\lambda_y)\varphi_{\underline{y}}(x_1) + \lambda_y \varphi_{\overline{y}}(t_{x,y}(x_1)) \\ &\text{s.t. } \max\{\beta\underline{x}, T_{x,y}(\overline{x})\} \leq x_1 \leq \min\{\overline{x}, T_{x,y}(\beta\underline{x})\}. \end{aligned} \tag{4.28}$$

We recall from Equation (4.17) that the unique solution of $F'_{x,y}(x_1) = 0$ is given by

$$x_{rr} = x \frac{\overline{y}^{\frac{1}{\alpha-1}}}{\lambda_y \underline{y}^{\frac{1}{\alpha-1}} + (1-\lambda_y)\overline{y}^{\frac{1}{\alpha-1}}}. \tag{4.29}$$

We now show that x_{rr} is always greater than the lower bound of x_1.

Proposition 4.4.3. *It holds* $\max\{\beta\underline{x}, T_{x,y}(\overline{x})\} \le x_{rr}$ *with* x_{rr} *defined in* (4.29).

Proof. We first prove $\beta\underline{x} \le x_{rr}$. The lower bound in case RR obviously satisfies $\beta\underline{x} \le x_1$ and $\beta\underline{x} \le t_{x,y}(x_1)$ and thus $\beta\underline{x} \le x$. From (4.29) and $\beta\underline{x} \le x$ follows

$$x_{rr} \ge \beta\underline{x} \frac{\overline{y}^{\frac{1}{\alpha-1}}}{\lambda_y \underline{y}^{\frac{1}{\alpha-1}} + (1-\lambda_y)\overline{y}^{\frac{1}{\alpha-1}}}.$$

Since $\overline{y}^{\frac{1}{\alpha-1}}/(\lambda_y \underline{y}^{\frac{1}{\alpha-1}}, +(1-\lambda_y)\overline{y}^{\frac{1}{\alpha-1}}) \ge 1$, it holds $\beta\underline{x} \le x_{rr}$. Furthermore, from Equation (4.23) directly follows $T_{x,y}(\overline{x}) \le x_{rr}$. □

According to Proposition 4.4.3, the mid-value of the lower and upper bound of x_1 and x_{rr} reduces to

$$\operatorname{mid}\Big\{ \min\{\beta\underline{x}, T_{x,y}(\overline{x})\}, x_{rr}, \min\{\overline{x}, T_{x,y}(\beta\underline{x})\}\Big\} = \min\{x_{rr}, \min\{\overline{x}, T_{x,y}(\beta\underline{x})\}\}.$$

So the question arises when x_{rr} is less or greater than $\min\{\overline{x}, T_{x,y}(\beta\underline{x})\}$. To answer this question, we derive a condition that only depends on the bounds of the problem, which we call, for this reason, the *bound condition.* The following definition formalizes the idea of the bound condition.

Definition 4.4.4. *If the inequality*

$$\frac{\overline{x}}{\beta\underline{x}} \le \left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}}$$

is satisfied, we say that the bound condition *holds.*

In the case that the bound condition holds, then from Remark 4.3.2 we already know the convex envelope:

$$\overline{x} \le \left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}} \beta\underline{x} \quad \Rightarrow \quad \overline{x} \le x_{rl} \overset{Remark\ 4.3.2}{\Rightarrow} \operatorname{vex}_D[f](x,y) = F_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x})\}).$$

In a similar vein, the *bound condition* decides on whether the optimal solution of Problem (4.28) is given by the upper bound of x_1 or by x_{rr}, as stated by the following proposition.

Proposition 4.4.5. *For all points* $(x,y) \in [\beta\underline{x}, \overline{x}] \times (\underline{y}, \overline{y})$ *holds*

$$\min\{\overline{x}, T_{x,y}(\beta\underline{x})\} \le x_{rr},$$

if and only if the bound condition (from Definition 4.4.4) is satisfied.

Proof. "$\Leftarrow$" We show that the bound condition implies $\min\{\overline{x}, T_{x,y}(\beta\underline{x})\} \leq x_{rr}$.

(i) Assume $\min\{\overline{x}, T_{x,y}(\beta\underline{x})\} = \overline{x}$. The assumption $\overline{x} \leq T_{x,y}(\beta\underline{x})$ is equivalent to $\overline{x} \leq t_{x,y}(\overline{x})$.

We need to show $\overline{x} \leq x_{rr}$, which, after applying function G from (4.22), is equivalent to $\overline{x} \leq t_{x,y}(\overline{x})\,(\overline{y}/\underline{y})^{\frac{1}{\alpha-1}}$. Then, from the bound condition, we deduce

$$\overline{x} \leq \beta\underline{x}\,(\overline{y}/\underline{y})^{\frac{1}{\alpha-1}} \quad \overset{\beta\underline{x} \leq t_{x,y}(\overline{x})}{\Longleftrightarrow} \quad \overline{x} \leq t_{x,y}(\overline{x})\,(\overline{y}/\underline{y})^{\frac{1}{\alpha-1}}.$$

(ii) Assume $\min\{\overline{x}, T_{x,y}(\beta\underline{x})\} = T_{x,y}(\beta\underline{x})$.

We need to show $T_{x,y}(\beta\underline{x}) \leq x_{rr}$, which, after applying G is equivalent to $T_{x,y}(\beta\underline{x}) \leq \beta\underline{x}\left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}}$. But this latter claim directly follows from the assumption $T_{x,y}(\beta\underline{x}) \leq \overline{x}$ and the *bound condition* $\overline{x} \leq \beta\underline{x}\left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}}$.

"$\Rightarrow$" Proof by contradiction. Assume that the *bound condition* is violated, i.e., it holds $\beta\underline{x}\,(\overline{y}/\underline{y})^{\frac{1}{\alpha-1}} < \overline{x}$. We show that there exists a point $(x, y) \in [\beta\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$, such that $\min\{\overline{x}, T_{x,y}(\beta\underline{x})\} > x_{rr}$. To this end, we consider the point

$$(x, y) := \left(\frac{\overline{x} + \beta\underline{x}}{2}, \frac{\overline{y} + \underline{y}}{2}\right),$$

which implies $\lambda_y = 1/2$. This point satisfies $T_{x,y}(\beta\underline{x}) = \overline{x}$. So, we need to show $x_{rr} < \overline{x}$. Starting with the violated *bound condition*, it follows:

$$\begin{aligned}
& & \beta\underline{x}\left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}} &< \overline{x} \\
&\Longleftrightarrow & \beta\underline{x}\left(\frac{\overline{y}^{1-\alpha}}{\underline{y}^{1-\alpha} + \overline{y}^{1-\alpha}}\right) &< \overline{x}\left(\frac{\underline{y}^{1-\alpha}}{\underline{y}^{1-\alpha} + \overline{y}^{1-\alpha}}\right) \\
&\Longleftrightarrow & \beta\underline{x}\left(\frac{\overline{y}^{1-\alpha}}{\underline{y}^{1-\alpha} + \overline{y}^{1-\alpha}}\right) &< \overline{x}\left(1 - \frac{\overline{y}^{1-\alpha}}{\underline{y}^{1-\alpha} + \overline{y}^{1-\alpha}}\right) \\
&\Longleftrightarrow & (\beta\underline{x} + \overline{x})\left(\frac{\overline{y}^{1-\alpha}}{\underline{y}^{1-\alpha} + \overline{y}^{1-\alpha}}\right) &< \overline{x} \\
&\Longleftrightarrow & \left(\frac{\beta\underline{x} + \overline{x}}{2}\right)\left(\frac{\overline{y}^{1-\alpha}}{1/2\,(\underline{y}^{1-\alpha} + \overline{y}^{1-\alpha})}\right) &< \overline{x} \\
&\overset{\lambda_y = 1/2}{\Longleftrightarrow} & x_{rr} &< \overline{x}.
\end{aligned}$$

□

In the following, we derive the optimal solution of Problem (4.28) for case RR depending on whether the bound condition holds or not. If the bound condition holds, Propositions 4.4.3 and 4.4.5 imply that the mid-value between the lower and upper bound of x_1 and x_{rr}, and thus the minimum of $F_{x,y}$ is achieved when x_1 is at its upper bound $\min\{\overline{x}, T_{x,y}(\beta\underline{x})\}$, see also Figure 4.8:

$$\min F_{x,y} = \begin{cases} F_{x,y}(T_{x,y}(\beta\underline{x})) & \text{if } T_{x,y}(\beta\underline{x}) \leq \overline{x} \\ F_{x,y}(\overline{x}) & \text{if } \overline{x} \leq T_{x,y}(\beta\underline{x}). \end{cases} \tag{4.30}$$

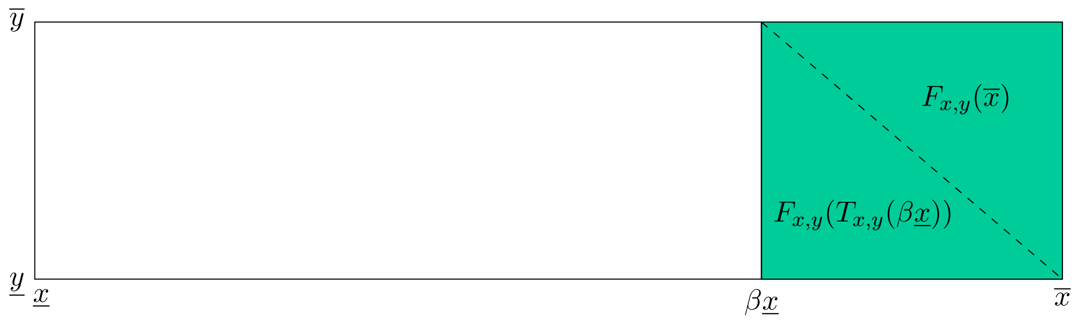


Figure 4.8: Optimal objective value of Problem (4.28) in case RR, when the bound condition holds. For any point (x, y) in the green area, the endpoints $(x_1, \underline{y})$ and $(t_{x,y}(x_1), \overline{y})$ of the line segment that contains (x, y) and minimizes (4.28) satisfy $x_1 \in [\beta\underline{x}, \overline{x}]$ and $t_{x,y}(x_1) \in [\beta\underline{x}, \overline{x}]$.

Otherwise, if the bound condition is violated, then the solution of Problem (4.28) is given by the minimum between x_{rr} and $\min\{\overline{x}, T_{x,y}(\beta\underline{x})\}$ leading to

$$\min F_{x,y} = \begin{cases} F_{x,y}(T_{x,y}(\beta\underline{x})) & \text{if } T_{x,y}(\beta\underline{x}) \leq \overline{x} \wedge T_{x,y}(\beta\underline{x}) \leq x_{rr} \\ F_{x,y}(\overline{x}) & \text{if } \overline{x} \leq T_{x,y}(\beta\underline{x}) \wedge \overline{x} \leq x_{rr} \\ F_{x,y}(x_{rr}) & \text{else.} \end{cases} \tag{4.31}$$

In fact, it is possible to further simplify this solution, since $T_{x,y}(\beta\underline{x}) \leq \overline{x}$ and $\overline{x} \leq T_{x,y}(\beta\underline{x})$ are redundant conditions: The redundancy of $T_{x,y}(\beta\underline{x}) \leq \overline{x}$ follows from

$$\begin{aligned} T_{x,y}(\beta\underline{x}) \leq x_{rr} \iff & \; G_{x,y}(T_{x,y}(\beta\underline{x})) \leq 0 \\ \iff & \; T_{x,y}(\beta\underline{x}) \leq \beta\underline{x} \left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}}, \end{aligned}$$

with $G_{x,y}$ defined in (4.22). Given that the bound condition is violated, this last inequality is true, and thus we conclude $T_{x,y}(\beta\underline{x}) \leq \beta\underline{x} \left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}} < \overline{x}$.

Following this train of thought, the redundancy of the other condition $\overline{x} \leq T_{x,y}(\beta\underline{x})$ follows from

$$\begin{aligned} \overline{x} \leq x_{rr} \iff & \; G_{x,y}(\overline{x}) \leq 0 \\ \iff & \; t_{x,y}(\overline{x}) \left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}} \leq \overline{x}. \end{aligned}$$

Then, from the violated bound condition, we deduce

$$t_{x,y}(\overline{x}) \left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}} \leq \overline{x} \leq \beta\underline{x} \left(\frac{\overline{y}}{\underline{y}}\right)^{\frac{1}{\alpha-1}}$$

$$\begin{aligned} &\iff \quad t_{x,y}(\overline{x}) \leq \beta \underline{x} \\ &\iff \quad \overline{x} \leq T_{x,y}(\beta \underline{x}). \end{aligned}$$

Finally, removing the redundant conditions in (4.31) yields

$$\min F_{x,y} = \begin{cases} F_{x,y}(T_{x,y}(\beta \underline{x})) & \text{if } T_{x,y}(\beta \underline{x}) \leq x_{rr} \\ F_{x,y}(\overline{x}) & \text{if } \overline{x} \leq x_{rr} \\ F_{x,y}(x_{rr}) & \text{else.} \end{cases} \tag{4.32}$$

see Figure 4.9.

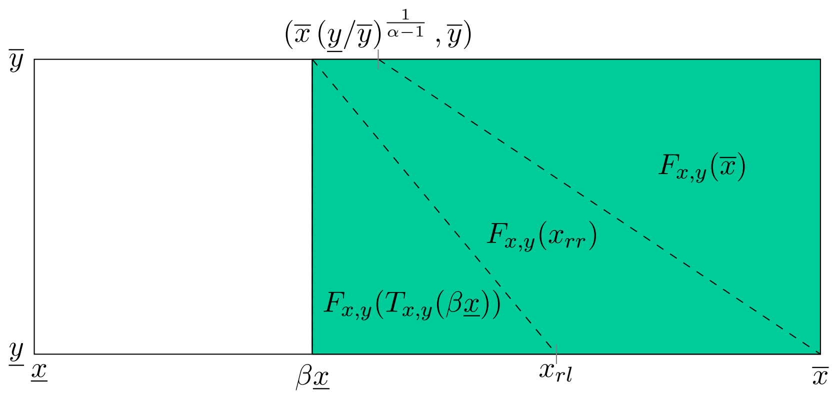


Figure 4.9: Optimal objective value of Problem (4.28) in case RR, when the bound condition is violated. For any point (x, y) in the green area, the endpoints $(x_1, \underline{y})$ and $(t_{x,y}(x_1), \overline{y})$ of the line segment that contains (x, y) and minimizes (4.28) satisfy $x_1 \in [\beta \underline{x}, \overline{x}]$ and $t_{x,y}(x_1) \in [\beta \underline{x}, \overline{x}]$.

Case RL. In this case, the inner minimization problem reads

$$\begin{aligned} \min_{x_1} \; & (1 - \lambda_y)\varphi_{\underline{y}}(x_1) + \lambda_y \varphi_{\overline{y}}(t_{x,y}(x_1)) \\ \text{s.t.} \; & \max\{\beta \underline{x}, T_{x,y}(\beta \underline{x})\} \leq x_1 \leq \min\{\overline{x}, T_{x,y}(\underline{x})\}. \end{aligned} \tag{4.33}$$

We recall from Equation (4.16) that the unique solution of $F'_{x,y}(x_1) = 0$ is given by

$$x_{rl} = \beta \underline{x} \left(\frac{\overline{y}}{\underline{y}} \right)^{\frac{1}{\alpha - 1}}.$$

Here the bound condition appears naturally in the sense that it implies $\overline{x} \leq x_{rl}$. The following proposition states that x_{rl} is greater than the upper bound of x_1.

Proposition 4.4.6. *If the bound condition holds, then*

$$\min\{\overline{x}, T_{x,y}(\underline{x})\} \leq x_{rl}.$$

Proof. The bound condition is equivalent to $\overline{x} \leq x_{rl}$. It follows $\min\{\overline{x}, T_{x,y}(\underline{x})\} \leq x_{rl}$. □

This proposition implies

$$\text{mid}\left\{ \max\{\beta\underline{x}, T_{x,y}(\beta\underline{x})\}, x_{rl}, \min\{\overline{x}, T_{x,y}(\underline{x})\}\right\} = \min\left\{\overline{x}, T_{x,y}(\underline{x})\right\}.$$

So, when the bound condition holds, the solution of Problem (4.33) is given by

$$\min F_{x,y} = \begin{cases} F_{x,y}(T_{x,y}(\underline{x})) & \text{if } T_{x,y}(\underline{x}) \leq \overline{x} \\ F_{x,y}(\overline{x}) & \text{if } \overline{x} \leq T_{x,y}(\underline{x}), \end{cases} \tag{4.34}$$

see Figure 4.10.

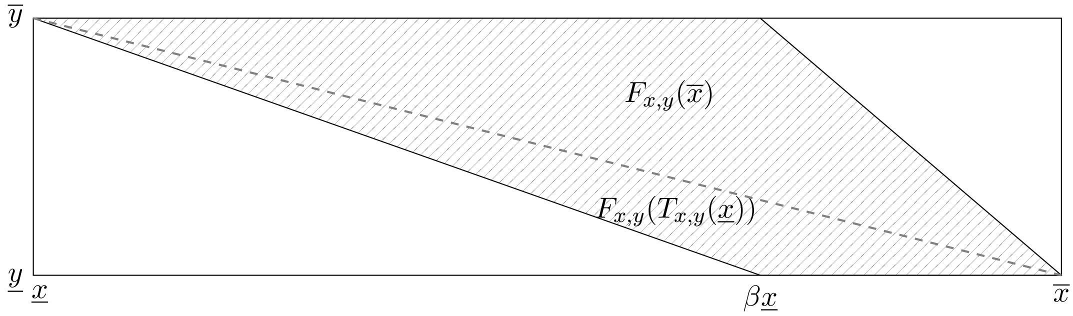


Figure 4.10: Optimal objective value in Problem (4.33) in case RL, when the bound condition holds. For any point (x, y) in the gray hatched area, the endpoints $(x_1, \underline{y})$ and $(t_{x,y}(x_1), \overline{y})$ of the line segment that contains (x, y) and minimizes (4.33) satisfy $x_1 \in [\beta\underline{x}, \overline{x}]$ and $t_{x,y}(x_1) \in [\underline{x}, \beta\underline{x}]$.

When the bound condition is violated, then $x_{rl} \leq \overline{x}$ holds by definition. In addition, $\beta\underline{x} \leq x_{rl}$ holds (with $\underline{y} \leq \overline{y}$), and thus

$$\text{mid}\{\max\{\beta\underline{x}, T_{x,y}(\beta\underline{x})\}, x_{rl}, \min\{\overline{x}, T_{x,y}(\underline{x})\}\} = \text{mid}\{T_{x,y}(\beta\underline{x}), x_{rl}, T_{x,y}(\underline{x})\}.$$

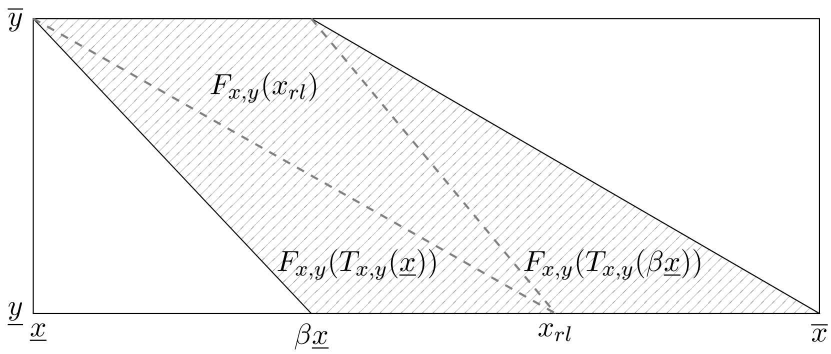


Figure 4.11: Optimal objective value of Problem (4.33) in case RL, when the bound condition is violated. For any point (x, y) in the gray hatched area, the endpoints $(x_1, \underline{y})$ and $(t_{x,y}(x_1), \overline{y})$ of the line segment that contains (x, y) and minimizes (4.33) satisfy $x_1 \in [\beta\underline{x}, \overline{x}]$ and $t_{x,y}(x_1) \in [\underline{x}, \beta\underline{x}]$.

Then, the solution of Problem (4.33) is given by

$$\min F_{x,y} = \begin{cases} F_{x,y}(T_{x,y}(\underline{x})) & \text{if } T_{x,y}(\underline{x}) \le x_{rl} \\ F_{x,y}(T_{x,y}(\beta\underline{x})) & \text{if } x_{rl} \le T_{x,y}(\beta\underline{x}) \\ F_{x,y}(x_{rl}) & \text{else,} \end{cases} \tag{4.35}$$

shown in Figure 4.11.

Case LL. In this case, the inner minimization problem reads

$$\begin{aligned} \min_{x_1} \ & (1-\lambda_y)\varphi_{\underline{y}}(x_1) + \lambda_y \varphi_{\overline{y}}(t_{x,y}(x_1)) \\ \text{s.t. } & \max\{\underline{x}, T_{x,y}(\beta\underline{x})\} \le x_1 \le \min\{\beta\underline{x}, T_{x,y}(\underline{x})\}. \end{aligned} \tag{4.36}$$

It holds $F'_{x,y}(x_1) < 0$ for all $x_1 \in LL$, since $F'_{x,y}(x_1) = (1-\lambda_y)\alpha(\beta\underline{x})^{\alpha-1}(\underline{y}-\overline{y}) < 0$, see Equation (4.14). This means that $F_{x,y}$ is monotone decreasing and the minimum of $F_{x,y}$ is obtained when x_1 is at its upper bound $\max\{\beta\underline{x}, T_{x,y}(\underline{x})\}$. Therefore, the solution of Problem (4.36) in case LL is given by

$$\min F_{x,y} = \begin{cases} F_{x,y}(T_{x,y}(\underline{x})) & \text{if } T_{x,y}(\underline{x}) \le \beta\underline{x} \\ F_{x,y}(\beta\underline{x}) & \text{if } \beta\underline{x} \le T_{x,y}(\underline{x}), \end{cases} \tag{4.37}$$

shown in Figure 4.12.

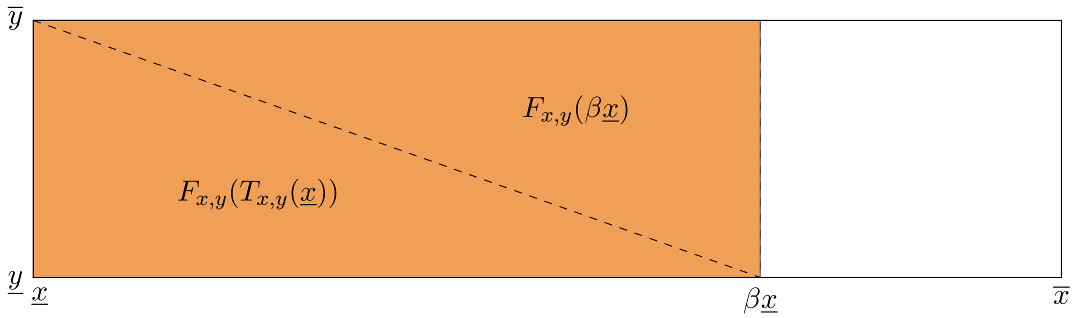


Figure 4.12: Optimal objective value of Problem (4.36) in case LL. For any point (x, y) in the orange area, the endpoints $(x_1, \underline{y})$ and $(t_{x,y}(x_1), \overline{y})$ of the line segment that contains (x, y) and minimizes (4.36) satisfy $x_1 \in [\underline{x}, \beta\underline{x}]$ and $t_{x,y}(x_1) \in [\underline{x}, \beta\underline{x}]$.

In the following, we show that $(x, y) \mapsto F_{x,y}(\beta\underline{x})$ and $(x, y) \mapsto F_{x,y}(T_{x,y}(\underline{x}))$ are linear hyperplanes in the LL case. Moreover, we deduce that $(x, y) \mapsto F_{x,y}(\beta\underline{x})$ is not a valid underestimator of $\varphi\colon \mathbb{R}^2 \supseteq [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}] \to \mathbb{R}$ with $\varphi(x, y) := \varphi_y(x)$ given in Equation (4.6). Hence, it is not a valid underestimator of $\text{vex}_D[f]$. In Section 4.5, we show that the convex envelope and the underestimator of SCIP coincide in the linear part $F_{x,y}(T_{x,y}(\underline{x}))$.

Lemma 4.4.7. *The functions $g_1, g_2\colon \mathbb{R}^2 \to \mathbb{R}$ with $g_1(x, y) = F_{x,y}(\beta\underline{x})$ and $g_2(x, y) = F_{x,y}(T_{x,y}(\underline{x}))$ from case LL are linear.*

Proof. We show that g_1 and g_2 have constant gradients. Since we are in case LL, it follows

$$\begin{aligned} g_1(x,y) &= (1-\lambda_y)\varphi_{\underline{y}}(\beta\underline{x}) + \lambda_y\varphi_{\overline{y}}(t_{x,y}(\beta\underline{x})) \\ &= (1-\lambda_y)\varphi_{\underline{y}}(\beta\underline{x}) + (\alpha(\beta\underline{x})^{\alpha-1}(x-(1-\lambda_y)\beta\underline{x})\,\overline{y}) + \lambda_y(1-\alpha)(\beta\underline{x})^{\alpha}, \quad (4.38) \end{aligned}$$

and

$$\begin{aligned} g_2(x,y) &= (1-\lambda_y)\varphi_{\underline{y}}(T_{x,y}(\underline{x})) + \lambda_y\varphi_{\overline{y}}(\underline{x}) \\ &= (1-\lambda_y)\Big(\alpha(\beta\underline{x})^{\alpha-1}T_{x,y}(\underline{x})\underline{y} + (1-\alpha)(\beta\underline{x})^{\alpha}\underline{y}\Big) + \lambda_y\overline{y}\underline{x}^{\alpha} \\ &= (1-\lambda_y)\Big(\alpha(\beta\underline{x})^{\alpha-1}\,\frac{x-\lambda_y\underline{x}}{1-\lambda_y}\,\underline{y} + (1-\alpha)(\beta\underline{x})^{\alpha}\underline{y})\Big) + \lambda_y\overline{y}\underline{x}^{\alpha} \\ &= \alpha(\beta\underline{x})^{\alpha-1}(x-\lambda_y\underline{x})\underline{y} + (1-\lambda_y)(1-\alpha)(\beta\underline{x})^{\alpha}\underline{y} + \lambda_y\overline{y}\underline{x}^{\alpha}. \quad (4.39) \end{aligned}$$

Given that x and y (in terms of λ_y) only appear linearly in (4.38) and (4.39), g_1 and g_2 are linear functions. □

In order to show that $(x,y)\mapsto F_{x,y}(\beta\underline{x})$ is not a valid underestimator of the convex envelope, we need the following theorem, which we state without proof here.

Theorem 4.4.8. *Let $f\colon\mathbb{R}^n\to\mathbb{R}$ be a differentiable function and $x_0\in\mathbb{R}^n$. If $f(x_0)=0$ and $\nabla f(x_0)\neq 0$, then for any $\varepsilon>0$ there exist $x_1,x_2\in B_\varepsilon(x_0)$ such that $f(x_1)<0$ and $f(x_2)>0$.*

Proposition 4.4.9. *The function $g_1\colon\mathbb{R}^2\supseteq[\underline{x},\overline{x}]\times[\underline{y},\overline{y}]\to\mathbb{R}$ from case LL, with $g_1(x,y):=F_{x,y}(\beta\underline{x})=(1-\lambda_y)\varphi_{\underline{y}}(\beta\underline{x})+\lambda_y\varphi_{\overline{y}}(t_{x,y}(\beta\underline{x}))$ is not a valid underestimator of $\varphi\colon\mathbb{R}^2\supseteq[\underline{x},\overline{x}]\times[\underline{y},\overline{y}]\to\mathbb{R}$ with $\varphi(x,y):=\varphi_y(x)$.*

Proof. First, we recall that $F_{x,y}(\beta\underline{x})=(1-\lambda_y)\underline{y}(\beta\underline{x})^{\alpha}+\lambda_y(\beta\underline{x})^{\alpha}$. Now the functions $(x,y)\mapsto\varphi(x,y)$ and $(x,y)\mapsto F_{x,y}(\beta\underline{x})$ coincide on the line segment $\{(\beta\underline{x},y)\colon y\in[\underline{y},\overline{y}]\}$. Let us consider the function $\Delta\colon\mathbb{R}^2\supseteq[\underline{x},\overline{x}]\times[\underline{y},\overline{y}]\to\mathbb{R}$ with $\Delta(x,y):=\varphi(x,y)-F_{x,y}(\beta\underline{x})$. We have that $\Delta\left(\beta\underline{x},\frac{\underline{y}+\overline{y}}{2}\right)=0$ and $\nabla\Delta\left(\beta\underline{x},\frac{\underline{y}+\overline{y}}{2}\right)\neq 0$, because

$$\begin{aligned} \frac{\partial}{\partial x}\Delta\left(\beta\underline{x},\frac{\underline{y}+\overline{y}}{2}\right) =& \frac{\partial}{\partial x}\varphi\left(\beta\underline{x},\frac{\underline{y}+\overline{y}}{2}\right) - \frac{\partial}{\partial x}F_{x,y}(\beta\underline{x}) \\ =& \alpha(\beta\underline{x})^{\alpha-1}\left(\frac{\underline{y}+\overline{y}}{2}\right) - \frac{1}{2}(\alpha(\beta\underline{x})^{\alpha-1}(\overline{y}-\underline{y})) \\ =& \alpha(\beta\underline{x})^{\alpha-1}\,\underline{y}\;\neq\;0. \end{aligned}$$

Since $\nabla\Delta\left(\beta\underline{x},\frac{\underline{y}+\overline{y}}{2}\right)\neq 0$, Theorem 4.4.8 implies that function Δ is negative for some point in the domain of φ. Hence, $(x,y)\mapsto F_{x,y}(\beta\underline{x})$ is not a valid underestimator of φ. □

Remark 4.4.10. *Proposition 4.4.9 implies that $\mathrm{vex}_D[f](x,y)$ is given by the value of $F_{x,y}(x,y)$ from case RL in the region $\{(x,y)\in[\underline{x},\overline{x}]\times[\underline{y},\overline{y}] : \beta\underline{x}\leq T_{x,y}(\underline{x})\}$, as $(x,y)\mapsto F_{x,y}(\beta\underline{x})$ from case LL is not a valid underestimator and RL is the only case that shares points with LL in this region.*

4.4.2 Solving the overall optimization problem

Now that we derived the optimal solution of the inner optimization Problem (4.24) for each of the cases LR, RR, RL and LL separately, we are ready to solve the overall Problem (4.24). To this end, we compare the optimal objective values of the inner optimization problem between the different cases according to the criteria, whether

– the *bound condition* holds, or

– the *bound condition* is violated.

For this comparison, we need the concept of "optimal line segments" as stated by the following theorem.

Theorem 4.4.11 (Optimal line segments)**.** *Consider a point* $(x,y) \in [\underline{x},\overline{x}] \times [\underline{y},\overline{y}]$ *and a line segment*

$$l = \Big\{(1-\mu)(x_1,\underline{y}) + \mu(t_{x,y}(x_1),\overline{y}) \mid 0 \le \mu \le 1\Big\},$$

with $x_1 \in [\underline{x},\overline{x}]$ *that is optimal for this point, i.e.,*

i) $(x,y) \in l$*,*

ii) x_1 *is the optimal solution of Problem* (4.24) *for* (x,y)*.*

Then, for all $(\hat{x},\hat{y}) \in l$*,* l *is the optimal line segment.*

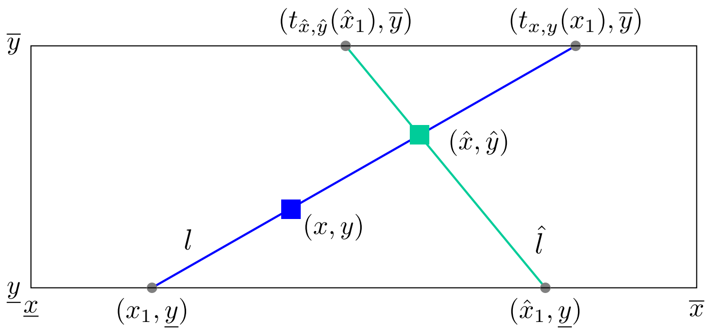


Figure 4.13: Illustration of Theorem 4.4.11

Proof. Proof by contradiction. For (x,y), let l be the optimal line segment and $F_{x,y}(x_1)$ be the optimal objective value of Problem (4.24). Assume that there exists a point $(\hat{x},\hat{y}) \neq (x,y)$ with $(\hat{x},\hat{y}) \in l$, which has a different optimal line segment $\hat{l}$, i.e., $(\hat{x},\hat{y}) \in \hat{l}$, where

$$\hat{l} = \Big\{(1-\mu)(\hat{x}_1,\underline{y}) + \mu(t_{\hat{x},\hat{y}}(\hat{x}_1),\overline{y}) \mid 0 \le \mu \le 1\Big\},$$

and $\hat{x}_1 \in [\underline{x},\overline{x}]$. Then, the optimal objective value of Problem (4.24) at $(\hat{x},\hat{y})$ satisfies $F_{\hat{x},\hat{y}}(\hat{x}_1) < F_{\hat{x},\hat{y}}(x_1)$. W.l.o.g. let us assume that $(\hat{x},\hat{y})$ is on the segment between the points (x, y) and $(t_{x,y}(x_1), \overline{y})$, see Figure 4.13. Then, the line segment containing the points $(x_1, \underline{y})$ and $(\hat{x}, \hat{y})$ yields an optimal objective value at point (x, y) that is smaller than the one induced by $F_{x,y}(x_1)$, which contradicts the assumption that $F_{x,y}(x_1)$ is the optimal objective value of Problem (4.24). □

The bound condition holds. Partitioning the domain according to the different cases LL (orange), RL (hatched), and RR (green) leads to the following regions **I** – **VI** in Figure 4.14 when the *bound condition* holds. By Remark 4.4.2, we neglect the division suggested from case LR. This means, we compare the solutions given in (4.30), (4.34) and (4.37).

We now provide a detailed analysis of the optimal solutions on the different regions. In particular, we compare the solutions on those regions that share common points. This allows us to deduce the optimal objective value to the overall Problem (4.24) yielding $\text{vex}_D[f]$.

The analysis works as follows: Some regions belong to a single case, for example, region **I** to case LL and **VI** to case RR. For points in these regions, the optimal objective value $F_{x,y}$ of the overall Problem (4.24) is determined by the optimal objective value of the inner minimization problem from the corresponding case. This also applies, for example, to the regions **II** and **IV** that partly belong to the two different cases LL and RL. Following Remark 4.4.10, the optimal objective value of the inner minimization problem from case LL does not induce a valid underestimator of $\text{vex}_D[f]$. As a consequence, the optimal solution of (4.24) is given by the solution to case RL. For the remaining cases, we apply the concept of "optimal line segments" from Theorem 4.4.11. It states that a line segment, which is optimal for a single point, is optimal for all points on that line segment. Given that optimal line segments of points in region **II** also traverse region **III** and the optimal solution of (4.24) is given by case RL, it follows from Theorem 4.4.11 that these line segments are also optimal for points in region **III**. The same applies to the regions **IV** and **V**.

In the following, we state the optimal objective value $F_{x,y}$ of Problem (4.24) on each particular region.

I: The only possible solution is given by case LL yielding $F_{x,y}(T_{x,y}(\underline{x}))$.

II: Using Remark 4.4.10, the only solution is given by case RL leading to $F_{x,y}(T_{x,y}(\underline{x}))$.

III: The solution for region **II** is $F_{x,y}(T_{x,y}(\underline{x}))$ from case RL. Then, from $x_1 = T_{x,y}(\underline{x})$ follows $x_2 = t_{x,y}(T_{x,y}(\underline{x})) = \underline{x}$. This implies that the optimal line segment for a point (x, y) in region **II** lies between the endpoints $(\underline{x}, \overline{y})$ and some point in $[\beta\underline{x}, \overline{x}] \times \underline{y}$. Therefore, the optimal line goes through region IV. Using Theorem 4.4.11, we conclude that the solution for this region is given by case RL yielding $F_{x,y}(T_{x,y}(\underline{x}))$.

IV: Following the same argument as in region **II** yields $F_{x,y}(\overline{x})$ from case RL.

V: Similar to the previous case, we conclude that the optimal objective value is given by case RL, namely $F_{x,y}(\overline{x})$.

VI: The only possible solution is given by case RR leading to $F_{x,y}(\overline{x})$.

Finally, when the bound condition holds, we have that the convex envelope is given by

$$\text{vex}_D[f](x,y) = \begin{cases} F_{x,y}(T_{x,y}(\underline{x})) & \text{if } T_{x,y}(\underline{x}) \leq \overline{x} \\ F_{x,y}(\overline{x}) & \text{if } \overline{x} \leq T_{x,y}(\underline{x}), \end{cases} \tag{4.40}$$

see Figure 4.15. From Remark 4.3.2 directly follows that (4.40) equals the representation of $\text{vex}_D[f](x,y)$ as given in the first solution approach.

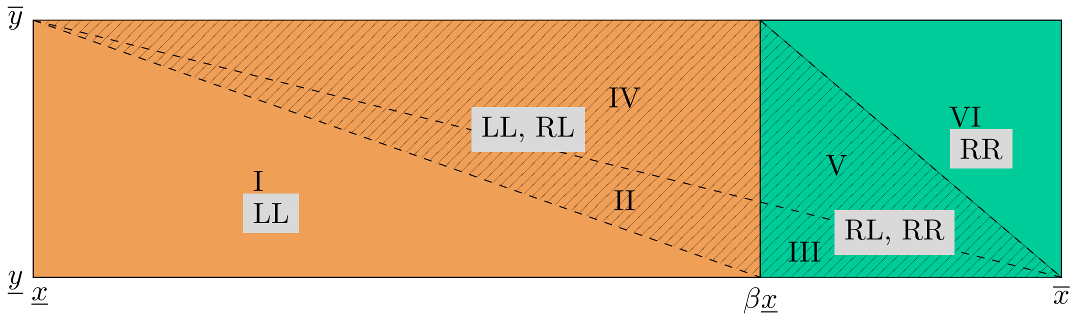


Figure 4.14: Domain partitioning, when the *bound condition* holds. The orange region represents case LL, the green region case RR, and the hatched region case RL. Dotted lines illustrate the domain partitions that arise from the piecewise-defined objective function $F_{x,y}$. The numbers mark the partial domains, where $\text{vex}_D[f](x,y)$ is determined by comparing the optimal solutions of the possibly multiple cases that are highlighted in gray, i.e.,region I (case LL), region II (cases LL, RL), region III (cases RL, RR), region IV (cases LL, RL), region V (cases RL, RR), and region VI (case RR).

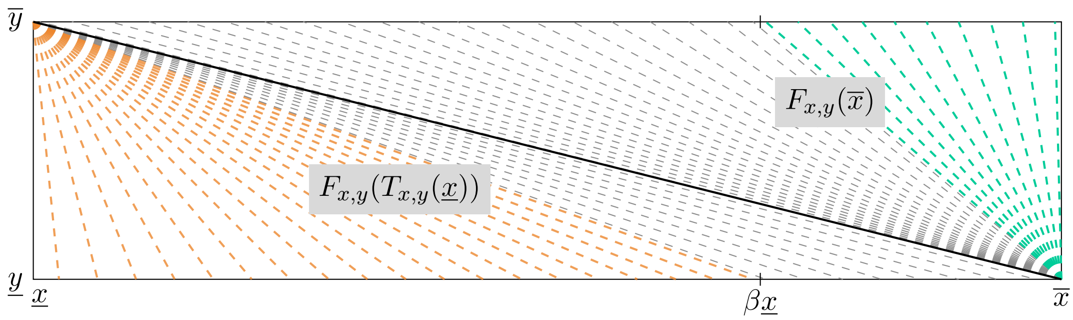


Figure 4.15: Representation of $\text{vex}_D[f](x,y)$ as given in (4.40), when the bound condition holds. The convex envelope is given by the values $F_{x,y}(T_{x,y}(\underline{x}))$ and $F_{x,y}(\overline{x})$ being separated by the bold line. Dashed lines represent the optimal line segments for points inside the particular regions. Here, optimal line segments are shown in orange for case LL, in gray for RL, and in green for RR.

The bound condition is violated. Partitioning the domain according to the different cases LL (orange), RL (hatched), and RR (green) leads to the following regions **I** – **VIII** in Figure 4.16 when the bound condition is violated. Again, by Remark 4.4.2, we neglect the division suggested from case LR. This means, we compare the solutions given in (4.32), (4.35) and (4.37).

The analysis follows the same argumentation as when the bound condition holds.

I: The only possible solution is given by case LL yielding $F_{x,y}(T_{x,y}(\underline{x}))$.

II: Using Remark 4.4.10, the only possible solution is given by case RL, i.e., $F_{x,y}(T_{x,y}(\underline{x}))$.

III: The solution for region **II** is $F_{x,y}(T_{x,y}(\underline{x}))$ from case RL. Then, from $x_1 = T_{x,y}(\underline{x})$ follows $x_2 = t_{x,y}(T_{x,y}(\underline{x})) = \underline{x}$. This implies that the optimal line segment for a point (x, y) in region **II** lies between the endpoints $(\underline{x}, \overline{y})$ and some point in $[\beta\underline{x}, \overline{x}] \times \underline{y}$. Therefore, the optimal line goes through region **IV**. Using Theorem 4.4.11, we conclude that the solution for this region is given by case RL, yielding $F_{x,y}(T_{x,y}(\underline{x}))$.

IV: Following the same argument as in region **II** yields $F_{x,y}(x_{rl})$ from case RL.

V: Similar to the previous case, we conclude that the optimal objective value is given by case RL, namely $F_{x,y}(x_{rl})$.

VI: The only possible solution is given by case RR leading to $F_{x,y}(\overline{x})$.

VII: The only possible solution is given by case RR yielding $F_{x,y}(x_{rr})$.

VIII: The solution of region **VII** is given by $F_{x,y}(x_{rr})$. For a point (x, y) in region **VII** with $x \in [\beta\underline{x}, \overline{x}\,(\underline{y}/\overline{y})^{1/(\alpha-1)}]$ and $y = \overline{y}$ follows $\lambda_y = 1$, which implies $x_{rr} = x(\overline{y}/\underline{y})^{1/(\alpha-1)}$. From $x \in [\beta\underline{x}, \overline{x}\,(\underline{y}/\overline{y})^{1/(\alpha-1)}]$ follows that $x_{rr} \in [\beta\underline{x}(\overline{y}/\underline{y})^{1/(\alpha-1)}, \overline{x}]$. So the optimal lines sweep the whole area of regions **VII** and **VIII**, which according to Theorem 4.4.11 implies that the optimal objective value for region **VIII** is also given by case RR, i.e., $F_{x,y}(x_{rr})$.

Finally, when the bound condition is violated, from the description of the solution in **I**–**VIII** follows that the convex envelope is given by

$$\text{vex}_D[f](x,y) = \begin{cases} F_{x,y}(T_{x,y}(\underline{x})) & \text{if } T_{x,y}(\underline{x}) \leq x_{rl} \\ F_{x,y}(x_{rl}) & \text{if } x_{rl} \leq T_{x,y}(\underline{x}) \text{ and } t_{x,y}(x_{rl}) \leq \beta\underline{x} \\ F_{x,y}(x_{rr}) & \text{if } x_{rr} \leq \overline{x} \text{ and } \beta\underline{x} \leq t_{x,y}(x_{rl}) \\ F_{x,y}(\overline{x}) & \text{if } \overline{x} \leq x_{rr}, \end{cases} \tag{4.41}$$

see Figure 4.17. As in the case when the bound condition holds, (4.41) coincides with the representation of $\text{vex}_D[f]$ from the first solution approach given in (4.19), i.e., $\text{vex}_D[f](x,y) = F_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x}), \max\{x_{rl}, x_{rr}\}\})$. The condition $t_{x,y}(x_{rl}) \leq \beta\underline{x}$ that is equivalent to $x_{rl} \geq x_{rr}$, see Equation (4.20), enables to deduce both representations from each other.

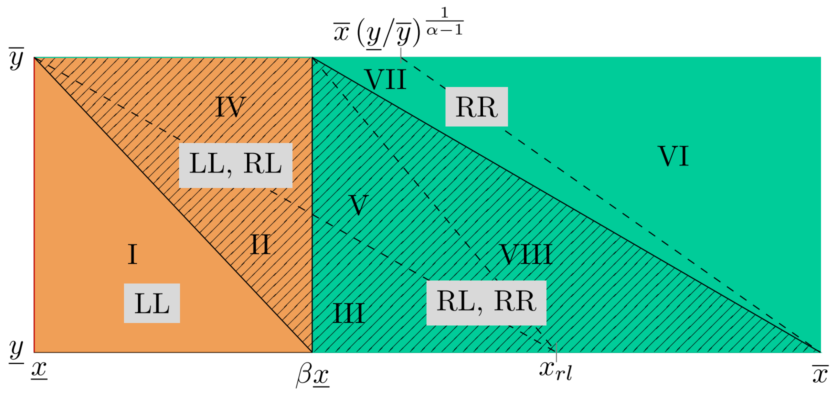


Figure 4.16: Domain partitioning, when the *bound condition* is violated. The orange region represents case *LL*, the green region case *RR*, and the hatched region case *RL*. Dotted lines illustrate the domain partitions that arise from the piecewise-defined objective function $F_{x,y}$. The numbers mark the partial domains, where $\text{vex}_D[f](x,y)$ is determined by comparing the optimal solutions of the possibly multiple cases that are highlighted in gray, i.e.,region I (case *LL*), region II (cases *LL*, *RL*), region III (cases *RL*, *RR*), region IV (cases *LL*, *RL*), region V (cases *RL*, *RR*), region VI (case *RR*), region VII (case *RR*), and region VIII (cases *RL*, *RR*).

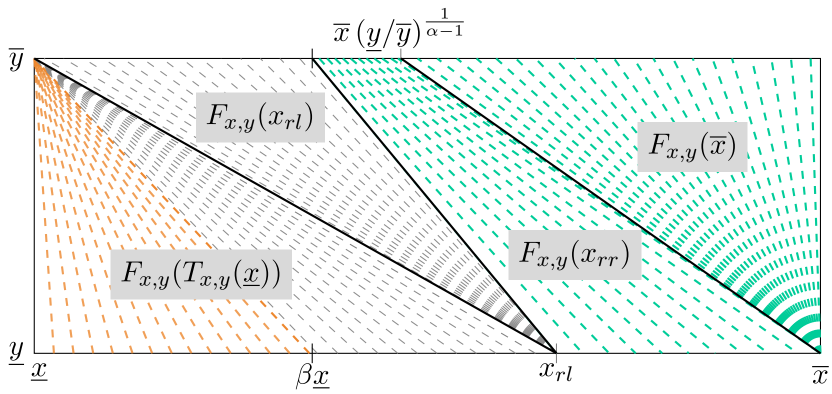


Figure 4.17: Representation of $\text{vex}_D[f](x,y)$ as given in (4.41), when the *bound condition* is violated. The convex envelope is given by the values $F_{x,y}(T_{x,y}(\underline{x})), F_{x,y}(\overline{x}), F_{x,y}(x_{rl})$ and $F_{x,y}(x_{rr})$ being separated by bold lines. Dashed lines represent the optimal line segments for points in the particular regions. Here, optimal line segments are shown in orange for case *LL*, in gray for *RL*, and in green for *RR*.

4.5 Convexification of $y\,sgn(x)|x|^\alpha$ in SCIP

In this section we compare the convex underestimator induced by SCIP with the convex envelope of $y\,\text{sgn}(x)|x|^\alpha$. To this end, we consider the epigraph of $y\,\text{sgn}(x)|x|^\alpha$,

$$\big\{(x,y,\theta) \in D \times \mathbb{R} : \theta \geq y\,\text{sgn}(x)|x|^\alpha\big\}. \tag{4.42}$$

By virtue of Theorem 4.1.5, any convex relaxation of (4.42) induces a convex underestimator of $y\,\text{sgn}(x)|x|^\alpha$. Therefore, we construct the convex underestimator of $y\,\text{sgn}(x)|x|^\alpha$

induced by SCIP's convex relaxation of (4.42) and compare it to the convex envelope of $y\,\mathrm{sgn}(x)|x|^\alpha$.

SCIP, as other MINLP solvers, use a standard reformulation procedure that allows to generate a convex relaxation for factorable MINLPs. The idea is to decompose a factorable constraint function into simpler functions for which convex underestimators or even convex envelopes are known. For general information about this reformulation method, we refer to Smith and Pantelides (1999), and for a description of how it works in SCIP to Vigerske (2012).

The decomposition is achieved by introducing additional variables forming, thus, an extended formulation of the original problem. In our case, SCIP decomposes the constraint $\theta \geq y\,\mathrm{sgn}(x)|x|^\alpha$ by introducing an auxiliary variable z into

$$\begin{aligned} &z = \mathrm{sgn}(x)|x|^\alpha, \\ &\theta \geq yz. \end{aligned}$$

Then, SCIP builds a relaxation by computing the convex envelopes of $f_1(x) = \mathrm{sgn}(x)|x|^\alpha$ and $f_2(y, z) = y\, z$. Note that by Proposition 4.2.2, the convex envelope of f_1 is φ_y with $y = 1$, which we denote by φ_1. Moreover, the convex envelope of f_2 is given by the well-known McCormick inequalities McCormick (1976),

$$\psi(y, z) := \max\left\{y\underline{z} + \underline{y}z - \underline{y}\underline{z},\ y\overline{z} + \overline{y}z - \overline{y}\,\overline{z}\right\},$$

where $\underline{z} = \min_{x\in[\underline{x},\overline{x}]} f_1(x)$ and $\overline{z} = \max_{x\in[\underline{x},\overline{x}]} f_1(x)$.

The convex relaxation of (4.42) constructed by SCIP is

$$C = \mathrm{Proj}_{x,y,\theta}\big\{(x, y, z, \theta) \colon z \geq \varphi_1(x), \theta \geq \psi(y, z)\big\}.$$

Hence, the underestimator induced by C is

$$\phi_S(x, y) := \min_{\theta,z}\big\{\theta : z \geq \varphi_1(x), \theta \geq \psi(y, z)\big\}. \tag{4.43}$$

Given that ψ is increasing with respect to z for every y, an optimal solution of Problem (4.43) satisfies $z = \varphi_1(x)$. Therefore, the convex underestimator induced by SCIP is $\phi_S(x, y) = \psi(y, \varphi_1(x))$.

Figure 4.18 shows the difference between $\mathrm{vex}_D[f]$ and ϕ_S over D. From the figure we can see that, for $D = [-100, 100] \times [0.01, 1]$, ϕ_S and $\mathrm{vex}_D[f]$ coincide in the region $\{(x, y) \in D : T_{x,y}(\underline{x}) \leq \beta\underline{x}\}$, see also Figure 4.3. Using $D = [-100, 20] \times [0.01, 1]$ instead, would show that ϕ_S and $\mathrm{vex}_D[f]$ coincide in the entire domain D. The next proposition explains this behavior.

Proposition 4.5.1. *If $\overline{x} \leq \beta\underline{x}$, then $vex_D[f](x, y) = \phi_S(x, y)$ for all $(x, y) \in D$.*

Proof. Proposition 4.2.5 implies that $\mathrm{vex}_D[f](x, y) = F_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x})\})$.

As $\overline{x} \leq \beta\underline{x}$, we have that $\min\{\overline{x}, T_{x,y}(\underline{x})\} \leq \beta\underline{x}$ and $t_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x})\}) \leq \beta\underline{x}$. Therefore, in $F_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x})\})$, the functions φ_y and $\varphi_{\overline{y}}$ are evaluated at points smaller than $\beta\underline{x}$, and so they are affine linear according to the definition of φ_y in (4.6).

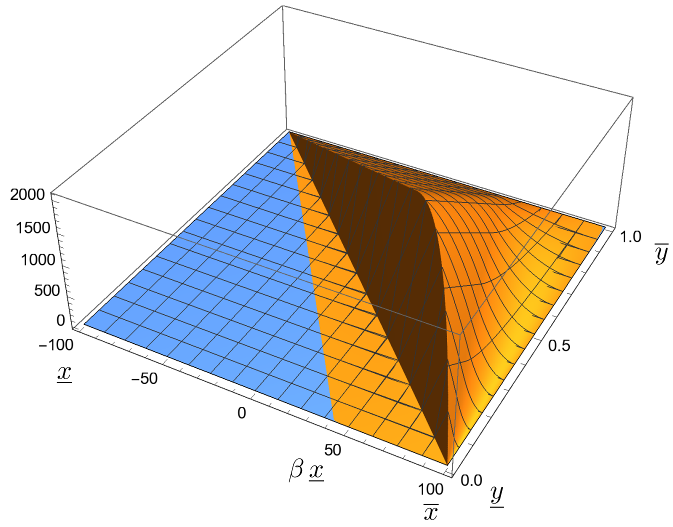


Figure 4.18: Difference $\text{vex}_D[f] - \phi_S$ on $D = [-100, 100] \times [0.01, 1]$. The region $A := \{(x, y) \in D : T_{x,y}(\underline{x}) \leq \beta\underline{x}\}$ (blue) shows where $\text{vex}_D[f] = \phi_S$, cf. Proposition 4.5.1. In this region, both $\text{vex}_D[f](x, y)$ and ϕ_S are linear, while the region in orange illustrates the nonlinear part of $\text{vex}_D[f]$ that is tighter than ϕ_S.

Assume that $\min\{\overline{x}, T_{x,y}(\underline{x})\} = T_{x,y}(\underline{x})$. Notice that $T_{x,y}(\underline{x})$ is an affine combination of x and $\underline{x}$ as $T_{x,y}(\underline{x}) = \frac{1}{1-\lambda_y}x + \frac{-\lambda_y}{1-\lambda_y}\underline{x}$ and $\frac{1}{1-\lambda_y} + \frac{-\lambda_y}{1-\lambda_y} = 1$. Since $\varphi_{\underline{y}}$ is affine linear, we have that $(1 - \lambda_y)\varphi_{\underline{y}}(T_{x,y}(\underline{x})) = \varphi_{\underline{y}}(x) - \lambda_y\varphi_{\underline{y}}(\underline{x})$. Therefore, $F_{x,y}(T_{x,y}(\underline{x})) = \varphi_{\underline{y}}(x) - \lambda_y\varphi_{\underline{y}}(\underline{x}) + \lambda_y\varphi_{\overline{y}}(\underline{x})$.

From $\varphi_y(x) = y\varphi_1(x)$ and the definition of λ_y, we obtain

$$F_{x,y}(T_{x,y}(\underline{x})) = \underline{y}\varphi_1(x) + y\varphi_1(\underline{x}) - \underline{y}\varphi_1(\underline{x}).$$

Notice that $\underline{z} = \varphi_1(\underline{x})$ (as φ_1 is the convex envelope of f_1) and so

$$F_{x,y}(T_{x,y}(\underline{x})) = \underline{y}\varphi_1(x) + y\underline{z} - \underline{y}\underline{z} \leq \max\left\{y\underline{z} + \underline{y}\varphi_1(x) - \underline{y}\underline{z},\ y\overline{z} + \overline{y}\varphi_1(x) - \overline{y}\,\overline{z}\right\} = \phi_S(x, y).$$

However, $F_{x,y}(T_{x,y}(\underline{x}))$ is the convex envelope of $y\,\text{sgn}(x)|x|^\alpha$, while $\phi_S(x, y)$ is just a convex underestimator, which implies that $\phi_S(x, y) \leq F_{x,y}(T_{x,y}(\underline{x}))$. From here we conclude that $F_{x,y}(T_{x,y}(\underline{x})) = \phi_S(x, y)$, and hence $\text{vex}_D[f](x, y) = \phi_S(x, y)$.

The case $\min\{\overline{x}, T_{x,y}(\underline{x})\} = \overline{x}$ follows a similar argumentation. □

We now explain the difference between $\text{vex}_D[f]$ and ϕ_S shown in Figure 4.18.

Proposition 4.5.2. *Assume that $\beta\underline{x} < \overline{x}$ and let $A := \{(x, y) \in D : T_{x,y}(\underline{x}) \leq \beta\underline{x}\}$. Then, for every $(x, y) \in A$,*

$$\phi_S(x, y) = \text{vex}_D[f](x, y).$$

Furthermore, $\text{vex}_D[f]$ is linear in A.

Proof. Given that $T_{x,y}(\underline{x}) \leq \beta\underline{x}$ and $\beta\underline{x} < \overline{x}$, we have that $\min\{\overline{x}, T_{x,y}(\underline{x})\} = T_{x,y}(\underline{x})$ and $t_{x,y}(\min\{\overline{x}, T_{x,y}(\underline{x})\}) = \underline{x} \leq \beta\underline{x}$. Thus, we can apply the same reasoning as the one in the proof of Proposition 4.5.1. □

Remark 4.5.3. *A similar behavior can be observed between the concave overestimator induced by SCIP and the concave envelope of* $y\,sgn(x)|x|^{\alpha}$.

4.6 Computational Study

In this section, we present a computational study that investigates the impact of the presented convex envelope on the performance of the stationary expansion planning problem from Chapter 3. For this, we extend SCIP with the convex envelope and compare it against default SCIP. We conduct two experiments to answer the following questions:

1. `ROOTGAP` : How much gap can be additionally closed in the root node of the branch-and-bound tree when using the envelopes of the constraint function $f(x, y) = y\,\mathrm{sgn}(x)|x|^{\alpha}$ with aggressive separation settings?

2. `TREE` : To which extent do the presented envelopes affect the performance of SCIP, both in running time and solvability?

4.6.1 Experimental setup

In order to measure the impact of the convex envelope over the standard relaxation in terms of improving the dual bound, we use an aggressive emphasis setting for the separation in the `ROOTGAP` experiment. To reduce the impact of side effects and performance variability (Lodi and Tramontani, 2013), we disable restarts[1], all primal heuristics, propagators[2] and we give the value of the best known primal solution.

In contrast to the `ROOTGAP` experiment, the `TREE` experiment compares SCIP default against SCIP with the additional separation routine that generates gradient cuts to the envelopes of f in every local node during the entire tree search. We remark that these gradient cuts are, in general, only locally valid since we compute them using the local bounds of the node.

In both experiments, the gradient cuts for the constraint $y\,\mathrm{sgn}(x)|x|^2 = \pi_v - \pi_w$ using the convex and concave envelope read, respectively,

$$\mathrm{vex}_D[f](x_0, y_0) + \nabla \mathrm{vex}_D[f](x_0, y_0)^T \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix} \le \pi_v - \pi_w, \tag{4.44}$$

and

$$\mathrm{cave}_D[f](x_0, y_0) + \nabla \mathrm{cave}_D[f](x_0, y_0)^T \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix} \ge \pi_v - \pi_w. \tag{4.45}$$

[1]In restarts, SCIP aborts the current search after encountering sufficient variable bound reductions and starts preprocessing the problem again. For more details about restarts in SCIP, we refer to Section 10.9 in Achterberg (2007).

[2]We used the following SCIP settings: `limits/totalnodes = 1`, `separation/emphasis/aggressive = True`, `limits/restarts = 0`, `heuristics/emphasis = Off` and `propagating/maxroundsroot = 0`.

Test sets. We conduct our experiments on a subset of instances that we used for the comparison of stationary expansion planning models in the computational experiments in Chapter 3. Here, we use Model E (3.13) as underlying model of all instances, since it performed best among the presented approaches from the previous chapter. Let us recall this model here, where the computational study concerns the convex relaxation of constraint (4.46a). As we use gas instances, the exponent α is set to $\alpha = 2$ in (4.46a):

$$\begin{aligned}
\min_{y,c,x,\pi} \quad & \textstyle\sum_{a\in\mathcal{A}} L_a c_a \\
\text{subject to} \quad & \pi_v - \pi_w = y_a \operatorname{sgn}(x_a)|x_a|^2 && \forall a \in \mathcal{A},\ (v,w) = r(a), \qquad (4.46a) \\
& \sum_{a\in\delta^+(v)} x_a - \sum_{a\in\delta^-(v)} x_a = b_v && \forall\, v \in \mathcal{V}, \\
& c_a \geq s_{a,i} y_a + t_{a,i} && \forall a \in A\ \forall i \in [k_a - 1], \\
& \underline{\pi}_v \leq \pi_v \leq \overline{\pi}_v && \forall\, v \in \mathcal{V}, \\
& \underline{x}_a \leq x_a \leq \overline{x}_a && \forall\, a \in \mathcal{A}, \\
& \underline{y}_a \leq y_a \leq \overline{y}_a && \forall\, a \in \mathcal{A}, \\
& c_a \geq 0 && \forall\, a \in \mathcal{V}.
\end{aligned}$$

In our experiments, we consider a vast amount of diverse instances (at all 6,500) from Section 3.6.1 that reflect manifold demand situations of different gas networks. We include all 2,000 instances of the *original Belgian* network (with a total demand of $\mathcal{B} = 100, 200, 500$ and 1,000), all 2,000 instances of the *GasLib-40* network (with a total demand of $\mathcal{B} = 50, 100, 500$ and 1,000), and all 2,500 instances of the *Belgian* network with $2, 4, 6, 8, 10$ additional arcs, called *Circuit rank*, where each instance is equipped with a total demand of $\mathcal{B} = 200$. In summary, the test sets *Belgium*, *GasLib-40* and *Circuit rank* contain 6,500 instances, since each particular data set comprises 500 individual scenarios.

Implementation. We extended a development version[3] of SCIP by a so-called nonlinear handler, where the separation is applied in the separation and enforcement callbacks. With this handler we add the cuts (4.44) and (4.45) in addition to the cuts that SCIP generates by default (see Section 4.5).

Given that the considered gas instances tend to be numerically unstable due to variables in different scales, we observed that many invalid cuts were generated in some preliminary experiments. The following two measures prevented the addition of invalid cuts to SCIP.

- We only add the cuts (4.44) and (4.45) to SCIP if they are violated at least by 10^{-4}.
- If the LP solution $(\hat{x}, \hat{y})$ satisfies $\hat{y} = \underline{y}$ or $\hat{y} = \overline{y}$, then we compute a slightly weakened gradient cut by considering the envelopes over the enlarged domain $[\underline{x}, \overline{x}] \times [\underline{y} - s, \overline{y}]$ or $[\underline{x}, \overline{x}] \times [\underline{y}, \overline{y} + s]$, respectively, with $s = 10^{-4}$.

[3]To be released in SCIP 8.

Moreover, Propositions 4.5.1 and 4.5.2 state conditions when $\phi_S(x, y) = \text{vex}_D[f](x, y)$. In those cases, we omit the generation of gradient cuts of $\text{vex}_D[f](x, y)$ because they will be generated in any case by SCIP. Specifically,

– if the variable bounds in the current LP relaxation satisfy $\overline{x} < \beta\underline{x}$, then we do not generate gradient cuts for $\text{vex}_D[f](x, y)$ (Proposition 4.5.1).

– if the current LP solution satisfies $T_{x,y}(\underline{x}) < \beta\underline{x}$ and $\beta\underline{x} < \overline{x}$, i.e., $(x, y) \in A$, then we do not generate gradient cuts for $\text{vex}_D[f](x, y)$ (Proposition 4.5.2).

Hardware and software. The experiments were conducted on a cluster of 64-bit Intel Xeon CPU E5-2670 v2 CPUs at 2.5 GHz with 25 MB cache and 128 GB main memory. In order to safeguard against a potential mutual slowdown of parallel processes, we bind the processes to specific cores and run at most one job per node at a time. We used a development version of SCIP 6.0.2.4 (Gleixner et al., 2018) with CPLEX 12.7.1.0 as LP solver (Cplex, IBM ILOG, 2019) and Ipopt 3.12.13 as NLP solver (Wächter and Biegler, 2006).

4.6.2 Computational results

In this section, we present the results for the `ROOTGAP` and `TREE` experiments. In the following, *Enabled* refers to adding the gradient cuts to the convex and concave envelopes, as opposed to *Disabled*, which refers to SCIP default settings.

`ROOTGAP` **Experiment.** From the 6,500 instances, we do not consider the ones that have been detected infeasible, no primal solution is known, or none of the versions could increase the dual bound by more than 10^{-6}. This restriction results in 3,074 instances for the `ROOTGAP` experiment.

To compare the dual bounds of *Enabled* and *Disabled* relative to a given primal bound, we use the following measure. For an instance let $d_1 \in \mathbb{R}$ be the dual bound of *Enabled*, and let $d_2 \in \mathbb{R}$ be the dual bound of *Disabled*. Furthermore let $p \in \mathbb{R}$ be a reference primal bound, for example the optimal or best known objective value of that instance. The function $GC\colon \mathbb{R}^3 \to [-1, 1]$ defined as

$$GC(d_1, d_2, p) := \begin{cases} 0 & \text{if } d_1 = d_2 \\ +1 - \frac{p-d_1}{p-d_2} & \text{if } d_1 > d_2 \\ -1 + \frac{p-d_2}{p-d_1} & \text{if } d_1 < d_2 \end{cases}$$

measures the improvement of the *gap closed* when comparing the dual bounds d_1 and d_2 relative to p, see also Müller et al. (2019). Note that a positive value $GC(d_1, d_2, p) > 0$ implies that the dual bound improved by adding the gradient cuts to the convex envelope $\text{vex}_D[f](x, y)$. Analogously, a negative value $GC(d_1, d_2, p) < 0$ indicates that the addition of the gradient cuts deteriorates the performance.

Aggregated results for the `ROOTGAP` experiment are shown in Figure 4.19 and Table 4.1 for all instances. Further aggregated results with respect to single data sets can be found in Figures B.1 – B.2b and Tables B.1 – B.3 in the appendix.

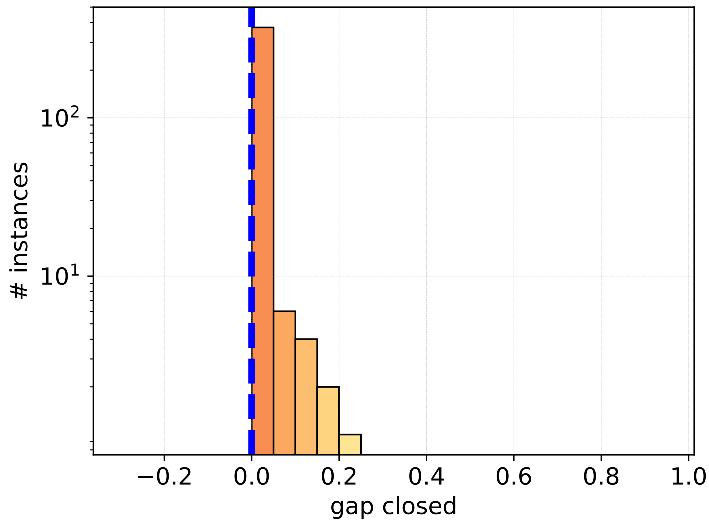


Figure 4.19: The *Gap closed* improvement, where each bar on the x-axis represents the relative improvement with the corresponding number of instances on the y-axis in logarithmic scale. Positive values on the x-axis represent improved dual bounds for *Enabled*, and negative values indicate better dual bounds for *Disabled*. An instance having a *gap closed* value of 0.5 (on the x-axis) implies that the gap could be closed by 50% using *Enabled* over *Disabled*. For all 3074 instances under consideration, *Enabled* yields better dual bounds on 720 instances and *Disabled* on 0 instances.

	# instances	gap closed (%)
	3074	0.33
> 0% `change`	720	1.39
> 0% `better`	720	1.39
> 0% `worse`	0	–
> 2% `change`	125	3.49
> 2% `better`	125	3.49
> 2% `worse`	0	–
> 10% `change`	7	15.21
> 10% `better`	7	15.21
> 10% `worse`	0	–

Table 4.1: Aggregated results for the *gap closed* of the `ROOTGAP` experiment. The *gap closed* values are depicted on average for `ALL` instances, and for those instances where *Enabled* yields an improvement or deterioration of more than 0%, 2% and 10% compared to *Disabled*, called `better` or `worse`. The category `change` corresponds to the union of `better` and `worse` instances.

Table 4.1 shows that the addition of the cuts closes more gap on 720 out of the 3074 instances. Additionally, the table presents the average *gap closed* for instances, where either *Enabled* improves (`better`) or deteriorates (`worse`) by more than 0%, 2%, or 10%, whereas `change` encompasses the union of those instances that are `better` or `worse` by more than 0%, 2%, or 10%.

Among all instances, the average gap closed improvement is 0.33%. However, if we consider instances for which the gap closed differs by 2% and 10%, then the gap closed improvement increases to 3.49% and 15.21%, respectively.

The presented results show that our convexification of the considered constraint function has a significant impact on the quality of the relaxation. Moreover, the dual bounds obtained by *Enabled* are always at least as good as the ones from *Disabled*, since we apply the gradient cuts on top of SCIP default.

`TREE` Experiment. Aggregated results for the `TREE` experiment are shown in Tables 4.2a – 4.2c for the three test sets under consideration. Further aggregated results with respect to single data sets can be found in Tables B.4 – B.16 in the appendix. There, we also present detailed instance-wise results for the `TREE` and `ROOTGAP` experiments in Tables B.17 – B.19. We compare the number of solved instances and the solver performance when activating the cut generation for the convex envelope (*Enabled*) with SCIP using default settings (*Disabled*). The number of solved instances and the computation time is split into different categories: `ALL`, `ALL OPT`, $[1, \texttt{tlim}]$, $[10, \texttt{tlim}]$, $[100, \texttt{tlim}]$, and $[1000, \texttt{tlim}]$. `ALL` consists of all instances. `ALL OPT` consists of the instances that are solved to optimality or detected to be infeasible by both settings. $[t, \texttt{tlim}]$ consists of the instances that could be solved by at least one of both settings *Enabled* or *Disabled*, in more than t seconds within the time limit of one hour. Note that the instances in $[t_1, \texttt{tlim}]$ are contained in $[t_2, \texttt{tlim}]$ whenever $t_1 \geq t_2$ and that one can consider the instances in $[t, \texttt{tlim}]$ to be harder to solve the larger t is.

Most importantly, on all test sets, enabling the cut generation for the convex envelope increases the number of solved instances and decreases the average solving time compared to SCIP default. Concerning the number of solved instances, *Enabled* solves 39 more instances on *Belgium*, 19 more instances on *GasLib-40*, and 68 more instances on *Circuit rank*. From the tables we deduce that all the instances that *Enabled* solves alone belong to the group $[1000, \texttt{tlim}]$. This means that our separation routine renders these instances solvable but not trivially solvable.

To compare the total running time between *Enabled* and *Disabled*, we use the shifted geometric mean (sgm) with a shift value of $s = 10$ for the average solving time in seconds. For instances that are solved to optimality by both settings (`ALL OPT`), we additionally present the sgm for the number of explored branch-and-bound nodes with a shift value of $s = 100$. As can be seen in Tables 4.2a – 4.2c, there is an improvement in the performance when using *Enabled* over SCIP default in all test sets and through all time categories. This improvement increases towards the more "difficult" instances to a speed-up of up to 58% on *Belgium*. Even for `ALL OPT`, there is throughout a speed-up of 19% (*Belgium*) and 12% (*GasLib-40* and *Circuit rank*) when using the *Enabled* separation routine. Likewise, the number of branch-and-bound nodes significantly decreases by 22%, 25%, and 14%

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
`ALL`	2000	1905	1866	1.19	-
`ALL OPT`	1832	1832	1832	1.19	1.22
[1, `tlim`]	1635	1601	1562	1.23	-
[10, `tlim`]	1039	1005	966	1.34	-
[100, `tlim`]	542	508	469	1.47	-
[1000, `tlim`]	237	203	164	1.58	-

(a) Aggregated results for the `TREE` experiment on *Belgium*

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
`ALL`	2000	1181	1162	1.05	-
`ALL OPT`	1108	1108	1108	1.12	1.25
[1, `tlim`]	1146	1092	1073	1.14	-
[10, `tlim`]	1083	1029	1010	1.15	-
[100, `tlim`]	719	665	646	1.21	-
[1000, `tlim`]	293	239	220	1.20	-

(b) Aggregated results for the `TREE` experiment on *GasLib-40*

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
`ALL`	2500	1041	973	1.08	-
`ALL OPT`	766	766	766	1.12	1.14
[1, `tlim`]	1248	1041	973	1.16	-
[10, `tlim`]	1248	1041	973	1.16	-
[100, `tlim`]	1096	889	821	1.18	-
[1000, `tlim`]	831	624	556	1.16	-

(c) Aggregated results for the `TREE` experiment on *Circuit rank*.

Table 4.2: Aggregated results for the `TREE` experiment on three different test sets. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

for ALL OPT on these three test sets.

Finally, we remark that the Wilcoxon signed-rank test described in Chapter 2 judges the performance improvements on the *Belgium*, *GasLib-40*, and *Circuit rank* test sets as highly statistical significant being consistently distributed in the shifted geometric mean across all three test sets.

4.7 Conclusion

In this chapter, we derived the convex envelope of the nonconvex and bivariate function $f(x, y) = y\,\mathrm{sgn}(x)|x|^{\alpha}$ for $\alpha > 1$. This contribution was motivated by the fact that strong relaxations of nonconvex constraint functions play a key component in solving MINLPs, see Grossmann and Kravanja (1997). As this function frequently occurs in stationary expansion planning problems of potential-driven networks, it is also of significant practical relevance. We performed an extensive computational study on real-world instances, which shows the benefit of having the convex envelope at hand compared to the standard relaxation applied by state-of-the-art MINLP solvers such as SCIP. Given that we provide a closed-form expression for the envelopes, our implementation in SCIP allows us to calculate and exploit the envelopes at every node in the branch-and-bound tree, which has a crucial impact on the solving process. Apart from yielding improved dual bounds that reduce the gap already in the root node, our procedure drastically boosts the solving process in the branch-and-bound tree. Our procedure significantly reduces the solving time on all test sets, for example, by up to 58% for hard instances on *Belgium*. Even more important, this tighter relaxation enables solving significantly more instances on all test sets. In summary, this corresponds to 9% more solved instances by activating our separation routine when aggregating the number of solved instances in $[1000, \texttt{tlim}]$ over all three test sets.

An interesting avenue for further research would be to calculate the convex envelope of function f on a polytope $D \subseteq [\underline{x}, \overline{y}] \times [\underline{y}, \overline{y}]$. So far, our methods restrict to box domains. However, extensions of the stationary expansion planning problem could induce further constraints that additionally link the flow variable x and the expansion variable y resulting in polyhedral domains when being projected onto the space of (x, y)-variables. To this end, it could be useful to build upon already existing approaches for the convexification of bivariate constraints on polyhedral domains. For related literature, we refer to Section 4.1.

Moreover, while we generated the convex envelope for each constraint function separately, a possibly tighter relaxation could be gained by convexifying multiple constraints simultaneously. In this fashion, Liers et al. (2021) already report promising results for quadratic functions with absolute value terms for gas network related problems.

Part II

Optimizing the Control of Transient Large-Scale Networks

Chapter 5

An MINLP Model for the Transient Control Problem

Natural gas is a major energy source in Europe. Nowadays, it is considered to be an important transit medium on the way to a more climate-friendly and sustainable energy system (DVGW, 2020). Until the phase-out of nuclear energy and fossil fuels, natural gas can be used to overcome shortcomings of intermittent renewable energy sources. Under the assumption that the planned energy turnaround increases the share of renewable energy in the future, such as wind and solar power, whose production is subject to uncertainty, gas network operations will be exposed to more volatile scenarios. Therefore, new challenges arise for gas network operators. In fact, gas network operators already face difficult transport situations as a consequence of the gas market liberalization in the EU (Bundesministerium der Justiz, 2005; Gotzes et al., 2015) that grants open network access to all gas trading companies resulting in more volatile demand scenarios. To make matters worse, gas typically moves slowly through pipelines, roughly speaking in the speed of a cyclist (Domschke et al., 2017; Hennings, 2018). Therefore, decisions on the transport paths have to be taken early enough in order to establish operations that guarantee secure gas supply.

In the real-world, dispatchers control gas networks from a central operating room by determining technical settings and operation modes of so-called *remote-controllable* elements. The different possible operation modes correspond to gas compression, pressure reduction, friction-less gas throughput, and blocking gas flows. Elements that allow realizing these modes are also referred to as *active elements* and comprise valves, control valves, and compressors. Obviously, viable control decisions base upon the current network situation that is obtained by monitoring the physical state of specific elements or network parts in real-time. In order to not stress the network unnecessarily, dispatchers aim at taking proactive decisions, for example, filling pipelines or pulling gas out of pipelines in the case of high predicted withdrawals or injections. Hence, the decision-making also depends on the demand forecast. Based on this information, operators determine control decisions, which allow routing the gas such that demands are satisfied, while respecting operational and physical restrictions. Thereby, the dispatchers' major task aims at obtaining stable and safe gas network operations. While in Germany, for example, feasible gas network operations are considered “systemrelevant” (Bundesnetzagentur, 2020), i.e., highly relevant for the basic infrastructure of society and economy, the German gas network was already allegedly at risk of a blackout in recent years (Die Welt, 2020), almost not being able to fulfill transport tasks and to satisfy crucial demands. Altogether, this motivates tackling the task that dispatchers face in practice as optimization problem: *controlling gas flows in transient large-scale networks.*

In this chapter, we mathematically formalize this problem, also referred to as *Transient Control Problem* in the remainder of this thesis. A solution for it is a plan of how to operate remotely controllable elements in the upcoming half day-ahead operations, i.e., for the next twelve hours. The objective is to minimize the number of operation mode changes over time. The rationale behind this objective is that practitioners, in general, seek stable gas flows and network states. Even though there is no mathematical definition for the *stability* of gas network operations, practitioners usually consider operations as stable if they require only few interventions on changing the operational settings of the active elements. While the meaning of a “few interventions” certainly depends on the particular network, it gives rise to this objective. The optimization problem contains several forms of complexity, such as the combinatorics arising from active elements, and the necessity to deal with nonlinearities, which are, as an example, due to the transient behavior in pipelines. That is, we model the *Transient Control Problem* as a genuine MINLP problem.

Starting with a literature overview about the optimization of transient gas flows, we subsequently derive an algebraic formulation for the gas physics in pipelines by discretizing the underlying PDE system in Section 5.1. We propose a slightly different discretization in contrast to the frequently used implicit Box-scheme in order to reduce the number of involved nonlinear terms. Finally, we present a detailed mathematical modeling description of all network elements in the remaining sections. Other than most of the existing optimization models for transient gas transport, we include the modeling of storages, which in our context represent aggregated subnetworks.

Literature overview. Research about gas transport can be classified on whether the system is in steady or transient state. For systems in steady state, one assumes that the physical entities do not change over time, whereas the transient state accounts for changes over time. Accordingly, two different strands of research exist: stationary and transient gas flow models. While a large body of the literature on gas network optimization is concerned with the stationary case, see Koch et al. (2015); Pfetsch et al. (2015) for optimization problems and state-of-the-art approaches, controlling and optimizing transient gas networks has become more prominent in recent years, even across different countries and continents. This can be seen by means of the different computational studies conducted on parts of the German gas network (Moritz, 2007), on the Finnish gas network (Aalto, 2015), on gas transport along the East Coast of the United States (Zlotnik et al., 2015b), and between West and East China (Liu et al., 2019).

Rather early approaches to optimization of transient gas networks use linear models. Kelling et al. (2000) approximate the nonlinearities by Taylor expansions in order to apply sequential linear programming, while Nowak and Westphalen (2003) use a simple linear *line-pack* model for the transient gas transport.

Different approaches explicitly incorporate the discrete nature of active elements leading to Mixed-Integer Programming models (Moritz, 2007; Mahlke et al., 2007, 2010). These models include the switching of operation modes of valves and compressors, and besides, account for the start-up and ramp-down of compressors. The papers approximate the problem's inherent nonlinearities by piecewise linear functions using special ordered sets (SOS) of type 4 (Beale and Tomlin, 1970) for the nonlinear power consumption in compressor drives and SOS conditions of type 2 and 3 to linearize the discretized Euler equations for pipes. Moritz (2007) solves the resulting MIP model using a branch-and-cut algorithm and thereby exploits the handling of SOS conditions with tailored preprocessing, branching and separation techniques. Finally, Moritz (2007); Mahlke et al. (2010) incorporate a Simulated Annealing based primal heuristic into the branch-and-cut framework that aims at finding good solutions in short running time. The authors report promising results on three test networks having 15, 25 and 86 nodes.

Several approaches neglect the discrete aspects and propose NLP model formulations for transient gas flow optimization problems, where the objective is to minimize operation (compression) costs (Ehrhardt and Steinbach, 2005; Steinbach, 2007; Zlotnik et al., 2015a; Mak et al., 2016). While Steinbach (2007) measures the solution accuracy based on the condition number of the underlying KKT systems, Zlotnik et al. (2015a) and Mak et al. (2016) validate the solutions using simulations.

Starting with Domschke et al. (2011), a strand of approaches emerged that use MINLP formulations for transient gas transport problems. Domschke et al. (2011), however, solve a MILP variant, where Delaunay triangulations are used to approximate nonlinearities by piecewise linear terms. In order to obtain solutions that satisfy the original nonlinear constraints, the authors establish an adaptive strategy that alternates between solving MILPs and NLPs. Here, the NLP models are obtained by fixing the binary variables to solutions of the MILP models. The solutions of the NLPs are then used to refine the approximations of the nonlinear constraint functions. Hahn et al. (2017) present an MINLP model, which aims at finding feasible network controls at minimal operation cost, while satisfying, in particular, fluctuating demand profiles. For the experiments on two rather small networks consisting of four and five network nodes, they customize the branch-and-bound solver Minotaur[1] by using special tailored branching rules that exploit the problem structure. Gugat et al. (2018) present an instantaneous control approach for solving transient gas transport problems, where the objective is to switch from a given initial state into a prescribed network state within a given time range under the requirement of retaining feasible network control. Thus, the objective differs from the previously mentioned approaches. Based on special discretization schemes of the underlying system of Euler equations, Gugat et al. (2018) decompose the problem on the time scale and solve the resulting MIP models for each time step, where the

[1] Ashutosh Mahajan, Sven Leyffer, Jeffrey T. Linderoth, James Luedtke, and Todd Munson. MINOTAUR: a Toolkit for Solving Mixed-Integer Nonlinear Optimization. `https://www.anl.gov/mcs/minotaur-toolkit-for-mixed-integer-nonlinear-optimization-problems`, accessed in November 2020.

respective control decisions look one time step ahead. The computational experiments are accomplished on a small network comprising 16 nodes and 17 arcs out of which 12 are pipes. However, they use a spatial discretization that results in 920 spatial intervals, and a time discretization of one minute leading to 180 time intervals. In a recent paper, Burlacu et al. (2019) study the problem of maximizing the storage capacity of gas networks, i.e., maximizing the amount of stored gas in the system for given initial conditions, while retaining feasible network operations. The authors propose a novel discretization scheme that introduces only one flow variable per pipe, as opposed to the frequently applied implicit Box-scheme, which requires two separate flow variables for modeling inflow and outflow of a pipeline. As a result, their discretization technique of the Euler equations preserves the algebraic formulation for the pressure loss in pipelines from stationary gas transport, also known as Weymouth equation. Finally, the problem is modeled as MINLP and solved to global optimality by alternating between solving MIP relaxations, where the nonlinear constraint functions are replaced by linear outer relaxations, and solving NLP models, where the discrete variables are fixed to the solutions of the MIP relaxations, see also Geißler (2011). The paper reports promising computational results on the *GasLib-11* test network (Schmidt et al., 2017a) with a time granularity of ten-minutes covering a time horizon of eight hours.

5.1 Transient Gas Physics in Pipelines

When modeling gas network operations, flows in pipelines are frequently considered as one-dimensional in space and pipelines are assumed to have cylindrical shapes. These assumptions enable modeling gas flows in pipelines by a hyperbolic system of nonlinear partial differential equations, the so-called Euler equations. This system describes the conservation of mass, momentum, and energy by virtue of the *continuity equation*, *momentum equation* and *energy equation* and reads

$$\partial_t \rho + \partial_x(\rho v) = 0, \tag{5.1a}$$

$$\partial_t(\rho v) + \partial_x(p + \rho v^2) + g\rho s + \frac{\lambda}{2\,D}\rho v|v| = 0, \tag{5.1b}$$

$$\partial_t\Big(\rho\left(\frac{1}{2}v^2 + e\right)\Big) + \partial_x\Big(\rho v\left(\frac{1}{2}v^2 + e\right) + p\,v\Big) + \frac{k_w}{D}(T - T_w) = 0, \tag{5.1c}$$

see, for example, LeVeque (2002) and Brouwer et al. (2011). Here, the symbols x and t denote the spatial and temporal coordinate of System (5.1). This means that for a local parametrization of the pipe in space from 0 to L, where L is the pipe length, the term x describes the distance to 0. Similarly, for a local parametrization of the pipe in time from t_0 to T, the term t describes the distance to t_0.

The unknown entities of System (5.1) are the density ρ, pressure p, velocity v, temperature T, and consequently the internal energy $e = c_v T + g\,h$ as sum of the thermal and potential energy with specific heat c_v and pipe height h at point x. The parameter D denotes the diameter, $s \in \mathbb{R}$ the slope, and λ the friction coefficient of the pipe, whereas g represents the gravitational constant, k_w the heat coefficient, and T_w the temperature at the pipe wall surface. In addition, we consider the *equation of state* for real gases, which describes the inner state of a thermo-dynamical system. It links the pressure, density, and temperature of the gas, and is given by

$$p = R_s \, \rho \, T \, z(p, T). \tag{5.2}$$

The equation of state for real gases introduces two new terms, the specific gas constant R_s and the compressibility factor $z(p, T)$, where the latter corresponds to a correction factor from the equation of state for ideal gases.

Friction. Flows in pipelines cause a friction-induced pressure drop. The impact of the friction on the pressure drop is modeled by the so-called friction factor λ, which itself depends on the flow, diameter D and a roughness coefficient k of the pipe, and the dynamic viscosity. Different approaches exist to approximate λ, which, for example, differ on whether the flow is *laminar* or *turbulent*. In the literature, the degree of turbulences is described by the so-called *Reynolds Number*, see, for example, Saleh (2002) and Lurie (2008). Here, we use the formula of *Nikuradse*, see Nikuradse (1950), which assumes infinite turbulences in the pipelines. It enables one to express λ independent of the amount of flow, only depending on the diameter and the roughness coefficient

$$\lambda = \left(2 \log_{10}\left(\frac{D}{k}\right) + 1.138\right)^{-2}.$$

Simplifications. In order to be able to solve the *Transient Control Problem* with MINLP solvers, we simplify System (5.1) in a similar fashion as done, for example, in Burlacu et al. (2019).

First, we consider constant gas temperature T, which enables neglecting the *energy Equation* (5.1c) resulting in the so-called system of isothermal Euler equations.

Second, using the speed of sound $c_s = \sqrt{p/\rho}$ enables us to reformulate the spatial derivative term in the momentum Equation (5.1b)

$$c_s = \sqrt{p/\rho} \quad \Rightarrow \quad p + \rho v^2 = p\left(1 + \frac{v^2}{{c_s}^2}\right). \tag{5.3}$$

Assuming that the velocity v of gas flow typically holds in practice $v \ll c_s$, then (5.3) enables us to neglect the term $\partial_x(\rho v^2)$ in Equation (5.1b), see also the section about semi-linear equations in Domschke et al. (2017), Burlacu et al. (2019), and Hennings et al. (2019). In the same spirit, Osiadacz (1996) proposes to approximate v^2/c^2 by 0.

Third, we assume a constant compressibility factor $z := z(p, T)$. Then, by virtue of the Equation of state (5.2), the speed of sound $c_s = \sqrt{p/\rho}$ transforms to ${c_s}^2 = R_s T z$, since

$$p = R_s \, \rho \, T \, z \quad \Longleftrightarrow \quad \frac{p}{\rho} = R_s \, T \, z \quad \Longleftrightarrow \quad c_s^2 = R_s \, T \, z, \tag{5.4}$$

cf. the nonlinear models in Bales et al. (2009) and Domschke et al. (2015). Given that we treat R_s, T and z as constants, then c_s is also constant. In line with different approaches in the literature (e.g., Hahn et al., 2017; Burlacu et al., 2019), we set the speed of sound to a constant value. In this thesis, we set $c_s := 340$ [m/s] for all pipes. In natural gas, the speed of sound is approximately given by this value, see Domschke et al. (2017).

Fourthly, we drop the term $\partial_t(\rho v)$, as it contributes insignificantly to the momentum Equation under normal operating conditions, see, for example, Wilkinson et al. (1964) and Ehrhardt and Steinbach (2005). In summary, all these simplifications together reduce System (5.1) to

$$\partial_t \rho + \partial_x(\rho v) = 0, \tag{5.5a}$$

$$\partial_x p + g\rho s + \frac{\lambda}{2D}\rho v|v| = 0. \tag{5.5b}$$

Finally, we reformulate Equations (5.5a) and (5.5b) in terms of the physical quantities that we use as state variables in the *Transient Control Problem*, namely pressure p and mass flow q. Provided that the pipelines have cylindrical shapes, the mass flow is given by $q = A\,\rho\,v$ with cross-sectional area $A = D^2\,\pi/4$, leading to

$$\frac{A}{{c_s}^2}\partial_t p + \partial_x q = 0, \tag{5.6a}$$

$$\partial_x p + \frac{gs}{{c_s}^2}p + \frac{\lambda\, {c_s}^2}{2DA^2}\frac{q|q|}{p} = 0. \tag{5.6b}$$

Note that System (5.6) is often referred to as *friction-dominated* model, see model variant (FD1) in Brouwer et al. (2011) and (ISO3) in Domschke et al. (2017).

Discretization in time and space We now discretize system (5.6) to obtain algebraic formulations. As mentioned above, let L be the length of a pipeline, which we locally parametrize in space from 0 to L. Associating the incident nodes ℓ and r of the pipe with the beginning and the end of the parametrized pipeline, we introduce the following notation: $q_\ell(t) := q(0,t)$, $q_r(t) := q(L,t)$ and $p_\ell(t) := p(0,t)$, $p_r(t) := p(L,t)$. Considering a particular point in time t_i, we further introduce the notation: $q_{\ell,t_i} := q(0,t_i)$, $q_{r,t_i} := q(L,t_i)$ and $p_{\ell,t_i} := p(0,t_i)$, $p_{r,t_i} := p(L,t_i)$.

Discretization of the continuity equation. For the discretization of the continuity equation

$$\frac{A}{{c_s}^2}\partial_t p(x,t) + \partial_x q(x,t) = 0,$$

we first integrate it over space and time, yielding

$$\frac{A}{{c_s}^2}\int_{t_0}^{t_1}\int_0^L \Big(\partial_t p(x,t) + \partial_x q(x,t)\Big) dx\, dt = 0. \tag{5.7}$$

To approximate the non-resolving integral of (5.7) in space, we use the "trapezoidal quadrature" rule, which reads

$$\int_0^L \partial_t p(x,t)dx \approx \frac{L}{2}\Big(\partial_t p_r(t) + \partial_t p_\ell(t)\Big).$$

The application of the trapezoidal quadrature rule transforms (5.7) to

$$\frac{A}{\mathrm{c_s}^2}\int_{t_0}^{t_1}\frac{L}{2}\Big(\partial_t p_r(t)+\partial_t p_\ell(t)\Big)dt+\int_{t_0}^{t_1}\Big(q_r(t)-q_\ell(t)\Big)dt=0. \tag{5.8}$$

To approximate the non-resolving integral of (5.8) in time, we use the "right-hand rectangle rule", also referred to as implicit Euler method, which reads

$$\int_{t_0}^{t_1}\Big(q_r(t)-q_\ell(t)\Big)dt \approx (q_{r,t_1}-q_{\ell,t_1})\,(t_1-t_0).$$

Here, the application of the quadrature rule transforms (5.8) to

$$\frac{A}{\mathrm{c_s}^2}\frac{L}{2}\Big(p_{r,t_1}-p_{r,t_0}+p_{\ell,t_1}-p_{\ell,t_0}\Big)+\Big(q_{r,t_1}-q_{\ell,t_1}\Big)(t_1-t_0)=0. \tag{5.9}$$

Setting

$$\alpha := \frac{\mathrm{c_s}^2}{A}\frac{2}{L} \quad \text{and} \quad \tau := (t_1-t_0), \tag{5.10}$$

then the discretized continuity equation reads

$$\boxed{p_{r,t_1}-p_{r,t_0}+p_{\ell,t_1}-p_{\ell,t_0}+\alpha\,\tau\Big(q_{r,t_1}-q_{\ell,t_1}\Big)=0}. \tag{5.11}$$

In the Appendix C.1, we provide an alternative discretization approach resulting likewise in Equation (5.11), which is based on finite differences instead of quadrature rules.

Discretization of the momentum equation. For the discretization of the momentum equation

$$\begin{aligned} \partial_x p(x,t)+\frac{gs}{\mathrm{c_s}^2}p(x,t)+\frac{\lambda\,\mathrm{c_s}^2}{2\,DA^2}\frac{q(x,t)|q(x,t)|}{p(x,t)} &= 0 \\ \iff \quad (\partial_x p(x,t))\,p(x,t)+\frac{gs}{\mathrm{c_s}^2}p^2(x,t)+\frac{\lambda\,\mathrm{c_s}^2}{2DA^2}q(x,t)|q(x,t)| &= 0, \end{aligned} \tag{5.12}$$

we first substitute the pressure based term $(\partial_x p(x,t))\,p(x,t)$ for $\partial_x\left(p(x,t)^2\right)/2$ in (5.12). This substitution is possible, since the product rule yields

$$\partial_x\Big(p^2(x,t)\Big)=\Big(\partial_x p(x,t)\Big)p(x,t)+p(x,t)\Big(\partial_x p(x,t)\Big)=2\,p(x,t)\,\Big(\partial_x p(x,t)\Big),$$

which allows to reformulate (5.12) to

$$\frac{\partial_x\left(p(x,t)^2\right)}{2}+\frac{gs}{\mathrm{c_s}^2}p(x,t)^2+\frac{\lambda\,\mathrm{c_s}^2}{2\,D\,A^2}q(x,t)|q(x,t)|=0. \tag{5.13}$$

Then, we integrate (5.13) over space, i.e.,

$$\int_0^L\left(\partial_x\left(p^2(x,t)\right)+\frac{2\,gs}{\mathrm{c_s}^2}p^2(x,t)+\frac{\lambda\,\mathrm{c_s}^2}{DA^2}q(x,t)|q(x,t)|\right)dx=0. \tag{5.14}$$

To approximate the non-resolving integral $\int_0^L p^2(x,t)dx$, we use the trapezoidal rule

$$\int_0^L p^2(x,t)dx \;\approx\; L\,\frac{p_r^2(t)+p_\ell^2(t)}{2}. \tag{5.15}$$

For the non-resolving integral $\int_0^L q(x,t)|q(x,t)|dx$, we use a modified version of the *mid-point rule*, where we first approximate $q(x,t)$ by the secant $q_\ell(t)+\frac{x}{L}(q_r(t)-q_\ell(t))$ before applying the *mid-point rule* yielding

$$\begin{aligned}\int_0^L q(x,t)|q(x,t)|dx \;&\approx\; \int_0^L \Big(q_\ell(t)+\frac{x}{L}\,(q_r(t)-q_\ell(t))\Big)\,\Big|q_\ell(t)+\frac{x}{L}\,(q_r(t)-q_\ell(t))\Big|dx\\ &\approx\; L\,\frac{q_r(t)+q_\ell(t)}{2}\,\frac{|q_r(t)+q_\ell(t)|}{2}.\end{aligned} \tag{5.16}$$

Let us briefly remark that a common approach to approximate the non-resolving integral is to apply the trapezoidal rule (Domschke et al., 2011) instead, i.e.,

$$\int_0^L q(x,t)|q(x,t)|dx \approx \frac{L}{2}\Big(q_\ell(t)|q_\ell(t)|+q_r(t)|q_r(t)|\Big).$$

However, the advantage of the modified *mid-point rule* is that it reduces the number of nonlinear terms. In view of using an MINLP solver for the *Transient Control Problem* with discretized pipe equations, reducing the number of absolute value terms is beneficial. Hence, the solver only generates one convex underestimator for $(q_r(t)+q_\ell(t))\,|q_r(t)+q_\ell(t)|$ compared to $q_\ell(t)|q_\ell(t)|+q_r(t)|q_r(t)|$, which requires generating a convex underestimator for each of both terms. Nevertheless, we remark that underestimators in general depend on variable bounds, which are tighter in the latter case, since the terms $q_\ell(t)|q_\ell(t)|$ and $q_r(t)|q_r(t)|$ are separable in $q_\ell(t)$ and $q_r(t)$.

Finally, applying the approximations (5.15) and (5.16) transforms (5.14) to

$$p_r^2(t)-p_\ell^2(t)+\frac{g\,s\,L}{\mathrm{c_s}^2}\left(p_r^2(t)+p_\ell^2(t)\right)+\frac{\lambda\,\mathrm{c_s}^2\,L}{4\,D\,A^2}\Big(q_r(t)+q_\ell(t)\Big)\Big|q_r(t)+q_\ell(t)\Big|=0.$$

Substituting the slope s for the difference of height with respect to the orientation of the pipe from node ℓ to node r

$$s=\frac{h_r-h_\ell}{L},$$

and setting $t:=t_1$ as done above in Equation (5.9), we get

$$p_{r,t_1}^2-p_{\ell,t_1}^2+\frac{g(h_r-h_\ell)}{\mathrm{c_s}^2}\left(p_{r,t_1}^2+p_{\ell,t_1}^2\right)+\frac{\lambda\,\mathrm{c_s}^2\,L}{4\,D\,A^2}\,(q_{r,t_1}+q_{\ell,t_1})\,|q_{r,t_1}+q_{\ell,t_1}|=0.$$

To simplify notation, let us define

$$\gamma:=\frac{\lambda\,\mathrm{c_s}^2\,L}{4\,D\,A^2}\qquad\text{and}\qquad\beta:=\frac{g\,(h_r-h_\ell)}{\mathrm{c_s}^2},$$

then the discretized momentum equation reads

$$\boxed{p_{r,t_1}^2-p_{\ell,t_1}^2+\beta\left(p_{r,t_1}^2+p_{\ell,t_1}^2\right)+\gamma\,(q_{r,t_1}+q_{\ell,t_1})\,|q_{r,t_1}+q_{\ell,t_1}|=0}. \tag{5.17}$$

5.2 Modeling Prerequisites

We model the topology of a gas network as directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ with node set $\mathcal{V}$ and arc set $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$, where we allow for multiple (anti-)parallel arcs between any pair of nodes.[2] The nodes consist of sources $\mathcal{V}^+ \subseteq \mathcal{V}$, sinks $\mathcal{V}^- \subseteq \mathcal{V}$, and transshipment nodes $\mathcal{V}^* := \mathcal{V} \setminus (\mathcal{V}^+ \cup \mathcal{V}^-)$. In this thesis, we also refer to sources and sinks as *boundary nodes*, and assume that $\mathcal{V}^+ \cap \mathcal{V}^- = \emptyset$ holds. Moreover, we are given a set of storage nodes $\mathcal{V}_{\text{sto}} \subseteq \mathcal{V}$. Storage nodes can be of any node type, i.e., for $v \in \mathcal{V}_{\text{sto}}$ holds $v \in \mathcal{V}^+ \cup \mathcal{V}^- \cup \mathcal{V}^*$. This means that storage nodes might also enable gas supply or withdrawal from outside of the network, apart from enabling flow into and out of the storage. The arc set is partitioned into $\mathcal{A} = \mathcal{A}_{pi} \dot\cup \mathcal{A}_{sc} \dot\cup \mathcal{A}_{va} \dot\cup \mathcal{A}_{cv} \dot\cup_{s \in \mathcal{S}} \mathcal{A}_s$ with a set of station arcs $\mathcal{A}_s$ for each network station $s \in \mathcal{S}$, where $\mathcal{A}_s = \mathcal{A}_{va,s} \dot\cup \mathcal{A}_{\text{rg,s}} \dot\cup \mathcal{A}_{cs,s}$ consist of station valves $\mathcal{A}_{va,s}$, station regulators $\mathcal{A}_{rg,s}$, and station compressors $\mathcal{A}_{cs,s}$, see also Section 5.8. The remaining sets represent arcs that are located outside of stations, here pipes $\mathcal{A}_{pi}$, short cuts $\mathcal{A}_{sc}$, valves $\mathcal{A}_{va}$, and control valves $\mathcal{A}_{cv}$. In the following, we associate station arcs with the additional subscript s in order to distinguish between station and non-station arcs.

Furthermore, we represent the time interval $[0, T]$ by a sequence of discretized points in time $0 < 1 < ... < k$, where 0 corresponds to the initial time point and $1, ..., k$ to the future time points. The difference between two successive points in time is given by τ. We define the following time sets $\mathcal{T}_0 := \{0, ..., k\}$ and $\mathcal{T} := \mathcal{T}_0 \setminus \{0\}$. A demand vector $b \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{T}|}$ is given for all nodes $v \in \mathcal{V}$ and for all time points $t \in \mathcal{T}$, where all sources $v \in \mathcal{V}^+$ hold $b_{v,t} \geq 0$, all sinks $v \in \mathcal{V}^+$ hold $b_{v,t} \leq 0$, and all transshipment nodes $v \in \mathcal{V}^*$ hold $b_{v,t} = 0$.

We model the physical state of the gas network using pressure in [bar] and mass flow in [kg/s]. More precisely, we associate a non-negative pressure variable $p_{v,t} \in [\underline{p}_v, \overline{p}_v]$ with every node $v \in \mathcal{V}$ and time point $t \in \mathcal{T}$, where the initial pressure value at $t = 0$ is given input data. For storage nodes $v \in \mathcal{V}_{\text{sto}}$, flow variables $q_{v,t} \in [\underline{q}_v, \overline{q}_v]$ represent storage inflow and outflow at time $t \in \mathcal{T}$. For each pipe $a = (\ell, r) \in \mathcal{A}_{pi}$ and for each $t \in \mathcal{T}$, we use two flow variables $q_{\ell,t}$ and $q_{r,t}$ describing the arcs' inflow and outflow at nodes ℓ and r. Storage nodes and pipelines have associated volumes and thus enable storing gas, which is often referred to as *line-pack*. In contrast, the remaining arcs $a = (\ell, r) \in \mathcal{A} \setminus \mathcal{A}_{pi}$ are considered to have short length and no associated volume, and thus, we use simply one variable $q_{a,t} \in [\underline{q}_a, \overline{q}_a]$ to describe mass flow at $t \in \mathcal{T}$. Moreover, we assume all variable bounds to be time-independent. Finally, let us remark that storages $\mathcal{V}_{\text{sto}}$ and short cuts $\mathcal{A}_{sc}$ are the only elements that do not occur as such in the network and are introduced as new element types in the aggregation.

In the following, we describe the model constraints induced by the particular network elements.

[2] Actually, we deal with multigraphs, but for the sake of simplicity, we restrict our notation to that of simple graphs. Where necessary, it will be evident from the context, to which of the different (anti-)parallel arcs is referred.

5.3 Modeling Pipelines

The gas physics in pipelines is represented by the discretized continuity and momentum Equations (5.11) and (5.17), which we repeat here for the sake of completeness. For all $a = (\ell, r) \in \mathcal{A}_{pi}$ and for all $t \in \mathcal{T}$ holds

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + \alpha_a \, \tau \Big(q_{r,t} - q_{\ell,t} \Big) = 0, \tag{5.18}$$

$$p_{r,t}^2 - p_{\ell,t}^2 + \beta_a \left(p_{r,t}^2 + p_{\ell,t}^2 \right) + \gamma_a \, (q_{r,t} + q_{\ell,t}) \, |q_{r,t} + q_{\ell,t}| = 0. \tag{5.19}$$

5.4 Modeling Storages

We use storages in order to represent aggregated subnetworks that can be considered as less significant for the network control, particularly distribution and passive tree subnetworks, see Chapter 6. Even though storages are not original elements of the networks considered in the computational study in Chapter 8, the model formulation presented in this section can be applied to networks containing real existing storages.

In our model, one can conceptually understand storages as modified nodes having an associated volume and thereby enabling gas storage. For this reason, we also call them *storage nodes*. Let $v \in \mathcal{V}_{\text{sto}}$ be a storage node. Similar to water tanks (D'Ambrosio et al., 2015), where storages are modeled by

$$A_v \, \partial_t \, p_v(t) + q_v(t) = 0,$$

with cross-sectional area A_v, and pressure p_v, which in water modeling is referred to as *hydraulic head* corresponding to the sum of velocity head, elevation head, and pressure head, we model gas storages by

$$\frac{vol_v}{\mathrm{c_s}} \, \partial_t \, p_v(t) + q_v(t) = 0.$$

Here, the speed of sound $\mathrm{c_s}$ reflects that gas is compressible in contrast to water. Moreover, vol_v denotes the volume, p_v the pressure, and q_v the inflow and outflow of the storage, depending on the sign. However, we remark that this can be done in alternative ways, for example, Tomasgard et al. (2007) consider inflow and outflow rates of a storage as dependent on the storage filling level.

In order to acquire an algebraic formulation, we first integrate the last equation over time, and then approximate the non-resolving integral $\int_{t_0}^{t_1} q(t)dt$ by using the implicit Euler method, i.e.,

$$\begin{aligned} & \int_{t_0}^{t_1} \left(\frac{vol_v}{\mathrm{c_s}} \, \partial_t \, p(t) + q(t) \right) dt = 0 \\ \Rightarrow \quad & \frac{vol_v}{\mathrm{c_s}} \, (p_{v,t_1} - p_{v,t_0}) + \tau \, q_{v,t_1} = 0, \end{aligned}$$

with $\tau = t_1 - t_0$. Finally, we model all storages nodes $v \in \mathcal{V}_{\text{sto}}$ for all $t \in \mathcal{T}$ by

$$p_{v,t} - p_{v,t-1} + \alpha_v \, \tau \, q_{v,t} = 0 \tag{5.20}$$

with $\alpha_v := \mathrm{c_s}\,/vol_v$ and $\tau = t_1 - t_0$.

Note that storage nodes can be thought of as modified pipelines $a = (\ell, r)$ having a "dead end". Let us draw the relation between modeling storages $\mathcal{V}_{\text{sto}}$ as done above and modeling storages as dead end pipelines $a = (\ell, r) \in \mathcal{A}_{pi}$. To this end, let node ℓ be the connection to the remaining network, and let node $r \in \mathcal{V}^*$ be the dead end, and let a be a horizontal pipeline, i.e., $h_\ell = h_r$ leading to $\beta_a = 0$. The property to store gas in pipelines is governed by the discretized continuity equation, which we keep for this purpose as given by (5.18). However, modifying the discretized momentum Equation (5.17) by setting the friction factor λ_a to zero yields $\gamma_a = 0$. These assumptions allow beaming flow along a and result in equal pressure values at the pipe's incident nodes ℓ and r. Moreover, the flow conservation at node r enforces $q_{r,t} = 0$ given that $r \in \mathcal{V}^*$ is a transshipment node having zero demand $b_{r,t} = 0$. For pipe a and a particular time step $t \in \mathcal{T}$ then holds

$$\begin{aligned} p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + \alpha_a\, \tau\, q_{\ell,t} &= 0, \quad \text{and} \\ p_{\ell,t} &= p_{r,t}. \end{aligned}$$

Combining these equations with $\alpha_a = 2\,\mathrm{c_s}^2\,/(L_a\,A_a)$, as defined in Equation (5.10) yields

$$p_{\ell,t} - p_{\ell,t-1} - \frac{\alpha_a}{2}\,\tau\, q_{\ell,t} = 0.$$

Finally, substituting α for $\tilde{\alpha}_a := 2\,\alpha_a$ in the previous equation implies that storage nodes can equivalently be modeled as pipelines having a dead end. However, we note that modeling such pipelines has the downside to require initializing new pressure values and determining new bounds at node r. For this reason, we decided to use storage nodes instead.

5.5 Modeling Short Cuts

Short cuts can be thought of as connections between two nodes that are conceptually of zero length, such that flow does not experience friction and no pressure loss occurs. We use the concept of short cuts in order to represent network points that can potentially be used as sources and sinks. We model such points as transshipment nodes and link them with short cuts to source and sink nodes, respectively. Let us remark that short cuts do not occur as such in original gas networks. Instead, they are introduced in the process of aggregating the network topology in Section 6.1, when contracting arcs and thereby merging sources and sinks.

For all $a = (\ell, r) \in \mathcal{A}_{sc}$ and for all $t \in \mathcal{T}$ holds

$$p_{\ell,t} = p_{r,t}. \tag{5.21}$$

5.6 Modeling Valves

Valves have two different operation modes, *open* and *closed*, which we model by using a binary variable $z_{a,t} \in \{0, 1\}$. An open valve, indicated by $z_{a,t} = 1$, allows for backward and forward flow, and moreover, induces the same pressure values at its incident nodes, i.e., $p_{\ell,t} = p_{r,t}$. In the case that the valve is closed, i.e., $z_{a,t} = 0$, the flow is forced to be zero and the pressures are decoupled at the valve's incident nodes. Inequalities (5.22) – (5.26) model this behavior for all valves $a = (\ell, r) \in \mathcal{A}_{va}$ and for all $t \in \mathcal{T}$

$$p_{\ell,t} - p_{r,t} \leq (\overline{p}_\ell - \underline{p}_r)(1 - z_{a,t}), \tag{5.22}$$

$$p_{r,t} - p_{\ell,t} \leq (\overline{p}_r - \underline{p}_\ell)(1 - z_{a,t}), \tag{5.23}$$

$$q_{a,t} \leq \overline{q}_a z_{a,t}, \tag{5.24}$$

$$\underline{q}_a z_{a,t} \leq q_{a,t}, \tag{5.25}$$

$$z_{a,t} \in \{0, 1\}. \tag{5.26}$$

5.7 Modeling Control Valves

Control valves enable to reduce the pressure along the flow direction. For this reason, they are also known as *pressure regulators*, see Fügenschuh et al. (2015). This reference also provides more information about the technical details of control valves. Similar to valves, control valves can be open or closed, which we model by using a binary variable $z_{a,t}$, where $z_{a,t} = 1$ ($z_{a,t} = 0$) indicates that the control valve is open (closed). However, in contrast to valves, control valves can also be partially open, where the degree of opening controls the amount of pressure reduction. Open control valves take one of the modes *active* and *bypass*. For an open control valve, constraint (5.27) enforces either the active mode, $z^{ac}_{a,t} = 1$, or the bypass mode, $z^{bp}_{a,t} = 1$, with binary variables $z^{ac}_{a,t}, z^{bp}_{a,t} \in \{0, 1\}$. An active control valve enables pressure loss along with gas flow towards the direction of the arc, which is modeled by constraints (5.28) and (5.29). Bypass mode allows for backward and forward flow, while no pressure loss takes place between its incident nodes, modeled by constraints (5.30) – (5.33). Finally, a closed control valve, i.e., $z_{a,t}, z^{ac}_{a,t}, z^{bp}_{a,t} = 0$, blocks flow and thereby decouples the pressure at its incident nodes.

For all $a = (\ell, r) \in \mathcal{A}_{cv}$ and for all $t \in \mathcal{T}$, the resulting constraints read

$$z_{a,t} = z^{ac}_{a,t} + z^{bp}_{a,t}, \tag{5.27}$$

$$p_{r,t} - p_{\ell,t} \leq (\overline{p}_r - \underline{p}_\ell)(1 - z^{ac}_{a,t}), \tag{5.28}$$

$$\underline{q}_a(1 - z^{ac}_{a,t}) \leq q_{a,t}, \tag{5.29}$$

$$p_{r,t} - p_{\ell,t} \leq (\overline{p}_r - \underline{p}_\ell)(1 - z^{bp}_{a,t}), \tag{5.30}$$

$$p_{\ell,t} - p_{r,t} \leq (\overline{p}_\ell - \underline{p}_r)(1 - z^{bp}_{a,t}), \tag{5.31}$$

$$\underline{q}_a z_{a,t} \leq q_{a,t}, \tag{5.32}$$

$$q_{a,t} \leq \overline{q}_a z_{a,t}, \tag{5.33}$$

$$z_{a,t}, z^{ac}_{a,t}, z^{bp}_{a,t} \in \{0, 1\}. \tag{5.34}$$

5.8 Modeling Stations

Important subnetworks are so-called *stations*, which are typically located between major transportation parts of the network. They assemble the majority of active network elements and thus enable the adjustment of pressure levels and to route gas flows between different network regions.

Station sets. Let $\mathcal{S}$ denote the set of all network stations, and let $(\mathcal{G}_s = (\mathcal{V}_s, \mathcal{A}_s))_{s\in\mathcal{S}}$ be the family of all directed station subgraphs with $\mathcal{G}_s \subseteq \mathcal{G}$. We assume that each subgraph is connected. The node and arc sets belonging to a particular station $s \in \mathcal{S}$ are given by $\mathcal{V}_s \subseteq \mathcal{V}$ and $\mathcal{A}_s := \{a = (\ell, r) \,|\, \ell, r \in \mathcal{V}_s\} \subseteq \mathcal{A}$, where each arc links a pair of different station nodes in $\mathcal{V}_s$. Moreover, we assume that each node $v \in \mathcal{V}_s$ is incident to at least one station arc in $\mathcal{A}_s$. Station arcs comprise three different types, *valves* $\mathcal{A}_{va,s} \subseteq \mathcal{A}_s$, *compressing arcs* $\mathcal{A}_{cs,s} \subseteq \mathcal{A}_s$, and *regulating arcs* $\mathcal{A}_{\mathrm{rg,s}} \subseteq \mathcal{A}_s$. The latter ones are also called *check valves* or *non-return control valves*, since they only allow for flow in forward direction, as opposed to control valves that are located in the exterior of stations, which allow for backward and forward flow, see Section 5.7. We denote station arcs throughout with subscript $s \in \mathcal{S}$ in order to indicate the corresponding station. In general, different operation modes might be realized between any pair of station nodes, such as compression, pressure reduction, frictionless flow throughput, or blocking gas flow. The exact possibilities are indicated by the possible operation modes of the existing station arcs, for an example see Figure 5.1.

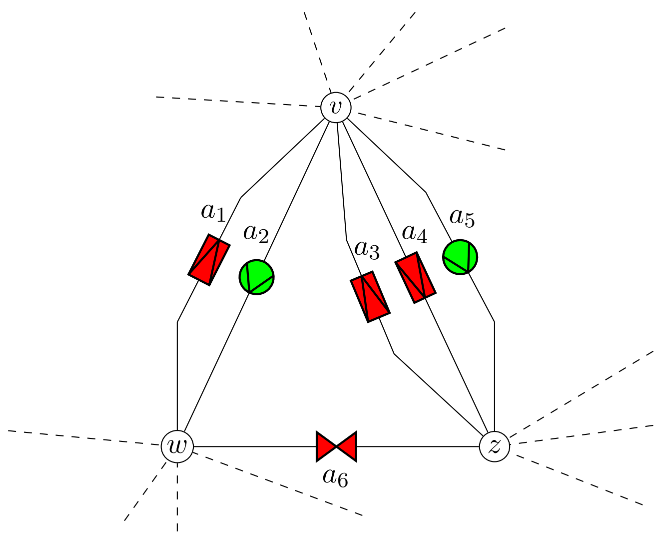


Figure 5.1: Representation of a station $s \in \mathcal{S}$, with station nodes $\mathcal{V}_s = \{v, w, z\}$ and station arcs $\mathcal{A}_s = \{a_1, a_2, a_3, a_4, a_5, a_6\}$, where dashed lines indicate linking arcs to the remaining network. Between any pair of station nodes, parallel and anti-parallel station arcs might exist. Here, for a certain time step, valve a_6 ( ) and the regulators a_1, a_3, a_4 ( ) in red are *closed*, whereas compressors a_2, a_5 ( ) in green are *open*.

Station variables. To model the physical state of a station $s \in \mathcal{S}$, we use the same physical quantities as described above, i.e., pressure at nodes $\mathcal{V}_s$ and mass flow in arcs $\mathcal{A}_s$. Compressing and regulating arcs $a = (\ell, r) \in \mathcal{A}_{cs,s} \cup \mathcal{A}_{\mathrm{rg,s}}$ in stations only allow for flow in the direction of the arc orientation from node ℓ to r, i.e., $q_{a,t} \geq 0$. In contrast, valves enable forward and backward flow, i.e., $q_{a,t} \in [\underline{q}_a, \overline{q}_a]$ with $\underline{q}_a < 0$. Besides, we introduce for each station arc and time step $t \in \mathcal{T}$ a control variable $z_{a,t} \in \{0, 1\}$ that (de)-activates the usage of the arc and its related operation modes. By setting $z_{a,t} = 1$, we model that arc a is used, which consequently activates the corresponding physical behavior. In the case of $z_{a,t} = 0$, the arc is closed, which implies zero flow along $a = (\ell, r)$ and decouples the pressures at its incident nodes.

In the following, we describe the constraints induced by station arcs.

5.8.1 Station valves

Station valves are modeled equivalently to non-station valves described in Section 5.6. For the sake of completeness, we present the modeling here again. For all $a \in \mathcal{A}_{va,s}$, for all $s \in \mathcal{S}$, and for all $t \in \mathcal{T}$, the constraints read

$$p_{\ell,t} - p_{r,t} \leq (\overline{p}_\ell - \underline{p}_r)\,(1 - z_{a,t}), \tag{5.35}$$

$$p_{r,t} - p_{\ell,t} \leq (\overline{p}_r - \underline{p}_\ell)\,(1 - z_{a,t}), \tag{5.36}$$

$$q_{a,t} \leq \overline{q}_a z_{a,t}, \tag{5.37}$$

$$\underline{q}_a z_{a,t} \leq q_{a,t}. \tag{5.38}$$

5.8.2 Station regulators

Station regulators are equipped with a flap trap that prevents backward flow. A closed station regulator, modeled by $z_{a,t} = 0$, forces zero flow and decouples the pressures at its incident nodes. On the other hand, an open mode, $z_{a,t} = 1$, enables pressure reduction $p_{\ell,t} \geq p_{r,t}$ along with non-negative flow $q_{a,t} \in [0, \overline{q}_a]$ from node ℓ to r. For all $a \in \mathcal{A}_{\mathrm{rg,s}}$, for all $s \in \mathcal{S}$, and for all $t \in \mathcal{T}$, the resulting constraints read

$$p_{\ell,t} - p_{r,t} \leq (\underline{p}_\ell - \overline{p}_r)\,(1 - z_{a,t}), \tag{5.39}$$

$$0 \leq q_{a,t} \leq \overline{q}_a\, z_{a,t}. \tag{5.40}$$

5.8.3 Station compressors

Compressors are key elements for the control of gas network operations, as they enable to increase the pressure along the flow direction. This pressure boost is required in order to overcome friction-based pressure losses due to gas transport in pipelines. Compressors can be closed, in bypass, or active. Closed compressor arcs are modeled in the same fashion as (control) valves by setting the control variable $z_{a,t} \in \{0, 1\}$ to zero, which disconnects the arcs' incident nodes and thereby decouples their pressures and prevents gas flow. The modes *bypass* and *active* are indicated by the value $z_{a,t} = 1$. Both cases entail non-negative flow, which is modeled by

$$0 \leq q_{a,t} \leq \overline{q}_a z_{a,t} \tag{5.41}$$

for all $a = (\ell, r) \in \mathcal{A}_{cs,s}$, for all $s \in \mathcal{S}$, and for all $t \in \mathcal{T}$. Additionally, compressor arcs in bypass hold $p_{\ell,t} = p_{r,t}$, whereas active compressors increase the pressure towards the flow direction from node ℓ to node r, both modeled by

$$p_{\ell,t} - p_{r,t} \leq (\overline{p}_\ell - \underline{p}_r)\,(1 - z_{a,t}). \tag{5.42}$$

Besides, active compressors have to obey technical restrictions on the compression ratio and on the upper power limit at disposal, which leads to additional constraints, explained in more detail in the following paragraphs.

Modeling compression ratio in compressor arcs. At first, active compressor arcs have to satisfy an upper limit on the compression ratio r_a of the out and ingoing pressure, i.e.,

$$p_{r,t}/p_{\ell,t} \leq r_a,$$

with $r_a \geq 1$, which is modeled by

$$p_{\ell,t} - p_{r,t}\, r_a \leq M_r\,(1 - z_{a,t}), \tag{5.43}$$

for all $a = (\ell, r) \in \mathcal{A}_{cs}$, for all $s \in \mathcal{S}$, and for all $t \in \mathcal{T}$, where $M_r = \overline{p}_{r,t} - r_a\,\underline{p}_{\ell,t}$. We remark that Bonami et al. (2015) report promising computational results when modeling the big-M term M_r as $M_r = p_{r,t} - r_a\,p_{\ell,t}$ instead, since M_r is locally tightened in the nodes of the branch-and-bound tree, even though constraint (5.43) becomes nonlinear.

Modeling power in compressor arcs. Running compressors to increase pressure in the direction of the gas flow requires a certain amount of power, which is consumed by the drives of the working compressor machines. For ease of notation, we drop the time index in the following. The relationship between the used power P_a, the compression ratio p_ℓ/p_r, and the flow q_a is given by

$$P_a \cdot \eta_{ad} = q_a \cdot H_{ad}.$$

In general, the feasible operating range of a compressor is described by the so-called *characteristic diagram* (H_{ad}, Q_a), which is given in terms of the *specific change in adiabatic enthalpy*

$$H_{ad} = \mathrm{c_s}^2\, \frac{\kappa}{\kappa - 1} \cdot \left(\left(\frac{p_r}{p_\ell} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right)$$

and the *volumetric flow* $Q_a = q_a\, p_\ell/(R_s\, T\, z_\ell)$. The term H_{ad} depends on $\mathrm{c_s}^2 = z_\ell\, R_s\, T$, see Equation (5.4), the *isentropic exponent* κ, and on the compressibility factor z_ℓ of the gas. We approximate z_ℓ by using the *Papay formula* (Papay, 1968; Saleh, 2002), and set $\kappa = 1.296$, as frequently done in the literature (Percell and Ryan, 1987; Fügenschuh et al., 2015; Schmidt et al., 2015). Moreover, the power constraint $P_a \cdot \eta_{ad} = q_a \cdot H_{ad}$ also depends on the *adiabatic efficiency* $\eta_{ad} \in [0, 1]$ of the compression process. For more information about feasible operation ranges of compressors, modeling the power constraint and its quantities involved, we refer to Fügenschuh et al. (2015).

In order to acquire a linear model, we approximate this highly nonlinear and non-convex power constraint $P_a \cdot \eta_{ad} = q_a \cdot H_{ad}$. For example, van der Hoeven (2004) approximates the power by

$$P_a \approx q_a \frac{p_r - p_\ell}{\frac{2}{3}p_r + \frac{1}{3}p_\ell},$$

and then linearizes this expression by using a first-order Taylor expansion, where the linearization point is obtained by the last iteration within a *Sequential Linear Programming* solution approach. In general, we could use a current working point $(p_{\ell,0}, p_{r,0}, q_{a,0})$ at time step $t = 0$, if available, to apply this linearization procedure here, but instead, we aim at finding a linear approximation that is reasonably accurate in the entire range of the considered variable bounds. To this end, we sample the region of feasible points, which additionally satisfy a given maximal compression ratio $p_r \leq r_a\, p_\ell$, i.e.,

$$\mathcal{S} := \left\{ (p_\ell, p_r, q_a) \in [\underline{p}_\ell, \overline{p}_\ell] \times p_r \in [\underline{p}_r, \overline{p}_r] \times q_a \in [\underline{q}_a, \overline{q}_a] \quad | \quad 1 \leq p_{r,t}/p_{\ell,t} \leq r_a \right\},$$

and calculate the actual power P_a^s for each sample $s \in \mathcal{S}$. To determine a linear approximation of the power P_a, we use an ordinary least-squares regression. In order to reduce the linearization error for larger power values close to, or at the maximal power bound, where the linearized power constraint (5.44) should be binding, we introduce sample weights. Here, we use the calculated power P_a^s of the samples as weights. As trade-off, larger errors arise for smaller power values, where, however, the linearized power constraint (5.44) should be inactive. Then, the regression determines coefficients $a_0, a_1, a_2, a_3 \in \mathbb{R}$ by solving

$$\min_{p_\ell^s, p_r^s, q_a^s} \sum_{s \in \mathcal{S}} \left(\frac{1}{P_a^s} \left(a_0\, p_\ell^s + a_1\, p_r^s + a_2\, q_a^s + a_3 - P_a^s \right) \right)^2.$$

Finally, we bound the approximated power expression by a given upper power bound $P_{\max}$ and add the following time-independent linear inequality for all $a = (\ell, r) \in \mathcal{A}_{cs,s}$, for all $s \in \mathcal{S}$, and for all $t \in \mathcal{T}$ to the model

$$a_0\, p_{\ell,t} + a_1\, p_{r,t} + a_2\, q_{a,t} + a_3 \leq P_{\max} + M_P\, (1 - z_{a,t}). \tag{5.44}$$

Here, the control variable $z_{a,t}$ activates the inequality by using a big-M term $M_P = a_0\, \underline{p}_\ell + a_1\, \overline{p}_r + a_2\, \overline{q}_{a,t} + a_3$.

A visualization of the real power and the linearized version is given in Figure 5.2, where the real power is displayed in level contours, and the linearized power is illustrated for different ingoing pressure values $p_{\ell,t}$.

5.9 Flow Conservation

The classical flow conservation holds for all network nodes $v \in \mathcal{V}$ and for all time steps $t \in \mathcal{T}$, ensuring that the amount of flow entering and leaving a node is the same.

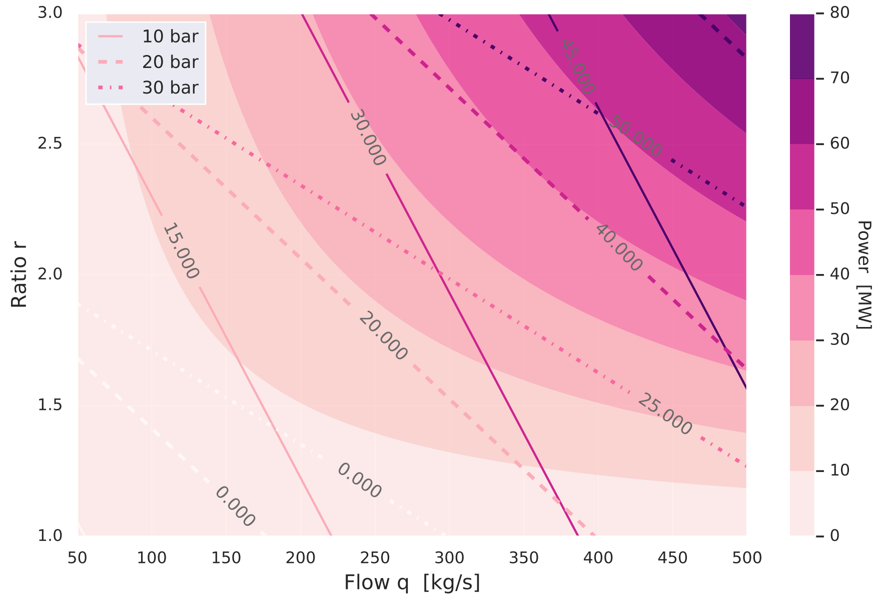


Figure 5.2: Linear approximation to the nonlinear power function. For different mass flow values (x-axis) and compression ratios (y-axis), the real used power $P_a \cdot \eta_{ad,t} = q_{a,t} \cdot H_{ad}$ is displayed in level sets, while the linearized power is illustrated for different values of the ingoing pressure $p_{\ell,t}$ (in bar), i.e., for $p_{\ell,t} = 10$ (solid lines), $p_{\ell,t} = 20$ (dashed lines), and $p_{\ell,t} = 30$ (dash dotted lines). Here, the efficiency is set to $\eta_{ad} = 1$.

For non-storage nodes $v \in \mathcal{V} \setminus \mathcal{V}_{\text{sto}}$, we have

$$\begin{aligned} &\sum_{a=(v,r)\in\mathcal{A}\setminus\mathcal{A}_{pi}} q_{a,t} - \sum_{a=(\ell,v)\in\mathcal{A}\setminus\mathcal{A}_{pi}} q_{a,t} \\ &+ \sum_{a=(v,r)\in\mathcal{A}_{pi}} q_{v,t} - \sum_{a=(\ell,v)\in\mathcal{A}_{pi}} q_{v,t} = b_{v,t}, \end{aligned} \tag{5.45}$$

and for storage nodes $v \in \mathcal{V}_{\text{sto}}$

$$\begin{aligned} q_{v,t} &+ \sum_{a=(v,r)\in\mathcal{A}\setminus\mathcal{A}_{pi}} q_{a,t} - \sum_{a=(\ell,v)\in\mathcal{A}\setminus\mathcal{A}_{pi}} q_{a,t} \\ &+ \sum_{a=(v,r)\in\mathcal{A}_{pi}} q_{v,t} - \sum_{a=(\ell,v)\in\mathcal{A}_{pi}} q_{v,t} = b_{v,t}. \end{aligned} \tag{5.46}$$

5.10 Objective Function

The objective is to minimize the amount of changes in network stations over time. This refers to minimizing the changes of operation modes, such as opening and closing valves, regulators, and compressors. The basic idea behind this objective is twofold. First, changing operation modes increases wear on the devices. In particular, compressors typically require keeping an operation mode for a certain amount of time in practice, before ramping up or down. Second, practitioners, in general, seek stable gas flows and stable network states. Even though there is no mathematical definition for the *stability* of gas network operations, practitioners usually consider operations as stable, if they require only few interventions on changing the operational settings of the active elements. While the meaning of a “few interventions” certainly depends on the particular network, it gives rise to using an objective function that minimizes the number of operation mode changes over time.

In the following, we weigh the changes of station modes in order to prioritize a constant control of compressors. Valves can be considered as the most flexible active elements that even can change their modes instantaneously. Besides, valves are less cost-intensive than control valves and compressors. Compressors are the most expensive elements due to the necessary equipment that enables realizing pressure increase. The penalty coefficients read $0 < \omega_a \leq \omega_b \leq \omega_c$ for all $a \in \mathcal{A}_{va,s}$, for all $b \in \mathcal{A}_{rg,s}$, for all $c \in \mathcal{A}_{cs,s}$, and for all $s \in \mathcal{S}$. Then, the objective function is given by

$$\begin{aligned}\min \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \Bigg(\sum_{a\in\mathcal{A}_{va,s}} \Big(\omega_a \left|z_{a,t-1} - z_{a,t}\right|\Big) + \sum_{a\in\mathcal{A}_{cv,s}} \Big(\omega_a \left|z_{a,t-1} - z_{a,t}\right|\Big) \\ + \sum_{a\in\mathcal{A}_{cs,s}} \Big(\omega_a \left|z_{a,t-1} - z_{a,t}\right|\Big) \Bigg).\end{aligned} \tag{5.47}$$

Model formulation for the *Transient Control Problem*. In summary, the model reads

	(5.47)	
s.t.	(5.18) – (5.19)	$\forall a = (\ell, r) \in \mathcal{A}_{pi},\ \forall t \in \mathcal{T}$,
	(5.20)	$\forall v \in \mathcal{V}_{\text{sto}},\ \forall t \in \mathcal{T}$,
	(5.21)	$\forall a = (\ell, r) \in \mathcal{A}_{sc},\ \forall t \in \mathcal{T}$,
	(5.22) – (5.26)	$\forall a = (\ell, r) \in \mathcal{A}_{va},\ \forall t \in \mathcal{T}$,
	(5.27) – (5.34)	$\forall a = (\ell, r) \in \mathcal{A}_{cv},\ \forall t \in \mathcal{T}$,
	(5.35) – (5.38)	$\forall s \in \mathcal{S}, \forall a = (\ell, r) \in \mathcal{A}_{va,s}, \forall t \in \mathcal{T}$,
	(5.39) – (5.40)	$\forall s \in \mathcal{S}, \forall a = (\ell, r) \in \mathcal{A}_{rg,s}, \forall t \in \mathcal{T}$,
	(5.41) – (5.44)	$\forall s \in \mathcal{S}, \forall a = (\ell, r) \in \mathcal{A}_{cs,s}, \forall t \in \mathcal{T}$,
	(5.45)	$\forall v \in \mathcal{V} \setminus \mathcal{V}_{\text{sto}},\ \forall t \in \mathcal{T}$,
	(5.46)	$\forall v \in \mathcal{V}_{\text{sto}},\ \forall t \in \mathcal{T}$.

Chapter 6

Aggregation of Transient Gas Networks

6.1 Introduction

In practical optimization problems, instances have to be solved since decades, whose sizes are beyond the capabilities of the present state-of-the-art solution techniques at hand. In addition, the problems are frequently $\mathcal{NP}$-hard, such as our *Transient Control Problem*, and thus the idea of aggregating data has throughout accompanied the process of solving large-scale instances, see Rogers et al. (1991) for a broad overview. The authors themselves propose a general framework for the *aggregation and disaggregation* that can be summarized as follows: (1) aggregate data of the original optimization problem, (2) solve an auxiliary model of reduced size relative to the original model, and (3) disaggregate the solution of the reduced auxiliary model in order to obtain a (possibly approximate) solution of the original model. Even though solution techniques and computer hardware improved since then, which enables problems to be solved that seemed impenetrable in that time, this approach has throughout stimulated numerous applications, see, e.g., Bärmann et al. (2015) for an application to network design problems, and Reuther (2017) for a coarse to fine approach in rolling stock rotations of train scheduling, just to name a few recent publications. For more information about aggregation of large-scale optimization we refer to the textbook of Litvinchev and Tsurkov (2013).

We now turn our attention to the aggregation of gas networks. For the operation of stationary gas transport, a few approaches already exist that reduce the network size, see Ríos-Mercado et al. (2002), Clees et al. (2018) and Groß et al. (2019). Ríos-Mercado et al. (2002) present a reduction technique, which shrinks passive subnetworks between compressing arcs to single nodes. The authors show that if the reduced graph is a tree, then the flows along the compressing arcs are fixed and the resulting flows on the original passive subnetworks are uniquely determined. In the case that the reduced graph is cyclic however, the authors suggest using a numerical approximation for the flow on the reduced graph. Clees et al. (2018) and Groß et al. (2019) apply the concept of *generalized series-parallel graphs*, see Korneyenko (1994), by successively merging pipes in parallel and series and shrinking leaf nodes, which reduces the size of the considered networks in both papers about roughly 70%. Indeed, generalized series-parallel graphs frequently occur in network applications, either directly or as substructures. Through the successive application of serial and parallel merges and leaf node reductions, they are reducible to a single arc, i.e., to the K_2. Moreover, generalized series-parallel graphs can be detected in linear time (Takamizawa et al., 1982). Besides the aggregation of gas networks, several approaches use specially tailored decomposition methods of the underlying graph to solve stationary gas network transport problems, see (e.g., Morsi, 2013; Stangl, 2014), and the references in Koch et al. (2015). While all of these approaches are restricted to stationary gas networks, we investigate the aggregation of transient gas networks in this chapter.

Our goal is to solve the *Transient Control Problem* from Chapter 5 on real-world networks. However, practically relevant gas networks frequently contain up to several thousands of elements, see e.g., Carvalho et al. (2009), Geißler et al. (2015), and Schmidt et al. (2017b). As a consequence of dealing with transient gas flow problems as MINLPs on large-scale networks, high-dimensional optimization problems arise. Such problems have not been successfully optimized so far, mainly for two reasons. On the one hand, research about transient gas flow problems is still in its early stages, but moreover the main reason concerns the limited capability of MINLP solvers at the current state of development. For example, solving the *Transient Control Problem* for a duration of twelve time steps on an underlying network that comprises 2,000 pipelines already yields 24,000 nonconvex quadratic constraints of type (5.17), which is by far out of scope for state-of-the-art MINLP solvers. However, the need to solve the *Transient Control Problem* opens the door to investigate a topological reduction of large-scale gas networks. In this chapter, we develop an aggregation procedure specially tailored to large-scale gas networks that reduces the size of the network thus far that transient gas flow problems on the aggregated structures can be solved with current state-of-the-art MINLP solvers. To the best of our knowledge, our approach is the first that aggregates transient gas networks.

In fact, also TSOs typically deal with an aggregated representation of the network, when making decisions about their operations. Using a coarser representation of gas networks also applies to other problems in practice, such as the evaluation of potential new contracts with clients, the validation of worst-case scenarios, or expansion planning. The decision-making of these applications is usually acquired by running simulation or optimization tools on networks, where certain parts, considered to be of minor importance for the respective decision process, are coarsely represented.

Concerning the operation of gas networks, dispatchers, in general, use active devices to control the physical state of the network. As the majority of control elements are located within network stations and the majority of pipes are located outside of stations, this encourages the development of an aggregation approach, which keeps stations and their control elements untouched and focuses on aggregating the exterior of stations. Moreover, restricting the aggregation to the exterior of stations enables the user to plug any desired level of station modeling into the transient gas flow model, which imparts a generic aspect to the aggregation.

By concept, specific network parts located in the exterior of stations are distribution subnetworks. Based on real-world data, we accomplish a study about distribution subnetworks, which suggests that they only slightly contribute to buffering network operations. That is, their inflow from the remaining network is rather instantaneously consumed within the distribution subnetwork, see Section 6.3. For this reason, distribution subnetworks correspond to potential sources of aggregation, as opposed to the transportation part, which forms the *backbone* of the network. While in practice, the representation of *backbone networks* is typically designed manually by engineers applying expert knowledge about the particular topology, we present an automated aggregation approach that is applicable to different gas networks. Apart from the topology, our approach even makes use of the current physical network state and the predicted demand. Provided that all of these ingredients change over time (i.e., the topology, the physical state and the demand), we understand the aggregation as a tool that is carried out for each particular optimization run. In this fashion, we utilize the aggregation approach when optimizing the *Transient Control Problem* on test set *Net 3* in the computational study in Chapter 8. There, we sequentially solve the control problem every hour over a total duration of two months, where a particular instance covers a time range of 12 hours with an hourly discretization. For each of these instances, we deploy the aggregation procedure, where each particular aggregation bases on the respective topology, its physical state, and future demand.

We already mentioned that we exclude stations from the aggregation process, as the majority of control decisions are located in stations. Concerning the control decisions outside of stations, our approach only aggregates those active elements that allow for a unique replacement by a structurally equivalent substitute element. Then, by solving the *Transient Control Problem* on the aggregated network, we acquire solutions for the operation modes of the many station elements and the (rather few) control decisions of possibly aggregated non-station elements. This enables us to omit the disaggregation step (3) of Rogers et al. (1991), even though the disaggregation would allow for a physical validation of the disaggregated solution. Nevertheless, the obtained modes can be interpreted as a recommendation and support for practitioners of how to operate the network daily. Finally, we remark that our aggregation approach has been successfully integrated into a decision-support system for gas network dispatchers that was developed within the GasLab of the Research Campus MODAL[1]. For further information about this project, we also refer to Hoppmann et al. (2019) and Hennings et al. (2019).

The remainder of this introduction is structured as follows: In Subsection 6.1.1, we present the overall aggregation approach. In Subsection 6.1.2, we characterize and categorize all deployed aggregation methods and introduce the concept of equivalent and heuristic subnetwork replacements. In order to evaluate the aggregation methods, we apply extensive computational studies that quantify the physical impact of the aggregated structures on the remaining network. These studies depend on the used real-world data and measures described in Subsection 6.1.3, but also on the deployed simulations described in Subsection 6.1.4. We conclude the introduction by outlining the investigation of all aggregation methods in Subsection 6.1.5.

[1] `http://forschungscampus-modal.de`

6.1.1 Aggregation scheme for transient gas networks

The aggregation approach is sketched in Algorithm 1. As input, it requires a network, its initial physical state and a future demand profile. The only user-defined input parameter is a threshold value $\mathscr{L}$ for the pipe length required for shrinking pipelines, see Section 6.5. The algorithm outputs the topologically reduced network with corresponding values for the aggregated initial state and demand.

Algorithm 1 Aggregation scheme of transient gas networks

Input: network, initial state, demand, pipe length threshold $\mathscr{L}$
Output: aggregated network, aggregated initial state, aggregated demand

```
1: Remove-closed-nrc-elements
2: Shrink-open-nrc-elements
3: Remove-parts-wo-boundary-nodes
4: Aggregate-distr-parts
5: Aggregate-passive-trees
6: Shrink-pipes-with-short-length
7: Remove-self-loops
8: Remove-degree-zero-nodes
9: isAggregationActive ← true
10: while isAggregationActive do
11:     hasAggregated ← Merge-parallel-pipes
12:     isAggregationActive ← hasAggregated
13:     hasAggregated ← Merge-parallel-act-elements
14:     isAggregationActive ← hasAggregated or isAggregationActive
15:     hasAggregated ← Merge-serial-pipes
16:     isAggregationActive ← hasAggregated or isAggregationActive
17:     hasAggregated ← Merge-serial-act-elements
18:     isAggregationActive ← hasAggregated or isAggregationActive
19: end while
20: Decompose-mixed-boundary-nodes
```

At first, we apply aggregation methods that allow to reduce the network only once successfully, see lines 1–8 in the algorithm. The following three methods remove and shrink elements without providing substitutes, where shrink corresponds to contracting arcs and combining their incident nodes. These methods can be understood as preprocessing steps that eliminate network structures, which are not under the direct control of dispatchers at the present time.

Remove-closed-nrc-elements removes closed non-remote controllable elements;

Shrink-open-nrc-elements shrinks open non-remote controllable elements;

Remove-parts-wo-boundary-nodes removes disconnected components without sources and sinks.

Then, we aggregate subnetworks that do not enable gas transit to other network regions and thus correspond to outbound zones of the network. The following procedures substitute such subnetworks for storage nodes:

Aggregate-distr-parts aggregates distribution subnetworks;

Aggregate-passive-trees aggregates tree subnetworks that only contain pipes.

Compared to passive tree subnetworks, distribution subnetworks might contain active elements, which are, in most cases non-remote controllable and thus have been aggregated beforehand. Hence, the contribution of both subnetwork types to control network flows can be considered as somewhat limited. Afterwards, we shrink pipelines with short length:

Shrink-pipes-with-short-length shrinks pipes having shorter lengths than a specified threshold value $\mathscr{L}$.

Note that distribution and passive tree subnetworks might also contain pipelines with short lengths. However, their aggregation methods provide substitute elements that enable it to adequately represent the storage capacity and gas volume of all aggregated pipes, as opposed to the latter method, which we apply for this reason later in the algorithm. Shrinking pipelines is a powerful tool to reduce the network size. Nevertheless, it should be used carefully in order to avoid shrinking entire transport paths, e.g., between different stations, see also the discussion in Section 6.5.

Given that all shrinking routines do not provide substitutes, they might lead to meaningless network structures, such as self-loops, which connect a node to itself. Moreover all methods deployed so far might yield nodes of degree zero. Thus, we tidy-up the network:

Remove-self-loops removes self-loops;

Remove-degree-zero-nodes removes degree zero nodes.

Afterwards, we successively run *series-parallel graph* methods, where the Boolean `hasAggregated` indicates whether a particular subroutine successfully reduced the network size. Provided that a successful reduction evolves new topological structures that might trigger further reductions, we apply the following *series-parallel graph* subroutines within a `while loop`, until none of them detects possible aggregation structures:

Merge-parallel-pipes merges parallel pipes;

Merge-serial-pipes merges serial pipes;

Merge-parallel-act-elements merges parallel active elements;

Merge-serial-act-elements merges serial active elements.

Finally, the algorithm terminates, if no further reductions are possible, indicated by the Boolean `isAggregationActive`.

Concerning the substitution process, our aggregation approach is tailored to the *Transient Control Problem*, as the corresponding procedures use the modeling of the elements involved. In particular, this refers to the modeling of pipes, valves, control valves and storage nodes. Moreover, we indicate methods that introduce substitute elements by the prefix *Aggregate* and *Merge*.

In the course of the aggregation, it can happen that different boundary nodes are merged. Therefore, we track the information of which nodes in the current network represent the aggregated nodes and relocate the demand accordingly. In the case of merging different kinds of boundary nodes, we handle such occurrences internally by introducing a new node type, called *mixed*. This occurs for example, when shrinking arcs and thereby combining a sink and source, or when aggregating subnetworks that contain different types of boundary nodes. For this reason, we apply as a last step the procedure:

Decompose-mixed-boundary-nodes decomposes mixed boundary nodes.

This subroutine replaces each mixed node by an inner node and introduces a pair of source and sink nodes. These source and sink nodes are then connected via so-called *short cuts* with the inner node. The short cuts can be understood as artificial links that establish the same pressure values at the inner node, source and sink. Finally, the demand of the mixed node with positive share is relocated to the source at the respective point in time. Under the same logic, the sink inherits the demand of the mixed node with a negative share. As the concept of *mixed* boundary nodes is rather an internal implementation detail, we omit the introduction of *mixed* boundary nodes in this thesis and with a little abuse of notation retain the notation of sinks and sources.

Finally, using the example of serial pipe merges, we point out that aggregating elements is not in the spirit of refining the spatial discretization which would be required in order to converge towards a PDE solution of the Euler Equations (5.1). Instead, this aggregation procedure enlarges the step size of the spatial discretization by removing the intermediate node and thus dealing with an entire pipe instead of two pipe segments. Hence, we apply the aggregation methods from the viewpoint of optimization with the intension to reduce the number of variables and constraints present in the model.

6.1.2 Characteristics of the aggregation methods

In this subsection, we characterize the aggregation methods with respect to algorithmic, topological and structure preserving aspects. An overview is provided in Table 6.1.

The network topology typically alters over time due to expansion and maintenance of network elements. Likewise its physical state and the predicted demand change over time. For this reason, we accomplish the aggregation for a particular optimization run and take the current *network*, *initial state* and *demand* into account. For sure, all methods depend on the network topology, while the demand and the initial state are additionally required for the detection of certain network structures, for contracting arcs and for the generation of substitute elements, see column *bases on*.

Explanation / Aggregation method	bases on	enables multiple reductions	provides substitutes	substitute type	replaces		preserve properties	evaluation
					equiv	heur		
Remove closed non-remote controllable elements REMOVE-CLOSED-NRC-ELEMENTS	network							
Shrink open non-remote controllable elements SHRINK-OPEN-NRC-ELEMENTS	network demand							
Removal of disconnected subnetworks without boundary nodes REMOVE-PARTS-WO-BOUNDARY-NODES	network demand							
Tidy-up network REMOVE-SELF-LOOPS REMOVE-DEGREE-ZERO-NODES	network							
Aggregation of distribution subnetworks AGGREGATE-DISTR-PARTS	network state demand		✓	storage		✓	(i) - (iii)	✓
Aggregation of passive tree subnetworks AGGREGATE-PASSIVE-TREES	network state demand		✓	storage		✓	(i) - (iii)	✓
Shrink pipes with short lengths SHRINK-PIPES-WITH-SHORT-LENGTH	network state demand $\mathscr{L}$					✓		
Merge parallel pipes MERGE-PARALLEL-PIPES	network	✓	✓	pipe	✓			✓
Merge parallel active elements MERGE-PARALLEL-ACT-ELEMENTS	network	✓	✓	act. element	✓			
Merge serial pipes MERGE-SERIAL-PIPES	network demand	✓	✓	pipe		✓	(i), (iv) - (v)	✓
Merge serial active elements MERGE-SERIAL-ACT-ELEMENTS	network demand	✓	✓	act. element	✓			

Table 6.1: Characteristics of the deployed aggregation methods. The column *aggregation method* denotes their abbreviations as used in Algorithm 1, while column *bases on* depicts their input parameters. The remaining columns illustrate whether a method *enables multiple reductions*, *provides substitute* structures with corresponding *substitute type* and whether it *equivalently* or *heuristically replaces* a given structure. Column *preserve properties* states principles for the heuristic generation of substitute elements, as explained on page 137. Column *evaluation* indicates whether the method is validated by using simulations based on real-world examples.

Equivalent subnetwork replacements. A major concern of the aggregation is that the control decisions acquired by solving the *Transient Control Problem* on the reduced network can also be used for the control of the original network. To this end, the *Transient Control Problem* should ideally be feasible on both the original and the aggregated network at the same time. In the following, we formalize this requirement most generally for a method that replaces a subnetwork $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{A}_C)$ with $\mathcal{V}_C \subseteq \mathcal{V}$, $\mathcal{A}_C \subseteq \mathcal{A}$ by an aggregated structure $\mathcal{G}_C^{\text{agg}} = (\mathcal{V}_C^{\text{agg}}, \mathcal{A}_C^{\text{agg}})$. We first introduce some notation. Even though using the notation for graphs, we recall that the aggregation does not only refer to the topology, but also to the corresponding initial state and demand.

Let the entire network after the aggregation of $\mathcal{G}_C$ be given by

$$\mathcal{G}^{\text{agg}} = (\mathcal{V}^{\text{agg}}, \mathcal{A}^{\text{agg}}) := \Big((\mathcal{V} \setminus \mathcal{V}_C) \cup \mathcal{V}_C^{\text{agg}}, (\mathcal{A} \setminus \mathcal{A}_C) \cup \mathcal{A}_C^{\text{agg}} \Big),$$

and let the part of the original network $\mathcal{G}$ that coincides with the aggregated network $\mathcal{G}^{\text{agg}}$ be given by

$$\mathcal{G} \cap \mathcal{G}^{\text{agg}} = \left(\mathcal{V} \cap \mathcal{V}^{\text{agg}}, \mathcal{A} \cap \mathcal{A}^{\text{agg}}\right).$$

Note that $\mathcal{G} \cap \mathcal{G}^{\text{agg}} = \mathcal{G} \setminus \mathcal{G}_C = \mathcal{G}^{\text{agg}} \setminus \mathcal{G}_C^{\text{agg}}$ holds.

In the following, consider a *Transient Control Problem* and let

$\mathcal{X}_{\mathcal{G}}$	be a solution for network $\mathcal{G}$ before aggregating $\mathcal{G}_C$,
$\mathcal{X}_{\mathcal{G}^{\text{agg}}}$	be a solution for network $\mathcal{G}^{\text{agg}}$ after aggregating $\mathcal{G}_C$,
$\mathcal{X}_{\mathcal{G}_C}$	the restriction of $\mathcal{X}_{\mathcal{G}}$ to the variables induced by $\mathcal{G}_C$, and
$\mathcal{X}_{\mathcal{G}_C^{\text{agg}}}$	the restriction of $\mathcal{X}_{\mathcal{G}^{\text{agg}}}$ to the variables induced by $\mathcal{G}_C^{\text{agg}}$.

We understand equivalent replacements of subnetworks in such a way that it is possible to extend a solution for the aggregated subnetwork to a solution for the non-aggregated subnetwork and vice versa. The following definition formalizes this concept.

Definition 6.1.1 (Equivalent subnetwork replacement)**.** *Let $\mathcal{G}_C^{agg} \cap \mathcal{G}_C \neq \emptyset$. We say that an aggregated subnetwork $\mathcal{G}_C^{agg}$* equivalently replaces *a non-aggregated subnetwork $\mathcal{G}_C$, if*

i) a solution $\mathcal{X}_{\mathcal{G}_C^{agg}}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}_C}$, and

ii) a solution $\mathcal{X}_{\mathcal{G}_C}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}_C^{agg}}$.

In other words, the first condition of the definition says that after solving the *Transient Control Problem* on the aggregated network and fixing the solution values of those variables that are induced by common elements of $\mathcal{G}_C^{\text{agg}}$ and $\mathcal{G}_C$, values can be found for the variables that are induced by the remaining elements in $\mathcal{G}_C \setminus (\mathcal{G}_C \cap \mathcal{G}_C^{\text{agg}})$ such that the corresponding constraints of the *Transient Control Problem* are satisfied. In this fashion, we utilize equivalent subnetwork replacements in the overall solution approach from Figure 1.2. In addition, the second condition addresses the reverse direction from original to aggregated subnetworks. Together, both conditions guarantee that there is a solution to the *Transient Control Problem* for the network before and after aggregation of $\mathcal{G}_C$, if it exists for either of them.

The following proposition states that a solution on the aggregated network $\mathcal{G}^{\text{agg}}$ can be extended to a solution on the non-aggregated network $\mathcal{G}$ and vice versa.

Proposition 6.1.2. *Assume that $\mathcal{G}_C^{agg}$ equivalently replaces a subnetwork $\mathcal{G}_C$ according to Definition 6.1.1. Then,*

i) a solution $\mathcal{X}_{\mathcal{G}^{agg}}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}}$, and

ii) a solution $\mathcal{X}_{\mathcal{G}}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}^{agg}}$.

Proof. We show that (i) a solution $\mathcal{X}_{\mathcal{G}^{\text{agg}}}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}}$. The second claim follows analogously.

Let $\mathcal{X}_{\mathcal{G}}$ be defined as follows: It is going to be the solution $\mathcal{X}_{\mathcal{G}^{\text{agg}}}$ for all elements in $\mathcal{G} \setminus \mathcal{G}_C$ and all elements in $\mathcal{G}_C \cap \mathcal{G}_C^{\text{agg}}$. It remains to define $\mathcal{X}_{\mathcal{G}}$ for the elements in $\mathcal{G}_C \setminus (\mathcal{G}_C \cap \mathcal{G}_C^{\text{agg}})$, but from Definition 6.1.1 follows that there exists an assignment that makes it feasible for $\mathcal{G}_C$. $\square$

When restricting to the possible (multiple) application of aggregation procedures that equivalently replace subnetworks, then from Proposition 6.1.2 follows that a solution for the entirely aggregated network can be extended to a solution for the original network. Consequently, one would like to apply aggregation methods that establish equivalent replacements. In our context, we show that this property holds for all merging procedures, but the serial pipe merge. For proofs, we refer to Sections 6.6 and D.3 in the appendix.

Heuristic subnetwork replacements. If an aggregation method does not equivalently replace a subnetwork, we call the procedure *heuristic*. The additional application of heuristic methods might be necessary in order to sufficiently reduce the network size. For sure, to keep the impact of heuristic procedures on the remaining network as small as possible, we aim at developing these methods such that for every solution $\mathcal{X}_{\mathcal{G}_C^{\text{agg}}}$, there is ideally a point close to it that can be extended to a solution $\mathcal{X}_{\mathcal{G}_C}$.

Preserving properties in heuristic subnetwork replacements. To achieve this, we figured out that preserving specific properties of the subnetwork and its physical state plays a crucial role. In the following, we discuss these properties for heuristic aggregation methods that introduce substitute elements. For the aggregation of distribution and passive tree subnetworks and serial pipe merges, we

i) maintain the volume of the subnetworks' pipes,

ii) maintain the initial amount of gas (line-pack).

In the case of distribution and passive tree subnetworks, we

iii) calculate tight pressure bounds at the storage node.

In the case of merging serial pipes, we

iv) minimize the error between the pressure and flow realizations of the subnetwork before and after the aggregation, and

v) maintain the difference in height induced by the original pipes.

The rationale for these properties is as follows: First, we generate substitute structures that have the same pipe volume as the subnetworks prior to aggregation, which enables storing a similar amount of gas. Second, if possible, we maintain the amount of gas from the non-aggregated subnetwork, called line-pack. Line-pack can be used as safety stock and to buffer peaks in gas flows. Given that line-pack can be described in terms of pressure, this requirement is closely related to determining initial pressure values at the substitute elements.

Third, when replacing subnetworks by storage nodes, the rationale for the calculation of pressure bounds at the storage nodes is as follows: When fulfilling the storage node's pressure bounds, all pressure restrictions at nodes of the non-aggregated subnetwork should ideally be satisfied. To this end, we approximate the maximal pressure loss inside the subnetwork, combine it with the pressure bounds at its nodes and apply the intersection of all these bound intervals to the new storage node.

Fourth, concerning serial pipe merging, we minimize the error between the pressure and flow realizations at common elements of the aggregated and non-aggregated structures. Fifth, we maintain the difference in height along the serial pipes in order to keep the height of the remaining network elements constant. Please note that procedures that equivalently replace subnetworks also fulfill all these properties.

Finally, for all heuristic aggregation methods that introduce substitute elements, we evaluate their physical impact on the remaining network by using simulations, see column *evaluation* in Table 6.1. Our intensive computational experiments suggest that following the described criteria keeps the impact of the aggregation on the remaining network reasonably small.

6.1.3 Computational studies using real-world data

In order to analyze the real-world behavior of certain network structures and to evaluate the heuristic aggregation methods, we perform the following computational studies in this chapter:

– *simulation-based evaluation of distribution and passive tree subnetwork aggregations, and serial pipe merges*, explained in more detail in Subsection 6.1.4;

– *analysis of line-pack in distribution subnetworks* in order to study their impact on daily gas network operations in Subsections 6.3.3 and D.1.1;

– *calculation of maximal pressure losses in distribution and passive tree subnetworks* in order to determine pressure bounds at new storages nodes in Subsections 6.3.6 and 6.4.3;

– *analysis of maximal pressure loss and line-pack values in real-world pipelines* in order to facilitate the selection of a threshold value $\mathscr{L}$, when shrinking pipelines with short lengths in Subsection 6.5.

For all computational experiments in this chapter, we use data provided by our industrial cooperation partner Open Grid Europe GmbH. The used real-world data concerns the network topology and historical data for the physical state over time. More precisely, the historical data comprises a selection of states, where a single state specifies nodal pressure and demand flow values, as well as arc flow values for a particular time step. In total, we are given a set of physical states covering an entire year split into three-minute steps. Due to this large time range, our studies account for seasonal demand fluctuations, as winters typically entail higher gas consumptions than summers. In practice, the historical data results from measurement devices that are installed in the network. Our industrial project partner completed this data by running simulations in order to cover the entire network and time range. Hence, this data might not correspond to exact physical values that appear in the network and might be error-prone. In order to diminish the impact of peaks due to possible measurement and simulation errors, we smoothen the pressure and flow values. To this end, we average the given historical data consisting of 175,200 single states (three-minute steps over an entire year, i.e., 365 days). More precisely, we aggregate five consecutive three-minute steps yielding 15-minute intervals resulting in 35,040 states for the whole year: Let $(x_1, ..., x_{175,200})$ be the corresponding values for a physical entity, such as pressure values in [bar] or mass flow values in [kg/s], then the averaging determines $(y_1, ..., y_{35,040})$, where a single element is given by

$$y_j = \sum_{i=1}^{5} \frac{x_{5\cdot(j-1)+i}}{5}, \quad \text{with} \quad j \in \{1, ..., 35040\}.$$

In all studies we selected the subnetworks such that they contain the same network elements over time. This is not self-evident, since network extensions, maintenance, or break downs due to malfunctions of elements typically modify the topology over time.

Measures. As measures, we decided to use *absolute deviation* (AD) and *mean absolute deviation* (MAD) (Leys et al., 2013). These measures allow interpreting the results in terms of their physical units, here pressure values in [bar] and mass flow values in [kg/s]. Since operative planning of gas networks typically comprises a time horizon of about an entire day (Ehrhardt and Steinbach, 2005; Steinbach, 2007), we average both measures over the total number of considered days. Following the above described averaging scheme, a single day $d = 1, ..., 365$ consists of $96 = 4 \times 24$ time steps with a granularity of 15-minute intervals. Taking averages in fact means that we consider the mean AD and mean MAD. Besides, we report the real absolute deviation AD.

$$\text{AD:} \quad \max_{d\in\{1,...,365\}} \max_{t\in\{1,...,96\}} \left| x_{d_t}^{\text{orig}} - x_{d_t}^{\text{agg}} \right|, \tag{6.1}$$

$$\text{mean AD:} \quad \frac{1}{365} \max_{t\in\{1,...,96\}} \left| x_t^{\text{orig}} - x_t^{\text{agg}} \right|, \quad \text{and} \tag{6.2}$$

$$\text{mean MAD:} \quad \frac{1}{365} \sum_{d=1}^{365} \left(\frac{1}{96} \cdot \sum_{t=1}^{96} \left| x_t^{\text{orig}} - x_t^{\text{agg}} \right| \right). \tag{6.3}$$

Here, x_t^{orig} and x_t^{agg} are placeholders for nodal pressure and arc flow values before and after the aggregation.

6.1.4 Simulation-based evaluation of aggregation methods

Under the assumption that the historical data might not necessarily represent real physical values, and accumulating five consecutive time steps to 15-minute intervals coarsens the time granularity, we decided to evaluate the simulation results of the aggregated structures against the simulation results of the non-aggregated structures, instead of comparing against historical data. In this subsection, we describe the simulation-based evaluation approach.

For the evaluation, we use a simulator for gas transportation networks that solves a differential-algebraic-system of equations. The simulator corresponds to an extended version of the one developed in Benner et al. (2019), where the further development was carried out within the research project of the GasLab as part of the Research Campus MODAL. Customized to our purposes, it enables the use of the discretization for modeling pipes and storages presented in Sections 5.3 and 5.4. We evaluate each of the methods AGGREGATE-DISTR-PARTS and AGGREGATE-PASSIVE-TREES and MERGE-SERIAL-PIPES on six different real-world examples. As the first two methods are coarse aggregation approaches that replace entire subnetworks by storages, the question arises as to what extent the remaining network is affected by these modifications. To investigate this question, we first generate surrounding networks of the subnetworks, which we simply refer to as *surroundings* in the remainder of this thesis. Then, we run simulations in order to validate the impact of the aggregation procedures on these surroundings. Ideally, the impact is local and small. Then, only the elements would be affected that are located in the direct neighborhood of the aggregated subnetwork and the impact decreases with more distant elements. Concerning the merging of serial pipes, instead, we directly evaluate the aggregation method by comparing the simulation results at common end nodes of the serial and merged pipes.

In summary, we run 365 simulations covering an entire year, where the time horizon of a particular simulation is one day. Using the described time discretization of 15-minute intervals, each particular simulation consists of 96 time steps. The required values of each simulation run for the initial state and future demand profiles are derived from historical values, see also the previous paragraph. Due to the considered large and fine-grained time range, we can assume that the simulation results provide us with a representative picture for the impact of the considered aggregation methods on the gas transport with respect to real-world scenarios.

Surrounding networks of distribution and passive tree subnetworks. Let us consider a subnetwork $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{A}_C)$ with $\mathcal{V}_C \subseteq \mathcal{V}$, $\mathcal{A}_C \subseteq \mathcal{A}$. In order to automatically generate a surrounding $\mathcal{G}_C^{\text{surr}} = (\mathcal{V}_C^{\text{surr}}, \mathcal{A}_C^{\text{surr}})$ of $\mathcal{G}_C$ with $\mathcal{V}_C^{\text{surr}} \subseteq \mathcal{V}$, $\mathcal{A}_C^{\text{surr}} \subseteq \mathcal{A}$, we run a Breadth-First-Search (BFS) in the complement graph $\mathcal{G} \setminus \mathcal{G}_C = (\mathcal{V} \setminus \mathcal{V}_C, \mathcal{A} \setminus \mathcal{A}_C)$ up to a predefined depth level, starting from every node in $\mathcal{V} \setminus \mathcal{V}_C$ that is adjacent to a node in $\mathcal{V}_C$. In summary, all nodes and arcs visited in the BFS form the surrounding $\mathcal{G}_C^{\text{surr}}$. By construction, $\mathcal{G}_C$ and $\mathcal{G}_C^{\text{surr}}$ are node-disjoint graphs. While the sources and sinks in $\mathcal{V}_C^{\text{surr}}$ and $\mathcal{V}_C$ are naturally endowed with a historical demand profile, nodes in $\mathcal{V}_C^{\text{surr}}$ corresponding to the last level of the BFS inherit the demand profile from the historical flow values of their incident arcs that connect the surrounding $\mathcal{G}_C^{\text{surr}}$ with the remaining network $\mathcal{G} \setminus \mathcal{G}_C^{\text{surr}}$. Then, we run two kinds of simulations:

i) simulations based on the original structure and its surrounding, i.e.,
$\mathcal{G}_C \cup \mathcal{G}_C^{\text{surr}} = (\mathcal{V}_C \cup \mathcal{V}_C^{\text{surr}}, \mathcal{A}_C \cup \mathcal{A}_C^{\text{surr}})$, and

ii) simulations based on the aggregated structure and its surrounding, i.e.,
$\mathcal{G}_C^{\text{agg}} \cup \mathcal{G}_C^{\text{surr}} = (\mathcal{V}_C^{\text{agg}} \cup \mathcal{V}_C^{\text{surr}}, \mathcal{A}_C^{\text{agg}} \cup \mathcal{A}_C^{\text{surr}})$.

To validate the impact of the aggregation on $\mathcal{G}_C^{\text{surr}}$, we compare the simulation results at nodes that are located at different distances to $\mathcal{G}_C$. As distance measure, we use the depth level of the BFS. In order to verify whether the aggregation error decreases with distant elements, we consider the following node sets:

- all nodes in $\mathcal{V}_C$ ($\mathcal{V}_C^{\text{agg}}$ resp.) linked to $\mathcal{V}_C^{\text{surr}}$;
- all nodes in $\mathcal{V}_C^{\text{surr}}$ linked to $\mathcal{V}_C$ ($\mathcal{V}_C^{\text{agg}}$ resp.) (so-called BFS *level 0 nodes*);
- all nodes in $\mathcal{V}_C^{\text{surr}}$;
- all nodes in $\mathcal{V}_C^{\text{surr}}$ that correspond to the last level nodes of the BFS.

For each of these node sets, we compare the resulting pressure values of both simulations (i) and (ii) with respect to Measures (6.1) – (6.3), where we set

$$\left|x_t^{\text{orig}} - x_t^{\text{agg}}\right| := \sum_v \left|p_{v,t}^{\text{orig}} - p_{v,t}^{\text{agg}}\right|.$$

We remark that for non-aggregated and aggregated subnetworks most likely holds $\mathcal{V}_C \neq \mathcal{V}_C^{\text{agg}}$, even though both have the same surrounding $\mathcal{G}_C^{\text{surr}}$. Hence, the comparison of pressure values between nodes in $\mathcal{V}_C$ and $\mathcal{V}_C^{\text{agg}}$ might not directly be possible. In this case we will use the maximal pressure difference between appropriate nodes instead. For more information, we refer to Section 6.3.7.

To compare the difference of inflows and outflows of subnetwork $\mathcal{G}_C$, we apply Measures (6.1) – (6.3) to the accumulated arc flow between nodes in $\mathcal{V}_C$ and $\mathcal{V}_C^{\text{surr}}$

$$x_t^{\text{orig}} := \sum_{\substack{a=(\ell,r)\in\mathcal{A}\setminus\mathcal{A}_{pi}\\ \ell\in\mathcal{V}_C,\\ r\notin\mathcal{V}_C}} q_{a,t}^{\text{orig}} + \sum_{\substack{a=(\ell,r)\in\mathcal{A}_{pi}\\ \ell\in\mathcal{V}_C,\\ r\notin\mathcal{V}_C}} q_{\ell,t}^{\text{orig}} - \sum_{\substack{a=(\ell,r)\in\mathcal{A}\setminus\mathcal{A}_{pi}\\ \ell\notin\mathcal{V}_C,\\ r\in\mathcal{V}_C}} q_{a,t}^{\text{orig}} - \sum_{\substack{a=(\ell,r)\in\mathcal{A}_{pi}\\ \ell\notin\mathcal{V}_C\\ r\in\mathcal{V}_C}} q_{r,t}^{\text{orig}}, \quad (6.4)$$

and the accumulated arc flow between nodes in $\mathcal{V}_C^{\text{agg}}$ and $\mathcal{V}_C^{\text{surr}}$

$$x_t^{\text{agg}} := \sum_{\substack{a=(\ell,r)\in\mathcal{A}\setminus\mathcal{A}_{pi}\\ \ell\in\mathcal{V}_C^{\text{agg}},\\ r\notin\mathcal{V}_C^{\text{agg}}}} q_{a,t}^{\text{agg}} + \sum_{\substack{a=(\ell,r)\in\mathcal{A}_{pi}\\ \ell\in\mathcal{V}_C^{\text{agg}},\\ r\notin\mathcal{V}_C^{\text{agg}}}} q_{\ell,t}^{\text{agg}} - \sum_{\substack{a=(\ell,r)\in\mathcal{A}\setminus\mathcal{A}_{pi}\\ \ell\notin\mathcal{V}_C^{\text{agg}},\\ r\in\mathcal{V}_C^{\text{agg}}}} q_{a,t}^{\text{agg}} - \sum_{\substack{a=(\ell,r)\in\mathcal{A}_{pi}\\ \ell\notin\mathcal{V}_C^{\text{agg}},\\ r\in\mathcal{V}_C^{\text{agg}}}} q_{r,t}^{\text{agg}}. \quad (6.5)$$

Further information on the applied measures can be found in the respective sections.

6.1.5 Outline

In the remainder of this chapter, we present the deployed aggregation methods in more detail, starting with preprocessing methods, such as removing *non-remote controllable elements* and removing *disconnected subnetworks without boundary nodes* in Section 6.2. In Sections 6.3 and 6.4, we introduce procedures for the aggregation of *distribution* and *passive tree subnetworks*. In Section 6.5, we discuss the *aggregation of pipes with short lengths*, followed by merging *parallel and serial pipes* in Sections 6.6 and 6.7 and *active elements* in Section 6.8. Finally, we provide a theoretical excursion to determining pressure bounds at aggregated nodes from a geometrical point of view in Section 6.9.

6.2 Aggregation Preprocessing

In this section, we present two aggregation methods that can be thought of as preprocessing steps. They eliminate network structures, which are not under the direct control of dispatchers at the present time of the decision-making.

6.2.1 Removal of non-remote controllable elements

Large-scale gas networks typically contain a large number of valves. Valves increase the option to route gas and to decouple different zones and their pressure levels from each other. In practice, two different types of valves can be distinguished: *non-remote controllable* and *remote controllable*. Non-remote controllable valves require being manually changed on-site by a human operator. Valves of this type most frequently serve as backup valves that enable alternative operations when certain elements or parts of the network are under construction or maintenance. Hence, the operation mode of non-remote controllable valves is rather changed once in a while, whereas valves with remote access can be controlled directly from the dispatcher. For this reason, we only account for remote controllable valves in the *Transient Control Problem* and fix non-remote controllable valves to their current state at the initial point in time of the optimization run, which is either closed or open. As a consequence, we remove all closed non-remote controllable valves and shrink open non-remote controllable valves $a = (v, w) \in \mathcal{A}_{va}$ by redirecting their incident arcs from, for example, node w to node v and finally removing node w and arc a.

Remark 6.2.1 (Shrinking non-remote controllable elements)**.** *Let $a = (\ell, r) \in \mathcal{A}_{va}$ be a non-remote controllable valve. Then, we contract valve a by combining nodes ℓ and r. W.l.o.g., let node r be retained. Then node r inherits the demand of the removed node ℓ, i.e., $b_{r,t} \leftarrow b_{\ell,t} + b_{r,t}$ for all $t \in \mathcal{T}$, and all incident arcs of node ℓ are relocated to node r.*

In general, gas networks might also contain non-remote controllable regulators, see e.g., Fügenschuh et al. (2015). Even though, these elements are not under direct control of the dispatchers, non-remote controllable regulators change their modes automatically, depending on the particular realization of the pressures. This means that non-remote controllable regulators can change their state from open to close and vice versa during the considered time horizon. Thus non-remote controllable regulators exhibit a dynamic behavior over time. For this reason, we exclude all non-remote controllable regulators from the aggregation process and treat them as remote-controllable elements.

6.2.2 Removal of disconnected components without boundary nodes

Removing non-remote controllable elements, as described in the previous section, is the only aggregation method under consideration that might decompose the network into several disconnected components. Here, we remove all disconnected components that do not contain at least one source and one sink, as required for their self-supply. Let us mention that this procedure might not necessarily re-establish a single connected graph. Even though for the data under consideration, this is throughout the case.

6.3 Aggregation of Distribution Subnetworks

When it comes to decisions about gas transport in large-scale networks, the transportation part, also called transmission part, typically forms the *backbone network*. By contrast, the distribution part represents, for example, regional areas, where gas is delivered to end-consumers. For this reason, distribution subnetworks tend to feature certain characteristics, such as comprising of pipes with smaller diameters or representing lower pressure zones. In line with the first aspect, Carvalho et al. (2009) classify distribution areas of the European gas network utilizing the pipes' diameter sizes. However, such characteristics might not appropriately identify distribution subnetworks, because depending on the particular network, the transportation part may also contain pipelines with small diameters and may additionally be operated at low-pressure levels. Even upper-pressure bounds may not allow for a recognition of distribution subnetworks. In fact, we encountered in several research projects that the quality of data delivered by industrial partners is frequently not appropriate. For example, upper-pressure bounds are often set to unreasonably high, and thus unusable, default values. Hence, such features might not be adequate for an automated detection and aggregation of distribution subnetworks. While in practice, backbone networks are generally created manually by engineers who apply expert knowledge about the particular topology, we present a generic approach that first splits the network into transport and distribution subnetworks and then substitutes the latter ones for storage nodes.

The remainder of this section is organized as follows. In Subsection 6.3.1, we first present a definition of *distribution subnetworks* and then based on this definition an approach to detecting transmission and distribution subnetworks in Subsection 6.3.2. In Subsection 6.3.3, we study the behavior of line-pack in six distribution subnetworks that result from running this detection algorithm on real-world data. The study suggests that distribution subnetworks hardly serve as gas storages, which encourages one to develop a coarse aggregation approach, presented in Subsection 6.3.4. Aggregating and substituting elements requires equipping the new nodes with pressure bounds and initial pressure values. In particular the task of determining appropriate pressure values at the initial point in time is strongly related to the task of retaining the stored amount of gas (line-pack) in the aggregated structure, see Subsection 6.3.5. In Subsection 6.3.6, we calculate pressure bounds at the storage node, which aim to be sufficiently tight, so that the realized pressure values would entail feasible operations inside the non-aggregated distribution subnetwork, even in the case of possible high pressure losses. Finally, we evaluate our aggregation procedure in Subsection 6.3.7 by using simulations.

6.3.1 Formal description of distribution subnetworks

In order to mathematically describe distribution subnetworks, we denote the set of all regulating arcs located inside and outside of stations by $\mathcal{A}_{rg} := \mathcal{A}_{cv} \dot\cup_{s\in\mathcal{S}} \mathcal{A}_{rg,s}$. Further, we partition the set of arcs $\mathcal{A} = \mathcal{A}^{\text{dir}} \dot\cup \mathcal{A}^{\text{undir}}$ into a set of directed and undirected arcs. The set of directed arcs $\mathcal{A}^{\text{dir}} = \mathcal{A}_{rg}$ contains all regulating arcs and the set of undirected arcs $\mathcal{A}^{\text{undir}} = \mathcal{A}_{pi} \dot\cup \mathcal{A}_{sc} \dot\cup \mathcal{A}_{va} \dot\cup_{s\in\mathcal{S}} \mathcal{A}_{va,s} \dot\cup_{s\in\mathcal{S}} \mathcal{A}_{cs,s}$ contains the remaining arcs. Graphs that contain both directed and undirected arcs simultaneously are called *mixed graphs*, see for example Beck et al. (2015). Let us briefly mention that *mixed graphs* appear in several applications, for example, in job-scheduling, where undirected arcs are used to model incompatible jobs, and directed arcs to model precedence of certain jobs, see e.g., Sotskov et al. (2001).

Here, we use the notation of directed and undirected arc sets to formalize *distribution subnetworks*. The Definition 6.3.1 is motivated by the rationale that pressure cannot be reduced from distribution to transmission subnetworks (by using regulators) nor gas compressed from transmission to distribution subnetworks (by using compressors). To this end, requirement (i) in the definition guarantees that only regulating arcs connect a distribution subnetwork with the remaining network. Requirement (ii)(a) prevents gas compression along any path starting from a distribution subnetwork node. Moreover, none of these paths is allowed to contain a source node, established by requirement (ii)(b). In practice, sources can be considered as part of the transport network, as they allow drawing in gas at higher pressure values and thereby enabling gas transport over long distances without the need for compression.

Definition 6.3.1 (Distribution subnetwork)**.** *Let $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ and $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{A}_C)$ with $\mathcal{V}_C \subseteq \mathcal{V}$ and $\mathcal{A}_C \subseteq \mathcal{A}$ be given, where $\mathcal{A} = \mathcal{A}^{dir} \,\dot\cup\, \mathcal{A}^{undir}$ is split into a directed and undirected arc set*

$$\begin{aligned} \mathcal{A}^{dir} &= \mathcal{A}_{rg}, \text{ and} \\ \mathcal{A}^{undir} &= \mathcal{A}_{pi} \,\dot\cup\, \mathcal{A}_{sc} \,\dot\cup\, \mathcal{A}_{va} \,\dot\cup_{s\in\mathcal{S}} \mathcal{A}_{va,s} \,\dot\cup_{s\in\mathcal{S}} \mathcal{A}_{cs,s}. \end{aligned}$$

We call $\mathcal{G}_C$ a distribution subnetwork of $\mathcal{G}$, if

i) for all $a = (v, w) \in \mathcal{A}$ and for all $a = (w, v) \in \mathcal{A}$ with $v \in \mathcal{V}_C$ and $w \in \mathcal{V} \setminus \mathcal{V}_C$ follows $a \in \mathcal{A}^{dir}$, and

ii) all paths $(v_0, v_1, ..., v_n)$ with $v_0 \in \mathcal{V}_C$ satisfy

(a) $(v_{i-1}, v_i) \in \mathcal{A} \setminus \dot\cup_{s\in\mathcal{S}} \mathcal{A}_{cs,s}$ for all $i \in \{1, ..., n\}$, and

(b) $v_i \in \mathcal{V} \setminus \mathcal{V}^+$ for all $i \in \{0, ..., n\}$.

6.3.2 Detection of transmission and distribution subnetworks

In the following, we explain in more detail how to split the network into transmission and distribution parts. At first, we decompose the network into several disjoint connected components. In line with Definition 6.3.1, this decomposition is accomplished alongside regulators $\mathcal{A}_{rg}$, since they potentially reduce the pressure from transmission to distribution components. For the decomposition, we consider the graph whose nodes and arcs are given by $\mathcal{V}$ and $\mathcal{A}\setminus\mathcal{A}_{rg}$ and denote its k-many connected components by $\mathcal{C} := \{C_1, ..., C_k\}$ with corresponding sub-graphs $\mathcal{G}_1 = (\mathcal{V}_1, \mathcal{A}_1), \ldots, \mathcal{G}_k = (\mathcal{V}_k, \mathcal{A}_k)$. In order to assign each component to the labels *distribution* or *transmission*, we build a simplified, but related graph $\mathcal{G}^{\text{cg}} := (\mathcal{V}^{\text{cg}}, \mathcal{A}^{\text{cg}})$, which we call *components graph* of $\mathcal{G}$: For each component $C_i \in \mathcal{C}$ we introduce a representative node c_i and set $\mathcal{V}^{\text{cg}} := (c_1, ..., c_k)$, while $\mathcal{A}^{\text{cg}}$ contains the (possibly multiple) connecting regulating arcs between the different components of $\mathcal{C}$, i.e.,

$$\mathcal{A}^{\mathrm{cg}} := \{(c_i, c_j) \mid \exists\, v \in \mathcal{V}_i, w \in \mathcal{V}_j : (v, w) \in \mathcal{A}_{rg},\ C_i, C_j \in \mathcal{C},\ i \neq j, \text{ and } c_i, c_j \in \mathcal{V}^{\mathrm{cg}}\}.$$

Algorithm 2 operates on $\mathcal{G}^{\mathrm{cg}}$ and uniquely assigns each $C \in \mathcal{C}$ to the label *distribution* or *transmission*. W.l.o.g., we assume that $\mathcal{V}^{\text{cg-tn}} := \{c_1, ..., c_r\} \subseteq \mathcal{V}^{\mathrm{cg}}$ with $r \leq k$ represent components that contain source nodes or compressing arcs. Right from the beginning, these components belong to the transport network due to conditions (ii)(a) and (ii)(b) in Definition 6.3.1. Here, the superscript cg-tn denotes *components graph - transport network*. In order to detect the remaining transport components, we reverse the direction of the regulators in $\mathcal{A}^{\mathrm{cg}}$, called $\mathcal{A}^{\mathrm{cg}}_{\leftarrow}$, and traverse the graph $\mathcal{G}^{\mathrm{cg}}_{\leftarrow} = (\mathcal{V}^{\mathrm{cg}}, \mathcal{A}^{\mathrm{cg}}_{\leftarrow})$ by a Breadth First Search (BFS) starting from all labeled nodes in $\mathcal{V}^{\text{cg-tn}}$. All components whose corresponding nodes in $\mathcal{V}^{\mathrm{cg}}$ can be reached by BFS are labeled as *transmission* components. Finally, the remaining unlabeled components form the distribution subnetworks. By construction, Algorithm 2 determines distribution subnetworks that are well-defined according to Definition 6.3.1.

Algorithm 2 Detection of transmission and distribution components

```
Input: G = (V, A)
Output: Transmission and distribution components of G
Decompose G along the regulating arcs A_rg
Generate components graph G^cg = (V^cg, A^cg)
Reverse direction of the regulating arcs, i.e., G^cg_← = (V^cg, A^cg_←)
Build V^cg-tn ⊆ V^cg
Label components represented by corresponding nodes in V^cg-tn ← Transmission
Run BFS in G^cg_← starting from all nodes in V^cg-tn
for each v ∈ V^cg do
    if node v is reached in BFS then
        Label component represented by node v ∈ V^cg ← Transmission
    end if
end for
Label components whose nodes v ∈ V^cg are not visited in BFS ← Distribution
```

While the algorithm classifies network elements to be either part of the transmission or distribution components, regulators remain basically unassigned. In general, a regulator could be considered as part of a distribution subnetwork, if there exists an alternative path between its incident nodes and all arcs of this path are in the distribution component. However, it is unlikely that distribution subnetworks contain regulating arcs (including both incident nodes), given that the network decomposition is done along regulators in order to guarantee the required pressure drop from transmission to distribution zones, or even between different distribution subnetworks.

6.3.3 The role of line-pack in distribution subnetworks

In gas transmission networks, pipelines primarily serve to transport gas between sources and sinks. However, due to the compressible nature of gas, they can also be used as storages. This process is called line-pack and plays an important role in the daily and mid-term operation of gas networks, especially in periods of volatile demands. For example, in the case of sudden high withdrawals, line-pack can be used as safety stock. For this reason, keeping sufficient storage levels in the network is crucial for the overall security of supply, in particular to hedge against unpredictable events, such as gas leakages. A typical pattern for line-packing of pipeline systems corresponds to "withdrawing larger amounts of gas from the system during periods of high demand and injecting more gas into the pipelines during off peak times" (Ríos-Mercado and Borraz-Sánchez, 2015). The optimization of this process is called *line-packing problem*. It has given rise to numerous approaches in the literature on maximizing storage capacity (Carter and Rachford Jr, 2003; Krishnaswami et al., 2004; Frimannslund and Haugland, 2008; Zavala, 2014; Burlacu et al., 2019; Kazda et al., 2020). Technically, the packing process is realized, e.g., by increasing pressure through upstream compressors and closing downstream (control) valves.

Network structures containing compressors, which allow realizing this packing process, by concept belong to the transportation part, see Algorithm 2. Furthermore, the transportation part comprises all network stations including their control elements that are significant for the network control. Hence, in practical applications the transportation part frequently forms the so-called *backbone* of the network, as explained in Section 6.1. By contrast, the distribution part is considered as less important for the decision-making, which might rely on the assumption that it is only used to a small extent as a network buffer compared to the transmission part. In the following, we investigate this assumption by studying the real-world behavior of line-pack in distribution subnetworks. In particular, we investigate whether distribution subnetworks are used as storages.

First, we introduce some notation. Let $\mathcal{V}_C^{in} \subseteq \mathcal{V}_C$ and $\mathcal{V}_C^{out} \subseteq \mathcal{V}_C$ be the node sets, where gas possibly enters and leaves a distribution subnetwork C through upstream regulators $\mathcal{A}_C^{in}$ and downstream regulators $\mathcal{A}_C^{out}$ respectively:

$$\begin{aligned}
\mathcal{V}_C^{in} &:= \{v \in V_C \mid \exists\, a = (w,v) \in \mathcal{A}_{rg} : w \notin \mathcal{V}_C,\, v \in \mathcal{V}_C\};\\
\mathcal{V}_C^{out} &:= \{v \in \mathcal{V}_C \mid \exists\, a = (v,w) \in \mathcal{A}_{rg} : v \in \mathcal{V}_C;\, w \notin \mathcal{V}_C\};\\
\mathcal{A}_C^{in} &:= \{a = (v,w) \in \mathcal{A}_{rg} \mid v \notin \mathcal{V}_C;\, w \in \mathcal{V}_C^{in}\};\\
\mathcal{A}_C^{out} &:= \{a = (v,w) \in \mathcal{A}_{rg} \mid v \in \mathcal{V}_C^{out};\; w \notin \mathcal{V}_C\};
\end{aligned}$$

and let $\mathcal{V}_C^- := \{v \in \mathcal{V}_C \mid v \in \mathcal{V}^-\}$ be the set of exit nodes in C.

In order to investigate whether line-pack is used in distribution subnetworks, we consider six distribution subnetworks $\{D_1, \ldots, D_6\}$ that result from the application of Algorithm 2 on a real-world network. The selected distribution subnetworks feature different characteristics, varying in the number of elements and regulators that are linked to the remaining network, see Table 6.2. We study the line-pack as a difference of gas flow that enters and leaves a component over time. Let this difference be described by

	Distr. subnetworks			Connections		Surrounding of distr. subnetworks			
	$\|\mathcal{V}\|$	$\|\mathcal{V}^-\|$	$\|\mathcal{A}\|$	$\|\mathcal{A}^{in}\|$	$\|\mathcal{A}^{out}\|$	$\|\mathcal{V}^{\text{surr}}\|$	$\|\mathcal{V}^{\pm\text{surr}}\|$	$\|\mathcal{A}^{\text{surr}}\|$	# Last level nodes
D_1	119	22	125	2	0	82	15	85	4
D_2	31	8	35	1	0	42	5	44	3
D_3	15	5	18	1	0	63	3	66	5
D_4	4	2	3	1	0	20	2	20	3
D_5	18	4	22	1	0	85	12	86	3
D_6	5	2	8	1	0	81	10	81	6

Table 6.2: Number of elements of distribution subnetworks $D_1, ..., D_6$ and their surroundings used in the computational experiments. The column *connections* describes the number of ingoing and outgoing regulators that link a distribution subnetwork with its surrounding. Here, the surroundings are generated with a BFS algorithm up to a depth level of 15. The *last level nodes* correspond to those nodes of the surroundings that are incident to the remaining network (i.e., nodes of depth level 15 in the BFS). Finally, $|\mathcal{V}^{\pm\text{surr}}| = |\mathcal{V}^{+\text{surr}} \cup \mathcal{V}^{-\text{surr}}|$ denotes the number of sources and sinks contained in the surrounding network.

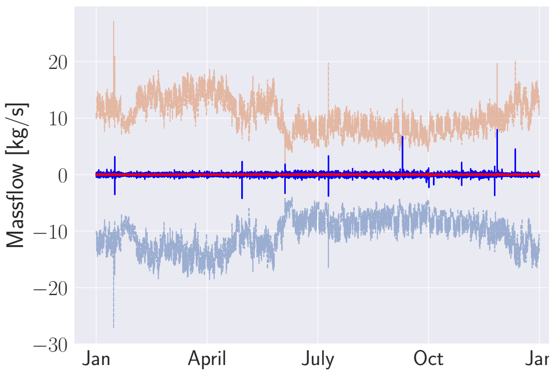
function $f\colon \mathcal{T} \to \mathbb{R}$ with

$$t \mapsto \sum_{\mathcal{A}_C^{in}} q_{a,t} - \sum_{\mathcal{A}_C^{out}} q_{a,t} + \sum_{v \in \mathcal{V}_C^+} b_{v,t} - \sum_{v \in \mathcal{V}_C^-} |b_{v,t}|.$$

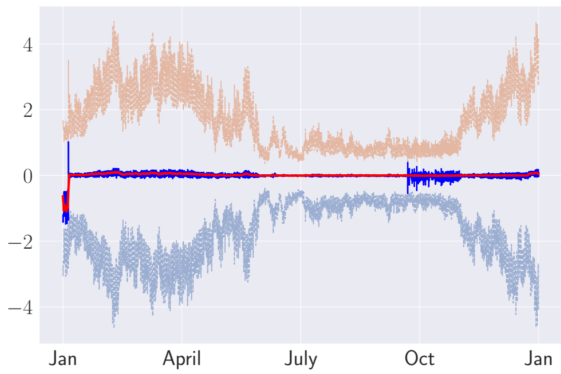
Here, the considered distribution subnetwork examples $C \in \{D_1, \ldots, D_6\}$ have no outgoing regulators and by construction no sources, i.e., $\mathcal{A}_C^{out} = \mathcal{V}_C^+ = \emptyset$. Thus, function f reduces to $t \mapsto \sum_{a \in \mathcal{A}_C^{in}} q_{a,t} - \sum_{v \in \mathcal{V}_C^-} |b_{v,t}|$. Note that $v \in \mathcal{V}_C^-$ implies $b_{v,t} \leq 0$.

In summary, we consider 365 consecutive days following the description in Section 6.1. To focus on the change of gas volume over time and to reduce the impact of short gas flow peaks, we additionally use a convolution (also called running mean). The convolution moves a window along the input function f and thereby smooths the difference of inflow and outflow values using an arithmetic mean. As we consider daily operations of gas networks, we select the length of the moving window such that it represents an entire day yielding $n = 96$ time steps with 15 minute time intervals. We use the implementation of the convolution method *convolve* available in the `numpy` package (Oliphant, 2006).

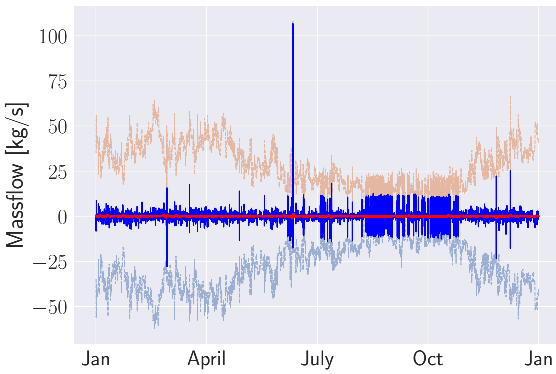
It can be seen in Figure 6.1 that the inflows (light orange) and outflows (light blue) of all components are symmetric with respect to the zero flow axis. Besides that, their differences are approximately zero, in particular for D_2, D_4, D_5, and D_6. This means that inflows are more or less consumed instantly yielding a balanced line-pack behavior over an entire day. Table D.1 in the appendix summarizes the numerical results, as shown in Figure 6.1. These values confirm that the convolution (red) smooths peaks, where function f (dark blue) has high absolute deviations. By way of explanation, see for example D_3, where the absolute deviation AD = 107 [kg/s] (function f) is reduced to AD = 1.3 [kg/s] (convolution). Hence, higher inflows are immediately adjusted in the



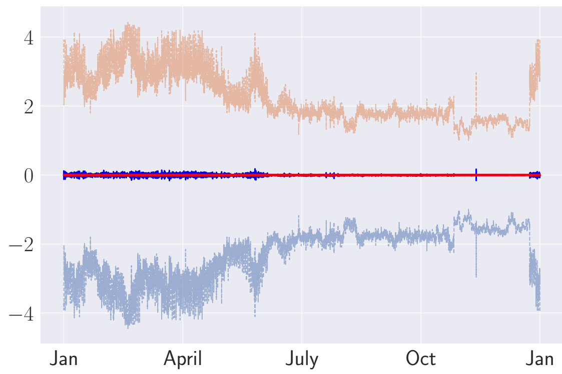
(a) Distribution subnetwork D_1



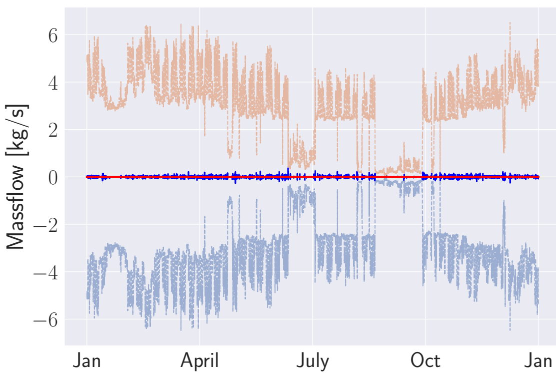
(b) Distribution subnetwork D_2



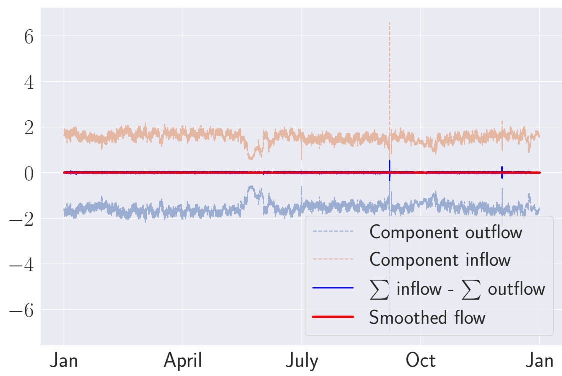
(c) Distribution subnetwork D_3



(d) Distribution subnetwork D_4



(e) Distribution subnetwork D_5



(f) Distribution subnetwork D_6

Figure 6.1: Inflow and outflow values of six distribution subnetworks $D_1, ..., D_6$ over an entire year with a time resolution of 15-minutes. The inflow (light orange) and the outflow values (light blue) of a subnetwork, and their difference (dark blue), as well as the smoothed values (red) are given as mass flow values in [kg/s] (y-axis) per time step (x-axis). Positive values correspond to more inflow into the subnetwork than outflow (exit consumption), and negative values vice versa.

subsequent time steps through higher outflow values. Moreover, both the real difference of inflows and outflows as given by function f and the convolution, are very small in the averaged measures *mean AD* and *mean MAD*, which confirms that the daily difference of the subnetworks' inflow and outflow is fairly constant over the days. A second study on whether distribution subnetworks at all provide the possibility to store gas can be found in the Appendix D.1.2.

In summary, we can state that the demand is consumed rather instantly within the distribution components, and thus, they are hardly used as storage in practice. This analysis suggests that line-pack in distribution subnetworks might, in general, play a minor role in daily operations.

6.3.4 Aggregation approach

Our study in the previous section indicates that line-packing in distribution subnetworks plays a negligible role in intraday operations, which suggests that the impact of distribution parts on the overall network control is rather small. This gives rise to the use of a coarse aggregation procedure. We decided to substitute entire distribution subnetworks for storages nodes, which allow absorbing the imbalances between their inflows and outflows.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ be a connected graph, where $\mathcal{C}^{dc}$ denotes the set of distribution components with corresponding subgraphs $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{A}_C)$ for $C \in \mathcal{C}^{dc}$ that result from Algorithm 2. We aggregate each $C \in \mathcal{C}^{dc}$ by (i) removing $\mathcal{V}_C$ and $\mathcal{A}_C$, and (ii) introducing the storage node $v_C \in \mathcal{V}_{\text{sto}}$. So, an aggregated distribution subnetwork reads $\mathcal{G}_C^{\text{agg}} = (\mathcal{V}_C^{\text{agg}}, \mathcal{A}_C^{\text{agg}})$ with $\mathcal{V}_C^{\text{agg}} = \{v_c\}$ and $\mathcal{A}_C^{\text{agg}} = \emptyset$. In addition, we redirect all ingoing and outgoing regulators that link $\mathcal{G}_C$ with the remaining network to node v_C. In the case that a distribution subnetwork contains exit nodes, node v_C inherits their accumulated demand

$$b_{v_C,t} := \sum_{v \in \mathcal{V}_C^-} b_{v,t} \quad \forall t \in \mathcal{T}.$$

In order to enable storing roughly the same amount of gas as within the original subnetwork, we equip the storage node with the pipe volume of C i.e.,

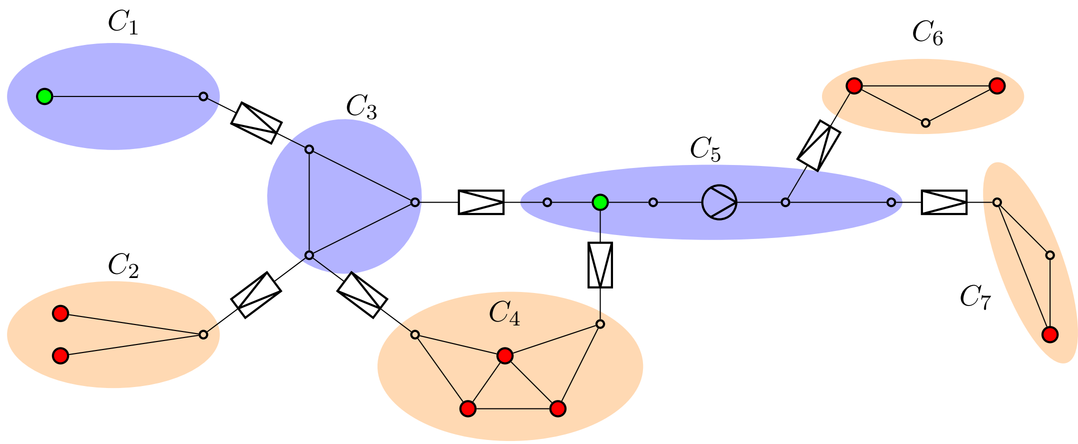
$$vol_{v_C} := vol_C, \tag{6.6}$$

where $vol_C := \sum_{a \in \mathcal{A}_C \cap \mathcal{A}_{pi}} vol_a$.

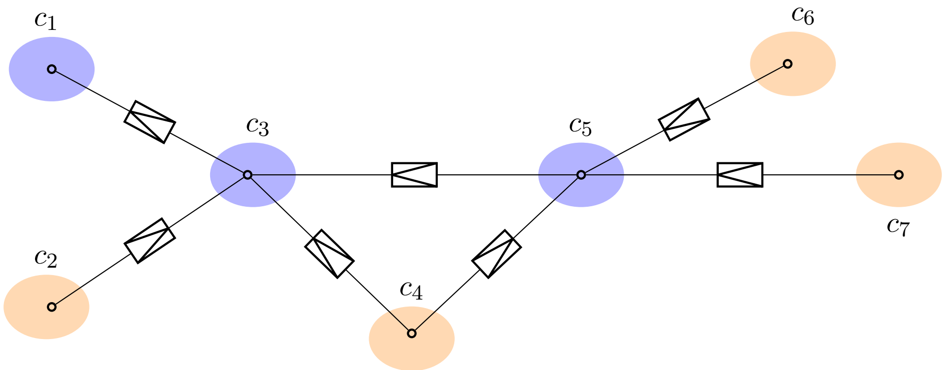
Figures 6.2a and 6.2c illustrate a network before and after the aggregation of the distribution subnetworks, while Figure 6.2b shows the corresponding *components graph* that is used for the detection of the distribution and transportation part in Algorithm 2.

6.3.5 Representing the initial amount of gas

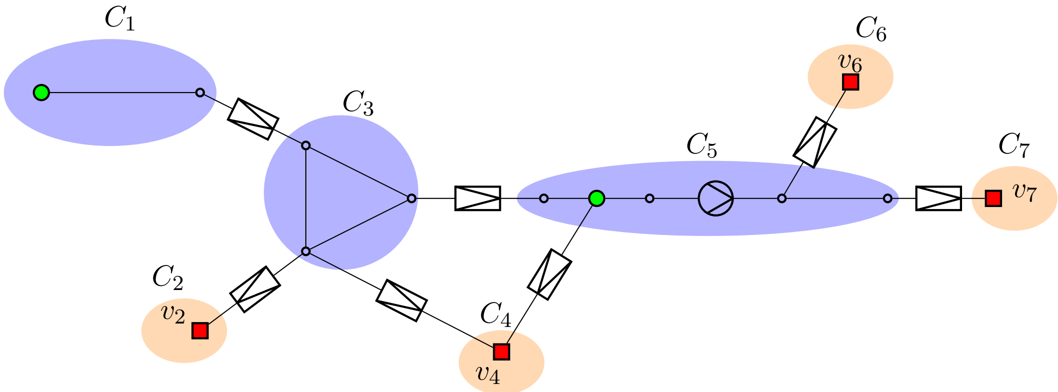
Substituting distribution subnetworks for storage nodes requires equipping the new nodes with initial pressure values. We determine these values such that the amount of line-pack from the original subnetworks is retained. Line-pack describes the amount of gas in a



(a) Algorithm 2 decomposes $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ along $\mathcal{A}_{rg}$ ( ) into distribution components C_2, C_4, C_6, C_7 (light orange) and transportation components C_1, C_3, C_5 (blue). Source and exit nodes are highlighted in green and red respectively. Here, the distribution components are connected with the remaining network by one or two ingoing regulating arcs, i.e., $|\mathcal{A}^{in}_{C_2}| = |\mathcal{A}^{in}_{C_6}| = |\mathcal{A}^{in}_{C_7}| = 1$, $|\mathcal{A}^{in}_{C_4}| = 2$ and no outgoing control valves, i.e., $|\mathcal{A}^{out}_{C_2}| = |\mathcal{A}^{out}_{C_4}| = |\mathcal{A}^{out}_{C_6}| = |\mathcal{A}^{out}_{C_7}| = 0$.



(b) Components graph with reversed regulating arcs $\mathcal{G}^{\text{cg}}_{\leftarrow} = (\mathcal{V}^{\text{cg}}, \mathcal{A}^{\text{cg}}_{\leftarrow})$, as used in Algorithm 2. The components C_1, C_5 are by default part of the transport network, i.e., $V^{\text{cg-tn}} = \{c_1, c_5\}$, since they contain source nodes and component C_5 additionally comprises a compressor arc ( ).



(c) Network after the aggregation of the distribution components. Each aggregated distribution component $C \in \mathcal{C}^{dc} = \{C_2, C_4, C_6, C_7\}$ consists of a storage node v_C (squared).

Figure 6.2: Detection and aggregation of distribution subnetworks

system at a given point in time, which we specify as mass denoted in $[kg]$. Let us illustrate the line-pack of a single pipeline $a = (\ell, r)$ at $t \in \mathcal{T}_0$. Starting with given pressure values $p_{\ell,t}$ and $p_{r,t}$, we first approximate a mean pressure $p_{a,t}^{mean}$ along the pipeline a using a closed-form expression from the stationary case, cf. Koch et al. (2015) or Saleh (2002):

$$p_{a,t}^{mean} = \frac{2}{3}\left(p_{\ell,t} + p_{r,t} - \frac{p_{\ell,t}\, p_{r,t}}{p_{\ell,t} + p_{r,t}}\right). \tag{6.7}$$

From the equation of state $p = \mathrm{c_s}^2\, \rho$, we then derive the mean density:

$$\rho_a^{mean} = p_a^{mean} / \,\mathrm{c}_\mathrm{s}^2,$$

which allows to describe the line-pack $LP_{a,t}$ of a single pipe in terms of the density $\rho_{a,t}^{mean}$ and pipe volume vol_a:

$$LP_{a,t} = \rho_{a,t}^{mean}\, vol_a = \frac{p_{a,t}^{mean}}{\mathrm{c_s}^2}\, vol_a. \tag{6.8}$$

With these preliminaries, it is possible to determine the amount of gas in a distribution subnetwork $C \in \mathcal{C}^{dc}$, which we state as the sum of line-pack values of all its pipes:

$$LP_{C,t} := \sum_{a \in \mathcal{A}_C \cap \mathcal{A}_{pi}} LP_{a,t}. \tag{6.9}$$

Similarly, we state the line-pack in a storage node v_C as

$$LP_{v_C,t} := \frac{p_{v_C,t}}{\mathrm{c_s}^2}\, vol_{v_C}. \tag{6.10}$$

In order to retain the given amount of mass as before the aggregation of C, we require that the line-pack of the storage node v_C holds

$$LP_{v_C,t} = LP_{C,t}.$$

By combining Equations (6.8) – (6.10), it is possible to determine an initial pressure value p_{v_C,t_0} at the storage node and time step t_0:

$$\begin{aligned}
& & LP_{v_C,t_0} &= LP_{C,t_0} \\
&\overset{(6.8),(6.9),(6.10)}{\Rightarrow} & \frac{p_{v_C,t_0}}{\mathrm{c_s}^2}\, vol_{v_C} &= \sum_{a \in \mathcal{A}_C \cap \mathcal{A}_{pi}} \frac{p_{a,t_0}^{mean}}{\mathrm{c_s}^2} vol_a \\
&\overset{(6.6)}{\Rightarrow} & p_{v_C,t_0} \sum_{a \in \mathcal{A}_C \cap \mathcal{A}_{pi}} vol_a &= \sum_{a \in \mathcal{A}_C \cap \mathcal{A}_{pi}} p_{a,t_0}^{mean} vol_a \\
&\Rightarrow & p_{v_C,t_0} &= \frac{\sum_{a \in \mathcal{A}_C \cap \mathcal{A}_{pi}} p_{a,t_0}^{mean}\, vol_a}{\sum_{a \in \mathcal{A}_C \cap \mathcal{A}_{pi}} vol_a}.
\end{aligned}$$

6.3.6 Determining pressure bounds at the storage node

A concern about distribution subnetworks is to meet the pressure limits at their exit nodes. In practice, upstream control valves are installed that enable satisfying their upper bounds. On the other hand, to satisfy their lower bounds even in the case of high-pressure losses, the pressure values at the connection nodes to the remaining network are required to be high enough. However, pressure losses inside the non-aggregated subnetwork $\mathcal{G}_C$ cannot be represented within the aggregated structure $\mathcal{G}_C^{\text{agg}}$, as it consists of a single storage node. Therefore, we equip the lower pressure bound of the storage node with a safety margin. Ideally, this safety margin covers the maximal possible pressure loss inside $\mathcal{G}_C$. Consequently, when fulfilling this pressure bound, it should be possible to satisfy all pressure restrictions at the nodes of the non-aggregated subnetwork.

In this section, we approximate the maximal pressure loss inside the non-aggregated distribution subnetwork by solving an auxiliary optimization model, which we call *Max Pressure Loss Problem*. The resulting objective value corresponds to the safety margin. Let us mention that our analysis about line-pack reveals that pressure levels in distribution subnetworks are hardly volatile, cf. Figure D.1 in the appendix, which shifts the need to determine such bounds to very few events of more volatile pressure profiles.

A possible approach to determine the maximal pressure loss could be to solve a variant of the *Transient Control Problem* restricted to the distribution subnetwork under consideration. Nevertheless the possible size of its topology can lead to a large and highly nonlinear optimization model that is most likely intractable using current state-of-the-art solvers. However, to be prepared for such rare events, we decided to restrict our approximation to stationary modeling and apply the knowledge and machinery from the stationary case, see e.g., Koch et al. (2015). As stationary models require balanced demand scenarios, we enforce

$$\sum_{a \in \mathcal{A}_C^{in}} q_{a,t} = \sum_{a \in \mathcal{A}_C^{out}} q_{a,t} + \sum_{v \in \mathcal{V}_C^-} |b_{v,t}|, \tag{6.11}$$

where the demand at the exits $\mathcal{V}_C^-$ is known. In fact, we will circumvent the need to derive flow values for every arc in $\mathcal{A}_C^{in}$ and $\mathcal{A}_C^{out}$ and deal with accumulated inflow and outflow values of a distribution subnetwork.

In general, $(|\mathcal{C}^{dc}| \times |\mathcal{T}|)$-many stationary optimization problems would need to be solved in order to determine the maximal pressure loss for all $C \in \mathcal{C}^{dc}$ and $t \in \mathcal{T}$ separately. However, we will solve exactly one problem for each subnetwork $C \in \mathcal{C}^{dc}$ by exploiting the fact that the maximal pressure loss Δ_C is most likely to occur at that particular point in time $\hat{t}_C \in \mathcal{T}$ with the highest demand, i.e.,

$$\hat{t}_C := \arg\max_{t \in \mathcal{T}} \left(\sum_{a \in \mathcal{A}_C^{out}} q_{a,t} + \sum_{v \in \mathcal{V}_C^-} |b_{v,t}| \right).$$

Here, taking the highest demand is motivated by the fact that pressure losses along pipelines are proportional to the squared amount of flows, see e.g., the Weymouth Equation (6.16b), and the higher the demand, the higher the flow. Note that $\hat{t}_C$ cannot

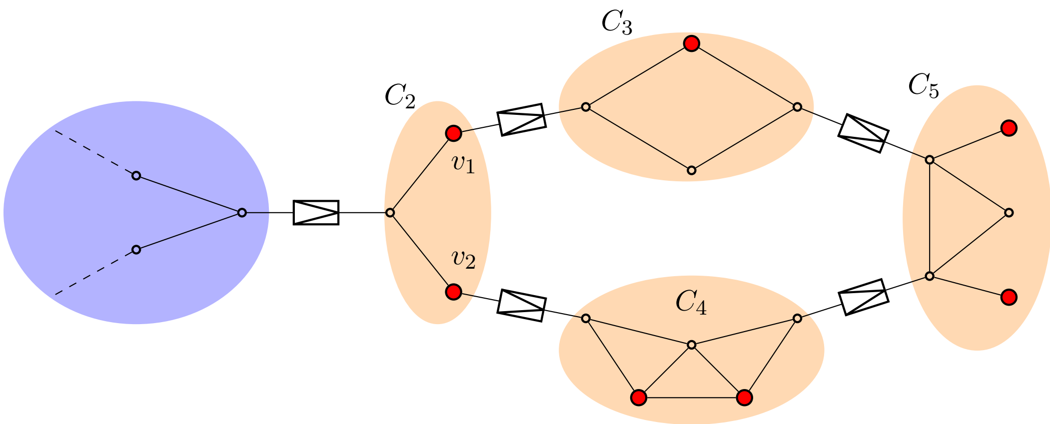


Figure 6.3: An example that requires balancing the demand of distribution subnetwork C_2 for the stationary *Max Pressure Loss Problem*, given that $|\mathcal{A}^{out}_{C_2}| \geq 1$ holds. The downstream components of C_2 are given by $\mathcal{C}^{ds-dc}_{C_2} = \{C_3, C_4, C_5\}$ and the outflow of C_2 corresponds to $\sum_{a \in \mathcal{A}^{out}_{C_2}} q_{a,t} = \sum_{D \in \{C_3,C_4,C_5\}} \sum_{v \in \mathcal{V}^-_D} |b_{v,t}|$ for $t \in \mathcal{T}$, as determined in Equation (6.12). Here, the distribution subnetworks are highlighted in light orange, the remaining transport subnetworks in blue and exit nodes in red.

canonically be determined in advance, because $\sum_{a \in \mathcal{A}^{out}_C} q_{a,t_C}$ (and also $\sum_{a \in \mathcal{A}^{in}_C} q_{a,t_C}$) in Equation (6.11) is unknown depending on the number of connections to the downstream and upstream components. To determine $\sum_{a \in \mathcal{A}^{out}_C} q_{a,t}$ and $\sum_{a \in \mathcal{A}^{in}_C} q_{a,t}$ while establishing a balanced scenario (6.11), we operate again on the components graph $\mathcal{G}^{\text{cg}} = (\mathcal{V}^{\text{cg}}, \mathcal{A}^{\text{cg}})$ and collect all distribution components that can be reached by a downstream path of regulators starting from the representative node of component C. Let $\mathcal{C}^{ds-dc}_C \subseteq \mathcal{C}^{dc}$ be the set of *downstream - distribution components* of C, for an example see Figure 6.3. Given that the demand at exit nodes in $D \in \mathcal{C}^{ds-dc}_C$ is known, i.e., $b_{v,t}$ is fixed for all $v \in \mathcal{V}^-_D$ and for all $D \in \mathcal{C}^{ds-dc}_C$, it enables us to calculate

$$\sum_{a \in \mathcal{A}^{out}_C} q_{a,t} := \sum_{D \in \mathcal{C}^{ds-dc}_C} \sum_{v \in \mathcal{V}^-_D} |b_{v,t}| \qquad \forall t \in \mathcal{T}, \tag{6.12}$$

$$\sum_{a \in \mathcal{A}^{in}_C} q_{a,t} := \sum_{a \in \mathcal{A}^{out}_C} q_{a,t} + \sum_{v \in \mathcal{V}^-_C} |b_{v,t}| \qquad \forall t \in \mathcal{T}. \tag{6.13}$$

Then, we use (6.12) to determine $\hat{t}_C := \arg\max_{t \in \mathcal{T}} (\sum_{a \in \mathcal{A}^{out}_C} q_{a,t} + \sum_{v \in \mathcal{V}^-_C} |b_{v,t}|)$ for all $C \in \mathcal{C}^{dc}$, as described above. This approach potentially overestimates the flow inside C and hence the maximal pressure loss Δ_C, because depending on the network structure, the demand of the downstream components might be fed by other components than the considered one. Even though we encountered that in practice typically holds $\mathcal{A}^{out}_C = \emptyset$. By way of explanation, see for example the selected distribution subnetworks in Table 6.2. Finally, we relocate the components' inflow and outflow to

$$\sum_{v \in \mathcal{V}^{in}_C} b_{v,\hat{t}_C} \leftarrow \sum_{v \in \mathcal{V}^{in}_C} b_{v,\hat{t}_C} + \sum_{a \in \mathcal{A}^{in}_C} q_{a,\hat{t}_C}, \tag{6.14}$$

$$\sum_{v \in \mathcal{V}_C^{out}} b_{v,\hat{t}_C} \leftarrow \sum_{v \in \mathcal{V}_C^{out}} b_{v,\hat{t}_C} + \sum_{a \in \mathcal{A}_C^{out}} q_{a,\hat{t}_C}. \tag{6.15}$$

Let us now turn to the *Max Pressure Loss Problem* that determines the maximal pressure loss Δ_C in component $C \in \mathcal{C}^{dc}$ at time step $\hat{t}_C$. Usually, the maximal pressure loss takes place between nodes in $\mathcal{V}_C^{in}$ and $\mathcal{V}_C^{out} \cup \mathcal{V}_C^{-}$, where gas enters and leaves the subnetwork. For this reason, we introduce a binary variable z_{vw} for each pair of nodes $(v,w) \in \mathcal{V}_C^{in} \times \left(\mathcal{V}_C^{out} \cup \mathcal{V}_C^{-}\right)$ that activates the corresponding pressure loss in the objective function (6.16a). To ensure that the pressure loss is maximized between precisely one such node pair, Equation (6.16d) selects exactly one binary variable z_{vw}. The pressure loss along the pipes is modeled by the stationary Weymouth Equation (6.16b) and the flow conservation by Equation (6.16c). Constraints (6.16e) – (6.16f) and (6.16g) – (6.16k) model the stationary behavior of valves and non-station control valves, as done in Humpola et al. (2015), and constraints (6.16l) – (6.16m) model station regulators. Finally, the *Max Pressure Loss Problem* reads:

$$\Delta_C := \max \sum_{v \in \mathcal{V}_C^{in}} \sum_{w \in \mathcal{V}_C^{out} \cup \mathcal{V}_C^{-}} |p_v - p_w| \, z_{vw} \tag{6.16a}$$

$$\text{s.t.} \quad p_v^2 - \beta_a \, p_w^2 = \gamma_a \, q_a |q_a| \qquad \forall a = (v,w) \in \mathcal{A}_C \cap A_{pi} \tag{6.16b}$$

$$\sum_{\substack{a \in \delta^+(v) \\ a \in \mathcal{A}_C}} q_a - \sum_{\substack{a \in \delta^-(v) \\ a \in \mathcal{A}_C}} q_a = b_{v,\hat{t}_C} \qquad \forall v \in \mathcal{V}_C \tag{6.16c}$$

$$\sum_{v,w \in \mathcal{V}_C^{in} \times \left(\mathcal{V}_C^{out} \cup \mathcal{V}_C^{-}\right)} z_{vw} = 1 \tag{6.16d}$$

$$\lambda_a = 0 \;\Rightarrow\; q_a = 0 \qquad \forall a \in \mathcal{A}_C \cap (\mathcal{A}_{va} \cup_{s \in \mathcal{S}} \mathcal{A}_{va,s}) \tag{6.16e}$$

$$\lambda_a = 1 \;\Rightarrow\; p_v = p_w \qquad \forall a \in \mathcal{A}_C \cap (\mathcal{A}_{va} \cup_{s \in \mathcal{S}} \mathcal{A}_{va,s}) \tag{6.16f}$$

$$s_a = 0 \;\Rightarrow\; q_a = 0 \qquad \forall a = (v,w) \in \mathcal{A}_C \cap \mathcal{A}_{cv} \tag{6.16g}$$

$$s_a^{ac} = 1 \;\Rightarrow 0 \le p_v - p_w \le \overline{p}_v - \underline{p}_w \qquad \forall a = (v,w) \in \mathcal{A}_C \cap \mathcal{A}_{cv} \tag{6.16h}$$

$$s_a^{ac} = 1 \;\Rightarrow\; q_a \ge 0 \qquad \forall a = (v,w) \in \mathcal{A}_C \cap \mathcal{A}_{cv} \tag{6.16i}$$

$$s_a^{bp} = 1 \;\Rightarrow\; p_v = p_w \qquad \forall a = (v,w) \in \mathcal{A}_C \cap \mathcal{A}_{cv} \tag{6.16j}$$

$$s_a^{ac} + s_a^{bp} = s_a \qquad \forall a = (v,w) \in \mathcal{A}_C \cap \mathcal{A}_{cv} \tag{6.16k}$$

$$p_{r,t} - p_{\ell,t} \le (\overline{p}_r - \underline{p}_\ell)\,(1 - z_{a,t}) \qquad \forall a = (v,w) \in \mathcal{A}_C \cap \mathcal{A}_{rg,s}, \forall s \in \mathcal{S} \tag{6.16l}$$

$$0 \le q_{a,t} \le \overline{q}_a \, z_{a,t} \qquad \forall a = (v,w) \in \mathcal{A}_C \cap \mathcal{A}_{rg,s}, \forall s \in \mathcal{S} \tag{6.16m}$$

$$z_{vw} \in \{0,1\} \qquad \forall v,w \in \mathcal{V}_C^{in} \times \left(\mathcal{V}_C^{out} \cup \mathcal{V}_C^{-}\right) \tag{6.16n}$$

$$p_v \in [\underline{p}_v, \overline{p}_v] \qquad \forall v \in \mathcal{V}_C \tag{6.16o}$$

$$q_a \in [\underline{q}_a, \overline{q}_a] \qquad \forall a \in \mathcal{A}_C \tag{6.16p}$$

$$\lambda_a \in \{0,1\} \qquad \forall a \in \mathcal{A}_C \cap (\mathcal{A}_{va} \cup_{s \in \mathcal{S}} \mathcal{A}_{va,s}) \tag{6.16q}$$

$$s_a, s_a^{ac}, s_a^{bp} \in \{0,1\} \qquad \forall a \in \mathcal{A}_C \cap \mathcal{A}_{cv} \tag{6.16r}$$

$$z_a \in \{0,1\} \qquad \forall a \in \mathcal{A}_C \cap \mathcal{A}_{rg,s}, \forall s \in \mathcal{S}. \tag{6.16s}$$

Then, we select the tightest pressure bounds among all nodes in C and increase the lower pressure bound at node v_C by the value Δ_C, i.e.,

$$\underline{p}_{v_C} := \max_{v \in \mathcal{V}_C} \underline{p}_v + \Delta_C, \tag{6.17}$$

$$\overline{p}_{v_C} := \min_{v \in \mathcal{V}_C} \overline{p}_v. \tag{6.18}$$

In our computational experiments, the *Max Pressure Loss Problem* could always be optimally solved for all considered subnetworks. This includes the resulting distribution subnetworks in the computational experiments in Chapter 8, and the six distribution subnetwork examples $C \in \{D_1, \ldots, D_6\}$ presented in Table 6.2. For the latter ones, Table 6.3 shows the maximal pressure loss values resulting from solving Problem (6.16) with respect to the particular time step $\hat{t}_C \in \{0, \ldots, 35000\}$ in the historical data, where the maximal amount of demand flow occurs.

Δ_{D_1}	Δ_{D_2}	Δ_{D_3}	Δ_{D_4}	Δ_{D_5}	Δ_{D_6}
2.073	0.011	1.471	0.567	0.003	4.402

Table 6.3: Results for the lower pressure bound increase Δ_{D_i} in [bar] at storage nodes of the six distribution subnetwork examples $D_1, ..., D_6$ from Table 6.2. The values Δ_{D_i} correspond to the safety margins given by the objective function value of Problem (6.16).

6.3.7 Evaluation of the aggregation approach

In this section, we investigate the impact of our aggregation approach on the six distribution subnetworks from Table 6.2. Following the description of the evaluation process in Section 6.1, we first determine a surrounding $\mathcal{G}_C^{\text{surr}} = (\mathcal{V}_C^{\text{surr}}, \mathcal{A}_C^{\text{surr}})$ of subnetwork $\mathcal{G}_C$ and then compare the results of two simulations: (i) a simulation of the non-aggregated distribution subnetwork and its surrounding, and (ii) a simulation of the aggregated distribution subnetwork and its surrounding.

To validate the impact of the aggregation on $\mathcal{G}_C^{\text{surr}}$, we compare the pressure values of both simulations at the node sets described in Section 6.1.4: subnetwork nodes linked to the surrounding, surrounding nodes linked to the subnetwork (BFS level 0 nodes), all surrounding nodes, and BFS last level nodes. Here, we have chosen a BFS last level of 15. For the subnetwork nodes that are linked to the surrounding, we actually compare the pressure values of the storage node v_C and the node $v \in \mathcal{V}_C^{in}$ yielding the highest deviation, i.e., $\max_{v \in \mathcal{V}_C^{in}} |p_{v_C,t} - p_{v,t}|$ given that $v_C \cap \mathcal{V}_C^{in} = \emptyset$ (col. *Storage node* in Table 6.4). For the comparison of inflows and outflows of the distribution subnetworks before and after the aggregation (col. *Connect. arcs*), the accumulated flows (6.4) – (6.5) reduce to the following values

$$x_{a,t}^{\text{orig}} = \sum_{\mathcal{A}_C^{in}} q_{a,t}^{\text{orig}} \qquad \text{and} \qquad x_{a,t}^{\text{agg}} = \sum_{\mathcal{A}_C^{in}} q_{a,t}^{\text{agg}},$$

since all subnetwork examples hold $|\mathcal{V}_C^{out}| = 0$. Throughout, we use the Measures (6.1) – (6.3).

Table 6.4 shows three major effects: At first, all six distribution subnetworks but D_2 feature a non-increasing pressure deviation the farther away the nodes are located from the subnetwork, i.e., starting from the *storage node* along the depth *level 0 nodes* over *all surrounding nodes* on average, up to the *last level nodes*. It is remarkable that already three out of six subnetworks (D_3, D_5, D_6) exhibit pressure profiles that only deviate at the storage node and are completely equal in the entire surrounding. Second, for all subnetworks but D_3, the deviation of pressure values is throughout remarkably small with at most 0.075 bar measured in the mean absolute deviation (mean MAD, D_2). Only D_3 has a mean MAD of approximately 0.2 bar and an AD of 0.85 bar. Third, the differences of flow values into the non-aggregated and aggregated subnetworks are apparently negligible with at most 0.32 [kg/s] (AD, D_2).

We analyzed these effects and identified the following major reason: A key role play the regulators, which link non-aggregated and aggregated distribution subnetworks with the remaining network. Their flow rates are determined by the pressure difference between their adjacent nodes. For the simulation, so-called set-point values are used that aim at influencing the regulators' pressure and flow values in such way that a predefined behavior of the regulator is enforced. These set-point values can be thought of as desired values for some, but not all pressure and flow variables that are used to model regulators in the simulator. The interesting question is whether the simulated pressures at nodes in $\mathcal{V}_C^{in}$ and v_C, i.e., before and after the aggregation of C, react equally to changes in the simulated gas flow through the regulator over time. This amount can be estimated in

Case	Measure	*Storage node* [bar]	*Level 0 nodes* [bar]	*All surr. nodes* [bar]	*Last level nodes* [bar]	*Connect. arcs* [kg/s]
D_1	AD	1.90e−01	4.69e−02	3.59e−02	0.0	1.08e−02
	mean AD	6.86e−02	3.87e−03	1.88e−03	0.0	2.81e−03
	mean MAD	4.53e−02	2.70e−03	1.31e−03	0.0	1.11e−03
D_2	AD	2.37e−01	1.75e−01	1.90e−01	1.89e−01	3.17e−01
	mean AD	9.90e−02	1.28e−02	2.15e−02	2.79e−02	7.39e−03
	mean MAD	7.46e−02	7.37e−03	1.51e−02	2.07e−02	3.04e−04
D_3	AD	8.54e−01	0.0	0.0	0.0	0.0
	mean AD	3.26e−01	0.0	0.0	0.0	0.0
	mean MAD	2.08e−01	0.0	0.0	0.0	0.0
D_4	AD	6.15e−03	3.90e−03	3.80e−03	3.71e−03	2.24e−01
	mean AD	9.31e−04	7.70e−04	7.37e−04	7.31e−04	2.57e−02
	mean MAD	1.59e−04	1.58e−04	1.55e−04	1.54e−04	2.47e−03
D_5	AD	3.79e−02	0.0	0.0	0.0	0.0
	mean AD	1.47e−02	0.0	0.0	0.0	0.0
	mean MAD	1.01e−02	0.0	0.0	0.0	0.0
D_6	AD	1.04e−02	0.0	0.0	0.0	0.0
	mean AD	5.56e−03	0.0	0.0	0.0	0.0
	mean MAD	3.68e−03	0.0	0.0	0.0	0.0

Table 6.4: Differences of simulation results before and after the aggregation of $D_1, ..., D_6$.

both cases separately, for example by a secant approximation $p_{r,t} - p_{r,t-1}/(q_{a,t} - q_{a,t-1})$ for a regulator $a = (\ell, r)$, where node r corresponds to $v \in \mathcal{V}_C^{in}$ and v_C, respectively. Our simulation results show that this amount is roughly the same in both cases, cf. the moderate deviations in simulated pressure (col. *Storage node*) and inflow values (col. *Connect. arcs*). The small difference in flow is partly a consequence of the fact that the inflows into the distribution subnetworks are more or less consumed instantly, as shown in our study in Section 6.3.3. By concept, this also applies to the storage node, as it is the only representative node of the aggregated subnetwork. Moreover, the non-aggregated distribution subnetworks and the storage nodes buffer the same amount of gas flows that arise from their surroundings. Concerning the small difference in pressure before and after the aggregation of C, it turns out that the chosen aggregation principles on maintaining the subnetworks' volume and its initial amount of line-pack (described in Section 6.3.5) yield an inertia that changes the storage node's pressure values in a similar way as for the non-aggregated nodes in $\mathcal{V}_C^{in}$.

In a nutshell, we performed simulations over 365 days for each of the six distribution subnetwork examples. Their overall small deviations between the aggregated and non-aggregated structures indicate that the physical impact of the aggregation is local and small. Hence, these results suggest that our approach of detecting and aggregating distribution subnetworks is appropriate in the context of modeling and optimizing operations of transient large-scale gas networks.

6.4 Aggregation of Passive Tree Subnetworks

Similar to distribution subnetworks, passive trees correspond to outbound zones of the network with the difference that they belong to the transmission part when Algorithm 1 invokes their aggregation. In practice, however, the options to control transient gas flows in passive trees are rather limited because they generally have default transport paths and serve as potential storage facilities. Therefore, in Section 6.4.1 we apply a coarse aggregation approach and replace entire passive trees that are located in the exterior of stations by storage nodes, as done for distribution subnetworks. Again, the aggregation follows the principles described in Section 6.1.2. That means, besides maintaining the pipe volume of the trees, it also keeps their initial amount of line-pack, see Section 6.4.2. Next, in Section 6.4.3 our procedure calculates tightened pressure bounds at the storage nodes. But other than for distribution subnetworks, we exploit the structure of the trees and approximate these bounds without solving an optimization model. Finally, in Section 6.4.4, we evaluate our aggregation procedure on six passive tree subnetworks based on real-world data.

6.4.1 Aggregation approach

At first, we formalize passive tree structures that are located outside of stations. For a given graph, Definition 6.4.1 allows identifying passive tree subgraphs and uniquely selecting their root nodes.

Definition 6.4.1 (Passive trees outside of stations)**.** *For a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, let $\mathcal{G}^{tr}$ be the set of subgraphs $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{A}_C)$, $\mathcal{V}_C \subseteq \mathcal{V}$, $\mathcal{A}_C \subseteq \mathcal{A}$, where $\mathcal{G}_C \in \mathcal{G}^{tr}$ is characterized as follows:*

i) $\mathcal{G}_C$ is a tree,

ii) for all $a \in \mathcal{A}_C \, : \, a \in \mathcal{A}_{pi}$,

iii) for all $a \in \mathcal{A}_C$ and for all $s \in \mathcal{S} \, : \, a \notin \mathcal{A}_s$,

iv) the root node r_C is the only node in $\mathcal{V}_C$ that can be adjacent to arcs in $\mathcal{A} \setminus \mathcal{A}_C$,

v) for all $\mathcal{G}_D = (\mathcal{V}_D, \mathcal{A}_D) \subseteq \mathcal{G}$ that satisfy (i) – (iv) $: \mathcal{G}_C \not\subset \mathcal{G}_D$.

We call $\mathcal{G}^{tr}$ the set of passive trees that are located in the exterior of stations.

Conditions (i) – (ii) imply that we consider passive trees, whose arcs are pipes and not, for example, active elements. Condition (iii) guarantees that all tree arcs are located outside of stations. Condition (iv) states that the trees have at most one connection node to the remaining network $\mathcal{G} \setminus \mathcal{G}_C$, which we designate as root node r_C. As a consequence of (iii) and (iv), only the root node r_C can be a station node, i.e., for all $v \in \mathcal{V}_C \setminus \{r_C\}$ and for all $s \in \mathcal{S}$ it follows that $v \notin \mathcal{V}_s$. Finally, condition (v) ensures that $\mathcal{G}_C$ is not a real subset of any other tree in $\mathcal{G}^{tr}$. Therefore, $\mathcal{G}_C$ is maximal with respect to the number of nodes and arcs. Altogether, these five conditions uniquely determine passive trees outside stations of maximum size. They also state a criterion for the unique selection of the root node r_C, at least when $\mathcal{G}$ is not a tree that itself fulfills conditions (i) – (v). Furthermore, no tree node $v \in \mathcal{V}_C$ and tree pipe $a \in \mathcal{A}_C$ is contained within any cycle of nodes and arcs in $\mathcal{G}$.

The following proposition states that all trees in $\mathcal{G}^{tr}$ are disjoint.

Proposition 6.4.2. *For a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, let $\mathcal{G}^{tr}$ be the set of trees that satisfy characteristics (i) – (v) from Definition 6.4.1. Then, all trees $\mathcal{G}_C \in \mathcal{G}^{tr}$ have pairwise disjoint nodes and arcs.*

Proof. Proof by contradiction. Consider $\mathcal{G}_C, \mathcal{G}_D \in \mathcal{G}^{tr}$ with $\mathcal{G}_C, \mathcal{G}_D \subseteq \mathcal{G}$ and $\mathcal{G}_C \neq \mathcal{G}_D$. Let us restrict to the non-trivial cases $\mathcal{G} \setminus \mathcal{G}_C \neq \emptyset$ and $\mathcal{G} \setminus \mathcal{G}_D \neq \emptyset$. Then, there exist nodes $r_C \in \mathcal{G}_C$ and $r_D \in \mathcal{G}_D$ that link $\mathcal{G}_C$ with $\mathcal{G} \setminus \mathcal{G}_C$ and $\mathcal{G}_D$ with $\mathcal{G} \setminus \mathcal{G}_D$ respectively. We assume $\mathcal{G}_C \cap \mathcal{G}_D \neq \emptyset$, i.e., there exists a node $\hat{v} \in \mathcal{V}_C \cap \mathcal{V}_D$. In the following, we consider three cases as dependent on whether $\hat{v}$ is the root node r_C or r_D of $\mathcal{G}_C$ and $\mathcal{G}_D$:

1. $\hat{v} = r_C$ and $\hat{v} = r_D$,
2. $\hat{v} \neq r_C$ and $\hat{v} \neq r_D$, and
3. $\hat{v} = r_C$ and $\hat{v} \neq r_D$ (or $\hat{v} \neq r_C$ and $\hat{v} = r_D$).

We show that in each case one of both trees violates one of the properties (i) to (v).

1. $\hat{v} = r_C$ and $\hat{v} = r_D$:
Since $\mathcal{G}_C, \mathcal{G}_D \in \mathcal{G}^{tr}$, it follows from condition (v) together with $\mathcal{G}_C, \mathcal{G}_D \subset \mathcal{G}_C \cup \mathcal{G}_D$ that $\mathcal{G}_C \cup \mathcal{G}_D \notin \mathcal{G}^{tr}$. However, from $\mathcal{G}_C, \mathcal{G}_D \in \mathcal{G}^{tr}$ directly follows that $\mathcal{G}_C \cup \mathcal{G}_D$ satisfies conditions (i) – (iv). Then, $\mathcal{G}_C \cup \mathcal{G}_D$ has to violate condition (v), which means that there exists $E \in \mathcal{G}^{tr}$ such that $\mathcal{G}_C \cup \mathcal{G}_D \subset \mathcal{G}_E$. But, $\mathcal{G}_C, \mathcal{G}_D \subset \mathcal{G}_C \cup \mathcal{G}_D \subset \mathcal{G}_E$ implies $\mathcal{G}_C, \mathcal{G}_D \subset \mathcal{G}_E$, which violates condition (v) for $\mathcal{G}_C, \mathcal{G}_D$ and thus $\mathcal{G}_C, \mathcal{G}_D \notin \mathcal{G}^{tr}$.

2. $\hat{v} \neq r_C$ and $\hat{v} \neq r_D$:
We distinguish the following two cases: a) $r_C \neq r_D$ and b) $r_C = r_D$.

a) $r_C \neq r_D$: From $\hat{v} \in \mathcal{V}_C \cap \mathcal{V}_D$ we deduce that there exist paths between nodes r_C and $\hat{v}$ in $\mathcal{G}_C$ and between nodes r_D and $\hat{v}$ in $\mathcal{G}_D$. According to condition (iv), r_C and r_D are the only nodes that connect $\mathcal{G}_C$ with $\mathcal{G} \setminus \mathcal{G}_C$ and $\mathcal{G}_D$ with $\mathcal{G} \setminus \mathcal{G}_D$ respectively. Then, it follows that $r_C \in \mathcal{V}_D$ and $r_D \in \mathcal{V}_C$.

   Let $\mathcal{G}_{r_C} := (r_c \cup \mathcal{V} \setminus \mathcal{V}_C, \mathcal{A} \setminus \mathcal{A}_C)$. Since $r_C \in \mathcal{V}_D$, $r_D \notin \mathcal{V}_{r_C}$ (because $r_D \in \mathcal{V}_C$ and $r_D \neq r_C$) and r_D is the only node that links $\mathcal{A}_D$ and $\mathcal{A} \setminus \mathcal{A}_D$ (because $\mathcal{G}_D \in \mathcal{G}^{tr}$), it follows that $\mathcal{G}_{r_C} \subset \mathcal{G}_D$. In the following, we show that $\mathcal{G}_C \cup \mathcal{G}_{r_C} \in \mathcal{G}^{tr}$, which contradicts the assumption $\mathcal{G}_C \in \mathcal{G}^{tr}$ because $\mathcal{G}_C \subset \mathcal{G}_C \cup \mathcal{G}_{r_C}$ violates condition (v). From $\mathcal{G}_{r_C} \subset \mathcal{G}_D$ follows that $\mathcal{G}_{r_C}$ is a tree. Since both trees $\mathcal{G}_C$ and $\mathcal{G}_{r_C}$ only share one common node r_C, it follows that $\mathcal{G}_C \cup \mathcal{G}_{r_C}$ is also a tree (condition (i)). Moreover, from $\mathcal{G}_{r_C} \subset \mathcal{G}_D$ follows that all $a \in \mathcal{A}_{r_C}$ hold $a \in \mathcal{A}_{pi}$ (condition (ii)) and all $a \in \mathcal{A}_{r_C}$ hold $a \notin \mathcal{A}_s$ for all $s \in \mathcal{S}$ (condition (iii)). Finally, since $\mathcal{G}_C \cup \mathcal{G}_{r_C} = \mathcal{G}$, there exists no node in $\mathcal{V}_C \cup \mathcal{V}_{r_C}$ that is adjacent to $\mathcal{V} \setminus (\mathcal{V}_C \cup \mathcal{V}_{r_C})$ (condition (iv)), and there exists no $\mathcal{G}_E \subseteq \mathcal{G}$ that satisfies (i) – (iv) and holds $\mathcal{G}_C \cup \mathcal{G}_{r_C} \subset \mathcal{G}_E$.

b) $r_C = r_D$: This case follows analogously to the first case by showing that $\mathcal{G}_C \cup \mathcal{G}_D$ satisfies conditions (i) to (iv).

3. $\hat{v} = r_C$ and $\hat{v} \neq r_D$:
Under the same logic follows the case $\hat{v} \neq r_C$ and $\hat{v} = r_D$. This case follows analogously to the case 2a) by showing that $\mathcal{G}_C \cup \mathcal{G}_{r_C} \in \mathcal{G}^{tr}$ with $\mathcal{G}_{r_C} := (r_c \cup \mathcal{V} \setminus \mathcal{V}_C, \mathcal{A} \setminus \mathcal{A}_C)$, contradicts the assumption $\mathcal{G}_C \in \mathcal{G}^{tr}$ because $\mathcal{G}_C \subset \mathcal{G}_C \cup \mathcal{G}_{r_C}$ violates condition (v). □

Let us consider an arbitrary $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{A}_C) \in \mathcal{G}^{tr}$, where the set of sources and sinks are given by $\mathcal{V}_C^+ \subseteq \mathcal{V}_C$ and $\mathcal{V}_C^- \subseteq \mathcal{V}_C$. We substitute $\mathcal{G}_C$ for a storage node, similarly as done for distribution subnetworks in Section 6.3. More precisely, we remove all nodes $\mathcal{V}_C \setminus \{r_C\}$ and arcs $\mathcal{A}_C$ and designate the root node r_C as storage node. So, an aggregated subnetwork reads $\mathcal{G}_C^{\text{agg}} = (\mathcal{V}_C^{\text{agg}}, \mathcal{A}_C^{\text{agg}})$ with $\mathcal{V}_C^{\text{agg}} = \{r_c\}$ and $\mathcal{A}_C^{\text{agg}} = \emptyset$. Then, we endow the storage node v_C with the volume of the aggregated tree, i.e.,

$$vol_{r_C} := \sum_{a \in \mathcal{A}_C \cap \mathcal{A}_{pi}} vol_a.$$

In the case that a tree contains exit or source nodes, the root node r_C inherits the accumulated demand of its boundary nodes:

$$b_{r_C,t} := \sum_{v \in \mathcal{V}_C^-} b_{v,t} + \sum_{v \in \mathcal{V}_C^+} b_{v,t} \quad \forall t \in T.$$

6.4.2 Representing the initial amount of gas

Here, we set the initial amount of gas in the storage node v_C to the amount stored in tree $\mathcal{G}_C \in \mathcal{G}^{tr}$. To this end, we directly apply the procedure from Section 6.3.5 where trees take the role of distribution subnetworks here. Again, we describe the line-pack LP_C of a tree as the accumulated line-pack of all pipes in $\mathcal{G}_C$ leading to

$$p_{v_C,t_0} = \frac{\sum_{a\in\mathcal{A}_C\cap\mathcal{A}_{pi}} p^{mean}_{a,t_0}\, vol_a}{\sum_{a\in\mathcal{A}_C\cap\mathcal{A}_{pi}} vol_a}.$$

6.4.3 Determining pressure bounds at the storage node

In this section, we calculate tightened pressure bounds at the storage node. Our goal is that when fulfilling the storage node's pressure bounds, we want to ensure, if possible, that all pressure restrictions at nodes of the non-aggregated tree are satisfied. To achieve this, we present an algorithm that approximates the maximal possible pressure loss in the tree, combine it with the pressure bounds of the tree nodes and apply the intersection of all these bound intervals to the root node. Similar to connection nodes $\mathcal{V}_C^{in}$ in distribution subnetworks from Section 6.3.4, the lower pressure bounds at tree root nodes are required to be high enough. In addition, their upper pressure bounds should be small enough because by the time Algorithm 1 invokes the tree aggregation, passive trees are part of the transport network and might also contain sources. As a consequence, high source flows might induce pressure losses inside trees, but in the reverse direction, i.e., towards the root nodes.

For the pressure bound tightening, we first approximate maximal pressure losses in the trees. To this end, we again apply stationary modeling. But in contrast to distribution subnetworks, it is possible here to exploit the structure of the trees and approximate these bounds without solving an optimization model, such as the *Max Pressure Loss Problem*, because stationary flows and pressure losses in trees are uniquely determined.

For a given tree $\mathcal{G}_C \in \mathcal{G}^{tr}$, we iteratively propagate the bounds of leaf nodes to the root node r_C, similar to Groß et al. (2019) for the leaf reduction of potential-based and stationary networks, with the only difference that we perform this propagation separately for each time step $t \in \mathcal{T}$. This results in $|T|$-many possible bounds for the root node r_C, from which we finally select the tightest one. Algorithm 3 illustrates this procedure in more detail.

Let $\mathcal{G}_C \in \mathcal{G}^{tr}$ be a tree. For a given time step $t \in \mathcal{T}$, we denote its graph by $\mathcal{G}_C^t = (\mathcal{V}_C^t, \mathcal{A}_C^t)$ and its set of leaf nodes by $\mathcal{V}_{C,leaf}^t \subseteq \mathcal{V}_C^t$. Further, let node u be adjacent to $v \in \mathcal{V}_{C,leaf}^t$ and w.l.o.g. let their connecting pipe $a = (u, v)$ be directed towards the leaf node v. In the current time iteration $t \in \mathcal{T}$, the demand $b_{v,t}$ induces a pressure loss along pipe a that we use to obtain new pressure bounds at node u by virtue of the potential loss function ϕ from Equation (3.2). In the case that v is a source node, the upper pressure bound at node u is obtained by setting $\overline{p}_u^t \leftarrow \min\{\, \overline{p}_u^t,\, ((\overline{p}_v^t)^2 + \phi(q_{a,t}))^{1/2}\,\}$ given that source flow $b_{v,t} \geq 0$ induces non-positive flow $q_{a,t} = -b_{v,t} \leq 0$ along $a = (u, v)$ according to its orientation. Analogously, for a sink v, the lower pressure bound at node u is obtained by setting $\underline{p}_u^t \leftarrow \max\{\, \underline{p}_u^t,\, ((\underline{p}_v^t)^2 + \phi(q_{a,t}))^{1/2}\,\}$ because sink flow $b_{v,t} \leq 0$

Algorithm 3 Pressure bound tightening at root nodes of passive trees.

Input: $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{A}_C) \in \mathcal{G}^{tr}$ and demand $(b_{v,t})_{v \in \mathcal{V}_C, t \in \mathcal{T}}$
Output: Tightened pressure bounds at root node r_C of tree $\mathcal{G}_C$

for each $t \in \mathcal{T}$ **do**
  Initialize graph $\mathcal{G}_C^t = \mathcal{G}_C$
  Initialize set of leaf nodes $\mathcal{V}_{C,leaf}^t = \{v \in \mathcal{V}_C^t \,|\, deg(v) = 1\}$
  while $\mathcal{V}_{C,leaf}^t \neq \{r_C\}$ **do**
    select $v \in \mathcal{V}_{C,leaf}^t$ and $a = (u,v)$
    if $b_{v,t} < 0$ **then** ▷ exit flows
      $\underline{p}_u^t \leftarrow \max\left\{ \underline{p}_u^t,\ \left((\underline{p}_v^t)^2 + \phi(q_{a,t})\right)^{1/2} \right\}$
    else if $b_{v,t} \geq 0$ **then** ▷ source flows
      $\overline{p}_u^t \leftarrow \min\left\{ \overline{p}_u^t,\ \left((\overline{p}_v^t)^2 + \phi(q_{a,t})\right)^{1/2} \right\}$
    end if
    $b_{u,t} \leftarrow b_{u,t} + b_{v,t}$
    if $deg(u) = 1$ and $u \neq r_C$ **then**
      $\mathcal{V}_{C,leaf}^t \leftarrow u$
    end if
    $\mathcal{V}_{C,leaf}^t = \mathcal{V}_{C,leaf}^t \setminus \{v\}$
    $\mathcal{G}_C^t = (\mathcal{V}_C^t \setminus \{v\}, \mathcal{A}_C^t \setminus \{a\})$
  end while
end for
$\underline{p}_{r_C} \leftarrow \max\{ \underline{p}_{r_C}^t \,|\, t \in \mathcal{T} \}$
$\overline{p}_{r_C} \leftarrow \min\{ \overline{p}_{r_C}^t \,|\, t \in \mathcal{T} \}$

induces non-negative flow $q_{a,t} = b_{v,t} \geq 0$. Note that we equip the notation of $\underline{p}_u^t, \overline{p}_u^t$ with index $t \in \mathcal{T}$ in order to indicate their dependence on the current time iteration. Then, we update the demand of node u by $b_{u,t} \leftarrow b_{u,t} + b_{v,t}$ and delete node v and arc $a = (u,v)$ from the graph, i.e., $\mathcal{G}_C^t = (\mathcal{V}_C^t \setminus \{v\}, \mathcal{A}_C^t \setminus \{a\})$. For a particular time step, this procedure is applied until $\mathcal{G}_C^t$ reduces to the root node, i.e., until $\mathcal{V}_C^t = r_C$ and $\mathcal{A}_C^t = \emptyset$. Finally, Algorithm 3 selects the tightest pressure bounds at the root node among all time steps:

$$\begin{aligned} \underline{p}_{r_C} &\leftarrow \max\{ \underline{p}_{r_C}^t \,|\, t \in \mathcal{T}\} \quad \text{and} \\ \overline{p}_{r_C} &\leftarrow \min\{ \overline{p}_{r_C}^t \,|\, t \in \mathcal{T} \}. \end{aligned}$$

Table 6.5 shows by how much Algorithm 3 tightens the pressure bounds at root nodes in six real-world tree examples $T_1, ..., T_6$. The considered trees are also used to evaluate the tree aggregation approach and thus feature different characteristics varying, e.g., in the number of nodes and arcs, see Table 6.5. Tree T_4 represents a *dead-end* tree not containing any boundary nodes, which prevents pressure bound tightening. All other trees contain exit nodes yielding tighter lower pressure bounds, except for T_1. Here, the source flow is throughout higher than the exit flows in each time iteration leading to a non-negative demand $b_{v,t} \geq 0$ at the current leaf nodes and thus Algorithm 3 tightens the upper pressure bounds instead.

	Passive trees						Surrounding of trees				
Trees	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{V}^+\rvert$	$\lvert\mathcal{V}^-\rvert$	$\lvert\mathcal{A}\rvert$	Δ_r^{low}	Δ_r^{up}	$\lvert\mathcal{V}^{\text{surr}}\rvert$	$\lvert\mathcal{V}^{+\text{surr}}\rvert$	$\lvert\mathcal{V}^{-\text{surr}}\rvert$	$\lvert\mathcal{A}^{\text{surr}}\rvert$	# Last level nodes
T_1	8	1	2	7	0.00	0.01	27	1	2	30	2
T_2	9	1	4	8	0.43	0.13	42	6	8	46	4
T_3	11	0	4	10	3.34	0.00	31	0	7	35	2
T_4	2	0	0	1	0.00	0.00	27	0	3	29	2
T_5	23	0	10	22	0.91	0.00	23	0	3	23	2
T_6	6	0	1	5	0.30	0.00	23	0	0	22	3

Table 6.5: Number of elements of the passive trees $T_1, ..., T_6$ and their surroundings used in the evaluation of the aggregation approach. Here, the surroundings are generated with a BFS algorithm up to a depth level of 10. The *last level nodes* correspond to those nodes of the surroundings that are incident to the remaining network (i.e., nodes of depth level 10 in the BFS). Besides, the values $\Delta_{r_{T_i}}^{\text{low}}$ and $\Delta_{r_{T_i}}^{\text{up}}$ (in [bar]) indicate by how much the lower and upper bounds of the root node r_{T_i} are tightened through Algorithm 3.

6.4.4 Evaluation of the aggregation approach

In this section, we evaluate the impact of the aggregation on six passive tree examples from Table 6.5 that are all located in the exterior of stations. We perform the evaluation in the same fashion as for distribution subnetworks. First, we determine a surrounding $\mathcal{G}_C^{\text{surr}} = (\mathcal{V}_C^{\text{surr}}, \mathcal{A}_C^{\text{surr}})$ of tree $\mathcal{G}_C$ and then compare the results of two simulations: (i) a simulation of the non-aggregated tree and its surrounding, and (ii) a simulation of the aggregated tree and its surrounding. To validate the impact of the aggregation on $\mathcal{G}_C^{\text{surr}}$, we compare the pressure values of both simulations at the root node r_C, surrounding nodes linked to the subnetwork (BFS level 0 nodes), all surrounding nodes, and BFS last level nodes, see the description in Section 6.1.4. Here, we have chosen a BFS last level of 10. To compare the difference of inflows and outflows at the root nodes (col. *Connecting arcs* in Table 6.6), we use the flow values specified in Equations (6.4) – (6.5).

Table 6.6 shows four major effects: At first, all six trees feature a non-increasing pressure deviation the farther away the nodes are located from the tree, i.e., starting from the *root node* over the depth *level 0 nodes*, then *all surrounding nodes* on average, up to the *last level nodes*. Second, the deviation of pressure values is throughout remarkably small for all trees with at most 0.045 bar (mean AD, T_1). Third, trees having no demand yield the least deviations, here *dead-end* tree T_4. Last, the difference between the inflow of the non-aggregated and aggregated trees is negligible with at most 0.14 [kg/s] (mean AD, T_1).

In a nutshell, we performed simulations over 365 days for each of the six passive tree examples. Their overall small deviations between the aggregated and non-aggregated structures indicate that the physical impact of the aggregation is small. Hence, these results suggest that our approach of detecting and aggregating distribution subnetworks is appropriate in the context of modeling and optimizing the control of transient gas flows in large-scale gas networks.

Trees	Measure	*Root node* [bar]	*Level 0 nodes* [bar]	*All surr. nodes* [bar]	*Last level nodes* [bar]	*Connecting arcs* [kg/s]
	AD	8.53e−02	8.43e−02	7.53e−02	0.0	8.50e−01
T_1	mean AD	4.50e−02	4.49e−02	1.50e−02	0.0	1.40e−01
	mean MAD	3.43e−02	3.43e−02	1.14e−02	0.0	1.30e−02
	AD	3.96e−01	3.96e−01	3.06e−01	1.20e−01	6.28e−02
T_2	mean AD	3.42e−02	3.42e−02	1.75e−02	1.22e−02	2.00e−03
	mean MAD	1.62e−02	1.62e−02	9.64e−03	8.15e−03	1.49e−03
	AD	2.09e−01	1.93e−01	1.62e−01	1.53e−01	2.77e−01
T_3	mean AD	1.33e−02	1.32e−02	1.06e−02	9.65e−03	9.77e−02
	mean MAD	7.65e−03	7.64e−03	6.50e−03	6.02e−03	8.61e−03
	AD	0.0	0.0	0.0	0.0	0.0
T_4	mean AD	0.0	0.0	0.0	0.0	0.0
	mean MAD	0.0	0.0	0.0	0.0	0.0
	AD	3.07e−03	3.03e−03	2.97e−03	2.54e−03	3.39e−02
T_5	mean AD	6.53e−04	6.44e−04	6.19e−04	6.08e−04	8.12e−03
	mean MAD	4.23e−04	4.23e−04	4.19e−04	4.15e−04	6.70e−04
	AD	2.05e−01	2.01e−01	1.95e−01	1.55e−01	1.35e−01
T_6	mean AD	3.05e−03	3.00e−03	2.56e−03	1.47e−03	1.20e−03
	mean MAD	3.52e−03	3.46e−03	2.95e−03	1.69e−03	1.10e−03

Table 6.6: Differences of the simulation results before and after the aggregation of trees $T_1, ..., T_6$.

6.5 Aggregation of Pipes with Short Lengths

Shrinking a pipeline by merging its end nodes and removing the arc itself has several effects on network operations. For example, the shrinking neglects the possibility of using the pipeline as storage and thereby disregards its current amount of stored gas (line-pack). Besides that, the shrinking discards a possible flow and pressure loss along the removed pipe, possibly resulting in different flow transport situations. As a consequence, this procedure has to be applied carefully, and needs a thorough network analysis beforehand, which avoids shrinking entire transportation paths, for example, between different network stations. For this reason, one would like to shrink the least possible amount of pipelines. To reduce (i) the impact on different pressure and flow distributions across the network and to keep (ii) the line-pack and storage volume of the network roughly constant, we prefer to shrink pipelines that entail small pressure losses and have small pipe volumes. While pipes with short lengths and thick diameters imply smaller pressure losses, pipes with short lengths and thin diameters yield smaller pipe volumes and line-pack values, see Equations (5.11) and (5.17). In total, the requirements on small pressure loss, small volumes and small line-pack values coincide in the aspect of *short length*. For this reason, we focus on shrinking pipes with short lengths. To this end, we introduce a threshold value $\mathscr{L}$ and shrink all pipelines (except those in stations) having shorter length than $\mathscr{L}$. For example, concerning the aggregation of the considered networks in our computational study in Chapter 8, we select the smallest possible threshold value. This value depends on whether the size of the network can be sufficiently reduced in combination with the other aggregation methods, such that the resulting *Transient Control Problem* (MINLP) becomes tractable with respect to the algorithmic developments and model improvements presented in Chapter 7. Let us remark that for all three networks used in our computational study, we set the *threshold* value for the smallest network under consideration to $\mathscr{L} = 0$ [m], for the middle-sized network to $\mathscr{L} = 20$ [m], and for the largest network to $\mathscr{L} = 1{,}000$ [m]. For a visualization of the large network and all pipes $a \in \mathcal{A}_{pi}$ having a length $L_a \leq 1{,}000$ [m], we refer to Figure 8.2b. Finally, the following remark formalizes the shrinking process.

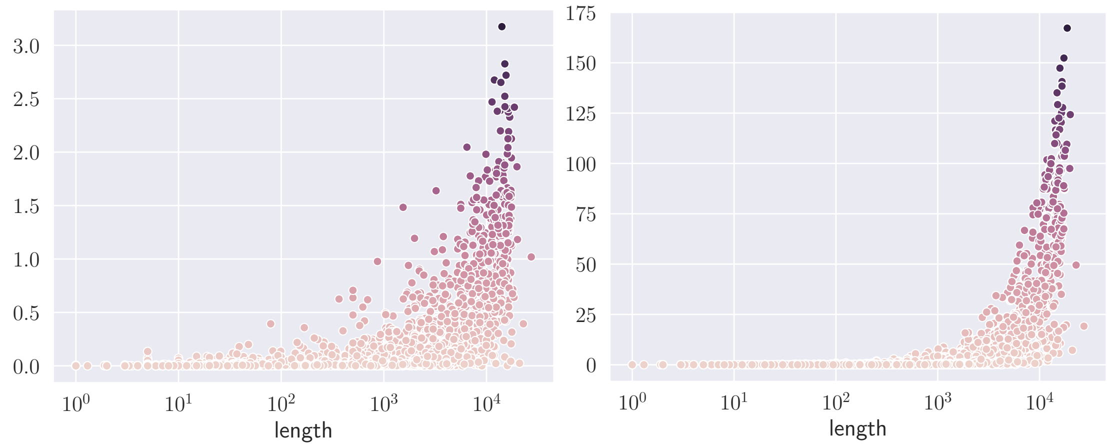
Remark 6.5.1 (Shrinking pipes)**.** *Let a threshold value $\mathscr{L}$ for the pipe length be given and let $a = (\ell, r) \in \mathcal{A}_{pi}$ be a pipe with length $L_a \leq \mathscr{L}$. Then, we contract pipe a by combining nodes ℓ and r. W.l.o.g., let node r be retained. Node r inherits the demand of the removed node ℓ, i.e., $b_{r,t} \leftarrow b_{\ell,t} + b_{r,t}$ and all incident arcs of node ℓ are relocated to node r. Finally, we set the initial pressure value of node r to $p_{r,t_0} \leftarrow 1/2\,(p_{\ell,t_0} + p_{r,t_0})$.*

To facilitate the selection of a threshold value $\mathscr{L}$, we empirically investigate the maximal pressure loss and maximal line-pack in real-world pipelines as dependent on the pipe length. These values can be used as an indicator for pipeline shrinking. In summary, we consider 5,091 pipelines taken from a network of our cooperation partner OGE, where each pipe is equipped with a history of pressure loss and line-pack realizations covering an entire year in 15-minute steps, see also Section 6.1.3. Table 6.7 summarizes the maximal pressure loss and line-pack values classified in groups of pipes with respect to the pipe length (col. *Length*). A visualization of the maximal pressure loss and line-pack values (*Max*) for each pipeline is given in Figure 6.4.

It can be seen that the maximal pressure loss and line-pack increase with the pipeline length. Besides that, the ratio Max/Avg_{max} is rather high for pipes with short length, e.g., 70 ($L_a \leq 20$ [m]) and 49 (20 [m] $\leq L_a \leq 100$ [m]), and steadily decreases to 3 with increasing length of the pipe. Thus, we deduce that maximal pressure loss values, such as 0.443 [bar] (*Max*) for pipelines with $L_a \leq 100$ [m], correspond to exceptions

Length L [m]	Pipes #	Max [bar]	Avg_{max} [bar]	$\frac{Max}{Avg_{max}}$ -	Max [1,000kg]	Avg_{max} [1,000kg]	$\frac{Max}{Avg_{max}}$ -
$0 < L \leq 20$	1,950	0.140	0.002	70	0.10	0.006	17
$20 < L \leq 100$	672	0.443	0.009	49	0.81	0.041	20
$100 < L \leq 500$	543	0.831	0.030	28	2.87	0.171	17
$500 < L \leq 1{,}000$	331	1.132	0.051	22	6.27	0.473	13
$1{,}000 < L \leq 2{,}000$	477	1.581	0.098	16	15.97	1.3823	12
$2{,}000 < L \leq 10{,}000$	852	2.182	0.341	6	80.41	10.7996	8
$L < 10{,}000$	266	3.459	1.143	3	167.21	62.1514	3

Table 6.7: Statistics on the amount of pressure loss and line-pack values in real-world pipelines partitioned into different categories with respect to the pipe length. *Max* describes the maximal pressure loss/line-pack per category and Avg_{max} represents the maximal pressure loss/line-pack on average for all pipes per category. The ratio of both measures is shown in column Max/Avg_{max}.



(a) Maximal pressure loss in [bar]

(b) Maximal line-pack in [1000 kg]

Figure 6.4: Maximal pressure loss and line-pack values over the pipe length in logarithmic scale for 5,091 real-world pipelines, where each dot corresponds to a particular pipeline.

and short pipelines, in general, entail small maximal pressure losses in practice, here for example around 0.009 [bar] or even less. According to our analysis, pipes with short lengths located in the exterior of stations (here $L_a \leq 100$ [m]), connect for example, entries and exits with the remaining network, or link trunks of parallel transmission pipelines, or belong to distribution zones.

6.6 Merging Parallel Pipes

In this section, we investigate parallel pipe merging under transient conditions. At first, we give a brief introduction about the equations involved. Then, we present a merging approach that equivalently replaces the parallel pipes by a merged pipe. We will see that the merging procedure satisfies all of the deployed aspects from Section 6.1.2, i.e., it preserves the volume, line-pack, difference in height and pressure and flow realizations resulting from the parallel pipes. We conclude this section by showing the analogy to the parallel merge from the stationary setting.

System of equations for the parallel and merged pipes. In the following, we consider two parallel pipes $a = (\ell, r)$ and $b = (\ell, r)$ as a stand-alone subnetwork. For both pipes, a system of equations $\mathcal{F}^{orig}$ represents the corresponding discretized continuity and momentum equations, flow conservation at nodes ℓ and r and variable bounds. The parallel merge substitutes both pipes for an artificial pipe $c = (\ell, r)$. Here, the merge replaces $\mathcal{F}_{orig}$ by another system $\mathcal{F}_{agg}$ that represents the discretized continuity and momentum equation of the merged pipe c, the flow conservation at its end nodes and variable bounds.

The system of equations $\mathcal{F}_{orig}$ for pipes a and b is given for all $t \in \mathcal{T}$ by

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r_a,t} - q_{\ell_a,t})\,\alpha_a\,\tau = 0, \tag{6.19}$$

$$p_{r,t}^2\,(1+\beta_a) - p_{\ell,t}^2\,(1-\beta_a) + (q_{r_a,t} + q_{\ell_a,t})|q_{r_a,t} + q_{\ell_a,t}|\gamma_a = 0, \tag{6.20}$$

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r_b,t} - q_{\ell_b,t})\,\alpha_b\,\tau = 0, \tag{6.21}$$

$$p_{r,t}^2\,(1+\beta_b) - p_{\ell,t}^2\,(1-\beta_b) + (q_{r_b,t} + q_{\ell_b,t})|q_{r_b,t} + q_{\ell_b,t}|\gamma_b = 0, \tag{6.22}$$

$$q_{\ell_a,t} + q_{\ell_b,t} = b_{\ell,t}, \tag{6.23}$$

$$q_{r_a,t} + q_{r_b,t} = b_{r,t}, \tag{6.24}$$

$$p_{\ell,t} \in [\underline{p}_\ell, \overline{p}_\ell],\; p_{r,t} \in [\underline{p}_r, \overline{p}_r], \tag{6.25}$$

$$q_{\ell_a,t} \in [\underline{q}_{\ell_a}, \overline{q}_{\ell_a}],\; q_{r_a,t} \in [\underline{q}_{r_a}, \overline{q}_{r_a}], q_{\ell_b,t} \in [\underline{q}_{\ell_b}, \overline{q}_{\ell_b}],\; q_{r_b,t} \in [\underline{q}_{r_b}, \overline{q}_{r_b}]. \tag{6.26}$$

The system of equations $\mathcal{F}_{agg}$ for the merged pipe c is given for all $t \in \mathcal{T}$ by

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r_c,t} - q_{\ell_c,t})\,\alpha_c\,\tau = 0, \tag{6.27}$$

$$p_{r,t}^2\,(1+\beta_c) - p_{\ell,t}^2\,(1-\beta_c) + (q_{r_c,t} + q_{\ell_c,t})|q_{r_c,t} + q_{\ell_c,t}|\gamma_c = 0, \tag{6.28}$$

$$q_{\ell_c,t} = b_{\ell,t}, \tag{6.29}$$

$$q_{r_c,t} = b_{r,t}, \tag{6.30}$$

$$p_{\ell,t} \in [\underline{p}_\ell, \overline{p}_\ell],\; p_{r,t} \in [\underline{p}_r, \overline{p}_r], q_{\ell_c,t} \in [\underline{q}_{\ell_c}, \overline{q}_{\ell_c}],\; q_{r_c,t} \in [\underline{q}_{r_c}, \overline{q}_{r_c}]. \tag{6.31}$$

We recall that the parameters of pipe a and pipe b are given by

$$\alpha_a := \frac{2\ \mathrm{c_s}^2}{A_a\,L_a}, \qquad \gamma_a := \frac{\lambda_a\ \mathrm{c_s}^2\ L_a}{4D_a A_a^2}, \qquad \beta_a := \frac{g\,(h_r - h_\ell)}{R_s T z},$$

$$\alpha_b := \frac{2\ \mathrm{c_s}^2}{A_b\,L_b}, \qquad \gamma_b := \frac{\lambda_b\ \mathrm{c_s}^2\ L_b}{4D_b A_b^2}, \qquad \beta_b := \frac{g\,(h_r - h_\ell)}{R_s T z},$$

with $\alpha_a, \alpha_b, \gamma_a, \gamma_b > 0$ and $\beta_a, \beta_b \in \mathbb{R}$.

An illustration of the parallel pipes a, b and the merged pipe c with corresponding variables for a single time step $t \in \mathcal{T}$ is provided in Figure 6.5. Considering multiple time steps, k-many parallel merges reduce the size of the model by $k \cdot |\mathcal{T}| \cdot 2$ variables (since only two flow variables are needed per time step instead of four) and $k \cdot |\mathcal{T}| \cdot 6$ constraints (one continuity equation and one momentum equation and lower and upper bounds of two flow variables), cf. Equations (6.19) – (6.31).

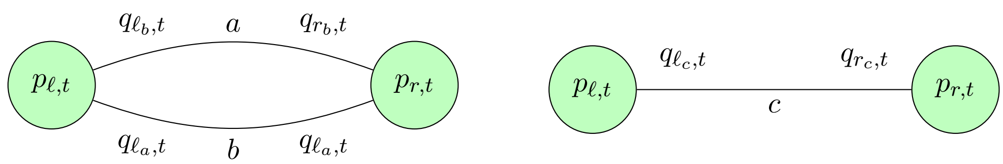


Figure 6.5: Two parallel pipes (left) and the merged pipe (right) at time step t with associated pressure variables $p_{\ell,t}, p_{r,t}$ and inflow and outflow variables $q_{\ell_a,t}, q_{r_a,t}$ and $q_{\ell_b,t}, q_{r_b,t}$ and $q_{\ell_c,t}, q_{r_c,t}$.

6.6.1 Merging approach

In this subsection, we present a merging approach that equivalently replaces the parallel pipes by the merged pipe. Equivalent replacements, as stated in Definition 6.1.1, means that a solution of the *Transient Control Problem* for the merged pipe c can be extended to a solution for the non-aggregated pipes a and b, and vice versa. Here, this translates to finding appropriate parameters $\alpha_c, \gamma_c > 0$ and $\beta_c \in \mathbb{R}$ such that $\mathcal{F}_{agg}$ and $\mathcal{F}_{orig}$ can be satisfied simultaneously.

Please note that the parallel – and also the serial – merging procedure might be applied to pipelines that already have been merged in the course of the aggregation and thus differ for example in length. For this reason, it is advantageous that we derive aggregated parameters $\alpha_c, \beta_c, \gamma_c$ instead of disaggregated values for each particular physical entity, like the diameter, length, and friction factor.

Let us start with the requirement to keep the difference in height along pipe $c = (\ell, r)$ as induced from pipes a and b leading to

$$\beta_c := \beta_a = \beta_b = g\,(h_r - h_\ell)/\,\mathrm{c}_\mathrm{s}^2\,.$$

On this basis, we determine α_c and γ_c such that the same pressure and flow realizations are obtained for the merged pipe as for the parallel pipes. Theorem 6.6.1 formalizes the concept of the parallel pipe merge.

Theorem 6.6.1. *Given two parallel pipes a and b between nodes ℓ and r, and pipe parameters $\alpha_a, \gamma_a > 0$ and $\alpha_b, \gamma_b > 0$. Under the assumption $\beta_c := \beta_a = \beta_b$, pipelines a and b can be equivalently replaced by a single pipe $c = (\ell, r)$ with*

$$\alpha_c = \frac{\alpha_a \cdot \alpha_b}{\alpha_a + \alpha_b}, \qquad \textit{and} \qquad \gamma_c = \frac{\gamma_b \cdot \gamma_a}{\left(\sqrt{\gamma_b} + \sqrt{\gamma_a}\right)^2}.$$

Proof. We restrict the proof to one time step. Applying the arguments sequentially to all time steps then completes the proof. The proof contains two steps. At first we deduce parameter α_c from the continuity Equations (6.19), (6.21) and (6.27) and secondly we derive parameter γ_c from the momentum Equations (6.20), (6.22) and (6.28). To this end, we deduce from Equations (6.23), (6.24) and (6.29), (6.30):

$$q_{\ell_c,t} = q_{\ell_a,t} + q_{\ell_b,t} \qquad \forall\, t \in \mathcal{T}, \quad \text{and} \tag{6.32}$$

$$q_{r_c,t} = q_{r_a,t} + q_{r_b,t} \qquad \forall\, t \in \mathcal{T}, \tag{6.33}$$

and set $\underline{q}_{c_a} := \underline{q}_{r_a} + \underline{q}_{r_b}$ and $\overline{q}_{c_a} := \overline{q}_{r_a} + \overline{q}_{r_b}$

Determine the pipe parameter α_c. Using Equations (6.32) and (6.33), we get

$$(q_{\ell_c,t} - q_{r_c,t})\,\alpha_c = (q_{\ell_a,t} + q_{\ell_b,t} - q_{r_a,t} - q_{r_b,t})\,\alpha_c. \tag{6.34}$$

All Equations (6.19), (6.21) and (6.27) contain the expression $p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1}$, hence we deduce from (6.19) and (6.21):

$$(q_{\ell_a,t} - q_{r_a,t})\,\alpha_a = (q_{\ell_b,t} - q_{r_b,t})\,\alpha_b \tag{6.35}$$

and from Equations (6.19) and (6.27):

$$\begin{aligned} & (q_{\ell_c,t} - q_{r_c,t})\,\alpha_c = (q_{\ell_a,t} - q_{r_a,t})\,\alpha_a \\ \overset{(6.34)}{\iff} \quad & (q_{\ell_a,t} - q_{r_a,t})\,\alpha_c + (q_{\ell_b,t} - q_{r_b,t})\,\alpha_c = (q_{\ell_a,t} - q_{r_a,t})\,\alpha_a \\ \iff \quad & (q_{\ell_a,t} - q_{r_a,t})\,(\alpha_a - \alpha_c) = (q_{\ell_b,t} - q_{r_b,t})\,\alpha_c. \end{aligned} \tag{6.36}$$

Finally, we conclude from Equations (6.35) and (6.36):

$$\begin{aligned} \Rightarrow \quad & \frac{\alpha_a - \alpha_c}{\alpha_c} = \frac{\alpha_a}{\alpha_b} \\ \Rightarrow \quad & \boxed{\alpha_c = \frac{\alpha_a \alpha_b}{\alpha_a + \alpha_b}}. \end{aligned}$$

Determine the pipe parameter γ_c. Given that $\beta_a = \beta_b$, it follows from Equations (6.20) and (6.22):

$$(q_{r_a,t} + q_{\ell_a,t})|q_{r_a,t} + q_{\ell_a,t}|\gamma_a = (q_{r_b,t} + q_{\ell_b,t})|q_{r_b,t} + q_{\ell_b,t}|\gamma_b. \tag{6.37}$$

From Equation (6.37) it follows together with $\gamma_a, \gamma_b > 0$ that

$$\operatorname{sgn}(q_{r_a,t} + q_{\ell_a,t}) = \operatorname{sgn}(q_{r_b,t} + q_{\ell_b,t}) \tag{6.38}$$

holds for all $t \in \mathcal{T}$. Using (6.38), then Equation (6.37) transforms to

$$q_{r_a,t} + q_{\ell_a,t} = \sqrt{\frac{\gamma_b}{\gamma_a}}(q_{r_b,t} + q_{\ell_b,t}). \tag{6.39}$$

Provided that $\beta_c = \beta_b$, we further deduce from Equations (6.22) and (6.28):

$$\begin{aligned} & (q_{r_b,t} + q_{\ell_b,t})|q_{r_b,t} + q_{\ell_b,t}|\gamma_b \\ = \quad & (q_{r_c,t} + q_{\ell_c,t})|q_{r_c,t} + q_{\ell_c,t}|\gamma_c \\ = \quad & (q_{r_a,t} + q_{r_b,t} + q_{\ell_a,t} + q_{\ell_b,t})|q_{r_a,t} + q_{r_b,t} + q_{\ell_a,t} + q_{\ell_b,t}|\gamma_c \\ \overset{(6.39)}{=} \quad & \left(\sqrt{\frac{\gamma_b}{\gamma_a}}(q_{r_b,t} + q_{\ell_b,t}) + q_{r_b,t} + q_{\ell_b,t}\right) \\ & \cdot \left| \left(\sqrt{\frac{\gamma_b}{\gamma_a}}(q_{r_b,t} + q_{\ell_b,t}) + q_{r_b,t} + q_{\ell_b,t}\right) \right| \gamma_c \\ \overset{\gamma_a,\gamma_b>0}{=} & (q_{r_b,t} + q_{\ell_b,t})|q_{r_b,t} + q_{\ell_b,t}| \left(1 + \sqrt{\frac{\gamma_b}{\gamma_a}}\right)^2 \gamma_c \\ \Rightarrow \quad & \gamma_c = \frac{\gamma_b}{\left(1 + \sqrt{\frac{\gamma_b}{\gamma_a}}\right)^2} \quad = \quad \frac{\gamma_a\,\gamma_b}{\gamma_a + \gamma_b + 2\,\sqrt{\gamma_a \gamma_b}} \\ \Rightarrow \quad & \boxed{\gamma_c = \frac{\gamma_a\,\gamma_b}{\left(\sqrt{\gamma_a} + \sqrt{\gamma_b}\right)^2}}. \end{aligned}$$

□

This merging procedure indeed preserves the accumulated volume and line-pack of both parallel pipes, as stated by the following corollary.

Corollary 6.6.2. *Given two parallel pipes a and b between nodes ℓ and r. Applying the parallel pipe merge from Theorem 6.6.1, then the volume vol_c and the line-pack $LP_{c,t}$ of the merged pipe $c = (\ell, r)$ hold*

i) $vol_c = vol_a + vol_b$, *and*

ii) $LP_{c,t} = LP_{a,t} + LP_{b,t}$.

Proof. (i) The volume of pipes a and b is given by

$$vol_a = L_a\, A_a \qquad \text{and} \qquad vol_b = L_b\, A_b.$$

By virtue of parameter $\alpha_c = 2\, \mathrm{c_s}^2 / vol_c$, we deduce from Theorem 6.6.1:

$$\begin{aligned}
\alpha_c = \frac{\alpha_a \cdot \alpha_b}{\alpha_a + \alpha_b} \quad &\Rightarrow \quad vol_c^{-1} = \frac{vol_a^{-1} \cdot vol_b^{-1}}{vol_a^{-1} + vol_b^{-1}} \\
&\Rightarrow \quad vol_c \ = \frac{vol_a^{-1} + vol_b^{-1}}{vol_a^{-1} \cdot vol_b^{-1}} \ = \ \frac{vol_b + vol_a}{vol_a \cdot vol_b} \cdot vol_a \cdot vol_b \\
&\Rightarrow \quad vol_c \ = \ vol_a \ + \ vol_b.
\end{aligned}$$

(ii) Using $vol_c = vol_a + vol_b$ and the line-pack formula (6.8), where the mean pressure is approximated by (6.7), then $p^{mean}_{c,t} = p^{mean}_{a,t} = p^{mean}_{b,t}$ holds, and consequently

$$LP_{c,t} = \frac{p^{mean}_{c,t}}{\mathrm{c_s}^2}\, vol_c = \frac{p^{mean}_{a,t}}{\mathrm{c_s}^2}\, vol_a + \frac{p^{mean}_{b,t}}{\mathrm{c_s}^2}\, vol_b = LP_{a,t} + LP_{b,t}.$$

□

6.6.2 Relation of transient and stationary parallel pipe merging

We conclude this section by showing the analogy between transient and stationary parallel pipe merges. Merging parallel pipelines in the stationary setting has been independently investigated in Lenz and Schwarz (2016) and Groß et al. (2019). For the sake of readability, we distinguish between the parameters $\gamma^{st}_a, \beta^{st}_a$ and $\gamma^{tr}_a, \beta^{tr}_a$ for the stationary and transient case in the remainder of this section. Let us recall that a common approximation for the interdependency of pressure and flow along a pipeline $a = (\ell, r)$ is governed by the stationary Weymouth equation

$$p_r^2 - \beta^{st}_a\, p_\ell^2 + \gamma^{st}\, q_a |q_a| = 0. \tag{6.40}$$

Without explicitly stating the parameters $\gamma^{st}_a, \beta^{st}_a > 0$ here, we mention that $\beta^{st}_a = 1$ represents a horizontal pipe in the stationary case and refer to Fügenschuh et al. (2015) for further information.

Proposition 6.6.3. *Given two parallel pipes a and b between nodes ℓ and r, and pipe characteristics $\gamma_a^{st}, \gamma_b^{st}, \beta_a^{st}, \beta_b^{st} > 0$. Then, both pipes can be equivalently replaced by a single pipe $c = (\ell, r)$ with*

$$\beta_c^{st} = \beta_a^{st} = \beta_b^{st} \quad \text{and} \quad \gamma_c^{st} = \frac{\gamma_b^{st}\,\gamma_a^{st}}{\left(\sqrt{\gamma_b^{st}} + \sqrt{\gamma_a^{st}}\right)^2}.$$

Proof. When two pipes appear parallel to each other, each one is represented by Equation (6.40), yielding:

$$p_\ell^2 - \beta_a^{st}\,p_r^2 = \gamma_a^{st}\,q_a\,|q_a|, \quad \text{and}$$
$$p_\ell^2 - \beta_b^{st}\,p_r^2 = \gamma_b^{st}\,q_b\,|q_b|.$$

We transform this into an equivalent single equation $p_\ell^2 - \beta_c^{st}\,p_r^2 = \gamma_c^{st}\,q_c\,|q_c|$ with respect to the aggregated flow $q_c = q_a + q_b$. Since all pipes have the same pressure difference, we get:

$$\beta_c^{st} = \beta_a^{st} = \beta_b^{st}$$

and

$$\gamma_a^{st}\,q_a\,|q_a| = \gamma_b^{st}\,q_b\,|q_b| = \gamma_c^{st}\,q_c\,|q_c|.$$

Provided that $\gamma_a^{st}, \gamma_b^{st} > 0$, both parallel pipes have the same flow direction, i.e., $\operatorname{sgn}(q_a) = \operatorname{sgn}(q_b)$. Therefore, it follows

$$\begin{aligned}
\gamma_a^{st}\,q_a^2 = \gamma_b^{st}\,q_b^2 \quad &\Rightarrow \quad q_a = \frac{\sqrt{\gamma_b^{st}}}{\sqrt{\gamma_a^{st}}}\,q_b \\
&\overset{q = q_a + q_b}{\Rightarrow} \quad q_c = \left(\frac{\sqrt{\gamma_b^{st}}}{\sqrt{\gamma_a^{st}}} + 1\right) q_b \\
\gamma_b^{st}\,q_b^2 = \gamma_c^{st}\,q_c^2 \quad &\Rightarrow \quad \gamma_c^{st} = \frac{\gamma_b^{st}\,q_b^2}{\left(\left(\frac{\sqrt{\gamma_b^{st}}}{\sqrt{\gamma_a^{st}}} + 1\right) q_b\right)^2} \\
&\Rightarrow \quad \gamma_c^{st} = \frac{\gamma_b^{st}}{\left(\frac{\sqrt{\gamma_b^{st}} + \sqrt{\gamma_a^{st}}}{\sqrt{\gamma_a^{st}}}\right)^2} \\
&\Rightarrow \quad \gamma_c^{st} = \frac{\gamma_b^{st}\,\gamma_a^{st}}{\left(\sqrt{\gamma_b^{st}} + \sqrt{\gamma_a^{st}}\right)^2}.
\end{aligned}$$

□

In summary, we can state that the parallel pipe merge is calculated in the same way as in the transient and stationary case by

$$\gamma_c^{tr} = \frac{\gamma_b^{tr}\,\gamma_a^{tr}}{\left(\sqrt{\gamma_b^{tr}} + \sqrt{\gamma_a^{tr}}\right)^2} \quad \text{and} \quad \gamma_c^{st} = \frac{\gamma_b^{st}\,\gamma_a^{st}}{\left(\sqrt{\gamma_b^{st}} + \sqrt{\gamma_a^{st}}\right)^2}.$$

6.7 Merging Serial Pipes

In this section, we investigate serial pipe merging under transient conditions. But other than the parallel merge, we show that it is not possible to equivalently replace two serial pipes by one pipe that throughout admits the same pressure and flow realizations. However, solving transient gas network optimization problems benefits from reducing the network size by applying serial merges. For this reason, we introduce a new method that approximates the feasible region of the original system of equations by using a sampling-based regression approach. More precisely, the approach samples the feasible region of the original system and determines a parameter of a new reduced, but structurally equivalent system that minimizes the error incurred through sampling. Afterwards, we evaluate this method on different pairs of serial pipes using real-world data. It turns out that this method performs tremendously well in the sense that it approximates the solution space of the original system very adequately. Finally, we bear an analogy to the serial merge from the stationary setting.

We start with a brief introduction about the orientation of serial pipes, followed by a presentation of the system of equations involved, before we state a counterexample to equivalent serial pipe merging.

Orientation of arcs. We shortly recall the modeling of flow variables from Chapter 5. In the remainder of this section, we consider two serial pipes $a = (\ell, m)$ and $b = (m, r)$ as a stand-alone subnetwork, where node $m \in \mathcal{V}^*$ is a transshipment node and has $deg(m) = 2$. Each pipeline induces two flow variables at time step $t \in \mathcal{T}$, which indicate the amount of flow entering and leaving the pipeline, see Figure 6.6. Given that we model flow variables towards the orientation of the arc with positive sign, the flow conservation constraints at these three nodes read

$$\begin{aligned} q_{\ell,t} &= b_{\ell,t}, \\ q_{m_b,t} - q_{m_a,t} &= b_{m,t}, \\ -q_{r,t} &= b_{r,t}, \end{aligned}$$

where demand values $b_{v,t} \geq 0$ correspond to inflow and $b_{v,t} \leq 0$ to outflow values from the remaining network at $v \in \{\ell, r\}$. Since $deg(m) = 2$ and $m \in \mathcal{V}^*$ (i.e., $b_{m,t} = 0$ for all $t \in \mathcal{T}$), it is possible to eliminate one flow variable by introducing $q_{m,t}$ and substituting $q_{m,t} := q_{m_a,t} = -q_{m_b,t}$. Considering multiple time steps, then, after eliminating the flow conservation constraint at node m and one flow variable, k-many serial merges reduce the size of the model by $k \cdot |\mathcal{T}| \cdot 2$ variables (where the two variables are given by $p_{m,t}$ and $q_{m,t}$) and $k \cdot |\mathcal{T}| \cdot 6$ constraints (one continuity equation and one momentum equation and lower and upper bounds of $p_{m,t}$ and $q_{m,t}$), cf. Equations (6.41) – (6.53).

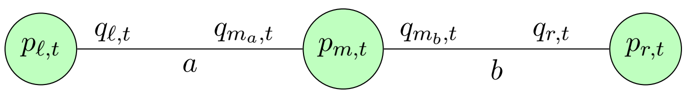


Figure 6.6: Illustration of the variables present at modeling serial pipes a and b at time step t.

System of equations for the serial and merged pipes. For both pipes $a = (\ell, m)$ and $b = (m, r)$, a system of equations $\mathcal{F}_{orig}$ represents the corresponding discretized continuity and momentum equations, flow conservation at nodes ℓ and r and variable bounds. Note that by using $q_{m,t}$ from above, the flow conservation at node m is already enforced. The serial merge substitutes both pipes for an artificial pipe $c = (\ell, r)$. Here, the merge replaces $\mathcal{F}_{orig}$ by another system $\mathcal{F}_{agg}$ that represents the discretized continuity and momentum equation of the merged pipe c, the flow conservation at its end nodes and variable bounds.

The system of equations $\mathcal{F}_{orig}$ for pipes a and b is given for all $t \in \mathcal{T}$ by

$$p_{m,t} - p_{m,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{m,t} - q_{\ell,t})\,\alpha_a\,\tau = 0, \tag{6.41}$$

$$p_{m,t}^2(1 + \beta_a) - p_{\ell,t}^2(1 - \beta_a) + (q_{m,t} + q_{\ell,t})\,|q_{m,t} + q_{\ell,t}|\gamma_a = 0, \tag{6.42}$$

$$p_{r,t} - p_{r,t-1} + p_{m,t} - p_{m,t-1} + (q_{r,t} - q_{m,t})\,\alpha_b\,\tau = 0, \tag{6.43}$$

$$p_{r,t}^2(1 + \beta_b) - p_{m,t}^2(1 - \beta_b) + (q_{r,t} + q_{m,t})\,|q_{r,t} + q_{m,t}|\gamma_b = 0, \tag{6.44}$$

$$q_{\ell,t} = b_{\ell,t}, \tag{6.45}$$

$$-q_{r,t} = b_{r,t}, \tag{6.46}$$

$$p_{\ell,t} \in [\underline{p}_\ell, \overline{p}_\ell],\ p_{m,t} \in [\underline{p}_m, \overline{p}_m],\ p_{r,t} \in [\underline{p}_r, \overline{p}_r], \tag{6.47}$$

$$q_{\ell,t} \in [\underline{q}_\ell, \overline{q}_\ell],\ q_{m,t} \in [\underline{q}_m, \overline{q}_m], q_{r,t} \in [\underline{q}_r, \overline{q}_r]. \tag{6.48}$$

The system of equations $\mathcal{F}_{agg}$ of the merged pipe $c = (\ell, r)$ is given for all $t \in \mathcal{T}$ by

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r,t} - q_{\ell,t})\,\alpha_c\,\tau = 0, \tag{6.49}$$

$$p_{r,t}^2(1 + \beta_c) - p_{\ell,t}^2(1 - \beta_c) + (q_{r,t} + q_{\ell,t})\,|q_{r,t} + q_{\ell,t}|\gamma_c = 0, \tag{6.50}$$

$$q_{\ell,t} = b_{\ell,t}, \tag{6.51}$$

$$-q_{r,t} = b_{r,t}, \tag{6.52}$$

$$p_{\ell,t} \in [\underline{p}_\ell, \overline{p}_\ell],\ p_{r,t} \in [\underline{p}_r, \overline{p}_r],\ q_{\ell,t} \in [\underline{q}_\ell, \overline{q}_\ell],\ q_{r,t} \in [\underline{q}_r, \overline{q}_r]. \tag{6.53}$$

We recall that the parameters of pipe a and pipe b are given by

$$\alpha_a := \frac{2\ \mathrm{c_s}^2}{A_a\, L_a}, \qquad \gamma_a := \frac{\lambda\, \mathrm{c_s}^2\ L_b}{4 D_b A_b^2}, \qquad \beta_a := \frac{g\,(h_m - h_\ell)}{\mathrm{c_s}^2},$$

$$\alpha_b := \frac{2\ \mathrm{c_s}^2}{A_b\, L_b}, \qquad \gamma_b := \frac{\lambda\, \mathrm{c_s}^2\ L_b}{4 D_b A_b^2}, \qquad \beta_b := \frac{g\,(h_r - h_m)}{\mathrm{c_s}^2},$$

with $\alpha_a, \alpha_b, \gamma_a, \gamma_b > 0$ and $\beta_a, \beta_b \in \mathbb{R}$.

As with the parallel merge, the serial merge requires determining parameters α_c, β_c and γ_c in system $\mathcal{F}_{agg}$. In the following, we show that α_c and β_c follow naturally from the conditions to preserve the volume and difference in height of the serial pipes. In contrast, the calculation of γ_c is more sophisticated and is part of the regression-based merging method in Subsection 6.7.2.

Concerning α_c: The property to keep the volume of both serial pipes reads $vol_c = vol_a + vol_b$. Given that the volume can be extracted from the parameters

$$\alpha_a = \frac{2\ \mathrm{c_s}^2}{vol_a} \quad \text{and} \quad \alpha_b = \frac{2\ \mathrm{c_s}^2}{vol_b},$$

it is possible to derive α_c from α_a and α_b:

$$vol_c = vol_a + vol_b \quad \Rightarrow \quad \alpha_c = \frac{2\ \mathrm{c_s}^2}{vol_a + vol_b} = \left(\frac{vol_a}{2\ \mathrm{c_s}^2} + \frac{vol_b}{2\ \mathrm{c_s}^2} \right)^{-1}$$

$$\Rightarrow \quad \boxed{\alpha_c = \left(\frac{1}{\alpha_a} + \frac{1}{\alpha_b} \right)^{-1}}. \tag{6.54}$$

Remark 6.7.1 (Serial pipes - line pack)**.** *Unlike parallel pipe merging, the serial merge might not necessarily retain the line-pack of the serial pipes, i.e., it might hold* $LP_{c,t} \neq LP_{a,t} + LP_{b,t}$*. Here, the line-pack is only preserved, if* $p^{mean}_{c,t} = p^{mean}_{a,t} + p^{mean}_{b,t}$*. This can be derived from the line-pack formula* (6.8) *together with* $vol_c = vol_a + vol_b$*.*

Concerning β_c: The difference in height $h_r - h_\ell$ along the serial pipes is modeled by the parameters β_a and β_b. Keeping this difference along the merged pipe c follows naturally by assuming that the parameters β_a and β_b are additive, i.e.,

$$\begin{aligned} & (h_r - h_m) + (h_m - h_\ell) = h_r - h_\ell \\ \Rightarrow \quad & \frac{g\,(h_r - h_m)}{\mathrm{c_s}^2} + \frac{g\,(h_m - h_\ell)}{\mathrm{c_s}^2} = \frac{g\,(h_r - h_\ell)}{\mathrm{c_s}^2}. \end{aligned}$$

By defining

$$\beta_c := \frac{g\,(h_r - h_\ell)}{\mathrm{c_s}^2},$$

then follows

$$\beta_c = \beta_a + \beta_b. \tag{6.55}$$

However, we remark that the geographic height at specific spatial coordinates might be different between the serial pipes and the merged pipe. This happens, for example, if both serial pipes have different slopes $s_a \neq s_b$.

6.7.1 Counterexample to equivalent transient serial pipe merging

In the previous subsection, we have seen that α_c and β_c are uniquely determined by retaining the volume and the difference in height of the serial pipes. Ideally and in addition to both requirements, the systems $\mathcal{F}_{orig}$ and $\mathcal{F}_{agg}$ can be satisfied at the same time. However, in the following we provide a counterexample to satisfying all these requirements simultaneously. To this end, we throughout consider horizontal pipes, which implies $\beta_a, \beta_b, \beta_c = 0$. To generate the counterexample, we first introduce some preliminary results about transient gas flows in a single pipe and in two serial pipes. We start by proving that the solution of system $\mathcal{F}_{agg}$ for a single horizontal pipe is unique, given an initial state and a demand. This fact is not self-evident, for example, Weltsch (2018) shows that the solution of the frequently used *implicit Box* discretization scheme is not necessarily unique for a single pipe.

Lemma 6.7.2 (Single horizontal pipe - uniqueness of solution)**.** *Let pipe $a = (\ell, r)$ with parameters $\alpha_a, \gamma_a > 0$, initial pressure values $p_{\ell,t_0}, p_{r,t_0} \geq 0$, and demand values $b_{\ell,t}$ and $b_{r,t}$ for all $t \in \mathcal{T}$ be given. Then, the following nonlinear system*

$$\begin{aligned} p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r,t} - q_{\ell,t})\,\alpha_a &= 0 \quad && \forall\, t \in \mathcal{T} \\ p_{r,t}^2 - p_{\ell,t}^2 + (q_{r,t} + q_{\ell,t})\,|q_{r,t} + q_{\ell,t}|\gamma_a &= 0 && \forall\, t \in \mathcal{T} \\ q_{\ell,t} &= b_{\ell,t} && \forall\, t \in \mathcal{T} \\ -q_{r,t} &= b_{r,t} && \forall\, t \in \mathcal{T} \end{aligned}$$

has a unique solution $(p_{\ell,t}, p_{r,t}, q_{\ell,t}, q_{r,t})_{t \in \mathcal{T}}$ with non-negative values $p_{\ell,t} \in [\underline{p}_\ell, \overline{p}_\ell]$ and $p_{r,t} \in [\underline{p}_r, \overline{p}_r]$, or it is infeasible.

Proof. We show that the pressure values $p_{\ell,t}, p_{r,t}$ are uniquely determined for given values $p_{\ell,t-1}, p_{r,t-1}$ and $b_{\ell,t}, b_{r,t}$ at an arbitrary time step $t \in \mathcal{T}$. Then, starting with the first time step $t = 1$ and applying this argument iteratively to two consecutive time steps via induction, completes the proof.

For a single step t, the discretized continuity and momentum equations reduce to

$$p_{r,t} = s_1 - p_{\ell,t}, \tag{6.56}$$

$$p_{r,t}^2 - p_{\ell,t}^2 + s_2 = 0, \tag{6.57}$$

where the parameters s_1, s_2 are given by

$$\begin{aligned} s_1 &:= p_{r,t-1} + p_{\ell,t-1} - (q_{r,t} - q_{\ell,t})\,\alpha_a, \\ s_2 &:= (q_{r,t} + q_{\ell,t})\,|q_{r,t} + q_{\ell,t}|\gamma_a. \end{aligned}$$

Combining Equations (6.56) and (6.57) yields

$$\begin{aligned} &\Rightarrow \quad (s_1 - p_{\ell,t})^2 - p_{\ell,t}^2 + s_2 = 0 \\ &\Rightarrow \quad p_{\ell,t} = \frac{s_1^2 + s_2}{2\,s_1}, \quad \text{if } s_1 \neq 0. \end{aligned}$$

Hence, $p_{\ell,t}$ is uniquely defined only depending on the parameters $s_1 \neq 0$ and s_2. Then, $p_{r,t}$ is also uniquely defined by virtue of Equation (6.57). On the other hand, if $s_1 = 0$, then (6.56) reduces to $p_{r,t} = -p_{\ell,t}$, which, according to the non-negativity requirement of the pressure variables is only the case, if $p_{r,t} = p_{\ell,t} = 0$. Finally, all uniquely determined values $p_{r,t}, p_{\ell,t}$ are either non-negative and within the bounds, or the system is infeasible. □

Let us now come back to two serial pipes. The following lemma characterizes stationary behavior in a time-dependent setting. More precisely, given a constant and balanced demand over time and initial pressure values in steady-state, then there exists a time-dependent solution in steady-state.

Lemma 6.7.3 (Serial horizontal pipes - stationary behavior in a time-dependent setting)**.** *Let two serial pipes $a = (\ell, m)$ and $b = (m, r)$ be given with $\beta_a, \beta_b = 0$ and $\gamma_a, \gamma_b > 0$ and let $deg(m) = 2$ and $b_{m,t} = 0$ for all $t \in \mathcal{T}_0$. Further, let be given*

i) a constant and balanced demand $b_{\ell,t} := \hat{q}$ and $b_{r,t} := -\hat{q}$ with $q_{\ell,t}, q_{r,t} := \hat{q}$ and $\underline{q}_\ell \leq q_{\ell,t} \leq \overline{q}_\ell$ and $\underline{q}_r \leq q_{r,t} \leq \overline{q}_r$ for all $t \in \mathcal{T}_0$, and

ii) initial pressure values $p_{\ell,t_0} \in [\underline{p}_\ell, \overline{p}_\ell]$ and $p_{m,t_0} \in [\underline{p}_m, \overline{p}_m]$ and $p_{r,t_0} \in [\underline{p}_r, \overline{p}_r]$ in steady-state, i.e.,

$$p_{m,t_0} = \left(p_{\ell,t_0}^2 - (q_{m,t_0} + q_{\ell,t_0})\,|q_{m,t_0} + q_{\ell,t_0}|\,\gamma_a\right)^{1/2}, \tag{6.58}$$

$$p_{r,t_0} = \left(p_{m,t_0}^2 - (q_{r,t_0} + q_{m,t_0})\,|q_{r,t_0} + q_{m,t_0}|\,\gamma_b\right)^{1/2}. \tag{6.59}$$

Then, $\left(p_{\ell,t}, p_{m,t}, p_{r,t}, q_{\ell,t}, q_{m,t}, q_{r,t}\right)_{t\in\mathcal{T}}$ with $q_{m,t} := q_{\ell,t}, q_{r,t}$ and $p_{\ell,t} := p_{\ell,t_0}$ and $p_{m,t} := p_{m,t_{t_0}}$ and $p_{r,t} := p_{r,t_0}$ is a solution of system $\mathcal{F}_{orig}$.

Proof. Setting $q_{m,t} = \hat{q}, p_{\ell,t} = p_{\ell,t_0}$, $p_{m,t} = p_{m,t_0}, p_{r,t} = p_{r,t_0}$ as given in (6.58) – (6.59) together with $q_{\ell,t}, q_{r,t} = \hat{q}$ for all $t \in \mathcal{T}$ implies that $\left(p_{\ell,t}, p_{m,t}, p_{r,t}, q_{\ell,t}, q_{m,t}, q_{r,t}\right)_{t\in\mathcal{T}}$ is a solution of system (6.41) – (6.48). □

Under the premise of Lemma 6.7.3, it is possible to derive γ_c such that the merged pipe and serial pipes entail the same pressure and flow realizations. The following corollary formalizes this claim and explicitly states a formula for γ_c.

Corollary 6.7.4. *Consider a horizontal pipe $c = (\ell, r)$ that is split into two pipe segments $a = (\ell, m)$ and $b = (m, r)$ with $\beta_a, \beta_b = 0$ and $\gamma_a, \gamma_b > 0$ and let $deg(m) = 2$ and $b_{m,t} = 0$ for all $t \in \mathcal{T}_0$. Further, let be given*

i) a constant and balanced demand $b_{\ell,t} := \hat{q}$ and $b_{r,t} := -\hat{q}$ with $q_{\ell,t}, q_{r,t} := \hat{q}$ and $\underline{q}_\ell \leq q_{\ell,t} \leq \overline{q}_\ell$ and $\underline{q}_r \leq q_{r,t} \leq \overline{q}_r$ for all $t \in \mathcal{T}_0$, and

ii) initial pressure values $p_{\ell,t_0} \in [\underline{p}_\ell, \overline{p}_\ell]$ and $p_{m,t_0} \in [\underline{p}_m, \overline{p}_m]$ and $p_{r,t_0} \in [\underline{p}_r, \overline{p}_r]$ in steady-state, i.e.,

$$p_{m,t_0} = \left(p_{\ell,t_0}^2 - (q_{m,t_0} + q_{\ell,t_0})\,|q_{m,t_0} + q_{\ell,t_0}|\gamma_a\right)^{1/2},$$
$$p_{r,t_0} = \left(p_{m,t_0}^2 - (q_{r,t_0} + q_{m,t_0})\,|q_{r,t_0} + q_{m,t_0}|\gamma_b\right)^{1/2}.$$

Then, for the solution of system $\mathcal{F}_{orig}$ stated in Lemma 6.7.3 follows

$$\gamma_c = \gamma_a + \gamma_b.$$

Proof. The momentum equations for pipes a, b and c reduce to

$$\begin{aligned}
&\begin{cases} p_{m,t}^2 - p_{\ell,t}^2 + 4\,\hat{q}^2\,\gamma_a = 0 & \forall t \in \mathcal{T}, \\ p_{r,t}^2 - p_{m,t}^2 + 4\,\hat{q}^2\,\gamma_b = 0 & \forall t \in \mathcal{T}, \end{cases} \\
\Rightarrow\quad & p_{r,t}^2 - p_{\ell,t}^2 + 4\,\hat{q}^2(\gamma_a + \gamma_b) = 0 \quad \forall t \in \mathcal{T}, \\
\Rightarrow\quad & \gamma_c = \gamma_a + \gamma_b.
\end{aligned}$$

□

In Subsection 6.7.4, we show that merging serial pipes in the stationary case also yields $\gamma_c = \gamma_a + \gamma_b$, cf. Equation (6.69). Nevertheless, it is not possible to establish this relation in general. It even suffices to slightly distort a balanced demand of a system in steady-state in order to generate a counterexample. The counterexample works as follows: We consider two equal horizontal serial pipes, i.e., $\alpha_a = \alpha_b$, $\gamma_a = \gamma_b$ and $\beta_a = \beta_b = 0$ and assume that the stationary assumptions from Corollary 6.7.4 (and Lemma 6.7.3) hold, i.e., (i) a constant and balanced demand over time, and (ii) initial pressure values in steady-state. As we know from Corollary 6.7.4, these assumptions allow us to derive a parameter γ_c that establishes the same pressure and flow realizations for the merged pipe c as for the serial pipes a and b. Afterwards, we slightly change the demand, which finally requires a different parameter $\tilde{\gamma_c} \neq \gamma_c$ in order to obtain the same pressure and flow profiles.

Example 6.7.5 (Counterexample to equivalent transient serial pipe merging)**.** *Consider a stand-alone network consisting of two horizontal pipes $a = (\ell, m)$ and $b = (m, r)$ in serial, with $\alpha_a, \alpha_b = 1$, $\gamma_a, \gamma_b = 1$ and $\beta_a, \beta_b = 0$ and $\tau = 1$ for a single time step from t_0 to t_1. Let a demand be given by $b_{\ell,t} = 5$, $b_{r,t} = -5$ and $b_{m,t} = 0$, which, for this network translates to $\hat{q}_{\ell,t_1}, \hat{q}_{r,t_1} = 5$. Further, let an initial state be given that fulfills stationary conditions*

$$\hat{p}_{\ell,t_0} \approx 50.00, \quad \hat{p}_{m,t_0} \approx 48.99, \quad \hat{p}_{r,t_0} \approx 47.96, \tag{6.60}$$

(rounded to two decimal places) with respect to flow values $\hat{q}_{\ell,t_0} = \hat{q}_{m,t_0} = \hat{q}_{r,t_0} = 5$. Then, from Lemma 6.7.3 follows that a solution of system $\mathcal{F}_{orig}$ for both serial pipes a and b is given by

$$\hat{p}_{\ell,t_1} = \hat{p}_{\ell,t_0}, \quad \hat{p}_{m,t_1} = \hat{p}_{m,t_0}, \quad \hat{p}_{r,t_1} = \hat{p}_{r,t_0}, \quad \textit{and} \quad \hat{q}_{m,t_1} = 5.$$

Besides, the assumptions of Corollary 6.7.4 are satisfied, and hence the parameter γ_c for the merged pipe c holds

$$\gamma_c = \gamma_a + \gamma_b = 2. \tag{6.61}$$

Then, for γ_c as given in (6.61) and any choice of $\alpha_c \geq 0$, $(\hat{p}_{\ell,t_1}, \hat{p}_{r,t_1}, \hat{q}_{\ell,t_1}, \hat{q}_{r,t_1})$ is the unique solution of system $\mathcal{F}_{agg}$ for the merged pipe c according to Lemma 6.7.2.

However, when changing the demand to

$$\tilde{q}_{\ell,t_1} = 10, \quad \tilde{q}_{r,t_1} = 5,$$

while keeping the initial conditions of both serial pipes from (6.60), the solution of system $\mathcal{F}_{orig}$ (for the serial pipes) changes to

$$\tilde{p}_{\ell,t_1} \approx 52.57, \quad \tilde{p}_{m,t_1} \approx 49.91, \quad \tilde{p}_{r,t_1} \approx 48.56, \quad \tilde{q}_{m,t_1} \approx 6.52.$$

But the solution $(\tilde{p}_{\ell,t_1}, \tilde{p}_{r,t_1} \tilde{q}_{\ell,t_1}, \tilde{q}_{r,t_1})$ cannot be realized with γ_c as determined in Equation (6.61), since Equation (6.50) together with given values $\tilde{p}_{\ell,t_1}, \tilde{p}_{r,t_1}, \tilde{q}_{\ell,t_1}$ and $\tilde{q}_{r,t_1}$ uniquely determines $\tilde{\gamma_c} \approx 1.8 \neq 2$.

6.7.2 Sampling based regression approach

Motivated by the counterexample above, we develop a sampling-based merging approach that minimizes the error incurred. At first, we sample the feasible region of system $\mathcal{F}_{orig}$. Then, we use a least-squares regression to acquire a linear model. The model returns a parameter γ_c that minimizes the violation of the samples with respect to system $\mathcal{F}_{agg}$.

Sampling procedure. We are given two serial pipes $a = (\ell, m)$ and $b = (m, r)$ with corresponding parameters $\alpha_a, \alpha_b, \beta_a, \beta_b, \gamma_a, \gamma_b$ and a time horizon $\mathcal{T}_0$. In the following, we generate a sampling set $\mathcal{S} := \{s_k \mid k \in \mathcal{K}\}$ that approximates the feasible region of the serial pipes' system $\mathcal{F}_{orig}$. Here, we use index $k \in \mathcal{K}$ to denote a particular sample s_k. A single sample $s_k \in \mathcal{S}$ reads

$$\left(p^k_{\ell,t_0}, ..., p^k_{\ell,t_n}, p^k_{m,t_0}, ..., p^k_{m,t_n}, p^k_{r,t_0}, ..., p^k_{r,t_n}, q^k_{\ell,t_1}, ..., q^k_{\ell,t_n}, q^k_{m,t_1}, ..., q^k_{m,t_n}, q^k_{r,t_1}, ..., q^k_{r,t_n}\right),$$

where its domain is given by

$$s_k \in [\underline{p}_\ell, \overline{p}_\ell]^{|\mathcal{T}_0|} \times [\underline{p}_m, \overline{p}_m]^{|\mathcal{T}_0|} \times [\underline{p}_r, \overline{p}_r]^{|\mathcal{T}_0|} \times [\underline{q}_\ell, \overline{q}_\ell]^{|\mathcal{T}|} \times [\underline{q}_m, \overline{q}_m]^{|\mathcal{T}|} \times [\underline{q}_r, \overline{q}_r]^{|\mathcal{T}|}.$$

We now describe the sampling procedure in more detail, as sketched in Algorithm 4. The generation of a particular sample s_k consists of the following three steps:

i) generate initial pressure values $p^k_{\ell,t_0}, p^k_{m,t_0}, p^k_{r,t_0}$ by GenerateInitialState,

ii) generate demand values $\left(q^k_{\ell,t}, q^k_{r,t}\right)_{t\in\mathcal{T}}$ by GenerateDemand,

iii) generate remaining values $\left(p^k_{\ell,t}, p^k_{m,t}, p^k_{r,t}, q^k_{m,t}\right)_{t\in\mathcal{T}}$ by CompleteSample.

In order to imitate a "more realistic" physical behavior than sampling arbitrary values within the bounds, we build each sample upon an initial state that fulfills stationary conditions. To this end, we select a random flow value $\hat{q}^k_{t_0} \in [\underline{q}, \overline{q}] := [\underline{q}_\ell, \overline{q}_\ell] \cap [\underline{q}_m, \overline{q}_m] \cap [\underline{q}_r, \overline{q}_r]$ and set $q^k_{\ell,t_0}, q^k_{m,t_0}, q^k_{r,t_0} := \hat{q}^k_{t_0}$ in GenerateInitialState. In this way, a flow direction along both pipes is predetermined at t_0, and thus, it either holds $p^k_{\ell,t_0} > p^k_{m,t_0} > p^k_{r,t_0}$ or $p^k_{r,t_0} > p^k_{m,t_0} > p^k_{\ell,t_0}$. To initialize these pressure values, we first generate a random value for the pressure variable that is supposed to have the highest value, i.e., $p^k_{\ell,t_0} \in [\underline{p}_\ell, \overline{p}_\ell]$ in the case of $\text{sgn}(\hat{q}^k_{t_0}) > 0$, and $p^k_{r,t_0} \in [\underline{p}_r, \overline{p}_r]$ in the case of $\text{sgn}(\hat{q}^k_{t_0}) < 0$. Then, we use the given flow value $\hat{q}^k_{t_0}$ together with the discretized momentum equation to successively determine the remaining initial pressure values, see Algorithm 4. This flow value also serves for the generation of the demand. Based on q^k_{ℓ,t_0} and q^k_{r,t_0}, we derive random flow values $q^k_{\ell,t}$ and $q^k_{r,t}$ for all time steps $t \in \mathcal{T}$ that are only allowed to vary within a predefined range from their respective predecessors, i.e., $q^k_{\ell,t} \in [q^k_{\ell,t-1} - q_\epsilon, q^k_{\ell,t-1} + q_\epsilon]$ and $q^k_{r,t} \in [q^k_{r,t-1} - q_\epsilon, q^k_{r,t-1} + q_\epsilon]$ in GenerateDemand. This assumption seems reasonable in order to avoid a completely arbitrary demand profile. In this context, we also refer to the upper Sub-figures 6.8 – 6.9, which illustrate historical inflow and outflow

Algorithm 4 Sampling generation in regression-based serial pipe merge

Generation of sampling set $\mathcal{S}$

$\mathcal{S} \leftarrow \emptyset$
while $k \leftarrow 1 < |\mathcal{K}|$ **do** ▷ the number of generated samples $|\mathcal{K}|$ is predefined
  $(p^k_{\ell,t_0}, p^k_{m,t_0}, p^k_{r,t_0}, q^k_{\ell,t_0}, q^k_{r,t_0}) \leftarrow$ GenerateInitialState
  $\left(q^k_{\ell,t}, q^k_{r,t}\right)_{t\in\mathcal{T}} \leftarrow$ GenerateDemand$(q^k_{\ell,t_0}, q^k_{r,t_0})$
  `isSampleValid` $\leftarrow$ CompleteSample$(p^k_{\ell,t_0}, p^k_{m,t_0}, p^k_{r,t_0}, q^k_{\ell,t}, q^k_{r,t})_{t\in\mathcal{T}}$
  if `isSampleValid` **then**
    $\mathcal{S} \leftarrow \left(p^k_{\ell,\tilde{t}}, p^k_{m,\tilde{t}}, p^k_{r,\tilde{t}}, q^k_{\ell,t}, q^k_{m,t}, q^k_{r,t}\right)_{\tilde{t}\in\mathcal{T}_0, t\in\mathcal{T}}$ ▷ add sample to sampling set
    $k \leftarrow k+1$
  end if
end while

Generation of an (i) initial state, (ii) demand and (iii) sample completion

function GenerateInitialState
  Select $\hat{q}^k_{t_0} \in [\underline{q}, \overline{q}]$ and set $q^k_{\ell,t_0}, q^k_{r,t_0} := \hat{q}^k_{t_0}$ ▷ select initial random flow in $[\underline{q}, \overline{q}]$
  if $q^k_{\ell,t_0}, q^k_{r,t_0} < 0$ **then** ▷ flow from node r to node ℓ
    Select p^k_{ℓ,t_0}
    $p^k_{m,t_0} \leftarrow \left(\left((p^k_{\ell,t_0})^2(1-\beta_a) + (2\cdot\hat{q}^k_{t_0})^2\,\gamma_a \right) / (1+\beta_a) \right)^{1/2}$
    $p^k_{r,t_0} \leftarrow \left(\left((p^k_{m,t_0})^2(1-\beta_b) + (2\cdot\hat{q}^k_{t_0})^2\,\gamma_b \right) / (1+\beta_b) \right)^{1/2}$
  else ▷ flow from node ℓ to node r
    Select p^k_{r,t_0}
    $p^k_{m,t_0} \leftarrow \left(\left((p^k_{r,t_0})^2(1+\beta_b) + (2\cdot\hat{q}^k_{t_0})^2\,\gamma_b \right) / (1-\beta_b) \right)^{1/2}$
    $p^k_{\ell,t_0} \leftarrow \left(\left((p^k_{m,t_0})^2(1+\beta_a) + (2\cdot\hat{q}^k_{t_0})^2\,\gamma_a \right) / (1-\beta_a) \right)^{1/2}$
  end if
  return $(p^k_{\ell,t_0}, p^k_{m,t_0}, p^k_{r,t_0}, q^k_{\ell,t_0}, q^k_{r,t_0})$
end function

function GenerateDemand$(q^k_{\ell,t_0}, q^k_{r,t_0})$
  for t $\leftarrow$ 1 to $|\mathcal{T}|$ **do**
    Select $q^k_{\ell,t} \in \left[q^k_{\ell,t-1} - q_\epsilon, q^k_{\ell,t-1} + q_\epsilon\right]$
    Select $q^k_{r,t} \in \left[q^k_{r,t-1} - q_\epsilon, q^k_{r,t-1} + q_\epsilon\right]$
  end for
  return $\left(q^k_{\ell,t}, q^k_{r,t}\right)_{t\in\mathcal{T}}$
end function

function CompleteSample$((p^k_{\ell,t_0}, p^k_{m,t_0}, p^k_{r,t_0}, q^k_{\ell,t}, q^k_{r,t})_{t\in\mathcal{T}})$
  solve system of Equations (6.41) – (6.48) ▷ for given values $(p^k_{\ell,t_0}, p^k_{m,t_0}, p^k_{r,t_0}, q^k_{\ell,t}, q^k_{r,t})_{t\in\mathcal{T}}$
  return Feasibility of system (6.41) – (6.48)
end function

values of different pairs of serial pipes taken from real-world data. These figures show that sudden peaks in the difference between the inflow and outflow of two serial pipes occur rarely. However, in general it is possible that flows are more volatile than being generated here.

So far, after the generation of (i) an initial state and (ii) a demand scenario at nodes ℓ and r, the values of the parameters in bold are fixed for a particular sample in $s_k \in \mathcal{S}$:

$$\left(\boldsymbol{p^k_{\ell,t_0}}, ..., p^k_{\ell,t_n}, \boldsymbol{p^k_{m,t_0}}, ..., p^k_{m,t_n}, \boldsymbol{p^k_{r,t_0}}, ..., p^k_{r,t_n}, \boldsymbol{q^k_{\ell,t_1}}, ..., \boldsymbol{q^k_{\ell,t_n}}, q^k_{m,t_1}, ..., q^k_{m,t_n}, \boldsymbol{q^k_{r,t_1}}, ..., \boldsymbol{q^k_{r,t_n}}\right).$$

To complete the sample, we run CompleteSample. The underlying system $\mathcal{F}_{orig}$ of the sample completion either admits a feasible solution $(p_{\ell,t}, p_{m,t}, p_{r,t}, q_{m,t})_{t\in\mathcal{T}}$ or is infeasible. Our preliminary computational experiments even pose the conjecture that the solution of this sample completion is unique. To check the feasibility of this system, we use SCIP. The sample completion is an NLP-Problem, which is solved in significantly less than one second for a reasonable amount of time steps $|\mathcal{T}|$. Nevertheless we set a time limit in order to compute thousands of samples on short notice that are needed for the merging procedure. In the case that $\mathcal{F}_{orig}$ is feasible with respect to sample s_k (indicated by the Boolean `isSampleValid`), we add s_k to the sampling set $\mathcal{S}$.

From the counterexample above, we already know that a sample is most likely not feasible for the merged pipe c. Therefore, after having generated $|\mathcal{K}|$-many samples, we determine γ_c by using linear regression. The regression minimizes the error of the samples in $\mathcal{S}$ with respect to their corresponding realizations in system $\mathcal{F}_{agg}$. In general, different forms of linear regression exist that mostly vary in the measure of the error using different norms. Based on preliminary computational tests, we decided to use an ordinary least-squares regression.

Regression. For a particular sample $s_k \in \mathcal{S}$, we introduce slack (residuum) values $r^k_t, \tilde{r}^k_t \in \mathbb{R}$ for all $t \in \mathcal{T}$ in the following equations present in $\mathcal{F}_{orig}$:

$$p^k_{r,t} - p^k_{r,t-1} + p^k_{\ell,t} - p^k_{\ell,t-1} + \left(q^k_{r,t} - q^k_{\ell,t}\right) \alpha_c \, \tau + r^k_t = 0 \tag{6.62}$$

$$(p^k_{r,t})^2(1+\beta_c) - (p^k_{\ell,t})^2(1-\beta_c) + \left(q^k_{r,t} + q^k_{\ell,t}\right) |q^k_{r,t} + q^k_{\ell,t}|\gamma_c + \tilde{r}^k_t = 0. \tag{6.63}$$

However, since α_c, β_c are uniquely determined through Equations (6.54) – (6.55) before sampling, and each sample together with α_c uniquely determines the residuum r^k_t in (6.62), the only remaining degree of freedom is γ_c in (6.63). That is, we minimize the slack values $\tilde{r}^k_t$ for all $t \in \mathcal{T}$ across all samples. Thereby, the regression allows to calculate γ_c in such a way that it might compensate for a possible inaccurate impact of the previously determined parameter $\beta_c = \beta_a + \beta_b$.

In the following, we determine γ_c analytically, given that a linear ordinary least-squares regression admits a closed-form solution, see also Hastie et al. (2009). To this end, we define a function $f\colon \mathbb{R} \to \mathbb{R}$ that describes the sum of squared slack values for all samples, which is to be minimized:

$$f(\gamma_c) := \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} \left((p^k_{r,t})^2\,(1+\beta_c) - (p^k_{\ell,t})^2(1-\beta_c) + \left(q^k_{r,t} + q^k_{\ell,t}\right)|q^k_{r,t} + q^k_{\ell,t}|\gamma_c\right)^2.$$

The minimum of f is obtained at $\partial_{\gamma_c} f = 0$ with

$$\partial_{\gamma_c} f = \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} 2\Big((p^k_{r,t})^2\,(1+\beta_c) - (p^k_{\ell,t})^2(1-\beta_c) \\ + \left(q^k_{r,t} + q^k_{\ell,t}\right)|q^k_{r,t} + q^k_{\ell,t}|\gamma_c\Big)\; \left(q^k_{r,t} + q^k_{\ell,t}\right)|q^k_{r,t} + q^k_{\ell,t}|,$$

finally yielding

$$\arg\min_{\gamma_c}\; f(\gamma_c) = -\frac{\sum_k \sum_t \left(\left((p^k_{r,t})^2\,(1+\beta_c) - (p^k_{\ell,t})^2(1-\beta_c)\right)\left(q^k_{r,t} + q^k_{\ell,t}\right)|q^k_{r,t} + q^k_{\ell,t}|\right)}{\left(\sum_k \sum_t \left(q^k_{r,t} + q^k_{\ell,t}\right)|q^k_{r,t} + q^k_{\ell,t}|\right)^2}. \tag{6.64}$$

Note that (6.64) is indeed the point where the minimum of f is achieved, since $\partial^2_{\gamma_c} f$ is non-negative, i.e., $\partial^2_{\gamma_c} f(\gamma_c) = \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} 2\left(\left(q^k_{r,t} + q^k_{\ell,t}\right)|q^k_{r,t} + q^k_{\ell,t}|\right)^2 \geq 0$.

6.7.3 Evaluation of the serial merging approach

In this section, we evaluate the regression-based merging approach on six different pairs of serial pipes taken from a real-world network. For each example, we generate a sampling set $\mathcal{S}$ that consists of 2,000 samples, where a particular sample comprises four time steps, i.e., $\mathcal{T} = \{t_1, ..., t_4\}$. For the sample completion, we use SCIP, see Chapter 8. The bounds of all flow variables are given by $\underline{q} = -500$ and $\overline{q} = 500$ in all six cases, and the volatility of the demand is determined by the parameter $q_\epsilon = 50$.

Evaluation based on real-world data. For the evaluation, we use six pairs of serial pipes $S_1, S_2, S_3, S_4, S_5, S_6$ that are located in different network parts and cover a wide range of possible pipe characteristics. For example, the serial pipelines S_1, S_3, S_4, and S_5 have pairwise similar lengths, while S_2 and S_6 consist of one long and one short pipeline. Besides, the pipelines in S_2, S_4 and S_6 have notably different slopes (more than 1%), while the pipes in S_5 have even opposite slopes of mild magnitude. Table 6.8 provides an overview of the pipe characteristics. For the sake of completeness, Table D.2 in the appendix illustrates the values of the corresponding parameters α, β and γ for the serial pipes a and b and for the merged pipe c that result from the sampling-based regression.

For each of these six pairs of serial pipes $a = (\ell, m)$ and $b = (m, r)$, we test the merging approach by running simulations, on the one hand, for the serial pipes, and on the other hand, for the merged pipe $c = (\ell, r)$. Afterwards, we compare the simulated pressure values at common nodes of the serial and merged pipes with respect to Measures (6.1) – (6.3). Following the description in Section 6.1, we run 365 simulations covering an entire year, where the time horizon of a particular simulation corresponds to one day consisting of 96 time intervals. A visualization of the demand scenarios $(b_{\ell,t}, b_{r,t})_{t\in\mathcal{T}}$ with $\mathcal{T} = \{1, \ldots, 35040\}$, can be found in the upper Sub-figures 6.7 – 6.9. Here, the demand values $b_{\ell,t} := q_{\ell,t}$ and $b_{r,t} := -q_{r,t}$ determine the inflow and outflow values at the left and right nodes ℓ and r of the pipes $a = (\ell, m)$ and $b = (m, r)$.

Cases	Pipe a				Pipe b			
	L_a [km]	D_a [m]	s_a %	λ_a -	L_b [km]	D_b [m]	s_b %	λ_b -
S_1	13.003	0.889	0.56	6.46e-03	12.607	0.889	0.55	6.46e-03
S_2	9.408	0.694	1.31	7.88e-03	0.487	0.694	0.04	7.88e-03
S_3	14.388	1.086	0.91	6.29e-03	15.386	1.086	0.61	6.29e-03
S_4	15.065	0.996	1.41	7.46e-03	14.853	0.996	0.07	7.46e-03
S_5	16.090	1.036	0.04	5.77e-03	16.183	1.036	-0.04	5.77e-03
S_6	17.500	0.982	0.37	6.38e-03	0.100	0.982	2.40	6.38e-03

Table 6.8: Characteristics of the serial pipes a and b used in the evaluation of the sampling-based merging approach, with length L, diameter D, slope s, and friction factor λ.

Results. Table 6.9 summarizes the differences of simulated pressure values at the common nodes ℓ and r of the serial and merged pipes. It can be seen that the deviations are very small for all examples but S_3, varying from 0.0006 to 0.04 [bar] in the absolute deviation (AD). Only for S_3 there is a difference of 0.5 [bar] (AD). However, when taking the average over all 365 maximal daily differences, it reduces to a value of 0.05 [bar] (mean AD) and likewise to 0.02 [bar] (mean MAD), which implies that a value of 0.5 in the absolute deviation is rather an exception.

The small differences in all three measures AD, mean AD and mean MAD suggest that our regression-based merging procedure suitably approximates the serial pipes. However, we remark that more severe demand situations would stress the pipes more.

Finally, the middle and lower Sub-figures 6.7 – 6.9 visualize the simulated pressure profiles of the serial pipes (red) and the merged pipe (green) at node ℓ (middle sub-figures) and node r (lower sub-figures). These figures additionally depict the historical pressure profiles at both nodes in cyan.

		S_1	S_2	S_3	S_4	S_5	S_6
node ℓ	AD	$4.07e{-}02$	$2.75e{-}03$	$5.02e{-}01$	$3.73e{-}02$	$4.20e{-}02$	$6.22e{-}04$
	mean AD	$8.10e{-}03$	$7.53e{-}04$	$5.10e{-}02$	$2.34e{-}02$	$6.03e{-}03$	$2.60e{-}04$
	mean MAD	$6.01e{-}03$	$5.92e{-}04$	$2.00e{-}02$	$2.16e{-}02$	$3.73e{-}03$	$2.23e{-}04$
node r	AD	$4.88e{-}02$	$2.72e{-}03$	$5.13e{-}01$	$3.73e{-}02$	$3.78e{-}02$	$8.36e{-}04$
	mean AD	$8.30e{-}03$	$7.44e{-}04$	$4.04e{-}02$	$2.29e{-}02$	$6.12e{-}03$	$3.53e{-}04$
	mean MAD	$6.16e{-}03$	$5.83e{-}04$	$2.10e{-}02$	$2.06e{-}02$	$3.81e{-}03$	$3.00e{-}04$

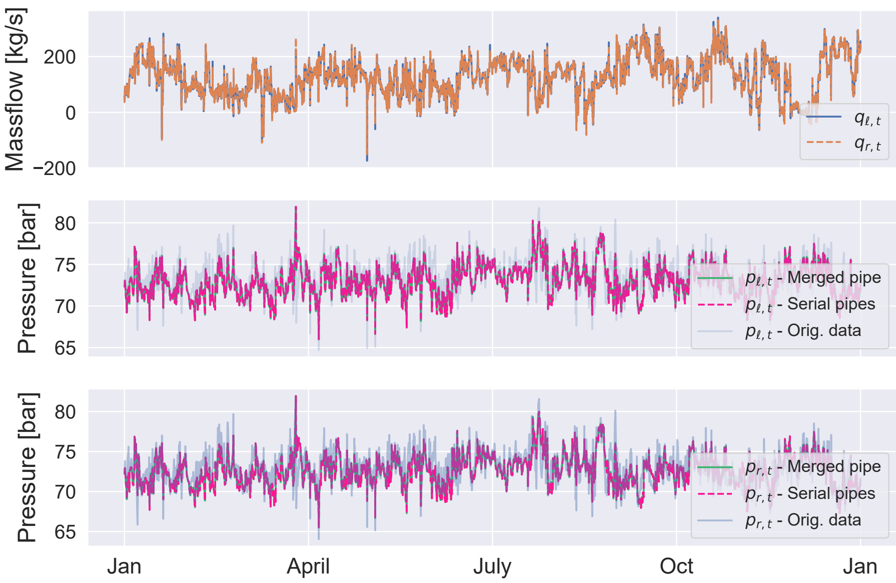
Table 6.9: Difference of simulated pressure values between the serial pipes and the merged pipe for all six examples $S_1, S_2, S_3, S_4, S_5, S_6$. The Measures (6.1) – (6.3) are stated in [bar].

(a) S_1: Comparison of simulated pressure profiles.

(b) S_2: Comparison of simulated pressure profiles.

Figure 6.7: Comparison of simulated pressure profiles of serial pipes (red) and merged pipe (green), and the historical pressure values (cyan) at node ℓ (middle sub-figures) and node r (lower sub-figures) for S_1 and S_2. The upper sub-figures illustrate the historical demand scenarios $(b_{\ell,t})_{t\in\mathcal{T}}$ (blue) and $(b_{r,t})_{t\in\mathcal{T}}$ (orange) at nodes ℓ and r of both serial pipes.

(a) S_3: Comparison of simulated pressure profiles.



(b) S_4: Comparison of simulated pressure profiles.

Figure 6.8: Comparison of simulated pressure profiles of serial pipes (red) and merged pipe (green), and the historical pressure values (cyan) at node ℓ (middle sub-figures) and node r (lower sub-figures) for S_3 and S_4. The upper sub-figures illustrate the historical demand scenarios $(b_{\ell,t})_{t\in\mathcal{T}}$ (blue) and $(b_{r,t})_{t\in\mathcal{T}}$ (orange) at nodes ℓ and r of both serial pipes.

(a) S_5: Comparison of simulated pressure profiles.

(b) S_6: Comparison of simulated pressure profiles.

Figure 6.9: Comparison of simulated pressure profiles of serial pipes (red) and merged pipe (green), and the historical pressure values (cyan) at node ℓ (middle sub-figures) and node r (lower sub-figures) for S_5 and S_6. The upper sub-figures illustrate the historical demand scenarios $(b_{\ell,t})_{t\in\mathcal{T}}$ (blue) and $(b_{r,t})_{t\in\mathcal{T}}$ (orange) at nodes ℓ and r of both serial pipes.

6.7.4 Relation of transient and stationary serial pipe merging

We know from Section 6.6 that the parallel merge allows determining pipe parameters for the merged pipe, which preserve the flow and pressure profiles of the parallel pipelines. This is possible in both, the stationary and transient case. However, when merging serial pipes in the transient case, this property is in general not true, as we have seen in Example 6.7.5. But, Corollary 6.7.4 and previous versions for serial pipe merging of horizontal pipes (Lenz and Schwarz, 2016; Groß et al., 2019) reveal that stationary assumptions enable determining such a pipe parameter. The following Proposition generalizes these results to non-horizontal pipelines in the stationary case. Again, we denote the parameters for the stationary case by γ^{st}, β^{st}, as done for parallel pipe merging in Section 6.6.

Proposition 6.7.6. *Given two serial pipes $a = (\ell, r)$ and $b = (\ell, r)$ with pipe characteristics $\gamma_a^{st}, \gamma_b^{st} > 0$ and $\beta_a^{st}, \beta_b^{st} \geq 0$ and $b_m = 0$, $deg(m) = 2$. Then, both pipes can equivalently be replaced by a single pipe $c = (\ell, r)$ with*

$$\beta_c^{st} = \beta_a^{st}\, \beta_b^{st} \quad \textit{and} \quad \gamma_c^{st} = \gamma_a^{st} + \beta_a^{st}\, \gamma_b^{st}.$$

Proof. From $b_m = 0$ and $\deg(m) = 2$ follows $q_c := q_a = q_b$. For two pipes in serial, each one is represented by a single Equation (6.40):

$$p_\ell^2 - \beta_a^{st}\, p_m^2 + \gamma_a^{st}\, q_c|q_c| = 0 \tag{6.65}$$

$$p_m^2 - \beta_b^{st}\, p_r^2 + \gamma_b^{st}\, q_c|q_c| = 0. \tag{6.66}$$

Then, it follows

$$\overset{(6.66)}{\Rightarrow} \quad \beta_a^{st}\, p_m^2 - \beta_a^{st}\, \beta_b^{st}\, p_r^2 + \beta_a^{st}\, \gamma_b q_c|q_c| = 0 \tag{6.67}$$

$$\overset{(6.65),(6.67)}{\Rightarrow} \quad p_\ell^2 - \beta_a^{st}\, p_m^2 + \beta_a^{st}\, p_m^2 - \beta_a^{st}\, \beta_b^{st}\, p_r^2 + (\gamma_a^{st} + \beta_a^{st}\, \gamma_b^{st})\, q_c|q_c| = 0$$

$$\Rightarrow \quad p_\ell^2 - \beta_a^{st}\, \beta_b^{st}\, p_r^2 + (\gamma_a^{st} + \beta_a^{st}\, \gamma_b^{st})\, q_c|q_c| = 0$$

$$\Rightarrow \quad \boxed{\beta_c^{st} = \beta_a^{st}\, \beta_b^{st}} \quad \text{and} \quad \boxed{\gamma_c^{st} = \gamma_a^{st} + \beta_a^{st}\, \gamma_b^{st}}. \tag{6.68}$$

□

In the stationary case, horizontal serial pipes hold $\beta_a^{st} = \beta_b^{st} = 1$, see Fügenschuh et al. (2015), then from (6.68) directly follows

$$\beta_c^{st} = 1 \quad \text{and} \quad \gamma_c^{st} = \gamma_a^{st} + \gamma_b^{st}, \tag{6.69}$$

which coincides with the formula stated in Corollary 6.7.4 for transient serial merges under "stationary" conditions.

6.8 Merging Active Elements

In the course of the aggregation Algorithm 1, new structures may arise that consist of parallel or serial active elements. Our modeling from Chapter 5 enables us to merge such active elements, similarly as done for pipelines. Following the description from Section 6.1, we exclude station arcs and restrict the merges to valves and control valves that are located outside of stations. For this reason, let us consider two parallel or serial active elements $a, b \in \mathcal{A}_{va} \cup \mathcal{A}_{cv}$ with $a, b \notin \mathcal{A}_{a,s}$ for any $s \in \mathcal{S}$.

We formalize parallel and serial merges of valves in Subsection 6.8.1, control valves in Subsection 6.8.2, and finally valves and control valves in Subsection 6.8.3. We present propositions, which state that the parallel and serial elements can be replaced by new structurally equivalent elements. The corresponding proofs of the propositions are rather technical and can be found in the Appendix D.3.

6.8.1 Merging valves

Proposition 6.8.1 (Merging parallel valves)**.** *Given two parallel valves a and b between nodes ℓ and r. Then a and b can be equivalently replaced by a new valve $c = (\ell, r)$.*

Proposition 6.8.2 (Merging serial valves)**.** *Let $a = (\ell, m)$ and $b = (m, r)$ be two serial valves with $deg(m) = 2$ and $b_{m,t} = 0$ for all $t \in \mathcal{T}$ and let $\underline{p}_\ell = \underline{p}_m = \underline{p}_r$ and $\overline{p}_\ell = \overline{p}_m = \overline{p}_r$. Then a and b can be equivalently replaced by a new valve $c = (\ell, r)$.*

6.8.2 Merging control valves

Compared to valves and pipes, the orientation of control valves is crucial, since control valves reduce pressure in the flow direction. Hence, we apply parallel and serial merges only to control valves that have the same orientation.

Proposition 6.8.3 (Merging parallel control valves)**.** *Given two control valves a and b between nodes ℓ and r. Then a and b can be equivalently replaced by a new control valve $c = (\ell, r)$.*

Proposition 6.8.4 (Merging serial control valves)**.** *Let $a = (\ell, m)$ and $b = (m, r)$ be two serial control valves with $deg(m) = 2$ and $b_{m,t} = 0$ for all $t \in \mathcal{T}$ and let $\underline{p}_\ell = \underline{p}_m = \underline{p}_r$ and $\overline{p}_\ell = \overline{p}_m = \overline{p}_r$. Then a and b can be equivalently replaced by a new control valve $c = (\ell, r)$.*

6.8.3 Merging valves and control valves

If a valve and control valve are linked, either in parallel or in serial, then the valve can be removed.

Proposition 6.8.5 (Merging parallel valves and control valves)**.** *Let a be a valve and let b be a control valve between nodes ℓ and r. Then a and b can be equivalently replaced by a new control valve $c = (\ell, r)$.*

Proposition 6.8.6 (Merging serial valves and control valves)**.** *Let $a = (\ell, m)$ be a valve and $b = (m, r)$ a control valve with $deg(m) = 2$ and $b_{m,t} = 0$ for all $t \in \mathcal{T}$ and let $\underline{p}_\ell = \underline{p}_m = \underline{p}_r$ and $\overline{p}_\ell = \overline{p}_m = \overline{p}_r$. Then a and b can be equivalently replaced by a new control valve $c = (\ell, r)$.*

6.9 Geometrical Excursion on Pressure Bounds

So far, the aggregation approach introduces storage nodes in order to replace distribution and passive tree subnetworks. In this section, we present an outlook to substituting an arbitrary subnetwork $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{A}_C)$ for a real pipe $a_C = (v_C, w_C) \in \mathcal{A}_{pi}$ that is only connected with the remaining network, e.g., through node v_C. Using pipes instead of storage nodes allows imitating a pressure loss inside $\mathcal{G}_C$ up to a certain extent. Naturally, such aggregation poses several questions, for example, when to apply it, and if so, how to determine new pipe parameters $\alpha_{a_C}, \beta_{a_C}, \gamma_{a_C}$, initial pressure values p_{v_C,t_0}, p_{w_C,t_0}, and new pressure bounds $\underline{p}_{v_C}, \overline{p}_{v_C}$ and $\underline{p}_{w_C}, \overline{p}_{w_C}$. While Sections 6.3.5, 6.3.6 and 6.4.3 already suggest possible solutions to some of these aspects, e.g., how to determine p_{v_C,t_0} and $\underline{p}_{v_C}, \overline{p}_{v_C}$ at node v_C that is linked to the remaining network, we propose here an approach to derive new pressure bounds at the remaining node w_C. Under the requirement that pipe a_C admits to store the same amount of gas as the entire subnetwork $\mathcal{G}_C$, it turns out that the calculation of these bounds has a nice geometrical interpretation. The new bounds arise from the intersection of lines and ellipses, where the lines correspond to variable bounds and the ellipses originate from the approximation of minimal and maximal possible line pack values of $\mathcal{G}_C$. Moreover, the presented technique can be used to uniquely determine an initial pressure value p_{w_C,t_0}. Even though we do not apply this procedure in the aggregation Algorithm 1, we present it as an alternative approach to determining these values by using the average, minimum, or maximum of given values.

Derivation of new pressure bounds at node w_C. First, assume that values $\underline{m}_C$ and $\overline{m}_C$ for the minimal and maximal possible line-pack of $\mathcal{G}_C$ are given. These values can be derived from nodal pressure bounds in $\mathcal{G}_C$, or can be approximated through the application of transient or stationary optimization models. Then, using $\underline{m}_C, \overline{m}_C$ as bounds for the line pack LP_{a_C} in pipe a_C leads to:

$$\begin{aligned} & \underline{m}_C \leq LP_{a_C} \leq \overline{m}_C \\ \overset{(6.8)}{\Rightarrow} \quad & \underline{m}_C \leq \frac{p^{mean}_{a_C}}{\mathrm{c_s}^2}\, vol_{a_C} \leq \overline{m}_C \\ \overset{(6.7)}{\Rightarrow} \quad & \underline{m}_C \leq \frac{1}{\mathrm{c_s}^2}\,\frac{2}{3}\left(p_{v_C} + p_{w_C} - \frac{p_{v_C}\, p_{w_C}}{p_{v_C} + p_{w_C}}\right) vol_{a_C} \leq \overline{m}_C, \end{aligned} \tag{6.70}$$

with $p_{v_C} \in [\underline{p}_{v_C}, \overline{p}_{v_C}]$, where the values $\underline{p}_{v_C}, \overline{p}_{v_C} \geq 0$ can be determined similarly to distribution subnetworks in (6.17) (6.18). For the sake of a simpler notation, we drop subscript C in the remainder of this subsection and define

$$\underline{m} := \frac{\mathrm{c_s}^2\; 3\,\underline{m}_C}{2\,vol_{a_C}} \geq 0 \qquad \text{and} \qquad \overline{m} := \frac{\mathrm{c_s}^2\; 3\,\overline{m}_C}{2\,vol_{a_C}} \geq 0,$$

assuming that we throughout refer to a particular component C. Moreover, we set $vol_{a_C} := vol_C$, as done for storage nodes in Equation (6.6), and assume that C contains at least one pipe, which implies $vol_C > 0$. Then, the Inequalities (6.70) read

$$\underline{m}\ (p_v + p_w) \le p_v^2 + p_w^2 + p_v\, p_w \le \overline{m}\ (p_v + p_w)\,. \tag{6.71}$$

Let us define the functions $f_{\underline{m}}, f_{\overline{m}} \colon \mathbb{R}^2 \to \mathbb{R}$ as

$$f_{\underline{m}}(p_v, p_w) \ := \ p_v^2 + p_w^2 + p_v\, p_w - \underline{m}\,(p_v + p_w), \quad \text{and} \tag{6.72}$$

$$f_{\overline{m}}(p_v, p_w) \ := \ p_v^2 + p_w^2 + p_v\, p_w - \overline{m}\,(p_v + p_w). \tag{6.73}$$

Then, $\overline{p}_w$ and $\underline{p}_w$ correspond to the optimal objective values of the following optimization problems

$$\overline{p}_w = \max_{p_v, p_w} \quad p_w \tag{6.74a}$$

$$\text{s.t.} \quad f_{\underline{m}}(p_v, p_w) \ge 0 \tag{6.74b}$$

$$f_{\overline{m}}(p_v, p_w) \le 0 \tag{6.74c}$$

$$p_v \in [\underline{p}_v, \overline{p}_v] \tag{6.74d}$$

$$p_w \ge 0 \tag{6.74e}$$

and

$$\underline{p}_w = \min_{p_v, p_w} \quad p_w \tag{6.75a}$$

$$\text{s.t.} \quad f_{\underline{m}}(p_v, p_w) \ge 0 \tag{6.75b}$$

$$f_{\overline{m}}(p_v, p_w) \le 0 \tag{6.75c}$$

$$p_v \in [\underline{p}_v, \overline{p}_v] \tag{6.75d}$$

$$p_w \ge 0. \tag{6.75e}$$

Now, we provide the reader with some intuition about the geometrical interpretation of these problems and the curves $f_{\overline{m}}(p_v, p_w) = 0$ and $f_{\underline{m}}(p_v, p_w) = 0$. More generally, we consider the family of functions $f_m(p_v, p_w) \ = \ p_v^2 + p_w^2 + p_v\, p_w - m\,(p_v + p_w)$ being parametrized by m, i.e., $\{f_m\}_{m \in [\underline{m}, \overline{m}]}$. The following Propositions 6.9.1 and 6.9.2 analyze the curves in more detail. A visualization of the curves is given in Figure 6.10.

Proposition 6.9.1 (Ellipse)**.** *Let $\underline{m}, \overline{m} \ge 0$. For each $m \in [\underline{m}, \overline{m}]$, the solution set of the equation $f_m(p_v, p_w) = 0$ describes an ellipse that*

i) is centered in $(m/3, m/3)$,

ii) is non-degenerate, i.e., it is not a single point, if $m > 0$,

iii) the length of the minor and major axes are given by $\lambda_{m,1} = \frac{m}{3}$ and $\lambda_{m,2} = \frac{m}{\sqrt{3}}$, and

iv) contains the points $(0, m)$ and $(m, 0)$.

Proof. To show that the solution set of $f_m(p_v, p_w) = 0$ describes an ellipse, we express $f_m(p_v, p_w) = 0$ as quadratic form

$$f_m(p_v, p_w) = (p_v, p_w, 1) \underbrace{\begin{pmatrix} 1 & \frac{1}{2} & -\frac{m}{2} \\ \frac{1}{2} & 1 & -\frac{m}{2} \\ -\frac{m}{2} & -\frac{m}{2} & 0 \end{pmatrix}}_{B} \begin{pmatrix} p_v \\ p_w \\ 1 \end{pmatrix}^T = 0.$$

Let

$$A = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$$

be the minor 2×2 sub-matrix of B. The function $f_m(p_v, p_w) = 0$ is an ellipse, if and only if, the sub-matrix A is positive definite. But this holds, since $\det A = 3/4 > 0$ and $A_{11} = 1 > 0$, (where A_{11} is the entry of the first row and column of matrix A).

(i) *Center:* The determinant $\det A = 3/4 \neq 0$ implies that a geometric center of the ellipse exists, see Pettofrezzo (1966). It can be shown that the center of an ellipse is characterized by the fact that the gradient of the function vanishes, see Ayoub (1993). Here, this condition leads to

$$\nabla f_m(p_v, p_w) = 0 \quad \Rightarrow \quad \begin{cases} \dfrac{\partial f_m}{\partial p_v} = 2p_v + p_w - m = 0 \\ \dfrac{\partial f_m}{\partial p_w} = 2p_w + p_v - m = 0. \end{cases}$$

Then, the unique solution of $\nabla f_m(p_v, p_w) = 0$ yields the center $(p_v, p_w) = \left(\frac{m}{3}, \frac{m}{3}\right)$ of the ellipse.

(ii) *Non-degeneracy:* The ellipse $f_m(p_v, p_w) = 0$ is non-degenerate if $\det B \neq 0$ according to Lawrence (2014). Here, it holds $\det B = -\frac{1}{4}m^2$. Assuming $m > 0$, it follows that $f_m(p_v, p_w) = 0$ is non-degenerate.

(iii) *Length of the major and minor axes:* According to the principal axis theorem, the major and minor axes point in the direction of the eigenvectors of matrix A. Matrix

$$A = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$$

has the following eigenvalues $\{\frac{3}{2}, \frac{1}{2}\}$ with corresponding eigenvectors $\{(1,1), (-1,1)\}$. The lengths $\lambda_{m,1}, \lambda_{m,2}$ of these axes are obtained by calculating the intersections of the ellipse with the minor and major axes, i.e., solving

$$f_m\left(\left(\frac{m}{3}, \frac{m}{3}\right) + \lambda_{m,1}(1,1)\right) = 0, \quad \text{and}$$
$$f_m\left(\left(\frac{m}{3}, \frac{m}{3}\right) + \lambda_{m,2}(-1,1)\right) = 0,$$

which yields $\lambda_{m,1} = m/3$ (along the minor axis) and $\lambda_{m,2} = m/\sqrt{3}$ (along the major axis).

(iv) From the definition of f_m follows directly $f_m(0, p_w) = m$ and $f_m(p_v, 0) = m$. □

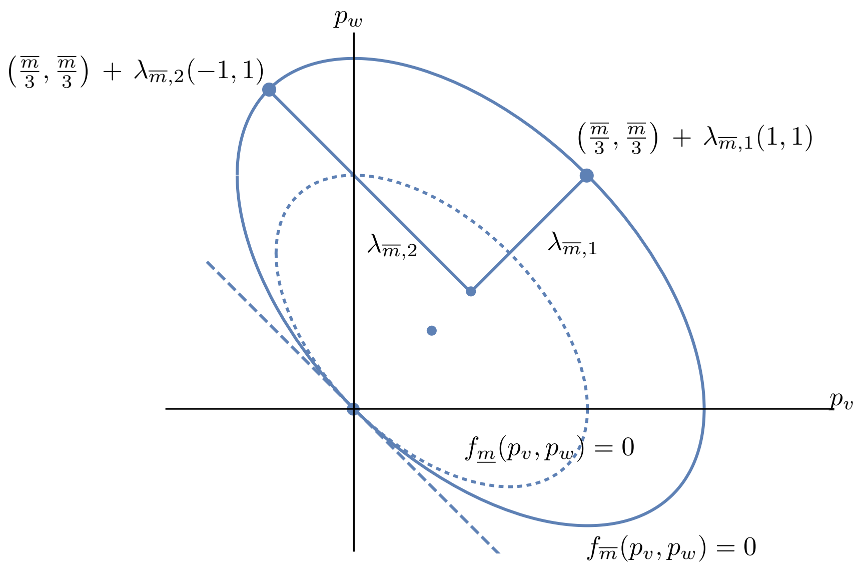


Figure 6.10: An illustration of the properties stated in Proposition 6.9.1. Small dots in blue represent the centers $(\underline{m}/3, \underline{m}/3)$ and $(\overline{m}/3, \overline{m}/3)$ of $f_{\overline{m}}(p_v, p_w) = 0$ (outer ellipse) and $f_{\underline{m}}(p_v, p_w) = 0$ (inner ellipse). The length of the semi-minor and semi-major axes of $f_{\overline{m}}(p_v, p_w) = 0$ is given by $\lambda_{\overline{m},1}$ and $\lambda_{\overline{m},2}$ along the eigenvectors $v_1 = (1, 1), v_2 = (-1, 1)$, where $\left(\frac{\overline{m}}{3}, \frac{\overline{m}}{3}\right) + \lambda_{\overline{m},1}(1, 1)$ and $\left(\frac{\overline{m}}{3}, \frac{\overline{m}}{3}\right) + \lambda_{\overline{m},2}(-1, 1)$ describe the intersection points of the ellipse $f_{\overline{m}}(p_v, p_w) = 0$ with the minor and major axes (big dots in blue). Both ellipses touch each other tangentially in $(0, 0)$ (Proposition 6.9.2) as indicated by the dashed tangent through the origin.

Let us mention that the ellipse $f_m(p_v, p_w) = 0$ is degenerate, i.e., $0 = \underline{m} = m$, if either the pressure is zero within the entire component, or the component has no storage volume, see the line-pack representation in Equation (6.8). However, this implies that the underlying component already has been somehow degenerated. In such case the ellipse collapses to a single point.

We now consider both ellipses $f_{\underline{m}}(p_v, p_w) = 0$ and $f_{\overline{m}}(p_v, p_w) = 0$ simultaneously, since they enclose together the feasible region of the optimization problems (6.74) and (6.75). A structural result about the common points of these ellipses is given by the following proposition, which allows us to conclude that there is an "inner" ellipse, $f_{\underline{m}}(p_v, p_w) = 0$, and an "outer" ellipse, $f_{\overline{m}}(p_v, p_w) = 0$.

Proposition 6.9.2. *Let* $0 \leq \underline{m} < \overline{m}$ *and* $f_{\underline{m}}, f_{\overline{m}}$ *be given as defined in* (6.72) *and* (6.73).

i) Then, it follows

$$\left\{(p_v, p_w) \in \mathbb{R}^2 \mid f_{\underline{m}}(p_v, p_w) \leq 0\right\} \subseteq \left\{(p_v, p_w) \in \mathbb{R}^2 \mid f_{\overline{m}}(p_v, p_w) \leq 0\right\}.$$

ii) Both ellipses $f_{\underline{m}}(p_v, p_w) = 0$ *and* $f_{\overline{m}}(p_v, p_w) = 0$ *touch each other tangentially in the point* $(p_v, p_w) = (0, 0)$. *In addition, it is the only common point of both ellipses.*

Proof. (i) Consider

$$f_{\overline{m}}(p_v, p_w) \leq 0 \iff p_v^2 + p_w^2 + p_v\, p_w - \overline{m}\,(p_v + p_w) \leq 0 \tag{6.76}$$

$$f_{\underline{m}}(p_v, p_w) \leq 0 \iff p_v^2 + p_w^2 + p_v\, p_w - \underline{m}\,(p_v + p_w) \leq 0. \tag{6.77}$$

Provided $\underline{m} < \overline{m}$, every point $(p_v, p_w) \in \mathbb{R}^2$ that satisfies (6.77) also satisfies (6.76).

(ii) From the definitions of $f_{\underline{m}}(p_v, p_w) = 0$ and $f_{\overline{m}}(p_v, p_w) = 0$ it directly follows that $(p_v, p_w) = (0, 0)$ is feasible for both ellipses and that it is the only point which fulfills $f_{\overline{m}}(p_v, p_w) = f_{\underline{m}}(p_v, p_w)$ for $\underline{m} \neq \overline{m}$. Moreover, both ellipses have the same tangents in $(p_v, p_w) = (0, 0)$, because their partial derivatives satisfy

$$\frac{\partial f_{\underline{m}}}{\partial p_v}(0,0) = \frac{\partial f_{\underline{m}}}{\partial p_w}(0,0) = -\underline{m}, \qquad \text{and} \qquad \frac{\partial f_{\overline{m}}}{\partial p_v}(0,0) = \frac{\partial f_{\overline{m}}}{\partial p_w}(0,0) = -\overline{m}.$$

Hence their normal vectors are parallel, which implies that both tangents are equal. □

As mentioned prior to the proposition, the common feasible region of the optimization Problems (6.74) and (6.75) is enclosed by $f_{\overline{m}}(p_v, p_w) = 0$ and $f_{\overline{m}}(p_v, p_w) = 0$. Given that $\underline{p}_v, \overline{p}_v, \underline{p}_w \geq 0$ holds, it suffices to restrict our considerations to the first quadrant of the (p_v, p_w)–space. Figure 6.11 illustrates the feasible region of Problems (6.74) and (6.75) in dark blue. The figure suggests how to determine their optimal solutions:

i) the tightest upper bound $\overline{p}_w$ is obtained by the intersection of $f_{\overline{m}}(p_v, p_w) = 0$ and $p_v = \underline{p}_v$ for $p_w \geq 0$,

ii) the tightest lower bound $\underline{p}_w$ is obtained by the intersection of $f_{\underline{m}}(p_v, p_w) = 0$ and $p_v = \overline{p}_v$ for $p_w \geq 0$.

These claims in fact require that the two-dimensional function $g\colon [0, m] \times [\underline{m}, \overline{m}] \to \mathbb{R}_{\geq 0}$ is monotone decreasing in p_v, where g describes the solution set of $f_m(p_v, p_w) = 0$:

$$\{(p_v, p_w) \in [0, m] \times [0, \infty] \mid p_w = g(p_v, m)\}.$$

More precisely, we require $g(p_v, \underline{m})$ and $g(p_v, \overline{m})$ to be monotone decreasing in p_v. Then, the monotonicity allows us to prove that the aforementioned intersections yield the optimal objective values of Problems (6.74) and (6.75). The function $g(p_v, m)$ reads

$$g(p_v, m) := \frac{1}{2}\Big(\sqrt{m^2 + 2mp_v - 3p_v^2} + m - p_v\Big). \tag{6.78}$$

Note that function g is well-defined, since the radicand is non-negative for $p_v \in [0, m]$. Indeed, we restrict our considerations to $p_v \in [0, m]$, since $f_m(m, 0) = 0$ is satisfied and function g is monotone decreasing in p_v on $[0, m]$, as the following lemma states.

Lemma 6.9.3. *The function* $g\colon [0, m] \times [\underline{m}, \overline{m}] \to \mathbb{R}_{\geq 0}$ *defined in* (6.78) *is*

i) monotone decreasing in p_v*, and*

ii) monotone increasing in m*.*

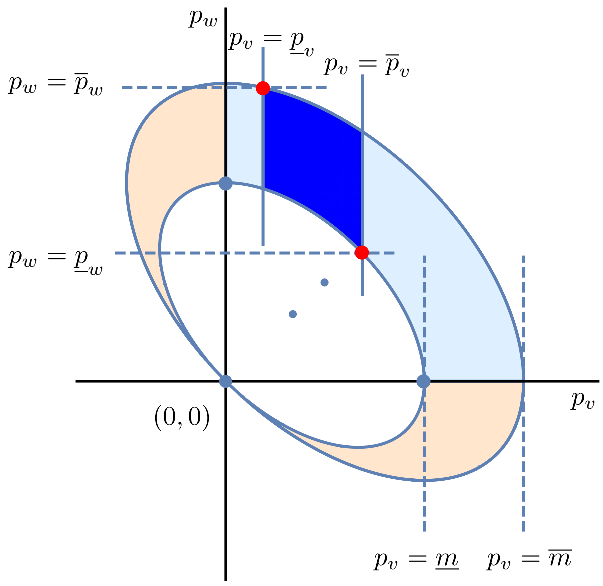


Figure 6.11: A geometrical interpretation of Problems (6.74) and (6.75) to analytically determine $\overline{p}_w$ and $\underline{p}_w$. Small dots in blue represent the centers $(\underline{m}/3, \underline{m}/3)$ and $(\overline{m}/3, \overline{m}/3)$ of $f_{\overline{m}}(p_v, p_w) = 0$ (outer ellipse) and $f_{\underline{m}}(p_v, p_w) = 0$ (inner ellipse). Big dots in blue illustrate the points $(p_v, p_w) = (0, \underline{m})$ and $(p_v, p_w) = (\underline{m}, 0)$, where the curve $f_{\underline{m}}(p_v, p_w) = 0$ takes the maximal values for p_w and p_v in the non-negative (p_v, p_w)–space. Red points depict the intersection of $p_v = \overline{p}_v$ with $f_{\overline{m}}(p_v, p_w) = 0$ and the intersection of $p_w = \underline{p}_v$ with $f_{\underline{m}}(p_v, p_w) = 0$ yielding the values for $\underline{p}_w$ and $\overline{p}_w$ we are interested in. The area in dark blue corresponds to the feasible region that is enclosed by both ellipses when being additionally restricted by the final bounds of p_v and p_w, while the additional light blue area corresponds to the region where the bounds of the variables p_v, p_w are non-negative, and the additional light orange area displays the region without any restrictions on the bounds of the variables.

Proof. (i) The function g is monotone decreasing in p_v:

For fixed $m \in [\underline{m}, \overline{m}]$, let us consider the first and second partial derivative of g:

$$\frac{\partial}{\partial p_v} g(p_v, m) = \frac{2m - 6p_v}{4(m^2 + 2mp_v - 3p_v^2)^{1/2}} - \frac{1}{2},$$
$$\frac{\partial^2}{\partial^2 p_v} g(p_v, m) = -\frac{2m^2}{(m^2 + 2mp_v - 3p_v^2)^{3/2}}.$$

First, $\partial g/\partial p_v$ is monotone decreasing in $p_v \in [0, m)$, since $(\partial^2 g/\partial^2 p_v)(p_v, m) \leq 0$ for all $p_v \in [0, m)$. Given that $(\partial g/\partial p_v)(0, m) = 0$ for any $m \in [\underline{m}, \overline{m}]$, it follows $(\partial g/\partial p_v)(p_v, m) \leq 0$ for all $p_v \in [0, m)$, which implies that g is monotone decreasing in p_v on $[0, m)$. Finally, g is monotone decreasing in p_v on the entire interval $[0, m]$, since $g(p_v, m) > 0$ holds for all $p_v < m$ and $g(m, m) = 0$.

(ii) The function g is monotone increasing in m:

Let $p_v \in [0, m_1]$ be fixed and let $\underline{m} \leq m_1 \leq m_2 \leq \overline{m}$. We consider the following function $\Delta\colon [0, m_1] \times [\underline{m}, \overline{m}] \times [\underline{m}, \overline{m}] \to \mathbb{R}_{\geq 0}$ with $\Delta(p_v, m_1, m_2) = g(p_v, m_2) - g(p_v, m_1)$, i.e.,

$$\Delta(p_v, m_1, m_2) = \frac{1}{2}\Big(\sqrt{m_2^2 + 2m_2p_v - 3p_v^2} - \sqrt{m_1^2 + 2m_1p_v - 3p_v^2} + m_2 - m_1\Big).$$

Note that Δ is only defined for $p_v \in [0, m_1]$. Obviously, it holds $\Delta(p_v, m_1, m_2) \geq 0$ for all $p_v \in [0, m_1]$ and $\underline{m} \leq m_1 \leq m_2 \leq \overline{m}$. □

With these preliminaries, we are ready to determine the bounds of p_w, as formalized by the following theorem.

Theorem 6.9.4. *Let* $0 < \underline{p}_v < \overline{p}_v < \underline{m} < \overline{m}$. *Then the optimal objective value of Problem* (6.74) *is given by the intersection of* $f_{\overline{m}}(p_v, p_w) = 0$ *and* $p_v = \underline{p}_v$ *for* $p_v, p_w \in [\underline{p}_v, \overline{p}_v] \times [0, \infty]$, *yielding*

$$\overline{p}_w := \frac{1}{2}\Big(\sqrt{\overline{m}^2 + 2\,\overline{m}\,\underline{p}_v - 3\,\underline{p}_v^2} + \overline{m} - \underline{p}_v\Big).$$

Accordingly, the optimal objective value of Problem (6.75) *is given by the intersection of* $f_{\underline{m}}(p_v, p_w) = 0$ *and* $p_v = \overline{p}_v$ *for* $p_v, p_w \in [\underline{p}_v, \overline{p}_v] \times [0, \infty]$, *yielding*

$$\underline{p}_w := \frac{1}{2}\Big(\sqrt{\underline{m}^2 + 2\,\underline{m}\,\overline{p}_v - 3\,\overline{p}_v^2} + \underline{m} - \overline{p}_v\Big).$$

Proof. We prove the claim for the optimal objective value $\overline{p}_w$ of Problem (6.74). Under the same logic follows the argumentation for the optimal objective value $\underline{p}_w$ of Problem (6.75).

The feasible region $\mathcal{X}$ of Problem (6.74) (and also Problem (6.75)) is given by

$$\begin{aligned} & \mathcal{X} = \Big\{(p_v, p_w) \in [\underline{p}_v, \overline{p}_v] \times [\underline{p}_w, \overline{p}_w] \mid f_{\underline{m}}(p_v, p_w) \geq 0,\ f_{\overline{m}}(p_v, p_w) \leq 0\Big\} \\ \iff\ & \mathcal{X} = \bigcup_{m \in [\underline{m}, \overline{m}]} \Big\{(p_v, p_w) \in [\underline{p}_v, \overline{p}_v] \times [\underline{p}_w, \overline{p}_w] \mid g(p_v, m) = p_w\Big\}. \end{aligned}$$

Naturally, for each point $(p_v, p_w) \in \mathcal{X}$ exists $m \in [\underline{m}, \overline{m}]$ with $g(p_v, m) = p_w$. In fact, for each point $(p_v, p_w) \in \mathcal{X}$ exists exactly one such $m \in [\underline{m}, \overline{m}]$. The uniqueness of m follows for a fixed point $(p_v, p_w) \in \mathcal{X}$ by resolving $p_w = g(p_v, m)$, as given in (6.78), for m, which yields the unique value

$$m = \frac{p_v^2 + p_vp_w + p_w^2}{p_v + p_w}.$$

Furthermore, function g is monotone decreasing in p_v and monotone increasing in m (Lemma 6.9.3). The facts that (i) for all $(p_v, p_w) \in \mathcal{X}$ exists exactly one $m \in [\underline{m}, \overline{m}]$ that holds $g(p_v, m) = p_w$, and (ii) $g(p_v, m)$ is monotone decreasing in p_v on $[0, m]$, and (iii) $g(p_v, m)$ is monotone increasing in m on $[\underline{m}, \overline{m}]$, allows us to determine the optimal objective value of Problem (6.74) by evaluating $g(p_v, \overline{m})$ at $p_v = \underline{p}_v$. In other words, $\overline{p}_w$ is given by the intersection of $p_v = \underline{p}_v$ and $p_w = g(p_v, \overline{m})$.

Finally, calculating these intersections yields

$$\overline{p}_w = \frac{1}{2}\Big(\sqrt{\overline{m}^2 + 2\,\overline{m}\,\underline{p}_v - 3\,\underline{p}_v^2} + \overline{m} - \underline{p}_v\Big), \quad \text{and}$$
$$\underline{p}_w = \frac{1}{2}\Big(\sqrt{\underline{m}^2 + 2\,\underline{m}\,\overline{p}_v - 3\,\overline{p}_v^2} + \underline{m} - \overline{p}_v\Big).$$

□

Concluding remarks. From Theorem 6.9.4 follow several remarks.

i) In the case that the assumptions in Theorem 6.9.4 are violated, i.e., $\underline{p}_v = 0$ or $\overline{p}_v \geq \underline{m}$ holds, then the values for the bounds of p_w read as follows:
   - if $\underline{p}_v = 0$, then $\overline{p}_w = \overline{m}$
   - if $\overline{p}_v \geq \underline{m}$, then $\underline{p}_w = 0$.

ii) Moreover, if $f_{\overline{m}}(p_v, p_w) = 0$ is degenerate, then the ellipse consists of a single point, namely $(0, 0)$, and the bounds hold: $\underline{p}_w = \overline{p}_w = 0$.

iii) As a consequence of Theorem 6.9.4, the following relationships hold (without proof):

$$\text{(i)} \qquad \overline{p}_v < \underline{m} \quad \Longleftrightarrow \quad \underline{p}_w > 0$$
$$\text{(ii)} \qquad 0 < \underline{p}_v \quad \Longleftrightarrow \quad \overline{p}_w < \overline{m}.$$

The conditions $\overline{p}_v < \underline{m}$ and $0 < \underline{p}_v$ state a criteria, when it is indeed possible to derive tightened bounds for p_w.

Finally, we remark that using function g defined in (6.78) allows us to uniquely determine pressure values $p_{w_C,t} = g(p_{v_C,t}, m)$ for given values $p_{v_C,t}$ and m. This is of particular interest when determining initial pressure values p_{w_C,t_0} as required input for the *Transient Control Problem.*

6.10 Conclusion

In this chapter, we studied the aggregation of transient gas networks, and presented, to the best of our knowledge for the first time, an aggregation scheme that is tailored to optimizing the control of transient large-scale gas networks. While in practice, the representation of aggregated networks is generally created manually by engineers applying expert knowledge about the particular network topology, our approach can easily be automated. It consists of merely one configuration possibility and thereby requires only little information about the particular network.

Motivated by the overall goal to find solutions to the *Transient Control Problem* that can be used to control the network in daily operations, the impact of the aggregation is to reduce the network size and thereby implicitly the size of the optimization model. To this end, we identified parts that can be considered as of minor importance for the network control and thus allow for a coarser representation. Taking the example of distribution subnetworks, our study about line-pack using real-world data suggests that distribution parts only slightly contribute to buffering flow imbalances and that their impact on controlling operations is rather small. To the best of our knowledge, we are the first to present a mathematical definition for distribution subnetworks and based on it, a subroutine for their detection and aggregation.

Moreover, we showed that specific network structures can be replaced equivalently by simpler structures. In order to benefit from further network reductions, we additionally developed heuristic aggregation methods aiming at reliably approximating the physical behavior of the original structures. Based on a huge history of fine-grained real-world data covering the time range of one year, we comprehensively evaluated all of the heuristic aggregation methods that introduce substitute elements by using simulations. Our evaluations show that the simulation results of non-aggregated and aggregated structures only differ insignificantly, which suggests that the impact of the heuristic aggregation methods on the physical behavior of the remaining network is inconsequential. Furthermore, whenever results for stationary aggregation procedures are already known, we highlighted their analogy to the corresponding transient aggregation procedures.

We designed our aggregation approach so that stations and their control elements remain untouched. This allows the user to model any desired level of station detail, while simultaneously applying our aggregation scheme. As a consequence of (i) the accurate approximations, and (ii) the non-aggregation of stations, solving the *Transient Control Problem* on the aggregated network allows, first, acquiring recommendations for the station control, and second, testing and implementing the solutions in practice. Even though all studies carried out in this chapter arise from natural gas applications due to the availability of data, our overall aggregation approach is not necessarily restricted to natural gas driven networks. Instead, it can be generalized to other networks that feature transient behavior, and capture similar physics, for example, other types of gas, such as hydrogen networks. Finally, we presented an excursion to determining pressure bounds at aggregated nodes, which, from a geometrical perspective arise from the intersection of lines and ellipses.

An interesting avenue for further research would be to introduce a measure for the error incurred in the aggregation. This would allow us to aggregate the network without exceeding a predefined aggregation error. This error measure could be customized in the spirit of Geißler (2011). Albeit in the context of generating piecewise relaxations to non-linear constraint functions, the author determines the number of required discretization points a priori in order to satisfy a given error tolerance.

From the application point of view, we plan to utilize the presented aggregation scheme to generate a network-based demand prognosis for all network nodes. To this end, we could customize our aggregation approach in order to cluster nodes that have a higher interdependency concerning the network topology.

Chapter 7

Solving the Transient Control Problem on Aggregated Networks

In this chapter, we present techniques that aim at finding high-quality solutions for the *Transient Control Problem* on aggregated networks. First, we exploit the disjunctive nature of modeling network stations. More precisely, we apply Balas's convex-hull reformulation (Balas, 1985) to constraints that arise from parallel station arcs. Second, we linearize the objective function in order to avoid absolute value terms. Third, we present a special-tailored moving horizon based primal heuristic for the *Transient Control Problem*.

7.1 Modeling (Anti-)Parallel Station Arcs using Disjunctions

In general, it can be possible to realize different operation modes between two station nodes, such as pressure reduction, compression, frictionless flow throughput, or blocking gas flow. Within the modeling of network stations presented in Section 5.8, these possibilities are implemented by using the concept of station arcs, possibly leading to multiple (anti-)parallel arcs. An example of (anti-)parallel station arcs is given in Figure 5.1, where several modes can be realized between nodes v and z, such as compression from v to z, or pressure reduction from one of both nodes towards the other. A reasonable assumption is that at most one of the (anti-)parallel arcs is active at a time, even though the model would allow for multiple active arcs, for example, by operating parallel compressors and regulators in bypass. However, in most cases, it is not possible to determine the required mode a priori. This means that it is not possible to decide beforehand, if any of the arcs is active, and if so, which one. Since multiple station arcs frequently exist between a given pair of station nodes and at most one of these arcs is active at a time, the station modeling possesses a disjunctive nature.

7.1.1 General disjunctive program

In the following, we present a disjunctive model formulation for the different possible modes between a given pair of station nodes $\ell, r \in \mathcal{V}_s$ with $s \in \mathcal{S}$. Here, one disjunction represents the closed connection between the two nodes, while the remaining disjunctions represent the different open modes, where each one is induced by exactly one corresponding arc. In order to formalize these disjunctions, let us denote the set of all parallel and anti-parallel arcs between two station nodes $\ell, r \in \mathcal{V}_s$ by

$$\mathcal{P}_{\ell,\mathrm{r}} := \{a \in \mathcal{A}_s \mid a = (\ell, r) \quad \text{or} \quad a = (r, \ell),\ s \in \mathcal{S}\}.$$

Then, a general disjunction reads

$$\left(A^{\mathrm{cl}} x \leq b^{\mathrm{cl}}\right) \bigvee_{a \in \mathcal{P}_{\ell,\mathrm{r}}} \left(A_a^{\mathrm{op}}\, x \leq b_a^{\mathrm{op}}\right), \tag{7.1}$$

with $x \in [\underline{x}, \overline{x}] \subseteq \mathbb{R}^n$ and $b^{\mathrm{cl}} \in \mathbb{R}^{m_1}, A^{\mathrm{cl}} \in \mathbb{R}^{m_1 \times n}$, $b_a^{\mathrm{op}} \in \mathbb{R}^{m_2}$, and $A_a^{\mathrm{op}} \in \mathbb{R}^{m_2 \times n}$. System $A^{\mathrm{cl}} x \leq b^{\mathrm{cl}}$ represents the closed connection between nodes ℓ and r, and the arcs in $\mathcal{P}_{\ell,\mathrm{r}}$ impose a system of inequalities $A_a^{\mathrm{op}}\, x \leq b_a^{\mathrm{op}}$ representing the open modes for the existing valves, regulators and compressors (including bypass and active modes). Here, the x-variables correspond to pressure and flow variables associated with the connection between ℓ and r. For an illustration of $\mathcal{P}_{\ell,\mathrm{r}}$, see Figure 5.1. In the example of the figure, the (anti-)parallel station arc sets of a network station $s \in \mathcal{S}$ are given by $\mathcal{P}_{v,w} = \{a_1, a_2\}$, $\mathcal{P}_{v,z} = \{a_3, a_4, a_5\}$, and $\mathcal{P}_{w,z} = \{a_6\}$ with station nodes $\mathcal{V}_s = \{v, w, z\}$ and station arcs $\mathcal{A}_s = \{a_1, a_2, a_3, a_4, a_5, a_6\}$.

7.1.2 Model formulation for disjunctive program

While we introduced in Chapter 5 the concept of station modeling by using big-M terms and binary variables $z_{a,t}$ for the sake of an intuitive formulation, preliminary computational tests revealed that the best model performance is achieved through Balas's convex-hull reformulation (Balas, 1985), where we model the station constraints (5.35) – (5.43) as disjunctions of the form (7.1).

By using binary variables $\lambda^{\mathrm{cl}}, \lambda_a^{\mathrm{op}}$ and copies $x^{\mathrm{cl}}, x_a^{\mathrm{op}} \in [\underline{x}, \overline{x}]$ of the x-variables for the closed and open connections, the disjunction (7.1) can be realized by the following general system of (in)equalities

$$\lambda^{\mathrm{cl}} + \sum_{a \in \mathcal{P}_{\ell,\mathrm{r}}} \lambda_a^{\mathrm{op}} = 1, \tag{7.2a}$$

$$A^{\mathrm{cl}} x^{\mathrm{cl}} \leq b^{\mathrm{cl}} \lambda^{\mathrm{cl}}, \tag{7.2b}$$

$$A_a^{\mathrm{op}}\, x_a^{\mathrm{op}} \leq b_a^{\mathrm{op}}\, \lambda_a^{\mathrm{op}} \qquad \forall a \in \mathcal{P}_{\ell,\mathrm{r}}, \tag{7.2c}$$

$$x = x^{\mathrm{cl}} + \sum_a x_a^{\mathrm{op}}, \tag{7.2d}$$

$$x, x^{\mathrm{cl}}, x_a^{\mathrm{op}} \in [\underline{x}, \overline{x}] \qquad \forall a \in \mathcal{P}_{\ell,\mathrm{r}}, \tag{7.2e}$$

$$\lambda^{\mathrm{cl}}, \lambda_a^{\mathrm{op}} \in \{0, 1\} \qquad \forall a \in \mathcal{P}_{\ell,\mathrm{r}}\,. \tag{7.2f}$$

Here, (7.2a) selects exactly one mode, (7.2b) and (7.2c) activate the corresponding variables and constraints needed to realize this mode, (7.2d) links the necessary continuous variables of the different modes.

Note that Model (7.2) is bounded by virtue of (7.2e) – (7.2f). Then, Balas's convex-hull reformulation is as tight as possible in terms of the LP relaxation. This means in our context, when relaxing the integrality condition on the binary variables, the feasible region of the x-variables corresponds to the convex hull of the regions for the different modes, i.e.,

$$\begin{aligned}
&\operatorname{Proj}_x \Big\{ \Big(x, x^{\mathrm{cl}}, x^{\mathrm{op}}_a, \lambda^{\mathrm{cl}}, \lambda^{\mathrm{op}}_a\Big) \;:\; \Big(x, x^{\mathrm{cl}}, x^{\mathrm{op}}_a, \lambda^{\mathrm{cl}}, \lambda^{\mathrm{op}}_a\Big) \text{ satisfy (7.2a) – (7.2e)} \\
&\qquad \text{and } \lambda^{\mathrm{cl}}, \lambda^{\mathrm{op}}_a \in [0,1] \;\; \forall a \in \mathcal{P}_{\ell,\mathrm{r}} \Big\} \\
&= \operatorname{conv}\Big(\Big\{ x \mid A^{\mathrm{cl}} x \le b^{\mathrm{cl}} \Big\} \bigcup_{a \in \mathcal{P}_{\ell,\mathrm{r}}} \Big\{ x \mid A^{\mathrm{op}}_a x \le b^{\mathrm{op}}_a \Big\} \Big).
\end{aligned}$$

In the following, we formulate the general disjunctive program (7.2) for a particular set of station arcs $\mathcal{P}_{\ell,\mathrm{r}} \subseteq \mathcal{A}_s, s \in \mathcal{S}$. The x-variables in (7.2) represent the pressures $p_{\ell,t} \in [\underline{p}_\ell, \overline{p}_\ell]$, $p_{r,t} \in [\underline{p}_r, \overline{p}_r]$ and the overall flow $q_{\mathcal{P}_{\ell,\mathrm{r}},t} \in [\min_{a \in \mathcal{P}_{\ell,\mathrm{r}}} \underline{q}_a, \max_{a \in \mathcal{P}_{\ell,\mathrm{r}}} \overline{q}_a]$ between nodes ℓ and r. Accordingly, the copies x^{cl} and x^{op}_a of the x-variables in (7.2) correspond to auxiliary pressure and flow variables for the different modes. More precisely, for the closed mode, we have auxiliary pressure variables and one binary variable for each time step $t \in \mathcal{T}$, and for the open modes, we have auxiliary pressure and flow variables as well as binary variables for all $a \in \mathcal{P}_{\ell,\mathrm{r}}$ and for all $t \in \mathcal{T}$, where subindices ℓ_a and r_a indicate the corresponding arc $a \in \mathcal{P}_{\ell,\mathrm{r}}$:

$$p^{\mathrm{cl}}_{\ell,t} \in [0, \overline{p}_\ell],\; p^{\mathrm{cl}}_{r,t} \in [0, \overline{p}_r],\; \lambda^{\mathrm{cl}}_{\mathcal{P}_{\ell,\mathrm{r}},t} \in \{0,1\} \qquad \text{closed mode variables;}$$
$$p^{\mathrm{op}}_{\ell_a,t} \in [0, \overline{p}_\ell],\; p^{\mathrm{op}}_{r_a,t} \in [0, \overline{p}_r],\; q^{\mathrm{op}}_{a,t} \in [\underline{q}_a, \overline{q}_a],\; \lambda^{\mathrm{op}}_{a,t} \in \{0,1\} \qquad \text{open mode variables.}$$

Note that we only have flow variables for the open modes, as the closed mode implies zero flow. Then, the model constraints read:

Selecting exactly one (anti-)parallel station mode. For all $t \in \mathcal{T}$ holds

$$\lambda^{\mathrm{cl}}_{\mathcal{P}_{\ell,\mathrm{r}},t} + \sum_{a \in \mathcal{P}_{\ell,\mathrm{r}}} \lambda^{\mathrm{op}}_{a,t} = 1. \tag{7.3}$$

Pressure coupling. For all $t \in \mathcal{T}$ holds

$$p_{\ell,t} = p^{\mathrm{cl}}_{\ell,t} + \sum_{a \in \mathcal{P}_{\ell,\mathrm{r}}} p^{\mathrm{op}}_{\ell_a,t}, \tag{7.4}$$

$$p_{r,t} = p^{\mathrm{cl}}_{r,t} + \sum_{a \in \mathcal{P}_{\ell,\mathrm{r}}} p^{\mathrm{op}}_{r_a,t}. \tag{7.5}$$

Flow coupling. For all $t \in \mathcal{T}$ holds

$$q_{\mathcal{P}_{\ell,\mathrm{r}},t} = \sum_{a \in \mathcal{P}_{\ell,\mathrm{r}}} q^{\mathrm{op}}_{a,t}. \tag{7.6}$$

Closed modes. For all $t \in \mathcal{T}$ holds

$$p^{\mathrm{cl}}_{\ell,t} \le \overline{p}_\ell \, \lambda^{\mathrm{cl}}_{\mathcal{P}_{\ell,\mathrm{r}},t}, \tag{7.7a}$$

$$p^{\mathrm{cl}}_{r,t} \le \overline{p}_r \, \lambda^{\mathrm{cl}}_{\mathcal{P}_{\ell,\mathrm{r}},t}. \tag{7.7b}$$

Open valves. For all $a = (\ell, r) \in \mathcal{P}_{\ell,\mathrm{r}} \cap \mathcal{A}_{va,s}$ and for all $t \in \mathcal{T}$ holds

$$p^{\mathrm{op}}_{\ell_a,t} - p^{\mathrm{op}}_{r_a,t} \le 0, \tag{7.8a}$$

$$p^{\mathrm{op}}_{r_a,t} - p^{\mathrm{op}}_{\ell_a,t} \le 0, \tag{7.8b}$$

$$p^{\mathrm{op}}_{\ell_a,t} \le \overline{p}_\ell \, \lambda^{\mathrm{op}}_{a,t}, \tag{7.8c}$$

$$p^{\mathrm{op}}_{r_a,t} \le \overline{p}_r \, \lambda^{\mathrm{op}}_{a,t}, \tag{7.8d}$$

$$q^{\mathrm{op}}_{a,t} \le \overline{q}_a \, \lambda^{\mathrm{op}}_{a,t}, \tag{7.8e}$$

$$-q^{\mathrm{op}}_{a,t} \le -\, \underline{q}_a \, \lambda^{\mathrm{op}}_{a,t}. \tag{7.8f}$$

Open regulators. For all $a = (\ell, r) \in \mathcal{P}_{\ell,\mathrm{r}} \cap \mathcal{A}_{rg,s}$ and for all $t \in \mathcal{T}$ holds

$$p^{\mathrm{op}}_{r_a,t} \le p^{\mathrm{op}}_{\ell,t}, \tag{7.9a}$$

$$p^{\mathrm{op}}_{\ell_a,t} \le \overline{p}_\ell \, \lambda^{\mathrm{op}}_{a,t}, \tag{7.9b}$$

$$p^{\mathrm{op}}_{r_a,t} \le \overline{p}_r \, \lambda^{\mathrm{op}}_{a,t}, \tag{7.9c}$$

$$q^{\mathrm{op}}_{a,t} \le \overline{q}_a \, \lambda^{\mathrm{op}}_{a,t}. \tag{7.9d}$$

Open compressors. For all $a = (\ell, r) \in \mathcal{P}_{\ell,\mathrm{r}} \cap \mathcal{A}_{cs,s}$ and for all $t \in \mathcal{T}$ holds

$$p^{\mathrm{op}}_{r_a,t} - p^{\mathrm{op}}_{\ell_a,t} \, r^{max}_a \le 0, \tag{7.10a}$$

$$a_0 \, p^{\mathrm{op}}_{\ell_a,t} + a_1 \, p^{\mathrm{op}}_{r_a,t} + a_2 \, q^{\mathrm{op}}_{a,t} \le (P^{\max}_a - a_3) \, \lambda^{\mathrm{op}}_{a,t}, \tag{7.10b}$$

$$p^{\mathrm{op}}_{\ell_a,t} \le \overline{p}_\ell \, \lambda^{\mathrm{op}}_{a,t}, \tag{7.10c}$$

$$p^{\mathrm{op}}_{r_a,t} \le \overline{p}_r \, \lambda^{\mathrm{op}}_{a,t}, \tag{7.10d}$$

$$q^{\mathrm{op}}_{a,t} \le \overline{q}_a \, \lambda^{\mathrm{op}}_{a,t}. \tag{7.10e}$$

Equation (7.3) selects exactly one operation mode, which is indicated by setting the corresponding binary variable to one. Then, the remaining binary variables equal zero, and as a consequence, the auxiliary pressure and flow variables of the non-selected modes are disabled by also setting them to zero in (7.7) – (7.10). For a pair of station nodes ℓ and r, (7.7) models the closed mode and (7.8) – (7.10) model the open modes induced by all valves, regulators and compressor arcs in $\mathcal{P}_{\ell,\mathrm{r}}$. By deactivating a mode, the upper bounds of the auxiliary pressure variables correspond to the ones of the pressure variables $p_{\ell,t}$ and $p_{r,t}$, while the lower bounds of the auxiliary pressure variables are zero by default. The bounds of the auxiliary flow variables equal the ones of the corresponding station arcs from Section 5.8. There, we assumed that the flow bounds are specified such that bi-directional flow in valves and one-directional flow in regulators and compressors is possible, i.e.,

$$\begin{aligned} &q^{\mathrm{op}}_{a,t} \in [\underline{q}_a, \overline{q}_a],\ \underline{q}_a < 0 && \forall a = (\ell, r) \in \mathcal{P}_{\ell,\mathrm{r}} \cap \mathcal{A}_{va,s},\ \forall t \in \mathcal{T}, \\ &q^{\mathrm{op}}_{a,t} \in [0, \overline{q}_a] && \forall a = (\ell, r) \in \mathcal{P}_{\ell,\mathrm{r}} \cap (\mathcal{A}_{rg,s} \cup \mathcal{A}_{cs,s}),\ \forall t \in \mathcal{T}. \end{aligned}$$

Equations (7.4) – (7.5) link all auxiliary pressure variables associated with the different modes and thereby guarantee that they equal in sum the pressure variables $p_{\ell,t}$ and $p_{r,t}$. Likewise in Equation (7.6), the overall flow $q_{\mathcal{P}_{\ell,\mathrm{r}},t}$ between two station nodes ℓ and r corresponds to the sum of the auxiliary flow variables. Finally, Systems (7.8), (7.9) and (7.10) establish the required pressure and flow behavior of open valves, regulators and compressors (including bypass and active modes). In total, the reformulated station modeling for a particular set of station arcs $\mathcal{P}_{\ell,\mathrm{r}} \subseteq \mathcal{A}_s$ with $s \in \mathcal{S}$ is given by constraints (7.3) – (7.10).

7.1.3 Linearization of the objective function

Due to our model reformulations, the objective function from Section 5.10, given by

$$\begin{aligned} \min \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \Bigg(&\sum_{a \in \mathcal{A}_{va,s}} \Big(\omega_a \left| z_{a,t-1} - z_{a,t} \right| \Big) + \sum_{a \in \mathcal{A}_{cv,s}} \Big(\omega_a \left| z_{a,t-1} - z_{a,t} \right| \Big) \\ &+ \sum_{a \in \mathcal{A}_{cs,s}} \Big(\omega_a \left| z_{a,t-1} - z_{a,t} \right| \Big) \Bigg), \end{aligned}$$

transforms to

$$\min \sum_{s \in \mathcal{S}} \sum_{\mathcal{P}_{\ell,\mathrm{r}} \subseteq \mathcal{A}_s} \sum_{a \in \mathcal{P}_{\ell,\mathrm{r}}} \sum_{t \in \mathcal{T}} \omega_a \left| \lambda^{\mathrm{op}}_{a,t-1} - \lambda^{\mathrm{op}}_{a,t} \right|.$$

In order to linearize this objective function, we introduce binary variables

$$\lambda_{a,t} \in \{0, 1\} \qquad \forall s \in \mathcal{S},\ \forall \mathcal{P}_{\ell,\mathrm{r}} \subseteq \mathcal{A}_s,\ \forall a \in \mathcal{P}_{\ell,\mathrm{r}},\ \forall t \in \mathcal{T}$$

that indicate mode changes of station arcs between time steps $t - 1$ and t. Then, we model the absolute value terms by adding the following side constraints to the model

$$\lambda^{\mathrm{op}}_{a,t-1} - \lambda^{\mathrm{op}}_{a,t} \le \lambda_{a,t} \tag{7.11}$$

$$-\lambda^{\mathrm{op}}_{a,t-1} + \lambda^{\mathrm{op}}_{a,t} \le \lambda_{a,t} \tag{7.12}$$

for all $s \in \mathcal{S}$, for all $\mathcal{P}_{\ell,\mathrm{r}} \subseteq \mathcal{A}_s$, for all $a \in \mathcal{P}_{\ell,\mathrm{r}}$, and for all $t \in \mathcal{T}$, and minimize over

$$\min \sum_{s \in \mathcal{S}} \sum_{\mathcal{P}_{\ell,\mathrm{r}} \subseteq \mathcal{A}_s} \sum_{a \in \mathcal{P}_{\ell,\mathrm{r}}} \sum_{t \in \mathcal{T}} \omega_a \, \lambda_{a,t}. \tag{7.13}$$

7.2 Final Model

With these reformulations, our final model $\mathscr{M}$ for the *Transient Control Problem* reads

$$
\begin{aligned}
& (7.13) && && (7.14a)\\
\text{s.t.}\ & (5.18) - (5.19) && \forall a \in \mathcal{A}_{pi},\ \forall t \in \mathcal{T}, && (7.14b)\\
& (5.20) && \forall v \in \mathcal{V}_{\text{sto}},\ \forall t \in \mathcal{T}, && (7.14c)\\
& (5.21) && \forall a \in \mathcal{A}_{sc},\ \forall t \in \mathcal{T}, && (7.14d)\\
& (5.22) - (5.26) && \forall a \in \mathcal{A}_{va},\ \forall t \in \mathcal{T}, && (7.14e)\\
& (5.27) - (5.34) && \forall a \in \mathcal{A}_{cv},\ \forall t \in \mathcal{T}, && (7.14f)\\
& (5.45) && \forall v \in \mathcal{V} \setminus \mathcal{V}_{\text{sto}},\ \forall t \in \mathcal{T}, && (7.14g)\\
& (5.46) && \forall v \in \mathcal{V}_{\text{sto}},\ \forall t \in \mathcal{T}, && (7.14h)\\
& (7.3) - (7.7) && \forall s \in \mathcal{S},\ \forall \mathcal{P}_{\ell,\text{r}} \subseteq \mathcal{A}_s,\ \forall t \in \mathcal{T}, && (7.14i)\\
& (7.8) && \forall s \in \mathcal{S},\ \forall \mathcal{P}_{\ell,\text{r}} \subseteq \mathcal{A}_s,\ \forall a \in \mathcal{P}_{\ell,\text{r}} \cap \mathcal{A}_{va,s},\ \forall t \in \mathcal{T}, && (7.14j)\\
& (7.9) && \forall s \in \mathcal{S},\ \forall \mathcal{P}_{\ell,\text{r}} \subseteq \mathcal{A}_s,\ \forall a \in \mathcal{P}_{\ell,\text{r}} \cap \mathcal{A}_{rg,s},\ \forall t \in \mathcal{T}, && (7.14k)\\
& (7.10) && \forall s \in \mathcal{S},\ \forall \mathcal{P}_{\ell,\text{r}} \subseteq \mathcal{A}_s,\ \forall a \in \mathcal{P}_{\ell,\text{r}} \cap \mathcal{A}_{cs,s},\ \forall t \in \mathcal{T}, && (7.14l)\\
& (7.11) - (7.12) && \forall s \in \mathcal{S},\ \forall \mathcal{P}_{\ell,\text{r}} \subseteq \mathcal{A}_s,\ \forall a \in \mathcal{P}_{\ell,\text{r}},\ \forall t \in \mathcal{T}. && (7.14m)
\end{aligned}
$$

7.3 Specialized Moving Horizon Heuristic

In the literature, a vast amount of heuristic approaches exist that apply moving horizon schemes to optimization problems, also referred to as rolling horizon. While the classical field of moving horizon applications concerns problems that involve uncertain data, it is also used to reduce the model size. Applications in line with the second aspect comprise, for example, scheduling of multi-purpose plants (Dimitriadis et al., 1997), the capacitated lot sizing problem (Mercé and Fontan, 2003), the aircraft sequencing problem (Furini et al., 2015) and room scheduling (Addis et al., 2016).

In the spirit of reducing the model size, the presented moving horizon heuristic, also called HEUR, splits Model $\mathscr{M}$ into $|\mathcal{T}|$-many MINLP sub-problems, resulting in a sub-model for each time step. The rationale of the heuristic is to deal with sub-models that are small enough and can be solved reliably in short computation time, ideally to global optimality. To facilitate their solving process, we fix the station modes of the Sub-MINLP-models to modes acquired by solving auxiliary MIP-models. Starting with the first time step, the algorithm moves forward in time and solves Sub-MINLP-models of consecutive time steps. However, in the case that a sub-model does not find a solution, it might be necessary to unfix these station modes or even move backwards in time in order to compensate disadvantageous decisions taken in preceding time steps. Finally, a solution for the original Model $\mathscr{M}$ can be gained by combining the solutions of the sub-models.

In the following, we present the Sub-MINLP-models and the auxiliary MIP models before explaining the heuristic in more detail.

Sub-MINLP models. For a particular time step $t \in \mathcal{T}$, let us denote the Sub-MINLP-model by $\texttt{SubMINLP}_t$. Its variables and constraints are induced by Model $\mathscr{M}$ restricted to time step t. Note that the sub-models are still transient, as they contain the continuity constraints for pipes $\mathcal{A}_{pi}$ and storages $\mathcal{V}_{\text{sto}}$ that couple pressures over time, here between time steps $t-1$ and t.

We note that the additional incorporation of station slacks supports finding even non-slack solutions to the Sub-MINLPs. For this reason, we introduce slack variables $p^+_{v,t}, p^-_{v,t} \geq 0$ and $q^+_{a,t}, q^-_{a,t} \geq 0$ for all station nodes $v \in \mathcal{V}_s$ and station arcs $a \in \mathcal{A}_s$, $s \in \mathcal{S}$, and then add the following constraints to the $\texttt{SubMINLP}_t$-models

$$\underline{p}_v - p^-_{v,t} \leq p_{v,t} \tag{7.15a}$$

$$p_{v,t} \leq \overline{p}_v + p^+_{v,t} \tag{7.15b}$$

$$\underline{q}_a - q^-_{a,t} \leq q_{a,t} \tag{7.15c}$$

$$q_{a,t} \leq \overline{q}_a + q^+_{a,t}. \tag{7.15d}$$

In return, the usage of the slack variables is penalized in the sub-models' objective function with coefficients $\omega_q, \omega_p \geq 0$, in addition to minimizing the number of operation mode changes in network stations with respect to the previous time step. The objective then reads for a particular time step $t \in \mathcal{T}$:

$$\begin{aligned} \min \; & \sum_{s \in \mathcal{S}} \sum_{\mathcal{P}_{\ell,\mathrm{r}} \subseteq \mathcal{A}_s} \sum_{a \in \mathcal{P}_{\ell,\mathrm{r}}} \omega_a \, \lambda_{a,t} \\ & + \sum_{s \in \mathcal{S}} \left(\omega_q \sum_{a \in \mathcal{A}_s} \left(q^+_{a,t} + q^-_{a,t} \right) + \omega_p \sum_{v \in \mathcal{V}_s} \left(p^+_{v,t} + p^-_{v,t} \right) \right). \end{aligned} \tag{7.16}$$

However, we only accept a solution of a $\texttt{SubMINLP}_t$-model as part of the overall solution to Model $\mathscr{M}$, if all slack values are zero. To avoid that the best solution found has positive slack values despite existence of a non-slack solution with a worse objective value, the coefficients ω_q, ω_p can be set to reasonable high values.

In summary, a particular $\texttt{SubMINLP}_t$-model is given by

$$\begin{aligned} & (7.16), \\ \text{s.t.} \; & (7.14\text{b}) - (7.14\text{m}), \\ & (7.15\text{a}) - (7.15\text{d}), \end{aligned}$$

where all constraints are restricted to the current time step $t \in \mathcal{T}$.

Although the $\texttt{SubMINLP}_t$-models are reduced in size, they might still be challenging to solve, depending on the number of variables and constraints, and the structure of the optimization model, which is induced by the underlying (aggregated) network. For this reason, we attempt to further alleviate the computational burden of the $\texttt{SubMINLP}_t$-models by fixing the binary variables $(\lambda^{\text{cl}}_{\mathcal{P}_{\ell,\mathrm{r}},t}, \lambda^{\text{op}}_{a,t})_{a \in \mathcal{A}_s, \mathcal{P}_{\ell,\mathrm{r}} \subseteq \mathcal{A}_s, s \in \mathcal{S}}$ to solution values gained by solving auxiliary MIP models. We recall that these binary variables indicate the station operation modes at the current time step $t \in \mathcal{T}$. If no solution to $\texttt{SubMINLP}_t$ is obtained in this way, the heuristic proceeds by solving $\texttt{SubMINLP}_t$ with variable station modes, indicated by the boolean `isStationFixed` in Algorithm 5.

Auxiliary MIP models. For a particular time step $t \in \mathcal{T}$, let us denote the auxiliary MIP model by $\texttt{MIP}_{t,\ldots,|\mathcal{T}|}$. It corresponds to the model formulation $\mathscr{M}$ from (7.14), with two differences: First, we include only those constraints adhering to the current and all future time steps $\mathcal{T}_t := \{t, \ldots, |\mathcal{T}|\}$, and second, we acquire a linear model by linearizing the momentum Equation (5.19). To this end, we set the absolute velocity $v = (\mathrm{c_s}/A)\,(q/p)$ to a constant value in the friction term, similarly as done in Hennings (2018). By applying the trapezoidal rule, this linearization technique yields

$$p_{r,t} - p_{\ell,t} + \beta_a\,(p_{r,t} + p_{\ell,t}) + \tilde{\gamma}_a\,(q_{r,t} + q_{\ell,t}) = 0, \tag{7.17}$$

with $\tilde{\gamma}_a = \gamma_a\,|v_a|\,A_a/\,\mathrm{c_s}^2$.

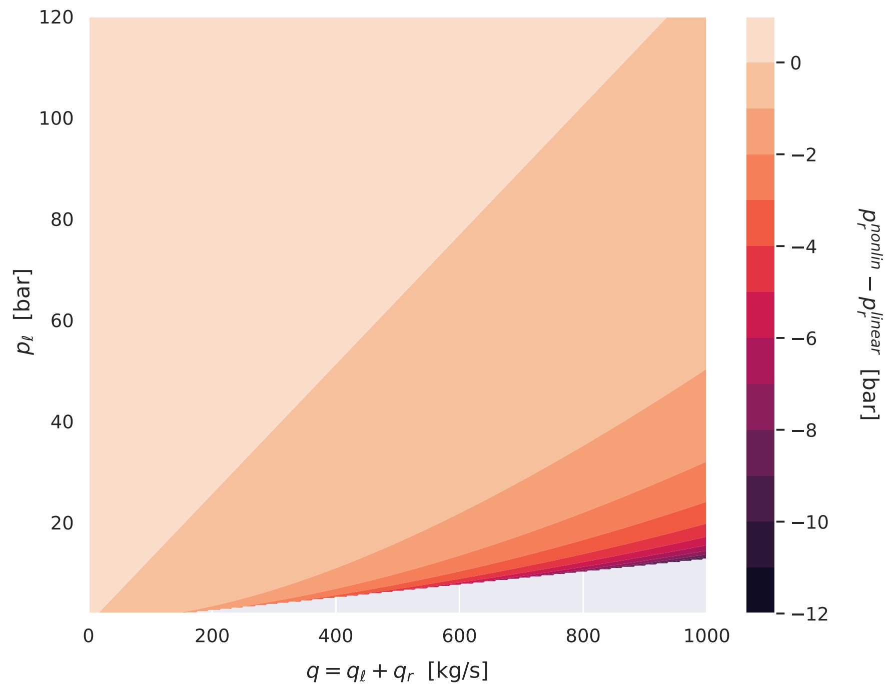


Figure 7.1: Difference between the nonlinear (5.19) and linearized (7.17) pipe equations for a horizontal pipe $a = (\ell, r)$, with parameters $\lambda_a = 8.6e{-}03$, $L_a = 10,000$ [m], $D_a = 1.2$ [m], $v_a = 4.0$ [m/s] and $\beta_a = 0.0$.

Figure 7.1 illustrates the difference between the nonlinear (5.19) and linearized (7.17) pipe equations for a single horizontal transmission pipeline. It can be seen that the difference increases with higher mass flow values at lower pressures levels. Besides that, in the example of the figure, the linearization overestimates the nonlinear pressure loss.

For each solution that occurs while solving $\texttt{MIP}_{t,\ldots,|\mathcal{T}|}$, we store the values of the binary variables

$$\left(\lambda^{\mathrm{cl}}_{\mathcal{P}_{\ell,\mathrm{r}},k}, \lambda^{\mathrm{op}}_{a,k}\right)_{a\in\mathcal{A}_s,\,\mathcal{P}_{\ell,\mathrm{r}}\subseteq\mathcal{A}_s,\,s\in\mathcal{S},\,k\in\{t,\ldots,|\mathcal{T}|\}}$$

in a solution pool, called $\texttt{MIPSols}_{t,\dots,|\mathcal{T}|}$. Note that each time step $t \in \mathcal{T}$ induces a different pool. Solutions in a particular pool $\texttt{MIPSols}_{t,\dots,|\mathcal{T}|}$ only contain station modes with respect to the current and future time steps $t, \dots, |\mathcal{T}|$ and are gained from that particular $\texttt{MIP}_{t,\dots,|\mathcal{T}|}$. As we use these solutions for possible fixings of station operation modes when solving $\texttt{SubMINLP}_t$-models, we aim at getting a variety of station solutions. Thus, we catch all solutions occurring in the branch-and-bound algorithm, even so-called poor solutions that do not improve the incumbent in the solving process. However, we only add solutions to the pool, if they are different than the stored ones. Subsequently, we order the solutions in $\texttt{MIPSols}_{t,\dots,|\mathcal{T}|}$ in non-descending order with respect to their objective value. Here, solutions with smaller objective values are preferred, as we deal with a minimization problem. Given that these station solutions are feasible for a linearized model, there is hope to assume that they are also feasible for the nonlinear sub-models. Nevertheless, the differences between the linearized and nonlinear pipe equations, as shown in Figure 7.1, might turn the physically more accurate nonlinear $\texttt{SubMINLP}_t$-models infeasible.

Non-empty solution pools $\texttt{MIPSols}_{t,\dots,|\mathcal{T}|}$ are a key ingredient of HEUR. Thus, in the case that no solution to a $\texttt{MIP}_{t,\dots,|\mathcal{T}|}$-model has been found within the given time limit, we decompose $\texttt{MIP}_{t,\dots,|\mathcal{T}|}$ by dividing the discretized time range $t, \dots, \mathcal{T}$ in halves. Then we solve the reduced MIP-models in sequential order with respect to progressing time, because a solution of a preceding model provides the required initial pressure values for the subsequent model. If no solution to one of the reduced models is found, then we continue the partitioning until all reduced MIP-models are feasible or no more partitioning is possible. In the latter case, at least one model with respect to the finest possible partition did not find a solution within the given time limit. Otherwise, all reduced MIP-models are feasible. Then, we add the station modes of the best available solution of each reduced MIP to $\texttt{MIPSols}_{t,\dots,|\mathcal{T}|}$.

Heuristic. Let us now explain the moving horizon based Algorithm 5 in more detail. It consists of the following major steps:

1. generate initial station modes for all time steps $t \in \mathcal{T}$ by solving $\texttt{MIP}_{t=1,\dots,|\mathcal{T}|}$;
2. move along the time horizon;
   a) decide on moving backwards;
   b) decide on generating new station modes;
3. solve consecutive $\texttt{SubMINLP}_t$-models and decide on moving forwards;
4. validate, whether the final solution is accepted for the original model $\mathscr{M}$.

The heuristic starts by solving $\texttt{MIP}_{t=1,\dots,|\mathcal{T}|}$ in order to generate station modes for the entire time horizon $\mathcal{T}$ (Step 1), which we refer to as *initial station mode fixings* in the following. In the case that no solution of $\texttt{MIP}_{t=1,\dots,|\mathcal{T}|}$ is found, even not by decomposing $\texttt{MIP}_{t=1,\dots,|\mathcal{T}|}$, then the heuristic aborts, assuming that solving the entire MINLP Model $\mathscr{M}$ is also too challenging. Otherwise, we add all available station solutions to the pool

Algorithm 5 Moving horizon heuristic (HEUR) for the *Transient Control Problem*

Input: Model formulation $\mathscr{M}$ for *Transient Control Problem*
Output: Solution `finalSol` of Model $\mathscr{M}$ or `NO SOL FOUND`

`finalSol` $\leftarrow \emptyset$
▷ Step 1: Generate initial station modes at time step $t = 1$
`MIPSols`$_{t=1,\dots,|\mathcal{T}|}$ $\leftarrow$ solve `MIP`$_{t=1,\dots,|\mathcal{T}|}$ ▷ collect all station solutions of initial MIP
if `MIPSols`$_{t=1,\dots,|\mathcal{T}|} = \emptyset$ **then**
  return `NO SOL FOUND` ▷ leave heuristic
end if
`stationSol`$_{t=1,\dots,|\mathcal{T}|}$ $\leftarrow$ `getNextBestMipSol`$_{t=1,\dots,|\mathcal{T}|}$ ▷ select best station solution
▷ Step 2: Start moving along the time horizon
`nrCurrentIterationCalls`$_t \leftarrow 0$ ▷ number of successive calls of current time iteration t
while $t \leftarrow 1 \leq |\mathcal{T}|$ **do**
  ▷ Step 2a: Decide on moving backwards
  `isStationFixed` $\leftarrow$ `nrCurrentIterationCalls`$_t \neq 1$ ▷ assign comparison result
  if `nrCurrentIterationCalls`$_t > 1$ **then**
    if no `MIPSols`$_{t,\dots,|\mathcal{T}|}$ available **then** ▷ all `MIPSols`$_{t,\dots,|\mathcal{T}|}$ used or `MIP`$_{t,\dots,|\mathcal{T}|}$ not solved yet
      if $t = 1$ **then**
        return `NO SOL FOUND` ▷ leave heuristic
      end if
      $t \leftarrow t - 1$ ▷ move backwards
    end if
    ▷ Step 2b: Get new station modes, eventually require solving a MIP
    if `MIP`$_{t,\dots,|\mathcal{T}|}$ not solved yet **then** ▷ MIP model for time steps in $\mathcal{T}_t$
      `MIPSols`$_{t,\dots,|\mathcal{T}|}$ $\leftarrow$ solve `MIP`$_{t,\dots,|\mathcal{T}|}$ ▷ collect all station solutions of MIP
    end if
    if `MIPSols`$_{t,\dots,|\mathcal{T}|} = \emptyset$ **then**
      return `NO SOL FOUND` ▷ leave heuristic
    end if
    `stationSol`$_{t,\dots,|\mathcal{T}|}$ $\leftarrow$ `getNextBestMipSol`$_{t,\dots,|\mathcal{T}|}$ ▷ get next station solution
  end if
  `nrCurrentIterationCalls`$_t$ = `nrCurrentIterationCalls`$_t + 1$
  ▷ Step 3: Solve current Sub-MINLP for time step t and update final solution
  if `SubMINLP`$_t$ has not been solved yet **then**
    solve `SubMINLP`$_t$ (`isStationFixed`, `stationSol`$_{t,\dots,|\mathcal{T}|}$, `finalSol`)
    if solution of `SubMINLP`$_t$ exists with non-zero slack **then** ▷ move forwards
      `finalSol` $\leftarrow$ update with solution of `SubMINLP`$_t$
      $t \leftarrow t + 1$
      `nrCurrentIterationCalls`$_t \leftarrow 0$
    end if
  end if
end while
▷ Step 4: Check if solution for $\mathscr{M}$ is accepted
if `finalSol` accepted **then**
  return `finalSol`
end if
return `NO SOL FOUND`

$\texttt{MIPSols}_{t=1,\dots,|\mathcal{T}|}$ (lines 2–5). Afterwards on line 6, $\texttt{getNextBestMipSol}_{t=1,\dots,|\mathcal{T}|}$ selects the best available and so far unused solution

$$\texttt{stationSol}_{t,\dots,|\mathcal{T}|} := \left(\lambda^{\mathrm{cl}}_{\mathcal{P}_{\ell,\mathrm{r}},k}, \lambda^{\mathrm{op}}_{a,k}\right)_{\forall a\in\mathcal{A}_s,\, \forall\, \mathcal{P}_{\ell,\mathrm{r}}\subseteq\mathcal{A}_s,\, \forall s\in\mathcal{S},\, \forall k\in\{t,\dots,|\mathcal{T}|\}}$$

from the pool, where the selection criterion is given by the objective function value.

From line 7 on, the algorithm enters the moving horizon phase (Step 2). Starting with the first time step $t = 1$, the heuristic moves along the time horizon up to and including $|\mathcal{T}|$. Let us denote by $\texttt{nrCurrentIterationCalls}_t$ the number of times the current time step t is called one after the other without moving to the next or previous time step, and let the boolean `isStationFixed` indicate whether $\texttt{SubMINLP}_t$ is solved with fixed station modes taken from the solution pool. Note that the current $\texttt{SubMINLP}_t$ only depends on time step t, however, $\texttt{stationSol}_{t,\dots,|\mathcal{T}|}$ additionally contains station modes for the future time steps $t+1,\dots,\mathcal{T}$, as they will be used for Sub-MINLP-solves at subsequent time steps, if not switching to other fixings.

Each $\texttt{SubMINLP}_t$ is either solved with fixed or variable station modes, specified on line 9: For a given time step t, we first solve $\texttt{SubMINLP}_t$ with fixed station modes, i.e., $\texttt{nrCurrentIterationCalls}_t = 0$. By fixing binary variables and solving Sub-MINLP-models with less integer variables, HEUR works in the fashion of the Sub-NLP heuristic (Vigerske, 2012), where all discrete variables are fixed and the resulting NLP-model is solved. However, when no solution of a $\texttt{SubMINLP}_t$ is found, or it is even infeasible for given station mode fixings, then we solve $\texttt{SubMINLP}_t$ again, but with variable station modes, assuming that these fixings may have been too restrictive. This event occurs when $\texttt{nrCurrentIterationCalls}_t = 1$. If still no solution for $\texttt{SubMINLP}_t$ exists, i.e., $\texttt{nrCurrentIterationCalls}_t > 1$, then HEUR either solves $\texttt{SubMINLP}_t$ with differently fixed station modes or moves backwards in time (line 15 in Step 2a). Moving backwards in time occurs, if the heuristic advanced to a time step $1 < t \leq |\mathcal{T}|$ and no solution in the current pool $\texttt{MIPSols}_{t,\dots,|\mathcal{T}|}$ is available. Note that the algorithm moves backwards to the preceding time step $t-1$ even though a solution to $\texttt{SubMINLP}_{t-1}$ has already been found. But solving $\texttt{SubMINLP}_{t-1}$ with differently fixed station modes yields a modified solution, which in return provides new initial pressure values for $\texttt{SubMINLP}_t$ at time step t. This is exactly the reason for moving back in time, as decisions that are taken at earlier stages within a moving heuristic approach might result in unfavorable or even infeasible pressure and flow situations at later time steps. In our context, we observed that fixed station modes $\texttt{stationSol}_{t,\dots,|\mathcal{T}|}$ leading to an infeasible $\texttt{SubMINLP}_t$-model at time step t are likely to yield infeasible sub-models at the subsequent time steps $t+1,\dots,|\mathcal{T}|$, if not switching to other station modes in earlier time steps.

In Step 2b, new station modes are fetched (line 23), either for the preceding time step $t-1$ in the case of a backward move, or for the current time step t. To this end, the corresponding MIP is solved (line 18), if not solved yet. Let us remark that the heuristic throughout aborts if all solutions of the current pool have been used, or solving a MIP-model did not yield any solution (lines 4, 13, 21).

In Step 3, a $\texttt{SubMINLP}_t$-model is solved (lines 26 – 33). However, to avoid redundant $\texttt{SubMINLP}_t$-runs, we ensure that the exact same $\texttt{SubMINLP}_t$-model is not solved multiple times. A particular $\texttt{SubMINLP}_t$-solve depends on the parameters `isStationFixed` and $\texttt{stationSol}_{t,\dots,|\mathcal{T}|}$ indicating whether and how to fix station modes. It also depends on `finalSol`, which contains solutions of preceding $\texttt{SubMINLP}_t$-runs and thereby the necessary initial values for the constraints that couple time steps $t-1$ and t. Whenever a $\texttt{SubMINLP}_t$-model is feasible and all slack values are zero, we add its best available solution to `finalSol` (line 29) and proceed with the next time step $t+1$ (line 30). After having solved $\texttt{SubMINLP}_t$ for all $t \in \mathcal{T}$, `finalSol` will be the solution of the original MINLP model $\mathscr{M}$. However, as long as all sub-models for the remaining time steps $t+1, \dots, |\mathcal{T}|$ have not been solved, the $\texttt{SubMINLP}_t$-solution is not finally confirmed for Model $\mathscr{M}$. This is because in the case that a sub-model finds no solution at a later time step $t+1, \dots, |\mathcal{T}|$, backward moves might discard previously obtained sub-model solutions. Then, we update `finalSol` with the solutions of the latest solved $\texttt{SubMINLP}_t$-models. Note that as long as $\texttt{SubMINLP}_t$-models are feasible at consecutive time steps without requiring fixing new station modes, i.e., without entering the if statement on line 10, the corresponding station modes originate from $\texttt{stationSol}_{\hat{t},\dots,|\mathcal{T}|}$ of an earlier time step $\hat{t} < t$. Ideally, all nonlinear sub-models provide a solution for the initial station mode fixings $\texttt{stationSol}_{t=1,\dots,|\mathcal{T}|}$, but this is frequently not the case, as the computational study in Chapter 8 shows.

Finally, in Step 4, when solutions to all $\texttt{SubMINLP}_t$-models are found, the algorithm verifies if `finalSol` is indeed a solution for the original model $\mathscr{M}$, since numerical issues and feasibility tolerances of the solver might classify the generated solution `finalSol` as invalid.

Chapter 8

Computational Study

In this chapter, we present a computational study that investigates whether the solution approach described in Chapters 6 – 7 is suited to find solutions for Model $\mathscr{M}$. We recall that the approach comprises three major steps, (i) the network aggregation from Chapter 6, (ii) the model reformulations and (iii) the moving horizon based primal heuristic HEUR from Chapter 7. These developments were driven by the goal to acquire a plan for the network control that can ideally be used as a recommendation for practitioners on how to operate remotely controllable elements. We benchmark our solution approach on three test sets, each containing a huge number of instances based on real-world data, and investigate whether it reliably generates solutions in acceptable computation time.

8.1 Experimental Setup

Networks. The computational study was carried out on three different networks that vary in size and structure. As starting point, we used a network, called *Net 1*, consisting of 351 nodes and 506 arcs. It has a simple, rather gunbarrel-shaped structure, even though it comprises multiple transmission lines in parallel. To test our approach on networks that are larger and more complex, we continued the computational experiments on the middle-sized network *Net 2* and large network *Net 3*, which are approximately three and ten times bigger than *Net 1* with respect to the number of nodes and arcs. The largest network represents a major part of the German high-calorific gas network and was provided by our project partner Open Grid Europe (OGE). Furthermore, *Net 1* and *Net 2* are subnetworks of *Net 3*. Figures 8.1 – 8.2 show all networks before and after the aggregation. For *Net 3*, we additionally visualize a backbone network, whose representation was manually created by experts of OGE. In this way, we have a sketch of how practitioners think about their network.

Table 8.1 provides detailed information about the number of elements of all three networks before, during, and after the aggregation. Our aggregation Algorithm 1 reduces the network sizes down to 16% (*Net 1*), 13% (*Net 2*), and 6% (*Net 3*) with respect to the number of network arcs. Having larger reduction factors with increasing network sizes is partly a consequence of the more aggressive threshold value that we selected for

Aggregation method	$\|\mathcal{V}\|$	$\|\mathcal{A}\|$	$\|\mathcal{V}^+\|$	$\|\mathcal{V}^-\|$	Vol	LP	Size
Original network	**351**	**506**	5	24	100.0	100.0	100.0
Remove non-remote controllable elements	268	254	-	-	-	-	50.20
Remove disconnected components	190	210	-	-	95.98	99.74	41.50
Aggregate distribution subnetworks	184	203	-	-	-	-	40.11
Aggregate passive trees	143	162	-	-	-	-	32.01
Shrink pipes (short lengths)	-	-	-	-	-	-	-
Tidy-up network	-	-	-	-	-	-	-
Merge parallel pipes	143	151	-	-	-	-	29.84
Merge parallel active elements	-	-	-	-	-	-	-
Merge serial pipes	58	75	-	-	-	99.69	14.82
Merge serial active elements	-	-	-	-	-	-	-
Decompose mixed boundary nodes	66	83	-	-	-	-	16.40
Aggregated network	**66**	**83**	5	22	95.98	99.69	16.40

(a) *Net 1*

Aggregation method	$\|\mathcal{V}\|$	$\|\mathcal{A}\|$	$\|\mathcal{V}^+\|$	$\|\mathcal{V}^-\|$	Vol	LP	Size
Original network	**1,190**	**1,498**	11	108	100.0	100.0	100.0
Remove non-remote controllable elements	831	936	-	-	-	-	62.48
Remove disconnected components	729	853	-	-	97.98	99.78	56.94
Aggregate distribution subnetworks	579	660	-	-	-	-	44.06
Aggregate passive trees	526	607	-	-	-	-	40.52
Shrink pipes (short lengths)	317	398	-	-	97.96	99.32	26.57
Tidy-up network	314	393	-	-	-		26.23
Merge parallel pipes	314	362	-	-	-	-	24.17
Merge parallel active elements	314	357	-	-	-	-	23.83
Merge serial pipes	129	172	-	-	-	97.59	11.48
Merge serial active elements	119	162	-	-	-	-	10.81
Decompose mixed boundary nodes	135	188	-	-	-	-	12.55
Aggregated network	**135**	**188**	10	60	97.96	97.59	12.55

(b) *Net 2*

Aggregation method	$\|\mathcal{V}\|$	$\|\mathcal{A}\|$	$\|\mathcal{V}^+\|$	$\|\mathcal{V}^-\|$	Vol	LP	Size
Original network	**3,729**	**4,507**	38	348	100.0	100.0	100.0
Remove non-remote controllable elements	2,696	2,854	-	-	-	-	63.32
Remove disconnected components	2,149	2,537	-	-	97.10	97.84	56.29
Aggregate distribution subnetworks	1,525	1,768	-	-	-	-	39.22
Aggregate passive trees	1,123	1,366	-	-	-	-	30.31
Shrink pipes (short lengths)	489	717	-	-	91.25	87.86	15.91
Tidy-up network	484	686	-	-	-		15.22
Merge parallel pipes	484	602	-	-	-	-	13.36
Merge parallel active elements	484	561	-	-	-	-	12.45
Merge serial pipes	194	271	-	-	-	88.22	6.01
Merge serial active elements	159	236	-	-	-	-	5.24
Decompose mixed boundary nodes	205	282	-	-	-	-	6.26
Aggregated network	**205**	**282**	30	89	91.25	88.22	6.26

(c) *Net 3*

Table 8.1: Impact of the aggregation methods on the network size. For example instances, the table shows the number of network elements. It also displays the total storage volume (col. *Vol*) and the total initial line-pack value $LP_{t=0}$ of the networks (col. *LP*), and the reduced arc size (col. *Size*) in relation to the corresponding values of the original network in [%]. The number of sources and sinks are only depicted for the original and final aggregated networks, as we use the concept of mixed boundary nodes during the aggregation process, see Section 6.1.1. For the *series-parallel graph* methods, the table reports summarized values, since they might be invoked multiple times within Algorithm 1.

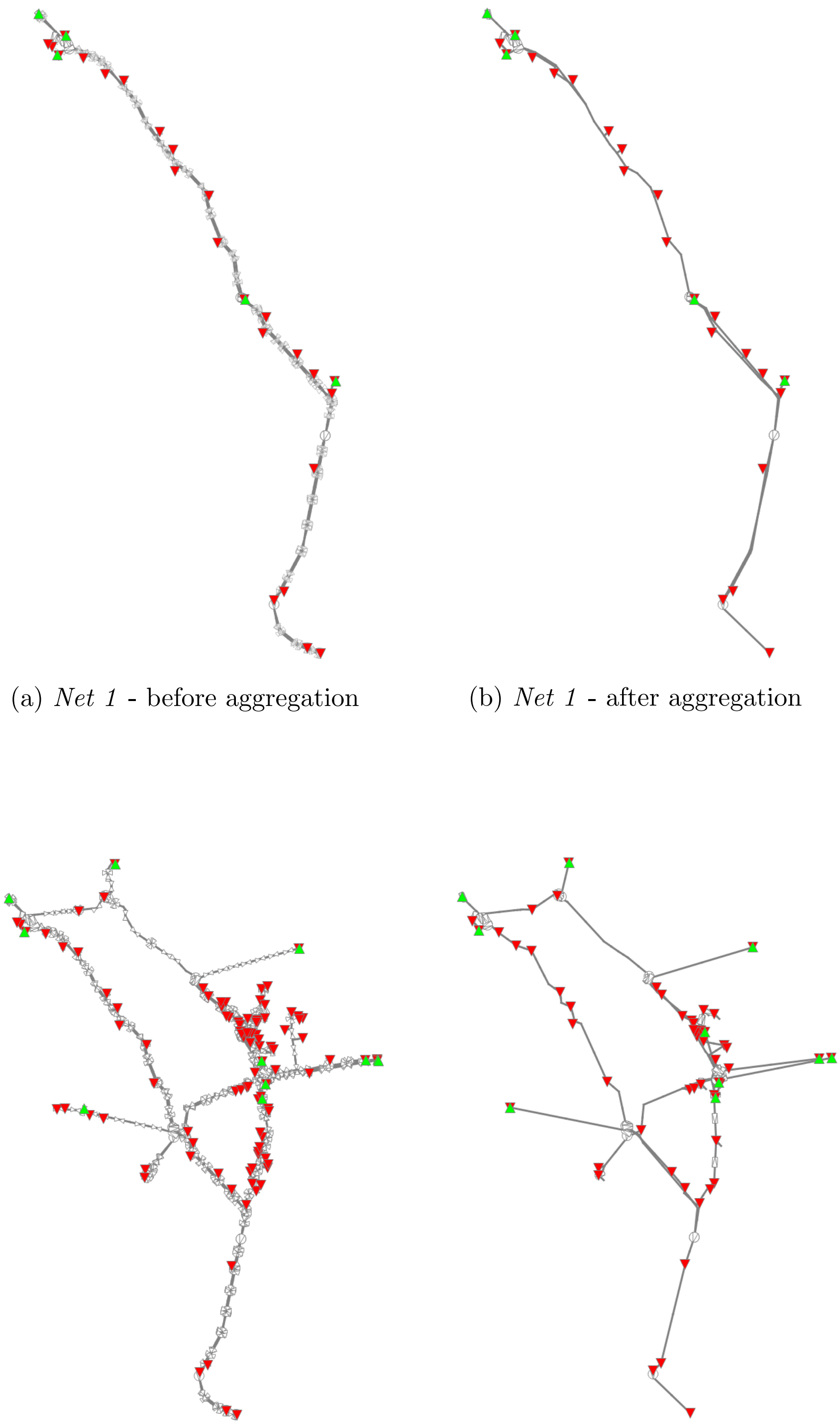
shrinking pipelines with short lengths, here $\mathscr{L} = 0$ [m] (*Net 1*), i.e., no pipe shrinking at all, $\mathscr{L} = 20$ [m] (*Net 2*), and $\mathscr{L} = 1{,}000$ [m] (*Net 3*). Besides that, it also arises from a higher proportion of distribution and passive tree subnetworks in *Net 3*. However, the strongest aggregation impact has *removing non-remote controllable elements*, which reduces the original size of the networks by up to 50%. This is due to the removal of almost all valves in the exterior of network stations. Let us note that serial pipe merging also has a strong impact, halving the size of the networks at the respective point of aggregation. In contrast, the impact of merging active elements is somewhat small, especially on *Net 1* and *Net 2*, because most of the active elements located outside of stations are non-remote controllable elements that have already been eliminated beforehand.

We designed our aggregation scheme such that the total volume and initial line-pack of the network are largely preserved during the reduction process. However, the only exception is shrinking pipelines in *Net 3*, which reduces the volume and line-pack by about 6% and 10%. This is caused by the high threshold value $\mathscr{L} = 1{,}000$ [m] that we chose in order to achieve a final network size that allows us to solve the $\texttt{SubMINLP}_t$-models in HEUR. Table 8.2 provides a detailed overview of the final number of network elements used in the computational experiments. The table shows that, in particular, *Net 3* has a higher circuit rank, which can be considered as an important indicator that the resulting optimization problems are more challenging to solve. The existence of cycles leads to more complex patterns of flow directions, see also the discussion of the computational results in Chapter 3.

	All		Nodes			Non-station arcs				Stations				Circuit rank
	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{V}^+\rvert$	$\lvert\mathcal{V}^-\rvert$	$\lvert\mathcal{V}_{\text{sto}}\rvert$	$\lvert\mathcal{A}_{pi}\rvert$	$\lvert\mathcal{A}_{sc}\rvert$	$\lvert\mathcal{A}_{va}\rvert$	$\lvert\mathcal{A}_{cv}\rvert$	$\lvert\mathcal{S}\rvert$	$\lvert\mathcal{V}_s\rvert$	$\lvert\mathcal{A}_s\rvert$	$\#\mathcal{A}_s^{\ell,r}$	$\lvert\mathcal{A}\rvert - \lvert\mathcal{V}\rvert + 1$
Net 1	66	83	5	22	23	51	8	0	4	4	12	20	7	18
Net 2	135	188	10	60	27	83	16	6	18	8	35	65	21	54
Net 3	205	282	30	89	46	85	46	19	35	19	51	97	30	78

Table 8.2: Characteristics of the aggregated network elements from Table 8.1.

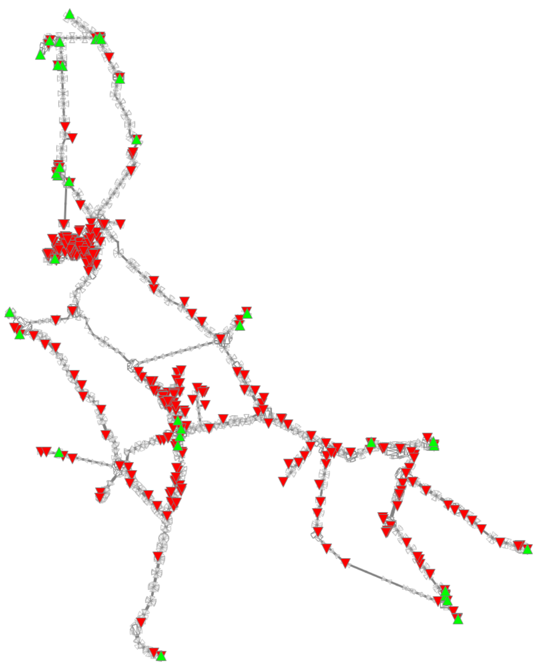
Demand. For each of the three real-world networks, we generated demand scenarios consisting of hourly flow values for a time range of $\mathcal{T} = 12$ hours. That is, we consider hourly and equidistant time steps yielding constant τ over time. For the networks *Net 1* and *Net 2*, we generated 500 instances, where the demand at sources and sinks corresponds to historical flow values. However, for *Net 3*, we used a time-series forecasting technique instead for the demand generation (Petkovic et al., 2019). Here, we consider a horizon of 60 consecutive days, where a particular scenario starts every hour and covers the upcoming 12 hours resulting in 1440 instances in total. This setting is of particular interest, as solving the 1440 instances allows us to obtain every hour updated station decisions for the next 12 hours, where the current station decisions are based upon the current network state and demand forecast. In this way, we can imitate embedding our solution approach into a decision-support system that continuously generates station decisions.



(a) *Net 1* - before aggregation (b) *Net 1* - after aggregation

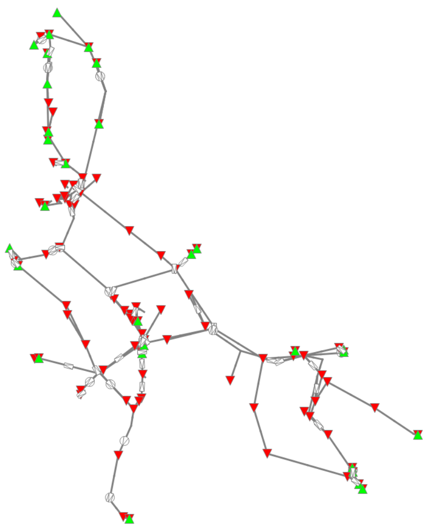
(c) *Net 2* - before aggregation (d) *Net 2* - after aggregation

Figure 8.1: *Net 1* and *Net 2* before and after aggregation

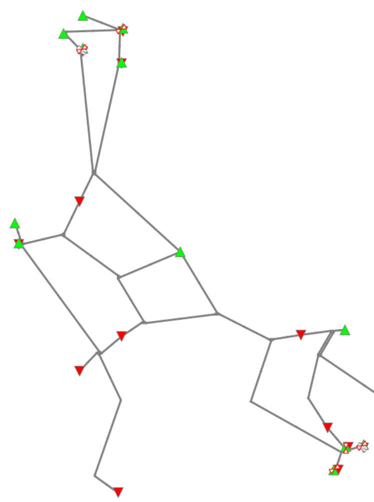


(a) *Net 3* - before aggregation

(b) *Net 3* - pipes (black) with $L_a \leq 1{,}000$ [m]



(c) *Net 3* - after aggregation



(d) *Net 3* - expert backbone network

Figure 8.2: *Net 3* (a) before aggregation, (b) indicating all pipes with a length shorter than the threshold value $\mathscr{L} = 1{,}000$ [m], (c) after aggregation, and (d) manually created backbone network by experts.

The forecasting method uses features, such as the weather forecast, the type of week day, a detailed gas flow history of the last months, and heuristically selects the most important ones by solving an MILP model. Afterwards, an LP-based regression determines future gas flow values at sources and sinks, where the selected features are used to approximate historical flow values, while minimizing outliers. In our setting, this forecast approach utilizes historical gas flow data of the previous 120 days. However, it might happen that this approach forecasts nodal gas flow values that are statistically out of range or hit predefined threshold values. Then, the forecasting method switches to a so-called baseline prognosis instead, which fixes the amount of future gas flow to the flow values at the corresponding hours of the previous day.

Finally, we use the names *Net 1*, *Net 2*, and *Net 3* to denote the networks as well as the benchmark test sets that contain the corresponding instances. Which ones are meant will become evident from the context.

Hardware and software. The experiments were conducted on a cluster of 64-bit Intel Xeon CPU E5-2670 v2 CPUs at 2.5 GHz with 25 MB cache and 128 GB main memory. In order to safeguard against a potential mutual slowdown of parallel processes, we bind the processes to specific cores and run at most one job per node at a time.

We implemented the network aggregation as part of a framework that is based on the programming language `Ada` and developed within the *Research Campus MODAL-GasLab*. For the implementation of Model $\mathscr{M}$ and the heuristic as plug-in, we used `PyScipOpt` 3.0.0 (Maher et al., 2016). `PyScipOpt` provides an interface to the SCIP Optimization Suite, where we used SCIP 7.0.0 (Gamrath et al., 2020) with SoPlex 5.0.0 as LP solver and Ipopt 3.12.13 as NLP solver (Wächter and Biegler, 2006).

8.2 Implementation Details

Model $\mathscr{M}$. To calculate the regression parameters of the linearized power equation in Section 5.8.3, we first generated 5,000 samples and then used the implementation of the least-squares method provided in the package `numpy` (Oliphant, 2006).

In the objective function, we set the weights to $\omega_a = 1$ for all $a \in \mathcal{A}_{va,s} \cup \mathcal{A}_{rg,s} \cup \mathcal{A}_{cs,s}$, and for all $s \in \mathcal{S}$. Then, the objective function value of the Models $\mathscr{M}$, $\texttt{SubMINLP}_t$ and $\texttt{MIP}_{t,\dots,|\mathcal{T}|}$ equal the total amount of station mode changes in the respective time horizons.

Network aggregation. For the pressure bound tightening at the distribution subnetworks' storage nodes from Section 6.3.6, we solved the *Max Pressure Loss Problem* (6.16) using SCIP.

In the serial pipe merging procedure from Section 6.7.2, we generated 2,000 samples for the sampling based regression approach, and utilized a time discretization of $|\mathcal{T}| = 4$ hourly steps for each particular sample. In order to solve the resulting NLPs of the involved sample completion (Algorithm 4), we used SCIP. We note that the sample completion throughout needs less than 0.1 seconds for each generated sample.

Heuristic. We call the heuristic with a frequency of 100 and a zero frequency offset, which means that the heuristic is invoked at every hundredth depth level of the branch-and-bound tree, starting in the root node after presolve. Given that HEUR is the most promising ingredient to find solutions for Model $\mathscr{M}$, we allow spending much solving time in the heuristic. More precisely, we set a time limit of 300 seconds for the initial $\texttt{MIP}_{t=1,\dots,|\mathcal{T}|}$-model, since a non-empty solution pool $\texttt{MIPSols}_{t=1,\dots,|\mathcal{T}|}$ is a key ingredient of the heuristic. For auxiliary models $\texttt{MIP}_{t>1,\dots,|\mathcal{T}|}$, we select a smaller time limit of 180 seconds, since the model size reduces with increasing time t. Under the same logic, we set the time limit to 120 seconds for all partitioned models $\texttt{MIP}_{t_1,\dots,t_2}(1 \leq t_1 < t_2 \leq |\mathcal{T}|)$. Finally, for the $\texttt{SubMINLP}_t$-models, we determine the time limit depending on the model size. As indicator, we use the number of arcs $|\mathcal{A}|$ of the underlying aggregated network, given that most of the model constraints are generated for arcs, apart from flow conservation and storage node constraints. Starting with 30 seconds for networks with up to 50 arcs, we increase the time limit by 15 seconds for every additional 50 arcs. To determine the time limits for arbitrary network sizes out of the box, we set

$$\texttt{limits/time} := 30 + rd\,(|\mathcal{A}|/50) \times 15, \tag{8.1}$$

where the operator $rd(.)$ rounds the value $|\mathcal{A}|/50$ to the nearest integer.

Concerning the example instances for *Net 1*, *Net 2*, and *Net 3* from Tables 8.1 – 8.2, (8.1) results in a time limit of 60, 90 and 120 seconds for the $\texttt{SubMINLP}_t$-models. For the overall Model $\mathscr{M}$, we set a time limit of 3,600 seconds.

For all $\texttt{MIP}_{t,\dots,|\mathcal{T}|}$-models and $\texttt{SubMINLP}_t$-models, we use SCIP's predefined emphasis setting for feasibility problems, which runs SCIP's default heuristics more aggressively than using SCIP default settings.[1] This choice is motivated by the fact that our heuristic first and foremost aims at finding solutions rather than improving incumbents. Besides, we disabled restarts due to their rather limited success on the heuristic. In each solution pool $\texttt{MIPSols}_{t,\dots,|\mathcal{T}|}$, we store at most 100 solutions, which is the default value for the number of solutions that SCIP collects in the solving process, including so-called poor solutions that do not improve the incumbents.

Finally, we approximate the absolute velocity term in Equation (7.17) by setting $|v_a| = 4$ [m/s] in the $\texttt{MIP}_{t,\dots,|\mathcal{T}|}$-models. This value lies within the range of approximate values $1.4-4.4$ [m/s] that Hennings (2018) states for the average velocity on six real-world pipes considered over two years.

8.3 General Results

In the following, *Enabled* refers to activating the heuristic HEUR, as opposed to *Disabled*, which refers to SCIP default settings. The impact of the heuristic can directly be seen in Table 8.3. For *Enabled* and *Disabled*, it illustrates the number of instances for which a solution was found, and the number of instances solved to global optimality. Most importantly, Model $\mathscr{M}$ is too challenging to solve for *Disabled*. When disabling HEUR, SCIP finds no solution for any of the instances across all test sets. In contrast, HEUR

[1] We use the following SCIP settings `emphasis/feasibility = True`

		Enabled		*Disabled*		
# instances	`All`	HEUR	solved	sol found	solved	exceed root node
Net 1	500	500	500	-	-	0
Net 2	500	490	253	-	-	11
Net 3	1,440	1,383	64	-	-	83

Table 8.3: Number of instances for which *Enabled* and *Disabled* found a solution to Model $\mathscr{M}$. `All` denotes the total number of instances per test set, among those, column HEUR depicts the number of instances for which the heuristic HEUR found a solution, column *sol found* denotes the number of instances, for which disabling the heuristic found a solution, whereas *solved* denotes the number of instances solved to global optimality. Finally, the table shows the number of instances that exceed the root node.

	Enabled					*Disabled*	
	`Affected`		`All`			`Affected`	`All`
Time to first	sgm	max	sgm	avg	σ	sgm	sgm
Net 1	12.8	14.9	12.8	12.8	0.4	-	3,600.0
Net 2	175.7	1,580.4	187.1	341.5	539.3	-	3,600.0
Net 3	1,246.0	3,594.4	1,270.8	1,527.1	777.5	-	3,600.0

Table 8.4: Running time to first solution in seconds of Model $\mathscr{M}$ for `Affected` and `All` instances, where *sgm* denotes the shifted geometric mean, *avg* the arithmetic mean, σ the standard deviation, and *max* the maximum runtime of a particular instance within the `Affected` instances.

finds solutions for all 500 instances in *Net 1*, for 490 out of 500 instances in *Net 2*, and for 1383 out of 1440 instances in *Net 3*, which corresponds to a ratio of 100%, 98% and 96% of all instances in the respective test sets. Notably, all solutions are found by HEUR. The high amount of feasible instances confirms the impact of the heuristic and shows that it reliably finds solutions to the vast majority of instances. In the following, we denote the set of feasible instances as `Affected`. Among the `Affected` instances, HEUR finds the optimal solution of 500 instances (*Net 1*), 253 instances (*Net 2*), and 64 instances (*Net 3*). We remark that instances are only optimally solved when the objective value is zero, requiring no station mode changes. The results suggest that it is easier to provide solutions with zero station mode changes for smaller and simpler structured networks.

Nearly all instances spend the entire solving time in the root node, only 11 instances (*Net 2*) and 83 instances (*Net 3*) exceed the root node. None of these 11 instances in *Net 2*, and only 2 out of these 83 instances in *Net 3* exceed 100 branch-and-bound nodes, thereby invoking a second call of the heuristic. However, all results presented in this chapter refer to the best solution obtained by any HEUR call. Analyzing the solver statistics revealed that basically two solver ingredients are responsible for the excessive running times in the root node: (i) intensive bound tightening by running OBBT (Gleixner and Weltge, 2013), and (ii) running HEUR after presolve.

Concerning the running time, Table 8.4 reports the *time to the first solution* of the `Affected` and `All` instances for *Enabled* and *Disabled*. The table shows that the running time (to the first solution) decreases with the complexity and size of the networks, and corresponds to 12.8 (*Net 1*), 175.7 (*Net 2*), and 1246.0 (*Net 3*) seconds in shifted geometric mean for the `Affected` instances. With respect to the arithmetic mean (avg), the average runtime is even higher for *Net 2* and *Net 3*, namely 341.5 and 1,527.1 seconds, which is a consequence of more instances hitting the time limit. A visualization of the running time to the first solution of `All` instances can be found in Figure 8.3. Detailed instance-wise results are reported in Tables E.3 – E.5.

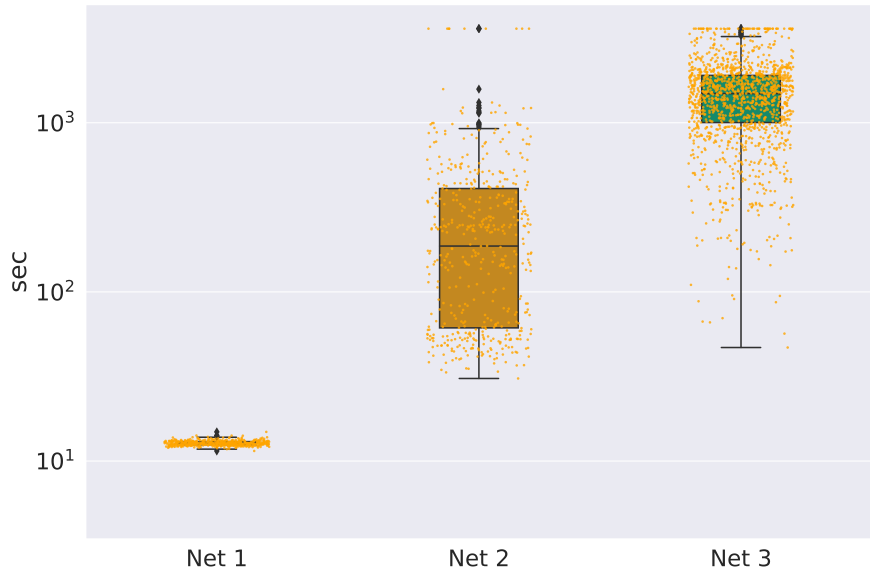


Figure 8.3: Running times of all instances in the different test sets. Each dot in orange corresponds to the runtime of a particular instance in seconds. The boxes represent the interquartile ranges between the 25th (Q1) and 75th (Q3) percentiles, where the line in between corresponds to the median. Black dots depict outliers, where the runtime is either smaller than the minimum (= Q1 - 1.5 (Q3-Q1)) or higher than the maximum (= Q3 + 1.5 (Q3-Q1)).

Apart from the average running time, the standard deviation σ increases with the network size, from 0.4 seconds (*Net 1*) over 539.3 seconds (*Net 2*) to 777.6 seconds (*Net 3*). Hence, the running times of the instances in *Net 2* and *Net 3* are rather widely spread, compared to *Net 1*. While the maximum running time to the first solution of an `Affected` instance in *Net 2* is 1,580.4 seconds, i.e., still far from the time limit of 3,600 seconds, it is 3,594.4 seconds in *Net 3*, which is close to the time limit. Thus, for *Net 3* the running time is very sensitive to the selected total time limit. Given that HEUR solves several sub-models, this sensitivity strongly depends on the imposed time limits for the sub-models, and thereby on the number of additional model solves when moving backwards and forwards on the time scale. In fact, this sensitivity is the main reason for our sophisticated time selection rule (8.1).

8.4 Results for the Primal Heuristic

In order to study the impact of the heuristic on the model performance and solvability of instances, we analyze the computational results with respect to HEUR-related key aspects, such as the number of `Affected` instances that are feasible (i) with initial station mode fixings acquired by solving $\texttt{MIP}_{t=1,\ldots,|\mathcal{T}|}$, (ii) by unfixing these initial station modes in $\texttt{SubMINLP}_t$-models at some $t \in \mathcal{T}$, and (iii) by requiring additional station solutions from the pools. Further, we investigate the occurrence of MIP-models that did not find any solution in the solving process and thus result in MIP-model partitioning. This is motivated by the fact that non-empty solution pools are essential for the success of the heuristic. Moreover, we investigate the highest achieved time step for `All` instances. In particular, for instances where no solution was found it provides information on how far HEUR moved along the time scale, and whether it was close to passing all time steps successfully.

Finally, the setup of test set *Net 3* allows us to analyze daily station mode changes over a duration of two months. In this way, we can verify whether our solution approach reliably generates a feasible network control in the long-term. If so, the approach can be understood as a step towards using MINLP models in practice by embedding it into a decision-support system that runs continuously and aims at automating the decision-making process of TSOs.

Feasible instances with initial station mode fixings. At first, we analyze the number of instances, where all $\texttt{SubMINLP}_t$-models are feasible with fixed station modes gained from the best available initial solution $\texttt{stationSol}_{t=1,\ldots,|\mathcal{T}|}$. While it concerns all 500 instances in *Net 1*, this number considerably decreases with the network size and corresponds to 214 and 48 instances of *Net 2* and *Net 3* (row # `Affected` instances, feasible with initial station mode fixings in Table 8.5). In total, these numbers correspond to 100% (*Net 1*), 44% (*Net 2*), and 4% (*Net 3*) of the `Affected` instances. That means, all instances in *Net 1* require only one $\texttt{SubMINLP}_t$-solve per time step $t \in \mathcal{T}$. This effect can be attributed to the fact that the linearized model $\texttt{MIP}_{t=1,\ldots,|\mathcal{T}|}$ finds fast optimal station solutions $\texttt{stationSol}_{t=1,\ldots,|\mathcal{T}|}$ that turn out to be also optimal for the nonlinear $\texttt{SubMINLP}_t$-models and for Model $\mathscr{M}$. Indeed, finding fast optimal station solutions for both linear and nonlinear models here is a consequence of the lower number of arc flow directions in the rather tree-like network *Net 1*. Taking into account the overall short runtime of 12.8 seconds in shifted geometric mean (*Net 1*) together with the length of the time horizon $|\mathcal{T}| = 12$, the average running time of a single $\texttt{SubMINLP}_t$-model corresponds to roughly one second. The significantly longer running times for *Net 2* and *Net 3*, each by an order of magnitude, are partly a consequence of higher solving times for the initial $\texttt{MIP}_{t=1,\ldots,|\mathcal{T}|}$-models. However, the main reason concerns the significant lower number of instances for which the $\texttt{SubMINLP}_t$-models are solved in the first iteration with fixed station modes. As a result, more instances in *Net 2* and *Net 3* require solving $\texttt{SubMINLP}_t$-models with unfixed station modes, or even utilize new station solutions by solving additional $\texttt{MIP}_{t,\ldots,|\mathcal{T}|}$-models and possibly require going back in time, see also the analysis in the following paragraphs.

		# instances		
		Net 1	*Net 2*	*Net 3*
	# `Affected` instances	500	490	1,383
feasible	with initial station mode fixings	500	214	48
	when unfixing the initial station modes	0	117	920
	when utilizing other than the initial station modes	0	159	415
objective values	$z_{\mathscr{M}} > z_{\texttt{MIP}_{t=1,\dots,\lvert\mathcal{T}\rvert}}$	0	232	1,227
	$z_{\mathscr{M}} < z_{\texttt{MIP}_{t=1,\dots,\lvert\mathcal{T}\rvert}}$	0	0	38
	$z_{\mathscr{M}} = z_{\texttt{MIP}_{t=1,\dots,\lvert\mathcal{T}\rvert}}$	500	258	118
Partitioning	of any $\texttt{MIP}_{t,\dots,\lvert\mathcal{T}\rvert}$	0	8 (8)	300 (292)
	of initial $\texttt{MIP}_{t=1,\dots,\lvert\mathcal{T}\rvert}$	0	8 (8)	242 (239)
	of $\texttt{MIP}_{t>1,\dots,\lvert\mathcal{T}\rvert}$	0	0	58 (53)

Table 8.5: HEUR specific statistics. The table shows the number of `Affected` instances, for which all $\texttt{SubMINLP}_t$-models are feasible with respect to the initial station mode fixings $\texttt{stationSol}_{t=1,\dots,\lvert\mathcal{T}\rvert}$. It shows the number of `Affected` instances that are feasible when unfixing the initial station modes in some $\texttt{SubMINLP}_t$-run, and that are feasible when utilizing another solution from any pool $\texttt{MIPSols}_{t,\dots,\lvert\mathcal{T}\rvert}$ than the initial station mode fixings. The table compares the objective value $z_{\mathscr{M}}$ of Model $\mathscr{M}$ with the objective value $z_{\texttt{MIP}_{t=1,\dots,\lvert\mathcal{T}\rvert}}$ of the initial $\texttt{MIP}_{t=1,\dots,\lvert\mathcal{T}\rvert}$ for `Affected` instances. The table lists the number of instances that require $\texttt{MIP}_{t,\dots,\lvert\mathcal{T}\rvert}$-model decompositions and the number of corresponding feasible instances in brackets, grouped by time steps $t = 1$ and $t > 1$.

Feasible instances requiring unfixings of the initial station modes. Let us now turn to those `Affected` instances that require unfixings of the initial station modes in some $\texttt{SubMINLP}_t$-run, but do not utilize other than the initial station mode fixings. These instances restrict their fixings to $\texttt{stationSol}_{t=1,\dots,\lvert\mathcal{T}\rvert}$ and hence do not move backwards. While the number of affected instances in *Net 1* is zero, it is 117 in *Net 2*, and 920 in *Net 3*, reported in row (`Affected` instances, feasible when unfixing the initial station modes) of Table 8.5. A detailed analysis reveals that these instances in *Net 2* and *Net 3* exhibit different characteristics. While 95 out of the 117 instances in *Net 2* are found with at most three $\texttt{SubMINLP}_t$-calls with unfixed station modes, among the 920 affected instances in *Net 3* throughout 48 to 100 instances require $\texttt{SubMINLP}_t$-calls with unfixed station modes in each time iteration $t \in \mathcal{T} = \{1, \dots, 12\}$, see Table E.1 in the appendix. This means, a considerable amount of feasible instances in *Net 3* need a second $\texttt{SubMINLP}_t$-solve before advancing to the next time iteration, requiring in the worst case up to $\lvert\mathcal{T}\rvert = 12$ additional $\texttt{SubMINLP}_t$-runs.

Feasible instances requiring additional MIP solutions. We now investigate those `Affected` instances that use additional MIP solutions. This concerns 159 instances of *Net 2* and 415 instances of *Net 3*, see row (`Affected` instances, feasible when utilizing other than the initial station modes) in Table 8.5. Please note that utilizing new MIP solutions does not necessarily imply moving backwards on the time scale. For example, in the first time step $t = 1$, HEUR might try multiple station fixings $\texttt{stationSol}_{t=1,\ldots,|\mathcal{T}|}$ before advancing to the next time step $t = 2$. Figure 8.4 visualizes the distribution of `Affected` instances concerning the number of moving backwards in time. The figure shows that HEUR moves backwards only once or twice for $136 = 113 + 23$ out of the 159 instances (*Net 2*, orange), whereas only 4 instances require going backwards more than that. In contrast, for *Net 3* (green), 390 ($= 194 + 84 + 41 + 31 + 17 + 14 + 9$) instances need one to seven backward moves, whereas 22 instances need at least eight and one instance even 20 such moves.

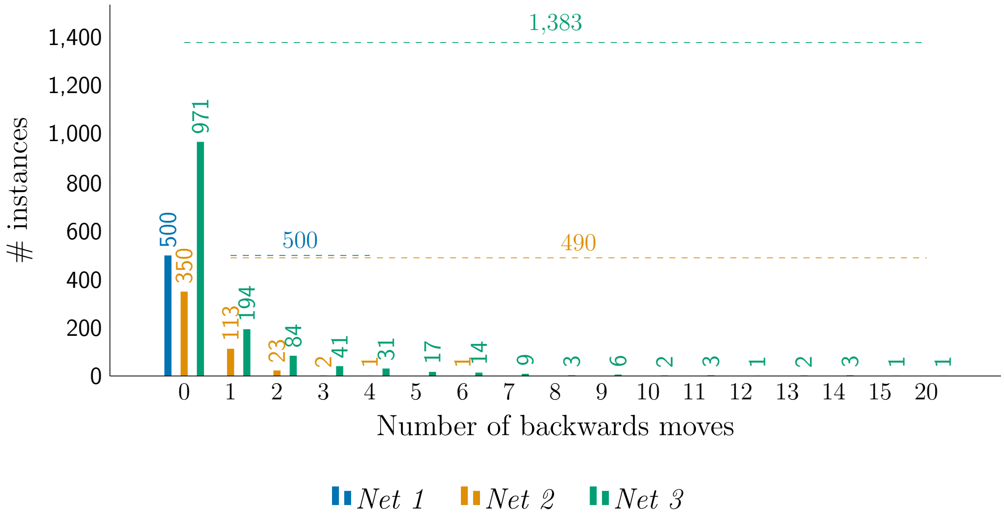


Figure 8.4: Distribution of backward moves in HEUR for `Affected` instances.

From the analysis carried out in the last two paragraphs, we deduce that the initial station mode fixings $\texttt{stationSol}_{t=1,\ldots,|\mathcal{T}|}$ are more useful for solving $\texttt{SubMINLP}_t$-models with respect to instances of *Net 1* and *Net 2* as compared to *Net 3*. We analyzed this effect and identified that first, demand scenarios belonging to *Net 3* have, in general, a higher degree of volatility, second a higher degree of imbalance between the overall network inflow and outflow, and third higher amounts of total network demand than the other networks. Altogether, this results in higher flow rates on some arcs of network *Net 3* yielding different pressure solutions for the $\texttt{MIP}_{t=1,\ldots,|\mathcal{T}|}$- and $\texttt{SubMINLP}_t$-models, as shown in Figure 7.1. These differences even increase with more distant time steps finally changing flow directions and leading to the need to switch station modes. Thereby, $\texttt{SubMINLP}_t$-models might run into the time limit without providing solutions or even become infeasible. Then, the algorithm continues by solving $\texttt{SubMINLP}_t$-models with unfixed station modes or by acquiring new station mode fixings and possibly moves backwards in time.

Instances requiring $\texttt{MIP}_{t,\ldots,|\mathcal{T}|}$-model partitioning. HEUR strongly relies on the existence of solutions in $\texttt{MIPSols}_{t,\ldots,|\mathcal{T}|}$. For this reason, the heuristic decomposes the time range and solves the corresponding reduced MIP-models in the case that solving the entire $\texttt{MIP}_{t,\ldots,|\mathcal{T}|}$-model did not find a solution. Table 8.5 provides additional information on the number of instances, where such $\texttt{MIP}_{t,\ldots,|\mathcal{T}|}$-partitioning occurs. For *Net 2*, in total 8 instances require decomposing MIP-models, all of which are feasible. In contrast, for 300 instances of *Net 3* occur MIP decompositions, out of which 292 instances are feasible. Hence, we deduce that MIP-models are even challenging to solve for about one-fifth of instances in *Net 3* (300 of 1440), but the decomposition procedure allows obtaining MIP-solutions that facilitate finding solutions to Model $\mathscr{M}$ for nearly all of them.

Instances without solutions. Besides investigating feasible instances, we also analyzed those instances for which HEUR did not find a solution. In total, this applies to 10 instances of *Net 2* and 57 instances of *Net 3*. Figure 8.5 shows that all 10 failing instances of *Net 2* did not pass the first two time steps. The main reason for not finding any solution to $\mathscr{M}$ is the unavailability of more solutions in the pools $\texttt{MIPSols}_{t,\ldots,|\mathcal{T}|}$, apart from the previously mentioned fact of failing initial MIP-solves. Table E.2 in the appendix shows that for most time steps $t \in \mathcal{T}$, the $\texttt{MIP}_{t,\ldots,|\mathcal{T}|}$-models find two or three solutions on average. However, all instances of *Net 2* should ideally be feasible, given that the corresponding demand scenarios represent historical flow values.

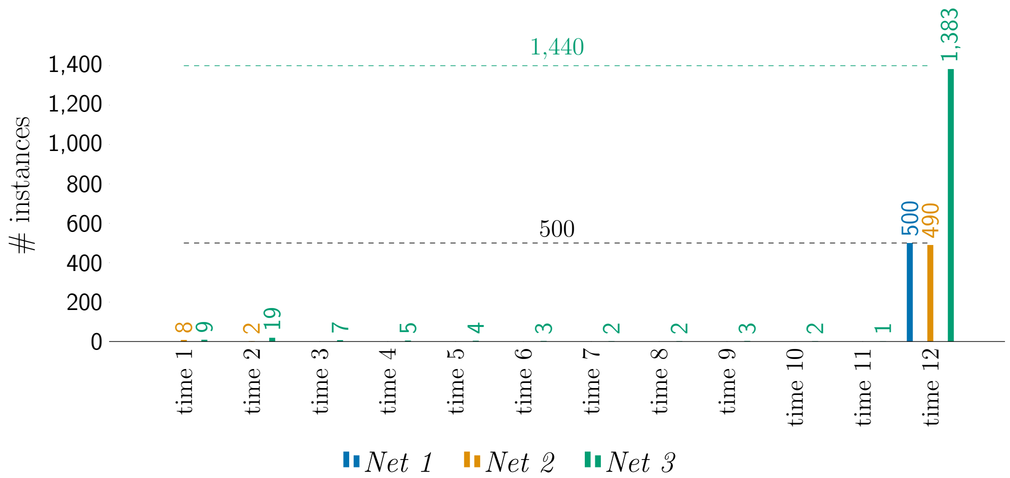


Figure 8.5: Distribution of highest achieved time step in HEUR for `All` instances.

For *Net 3*, in sum 28 (9+19) failing instances did not pass the first two time steps, while the remaining 29 instances advanced to higher time steps. Here, it is basically unknown whether a solution exists, since the corresponding demand profiles are generated by forecasts and do not correspond to historical flow values that occurred in the real-world network. Certainly, there are numerous possibilities of how to extend HEUR in order to continue the search for solutions. However, we remark that it could be worthwhile to add conflicting constraints generated from station arcs instead, which aim at increasing the dual bounds and support detecting possible infeasible instances. We remark that the dual bound of all instances across the three test stets is zero. Likewise, $\mathtt{MIP}_{t=1,\ldots,|\mathcal{T}|}$-models are only solved to global optimality when the objective value is zero.

Analysis of daily operation mode changes. Figure 8.6 shows the number of daily operation mode changes for 60 consecutive days of *Net 3*. Here, the changes are given by the solutions to the MINLP Model $\mathscr{M}$ (blue), or rather by the solutions to the initial $\mathtt{MIP}_{t=1,\ldots,|\mathcal{T}|}$-model (red). All 1440 instances are chronologically ordered, where the start time of consecutive instances is one hour ahead. In this way, the daily changes are compounded by taking the changes from the first hour of 24 successive instances. If for an instance, no solution to Model $\mathscr{M}$ or Model $\mathtt{MIP}_{t=1,\ldots,|\mathcal{T}|}$ is found, then the number of mode changes for the corresponding hour is taken from the solution of the previous instance.

The figure allows two major observations: First, the number of changes is considerably higher for $\mathscr{M}$, requiring roughly between 10 and 30 daily station mode changes out of 97 possible ones, as compared to $\mathtt{MIP}_{t=1,\ldots,|\mathcal{T}|}$, which mostly needs less than 10 daily station mode changes. Second, the pattern of mode changes is more volatile for $\mathscr{M}$. These effects mainly go back to the nature of the moving horizon heuristic. While the $\mathtt{MIP}_{t=1,\ldots,|\mathcal{T}|}$-models minimize station mode changes for the entire time horizon, HEUR moves along the time steps, only minimizing the number of changes with respect to the previous time step.

Even though Model $\mathscr{M}$ has clearly more station mode changes per day, this does not prevent Model $\mathscr{M}$ from having better objective values for 38 instances, see Table 8.5.

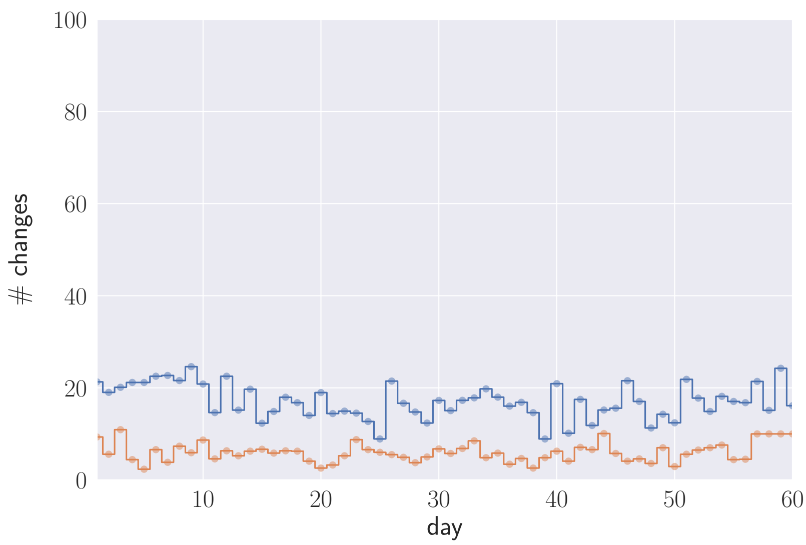


Figure 8.6: Number of daily operation mode changes over 60 consecutive days for *Net 3*. Here, the number of daily changes are given by the solutions to the MINLP Model $\mathscr{M}$ (blue), or rather by the solutions to the initial $\mathtt{MIP}_{t=1,\ldots,|\mathcal{T}|}$-model (red).

8.5 Conclusion

In the second part of this thesis, we introduced the *Transient Control Problem*. Motivated by the core objective of TSOs to acquire a stable network control, the problem aims at making half day-ahead control decisions that enable feasible gas transport in large-scale networks. To generate these decisions, we proposed a specially-tailored solution approach. It first aggregates network parts that are considered as less important for the decision process. For the deployed real-world networks, our aggregation scheme reduces the size of the networks down to approximately 16% (*Net 1*), 13% (*Net 2*), and 6% (*Net 3*). As finding solutions to the MINLP model on the aggregated networks is still challenging, we further applied model reformulations that exploit the disjunctive nature of the presented network station modeling and developed a moving horizon based heuristic. Our computational study showed that these improvements reliably generated solutions for 100%, 98% and 96% of the real-world instances across all three test sets.

Ideally, the acquired control decisions can be used as recommendations for dispatchers. To verify that these decisions indeed yield feasible operations on the original network, future research concerns the development of a strategy that validates whether a solution of the aggregated network can be extended to a solution of the original network. A promising step in this direction could be to first recover the full network by disaggregating all aggregated structures. For this purpose, the disaggregation procedure can easily be retrieved from the aggregation history by mapping aggregated to original structures. Then, by running simulations for each disaggregated structure individually, it could be verified whether the disaggregated structures allow for physically valid operations. Given that such approach would not consider the network as a monolithic system, but rather as multiple disaggregated structures, it even allows to run simulations in parallel. Finally, a mechanism would be required that establishes feasibility when coupling the individual disaggregated systems.

Obviously, there exists further potential for improving the solving process using MINLP solvers. Given that the dual bounds are zero for all instances, they could possibly be improved for some instances by deriving conflicting constraints for incompatible station modes. Moreover, the *Transient Control Problem* could be extended to include more details and features, in particular for the station modeling, such as ramp-up and ramp-down times, and the operation of compressor machines in parallel and serial. Besides that, the used approximation of the power consumption in compressor drives could be substituted for the actual nonlinear constraint function, while enriching the solving process with its convex envelope that could possibly be developed in a similar fashion as done in Chapter 4 for function $f(x, y) = y \operatorname{sgn}(x)|x|^\alpha$, but adhering to higher dimensions here.

Chapter 9

Conclusion

In this thesis, we studied two practically highly relevant problems: the optimization of stationary network expansions and the optimization of operations in transient large-scale energy networks. Both problems are inherently located in the field of Mixed-Integer Nonlinear Programming. However, global state-of-the-art MINLP solvers are not able to solve or even find primal solutions on real-world instances, since the resulting degree of nonlinearity and nonconvexity pose principal difficulties. In order to overcome these challenges, we developed new algorithmic techniques and model formulations that significantly improve the performance of the global MINLP solver SCIP and allow us, for the first time, to reliably compute primal feasible solutions for both problems.

The first part of the thesis was devoted to the *stationary expansion planning problem of potential-driven networks by using loops*. In Chapter 3, we showed properties of the looping problem, such as its non-convexity, and stated conditions that guarantee feasible loop expansions. Building on an existing method of selecting cost-minimal loop diameters a priori, we presented a model reduction approach for multiple loops. On this basis, we introduced new models for both discrete and split-pipe looping and contrasted these with existing models for the looping problem and related problems in the literature, both theoretically and experimentally. This was also the first time that discrete and split-pipe approaches were compared for networks of a practically relevant size and complexity. The performance of the models was analyzed in an extensive computational study with a large set of diverse demand vectors and networks of different sizes and topologies, including network variations for different circuit ranks, and led to recommendations regarding the use of the different models. In particular, our experiments showed that overall our new split-pipe model tremendously outperforms the existing models with respect to computational time, the number of solutions found, the number of instances solved and cost savings.

Motivated by the fact that strong relaxations of nonconvex constraint functions play an essential role in global MINLP solvers, one of our main theoretical contributions in this thesis is the derivation of the convex envelope for the nonconvex and bivariate function $f(x, y) = y \operatorname{sgn}(x)|x|^{\alpha}$ with $\alpha > 1$, which is a key constraint function in stationary expansion planning problems. In Chapter 4, we presented two different ways to obtain an explicit algebraic solution for the convex envelope of f. We utilized this algebraic description to implement a separation procedure in SCIP. Dealing with a closed-form expression allowed us to quickly calculate and exploit the envelopes at every node in the branch-and-bound tree. Our extensive computational study on real-world instances showed the benefit of the convex envelope over the standard relaxation that state-of-the-art MINLP solvers apply. In fact, it significantly reduced the solving time on all test sets up to 58% in shifted geometric mean running time for difficult instances. Even more, it enabled us to solve significantly more instances on all test sets.

The second part of the thesis was concerned with the *optimization of operations in transient large-scale gas networks*. First, we introduced an MINLP formulation for the *Transient Control Problem* in Chapter 5. Motivated by the guiding objective of TSOs to guarantee security of supply, the problem aims at making half day-ahead control decisions that enable feasible gas transport. To acquire these decisions, we proposed a specially-tailored solution approach. At first, we aggregated network parts that are considered as less important for the decision process in Chapter 6. Given that the MINLP-model is induced by the underlying graph, the rationale of the aggregation was to reduce the model size by decreasing the number of network elements. For the aggregation, we formalized the concept of *equivalent subnetwork replacements*, and based on this definition, we developed methods that equivalently replace subnetworks. In the case that equivalent replacements were not possible, we further reduced the network size by applying heuristic procedures.

Our extensive computational studies based on real-world data suggest that the deployed heuristic methods appropriately approximate the physical behavior of the original structures. However, as it turned out that finding solutions on aggregated representations of real-world and large-scale networks is still challenging, we presented further improvements. In Chapter 7, we applied model reformulations that exploit the disjunctive nature of network station modeling and developed a moving horizon based primal heuristic that solves MINLP sub-models and thereby moves forward and backward on the time scale in order to compensate possibly disadvantageous decisions taken at earlier stages. Our computational study in Chapter 8 shows that this approach reliably generates solutions for 96% to 100% of the instances across all three test sets that are based upon real-world data.

In the introduction, we have mentioned that practitioners, if at all, use computer-aided decision-making tools that typically resort to complete linearized models due to the advanced technology of MIP solvers. However, to account for the genuine nonlinear nature of the problems, we modeled the expansion planning and daily network control problems as MINLPs. We developed special-tailored solution approaches for both problems that exploit specific structures of the models, such as the nonconvex nonlinearities and the time expansion. In this way, we were able to reliably find primal solutions for real-world instances of size and complexity encountered in practice. Our results in this thesis show that general-purpose MINLP solvers customized with the presented techniques already provide nowadays the possibility to be used in practical applications. Following this line of thought, we strongly believe that this thesis constitutes a major step towards using MINLPs in industrial decision-support systems.

Further research aims at developing an integrated approach that solves the expansion planning and the daily network control problem at once.

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Appendix A

Stationary Expansion Planning of Potential Driven Networks

A.1 Equivalent Diameters in Parallel Pipe Merging

Proposition A.1.1. *Given two parallel pipes $a = (\ell, r)$ and $b = (\ell, r)$ with diameters $d_a, d_b > 0$ respectively. If both pipes have the same length $L := L_a = L_b$ and physical parameter $R := R_a = R_b$, then it is possible to express both parallel pipes by a single pipe $c = (\ell, r)$ with equivalent diameter*

$$D_c^{\beta/\alpha} = d_a^{\beta/\alpha} + d_b^{\beta/\alpha}.$$

Proof. Let

$$\gamma_a = \frac{RL}{d_a^\beta}, \qquad \gamma_b = \frac{RL}{d_b^\beta}, \qquad \gamma_c = \frac{RL}{D_c^\beta},$$

then from Proposition 6.6.3 follows

$$\begin{aligned}
&\gamma_c = \frac{\alpha\,\beta}{(\sqrt{\alpha} + \sqrt{\beta})^2} \\
\iff\quad &\frac{1}{D_c^\beta} = \frac{\frac{1}{d_a^\beta d_b^\beta}}{\left(\frac{1}{d_a^{\beta/\alpha}} + \frac{1}{d_b^{\beta/\alpha}}\right)^\alpha} \\
\iff\quad &D_c^\beta = (d_a^\beta d_b^\beta)\left(\frac{1}{d_a^{\beta/\alpha}} + \frac{1}{d_b^{\beta/\alpha}}\right)^\alpha \\
\iff\quad &D_c^{\beta/\alpha} = d_a^{\beta/\alpha} + d_b^{\beta/\alpha}.
\end{aligned}$$

□

A.2 Detailed Results of Expansion Planning Models

Table A.1: Detailed results of the discrete models on *Belgium* with $\mathcal{B} = 100$ as summarized in Figure 3.7a and Table 3.3a. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_1	218.1k	218.0k	0.1	0.3	218.1k	218.1k	0.0	16.0	218.1k	218.1k	0.0	1.1
dem_100_belg_orig_2	184.5k	184.5k	0.0	0.2	184.5k	184.5k	0.0	21.5	184.5k	184.5k	0.0	0.4
dem_100_belg_orig_3	200.0k	200.0k	0.0	0.4	200.0k	200.0k	0.0	15.8	200.0k	199.9k	0.1	0.6
dem_100_belg_orig_4	462.9k	462.9k	0.0	274.8	462.9k	462.9k	0.0	74.1	462.9k	462.9k	0.0	1177.4
dem_100_belg_orig_5	183.3k	183.2k	0.0	0.6	183.3k	183.3k	0.0	20.9	183.3k	183.3k	0.0	1.2
dem_100_belg_orig_6	241.8k	241.8k	0.0	1.1	241.8k	241.8k	0.0	13.5	241.8k	241.6k	0.1	0.5
dem_100_belg_orig_7	526.3k	525.9k	0.1	2.8	526.0k	526.0k	0.0	232.7	526.0k	526.0k	0.0	23.4
dem_100_belg_orig_8	252.9k	252.9k	0.0	10.8	252.9k	252.9k	0.0	30.2	252.9k	252.9k	0.0	12.6
dem_100_belg_orig_9	466.3k	466.3k	0.0	3443.4	466.3k	466.3k	0.0	523.5	466.3k	444.8k	4.8	14400.0
dem_100_belg_orig_10	130.5k	130.5k	0.0	8.3	130.5k	130.5k	0.0	8.3	130.5k	130.5k	0.0	6.8
dem_100_belg_orig_11	160.4k	160.4k	0.0	223.1	160.4k	160.4k	0.0	290.5	160.4k	160.4k	0.1	625.3
dem_100_belg_orig_12	309.2k	309.2k	0.0	45.5	309.2k	309.2k	0.0	181.9	309.2k	309.2k	0.0	1149.4
dem_100_belg_orig_13	187.4k	187.3k	0.0	0.3	187.4k	187.2k	0.1	22.6	187.4k	187.2k	0.1	0.6
dem_100_belg_orig_14	200.9k	200.9k	0.0	0.2	200.9k	200.9k	0.0	10.1	200.9k	200.9k	0.0	0.4
dem_100_belg_orig_15	338.9k	338.9k	0.0	58.9	338.9k	338.9k	0.0	97.9	338.9k	338.9k	0.0	681.8
dem_100_belg_orig_16	254.1k	254.1k	0.0	39.4	254.1k	254.0k	0.0	75.2	254.1k	254.1k	0.0	1750.3
dem_100_belg_orig_17	278.1k	278.1k	0.0	1.5	278.1k	278.1k	0.0	41.4	278.1k	277.8k	0.1	8.1
dem_100_belg_orig_18	334.3k	334.3k	0.0	0.3	334.3k	334.3k	0.0	26.7	334.3k	334.1k	0.1	0.4
dem_100_belg_orig_19	185.1k	185.0k	0.0	0.2	185.1k	185.1k	0.0	9.9	185.1k	184.9k	0.1	1.0
dem_100_belg_orig_20	286.2k	286.1k	0.0	0.1	286.2k	286.1k	0.0	29.5	286.2k	286.1k	0.0	0.3
dem_100_belg_orig_21	300.1k	300.1k	0.0	3.0	300.1k	300.1k	0.0	44.2	300.1k	300.0k	0.0	19.2
dem_100_belg_orig_22	194.6k	194.6k	0.0	0.2	194.6k	194.6k	0.0	14.9	194.6k	194.5k	0.0	0.5
dem_100_belg_orig_23	183.5k	183.4k	0.1	0.5	183.5k	183.5k	0.0	16.9	183.5k	183.5k	0.0	8.3
dem_100_belg_orig_24	185.8k	185.7k	0.1	0.4	185.8k	185.8k	0.0	14.2	185.8k	185.8k	0.0	1.3
dem_100_belg_orig_25	341.5k	341.5k	0.0	24.8	341.5k	341.5k	0.0	27.0	341.5k	341.3k	0.0	11.3
dem_100_belg_orig_26	207.1k	207.0k	0.0	9.5	207.1k	207.1k	0.0	30.6	207.1k	207.1k	0.0	23.9
dem_100_belg_orig_27	327.2k	327.0k	0.1	0.6	327.2k	327.2k	0.0	51.0	327.2k	327.2k	0.0	1.4
dem_100_belg_orig_28	437.3k	437.3k	0.0	905.3	437.3k	437.0k	0.1	215.7	437.3k	428.2k	2.1	14400.0
dem_100_belg_orig_29	192.8k	192.8k	0.0	0.3	192.8k	192.8k	0.0	24.3	192.8k	192.8k	0.0	0.4
dem_100_belg_orig_30	348.2k	348.2k	0.0	3.2	348.2k	348.2k	0.0	53.7	348.2k	348.2k	0.0	57.3
dem_100_belg_orig_31	128.3k	128.3k	0.0	0.3	128.3k	128.3k	0.0	10.1	128.3k	128.3k	0.0	7.5
dem_100_belg_orig_32	118.4k	118.3k	0.1	4.2	118.4k	118.4k	0.0	14.4	118.4k	118.4k	0.0	0.5
dem_100_belg_orig_33	202.1k	202.1k	0.0	0.2	202.1k	201.9k	0.1	17.6	202.1k	201.9k	0.1	0.3
dem_100_belg_orig_34	126.3k	126.3k	0.0	8.6	126.3k	126.2k	0.1	36.7	126.3k	126.3k	0.0	6.4
dem_100_belg_orig_35	367.2k	367.2k	0.0	16.5	367.2k	367.2k	0.0	64.4	367.2k	367.2k	0.0	30.0
dem_100_belg_orig_36	155.6k	155.6k	0.0	0.3	155.6k	155.6k	0.0	10.8	155.6k	155.6k	0.0	9.9
dem_100_belg_orig_37	148.1k	148.1k	0.0	0.1	148.1k	148.1k	0.0	7.4	148.1k	148.1k	0.0	0.4
dem_100_belg_orig_38	263.9k	263.9k	0.0	55.0	263.9k	263.9k	0.0	102.3	263.9k	263.9k	0.0	743.0
dem_100_belg_orig_39	177.8k	177.8k	0.0	0.2	177.8k	177.8k	0.0	9.2	177.8k	177.8k	0.0	0.4
dem_100_belg_orig_40	227.6k	227.6k	0.0	7.7	227.6k	227.6k	0.0	37.5	227.6k	227.6k	0.0	6.6
dem_100_belg_orig_41	146.9k	146.8k	0.1	0.3	146.9k	146.9k	0.0	17.3	146.9k	146.8k	0.1	0.5
dem_100_belg_orig_42	165.1k	165.1k	0.0	0.5	165.1k	165.0k	0.1	26.1	165.1k	165.0k	0.1	1.2
dem_100_belg_orig_43	170.5k	170.5k	0.0	0.3	170.5k	170.5k	0.0	24.9	170.5k	170.5k	0.0	1.2
dem_100_belg_orig_44	397.5k	397.5k	0.0	76.8	397.5k	397.2k	0.1	323.0	397.5k	397.5k	0.0	144.2
dem_100_belg_orig_45	193.7k	193.7k	0.0	0.3	193.7k	193.7k	0.0	26.4	193.7k	193.7k	0.0	0.5
dem_100_belg_orig_46	183.5k	183.3k	0.1	0.3	183.5k	183.5k	0.0	17.5	183.5k	183.5k	0.0	0.5
dem_100_belg_orig_47	129.9k	129.7k	0.1	0.4	129.9k	129.9k	0.0	8.2	129.9k	129.9k	0.0	4.2
dem_100_belg_orig_48	210.4k	210.2k	0.1	0.3	210.4k	210.4k	0.0	14.3	210.4k	210.2k	0.1	0.7
dem_100_belg_orig_49	174.9k	174.8k	0.1	0.3	174.9k	174.9k	0.0	14.6	174.9k	174.9k	0.0	0.4
dem_100_belg_orig_50	116.5k	116.5k	0.0	0.7	116.5k	116.5k	0.0	20.2	116.5k	116.5k	0.0	0.4
dem_100_belg_orig_51	184.3k	184.2k	0.0	0.3	184.3k	184.3k	0.0	19.1	184.3k	184.3k	0.0	0.7
dem_100_belg_orig_52	151.0k	151.0k	0.0	0.1	151.0k	151.0k	0.0	11.1	151.0k	151.0k	0.0	0.3
dem_100_belg_orig_53	98.4k	98.3k	0.1	0.2	98.4k	98.4k	0.0	8.4	98.4k	98.4k	0.0	0.8
dem_100_belg_orig_54	264.1k	264.1k	0.0	0.3	264.1k	264.1k	0.0	14.6	264.1k	263.8k	0.1	0.9
dem_100_belg_orig_55	142.0k	141.9k	0.1	0.5	142.0k	142.0k	0.0	18.5	142.0k	142.0k	0.0	1.4
dem_100_belg_orig_56	135.2k	135.2k	0.0	0.3	135.2k	135.2k	0.0	11.2	135.2k	135.2k	0.0	0.6
dem_100_belg_orig_57	213.4k	213.2k	0.1	0.1	213.4k	213.2k	0.1	13.8	213.4k	213.2k	0.1	0.5
dem_100_belg_orig_58	326.2k	326.2k	0.0	0.3	326.2k	325.9k	0.1	25.6	326.2k	326.0k	0.1	0.7
dem_100_belg_orig_59	198.1k	198.1k	0.0	0.3	198.1k	198.1k	0.0	25.9	198.1k	198.1k	0.0	0.6
dem_100_belg_orig_60	335.7k	335.7k	0.0	7.3	335.7k	335.5k	0.1	65.0	335.7k	335.7k	0.0	15.4
dem_100_belg_orig_61	216.6k	216.5k	0.1	0.4	216.6k	216.6k	0.0	13.2	216.6k	216.4k	0.1	0.5
dem_100_belg_orig_62	151.0k	151.0k	0.0	0.2	151.0k	151.0k	0.0	10.6	151.0k	151.0k	0.0	6.4
dem_100_belg_orig_63	271.1k	271.1k	0.0	8.9	271.1k	271.1k	0.0	36.0	271.1k	271.1k	0.0	1.9
dem_100_belg_orig_64	149.7k	149.5k	0.1	0.2	149.7k	149.7k	0.0	9.2	149.7k	149.6k	0.0	0.8
dem_100_belg_orig_65	210.8k	210.6k	0.1	0.5	210.8k	210.6k	0.1	18.6	210.8k	210.6k	0.1	0.8
dem_100_belg_orig_66	165.4k	165.4k	0.0	8.2	165.4k	165.4k	0.0	29.6	165.4k	165.4k	0.0	12.2
dem_100_belg_orig_67	204.8k	204.8k	0.0	0.8	204.8k	204.8k	0.0	18.9	204.8k	204.8k	0.0	0.6
dem_100_belg_orig_68	186.8k	186.7k	0.0	3.9	186.7k	186.7k	0.0	22.8	186.7k	186.7k	0.0	11.4
dem_100_belg_orig_69	360.8k	360.8k	0.0	30.4	360.8k	360.6k	0.1	48.7	360.8k	360.8k	0.0	169.1
dem_100_belg_orig_70	306.2k	306.2k	0.0	0.5	306.2k	306.2k	0.0	23.3	306.2k	306.0k	0.1	1.5

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Table A.1: Comparison of discrete models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_71	188.8k	188.6k	0.1	4.5	188.8k	188.8k	0.0	14.4	188.8k	188.6k	0.1	3.0
dem_100_belg_orig_72	168.1k	168.1k	0.0	4.9	168.1k	168.1k	0.0	14.7	168.1k	168.1k	0.0	0.6
dem_100_belg_orig_73	64.3k	64.3k	0.0	1.7	64.3k	64.3k	0.0	5.6	64.3k	64.3k	0.0	0.4
dem_100_belg_orig_74	345.9k	345.9k	0.0	481.4	345.9k	345.8k	0.0	108.4	345.9k	345.9k	0.0	5102.4
dem_100_belg_orig_75	495.9k	495.9k	0.0	268.3	495.9k	495.9k	0.0	296.3	495.9k	495.9k	0.0	3894.8
dem_100_belg_orig_76	335.1k	335.1k	0.0	8.7	335.1k	335.1k	0.0	52.8	335.1k	335.1k	0.0	35.4
dem_100_belg_orig_77	397.7k	397.4k	0.1	8.9	397.7k	397.7k	0.0	146.9	397.7k	397.7k	0.0	67.3
dem_100_belg_orig_78	162.8k	162.8k	0.0	0.2	162.8k	162.8k	0.0	11.7	162.8k	162.7k	0.0	0.5
dem_100_belg_orig_79	187.9k	187.9k	0.0	0.3	187.9k	187.9k	0.0	17.3	187.9k	187.9k	0.0	0.6
dem_100_belg_orig_80	149.4k	149.4k	0.0	0.2	149.4k	149.3k	0.1	22.3	149.4k	149.4k	0.0	0.3
dem_100_belg_orig_81	263.3k	263.3k	0.0	3.8	263.3k	263.3k	0.0	49.3	263.3k	263.3k	0.0	76.0
dem_100_belg_orig_82	262.6k	262.6k	0.0	0.7	262.6k	262.6k	0.0	40.0	262.6k	262.6k	0.0	2.0
dem_100_belg_orig_83	256.8k	256.6k	0.1	0.7	256.8k	256.8k	0.0	15.7	256.8k	256.6k	0.1	0.7
dem_100_belg_orig_84	361.2k	361.2k	0.0	17.2	361.2k	361.2k	0.0	55.9	361.2k	361.2k	0.0	24.0
dem_100_belg_orig_85	286.4k	286.4k	0.0	152.3	286.4k	286.4k	0.0	439.3	286.4k	286.4k	0.0	1002.4
dem_100_belg_orig_86	230.6k	230.6k	0.0	0.4	230.6k	230.6k	0.0	32.8	230.6k	230.6k	0.0	20.2
dem_100_belg_orig_87	568.5k	568.5k	0.0	10.9	568.5k	568.5k	0.0	168.8	568.5k	568.3k	0.0	3.2
dem_100_belg_orig_88	54.1k	54.1k	0.0	1.3	54.1k	54.1k	0.0	33.4	54.1k	54.1k	0.0	9.4
dem_100_belg_orig_89	94.1k	94.1k	0.0	0.3	94.1k	94.1k	0.0	11.7	94.1k	94.1k	0.0	0.5
dem_100_belg_orig_90	263.5k	263.4k	0.0	0.3	263.5k	263.5k	0.0	51.5	263.5k	263.5k	0.0	0.7
dem_100_belg_orig_91	198.2k	198.1k	0.1	0.3	198.2k	198.2k	0.0	20.8	198.2k	198.1k	0.1	0.5
dem_100_belg_orig_92	202.5k	202.5k	0.0	0.5	202.5k	202.5k	0.0	21.7	202.5k	202.5k	0.0	1.2
dem_100_belg_orig_93	182.7k	182.7k	0.0	0.5	182.7k	182.7k	0.0	18.1	182.7k	182.5k	0.1	0.7
dem_100_belg_orig_94	532.8k	532.4k	0.1	4.6	532.8k	532.8k	0.0	187.8	532.8k	532.5k	0.1	4.5
dem_100_belg_orig_95	379.3k	379.3k	0.0	53.4	379.3k	379.3k	0.0	375.6	379.3k	379.3k	0.0	79.5
dem_100_belg_orig_96	343.9k	343.6k	0.1	1.0	343.9k	343.8k	0.0	40.1	343.9k	343.7k	0.1	0.6
dem_100_belg_orig_97	144.8k	144.8k	0.0	0.3	144.8k	144.8k	0.0	9.5	144.8k	144.8k	0.0	0.8
dem_100_belg_orig_98	281.7k	281.7k	0.0	0.2	281.7k	281.7k	0.0	48.5	281.7k	281.7k	0.0	0.4
dem_100_belg_orig_99	259.9k	259.9k	0.0	12.7	259.9k	259.9k	0.0	67.6	259.9k	259.9k	0.0	38.3
dem_100_belg_orig_100	314.4k	314.4k	0.0	78.6	314.4k	314.4k	0.0	59.7	314.4k	314.4k	0.0	308.4
dem_100_belg_orig_101	444.4k	444.4k	0.0	126.8	444.4k	444.4k	0.0	56.5	444.4k	444.0k	0.1	2496.4
dem_100_belg_orig_102	339.8k	339.8k	0.0	0.5	339.8k	339.8k	0.0	31.7	339.8k	339.6k	0.1	0.8
dem_100_belg_orig_103	414.0k	414.0k	0.0	3.2	414.0k	413.9k	0.0	50.0	414.0k	414.0k	0.0	3.1
dem_100_belg_orig_104	408.7k	408.7k	0.0	228.8	408.7k	408.7k	0.0	159.4	408.7k	408.7k	0.0	863.2
dem_100_belg_orig_105	124.2k	124.2k	0.0	0.4	124.2k	124.2k	0.0	14.2	124.2k	124.2k	0.0	0.6
dem_100_belg_orig_106	341.8k	341.4k	0.1	0.6	341.8k	341.8k	0.0	34.5	341.9k	341.5k	0.1	0.8
dem_100_belg_orig_107	189.5k	189.4k	0.1	0.3	189.5k	189.4k	0.1	14.3	189.5k	189.5k	0.0	0.4
dem_100_belg_orig_108	314.6k	314.3k	0.1	14.8	314.6k	314.6k	0.0	39.1	314.6k	314.6k	0.0	20.3
dem_100_belg_orig_109	254.5k	254.3k	0.1	0.4	254.5k	254.5k	0.0	26.9	254.5k	254.5k	0.0	0.8
dem_100_belg_orig_110	183.5k	183.5k	0.0	0.3	183.5k	183.5k	0.0	11.8	183.5k	183.5k	0.0	0.8
dem_100_belg_orig_111	169.6k	169.5k	0.1	0.2	169.6k	169.6k	0.0	20.0	169.6k	169.6k	0.0	0.4
dem_100_belg_orig_112	105.1k	105.0k	0.0	0.3	105.1k	105.1k	0.0	19.6	105.1k	105.1k	0.0	0.6
dem_100_belg_orig_113	183.5k	183.4k	0.0	0.7	183.5k	183.5k	0.0	11.0	183.5k	183.5k	0.0	0.9
dem_100_belg_orig_114	276.6k	276.4k	0.1	0.8	276.6k	276.6k	0.0	32.5	276.6k	276.6k	0.0	1.2
dem_100_belg_orig_115	58.8k	58.8k	0.0	0.1	58.8k	58.8k	0.0	6.2	58.8k	58.8k	0.0	0.3
dem_100_belg_orig_116	348.9k	348.9k	0.0	213.1	348.9k	348.7k	0.0	122.6	348.9k	348.9k	0.0	2435.6
dem_100_belg_orig_117	193.9k	193.7k	0.1	0.6	193.8k	193.6k	0.1	24.5	193.8k	193.8k	0.0	0.7
dem_100_belg_orig_118	439.4k	439.4k	0.0	58.7	439.4k	439.4k	0.0	216.3	439.4k	439.4k	0.0	502.8
dem_100_belg_orig_119	221.5k	221.5k	0.0	13.9	221.5k	221.3k	0.1	45.1	221.5k	221.5k	0.0	49.2
dem_100_belg_orig_120	236.4k	236.4k	0.0	13.0	236.4k	236.4k	0.0	133.2	236.4k	236.4k	0.0	25.1
dem_100_belg_orig_121	211.3k	211.2k	0.1	0.3	211.3k	211.3k	0.0	10.6	211.3k	211.2k	0.1	0.5
dem_100_belg_orig_122	117.4k	117.3k	0.0	0.3	117.4k	117.4k	0.0	12.5	117.4k	117.4k	0.0	0.6
dem_100_belg_orig_123	245.7k	245.7k	0.0	681.9	245.7k	245.7k	0.0	333.4	245.7k	245.7k	0.0	246.1
dem_100_belg_orig_124	185.8k	185.8k	0.0	0.3	185.8k	185.8k	0.0	11.4	185.8k	185.8k	0.0	0.5
dem_100_belg_orig_125	182.4k	182.3k	0.1	0.3	182.4k	182.4k	0.0	22.3	182.4k	182.4k	0.0	1.0
dem_100_belg_orig_126	405.0k	405.0k	0.0	163.3	405.0k	404.7k	0.1	90.8	405.0k	405.0k	0.0	626.9
dem_100_belg_orig_127	239.9k	239.9k	0.0	5.9	239.9k	239.9k	0.0	57.7	239.9k	239.9k	0.0	29.1
dem_100_belg_orig_128	474.7k	474.7k	0.0	3.1	474.7k	474.7k	0.0	108.7	474.7k	474.7k	0.0	37.2
dem_100_belg_orig_129	170.8k	170.8k	0.0	164.4	170.8k	170.8k	0.0	347.1	170.8k	170.8k	0.0	713.0
dem_100_belg_orig_130	350.5k	350.5k	0.0	489.3	350.5k	350.5k	0.0	316.8	350.5k	337.7k	3.8	14400.0
dem_100_belg_orig_131	345.0k	345.0k	0.0	57.2	345.0k	345.0k	0.0	430.0	345.0k	345.0k	0.0	635.5
dem_100_belg_orig_132	405.6k	405.5k	0.0	0.3	405.6k	405.6k	0.0	55.6	405.6k	405.3k	0.1	0.5
dem_100_belg_orig_133	308.3k	308.2k	0.0	1.0	308.3k	308.3k	0.0	25.1	308.3k	308.0k	0.1	3.1
dem_100_belg_orig_134	521.5k	521.0k	0.1	233.9	521.5k	521.1k	0.1	232.0	521.5k	521.5k	0.0	196.7
dem_100_belg_orig_135	172.5k	172.5k	0.0	0.3	172.5k	172.5k	0.0	18.1	172.5k	172.5k	0.0	2.7
dem_100_belg_orig_136	326.1k	326.1k	0.0	80.7	326.1k	325.8k	0.1	51.8	326.1k	326.1k	0.0	159.1
dem_100_belg_orig_137	343.6k	343.6k	0.0	80.2	343.6k	343.5k	0.0	162.3	343.6k	343.6k	0.0	386.2
dem_100_belg_orig_138	171.9k	171.8k	0.1	0.3	171.9k	171.9k	0.0	12.4	171.9k	171.9k	0.0	5.4
dem_100_belg_orig_139	197.2k	197.2k	0.0	0.2	197.2k	197.0k	0.1	18.0	197.2k	197.2k	0.0	0.3
dem_100_belg_orig_140	153.3k	153.3k	0.0	4.0	153.3k	153.3k	0.0	31.1	153.3k	153.3k	0.0	13.9
dem_100_belg_orig_141	321.5k	321.5k	0.0	356.7	321.5k	321.5k	0.0	528.8	321.5k	321.5k	0.0	7848.4
dem_100_belg_orig_142	183.5k	183.4k	0.1	0.4	183.5k	183.5k	0.0	20.9	183.5k	183.5k	0.0	0.7
dem_100_belg_orig_143	500.0k	499.6k	0.1	94.9	500.0k	500.0k	0.0	540.1	500.0k	500.0k	0.0	241.8
dem_100_belg_orig_144	512.0k	512.0k	0.0	66.6	512.0k	512.0k	0.0	444.0	512.0k	512.0k	0.0	136.0
dem_100_belg_orig_145	427.0k	427.0k	0.0	599.8	427.0k	426.7k	0.1	262.3	427.0k	427.0k	0.0	2562.7
dem_100_belg_orig_146	271.6k	271.6k	0.0	1.7	271.6k	271.6k	0.0	17.9	271.6k	271.4k	0.1	2.7
dem_100_belg_orig_147	237.7k	237.7k	0.0	12.8	237.7k	237.5k	0.1	43.8	237.7k	237.7k	0.0	23.8
dem_100_belg_orig_148	240.2k	240.1k	0.0	0.5	240.2k	240.2k	0.0	19.5	240.2k	240.2k	0.0	1.5

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Table A.1: Comparison of discrete models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_149	387.5k	387.5k	0.0	9.4	387.5k	387.5k	0.0	72.5	387.5k	387.5k	0.0	8.1
dem_100_belg_orig_150	353.0k	352.7k	0.1	0.5	353.0k	353.0k	0.0	39.9	353.0k	352.7k	0.1	1.6
dem_100_belg_orig_151	169.6k	169.6k	0.0	0.2	169.6k	169.6k	0.0	12.3	169.6k	169.5k	0.0	0.4
dem_100_belg_orig_152	156.7k	156.7k	0.1	0.3	156.7k	156.7k	0.0	26.3	156.7k	156.7k	0.0	0.5
dem_100_belg_orig_153	384.4k	384.4k	0.0	23.3	384.4k	384.4k	0.0	84.8	384.4k	384.4k	0.0	63.8
dem_100_belg_orig_154	254.9k	254.7k	0.1	0.3	254.9k	254.9k	0.0	22.8	254.9k	254.7k	0.1	0.7
dem_100_belg_orig_155	161.6k	161.6k	0.0	0.3	161.6k	161.6k	0.0	9.0	161.6k	161.5k	0.1	0.9
dem_100_belg_orig_156	205.4k	205.3k	0.1	0.4	205.4k	205.4k	0.0	21.4	205.4k	205.4k	0.0	3.0
dem_100_belg_orig_157	393.2k	393.2k	0.0	42.5	393.2k	393.2k	0.0	125.6	393.2k	393.2k	0.0	1264.6
dem_100_belg_orig_158	330.1k	330.1k	0.0	28.1	330.1k	330.0k	0.0	37.5	330.1k	330.0k	0.0	16.1
dem_100_belg_orig_159	255.6k	255.6k	0.0	0.4	255.6k	255.6k	0.0	30.8	255.6k	255.6k	0.0	1.1
dem_100_belg_orig_160	208.6k	208.6k	0.0	0.3	208.6k	208.6k	0.0	13.8	208.6k	208.5k	0.0	0.5
dem_100_belg_orig_161	598.6k	598.6k	0.0	0.3	598.8k	598.3k	0.1	30.0	598.6k	598.6k	0.0	0.7
dem_100_belg_orig_162	259.3k	259.1k	0.1	0.5	259.3k	259.3k	0.0	24.1	259.3k	259.3k	0.0	0.6
dem_100_belg_orig_163	499.0k	498.6k	0.1	0.2	499.0k	499.0k	0.0	66.3	499.0k	498.5k	0.1	0.9
dem_100_belg_orig_164	293.5k	293.5k	0.0	35.1	293.5k	293.5k	0.0	77.8	293.5k	293.5k	0.0	29.9
dem_100_belg_orig_165	171.9k	171.9k	0.0	8.9	171.9k	171.9k	0.0	11.9	171.9k	171.9k	0.0	0.9
dem_100_belg_orig_166	231.1k	231.1k	0.0	6.8	231.1k	231.1k	0.0	39.2	231.1k	231.1k	0.0	19.4
dem_100_belg_orig_167	188.4k	188.3k	0.1	1.0	188.4k	188.4k	0.0	20.2	188.4k	188.3k	0.1	3.9
dem_100_belg_orig_168	272.3k	272.2k	0.1	1.2	272.3k	272.3k	0.0	61.3	272.3k	272.3k	0.0	6.0
dem_100_belg_orig_169	277.4k	277.4k	0.0	0.3	277.4k	277.3k	0.0	31.4	277.4k	277.3k	0.0	0.6
dem_100_belg_orig_170	387.0k	386.7k	0.1	0.4	387.0k	387.0k	0.0	28.9	387.0k	386.6k	0.1	0.7
dem_100_belg_orig_171	287.8k	287.5k	0.1	0.3	287.8k	287.8k	0.0	33.3	287.8k	287.5k	0.1	0.8
dem_100_belg_orig_172	148.5k	148.5k	0.0	8.1	148.5k	148.5k	0.0	25.0	148.5k	148.5k	0.0	4.2
dem_100_belg_orig_173	308.6k	308.3k	0.1	0.5	308.4k	308.2k	0.1	30.7	308.6k	308.4k	0.1	3.0
dem_100_belg_orig_174	90.2k	90.2k	0.0	0.2	90.2k	90.2k	0.0	6.3	90.2k	90.2k	0.0	6.3
dem_100_belg_orig_175	176.3k	176.2k	0.1	4.0	176.3k	176.3k	0.0	20.1	176.3k	176.3k	0.0	4.3
dem_100_belg_orig_176	433.0k	432.6k	0.1	1.5	432.7k	432.7k	0.0	43.8	432.7k	432.3k	0.1	0.7
dem_100_belg_orig_177	282.5k	282.5k	0.0	35.9	282.5k	282.5k	0.0	276.1	282.5k	282.5k	0.0	734.6
dem_100_belg_orig_178	315.8k	315.5k	0.1	0.4	315.8k	315.8k	0.0	38.1	315.8k	315.4k	0.1	0.8
dem_100_belg_orig_179	227.8k	227.8k	0.0	0.8	227.8k	227.8k	0.0	23.6	227.8k	227.7k	0.0	1.0
dem_100_belg_orig_180	228.9k	228.9k	0.0	0.7	228.9k	228.9k	0.0	34.3	228.9k	228.8k	0.0	0.7
dem_100_belg_orig_181	211.3k	211.3k	0.0	2.4	211.3k	211.3k	0.0	52.2	211.3k	211.3k	0.0	9.8
dem_100_belg_orig_182	547.5k	547.5k	0.0	23.8	547.5k	547.5k	0.0	332.8	547.5k	547.5k	0.0	32.7
dem_100_belg_orig_183	339.9k	339.5k	0.1	0.3	339.9k	339.6k	0.1	21.5	339.9k	339.5k	0.1	0.9
dem_100_belg_orig_184	98.4k	98.3k	0.1	0.3	98.4k	98.4k	0.0	10.1	98.4k	98.4k	0.0	0.5
dem_100_belg_orig_185	286.6k	286.4k	0.1	1.2	286.6k	286.5k	0.1	34.2	286.6k	286.5k	0.0	1.4
dem_100_belg_orig_186	125.9k	125.9k	0.0	0.3	125.9k	125.9k	0.0	11.7	125.9k	125.9k	0.0	0.4
dem_100_belg_orig_187	113.8k	113.8k	0.0	0.3	113.8k	113.8k	0.0	14.8	113.8k	113.8k	0.0	6.1
dem_100_belg_orig_188	431.5k	431.5k	0.0	24.2	431.5k	431.5k	0.0	123.4	431.5k	431.5k	0.0	724.4
dem_100_belg_orig_189	183.5k	183.5k	0.0	0.2	183.5k	183.5k	0.0	17.4	183.5k	183.5k	0.0	0.5
dem_100_belg_orig_190	242.5k	242.5k	0.0	10.1	242.5k	242.5k	0.0	36.2	242.5k	242.5k	0.0	18.1
dem_100_belg_orig_191	330.1k	330.1k	0.0	6.5	330.1k	330.1k	0.0	48.8	330.1k	329.9k	0.0	3.7
dem_100_belg_orig_192	202.1k	202.1k	0.0	0.3	202.1k	202.1k	0.0	32.6	202.1k	202.0k	0.1	0.5
dem_100_belg_orig_193	186.2k	186.2k	0.0	0.5	186.2k	186.2k	0.0	32.6	186.2k	186.2k	0.0	0.6
dem_100_belg_orig_194	217.5k	217.3k	0.1	0.7	217.5k	217.4k	0.0	18.4	217.5k	217.5k	0.0	6.1
dem_100_belg_orig_195	350.1k	349.8k	0.1	8.0	350.1k	350.1k	0.0	30.6	350.1k	350.1k	0.0	17.6
dem_100_belg_orig_196	174.9k	174.7k	0.1	4.2	174.9k	174.9k	0.0	16.1	174.9k	174.9k	0.0	8.0
dem_100_belg_orig_197	350.3k	350.0k	0.1	1.0	350.3k	350.0k	0.1	53.8	350.3k	350.0k	0.1	0.6
dem_100_belg_orig_198	223.1k	223.1k	0.0	8.0	223.1k	223.1k	0.0	50.4	223.1k	223.1k	0.0	8.0
dem_100_belg_orig_199	195.4k	195.4k	0.0	0.5	195.4k	195.4k	0.0	28.0	195.4k	195.4k	0.0	9.0
dem_100_belg_orig_200	232.0k	231.8k	0.1	1.3	232.0k	232.0k	0.0	43.8	232.0k	232.0k	0.0	1.9
dem_100_belg_orig_201	381.6k	381.6k	0.0	84.6	381.6k	381.4k	0.0	643.6	381.6k	381.6k	0.0	553.5
dem_100_belg_orig_202	368.7k	368.7k	0.0	4.6	368.7k	368.7k	0.0	68.0	368.7k	368.7k	0.0	78.0
dem_100_belg_orig_203	531.3k	531.3k	0.0	1402.4	531.3k	531.2k	0.0	127.9	531.3k	531.3k	0.0	10845.0
dem_100_belg_orig_204	339.6k	339.5k	0.0	0.6	339.6k	339.6k	0.0	41.7	339.6k	339.5k	0.0	5.6
dem_100_belg_orig_205	169.6k	169.5k	0.1	0.1	169.6k	169.6k	0.0	8.4	169.6k	169.6k	0.0	0.5
dem_100_belg_orig_206	255.3k	255.1k	0.1	0.4	255.3k	255.3k	0.0	37.5	255.3k	255.2k	0.0	1.1
dem_100_belg_orig_207	346.0k	346.0k	0.0	159.0	346.0k	346.0k	0.0	37.6	346.0k	346.0k	0.0	268.5
dem_100_belg_orig_208	288.6k	288.6k	0.0	103.0	288.6k	288.4k	0.1	77.4	288.6k	288.6k	0.0	183.4
dem_100_belg_orig_209	374.5k	374.5k	0.0	8.4	374.5k	374.5k	0.0	54.3	374.5k	374.5k	0.0	22.1
dem_100_belg_orig_210	230.7k	230.7k	0.0	41.8	230.7k	230.7k	0.0	678.3	230.7k	230.7k	0.0	386.6
dem_100_belg_orig_211	97.7k	97.7k	0.0	0.3	97.7k	97.7k	0.0	15.2	97.7k	97.7k	0.0	0.5
dem_100_belg_orig_212	335.1k	335.1k	0.0	13.7	335.1k	335.1k	0.0	38.5	335.1k	335.1k	0.0	33.7
dem_100_belg_orig_213	168.5k	168.4k	0.0	0.4	168.5k	168.4k	0.0	25.5	168.5k	168.5k	0.0	0.8
dem_100_belg_orig_214	199.9k	199.9k	0.0	5.5	199.9k	199.9k	0.0	20.6	199.9k	199.9k	0.0	9.2
dem_100_belg_orig_215	167.5k	167.3k	0.1	0.2	167.5k	167.5k	0.0	8.2	167.5k	167.5k	0.0	0.2
dem_100_belg_orig_216	307.2k	307.2k	0.0	22.7	307.2k	307.2k	0.0	52.3	307.2k	307.0k	0.1	43.0
dem_100_belg_orig_217	166.5k	166.5k	0.0	0.3	166.5k	166.5k	0.0	12.7	166.5k	166.5k	0.0	0.5
dem_100_belg_orig_218	237.5k	237.5k	0.0	0.4	237.5k	237.2k	0.1	18.9	237.5k	237.4k	0.0	0.5
dem_100_belg_orig_219	334.1k	334.1k	0.0	239.2	334.1k	334.1k	0.0	112.0	334.1k	334.0k	0.0	10242.8
dem_100_belg_orig_220	148.9k	148.9k	0.0	0.4	148.9k	148.9k	0.0	30.9	148.9k	148.9k	0.0	2.8
dem_100_belg_orig_221	376.9k	376.9k	0.0	33.3	376.9k	376.9k	0.0	55.2	376.9k	376.9k	0.0	166.4
dem_100_belg_orig_222	250.3k	250.3k	0.0	0.2	250.3k	250.3k	0.0	25.6	250.3k	250.3k	0.0	0.6
dem_100_belg_orig_223	228.6k	228.6k	0.0	0.5	228.6k	228.5k	0.0	14.4	228.6k	228.6k	0.0	0.6
dem_100_belg_orig_224	224.8k	224.8k	0.0	0.3	224.8k	224.6k	0.1	29.2	224.8k	224.6k	0.1	0.6
dem_100_belg_orig_225	428.9k	428.9k	0.0	2051.5	428.9k	428.9k	0.0	239.1	428.9k	406.4k	5.5	14400.0
dem_100_belg_orig_226	537.9k	537.9k	0.0	17.6	537.9k	537.9k	0.0	325.7	537.9k	537.9k	0.0	40.9

continued on next page

Table A.1: Comparison of discrete models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_227	399.9k	399.9k	0.0	61.4	399.9k	399.9k	0.0	109.0	399.9k	399.9k	0.0	18.5
dem_100_belg_orig_228	211.3k	211.1k	0.1	0.8	211.3k	211.3k	0.0	19.0	211.3k	211.3k	0.0	0.8
dem_100_belg_orig_229	187.3k	187.3k	0.0	0.3	187.3k	187.3k	0.0	20.1	187.3k	187.3k	0.0	0.5
dem_100_belg_orig_230	218.6k	218.6k	0.0	0.4	218.6k	218.6k	0.0	22.2	218.6k	218.6k	0.0	1.7
dem_100_belg_orig_231	184.3k	184.2k	0.0	0.2	184.3k	184.3k	0.0	16.9	184.3k	184.3k	0.0	0.5
dem_100_belg_orig_232	151.0k	150.9k	0.1	0.1	151.0k	151.0k	0.0	6.9	151.0k	151.0k	0.0	0.5
dem_100_belg_orig_233	169.6k	169.6k	0.0	0.3	169.6k	169.6k	0.0	10.2	169.6k	169.6k	0.0	0.5
dem_100_belg_orig_234	192.8k	192.8k	0.0	0.3	192.8k	192.8k	0.0	21.2	192.8k	192.8k	0.0	3.9
dem_100_belg_orig_235	215.6k	215.6k	0.0	10.2	215.6k	215.6k	0.0	38.7	215.6k	215.6k	0.0	9.9
dem_100_belg_orig_236	400.4k	400.1k	0.1	4.9	400.4k	400.4k	0.0	61.4	400.4k	400.4k	0.0	309.3
dem_100_belg_orig_237	265.1k	264.9k	0.1	0.6	265.1k	265.1k	0.0	40.9	265.1k	265.1k	0.0	4.0
dem_100_belg_orig_238	225.3k	225.1k	0.1	0.4	225.3k	225.3k	0.0	19.0	225.3k	225.3k	0.0	0.8
dem_100_belg_orig_239	171.2k	171.2k	0.0	0.3	171.2k	171.2k	0.0	20.9	171.2k	171.2k	0.0	0.5
dem_100_belg_orig_240	251.2k	251.0k	0.1	0.5	251.2k	251.2k	0.0	61.4	251.2k	251.2k	0.0	0.7
dem_100_belg_orig_241	298.1k	297.9k	0.1	12.1	298.1k	297.9k	0.1	71.3	298.1k	298.1k	0.0	301.3
dem_100_belg_orig_242	206.2k	206.2k	0.0	0.5	206.2k	206.2k	0.0	24.2	206.2k	206.0k	0.1	0.8
dem_100_belg_orig_243	231.2k	231.2k	0.0	8.0	231.2k	231.2k	0.0	94.1	231.2k	231.2k	0.0	80.0
dem_100_belg_orig_244	473.5k	473.5k	0.0	16.7	473.5k	473.5k	0.0	465.2	473.5k	473.5k	0.0	2010.0
dem_100_belg_orig_245	399.0k	399.0k	0.0	33.2	399.0k	398.7k	0.1	129.2	399.0k	399.0k	0.0	89.2
dem_100_belg_orig_246	236.9k	236.9k	0.0	9.0	236.9k	236.9k	0.0	29.4	236.9k	236.9k	0.0	9.3
dem_100_belg_orig_247	371.1k	371.1k	0.0	44.4	371.1k	371.1k	0.0	111.6	371.1k	371.0k	0.0	74.8
dem_100_belg_orig_248	350.1k	350.1k	0.0	1.0	350.1k	350.1k	0.0	61.5	350.1k	349.7k	0.1	2.0
dem_100_belg_orig_249	355.6k	355.2k	0.1	2.0	355.6k	355.6k	0.0	82.3	355.6k	355.6k	0.0	23.9
dem_100_belg_orig_250	265.2k	265.2k	0.0	19.1	265.2k	265.2k	0.0	71.5	265.2k	265.2k	0.0	56.2
dem_100_belg_orig_251	329.7k	329.7k	0.0	7.8	329.7k	329.7k	0.0	104.2	329.7k	329.7k	0.0	38.8
dem_100_belg_orig_252	171.9k	171.9k	0.0	0.1	171.9k	171.9k	0.0	23.0	171.9k	171.9k	0.0	0.4
dem_100_belg_orig_253	206.9k	206.9k	0.0	0.3	206.9k	206.9k	0.0	13.6	206.9k	206.9k	0.0	0.5
dem_100_belg_orig_254	274.3k	274.3k	0.0	1.0	274.3k	274.3k	0.0	57.4	274.3k	274.3k	0.0	2.4
dem_100_belg_orig_255	293.7k	293.4k	0.1	0.7	293.6k	293.6k	0.0	45.7	293.6k	293.4k	0.1	0.5
dem_100_belg_orig_256	247.0k	246.9k	0.1	0.3	247.0k	247.0k	0.0	15.1	247.0k	246.8k	0.1	0.9
dem_100_belg_orig_257	267.3k	267.1k	0.1	1.5	267.3k	267.3k	0.0	32.1	267.3k	267.2k	0.1	1.1
dem_100_belg_orig_258	321.3k	321.3k	0.0	14.4	321.3k	321.3k	0.0	61.2	321.3k	321.3k	0.0	70.5
dem_100_belg_orig_259	291.5k	291.5k	0.0	14.1	291.5k	291.5k	0.0	34.8	291.5k	291.5k	0.0	16.6
dem_100_belg_orig_260	169.6k	169.6k	0.0	0.1	169.6k	169.6k	0.0	8.4	169.6k	169.6k	0.0	0.5
dem_100_belg_orig_261	166.4k	166.4k	0.0	0.2	166.4k	166.4k	0.0	10.0	166.4k	166.4k	0.0	0.4
dem_100_belg_orig_262	210.4k	210.4k	0.0	0.5	210.4k	210.4k	0.0	52.2	210.4k	210.4k	0.0	13.8
dem_100_belg_orig_263	278.3k	278.3k	0.0	6.0	278.3k	278.3k	0.0	38.0	278.3k	278.1k	0.1	4.7
dem_100_belg_orig_264	307.2k	307.2k	0.0	27.1	307.2k	307.2k	0.0	31.3	307.2k	307.2k	0.0	56.0
dem_100_belg_orig_265	391.4k	391.4k	0.0	101.9	391.4k	391.4k	0.0	116.0	391.4k	391.4k	0.0	835.2
dem_100_belg_orig_266	454.6k	454.6k	0.0	54.5	454.6k	454.6k	0.0	244.2	454.6k	454.4k	0.1	724.7
dem_100_belg_orig_267	263.2k	263.0k	0.1	0.6	263.2k	263.0k	0.1	24.1	263.2k	263.0k	0.1	0.6
dem_100_belg_orig_268	179.6k	179.5k	0.1	0.3	179.6k	179.6k	0.0	12.8	179.6k	179.6k	0.0	0.5
dem_100_belg_orig_269	192.1k	192.1k	0.0	0.3	192.1k	192.1k	0.0	17.6	192.1k	192.1k	0.0	0.7
dem_100_belg_orig_270	158.1k	158.1k	0.0	5.0	158.1k	158.1k	0.0	18.3	158.1k	158.1k	0.0	9.0
dem_100_belg_orig_271	212.0k	212.0k	0.0	8.3	212.0k	212.0k	0.0	42.5	212.0k	212.0k	0.0	29.6
dem_100_belg_orig_272	112.9k	112.9k	0.1	0.5	112.9k	112.9k	0.0	21.6	112.9k	112.8k	0.1	1.3
dem_100_belg_orig_273	173.5k	173.4k	0.1	1.1	173.5k	173.5k	0.0	12.4	173.5k	173.5k	0.0	1.7
dem_100_belg_orig_274	319.6k	319.5k	0.0	0.3	319.6k	319.6k	0.0	41.6	319.6k	319.5k	0.0	0.8
dem_100_belg_orig_275	174.3k	174.3k	0.0	0.2	174.3k	174.3k	0.0	13.4	174.3k	174.2k	0.1	0.3
dem_100_belg_orig_276	293.7k	293.4k	0.1	0.6	293.7k	293.7k	0.0	23.5	293.7k	293.5k	0.1	0.6
dem_100_belg_orig_277	207.7k	207.7k	0.0	0.1	207.7k	207.7k	0.0	19.4	207.7k	207.5k	0.1	0.4
dem_100_belg_orig_278	243.6k	243.6k	0.0	66.4	243.6k	243.6k	0.0	42.3	243.6k	243.6k	0.0	43.7
dem_100_belg_orig_279	255.6k	255.6k	0.0	0.3	255.6k	255.6k	0.0	31.8	255.6k	255.6k	0.0	0.6
dem_100_belg_orig_280	258.2k	258.2k	0.0	2.0	258.2k	258.0k	0.1	29.6	258.2k	258.2k	0.0	0.9
dem_100_belg_orig_281	389.9k	389.9k	0.0	24.1	389.9k	389.9k	0.0	61.5	389.9k	389.9k	0.0	126.8
dem_100_belg_orig_282	262.8k	262.5k	0.1	0.5	262.8k	262.8k	0.0	22.8	262.8k	262.6k	0.0	0.6
dem_100_belg_orig_283	347.6k	347.6k	0.0	114.8	347.6k	347.6k	0.0	53.6	347.6k	347.6k	0.0	891.9
dem_100_belg_orig_284	319.4k	319.1k	0.1	0.9	319.4k	319.4k	0.0	42.0	319.4k	319.4k	0.0	1.6
dem_100_belg_orig_285	372.3k	372.3k	0.0	11.7	372.3k	372.3k	0.0	59.9	372.3k	372.3k	0.0	12.2
dem_100_belg_orig_286	410.3k	410.3k	0.0	18.0	410.3k	410.3k	0.0	76.8	410.3k	410.0k	0.1	95.9
dem_100_belg_orig_287	228.1k	227.9k	0.1	0.3	228.1k	228.1k	0.0	23.1	228.1k	228.1k	0.0	3.0
dem_100_belg_orig_288	290.5k	290.5k	0.0	88.5	290.5k	290.5k	0.0	82.4	290.5k	290.4k	0.1	2249.2
dem_100_belg_orig_289	206.0k	206.0k	0.0	0.3	206.0k	206.0k	0.0	15.6	206.0k	206.0k	0.0	0.5
dem_100_belg_orig_290	167.1k	167.0k	0.0	0.8	167.1k	167.1k	0.0	14.3	167.1k	167.1k	0.0	0.4
dem_100_belg_orig_291	311.2k	311.2k	0.0	3096.7	311.2k	311.2k	0.0	360.8	311.3k	276.8k	12.4	14400.0
dem_100_belg_orig_292	335.4k	335.2k	0.1	1.4	335.4k	335.4k	0.0	42.3	335.4k	335.4k	0.0	2.3
dem_100_belg_orig_293	269.3k	269.0k	0.1	0.6	269.3k	269.3k	0.0	25.9	269.3k	269.0k	0.1	1.2
dem_100_belg_orig_294	218.6k	218.4k	0.1	1.5	218.6k	218.6k	0.0	21.3	218.6k	218.4k	0.1	0.5
dem_100_belg_orig_295	143.4k	143.4k	0.0	0.3	143.4k	143.4k	0.0	16.5	143.4k	143.4k	0.0	1.4
dem_100_belg_orig_296	250.0k	250.0k	0.0	1.8	250.0k	250.0k	0.0	82.4	250.0k	249.8k	0.1	9.7
dem_100_belg_orig_297	237.3k	237.2k	0.0	1.9	237.3k	237.2k	0.1	71.5	237.3k	237.3k	0.0	8.0
dem_100_belg_orig_298	212.9k	212.8k	0.1	0.5	212.9k	212.8k	0.1	28.1	212.9k	212.9k	0.0	0.5
dem_100_belg_orig_299	184.3k	184.3k	0.0	0.5	184.3k	184.3k	0.0	26.5	184.3k	184.3k	0.0	7.8
dem_100_belg_orig_300	423.8k	423.8k	0.0	220.1	423.8k	423.8k	0.0	230.4	423.8k	423.8k	0.0	998.2
dem_100_belg_orig_301	239.9k	239.9k	0.0	14.7	239.9k	239.8k	0.0	53.7	239.9k	239.9k	0.0	14.0
dem_100_belg_orig_302	381.0k	381.0k	0.0	0.7	381.0k	381.0k	0.0	74.5	381.0k	381.0k	0.0	6.1
dem_100_belg_orig_303	297.0k	297.0k	0.0	0.4	297.0k	297.0k	0.0	51.1	297.0k	297.0k	0.0	0.4
dem_100_belg_orig_304	160.0k	160.0k	0.0	0.3	160.0k	160.0k	0.0	11.4	160.0k	160.0k	0.0	0.6

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Table A.1: Comparison of discrete models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_305	388.9k	388.9k	0.0	4.5	388.9k	388.9k	0.0	65.7	388.9k	388.9k	0.0	99.0
dem_100_belg_orig_306	216.6k	216.6k	0.0	0.3	216.6k	216.6k	0.0	20.3	216.6k	216.6k	0.0	0.6
dem_100_belg_orig_307	124.8k	124.8k	0.0	0.2	124.8k	124.8k	0.0	5.8	124.8k	124.8k	0.0	0.5
dem_100_belg_orig_308	160.2k	160.1k	0.1	6.2	160.2k	160.2k	0.0	12.8	160.2k	160.2k	0.0	8.2
dem_100_belg_orig_309	166.3k	166.2k	0.1	0.3	166.3k	166.2k	0.1	21.3	166.3k	166.2k	0.1	0.6
dem_100_belg_orig_310	146.7k	146.7k	0.0	0.2	146.7k	146.7k	0.0	5.8	146.7k	146.7k	0.0	0.5
dem_100_belg_orig_311	260.1k	260.1k	0.0	194.1	260.1k	260.1k	0.0	101.5	260.1k	260.1k	0.0	62.0
dem_100_belg_orig_312	200.9k	200.8k	0.1	0.2	200.9k	200.9k	0.0	13.8	200.9k	200.8k	0.1	0.4
dem_100_belg_orig_313	427.4k	427.4k	0.0	591.5	427.4k	427.4k	0.0	64.7	427.4k	427.4k	0.0	2022.3
dem_100_belg_orig_314	363.7k	363.7k	0.0	140.6	363.7k	363.7k	0.0	128.4	363.7k	363.7k	0.0	665.1
dem_100_belg_orig_315	180.8k	180.6k	0.1	0.4	180.8k	180.8k	0.0	11.8	180.8k	180.8k	0.0	0.7
dem_100_belg_orig_316	194.6k	194.6k	0.0	5.4	194.6k	194.6k	0.0	19.5	194.6k	194.6k	0.0	7.9
dem_100_belg_orig_317	323.9k	323.6k	0.1	36.0	323.9k	323.9k	0.0	48.9	323.9k	323.9k	0.0	24.2
dem_100_belg_orig_318	389.9k	374.4k	4.2	14400.0	389.9k	389.9k	0.0	363.7	389.9k	356.6k	9.4	14400.0
dem_100_belg_orig_319	305.2k	305.2k	0.0	41.2	305.2k	305.2k	0.0	81.5	305.2k	305.2k	0.0	416.9
dem_100_belg_orig_320	173.4k	173.4k	0.0	0.6	173.4k	173.4k	0.0	13.1	173.4k	173.4k	0.0	5.8
dem_100_belg_orig_321	227.2k	227.1k	0.0	0.1	227.2k	227.1k	0.0	20.0	227.2k	227.1k	0.0	0.3
dem_100_belg_orig_322	281.6k	281.6k	0.0	0.4	281.6k	281.6k	0.0	25.6	281.6k	281.5k	0.0	0.6
dem_100_belg_orig_323	161.4k	161.4k	0.0	0.3	161.4k	161.4k	0.0	11.5	161.4k	161.4k	0.0	0.6
dem_100_belg_orig_324	176.2k	176.2k	0.0	0.3	176.2k	176.2k	0.0	26.8	176.2k	176.2k	0.0	0.5
dem_100_belg_orig_325	255.4k	255.4k	0.0	0.6	255.4k	255.4k	0.0	19.7	255.4k	255.4k	0.0	1.1
dem_100_belg_orig_326	129.0k	129.0k	0.0	0.3	129.0k	129.0k	0.0	9.5	129.0k	129.0k	0.0	1.7
dem_100_belg_orig_327	286.3k	286.0k	0.1	0.6	286.3k	286.3k	0.0	23.9	286.3k	286.1k	0.1	1.1
dem_100_belg_orig_328	217.7k	217.7k	0.0	389.7	217.7k	217.7k	0.0	56.0	217.7k	217.7k	0.0	15.8
dem_100_belg_orig_329	187.9k	187.8k	0.1	0.3	187.9k	187.9k	0.0	24.4	187.9k	187.9k	0.0	6.9
dem_100_belg_orig_330	432.4k	432.4k	0.0	196.6	432.4k	432.4k	0.0	53.8	432.4k	432.1k	0.1	963.8
dem_100_belg_orig_331	221.1k	221.1k	0.0	0.7	221.1k	221.1k	0.0	17.8	221.1k	221.0k	0.0	0.5
dem_100_belg_orig_332	210.4k	210.4k	0.0	0.3	210.4k	210.4k	0.0	16.8	210.4k	210.2k	0.1	0.9
dem_100_belg_orig_333	115.2k	115.2k	0.0	0.4	115.2k	115.2k	0.0	12.0	115.2k	115.2k	0.0	1.8
dem_100_belg_orig_334	168.0k	168.0k	0.0	0.2	168.0k	168.0k	0.0	10.1	168.0k	168.0k	0.0	0.4
dem_100_belg_orig_335	281.5k	281.5k	0.0	8.2	281.5k	281.5k	0.0	54.9	281.5k	281.5k	0.0	12.9
dem_100_belg_orig_336	319.6k	319.6k	0.0	1.1	319.6k	319.6k	0.0	55.0	319.6k	319.6k	0.0	58.5
dem_100_belg_orig_337	177.1k	177.1k	0.0	6.2	177.1k	177.1k	0.0	44.9	177.1k	177.1k	0.0	17.1
dem_100_belg_orig_338	252.5k	252.5k	0.0	0.2	252.5k	252.5k	0.0	25.9	252.5k	252.5k	0.0	1.3
dem_100_belg_orig_339	408.8k	408.8k	0.0	1105.2	408.8k	408.8k	0.0	1093.2	408.8k	401.6k	1.8	14400.0
dem_100_belg_orig_340	387.9k	387.6k	0.1	6.2	387.9k	387.5k	0.1	49.8	387.9k	387.9k	0.0	4.8
dem_100_belg_orig_341	318.3k	318.3k	0.0	0.2	318.3k	318.3k	0.0	27.0	318.3k	318.3k	0.0	0.5
dem_100_belg_orig_342	164.3k	164.2k	0.1	0.6	164.3k	164.3k	0.0	48.8	164.3k	164.3k	0.0	21.9
dem_100_belg_orig_343	193.7k	193.7k	0.0	0.2	193.7k	193.7k	0.0	51.9	193.7k	193.7k	0.0	0.4
dem_100_belg_orig_344	164.4k	164.3k	0.0	0.5	164.4k	164.4k	0.0	16.3	164.4k	164.4k	0.0	0.8
dem_100_belg_orig_345	184.5k	184.4k	0.0	0.3	184.5k	184.5k	0.0	14.0	184.5k	184.5k	0.0	1.0
dem_100_belg_orig_346	226.4k	226.2k	0.1	0.4	226.4k	226.4k	0.0	28.5	226.4k	226.2k	0.1	2.1
dem_100_belg_orig_347	355.9k	355.9k	0.0	21.3	355.9k	355.9k	0.0	79.6	355.9k	355.7k	0.1	23.1
dem_100_belg_orig_348	222.4k	222.4k	0.0	10.4	222.4k	222.4k	0.0	36.9	222.4k	222.4k	0.0	22.1
dem_100_belg_orig_349	266.1k	265.9k	0.1	3.3	266.1k	266.1k	0.0	70.3	266.1k	265.8k	0.1	0.8
dem_100_belg_orig_350	213.4k	213.4k	0.0	0.3	213.4k	213.4k	0.0	22.5	213.4k	213.4k	0.0	0.6
dem_100_belg_orig_351	213.8k	213.8k	0.0	0.2	213.8k	213.8k	0.0	17.3	213.8k	213.8k	0.0	0.5
dem_100_belg_orig_352	164.9k	164.7k	0.1	0.1	164.9k	164.9k	0.0	9.5	164.9k	164.9k	0.0	0.5
dem_100_belg_orig_353	146.8k	146.8k	0.0	0.1	146.8k	146.8k	0.0	11.9	146.8k	146.7k	0.1	0.3
dem_100_belg_orig_354	144.2k	144.2k	0.0	0.2	144.2k	144.1k	0.1	12.6	144.2k	144.2k	0.0	0.4
dem_100_belg_orig_355	213.8k	213.8k	0.0	4.8	213.8k	213.8k	0.0	37.9	213.8k	213.8k	0.0	4.2
dem_100_belg_orig_356	286.8k	286.8k	0.0	14.0	286.8k	286.6k	0.1	76.2	286.8k	286.8k	0.0	9.4
dem_100_belg_orig_357	341.2k	341.2k	0.0	21.6	341.2k	341.2k	0.0	98.2	341.2k	341.2k	0.0	52.2
dem_100_belg_orig_358	206.8k	206.8k	0.0	78.4	206.8k	206.8k	0.0	96.5	206.8k	206.8k	0.0	26.8
dem_100_belg_orig_359	241.0k	240.9k	0.1	2.2	241.0k	241.0k	0.0	41.1	241.0k	241.0k	0.0	12.1
dem_100_belg_orig_360	354.8k	354.6k	0.1	9.1	354.8k	354.8k	0.0	53.2	354.8k	354.8k	0.0	125.6
dem_100_belg_orig_361	165.7k	165.7k	0.0	0.3	165.7k	165.7k	0.0	9.5	165.7k	165.7k	0.0	2.8
dem_100_belg_orig_362	238.7k	238.7k	0.0	14.8	238.7k	238.7k	0.0	143.3	238.7k	238.7k	0.0	54.8
dem_100_belg_orig_363	223.7k	223.5k	0.1	0.4	223.7k	223.7k	0.0	17.0	223.7k	223.6k	0.1	1.1
dem_100_belg_orig_364	296.9k	296.9k	0.0	14.8	296.9k	296.6k	0.1	52.2	296.9k	296.9k	0.0	28.5
dem_100_belg_orig_365	193.9k	193.9k	0.0	2.6	193.9k	193.9k	0.0	18.8	193.9k	193.9k	0.0	2.7
dem_100_belg_orig_366	175.2k	175.2k	0.0	1.4	175.2k	175.2k	0.0	30.0	175.2k	175.2k	0.0	34.1
dem_100_belg_orig_367	108.0k	108.0k	0.0	0.2	108.0k	108.0k	0.0	7.8	108.0k	108.0k	0.0	0.5
dem_100_belg_orig_368	234.7k	234.6k	0.0	0.3	234.7k	234.7k	0.0	20.5	234.7k	234.6k	0.0	0.5
dem_100_belg_orig_369	305.2k	305.0k	0.1	10.5	305.0k	305.0k	0.0	38.0	305.0k	305.0k	0.0	67.6
dem_100_belg_orig_370	459.5k	459.5k	0.0	205.3	459.5k	459.5k	0.0	1138.9	459.5k	459.5k	0.0	4975.4
dem_100_belg_orig_371	243.8k	243.8k	0.0	0.7	243.8k	243.8k	0.0	17.5	243.8k	243.7k	0.1	0.5
dem_100_belg_orig_372	244.3k	244.1k	0.1	0.4	244.2k	244.2k	0.0	17.9	244.3k	244.1k	0.1	0.6
dem_100_belg_orig_373	243.2k	243.2k	0.0	146.1	243.2k	243.2k	0.0	104.2	243.2k	243.2k	0.0	410.6
dem_100_belg_orig_374	164.4k	164.2k	0.1	0.3	164.4k	164.4k	0.0	11.6	164.4k	164.3k	0.0	0.5
dem_100_belg_orig_375	307.3k	307.3k	0.0	21.3	307.3k	307.2k	0.0	62.2	307.3k	307.3k	0.0	101.4
dem_100_belg_orig_376	314.1k	314.1k	0.0	344.4	314.1k	314.1k	0.0	75.2	314.1k	314.1k	0.0	6078.2
dem_100_belg_orig_377	191.9k	191.9k	0.0	0.3	191.9k	191.9k	0.0	32.4	191.9k	191.9k	0.0	0.5
dem_100_belg_orig_378	237.8k	237.8k	0.0	7.2	237.8k	237.7k	0.1	65.7	237.8k	237.8k	0.0	44.1
dem_100_belg_orig_379	201.1k	200.9k	0.1	0.4	201.1k	201.1k	0.0	21.0	201.1k	201.0k	0.1	0.8
dem_100_belg_orig_380	148.3k	148.3k	0.0	0.2	148.3k	148.3k	0.0	10.0	148.3k	148.3k	0.0	0.3
dem_100_belg_orig_381	224.8k	224.6k	0.1	0.3	224.8k	224.8k	0.0	19.5	224.8k	224.6k	0.1	0.6
dem_100_belg_orig_382	259.7k	259.6k	0.1	0.1	259.7k	259.7k	0.0	17.4	259.7k	259.5k	0.1	0.5

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Table A.1: Comparison of discrete models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_383	203.3k	203.1k	0.1	0.3	203.3k	203.3k	0.0	11.5	203.3k	203.1k	0.1	0.3
dem_100_belg_orig_384	203.4k	203.4k	0.0	0.3	203.4k	203.4k	0.0	21.1	203.4k	203.3k	0.0	0.5
dem_100_belg_orig_385	327.6k	327.5k	0.0	27.2	327.6k	327.3k	0.1	87.6	327.6k	327.6k	0.0	333.1
dem_100_belg_orig_386	139.3k	139.3k	0.0	0.3	139.3k	139.3k	0.0	10.8	139.3k	139.3k	0.0	0.9
dem_100_belg_orig_387	270.6k	270.6k	0.0	38.9	270.6k	270.6k	0.0	511.5	270.6k	270.6k	0.0	570.1
dem_100_belg_orig_388	189.5k	189.5k	0.0	0.2	189.5k	189.3k	0.1	11.5	189.5k	189.3k	0.1	0.5
dem_100_belg_orig_389	432.8k	432.6k	0.0	54.2	432.8k	432.8k	0.0	118.2	432.8k	432.8k	0.0	462.3
dem_100_belg_orig_390	286.7k	286.7k	0.0	85.9	286.7k	286.7k	0.0	47.2	286.7k	286.7k	0.0	520.3
dem_100_belg_orig_391	386.8k	386.8k	0.0	36.4	386.8k	386.8k	0.0	229.8	386.8k	386.5k	0.1	415.3
dem_100_belg_orig_392	267.8k	267.8k	0.0	14.3	267.8k	267.8k	0.0	91.2	267.8k	267.8k	0.0	50.4
dem_100_belg_orig_393	339.4k	339.4k	0.0	0.3	339.4k	339.4k	0.0	55.1	339.4k	339.1k	0.1	0.4
dem_100_belg_orig_394	202.6k	202.4k	0.1	0.2	202.6k	202.4k	0.1	14.7	202.6k	202.5k	0.0	0.6
dem_100_belg_orig_395	336.0k	336.0k	0.0	35.8	336.0k	336.0k	0.0	54.2	336.0k	336.0k	0.0	205.4
dem_100_belg_orig_396	201.4k	201.4k	0.0	5.1	201.4k	201.4k	0.0	48.5	201.4k	201.4k	0.0	9.8
dem_100_belg_orig_397	351.6k	351.6k	0.0	1.5	351.6k	351.6k	0.0	56.4	351.6k	351.3k	0.1	6.9
dem_100_belg_orig_398	76.8k	76.8k	0.0	6.0	76.8k	76.8k	0.0	11.9	76.8k	76.8k	0.0	2.6
dem_100_belg_orig_399	203.4k	203.3k	0.0	0.5	203.4k	203.4k	0.0	16.4	203.4k	203.4k	0.0	1.0
dem_100_belg_orig_400	238.4k	238.4k	0.0	0.5	238.4k	238.4k	0.0	15.8	238.4k	238.4k	0.0	0.7
dem_100_belg_orig_401	145.3k	145.3k	0.0	0.3	145.3k	145.3k	0.0	9.4	145.3k	145.3k	0.0	0.7
dem_100_belg_orig_402	369.4k	369.4k	0.0	71.0	369.4k	369.1k	0.1	55.6	369.4k	369.1k	0.1	51.9
dem_100_belg_orig_403	210.4k	210.3k	0.1	1.0	210.4k	210.4k	0.0	49.8	210.4k	210.4k	0.0	2.7
dem_100_belg_orig_404	159.0k	159.0k	0.0	0.3	159.0k	159.0k	0.0	24.6	159.0k	158.9k	0.0	0.5
dem_100_belg_orig_405	239.6k	239.6k	0.0	5.8	239.6k	239.6k	0.0	62.8	239.6k	239.6k	0.0	13.1
dem_100_belg_orig_406	226.9k	226.9k	0.0	13.4	226.9k	226.9k	0.0	82.7	226.9k	226.9k	0.0	14.2
dem_100_belg_orig_407	185.0k	185.0k	0.0	4.1	185.0k	185.0k	0.0	14.8	185.0k	185.0k	0.0	7.2
dem_100_belg_orig_408	207.7k	207.7k	0.0	18.7	207.7k	207.6k	0.0	78.2	207.7k	207.7k	0.0	14.5
dem_100_belg_orig_409	158.6k	158.5k	0.1	0.2	158.6k	158.6k	0.0	6.5	158.6k	158.5k	0.1	0.3
dem_100_belg_orig_410	147.4k	147.3k	0.1	0.2	147.4k	147.4k	0.0	9.2	147.4k	147.4k	0.0	0.4
dem_100_belg_orig_411	439.5k	439.4k	0.0	1082.7	439.5k	439.4k	0.0	75.6	439.5k	439.5k	0.0	9281.1
dem_100_belg_orig_412	248.3k	248.1k	0.1	0.8	248.3k	248.3k	0.0	18.9	248.3k	248.3k	0.0	0.6
dem_100_belg_orig_413	170.5k	170.5k	0.0	5.6	170.5k	170.5k	0.0	102.6	170.5k	170.5k	0.0	11.1
dem_100_belg_orig_414	231.5k	231.5k	0.0	1.9	231.5k	231.5k	0.0	35.0	231.5k	231.5k	0.0	6.9
dem_100_belg_orig_415	266.4k	266.2k	0.1	0.1	266.4k	266.4k	0.0	24.0	266.4k	266.1k	0.1	0.4
dem_100_belg_orig_416	310.1k	309.9k	0.1	9.1	310.1k	310.1k	0.0	33.6	310.1k	310.1k	0.0	10.1
dem_100_belg_orig_417	86.1k	86.1k	0.0	0.2	86.1k	86.1k	0.0	5.3	86.1k	86.1k	0.0	0.4
dem_100_belg_orig_418	237.5k	237.3k	0.1	0.3	237.5k	237.5k	0.0	32.2	237.5k	237.3k	0.1	0.4
dem_100_belg_orig_419	160.9k	160.9k	0.0	0.3	160.9k	160.9k	0.0	11.8	160.9k	160.9k	0.0	0.6
dem_100_belg_orig_420	251.9k	251.6k	0.1	1.5	251.9k	251.6k	0.1	23.0	251.9k	251.9k	0.0	0.9
dem_100_belg_orig_421	345.6k	345.6k	0.0	2.1	345.6k	345.6k	0.0	77.8	345.6k	345.4k	0.0	8.3
dem_100_belg_orig_422	320.4k	320.4k	0.0	19.9	320.4k	320.4k	0.0	62.7	320.4k	320.4k	0.0	104.2
dem_100_belg_orig_423	426.4k	426.4k	0.0	27.3	426.4k	426.4k	0.0	115.9	426.4k	426.4k	0.0	66.5
dem_100_belg_orig_424	401.1k	401.1k	0.0	152.5	401.1k	401.1k	0.0	1041.4	401.1k	401.1k	0.0	1654.0
dem_100_belg_orig_425	160.9k	160.8k	0.0	0.2	160.9k	160.8k	0.0	21.5	160.9k	160.8k	0.0	0.3
dem_100_belg_orig_426	329.9k	329.9k	0.0	166.7	329.9k	329.9k	0.0	47.3	329.9k	329.9k	0.0	987.9
dem_100_belg_orig_427	99.8k	99.8k	0.0	0.1	99.8k	99.8k	0.0	5.8	99.8k	99.8k	0.0	0.3
dem_100_belg_orig_428	476.5k	476.5k	0.0	172.8	476.5k	476.5k	0.0	161.3	476.5k	476.5k	0.0	283.5
dem_100_belg_orig_429	411.6k	411.6k	0.0	34.6	411.6k	411.2k	0.1	52.9	411.6k	411.6k	0.0	216.5
dem_100_belg_orig_430	109.0k	109.0k	0.0	0.2	109.0k	109.0k	0.0	6.8	109.0k	109.0k	0.0	0.4
dem_100_belg_orig_431	255.8k	255.7k	0.1	0.3	255.8k	255.6k	0.1	22.6	255.8k	255.8k	0.0	0.6
dem_100_belg_orig_432	148.9k	148.9k	0.0	0.4	148.9k	148.9k	0.0	18.1	148.9k	148.9k	0.0	2.4
dem_100_belg_orig_433	395.5k	395.5k	0.0	19.3	395.5k	395.5k	0.0	44.4	395.5k	395.2k	0.1	20.7
dem_100_belg_orig_434	379.8k	379.8k	0.0	21.2	379.8k	379.8k	0.0	148.4	379.8k	379.8k	0.0	78.2
dem_100_belg_orig_435	184.3k	184.1k	0.1	0.4	184.3k	184.3k	0.0	15.3	184.3k	184.3k	0.0	6.3
dem_100_belg_orig_436	145.3k	145.3k	0.0	5.8	145.3k	145.3k	0.0	24.5	145.3k	145.3k	0.0	5.1
dem_100_belg_orig_437	209.7k	209.6k	0.1	4.9	209.7k	209.6k	0.0	51.3	209.7k	209.7k	0.0	17.3
dem_100_belg_orig_438	165.7k	165.7k	0.0	0.3	165.7k	165.7k	0.0	15.8	165.7k	165.6k	0.1	0.7
dem_100_belg_orig_439	183.5k	183.5k	0.0	7.4	183.5k	183.5k	0.0	21.4	183.5k	183.5k	0.0	7.1
dem_100_belg_orig_440	120.4k	120.4k	0.0	4.1	120.4k	120.4k	0.0	47.8	120.4k	120.4k	0.0	13.3
dem_100_belg_orig_441	188.8k	188.8k	0.0	0.3	188.8k	188.8k	0.0	11.5	188.8k	188.8k	0.0	0.5
dem_100_belg_orig_442	160.1k	160.1k	0.0	0.3	160.1k	160.1k	0.0	14.5	160.1k	160.0k	0.1	1.1
dem_100_belg_orig_443	209.4k	209.2k	0.1	0.7	209.4k	209.4k	0.0	18.5	209.4k	209.4k	0.0	1.1
dem_100_belg_orig_444	317.6k	317.2k	0.1	0.4	317.6k	317.6k	0.0	26.4	317.6k	317.4k	0.1	0.5
dem_100_belg_orig_445	184.9k	184.9k	0.0	0.5	184.9k	184.7k	0.1	41.1	184.9k	184.9k	0.0	0.9
dem_100_belg_orig_446	191.9k	191.9k	0.0	0.2	191.9k	191.9k	0.0	17.6	191.9k	191.9k	0.0	0.3
dem_100_belg_orig_447	388.4k	388.4k	0.0	259.1	388.4k	388.4k	0.0	294.8	388.4k	388.4k	0.0	5528.1
dem_100_belg_orig_448	221.3k	221.3k	0.0	0.6	221.3k	221.3k	0.0	20.1	221.3k	221.2k	0.0	0.6
dem_100_belg_orig_449	311.3k	311.0k	0.1	0.6	311.3k	311.3k	0.0	26.1	311.3k	311.0k	0.1	0.9
dem_100_belg_orig_450	119.7k	119.7k	0.0	12.8	119.7k	119.7k	0.0	61.3	119.7k	119.7k	0.0	62.6
dem_100_belg_orig_451	257.7k	257.4k	0.1	0.3	257.7k	257.7k	0.0	37.0	257.7k	257.7k	0.0	0.7
dem_100_belg_orig_452	345.8k	345.8k	0.0	1.4	345.8k	345.8k	0.0	50.2	345.8k	345.8k	0.0	13.3
dem_100_belg_orig_453	209.8k	209.8k	0.0	3.2	209.8k	209.8k	0.0	410.5	209.8k	209.8k	0.0	106.2
dem_100_belg_orig_454	301.0k	301.0k	0.0	1.3	301.0k	301.0k	0.0	87.6	301.0k	301.0k	0.0	14.2
dem_100_belg_orig_455	316.6k	316.6k	0.0	0.6	316.6k	316.6k	0.0	52.1	316.6k	316.6k	0.0	13.4
dem_100_belg_orig_456	180.7k	180.7k	0.0	0.3	180.7k	180.7k	0.0	20.4	180.7k	180.6k	0.1	0.7
dem_100_belg_orig_457	417.5k	417.5k	0.0	24.6	417.5k	417.5k	0.0	54.0	417.5k	417.5k	0.0	149.0
dem_100_belg_orig_458	166.4k	166.4k	0.0	0.3	166.4k	166.4k	0.0	13.7	166.4k	166.4k	0.0	0.9
dem_100_belg_orig_459	508.6k	508.6k	0.0	0.5	508.6k	508.6k	0.0	142.6	508.6k	508.6k	0.0	1.3
dem_100_belg_orig_460	187.2k	187.2k	0.0	0.3	187.2k	187.2k	0.0	13.1	187.2k	187.2k	0.0	0.6

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Table A.1: Comparison of discrete models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_461	122.1k	122.1k	0.0	0.2	122.1k	122.1k	0.0	5.2	122.1k	122.1k	0.0	0.3
dem_100_belg_orig_462	251.5k	251.5k	0.0	9.0	251.5k	251.3k	0.1	34.1	251.5k	251.5k	0.0	10.8
dem_100_belg_orig_463	515.9k	515.9k	0.0	0.7	515.9k	515.9k	0.0	87.0	515.9k	515.9k	0.0	3.9
dem_100_belg_orig_464	207.6k	207.6k	0.0	4.1	207.6k	207.6k	0.0	37.3	207.6k	207.6k	0.0	31.1
dem_100_belg_orig_465	169.6k	169.6k	0.0	0.2	169.6k	169.6k	0.0	10.5	169.6k	169.5k	0.1	0.4
dem_100_belg_orig_466	189.5k	189.4k	0.1	0.2	189.5k	189.5k	0.0	12.0	189.5k	189.5k	0.0	0.3
dem_100_belg_orig_467	158.2k	158.2k	0.0	0.1	158.2k	158.1k	0.0	19.2	158.2k	158.0k	0.1	0.2
dem_100_belg_orig_468	170.3k	170.3k	0.0	0.3	170.3k	170.3k	0.0	25.5	170.4k	170.3k	0.0	0.7
dem_100_belg_orig_469	164.1k	164.1k	0.0	0.2	164.1k	164.1k	0.0	20.5	164.1k	164.1k	0.0	0.5
dem_100_belg_orig_470	402.0k	402.0k	0.0	3.6	402.0k	402.0k	0.0	57.1	402.0k	402.0k	0.0	7.9
dem_100_belg_orig_471	221.0k	221.0k	0.0	0.2	221.0k	221.0k	0.0	13.9	221.0k	220.8k	0.1	0.6
dem_100_belg_orig_472	146.4k	146.4k	0.0	0.3	146.4k	146.4k	0.0	9.1	146.4k	146.4k	0.0	0.8
dem_100_belg_orig_473	309.2k	309.2k	0.0	3.9	309.2k	309.2k	0.0	51.8	309.2k	309.2k	0.0	1.6
dem_100_belg_orig_474	214.1k	214.0k	0.0	0.5	214.1k	214.1k	0.0	26.3	214.1k	214.1k	0.0	0.8
dem_100_belg_orig_475	203.7k	203.5k	0.1	0.6	203.7k	203.7k	0.0	26.5	203.7k	203.7k	0.0	1.2
dem_100_belg_orig_476	296.5k	296.5k	0.0	0.5	296.5k	296.5k	0.0	41.0	296.5k	296.5k	0.0	10.6
dem_100_belg_orig_477	209.1k	209.1k	0.0	13.9	209.1k	209.1k	0.0	101.6	209.1k	209.1k	0.0	25.1
dem_100_belg_orig_478	412.3k	411.9k	0.1	91.0	412.3k	412.3k	0.0	342.2	412.3k	412.3k	0.0	1730.3
dem_100_belg_orig_479	318.2k	318.2k	0.0	216.6	318.2k	318.2k	0.0	88.3	318.2k	318.2k	0.0	4153.2
dem_100_belg_orig_480	255.0k	254.8k	0.1	0.3	255.0k	255.0k	0.0	25.5	255.0k	254.8k	0.1	0.9
dem_100_belg_orig_481	291.1k	290.9k	0.1	5.9	291.1k	291.1k	0.0	125.3	291.1k	291.1k	0.0	66.0
dem_100_belg_orig_482	301.8k	301.8k	0.0	131.7	301.8k	301.8k	0.0	44.5	301.8k	301.6k	0.1	749.2
dem_100_belg_orig_483	176.5k	176.3k	0.1	0.3	176.3k	176.3k	0.0	13.2	176.3k	176.3k	0.0	9.8
dem_100_belg_orig_484	213.2k	213.2k	0.0	7.9	213.2k	213.2k	0.0	56.6	213.2k	213.2k	0.0	29.6
dem_100_belg_orig_485	199.9k	199.7k	0.1	0.2	199.9k	199.9k	0.0	15.5	199.9k	199.7k	0.1	0.5
dem_100_belg_orig_486	285.8k	285.6k	0.1	0.4	285.8k	285.8k	0.0	34.3	285.8k	285.6k	0.1	0.7
dem_100_belg_orig_487	188.6k	188.6k	0.0	0.6	188.6k	188.6k	0.0	23.6	188.6k	188.4k	0.1	0.5
dem_100_belg_orig_488	261.2k	261.2k	0.0	11.2	261.2k	261.2k	0.0	47.9	261.2k	261.2k	0.0	47.2
dem_100_belg_orig_489	176.5k	176.4k	0.1	0.3	176.5k	176.5k	0.0	11.5	176.5k	176.5k	0.0	0.6
dem_100_belg_orig_490	195.9k	195.9k	0.0	0.5	195.9k	195.8k	0.0	18.0	195.9k	195.8k	0.1	1.1
dem_100_belg_orig_491	216.6k	216.4k	0.1	0.5	216.6k	216.6k	0.0	16.3	216.6k	216.6k	0.0	0.8
dem_100_belg_orig_492	243.1k	243.1k	0.0	0.4	243.1k	243.1k	0.0	27.2	243.1k	242.9k	0.1	0.5
dem_100_belg_orig_493	405.5k	405.5k	0.0	185.5	405.5k	405.5k	0.0	259.8	405.5k	405.5k	0.0	4411.4
dem_100_belg_orig_494	241.3k	241.3k	0.0	54.8	241.3k	241.3k	0.0	1035.5	241.3k	241.3k	0.0	487.1
dem_100_belg_orig_495	298.5k	298.5k	0.0	2.7	298.5k	298.5k	0.0	50.7	298.5k	298.5k	0.0	4.3
dem_100_belg_orig_496	503.7k	503.7k	0.0	0.3	503.7k	503.7k	0.0	61.9	503.7k	503.2k	0.1	0.7
dem_100_belg_orig_497	430.8k	430.6k	0.1	33.5	430.8k	430.8k	0.0	81.6	430.8k	430.8k	0.0	27.5
dem_100_belg_orig_498	237.4k	237.1k	0.1	0.3	237.4k	237.4k	0.0	21.6	237.4k	237.1k	0.1	0.7
dem_100_belg_orig_499	255.3k	255.3k	0.0	0.4	255.3k	255.3k	0.0	25.5	255.3k	255.2k	0.0	0.8
dem_100_belg_orig_500	165.4k	165.4k	0.0	0.2	165.4k	165.4k	0.0	10.3	165.4k	165.4k	0.0	0.4

Table A.2: Detailed results of the split-pipe models on *Belgium* with $\mathcal{B} = 100$ as summarized in Figure 3.13a and Table 3.4a. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_1	217.7k	217.7k	0.0	1.4	217.7k	217.7k	0.0	0.5
dem_100_belg_orig_2	184.1k	184.1k	0.0	0.3	184.1k	184.1k	0.0	0.5
dem_100_belg_orig_3	197.3k	197.3k	0.0	0.5	197.3k	197.3k	0.0	0.2
dem_100_belg_orig_4	456.4k	455.9k	0.1	0.9	456.4k	456.4k	0.0	0.8
dem_100_belg_orig_5	178.0k	178.0k	0.0	0.5	178.0k	178.0k	0.0	0.5
dem_100_belg_orig_6	241.2k	241.1k	0.0	2.8	241.1k	241.1k	0.0	0.5
dem_100_belg_orig_7	524.4k	524.2k	0.0	2.0	524.4k	524.2k	0.0	0.4
dem_100_belg_orig_8	246.9k	246.7k	0.1	0.8	246.9k	246.7k	0.1	0.8
dem_100_belg_orig_9	460.8k	460.8k	0.0	0.8	460.8k	460.7k	0.0	2.4
dem_100_belg_orig_10	129.7k	129.6k	0.1	0.8	129.7k	129.6k	0.1	0.4
dem_100_belg_orig_11	158.4k	158.3k	0.1	1.5	158.4k	158.4k	0.0	2.6
dem_100_belg_orig_12	291.4k	291.4k	0.0	3.3	291.4k	291.1k	0.1	1.0
dem_100_belg_orig_13	186.7k	186.7k	0.0	0.6	186.7k	186.7k	0.0	0.5
dem_100_belg_orig_14	193.1k	193.1k	0.0	0.6	193.1k	193.1k	0.0	0.2
dem_100_belg_orig_15	334.9k	334.8k	0.0	1.4	334.9k	334.9k	0.0	0.8
dem_100_belg_orig_16	244.8k	244.6k	0.1	1.0	244.8k	244.6k	0.1	0.6
dem_100_belg_orig_17	274.8k	274.6k	0.1	1.1	274.8k	274.8k	0.0	0.2
dem_100_belg_orig_18	334.0k	334.0k	0.0	0.6	334.0k	334.0k	0.0	0.3
dem_100_belg_orig_19	183.9k	183.9k	0.0	0.9	183.9k	183.9k	0.0	0.5
dem_100_belg_orig_20	286.1k	286.1k	0.0	0.9	286.1k	286.1k	0.0	0.2
dem_100_belg_orig_21	285.5k	285.3k	0.1	0.8	285.5k	285.3k	0.1	0.4
dem_100_belg_orig_22	193.7k	193.7k	0.0	0.7	193.7k	193.7k	0.0	0.6
dem_100_belg_orig_23	182.0k	181.9k	0.1	0.9	182.0k	182.0k	0.0	0.6

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Table A.2: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_24	185.3k	185.3k	0.0	0.4	185.3k	185.3k	0.0	0.4
dem_100_belg_orig_25	334.9k	334.7k	0.1	0.8	334.9k	334.7k	0.1	0.7
dem_100_belg_orig_26	197.2k	197.1k	0.1	0.4	197.2k	197.2k	0.0	0.5
dem_100_belg_orig_27	323.5k	323.5k	0.0	0.3	323.5k	323.5k	0.0	0.3
dem_100_belg_orig_28	426.0k	425.6k	0.1	1.5	426.0k	425.6k	0.1	2.0
dem_100_belg_orig_29	192.2k	192.2k	0.0	0.7	192.2k	192.2k	0.0	0.4
dem_100_belg_orig_30	338.3k	338.0k	0.1	0.8	338.3k	338.2k	0.0	0.4
dem_100_belg_orig_31	127.5k	127.4k	0.0	1.1	127.5k	127.3k	0.1	0.5
dem_100_belg_orig_32	111.0k	111.0k	0.0	0.9	111.0k	111.0k	0.0	0.8
dem_100_belg_orig_33	201.9k	201.9k	0.0	0.6	201.9k	201.9k	0.0	0.8
dem_100_belg_orig_34	123.2k	123.1k	0.1	0.8	123.2k	123.2k	0.0	0.8
dem_100_belg_orig_35	363.5k	363.4k	0.0	1.2	363.5k	363.5k	0.0	0.7
dem_100_belg_orig_36	155.2k	155.2k	0.0	0.6	155.2k	155.2k	0.0	0.5
dem_100_belg_orig_37	147.7k	147.7k	0.0	1.0	147.7k	147.7k	0.0	0.4
dem_100_belg_orig_38	256.5k	256.5k	0.0	0.9	256.5k	256.5k	0.0	0.5
dem_100_belg_orig_39	175.4k	175.4k	0.0	1.1	175.4k	175.4k	0.0	0.3
dem_100_belg_orig_40	222.0k	221.9k	0.1	0.6	222.0k	221.9k	0.1	0.4
dem_100_belg_orig_41	146.4k	146.4k	0.0	3.3	146.4k	146.4k	0.0	0.5
dem_100_belg_orig_42	164.0k	164.0k	0.0	0.4	164.0k	164.0k	0.0	0.5
dem_100_belg_orig_43	162.5k	162.4k	0.0	2.8	162.4k	162.4k	0.0	0.2
dem_100_belg_orig_44	389.7k	389.4k	0.1	0.7	389.7k	389.4k	0.1	1.1
dem_100_belg_orig_45	192.7k	192.7k	0.0	0.7	192.7k	192.7k	0.0	0.3
dem_100_belg_orig_46	180.5k	180.5k	0.0	2.3	180.5k	180.5k	0.0	0.2
dem_100_belg_orig_47	122.3k	122.3k	0.0	0.3	122.3k	122.3k	0.0	0.5
dem_100_belg_orig_48	209.6k	209.6k	0.0	1.2	209.6k	209.6k	0.0	0.2
dem_100_belg_orig_49	174.3k	174.3k	0.0	0.6	174.3k	174.3k	0.0	0.4
dem_100_belg_orig_50	112.1k	112.1k	0.0	1.3	112.1k	112.1k	0.0	0.8
dem_100_belg_orig_51	182.5k	182.5k	0.0	0.4	182.5k	182.5k	0.0	0.3
dem_100_belg_orig_52	150.4k	150.3k	0.1	2.2	150.3k	150.3k	0.0	0.5
dem_100_belg_orig_53	94.8k	94.8k	0.0	0.3	94.8k	94.8k	0.0	0.3
dem_100_belg_orig_54	263.0k	263.0k	0.0	0.6	263.0k	263.0k	0.0	0.4
dem_100_belg_orig_55	131.7k	131.6k	0.1	0.9	131.7k	131.6k	0.1	0.3
dem_100_belg_orig_56	133.3k	133.3k	0.0	0.6	133.3k	133.3k	0.0	0.4
dem_100_belg_orig_57	207.8k	207.8k	0.0	0.3	207.8k	207.8k	0.0	0.4
dem_100_belg_orig_58	325.4k	325.4k	0.0	0.8	325.4k	325.4k	0.0	0.3
dem_100_belg_orig_59	197.3k	197.3k	0.0	0.5	197.3k	197.3k	0.0	0.3
dem_100_belg_orig_60	321.8k	321.8k	0.0	1.1	321.8k	321.8k	0.0	0.4
dem_100_belg_orig_61	216.0k	216.0k	0.0	1.0	216.0k	216.0k	0.0	0.6
dem_100_belg_orig_62	150.7k	150.6k	0.1	0.5	150.7k	150.6k	0.1	0.3
dem_100_belg_orig_63	267.8k	267.6k	0.0	0.8	267.8k	267.6k	0.0	0.5
dem_100_belg_orig_64	148.7k	148.7k	0.0	0.5	148.7k	148.7k	0.0	0.5
dem_100_belg_orig_65	208.4k	208.4k	0.0	0.3	208.4k	208.4k	0.0	0.6
dem_100_belg_orig_66	160.4k	160.3k	0.1	2.0	160.4k	160.3k	0.1	0.8
dem_100_belg_orig_67	203.4k	203.4k	0.0	1.0	203.4k	203.4k	0.0	0.2
dem_100_belg_orig_68	185.2k	185.2k	0.0	0.4	185.2k	185.1k	0.0	0.5
dem_100_belg_orig_69	352.2k	352.1k	0.0	0.9	352.2k	352.1k	0.0	0.5
dem_100_belg_orig_70	304.5k	304.5k	0.0	0.8	304.5k	304.5k	0.0	0.6
dem_100_belg_orig_71	186.0k	185.9k	0.0	0.5	186.0k	186.0k	0.0	0.6
dem_100_belg_orig_72	164.5k	164.5k	0.0	1.4	164.5k	164.5k	0.0	0.3
dem_100_belg_orig_73	34.6k	34.6k	0.0	0.5	34.6k	34.6k	0.0	0.2
dem_100_belg_orig_74	338.1k	337.8k	0.1	0.8	338.1k	338.0k	0.0	0.7
dem_100_belg_orig_75	492.5k	492.5k	0.0	1.2	492.5k	492.3k	0.0	0.5
dem_100_belg_orig_76	327.3k	327.0k	0.1	0.8	327.3k	327.1k	0.1	0.5
dem_100_belg_orig_77	392.2k	391.9k	0.1	0.6	392.2k	391.9k	0.1	1.7
dem_100_belg_orig_78	159.6k	159.6k	0.0	0.5	159.6k	159.6k	0.0	0.6
dem_100_belg_orig_79	186.3k	186.3k	0.0	0.5	186.3k	186.3k	0.0	0.3
dem_100_belg_orig_80	149.2k	149.2k	0.0	0.3	149.2k	149.2k	0.0	0.3
dem_100_belg_orig_81	254.1k	254.1k	0.0	0.5	254.1k	253.9k	0.1	0.5
dem_100_belg_orig_82	260.2k	260.1k	0.0	1.2	260.2k	260.2k	0.0	0.8
dem_100_belg_orig_83	255.2k	255.2k	0.0	1.0	255.2k	255.2k	0.0	0.3
dem_100_belg_orig_84	359.3k	359.0k	0.1	0.5	359.3k	359.0k	0.1	0.4
dem_100_belg_orig_85	260.3k	260.1k	0.1	1.0	260.3k	260.1k	0.1	0.9
dem_100_belg_orig_86	230.0k	230.0k	0.0	1.5	230.0k	230.0k	0.0	0.6
dem_100_belg_orig_87	566.6k	566.4k	0.0	0.8	566.6k	566.5k	0.0	0.9
dem_100_belg_orig_88	46.2k	46.2k	0.0	1.6	46.2k	46.1k	0.1	0.5
dem_100_belg_orig_89	85.3k	85.3k	0.1	2.3	85.3k	85.3k	0.0	0.4
dem_100_belg_orig_90	262.5k	262.4k	0.1	0.4	262.5k	262.4k	0.1	0.2
dem_100_belg_orig_91	196.4k	196.4k	0.0	0.4	196.4k	196.4k	0.0	0.2
dem_100_belg_orig_92	197.7k	197.7k	0.0	1.0	197.7k	197.7k	0.0	0.3
dem_100_belg_orig_93	177.9k	177.9k	0.0	0.7	177.9k	177.9k	0.0	0.7
dem_100_belg_orig_94	530.6k	530.5k	0.0	0.9	530.6k	530.5k	0.0	0.8
dem_100_belg_orig_95	376.1k	375.8k	0.1	4.1	376.1k	376.1k	0.0	0.6
dem_100_belg_orig_96	341.5k	341.5k	0.0	1.4	341.5k	341.5k	0.0	0.2
dem_100_belg_orig_97	144.5k	144.5k	0.0	1.0	144.5k	144.5k	0.0	0.4
dem_100_belg_orig_98	281.1k	281.1k	0.0	1.1	281.1k	281.1k	0.0	0.2
dem_100_belg_orig_99	256.2k	255.9k	0.1	1.2	256.2k	255.9k	0.1	0.5
dem_100_belg_orig_100	300.7k	300.5k	0.1	1.7	300.7k	300.7k	0.0	0.8
dem_100_belg_orig_101	435.5k	435.3k	0.1	1.6	435.5k	435.3k	0.1	0.9

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Table A.2: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_102	338.8k	338.5k	0.1	0.5	338.8k	338.5k	0.1	0.8
dem_100_belg_orig_103	410.4k	410.2k	0.0	0.7	410.4k	410.2k	0.0	0.7
dem_100_belg_orig_104	406.1k	405.8k	0.1	0.6	406.1k	406.0k	0.0	0.9
dem_100_belg_orig_105	112.1k	112.1k	0.0	1.1	112.1k	112.1k	0.0	0.4
dem_100_belg_orig_106	340.6k	340.3k	0.1	1.5	340.6k	340.4k	0.0	0.9
dem_100_belg_orig_107	189.1k	189.1k	0.0	0.5	189.1k	189.1k	0.0	0.6
dem_100_belg_orig_108	312.1k	311.9k	0.1	0.8	312.1k	311.9k	0.1	0.8
dem_100_belg_orig_109	252.5k	252.5k	0.0	0.5	252.5k	252.5k	0.0	0.4
dem_100_belg_orig_110	181.9k	181.9k	0.0	0.5	181.9k	181.9k	0.0	0.8
dem_100_belg_orig_111	168.9k	168.9k	0.0	0.7	168.9k	168.9k	0.0	0.5
dem_100_belg_orig_112	94.4k	94.4k	0.0	0.8	94.4k	94.4k	0.0	0.5
dem_100_belg_orig_113	182.2k	182.1k	0.1	1.0	182.2k	182.1k	0.1	0.7
dem_100_belg_orig_114	274.9k	274.7k	0.1	0.8	274.9k	274.8k	0.0	0.7
dem_100_belg_orig_115	50.4k	50.3k	0.1	0.5	50.4k	50.4k	0.0	0.6
dem_100_belg_orig_116	334.5k	334.2k	0.1	1.9	334.5k	334.2k	0.1	0.5
dem_100_belg_orig_117	191.0k	190.8k	0.1	0.7	191.0k	190.8k	0.1	0.4
dem_100_belg_orig_118	435.0k	434.9k	0.0	0.8	435.0k	435.0k	0.0	0.8
dem_100_belg_orig_119	208.9k	208.8k	0.1	1.0	208.9k	208.8k	0.1	0.9
dem_100_belg_orig_120	231.0k	230.9k	0.0	0.7	231.0k	231.0k	0.0	0.8
dem_100_belg_orig_121	210.8k	210.8k	0.0	0.8	210.8k	210.8k	0.0	0.3
dem_100_belg_orig_122	116.6k	116.6k	0.0	0.9	116.6k	116.6k	0.0	0.6
dem_100_belg_orig_123	244.1k	244.1k	0.0	7.5	244.1k	243.9k	0.1	7.0
dem_100_belg_orig_124	185.6k	185.6k	0.0	0.3	185.6k	185.6k	0.0	0.7
dem_100_belg_orig_125	179.3k	179.3k	0.0	0.5	179.3k	179.3k	0.0	0.2
dem_100_belg_orig_126	391.6k	391.3k	0.1	0.9	391.6k	391.5k	0.0	0.8
dem_100_belg_orig_127	227.2k	227.2k	0.0	1.2	227.2k	227.2k	0.0	0.9
dem_100_belg_orig_128	472.2k	471.8k	0.1	0.7	472.2k	471.8k	0.1	0.8
dem_100_belg_orig_129	162.2k	162.1k	0.1	4.1	162.2k	162.0k	0.1	2.2
dem_100_belg_orig_130	346.5k	346.2k	0.1	1.5	346.5k	346.5k	0.0	1.0
dem_100_belg_orig_131	336.4k	336.2k	0.1	0.7	336.4k	336.2k	0.1	1.1
dem_100_belg_orig_132	404.1k	403.8k	0.1	0.5	404.1k	403.8k	0.1	0.8
dem_100_belg_orig_133	298.9k	298.7k	0.1	0.8	298.9k	298.7k	0.1	0.7
dem_100_belg_orig_134	518.0k	517.7k	0.1	1.0	518.0k	517.9k	0.0	0.5
dem_100_belg_orig_135	169.1k	169.1k	0.0	1.3	169.1k	169.1k	0.0	0.7
dem_100_belg_orig_136	322.9k	322.6k	0.1	1.1	322.9k	322.6k	0.1	0.9
dem_100_belg_orig_137	330.3k	330.2k	0.0	1.8	330.3k	330.0k	0.1	0.5
dem_100_belg_orig_138	171.6k	171.5k	0.1	0.4	171.6k	171.5k	0.1	0.4
dem_100_belg_orig_139	196.5k	196.5k	0.0	0.5	196.5k	196.5k	0.0	0.3
dem_100_belg_orig_140	150.5k	150.3k	0.1	0.8	150.5k	150.4k	0.1	0.5
dem_100_belg_orig_141	317.1k	317.1k	0.0	0.6	317.1k	317.1k	0.0	0.7
dem_100_belg_orig_142	182.2k	182.0k	0.1	0.8	182.2k	182.0k	0.1	0.7
dem_100_belg_orig_143	495.2k	494.8k	0.1	1.0	495.2k	494.8k	0.1	0.4
dem_100_belg_orig_144	508.9k	508.9k	0.0	0.7	508.9k	508.9k	0.0	0.9
dem_100_belg_orig_145	418.6k	418.6k	0.0	3.2	418.6k	418.5k	0.0	0.5
dem_100_belg_orig_146	266.6k	266.5k	0.0	0.8	266.6k	266.5k	0.0	1.0
dem_100_belg_orig_147	218.7k	218.6k	0.0	0.5	218.7k	218.6k	0.0	0.6
dem_100_belg_orig_148	230.3k	230.3k	0.0	1.3	230.3k	230.3k	0.0	0.4
dem_100_belg_orig_149	386.8k	386.8k	0.0	0.5	386.8k	386.8k	0.0	0.4
dem_100_belg_orig_150	344.9k	344.9k	0.0	0.6	344.9k	344.9k	0.0	0.6
dem_100_belg_orig_151	169.0k	169.0k	0.0	0.6	169.0k	169.0k	0.0	0.6
dem_100_belg_orig_152	149.8k	149.8k	0.0	0.7	149.8k	149.8k	0.0	0.2
dem_100_belg_orig_153	382.6k	382.5k	0.0	0.6	382.6k	382.6k	0.0	0.9
dem_100_belg_orig_154	253.0k	253.0k	0.0	0.3	253.0k	253.0k	0.0	0.3
dem_100_belg_orig_155	158.6k	158.6k	0.0	0.6	158.6k	158.6k	0.0	0.3
dem_100_belg_orig_156	203.0k	203.0k	0.0	0.7	203.0k	203.0k	0.0	0.5
dem_100_belg_orig_157	389.3k	388.9k	0.1	1.0	389.3k	389.3k	0.0	1.1
dem_100_belg_orig_158	312.5k	312.5k	0.0	0.8	312.5k	312.2k	0.1	0.4
dem_100_belg_orig_159	242.1k	242.1k	0.0	1.2	242.1k	242.1k	0.0	0.3
dem_100_belg_orig_160	208.0k	208.0k	0.0	0.6	208.0k	208.0k	0.0	0.6
dem_100_belg_orig_161	598.1k	598.1k	0.0	0.7	598.1k	598.1k	0.0	0.4
dem_100_belg_orig_162	258.2k	258.2k	0.0	0.6	258.2k	258.2k	0.0	0.7
dem_100_belg_orig_163	498.2k	498.2k	0.0	1.2	498.2k	498.2k	0.0	0.3
dem_100_belg_orig_164	275.8k	275.8k	0.0	1.7	275.8k	275.6k	0.1	1.0
dem_100_belg_orig_165	171.2k	171.1k	0.1	0.8	171.2k	171.0k	0.1	0.6
dem_100_belg_orig_166	228.2k	228.2k	0.0	1.4	228.2k	228.2k	0.0	0.9
dem_100_belg_orig_167	175.3k	175.2k	0.0	1.5	175.2k	175.2k	0.0	0.6
dem_100_belg_orig_168	269.5k	269.4k	0.0	1.9	269.5k	269.3k	0.1	0.6
dem_100_belg_orig_169	274.5k	274.4k	0.1	0.7	274.5k	274.4k	0.1	0.5
dem_100_belg_orig_170	386.2k	386.2k	0.0	0.8	386.2k	386.2k	0.0	0.8
dem_100_belg_orig_171	286.1k	286.1k	0.0	0.8	286.1k	286.1k	0.0	0.3
dem_100_belg_orig_172	148.1k	148.1k	0.0	1.0	148.1k	148.1k	0.0	0.7
dem_100_belg_orig_173	305.7k	305.7k	0.0	0.4	305.7k	305.7k	0.0	0.3
dem_100_belg_orig_174	89.7k	89.7k	0.0	0.7	89.7k	89.7k	0.0	0.7
dem_100_belg_orig_175	175.8k	175.7k	0.0	1.0	175.8k	175.8k	0.0	0.8
dem_100_belg_orig_176	431.7k	431.7k	0.0	0.6	431.7k	431.7k	0.0	0.5
dem_100_belg_orig_177	269.4k	269.4k	0.0	1.4	269.4k	269.1k	0.1	0.8
dem_100_belg_orig_178	313.4k	313.2k	0.0	0.7	313.4k	313.2k	0.0	0.4
dem_100_belg_orig_179	225.6k	225.5k	0.0	1.6	225.5k	225.3k	0.1	0.3

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Table A.2: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_180	225.6k	225.6k	0.0	0.6	225.6k	225.6k	0.0	0.3
dem_100_belg_orig_181	195.7k	195.6k	0.0	0.8	195.7k	195.7k	0.0	0.4
dem_100_belg_orig_182	545.1k	544.6k	0.1	1.7	544.8k	544.5k	0.0	0.7
dem_100_belg_orig_183	338.0k	337.7k	0.1	1.5	337.7k	337.7k	0.0	0.3
dem_100_belg_orig_184	97.1k	97.1k	0.0	0.5	97.1k	97.1k	0.0	0.8
dem_100_belg_orig_185	284.2k	284.1k	0.0	1.4	284.1k	284.1k	0.0	0.5
dem_100_belg_orig_186	118.0k	118.0k	0.0	0.6	118.0k	118.0k	0.0	0.4
dem_100_belg_orig_187	113.4k	113.4k	0.0	0.8	113.4k	113.4k	0.0	0.7
dem_100_belg_orig_188	424.2k	424.1k	0.0	0.9	424.2k	423.8k	0.1	0.6
dem_100_belg_orig_189	182.6k	182.6k	0.0	2.1	182.6k	182.6k	0.0	0.2
dem_100_belg_orig_190	241.1k	241.1k	0.0	0.8	241.1k	241.1k	0.0	0.9
dem_100_belg_orig_191	329.7k	329.4k	0.1	0.5	329.7k	329.5k	0.1	0.4
dem_100_belg_orig_192	201.5k	201.5k	0.0	0.4	201.5k	201.5k	0.0	0.4
dem_100_belg_orig_193	180.4k	180.4k	0.0	0.8	180.4k	180.4k	0.0	0.3
dem_100_belg_orig_194	214.9k	214.8k	0.0	1.1	214.9k	214.8k	0.0	0.4
dem_100_belg_orig_195	341.1k	340.8k	0.1	1.5	341.1k	340.8k	0.1	0.7
dem_100_belg_orig_196	167.2k	167.1k	0.1	0.5	167.2k	167.2k	0.0	0.4
dem_100_belg_orig_197	349.6k	349.4k	0.1	1.0	349.6k	349.4k	0.1	0.8
dem_100_belg_orig_198	206.0k	205.8k	0.1	0.8	206.0k	205.8k	0.1	0.5
dem_100_belg_orig_199	194.3k	194.3k	0.0	1.0	194.3k	194.3k	0.0	0.6
dem_100_belg_orig_200	227.4k	227.3k	0.1	1.2	227.4k	227.3k	0.1	0.5
dem_100_belg_orig_201	375.8k	375.7k	0.0	0.7	375.8k	375.7k	0.0	1.1
dem_100_belg_orig_202	360.4k	360.1k	0.1	1.0	360.4k	360.3k	0.0	0.4
dem_100_belg_orig_203	529.0k	529.0k	0.0	1.2	529.0k	528.8k	0.0	0.9
dem_100_belg_orig_204	329.6k	329.3k	0.1	0.7	329.6k	329.3k	0.1	0.8
dem_100_belg_orig_205	169.0k	168.9k	0.1	0.9	169.0k	168.9k	0.1	0.7
dem_100_belg_orig_206	252.6k	252.5k	0.1	0.4	252.6k	252.5k	0.0	0.6
dem_100_belg_orig_207	343.8k	343.6k	0.0	0.9	343.8k	343.7k	0.0	0.9
dem_100_belg_orig_208	284.6k	284.4k	0.1	1.0	284.6k	284.4k	0.1	0.7
dem_100_belg_orig_209	366.8k	366.4k	0.1	0.9	366.8k	366.4k	0.1	0.4
dem_100_belg_orig_210	221.0k	220.8k	0.1	1.3	221.0k	221.0k	0.0	1.0
dem_100_belg_orig_211	91.1k	91.0k	0.1	0.5	91.1k	91.0k	0.1	0.4
dem_100_belg_orig_212	322.1k	322.1k	0.0	1.0	322.1k	321.8k	0.1	0.9
dem_100_belg_orig_213	158.5k	158.5k	0.0	0.5	158.5k	158.5k	0.0	0.3
dem_100_belg_orig_214	198.4k	198.3k	0.1	0.5	198.4k	198.3k	0.1	0.9
dem_100_belg_orig_215	166.6k	166.6k	0.0	0.3	166.6k	166.6k	0.0	0.2
dem_100_belg_orig_216	300.2k	300.1k	0.0	0.8	300.2k	300.1k	0.0	0.8
dem_100_belg_orig_217	165.6k	165.6k	0.0	0.6	165.6k	165.6k	0.0	0.5
dem_100_belg_orig_218	237.0k	237.0k	0.0	0.7	237.0k	237.0k	0.0	0.6
dem_100_belg_orig_219	330.6k	330.3k	0.1	0.9	330.6k	330.4k	0.1	2.4
dem_100_belg_orig_220	148.7k	148.7k	0.0	1.1	148.7k	148.6k	0.1	1.0
dem_100_belg_orig_221	372.2k	371.9k	0.1	2.8	372.2k	371.9k	0.1	0.5
dem_100_belg_orig_222	249.1k	249.1k	0.0	1.1	249.1k	249.1k	0.0	0.5
dem_100_belg_orig_223	226.1k	226.1k	0.0	1.0	226.1k	226.1k	0.0	0.2
dem_100_belg_orig_224	222.4k	222.4k	0.0	0.8	222.4k	222.4k	0.0	0.3
dem_100_belg_orig_225	416.9k	416.5k	0.1	2.5	416.9k	416.5k	0.1	24.5
dem_100_belg_orig_226	535.1k	534.8k	0.1	1.1	535.1k	534.8k	0.1	0.7
dem_100_belg_orig_227	398.5k	398.4k	0.0	0.7	398.5k	398.5k	0.0	0.8
dem_100_belg_orig_228	210.0k	209.9k	0.0	2.2	209.9k	209.9k	0.0	0.5
dem_100_belg_orig_229	186.9k	186.9k	0.0	0.5	186.9k	186.9k	0.0	0.6
dem_100_belg_orig_230	212.2k	212.2k	0.0	1.6	212.2k	212.2k	0.0	0.3
dem_100_belg_orig_231	182.3k	182.3k	0.0	0.6	182.3k	182.3k	0.0	0.3
dem_100_belg_orig_232	150.9k	150.9k	0.0	0.5	150.9k	150.9k	0.0	0.4
dem_100_belg_orig_233	169.0k	169.0k	0.0	1.6	169.0k	169.0k	0.0	0.3
dem_100_belg_orig_234	192.1k	192.0k	0.0	0.8	192.1k	192.1k	0.0	0.7
dem_100_belg_orig_235	215.2k	215.2k	0.0	1.1	215.2k	215.1k	0.1	0.5
dem_100_belg_orig_236	393.0k	392.7k	0.1	1.0	393.0k	392.8k	0.1	1.0
dem_100_belg_orig_237	260.7k	260.6k	0.1	0.5	260.7k	260.6k	0.1	0.5
dem_100_belg_orig_238	223.6k	223.6k	0.0	0.5	223.6k	223.6k	0.0	0.2
dem_100_belg_orig_239	170.8k	170.8k	0.0	0.6	170.8k	170.8k	0.0	0.6
dem_100_belg_orig_240	249.7k	249.7k	0.0	0.9	249.7k	249.7k	0.0	0.7
dem_100_belg_orig_241	297.5k	297.2k	0.1	1.1	297.5k	297.3k	0.0	0.8
dem_100_belg_orig_242	203.4k	203.4k	0.0	0.9	203.4k	203.4k	0.0	0.2
dem_100_belg_orig_243	203.6k	203.4k	0.1	0.6	203.6k	203.4k	0.1	0.6
dem_100_belg_orig_244	465.7k	465.6k	0.0	1.4	465.7k	465.6k	0.0	0.6
dem_100_belg_orig_245	391.8k	391.8k	0.0	1.4	391.8k	391.5k	0.1	0.5
dem_100_belg_orig_246	228.6k	228.6k	0.0	0.8	228.6k	228.6k	0.0	0.5
dem_100_belg_orig_247	365.7k	365.7k	0.0	1.0	365.7k	365.7k	0.0	0.7
dem_100_belg_orig_248	345.8k	345.5k	0.1	3.8	345.7k	345.6k	0.0	0.4
dem_100_belg_orig_249	352.2k	352.1k	0.0	0.5	352.2k	352.1k	0.0	0.8
dem_100_belg_orig_250	249.0k	248.9k	0.0	1.1	249.0k	248.9k	0.0	0.7
dem_100_belg_orig_251	326.4k	326.2k	0.1	0.8	326.4k	326.2k	0.1	0.7
dem_100_belg_orig_252	171.4k	171.4k	0.0	0.4	171.4k	171.4k	0.0	0.8
dem_100_belg_orig_253	206.1k	206.1k	0.0	0.6	206.1k	206.1k	0.0	0.3
dem_100_belg_orig_254	271.4k	271.2k	0.1	0.5	271.4k	271.2k	0.1	0.6
dem_100_belg_orig_255	290.7k	290.7k	0.0	0.6	290.7k	290.7k	0.0	0.2
dem_100_belg_orig_256	245.8k	245.8k	0.0	0.6	245.8k	245.8k	0.0	0.4
dem_100_belg_orig_257	264.9k	264.9k	0.0	0.6	264.9k	264.9k	0.0	0.3

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Table A.2: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_258	314.7k	314.4k	0.1	0.6	314.7k	314.5k	0.1	0.5
dem_100_belg_orig_259	286.3k	286.1k	0.0	0.8	286.3k	286.1k	0.1	0.5
dem_100_belg_orig_260	169.3k	169.3k	0.0	0.7	169.3k	169.3k	0.0	0.2
dem_100_belg_orig_261	166.2k	166.2k	0.0	0.5	166.2k	166.2k	0.0	0.7
dem_100_belg_orig_262	209.7k	209.7k	0.0	2.2	209.7k	209.7k	0.0	0.4
dem_100_belg_orig_263	269.2k	268.9k	0.1	0.5	269.2k	269.0k	0.1	0.7
dem_100_belg_orig_264	295.6k	295.4k	0.1	0.8	295.6k	295.6k	0.0	0.7
dem_100_belg_orig_265	387.3k	387.1k	0.0	2.0	387.3k	387.1k	0.0	1.0
dem_100_belg_orig_266	452.1k	452.1k	0.0	0.7	452.1k	452.1k	0.0	0.9
dem_100_belg_orig_267	262.3k	262.1k	0.1	1.6	262.3k	262.1k	0.1	0.8
dem_100_belg_orig_268	173.6k	173.6k	0.0	0.2	173.6k	173.6k	0.0	0.4
dem_100_belg_orig_269	182.0k	182.0k	0.0	0.3	182.0k	182.0k	0.0	0.3
dem_100_belg_orig_270	156.8k	156.8k	0.0	1.2	156.8k	156.8k	0.0	0.3
dem_100_belg_orig_271	199.2k	199.0k	0.1	1.2	199.2k	199.0k	0.1	0.6
dem_100_belg_orig_272	103.5k	103.5k	0.0	0.4	103.5k	103.5k	0.0	0.4
dem_100_belg_orig_273	161.4k	161.3k	0.0	0.4	161.4k	161.3k	0.0	0.3
dem_100_belg_orig_274	316.1k	316.1k	0.0	0.3	316.1k	316.1k	0.0	0.3
dem_100_belg_orig_275	173.5k	173.5k	0.0	0.7	173.5k	173.5k	0.0	0.2
dem_100_belg_orig_276	291.9k	291.9k	0.0	0.7	291.9k	291.9k	0.0	0.6
dem_100_belg_orig_277	207.2k	207.2k	0.0	0.4	207.2k	207.2k	0.0	0.3
dem_100_belg_orig_278	228.4k	228.2k	0.1	1.1	228.4k	228.2k	0.1	0.4
dem_100_belg_orig_279	249.0k	249.0k	0.0	1.0	249.0k	249.0k	0.0	0.5
dem_100_belg_orig_280	255.6k	255.4k	0.1	0.4	255.6k	255.4k	0.1	0.4
dem_100_belg_orig_281	387.3k	387.0k	0.1	0.6	387.3k	387.0k	0.1	0.9
dem_100_belg_orig_282	261.8k	261.8k	0.0	0.8	261.8k	261.8k	0.0	0.6
dem_100_belg_orig_283	335.3k	335.1k	0.0	0.7	335.3k	335.2k	0.0	0.9
dem_100_belg_orig_284	316.2k	315.9k	0.1	1.6	316.2k	315.9k	0.1	0.8
dem_100_belg_orig_285	366.3k	366.0k	0.1	0.7	366.3k	366.0k	0.1	0.5
dem_100_belg_orig_286	404.7k	404.6k	0.0	0.9	404.7k	404.6k	0.0	0.4
dem_100_belg_orig_287	220.9k	220.9k	0.0	0.4	220.9k	220.9k	0.0	0.3
dem_100_belg_orig_288	281.3k	281.1k	0.1	0.9	281.3k	281.1k	0.1	1.0
dem_100_belg_orig_289	198.7k	198.7k	0.0	0.3	198.7k	198.7k	0.0	0.4
dem_100_belg_orig_290	164.2k	164.2k	0.0	1.2	164.2k	164.2k	0.0	0.6
dem_100_belg_orig_291	296.1k	295.8k	0.1	1.2	296.1k	295.9k	0.1	13.2
dem_100_belg_orig_292	331.9k	331.8k	0.0	1.2	331.9k	331.8k	0.0	0.4
dem_100_belg_orig_293	268.2k	268.2k	0.0	0.9	268.2k	268.2k	0.0	0.4
dem_100_belg_orig_294	213.4k	213.4k	0.0	0.3	213.4k	213.4k	0.0	0.3
dem_100_belg_orig_295	140.9k	140.9k	0.0	1.9	140.9k	140.9k	0.0	0.7
dem_100_belg_orig_296	237.0k	236.9k	0.0	1.5	237.0k	236.8k	0.1	0.5
dem_100_belg_orig_297	234.0k	233.8k	0.1	0.5	234.0k	234.0k	0.0	0.8
dem_100_belg_orig_298	211.1k	211.1k	0.0	1.1	211.1k	211.1k	0.0	0.2
dem_100_belg_orig_299	182.4k	182.4k	0.0	0.7	182.4k	182.4k	0.0	0.7
dem_100_belg_orig_300	418.4k	418.0k	0.1	2.0	418.4k	418.4k	0.0	1.1
dem_100_belg_orig_301	233.6k	233.6k	0.0	1.9	233.6k	233.6k	0.0	0.5
dem_100_belg_orig_302	376.7k	376.7k	0.0	0.4	376.7k	376.6k	0.0	1.0
dem_100_belg_orig_303	287.8k	287.8k	0.0	0.3	287.8k	287.8k	0.0	0.5
dem_100_belg_orig_304	158.2k	158.2k	0.0	0.3	158.2k	158.2k	0.0	0.5
dem_100_belg_orig_305	385.7k	385.3k	0.1	0.7	385.7k	385.5k	0.0	0.5
dem_100_belg_orig_306	215.8k	215.8k	0.0	0.6	215.8k	215.8k	0.0	0.3
dem_100_belg_orig_307	124.1k	124.1k	0.0	0.6	124.1k	124.1k	0.0	0.5
dem_100_belg_orig_308	158.0k	157.9k	0.1	1.4	158.0k	158.0k	0.0	0.7
dem_100_belg_orig_309	160.4k	160.4k	0.0	0.8	160.4k	160.4k	0.0	0.5
dem_100_belg_orig_310	146.2k	146.2k	0.0	0.3	146.2k	146.2k	0.0	0.3
dem_100_belg_orig_311	244.0k	243.9k	0.1	0.6	244.0k	243.8k	0.1	0.9
dem_100_belg_orig_312	199.6k	199.6k	0.0	0.8	199.6k	199.6k	0.0	0.5
dem_100_belg_orig_313	422.0k	421.7k	0.1	2.1	422.0k	421.7k	0.1	0.9
dem_100_belg_orig_314	355.7k	355.6k	0.0	2.6	355.7k	355.6k	0.0	1.2
dem_100_belg_orig_315	178.0k	178.0k	0.0	0.3	178.0k	178.0k	0.0	0.3
dem_100_belg_orig_316	193.9k	193.8k	0.0	1.1	193.9k	193.9k	0.0	0.6
dem_100_belg_orig_317	304.0k	303.8k	0.1	0.8	304.0k	303.8k	0.1	0.5
dem_100_belg_orig_318	383.5k	383.5k	0.0	1.9	383.5k	383.2k	0.1	1.2
dem_100_belg_orig_319	293.0k	292.9k	0.0	0.7	293.0k	292.7k	0.1	0.8
dem_100_belg_orig_320	172.9k	172.8k	0.0	2.7	172.9k	172.8k	0.0	0.8
dem_100_belg_orig_321	224.8k	224.8k	0.0	0.7	224.8k	224.8k	0.0	0.5
dem_100_belg_orig_322	280.9k	280.9k	0.0	1.1	280.9k	280.9k	0.0	0.4
dem_100_belg_orig_323	154.9k	154.9k	0.0	0.8	154.9k	154.9k	0.0	0.5
dem_100_belg_orig_324	174.4k	174.4k	0.0	0.3	174.4k	174.4k	0.0	0.3
dem_100_belg_orig_325	254.5k	254.5k	0.0	1.1	254.5k	254.5k	0.0	0.3
dem_100_belg_orig_326	121.5k	121.5k	0.0	1.2	121.5k	121.5k	0.0	0.3
dem_100_belg_orig_327	284.7k	284.7k	0.0	0.7	284.7k	284.7k	0.0	0.3
dem_100_belg_orig_328	213.4k	213.4k	0.0	1.1	213.4k	213.4k	0.0	0.6
dem_100_belg_orig_329	187.2k	187.2k	0.0	0.5	187.2k	187.2k	0.0	0.9
dem_100_belg_orig_330	425.1k	424.8k	0.1	1.0	425.1k	424.8k	0.1	0.5
dem_100_belg_orig_331	217.7k	217.5k	0.1	1.8	217.5k	217.5k	0.0	0.3
dem_100_belg_orig_332	209.3k	209.3k	0.0	0.7	209.3k	209.3k	0.0	0.4
dem_100_belg_orig_333	115.2k	115.1k	0.1	1.4	115.2k	115.1k	0.1	0.7
dem_100_belg_orig_334	166.8k	166.8k	0.0	0.7	166.8k	166.8k	0.0	0.5
dem_100_belg_orig_335	278.9k	278.7k	0.1	0.4	278.9k	278.7k	0.1	0.6

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Table A.2: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_336	308.8k	308.8k	0.0	0.4	308.8k	308.8k	0.0	0.2
dem_100_belg_orig_337	168.7k	168.6k	0.1	1.1	168.7k	168.7k	0.0	0.7
dem_100_belg_orig_338	252.2k	252.2k	0.0	0.4	252.2k	252.2k	0.0	0.2
dem_100_belg_orig_339	403.1k	402.9k	0.0	2.7	403.1k	403.0k	0.0	1.0
dem_100_belg_orig_340	387.4k	387.4k	0.0	1.7	387.4k	387.3k	0.0	0.5
dem_100_belg_orig_341	318.2k	318.2k	0.0	0.3	318.2k	318.2k	0.0	0.6
dem_100_belg_orig_342	154.3k	154.2k	0.1	0.7	154.3k	154.2k	0.1	0.6
dem_100_belg_orig_343	193.3k	193.3k	0.0	1.8	193.3k	193.3k	0.0	0.7
dem_100_belg_orig_344	161.4k	161.2k	0.1	0.8	161.4k	161.2k	0.1	0.3
dem_100_belg_orig_345	182.2k	182.2k	0.0	0.7	182.2k	182.2k	0.0	0.2
dem_100_belg_orig_346	223.0k	223.0k	0.0	0.7	223.0k	223.0k	0.0	0.6
dem_100_belg_orig_347	353.8k	353.6k	0.1	0.5	353.8k	353.8k	0.0	0.5
dem_100_belg_orig_348	202.6k	202.4k	0.1	0.9	202.6k	202.4k	0.1	0.5
dem_100_belg_orig_349	258.0k	257.8k	0.1	1.0	258.0k	257.8k	0.1	0.5
dem_100_belg_orig_350	212.3k	212.3k	0.0	0.6	212.3k	212.3k	0.0	0.4
dem_100_belg_orig_351	213.4k	213.4k	0.0	0.5	213.4k	213.4k	0.0	0.4
dem_100_belg_orig_352	164.3k	164.3k	0.0	1.4	164.3k	164.3k	0.0	0.2
dem_100_belg_orig_353	145.8k	145.8k	0.0	0.4	145.8k	145.8k	0.0	0.4
dem_100_belg_orig_354	143.7k	143.7k	0.0	0.5	143.7k	143.7k	0.0	1.0
dem_100_belg_orig_355	208.3k	208.1k	0.1	0.7	208.3k	208.1k	0.1	0.6
dem_100_belg_orig_356	271.4k	271.1k	0.1	1.7	271.4k	271.1k	0.1	0.8
dem_100_belg_orig_357	336.8k	336.5k	0.1	0.5	336.8k	336.5k	0.1	0.4
dem_100_belg_orig_358	204.0k	204.0k	0.0	1.5	204.0k	204.0k	0.0	2.0
dem_100_belg_orig_359	236.6k	236.5k	0.0	1.1	236.6k	236.5k	0.0	0.4
dem_100_belg_orig_360	351.6k	351.4k	0.1	0.5	351.6k	351.4k	0.1	0.7
dem_100_belg_orig_361	165.3k	165.3k	0.0	0.7	165.3k	165.3k	0.0	0.2
dem_100_belg_orig_362	235.0k	235.0k	0.0	0.9	235.0k	234.8k	0.1	0.9
dem_100_belg_orig_363	222.6k	222.6k	0.0	1.4	222.6k	222.6k	0.0	0.3
dem_100_belg_orig_364	292.2k	292.2k	0.0	2.8	292.2k	292.1k	0.0	0.9
dem_100_belg_orig_365	192.5k	192.4k	0.1	0.9	192.5k	192.4k	0.1	0.8
dem_100_belg_orig_366	162.2k	162.1k	0.0	1.7	162.2k	162.1k	0.0	0.4
dem_100_belg_orig_367	106.7k	106.6k	0.1	0.5	106.7k	106.6k	0.1	0.5
dem_100_belg_orig_368	234.2k	234.2k	0.0	0.8	234.2k	234.2k	0.0	0.4
dem_100_belg_orig_369	302.9k	302.7k	0.1	0.5	302.9k	302.8k	0.0	0.4
dem_100_belg_orig_370	454.0k	453.9k	0.0	0.8	454.0k	454.0k	0.0	1.2
dem_100_belg_orig_371	242.5k	242.5k	0.0	0.4	242.5k	242.5k	0.0	0.3
dem_100_belg_orig_372	242.7k	242.7k	0.0	0.7	242.7k	242.7k	0.0	0.3
dem_100_belg_orig_373	239.1k	238.9k	0.1	1.0	239.1k	239.0k	0.1	0.6
dem_100_belg_orig_374	163.1k	163.1k	0.0	1.1	163.1k	163.1k	0.0	0.4
dem_100_belg_orig_375	293.9k	293.7k	0.1	0.9	293.9k	293.7k	0.0	0.8
dem_100_belg_orig_376	302.4k	302.1k	0.1	0.8	302.4k	302.3k	0.0	1.1
dem_100_belg_orig_377	191.1k	191.1k	0.0	0.7	191.1k	191.1k	0.0	0.7
dem_100_belg_orig_378	221.4k	221.2k	0.1	0.8	221.4k	221.2k	0.1	0.4
dem_100_belg_orig_379	195.6k	195.6k	0.0	1.1	195.6k	195.6k	0.0	0.2
dem_100_belg_orig_380	147.0k	147.0k	0.0	0.7	147.0k	147.0k	0.0	0.4
dem_100_belg_orig_381	223.3k	223.3k	0.0	0.8	223.3k	223.3k	0.0	0.3
dem_100_belg_orig_382	259.1k	259.1k	0.0	0.4	259.1k	259.1k	0.0	0.4
dem_100_belg_orig_383	202.4k	202.4k	0.0	1.1	202.4k	202.4k	0.0	0.3
dem_100_belg_orig_384	201.1k	201.1k	0.0	1.2	201.1k	201.1k	0.0	0.3
dem_100_belg_orig_385	317.2k	316.9k	0.1	0.8	317.2k	317.2k	0.0	1.3
dem_100_belg_orig_386	138.0k	137.9k	0.0	2.9	137.9k	137.8k	0.1	0.6
dem_100_belg_orig_387	254.4k	254.4k	0.0	0.9	254.4k	254.3k	0.1	1.3
dem_100_belg_orig_388	189.2k	189.2k	0.0	0.6	189.2k	189.2k	0.0	0.2
dem_100_belg_orig_389	426.1k	425.9k	0.0	1.2	426.1k	425.9k	0.0	1.7
dem_100_belg_orig_390	278.5k	278.5k	0.0	0.6	278.5k	278.5k	0.0	0.5
dem_100_belg_orig_391	381.2k	381.2k	0.0	0.5	381.2k	380.9k	0.1	0.5
dem_100_belg_orig_392	248.9k	248.8k	0.1	0.8	248.9k	248.8k	0.1	0.7
dem_100_belg_orig_393	338.8k	338.8k	0.0	0.7	338.8k	338.8k	0.0	0.7
dem_100_belg_orig_394	200.3k	200.3k	0.0	0.7	200.3k	200.3k	0.0	0.3
dem_100_belg_orig_395	333.1k	332.8k	0.1	1.8	333.1k	332.8k	0.1	0.5
dem_100_belg_orig_396	188.5k	188.4k	0.1	0.7	188.5k	188.4k	0.1	0.5
dem_100_belg_orig_397	350.8k	350.6k	0.0	0.5	350.8k	350.5k	0.1	0.8
dem_100_belg_orig_398	71.7k	71.7k	0.0	0.8	71.7k	71.7k	0.0	0.6
dem_100_belg_orig_399	200.6k	200.5k	0.1	0.6	200.6k	200.5k	0.1	0.6
dem_100_belg_orig_400	232.7k	232.7k	0.0	0.5	232.7k	232.7k	0.0	0.2
dem_100_belg_orig_401	144.6k	144.5k	0.1	0.3	144.6k	144.5k	0.1	0.4
dem_100_belg_orig_402	367.8k	367.5k	0.1	0.6	367.8k	367.5k	0.1	0.8
dem_100_belg_orig_403	210.0k	210.0k	0.0	0.8	210.0k	210.0k	0.0	0.4
dem_100_belg_orig_404	157.7k	157.7k	0.0	1.1	157.7k	157.7k	0.0	0.3
dem_100_belg_orig_405	227.3k	227.3k	0.0	1.5	227.3k	227.3k	0.0	0.2
dem_100_belg_orig_406	222.6k	222.6k	0.0	1.1	222.6k	222.6k	0.0	1.4
dem_100_belg_orig_407	184.5k	184.5k	0.0	1.8	184.5k	184.5k	0.0	0.7
dem_100_belg_orig_408	199.4k	199.4k	0.0	0.9	199.4k	199.3k	0.1	0.9
dem_100_belg_orig_409	157.3k	157.3k	0.0	0.7	157.3k	157.3k	0.0	0.4
dem_100_belg_orig_410	146.8k	146.8k	0.0	0.6	146.8k	146.8k	0.0	0.5
dem_100_belg_orig_411	433.9k	433.8k	0.0	0.7	433.9k	433.6k	0.1	0.6
dem_100_belg_orig_412	247.7k	247.7k	0.0	0.6	247.7k	247.7k	0.0	0.3
dem_100_belg_orig_413	156.0k	155.8k	0.1	1.5	156.0k	155.8k	0.1	0.5

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Table A.2: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_414	230.4k	230.4k	0.0	1.9	230.4k	230.4k	0.0	1.6
dem_100_belg_orig_415	265.9k	265.9k	0.0	1.5	265.9k	265.9k	0.0	0.3
dem_100_belg_orig_416	309.5k	309.5k	0.0	0.5	309.5k	309.5k	0.0	0.4
dem_100_belg_orig_417	82.9k	82.9k	0.0	0.7	82.9k	82.9k	0.0	0.2
dem_100_belg_orig_418	235.0k	235.0k	0.0	0.9	235.0k	235.0k	0.0	0.2
dem_100_belg_orig_419	158.5k	158.5k	0.0	0.6	158.5k	158.5k	0.0	0.6
dem_100_belg_orig_420	249.3k	249.1k	0.1	2.8	249.3k	249.1k	0.1	0.5
dem_100_belg_orig_421	342.9k	342.6k	0.1	0.5	342.9k	342.7k	0.0	0.5
dem_100_belg_orig_422	317.0k	316.9k	0.0	0.6	317.0k	317.0k	0.0	0.5
dem_100_belg_orig_423	423.7k	423.7k	0.0	1.2	423.7k	423.4k	0.1	0.4
dem_100_belg_orig_424	392.2k	391.8k	0.1	2.8	392.2k	391.9k	0.1	0.5
dem_100_belg_orig_425	160.8k	160.8k	0.0	0.5	160.8k	160.8k	0.0	0.5
dem_100_belg_orig_426	326.2k	326.1k	0.0	0.6	326.2k	326.1k	0.0	0.8
dem_100_belg_orig_427	85.7k	85.7k	0.0	0.8	85.7k	85.7k	0.0	0.7
dem_100_belg_orig_428	469.7k	469.5k	0.1	2.1	469.7k	469.5k	0.1	0.5
dem_100_belg_orig_429	407.5k	407.4k	0.0	1.8	407.4k	407.3k	0.0	0.7
dem_100_belg_orig_430	108.8k	108.8k	0.0	0.3	108.8k	108.8k	0.0	1.0
dem_100_belg_orig_431	255.1k	255.1k	0.0	0.9	255.1k	255.1k	0.0	0.4
dem_100_belg_orig_432	148.7k	148.7k	0.0	0.9	148.7k	148.7k	0.0	0.4
dem_100_belg_orig_433	392.9k	392.5k	0.1	1.0	392.9k	392.6k	0.1	0.7
dem_100_belg_orig_434	370.6k	370.6k	0.0	0.9	370.6k	370.6k	0.0	0.7
dem_100_belg_orig_435	183.4k	183.3k	0.0	0.5	183.4k	183.3k	0.0	0.4
dem_100_belg_orig_436	140.9k	140.9k	0.0	0.4	140.9k	140.9k	0.0	0.3
dem_100_belg_orig_437	185.9k	185.9k	0.0	1.1	185.9k	185.8k	0.1	0.7
dem_100_belg_orig_438	165.4k	165.4k	0.0	1.1	165.4k	165.4k	0.0	0.6
dem_100_belg_orig_439	180.8k	180.6k	0.1	0.7	180.8k	180.6k	0.1	0.5
dem_100_belg_orig_440	111.1k	111.0k	0.0	1.5	111.1k	111.1k	0.0	0.6
dem_100_belg_orig_441	188.0k	188.0k	0.0	0.7	188.0k	188.0k	0.0	0.6
dem_100_belg_orig_442	155.9k	155.9k	0.0	0.6	155.9k	155.9k	0.0	0.4
dem_100_belg_orig_443	208.8k	208.8k	0.0	0.4	208.8k	208.8k	0.0	0.6
dem_100_belg_orig_444	313.4k	313.4k	0.0	0.5	313.4k	313.4k	0.0	0.4
dem_100_belg_orig_445	181.3k	181.3k	0.0	0.4	181.3k	181.3k	0.0	0.2
dem_100_belg_orig_446	191.1k	191.1k	0.0	0.5	191.1k	191.1k	0.0	0.3
dem_100_belg_orig_447	378.7k	378.7k	0.0	1.5	378.7k	378.7k	0.0	0.6
dem_100_belg_orig_448	220.4k	220.4k	0.0	1.8	220.4k	220.4k	0.0	0.4
dem_100_belg_orig_449	310.2k	310.2k	0.0	0.6	310.2k	310.2k	0.0	0.2
dem_100_belg_orig_450	109.5k	109.4k	0.1	0.8	109.5k	109.4k	0.0	0.5
dem_100_belg_orig_451	256.5k	256.5k	0.0	0.8	256.5k	256.5k	0.0	0.4
dem_100_belg_orig_452	343.8k	343.6k	0.0	0.5	343.8k	343.7k	0.0	0.5
dem_100_belg_orig_453	192.5k	192.5k	0.0	1.0	192.5k	192.5k	0.0	0.5
dem_100_belg_orig_454	297.4k	297.2k	0.1	1.2	297.4k	297.2k	0.1	0.8
dem_100_belg_orig_455	309.6k	309.3k	0.1	0.5	309.6k	309.4k	0.1	0.4
dem_100_belg_orig_456	179.5k	179.5k	0.0	2.0	179.5k	179.5k	0.0	0.4
dem_100_belg_orig_457	415.9k	415.5k	0.1	1.4	415.9k	415.6k	0.1	0.5
dem_100_belg_orig_458	165.7k	165.7k	0.0	0.9	165.7k	165.7k	0.0	0.5
dem_100_belg_orig_459	508.4k	508.4k	0.0	0.4	508.4k	508.4k	0.0	0.2
dem_100_belg_orig_460	186.9k	186.7k	0.1	0.6	186.9k	186.9k	0.0	0.3
dem_100_belg_orig_461	121.5k	121.4k	0.1	1.3	121.4k	121.3k	0.1	0.8
dem_100_belg_orig_462	249.4k	249.2k	0.1	0.5	249.4k	249.2k	0.1	0.8
dem_100_belg_orig_463	514.5k	514.1k	0.1	0.8	514.5k	514.1k	0.1	0.5
dem_100_belg_orig_464	204.9k	204.9k	0.0	1.5	204.9k	204.9k	0.0	0.6
dem_100_belg_orig_465	168.3k	168.3k	0.0	0.3	168.3k	168.3k	0.0	0.2
dem_100_belg_orig_466	189.4k	189.4k	0.0	0.5	189.4k	189.4k	0.0	0.4
dem_100_belg_orig_467	149.8k	149.8k	0.0	0.8	149.8k	149.8k	0.0	0.2
dem_100_belg_orig_468	169.8k	169.6k	0.1	0.5	169.8k	169.8k	0.0	0.8
dem_100_belg_orig_469	163.2k	163.2k	0.0	0.5	163.2k	163.2k	0.0	0.4
dem_100_belg_orig_470	400.9k	400.5k	0.1	0.8	400.9k	400.5k	0.1	0.5
dem_100_belg_orig_471	220.1k	220.1k	0.0	0.2	220.1k	220.1k	0.0	0.2
dem_100_belg_orig_472	146.0k	145.9k	0.1	0.3	146.0k	145.9k	0.1	0.4
dem_100_belg_orig_473	308.0k	307.9k	0.0	1.0	308.0k	307.9k	0.0	0.8
dem_100_belg_orig_474	207.6k	207.5k	0.0	1.6	207.5k	207.5k	0.0	0.4
dem_100_belg_orig_475	201.3k	201.3k	0.0	0.4	201.3k	201.3k	0.0	0.4
dem_100_belg_orig_476	292.7k	292.4k	0.1	0.7	292.7k	292.4k	0.1	0.6
dem_100_belg_orig_477	199.9k	199.9k	0.0	1.6	199.9k	199.8k	0.1	0.6
dem_100_belg_orig_478	406.6k	406.4k	0.1	1.0	406.6k	406.4k	0.1	0.6
dem_100_belg_orig_479	309.9k	309.9k	0.0	0.8	309.9k	309.9k	0.0	0.7
dem_100_belg_orig_480	252.1k	252.1k	0.0	0.7	252.1k	252.1k	0.0	0.7
dem_100_belg_orig_481	284.4k	284.1k	0.1	0.5	284.4k	284.4k	0.0	0.7
dem_100_belg_orig_482	299.1k	299.1k	0.0	1.0	299.1k	299.1k	0.0	0.9
dem_100_belg_orig_483	175.8k	175.6k	0.1	0.6	175.8k	175.6k	0.1	0.4
dem_100_belg_orig_484	207.1k	207.1k	0.0	1.1	207.1k	207.1k	0.0	0.7
dem_100_belg_orig_485	197.6k	197.6k	0.0	1.8	197.6k	197.6k	0.0	0.6
dem_100_belg_orig_486	282.7k	282.4k	0.1	0.4	282.7k	282.5k	0.1	0.5
dem_100_belg_orig_487	186.0k	186.0k	0.0	1.7	186.0k	186.0k	0.0	0.2
dem_100_belg_orig_488	252.3k	252.1k	0.1	1.8	252.3k	252.1k	0.1	0.5
dem_100_belg_orig_489	169.7k	169.7k	0.0	0.8	169.7k	169.7k	0.0	0.5
dem_100_belg_orig_490	194.1k	193.9k	0.1	0.8	194.1k	193.9k	0.1	0.2
dem_100_belg_orig_491	214.2k	214.1k	0.0	0.3	214.2k	214.1k	0.0	0.3

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Table A.2: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_belg_orig_492	242.0k	242.0k	0.0	1.1	242.0k	242.0k	0.0	0.4
dem_100_belg_orig_493	398.3k	398.3k	0.0	0.9	398.3k	398.3k	0.0	0.7
dem_100_belg_orig_494	226.0k	226.0k	0.0	2.4	226.0k	225.8k	0.1	0.9
dem_100_belg_orig_495	295.6k	295.5k	0.0	0.5	295.6k	295.5k	0.0	0.6
dem_100_belg_orig_496	502.4k	502.4k	0.0	0.3	502.4k	502.4k	0.0	0.5
dem_100_belg_orig_497	428.1k	428.1k	0.0	1.3	428.1k	428.1k	0.0	0.5
dem_100_belg_orig_498	235.5k	235.5k	0.0	0.7	235.5k	235.5k	0.0	0.4
dem_100_belg_orig_499	253.2k	253.2k	0.0	0.3	253.2k	253.2k	0.0	0.4
dem_100_belg_orig_500	164.6k	164.6k	0.0	0.7	164.6k	164.6k	0.0	0.6

Table A.3: Detailed results of the discrete models on *Belgium* with $\mathcal{B} = 200$ as summarized in Figure 3.7b and Table 3.3a. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_1	761.4k	761.4k	0.0	6.0	761.4k	761.4k	0.0	65.6	761.4k	761.4k	0.0	41.1
dem_200_belg_orig_2	679.9k	679.3k	0.1	6.9	679.9k	679.6k	0.0	80.0	679.9k	679.9k	0.0	33.8
dem_200_belg_orig_3	519.8k	519.8k	0.0	12.8	519.8k	519.8k	0.0	79.9	519.8k	519.8k	0.0	21.5
dem_200_belg_orig_4	797.9k	797.9k	0.0	16.6	797.9k	797.1k	0.1	62.7	797.9k	797.1k	0.1	9.4
dem_200_belg_orig_5	737.9k	737.9k	0.0	32.7	737.9k	737.9k	0.0	104.5	737.9k	737.9k	0.0	485.8
dem_200_belg_orig_6	333.6k	333.6k	0.0	0.2	333.6k	333.6k	0.0	27.6	333.6k	333.6k	0.0	0.7
dem_200_belg_orig_7	859.6k	859.6k	0.0	939.9	859.6k	859.5k	0.0	1122.8	859.6k	859.6k	0.0	5782.9
dem_200_belg_orig_8	220.9k	220.9k	0.0	1270.5	220.9k	220.9k	0.0	191.0	220.9k	220.9k	0.0	7057.9
dem_200_belg_orig_9	722.1k	722.1k	0.0	1420.0	722.1k	721.4k	0.1	244.3	722.1k	708.2k	2.0	14400.0
dem_200_belg_orig_10	739.8k	739.3k	0.1	47.3	739.8k	739.8k	0.0	74.8	739.8k	739.4k	0.1	320.1
dem_200_belg_orig_11	465.5k	465.5k	0.0	0.9	465.5k	465.5k	0.0	61.0	465.5k	465.1k	0.1	1.0
dem_200_belg_orig_12	354.7k	354.6k	0.0	555.8	354.7k	354.7k	0.0	2505.7	354.7k	354.7k	0.0	1979.8
dem_200_belg_orig_13	629.0k	628.5k	0.1	0.5	629.0k	628.4k	0.1	33.0	629.0k	628.4k	0.1	0.5
dem_200_belg_orig_14	795.2k	794.8k	0.0	0.8	795.2k	794.6k	0.1	42.2	795.2k	794.4k	0.1	0.7
dem_200_belg_orig_15	464.9k	464.9k	0.0	1165.3	464.9k	464.9k	0.0	2622.2	464.9k	458.3k	1.4	14400.0
dem_200_belg_orig_16	840.0k	840.0k	0.0	10018.8	840.0k	839.5k	0.1	597.8	840.0k	779.4k	7.8	14400.0
dem_200_belg_orig_17	433.1k	433.1k	0.0	10.9	433.1k	433.1k	0.0	34.6	433.1k	433.1k	0.0	57.1
dem_200_belg_orig_18	789.2k	789.2k	0.0	292.8	789.2k	788.4k	0.1	2704.4	789.2k	789.2k	0.0	1069.7
dem_200_belg_orig_19	429.6k	429.6k	0.0	21.4	429.6k	429.6k	0.0	471.0	429.6k	429.6k	0.0	53.4
dem_200_belg_orig_20	572.5k	572.5k	0.0	730.1	572.5k	572.5k	0.0	1454.0	572.5k	572.0k	0.1	1798.8
dem_200_belg_orig_21	199.4k	199.4k	0.0	0.3	199.4k	199.4k	0.0	21.4	199.4k	199.4k	0.0	0.5
dem_200_belg_orig_22	568.4k	568.4k	0.0	2882.9	568.4k	568.4k	0.0	1645.6	568.5k	540.6k	5.2	14400.0
dem_200_belg_orig_23	458.0k	458.0k	0.0	60.0	458.0k	457.7k	0.1	91.5	458.0k	458.0k	0.0	51.3
dem_200_belg_orig_24	506.2k	506.1k	0.0	0.2	506.2k	506.2k	0.0	48.4	506.2k	505.7k	0.1	0.6
dem_200_belg_orig_25	514.7k	514.7k	0.0	237.7	514.7k	514.5k	0.0	318.5	514.7k	514.7k	0.0	2551.4
dem_200_belg_orig_26	659.5k	658.9k	0.1	0.3	659.9k	659.3k	0.1	30.4	659.5k	658.9k	0.1	0.5
dem_200_belg_orig_27	364.6k	364.3k	0.1	0.3	364.6k	364.6k	0.0	37.8	364.6k	364.3k	0.1	0.8
dem_200_belg_orig_28	598.7k	598.7k	0.0	18.1	598.7k	598.7k	0.0	216.9	598.7k	598.7k	0.0	20.2
dem_200_belg_orig_29	468.7k	468.6k	0.0	0.3	468.7k	468.5k	0.0	28.8	468.7k	468.7k	0.0	0.7
dem_200_belg_orig_30	514.4k	514.4k	0.0	270.0	514.4k	514.4k	0.0	624.0	514.4k	514.4k	0.0	1381.9
dem_200_belg_orig_31	397.0k	397.0k	0.0	25.7	397.0k	396.6k	0.1	49.4	397.0k	397.0k	0.0	223.0
dem_200_belg_orig_32	652.3k	652.3k	0.0	1.0	652.3k	652.3k	0.0	57.9	652.3k	652.3k	0.0	4.8
dem_200_belg_orig_33	305.3k	305.3k	0.0	514.8	305.3k	305.3k	0.0	380.3	305.3k	305.3k	0.0	183.2
dem_200_belg_orig_34	300.1k	300.1k	0.0	528.6	300.1k	300.0k	0.1	1392.7	300.1k	300.1k	0.0	5279.7
dem_200_belg_orig_35	449.4k	449.0k	0.1	0.6	449.4k	449.4k	0.0	116.7	449.4k	449.2k	0.0	0.4
dem_200_belg_orig_36	741.9k	741.9k	0.0	0.3	741.9k	741.4k	0.1	28.8	741.9k	741.7k	0.0	0.5
dem_200_belg_orig_37	320.8k	320.8k	0.0	2165.5	320.8k	320.8k	0.0	1064.9	320.8k	320.8k	0.0	1440.1
dem_200_belg_orig_38	725.3k	725.3k	0.0	2114.1	725.3k	711.9k	1.9	14400.0	725.5k	690.4k	5.1	14400.0
dem_200_belg_orig_39	357.7k	357.7k	0.0	52.7	357.7k	357.7k	0.0	304.4	357.7k	357.7k	0.0	645.4
dem_200_belg_orig_40	495.9k	495.9k	0.0	3702.7	495.9k	495.5k	0.1	3246.6	495.9k	467.0k	6.2	14400.0
dem_200_belg_orig_41	625.1k	625.1k	0.0	1232.7	625.1k	625.1k	0.0	4929.4	625.1k	625.1k	0.0	7697.4
dem_200_belg_orig_42	480.4k	480.4k	0.0	687.1	480.4k	480.4k	0.0	593.7	480.4k	480.1k	0.1	485.8
dem_200_belg_orig_43	634.0k	634.0k	0.0	92.7	634.0k	633.8k	0.0	156.5	634.0k	634.0k	0.0	359.0
dem_200_belg_orig_44	439.8k	439.8k	0.0	206.0	439.8k	439.5k	0.1	191.6	439.8k	439.8k	0.0	330.3
dem_200_belg_orig_45	217.0k	217.0k	0.0	44.2	217.0k	217.0k	0.0	268.3	217.0k	217.0k	0.0	457.3
dem_200_belg_orig_46	689.7k	689.7k	0.0	36.1	689.7k	689.7k	0.0	52.0	689.7k	689.7k	0.0	63.9
dem_200_belg_orig_47	415.9k	415.9k	0.0	21.8	415.9k	415.6k	0.1	57.4	415.9k	415.9k	0.0	19.4
dem_200_belg_orig_48	396.4k	396.4k	0.0	27.3	396.4k	396.3k	0.0	85.9	396.4k	396.4k	0.0	21.1
dem_200_belg_orig_49	317.5k	317.5k	0.0	148.1	317.5k	317.5k	0.0	326.5	317.5k	317.5k	0.0	416.1
dem_200_belg_orig_50	636.7k	636.1k	0.1	1.2	636.7k	636.5k	0.0	69.4	636.7k	636.7k	0.0	21.2
dem_200_belg_orig_51	346.1k	345.8k	0.1	14.2	346.1k	346.1k	0.0	181.8	346.1k	346.1k	0.0	46.7
dem_200_belg_orig_52	568.2k	568.2k	0.0	78.2	568.2k	568.1k	0.0	127.7	568.2k	568.2k	0.0	2080.2
dem_200_belg_orig_53	395.0k	394.6k	0.1	1.0	395.0k	395.0k	0.0	45.2	395.0k	394.7k	0.1	0.7
dem_200_belg_orig_54	610.0k	610.0k	0.0	3.7	610.0k	610.0k	0.0	91.9	610.0k	609.5k	0.1	3.4

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Table A.3: Comparison of discrete models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_55	408.5k	408.5k	0.0	96.6	408.5k	408.3k	0.1	334.9	408.5k	408.5k	0.0	710.9
dem_200_belg_orig_56	580.1k	580.1k	0.0	6.3	580.1k	580.1k	0.0	89.9	580.1k	580.1k	0.0	6.3
dem_200_belg_orig_57	486.7k	486.7k	0.0	66.3	486.7k	486.7k	0.0	377.4	486.7k	486.7k	0.0	2027.8
dem_200_belg_orig_58	606.4k	606.4k	0.0	219.8	606.4k	606.4k	0.0	346.9	606.4k	606.4k	0.0	7036.9
dem_200_belg_orig_59	276.4k	276.4k	0.0	13.4	276.4k	276.4k	0.0	32.2	276.4k	276.4k	0.0	21.4
dem_200_belg_orig_60	570.0k	569.4k	0.1	1.0	570.0k	570.0k	0.0	295.3	570.0k	569.5k	0.1	1.5
dem_200_belg_orig_61	601.0k	600.4k	0.1	0.9	601.0k	600.5k	0.1	67.1	601.0k	601.0k	0.0	4.5
dem_200_belg_orig_62	740.6k	740.6k	0.0	9.5	740.6k	740.6k	0.0	64.2	740.6k	740.6k	0.0	13.8
dem_200_belg_orig_63	483.5k	483.5k	0.0	84.1	483.5k	483.5k	0.0	120.0	483.5k	483.5k	0.0	642.2
dem_200_belg_orig_64	288.5k	288.5k	0.0	1.1	288.5k	288.5k	0.0	49.2	288.5k	288.5k	0.0	10.4
dem_200_belg_orig_65	748.0k	748.0k	0.0	6.8	748.0k	748.0k	0.0	304.0	748.0k	748.0k	0.0	15.6
dem_200_belg_orig_66	632.9k	632.9k	0.0	16.4	632.9k	632.7k	0.0	69.8	632.9k	632.9k	0.0	50.3
dem_200_belg_orig_67	528.7k	528.7k	0.0	13.9	528.7k	528.7k	0.0	81.8	528.7k	528.3k	0.1	27.4
dem_200_belg_orig_68	575.0k	575.0k	0.0	30.2	575.0k	574.8k	0.0	183.9	575.0k	575.0k	0.0	228.4
dem_200_belg_orig_69	764.6k	764.6k	0.0	46.9	764.6k	764.0k	0.1	205.7	764.6k	764.6k	0.0	219.2
dem_200_belg_orig_70	581.9k	581.9k	0.0	0.2	581.9k	581.7k	0.0	29.0	581.9k	581.9k	0.0	0.3
dem_200_belg_orig_71	652.9k	652.9k	0.0	20.6	652.9k	652.7k	0.0	40.8	652.9k	652.9k	0.0	19.9
dem_200_belg_orig_72	378.1k	378.1k	0.0	62.5	378.1k	378.0k	0.0	859.1	378.1k	378.1k	0.0	1393.9
dem_200_belg_orig_73	266.6k	266.4k	0.1	1.4	266.6k	266.6k	0.0	28.6	266.6k	266.3k	0.1	1.4
dem_200_belg_orig_74	643.5k	643.5k	0.0	6.3	643.5k	643.3k	0.0	45.7	643.5k	643.5k	0.0	10.2
dem_200_belg_orig_75	605.4k	605.4k	0.0	26.9	605.4k	605.4k	0.0	379.1	605.4k	605.4k	0.0	85.6
dem_200_belg_orig_76	271.0k	271.0k	0.0	75.0	271.0k	270.7k	0.1	256.8	271.0k	271.0k	0.0	124.2
dem_200_belg_orig_77	494.0k	493.7k	0.0	0.3	494.0k	494.0k	0.0	71.8	494.0k	493.7k	0.1	0.4
dem_200_belg_orig_78	337.5k	337.5k	0.0	1.4	337.5k	337.5k	0.0	67.2	337.5k	337.5k	0.0	17.8
dem_200_belg_orig_79	369.7k	369.7k	0.0	7.8	369.7k	369.7k	0.0	355.4	369.7k	369.7k	0.0	65.3
dem_200_belg_orig_80	599.9k	599.6k	0.1	1.8	599.9k	599.9k	0.0	159.9	599.9k	599.9k	0.0	1.6
dem_200_belg_orig_81	508.3k	508.3k	0.0	0.1	508.3k	508.3k	0.0	33.4	508.3k	508.3k	0.0	0.3
dem_200_belg_orig_82	218.7k	218.7k	0.0	15.1	218.7k	218.7k	0.0	222.6	218.7k	218.7k	0.0	97.6
dem_200_belg_orig_83	566.1k	565.6k	0.1	0.1	566.1k	566.1k	0.0	37.5	566.1k	565.6k	0.1	0.2
dem_200_belg_orig_84	591.9k	591.9k	0.0	1.5	591.9k	591.9k	0.0	111.5	591.9k	591.9k	0.0	5.5
dem_200_belg_orig_85	600.9k	600.7k	0.0	2.9	600.9k	600.9k	0.0	71.5	600.9k	600.9k	0.0	14.7
dem_200_belg_orig_86	563.3k	563.3k	0.0	1.7	563.3k	562.7k	0.1	54.1	563.3k	563.3k	0.0	9.2
dem_200_belg_orig_87	412.2k	412.2k	0.0	4.8	412.2k	412.2k	0.0	70.3	412.2k	412.2k	0.0	14.4
dem_200_belg_orig_88	529.9k	529.4k	0.1	0.3	529.9k	529.9k	0.0	32.0	529.9k	529.5k	0.1	0.6
dem_200_belg_orig_89	1110.9k	1110.0k	0.1	1687.5	1110.9k	1110.9k	0.0	1007.8	1110.9k	1110.9k	0.0	3267.5
dem_200_belg_orig_90	549.3k	549.3k	0.0	1.6	549.3k	549.3k	0.0	75.4	549.3k	549.3k	0.0	4.0
dem_200_belg_orig_91	658.7k	658.7k	0.0	30.9	658.7k	658.7k	0.0	69.3	658.7k	658.7k	0.0	25.2
dem_200_belg_orig_92	877.8k	877.8k	0.0	385.4	877.8k	877.1k	0.1	78.5	877.8k	877.8k	0.0	504.8
dem_200_belg_orig_93	961.3k	961.1k	0.0	0.5	961.3k	960.5k	0.1	22.6	961.3k	960.8k	0.0	0.8
dem_200_belg_orig_94	473.9k	473.9k	0.0	74.4	473.9k	473.9k	0.0	702.3	473.9k	473.9k	0.0	1004.7
dem_200_belg_orig_95	367.8k	367.8k	0.0	643.1	367.8k	367.7k	0.0	434.2	367.8k	367.5k	0.1	8271.4
dem_200_belg_orig_96	389.5k	389.5k	0.0	4.2	389.5k	389.1k	0.1	34.2	389.5k	389.5k	0.0	0.6
dem_200_belg_orig_97	301.1k	301.0k	0.0	0.2	301.1k	301.1k	0.0	32.2	301.1k	300.9k	0.1	0.8
dem_200_belg_orig_98	585.2k	585.2k	0.0	9.0	585.2k	585.2k	0.0	323.5	585.2k	585.0k	0.0	61.4
dem_200_belg_orig_99	619.7k	619.7k	0.0	34.0	619.7k	619.3k	0.1	229.6	619.7k	619.6k	0.0	206.2
dem_200_belg_orig_100	609.5k	609.5k	0.0	18.2	609.5k	609.0k	0.1	391.9	609.5k	609.5k	0.0	17.0
dem_200_belg_orig_101	452.9k	452.9k	0.0	5.9	452.9k	452.5k	0.1	46.3	452.9k	452.9k	0.0	5.3
dem_200_belg_orig_102	726.9k	726.7k	0.0	0.4	726.9k	726.3k	0.1	42.0	726.9k	726.7k	0.0	1.0
dem_200_belg_orig_103	370.4k	370.4k	0.0	4020.3	370.4k	370.4k	0.0	490.6	370.4k	370.1k	0.1	5785.5
dem_200_belg_orig_104	431.5k	431.5k	0.0	812.5	431.5k	431.2k	0.1	1591.3	431.5k	431.2k	0.1	3684.2
dem_200_belg_orig_105	310.8k	310.8k	0.0	0.6	310.8k	310.8k	0.0	42.8	310.9k	310.6k	0.1	1.1
dem_200_belg_orig_106	498.2k	498.2k	0.0	30.4	498.2k	497.8k	0.1	117.0	498.2k	498.2k	0.0	34.4
dem_200_belg_orig_107	658.1k	658.1k	0.0	20.8	658.1k	658.1k	0.0	300.6	658.1k	658.0k	0.0	19.5
dem_200_belg_orig_108	248.6k	248.6k	0.0	70.6	248.6k	248.6k	0.0	1602.5	248.6k	248.6k	0.0	52.3
dem_200_belg_orig_109	668.3k	668.3k	0.0	225.7	668.3k	668.2k	0.0	119.3	668.3k	667.9k	0.1	1638.8
dem_200_belg_orig_110	729.8k	729.6k	0.0	0.4	729.8k	729.1k	0.1	29.7	729.8k	729.4k	0.0	0.6
dem_200_belg_orig_111	521.0k	521.0k	0.0	7.3	521.0k	520.5k	0.1	68.8	521.0k	521.0k	0.0	9.4
dem_200_belg_orig_112	277.8k	277.6k	0.1	0.3	277.8k	277.8k	0.0	44.6	277.8k	277.8k	0.0	0.5
dem_200_belg_orig_113	293.3k	293.3k	0.0	69.6	293.3k	293.3k	0.0	405.1	293.3k	293.3k	0.0	1623.5
dem_200_belg_orig_114	284.3k	284.1k	0.1	0.4	284.3k	284.3k	0.0	33.4	284.3k	284.1k	0.1	1.0
dem_200_belg_orig_115	642.4k	642.3k	0.0	152.8	642.4k	641.8k	0.1	347.4	642.4k	642.4k	0.0	3378.3
dem_200_belg_orig_116	473.9k	473.7k	0.0	0.6	473.9k	473.9k	0.0	30.7	473.9k	473.9k	0.0	1.1
dem_200_belg_orig_117	377.5k	377.5k	0.0	543.2	377.5k	369.0k	2.3	14400.0	377.5k	377.5k	0.0	1884.2
dem_200_belg_orig_118	558.8k	558.8k	0.0	3.3	558.8k	558.8k	0.0	209.0	558.8k	558.8k	0.0	7.5
dem_200_belg_orig_119	610.7k	610.7k	0.0	108.9	610.7k	610.3k	0.1	119.9	610.7k	610.4k	0.1	324.6
dem_200_belg_orig_120	427.9k	427.9k	0.0	1271.8	427.9k	427.6k	0.1	331.4	427.9k	405.6k	5.5	14400.0
dem_200_belg_orig_121	368.1k	368.1k	0.0	5.8	368.1k	368.1k	0.0	111.6	368.1k	368.1k	0.0	23.8
dem_200_belg_orig_122	608.0k	607.5k	0.1	2.3	608.0k	608.0k	0.0	153.8	608.0k	608.0k	0.0	3.4
dem_200_belg_orig_123	388.7k	388.7k	0.0	1.4	388.7k	388.7k	0.0	43.9	388.7k	388.7k	0.0	63.7
dem_200_belg_orig_124	671.6k	671.6k	0.0	25.1	671.6k	671.0k	0.1	41.9	671.6k	671.6k	0.0	27.7
dem_200_belg_orig_125	276.6k	276.6k	0.0	8.2	276.6k	276.6k	0.0	45.4	276.6k	276.4k	0.1	20.6
dem_200_belg_orig_126	437.9k	437.9k	0.0	179.7	437.9k	437.8k	0.0	158.7	437.9k	437.9k	0.0	2454.1
dem_200_belg_orig_127	552.2k	552.2k	0.0	169.9	552.2k	551.8k	0.1	124.9	552.2k	552.2k	0.0	1626.5
dem_200_belg_orig_128	628.8k	628.6k	0.0	7.2	628.8k	628.3k	0.1	46.0	628.8k	628.2k	0.1	153.9
dem_200_belg_orig_129	341.6k	341.6k	0.0	2105.0	341.6k	341.6k	0.0	1043.0	341.6k	341.3k	0.1	14067.3
dem_200_belg_orig_130	705.3k	705.0k	0.0	7.6	705.3k	704.7k	0.1	95.7	705.3k	705.3k	0.0	5.9
dem_200_belg_orig_131	463.6k	463.3k	0.1	0.4	463.6k	463.6k	0.0	31.6	463.6k	463.6k	0.0	0.8
dem_200_belg_orig_132	344.2k	344.1k	0.0	0.3	344.2k	343.9k	0.1	35.9	344.2k	344.2k	0.0	0.7

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Table A.3: Comparison of discrete models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_133	607.3k	607.3k	0.0	744.2	607.3k	607.3k	0.0	2683.3	607.3k	607.2k	0.0	11944.0
dem_200_belg_orig_134	898.7k	898.1k	0.1	0.6	898.7k	898.7k	0.0	38.1	898.7k	897.9k	0.1	0.9
dem_200_belg_orig_135	384.9k	384.9k	0.0	17.3	384.9k	384.9k	0.0	100.1	384.9k	384.9k	0.0	33.0
dem_200_belg_orig_136	1133.9k	1133.9k	0.0	147.3	1133.9k	1132.7k	0.1	839.3	1133.9k	1133.9k	0.0	6590.6
dem_200_belg_orig_137	827.8k	827.8k	0.0	26.5	827.8k	827.8k	0.0	117.3	827.8k	827.5k	0.0	45.0
dem_200_belg_orig_138	692.9k	692.2k	0.1	0.9	692.6k	691.9k	0.1	53.1	692.9k	692.3k	0.1	1.8
dem_200_belg_orig_139	372.9k	372.9k	0.0	1.4	372.9k	372.9k	0.0	59.0	372.9k	372.9k	0.0	7.7
dem_200_belg_orig_140	447.8k	447.8k	0.0	8479.8	447.8k	447.5k	0.1	649.9	447.8k	408.1k	9.7	14400.0
dem_200_belg_orig_141	629.5k	629.5k	0.0	7.2	629.5k	629.5k	0.0	745.6	629.5k	629.5k	0.0	277.9
dem_200_belg_orig_142	702.7k	702.7k	0.0	12.3	702.7k	702.7k	0.0	72.4	702.7k	702.7k	0.0	65.7
dem_200_belg_orig_143	271.6k	271.6k	0.0	0.2	271.6k	271.6k	0.0	35.6	271.6k	271.6k	0.0	0.4
dem_200_belg_orig_144	310.7k	310.7k	0.0	89.5	310.7k	310.7k	0.0	263.9	310.7k	310.7k	0.0	90.5
dem_200_belg_orig_145	547.2k	547.2k	0.0	145.1	547.2k	547.2k	0.0	226.5	547.2k	547.2k	0.0	834.1
dem_200_belg_orig_146	397.4k	397.4k	0.0	9.7	397.4k	397.1k	0.1	75.7	397.4k	397.4k	0.0	10.2
dem_200_belg_orig_147	601.0k	600.4k	0.1	0.9	601.0k	601.0k	0.0	44.6	601.0k	600.4k	0.1	0.8
dem_200_belg_orig_148	561.3k	561.3k	0.0	123.0	561.3k	560.7k	0.1	81.0	561.3k	561.3k	0.0	2095.4
dem_200_belg_orig_149	413.9k	413.9k	0.0	1212.4	413.9k	413.9k	0.0	577.5	413.9k	413.9k	0.0	5942.3
dem_200_belg_orig_150	833.8k	833.1k	0.1	1.7	833.8k	833.8k	0.0	60.2	833.8k	833.8k	0.0	4.5
dem_200_belg_orig_151	362.5k	362.5k	0.0	22.0	362.5k	362.5k	0.0	77.4	362.5k	362.5k	0.0	43.2
dem_200_belg_orig_152	370.2k	370.2k	0.0	4.5	370.2k	370.2k	0.0	63.8	370.2k	370.2k	0.0	8.2
dem_200_belg_orig_153	658.3k	657.7k	0.1	5.8	658.3k	658.1k	0.0	446.4	658.3k	658.1k	0.0	16.6
dem_200_belg_orig_154	448.9k	448.6k	0.1	62.4	448.9k	448.5k	0.1	98.5	448.9k	448.9k	0.0	641.7
dem_200_belg_orig_155	675.7k	675.7k	0.0	2.0	675.7k	675.7k	0.0	52.2	675.7k	675.7k	0.0	7.8
dem_200_belg_orig_156	370.4k	370.4k	0.0	11.1	370.4k	370.4k	0.0	82.7	370.4k	370.4k	0.0	15.8
dem_200_belg_orig_157	577.9k	577.9k	0.0	16.1	577.9k	577.7k	0.0	269.6	577.9k	577.9k	0.0	43.9
dem_200_belg_orig_158	640.0k	640.0k	0.0	903.5	640.0k	640.0k	0.0	1162.5	640.0k	630.0k	1.6	14400.0
dem_200_belg_orig_159	397.3k	397.3k	0.0	0.7	397.3k	397.1k	0.1	43.5	397.3k	397.3k	0.0	3.6
dem_200_belg_orig_160	565.6k	565.6k	0.0	12723.9	565.6k	565.6k	0.0	96.1	567.1k	513.6k	10.4	14400.0
dem_200_belg_orig_161	551.5k	551.3k	0.0	18.4	551.5k	551.5k	0.0	82.0	551.5k	551.5k	0.0	42.4
dem_200_belg_orig_162	649.5k	649.5k	0.0	16.1	649.5k	649.5k	0.0	66.5	649.5k	649.0k	0.1	30.8
dem_200_belg_orig_163	781.8k	781.5k	0.0	2.6	781.8k	781.8k	0.0	66.8	781.8k	781.8k	0.0	6.9
dem_200_belg_orig_164	447.5k	447.4k	0.0	2.1	447.5k	447.5k	0.0	479.1	447.5k	447.3k	0.1	42.4
dem_200_belg_orig_165	394.4k	394.4k	0.0	13.7	394.4k	394.4k	0.0	121.3	394.4k	394.4k	0.0	42.1
dem_200_belg_orig_166	335.4k	335.1k	0.1	0.4	335.2k	335.2k	0.0	20.0	335.2k	335.0k	0.0	1.3
dem_200_belg_orig_167	717.2k	717.2k	0.0	62.1	717.2k	717.2k	0.0	77.2	717.2k	716.6k	0.1	6.3
dem_200_belg_orig_168	567.1k	566.7k	0.1	0.4	567.1k	566.8k	0.1	28.6	567.1k	566.9k	0.0	0.6
dem_200_belg_orig_169	328.7k	328.7k	0.0	2176.7	328.7k	328.7k	0.0	369.0	328.7k	328.7k	0.0	3170.2
dem_200_belg_orig_170	550.6k	550.6k	0.0	4.5	550.6k	550.3k	0.0	1073.8	550.6k	550.6k	0.0	84.4
dem_200_belg_orig_171	398.0k	398.0k	0.0	0.3	398.0k	397.8k	0.1	20.9	398.0k	397.8k	0.0	0.6
dem_200_belg_orig_172	628.4k	600.8k	4.6	14400.0	628.4k	628.4k	0.0	377.1	628.4k	592.5k	6.1	14400.0
dem_200_belg_orig_173	798.0k	797.8k	0.0	0.1	798.0k	797.5k	0.1	19.7	798.0k	797.8k	0.0	0.3
dem_200_belg_orig_174	530.0k	529.8k	0.0	0.2	530.0k	530.0k	0.0	126.4	530.0k	529.8k	0.0	0.3
dem_200_belg_orig_175	233.3k	233.3k	0.0	0.2	233.3k	233.3k	0.0	16.3	233.3k	233.1k	0.1	0.3
dem_200_belg_orig_176	646.3k	646.3k	0.0	1356.7	646.3k	646.3k	0.0	60.6	646.3k	619.7k	4.3	14400.0
dem_200_belg_orig_177	728.1k	728.1k	0.0	4618.6	728.1k	727.4k	0.1	12008.8	728.1k	687.2k	6.0	14400.0
dem_200_belg_orig_178	363.0k	362.7k	0.1	0.4	363.0k	363.0k	0.0	57.0	363.0k	362.6k	0.1	0.6
dem_200_belg_orig_179	430.2k	429.8k	0.1	11.0	430.2k	430.2k	0.0	71.2	430.2k	430.2k	0.0	19.4
dem_200_belg_orig_180	341.2k	341.0k	0.0	9.2	341.2k	341.2k	0.0	267.4	341.2k	341.2k	0.0	91.2
dem_200_belg_orig_181	313.5k	313.3k	0.1	0.3	313.5k	313.5k	0.0	22.8	313.5k	313.4k	0.0	0.5
dem_200_belg_orig_182	574.8k	574.8k	0.0	63.2	574.8k	574.5k	0.1	78.8	574.8k	574.8k	0.0	122.4
dem_200_belg_orig_183	311.6k	311.6k	0.0	629.5	311.6k	311.5k	0.0	2578.1	311.6k	311.6k	0.0	449.4
dem_200_belg_orig_184	320.5k	320.4k	0.0	587.7	320.5k	320.5k	0.0	1723.1	320.5k	320.5k	0.0	2680.1
dem_200_belg_orig_185	382.6k	382.5k	0.0	0.3	382.6k	382.2k	0.1	35.4	382.6k	382.4k	0.0	0.7
dem_200_belg_orig_186	591.5k	591.0k	0.1	0.5	591.5k	591.1k	0.1	64.4	591.5k	591.5k	0.0	5.2
dem_200_belg_orig_187	833.9k	833.9k	0.0	9.6	833.9k	833.9k	0.0	118.8	833.9k	833.9k	0.0	16.4
dem_200_belg_orig_188	265.8k	265.6k	0.1	0.4	265.8k	265.8k	0.0	34.9	265.8k	265.8k	0.0	0.8
dem_200_belg_orig_189	590.2k	589.9k	0.0	743.2	590.2k	589.6k	0.1	436.9	590.2k	590.2k	0.0	3661.1
dem_200_belg_orig_190	406.4k	358.8k	13.3	14400.0	406.4k	406.4k	0.0	9392.8	417.8k	324.4k	28.8	14400.0
dem_200_belg_orig_191	670.7k	670.7k	0.0	25.7	670.7k	670.7k	0.0	863.9	670.7k	670.7k	0.0	211.8
dem_200_belg_orig_192	615.5k	615.2k	0.0	0.5	615.5k	615.5k	0.0	21.6	615.5k	615.0k	0.1	1.3
dem_200_belg_orig_193	470.7k	470.7k	0.0	284.6	470.7k	470.7k	0.0	1384.1	470.7k	470.7k	0.0	814.6
dem_200_belg_orig_194	560.4k	560.0k	0.1	1.7	560.4k	560.4k	0.0	40.8	560.4k	560.4k	0.0	11.3
dem_200_belg_orig_195	505.1k	505.1k	0.0	156.9	505.1k	505.1k	0.0	867.2	505.1k	505.1k	0.0	1430.7
dem_200_belg_orig_196	641.2k	641.2k	0.0	214.9	641.2k	641.2k	0.0	254.2	641.2k	641.2k	0.0	739.2
dem_200_belg_orig_197	882.6k	882.6k	0.0	4.5	882.6k	882.3k	0.0	65.2	882.6k	881.8k	0.1	10.1
dem_200_belg_orig_198	473.2k	473.2k	0.0	53.1	473.2k	473.2k	0.0	1291.1	473.2k	473.2k	0.0	136.8
dem_200_belg_orig_199	279.6k	279.6k	0.0	9.4	279.6k	279.6k	0.0	82.1	279.6k	279.6k	0.0	65.5
dem_200_belg_orig_200	654.0k	653.5k	0.1	0.7	654.0k	654.0k	0.0	50.6	654.0k	653.9k	0.0	3.0
dem_200_belg_orig_201	326.1k	326.1k	0.0	9616.4	326.1k	326.1k	0.0	3657.7	326.1k	257.2k	26.8	14400.0
dem_200_belg_orig_202	466.8k	466.8k	0.0	32.5	466.8k	466.8k	0.0	470.2	466.8k	466.8k	0.0	85.7
dem_200_belg_orig_203	523.3k	523.3k	0.0	91.0	523.3k	523.0k	0.1	95.0	523.3k	523.3k	0.0	17.5
dem_200_belg_orig_204	562.3k	562.3k	0.0	1236.5	562.3k	562.3k	0.0	1110.8	562.3k	546.2k	2.9	14400.0
dem_200_belg_orig_205	626.5k	626.5k	0.0	396.8	626.5k	626.5k	0.0	2504.2	626.5k	626.5k	0.0	2516.8
dem_200_belg_orig_206	306.7k	306.7k	0.0	90.5	306.7k	306.7k	0.0	174.5	306.7k	306.7k	0.0	136.2
dem_200_belg_orig_207	577.1k	577.1k	0.0	20.9	577.1k	577.1k	0.0	87.9	577.1k	577.1k	0.0	42.3
dem_200_belg_orig_208	559.2k	559.2k	0.0	5.8	559.2k	558.8k	0.1	72.6	559.2k	559.2k	0.0	17.3
dem_200_belg_orig_209	319.0k	319.0k	0.0	2703.9	319.0k	300.3k	6.3	14400.0	319.0k	319.0k	0.0	672.7
dem_200_belg_orig_210	609.8k	609.8k	0.0	0.2	609.8k	609.8k	0.0	68.2	609.8k	609.8k	0.0	0.5

continued on next page

Table A.3: Comparison of discrete models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_211	725.7k	725.7k	0.0	655.5	725.7k	725.7k	0.0	142.3	725.7k	718.5k	1.0	14400.0
dem_200_belg_orig_212	368.5k	368.5k	0.0	87.2	368.5k	368.5k	0.0	601.5	368.5k	368.5k	0.0	707.4
dem_200_belg_orig_213	750.5k	750.5k	0.0	7.1	750.5k	750.2k	0.0	70.1	750.5k	750.5k	0.0	13.4
dem_200_belg_orig_214	417.0k	417.0k	0.0	139.2	417.0k	417.0k	0.0	244.1	417.0k	417.0k	0.0	1287.0
dem_200_belg_orig_215	364.3k	364.3k	0.0	0.2	364.3k	364.3k	0.0	22.1	364.3k	364.3k	0.0	0.4
dem_200_belg_orig_216	593.0k	593.0k	0.0	0.2	593.0k	593.0k	0.0	58.1	593.1k	592.7k	0.1	0.4
dem_200_belg_orig_217	519.9k	519.7k	0.0	0.9	519.9k	519.9k	0.0	57.3	519.9k	519.4k	0.1	1.1
dem_200_belg_orig_218	879.0k	879.0k	0.0	51.0	879.0k	878.1k	0.1	126.9	879.0k	879.0k	0.0	137.4
dem_200_belg_orig_219	640.1k	640.1k	0.0	16.4	640.1k	640.0k	0.0	3257.3	640.1k	640.1k	0.0	27.6
dem_200_belg_orig_220	457.2k	457.2k	0.0	25.5	457.2k	457.2k	0.0	75.9	457.2k	457.2k	0.0	45.4
dem_200_belg_orig_221	290.0k	290.0k	0.0	18.0	290.0k	290.0k	0.0	78.2	290.0k	289.7k	0.1	35.5
dem_200_belg_orig_222	492.5k	492.5k	0.0	11.6	492.5k	492.5k	0.0	82.3	492.5k	492.5k	0.0	42.7
dem_200_belg_orig_223	428.3k	428.0k	0.1	0.4	428.3k	428.3k	0.0	63.5	428.3k	428.3k	0.0	1.0
dem_200_belg_orig_224	588.0k	588.0k	0.0	58.9	588.0k	588.0k	0.0	86.0	588.0k	588.0k	0.0	373.5
dem_200_belg_orig_225	502.7k	502.7k	0.0	19.4	502.7k	502.3k	0.1	429.0	502.7k	502.6k	0.0	26.4
dem_200_belg_orig_226	606.7k	606.1k	0.1	37.5	606.7k	606.5k	0.0	594.4	606.7k	606.7k	0.0	60.6
dem_200_belg_orig_227	282.3k	282.3k	0.0	3.8	282.3k	282.3k	0.0	18.2	282.3k	282.3k	0.0	11.8
dem_200_belg_orig_228	531.7k	531.3k	0.1	0.3	531.7k	531.7k	0.0	33.5	531.7k	531.5k	0.0	0.6
dem_200_belg_orig_229	720.6k	720.6k	0.0	112.1	720.6k	720.6k	0.0	63.4	720.6k	720.6k	0.0	1143.6
dem_200_belg_orig_230	630.9k	630.9k	0.0	12.3	630.9k	630.5k	0.1	144.8	630.9k	630.3k	0.1	69.5
dem_200_belg_orig_231	416.0k	415.6k	0.1	0.6	416.0k	416.0k	0.0	61.1	416.1k	415.7k	0.1	0.9
dem_200_belg_orig_232	300.2k	300.2k	0.0	5.2	300.2k	300.0k	0.1	29.5	300.2k	300.2k	0.0	10.4
dem_200_belg_orig_233	543.5k	543.5k	0.0	60.0	543.5k	543.1k	0.1	110.0	543.5k	543.5k	0.0	350.9
dem_200_belg_orig_234	263.9k	263.7k	0.1	0.3	263.9k	263.9k	0.0	26.5	263.9k	263.8k	0.0	0.7
dem_200_belg_orig_235	361.7k	361.7k	0.0	1.0	361.7k	361.7k	0.0	44.6	361.7k	361.7k	0.0	3.1
dem_200_belg_orig_236	569.2k	569.2k	0.0	180.4	569.2k	569.0k	0.0	98.0	569.2k	569.2k	0.0	1668.7
dem_200_belg_orig_237	529.7k	529.7k	0.0	45.0	529.7k	529.7k	0.0	179.2	529.7k	529.7k	0.0	48.9
dem_200_belg_orig_238	536.1k	536.1k	0.0	0.4	536.1k	536.1k	0.0	65.5	536.1k	536.0k	0.0	0.6
dem_200_belg_orig_239	713.4k	713.1k	0.1	1.1	713.4k	713.4k	0.0	66.8	713.4k	712.8k	0.1	2.4
dem_200_belg_orig_240	746.9k	746.2k	0.1	11.7	746.9k	746.9k	0.0	166.1	746.9k	746.9k	0.0	591.5
dem_200_belg_orig_241	674.1k	674.1k	0.0	4.0	674.1k	673.8k	0.1	75.5	674.1k	674.1k	0.0	6.8
dem_200_belg_orig_242	462.3k	462.3k	0.0	27.0	462.3k	462.3k	0.0	57.7	462.3k	462.3k	0.0	61.8
dem_200_belg_orig_243	587.3k	587.1k	0.0	206.5	587.3k	587.0k	0.0	1107.8	587.3k	587.3k	0.0	640.4
dem_200_belg_orig_244	496.0k	496.0k	0.0	586.2	496.0k	495.6k	0.1	343.8	496.0k	496.0k	0.0	354.3
dem_200_belg_orig_245	300.6k	300.6k	0.0	3441.1	300.6k	300.6k	0.0	2807.1	311.2k	240.6k	29.3	14400.0
dem_200_belg_orig_246	681.0k	680.5k	0.1	0.2	681.0k	680.3k	0.1	34.0	681.0k	680.5k	0.1	0.4
dem_200_belg_orig_247	431.0k	430.9k	0.0	964.4	431.0k	431.0k	0.0	1094.6	431.0k	413.6k	4.2	14400.0
dem_200_belg_orig_248	864.8k	864.8k	0.0	20.8	864.8k	864.2k	0.1	138.6	864.8k	864.8k	0.0	49.1
dem_200_belg_orig_249	354.7k	354.7k	0.0	8.0	354.7k	354.7k	0.0	41.5	354.7k	354.4k	0.1	1.6
dem_200_belg_orig_250	449.8k	449.8k	0.0	6.8	449.8k	449.7k	0.0	65.0	449.8k	449.8k	0.0	11.0
dem_200_belg_orig_251	548.5k	548.1k	0.1	0.2	548.1k	547.8k	0.1	28.8	548.1k	548.1k	0.0	0.7
dem_200_belg_orig_252	357.1k	357.1k	0.0	61.5	357.1k	357.1k	0.0	113.3	357.1k	357.1k	0.0	40.8
dem_200_belg_orig_253	816.7k	816.7k	0.0	4.1	816.7k	816.1k	0.1	38.1	816.7k	816.7k	0.0	9.7
dem_200_belg_orig_254	743.0k	743.0k	0.0	55.2	743.0k	743.0k	0.0	216.6	743.0k	743.0k	0.0	147.0
dem_200_belg_orig_255	194.1k	194.1k	0.0	7.5	194.1k	194.1k	0.0	58.0	194.1k	193.9k	0.1	30.2
dem_200_belg_orig_256	645.3k	644.9k	0.1	0.6	645.3k	645.3k	0.0	184.1	645.3k	645.3k	0.0	6.4
dem_200_belg_orig_257	453.5k	412.8k	9.8	14400.0	452.4k	452.4k	0.0	553.5	453.5k	391.6k	15.8	14400.0
dem_200_belg_orig_258	318.3k	318.0k	0.1	0.3	318.3k	318.3k	0.0	37.3	318.3k	318.1k	0.1	0.5
dem_200_belg_orig_259	490.6k	490.6k	0.0	84.1	490.6k	490.6k	0.0	565.6	490.6k	490.6k	0.0	264.4
dem_200_belg_orig_260	645.3k	645.3k	0.0	2803.0	645.3k	645.3k	0.0	1217.3	645.3k	625.6k	3.2	14400.0
dem_200_belg_orig_261	146.1k	146.1k	0.0	0.3	146.1k	146.1k	0.0	9.7	146.1k	146.1k	0.0	0.5
dem_200_belg_orig_262	530.2k	530.2k	0.0	54.0	530.2k	530.2k	0.0	65.9	530.2k	530.2k	0.0	672.5
dem_200_belg_orig_263	683.2k	682.8k	0.0	0.2	683.2k	682.5k	0.1	47.8	683.2k	682.9k	0.0	0.5
dem_200_belg_orig_264	600.8k	600.8k	0.0	1.3	600.8k	600.3k	0.1	56.4	600.9k	600.3k	0.1	5.4
dem_200_belg_orig_265	915.8k	915.8k	0.0	18.0	915.8k	915.3k	0.1	136.3	915.8k	915.8k	0.0	25.3
dem_200_belg_orig_266	532.6k	532.6k	0.0	2.1	532.6k	532.5k	0.0	75.2	532.6k	532.1k	0.1	2.4
dem_200_belg_orig_267	823.2k	823.2k	0.0	21.0	823.2k	822.6k	0.1	102.8	823.2k	823.2k	0.0	28.9
dem_200_belg_orig_268	543.6k	543.6k	0.0	2219.9	543.6k	543.2k	0.1	3001.9	543.6k	543.6k	0.0	5923.4
dem_200_belg_orig_269	490.7k	490.7k	0.0	26.2	490.7k	490.7k	0.0	246.6	490.7k	490.5k	0.1	613.9
dem_200_belg_orig_270	358.0k	358.0k	0.0	348.2	358.0k	357.7k	0.1	1725.5	358.0k	358.0k	0.0	566.0
dem_200_belg_orig_271	682.3k	682.1k	0.0	0.3	682.3k	682.3k	0.0	26.9	682.3k	682.2k	0.0	1.3
dem_200_belg_orig_272	803.6k	803.6k	0.0	22.9	803.6k	802.8k	0.1	47.0	803.6k	803.6k	0.0	67.2
dem_200_belg_orig_273	783.1k	783.1k	0.0	21.4	783.1k	782.8k	0.0	91.2	783.1k	782.7k	0.1	37.8
dem_200_belg_orig_274	454.5k	454.1k	0.1	31.6	454.5k	454.5k	0.0	292.4	454.5k	454.5k	0.0	55.1
dem_200_belg_orig_275	374.3k	374.0k	0.1	7.3	374.3k	374.3k	0.0	107.7	374.3k	374.3k	0.0	26.7
dem_200_belg_orig_276	254.2k	254.2k	0.0	14.0	254.2k	254.0k	0.1	76.1	254.2k	254.2k	0.0	16.8
dem_200_belg_orig_277	411.1k	411.1k	0.0	88.7	411.1k	411.1k	0.0	96.4	411.1k	411.1k	0.0	252.4
dem_200_belg_orig_278	366.4k	366.0k	0.1	0.7	366.4k	366.4k	0.0	31.4	366.4k	366.0k	0.1	1.0
dem_200_belg_orig_279	512.8k	512.8k	0.0	175.2	512.8k	512.8k	0.0	860.9	512.8k	512.8k	0.0	914.9
dem_200_belg_orig_280	718.6k	718.4k	0.0	0.3	719.0k	718.3k	0.1	23.8	718.6k	718.5k	0.0	0.5
dem_200_belg_orig_281	389.8k	389.5k	0.1	0.3	389.8k	389.8k	0.0	46.1	389.8k	389.6k	0.1	0.6
dem_200_belg_orig_282	634.7k	634.7k	0.0	8.8	634.7k	634.7k	0.0	110.6	634.7k	634.7k	0.0	16.1
dem_200_belg_orig_283	445.5k	445.5k	0.0	901.9	445.5k	445.5k	0.0	2831.7	445.5k	445.5k	0.0	4954.2
dem_200_belg_orig_284	494.3k	494.3k	0.0	36.3	494.3k	494.3k	0.0	316.2	494.3k	494.3k	0.0	131.8
dem_200_belg_orig_285	466.9k	466.9k	0.0	682.8	466.9k	466.9k	0.0	6377.1	466.9k	466.9k	0.0	5683.1
dem_200_belg_orig_286	528.3k	528.2k	0.0	31.3	528.3k	528.2k	0.0	81.0	528.3k	528.3k	0.0	80.0
dem_200_belg_orig_287	461.2k	461.2k	0.0	1776.3	461.2k	461.2k	0.0	1487.6	461.2k	461.2k	0.0	5287.2
dem_200_belg_orig_288	430.9k	430.9k	0.0	25.7	430.9k	430.9k	0.0	104.6	430.9k	430.9k	0.0	94.5

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Table A.3: Comparison of discrete models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_289	524.9k	524.9k	0.0	2279.4	524.9k	524.9k	0.0	1585.1	524.9k	524.9k	0.0	9765.7
dem_200_belg_orig_290	681.5k	681.5k	0.0	31.8	681.5k	681.2k	0.0	100.9	681.5k	681.2k	0.0	288.0
dem_200_belg_orig_291	591.4k	591.4k	0.0	245.4	591.4k	591.3k	0.0	233.5	591.4k	591.4k	0.0	6139.7
dem_200_belg_orig_292	355.5k	355.5k	0.0	15.2	355.5k	355.5k	0.0	68.8	355.5k	355.5k	0.0	69.2
dem_200_belg_orig_293	337.3k	337.3k	0.0	180.2	337.3k	337.3k	0.0	642.4	337.3k	337.0k	0.1	885.4
dem_200_belg_orig_294	470.1k	470.1k	0.0	48.6	470.1k	470.1k	0.0	884.3	470.1k	470.1k	0.0	454.8
dem_200_belg_orig_295	674.7k	674.7k	0.0	32.1	674.7k	674.2k	0.1	49.9	674.7k	674.7k	0.0	137.5
dem_200_belg_orig_296	582.4k	581.9k	0.1	0.6	582.4k	581.8k	0.1	34.5	582.4k	582.0k	0.1	4.9
dem_200_belg_orig_297	359.0k	359.0k	0.0	69.0	359.0k	359.0k	0.0	239.5	359.0k	359.0k	0.0	613.8
dem_200_belg_orig_298	346.7k	346.7k	0.0	7.1	346.7k	346.7k	0.0	49.8	346.7k	346.7k	0.0	5.0
dem_200_belg_orig_299	731.2k	731.2k	0.0	31.2	731.2k	730.5k	0.1	79.0	731.2k	731.2k	0.0	552.6
dem_200_belg_orig_300	330.8k	330.8k	0.0	325.9	330.8k	330.8k	0.0	258.4	330.8k	330.6k	0.0	432.4
dem_200_belg_orig_301	408.4k	408.1k	0.1	6.0	408.4k	408.4k	0.0	167.8	408.4k	408.2k	0.1	28.5
dem_200_belg_orig_302	572.9k	572.5k	0.1	1.0	572.9k	572.9k	0.0	63.9	572.9k	572.5k	0.1	0.9
dem_200_belg_orig_303	862.9k	862.9k	0.0	341.8	862.9k	862.9k	0.0	76.9	862.9k	862.9k	0.0	1195.8
dem_200_belg_orig_304	679.8k	679.8k	0.0	27.1	679.8k	679.2k	0.1	80.4	679.8k	679.8k	0.0	41.5
dem_200_belg_orig_305	473.6k	473.6k	0.0	0.8	473.6k	473.6k	0.0	31.4	473.9k	473.6k	0.1	1.2
dem_200_belg_orig_306	625.0k	624.7k	0.0	7.8	625.0k	624.4k	0.1	143.9	625.0k	625.0k	0.0	63.3
dem_200_belg_orig_307	297.4k	297.3k	0.0	0.3	297.4k	297.3k	0.0	28.5	297.4k	297.3k	0.0	0.4
dem_200_belg_orig_308	451.4k	451.4k	0.0	9062.5	451.4k	451.4k	0.0	2095.3	452.4k	413.8k	9.3	14400.0
dem_200_belg_orig_309	541.5k	541.5k	0.0	2853.0	541.5k	541.5k	0.0	1557.0	541.5k	541.5k	0.0	13268.1
dem_200_belg_orig_310	306.4k	306.4k	0.0	17.6	306.4k	306.4k	0.0	207.3	306.4k	306.4k	0.0	58.6
dem_200_belg_orig_311	426.1k	426.1k	0.0	17.0	426.1k	426.1k	0.0	128.1	426.1k	426.1k	0.0	13.2
dem_200_belg_orig_312	322.5k	322.5k	0.0	19.3	322.5k	322.2k	0.1	1766.0	322.5k	322.5k	0.0	37.7
dem_200_belg_orig_313	750.1k	750.1k	0.0	9.9	750.1k	750.1k	0.0	63.2	750.1k	750.1k	0.0	155.2
dem_200_belg_orig_314	854.2k	854.2k	0.0	15.5	854.2k	854.2k	0.0	72.5	854.2k	854.2k	0.0	61.9
dem_200_belg_orig_315	665.7k	665.1k	0.1	0.5	665.7k	665.2k	0.1	24.6	665.9k	665.3k	0.1	1.1
dem_200_belg_orig_316	353.8k	353.8k	0.0	0.3	353.8k	353.8k	0.0	23.4	353.8k	353.8k	0.0	0.6
dem_200_belg_orig_317	600.7k	600.2k	0.1	81.2	600.7k	600.2k	0.1	149.5	600.7k	600.7k	0.0	1449.1
dem_200_belg_orig_318	290.0k	289.8k	0.1	1.3	290.0k	290.0k	0.0	26.3	290.0k	289.7k	0.1	3.2
dem_200_belg_orig_319	324.0k	324.0k	0.0	28.5	324.0k	324.0k	0.0	104.6	324.0k	323.7k	0.1	91.0
dem_200_belg_orig_320	461.1k	461.1k	0.0	20.0	461.1k	460.7k	0.1	270.6	461.1k	461.1k	0.0	67.0
dem_200_belg_orig_321	314.7k	314.7k	0.0	27.4	314.7k	314.7k	0.0	303.9	314.7k	314.7k	0.0	103.2
dem_200_belg_orig_322	453.8k	453.8k	0.0	1122.3	453.8k	453.6k	0.0	3431.6	453.8k	453.5k	0.1	10582.3
dem_200_belg_orig_323	365.5k	365.5k	0.0	1.0	365.5k	365.5k	0.0	83.6	365.5k	365.3k	0.1	1.6
dem_200_belg_orig_324	559.3k	559.3k	0.0	79.0	559.3k	559.3k	0.0	614.0	559.3k	559.3k	0.0	189.2
dem_200_belg_orig_325	693.9k	693.3k	0.1	0.4	693.9k	693.9k	0.0	90.3	693.9k	693.3k	0.1	8.2
dem_200_belg_orig_326	589.2k	588.7k	0.1	8.0	589.2k	589.0k	0.0	78.7	589.2k	589.2k	0.0	107.1
dem_200_belg_orig_327	366.4k	366.4k	0.0	14.8	366.4k	366.4k	0.0	103.8	366.4k	366.4k	0.0	38.1
dem_200_belg_orig_328	472.2k	471.9k	0.1	2.6	472.2k	472.2k	0.0	554.4	472.2k	472.2k	0.0	19.8
dem_200_belg_orig_329	452.6k	452.3k	0.1	0.2	452.6k	452.1k	0.1	38.8	452.6k	452.3k	0.1	0.8
dem_200_belg_orig_330	401.3k	401.3k	0.0	5.9	401.3k	400.9k	0.1	69.7	401.3k	401.3k	0.0	16.6
dem_200_belg_orig_331	395.1k	395.1k	0.0	12.3	395.1k	394.9k	0.1	250.8	395.1k	395.1k	0.0	55.3
dem_200_belg_orig_332	879.4k	878.9k	0.1	102.7	879.4k	879.1k	0.0	362.0	879.4k	879.4k	0.0	1510.3
dem_200_belg_orig_333	414.7k	414.6k	0.0	0.4	414.7k	414.4k	0.1	21.5	414.7k	414.5k	0.1	1.1
dem_200_belg_orig_334	610.6k	610.6k	0.0	49.5	610.6k	610.6k	0.0	628.8	610.6k	610.6k	0.0	1050.5
dem_200_belg_orig_335	554.7k	554.7k	0.0	0.4	554.7k	554.7k	0.0	38.8	554.7k	554.3k	0.1	0.6
dem_200_belg_orig_336	461.4k	461.0k	0.1	6.3	461.4k	461.4k	0.0	262.6	461.4k	461.4k	0.0	18.4
dem_200_belg_orig_337	346.3k	346.0k	0.1	0.9	346.3k	346.3k	0.0	68.3	346.3k	346.3k	0.0	2.7
dem_200_belg_orig_338	325.9k	325.9k	0.0	0.2	325.9k	325.9k	0.0	19.6	325.9k	325.5k	0.1	0.7
dem_200_belg_orig_339	839.2k	838.4k	0.1	2.2	839.2k	839.2k	0.0	106.0	839.2k	839.2k	0.0	17.3
dem_200_belg_orig_340	872.1k	872.1k	0.0	2125.6	872.1k	871.4k	0.1	194.4	872.4k	828.5k	5.3	14400.0
dem_200_belg_orig_341	343.9k	343.9k	0.0	194.7	343.9k	343.9k	0.0	672.6	343.9k	343.9k	0.0	627.9
dem_200_belg_orig_342	489.8k	396.9k	23.4	14400.0	487.0k	487.0k	0.0	1444.6	489.5k	385.3k	27.0	14400.0
dem_200_belg_orig_343	433.2k	432.8k	0.1	0.2	433.0k	433.0k	0.0	32.2	433.0k	432.7k	0.1	0.6
dem_200_belg_orig_344	340.3k	340.3k	0.0	430.5	340.3k	340.0k	0.1	1432.1	340.3k	340.3k	0.0	1370.2
dem_200_belg_orig_345	697.2k	696.5k	0.1	1.4	697.1k	696.4k	0.1	32.5	697.1k	696.4k	0.1	0.8
dem_200_belg_orig_346	438.9k	438.9k	0.0	2.8	438.9k	438.9k	0.0	56.3	438.9k	438.9k	0.0	1.4
dem_200_belg_orig_347	584.4k	584.4k	0.0	8.7	584.4k	583.8k	0.1	82.2	584.4k	584.4k	0.0	0.8
dem_200_belg_orig_348	331.8k	331.5k	0.1	0.1	331.8k	331.8k	0.0	23.4	331.8k	331.5k	0.1	0.3
dem_200_belg_orig_349	305.3k	305.3k	0.0	586.8	305.3k	305.3k	0.0	268.7	305.3k	305.3k	0.0	1125.4
dem_200_belg_orig_350	658.0k	658.0k	0.0	1615.8	658.0k	658.0k	0.0	898.8	658.0k	638.6k	3.0	14400.0
dem_200_belg_orig_351	500.4k	500.4k	0.0	0.8	500.4k	500.4k	0.0	33.5	500.4k	500.0k	0.1	1.0
dem_200_belg_orig_352	633.5k	632.9k	0.1	38.7	633.5k	633.3k	0.0	109.0	633.5k	633.0k	0.1	326.6
dem_200_belg_orig_353	603.6k	603.2k	0.1	0.1	603.6k	603.2k	0.1	19.7	603.6k	603.2k	0.1	0.3
dem_200_belg_orig_354	429.4k	429.0k	0.1	0.4	429.4k	429.4k	0.0	39.0	429.4k	429.4k	0.0	0.6
dem_200_belg_orig_355	246.6k	246.6k	0.0	21.2	246.6k	246.6k	0.0	119.6	246.6k	246.6k	0.0	37.4
dem_200_belg_orig_356	538.4k	538.4k	0.0	0.9	538.4k	538.4k	0.0	57.4	538.4k	538.4k	0.0	5.0
dem_200_belg_orig_357	424.2k	424.2k	0.0	13.7	424.2k	424.2k	0.0	178.4	424.2k	424.2k	0.0	39.9
dem_200_belg_orig_358	888.1k	888.1k	0.0	2.4	888.1k	887.5k	0.1	46.1	888.1k	888.1k	0.0	17.2
dem_200_belg_orig_359	574.9k	574.9k	0.0	59.5	574.9k	574.9k	0.0	133.6	574.9k	574.7k	0.0	896.4
dem_200_belg_orig_360	345.9k	345.9k	0.0	0.7	345.9k	345.9k	0.0	36.1	345.9k	345.9k	0.0	3.4
dem_200_belg_orig_361	805.9k	805.9k	0.0	11.6	805.9k	805.9k	0.0	62.2	805.9k	805.2k	0.1	19.7
dem_200_belg_orig_362	374.8k	374.8k	0.0	44.1	374.8k	374.8k	0.0	79.2	374.8k	374.8k	0.0	335.9
dem_200_belg_orig_363	626.8k	626.7k	0.0	1.4	626.8k	626.8k	0.0	261.4	626.8k	626.8k	0.0	13.3
dem_200_belg_orig_364	614.1k	614.1k	0.0	107.3	614.1k	614.1k	0.0	190.9	614.1k	614.1k	0.0	4779.4
dem_200_belg_orig_365	692.5k	692.5k	0.0	23.0	692.5k	691.8k	0.1	69.2	692.5k	692.5k	0.0	85.8
dem_200_belg_orig_366	769.7k	769.7k	0.0	881.0	769.7k	769.7k	0.0	407.4	769.7k	769.7k	0.0	4897.3

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Table A.3: Comparison of discrete models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_367	461.5k	461.5k	0.0	1280.0	461.5k	461.5k	0.0	3631.2	461.5k	461.5k	0.0	12550.8
dem_200_belg_orig_368	369.1k	369.1k	0.0	207.4	369.1k	368.8k	0.1	192.7	369.1k	369.1k	0.0	2833.2
dem_200_belg_orig_369	257.7k	257.5k	0.1	13.6	257.7k	257.5k	0.1	76.1	257.7k	257.7k	0.0	42.2
dem_200_belg_orig_370	294.1k	294.1k	0.0	12.5	294.1k	293.8k	0.1	75.8	294.1k	294.1k	0.0	32.0
dem_200_belg_orig_371	731.2k	731.2k	0.0	10.6	731.2k	730.6k	0.1	96.8	731.2k	730.5k	0.1	56.4
dem_200_belg_orig_372	541.5k	541.5k	0.0	0.2	541.5k	541.4k	0.0	45.2	541.5k	541.5k	0.0	0.5
dem_200_belg_orig_373	724.5k	724.3k	0.0	41.5	724.5k	724.0k	0.1	81.1	724.5k	724.5k	0.0	399.7
dem_200_belg_orig_374	314.8k	314.8k	0.0	62.4	314.8k	314.8k	0.0	117.8	314.8k	314.8k	0.0	23.9
dem_200_belg_orig_375	490.7k	490.7k	0.0	61.0	490.7k	490.7k	0.0	528.5	490.7k	490.2k	0.1	306.5
dem_200_belg_orig_376	1080.6k	1079.7k	0.1	0.3	1080.6k	1079.9k	0.1	28.6	1080.6k	1079.8k	0.1	0.4
dem_200_belg_orig_377	404.4k	364.8k	10.9	14400.0	403.2k	403.2k	0.0	909.3	404.6k	356.4k	13.5	14400.0
dem_200_belg_orig_378	361.9k	361.9k	0.0	8.5	361.9k	361.9k	0.0	55.1	361.9k	361.9k	0.0	12.1
dem_200_belg_orig_379	667.8k	667.8k	0.0	24.8	667.8k	667.3k	0.1	86.9	667.8k	667.8k	0.0	54.0
dem_200_belg_orig_380	486.5k	486.4k	0.0	0.3	486.5k	486.1k	0.1	66.1	486.5k	486.2k	0.1	0.7
dem_200_belg_orig_381	611.0k	610.5k	0.1	85.0	611.0k	610.5k	0.1	329.8	611.0k	610.7k	0.1	1097.3
dem_200_belg_orig_382	766.8k	766.8k	0.0	9.2	766.8k	766.2k	0.1	53.6	766.8k	766.8k	0.0	6.1
dem_200_belg_orig_383	335.0k	335.0k	0.0	1743.1	335.0k	335.0k	0.0	2777.4	335.0k	335.0k	0.0	5643.9
dem_200_belg_orig_384	419.7k	419.7k	0.0	15.0	419.7k	419.7k	0.0	106.2	419.7k	419.7k	0.0	31.4
dem_200_belg_orig_385	537.3k	537.3k	0.0	19.6	537.3k	536.9k	0.1	96.9	537.3k	537.3k	0.0	161.9
dem_200_belg_orig_386	696.2k	696.2k	0.0	1003.4	696.5k	686.5k	1.5	14400.0	696.2k	695.8k	0.1	4517.2
dem_200_belg_orig_387	356.6k	356.6k	0.0	225.7	356.6k	356.6k	0.0	723.2	356.6k	356.4k	0.1	158.8
dem_200_belg_orig_388	302.5k	302.5k	0.0	506.1	302.5k	302.5k	0.0	607.2	302.5k	302.5k	0.0	4380.1
dem_200_belg_orig_389	348.6k	348.4k	0.1	2.0	348.6k	348.6k	0.0	85.3	348.6k	348.6k	0.0	29.8
dem_200_belg_orig_390	454.2k	454.2k	0.0	1.4	454.2k	454.2k	0.0	59.2	454.2k	454.2k	0.0	1.8
dem_200_belg_orig_391	417.4k	417.4k	0.0	38.8	417.4k	417.4k	0.0	68.5	417.4k	417.2k	0.0	32.7
dem_200_belg_orig_392	384.5k	384.3k	0.1	1.1	384.5k	384.2k	0.1	33.3	384.5k	384.5k	0.0	0.4
dem_200_belg_orig_393	502.4k	502.4k	0.0	73.8	502.4k	502.4k	0.0	123.1	502.4k	502.4k	0.0	144.1
dem_200_belg_orig_394	390.5k	390.4k	0.0	0.3	390.5k	390.5k	0.0	35.9	390.5k	390.4k	0.0	0.6
dem_200_belg_orig_395	504.3k	503.8k	0.1	0.7	504.0k	503.7k	0.0	42.6	504.3k	503.9k	0.1	1.8
dem_200_belg_orig_396	346.2k	346.2k	0.0	2109.5	346.2k	346.2k	0.0	403.9	346.2k	346.2k	0.0	9270.8
dem_200_belg_orig_397	840.2k	840.2k	0.0	34.8	840.2k	840.2k	0.0	75.9	840.2k	840.2k	0.0	27.0
dem_200_belg_orig_398	578.1k	578.1k	0.0	42.0	578.1k	578.1k	0.0	135.4	578.1k	577.8k	0.1	444.8
dem_200_belg_orig_399	519.4k	519.4k	0.0	10.8	519.4k	519.4k	0.0	453.9	519.4k	519.4k	0.0	41.3
dem_200_belg_orig_400	350.5k	350.5k	0.0	7.7	350.5k	350.5k	0.0	49.9	350.5k	350.5k	0.0	30.8
dem_200_belg_orig_401	591.2k	591.2k	0.0	5.9	591.2k	591.2k	0.0	76.8	591.2k	591.2k	0.0	82.0
dem_200_belg_orig_402	527.3k	527.3k	0.0	56.7	527.3k	527.3k	0.0	621.4	527.3k	527.3k	0.0	273.0
dem_200_belg_orig_403	481.2k	481.2k	0.0	641.5	481.2k	481.2k	0.0	320.7	481.2k	481.2k	0.0	1279.1
dem_200_belg_orig_404	640.6k	640.1k	0.1	1.8	640.6k	640.4k	0.0	60.0	640.6k	640.6k	0.0	4.3
dem_200_belg_orig_405	870.5k	870.5k	0.0	2.1	870.5k	870.5k	0.0	69.7	870.5k	870.5k	0.0	30.1
dem_200_belg_orig_406	279.2k	279.2k	0.0	34.8	279.2k	279.2k	0.0	83.0	279.2k	279.2k	0.0	50.3
dem_200_belg_orig_407	666.7k	666.7k	0.0	28.9	666.7k	666.4k	0.0	97.5	666.7k	666.7k	0.0	109.2
dem_200_belg_orig_408	199.5k	199.3k	0.1	0.4	199.5k	199.5k	0.0	10.7	199.5k	199.5k	0.0	0.6
dem_200_belg_orig_409	386.7k	386.7k	0.0	24.0	386.7k	386.7k	0.0	428.6	386.7k	386.5k	0.1	245.9
dem_200_belg_orig_410	389.6k	389.4k	0.0	541.5	389.6k	389.6k	0.0	1129.2	389.6k	389.6k	0.0	3803.9
dem_200_belg_orig_411	503.3k	503.1k	0.0	0.5	503.3k	503.3k	0.0	44.9	503.3k	503.3k	0.0	0.5
dem_200_belg_orig_412	404.9k	404.9k	0.0	28.2	404.9k	404.9k	0.0	768.6	404.9k	404.9k	0.0	77.0
dem_200_belg_orig_413	342.7k	342.7k	0.0	9.5	342.7k	342.7k	0.0	49.4	342.7k	342.7k	0.0	7.1
dem_200_belg_orig_414	381.1k	381.1k	0.0	166.2	381.1k	381.1k	0.0	765.4	381.1k	381.1k	0.0	428.4
dem_200_belg_orig_415	261.6k	261.6k	0.0	13.0	261.6k	261.6k	0.0	146.2	261.6k	261.6k	0.0	112.4
dem_200_belg_orig_416	669.7k	669.7k	0.0	1.1	669.7k	669.3k	0.1	140.3	669.7k	669.7k	0.0	1.1
dem_200_belg_orig_417	508.2k	508.2k	0.0	105.4	508.2k	508.2k	0.0	2662.9	508.2k	508.2k	0.0	1966.0
dem_200_belg_orig_418	658.8k	658.8k	0.0	38.9	658.8k	658.8k	0.0	147.7	658.8k	658.8k	0.0	394.7
dem_200_belg_orig_419	336.1k	336.1k	0.0	0.4	336.1k	335.9k	0.1	46.9	336.1k	335.9k	0.1	0.4
dem_200_belg_orig_420	513.1k	513.1k	0.0	22.0	513.1k	513.1k	0.0	362.4	513.1k	513.1k	0.0	97.6
dem_200_belg_orig_421	349.4k	349.4k	0.0	3.6	349.4k	349.3k	0.0	59.3	349.4k	349.4k	0.0	4.7
dem_200_belg_orig_422	500.3k	499.9k	0.1	0.7	500.3k	500.3k	0.0	41.1	500.3k	500.3k	0.0	5.0
dem_200_belg_orig_423	353.4k	353.4k	0.0	1001.6	353.4k	353.4k	0.0	5240.2	353.4k	353.1k	0.1	902.7
dem_200_belg_orig_424	601.1k	601.1k	0.0	130.3	601.1k	601.1k	0.0	2052.8	601.1k	601.1k	0.0	1185.0
dem_200_belg_orig_425	412.5k	412.2k	0.1	0.4	412.5k	412.4k	0.0	44.8	412.5k	412.2k	0.1	0.6
dem_200_belg_orig_426	456.9k	400.4k	14.1	14400.0	455.6k	455.6k	0.0	2677.0	468.8k	356.5k	31.5	14400.0
dem_200_belg_orig_427	401.0k	400.7k	0.1	6.1	401.0k	401.0k	0.0	156.3	401.0k	401.0k	0.0	23.4
dem_200_belg_orig_428	277.5k	277.3k	0.1	0.5	277.5k	277.5k	0.0	43.3	277.5k	277.5k	0.0	1.6
dem_200_belg_orig_429	714.0k	713.5k	0.1	1.0	714.0k	713.3k	0.1	42.3	714.0k	713.3k	0.1	0.8
dem_200_belg_orig_430	566.4k	566.4k	0.0	88.8	566.4k	566.4k	0.0	129.7	566.4k	566.4k	0.0	1282.9
dem_200_belg_orig_431	841.4k	841.4k	0.0	12.6	841.4k	841.4k	0.0	81.2	841.4k	841.4k	0.0	44.9
dem_200_belg_orig_432	427.5k	427.5k	0.0	10.1	427.5k	427.5k	0.0	96.7	427.5k	427.5k	0.0	10.1
dem_200_belg_orig_433	352.5k	352.5k	0.0	28.8	352.5k	352.5k	0.0	54.6	352.5k	352.5k	0.0	154.4
dem_200_belg_orig_434	727.7k	727.3k	0.0	1.2	727.7k	727.0k	0.1	25.4	727.7k	727.1k	0.1	1.6
dem_200_belg_orig_435	567.5k	567.5k	0.0	46.5	567.5k	567.5k	0.0	284.4	567.5k	567.4k	0.0	144.5
dem_200_belg_orig_436	404.3k	404.3k	0.0	288.5	404.3k	404.3k	0.0	946.4	404.3k	404.3k	0.0	504.0
dem_200_belg_orig_437	521.0k	521.0k	0.0	90.1	521.0k	521.0k	0.0	416.6	521.0k	520.7k	0.1	2999.3
dem_200_belg_orig_438	537.5k	537.5k	0.0	40.6	537.5k	537.5k	0.0	410.8	537.5k	537.5k	0.0	248.4
dem_200_belg_orig_439	636.8k	636.8k	0.0	70.1	636.8k	636.8k	0.0	325.7	636.8k	636.8k	0.0	1741.6
dem_200_belg_orig_440	774.0k	773.4k	0.1	153.0	774.0k	774.0k	0.0	160.0	774.0k	774.0k	0.0	351.2
dem_200_belg_orig_441	418.8k	418.4k	0.1	0.8	418.8k	418.8k	0.0	62.8	418.8k	418.8k	0.0	3.6
dem_200_belg_orig_442	396.1k	396.1k	0.0	33.0	396.1k	395.9k	0.1	101.8	396.1k	396.1k	0.0	57.1
dem_200_belg_orig_443	615.8k	615.8k	0.0	0.4	615.8k	615.8k	0.0	25.9	615.9k	615.8k	0.0	1.6
dem_200_belg_orig_444	301.3k	301.3k	0.0	134.3	301.3k	301.3k	0.0	278.4	301.3k	301.3k	0.0	220.5

continued on next page

Table A.3: Comparison of discrete models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_445	867.6k	867.6k	0.0	29.1	867.6k	867.6k	0.0	125.0	867.6k	867.6k	0.0	77.0
dem_200_belg_orig_446	407.4k	407.1k	0.1	0.4	407.4k	407.4k	0.0	25.6	407.4k	407.4k	0.0	0.8
dem_200_belg_orig_447	426.4k	426.1k	0.1	0.5	426.4k	426.4k	0.0	34.5	426.4k	426.4k	0.0	0.8
dem_200_belg_orig_448	436.9k	436.9k	0.0	3924.8	436.9k	436.9k	0.0	1985.8	438.6k	386.5k	13.5	14400.0
dem_200_belg_orig_449	594.3k	593.7k	0.1	1.3	594.3k	594.3k	0.0	149.7	594.4k	593.8k	0.1	1.0
dem_200_belg_orig_450	415.4k	415.4k	0.0	106.8	415.4k	415.4k	0.0	382.0	415.4k	415.1k	0.1	2757.2
dem_200_belg_orig_451	400.0k	400.0k	0.0	3226.3	400.0k	400.0k	0.0	3168.1	400.0k	369.0k	8.4	14400.0
dem_200_belg_orig_452	510.4k	510.4k	0.0	36.0	510.4k	510.4k	0.0	357.2	510.4k	510.4k	0.0	385.9
dem_200_belg_orig_453	742.3k	742.3k	0.0	392.0	742.3k	742.3k	0.0	268.1	742.3k	741.9k	0.1	2889.8
dem_200_belg_orig_454	416.7k	416.7k	0.0	135.9	416.7k	416.7k	0.0	797.0	416.7k	416.7k	0.0	1891.3
dem_200_belg_orig_455	299.0k	299.0k	0.0	4.8	299.0k	298.8k	0.1	35.6	299.0k	299.0k	0.0	126.9
dem_200_belg_orig_456	584.9k	584.9k	0.0	270.3	584.9k	584.9k	0.0	607.2	584.9k	584.9k	0.0	13421.9
dem_200_belg_orig_457	468.7k	468.7k	0.0	4.5	468.7k	468.7k	0.0	46.4	468.7k	468.6k	0.0	0.2
dem_200_belg_orig_458	906.7k	866.6k	4.6	14400.0	904.0k	904.0k	0.0	3489.3	905.1k	841.3k	7.6	14400.0
dem_200_belg_orig_459	215.3k	215.3k	0.0	0.3	215.3k	215.2k	0.1	14.8	215.3k	215.3k	0.0	0.4
dem_200_belg_orig_460	393.2k	393.2k	0.0	91.8	393.2k	393.2k	0.0	186.6	393.2k	393.1k	0.0	297.6
dem_200_belg_orig_461	416.6k	416.6k	0.0	2.4	416.6k	416.6k	0.0	78.6	416.6k	416.4k	0.0	0.3
dem_200_belg_orig_462	527.7k	527.4k	0.1	53.2	527.7k	527.7k	0.0	804.9	527.7k	527.7k	0.0	118.8
dem_200_belg_orig_463	418.3k	418.3k	0.0	1.3	418.3k	418.3k	0.0	41.3	418.3k	417.9k	0.1	1.4
dem_200_belg_orig_464	367.9k	367.6k	0.1	0.1	367.9k	367.9k	0.0	30.9	367.9k	367.6k	0.1	0.3
dem_200_belg_orig_465	478.2k	477.9k	0.1	0.5	478.2k	477.8k	0.1	40.6	478.2k	477.7k	0.1	1.4
dem_200_belg_orig_466	486.2k	486.2k	0.0	1.4	486.2k	486.0k	0.0	81.4	486.2k	486.2k	0.0	42.9
dem_200_belg_orig_467	684.4k	684.1k	0.0	0.3	684.4k	684.4k	0.0	39.9	684.4k	684.1k	0.1	0.6
dem_200_belg_orig_468	490.0k	417.3k	17.4	14400.0	485.9k	485.9k	0.0	820.6	490.1k	404.6k	21.1	14400.0
dem_200_belg_orig_469	538.9k	538.8k	0.0	112.8	538.9k	538.9k	0.0	342.2	538.9k	538.4k	0.1	243.6
dem_200_belg_orig_470	419.1k	419.1k	0.0	841.9	419.1k	419.1k	0.0	418.7	419.1k	394.5k	6.2	14400.0
dem_200_belg_orig_471	544.7k	544.7k	0.0	35.5	544.7k	544.7k	0.0	1401.8	544.7k	544.7k	0.0	361.4
dem_200_belg_orig_472	295.3k	295.3k	0.0	0.3	295.3k	295.3k	0.0	65.4	295.3k	295.3k	0.0	0.6
dem_200_belg_orig_473	652.5k	652.3k	0.0	9.6	652.5k	652.5k	0.0	131.9	652.5k	652.5k	0.0	9.2
dem_200_belg_orig_474	329.1k	329.1k	0.0	443.9	329.1k	329.1k	0.0	415.1	329.1k	329.1k	0.0	731.5
dem_200_belg_orig_475	335.7k	335.7k	0.0	13.7	335.7k	335.7k	0.0	402.4	335.7k	335.7k	0.0	24.3
dem_200_belg_orig_476	586.2k	586.2k	0.0	60.0	586.2k	586.2k	0.0	87.9	586.2k	586.2k	0.0	406.9
dem_200_belg_orig_477	573.9k	573.9k	0.0	287.4	573.9k	573.9k	0.0	2414.8	573.9k	573.9k	0.0	3117.9
dem_200_belg_orig_478	547.0k	547.0k	0.0	109.2	547.0k	546.8k	0.0	273.0	547.0k	547.0k	0.0	1422.7
dem_200_belg_orig_479	639.1k	638.6k	0.1	2.2	639.1k	638.6k	0.1	26.3	639.1k	638.7k	0.1	1.6
dem_200_belg_orig_480	360.4k	360.4k	0.0	3350.5	360.4k	360.4k	0.0	1482.8	361.5k	323.0k	11.9	14400.0
dem_200_belg_orig_481	491.7k	491.7k	0.0	5.8	491.7k	491.6k	0.0	74.5	491.7k	491.7k	0.0	8.5
dem_200_belg_orig_482	438.8k	438.6k	0.1	40.5	438.8k	438.8k	0.0	436.2	438.8k	438.8k	0.0	197.3
dem_200_belg_orig_483	294.3k	294.2k	0.0	0.4	294.3k	294.3k	0.0	36.4	294.3k	294.1k	0.1	0.7
dem_200_belg_orig_484	309.5k	309.5k	0.0	14.5	309.5k	309.5k	0.0	203.7	309.5k	309.5k	0.0	66.7
dem_200_belg_orig_485	275.0k	275.0k	0.0	13.5	275.0k	275.0k	0.0	60.8	275.0k	275.0k	0.0	7.2
dem_200_belg_orig_486	729.7k	729.2k	0.1	11.2	729.7k	729.7k	0.0	59.3	729.7k	729.6k	0.0	3.8
dem_200_belg_orig_487	347.5k	347.5k	0.0	8.5	347.5k	347.5k	0.0	138.5	347.5k	347.5k	0.0	69.2
dem_200_belg_orig_488	628.8k	628.8k	0.0	8.3	628.8k	628.2k	0.1	51.8	628.8k	628.8k	0.0	3.4
dem_200_belg_orig_489	723.5k	723.5k	0.0	19.7	723.5k	723.5k	0.0	74.1	723.5k	723.5k	0.0	11.3
dem_200_belg_orig_490	334.0k	334.0k	0.0	30.2	334.0k	334.0k	0.0	157.6	334.0k	334.0k	0.0	90.4
dem_200_belg_orig_491	429.9k	429.9k	0.0	202.8	429.9k	429.5k	0.1	175.0	429.9k	429.9k	0.0	9374.8
dem_200_belg_orig_492	303.4k	303.4k	0.0	80.5	303.4k	303.2k	0.0	852.9	303.4k	303.4k	0.0	45.8
dem_200_belg_orig_493	490.7k	490.7k	0.0	27.8	490.7k	490.7k	0.0	181.2	490.7k	490.7k	0.0	70.6
dem_200_belg_orig_494	540.0k	540.0k	0.0	6.2	540.0k	540.0k	0.0	146.4	540.0k	540.0k	0.0	14.0
dem_200_belg_orig_495	639.0k	639.0k	0.0	15.3	639.0k	639.0k	0.0	100.9	639.0k	638.5k	0.1	41.0
dem_200_belg_orig_496	417.5k	417.5k	0.0	74.0	417.5k	417.5k	0.0	886.2	417.5k	417.3k	0.0	320.5
dem_200_belg_orig_497	596.3k	595.8k	0.1	11.6	596.3k	595.8k	0.1	105.3	596.3k	596.3k	0.0	1.5
dem_200_belg_orig_498	590.8k	590.4k	0.1	0.3	590.8k	590.8k	0.0	27.4	590.8k	590.3k	0.1	0.2
dem_200_belg_orig_499	489.6k	489.6k	0.0	108.8	489.6k	489.6k	0.0	968.1	489.6k	489.6k	0.0	634.0
dem_200_belg_orig_500	330.3k	330.3k	0.0	380.7	330.3k	330.3k	0.0	379.0	330.3k	330.3k	0.0	801.2

Table A.4: Detailed results of the split-pipe models on *Belgium* with $\mathcal{B} = 200$ as summarized in Figure 3.13b and Table 3.4a. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_1	750.2k	750.2k	0.0	0.8	750.2k	750.2k	0.0	0.9
dem_200_belg_orig_2	674.5k	674.5k	0.0	1.1	674.5k	674.5k	0.0	0.6
dem_200_belg_orig_3	515.3k	515.3k	0.0	1.1	515.3k	515.3k	0.0	1.6
dem_200_belg_orig_4	793.4k	792.7k	0.1	1.2	793.4k	792.7k	0.1	0.8
dem_200_belg_orig_5	728.4k	727.7k	0.1	12.1	728.4k	727.7k	0.1	2.9
dem_200_belg_orig_6	326.2k	326.2k	0.0	0.9	326.2k	326.2k	0.0	0.6
dem_200_belg_orig_7	858.6k	857.8k	0.1	13.0	858.6k	857.8k	0.1	57.4

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Table A.4: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_8	206.3k	206.1k	0.1	30.8	206.3k	206.1k	0.1	207.2
dem_200_belg_orig_9	717.3k	716.8k	0.1	6.9	717.3k	716.7k	0.1	4.3
dem_200_belg_orig_10	736.4k	736.4k	0.0	1.1	736.4k	736.4k	0.0	0.9
dem_200_belg_orig_11	454.8k	454.8k	0.0	1.9	454.8k	454.8k	0.0	0.4
dem_200_belg_orig_12	349.7k	349.3k	0.1	11811.3	349.7k	349.3k	0.1	8988.0
dem_200_belg_orig_13	628.3k	628.1k	0.0	2.3	628.1k	628.1k	0.0	0.5
dem_200_belg_orig_14	777.5k	777.5k	0.0	0.8	777.5k	777.5k	0.0	0.6
dem_200_belg_orig_15	447.8k	447.4k	0.1	1.0	447.8k	447.5k	0.1	0.9
dem_200_belg_orig_16	830.1k	829.3k	0.1	299.9	830.1k	829.3k	0.1	54.0
dem_200_belg_orig_17	420.8k	420.8k	0.0	1.4	420.8k	420.8k	0.0	0.9
dem_200_belg_orig_18	782.2k	781.6k	0.1	2.0	782.4k	781.9k	0.1	1.5
dem_200_belg_orig_19	425.0k	425.0k	0.0	1.8	425.0k	425.0k	0.0	1.4
dem_200_belg_orig_20	563.8k	563.3k	0.1	2.5	563.8k	563.5k	0.1	2.8
dem_200_belg_orig_21	190.3k	190.3k	0.0	0.7	190.3k	190.3k	0.0	0.7
dem_200_belg_orig_22	557.9k	557.4k	0.1	15.9	557.9k	557.3k	0.1	6.8
dem_200_belg_orig_23	450.0k	449.8k	0.1	0.9	450.0k	449.7k	0.1	0.8
dem_200_belg_orig_24	502.0k	502.0k	0.0	1.3	502.0k	502.0k	0.0	0.7
dem_200_belg_orig_25	508.0k	507.5k	0.1	110.5	508.0k	507.5k	0.1	2971.9
dem_200_belg_orig_26	658.3k	658.3k	0.0	1.6	658.3k	658.3k	0.0	0.7
dem_200_belg_orig_27	357.7k	357.7k	0.0	1.4	357.7k	357.7k	0.0	0.4
dem_200_belg_orig_28	591.4k	590.9k	0.1	1.4	591.4k	591.0k	0.1	1.6
dem_200_belg_orig_29	467.8k	467.8k	0.0	1.5	467.8k	467.8k	0.0	0.3
dem_200_belg_orig_30	510.8k	510.6k	0.1	4.2	510.8k	510.4k	0.1	1.4
dem_200_belg_orig_31	392.1k	392.1k	0.0	1.6	392.1k	392.1k	0.0	0.8
dem_200_belg_orig_32	647.3k	647.3k	0.0	1.1	647.3k	647.3k	0.0	1.0
dem_200_belg_orig_33	302.7k	293.5k	3.1	14400.0	302.7k	302.4k	0.1	8.9
dem_200_belg_orig_34	289.8k	289.8k	0.0	12.4	289.8k	289.8k	0.0	58.6
dem_200_belg_orig_35	439.8k	439.8k	0.0	0.6	439.8k	439.8k	0.0	0.5
dem_200_belg_orig_36	741.1k	741.1k	0.0	1.6	741.1k	741.1k	0.0	0.7
dem_200_belg_orig_37	313.3k	313.0k	0.1	2.7	313.3k	313.0k	0.1	5.0
dem_200_belg_orig_38	702.4k	701.7k	0.1	30.6	702.4k	701.7k	0.1	307.3
dem_200_belg_orig_39	338.8k	338.4k	0.1	292.0	338.8k	338.4k	0.1	9.2
dem_200_belg_orig_40	486.6k	486.2k	0.1	7.0	486.6k	486.1k	0.1	5.7
dem_200_belg_orig_41	606.3k	605.8k	0.1	298.6	606.3k	605.7k	0.1	12.8
dem_200_belg_orig_42	472.4k	472.1k	0.1	4.3	472.4k	472.0k	0.1	3.8
dem_200_belg_orig_43	628.9k	628.6k	0.1	1.3	628.9k	628.3k	0.1	0.9
dem_200_belg_orig_44	435.6k	435.2k	0.1	1.7	435.6k	435.2k	0.1	2.0
dem_200_belg_orig_45	193.7k	193.6k	0.1	1.0	193.7k	193.5k	0.1	0.8
dem_200_belg_orig_46	684.3k	684.1k	0.0	0.9	684.3k	684.1k	0.0	0.8
dem_200_belg_orig_47	409.3k	408.9k	0.1	1.0	409.5k	409.1k	0.1	0.8
dem_200_belg_orig_48	382.0k	381.6k	0.1	1.1	382.0k	381.6k	0.1	0.7
dem_200_belg_orig_49	311.7k	311.4k	0.1	574.3	311.7k	311.4k	0.1	149.8
dem_200_belg_orig_50	635.2k	635.2k	0.0	1.1	635.2k	635.1k	0.0	0.9
dem_200_belg_orig_51	338.4k	338.3k	0.0	1.7	338.4k	338.3k	0.0	0.5
dem_200_belg_orig_52	556.0k	556.0k	0.0	1.2	556.0k	556.0k	0.0	1.4
dem_200_belg_orig_53	385.5k	385.5k	0.0	0.4	385.5k	385.5k	0.0	0.5
dem_200_belg_orig_54	608.0k	607.5k	0.1	1.3	608.0k	607.7k	0.1	0.9
dem_200_belg_orig_55	402.4k	402.2k	0.0	11.5	402.4k	402.0k	0.1	5.2
dem_200_belg_orig_56	577.0k	576.8k	0.0	1.1	577.0k	576.8k	0.0	0.4
dem_200_belg_orig_57	484.3k	484.1k	0.0	3.3	484.3k	484.0k	0.1	4.3
dem_200_belg_orig_58	604.1k	603.6k	0.1	1.5	604.1k	604.1k	0.0	1.9
dem_200_belg_orig_59	271.9k	271.9k	0.0	0.9	271.9k	271.9k	0.0	0.8
dem_200_belg_orig_60	565.6k	565.6k	0.0	1.2	565.6k	565.6k	0.0	0.3
dem_200_belg_orig_61	595.4k	595.3k	0.0	2.6	595.4k	595.0k	0.1	0.8
dem_200_belg_orig_62	737.6k	736.9k	0.1	1.7	737.6k	736.9k	0.1	1.2
dem_200_belg_orig_63	482.1k	481.6k	0.1	4.4	482.1k	481.7k	0.1	5.6
dem_200_belg_orig_64	276.8k	276.8k	0.0	3.5	276.8k	276.7k	0.0	1.2
dem_200_belg_orig_65	744.2k	744.2k	0.0	1.3	744.2k	744.2k	0.0	0.9
dem_200_belg_orig_66	624.8k	624.4k	0.1	0.8	624.8k	624.4k	0.1	0.6
dem_200_belg_orig_67	525.3k	525.3k	0.0	1.1	525.3k	525.3k	0.0	1.2
dem_200_belg_orig_68	572.2k	571.8k	0.1	3.2	572.2k	571.9k	0.1	1.5
dem_200_belg_orig_69	760.0k	760.0k	0.0	0.9	760.0k	759.5k	0.1	0.9
dem_200_belg_orig_70	581.6k	581.6k	0.0	1.2	581.6k	581.6k	0.0	0.4
dem_200_belg_orig_71	650.0k	649.6k	0.1	2.3	650.0k	649.6k	0.1	0.7
dem_200_belg_orig_72	360.9k	360.6k	0.1	13.1	360.9k	360.6k	0.1	9.6
dem_200_belg_orig_73	264.2k	264.1k	0.0	1.3	264.2k	264.1k	0.0	0.8
dem_200_belg_orig_74	635.5k	635.1k	0.1	1.2	635.5k	635.1k	0.1	1.1
dem_200_belg_orig_75	600.3k	600.3k	0.0	0.7	600.3k	600.3k	0.0	1.0
dem_200_belg_orig_76	261.8k	261.6k	0.1	14.5	261.8k	261.6k	0.1	17.8
dem_200_belg_orig_77	491.0k	491.0k	0.0	0.9	491.0k	491.0k	0.0	0.6
dem_200_belg_orig_78	330.0k	330.0k	0.0	1.1	330.0k	330.0k	0.0	0.9
dem_200_belg_orig_79	361.3k	361.2k	0.0	0.9	361.3k	361.3k	0.0	0.6
dem_200_belg_orig_80	596.0k	596.0k	0.0	0.7	596.0k	596.0k	0.0	0.3
dem_200_belg_orig_81	505.4k	505.4k	0.0	0.7	505.4k	505.4k	0.0	0.6
dem_200_belg_orig_82	195.8k	195.7k	0.1	2.4	195.8k	195.6k	0.1	2.6
dem_200_belg_orig_83	562.1k	562.1k	0.0	1.0	562.1k	562.1k	0.0	0.5
dem_200_belg_orig_84	575.9k	575.7k	0.0	0.9	575.9k	575.7k	0.0	1.0
dem_200_belg_orig_85	598.3k	598.3k	0.0	1.4	598.3k	598.3k	0.0	0.8

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Table A.4: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_86	561.9k	561.9k	0.0	1.2	561.9k	561.9k	0.0	1.0
dem_200_belg_orig_87	386.9k	386.9k	0.0	1.3	387.2k	386.9k	0.1	1.0
dem_200_belg_orig_88	526.8k	526.8k	0.0	0.7	526.8k	526.8k	0.0	0.5
dem_200_belg_orig_89	1107.0k	1105.9k	0.1	3.7	1106.8k	1105.7k	0.1	12.1
dem_200_belg_orig_90	546.2k	546.1k	0.0	1.0	546.2k	546.1k	0.0	0.9
dem_200_belg_orig_91	647.9k	647.5k	0.1	1.1	647.9k	647.5k	0.1	0.8
dem_200_belg_orig_92	870.3k	869.7k	0.1	1.7	870.3k	869.7k	0.1	1.1
dem_200_belg_orig_93	955.2k	955.2k	0.0	2.0	955.2k	955.2k	0.0	0.7
dem_200_belg_orig_94	469.8k	469.8k	0.0	2.8	469.8k	469.8k	0.0	3.4
dem_200_belg_orig_95	358.7k	356.4k	0.6	14400.0	358.7k	357.1k	0.5	14400.0
dem_200_belg_orig_96	388.1k	388.0k	0.0	0.5	388.1k	388.0k	0.0	0.2
dem_200_belg_orig_97	292.0k	292.0k	0.0	0.6	292.0k	292.0k	0.0	0.5
dem_200_belg_orig_98	565.3k	565.3k	0.0	0.8	565.3k	565.3k	0.0	0.6
dem_200_belg_orig_99	612.0k	611.4k	0.1	3.4	612.0k	611.4k	0.1	2.2
dem_200_belg_orig_100	602.0k	601.5k	0.1	1.7	602.0k	601.5k	0.1	0.7
dem_200_belg_orig_101	436.3k	436.2k	0.0	0.7	436.3k	436.2k	0.0	0.6
dem_200_belg_orig_102	725.6k	725.2k	0.1	1.2	725.6k	725.2k	0.1	0.6
dem_200_belg_orig_103	365.8k	365.8k	0.0	2.5	365.8k	365.6k	0.0	77.7
dem_200_belg_orig_104	429.3k	428.8k	0.1	26.8	429.3k	428.9k	0.1	39.8
dem_200_belg_orig_105	304.3k	304.1k	0.1	1.7	304.3k	304.2k	0.1	0.7
dem_200_belg_orig_106	496.1k	496.1k	0.0	4.7	496.1k	496.0k	0.0	1.1
dem_200_belg_orig_107	656.8k	656.2k	0.1	1.2	657.0k	656.4k	0.1	0.9
dem_200_belg_orig_108	233.9k	233.7k	0.1	1.7	233.9k	233.7k	0.1	2.3
dem_200_belg_orig_109	662.8k	662.8k	0.0	1.9	662.8k	662.8k	0.0	1.0
dem_200_belg_orig_110	728.9k	728.9k	0.0	1.2	728.9k	728.9k	0.0	0.6
dem_200_belg_orig_111	516.4k	516.1k	0.1	1.1	516.4k	516.1k	0.1	1.0
dem_200_belg_orig_112	276.8k	276.8k	0.0	1.3	276.8k	276.8k	0.0	0.6
dem_200_belg_orig_113	282.9k	282.9k	0.0	4.5	282.9k	282.9k	0.0	2.0
dem_200_belg_orig_114	279.7k	279.7k	0.0	1.0	279.7k	279.7k	0.0	0.6
dem_200_belg_orig_115	626.3k	626.3k	0.0	3.1	626.3k	625.9k	0.1	1.4
dem_200_belg_orig_116	471.1k	471.1k	0.0	1.7	471.1k	471.1k	0.0	0.8
dem_200_belg_orig_117	369.7k	369.4k	0.1	1.6	369.7k	369.4k	0.1	1.8
dem_200_belg_orig_118	555.8k	555.4k	0.1	1.3	555.8k	555.4k	0.1	0.6
dem_200_belg_orig_119	605.5k	604.9k	0.1	874.0	605.5k	604.9k	0.1	201.1
dem_200_belg_orig_120	422.5k	422.1k	0.1	11.9	422.5k	422.1k	0.1	13.7
dem_200_belg_orig_121	353.3k	353.0k	0.1	1.2	353.1k	352.8k	0.1	1.2
dem_200_belg_orig_122	605.2k	605.1k	0.0	1.0	605.2k	605.1k	0.0	0.6
dem_200_belg_orig_123	376.0k	375.6k	0.1	0.9	376.0k	375.9k	0.0	0.7
dem_200_belg_orig_124	664.3k	663.8k	0.1	1.5	664.3k	663.9k	0.1	0.9
dem_200_belg_orig_125	271.2k	271.2k	0.0	1.1	271.2k	271.2k	0.0	0.8
dem_200_belg_orig_126	427.8k	427.4k	0.1	1.6	427.8k	427.4k	0.1	2.4
dem_200_belg_orig_127	540.3k	540.2k	0.0	1.2	540.3k	540.3k	0.0	1.2
dem_200_belg_orig_128	624.4k	624.4k	0.0	1.6	624.4k	624.4k	0.0	0.8
dem_200_belg_orig_129	330.3k	330.0k	0.1	318.8	330.3k	321.1k	2.9	14400.0
dem_200_belg_orig_130	701.4k	701.1k	0.0	1.3	701.4k	701.1k	0.0	0.8
dem_200_belg_orig_131	453.2k	453.2k	0.0	2.0	453.2k	453.2k	0.0	0.7
dem_200_belg_orig_132	341.2k	341.2k	0.0	0.6	341.2k	341.2k	0.0	0.6
dem_200_belg_orig_133	600.3k	599.8k	0.1	2.9	600.3k	599.7k	0.1	8.5
dem_200_belg_orig_134	889.1k	889.1k	0.0	1.0	889.1k	889.1k	0.0	0.5
dem_200_belg_orig_135	383.5k	383.5k	0.0	0.9	383.5k	383.5k	0.0	2.1
dem_200_belg_orig_136	1128.6k	1127.6k	0.1	18.0	1128.6k	1127.5k	0.1	4.0
dem_200_belg_orig_137	814.6k	813.9k	0.1	2.5	813.9k	813.9k	0.0	1.2
dem_200_belg_orig_138	690.9k	690.9k	0.0	0.9	690.9k	690.9k	0.0	0.7
dem_200_belg_orig_139	365.5k	365.1k	0.1	3.0	365.5k	365.4k	0.0	0.9
dem_200_belg_orig_140	435.8k	435.1k	0.2	14400.0	435.8k	435.4k	0.1	6679.0
dem_200_belg_orig_141	622.8k	622.6k	0.0	0.8	622.8k	622.6k	0.0	0.6
dem_200_belg_orig_142	699.1k	699.1k	0.0	2.2	699.1k	699.1k	0.0	1.0
dem_200_belg_orig_143	259.8k	259.6k	0.1	1.7	259.8k	259.7k	0.1	0.7
dem_200_belg_orig_144	307.0k	307.0k	0.0	3.7	307.0k	307.0k	0.0	3.3
dem_200_belg_orig_145	540.9k	540.4k	0.1	7.8	540.9k	540.4k	0.1	98.5
dem_200_belg_orig_146	383.6k	383.6k	0.0	2.1	383.6k	383.6k	0.0	0.7
dem_200_belg_orig_147	598.4k	598.4k	0.0	1.2	598.4k	598.4k	0.0	0.5
dem_200_belg_orig_148	557.2k	556.9k	0.1	4.8	557.2k	556.9k	0.1	5.7
dem_200_belg_orig_149	410.9k	410.5k	0.1	62.4	410.9k	410.5k	0.1	3.7
dem_200_belg_orig_150	825.9k	825.7k	0.0	0.9	825.9k	825.7k	0.0	0.9
dem_200_belg_orig_151	353.0k	353.0k	0.0	0.9	353.0k	353.0k	0.0	1.1
dem_200_belg_orig_152	358.6k	358.5k	0.0	0.8	358.6k	358.6k	0.0	0.7
dem_200_belg_orig_153	654.0k	653.7k	0.0	0.9	654.0k	653.7k	0.0	1.6
dem_200_belg_orig_154	438.0k	437.8k	0.1	1.6	437.9k	437.5k	0.1	1.5
dem_200_belg_orig_155	672.6k	672.6k	0.0	1.8	672.6k	672.6k	0.0	1.0
dem_200_belg_orig_156	353.6k	353.6k	0.0	0.6	353.6k	353.6k	0.0	0.8
dem_200_belg_orig_157	574.5k	574.5k	0.0	1.2	574.5k	574.5k	0.0	0.9
dem_200_belg_orig_158	631.1k	630.6k	0.1	179.6	631.1k	630.5k	0.1	338.1
dem_200_belg_orig_159	385.0k	384.9k	0.0	0.8	385.0k	384.9k	0.0	0.8
dem_200_belg_orig_160	562.6k	562.5k	0.0	4.5	562.6k	562.4k	0.0	1.3
dem_200_belg_orig_161	534.8k	534.8k	0.0	1.6	535.2k	534.8k	0.1	1.0
dem_200_belg_orig_162	644.2k	644.1k	0.0	1.0	644.2k	644.1k	0.0	0.9
dem_200_belg_orig_163	780.7k	780.0k	0.1	1.4	780.7k	780.0k	0.1	0.9

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Table A.4: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_164	442.0k	441.6k	0.1	1.9	442.0k	441.6k	0.1	0.8
dem_200_belg_orig_165	376.4k	376.0k	0.1	2.5	376.4k	376.0k	0.1	1.0
dem_200_belg_orig_166	333.5k	333.5k	0.0	0.5	333.5k	333.5k	0.0	0.5
dem_200_belg_orig_167	713.6k	713.6k	0.0	1.2	713.6k	713.6k	0.0	1.1
dem_200_belg_orig_168	560.9k	560.9k	0.0	1.2	560.9k	560.9k	0.0	0.7
dem_200_belg_orig_169	322.6k	320.6k	0.6	14400.0	322.6k	322.6k	0.0	3.8
dem_200_belg_orig_170	545.7k	545.6k	0.0	1.6	545.7k	545.6k	0.0	0.8
dem_200_belg_orig_171	395.9k	395.9k	0.0	0.5	395.9k	395.9k	0.0	0.7
dem_200_belg_orig_172	624.9k	624.4k	0.1	1.8	624.9k	624.7k	0.0	1.2
dem_200_belg_orig_173	797.8k	797.8k	0.0	1.3	797.8k	797.8k	0.0	0.5
dem_200_belg_orig_174	528.3k	528.3k	0.0	0.5	528.3k	528.3k	0.0	0.3
dem_200_belg_orig_175	223.3k	223.3k	0.0	0.6	223.3k	223.3k	0.0	0.2
dem_200_belg_orig_176	640.0k	639.9k	0.0	1.1	640.0k	640.0k	0.0	1.2
dem_200_belg_orig_177	718.8k	718.8k	0.0	4.2	718.8k	718.2k	0.1	3.4
dem_200_belg_orig_178	351.8k	351.8k	0.0	1.0	351.8k	351.8k	0.0	0.8
dem_200_belg_orig_179	419.9k	419.6k	0.1	2.1	420.1k	419.7k	0.1	1.2
dem_200_belg_orig_180	334.9k	334.9k	0.0	1.0	334.9k	334.8k	0.1	0.9
dem_200_belg_orig_181	313.0k	313.0k	0.0	0.7	313.0k	313.0k	0.0	0.7
dem_200_belg_orig_182	568.9k	568.8k	0.0	1.7	568.9k	568.9k	0.0	0.8
dem_200_belg_orig_183	310.2k	309.9k	0.1	1744.7	310.2k	309.9k	0.1	17.9
dem_200_belg_orig_184	313.0k	312.7k	0.1	2.4	313.0k	313.0k	0.0	46.5
dem_200_belg_orig_185	381.2k	381.2k	0.0	0.5	381.2k	381.2k	0.0	0.8
dem_200_belg_orig_186	590.5k	590.3k	0.0	0.9	590.5k	590.3k	0.0	0.5
dem_200_belg_orig_187	818.2k	817.4k	0.1	1.3	818.2k	817.5k	0.1	1.1
dem_200_belg_orig_188	252.1k	252.1k	0.0	2.2	252.1k	252.1k	0.0	0.5
dem_200_belg_orig_189	579.9k	579.5k	0.1	2.0	579.9k	579.5k	0.1	1.3
dem_200_belg_orig_190	385.2k	384.9k	0.1	72.3	385.2k	384.8k	0.1	111.1
dem_200_belg_orig_191	663.9k	663.7k	0.0	0.9	663.9k	663.7k	0.0	0.7
dem_200_belg_orig_192	614.7k	614.6k	0.0	1.5	614.7k	614.6k	0.0	0.5
dem_200_belg_orig_193	458.9k	458.5k	0.1	2.6	458.9k	458.5k	0.1	2.1
dem_200_belg_orig_194	549.5k	549.1k	0.1	0.5	549.5k	549.0k	0.1	0.8
dem_200_belg_orig_195	488.3k	487.9k	0.1	2.3	488.4k	488.1k	0.1	2.0
dem_200_belg_orig_196	636.7k	636.0k	0.1	1578.6	636.7k	636.0k	0.1	856.3
dem_200_belg_orig_197	878.8k	878.0k	0.1	1.0	878.8k	877.9k	0.1	0.9
dem_200_belg_orig_198	468.7k	468.5k	0.0	1.7	468.7k	468.5k	0.0	0.9
dem_200_belg_orig_199	276.4k	276.1k	0.1	0.8	276.4k	276.4k	0.0	1.1
dem_200_belg_orig_200	651.8k	651.5k	0.0	1.1	651.8k	651.5k	0.0	0.8
dem_200_belg_orig_201	310.4k	310.4k	0.0	91.3	310.4k	310.1k	0.1	46.3
dem_200_belg_orig_202	460.9k	460.9k	0.0	0.8	460.9k	460.9k	0.0	1.6
dem_200_belg_orig_203	516.8k	516.7k	0.0	1.0	516.8k	516.7k	0.0	0.8
dem_200_belg_orig_204	559.5k	559.1k	0.1	1.7	559.5k	559.1k	0.1	1.1
dem_200_belg_orig_205	623.0k	622.8k	0.0	6.2	623.0k	622.4k	0.1	1.4
dem_200_belg_orig_206	305.7k	305.4k	0.1	4.0	305.7k	305.5k	0.1	2.7
dem_200_belg_orig_207	573.9k	573.5k	0.1	0.7	573.9k	573.9k	0.0	1.0
dem_200_belg_orig_208	555.9k	555.9k	0.0	0.6	555.9k	555.9k	0.0	0.6
dem_200_belg_orig_209	313.0k	302.5k	3.5	14400.0	313.0k	312.7k	0.1	6.3
dem_200_belg_orig_210	608.8k	608.8k	0.0	1.1	608.8k	608.8k	0.0	0.3
dem_200_belg_orig_211	722.1k	721.5k	0.1	0.9	722.1k	721.6k	0.1	1.1
dem_200_belg_orig_212	355.2k	354.8k	0.1	1.4	355.2k	354.9k	0.1	1.3
dem_200_belg_orig_213	748.4k	748.4k	0.0	1.0	748.4k	748.4k	0.0	0.8
dem_200_belg_orig_214	394.2k	393.8k	0.1	39.9	394.2k	393.9k	0.1	49.6
dem_200_belg_orig_215	361.9k	361.9k	0.0	0.7	361.9k	361.9k	0.0	0.8
dem_200_belg_orig_216	592.4k	592.4k	0.0	0.9	592.4k	592.4k	0.0	0.3
dem_200_belg_orig_217	518.1k	518.1k	0.0	1.5	518.1k	518.1k	0.0	0.9
dem_200_belg_orig_218	874.6k	873.7k	0.1	40.9	874.6k	873.7k	0.1	8.3
dem_200_belg_orig_219	636.9k	636.9k	0.0	1.3	636.9k	636.9k	0.0	0.9
dem_200_belg_orig_220	451.4k	451.0k	0.1	0.9	451.4k	451.1k	0.1	0.9
dem_200_belg_orig_221	288.9k	288.9k	0.0	0.9	288.9k	288.9k	0.0	1.0
dem_200_belg_orig_222	487.8k	487.6k	0.0	1.7	487.8k	487.6k	0.0	0.8
dem_200_belg_orig_223	421.2k	421.2k	0.0	0.8	421.2k	421.2k	0.0	0.7
dem_200_belg_orig_224	583.4k	583.4k	0.0	1.8	583.4k	583.4k	0.0	1.6
dem_200_belg_orig_225	500.5k	500.1k	0.1	1.6	500.5k	500.1k	0.1	0.8
dem_200_belg_orig_226	602.9k	602.9k	0.0	0.8	602.9k	602.9k	0.0	0.5
dem_200_belg_orig_227	262.0k	261.8k	0.1	0.8	262.0k	261.8k	0.1	0.8
dem_200_belg_orig_228	528.4k	528.4k	0.0	1.6	528.4k	528.4k	0.0	0.5
dem_200_belg_orig_229	713.8k	713.8k	0.0	1.1	713.8k	713.8k	0.0	0.9
dem_200_belg_orig_230	625.6k	625.6k	0.0	1.4	625.6k	625.6k	0.0	1.0
dem_200_belg_orig_231	410.8k	410.5k	0.1	1.9	410.7k	410.5k	0.0	0.5
dem_200_belg_orig_232	298.5k	298.2k	0.1	1.3	298.6k	298.5k	0.1	1.1
dem_200_belg_orig_233	537.2k	537.1k	0.0	1.2	537.2k	537.2k	0.0	1.1
dem_200_belg_orig_234	253.2k	253.2k	0.0	0.9	253.2k	253.2k	0.0	0.5
dem_200_belg_orig_235	345.5k	345.4k	0.0	1.0	345.5k	345.5k	0.0	0.9
dem_200_belg_orig_236	560.3k	560.3k	0.0	2.7	560.3k	560.3k	0.0	3.1
dem_200_belg_orig_237	527.4k	527.4k	0.0	0.9	527.4k	527.4k	0.0	0.8
dem_200_belg_orig_238	525.2k	525.2k	0.0	1.0	525.2k	525.2k	0.0	0.7
dem_200_belg_orig_239	707.7k	707.3k	0.1	0.3	707.7k	707.3k	0.1	0.6
dem_200_belg_orig_240	739.4k	738.9k	0.1	1.7	739.4k	738.9k	0.1	1.0
dem_200_belg_orig_241	672.6k	672.0k	0.1	1.6	672.6k	672.0k	0.1	0.9

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Table A.4: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_242	455.4k	455.4k	0.0	1.6	455.4k	455.1k	0.1	0.8
dem_200_belg_orig_243	575.9k	575.6k	0.1	1.4	575.9k	575.4k	0.1	1.4
dem_200_belg_orig_244	489.8k	489.4k	0.1	3.8	489.8k	489.4k	0.1	3.8
dem_200_belg_orig_245	290.3k	290.0k	0.1	394.0	290.3k	290.0k	0.1	7.1
dem_200_belg_orig_246	678.7k	678.7k	0.0	1.8	678.7k	678.7k	0.0	0.6
dem_200_belg_orig_247	416.7k	416.3k	0.1	27.8	416.7k	416.3k	0.1	7.3
dem_200_belg_orig_248	857.6k	857.6k	0.0	1.5	857.6k	856.7k	0.1	1.0
dem_200_belg_orig_249	347.5k	347.3k	0.1	1.5	347.5k	347.4k	0.1	0.9
dem_200_belg_orig_250	444.6k	444.5k	0.0	1.1	444.6k	444.5k	0.0	0.7
dem_200_belg_orig_251	547.4k	547.4k	0.0	0.7	547.4k	547.4k	0.0	0.5
dem_200_belg_orig_252	350.9k	350.5k	0.1	8.5	350.9k	350.5k	0.1	1.7
dem_200_belg_orig_253	815.7k	815.0k	0.1	0.9	815.7k	815.0k	0.1	0.9
dem_200_belg_orig_254	740.3k	739.6k	0.1	0.9	740.3k	739.6k	0.1	0.9
dem_200_belg_orig_255	185.3k	185.3k	0.0	1.9	185.3k	185.3k	0.0	1.0
dem_200_belg_orig_256	644.7k	644.6k	0.0	1.6	644.7k	644.6k	0.0	0.9
dem_200_belg_orig_257	440.3k	440.1k	0.0	1.9	440.3k	439.9k	0.1	44.6
dem_200_belg_orig_258	309.1k	309.1k	0.0	1.0	309.1k	309.1k	0.0	0.7
dem_200_belg_orig_259	487.0k	486.7k	0.1	2.1	487.0k	486.7k	0.1	2.6
dem_200_belg_orig_260	636.6k	636.1k	0.1	3.4	636.6k	636.3k	0.1	1.1
dem_200_belg_orig_261	144.1k	144.1k	0.0	2.2	144.1k	144.1k	0.0	0.4
dem_200_belg_orig_262	525.4k	525.2k	0.0	1.0	525.4k	525.0k	0.1	0.9
dem_200_belg_orig_263	682.0k	682.0k	0.0	2.2	682.0k	682.0k	0.0	0.6
dem_200_belg_orig_264	597.5k	597.5k	0.0	1.2	597.5k	597.5k	0.0	0.7
dem_200_belg_orig_265	914.3k	913.5k	0.1	3.8	913.8k	913.0k	0.1	1.4
dem_200_belg_orig_266	530.7k	530.6k	0.0	1.3	530.7k	530.6k	0.0	0.8
dem_200_belg_orig_267	813.1k	813.1k	0.0	1.4	813.1k	813.1k	0.0	1.1
dem_200_belg_orig_268	529.5k	529.0k	0.1	26.9	529.5k	529.0k	0.1	10.0
dem_200_belg_orig_269	488.6k	488.4k	0.0	3.0	488.6k	488.1k	0.1	3.7
dem_200_belg_orig_270	345.4k	345.0k	0.1	6358.3	345.4k	345.4k	0.0	3.6
dem_200_belg_orig_271	680.5k	680.5k	0.0	1.0	680.5k	680.5k	0.0	0.6
dem_200_belg_orig_272	798.1k	797.5k	0.1	1.7	798.1k	797.5k	0.1	1.0
dem_200_belg_orig_273	780.8k	780.7k	0.0	1.4	780.8k	780.7k	0.0	0.8
dem_200_belg_orig_274	447.0k	447.0k	0.0	1.6	447.0k	447.0k	0.0	0.7
dem_200_belg_orig_275	372.9k	372.9k	0.0	1.2	372.9k	372.9k	0.0	0.8
dem_200_belg_orig_276	241.7k	241.6k	0.0	1.0	241.7k	241.7k	0.0	0.8
dem_200_belg_orig_277	389.0k	388.8k	0.1	3.7	389.0k	388.6k	0.1	7.0
dem_200_belg_orig_278	364.7k	364.7k	0.0	0.6	364.7k	364.7k	0.0	0.5
dem_200_belg_orig_279	507.5k	507.3k	0.0	1.6	507.5k	507.2k	0.1	1.1
dem_200_belg_orig_280	717.8k	717.8k	0.0	2.1	717.8k	717.8k	0.0	0.6
dem_200_belg_orig_281	381.3k	381.3k	0.0	1.7	381.3k	381.3k	0.0	0.3
dem_200_belg_orig_282	628.4k	627.8k	0.1	8.1	628.2k	627.7k	0.1	1.3
dem_200_belg_orig_283	434.3k	433.9k	0.1	9.9	434.3k	433.9k	0.1	5.9
dem_200_belg_orig_284	489.1k	489.0k	0.0	1.2	489.1k	489.1k	0.0	1.4
dem_200_belg_orig_285	459.6k	459.4k	0.0	1.5	459.6k	459.2k	0.1	1.1
dem_200_belg_orig_286	507.2k	507.1k	0.0	1.3	507.2k	507.1k	0.0	0.8
dem_200_belg_orig_287	450.6k	450.2k	0.1	81.6	450.6k	450.3k	0.1	146.6
dem_200_belg_orig_288	417.6k	417.2k	0.1	1.0	417.6k	417.6k	0.0	1.9
dem_200_belg_orig_289	519.4k	519.4k	0.0	4.6	519.4k	518.9k	0.1	1.6
dem_200_belg_orig_290	679.5k	679.5k	0.0	6.4	679.5k	679.5k	0.0	4.9
dem_200_belg_orig_291	583.0k	582.4k	0.1	224.8	583.0k	582.5k	0.1	271.5
dem_200_belg_orig_292	348.8k	348.8k	0.0	0.9	349.1k	348.8k	0.1	0.8
dem_200_belg_orig_293	327.3k	327.0k	0.1	5.1	327.3k	327.0k	0.1	5.0
dem_200_belg_orig_294	464.7k	464.5k	0.0	1.4	464.7k	464.6k	0.0	2.3
dem_200_belg_orig_295	672.7k	672.5k	0.0	1.5	672.7k	672.6k	0.0	0.8
dem_200_belg_orig_296	580.8k	580.7k	0.0	2.0	580.8k	580.7k	0.0	0.6
dem_200_belg_orig_297	351.9k	351.6k	0.1	2.0	351.9k	351.6k	0.1	1.8
dem_200_belg_orig_298	345.4k	345.4k	0.0	1.4	345.4k	345.4k	0.0	0.6
dem_200_belg_orig_299	720.1k	719.3k	0.1	2.7	720.1k	719.3k	0.1	1.5
dem_200_belg_orig_300	326.1k	325.8k	0.1	244.1	326.1k	325.8k	0.1	8.6
dem_200_belg_orig_301	401.1k	401.1k	0.0	1.2	401.1k	401.1k	0.0	0.9
dem_200_belg_orig_302	569.1k	569.1k	0.0	0.8	569.1k	569.1k	0.0	0.6
dem_200_belg_orig_303	856.4k	855.5k	0.1	180.5	856.4k	855.6k	0.1	1.4
dem_200_belg_orig_304	679.1k	678.5k	0.1	1.0	679.1k	678.5k	0.1	0.8
dem_200_belg_orig_305	458.2k	458.2k	0.0	0.6	458.2k	458.2k	0.0	0.4
dem_200_belg_orig_306	619.4k	619.4k	0.0	1.0	619.4k	619.4k	0.0	0.9
dem_200_belg_orig_307	286.9k	286.9k	0.0	0.9	286.9k	286.9k	0.0	0.3
dem_200_belg_orig_308	433.4k	433.1k	0.1	6.8	433.4k	433.0k	0.1	32.9
dem_200_belg_orig_309	535.1k	534.6k	0.1	5.3	534.9k	534.3k	0.1	328.5
dem_200_belg_orig_310	302.9k	302.7k	0.1	5.5	303.0k	302.7k	0.1	1.9
dem_200_belg_orig_311	423.3k	422.9k	0.1	1.1	423.3k	422.9k	0.1	0.8
dem_200_belg_orig_312	321.6k	321.3k	0.1	2.5	321.6k	321.3k	0.1	2.0
dem_200_belg_orig_313	741.0k	740.5k	0.1	1.1	741.0k	741.0k	0.0	1.7
dem_200_belg_orig_314	847.6k	847.0k	0.1	1.8	847.6k	847.6k	0.0	1.7
dem_200_belg_orig_315	662.6k	662.6k	0.0	1.0	662.6k	662.6k	0.0	0.6
dem_200_belg_orig_316	347.4k	347.4k	0.0	1.6	347.4k	347.4k	0.0	0.5
dem_200_belg_orig_317	596.9k	596.6k	0.1	1.9	596.9k	596.5k	0.1	1.9
dem_200_belg_orig_318	283.2k	283.0k	0.0	0.7	283.2k	283.0k	0.0	0.6
dem_200_belg_orig_319	307.0k	306.7k	0.1	8.7	307.0k	306.6k	0.1	8.4

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Table A.4: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_320	449.4k	449.4k	0.0	1.3	449.4k	449.4k	0.0	1.0
dem_200_belg_orig_321	309.8k	309.5k	0.1	143.1	309.8k	309.5k	0.1	586.0
dem_200_belg_orig_322	446.9k	446.6k	0.1	2.8	446.8k	446.5k	0.1	4.2
dem_200_belg_orig_323	362.2k	362.2k	0.0	2.6	362.2k	362.2k	0.0	0.8
dem_200_belg_orig_324	555.9k	555.7k	0.0	1.0	555.9k	555.7k	0.0	0.9
dem_200_belg_orig_325	692.8k	692.2k	0.1	2.0	692.8k	692.2k	0.1	0.5
dem_200_belg_orig_326	581.5k	581.2k	0.0	0.9	581.5k	581.2k	0.0	1.0
dem_200_belg_orig_327	361.8k	361.8k	0.0	0.8	361.8k	361.8k	0.0	1.7
dem_200_belg_orig_328	469.1k	469.1k	0.0	1.4	469.1k	469.1k	0.0	0.9
dem_200_belg_orig_329	447.8k	447.8k	0.0	0.6	447.8k	447.8k	0.0	0.6
dem_200_belg_orig_330	387.8k	387.7k	0.0	2.1	387.8k	387.7k	0.0	1.4
dem_200_belg_orig_331	389.5k	389.5k	0.0	1.2	389.5k	389.5k	0.0	0.9
dem_200_belg_orig_332	874.7k	873.9k	0.1	3.2	874.1k	873.6k	0.1	6.8
dem_200_belg_orig_333	413.3k	413.3k	0.0	0.8	413.3k	413.3k	0.0	0.4
dem_200_belg_orig_334	601.6k	601.4k	0.0	2.4	601.6k	601.1k	0.1	1.7
dem_200_belg_orig_335	550.5k	550.5k	0.0	0.6	550.5k	550.5k	0.0	0.3
dem_200_belg_orig_336	453.9k	453.6k	0.1	1.5	453.9k	453.6k	0.1	0.9
dem_200_belg_orig_337	342.5k	342.5k	0.0	1.1	342.5k	342.5k	0.0	0.6
dem_200_belg_orig_338	324.8k	324.8k	0.0	1.4	324.8k	324.8k	0.0	0.6
dem_200_belg_orig_339	835.8k	835.8k	0.0	1.8	835.8k	835.8k	0.0	0.8
dem_200_belg_orig_340	847.8k	847.5k	0.0	1.2	847.8k	846.9k	0.1	1.0
dem_200_belg_orig_341	323.2k	322.8k	0.1	100.7	323.2k	322.9k	0.1	16.4
dem_200_belg_orig_342	476.1k	476.1k	0.0	42.6	476.1k	475.7k	0.1	3.0
dem_200_belg_orig_343	423.7k	423.7k	0.0	1.8	423.7k	423.7k	0.0	0.4
dem_200_belg_orig_344	329.4k	329.1k	0.1	4.3	329.4k	329.2k	0.1	1.5
dem_200_belg_orig_345	694.9k	694.9k	0.0	0.9	694.9k	694.9k	0.0	0.6
dem_200_belg_orig_346	432.6k	432.5k	0.0	1.5	432.6k	432.5k	0.0	0.7
dem_200_belg_orig_347	580.0k	579.7k	0.0	1.2	580.0k	579.7k	0.0	0.8
dem_200_belg_orig_348	331.3k	331.3k	0.0	1.5	331.3k	331.3k	0.0	0.6
dem_200_belg_orig_349	301.0k	300.7k	0.1	181.0	301.0k	300.8k	0.1	2.6
dem_200_belg_orig_350	650.2k	649.9k	0.0	1.9	650.2k	650.2k	0.0	1.2
dem_200_belg_orig_351	478.4k	478.4k	0.0	0.7	478.4k	478.4k	0.0	0.7
dem_200_belg_orig_352	628.2k	627.6k	0.1	1.1	628.2k	628.2k	0.0	1.4
dem_200_belg_orig_353	603.2k	603.2k	0.0	1.1	603.2k	603.2k	0.0	0.4
dem_200_belg_orig_354	426.6k	426.6k	0.0	1.2	426.6k	426.6k	0.0	0.5
dem_200_belg_orig_355	243.8k	243.7k	0.0	128.8	243.7k	243.5k	0.1	3.0
dem_200_belg_orig_356	527.4k	527.4k	0.0	0.5	527.4k	527.4k	0.0	0.4
dem_200_belg_orig_357	413.8k	413.8k	0.0	1.0	413.8k	413.8k	0.0	1.7
dem_200_belg_orig_358	880.4k	879.8k	0.1	1.5	880.4k	879.8k	0.1	0.8
dem_200_belg_orig_359	565.8k	565.8k	0.0	1.0	565.8k	565.8k	0.0	0.5
dem_200_belg_orig_360	329.4k	329.2k	0.1	1.8	329.4k	329.2k	0.1	0.5
dem_200_belg_orig_361	802.3k	801.6k	0.1	1.2	802.3k	801.6k	0.1	0.7
dem_200_belg_orig_362	359.3k	359.1k	0.0	1.5	359.3k	359.1k	0.0	0.7
dem_200_belg_orig_363	623.7k	623.7k	0.0	2.1	623.7k	623.7k	0.0	0.9
dem_200_belg_orig_364	608.7k	608.1k	0.1	1.7	608.7k	608.4k	0.1	1.1
dem_200_belg_orig_365	685.7k	685.3k	0.1	2.4	685.7k	685.7k	0.0	1.1
dem_200_belg_orig_366	766.1k	765.3k	0.1	705.1	766.1k	765.4k	0.1	3.7
dem_200_belg_orig_367	451.3k	451.1k	0.0	50.0	451.3k	450.9k	0.1	13.7
dem_200_belg_orig_368	363.3k	363.3k	0.0	1.9	363.3k	362.9k	0.1	2.4
dem_200_belg_orig_369	252.8k	252.8k	0.0	1.4	252.8k	252.8k	0.0	1.7
dem_200_belg_orig_370	291.4k	291.1k	0.1	9.5	291.4k	291.4k	0.0	4.3
dem_200_belg_orig_371	729.1k	728.5k	0.1	0.9	729.1k	728.5k	0.1	1.0
dem_200_belg_orig_372	539.8k	539.8k	0.0	1.6	539.8k	539.8k	0.0	0.6
dem_200_belg_orig_373	720.8k	720.4k	0.1	2.9	720.4k	720.4k	0.0	0.9
dem_200_belg_orig_374	306.7k	306.7k	0.0	2.9	306.7k	306.7k	0.0	0.9
dem_200_belg_orig_375	483.1k	482.7k	0.1	2.8	483.1k	482.7k	0.1	6.0
dem_200_belg_orig_376	1077.6k	1077.6k	0.0	0.8	1077.6k	1077.6k	0.0	0.3
dem_200_belg_orig_377	398.5k	398.3k	0.0	6.7	398.4k	398.1k	0.1	3.7
dem_200_belg_orig_378	356.4k	356.2k	0.1	1.2	356.4k	356.2k	0.1	0.7
dem_200_belg_orig_379	664.4k	664.4k	0.0	1.1	664.4k	664.4k	0.0	2.0
dem_200_belg_orig_380	484.4k	484.4k	0.0	1.1	484.4k	484.4k	0.0	0.5
dem_200_belg_orig_381	609.5k	608.9k	0.1	13.6	609.5k	608.9k	0.1	11.5
dem_200_belg_orig_382	765.6k	765.6k	0.0	1.2	765.6k	765.6k	0.0	0.8
dem_200_belg_orig_383	323.1k	322.8k	0.1	46.3	323.1k	322.8k	0.1	3109.0
dem_200_belg_orig_384	411.2k	410.8k	0.1	1.4	411.2k	410.8k	0.1	1.0
dem_200_belg_orig_385	515.2k	514.7k	0.1	1.4	515.2k	514.7k	0.1	1.2
dem_200_belg_orig_386	682.2k	681.5k	0.1	54.6	682.2k	673.0k	1.4	14400.0
dem_200_belg_orig_387	352.0k	352.0k	0.0	1.7	352.1k	351.7k	0.1	1.3
dem_200_belg_orig_388	297.4k	297.1k	0.1	9442.0	297.4k	297.2k	0.1	3.8
dem_200_belg_orig_389	341.1k	341.1k	0.0	1.4	341.1k	341.1k	0.0	1.0
dem_200_belg_orig_390	445.3k	445.3k	0.0	0.8	445.3k	445.3k	0.0	0.8
dem_200_belg_orig_391	396.4k	396.1k	0.1	0.9	396.6k	396.3k	0.1	0.9
dem_200_belg_orig_392	384.3k	384.3k	0.0	1.4	384.3k	384.3k	0.0	0.7
dem_200_belg_orig_393	491.5k	491.5k	0.0	1.8	491.6k	491.4k	0.0	1.7
dem_200_belg_orig_394	375.2k	375.0k	0.1	1.2	375.2k	374.9k	0.1	0.7
dem_200_belg_orig_395	499.1k	498.8k	0.1	1.3	499.1k	498.8k	0.1	0.7
dem_200_belg_orig_396	342.3k	342.0k	0.1	3.1	342.3k	342.0k	0.1	2.5
dem_200_belg_orig_397	837.1k	836.5k	0.1	1.3	837.1k	836.5k	0.1	0.8

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Table A.4: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_398	572.3k	572.3k	0.0	1.4	572.3k	572.3k	0.0	1.1
dem_200_belg_orig_399	518.1k	518.0k	0.0	1.0	518.1k	518.0k	0.0	0.9
dem_200_belg_orig_400	311.4k	311.2k	0.1	1.3	311.4k	311.2k	0.1	1.0
dem_200_belg_orig_401	585.1k	585.1k	0.0	0.8	585.1k	585.1k	0.0	0.9
dem_200_belg_orig_402	521.0k	521.0k	0.0	1.0	521.0k	521.0k	0.0	1.0
dem_200_belg_orig_403	467.1k	467.1k	0.0	6.7	467.1k	467.1k	0.0	8.4
dem_200_belg_orig_404	637.6k	637.6k	0.0	3.1	637.6k	637.6k	0.0	0.9
dem_200_belg_orig_405	858.1k	857.4k	0.1	2.3	857.4k	857.4k	0.0	0.6
dem_200_belg_orig_406	271.0k	270.8k	0.1	21.7	271.1k	270.9k	0.1	3.0
dem_200_belg_orig_407	660.2k	660.2k	0.0	0.8	660.2k	660.2k	0.0	0.8
dem_200_belg_orig_408	195.6k	195.6k	0.0	0.4	195.6k	195.6k	0.0	0.5
dem_200_belg_orig_409	379.1k	378.7k	0.1	2.6	379.3k	379.0k	0.1	48.3
dem_200_belg_orig_410	387.0k	378.0k	2.4	14400.0	387.0k	377.4k	2.5	14400.0
dem_200_belg_orig_411	501.1k	501.1k	0.0	1.0	501.1k	501.1k	0.0	0.4
dem_200_belg_orig_412	400.6k	400.6k	0.0	1.8	400.6k	400.6k	0.0	1.0
dem_200_belg_orig_413	341.6k	341.6k	0.0	1.0	341.6k	341.6k	0.0	0.7
dem_200_belg_orig_414	377.3k	377.1k	0.0	3.2	377.3k	376.9k	0.1	3.1
dem_200_belg_orig_415	250.9k	250.6k	0.1	3.9	250.9k	250.9k	0.0	1.7
dem_200_belg_orig_416	664.0k	664.0k	0.0	0.5	664.0k	664.0k	0.0	0.4
dem_200_belg_orig_417	493.0k	492.6k	0.1	1.4	493.0k	493.0k	0.0	1.2
dem_200_belg_orig_418	656.3k	655.7k	0.1	16.5	656.3k	655.7k	0.1	56.4
dem_200_belg_orig_419	334.2k	334.2k	0.0	0.9	334.2k	334.2k	0.0	0.3
dem_200_belg_orig_420	504.3k	504.3k	0.0	1.2	504.3k	504.3k	0.0	0.7
dem_200_belg_orig_421	347.4k	347.4k	0.0	0.5	347.4k	347.4k	0.0	0.6
dem_200_belg_orig_422	498.3k	497.9k	0.1	0.8	498.3k	498.0k	0.1	0.3
dem_200_belg_orig_423	334.3k	334.0k	0.1	2258.0	334.3k	334.0k	0.1	42.5
dem_200_belg_orig_424	582.7k	582.3k	0.1	66.8	582.7k	582.2k	0.1	4.9
dem_200_belg_orig_425	396.7k	396.6k	0.0	0.6	396.7k	396.6k	0.0	0.6
dem_200_belg_orig_426	451.8k	451.5k	0.1	138.1	451.8k	451.4k	0.1	45.5
dem_200_belg_orig_427	397.8k	397.8k	0.0	1.4	397.8k	397.8k	0.0	0.9
dem_200_belg_orig_428	269.2k	269.2k	0.0	0.6	269.2k	269.2k	0.0	0.6
dem_200_belg_orig_429	710.7k	710.7k	0.0	1.1	710.7k	710.7k	0.0	0.6
dem_200_belg_orig_430	561.4k	561.4k	0.0	5.1	561.4k	561.3k	0.0	1.3
dem_200_belg_orig_431	822.5k	822.4k	0.0	2.0	822.5k	822.4k	0.0	1.4
dem_200_belg_orig_432	425.2k	425.0k	0.0	1.4	425.2k	425.0k	0.0	0.7
dem_200_belg_orig_433	338.6k	338.4k	0.1	0.9	338.6k	338.4k	0.0	0.7
dem_200_belg_orig_434	726.5k	726.4k	0.0	1.3	726.5k	726.4k	0.0	0.3
dem_200_belg_orig_435	565.5k	565.5k	0.0	1.5	565.5k	565.5k	0.0	1.6
dem_200_belg_orig_436	397.3k	396.9k	0.1	36.7	397.3k	397.3k	0.0	134.0
dem_200_belg_orig_437	516.2k	515.7k	0.1	8.7	516.2k	515.7k	0.1	6.0
dem_200_belg_orig_438	528.7k	528.7k	0.0	1.3	528.7k	528.7k	0.0	1.0
dem_200_belg_orig_439	629.1k	628.5k	0.1	9.3	629.1k	628.5k	0.1	8.5
dem_200_belg_orig_440	768.0k	768.0k	0.0	1.7	768.0k	767.8k	0.0	0.9
dem_200_belg_orig_441	410.9k	410.6k	0.1	0.9	410.9k	410.7k	0.1	0.7
dem_200_belg_orig_442	392.3k	392.1k	0.0	0.9	392.3k	392.3k	0.0	0.8
dem_200_belg_orig_443	612.9k	612.9k	0.0	1.9	612.9k	612.9k	0.0	0.4
dem_200_belg_orig_444	291.7k	291.6k	0.1	4.3	291.8k	291.5k	0.1	2.9
dem_200_belg_orig_445	859.7k	859.1k	0.1	1.7	859.7k	859.6k	0.0	0.8
dem_200_belg_orig_446	404.4k	404.4k	0.0	0.5	404.4k	404.4k	0.0	0.8
dem_200_belg_orig_447	417.6k	417.6k	0.0	1.1	417.6k	417.6k	0.0	0.3
dem_200_belg_orig_448	421.8k	421.5k	0.1	70.6	421.8k	421.8k	0.0	178.2
dem_200_belg_orig_449	588.7k	588.5k	0.0	0.8	588.7k	588.5k	0.0	0.5
dem_200_belg_orig_450	414.7k	414.2k	0.1	2.0	414.7k	414.2k	0.1	1.8
dem_200_belg_orig_451	394.4k	394.0k	0.1	5.9	394.4k	394.0k	0.1	3.6
dem_200_belg_orig_452	507.4k	507.0k	0.1	1.8	507.5k	507.1k	0.1	2.5
dem_200_belg_orig_453	726.5k	725.8k	0.1	2.1	726.5k	725.8k	0.1	36.1
dem_200_belg_orig_454	414.4k	414.0k	0.1	2.3	414.4k	414.0k	0.1	2.2
dem_200_belg_orig_455	291.4k	291.3k	0.0	1.7	291.4k	291.2k	0.1	0.9
dem_200_belg_orig_456	573.4k	572.8k	0.1	4.2	573.5k	573.2k	0.1	2.8
dem_200_belg_orig_457	468.6k	468.6k	0.0	0.7	468.6k	468.6k	0.0	0.5
dem_200_belg_orig_458	899.7k	898.8k	0.1	118.8	899.7k	898.9k	0.1	48.5
dem_200_belg_orig_459	213.7k	213.7k	0.0	0.6	213.7k	213.7k	0.0	0.5
dem_200_belg_orig_460	384.8k	384.8k	0.0	1.4	384.8k	384.8k	0.0	0.9
dem_200_belg_orig_461	401.5k	401.5k	0.0	0.4	401.5k	401.5k	0.0	0.5
dem_200_belg_orig_462	518.3k	518.3k	0.0	2.6	518.3k	518.3k	0.0	2.2
dem_200_belg_orig_463	410.3k	410.3k	0.0	2.3	410.3k	410.3k	0.0	0.6
dem_200_belg_orig_464	366.6k	366.6k	0.0	1.4	366.6k	366.6k	0.0	0.3
dem_200_belg_orig_465	476.8k	476.8k	0.0	0.8	476.8k	476.8k	0.0	1.0
dem_200_belg_orig_466	484.4k	484.4k	0.0	0.6	484.4k	484.4k	0.0	0.5
dem_200_belg_orig_467	683.8k	683.8k	0.0	1.0	683.8k	683.8k	0.0	0.5
dem_200_belg_orig_468	480.6k	480.5k	0.0	100.1	480.6k	480.3k	0.1	88.7
dem_200_belg_orig_469	534.7k	534.7k	0.0	1.1	534.7k	534.7k	0.0	0.9
dem_200_belg_orig_470	411.1k	410.7k	0.1	31.8	411.1k	410.7k	0.1	17.4
dem_200_belg_orig_471	541.4k	541.4k	0.0	1.7	541.4k	541.4k	0.0	1.1
dem_200_belg_orig_472	291.4k	291.4k	0.0	1.6	291.4k	291.4k	0.0	0.7
dem_200_belg_orig_473	649.6k	649.6k	0.0	1.1	649.6k	649.5k	0.0	0.6
dem_200_belg_orig_474	326.5k	326.2k	0.1	6.7	326.5k	326.2k	0.1	8.1
dem_200_belg_orig_475	327.2k	327.2k	0.0	1.9	327.2k	327.2k	0.0	0.7

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Table A.4: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_belg_orig_476	575.4k	575.3k	0.0	0.9	575.4k	575.3k	0.0	0.5
dem_200_belg_orig_477	563.6k	563.1k	0.1	2.5	563.6k	563.0k	0.1	1.5
dem_200_belg_orig_478	542.4k	541.9k	0.1	5.6	542.4k	541.8k	0.1	5.5
dem_200_belg_orig_479	631.9k	631.9k	0.0	1.2	631.9k	631.9k	0.0	0.4
dem_200_belg_orig_480	355.4k	355.1k	0.1	13.1	355.5k	355.2k	0.1	9.2
dem_200_belg_orig_481	489.3k	489.2k	0.0	1.1	489.3k	489.2k	0.0	0.6
dem_200_belg_orig_482	433.7k	433.3k	0.1	1.1	433.7k	433.2k	0.1	0.8
dem_200_belg_orig_483	288.8k	288.8k	0.0	0.4	288.8k	288.8k	0.0	0.5
dem_200_belg_orig_484	306.6k	306.4k	0.1	1.1	306.6k	306.6k	0.0	0.8
dem_200_belg_orig_485	265.7k	265.6k	0.0	1.2	265.7k	265.7k	0.0	0.5
dem_200_belg_orig_486	724.6k	724.6k	0.0	1.2	724.6k	724.0k	0.1	0.7
dem_200_belg_orig_487	326.8k	326.8k	0.0	2.1	326.8k	326.8k	0.0	0.9
dem_200_belg_orig_488	625.8k	625.8k	0.0	0.8	625.8k	625.8k	0.0	0.6
dem_200_belg_orig_489	721.0k	720.2k	0.1	2.1	721.0k	720.2k	0.1	0.8
dem_200_belg_orig_490	317.6k	317.6k	0.0	1.4	317.6k	317.6k	0.0	1.0
dem_200_belg_orig_491	422.4k	422.0k	0.1	4.1	422.4k	422.0k	0.1	2.2
dem_200_belg_orig_492	283.2k	283.2k	0.0	1.6	283.2k	283.1k	0.0	0.5
dem_200_belg_orig_493	481.7k	481.7k	0.0	1.2	481.7k	481.4k	0.1	1.0
dem_200_belg_orig_494	537.7k	537.7k	0.0	1.4	537.7k	537.5k	0.0	0.8
dem_200_belg_orig_495	636.4k	635.9k	0.1	1.3	636.4k	635.9k	0.1	0.6
dem_200_belg_orig_496	400.7k	400.7k	0.0	2.4	400.7k	400.7k	0.0	1.3
dem_200_belg_orig_497	593.6k	593.6k	0.0	1.3	593.6k	593.6k	0.0	0.2
dem_200_belg_orig_498	588.8k	588.5k	0.1	2.6	588.5k	588.5k	0.0	0.5
dem_200_belg_orig_499	483.0k	482.5k	0.1	108.2	483.0k	483.0k	0.0	8.5
dem_200_belg_orig_500	324.5k	324.5k	0.0	885.3	324.5k	324.5k	0.0	3.9

Table A.5: Detailed results of the discrete models on *Belgium* with $\mathcal{B} = 500$ as summarized in Figure 3.7c and Table 3.3a. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_1	626.0k	626.0k	0.0	410.7	626.0k	626.0k	0.0	478.2	626.0k	626.0k	0.0	492.8
dem_500_belg_orig_2	1310.7k	1310.7k	0.0	2157.9	1310.7k	1298.6k	0.9	14400.0	1311.2k	1274.2k	2.9	14400.0
dem_500_belg_orig_3	1236.3k	1236.3k	0.0	3600.6	1236.3k	1236.3k	0.0	1215.8	1239.1k	1194.5k	3.7	14400.0
dem_500_belg_orig_4	1639.5k	1611.2k	1.8	14400.0	1639.5k	1607.6k	2.0	14400.0	1641.9k	1584.9k	3.6	14400.0
dem_500_belg_orig_5	1618.5k	1618.2k	0.0	138.2	1618.5k	1617.8k	0.0	1025.2	1618.5k	1618.5k	0.0	2238.7
dem_500_belg_orig_6	1217.7k	1217.7k	0.0	5001.7	1217.7k	1203.4k	1.2	14400.0	1218.2k	1154.1k	5.6	14400.0
dem_500_belg_orig_7	776.9k	776.9k	0.0	8590.9	776.9k	776.9k	0.0	7178.2	794.2k	650.1k	22.2	14400.0
dem_500_belg_orig_8	1615.8k	1615.8k	0.0	350.4	1615.8k	1614.2k	0.1	2946.0	1615.8k	1615.8k	0.0	3859.0
dem_500_belg_orig_9	1569.8k	1568.8k	0.1	149.9	1569.8k	1569.0k	0.0	1186.8	1569.8k	1568.3k	0.1	2243.6
dem_500_belg_orig_10	1646.0k	1608.2k	2.3	14400.0	1646.0k	1637.7k	0.5	14400.0	1646.0k	1549.8k	6.2	14400.0
dem_500_belg_orig_11	1979.7k	1979.7k	0.0	4.7	1979.7k	1977.8k	0.1	39.0	1979.7k	1979.7k	0.0	9.0
dem_500_belg_orig_12	1981.5k	1981.5k	0.0	15.5	1981.5k	1979.9k	0.1	75.1	1981.5k	1981.5k	0.0	41.8
dem_500_belg_orig_13	1090.0k	1039.5k	4.9	14400.0	1088.0k	1081.8k	0.6	14400.0	1098.7k	1013.7k	8.4	14400.0
dem_500_belg_orig_14	783.3k	652.9k	20.0	14400.0	768.8k	768.2k	0.1	4848.5	783.9k	642.7k	22.0	14400.0
dem_500_belg_orig_15	1583.5k	1583.5k	0.0	5516.5	1583.5k	1570.9k	0.8	14400.0	1583.5k	1540.6k	2.8	14400.0
dem_500_belg_orig_16	1810.2k	1810.2k	0.0	223.1	1810.2k	1808.5k	0.1	1040.3	1810.2k	1809.0k	0.1	10058.6
dem_500_belg_orig_17	2041.7k	2041.7k	0.0	30.7	2041.7k	2040.8k	0.0	162.6	2041.7k	2040.3k	0.1	885.6
dem_500_belg_orig_18	789.7k	735.9k	7.3	14400.0	784.6k	769.0k	2.0	14400.0	790.4k	715.2k	10.5	14400.0
dem_500_belg_orig_19	1291.6k	1291.6k	0.0	1707.8	1291.6k	1290.6k	0.1	806.3	1291.6k	1261.9k	2.4	14400.0
dem_500_belg_orig_20	1012.2k	929.9k	8.9	14400.0	1012.1k	1004.5k	0.8	14400.0	1016.5k	904.0k	12.4	14400.0
dem_500_belg_orig_21	835.6k	788.1k	6.0	14400.0	835.6k	835.6k	0.0	1001.4	838.5k	789.1k	6.3	14400.0
dem_500_belg_orig_22	1121.2k	1121.2k	0.0	109.1	1121.2k	1120.5k	0.1	564.4	1121.2k	1120.8k	0.0	612.8
dem_500_belg_orig_23	1301.1k	1301.1k	0.0	10478.0	1301.1k	1301.0k	0.0	5905.1	1301.1k	1247.6k	4.3	14400.0
dem_500_belg_orig_24	1354.0k	1316.5k	2.9	14400.0	1353.6k	1352.9k	0.1	3297.8	1354.0k	1325.7k	2.1	14400.0
dem_500_belg_orig_25	1479.2k	1479.2k	0.0	937.3	1479.2k	1478.7k	0.0	1846.3	1479.2k	1478.6k	0.0	2627.6
dem_500_belg_orig_26	1466.7k	1398.9k	4.8	14400.0	1467.3k	1444.7k	1.6	14400.0	1468.3k	1347.5k	9.0	14400.0
dem_500_belg_orig_27	1800.1k	1800.1k	0.0	238.1	1800.1k	1799.2k	0.0	2216.7	1800.1k	1800.1k	0.0	1887.9
dem_500_belg_orig_28	1065.0k	1021.7k	4.2	14400.0	1063.6k	1048.1k	1.5	14400.0	1069.7k	950.5k	12.5	14400.0
dem_500_belg_orig_29	1910.1k	1910.1k	0.0	870.8	1910.1k	1910.1k	0.0	111.0	1910.1k	1872.7k	2.0	14400.0
dem_500_belg_orig_30	1303.1k	1303.1k	0.0	276.1	1303.1k	1302.5k	0.0	1786.5	1303.1k	1303.1k	0.0	4759.9
dem_500_belg_orig_31	1146.8k	1146.8k	0.0	1481.3	1146.8k	1146.8k	0.0	748.1	1146.8k	1146.8k	0.0	14141.9
dem_500_belg_orig_32	1021.6k	1021.6k	0.0	797.4	1021.6k	1021.6k	0.0	1199.5	1021.6k	1020.7k	0.1	6129.1
dem_500_belg_orig_33	1261.7k	1045.1k	20.7	14400.0	1245.2k	1244.2k	0.1	6459.7	1255.4k	1045.9k	20.0	14400.0
dem_500_belg_orig_34	1070.4k	976.6k	9.6	14400.0	1062.3k	1040.2k	2.1	14400.0	1081.8k	948.6k	14.0	14400.0
dem_500_belg_orig_35	1165.9k	1165.3k	0.0	6276.2	1165.9k	1165.9k	0.0	3766.4	1169.1k	1133.4k	3.2	14400.0
dem_500_belg_orig_36	1819.6k	1818.8k	0.0	4592.8	1820.2k	1805.7k	0.8	14400.0	1820.7k	1790.7k	1.7	14400.0
dem_500_belg_orig_37	731.6k	596.0k	22.8	14400.0	716.4k	716.4k	0.0	8335.2	725.2k	603.3k	20.2	14400.0
dem_500_belg_orig_38	865.8k	831.1k	4.2	14400.0	865.7k	854.8k	1.3	14400.0	866.6k	840.4k	3.1	14400.0

continued on next page

Table A.5: Comparison of discrete models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_39	1967.2k	1911.2k	2.9	14400.0	1966.2k	1965.6k	0.0	4556.0	1966.2k	1880.7k	4.5	14400.0
dem_500_belg_orig_40	1880.5k	1830.4k	2.7	14400.0	1880.4k	1857.8k	1.2	14400.0	1882.8k	1790.1k	5.2	14400.0
dem_500_belg_orig_41	1141.6k	1084.0k	5.3	14400.0	1142.9k	1111.9k	2.8	14400.0	1143.6k	1071.0k	6.8	14400.0
dem_500_belg_orig_42	1545.5k	1545.5k	0.0	7609.6	1547.5k	1520.3k	1.8	14400.0	1546.4k	1488.2k	3.9	14400.0
dem_500_belg_orig_43	682.8k	682.8k	0.0	1082.6	682.8k	682.4k	0.1	2773.0	682.8k	667.2k	2.3	14400.0
dem_500_belg_orig_44	1710.3k	1710.3k	0.0	223.2	1710.3k	1708.7k	0.1	12831.2	1710.3k	1710.3k	0.0	2274.4
dem_500_belg_orig_45	1407.6k	1407.6k	0.0	642.6	1407.6k	1406.3k	0.1	2037.3	1407.6k	1407.6k	0.0	8975.8
dem_500_belg_orig_46	1568.5k	1568.5k	0.0	22.3	1568.5k	1567.1k	0.1	205.2	1568.5k	1568.5k	0.0	23.4
dem_500_belg_orig_47	1352.7k	1352.0k	0.0	618.8	1352.7k	1351.4k	0.1	6991.5	1352.7k	1352.7k	0.0	7613.2
dem_500_belg_orig_48	671.9k	671.9k	0.0	13874.6	671.9k	671.3k	0.1	5798.5	672.9k	631.4k	6.6	14400.0
dem_500_belg_orig_49	684.7k	592.1k	15.6	14400.0	692.0k	648.4k	6.7	14400.0	698.6k	546.8k	27.8	14400.0
dem_500_belg_orig_50	1045.5k	911.0k	14.8	14400.0	1021.2k	994.6k	2.7	14400.0	1e+20	870.8k	–	14400.0
dem_500_belg_orig_51	1938.1k	1938.1k	0.0	8361.3	1938.1k	1936.5k	0.1	11714.7	1938.1k	1904.7k	1.7	14400.0
dem_500_belg_orig_52	1362.9k	1362.9k	0.0	408.4	1362.9k	1362.0k	0.1	2500.2	1362.9k	1362.9k	0.0	8172.9
dem_500_belg_orig_53	1435.6k	1382.2k	3.9	14400.0	1435.6k	1434.5k	0.1	3092.5	1451.5k	1336.8k	8.6	14400.0
dem_500_belg_orig_54	841.2k	711.6k	18.2	14400.0	839.2k	801.3k	4.7	14400.0	839.3k	741.3k	13.2	14400.0
dem_500_belg_orig_55	1452.4k	1452.4k	0.0	709.1	1452.4k	1451.0k	0.1	1586.5	1452.4k	1421.3k	2.2	14400.0
dem_500_belg_orig_56	599.3k	599.3k	0.0	12306.3	599.3k	598.8k	0.1	11313.8	1e+20	556.1k	–	14400.0
dem_500_belg_orig_57	1887.7k	1887.4k	0.0	14.9	1887.7k	1886.5k	0.1	113.8	1887.7k	1887.7k	0.0	90.3
dem_500_belg_orig_58	1678.7k	1678.7k	0.0	201.3	1678.7k	1678.7k	0.0	994.2	1678.7k	1678.7k	0.0	4686.3
dem_500_belg_orig_59	2039.0k	2037.7k	0.1	103.3	2039.0k	2039.0k	0.0	1525.1	2039.0k	2039.0k	0.0	3358.2
dem_500_belg_orig_60	1018.5k	1018.5k	0.0	11505.5	1018.5k	1018.5k	0.0	1562.0	1018.5k	936.6k	8.7	14400.0
dem_500_belg_orig_61	732.7k	732.7k	0.0	57.5	732.7k	732.2k	0.1	2772.4	732.7k	732.7k	0.0	1174.1
dem_500_belg_orig_62	1420.3k	1420.3k	0.0	9488.6	1420.3k	1420.3k	0.0	2873.7	1420.3k	1374.9k	3.3	14400.0
dem_500_belg_orig_63	1067.2k	1067.2k	0.0	9.2	1067.2k	1067.2k	0.0	89.2	1067.2k	1067.2k	0.0	38.0
dem_500_belg_orig_64	2041.0k	2041.0k	0.0	6562.7	2045.3k	2014.4k	1.5	14400.0	1e+20	1971.5k	–	14400.0
dem_500_belg_orig_65	1762.0k	1760.5k	0.1	48.5	1762.0k	1760.7k	0.1	264.2	1762.0k	1760.8k	0.1	73.4
dem_500_belg_orig_66	2113.7k	2113.7k	0.0	2865.0	2113.7k	2112.3k	0.1	2999.2	2113.7k	2014.4k	4.9	14400.0
dem_500_belg_orig_67	1493.4k	1493.4k	0.0	36.6	1493.4k	1492.2k	0.1	481.0	1493.4k	1493.4k	0.0	181.8
dem_500_belg_orig_68	559.7k	559.7k	0.0	12073.9	559.7k	559.7k	0.0	13417.3	559.7k	559.7k	0.0	7743.4
dem_500_belg_orig_69	910.2k	910.2k	0.0	2616.9	910.2k	910.2k	0.0	7695.4	910.2k	851.6k	6.9	14400.0
dem_500_belg_orig_70	1892.3k	1890.9k	0.1	85.5	1892.3k	1892.3k	0.0	505.9	1892.3k	1892.3k	0.0	2193.5
dem_500_belg_orig_71	1801.1k	1801.1k	0.0	1901.2	1801.1k	1793.2k	0.4	14400.0	1803.6k	1774.9k	1.6	14400.0
dem_500_belg_orig_72	2398.5k	2398.5k	0.0	4.9	2398.5k	2396.2k	0.1	53.2	2398.5k	2398.5k	0.0	14.1
dem_500_belg_orig_73	763.3k	626.0k	21.9	14400.0	758.5k	758.5k	0.0	735.3	760.4k	611.6k	24.3	14400.0
dem_500_belg_orig_74	702.2k	702.2k	0.0	16.5	702.2k	702.2k	0.0	316.3	702.2k	702.1k	0.0	57.0
dem_500_belg_orig_75	1770.7k	1770.7k	0.0	3542.9	1770.7k	1738.0k	1.9	14400.0	1770.7k	1711.1k	3.5	14400.0
dem_500_belg_orig_76	1149.6k	1148.6k	0.1	8088.9	1149.6k	1148.9k	0.1	6565.7	1150.1k	1115.7k	3.1	14400.0
dem_500_belg_orig_77	1072.6k	1072.6k	0.0	7172.5	1072.6k	1071.5k	0.1	5764.9	1072.6k	1036.8k	3.4	14400.0
dem_500_belg_orig_78	1067.7k	1067.7k	0.0	4481.5	1067.7k	1067.7k	0.0	3816.7	1072.5k	1018.2k	5.3	14400.0
dem_500_belg_orig_79	1696.1k	1665.4k	1.8	14400.0	1693.8k	1674.8k	1.1	14400.0	1702.6k	1627.0k	4.7	14400.0
dem_500_belg_orig_80	1223.7k	1223.7k	0.0	3570.1	1223.7k	1222.8k	0.1	2232.7	1223.7k	1182.7k	3.5	14400.0
dem_500_belg_orig_81	904.5k	903.6k	0.1	10720.9	904.5k	904.5k	0.0	8924.1	904.5k	840.5k	7.6	14400.0
dem_500_belg_orig_82	1573.1k	1573.1k	0.0	4448.7	1573.1k	1571.8k	0.1	3202.6	1573.1k	1514.5k	3.9	14400.0
dem_500_belg_orig_83	752.1k	752.1k	0.0	203.9	752.1k	752.1k	0.0	1119.5	752.1k	751.8k	0.0	5366.9
dem_500_belg_orig_84	2328.9k	2327.5k	0.1	396.9	2328.9k	2311.7k	0.7	14400.0	2328.9k	2308.8k	0.9	14400.0
dem_500_belg_orig_85	1372.2k	1371.6k	0.0	15.8	1372.2k	1372.2k	0.0	195.8	1372.2k	1370.9k	0.1	138.3
dem_500_belg_orig_86	1527.5k	1526.5k	0.1	609.0	1527.5k	1527.5k	0.0	3111.4	1527.5k	1509.6k	1.2	14400.0
dem_500_belg_orig_87	1204.3k	1158.2k	4.0	14400.0	1203.9k	1170.6k	2.8	14400.0	1199.7k	1115.0k	7.6	14400.0
dem_500_belg_orig_88	1617.7k	1617.7k	0.0	50.6	1617.7k	1617.7k	0.0	350.8	1617.7k	1617.7k	0.0	189.6
dem_500_belg_orig_89	1322.2k	1322.2k	0.0	342.2	1322.2k	1322.2k	0.0	1103.5	1322.2k	1322.2k	0.0	7977.3
dem_500_belg_orig_90	546.0k	466.0k	17.2	14400.0	535.1k	535.1k	0.0	12690.6	547.4k	447.6k	22.3	14400.0
dem_500_belg_orig_91	1656.9k	1656.5k	0.0	650.7	1656.9k	1655.4k	0.1	6330.7	1656.9k	1649.9k	0.4	14400.0
dem_500_belg_orig_92	1063.9k	1063.9k	0.0	564.9	1063.9k	1063.9k	0.0	10558.1	1063.9k	1063.8k	0.0	8942.5
dem_500_belg_orig_93	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	23.9	1e+20	1e+20	0.0	0.2
dem_500_belg_orig_94	886.6k	769.0k	15.3	14400.0	872.8k	872.8k	0.0	8485.3	886.2k	731.6k	21.1	14400.0
dem_500_belg_orig_95	1560.3k	1527.7k	2.1	14400.0	1560.3k	1547.9k	0.8	14400.0	1560.3k	1509.0k	3.4	14400.0
dem_500_belg_orig_96	790.4k	790.4k	0.0	7285.6	790.4k	786.7k	0.5	14400.0	790.4k	719.1k	9.9	14400.0
dem_500_belg_orig_97	2042.3k	2042.3k	0.0	438.7	2042.3k	2041.1k	0.1	4604.3	2042.3k	2042.3k	0.0	4406.1
dem_500_belg_orig_98	1963.3k	1963.3k	0.0	6362.5	1963.3k	1951.0k	0.6	14400.0	1971.0k	1897.1k	3.9	14400.0
dem_500_belg_orig_99	2152.9k	2150.9k	0.1	11832.8	2152.9k	2151.3k	0.1	2865.8	2155.2k	2094.2k	2.9	14400.0
dem_500_belg_orig_100	932.9k	830.8k	12.3	14400.0	931.9k	898.2k	3.7	14400.0	968.7k	818.7k	18.3	14400.0
dem_500_belg_orig_101	1107.8k	1106.9k	0.1	11.2	1107.8k	1107.8k	0.0	102.5	1107.8k	1107.8k	0.0	27.6
dem_500_belg_orig_102	1608.2k	1608.2k	0.0	1722.3	1608.5k	1586.8k	1.4	14400.0	1608.4k	1547.4k	3.9	14400.0
dem_500_belg_orig_103	1263.7k	1263.4k	0.0	6260.8	1263.7k	1263.7k	0.0	2729.2	1264.5k	1238.3k	2.1	14400.0
dem_500_belg_orig_104	1418.3k	1417.3k	0.1	52.7	1418.3k	1416.9k	0.1	62.1	1418.3k	1418.3k	0.0	30.9
dem_500_belg_orig_105	1205.1k	1167.5k	3.2	14400.0	1204.7k	1201.1k	0.3	14400.0	1207.2k	1151.7k	4.8	14400.0
dem_500_belg_orig_106	798.1k	798.1k	0.0	371.8	798.1k	798.1k	0.0	2419.9	798.1k	798.1k	0.0	380.4
dem_500_belg_orig_107	1989.6k	1989.6k	0.0	14043.5	1995.6k	1946.3k	2.5	14400.0	1992.8k	1913.6k	4.1	14400.0
dem_500_belg_orig_108	1284.3k	1284.3k	0.0	6846.5	1284.3k	1283.0k	0.1	6254.9	1285.9k	1224.3k	5.0	14400.0
dem_500_belg_orig_109	1279.8k	1171.5k	9.2	14400.0	1278.7k	1278.7k	0.0	6166.5	1288.0k	1145.8k	12.4	14400.0
dem_500_belg_orig_110	1350.1k	1314.6k	2.7	14400.0	1346.6k	1346.6k	0.0	11311.6	1347.6k	1291.7k	4.3	14400.0
dem_500_belg_orig_111	1270.8k	1270.8k	0.0	747.1	1270.8k	1269.6k	0.1	13930.4	1270.8k	1270.8k	0.0	5970.3
dem_500_belg_orig_112	1072.9k	1072.9k	0.0	2152.3	1072.9k	1072.9k	0.0	1749.4	1073.2k	1044.9k	2.7	14400.0
dem_500_belg_orig_113	1118.4k	1117.7k	0.1	5222.6	1118.4k	1117.3k	0.1	2755.4	1118.4k	1100.1k	1.7	14400.0
dem_500_belg_orig_114	1245.5k	1245.5k	0.0	737.5	1245.5k	1244.7k	0.1	4186.0	1245.5k	1245.5k	0.0	3707.5
dem_500_belg_orig_115	1377.7k	1322.3k	4.2	14400.0	1377.7k	1365.0k	0.9	14400.0	1382.9k	1263.3k	9.5	14400.0
dem_500_belg_orig_116	1841.5k	1840.2k	0.1	386.1	1841.5k	1839.7k	0.1	2495.0	1841.5k	1839.9k	0.1	10520.5

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Table A.5: Comparison of discrete models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_117	1717.7k	1716.1k	0.1	40.6	1717.7k	1716.2k	0.1	184.7	1717.7k	1717.7k	0.0	259.5
dem_500_belg_orig_118	1473.0k	1473.0k	0.0	3324.9	1473.0k	1460.2k	0.9	14400.0	1473.0k	1446.9k	1.8	14400.0
dem_500_belg_orig_119	1446.2k	1446.2k	0.0	104.7	1446.2k	1446.2k	0.0	361.5	1446.2k	1446.2k	0.0	2544.7
dem_500_belg_orig_120	1934.3k	1895.2k	2.1	14400.0	1934.3k	1913.9k	1.1	14400.0	1942.0k	1833.7k	5.9	14400.0
dem_500_belg_orig_121	2070.4k	2070.4k	0.0	9657.2	2070.4k	2068.5k	0.1	3401.0	2071.7k	2019.2k	2.6	14400.0
dem_500_belg_orig_122	810.8k	810.2k	0.1	275.8	810.8k	810.8k	0.0	1312.0	810.8k	810.1k	0.1	753.1
dem_500_belg_orig_123	2118.1k	2118.0k	0.0	27.0	2118.1k	2116.2k	0.1	129.6	2118.1k	2118.1k	0.0	2096.2
dem_500_belg_orig_124	1796.3k	1796.3k	0.0	644.1	1796.3k	1796.3k	0.0	774.8	1796.3k	1796.3k	0.0	4869.8
dem_500_belg_orig_125	1788.3k	1768.9k	1.1	14400.0	1788.3k	1767.2k	1.2	14400.0	1788.5k	1736.7k	3.0	14400.0
dem_500_belg_orig_126	1468.3k	1466.8k	0.1	1.5	1468.3k	1467.5k	0.1	38.8	1468.3k	1467.0k	0.1	1.2
dem_500_belg_orig_127	1469.1k	1469.1k	0.0	640.8	1469.1k	1467.7k	0.1	8744.0	1469.1k	1469.1k	0.0	1426.4
dem_500_belg_orig_128	1620.1k	1620.1k	0.0	904.7	1620.1k	1620.1k	0.0	232.9	1620.1k	1603.4k	1.0	14400.0
dem_500_belg_orig_129	1190.3k	1190.3k	0.0	443.0	1190.3k	1189.7k	0.0	1623.7	1190.3k	1189.5k	0.1	190.3
dem_500_belg_orig_130	981.3k	981.3k	0.0	9300.2	981.3k	981.3k	0.0	8913.1	985.4k	917.3k	7.4	14400.0
dem_500_belg_orig_131	1261.6k	1261.6k	0.0	108.2	1261.6k	1261.6k	0.0	541.7	1261.6k	1261.2k	0.0	174.3
dem_500_belg_orig_132	1033.1k	958.3k	7.8	14400.0	1031.7k	1016.5k	1.5	14400.0	1032.9k	941.1k	9.8	14400.0
dem_500_belg_orig_133	1309.2k	1309.2k	0.0	41.8	1309.2k	1309.2k	0.0	345.7	1309.2k	1309.2k	0.0	529.8
dem_500_belg_orig_134	1333.1k	1333.1k	0.0	206.9	1333.1k	1331.8k	0.1	655.4	1333.1k	1333.1k	0.0	1717.9
dem_500_belg_orig_135	1061.5k	1019.3k	4.1	14400.0	1060.4k	1053.3k	0.7	14400.0	1063.9k	985.0k	8.0	14400.0
dem_500_belg_orig_136	708.5k	650.4k	8.9	14400.0	708.5k	707.8k	0.1	831.3	708.5k	651.8k	8.7	14400.0
dem_500_belg_orig_137	699.5k	699.5k	0.0	385.0	699.5k	698.9k	0.1	623.7	699.5k	699.5k	0.0	1373.2
dem_500_belg_orig_138	1132.6k	1132.6k	0.0	159.1	1132.6k	1131.5k	0.1	656.7	1132.6k	1132.4k	0.0	3243.6
dem_500_belg_orig_139	911.5k	872.2k	4.5	14400.0	909.4k	894.3k	1.7	14400.0	911.9k	842.1k	8.3	14400.0
dem_500_belg_orig_140	1268.7k	1268.7k	0.0	6234.2	1268.7k	1268.7k	0.0	468.8	1268.7k	1267.6k	0.1	9808.4
dem_500_belg_orig_141	1611.8k	1581.0k	1.9	14400.0	1610.5k	1610.5k	0.0	2347.6	1621.6k	1548.8k	4.7	14400.0
dem_500_belg_orig_142	1079.0k	1079.0k	0.0	101.1	1079.0k	1079.0k	0.0	903.8	1079.0k	1078.0k	0.1	555.1
dem_500_belg_orig_143	1678.0k	1677.8k	0.0	52.2	1678.0k	1678.0k	0.0	356.1	1678.0k	1677.2k	0.1	120.0
dem_500_belg_orig_144	1308.4k	1308.4k	0.0	3538.4	1308.4k	1307.2k	0.1	8174.0	1308.4k	1307.5k	0.1	13087.0
dem_500_belg_orig_145	2613.8k	2569.8k	1.7	14400.0	2613.7k	2612.0k	0.1	11386.0	2613.7k	2501.9k	4.5	14400.0
dem_500_belg_orig_146	1183.4k	1183.4k	0.0	1361.2	1183.4k	1172.4k	0.9	14400.0	1192.6k	1144.6k	4.2	14400.0
dem_500_belg_orig_147	1532.9k	1446.2k	6.0	14400.0	1530.1k	1530.1k	0.0	11160.6	1531.9k	1404.8k	9.0	14400.0
dem_500_belg_orig_148	1303.7k	1303.7k	0.0	274.8	1303.7k	1302.5k	0.1	675.0	1303.7k	1302.6k	0.1	1360.8
dem_500_belg_orig_149	1341.3k	1340.3k	0.1	119.9	1341.3k	1341.3k	0.0	372.0	1341.3k	1341.3k	0.0	299.0
dem_500_belg_orig_150	1642.4k	1642.4k	0.0	8.1	1642.4k	1641.4k	0.1	178.0	1642.4k	1641.1k	0.1	233.3
dem_500_belg_orig_151	1239.2k	1239.2k	0.0	2150.9	1239.2k	1239.2k	0.0	2146.3	1239.2k	1212.6k	2.2	14400.0
dem_500_belg_orig_152	1927.0k	1892.7k	1.8	14400.0	1927.0k	1918.1k	0.5	14400.0	1929.1k	1878.4k	2.7	14400.0
dem_500_belg_orig_153	1659.7k	1659.7k	0.0	1846.2	1659.7k	1651.8k	0.5	14400.0	1659.7k	1658.0k	0.1	7209.7
dem_500_belg_orig_154	1934.5k	1932.7k	0.1	3013.4	1934.5k	1929.7k	0.2	14400.0	1935.4k	1884.5k	2.7	14400.0
dem_500_belg_orig_155	1825.3k	1825.3k	0.0	12130.3	1825.3k	1825.3k	0.0	3664.5	1838.7k	1765.9k	4.1	14400.0
dem_500_belg_orig_156	1144.1k	1144.1k	0.0	1262.5	1144.1k	1143.5k	0.0	1530.2	1144.1k	1144.1k	0.0	4197.7
dem_500_belg_orig_157	1117.9k	1065.7k	4.9	14400.0	1117.9k	1117.5k	0.0	4567.8	1130.9k	1048.8k	7.8	14400.0
dem_500_belg_orig_158	917.3k	917.3k	0.0	2062.9	917.3k	917.3k	0.0	2293.1	917.3k	917.3k	0.0	224.6
dem_500_belg_orig_159	1740.7k	1693.8k	2.8	14400.0	1740.6k	1714.4k	1.5	14400.0	1749.3k	1589.1k	10.1	14400.0
dem_500_belg_orig_160	2016.6k	2014.8k	0.1	85.4	2016.6k	2016.6k	0.0	1371.3	2016.6k	2014.7k	0.1	434.2
dem_500_belg_orig_161	2050.6k	2050.6k	0.0	172.7	2050.6k	2049.0k	0.1	6262.8	2050.6k	2048.9k	0.1	3323.8
dem_500_belg_orig_162	1789.6k	1789.6k	0.0	1908.2	1789.6k	1787.9k	0.1	6870.3	1789.6k	1789.6k	0.0	9521.8
dem_500_belg_orig_163	1222.6k	1067.7k	14.5	14400.0	1216.7k	1216.7k	0.0	2196.0	1218.5k	1052.9k	15.7	14400.0
dem_500_belg_orig_164	1402.6k	1325.5k	5.8	14400.0	1401.8k	1401.8k	0.0	1519.6	1404.7k	1323.8k	6.1	14400.0
dem_500_belg_orig_165	1091.3k	1024.6k	6.5	14400.0	1089.3k	1078.0k	1.0	14400.0	1125.5k	997.6k	12.8	14400.0
dem_500_belg_orig_166	1562.5k	1373.3k	13.8	14400.0	1559.0k	1543.7k	1.0	14400.0	1563.1k	1355.5k	15.3	14400.0
dem_500_belg_orig_167	1189.0k	1131.5k	5.1	14400.0	1189.0k	1166.3k	2.0	14400.0	1206.9k	1086.7k	11.1	14400.0
dem_500_belg_orig_168	1097.5k	1096.9k	0.0	26.7	1097.5k	1096.7k	0.1	81.7	1097.5k	1097.5k	0.0	43.1
dem_500_belg_orig_169	1167.0k	1085.5k	7.5	14400.0	1165.0k	1130.1k	3.1	14400.0	1177.5k	1043.6k	12.8	14400.0
dem_500_belg_orig_170	1176.7k	1175.9k	0.1	93.8	1176.7k	1176.7k	0.0	450.2	1176.7k	1176.7k	0.0	119.0
dem_500_belg_orig_171	914.8k	914.8k	0.0	910.8	914.8k	914.8k	0.0	199.2	914.8k	914.8k	0.0	5653.0
dem_500_belg_orig_172	1939.3k	1939.3k	0.0	124.9	1939.3k	1937.4k	0.1	2606.2	1939.3k	1937.4k	0.1	1471.6
dem_500_belg_orig_173	1353.4k	1353.4k	0.0	12480.3	1353.4k	1336.6k	1.3	14400.0	1355.1k	1268.1k	6.9	14400.0
dem_500_belg_orig_174	1344.0k	1290.6k	4.1	14400.0	1344.0k	1342.8k	0.1	9468.2	1353.6k	1272.3k	6.4	14400.0
dem_500_belg_orig_175	1266.2k	1197.3k	5.8	14400.0	1270.7k	1229.0k	3.4	14400.0	1306.6k	1123.6k	16.3	14400.0
dem_500_belg_orig_176	1361.1k	1361.1k	0.0	20.7	1361.1k	1361.1k	0.0	138.8	1361.1k	1361.1k	0.0	52.2
dem_500_belg_orig_177	2736.9k	2736.9k	0.0	7.2	2736.9k	2734.6k	0.1	80.7	2736.9k	2734.4k	0.1	43.9
dem_500_belg_orig_178	740.6k	740.6k	0.0	6738.9	740.6k	728.8k	1.6	14400.0	740.8k	693.8k	6.8	14400.0
dem_500_belg_orig_179	1314.8k	1314.2k	0.0	0.8	1314.7k	1314.7k	0.0	59.7	1314.8k	1313.9k	0.1	1.5
dem_500_belg_orig_180	1715.4k	1715.4k	0.0	38.4	1715.4k	1713.9k	0.1	198.0	1715.4k	1715.4k	0.0	362.3
dem_500_belg_orig_181	1182.2k	1142.3k	3.5	14400.0	1182.2k	1181.1k	0.1	5783.1	1195.1k	1098.2k	8.8	14400.0
dem_500_belg_orig_182	1307.0k	1307.0k	0.0	2358.3	1307.0k	1305.7k	0.1	953.8	1309.5k	1269.7k	3.1	14400.0
dem_500_belg_orig_183	1328.0k	1280.9k	3.7	14400.0	1327.7k	1327.7k	0.0	12774.3	1328.7k	1237.8k	7.3	14400.0
dem_500_belg_orig_184	859.7k	859.7k	0.0	11407.4	859.7k	859.7k	0.0	5148.6	864.3k	806.9k	7.1	14400.0
dem_500_belg_orig_185	776.4k	776.4k	0.0	257.2	776.4k	776.4k	0.0	2506.0	776.4k	776.4k	0.0	1484.5
dem_500_belg_orig_186	2102.5k	2101.8k	0.0	16.5	2102.5k	2100.9k	0.1	156.9	2102.5k	2102.0k	0.0	225.0
dem_500_belg_orig_187	1936.1k	1906.3k	1.6	14400.0	1936.4k	1915.5k	1.1	14400.0	1940.6k	1873.2k	3.6	14400.0
dem_500_belg_orig_188	1110.1k	1110.1k	0.0	12.3	1110.1k	1110.1k	0.0	51.7	1110.1k	1110.1k	0.0	30.8
dem_500_belg_orig_189	1515.4k	1461.5k	3.7	14400.0	1515.4k	1514.2k	0.1	6565.9	1515.4k	1450.9k	4.4	14400.0
dem_500_belg_orig_190	1304.3k	1304.3k	0.0	4674.7	1304.3k	1303.1k	0.1	11521.8	1304.3k	1287.1k	1.3	14400.0
dem_500_belg_orig_191	1555.9k	1502.5k	3.6	14400.0	1552.1k	1552.1k	0.0	3099.8	1553.6k	1480.5k	4.9	14400.0
dem_500_belg_orig_192	1523.3k	1523.3k	0.0	1316.1	1523.3k	1523.3k	0.0	1643.9	1524.1k	1499.9k	1.6	14400.0
dem_500_belg_orig_193	1715.2k	1675.9k	2.3	14400.0	1719.3k	1685.3k	2.0	14400.0	1715.2k	1619.9k	5.9	14400.0
dem_500_belg_orig_194	1277.2k	1277.2k	0.0	814.8	1277.2k	1276.0k	0.1	215.9	1277.2k	1277.2k	0.0	1291.8

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Table A.5: Comparison of discrete models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_195	1407.6k	1348.9k	4.3	14400.0	1410.3k	1375.2k	2.6	14400.0	1407.2k	1324.5k	6.2	14400.0
dem_500_belg_orig_196	1911.8k	1911.8k	0.0	61.1	1911.8k	1910.3k	0.1	323.5	1911.8k	1911.8k	0.0	504.7
dem_500_belg_orig_197	1172.3k	1126.0k	4.1	14400.0	1172.2k	1141.1k	2.7	14400.0	1180.5k	1089.1k	8.4	14400.0
dem_500_belg_orig_198	555.2k	555.2k	0.0	9121.2	555.2k	555.2k	0.0	1377.8	558.0k	483.9k	15.3	14400.0
dem_500_belg_orig_199	1180.7k	1179.9k	0.1	949.9	1180.7k	1172.9k	0.7	14400.0	1180.7k	1179.5k	0.1	10460.2
dem_500_belg_orig_200	524.0k	524.0k	0.0	768.7	524.0k	524.0k	0.0	595.9	524.0k	523.8k	0.0	9121.4
dem_500_belg_orig_201	1443.2k	1443.2k	0.0	10862.2	1443.2k	1443.2k	0.0	2008.5	1447.9k	1384.0k	4.6	14400.0
dem_500_belg_orig_202	1324.1k	1324.1k	0.0	12.2	1324.1k	1323.1k	0.1	92.6	1324.1k	1324.1k	0.0	92.5
dem_500_belg_orig_203	2496.8k	2494.3k	0.1	13996.6	2496.8k	2489.8k	0.3	14400.0	2502.6k	2449.9k	2.2	14400.0
dem_500_belg_orig_204	1026.0k	1026.0k	0.0	877.8	1026.0k	1025.1k	0.1	499.2	1026.0k	1025.1k	0.1	2027.2
dem_500_belg_orig_205	1542.7k	1498.4k	3.0	14400.0	1542.7k	1521.9k	1.4	14400.0	1557.8k	1439.8k	8.2	14400.0
dem_500_belg_orig_206	2346.6k	2346.6k	0.0	107.5	2346.6k	2344.5k	0.1	1065.6	2346.6k	2345.4k	0.1	1557.3
dem_500_belg_orig_207	1319.5k	1319.5k	0.0	1699.5	1319.5k	1318.3k	0.1	1859.5	1319.5k	1294.9k	1.9	14400.0
dem_500_belg_orig_208	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	32.0	1e+20	1e+20	0.0	0.2
dem_500_belg_orig_209	2035.7k	1980.8k	2.8	14400.0	2035.7k	2034.3k	0.1	2172.8	2040.2k	1964.5k	3.9	14400.0
dem_500_belg_orig_210	1138.8k	1075.6k	5.9	14400.0	1133.3k	1133.3k	0.0	5237.9	1136.5k	1059.6k	7.3	14400.0
dem_500_belg_orig_211	634.1k	634.1k	0.0	0.4	634.1k	634.0k	0.0	332.5	634.1k	633.8k	0.1	0.5
dem_500_belg_orig_212	1861.3k	1861.3k	0.0	313.9	1861.3k	1851.5k	0.5	14400.0	1861.3k	1861.3k	0.0	1784.3
dem_500_belg_orig_213	960.7k	960.7k	0.0	10063.3	960.7k	960.7k	0.0	300.7	965.2k	908.8k	6.2	14400.0
dem_500_belg_orig_214	978.0k	978.0k	0.0	11641.9	978.0k	977.6k	0.0	4546.9	986.5k	932.9k	5.8	14400.0
dem_500_belg_orig_215	381.2k	381.2k	0.0	3251.3	381.2k	381.2k	0.0	3292.2	381.2k	353.1k	8.0	14400.0
dem_500_belg_orig_216	1060.3k	1059.7k	0.1	25.8	1060.3k	1059.9k	0.0	637.1	1060.3k	1059.4k	0.1	68.6
dem_500_belg_orig_217	1131.5k	1131.5k	0.0	233.1	1131.5k	1130.5k	0.1	835.0	1131.5k	1131.5k	0.0	296.4
dem_500_belg_orig_218	1598.6k	1558.4k	2.6	14400.0	1597.9k	1590.6k	0.5	14400.0	1597.9k	1544.3k	3.5	14400.0
dem_500_belg_orig_219	1225.4k	1225.4k	0.0	1554.2	1225.4k	1219.7k	0.5	14400.0	1225.4k	1198.4k	2.3	14400.0
dem_500_belg_orig_220	952.1k	952.1k	0.0	7400.2	952.1k	951.7k	0.0	7417.8	956.3k	919.0k	4.1	14400.0
dem_500_belg_orig_221	988.0k	939.6k	5.1	14400.0	988.0k	988.0k	0.0	2903.1	988.0k	939.5k	5.2	14400.0
dem_500_belg_orig_222	1456.7k	1415.9k	2.9	14400.0	1456.7k	1456.7k	0.0	470.6	1460.8k	1374.7k	6.3	14400.0
dem_500_belg_orig_223	1706.5k	1706.2k	0.0	11870.7	1706.5k	1704.8k	0.1	6201.2	1706.5k	1656.4k	3.0	14400.0
dem_500_belg_orig_224	1574.4k	1574.4k	0.0	2361.0	1574.4k	1574.4k	0.0	2344.3	1574.4k	1551.6k	1.5	14400.0
dem_500_belg_orig_225	832.2k	788.6k	5.5	14400.0	826.5k	811.3k	1.9	14400.0	847.2k	767.4k	10.4	14400.0
dem_500_belg_orig_226	1328.8k	1152.5k	15.3	14400.0	1293.6k	1292.8k	0.1	4800.1	1330.4k	1126.9k	18.1	14400.0
dem_500_belg_orig_227	434.6k	434.2k	0.1	0.1	434.6k	434.3k	0.1	41.1	434.6k	434.4k	0.1	0.6
dem_500_belg_orig_228	1148.9k	1148.1k	0.1	583.5	1148.9k	1148.9k	0.0	3624.0	1148.9k	1135.3k	1.2	14400.0
dem_500_belg_orig_229	1615.0k	1615.0k	0.0	8115.1	1615.0k	1615.0k	0.0	1009.4	1616.5k	1575.4k	2.6	14400.0
dem_500_belg_orig_230	1309.6k	1309.6k	0.0	11.3	1309.6k	1308.5k	0.1	59.9	1309.6k	1309.6k	0.0	20.3
dem_500_belg_orig_231	1628.8k	1568.4k	3.9	14400.0	1628.8k	1628.8k	0.0	1840.0	1636.1k	1554.0k	5.3	14400.0
dem_500_belg_orig_232	1989.9k	1989.9k	0.0	75.6	1989.9k	1989.0k	0.0	1233.4	1989.9k	1987.9k	0.1	1277.6
dem_500_belg_orig_233	1757.3k	1755.5k	0.1	1682.3	1757.3k	1755.8k	0.1	2795.5	1757.3k	1755.6k	0.1	2967.2
dem_500_belg_orig_234	1185.0k	1008.9k	17.5	14400.0	1162.9k	1162.9k	0.0	1043.1	1186.3k	963.9k	23.1	14400.0
dem_500_belg_orig_235	917.5k	917.5k	0.0	13703.9	917.6k	897.5k	2.2	14400.0	917.5k	870.1k	5.4	14400.0
dem_500_belg_orig_236	1398.2k	1398.2k	0.0	267.0	1398.2k	1398.1k	0.0	860.5	1398.2k	1398.2k	0.0	2069.0
dem_500_belg_orig_237	1913.9k	1912.0k	0.1	48.7	1913.9k	1912.1k	0.1	622.8	1913.9k	1913.9k	0.0	136.4
dem_500_belg_orig_238	1090.3k	1089.7k	0.1	101.7	1090.3k	1090.3k	0.0	358.4	1090.3k	1090.0k	0.0	1091.4
dem_500_belg_orig_239	2364.6k	2364.6k	0.0	832.2	2364.6k	2359.3k	0.2	14400.0	2364.6k	2301.6k	2.7	14400.0
dem_500_belg_orig_240	1737.7k	1736.4k	0.1	2344.2	1737.7k	1723.3k	0.8	14400.0	1737.7k	1736.1k	0.1	6130.1
dem_500_belg_orig_241	2556.4k	2556.4k	0.0	1493.7	2556.4k	2554.6k	0.1	1775.6	2556.4k	2530.0k	1.0	14400.0
dem_500_belg_orig_242	897.6k	897.0k	0.1	121.7	897.6k	897.2k	0.0	681.9	897.6k	897.6k	0.0	285.9
dem_500_belg_orig_243	1543.6k	1477.8k	4.5	14400.0	1543.6k	1531.1k	0.8	14400.0	1544.1k	1481.0k	4.3	14400.0
dem_500_belg_orig_244	983.8k	918.4k	7.1	14400.0	979.5k	979.5k	0.0	12410.6	980.5k	930.9k	5.3	14400.0
dem_500_belg_orig_245	640.2k	640.2k	0.0	25.4	640.2k	640.2k	0.0	101.1	640.2k	640.2k	0.0	336.1
dem_500_belg_orig_246	2132.8k	2130.9k	0.1	13.1	2132.8k	2132.8k	0.0	104.0	2132.8k	2131.8k	0.0	163.5
dem_500_belg_orig_247	2095.6k	2095.6k	0.0	1975.3	2095.6k	2093.8k	0.1	2996.9	2095.6k	2095.6k	0.0	7729.4
dem_500_belg_orig_248	1703.9k	1703.9k	0.0	71.5	1703.9k	1703.0k	0.1	652.0	1703.9k	1703.9k	0.0	4124.5
dem_500_belg_orig_249	1377.9k	1341.9k	2.7	14400.0	1376.6k	1376.6k	0.0	3179.8	1382.5k	1335.8k	3.5	14400.0
dem_500_belg_orig_250	1514.7k	1473.3k	2.8	14400.0	1511.0k	1493.5k	1.2	14400.0	1515.9k	1406.7k	7.8	14400.0
dem_500_belg_orig_251	641.4k	641.4k	0.0	29.5	641.4k	641.4k	0.0	204.4	641.4k	641.4k	0.0	31.8
dem_500_belg_orig_252	1648.8k	1648.8k	0.0	55.7	1648.8k	1647.2k	0.1	330.8	1648.8k	1647.3k	0.1	166.2
dem_500_belg_orig_253	1317.7k	1317.7k	0.0	2788.5	1317.7k	1309.3k	0.6	14400.0	1323.3k	1265.2k	4.6	14400.0
dem_500_belg_orig_254	1363.3k	1341.0k	1.7	14400.0	1363.4k	1344.4k	1.4	14400.0	1365.5k	1317.0k	3.7	14400.0
dem_500_belg_orig_255	1682.5k	1682.5k	0.0	2890.6	1682.5k	1682.1k	0.0	646.4	1683.0k	1647.9k	2.1	14400.0
dem_500_belg_orig_256	807.0k	807.0k	0.0	393.6	807.0k	807.0k	0.0	862.2	807.0k	807.0k	0.0	1757.8
dem_500_belg_orig_257	2077.4k	2077.4k	0.0	18.7	2077.4k	2075.5k	0.1	212.6	2077.4k	2077.4k	0.0	52.9
dem_500_belg_orig_258	882.6k	810.9k	8.8	14400.0	882.6k	861.8k	2.4	14400.0	915.0k	799.5k	14.4	14400.0
dem_500_belg_orig_259	1036.2k	982.0k	5.5	14400.0	1031.6k	1030.7k	0.1	2355.6	1031.6k	949.0k	8.7	14400.0
dem_500_belg_orig_260	1069.6k	1044.0k	2.4	14400.0	1069.6k	1069.6k	0.0	5580.4	1069.6k	1068.6k	0.1	6693.2
dem_500_belg_orig_261	1087.4k	925.6k	17.5	14400.0	1082.0k	1056.2k	2.4	14400.0	1087.3k	897.1k	21.2	14400.0
dem_500_belg_orig_262	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	35.1	1e+20	1e+20	0.0	0.2
dem_500_belg_orig_263	2111.1k	2111.1k	0.0	1393.5	2111.1k	2111.1k	0.0	3251.3	2111.2k	2073.9k	1.8	14400.0
dem_500_belg_orig_264	920.0k	877.4k	4.9	14400.0	919.3k	919.3k	0.0	1970.3	945.0k	841.6k	12.3	14400.0
dem_500_belg_orig_265	1730.2k	1730.2k	0.0	10.8	1730.2k	1730.2k	0.0	58.0	1730.2k	1730.1k	0.0	11.9
dem_500_belg_orig_266	1703.2k	1703.2k	0.0	61.0	1703.2k	1703.2k	0.0	232.7	1703.2k	1703.2k	0.0	171.9
dem_500_belg_orig_267	1783.4k	1697.4k	5.1	14400.0	1769.9k	1769.9k	0.0	6657.9	1787.5k	1676.7k	6.6	14400.0
dem_500_belg_orig_268	1708.5k	1708.0k	0.0	1664.8	1708.5k	1707.7k	0.0	800.5	1709.1k	1683.7k	1.5	14400.0
dem_500_belg_orig_269	1767.1k	1767.1k	0.0	799.6	1767.1k	1765.8k	0.1	3519.3	1767.1k	1765.8k	0.1	4103.1
dem_500_belg_orig_270	932.2k	865.4k	7.7	14400.0	930.6k	930.6k	0.0	11896.3	951.0k	826.0k	15.1	14400.0
dem_500_belg_orig_271	980.5k	917.8k	6.8	14400.0	975.2k	975.2k	0.0	6165.6	977.4k	906.9k	7.8	14400.0
dem_500_belg_orig_272	777.2k	777.2k	0.0	20.6	777.2k	776.4k	0.1	181.9	777.2k	777.2k	0.0	14.6

continued on next page

Table A.5: Comparison of discrete models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_273	1681.3k	1681.1k	0.0	2904.4	1681.3k	1680.1k	0.1	10421.3	1681.4k	1653.4k	1.7	14400.0
dem_500_belg_orig_274	1628.7k	1628.7k	0.0	46.8	1628.7k	1627.5k	0.1	268.7	1628.7k	1628.7k	0.0	180.6
dem_500_belg_orig_275	1009.7k	1008.8k	0.1	5584.4	1009.7k	1009.1k	0.1	6128.3	1011.3k	979.4k	3.2	14400.0
dem_500_belg_orig_276	680.0k	680.0k	0.0	3656.7	680.0k	679.9k	0.0	985.3	680.0k	680.0k	0.0	119.9
dem_500_belg_orig_277	1698.1k	1669.9k	1.7	14400.0	1697.2k	1695.6k	0.1	13748.7	1698.0k	1638.2k	3.7	14400.0
dem_500_belg_orig_278	1478.6k	1478.6k	0.0	22.1	1478.6k	1478.6k	0.0	282.4	1478.6k	1477.5k	0.1	219.8
dem_500_belg_orig_279	860.5k	860.5k	0.0	534.0	860.5k	860.5k	0.0	708.7	860.5k	860.5k	0.0	4047.9
dem_500_belg_orig_280	1807.7k	1806.3k	0.1	0.3	1807.7k	1806.4k	0.1	34.1	1807.7k	1805.9k	0.1	1.8
dem_500_belg_orig_281	682.7k	628.3k	8.7	14400.0	682.7k	682.2k	0.1	2281.5	682.7k	602.0k	13.4	14400.0
dem_500_belg_orig_282	1866.3k	1866.3k	0.0	397.0	1866.3k	1866.1k	0.0	2441.6	1866.3k	1864.6k	0.1	6527.3
dem_500_belg_orig_283	870.9k	870.9k	0.0	1960.4	870.9k	870.9k	0.0	661.6	870.9k	870.9k	0.0	3471.7
dem_500_belg_orig_284	1050.5k	996.9k	5.4	14400.0	1046.7k	1024.4k	2.2	14400.0	1057.4k	961.6k	10.0	14400.0
dem_500_belg_orig_285	2276.9k	2276.9k	0.0	6.3	2276.9k	2274.7k	0.1	109.8	2276.9k	2276.9k	0.0	15.0
dem_500_belg_orig_286	1185.6k	1185.6k	0.0	353.0	1185.6k	1185.6k	0.0	366.6	1185.6k	1185.6k	0.0	1708.6
dem_500_belg_orig_287	1616.6k	1548.5k	4.4	14400.0	1616.6k	1616.6k	0.0	7883.2	1620.4k	1498.3k	8.1	14400.0
dem_500_belg_orig_288	765.8k	667.6k	14.7	14400.0	758.3k	758.3k	0.0	14122.5	1e+20	628.5k	–	14400.0
dem_500_belg_orig_289	900.5k	881.7k	2.1	14400.0	900.5k	887.9k	1.4	14400.0	902.0k	839.5k	7.4	14400.0
dem_500_belg_orig_290	1696.7k	1696.7k	0.0	11881.0	1697.9k	1681.9k	1.0	14400.0	1706.6k	1611.8k	5.9	14400.0
dem_500_belg_orig_291	2080.0k	2080.0k	0.0	1571.2	2080.0k	2068.8k	0.5	14400.0	2080.0k	2049.2k	1.5	14400.0
dem_500_belg_orig_292	1365.8k	1364.5k	0.1	30.5	1365.8k	1364.5k	0.1	74.3	1365.8k	1365.8k	0.0	19.8
dem_500_belg_orig_293	684.0k	667.5k	2.5	14400.0	684.8k	661.7k	3.5	14400.0	684.2k	658.0k	4.0	14400.0
dem_500_belg_orig_294	993.4k	947.9k	4.8	14400.0	992.6k	968.7k	2.5	14400.0	1000.8k	932.1k	7.4	14400.0
dem_500_belg_orig_295	983.0k	982.3k	0.1	73.5	983.0k	982.6k	0.0	110.2	983.0k	983.0k	0.0	35.0
dem_500_belg_orig_296	1265.8k	1175.5k	7.7	14400.0	1264.8k	1264.8k	0.0	1160.7	1264.8k	1112.9k	13.7	14400.0
dem_500_belg_orig_297	1674.6k	1674.6k	0.0	1028.3	1674.6k	1673.1k	0.1	3587.4	1674.6k	1674.6k	0.0	3888.3
dem_500_belg_orig_298	635.0k	635.0k	0.0	4184.1	635.0k	635.0k	0.0	621.5	635.0k	635.0k	0.0	9513.5
dem_500_belg_orig_299	1326.9k	1094.1k	21.3	14400.0	1313.4k	1313.4k	0.0	10789.5	1341.4k	1070.3k	25.3	14400.0
dem_500_belg_orig_300	1372.9k	1285.3k	6.8	14400.0	1366.3k	1350.4k	1.2	14400.0	1368.9k	1237.0k	10.7	14400.0
dem_500_belg_orig_301	1240.4k	1240.4k	0.0	487.5	1240.4k	1240.4k	0.0	1678.9	1240.4k	1222.1k	1.5	14400.0
dem_500_belg_orig_302	746.1k	644.8k	15.7	14400.0	738.2k	738.2k	0.0	1735.0	741.0k	619.1k	19.7	14400.0
dem_500_belg_orig_303	1199.7k	1199.7k	0.0	3098.0	1199.7k	1199.7k	0.0	11305.4	1203.2k	1150.7k	4.6	14400.0
dem_500_belg_orig_304	1774.0k	1774.0k	0.0	43.1	1774.0k	1772.6k	0.1	550.6	1774.0k	1773.0k	0.1	202.9
dem_500_belg_orig_305	1854.1k	1854.1k	0.0	185.4	1854.1k	1852.2k	0.1	3175.3	1854.1k	1854.1k	0.0	2221.4
dem_500_belg_orig_306	1559.8k	1465.6k	6.4	14400.0	1559.2k	1534.9k	1.6	14400.0	1592.1k	1426.9k	11.6	14400.0
dem_500_belg_orig_307	2341.1k	2341.1k	0.0	710.0	2341.1k	2338.8k	0.1	4310.7	2341.1k	2341.1k	0.0	6299.1
dem_500_belg_orig_308	910.5k	910.5k	0.0	184.4	910.5k	909.8k	0.1	1186.0	910.5k	910.5k	0.0	2327.5
dem_500_belg_orig_309	1646.9k	1646.9k	0.0	21.1	1646.9k	1646.2k	0.0	124.2	1646.9k	1646.5k	0.0	168.2
dem_500_belg_orig_310	1313.1k	1272.4k	3.2	14400.0	1309.8k	1308.9k	0.1	5697.2	1331.1k	1247.4k	6.7	14400.0
dem_500_belg_orig_311	1138.0k	1044.2k	9.0	14400.0	1129.2k	1129.2k	0.0	3434.6	1158.4k	1022.4k	13.3	14400.0
dem_500_belg_orig_312	838.6k	838.6k	0.0	11462.0	838.6k	833.4k	0.6	14400.0	839.7k	788.6k	6.5	14400.0
dem_500_belg_orig_313	1255.3k	1255.3k	0.0	18.4	1255.3k	1254.7k	0.1	327.7	1255.3k	1254.2k	0.1	81.3
dem_500_belg_orig_314	1103.7k	1076.0k	2.6	14400.0	1103.7k	1102.7k	0.1	10603.1	1104.5k	1059.4k	4.3	14400.0
dem_500_belg_orig_315	1193.5k	1127.4k	5.9	14400.0	1175.7k	1162.9k	1.1	14400.0	1185.1k	1104.8k	7.3	14400.0
dem_500_belg_orig_316	1538.1k	1514.3k	1.6	14400.0	1538.1k	1495.0k	2.9	14400.0	1538.1k	1422.6k	8.1	14400.0
dem_500_belg_orig_317	626.2k	626.1k	0.0	9878.8	626.2k	620.6k	0.9	14400.0	636.3k	578.4k	10.0	14400.0
dem_500_belg_orig_318	1028.3k	1027.2k	0.1	20.5	1028.3k	1027.3k	0.1	146.3	1028.3k	1028.3k	0.0	53.6
dem_500_belg_orig_319	1119.4k	1119.4k	0.0	11164.9	1119.4k	1104.1k	1.4	14400.0	1119.4k	1085.3k	3.1	14400.0
dem_500_belg_orig_320	1963.6k	1913.2k	2.6	14400.0	1963.4k	1961.4k	0.1	3116.3	1963.6k	1931.2k	1.7	14400.0
dem_500_belg_orig_321	522.9k	522.9k	0.0	28.0	522.9k	522.9k	0.0	313.1	522.9k	522.9k	0.0	104.3
dem_500_belg_orig_322	1947.3k	1947.3k	0.0	27.6	1947.3k	1947.3k	0.0	141.5	1947.3k	1947.3k	0.0	403.4
dem_500_belg_orig_323	1639.1k	1639.1k	0.0	518.0	1639.1k	1638.4k	0.0	8153.6	1639.1k	1637.5k	0.1	6870.5
dem_500_belg_orig_324	1281.3k	1281.3k	0.0	9.0	1281.3k	1280.8k	0.0	54.6	1281.3k	1281.3k	0.0	47.0
dem_500_belg_orig_325	1357.1k	1356.2k	0.1	13783.1	1357.1k	1357.1k	0.0	2624.2	1377.0k	1274.9k	8.0	14400.0
dem_500_belg_orig_326	1835.4k	1788.8k	2.6	14400.0	1830.7k	1829.9k	0.0	3947.5	1844.8k	1774.4k	4.0	14400.0
dem_500_belg_orig_327	2332.8k	2330.5k	0.1	167.1	2332.8k	2330.6k	0.1	374.7	2334.8k	2266.3k	3.0	14400.0
dem_500_belg_orig_328	1834.9k	1834.9k	0.0	4412.8	1835.4k	1816.7k	1.0	14400.0	1835.5k	1749.4k	4.9	14400.0
dem_500_belg_orig_329	745.6k	745.6k	0.0	180.3	745.6k	745.4k	0.0	569.1	745.6k	745.6k	0.0	2668.3
dem_500_belg_orig_330	2004.6k	2004.6k	0.0	2280.3	2004.6k	1999.4k	0.3	14400.0	2004.6k	2004.6k	0.0	8714.7
dem_500_belg_orig_331	891.9k	891.9k	0.0	923.4	891.9k	891.1k	0.1	3868.2	891.9k	891.9k	0.0	1832.6
dem_500_belg_orig_332	1118.8k	1117.8k	0.1	1661.0	1118.8k	1118.8k	0.0	4346.6	1118.8k	1118.8k	0.0	6535.0
dem_500_belg_orig_333	1664.9k	1663.8k	0.1	4554.1	1664.9k	1663.4k	0.1	12791.8	1664.9k	1630.1k	2.1	14400.0
dem_500_belg_orig_334	1098.1k	1098.1k	0.0	430.1	1098.1k	1098.1k	0.0	992.8	1098.1k	1098.1k	0.0	496.7
dem_500_belg_orig_335	1454.6k	1388.6k	4.7	14400.0	1450.7k	1450.7k	0.0	2228.6	1454.3k	1317.3k	10.4	14400.0
dem_500_belg_orig_336	1738.8k	1738.4k	0.0	346.1	1738.8k	1737.1k	0.1	2506.8	1738.8k	1738.8k	0.0	966.3
dem_500_belg_orig_337	1114.0k	998.1k	11.6	14400.0	1104.2k	1103.6k	0.1	6552.0	1108.5k	984.0k	12.6	14400.0
dem_500_belg_orig_338	1441.2k	1419.9k	1.5	14400.0	1441.2k	1424.7k	1.2	14400.0	1456.1k	1358.8k	7.2	14400.0
dem_500_belg_orig_339	2186.7k	2184.6k	0.1	1445.8	2186.7k	2184.6k	0.1	5065.1	2186.7k	2185.2k	0.1	7391.1
dem_500_belg_orig_340	582.9k	548.3k	6.3	14400.0	580.7k	580.5k	0.0	4306.8	580.7k	530.8k	9.4	14400.0
dem_500_belg_orig_341	946.1k	946.1k	0.0	1586.5	946.1k	946.1k	0.0	1390.3	946.1k	946.1k	0.0	9793.8
dem_500_belg_orig_342	1554.1k	1554.1k	0.0	2175.0	1554.9k	1525.1k	2.0	14400.0	1554.1k	1522.9k	2.1	14400.0
dem_500_belg_orig_343	1888.3k	1886.5k	0.1	14178.1	1891.1k	1871.4k	1.1	14400.0	1889.7k	1846.6k	2.3	14400.0
dem_500_belg_orig_344	2391.1k	2391.1k	0.0	5.0	2391.1k	2388.7k	0.1	55.5	2391.1k	2389.7k	0.1	16.0
dem_500_belg_orig_345	1389.8k	1282.4k	8.4	14400.0	1389.8k	1365.0k	1.8	14400.0	1409.8k	1255.7k	12.3	14400.0
dem_500_belg_orig_346	1844.0k	1844.0k	0.0	7767.8	1844.0k	1842.2k	0.1	13352.3	1846.7k	1808.2k	2.1	14400.0
dem_500_belg_orig_347	1850.0k	1849.5k	0.0	3699.2	1850.0k	1845.7k	0.2	14400.0	1855.0k	1814.4k	2.2	14400.0
dem_500_belg_orig_348	1717.1k	1717.1k	0.0	5638.9	1718.6k	1697.5k	1.2	14400.0	1720.2k	1641.8k	4.8	14400.0
dem_500_belg_orig_349	2622.9k	2620.8k	0.1	4493.7	2622.9k	2617.6k	0.2	14400.0	2637.6k	2567.2k	2.7	14400.0
dem_500_belg_orig_350	1680.2k	1680.2k	0.0	912.4	1680.2k	1678.7k	0.1	2208.7	1680.2k	1642.3k	2.3	14400.0

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Table A.5: Comparison of discrete models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_351	1573.6k	1573.6k	0.0	3460.6	1573.6k	1572.3k	0.1	9835.2	1575.6k	1529.1k	3.0	14400.0
dem_500_belg_orig_352	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	27.2	1e+20	1e+20	0.0	0.1
dem_500_belg_orig_353	2034.7k	2034.7k	0.0	10557.3	2035.0k	2010.5k	1.2	14400.0	2036.5k	1961.0k	3.8	14400.0
dem_500_belg_orig_354	1051.7k	1050.7k	0.1	52.7	1051.7k	1051.7k	0.0	723.4	1051.7k	1051.7k	0.0	485.4
dem_500_belg_orig_355	1185.6k	1108.2k	7.0	14400.0	1185.2k	1161.3k	2.1	14400.0	1243.1k	1051.6k	18.2	14400.0
dem_500_belg_orig_356	956.3k	955.8k	0.1	694.3	956.3k	956.3k	0.0	813.8	956.3k	955.7k	0.1	886.4
dem_500_belg_orig_357	1354.0k	1354.0k	0.0	3458.3	1354.0k	1345.1k	0.7	14400.0	1355.1k	1319.3k	2.7	14400.0
dem_500_belg_orig_358	1171.1k	1171.1k	0.0	14302.1	1172.1k	1135.5k	3.2	14400.0	1171.9k	1119.8k	4.6	14400.0
dem_500_belg_orig_359	876.2k	876.0k	0.0	5821.9	876.2k	876.2k	0.0	6887.5	876.2k	825.7k	6.1	14400.0
dem_500_belg_orig_360	1339.6k	1339.5k	0.0	8.5	1339.6k	1339.6k	0.0	84.3	1339.6k	1339.6k	0.0	42.5
dem_500_belg_orig_361	2060.0k	2059.1k	0.0	95.2	2060.0k	2058.1k	0.1	1226.1	2060.0k	2060.0k	0.0	1945.6
dem_500_belg_orig_362	1743.5k	1743.5k	0.0	8458.5	1743.5k	1743.5k	0.0	5300.4	1744.7k	1672.6k	4.3	14400.0
dem_500_belg_orig_363	1640.8k	1640.8k	0.0	14138.5	1640.8k	1639.4k	0.1	9065.5	1643.1k	1603.9k	2.4	14400.0
dem_500_belg_orig_364	965.7k	912.5k	5.8	14400.0	978.0k	944.4k	3.6	14400.0	964.4k	891.2k	8.2	14400.0
dem_500_belg_orig_365	1806.8k	1785.8k	1.2	14400.0	1807.1k	1783.5k	1.3	14400.0	1812.5k	1732.2k	4.6	14400.0
dem_500_belg_orig_366	1171.6k	1171.6k	0.0	991.2	1171.6k	1160.6k	0.9	14400.0	1171.6k	1171.6k	0.0	7010.3
dem_500_belg_orig_367	1277.0k	1277.0k	0.0	5107.5	1277.0k	1258.7k	1.5	14400.0	1277.0k	1259.1k	1.4	14400.0
dem_500_belg_orig_368	1079.4k	1079.4k	0.0	3358.0	1079.4k	1078.7k	0.1	1703.2	1079.4k	1079.4k	0.0	13031.9
dem_500_belg_orig_369	1158.1k	1158.1k	0.0	2356.2	1158.1k	1158.1k	0.0	4152.4	1158.3k	1130.8k	2.4	14400.0
dem_500_belg_orig_370	1012.4k	926.6k	9.3	14400.0	1007.3k	966.5k	4.2	14400.0	1018.4k	873.2k	16.6	14400.0
dem_500_belg_orig_371	1172.0k	1172.0k	0.0	1454.6	1172.0k	1171.8k	0.0	1626.2	1172.0k	1172.0k	0.0	6259.2
dem_500_belg_orig_372	842.0k	687.2k	22.5	14400.0	840.1k	840.1k	0.0	7602.6	844.2k	678.3k	24.5	14400.0
dem_500_belg_orig_373	1469.4k	1469.4k	0.0	723.6	1469.4k	1467.9k	0.1	4591.5	1469.4k	1468.1k	0.1	7063.9
dem_500_belg_orig_374	1042.9k	1042.1k	0.1	573.9	1042.9k	1042.5k	0.0	686.8	1042.9k	1042.9k	0.0	4228.2
dem_500_belg_orig_375	821.7k	821.2k	0.1	180.6	821.7k	821.7k	0.0	584.6	821.7k	821.4k	0.0	861.7
dem_500_belg_orig_376	1454.2k	1454.2k	0.0	8984.6	1454.2k	1439.6k	1.0	14400.0	1454.2k	1398.8k	4.0	14400.0
dem_500_belg_orig_377	2105.9k	2105.9k	0.0	4.5	2105.9k	2104.5k	0.1	82.8	2105.9k	2104.2k	0.1	16.5
dem_500_belg_orig_378	1336.3k	1336.3k	0.0	748.9	1336.3k	1334.9k	0.1	7254.7	1336.3k	1336.3k	0.0	3593.6
dem_500_belg_orig_379	2200.9k	2200.9k	0.0	21.0	2200.9k	2198.8k	0.1	235.3	2200.9k	2199.0k	0.1	228.8
dem_500_belg_orig_380	1969.3k	1969.3k	0.0	327.9	1969.3k	1967.4k	0.1	1635.4	1969.3k	1952.0k	0.9	14400.0
dem_500_belg_orig_381	1189.4k	1189.4k	0.0	235.4	1189.4k	1189.4k	0.0	695.3	1189.4k	1189.4k	0.0	500.8
dem_500_belg_orig_382	770.9k	770.9k	0.0	1750.5	770.9k	770.6k	0.0	2717.0	770.9k	754.7k	2.2	14400.0
dem_500_belg_orig_383	1333.2k	1333.2k	0.0	416.9	1333.2k	1332.0k	0.1	1159.9	1333.2k	1333.2k	0.0	8507.3
dem_500_belg_orig_384	1323.2k	1323.2k	0.0	60.2	1323.2k	1322.1k	0.1	139.9	1323.2k	1322.2k	0.1	107.3
dem_500_belg_orig_385	2209.6k	2209.6k	0.0	10601.8	2209.6k	2195.3k	0.7	14400.0	2215.5k	2120.7k	4.5	14400.0
dem_500_belg_orig_386	1117.4k	1117.4k	0.0	473.0	1117.4k	1116.3k	0.1	7205.9	1117.4k	1117.4k	0.0	3163.1
dem_500_belg_orig_387	712.8k	712.8k	0.0	58.7	712.8k	712.8k	0.0	8624.5	712.8k	712.2k	0.1	955.6
dem_500_belg_orig_388	679.4k	679.4k	0.0	3183.5	679.4k	679.4k	0.0	1851.8	681.2k	634.4k	7.4	14400.0
dem_500_belg_orig_389	1858.1k	1774.9k	4.7	14400.0	1858.1k	1858.1k	0.0	2877.7	1860.5k	1680.1k	10.7	14400.0
dem_500_belg_orig_390	1219.9k	1068.2k	14.2	14400.0	1181.7k	1159.7k	1.9	14400.0	1190.1k	1041.0k	14.3	14400.0
dem_500_belg_orig_391	1686.3k	1685.6k	0.0	255.5	1686.3k	1686.1k	0.0	136.4	1686.3k	1685.6k	0.0	511.4
dem_500_belg_orig_392	1049.0k	959.4k	9.3	14400.0	1040.7k	1040.7k	0.0	417.8	1053.0k	939.2k	12.1	14400.0
dem_500_belg_orig_393	1561.2k	1561.2k	0.0	12491.6	1561.2k	1541.0k	1.3	14400.0	1567.7k	1515.2k	3.5	14400.0
dem_500_belg_orig_394	1755.6k	1755.6k	0.0	2336.5	1755.6k	1753.9k	0.1	3671.4	1755.6k	1753.9k	0.1	4679.7
dem_500_belg_orig_395	1940.7k	1940.7k	0.0	568.0	1940.7k	1939.3k	0.1	1841.6	1940.7k	1940.7k	0.0	11262.2
dem_500_belg_orig_396	1669.2k	1669.2k	0.0	106.5	1669.2k	1669.2k	0.0	1703.8	1669.2k	1668.1k	0.1	1569.3
dem_500_belg_orig_397	978.1k	978.1k	0.0	13728.0	978.1k	978.1k	0.0	6072.7	978.1k	916.7k	6.7	14400.0
dem_500_belg_orig_398	1334.5k	1334.5k	0.0	3695.3	1334.5k	1333.5k	0.1	1006.9	1334.5k	1334.5k	0.0	12175.1
dem_500_belg_orig_399	1279.2k	1278.0k	0.1	5270.2	1279.2k	1272.5k	0.5	14400.0	1280.1k	1242.8k	3.0	14400.0
dem_500_belg_orig_400	1361.5k	1343.7k	1.3	14400.0	1361.5k	1348.4k	1.0	14400.0	1361.5k	1330.8k	2.3	14400.0
dem_500_belg_orig_401	1348.2k	1348.2k	0.0	423.7	1349.0k	1337.1k	0.9	14400.0	1348.2k	1348.2k	0.0	10501.1
dem_500_belg_orig_402	848.4k	848.2k	0.0	170.1	848.4k	848.1k	0.0	1160.5	848.4k	848.4k	0.0	994.1
dem_500_belg_orig_403	1757.9k	1757.7k	0.0	15.7	1757.9k	1756.5k	0.1	92.1	1757.9k	1757.9k	0.0	24.6
dem_500_belg_orig_404	1729.9k	1729.9k	0.0	3659.9	1729.9k	1728.2k	0.1	9823.1	1729.9k	1700.3k	1.7	14400.0
dem_500_belg_orig_405	916.2k	916.2k	0.0	355.9	916.2k	916.2k	0.0	2077.1	916.2k	916.2k	0.0	1462.1
dem_500_belg_orig_406	1184.4k	1126.7k	5.1	14400.0	1183.5k	1183.5k	0.0	5190.9	1186.3k	1094.4k	8.4	14400.0
dem_500_belg_orig_407	1014.7k	963.6k	5.3	14400.0	1014.0k	1014.0k	0.0	7207.6	1022.7k	941.3k	8.7	14400.0
dem_500_belg_orig_408	834.1k	656.5k	27.0	14400.0	831.9k	791.1k	5.1	14400.0	839.3k	634.9k	32.2	14400.0
dem_500_belg_orig_409	2102.7k	2102.7k	0.0	25.9	2102.7k	2100.9k	0.1	611.3	2102.7k	2100.8k	0.1	88.2
dem_500_belg_orig_410	1303.7k	1303.7k	0.0	439.9	1303.7k	1303.1k	0.0	1002.0	1303.7k	1303.7k	0.0	4428.5
dem_500_belg_orig_411	832.5k	831.8k	0.1	370.3	832.5k	832.5k	0.0	411.8	832.5k	816.8k	1.9	14400.0
dem_500_belg_orig_412	1713.0k	1712.3k	0.0	268.6	1713.0k	1711.5k	0.1	4004.9	1713.0k	1711.6k	0.1	2291.0
dem_500_belg_orig_413	1123.8k	958.5k	17.2	14400.0	1121.1k	1121.1k	0.0	2578.2	1124.2k	1041.0k	8.0	14400.0
dem_500_belg_orig_414	885.7k	885.7k	0.0	385.8	885.7k	885.2k	0.0	479.6	885.7k	885.7k	0.0	4253.9
dem_500_belg_orig_415	1399.7k	1326.1k	5.6	14400.0	1399.7k	1399.7k	0.0	8270.2	1428.1k	1280.8k	11.5	14400.0
dem_500_belg_orig_416	870.0k	869.4k	0.1	99.5	870.0k	869.2k	0.1	428.4	870.0k	870.0k	0.0	319.7
dem_500_belg_orig_417	1685.8k	1684.5k	0.1	26.1	1685.8k	1684.2k	0.1	135.4	1685.8k	1684.3k	0.1	193.0
dem_500_belg_orig_418	1692.4k	1692.4k	0.0	171.9	1692.4k	1692.4k	0.0	468.8	1692.4k	1692.4k	0.0	2442.2
dem_500_belg_orig_419	1519.1k	1519.1k	0.0	48.0	1519.1k	1519.1k	0.0	432.7	1519.1k	1517.8k	0.1	129.1
dem_500_belg_orig_420	1144.8k	949.9k	20.5	14400.0	1141.1k	1141.1k	0.0	2907.9	1147.6k	925.3k	24.0	14400.0
dem_500_belg_orig_421	1679.3k	1628.2k	3.1	14400.0	1679.5k	1631.5k	2.9	14400.0	1677.1k	1577.5k	6.3	14400.0
dem_500_belg_orig_422	1519.6k	1470.0k	3.4	14400.0	1519.6k	1490.5k	2.0	14400.0	1521.1k	1445.3k	5.2	14400.0
dem_500_belg_orig_423	1263.5k	1229.9k	2.7	14400.0	1263.5k	1263.5k	0.0	10721.6	1e+20	1178.6k	–	14400.0
dem_500_belg_orig_424	1841.6k	1841.6k	0.0	49.6	1841.6k	1841.6k	0.0	206.2	1841.6k	1840.3k	0.1	805.3
dem_500_belg_orig_425	1177.6k	1142.3k	3.1	14400.0	1176.6k	1153.1k	2.0	14400.0	1176.6k	1055.2k	11.5	14400.0
dem_500_belg_orig_426	1271.4k	1205.8k	5.4	14400.0	1269.1k	1246.0k	1.9	14400.0	1285.8k	1159.6k	10.9	14400.0
dem_500_belg_orig_427	1003.7k	1003.5k	0.0	8.3	1003.7k	1003.7k	0.0	103.6	1003.7k	1003.3k	0.0	15.9
dem_500_belg_orig_428	1589.7k	1589.5k	0.0	6319.1	1589.7k	1588.1k	0.1	1304.5	1590.4k	1549.5k	2.6	14400.0

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Table A.5: Comparison of discrete models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_429	1557.6k	1527.8k	1.9	14400.0	1557.6k	1556.4k	0.1	4646.3	1565.7k	1458.8k	7.3	14400.0
dem_500_belg_orig_430	1792.7k	1706.8k	5.0	14400.0	1787.6k	1786.3k	0.1	2877.0	1787.6k	1655.8k	8.0	14400.0
dem_500_belg_orig_431	879.0k	879.0k	0.0	2496.2	879.0k	879.0k	0.0	4502.3	1e+20	838.2k	–	14400.0
dem_500_belg_orig_432	1502.6k	1502.6k	0.0	266.9	1502.6k	1501.4k	0.1	1539.1	1502.6k	1502.6k	0.0	6792.2
dem_500_belg_orig_433	1095.6k	1095.6k	0.0	7332.9	1095.6k	1071.3k	2.3	14400.0	1104.7k	1006.0k	9.8	14400.0
dem_500_belg_orig_434	1102.5k	1102.5k	0.0	1.9	1102.5k	1102.1k	0.0	54.5	1102.5k	1101.9k	0.1	4.0
dem_500_belg_orig_435	1404.2k	1404.2k	0.0	2191.5	1404.2k	1403.2k	0.1	1525.1	1404.2k	1375.3k	2.1	14400.0
dem_500_belg_orig_436	1242.2k	1242.2k	0.0	2228.4	1242.2k	1241.0k	0.1	12267.4	1245.6k	1189.2k	4.7	14400.0
dem_500_belg_orig_437	1845.4k	1845.4k	0.0	244.2	1845.4k	1843.7k	0.1	1392.7	1845.4k	1845.4k	0.0	1772.2
dem_500_belg_orig_438	1045.3k	819.7k	27.5	14400.0	1045.3k	997.8k	4.8	14400.0	1075.7k	815.2k	31.9	14400.0
dem_500_belg_orig_439	1307.2k	1307.2k	0.0	33.9	1307.2k	1307.2k	0.0	103.2	1307.2k	1307.2k	0.0	162.9
dem_500_belg_orig_440	1658.5k	1658.5k	0.0	29.9	1658.5k	1658.2k	0.0	81.5	1658.5k	1658.5k	0.0	505.1
dem_500_belg_orig_441	591.5k	591.5k	0.0	1009.8	591.5k	591.5k	0.0	852.1	591.5k	591.5k	0.0	3903.0
dem_500_belg_orig_442	546.2k	546.2k	0.0	445.3	546.2k	545.7k	0.1	1094.5	546.2k	546.2k	0.0	688.1
dem_500_belg_orig_443	1598.9k	1598.9k	0.0	81.8	1598.9k	1598.9k	0.0	1280.8	1598.9k	1598.9k	0.0	528.1
dem_500_belg_orig_444	1086.3k	1024.8k	6.0	14400.0	1089.6k	1066.3k	2.2	14400.0	1092.5k	1005.6k	8.6	14400.0
dem_500_belg_orig_445	2095.7k	2095.7k	0.0	1636.0	2095.7k	2073.8k	1.1	14400.0	2095.7k	2095.6k	0.0	14092.5
dem_500_belg_orig_446	1450.5k	1421.9k	2.0	14400.0	1450.5k	1450.5k	0.0	6074.6	1451.9k	1409.7k	3.0	14400.0
dem_500_belg_orig_447	1824.6k	1824.6k	0.0	16.3	1824.6k	1822.9k	0.1	58.4	1824.6k	1824.6k	0.0	15.2
dem_500_belg_orig_448	1451.1k	1451.1k	0.0	1227.6	1451.1k	1450.5k	0.0	5351.2	1451.1k	1450.2k	0.1	5375.2
dem_500_belg_orig_449	1632.9k	1631.3k	0.1	108.8	1632.9k	1631.3k	0.1	544.8	1632.9k	1631.6k	0.1	551.4
dem_500_belg_orig_450	1529.8k	1529.8k	0.0	131.5	1529.8k	1528.2k	0.1	137.1	1529.8k	1529.8k	0.0	41.0
dem_500_belg_orig_451	995.3k	995.3k	0.0	344.4	995.3k	995.3k	0.0	74.4	995.3k	994.4k	0.1	414.4
dem_500_belg_orig_452	1000.8k	1000.8k	0.0	1716.8	1000.8k	1000.8k	0.0	3893.3	1000.8k	991.5k	0.9	14400.0
dem_500_belg_orig_453	1061.9k	1061.9k	0.0	947.7	1061.9k	1061.9k	0.0	758.5	1061.9k	1060.9k	0.1	1209.8
dem_500_belg_orig_454	1606.0k	1558.1k	3.1	14400.0	1606.1k	1582.6k	1.5	14400.0	1608.1k	1560.1k	3.1	14400.0
dem_500_belg_orig_455	1183.1k	1183.1k	0.0	54.1	1183.1k	1182.0k	0.1	215.8	1183.1k	1183.1k	0.0	262.3
dem_500_belg_orig_456	848.2k	773.9k	9.6	14400.0	845.4k	812.8k	4.0	14400.0	846.3k	744.8k	13.6	14400.0
dem_500_belg_orig_457	1538.5k	1538.5k	0.0	248.5	1538.5k	1537.0k	0.1	475.8	1538.5k	1538.5k	0.0	1037.2
dem_500_belg_orig_458	753.8k	753.8k	0.0	11353.6	753.8k	727.5k	3.6	14400.0	753.9k	732.6k	2.9	14400.0
dem_500_belg_orig_459	1773.8k	1773.8k	0.0	255.1	1773.8k	1772.0k	0.1	2028.6	1773.8k	1773.8k	0.0	1201.6
dem_500_belg_orig_460	1091.1k	973.3k	12.1	14400.0	1084.7k	1084.7k	0.0	2751.6	1094.0k	961.2k	13.8	14400.0
dem_500_belg_orig_461	1440.7k	1218.4k	18.2	14400.0	1436.7k	1436.7k	0.0	4275.4	1440.9k	1267.2k	13.7	14400.0
dem_500_belg_orig_462	1466.8k	1466.8k	0.0	331.3	1466.8k	1466.8k	0.0	6160.0	1466.8k	1466.3k	0.0	8823.9
dem_500_belg_orig_463	897.5k	897.5k	0.0	2805.9	897.5k	897.5k	0.0	3011.7	900.1k	868.2k	3.7	14400.0
dem_500_belg_orig_464	1052.3k	1052.3k	0.0	217.1	1052.3k	1051.7k	0.1	1908.2	1052.3k	1052.3k	0.0	2773.1
dem_500_belg_orig_465	1630.4k	1629.5k	0.1	3097.6	1630.4k	1628.8k	0.1	10323.0	1630.4k	1629.6k	0.1	11611.4
dem_500_belg_orig_466	1997.5k	1997.5k	0.0	85.0	1997.5k	1997.0k	0.0	868.7	1997.5k	1997.5k	0.0	1529.3
dem_500_belg_orig_467	2156.1k	2156.1k	0.0	2803.6	2156.1k	2154.1k	0.1	2302.6	2156.1k	2142.5k	0.6	14400.0
dem_500_belg_orig_468	1717.7k	1717.7k	0.0	12.9	1717.7k	1716.2k	0.1	82.1	1717.7k	1717.7k	0.0	33.2
dem_500_belg_orig_469	1316.7k	1316.7k	0.0	12.7	1316.7k	1316.7k	0.0	120.1	1316.7k	1316.7k	0.0	116.8
dem_500_belg_orig_470	1339.4k	1266.2k	5.8	14400.0	1339.2k	1298.5k	3.1	14400.0	1350.5k	1229.2k	9.9	14400.0
dem_500_belg_orig_471	1713.7k	1625.9k	5.4	14400.0	1706.2k	1671.1k	2.1	14400.0	1723.5k	1553.2k	11.0	14400.0
dem_500_belg_orig_472	1005.1k	1005.1k	0.0	1731.4	1005.1k	1004.4k	0.1	1076.1	1005.1k	1005.1k	0.0	2830.3
dem_500_belg_orig_473	1325.1k	1234.3k	7.4	14400.0	1323.3k	1296.2k	2.1	14400.0	1324.3k	1190.3k	11.3	14400.0
dem_500_belg_orig_474	1296.5k	1257.0k	3.1	14400.0	1295.7k	1294.9k	0.1	6520.2	1296.5k	1208.4k	7.3	14400.0
dem_500_belg_orig_475	1075.9k	1013.2k	6.2	14400.0	1075.9k	1075.9k	0.0	2007.8	1103.2k	980.0k	12.6	14400.0
dem_500_belg_orig_476	1287.2k	1287.2k	0.0	9471.9	1287.2k	1287.0k	0.0	7573.6	1289.0k	1201.8k	7.3	14400.0
dem_500_belg_orig_477	1188.4k	1188.4k	0.0	517.2	1188.4k	1188.4k	0.0	1276.1	1188.4k	1188.4k	0.0	855.8
dem_500_belg_orig_478	1027.9k	996.0k	3.2	14400.0	1027.9k	1011.8k	1.6	14400.0	1061.7k	972.2k	9.2	14400.0
dem_500_belg_orig_479	1660.4k	1659.7k	0.0	2093.8	1660.4k	1658.8k	0.1	10864.1	1660.4k	1660.4k	0.0	12107.4
dem_500_belg_orig_480	1510.2k	1510.0k	0.0	22.6	1510.2k	1509.8k	0.0	178.3	1510.2k	1510.2k	0.0	197.7
dem_500_belg_orig_481	1046.6k	1046.6k	0.0	8.3	1046.6k	1046.6k	0.0	283.9	1046.6k	1046.6k	0.0	74.9
dem_500_belg_orig_482	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	18.5	1e+20	1e+20	0.0	0.2
dem_500_belg_orig_483	1045.4k	901.3k	16.0	14400.0	1045.4k	1022.6k	2.2	14400.0	1047.6k	858.7k	22.0	14400.0
dem_500_belg_orig_484	1347.7k	1347.7k	0.0	325.5	1347.7k	1347.7k	0.0	1378.9	1347.7k	1335.3k	0.9	14400.0
dem_500_belg_orig_485	894.8k	894.8k	0.0	1200.2	894.8k	894.4k	0.0	4081.2	894.8k	894.0k	0.1	1998.5
dem_500_belg_orig_486	1077.6k	1052.9k	2.3	14400.0	1076.7k	1075.7k	0.1	14054.3	1087.9k	1024.6k	6.2	14400.0
dem_500_belg_orig_487	1252.6k	1204.4k	4.0	14400.0	1247.0k	1247.0k	0.0	2171.0	1263.0k	1183.1k	6.8	14400.0
dem_500_belg_orig_488	1749.5k	1749.5k	0.0	7.3	1749.5k	1747.9k	0.1	75.6	1749.5k	1749.5k	0.0	11.4
dem_500_belg_orig_489	1160.6k	1160.0k	0.1	337.0	1160.6k	1159.5k	0.1	1479.1	1160.6k	1160.6k	0.0	540.5
dem_500_belg_orig_490	1007.7k	888.8k	13.4	14400.0	1007.4k	1007.4k	0.0	1808.9	1014.9k	830.2k	22.2	14400.0
dem_500_belg_orig_491	1134.4k	1042.2k	8.8	14400.0	1128.6k	1100.8k	2.5	14400.0	1129.1k	1007.1k	12.1	14400.0
dem_500_belg_orig_492	2190.3k	2188.7k	0.1	27.9	2190.3k	2190.3k	0.0	156.8	2190.3k	2190.3k	0.0	53.2
dem_500_belg_orig_493	1640.7k	1640.0k	0.0	8675.0	1640.7k	1640.7k	0.0	7564.3	1640.7k	1639.3k	0.1	11916.5
dem_500_belg_orig_494	1802.6k	1802.6k	0.0	10355.9	1802.6k	1801.9k	0.0	4966.9	1802.6k	1743.6k	3.4	14400.0
dem_500_belg_orig_495	1382.1k	1339.8k	3.2	14400.0	1385.0k	1355.0k	2.2	14400.0	1382.1k	1319.3k	4.8	14400.0
dem_500_belg_orig_496	1279.3k	1256.2k	1.8	14400.0	1279.3k	1278.8k	0.0	7066.1	1286.2k	1218.2k	5.6	14400.0
dem_500_belg_orig_497	1221.7k	1221.7k	0.0	1899.8	1221.7k	1221.7k	0.0	2178.8	1221.7k	1220.9k	0.1	7448.4
dem_500_belg_orig_498	1670.7k	1669.4k	0.1	1381.9	1670.7k	1648.1k	1.4	14400.0	1670.7k	1652.6k	1.1	14400.0
dem_500_belg_orig_499	962.9k	962.9k	0.0	1864.2	962.9k	962.9k	0.0	2848.9	962.9k	962.9k	0.0	4032.1
dem_500_belg_orig_500	1220.7k	1152.6k	5.9	14400.0	1215.7k	1185.8k	2.5	14400.0	1217.6k	1079.4k	12.8	14400.0

Table A.6: Detailed results of the split-pipe models on *Belgium* with $\mathcal{B} = 500$ as summarized in Figure 3.13c and Table 3.4a. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_1	612.2k	611.6k	0.1	16.9	612.2k	611.6k	0.1	21.9
dem_500_belg_orig_2	1307.0k	1305.7k	0.1	2187.3	1307.0k	1305.7k	0.1	3066.7
dem_500_belg_orig_3	1214.2k	1213.0k	0.1	11672.0	1214.2k	1213.0k	0.1	14340.0
dem_500_belg_orig_4	1603.6k	1552.8k	3.3	14400.0	1603.6k	1553.0k	3.3	14400.0
dem_500_belg_orig_5	1587.7k	1586.1k	0.1	27.3	1587.7k	1586.1k	0.1	30.2
dem_500_belg_orig_6	1205.3k	1204.1k	0.1	2489.3	1205.3k	1204.1k	0.1	1717.0
dem_500_belg_orig_7	754.2k	753.4k	0.1	1792.2	754.2k	753.6k	0.1	9535.4
dem_500_belg_orig_8	1601.8k	1600.2k	0.1	323.9	1601.8k	1596.0k	0.4	14400.0
dem_500_belg_orig_9	1562.9k	1561.4k	0.1	11.2	1562.9k	1561.3k	0.1	16.9
dem_500_belg_orig_10	1638.5k	1636.2k	0.1	14400.0	1638.5k	1633.7k	0.3	14400.0
dem_500_belg_orig_11	1975.1k	1974.7k	0.0	1.9	1974.9k	1973.1k	0.1	0.4
dem_500_belg_orig_12	1975.7k	1974.7k	0.1	1.2	1975.1k	1973.5k	0.1	0.6
dem_500_belg_orig_13	1053.2k	1049.4k	0.4	14400.0	1053.2k	1036.1k	1.6	14400.0
dem_500_belg_orig_14	738.3k	725.4k	1.8	14400.0	738.3k	723.5k	2.0	14400.0
dem_500_belg_orig_15	1575.9k	1574.3k	0.1	75.6	1575.9k	1574.3k	0.1	177.9
dem_500_belg_orig_16	1794.4k	1791.1k	0.2	14400.0	1794.4k	1791.1k	0.2	14400.0
dem_500_belg_orig_17	2032.9k	2031.7k	0.1	4.3	2032.6k	2030.6k	0.1	7.2
dem_500_belg_orig_18	778.1k	769.8k	1.1	14400.0	778.7k	701.1k	11.1	14400.0
dem_500_belg_orig_19	1277.2k	1276.0k	0.1	9.7	1277.2k	1275.9k	0.1	23.6
dem_500_belg_orig_20	999.2k	986.7k	1.3	14400.0	999.2k	998.2k	0.1	171.8
dem_500_belg_orig_21	821.3k	812.3k	1.1	14400.0	821.3k	820.5k	0.1	599.9
dem_500_belg_orig_22	1104.7k	1099.4k	0.5	14400.0	1104.7k	1102.7k	0.2	14400.0
dem_500_belg_orig_23	1279.6k	1226.5k	4.3	14400.0	1279.3k	1265.1k	1.1	14400.0
dem_500_belg_orig_24	1349.3k	1347.9k	0.1	1074.3	1349.3k	1327.4k	1.6	14400.0
dem_500_belg_orig_25	1473.1k	1471.6k	0.1	77.4	1473.1k	1471.6k	0.1	210.1
dem_500_belg_orig_26	1450.7k	1434.5k	1.1	14400.0	1450.7k	1388.2k	4.5	14400.0
dem_500_belg_orig_27	1785.7k	1778.5k	0.4	14400.0	1785.7k	1774.8k	0.6	14400.0
dem_500_belg_orig_28	1040.6k	1039.6k	0.1	2954.8	1040.6k	969.8k	7.3	14400.0
dem_500_belg_orig_29	1899.9k	1898.1k	0.1	0.9	1899.9k	1898.4k	0.1	0.8
dem_500_belg_orig_30	1286.5k	1253.3k	2.6	14400.0	1286.5k	1253.5k	2.6	14400.0
dem_500_belg_orig_31	1139.4k	1138.2k	0.1	8452.9	1139.4k	1138.9k	0.0	1422.7
dem_500_belg_orig_32	1014.8k	1013.8k	0.1	25.2	1014.8k	1013.8k	0.1	465.6
dem_500_belg_orig_33	1238.6k	1237.4k	0.1	44.4	1238.6k	1237.4k	0.1	20.0
dem_500_belg_orig_34	1052.9k	1041.7k	1.1	14400.0	1052.9k	1048.0k	0.5	14400.0
dem_500_belg_orig_35	1158.8k	1157.7k	0.1	246.7	1158.8k	1157.7k	0.1	9.3
dem_500_belg_orig_36	1783.5k	1780.3k	0.2	14400.0	1783.5k	1772.9k	0.6	14400.0
dem_500_belg_orig_37	705.9k	705.4k	0.1	1179.8	705.9k	705.2k	0.1	2946.3
dem_500_belg_orig_38	836.6k	835.8k	0.1	1749.6	836.6k	835.8k	0.1	130.8
dem_500_belg_orig_39	1953.1k	1895.9k	3.0	14400.0	1952.6k	1894.5k	3.1	14400.0
dem_500_belg_orig_40	1849.7k	1770.5k	4.5	14400.0	1849.7k	1847.9k	0.1	182.3
dem_500_belg_orig_41	1120.8k	1110.3k	0.9	14400.0	1120.8k	1116.3k	0.4	14400.0
dem_500_belg_orig_42	1510.5k	1453.3k	3.9	14400.0	1510.5k	1506.9k	0.2	14400.0
dem_500_belg_orig_43	675.7k	675.1k	0.1	26.5	675.7k	675.1k	0.1	24.1
dem_500_belg_orig_44	1686.9k	1670.0k	1.0	14400.0	1686.9k	1670.4k	1.0	14400.0
dem_500_belg_orig_45	1396.0k	1394.7k	0.1	9.6	1396.1k	1394.7k	0.1	7.6
dem_500_belg_orig_46	1562.9k	1561.6k	0.1	1.6	1562.9k	1561.4k	0.1	2.4
dem_500_belg_orig_47	1341.1k	1339.7k	0.1	7115.6	1341.1k	1339.7k	0.1	3387.1
dem_500_belg_orig_48	658.0k	646.1k	1.8	14400.0	658.0k	647.7k	1.6	14400.0
dem_500_belg_orig_49	668.2k	656.7k	1.8	14400.0	668.2k	667.5k	0.1	5814.8
dem_500_belg_orig_50	1006.1k	1005.1k	0.1	1660.8	1006.1k	1005.1k	0.1	2892.2
dem_500_belg_orig_51	1930.1k	1928.1k	0.1	9421.3	1930.1k	1928.1k	0.1	7636.6
dem_500_belg_orig_52	1348.4k	1347.0k	0.1	279.9	1348.4k	1335.3k	1.0	14400.0
dem_500_belg_orig_53	1429.9k	1420.8k	0.6	14400.0	1429.9k	1428.5k	0.1	791.1
dem_500_belg_orig_54	826.1k	805.8k	2.5	14400.0	826.1k	805.3k	2.6	14400.0
dem_500_belg_orig_55	1427.3k	1423.9k	0.2	14400.0	1427.3k	1425.9k	0.1	2864.7
dem_500_belg_orig_56	583.4k	574.9k	1.5	14400.0	583.4k	574.8k	1.5	14400.0
dem_500_belg_orig_57	1877.8k	1876.1k	0.1	2.4	1877.8k	1876.1k	0.1	1.3
dem_500_belg_orig_58	1655.3k	1641.9k	0.8	14400.0	1655.3k	1643.1k	0.7	14400.0
dem_500_belg_orig_59	2021.4k	2019.4k	0.1	751.7	2021.4k	2019.4k	0.1	816.3
dem_500_belg_orig_60	1010.4k	1010.4k	0.0	252.4	1010.4k	1009.4k	0.1	10.8
dem_500_belg_orig_61	722.1k	721.4k	0.1	3522.1	722.1k	721.4k	0.1	796.5
dem_500_belg_orig_62	1406.9k	1405.5k	0.1	41.0	1406.9k	1405.5k	0.1	94.4
dem_500_belg_orig_63	1049.5k	1048.6k	0.1	2.4	1049.5k	1048.5k	0.1	1.3
dem_500_belg_orig_64	2031.2k	1996.9k	1.7	14400.0	2031.2k	1997.1k	1.7	14400.0
dem_500_belg_orig_65	1748.1k	1746.8k	0.1	3.8	1748.1k	1746.6k	0.1	1.4
dem_500_belg_orig_66	2095.2k	2069.1k	1.3	14400.0	2091.8k	1959.6k	6.8	14400.0
dem_500_belg_orig_67	1489.3k	1487.8k	0.1	1.5	1488.8k	1487.8k	0.1	0.9
dem_500_belg_orig_68	546.3k	531.6k	2.8	14400.0	546.3k	543.3k	0.6	14400.0
dem_500_belg_orig_69	904.0k	904.0k	0.0	1695.9	904.0k	819.5k	10.3	14400.0
dem_500_belg_orig_70	1866.3k	1864.5k	0.1	10.6	1866.3k	1864.4k	0.1	1922.9
dem_500_belg_orig_71	1789.6k	1787.8k	0.1	2316.5	1789.6k	1765.0k	1.4	14400.0
dem_500_belg_orig_72	2385.5k	2383.5k	0.1	0.6	2385.5k	2383.7k	0.1	0.5
dem_500_belg_orig_73	753.6k	752.8k	0.1	95.9	753.6k	752.9k	0.1	115.3
dem_500_belg_orig_74	680.5k	679.8k	0.1	1.1	680.6k	680.0k	0.1	1.0

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Table A.6: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_75	1748.0k	1746.3k	0.1	114.5	1748.0k	1746.3k	0.1	107.6
dem_500_belg_orig_76	1115.9k	1114.4k	0.1	14400.0	1115.9k	1102.3k	1.2	14400.0
dem_500_belg_orig_77	1065.7k	1064.7k	0.1	472.5	1065.7k	1064.2k	0.1	14400.0
dem_500_belg_orig_78	1064.7k	1046.7k	1.7	14400.0	1061.5k	1044.8k	1.6	14400.0
dem_500_belg_orig_79	1689.6k	1664.9k	1.5	14400.0	1689.6k	1669.7k	1.2	14400.0
dem_500_belg_orig_80	1204.1k	1189.6k	1.2	14400.0	1204.1k	1170.7k	2.9	14400.0
dem_500_belg_orig_81	856.5k	854.6k	0.2	14400.0	856.5k	855.6k	0.1	318.3
dem_500_belg_orig_82	1563.7k	1562.2k	0.1	461.0	1563.7k	1561.3k	0.2	14400.0
dem_500_belg_orig_83	740.9k	740.1k	0.1	129.2	740.9k	738.5k	0.3	14400.0
dem_500_belg_orig_84	2315.9k	2291.3k	1.1	14400.0	2315.9k	2290.4k	1.1	14400.0
dem_500_belg_orig_85	1351.5k	1351.0k	0.0	1.4	1351.5k	1350.3k	0.1	1.0
dem_500_belg_orig_86	1515.0k	1513.5k	0.1	175.5	1515.0k	1513.5k	0.1	983.4
dem_500_belg_orig_87	1186.6k	1156.2k	2.6	14400.0	1186.6k	1154.8k	2.8	14400.0
dem_500_belg_orig_88	1614.9k	1613.3k	0.1	820.3	1614.9k	1613.3k	0.1	1539.9
dem_500_belg_orig_89	1308.7k	1307.4k	0.1	54.6	1308.8k	1307.5k	0.1	17.3
dem_500_belg_orig_90	531.5k	515.5k	3.1	14400.0	531.5k	519.7k	2.3	14400.0
dem_500_belg_orig_91	1650.9k	1649.3k	0.1	1510.7	1650.9k	1649.3k	0.1	1889.1
dem_500_belg_orig_92	1058.0k	1029.1k	2.8	14400.0	1057.8k	1030.3k	2.7	14400.0
dem_500_belg_orig_93	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_belg_orig_94	839.2k	834.8k	0.5	14400.0	839.2k	833.2k	0.7	14400.0
dem_500_belg_orig_95	1537.3k	1524.4k	0.8	14400.0	1537.4k	1516.6k	1.4	14400.0
dem_500_belg_orig_96	783.3k	774.1k	1.2	14400.0	783.3k	772.9k	1.3	14400.0
dem_500_belg_orig_97	2030.2k	2021.6k	0.4	14400.0	2030.2k	2028.2k	0.1	2550.1
dem_500_belg_orig_98	1952.5k	1943.1k	0.5	14400.0	1952.8k	1911.4k	2.2	14400.0
dem_500_belg_orig_99	2138.9k	2137.2k	0.1	493.6	2138.9k	2081.6k	2.8	14400.0
dem_500_belg_orig_100	910.2k	907.1k	0.3	14400.0	910.2k	886.9k	2.6	14400.0
dem_500_belg_orig_101	1092.4k	1091.9k	0.0	1.5	1092.4k	1091.9k	0.0	0.5
dem_500_belg_orig_102	1578.7k	1577.1k	0.1	1237.6	1578.7k	1574.6k	0.3	14400.0
dem_500_belg_orig_103	1256.7k	1256.7k	0.0	9.4	1256.7k	1255.5k	0.1	51.2
dem_500_belg_orig_104	1416.3k	1414.9k	0.1	1.5	1416.3k	1415.4k	0.1	1.2
dem_500_belg_orig_105	1197.9k	1196.7k	0.1	1426.9	1197.9k	1196.7k	0.1	2789.7
dem_500_belg_orig_106	771.8k	771.1k	0.1	3104.3	771.8k	771.1k	0.1	16.9
dem_500_belg_orig_107	1976.5k	1934.2k	2.2	14400.0	1976.4k	1934.1k	2.2	14400.0
dem_500_belg_orig_108	1270.1k	1247.6k	1.8	14400.0	1270.1k	1246.7k	1.9	14400.0
dem_500_belg_orig_109	1254.6k	1253.4k	0.1	576.5	1254.6k	1253.4k	0.1	587.1
dem_500_belg_orig_110	1341.0k	1339.6k	0.1	1530.6	1341.0k	1339.7k	0.1	706.4
dem_500_belg_orig_111	1261.5k	1260.2k	0.1	3191.8	1261.5k	1260.2k	0.1	389.9
dem_500_belg_orig_112	1062.3k	1052.1k	1.0	14400.0	1062.3k	1053.3k	0.9	14400.0
dem_500_belg_orig_113	1108.3k	1098.4k	0.9	14400.0	1108.3k	1096.7k	1.1	14400.0
dem_500_belg_orig_114	1236.7k	1235.5k	0.1	10171.3	1236.7k	1235.5k	0.1	2067.3
dem_500_belg_orig_115	1363.3k	1353.3k	0.7	14400.0	1363.3k	1361.9k	0.1	7295.8
dem_500_belg_orig_116	1834.8k	1832.9k	0.1	258.2	1834.8k	1826.9k	0.4	14400.0
dem_500_belg_orig_117	1706.2k	1704.6k	0.1	5.3	1706.2k	1704.5k	0.1	19.9
dem_500_belg_orig_118	1457.2k	1455.6k	0.1	14400.0	1457.2k	1427.6k	2.1	14400.0
dem_500_belg_orig_119	1439.8k	1438.4k	0.1	16.1	1439.8k	1438.4k	0.1	22.4
dem_500_belg_orig_120	1914.9k	1898.2k	0.9	14400.0	1914.9k	1900.0k	0.8	14400.0
dem_500_belg_orig_121	2062.0k	2059.9k	0.1	1277.3	2062.0k	2057.7k	0.2	14400.0
dem_500_belg_orig_122	782.7k	782.0k	0.1	72.4	782.7k	782.0k	0.1	4.7
dem_500_belg_orig_123	2104.9k	2104.2k	0.0	5.8	2104.9k	2102.8k	0.1	5.0
dem_500_belg_orig_124	1791.6k	1789.8k	0.1	266.2	1791.6k	1789.8k	0.1	68.5
dem_500_belg_orig_125	1753.9k	1752.2k	0.1	2075.7	1753.9k	1746.4k	0.4	14400.0
dem_500_belg_orig_126	1462.3k	1462.3k	0.0	1.2	1462.3k	1462.3k	0.0	0.2
dem_500_belg_orig_127	1455.2k	1432.0k	1.6	14400.0	1455.2k	1432.0k	1.6	14400.0
dem_500_belg_orig_128	1609.8k	1608.4k	0.1	6.3	1609.8k	1608.6k	0.1	18.4
dem_500_belg_orig_129	1160.8k	1157.3k	0.3	14400.0	1160.8k	1159.7k	0.1	85.9
dem_500_belg_orig_130	972.2k	923.3k	5.3	14400.0	970.7k	953.4k	1.8	14400.0
dem_500_belg_orig_131	1246.8k	1246.8k	0.0	38.1	1246.8k	1245.5k	0.1	35.5
dem_500_belg_orig_132	1011.7k	1010.7k	0.1	73.4	1011.7k	1010.7k	0.1	2666.3
dem_500_belg_orig_133	1297.0k	1295.7k	0.1	9.7	1297.0k	1295.7k	0.1	13.9
dem_500_belg_orig_134	1331.1k	1329.8k	0.1	37.1	1331.1k	1329.8k	0.1	17.5
dem_500_belg_orig_135	1042.6k	1030.8k	1.1	14400.0	1042.6k	1015.3k	2.7	14400.0
dem_500_belg_orig_136	685.4k	655.0k	4.6	14400.0	685.4k	684.8k	0.1	2.8
dem_500_belg_orig_137	670.1k	669.6k	0.1	3.0	670.1k	669.4k	0.1	10.0
dem_500_belg_orig_138	1120.2k	1119.1k	0.1	3.6	1120.2k	1119.2k	0.1	3.6
dem_500_belg_orig_139	894.6k	893.7k	0.1	11722.1	894.6k	875.3k	2.2	14400.0
dem_500_belg_orig_140	1262.5k	1262.0k	0.0	6.6	1262.5k	1261.4k	0.1	2.6
dem_500_belg_orig_141	1599.1k	1597.5k	0.1	12364.6	1599.1k	1597.5k	0.1	4579.2
dem_500_belg_orig_142	1059.2k	1057.5k	0.2	14400.0	1059.2k	1057.6k	0.1	14400.0
dem_500_belg_orig_143	1670.9k	1669.3k	0.1	2.8	1671.3k	1669.8k	0.1	2.0
dem_500_belg_orig_144	1298.3k	1297.0k	0.1	27.8	1298.1k	1296.8k	0.1	53.8
dem_500_belg_orig_145	2595.3k	2592.7k	0.1	102.8	2595.3k	2536.8k	2.3	14400.0
dem_500_belg_orig_146	1173.7k	1153.7k	1.7	14400.0	1173.7k	1153.6k	1.7	14400.0
dem_500_belg_orig_147	1518.7k	1513.1k	0.4	14400.0	1518.7k	1501.5k	1.2	14400.0
dem_500_belg_orig_148	1275.9k	1274.7k	0.1	3.7	1275.9k	1274.7k	0.1	6.8
dem_500_belg_orig_149	1327.4k	1326.5k	0.1	9.8	1327.4k	1326.1k	0.1	20.0
dem_500_belg_orig_150	1634.3k	1632.6k	0.1	818.0	1634.3k	1632.7k	0.1	6.2
dem_500_belg_orig_151	1217.0k	1212.1k	0.4	14400.0	1217.0k	1215.8k	0.1	5043.6
dem_500_belg_orig_152	1912.6k	1910.7k	0.1	1452.7	1912.6k	1910.7k	0.1	3048.6

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Table A.6: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_153	1641.3k	1618.4k	1.4	14400.0	1641.4k	1617.5k	1.5	14400.0
dem_500_belg_orig_154	1929.7k	1926.9k	0.1	14400.0	1929.7k	1923.7k	0.3	14400.0
dem_500_belg_orig_155	1811.2k	1797.3k	0.8	14400.0	1811.2k	1807.5k	0.2	14400.0
dem_500_belg_orig_156	1120.3k	1119.1k	0.1	168.8	1120.3k	1119.1k	0.1	118.0
dem_500_belg_orig_157	1111.6k	1099.0k	1.1	14400.0	1111.6k	1098.5k	1.2	14400.0
dem_500_belg_orig_158	903.4k	902.5k	0.1	342.9	903.4k	902.5k	0.1	154.4
dem_500_belg_orig_159	1722.5k	1703.5k	1.1	14400.0	1722.5k	1681.7k	2.4	14400.0
dem_500_belg_orig_160	2003.2k	2001.3k	0.1	17.8	2003.2k	2001.2k	0.1	27.5
dem_500_belg_orig_161	2017.9k	2015.9k	0.1	16.2	2017.9k	2015.9k	0.1	14400.0
dem_500_belg_orig_162	1779.8k	1778.1k	0.1	117.0	1779.8k	1778.1k	0.1	324.2
dem_500_belg_orig_163	1201.7k	1200.5k	0.1	33.3	1201.7k	1200.5k	0.1	54.9
dem_500_belg_orig_164	1394.4k	1375.2k	1.4	14400.0	1394.4k	1393.0k	0.1	25.3
dem_500_belg_orig_165	1066.6k	1050.9k	1.5	14400.0	1066.6k	1050.9k	1.5	14400.0
dem_500_belg_orig_166	1536.9k	1535.5k	0.1	67.6	1536.9k	1535.4k	0.1	65.5
dem_500_belg_orig_167	1174.2k	1174.2k	0.0	5730.4	1174.4k	1154.4k	1.7	14400.0
dem_500_belg_orig_168	1089.2k	1088.2k	0.1	2.0	1089.2k	1088.4k	0.1	1.2
dem_500_belg_orig_169	1152.3k	1145.6k	0.6	14400.0	1152.3k	1146.2k	0.5	14400.0
dem_500_belg_orig_170	1169.3k	1168.2k	0.1	633.3	1169.3k	1168.2k	0.1	5394.6
dem_500_belg_orig_171	912.2k	911.3k	0.1	14.9	912.2k	911.3k	0.1	25.7
dem_500_belg_orig_172	1928.0k	1926.2k	0.1	9.1	1928.0k	1926.1k	0.1	12.8
dem_500_belg_orig_173	1342.1k	1327.9k	1.1	14400.0	1342.1k	1328.7k	1.0	14400.0
dem_500_belg_orig_174	1330.2k	1330.2k	0.0	4597.3	1330.2k	1327.2k	0.2	14400.0
dem_500_belg_orig_175	1248.3k	1237.3k	0.9	14400.0	1248.7k	1186.8k	5.2	14400.0
dem_500_belg_orig_176	1348.1k	1346.8k	0.1	17.0	1348.1k	1346.8k	0.1	1.3
dem_500_belg_orig_177	2724.4k	2721.7k	0.1	7.4	2724.3k	2721.7k	0.1	6.9
dem_500_belg_orig_178	734.6k	734.0k	0.1	872.2	734.9k	702.8k	4.6	14400.0
dem_500_belg_orig_179	1302.1k	1301.8k	0.0	0.8	1302.1k	1301.8k	0.0	0.2
dem_500_belg_orig_180	1710.3k	1708.9k	0.1	2.5	1710.3k	1709.8k	0.0	2.2
dem_500_belg_orig_181	1168.2k	1124.6k	3.9	14400.0	1168.2k	1112.7k	5.0	14400.0
dem_500_belg_orig_182	1294.2k	1292.9k	0.1	1083.4	1294.2k	1292.9k	0.1	1063.8
dem_500_belg_orig_183	1317.6k	1300.0k	1.4	14400.0	1317.5k	1305.9k	0.9	14400.0
dem_500_belg_orig_184	835.7k	832.3k	0.4	14400.0	835.7k	834.8k	0.1	16.1
dem_500_belg_orig_185	771.0k	770.3k	0.1	31.1	771.0k	770.2k	0.1	155.0
dem_500_belg_orig_186	2094.8k	2092.8k	0.1	150.1	2094.8k	2092.7k	0.1	368.4
dem_500_belg_orig_187	1926.6k	1915.6k	0.6	14400.0	1926.6k	1915.9k	0.6	14400.0
dem_500_belg_orig_188	1098.8k	1097.7k	0.1	1.4	1098.8k	1097.7k	0.1	0.8
dem_500_belg_orig_189	1505.4k	1503.6k	0.1	14400.0	1505.4k	1503.9k	0.1	3536.8
dem_500_belg_orig_190	1288.4k	1286.9k	0.1	14400.0	1288.4k	1287.1k	0.1	724.0
dem_500_belg_orig_191	1535.1k	1484.6k	3.4	14400.0	1536.5k	1433.0k	7.2	14400.0
dem_500_belg_orig_192	1506.3k	1451.0k	3.8	14400.0	1506.3k	1504.8k	0.1	464.5
dem_500_belg_orig_193	1685.2k	1683.5k	0.1	6858.9	1685.2k	1629.9k	3.4	14400.0
dem_500_belg_orig_194	1256.9k	1255.7k	0.1	3.7	1256.9k	1255.7k	0.1	3.3
dem_500_belg_orig_195	1393.7k	1384.0k	0.7	14400.0	1393.7k	1384.6k	0.7	14400.0
dem_500_belg_orig_196	1893.4k	1891.6k	0.1	2.6	1892.8k	1891.0k	0.1	1.3
dem_500_belg_orig_197	1163.6k	1155.0k	0.7	14400.0	1163.8k	1128.6k	3.1	14400.0
dem_500_belg_orig_198	545.8k	545.8k	0.0	18.7	545.8k	545.3k	0.1	19.9
dem_500_belg_orig_199	1168.5k	1167.3k	0.1	10170.1	1168.5k	1158.7k	0.8	14400.0
dem_500_belg_orig_200	518.2k	517.7k	0.1	52.0	518.2k	517.8k	0.1	2.2
dem_500_belg_orig_201	1427.3k	1420.9k	0.5	14400.0	1427.3k	1425.9k	0.1	363.2
dem_500_belg_orig_202	1320.6k	1320.2k	0.0	3.6	1320.6k	1319.5k	0.1	2.9
dem_500_belg_orig_203	2479.5k	2446.8k	1.3	14400.0	2479.5k	2477.1k	0.1	18.4
dem_500_belg_orig_204	1007.1k	1006.4k	0.1	812.6	1007.1k	1006.1k	0.1	26.3
dem_500_belg_orig_205	1534.9k	1519.9k	1.0	14400.0	1534.9k	1533.4k	0.1	1571.1
dem_500_belg_orig_206	2337.0k	2334.7k	0.1	57.3	2337.0k	2334.6k	0.1	832.7
dem_500_belg_orig_207	1311.4k	1311.4k	0.0	1007.2	1311.4k	1287.3k	1.9	14400.0
dem_500_belg_orig_208	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_belg_orig_209	2022.3k	2020.3k	0.1	7319.1	2022.3k	2020.3k	0.1	13754.4
dem_500_belg_orig_210	1122.6k	1121.5k	0.1	54.0	1122.6k	1111.1k	1.0	14400.0
dem_500_belg_orig_211	633.1k	633.1k	0.0	0.6	633.1k	633.1k	0.0	0.4
dem_500_belg_orig_212	1843.8k	1820.8k	1.3	14400.0	1843.8k	1821.2k	1.2	14400.0
dem_500_belg_orig_213	954.3k	953.4k	0.1	85.7	954.3k	953.4k	0.1	502.9
dem_500_belg_orig_214	964.1k	963.1k	0.1	854.3	964.1k	958.8k	0.6	14400.0
dem_500_belg_orig_215	372.6k	372.3k	0.1	276.6	372.6k	372.3k	0.1	266.8
dem_500_belg_orig_216	1043.1k	1042.1k	0.1	2.5	1042.7k	1041.7k	0.1	0.8
dem_500_belg_orig_217	1121.1k	1120.0k	0.1	125.7	1121.1k	1120.0k	0.1	15.5
dem_500_belg_orig_218	1586.4k	1584.8k	0.1	187.9	1586.4k	1584.8k	0.1	68.1
dem_500_belg_orig_219	1216.5k	1215.2k	0.1	3837.1	1216.5k	1215.2k	0.1	6482.0
dem_500_belg_orig_220	922.0k	921.1k	0.1	510.9	922.0k	921.1k	0.1	5093.3
dem_500_belg_orig_221	976.8k	962.7k	1.5	14400.0	976.8k	961.0k	1.6	14400.0
dem_500_belg_orig_222	1425.3k	1423.9k	0.1	3.0	1425.3k	1423.9k	0.1	8.1
dem_500_belg_orig_223	1694.2k	1692.5k	0.1	695.6	1694.2k	1692.5k	0.1	24.3
dem_500_belg_orig_224	1563.3k	1561.7k	0.1	15.7	1563.3k	1561.8k	0.1	11.1
dem_500_belg_orig_225	814.6k	805.3k	1.2	14400.0	814.6k	741.4k	9.9	14400.0
dem_500_belg_orig_226	1282.0k	1280.7k	0.1	4735.4	1282.0k	1280.7k	0.1	5686.5
dem_500_belg_orig_227	423.2k	423.2k	0.0	0.8	423.2k	423.2k	0.0	0.2
dem_500_belg_orig_228	1144.7k	1133.8k	1.0	14400.0	1144.7k	1119.0k	2.3	14400.0
dem_500_belg_orig_229	1606.6k	1605.1k	0.1	58.5	1606.6k	1605.1k	0.1	5.9
dem_500_belg_orig_230	1304.5k	1303.2k	0.1	1.4	1304.5k	1303.4k	0.1	0.9

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Table A.6: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_231	1620.3k	1618.6k	0.1	13964.4	1620.3k	1611.0k	0.6	14400.0
dem_500_belg_orig_232	1969.4k	1967.6k	0.1	22.8	1969.4k	1967.4k	0.1	4.6
dem_500_belg_orig_233	1752.2k	1750.5k	0.1	54.3	1752.3k	1750.5k	0.1	93.1
dem_500_belg_orig_234	1149.4k	1148.3k	0.1	68.6	1149.4k	1148.3k	0.1	68.4
dem_500_belg_orig_235	899.1k	898.2k	0.1	1099.8	899.1k	898.2k	0.1	706.0
dem_500_belg_orig_236	1395.0k	1393.6k	0.1	70.4	1395.0k	1393.6k	0.1	117.3
dem_500_belg_orig_237	1908.7k	1906.8k	0.1	128.7	1908.7k	1906.9k	0.1	4.9
dem_500_belg_orig_238	1071.5k	1070.9k	0.1	1.9	1071.5k	1070.5k	0.1	10.0
dem_500_belg_orig_239	2358.6k	2318.8k	1.7	14400.0	2358.6k	2329.9k	1.2	14400.0
dem_500_belg_orig_240	1710.8k	1703.3k	0.4	14400.0	1710.8k	1709.1k	0.1	7463.3
dem_500_belg_orig_241	2530.9k	2528.3k	0.1	856.6	2530.9k	2509.1k	0.9	14400.0
dem_500_belg_orig_242	890.3k	889.5k	0.1	2.1	890.3k	889.4k	0.1	64.3
dem_500_belg_orig_243	1519.8k	1501.3k	1.2	14400.0	1519.8k	1502.0k	1.2	14400.0
dem_500_belg_orig_244	966.9k	959.3k	0.8	14400.0	966.9k	965.3k	0.2	14400.0
dem_500_belg_orig_245	617.8k	617.2k	0.1	1.1	617.8k	617.8k	0.0	1.3
dem_500_belg_orig_246	2108.5k	2102.3k	0.3	14400.0	2108.5k	2106.5k	0.1	12.1
dem_500_belg_orig_247	2093.9k	2091.8k	0.1	834.0	2093.9k	2091.8k	0.1	5533.3
dem_500_belg_orig_248	1696.0k	1694.3k	0.1	147.7	1696.0k	1694.3k	0.1	220.9
dem_500_belg_orig_249	1372.8k	1371.4k	0.1	14.1	1372.8k	1371.5k	0.1	20.3
dem_500_belg_orig_250	1472.4k	1463.6k	0.6	14400.0	1472.4k	1457.7k	1.0	14400.0
dem_500_belg_orig_251	633.0k	632.4k	0.1	7.2	633.0k	632.5k	0.1	1.4
dem_500_belg_orig_252	1644.7k	1643.4k	0.1	1.6	1643.6k	1642.0k	0.1	26.8
dem_500_belg_orig_253	1307.2k	1305.9k	0.1	11350.0	1307.2k	1305.1k	0.2	14400.0
dem_500_belg_orig_254	1341.5k	1331.7k	0.7	14400.0	1341.5k	1329.4k	0.9	14400.0
dem_500_belg_orig_255	1667.8k	1666.4k	0.1	3.8	1667.5k	1665.9k	0.1	17.9
dem_500_belg_orig_256	782.9k	782.1k	0.1	11.4	782.9k	782.1k	0.1	13.7
dem_500_belg_orig_257	2072.1k	2070.6k	0.1	1.6	2072.3k	2070.3k	0.1	1.1
dem_500_belg_orig_258	851.1k	834.0k	2.0	14400.0	851.1k	847.0k	0.5	14400.0
dem_500_belg_orig_259	1002.7k	920.1k	9.0	14400.0	1002.7k	1002.7k	0.0	102.8
dem_500_belg_orig_260	1056.1k	1055.1k	0.1	44.5	1056.1k	1055.0k	0.1	2257.0
dem_500_belg_orig_261	1066.3k	1065.3k	0.1	1144.5	1066.3k	1065.3k	0.1	1193.1
dem_500_belg_orig_262	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_belg_orig_263	2102.9k	2059.5k	2.1	14400.0	2102.9k	2061.1k	2.0	14400.0
dem_500_belg_orig_264	909.1k	897.2k	1.3	14400.0	909.1k	897.7k	1.3	14400.0
dem_500_belg_orig_265	1722.2k	1722.2k	0.0	1.8	1722.2k	1721.1k	0.1	0.5
dem_500_belg_orig_266	1693.5k	1692.6k	0.1	3.7	1692.9k	1691.6k	0.1	1.9
dem_500_belg_orig_267	1757.6k	1755.9k	0.1	865.8	1757.6k	1755.9k	0.1	935.7
dem_500_belg_orig_268	1688.8k	1631.9k	3.5	14400.0	1688.8k	1687.1k	0.1	32.3
dem_500_belg_orig_269	1742.6k	1734.2k	0.5	14400.0	1742.6k	1740.8k	0.1	1587.7
dem_500_belg_orig_270	916.8k	866.1k	5.9	14400.0	917.4k	823.3k	11.4	14400.0
dem_500_belg_orig_271	967.1k	965.1k	0.2	14400.0	967.1k	966.1k	0.1	1724.7
dem_500_belg_orig_272	764.3k	763.9k	0.1	1.4	764.3k	763.9k	0.1	0.8
dem_500_belg_orig_273	1675.0k	1642.3k	2.0	14400.0	1675.2k	1644.0k	1.9	14400.0
dem_500_belg_orig_274	1604.9k	1603.5k	0.1	0.9	1604.9k	1603.8k	0.1	1.6
dem_500_belg_orig_275	998.0k	987.2k	1.1	14400.0	998.0k	997.0k	0.1	838.2
dem_500_belg_orig_276	666.5k	665.9k	0.1	1.0	666.5k	665.8k	0.1	1.1
dem_500_belg_orig_277	1676.0k	1674.3k	0.1	3735.9	1676.3k	1593.2k	5.2	14400.0
dem_500_belg_orig_278	1470.7k	1469.2k	0.1	93.5	1470.7k	1469.2k	0.1	65.7
dem_500_belg_orig_279	827.5k	825.6k	0.2	14400.0	827.5k	825.6k	0.2	14400.0
dem_500_belg_orig_280	1796.7k	1796.7k	0.0	0.8	1796.7k	1796.7k	0.0	0.2
dem_500_belg_orig_281	669.9k	669.9k	0.0	11.4	669.9k	669.4k	0.1	0.9
dem_500_belg_orig_282	1842.9k	1841.1k	0.1	8.7	1842.9k	1841.1k	0.1	10.1
dem_500_belg_orig_283	831.0k	830.2k	0.1	998.2	831.0k	830.2k	0.1	49.3
dem_500_belg_orig_284	1040.8k	1028.8k	1.2	14400.0	1040.8k	950.1k	9.6	14400.0
dem_500_belg_orig_285	2272.1k	2269.8k	0.1	33.2	2272.8k	2270.7k	0.1	0.5
dem_500_belg_orig_286	1180.5k	1179.3k	0.1	83.6	1180.5k	1179.3k	0.1	79.6
dem_500_belg_orig_287	1592.7k	1591.1k	0.1	1399.6	1592.7k	1591.1k	0.1	254.8
dem_500_belg_orig_288	750.0k	749.2k	0.1	152.6	749.9k	749.2k	0.1	621.1
dem_500_belg_orig_289	888.2k	817.0k	8.7	14400.0	888.1k	817.5k	8.6	14400.0
dem_500_belg_orig_290	1674.1k	1660.0k	0.8	14400.0	1674.1k	1657.9k	1.0	14400.0
dem_500_belg_orig_291	2069.4k	2067.3k	0.1	5240.2	2069.4k	2067.3k	0.1	1418.6
dem_500_belg_orig_292	1359.8k	1358.5k	0.1	3.7	1359.8k	1358.6k	0.1	1.5
dem_500_belg_orig_293	671.1k	670.4k	0.1	1823.5	671.1k	670.4k	0.1	7620.0
dem_500_belg_orig_294	981.9k	980.9k	0.1	704.4	981.9k	973.6k	0.9	14400.0
dem_500_belg_orig_295	979.3k	978.4k	0.1	38.4	979.3k	978.5k	0.1	1.1
dem_500_belg_orig_296	1225.4k	1224.1k	0.1	3141.5	1225.4k	1224.1k	0.1	81.1
dem_500_belg_orig_297	1667.4k	1665.8k	0.1	2351.8	1667.4k	1665.8k	0.1	11348.7
dem_500_belg_orig_298	625.1k	624.6k	0.1	43.9	625.1k	624.7k	0.1	6897.5
dem_500_belg_orig_299	1284.5k	1283.3k	0.1	1455.9	1284.5k	1283.3k	0.1	181.9
dem_500_belg_orig_300	1356.1k	1354.7k	0.1	5203.1	1356.2k	1335.6k	1.5	14400.0
dem_500_belg_orig_301	1221.7k	1220.5k	0.1	44.8	1221.7k	1220.5k	0.1	362.4
dem_500_belg_orig_302	716.4k	715.7k	0.1	84.9	716.4k	715.7k	0.1	2552.9
dem_500_belg_orig_303	1194.9k	1193.7k	0.1	8546.2	1194.9k	1193.7k	0.1	6623.0
dem_500_belg_orig_304	1768.5k	1766.8k	0.1	21.4	1768.5k	1766.8k	0.1	156.6
dem_500_belg_orig_305	1847.8k	1843.3k	0.2	14400.0	1847.8k	1846.0k	0.1	74.4
dem_500_belg_orig_306	1551.4k	1524.3k	1.8	14400.0	1551.4k	1508.4k	2.9	14400.0
dem_500_belg_orig_307	2327.9k	2325.6k	0.1	1118.2	2327.9k	2325.6k	0.1	273.8
dem_500_belg_orig_308	900.7k	900.7k	0.0	14.0	900.7k	899.8k	0.1	410.8

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Table A.6: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_309	1636.8k	1636.0k	0.0	1.6	1636.7k	1635.2k	0.1	0.5
dem_500_belg_orig_310	1283.1k	1271.4k	0.9	14400.0	1283.1k	1281.8k	0.1	6226.3
dem_500_belg_orig_311	1119.3k	1118.2k	0.1	2959.8	1119.3k	1118.2k	0.1	162.9
dem_500_belg_orig_312	820.3k	777.3k	5.5	14400.0	820.3k	775.9k	5.7	14400.0
dem_500_belg_orig_313	1240.8k	1239.6k	0.1	3.8	1240.6k	1239.5k	0.1	7.3
dem_500_belg_orig_314	1088.4k	1075.6k	1.2	14400.0	1088.4k	1029.9k	5.7	14400.0
dem_500_belg_orig_315	1166.3k	1164.9k	0.1	14400.0	1166.3k	1165.1k	0.1	2649.9
dem_500_belg_orig_316	1522.2k	1515.2k	0.5	14400.0	1522.2k	1464.1k	4.0	14400.0
dem_500_belg_orig_317	621.1k	620.0k	0.2	14400.0	621.1k	621.0k	0.0	895.5
dem_500_belg_orig_318	1018.7k	1018.4k	0.0	3.1	1018.7k	1018.3k	0.0	1.7
dem_500_belg_orig_319	1109.6k	1054.7k	5.2	14400.0	1108.6k	1107.5k	0.1	95.0
dem_500_belg_orig_320	1952.5k	1950.6k	0.1	7927.1	1952.5k	1950.5k	0.1	13428.3
dem_500_belg_orig_321	515.3k	514.9k	0.1	1.2	515.3k	514.8k	0.1	0.8
dem_500_belg_orig_322	1938.8k	1938.0k	0.0	3.5	1938.8k	1936.9k	0.1	2.4
dem_500_belg_orig_323	1628.3k	1604.2k	1.5	14400.0	1628.3k	1604.7k	1.5	14400.0
dem_500_belg_orig_324	1262.4k	1261.1k	0.1	1.6	1261.5k	1260.7k	0.1	0.6
dem_500_belg_orig_325	1341.8k	1297.9k	3.4	14400.0	1341.8k	1297.6k	3.4	14400.0
dem_500_belg_orig_326	1814.6k	1811.0k	0.2	14400.0	1814.6k	1806.5k	0.4	14400.0
dem_500_belg_orig_327	2325.2k	2324.4k	0.0	5.0	2325.0k	2322.7k	0.1	123.6
dem_500_belg_orig_328	1826.7k	1824.8k	0.1	1691.2	1826.7k	1814.3k	0.7	14400.0
dem_500_belg_orig_329	740.5k	739.7k	0.1	15.3	740.5k	739.7k	0.1	109.0
dem_500_belg_orig_330	1998.1k	1974.6k	1.2	14400.0	1998.1k	1974.2k	1.2	14400.0
dem_500_belg_orig_331	867.9k	866.1k	0.2	14400.0	867.9k	867.0k	0.1	3885.3
dem_500_belg_orig_332	1104.0k	1102.9k	0.1	31.0	1104.0k	1102.9k	0.1	1.9
dem_500_belg_orig_333	1660.3k	1658.6k	0.1	4973.4	1660.3k	1658.6k	0.1	308.0
dem_500_belg_orig_334	1088.5k	1086.9k	0.1	14400.0	1088.5k	1087.5k	0.1	115.2
dem_500_belg_orig_335	1415.6k	1414.3k	0.1	16.0	1415.6k	1414.1k	0.1	139.0
dem_500_belg_orig_336	1708.6k	1708.6k	0.0	1.5	1708.6k	1708.6k	0.0	1.9
dem_500_belg_orig_337	1085.2k	976.4k	11.1	14400.0	1084.2k	979.5k	10.7	14400.0
dem_500_belg_orig_338	1434.6k	1429.9k	0.3	14400.0	1434.9k	1393.9k	2.9	14400.0
dem_500_belg_orig_339	2181.7k	2179.5k	0.1	3632.5	2181.7k	2179.5k	0.1	3731.3
dem_500_belg_orig_340	573.4k	572.8k	0.1	2815.3	573.4k	572.8k	0.1	99.1
dem_500_belg_orig_341	932.5k	931.6k	0.1	23.7	932.5k	931.6k	0.1	40.0
dem_500_belg_orig_342	1545.8k	1544.3k	0.1	373.9	1545.8k	1544.4k	0.1	7.7
dem_500_belg_orig_343	1861.0k	1836.1k	1.4	14400.0	1861.0k	1842.4k	1.0	14400.0
dem_500_belg_orig_344	2376.3k	2374.6k	0.1	0.7	2376.3k	2374.6k	0.1	0.5
dem_500_belg_orig_345	1375.4k	1364.6k	0.8	14400.0	1375.4k	1374.0k	0.1	11901.2
dem_500_belg_orig_346	1806.9k	1805.1k	0.1	580.6	1806.9k	1805.1k	0.1	2470.3
dem_500_belg_orig_347	1832.7k	1830.9k	0.1	205.2	1832.7k	1830.9k	0.1	136.2
dem_500_belg_orig_348	1706.2k	1678.3k	1.7	14400.0	1706.2k	1704.4k	0.1	1308.5
dem_500_belg_orig_349	2614.3k	2614.3k	0.0	4897.0	2614.3k	2611.7k	0.1	4079.2
dem_500_belg_orig_350	1675.5k	1673.8k	0.1	2613.4	1675.5k	1673.1k	0.1	14400.0
dem_500_belg_orig_351	1543.0k	1505.4k	2.5	14400.0	1543.0k	1505.4k	2.5	14400.0
dem_500_belg_orig_352	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_belg_orig_353	2030.6k	2028.6k	0.1	5816.1	2030.6k	2028.6k	0.1	3144.2
dem_500_belg_orig_354	1044.3k	1043.3k	0.1	1.6	1043.8k	1042.8k	0.1	12.3
dem_500_belg_orig_355	1173.1k	1164.1k	0.8	14400.0	1173.1k	1167.8k	0.5	14400.0
dem_500_belg_orig_356	951.3k	950.4k	0.1	33.4	951.3k	950.4k	0.1	2.3
dem_500_belg_orig_357	1349.0k	1340.5k	0.6	14400.0	1349.0k	1340.4k	0.6	14400.0
dem_500_belg_orig_358	1161.9k	1160.7k	0.1	1883.9	1162.0k	1130.1k	2.8	14400.0
dem_500_belg_orig_359	871.2k	860.6k	1.2	14400.0	871.2k	828.8k	5.1	14400.0
dem_500_belg_orig_360	1326.6k	1325.8k	0.1	2.2	1325.8k	1324.6k	0.1	0.6
dem_500_belg_orig_361	2040.4k	2040.0k	0.0	20.3	2040.4k	2038.3k	0.1	4.5
dem_500_belg_orig_362	1730.3k	1728.6k	0.1	306.1	1730.3k	1718.2k	0.7	14400.0
dem_500_belg_orig_363	1620.3k	1618.7k	0.1	9963.0	1620.3k	1596.4k	1.5	14400.0
dem_500_belg_orig_364	950.8k	949.8k	0.1	1161.0	950.8k	878.6k	8.2	14400.0
dem_500_belg_orig_365	1784.1k	1770.2k	0.8	14400.0	1784.2k	1763.3k	1.2	14400.0
dem_500_belg_orig_366	1146.9k	1145.8k	0.1	841.9	1146.9k	1140.8k	0.5	14400.0
dem_500_belg_orig_367	1257.5k	1243.3k	1.1	14400.0	1257.5k	1243.3k	1.1	14400.0
dem_500_belg_orig_368	1076.1k	1066.1k	0.9	14400.0	1076.1k	1036.2k	3.9	14400.0
dem_500_belg_orig_369	1153.5k	1152.4k	0.1	330.2	1153.5k	1152.4k	0.1	558.8
dem_500_belg_orig_370	990.8k	977.1k	1.4	14400.0	990.7k	960.5k	3.1	14400.0
dem_500_belg_orig_371	1165.5k	1164.3k	0.1	25.1	1165.5k	1164.3k	0.1	12.1
dem_500_belg_orig_372	826.6k	793.0k	4.2	14400.0	826.6k	791.4k	4.5	14400.0
dem_500_belg_orig_373	1465.1k	1463.6k	0.1	234.3	1465.1k	1463.6k	0.1	43.0
dem_500_belg_orig_374	1031.8k	1029.2k	0.3	14400.0	1031.8k	1029.2k	0.3	14400.0
dem_500_belg_orig_375	793.0k	792.2k	0.1	45.2	793.0k	792.2k	0.1	4.8
dem_500_belg_orig_376	1437.6k	1404.1k	2.4	14400.0	1437.6k	1404.0k	2.4	14400.0
dem_500_belg_orig_377	2077.9k	2076.9k	0.1	0.7	2077.9k	2076.9k	0.0	0.4
dem_500_belg_orig_378	1328.8k	1322.2k	0.5	14400.0	1328.8k	1319.1k	0.7	14400.0
dem_500_belg_orig_379	2195.1k	2194.1k	0.0	4.3	2195.1k	2192.9k	0.1	5.8
dem_500_belg_orig_380	1956.0k	1940.0k	0.8	14400.0	1956.0k	1934.8k	1.1	14400.0
dem_500_belg_orig_381	1172.5k	1171.4k	0.1	29.6	1172.5k	1171.4k	0.1	25.3
dem_500_belg_orig_382	751.5k	750.8k	0.1	52.4	751.5k	751.5k	0.0	182.2
dem_500_belg_orig_383	1328.9k	1327.6k	0.1	11.1	1328.9k	1327.6k	0.1	2.8
dem_500_belg_orig_384	1309.6k	1308.5k	0.1	1.5	1309.6k	1308.5k	0.1	0.9
dem_500_belg_orig_385	2185.3k	2174.4k	0.5	14400.0	2184.9k	2182.7k	0.1	2440.4
dem_500_belg_orig_386	1104.5k	1103.4k	0.1	403.5	1104.5k	1103.4k	0.1	12276.1

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Table A.6: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_387	709.2k	708.5k	0.1	2.9	709.2k	708.5k	0.1	9.8
dem_500_belg_orig_388	676.5k	675.8k	0.1	124.2	676.5k	675.8k	0.1	918.3
dem_500_belg_orig_389	1842.1k	1786.4k	3.1	14400.0	1842.1k	1813.9k	1.6	14400.0
dem_500_belg_orig_390	1173.7k	1172.6k	0.1	402.0	1173.7k	1172.7k	0.1	531.2
dem_500_belg_orig_391	1675.0k	1673.4k	0.1	3.2	1673.8k	1672.3k	0.1	1.9
dem_500_belg_orig_392	1035.2k	1034.2k	0.1	7793.1	1035.2k	1034.2k	0.1	9147.7
dem_500_belg_orig_393	1544.8k	1543.5k	0.1	4536.1	1544.8k	1543.3k	0.1	25.7
dem_500_belg_orig_394	1746.2k	1726.5k	1.1	14400.0	1746.1k	1736.9k	0.5	14400.0
dem_500_belg_orig_395	1931.7k	1929.8k	0.1	5427.4	1931.7k	1913.8k	0.9	14400.0
dem_500_belg_orig_396	1658.7k	1657.2k	0.1	3.8	1658.7k	1657.0k	0.1	19.9
dem_500_belg_orig_397	963.8k	962.9k	0.1	443.0	963.8k	938.0k	2.8	14400.0
dem_500_belg_orig_398	1321.2k	1320.4k	0.1	46.9	1321.2k	1319.9k	0.1	70.3
dem_500_belg_orig_399	1273.4k	1263.1k	0.8	14400.0	1273.4k	1263.6k	0.8	14400.0
dem_500_belg_orig_400	1333.2k	1332.4k	0.1	8341.8	1333.2k	1298.1k	2.7	14400.0
dem_500_belg_orig_401	1343.0k	1341.7k	0.1	192.7	1343.0k	1341.7k	0.1	123.3
dem_500_belg_orig_402	828.1k	828.0k	0.0	17.1	828.1k	827.3k	0.1	346.3
dem_500_belg_orig_403	1738.9k	1737.8k	0.1	1.5	1738.9k	1737.8k	0.1	0.8
dem_500_belg_orig_404	1712.4k	1710.7k	0.1	74.3	1712.4k	1710.7k	0.1	6542.6
dem_500_belg_orig_405	900.3k	896.4k	0.4	14400.0	900.3k	896.5k	0.4	14400.0
dem_500_belg_orig_406	1177.2k	1170.7k	0.6	14400.0	1177.2k	1175.5k	0.1	14400.0
dem_500_belg_orig_407	995.2k	995.2k	0.0	111.9	995.2k	994.2k	0.1	100.4
dem_500_belg_orig_408	805.9k	805.1k	0.1	100.8	805.9k	805.2k	0.1	11.9
dem_500_belg_orig_409	2083.2k	2081.7k	0.1	3.1	2082.9k	2080.9k	0.1	0.6
dem_500_belg_orig_410	1297.1k	1295.8k	0.1	1709.8	1297.1k	1295.8k	0.1	33.4
dem_500_belg_orig_411	826.3k	825.5k	0.1	118.6	826.3k	825.5k	0.1	40.4
dem_500_belg_orig_412	1698.0k	1696.3k	0.1	382.0	1698.0k	1696.3k	0.1	5742.0
dem_500_belg_orig_413	1110.9k	1109.8k	0.1	314.5	1110.9k	1109.8k	0.1	170.0
dem_500_belg_orig_414	870.5k	869.6k	0.1	47.1	870.5k	869.6k	0.1	9.8
dem_500_belg_orig_415	1381.5k	1362.8k	1.4	14400.0	1381.5k	1380.1k	0.1	1616.6
dem_500_belg_orig_416	852.5k	852.4k	0.0	3.8	852.5k	851.8k	0.1	5.9
dem_500_belg_orig_417	1678.6k	1677.1k	0.1	2.7	1679.6k	1678.2k	0.1	2.0
dem_500_belg_orig_418	1687.5k	1686.1k	0.1	154.0	1687.5k	1685.8k	0.1	235.2
dem_500_belg_orig_419	1513.8k	1512.7k	0.1	1.6	1514.0k	1512.8k	0.1	0.9
dem_500_belg_orig_420	1128.1k	1127.0k	0.1	586.0	1128.1k	1127.0k	0.1	653.1
dem_500_belg_orig_421	1670.9k	1660.3k	0.6	14400.0	1670.9k	1659.8k	0.7	14400.0
dem_500_belg_orig_422	1500.7k	1483.8k	1.1	14400.0	1500.7k	1484.5k	1.1	14400.0
dem_500_belg_orig_423	1257.4k	1247.3k	0.8	14400.0	1256.8k	1250.9k	0.5	14400.0
dem_500_belg_orig_424	1824.3k	1822.5k	0.1	3.1	1824.3k	1822.5k	0.1	1.6
dem_500_belg_orig_425	1167.7k	1140.9k	2.3	14400.0	1167.7k	1151.9k	1.4	14400.0
dem_500_belg_orig_426	1256.2k	1239.9k	1.3	14400.0	1256.2k	1240.3k	1.3	14400.0
dem_500_belg_orig_427	979.7k	979.7k	0.0	1.0	979.7k	979.7k	0.0	0.5
dem_500_belg_orig_428	1578.8k	1577.2k	0.1	1329.8	1578.8k	1577.2k	0.1	118.5
dem_500_belg_orig_429	1543.4k	1513.2k	2.0	14400.0	1543.4k	1519.4k	1.6	14400.0
dem_500_belg_orig_430	1764.8k	1763.0k	0.1	96.8	1764.8k	1763.0k	0.1	109.5
dem_500_belg_orig_431	871.3k	870.5k	0.1	1525.2	871.3k	870.6k	0.1	5594.9
dem_500_belg_orig_432	1483.9k	1482.5k	0.1	4.9	1483.9k	1482.4k	0.1	454.2
dem_500_belg_orig_433	1074.5k	1062.7k	1.1	14400.0	1074.5k	995.7k	7.9	14400.0
dem_500_belg_orig_434	1094.1k	1094.1k	0.0	0.6	1094.1k	1094.1k	0.0	0.6
dem_500_belg_orig_435	1398.3k	1396.9k	0.1	170.3	1398.3k	1365.0k	2.4	14400.0
dem_500_belg_orig_436	1222.3k	1198.1k	2.0	14400.0	1222.3k	1199.1k	1.9	14400.0
dem_500_belg_orig_437	1831.2k	1819.5k	0.6	14400.0	1831.2k	1818.6k	0.7	14400.0
dem_500_belg_orig_438	1013.9k	984.9k	2.9	14400.0	1013.9k	1013.6k	0.0	610.0
dem_500_belg_orig_439	1273.7k	1273.4k	0.0	1.1	1273.7k	1272.5k	0.1	1.2
dem_500_belg_orig_440	1644.4k	1642.7k	0.1	5.2	1644.4k	1642.9k	0.1	5.6
dem_500_belg_orig_441	570.4k	569.8k	0.1	427.1	570.4k	569.9k	0.1	4.8
dem_500_belg_orig_442	534.6k	534.1k	0.1	3.6	534.6k	534.0k	0.1	33.6
dem_500_belg_orig_443	1584.0k	1582.4k	0.1	104.8	1584.0k	1582.5k	0.1	142.8
dem_500_belg_orig_444	1071.6k	1070.6k	0.1	5457.4	1071.6k	1070.6k	0.1	4953.6
dem_500_belg_orig_445	2079.6k	2063.5k	0.8	14400.0	2079.6k	2077.5k	0.1	1695.5
dem_500_belg_orig_446	1439.9k	1439.9k	0.0	264.4	1439.9k	1434.9k	0.4	14400.0
dem_500_belg_orig_447	1817.0k	1815.7k	0.1	1.7	1817.0k	1815.3k	0.1	0.8
dem_500_belg_orig_448	1441.6k	1440.1k	0.1	10.0	1441.6k	1440.1k	0.1	132.7
dem_500_belg_orig_449	1616.3k	1614.7k	0.1	1733.2	1616.3k	1615.4k	0.1	27.3
dem_500_belg_orig_450	1527.8k	1526.3k	0.1	1.6	1527.8k	1526.3k	0.1	0.9
dem_500_belg_orig_451	973.7k	973.0k	0.1	1.2	973.7k	972.8k	0.1	0.5
dem_500_belg_orig_452	994.4k	993.4k	0.1	2141.9	994.4k	993.4k	0.1	3754.3
dem_500_belg_orig_453	1043.3k	1042.6k	0.1	25.9	1043.3k	1042.2k	0.1	20.8
dem_500_belg_orig_454	1589.6k	1572.4k	1.1	14400.0	1589.6k	1571.9k	1.1	14400.0
dem_500_belg_orig_455	1169.7k	1168.6k	0.1	2.6	1169.7k	1168.9k	0.1	3.8
dem_500_belg_orig_456	830.4k	824.6k	0.7	14400.0	830.4k	829.6k	0.1	7019.0
dem_500_belg_orig_457	1523.5k	1522.0k	0.1	147.8	1523.5k	1522.0k	0.1	147.8
dem_500_belg_orig_458	738.1k	728.3k	1.3	14400.0	737.7k	737.0k	0.1	9453.9
dem_500_belg_orig_459	1766.3k	1764.5k	0.1	8.7	1766.3k	1764.5k	0.1	34.8
dem_500_belg_orig_460	1079.2k	1048.4k	2.9	14400.0	1079.2k	1048.1k	3.0	14400.0
dem_500_belg_orig_461	1424.9k	1423.7k	0.1	192.8	1424.9k	1424.9k	0.0	263.5
dem_500_belg_orig_462	1433.3k	1431.9k	0.1	12484.5	1433.3k	1427.9k	0.4	14400.0
dem_500_belg_orig_463	882.8k	882.8k	0.0	1614.4	882.8k	882.0k	0.1	2382.7
dem_500_belg_orig_464	1044.7k	1043.6k	0.1	498.3	1044.7k	1043.6k	0.1	2437.5

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Table A.6: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_belg_orig_465	1617.5k	1615.9k	0.1	644.3	1617.5k	1608.9k	0.5	14400.0
dem_500_belg_orig_466	1991.3k	1989.4k	0.1	5.0	1991.3k	1989.4k	0.1	111.7
dem_500_belg_orig_467	2148.9k	2148.9k	0.0	161.2	2148.9k	2146.8k	0.1	151.8
dem_500_belg_orig_468	1715.0k	1714.2k	0.1	1.4	1714.2k	1714.2k	0.0	0.5
dem_500_belg_orig_469	1303.8k	1302.6k	0.1	1.1	1303.4k	1302.3k	0.1	0.6
dem_500_belg_orig_470	1330.6k	1313.6k	1.3	14400.0	1330.6k	1312.8k	1.4	14400.0
dem_500_belg_orig_471	1689.7k	1684.8k	0.3	14400.0	1689.7k	1591.6k	6.2	14400.0
dem_500_belg_orig_472	985.6k	984.6k	0.1	2.1	985.6k	984.6k	0.1	2.0
dem_500_belg_orig_473	1314.9k	1291.7k	1.8	14400.0	1314.9k	1299.6k	1.2	14400.0
dem_500_belg_orig_474	1288.6k	1277.6k	0.9	14400.0	1288.6k	1243.9k	3.6	14400.0
dem_500_belg_orig_475	1068.6k	1067.6k	0.1	3294.8	1068.6k	1067.6k	0.1	3562.0
dem_500_belg_orig_476	1278.3k	1267.2k	0.9	14400.0	1278.3k	1277.1k	0.1	1483.5
dem_500_belg_orig_477	1176.0k	1174.8k	0.1	10024.5	1176.0k	1174.8k	0.1	161.2
dem_500_belg_orig_478	1005.8k	991.0k	1.5	14400.0	1005.8k	949.8k	5.9	14400.0
dem_500_belg_orig_479	1635.6k	1634.0k	0.1	1083.8	1635.6k	1603.9k	2.0	14400.0
dem_500_belg_orig_480	1490.6k	1489.4k	0.1	1.3	1489.7k	1489.4k	0.0	5.0
dem_500_belg_orig_481	1037.0k	1036.8k	0.0	1.2	1036.9k	1036.2k	0.1	0.5
dem_500_belg_orig_482	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_belg_orig_483	1010.7k	1009.7k	0.1	60.0	1010.7k	1009.8k	0.1	101.6
dem_500_belg_orig_484	1339.7k	1338.4k	0.1	27.1	1339.7k	1338.4k	0.1	6.0
dem_500_belg_orig_485	885.9k	885.0k	0.1	1320.0	885.9k	885.0k	0.1	421.5
dem_500_belg_orig_486	1066.0k	1064.9k	0.1	2925.2	1066.0k	1052.8k	1.2	14400.0
dem_500_belg_orig_487	1237.1k	1224.9k	1.0	14400.0	1237.1k	1235.9k	0.1	43.2
dem_500_belg_orig_488	1743.9k	1743.0k	0.1	0.6	1743.9k	1743.1k	0.0	0.6
dem_500_belg_orig_489	1152.5k	1152.2k	0.0	62.8	1152.5k	1151.4k	0.1	215.8
dem_500_belg_orig_490	996.6k	995.7k	0.1	101.7	996.6k	995.7k	0.1	110.4
dem_500_belg_orig_491	1122.2k	1113.1k	0.8	14400.0	1122.2k	1112.5k	0.9	14400.0
dem_500_belg_orig_492	2181.9k	2179.9k	0.1	4.4	2181.9k	2180.0k	0.1	0.6
dem_500_belg_orig_493	1621.7k	1604.2k	1.1	14400.0	1621.7k	1604.5k	1.1	14400.0
dem_500_belg_orig_494	1797.0k	1783.3k	0.8	14400.0	1797.0k	1783.7k	0.7	14400.0
dem_500_belg_orig_495	1368.9k	1364.0k	0.4	14400.0	1368.9k	1345.2k	1.8	14400.0
dem_500_belg_orig_496	1273.6k	1272.3k	0.1	7168.1	1273.6k	1257.4k	1.3	14400.0
dem_500_belg_orig_497	1212.5k	1211.3k	0.1	34.2	1212.5k	1209.1k	0.3	14400.0
dem_500_belg_orig_498	1650.4k	1639.1k	0.7	14400.0	1650.4k	1640.9k	0.6	14400.0
dem_500_belg_orig_499	951.5k	950.6k	0.1	66.1	951.5k	950.6k	0.1	16.8
dem_500_belg_orig_500	1206.0k	1192.8k	1.1	14400.0	1206.0k	1204.8k	0.1	1737.0

Table A.7: Detailed results of the discrete models on *Belgium* with $\mathcal{B} = 1000$ as summarized in Figure 3.7d and Table 3.3a. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_1	1843.6k	1629.5k	13.1	14400.0	1832.0k	1806.2k	1.4	14400.0	1832.5k	1791.3k	2.3	14400.0
dem_1000_belg_orig_2	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_3	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_4	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_5	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_6	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	53.8	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_7	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	38.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_8	1751.4k	1701.4k	2.9	14400.0	1752.8k	1706.0k	2.7	14400.0	1753.0k	1641.8k	6.8	14400.0
dem_1000_belg_orig_9	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	35.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_10	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_11	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	40.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_12	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_13	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	41.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_14	1668.9k	1480.1k	12.8	14400.0	1667.3k	1667.3k	0.0	3929.3	1667.3k	1667.3k	0.0	1004.5
dem_1000_belg_orig_15	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_16	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_17	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	29.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_18	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	31.3	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_19	2479.5k	2479.5k	0.0	440.8	2479.5k	2479.5k	0.0	628.0	2479.5k	2479.5k	0.0	4144.9
dem_1000_belg_orig_20	1809.8k	1739.4k	4.0	14400.0	1798.2k	1798.2k	0.0	5880.1	1798.2k	1780.3k	1.0	14400.0
dem_1000_belg_orig_21	1289.7k	1223.9k	5.4	14400.0	1289.7k	1289.7k	0.0	5137.8	1291.6k	1136.1k	13.7	14400.0
dem_1000_belg_orig_22	2810.0k	2810.0k	0.0	596.6	2810.0k	2807.7k	0.1	4015.7	2810.0k	2807.2k	0.1	7624.7
dem_1000_belg_orig_23	2353.7k	2325.5k	1.2	14400.0	2356.5k	2323.3k	1.4	14400.0	2354.3k	2306.4k	2.1	14400.0
dem_1000_belg_orig_24	2135.6k	2135.6k	0.0	2682.9	2135.6k	2135.3k	0.0	677.4	2135.6k	2133.7k	0.1	487.8
dem_1000_belg_orig_25	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	25.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_26	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_27	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	11.9	1e+20	1e+20	0.0	0.1

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Table A.7: Comparison of discrete models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_28	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_29	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_30	1605.7k	1346.3k	19.3	14400.0	1611.3k	1529.7k	5.3	14400.0	1609.6k	1296.9k	24.1	14400.0
dem_1000_belg_orig_31	1775.1k	1774.4k	0.0	8.2	1775.1k	1773.4k	0.1	101.9	1775.1k	1773.4k	0.1	11.1
dem_1000_belg_orig_32	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_33	1802.9k	1704.7k	5.8	14400.0	1800.9k	1759.3k	2.4	14400.0	1804.3k	1619.1k	11.4	14400.0
dem_1000_belg_orig_34	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_35	1887.5k	1744.1k	8.2	14400.0	1871.5k	1824.0k	2.6	14400.0	1895.1k	1698.7k	11.6	14400.0
dem_1000_belg_orig_36	2039.4k	2038.3k	0.1	1059.0	2039.4k	2037.4k	0.1	1511.6	2039.4k	2037.3k	0.1	3643.4
dem_1000_belg_orig_37	2535.8k	2443.8k	3.8	14400.0	2535.8k	2487.2k	2.0	14400.0	2535.8k	2413.7k	5.1	14400.0
dem_1000_belg_orig_38	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	20.3	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_39	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	37.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_40	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_41	1e+20	2059.0k	–	14400.0	1e+20	1e+20	0.0	644.6	1e+20	2036.1k	–	14400.0
dem_1000_belg_orig_42	2355.6k	2353.6k	0.1	1.3	2355.6k	2353.3k	0.1	32.1	2355.6k	2355.6k	0.0	7.1
dem_1000_belg_orig_43	2059.9k	2059.9k	0.0	187.1	2059.9k	2058.0k	0.1	2623.6	2059.9k	2059.9k	0.0	5901.3
dem_1000_belg_orig_44	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_45	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_46	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	31.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_47	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	58.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_48	2375.2k	2373.0k	0.1	3106.6	2379.3k	2304.2k	3.3	14400.0	2376.4k	2322.0k	2.3	14400.0
dem_1000_belg_orig_49	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	32.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_50	1478.6k	1366.9k	8.2	14400.0	1478.6k	1451.7k	1.9	14400.0	1487.1k	1324.1k	12.3	14400.0
dem_1000_belg_orig_51	1996.6k	1956.8k	2.0	14400.0	1993.4k	1991.9k	0.1	5538.6	1995.9k	1934.7k	3.2	14400.0
dem_1000_belg_orig_52	2073.7k	1983.6k	4.5	14400.0	2071.6k	2040.7k	1.5	14400.0	2072.2k	1950.6k	6.2	14400.0
dem_1000_belg_orig_53	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_54	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	45.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_55	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_56	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	57.5	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_57	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_58	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_59	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_60	2139.2k	2064.2k	3.6	14400.0	2139.2k	2137.9k	0.1	7615.9	2150.7k	1966.0k	9.4	14400.0
dem_1000_belg_orig_61	1555.3k	1555.3k	0.0	5600.8	1555.3k	1553.8k	0.1	10165.1	1555.7k	1530.1k	1.7	14400.0
dem_1000_belg_orig_62	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_63	2681.1k	2571.5k	4.3	14400.0	2675.6k	2651.2k	0.9	14400.0	2682.3k	2556.4k	4.9	14400.0
dem_1000_belg_orig_64	2495.6k	2495.6k	0.0	934.2	2495.6k	2493.1k	0.1	5856.7	2495.6k	2493.4k	0.1	8875.6
dem_1000_belg_orig_65	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_66	984.3k	865.5k	13.7	14400.0	977.1k	977.1k	0.0	9833.3	979.7k	839.6k	16.7	14400.0
dem_1000_belg_orig_67	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_68	1848.1k	1792.0k	3.1	14400.0	1846.0k	1845.5k	0.0	10942.5	1847.0k	1730.8k	6.7	14400.0
dem_1000_belg_orig_69	2450.9k	2153.6k	13.8	14400.0	2406.2k	2406.2k	0.0	2679.7	2406.2k	2405.0k	0.0	6045.4
dem_1000_belg_orig_70	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_71	2009.7k	1966.7k	2.2	14400.0	2009.7k	1984.9k	1.3	14400.0	2012.0k	1961.3k	2.6	14400.0
dem_1000_belg_orig_72	2047.5k	2047.5k	0.0	2097.7	2047.5k	2047.5k	0.0	2028.0	2049.2k	2023.8k	1.3	14400.0
dem_1000_belg_orig_73	1951.1k	1828.9k	6.7	14400.0	1949.3k	1915.0k	1.8	14400.0	1957.0k	1823.9k	7.3	14400.0
dem_1000_belg_orig_74	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_75	2288.4k	2174.0k	5.3	14400.0	2291.6k	2213.7k	3.5	14400.0	2304.3k	2131.3k	8.1	14400.0
dem_1000_belg_orig_76	1604.8k	1413.0k	13.6	14400.0	1590.5k	1589.9k	0.0	8067.9	1590.5k	1566.3k	1.5	14400.0
dem_1000_belg_orig_77	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	62.6	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_78	2078.1k	2052.2k	1.3	14400.0	2078.1k	2078.1k	0.0	6900.5	2080.3k	1998.9k	4.1	14400.0
dem_1000_belg_orig_79	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_80	1829.4k	1829.4k	0.0	4011.6	1829.4k	1817.8k	0.6	14400.0	1829.4k	1828.2k	0.1	11110.7
dem_1000_belg_orig_81	1527.0k	1459.6k	4.6	14400.0	1523.3k	1523.3k	0.0	10845.9	1529.4k	1420.6k	7.7	14400.0
dem_1000_belg_orig_82	2435.9k	2433.7k	0.1	1099.7	2435.9k	2433.5k	0.1	2557.8	2435.9k	2405.4k	1.3	14400.0
dem_1000_belg_orig_83	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	40.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_84	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_85	1875.5k	1875.5k	0.0	535.5	1875.5k	1874.5k	0.1	3293.7	1875.5k	1875.5k	0.0	9834.0
dem_1000_belg_orig_86	2353.3k	2352.8k	0.0	6.2	2353.3k	2353.3k	0.0	189.3	2353.3k	2303.9k	2.1	14400.0
dem_1000_belg_orig_87	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_88	1766.1k	1766.1k	0.0	12891.6	1766.1k	1757.3k	0.5	14400.0	1766.1k	1728.8k	2.2	14400.0
dem_1000_belg_orig_89	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_90	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_91	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	14.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_92	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_93	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_94	2543.1k	2489.7k	2.1	14400.0	2545.3k	2506.3k	1.6	14400.0	2545.3k	2478.3k	2.7	14400.0
dem_1000_belg_orig_95	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	60.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_96	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_97	2099.8k	2097.9k	0.1	205.6	2099.8k	2099.1k	0.0	1386.9	2099.8k	2099.8k	0.0	1670.5
dem_1000_belg_orig_98	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_99	2055.0k	2053.0k	0.1	4794.8	2055.0k	2054.3k	0.0	13530.3	2055.0k	2026.5k	1.4	14400.0
dem_1000_belg_orig_100	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_101	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_102	1829.8k	1747.9k	4.7	14400.0	1827.1k	1788.3k	2.2	14400.0	1833.8k	1711.5k	7.1	14400.0
dem_1000_belg_orig_103	1983.3k	1983.3k	0.0	139.4	1983.3k	1982.8k	0.0	1020.6	1983.3k	1982.0k	0.1	4494.6
dem_1000_belg_orig_104	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	39.2	1e+20	1e+20	0.0	10462.5
dem_1000_belg_orig_105	1145.7k	923.0k	24.1	14400.0	1124.8k	1032.8k	8.9	14400.0	1128.9k	906.4k	24.5	14400.0

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Table A.7: Comparison of discrete models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_106	2462.8k	2460.9k	0.1	1082.6	2462.8k	2461.1k	0.1	13382.4	2462.8k	2461.0k	0.1	6769.1
dem_1000_belg_orig_107	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_108	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	45.6	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_109	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_110	1694.2k	1693.8k	0.0	235.9	1694.2k	1692.6k	0.1	2400.8	1694.2k	1692.6k	0.1	372.6
dem_1000_belg_orig_111	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_112	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	51.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_113	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	15.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_114	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	34.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_115	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	29.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_116	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	17.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_117	2326.5k	2326.5k	0.0	219.1	2326.5k	2324.8k	0.1	557.4	2326.5k	2325.3k	0.1	1038.8
dem_1000_belg_orig_118	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_119	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_120	2420.0k	2418.0k	0.1	242.8	2420.0k	2420.0k	0.0	501.2	2420.0k	2418.7k	0.1	141.3
dem_1000_belg_orig_121	2347.3k	2277.3k	3.1	14400.0	2346.9k	2317.2k	1.3	14400.0	2350.5k	2258.4k	4.1	14400.0
dem_1000_belg_orig_122	2483.1k	2336.5k	6.3	14400.0	2483.7k	2412.1k	3.0	14400.0	2482.7k	2413.6k	2.9	14400.0
dem_1000_belg_orig_123	2527.6k	2504.1k	0.9	14400.0	2527.6k	2527.6k	0.0	6868.0	2528.9k	2479.4k	2.0	14400.0
dem_1000_belg_orig_124	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	42.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_125	2005.0k	1932.5k	3.8	14400.0	2004.3k	2003.3k	0.1	5213.4	2004.3k	2004.3k	0.0	7129.2
dem_1000_belg_orig_126	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	27.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_127	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_128	1956.7k	1956.5k	0.0	685.2	1956.7k	1955.9k	0.0	5098.5	1956.7k	1956.7k	0.0	7818.0
dem_1000_belg_orig_129	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_130	1695.3k	1695.3k	0.0	4.0	1695.3k	1695.3k	0.0	98.3	1695.3k	1694.0k	0.1	7.9
dem_1000_belg_orig_131	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_132	1798.8k	1798.8k	0.0	1384.0	1798.8k	1797.1k	0.1	3682.8	1798.8k	1774.6k	1.4	14400.0
dem_1000_belg_orig_133	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_134	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	28.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_135	1579.3k	1496.6k	5.5	14400.0	1577.6k	1526.6k	3.3	14400.0	1e+20	1416.0k	–	14400.0
dem_1000_belg_orig_136	2946.2k	2943.9k	0.1	695.4	2946.2k	2943.3k	0.1	5895.1	2946.2k	2945.9k	0.0	5209.6
dem_1000_belg_orig_137	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	48.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_138	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	29.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_139	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_140	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_141	2402.6k	2402.6k	0.0	2039.6	2402.6k	2400.3k	0.1	3867.5	2402.6k	2336.5k	2.8	14400.0
dem_1000_belg_orig_142	1135.5k	1135.5k	0.0	307.3	1135.5k	1134.5k	0.1	765.3	1135.5k	1135.5k	0.0	560.1
dem_1000_belg_orig_143	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	21.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_144	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_145	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	24.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_146	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_147	1800.7k	1784.0k	0.9	14400.0	1800.7k	1781.3k	1.1	14400.0	1801.2k	1738.8k	3.6	14400.0
dem_1000_belg_orig_148	1807.0k	1607.4k	12.4	14400.0	1791.1k	1762.9k	1.6	14400.0	1791.1k	1761.2k	1.7	14400.0
dem_1000_belg_orig_149	2151.1k	2151.1k	0.0	1786.7	2151.1k	2151.1k	0.0	341.4	2151.1k	2149.0k	0.1	6207.9
dem_1000_belg_orig_150	1741.0k	1702.1k	2.3	14400.0	1741.0k	1708.1k	1.9	14400.0	1747.6k	1684.2k	3.8	14400.0
dem_1000_belg_orig_151	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_152	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	5.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_153	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_154	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	28.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_155	2003.2k	2003.2k	0.0	530.2	2003.2k	2002.9k	0.0	11600.9	2003.2k	2003.2k	0.0	3322.8
dem_1000_belg_orig_156	1489.5k	1227.1k	21.4	14400.0	1472.6k	1472.6k	0.0	1299.5	1486.7k	1214.6k	22.4	14400.0
dem_1000_belg_orig_157	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_158	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_159	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	37.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_160	2111.6k	2110.5k	0.1	787.4	2111.6k	2111.6k	0.0	7527.1	2111.9k	2081.7k	1.5	14400.0
dem_1000_belg_orig_161	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	5.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_162	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_163	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	12.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_164	1149.5k	956.7k	20.2	14400.0	1143.5k	1086.2k	5.3	14400.0	1149.4k	934.4k	23.0	14400.0
dem_1000_belg_orig_165	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	32.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_166	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_167	1749.4k	1749.4k	0.0	4327.4	1749.4k	1749.2k	0.0	1554.3	1749.4k	1674.7k	4.5	14400.0
dem_1000_belg_orig_168	1409.8k	1409.8k	0.0	447.4	1409.8k	1409.6k	0.0	5834.0	1409.8k	1408.7k	0.1	7410.8
dem_1000_belg_orig_169	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	36.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_170	1367.2k	1367.2k	0.0	240.0	1367.2k	1367.2k	0.0	985.4	1367.2k	1367.2k	0.0	9885.9
dem_1000_belg_orig_171	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	17.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_172	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_173	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_174	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	32.1	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_175	1709.8k	1665.0k	2.7	14400.0	1716.5k	1675.2k	2.5	14400.0	1709.8k	1669.0k	2.4	14400.0
dem_1000_belg_orig_176	1265.7k	1148.0k	10.2	14400.0	1255.6k	1255.6k	0.0	7416.9	1257.6k	1109.4k	13.4	14400.0
dem_1000_belg_orig_177	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_178	1347.3k	1346.6k	0.0	24.2	1347.3k	1346.7k	0.0	141.3	1347.3k	1347.3k	0.0	11.5
dem_1000_belg_orig_179	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_180	2253.7k	2014.9k	11.9	14400.0	2248.5k	2225.5k	1.0	14400.0	2250.4k	2208.1k	1.9	14400.0
dem_1000_belg_orig_181	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	44.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_182	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_183	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1

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Table A.7: Comparison of discrete models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_184	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_185	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_186	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	29.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_187	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_188	1696.2k	1583.8k	7.1	14400.0	1694.3k	1651.3k	2.6	14400.0	1704.8k	1610.2k	5.9	14400.0
dem_1000_belg_orig_189	1439.5k	1439.5k	0.0	7118.1	1439.5k	1435.3k	0.3	14400.0	1440.2k	1403.6k	2.6	14400.0
dem_1000_belg_orig_190	2796.3k	2796.3k	0.0	7201.3	2796.3k	2793.6k	0.1	13360.1	2796.6k	2733.9k	2.3	14400.0
dem_1000_belg_orig_191	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	25.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_192	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_193	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_194	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_195	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_196	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	18.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_197	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	34.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_198	1959.0k	1924.5k	1.8	14400.0	1959.0k	1949.7k	0.5	14400.0	1959.3k	1880.3k	4.2	14400.0
dem_1000_belg_orig_199	2484.9k	2484.9k	0.0	535.2	2484.9k	2483.5k	0.1	1856.3	2484.9k	2484.9k	0.0	614.1
dem_1000_belg_orig_200	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	24.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_201	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_202	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_203	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	46.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_204	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	43.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_205	1764.4k	1717.2k	2.7	14400.0	1764.4k	1764.1k	0.0	1197.0	1765.5k	1679.2k	5.1	14400.0
dem_1000_belg_orig_206	1690.9k	1600.1k	5.7	14400.0	1690.9k	1661.7k	1.8	14400.0	1701.9k	1557.3k	9.3	14400.0
dem_1000_belg_orig_207	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_208	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_209	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_210	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_211	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	33.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_212	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_213	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_214	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_215	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_216	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_217	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_218	2352.0k	2352.0k	0.0	17.9	2352.0k	2350.3k	0.1	215.2	2352.0k	2351.2k	0.0	123.3
dem_1000_belg_orig_219	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_220	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	30.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_221	2095.0k	1938.2k	8.1	14400.0	2093.4k	1992.4k	5.1	14400.0	2102.1k	1983.8k	6.0	14400.0
dem_1000_belg_orig_222	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_223	1154.6k	982.4k	17.5	14400.0	1150.8k	1123.4k	2.4	14400.0	1173.2k	956.7k	22.6	14400.0
dem_1000_belg_orig_224	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	55.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_225	1814.8k	1726.2k	5.1	14400.0	1812.4k	1778.9k	1.9	14400.0	1819.1k	1698.4k	7.1	14400.0
dem_1000_belg_orig_226	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	18.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_227	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_228	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_229	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_230	1974.9k	1937.6k	1.9	14400.0	1974.9k	1949.5k	1.3	14400.0	1979.2k	1893.3k	4.5	14400.0
dem_1000_belg_orig_231	1880.1k	1749.9k	7.4	14400.0	1870.6k	1870.6k	0.0	3354.0	1870.6k	1870.6k	0.0	4826.5
dem_1000_belg_orig_232	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_233	1877.3k	1876.3k	0.1	1669.7	1877.3k	1877.3k	0.0	904.6	1877.3k	1876.3k	0.1	256.8
dem_1000_belg_orig_234	2088.3k	2088.3k	0.0	6699.5	2088.3k	2088.3k	0.0	6216.7	2088.3k	2086.3k	0.1	1008.1
dem_1000_belg_orig_235	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	79.6	1e+20	1e+20	0.0	0.3
dem_1000_belg_orig_236	2103.5k	2103.4k	0.0	1181.3	2103.5k	2102.5k	0.1	2847.8	2103.5k	2101.5k	0.1	12450.5
dem_1000_belg_orig_237	2707.3k	2666.6k	1.5	14400.0	2707.3k	2682.9k	0.9	14400.0	2707.3k	2657.8k	1.9	14400.0
dem_1000_belg_orig_238	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_239	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	34.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_240	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_241	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_242	2159.4k	2009.0k	7.5	14400.0	2159.4k	2113.3k	2.2	14400.0	2166.7k	1974.8k	9.7	14400.0
dem_1000_belg_orig_243	2501.7k	2357.9k	6.1	14400.0	2500.7k	2433.8k	2.7	14400.0	2507.0k	2334.2k	7.4	14400.0
dem_1000_belg_orig_244	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_245	1794.3k	1746.0k	2.8	14400.0	1794.0k	1777.9k	0.9	14400.0	1797.2k	1658.2k	8.4	14400.0
dem_1000_belg_orig_246	2924.7k	2924.7k	0.0	5156.4	2924.7k	2922.6k	0.1	10338.3	2924.7k	2906.2k	0.6	14400.0
dem_1000_belg_orig_247	1277.0k	1277.0k	0.0	1665.3	1277.0k	1277.0k	0.0	3822.5	1277.0k	1253.3k	1.9	14400.0
dem_1000_belg_orig_248	2628.1k	2590.1k	1.5	14400.0	2628.1k	2596.3k	1.2	14400.0	2663.4k	2613.1k	1.9	14400.0
dem_1000_belg_orig_249	1e+20	1e+20	0.0	0.0	1e+20	2831.4k	–	29.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_250	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_251	1646.5k	1482.8k	11.0	14400.0	1636.8k	1609.8k	1.7	14400.0	1647.3k	1436.9k	14.6	14400.0
dem_1000_belg_orig_252	2111.9k	1857.2k	13.7	14400.0	2094.1k	1988.4k	5.3	14400.0	2132.0k	1881.2k	13.3	14400.0
dem_1000_belg_orig_253	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_254	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_255	2752.7k	2730.3k	0.8	14400.0	2752.7k	2750.1k	0.1	9030.3	2779.0k	2710.3k	2.5	14400.0
dem_1000_belg_orig_256	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_257	1689.7k	1689.7k	0.0	5692.9	1689.7k	1656.5k	2.0	14400.0	1690.3k	1645.9k	2.7	14400.0
dem_1000_belg_orig_258	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	53.4	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_259	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	18.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_260	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_261	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	0.1

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Table A.7: Comparison of discrete models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_262	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_263	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	30.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_264	2178.8k	2032.1k	7.2	14400.0	2178.3k	2178.3k	0.0	6724.1	2202.3k	1978.2k	11.3	14400.0
dem_1000_belg_orig_265	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	27.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_266	2206.8k	2205.7k	0.0	15.2	2206.8k	2206.8k	0.0	447.8	2206.8k	2205.7k	0.0	129.2
dem_1000_belg_orig_267	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_268	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_269	2255.4k	2135.6k	5.6	14400.0	2252.5k	2252.5k	0.0	4188.3	2254.3k	2183.4k	3.2	14400.0
dem_1000_belg_orig_270	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	29.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_271	1843.6k	1842.2k	0.1	1202.5	1843.6k	1843.6k	0.0	1178.5	1843.6k	1817.4k	1.4	14400.0
dem_1000_belg_orig_272	2559.8k	2559.8k	0.0	216.9	2559.8k	2557.2k	0.1	2585.3	2559.8k	2557.2k	0.1	2206.8
dem_1000_belg_orig_273	1877.7k	1678.5k	11.9	14400.0	1862.5k	1755.4k	6.1	14400.0	1863.7k	1669.2k	11.6	14400.0
dem_1000_belg_orig_274	2410.7k	2409.3k	0.1	3594.5	2410.7k	2408.3k	0.1	10109.4	2411.7k	2352.4k	2.5	14400.0
dem_1000_belg_orig_275	1886.6k	1867.2k	1.0	14400.0	1886.6k	1886.6k	0.0	3188.4	1889.8k	1839.5k	2.7	14400.0
dem_1000_belg_orig_276	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	23.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_277	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_278	2361.7k	2281.0k	3.5	14400.0	2359.4k	2359.4k	0.0	10540.6	2359.4k	2223.1k	6.1	14400.0
dem_1000_belg_orig_279	1678.5k	1678.0k	0.0	1619.6	1678.5k	1677.7k	0.0	7574.1	1679.4k	1636.7k	2.6	14400.0
dem_1000_belg_orig_280	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_281	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	47.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_282	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_283	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	20.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_284	2698.8k	2622.5k	2.9	14400.0	2698.8k	2671.5k	1.0	14400.0	2711.6k	2600.2k	4.3	14400.0
dem_1000_belg_orig_285	2082.3k	2002.7k	4.0	14400.0	2074.3k	2032.9k	2.0	14400.0	2080.4k	1989.4k	4.6	14400.0
dem_1000_belg_orig_286	2387.8k	2365.4k	0.9	14400.0	2387.8k	2386.1k	0.1	4421.7	2393.1k	2308.1k	3.7	14400.0
dem_1000_belg_orig_287	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_288	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	25.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_289	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_290	2526.5k	2405.6k	5.0	14400.0	2520.5k	2519.7k	0.0	7901.4	2532.8k	2383.8k	6.3	14400.0
dem_1000_belg_orig_291	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_292	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_293	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_294	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_295	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_296	1924.3k	1924.3k	0.0	28.2	1924.3k	1924.3k	0.0	78.7	1924.3k	1924.3k	0.0	9.1
dem_1000_belg_orig_297	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_298	2496.5k	2333.0k	7.0	14400.0	2486.8k	2486.8k	0.0	4930.1	2496.2k	2385.1k	4.7	14400.0
dem_1000_belg_orig_299	1558.2k	1557.2k	0.1	8971.9	1558.2k	1558.2k	0.0	2408.3	1558.2k	1514.7k	2.9	14400.0
dem_1000_belg_orig_300	1494.0k	1492.9k	0.1	69.0	1494.0k	1493.3k	0.0	192.1	1494.0k	1493.4k	0.0	18.4
dem_1000_belg_orig_301	2452.0k	2450.9k	0.0	13612.9	2452.0k	2450.6k	0.1	9510.5	2455.4k	2402.5k	2.2	14400.0
dem_1000_belg_orig_302	2333.7k	2247.3k	3.8	14400.0	2333.7k	2325.8k	0.3	14400.0	2338.8k	2172.7k	7.6	14400.0
dem_1000_belg_orig_303	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	41.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_304	2156.8k	2156.8k	0.0	1312.7	2156.8k	2156.8k	0.0	5347.9	2156.8k	2156.8k	0.0	3276.2
dem_1000_belg_orig_305	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	26.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_306	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	14.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_307	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_308	2603.9k	2603.9k	0.0	301.9	2603.9k	2601.4k	0.1	1095.3	2603.9k	2603.9k	0.0	214.0
dem_1000_belg_orig_309	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_310	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	28.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_311	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_312	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_313	1913.0k	1732.2k	10.4	14400.0	1912.4k	1862.3k	2.7	14400.0	1974.5k	1666.9k	18.4	14400.0
dem_1000_belg_orig_314	1190.6k	1110.4k	7.2	14400.0	1186.8k	1168.5k	1.6	14400.0	1199.2k	1085.2k	10.5	14400.0
dem_1000_belg_orig_315	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_316	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_317	1969.7k	1967.8k	0.1	1410.1	1969.7k	1967.8k	0.1	13650.7	1969.7k	1967.9k	0.1	13326.4
dem_1000_belg_orig_318	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_319	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_320	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_321	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	100.6	1e+20	1e+20	0.0	3.9
dem_1000_belg_orig_322	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_323	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_324	2364.2k	2343.1k	0.9	14400.0	2364.2k	2338.2k	1.1	14400.0	2380.7k	2238.5k	6.4	14400.0
dem_1000_belg_orig_325	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_326	2426.7k	2426.7k	0.0	1796.3	2426.7k	2424.3k	0.1	5085.9	2426.7k	2407.5k	0.8	14400.0
dem_1000_belg_orig_327	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_328	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_329	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	20.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_330	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_331	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_332	1938.5k	1900.2k	2.0	14400.0	1938.5k	1915.8k	1.2	14400.0	1938.5k	1838.1k	5.5	14400.0
dem_1000_belg_orig_333	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	19.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_334	2344.6k	2008.8k	16.7	14400.0	2341.6k	2341.6k	0.0	12346.2	2347.7k	2213.0k	6.1	14400.0
dem_1000_belg_orig_335	1552.0k	1487.3k	4.4	14400.0	1551.1k	1548.2k	0.2	14400.0	1551.5k	1453.5k	6.7	14400.0
dem_1000_belg_orig_336	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	60.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_337	2062.8k	2061.8k	0.0	634.0	2062.8k	2060.8k	0.1	3565.7	2062.8k	2060.8k	0.1	11350.8
dem_1000_belg_orig_338	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	28.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_339	1032.1k	992.0k	4.0	14400.0	1032.1k	1014.7k	1.7	14400.0	1041.3k	968.8k	7.5	14400.0

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Table A.7: Comparison of discrete models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_340	1788.8k	1462.5k	22.3	14400.0	1789.7k	1725.5k	3.7	14400.0	1783.9k	1712.5k	4.2	14400.0
dem_1000_belg_orig_341	2615.3k	2613.2k	0.1	1717.1	2615.3k	2582.6k	1.3	14400.0	2615.3k	2550.7k	2.5	14400.0
dem_1000_belg_orig_342	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_343	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_344	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_345	2602.6k	2600.9k	0.1	389.2	2604.6k	2602.1k	0.1	3580.4	2604.6k	2602.0k	0.1	2261.1
dem_1000_belg_orig_346	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	22.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_347	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_348	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_349	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_350	2312.8k	2198.2k	5.2	14400.0	2305.5k	2276.4k	1.3	14400.0	2315.9k	2166.2k	6.9	14400.0
dem_1000_belg_orig_351	2330.7k	2330.7k	0.0	5707.9	2330.7k	2328.8k	0.1	10849.6	2334.3k	2265.1k	3.1	14400.0
dem_1000_belg_orig_352	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_353	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_354	2197.6k	1783.1k	23.2	14400.0	2181.5k	2179.6k	0.1	7720.9	2183.1k	2070.6k	5.4	14400.0
dem_1000_belg_orig_355	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_356	2553.0k	2506.6k	1.9	14400.0	2553.0k	2533.7k	0.8	14400.0	2553.7k	2425.0k	5.3	14400.0
dem_1000_belg_orig_357	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_358	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_359	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	31.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_360	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_361	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	21.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_362	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_363	1886.7k	1795.7k	5.1	14400.0	1886.7k	1886.7k	0.0	9550.5	1887.2k	1835.0k	2.8	14400.0
dem_1000_belg_orig_364	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_365	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	31.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_366	2070.9k	1961.4k	5.6	14400.0	2070.6k	2005.0k	3.3	14400.0	2073.2k	1888.3k	9.8	14400.0
dem_1000_belg_orig_367	1535.4k	1323.9k	16.0	14400.0	1522.5k	1522.5k	0.0	9292.8	1552.6k	1257.3k	23.5	14400.0
dem_1000_belg_orig_368	2194.4k	2192.4k	0.1	2954.5	2194.4k	2194.4k	0.0	2372.4	2194.4k	2148.5k	2.1	14400.0
dem_1000_belg_orig_369	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_370	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_371	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_372	1119.6k	977.1k	14.6	14400.0	1096.1k	1096.1k	0.0	3175.0	1181.8k	957.9k	23.4	14400.0
dem_1000_belg_orig_373	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_374	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_375	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_376	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	46.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_377	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_378	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_379	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_380	1754.4k	1690.2k	3.8	14400.0	1754.4k	1714.4k	2.3	14400.0	1755.2k	1677.6k	4.6	14400.0
dem_1000_belg_orig_381	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	22.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_382	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	23.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_383	861.3k	861.3k	0.0	957.2	861.3k	861.3k	0.0	1147.7	861.3k	861.3k	0.0	4473.7
dem_1000_belg_orig_384	2268.2k	2268.2k	0.0	3475.2	2268.2k	2265.9k	0.1	8542.7	2268.3k	2252.4k	0.7	14400.0
dem_1000_belg_orig_385	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_386	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	91.4	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_387	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	20.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_388	1924.0k	1508.9k	27.5	14400.0	1920.9k	1892.9k	1.5	14400.0	1922.0k	1798.2k	6.9	14400.0
dem_1000_belg_orig_389	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_390	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_391	2721.6k	2677.1k	1.7	14400.0	2721.1k	2719.4k	0.1	8530.3	2727.3k	2630.6k	3.7	14400.0
dem_1000_belg_orig_392	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_393	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_394	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_395	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_396	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	63.8	1e+20	1e+20	0.0	3.7
dem_1000_belg_orig_397	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	27.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_398	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_399	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	44.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_400	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_401	1716.1k	1716.1k	0.0	1037.4	1716.1k	1714.4k	0.1	4909.9	1716.1k	1714.9k	0.1	8917.1
dem_1000_belg_orig_402	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_403	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	15.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_404	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_405	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_406	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	72.0	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_407	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	20.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_408	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_409	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_410	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	23.1	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_411	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_412	2380.5k	2380.5k	0.0	509.1	2380.5k	2378.1k	0.1	3742.8	2380.5k	2378.2k	0.1	8271.7
dem_1000_belg_orig_413	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	35.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_414	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_415	952.3k	879.8k	8.2	14400.0	952.3k	925.8k	2.9	14400.0	953.5k	854.7k	11.6	14400.0
dem_1000_belg_orig_416	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_417	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	44.6	1e+20	1e+20	0.0	0.1

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Table A.7: Comparison of discrete models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_418	1838.7k	1837.4k	0.1	190.7	1838.7k	1836.9k	0.1	1268.7	1838.7k	1838.7k	0.0	785.5
dem_1000_belg_orig_419	1296.7k	1203.8k	7.7	14400.0	1287.6k	1286.4k	0.1	3677.2	1290.3k	1097.2k	17.6	14400.0
dem_1000_belg_orig_420	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_421	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	10.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_422	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_423	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	40.6	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_424	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	27.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_425	2192.5k	2191.5k	0.0	64.9	2192.5k	2192.5k	0.0	678.1	2192.5k	2190.4k	0.1	272.8
dem_1000_belg_orig_426	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	39.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_427	1914.4k	1809.9k	5.8	14400.0	1920.8k	1865.0k	3.0	14400.0	1924.2k	1806.8k	6.5	14400.0
dem_1000_belg_orig_428	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_429	2315.5k	2315.5k	0.0	47.1	2315.5k	2313.3k	0.1	402.8	2315.5k	2313.7k	0.1	488.6
dem_1000_belg_orig_430	1884.7k	1724.8k	9.3	14400.0	1880.4k	1808.3k	4.0	14400.0	1884.9k	1763.3k	6.9	14400.0
dem_1000_belg_orig_431	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_432	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_433	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_434	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	34.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_435	1862.6k	1862.6k	0.0	1102.8	1862.6k	1854.1k	0.5	14400.0	1862.6k	1862.6k	0.0	246.5
dem_1000_belg_orig_436	1843.4k	1815.9k	1.5	14400.0	1843.4k	1843.4k	0.0	7328.2	1848.0k	1712.2k	7.9	14400.0
dem_1000_belg_orig_437	1922.9k	1745.3k	10.2	14400.0	1931.1k	1826.6k	5.7	14400.0	1928.7k	1740.8k	10.8	14400.0
dem_1000_belg_orig_438	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	21.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_439	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_440	2598.1k	2595.6k	0.1	4.0	2598.1k	2596.4k	0.1	44.4	2598.1k	2598.1k	0.0	9.0
dem_1000_belg_orig_441	1504.5k	1504.5k	0.0	5854.5	1504.5k	1504.5k	0.0	1072.7	1504.5k	1504.0k	0.0	673.3
dem_1000_belg_orig_442	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_443	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	21.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_444	1848.0k	1786.4k	3.4	14400.0	1848.0k	1848.0k	0.0	1317.0	1851.8k	1793.2k	3.3	14400.0
dem_1000_belg_orig_445	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_446	1979.1k	1979.1k	0.0	11961.0	1979.1k	1977.6k	0.1	1889.2	2005.6k	1850.8k	8.4	14400.0
dem_1000_belg_orig_447	2245.7k	2245.7k	0.0	1579.6	2245.7k	2243.5k	0.1	4331.6	2245.7k	2216.9k	1.3	14400.0
dem_1000_belg_orig_448	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	47.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_449	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	23.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_450	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	37.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_451	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	41.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_452	2444.3k	2443.5k	0.0	6460.2	2444.3k	2382.8k	2.6	14400.0	2444.3k	2337.9k	4.6	14400.0
dem_1000_belg_orig_453	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_454	1355.0k	1231.7k	10.0	14400.0	1349.1k	1296.2k	4.1	14400.0	1352.2k	1180.5k	14.5	14400.0
dem_1000_belg_orig_455	1727.0k	1725.5k	0.1	7591.8	1727.0k	1708.2k	1.1	14400.0	1732.2k	1672.8k	3.6	14400.0
dem_1000_belg_orig_456	1594.0k	1593.1k	0.1	857.4	1594.0k	1592.7k	0.1	3182.4	1594.0k	1570.6k	1.5	14400.0
dem_1000_belg_orig_457	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	7.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_458	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	32.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_459	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	49.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_460	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	44.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_461	1801.1k	1741.6k	3.4	14400.0	1802.0k	1760.9k	2.3	14400.0	1811.0k	1671.7k	8.3	14400.0
dem_1000_belg_orig_462	2221.1k	2040.9k	8.8	14400.0	2215.8k	2171.7k	2.0	14400.0	2221.8k	2086.0k	6.5	14400.0
dem_1000_belg_orig_463	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	33.6	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_464	2535.0k	2532.8k	0.1	802.4	2535.0k	2532.5k	0.1	7485.3	2535.0k	2534.7k	0.0	4651.8
dem_1000_belg_orig_465	1994.4k	1669.1k	19.5	14400.0	1967.3k	1967.3k	0.0	1722.1	1967.3k	1967.3k	0.0	1792.6
dem_1000_belg_orig_466	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_467	2931.0k	2902.0k	1.0	14400.0	2931.0k	2906.9k	0.8	14400.0	2940.6k	2764.2k	6.4	14400.0
dem_1000_belg_orig_468	1799.2k	1552.1k	15.9	14400.0	1785.7k	1785.7k	0.0	1626.8	1785.7k	1753.7k	1.8	14400.0
dem_1000_belg_orig_469	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	20.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_470	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_471	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_472	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	21.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_473	1567.3k	1459.2k	7.4	14400.0	1564.0k	1508.2k	3.7	14400.0	1602.3k	1422.6k	12.6	14400.0
dem_1000_belg_orig_474	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	1.7	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_475	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	77.8	1e+20	2712.6k	–	2.1
dem_1000_belg_orig_476	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	22.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_477	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_478	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	26.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_479	2353.3k	2186.9k	7.6	14400.0	2342.0k	2342.0k	0.0	9449.6	2342.8k	2272.1k	3.1	14400.0
dem_1000_belg_orig_480	2685.5k	2683.7k	0.1	9413.5	2685.5k	2685.5k	0.0	10206.9	2685.5k	2606.8k	3.0	14400.0
dem_1000_belg_orig_481	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_482	1946.3k	1661.1k	17.2	14400.0	1944.6k	1898.9k	2.4	14400.0	1926.2k	1854.7k	3.9	14400.0
dem_1000_belg_orig_483	2977.9k	2977.9k	0.0	13024.1	2977.9k	2954.0k	0.8	14400.0	2977.9k	2923.1k	1.9	14400.0
dem_1000_belg_orig_484	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	31.8	1e+20	1e+20	0.0	0.2
dem_1000_belg_orig_485	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_486	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	33.9	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_487	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	39.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_488	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_489	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	36.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_490	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	37.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_491	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	2.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_492	2403.0k	2229.5k	7.8	14400.0	2406.1k	2315.6k	3.9	14400.0	2410.4k	2287.6k	5.4	14400.0
dem_1000_belg_orig_493	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	30.4	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_494	1421.7k	1421.7k	0.0	435.3	1421.7k	1420.8k	0.1	1520.1	1421.7k	1421.7k	0.0	1659.3
dem_1000_belg_orig_495	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.1

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Table A.7: Comparison of discrete models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_496	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_497	2068.4k	2068.4k	0.0	10576.0	2068.7k	2048.0k	1.0	14400.0	2081.1k	1991.2k	4.5	14400.0
dem_1000_belg_orig_498	1924.4k	1747.1k	10.2	14400.0	1920.2k	1919.3k	0.0	12700.5	1955.4k	1756.2k	11.3	14400.0
dem_1000_belg_orig_499	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	36.5	1e+20	1e+20	0.0	0.1
dem_1000_belg_orig_500	2493.1k	2381.0k	4.7	14400.0	2489.9k	2479.1k	0.4	14400.0	2496.4k	2337.3k	6.8	14400.0

Table A.8: Detailed results of the split-pipe models on *Belgium* with $\mathcal{B} = 1000$ as summarized in Figure 3.13d and Table 3.4a. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_1	1801.1k	1799.3k	0.1	3388.6	1801.1k	1799.3k	0.1	2783.2
dem_1000_belg_orig_2	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_3	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_4	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_5	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_6	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_7	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_8	1711.0k	1700.0k	0.6	14400.0	1711.0k	1678.8k	1.9	14400.0
dem_1000_belg_orig_9	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_10	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_11	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_12	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_13	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_14	1621.0k	1620.5k	0.0	98.2	1620.9k	1619.3k	0.1	64.6
dem_1000_belg_orig_15	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_16	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_17	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_18	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_19	2452.2k	2450.0k	0.1	3059.9	2452.2k	2449.8k	0.1	2247.3
dem_1000_belg_orig_20	1778.5k	1776.6k	0.1	14400.0	1777.7k	1775.9k	0.1	583.4
dem_1000_belg_orig_21	1244.0k	1242.8k	0.1	40.0	1244.6k	1243.3k	0.1	53.4
dem_1000_belg_orig_22	2768.7k	2757.5k	0.4	14400.0	2768.7k	2764.3k	0.2	14400.0
dem_1000_belg_orig_23	2334.3k	2332.0k	0.1	1587.8	2334.3k	2330.9k	0.1	14400.0
dem_1000_belg_orig_24	2115.9k	2108.9k	0.3	14400.0	2114.4k	2108.9k	0.3	14400.0
dem_1000_belg_orig_25	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_26	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_27	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_28	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_29	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_30	1563.6k	1540.7k	1.5	14400.0	1563.6k	1549.0k	0.9	14400.0
dem_1000_belg_orig_31	1767.2k	1766.3k	0.1	0.9	1767.2k	1766.3k	0.1	0.9
dem_1000_belg_orig_32	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_33	1787.2k	1744.6k	2.4	14400.0	1787.6k	1742.9k	2.6	14400.0
dem_1000_belg_orig_34	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_35	1841.3k	1829.7k	0.6	14400.0	1841.3k	1828.9k	0.7	14400.0
dem_1000_belg_orig_36	1981.5k	1979.5k	0.1	892.0	1981.5k	1979.5k	0.1	128.2
dem_1000_belg_orig_37	2487.9k	2455.8k	1.3	14400.0	2487.9k	2453.1k	1.4	14400.0
dem_1000_belg_orig_38	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_39	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_40	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_41	1e+20	2053.6k	–	14400.0	1e+20	2052.9k	–	14400.0
dem_1000_belg_orig_42	2340.9k	2339.8k	0.0	1.5	2340.9k	2340.9k	0.0	0.4
dem_1000_belg_orig_43	2008.8k	1994.4k	0.7	14400.0	2008.9k	1957.3k	2.6	14400.0
dem_1000_belg_orig_44	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_45	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_46	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_47	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_48	2366.8k	2354.9k	0.5	14400.0	2366.8k	2364.4k	0.1	2003.8
dem_1000_belg_orig_49	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_50	1462.7k	1442.5k	1.4	14400.0	1462.7k	1442.7k	1.4	14400.0
dem_1000_belg_orig_51	1958.1k	1956.2k	0.1	45.1	1958.1k	1956.2k	0.1	361.6
dem_1000_belg_orig_52	2055.4k	2049.6k	0.3	14400.0	2055.5k	2053.4k	0.1	7279.5
dem_1000_belg_orig_53	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_54	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_55	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_56	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_57	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_58	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.8: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_59	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_60	2091.3k	2089.2k	0.1	13912.9	2091.3k	2074.7k	0.8	14400.0
dem_1000_belg_orig_61	1540.9k	1515.8k	1.7	14400.0	1541.2k	1488.5k	3.5	14400.0
dem_1000_belg_orig_62	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_63	2640.5k	2613.2k	1.0	14400.0	2640.5k	2614.3k	1.0	14400.0
dem_1000_belg_orig_64	2481.7k	2479.2k	0.1	12415.8	2481.7k	2479.2k	0.1	379.5
dem_1000_belg_orig_65	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_66	957.9k	955.8k	0.2	14400.0	957.9k	957.0k	0.1	2011.2
dem_1000_belg_orig_67	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_68	1835.1k	1825.4k	0.5	14400.0	1835.1k	1825.4k	0.5	14400.0
dem_1000_belg_orig_69	2391.3k	2384.8k	0.3	14400.0	2391.3k	2387.6k	0.2	14400.0
dem_1000_belg_orig_70	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_71	1999.5k	1985.3k	0.7	14400.0	1999.5k	1982.6k	0.9	14400.0
dem_1000_belg_orig_72	2001.5k	1989.2k	0.6	14400.0	2001.5k	1924.1k	4.0	14400.0
dem_1000_belg_orig_73	1929.5k	1926.1k	0.2	14400.0	1929.5k	1927.5k	0.1	13869.7
dem_1000_belg_orig_74	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_75	2273.8k	2271.5k	0.1	4758.6	2273.8k	2271.5k	0.1	9282.4
dem_1000_belg_orig_76	1555.4k	1549.7k	0.4	14400.0	1554.4k	1548.3k	0.4	14400.0
dem_1000_belg_orig_77	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_78	2058.4k	2028.0k	1.5	14400.0	2058.4k	2022.5k	1.8	14400.0
dem_1000_belg_orig_79	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_80	1821.9k	1733.5k	5.1	14400.0	1821.2k	1787.8k	1.9	14400.0
dem_1000_belg_orig_81	1492.0k	1489.0k	0.2	14400.0	1492.0k	1478.9k	0.9	14400.0
dem_1000_belg_orig_82	2415.8k	2405.5k	0.4	14400.0	2415.8k	2408.2k	0.3	14400.0
dem_1000_belg_orig_83	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_84	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_85	1856.8k	1846.6k	0.5	14400.0	1856.8k	1845.2k	0.6	14400.0
dem_1000_belg_orig_86	2345.1k	2342.9k	0.1	3.6	2345.1k	2342.8k	0.1	1.8
dem_1000_belg_orig_87	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_88	1711.9k	1659.5k	3.2	14400.0	1711.9k	1709.6k	0.1	14400.0
dem_1000_belg_orig_89	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_90	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_91	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_92	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_93	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_94	2521.0k	2518.5k	0.1	3562.2	2521.0k	2503.5k	0.7	14400.0
dem_1000_belg_orig_95	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_96	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_97	2086.8k	2078.1k	0.4	14400.0	2086.8k	2078.1k	0.4	14400.0
dem_1000_belg_orig_98	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_99	2044.4k	2032.9k	0.6	14400.0	2044.4k	2042.4k	0.1	6633.1
dem_1000_belg_orig_100	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_101	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_102	1800.8k	1793.1k	0.4	14400.0	1800.8k	1799.0k	0.1	6443.5
dem_1000_belg_orig_103	1969.7k	1967.8k	0.1	17.0	1969.7k	1967.8k	0.1	21.8
dem_1000_belg_orig_104	1e+20	1e+20	0.0	1.4	1e+20	1e+20	0.0	1.2
dem_1000_belg_orig_105	1106.7k	1089.1k	1.6	14400.0	1105.7k	1084.3k	2.0	14400.0
dem_1000_belg_orig_106	2446.8k	2429.9k	0.7	14400.0	2446.8k	2433.6k	0.5	14400.0
dem_1000_belg_orig_107	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_108	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_109	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_110	1639.1k	1637.5k	0.1	292.5	1639.1k	1637.5k	0.1	258.3
dem_1000_belg_orig_111	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_112	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_113	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_114	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_115	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_116	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_117	2317.6k	2316.0k	0.1	24.9	2316.8k	2314.6k	0.1	5.6
dem_1000_belg_orig_118	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_119	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_120	2391.5k	2389.9k	0.1	10.8	2392.2k	2390.4k	0.1	7.0
dem_1000_belg_orig_121	2327.3k	2324.9k	0.1	12466.5	2327.3k	2325.0k	0.1	9010.2
dem_1000_belg_orig_122	2466.4k	2432.1k	1.4	14400.0	2466.4k	2452.7k	0.6	14400.0
dem_1000_belg_orig_123	2501.3k	2487.4k	0.6	14400.0	2501.3k	2496.1k	0.2	14400.0
dem_1000_belg_orig_124	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_125	1963.2k	1961.2k	0.1	122.9	1963.2k	1961.2k	0.1	130.1
dem_1000_belg_orig_126	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_127	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_128	1936.3k	1934.4k	0.1	234.7	1936.3k	1934.4k	0.1	326.0
dem_1000_belg_orig_129	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_130	1674.9k	1673.3k	0.1	1.4	1674.9k	1673.3k	0.1	2.1
dem_1000_belg_orig_131	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_132	1793.2k	1789.5k	0.2	14400.0	1793.2k	1791.4k	0.1	2315.5
dem_1000_belg_orig_133	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_134	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_135	1549.2k	1544.0k	0.3	14400.0	1549.2k	1547.7k	0.1	13447.0
dem_1000_belg_orig_136	2926.1k	2923.2k	0.1	509.6	2926.1k	2923.2k	0.1	449.8

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Table A.8: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_137	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_138	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_139	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_140	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_141	2367.2k	2327.7k	1.7	14400.0	2367.3k	2364.9k	0.1	14372.7
dem_1000_belg_orig_142	1103.5k	1102.5k	0.1	3.5	1103.4k	1102.4k	0.1	3.9
dem_1000_belg_orig_143	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_144	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_145	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_146	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_147	1769.7k	1768.0k	0.1	1961.7	1769.7k	1768.0k	0.1	890.5
dem_1000_belg_orig_148	1776.3k	1775.2k	0.1	342.0	1777.1k	1775.3k	0.1	76.3
dem_1000_belg_orig_149	2143.6k	2142.5k	0.1	18.3	2144.5k	2142.4k	0.1	23.2
dem_1000_belg_orig_150	1726.1k	1712.3k	0.8	14400.0	1726.1k	1712.0k	0.8	14400.0
dem_1000_belg_orig_151	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_152	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_153	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_154	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_155	1969.7k	1954.1k	0.8	14400.0	1969.7k	1965.4k	0.2	14400.0
dem_1000_belg_orig_156	1461.0k	1459.5k	0.1	583.2	1461.0k	1459.5k	0.1	1107.2
dem_1000_belg_orig_157	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_158	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_159	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_160	2082.5k	2080.4k	0.1	387.5	2082.5k	2080.1k	0.1	14400.0
dem_1000_belg_orig_161	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_162	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_163	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_164	1096.7k	1078.2k	1.7	14400.0	1095.7k	1082.3k	1.2	14400.0
dem_1000_belg_orig_165	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_166	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_167	1737.9k	1736.2k	0.1	23.9	1737.9k	1736.2k	0.1	18.4
dem_1000_belg_orig_168	1372.6k	1363.0k	0.7	14400.0	1372.6k	1367.3k	0.4	14400.0
dem_1000_belg_orig_169	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_170	1333.4k	1332.9k	0.0	77.1	1333.4k	1333.4k	0.0	4.1
dem_1000_belg_orig_171	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_172	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_173	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_174	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_175	1677.1k	1676.6k	0.0	48.2	1677.1k	1677.1k	0.0	197.4
dem_1000_belg_orig_176	1238.0k	1236.5k	0.1	14400.0	1238.0k	1236.8k	0.1	6209.8
dem_1000_belg_orig_177	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_178	1328.6k	1327.4k	0.1	1.8	1328.6k	1327.4k	0.1	2.3
dem_1000_belg_orig_179	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_180	2232.8k	2227.2k	0.3	14400.0	2232.8k	2228.0k	0.2	14400.0
dem_1000_belg_orig_181	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_182	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_183	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_184	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_185	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_186	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_187	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_188	1673.0k	1671.3k	0.1	2358.5	1673.0k	1663.8k	0.6	14400.0
dem_1000_belg_orig_189	1432.5k	1422.9k	0.7	14400.0	1432.5k	1362.3k	5.2	14400.0
dem_1000_belg_orig_190	2786.4k	2778.8k	0.3	14400.0	2786.4k	2783.6k	0.1	10006.2
dem_1000_belg_orig_191	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_192	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_193	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_194	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_195	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_196	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_197	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_198	1938.2k	1909.4k	1.5	14400.0	1938.2k	1905.4k	1.7	14400.0
dem_1000_belg_orig_199	2449.2k	2446.8k	0.1	40.2	2450.1k	2448.1k	0.1	1525.9
dem_1000_belg_orig_200	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_201	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_202	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_203	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_204	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_205	1751.2k	1749.4k	0.1	7271.4	1751.2k	1661.5k	5.4	14400.0
dem_1000_belg_orig_206	1669.8k	1668.1k	0.1	2592.8	1669.8k	1663.3k	0.4	14400.0
dem_1000_belg_orig_207	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_208	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_209	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_210	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_211	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_212	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_213	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_214	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.8: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_215	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_216	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_217	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_218	2339.6k	2337.2k	0.1	15.2	2339.6k	2337.3k	0.1	509.0
dem_1000_belg_orig_219	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_220	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_221	2033.7k	2012.0k	1.1	14400.0	2033.7k	2022.3k	0.6	14400.0
dem_1000_belg_orig_222	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_223	1126.6k	1115.3k	1.0	14400.0	1126.6k	1115.5k	1.0	14400.0
dem_1000_belg_orig_224	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_225	1795.4k	1785.0k	0.6	14400.0	1795.4k	1791.1k	0.2	14400.0
dem_1000_belg_orig_226	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_227	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_228	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_229	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_230	1964.6k	1914.2k	2.6	14400.0	1963.6k	1952.1k	0.6	14400.0
dem_1000_belg_orig_231	1863.3k	1861.5k	0.1	144.6	1863.3k	1861.5k	0.1	59.4
dem_1000_belg_orig_232	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_233	1860.1k	1858.3k	0.1	52.5	1860.1k	1858.3k	0.1	23.1
dem_1000_belg_orig_234	2039.4k	2034.2k	0.3	14400.0	2039.4k	2034.3k	0.3	14400.0
dem_1000_belg_orig_235	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_236	2097.7k	2058.0k	1.9	14400.0	2098.1k	2054.5k	2.1	14400.0
dem_1000_belg_orig_237	2695.8k	2693.1k	0.1	1753.6	2695.8k	2693.1k	0.1	2741.7
dem_1000_belg_orig_238	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_239	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_240	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_241	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_242	2132.9k	2119.7k	0.6	14400.0	2132.9k	2130.8k	0.1	8031.5
dem_1000_belg_orig_243	2470.5k	2464.5k	0.2	14400.0	2470.5k	2465.4k	0.2	14400.0
dem_1000_belg_orig_244	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_245	1780.7k	1753.5k	1.5	14400.0	1780.6k	1778.0k	0.1	14400.0
dem_1000_belg_orig_246	2869.9k	2856.5k	0.5	14400.0	2869.9k	2864.7k	0.2	14400.0
dem_1000_belg_orig_247	1255.3k	1253.7k	0.1	14400.0	1255.3k	1205.9k	4.1	14400.0
dem_1000_belg_orig_248	2584.0k	2540.7k	1.7	14400.0	2584.0k	2574.7k	0.4	14400.0
dem_1000_belg_orig_249	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_250	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_251	1620.2k	1613.5k	0.4	14400.0	1620.2k	1604.0k	1.0	14400.0
dem_1000_belg_orig_252	2031.1k	2023.0k	0.4	14400.0	2031.1k	2023.4k	0.4	14400.0
dem_1000_belg_orig_253	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_254	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_255	2726.3k	2716.6k	0.4	14400.0	2726.3k	2714.1k	0.5	14400.0
dem_1000_belg_orig_256	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_257	1646.9k	1639.3k	0.5	14400.0	1646.9k	1645.2k	0.1	2867.0
dem_1000_belg_orig_258	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_259	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_260	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_261	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_262	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_263	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_264	2158.8k	2133.1k	1.2	14400.0	2159.0k	2140.7k	0.9	14400.0
dem_1000_belg_orig_265	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_266	2176.7k	2174.5k	0.1	10.2	2176.7k	2174.5k	0.1	5.0
dem_1000_belg_orig_267	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_268	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_269	2185.2k	2179.5k	0.3	14400.0	2185.2k	2180.2k	0.2	14400.0
dem_1000_belg_orig_270	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_271	1802.8k	1801.0k	0.1	63.8	1802.8k	1801.0k	0.1	239.1
dem_1000_belg_orig_272	2535.5k	2532.9k	0.1	14327.5	2535.5k	2532.9k	0.1	9223.5
dem_1000_belg_orig_273	1807.5k	1802.3k	0.3	14400.0	1807.5k	1797.0k	0.6	14400.0
dem_1000_belg_orig_274	2399.7k	2397.3k	0.1	1630.4	2399.7k	2397.3k	0.1	861.9
dem_1000_belg_orig_275	1863.1k	1861.3k	0.1	48.0	1863.1k	1861.3k	0.1	53.0
dem_1000_belg_orig_276	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_277	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_278	2326.4k	2322.3k	0.2	14400.0	2326.4k	2314.8k	0.5	14400.0
dem_1000_belg_orig_279	1652.6k	1613.1k	2.5	14400.0	1652.6k	1613.1k	2.4	14400.0
dem_1000_belg_orig_280	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_281	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_282	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_283	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_284	2649.7k	2647.0k	0.1	4841.4	2649.7k	2644.8k	0.2	14400.0
dem_1000_belg_orig_285	2060.7k	2050.6k	0.5	14400.0	2060.7k	2058.6k	0.1	3024.3
dem_1000_belg_orig_286	2349.6k	2320.3k	1.3	14400.0	2349.6k	2327.3k	1.0	14400.0
dem_1000_belg_orig_287	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_288	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_289	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_290	2506.9k	2504.4k	0.1	6427.1	2506.9k	2504.4k	0.1	5207.5
dem_1000_belg_orig_291	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_292	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.8: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_293	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_294	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_295	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_296	1887.3k	1885.9k	0.1	1.7	1887.8k	1886.1k	0.1	0.7
dem_1000_belg_orig_297	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_298	2478.5k	2476.1k	0.1	11011.0	2478.5k	2476.1k	0.1	722.9
dem_1000_belg_orig_299	1517.2k	1515.7k	0.1	20.6	1517.2k	1515.7k	0.1	121.5
dem_1000_belg_orig_300	1481.0k	1479.6k	0.1	0.9	1480.9k	1479.5k	0.1	1.5
dem_1000_belg_orig_301	2427.6k	2417.3k	0.4	14400.0	2427.6k	2421.9k	0.2	14400.0
dem_1000_belg_orig_302	2313.7k	2311.4k	0.1	5002.3	2313.7k	2307.0k	0.3	14400.0
dem_1000_belg_orig_303	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_304	2130.7k	2072.0k	2.8	14400.0	2130.7k	2128.6k	0.1	3269.6
dem_1000_belg_orig_305	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_306	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_307	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_308	2583.7k	2570.1k	0.5	14400.0	2583.6k	2581.0k	0.1	78.1
dem_1000_belg_orig_309	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_310	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_311	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_312	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_313	1867.0k	1865.1k	0.1	6083.5	1867.0k	1865.1k	0.1	14033.8
dem_1000_belg_orig_314	1151.5k	1145.9k	0.5	14400.0	1151.5k	1141.0k	0.9	14400.0
dem_1000_belg_orig_315	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_316	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_317	1943.5k	1890.5k	2.8	14400.0	1943.5k	1904.1k	2.1	14400.0
dem_1000_belg_orig_318	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_319	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_320	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_321	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_322	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_323	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_324	2351.7k	2349.3k	0.1	14400.0	2351.7k	2344.2k	0.3	14400.0
dem_1000_belg_orig_325	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_326	2418.6k	2412.1k	0.3	14400.0	2418.6k	2416.2k	0.1	3583.2
dem_1000_belg_orig_327	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_328	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_329	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_330	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_331	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_332	1918.2k	1912.9k	0.3	14400.0	1918.2k	1874.7k	2.3	14400.0
dem_1000_belg_orig_333	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_334	2315.5k	2284.3k	1.4	14400.0	2313.7k	2301.0k	0.6	14400.0
dem_1000_belg_orig_335	1548.1k	1537.8k	0.7	14400.0	1548.1k	1537.5k	0.7	14400.0
dem_1000_belg_orig_336	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_337	2039.1k	2030.2k	0.4	14400.0	2039.1k	2037.1k	0.1	2259.3
dem_1000_belg_orig_338	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_339	1020.4k	1019.4k	0.1	5524.1	1020.4k	1019.4k	0.1	1873.8
dem_1000_belg_orig_340	1768.4k	1766.6k	0.1	8456.7	1768.4k	1763.6k	0.3	14400.0
dem_1000_belg_orig_341	2603.7k	2601.1k	0.1	4291.4	2603.7k	2601.1k	0.1	151.4
dem_1000_belg_orig_342	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_343	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_344	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_345	2564.0k	2534.6k	1.2	14400.0	2564.0k	2534.5k	1.2	14400.0
dem_1000_belg_orig_346	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_347	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_348	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_349	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_350	2245.5k	2243.3k	0.1	8286.6	2245.5k	2239.9k	0.2	14400.0
dem_1000_belg_orig_351	2303.5k	2295.1k	0.4	14400.0	2303.5k	2301.2k	0.1	721.8
dem_1000_belg_orig_352	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_353	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_354	2173.3k	2171.1k	0.1	12240.3	2173.0k	2156.4k	0.8	14400.0
dem_1000_belg_orig_355	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_356	2516.9k	2514.3k	0.1	1181.1	2516.9k	2505.0k	0.5	14400.0
dem_1000_belg_orig_357	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_358	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_359	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_360	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_361	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_362	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_363	1855.1k	1852.4k	0.1	14400.0	1855.1k	1850.5k	0.3	14400.0
dem_1000_belg_orig_364	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_365	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_366	2026.7k	2021.5k	0.3	14400.0	2026.7k	1997.7k	1.5	14400.0
dem_1000_belg_orig_367	1484.7k	1473.4k	0.8	14400.0	1484.7k	1470.9k	0.9	14400.0
dem_1000_belg_orig_368	2171.3k	2169.1k	0.1	29.1	2171.3k	2169.1k	0.1	34.1
dem_1000_belg_orig_369	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_370	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.8: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_371	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_372	1062.0k	1044.4k	1.7	14400.0	1062.0k	1048.2k	1.3	14400.0
dem_1000_belg_orig_373	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_374	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_375	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_376	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_377	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_378	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_379	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_380	1736.2k	1724.6k	0.7	14400.0	1736.2k	1724.0k	0.7	14400.0
dem_1000_belg_orig_381	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_382	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_383	821.8k	808.0k	1.7	14400.0	821.8k	808.6k	1.6	14400.0
dem_1000_belg_orig_384	2233.4k	2224.8k	0.4	14400.0	2233.5k	2216.7k	0.8	14400.0
dem_1000_belg_orig_385	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_386	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_387	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_388	1901.3k	1877.8k	1.3	14400.0	1901.3k	1871.7k	1.6	14400.0
dem_1000_belg_orig_389	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_390	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_391	2675.0k	2672.3k	0.1	6947.5	2675.0k	2672.3k	0.1	4874.4
dem_1000_belg_orig_392	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_393	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_394	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_395	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_396	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_397	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_398	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_399	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_400	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_401	1705.0k	1703.3k	0.1	9140.3	1705.0k	1702.1k	0.2	14400.0
dem_1000_belg_orig_402	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_403	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_404	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_405	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_406	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_407	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_408	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_409	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_410	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_411	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_412	2361.4k	2359.1k	0.1	101.6	2361.4k	2353.4k	0.3	14400.0
dem_1000_belg_orig_413	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_414	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_415	897.8k	884.6k	1.5	14400.0	897.7k	893.5k	0.5	14400.0
dem_1000_belg_orig_416	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_417	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_418	1743.1k	1743.1k	0.0	2.2	1743.1k	1742.6k	0.0	0.5
dem_1000_belg_orig_419	1277.0k	1275.8k	0.1	1673.8	1277.0k	1275.8k	0.1	1563.2
dem_1000_belg_orig_420	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_421	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_422	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_423	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_424	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_425	2157.0k	2154.9k	0.1	15.3	2157.0k	2154.9k	0.1	39.3
dem_1000_belg_orig_426	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_427	1872.8k	1870.9k	0.1	1416.0	1872.8k	1870.9k	0.1	6096.4
dem_1000_belg_orig_428	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_429	2284.4k	2282.1k	0.1	5334.7	2284.5k	2272.7k	0.5	14400.0
dem_1000_belg_orig_430	1865.4k	1838.0k	1.5	14400.0	1865.4k	1827.5k	2.1	14400.0
dem_1000_belg_orig_431	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_432	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_433	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_434	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_435	1854.1k	1852.3k	0.1	28.8	1854.2k	1837.0k	0.9	14400.0
dem_1000_belg_orig_436	1803.3k	1747.3k	3.2	14400.0	1803.3k	1749.7k	3.1	14400.0
dem_1000_belg_orig_437	1866.3k	1841.4k	1.3	14400.0	1866.3k	1848.0k	1.0	14400.0
dem_1000_belg_orig_438	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_439	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_440	2584.3k	2582.7k	0.1	1.3	2584.6k	2583.1k	0.1	0.9
dem_1000_belg_orig_441	1492.2k	1490.8k	0.1	45.7	1492.2k	1490.9k	0.1	40.2
dem_1000_belg_orig_442	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_443	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_444	1841.6k	1839.8k	0.1	3385.8	1841.6k	1839.8k	0.1	3983.5
dem_1000_belg_orig_445	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_446	1953.9k	1950.1k	0.2	14400.0	1953.9k	1951.3k	0.1	14400.0
dem_1000_belg_orig_447	2237.1k	2228.4k	0.4	14400.0	2237.1k	2228.2k	0.4	14400.0
dem_1000_belg_orig_448	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.8: Comparison of split-pipe models on *Belgium* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_belg_orig_449	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_450	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_451	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_452	2418.9k	2411.0k	0.3	14400.0	2418.9k	2409.8k	0.4	14400.0
dem_1000_belg_orig_453	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_454	1325.3k	1295.1k	2.3	14400.0	1325.3k	1311.1k	1.1	14400.0
dem_1000_belg_orig_455	1707.0k	1700.5k	0.4	14400.0	1707.0k	1700.5k	0.4	14400.0
dem_1000_belg_orig_456	1576.5k	1537.2k	2.6	14400.0	1576.5k	1574.9k	0.1	1081.5
dem_1000_belg_orig_457	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_458	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_459	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_460	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_461	1783.8k	1765.1k	1.1	14400.0	1783.8k	1764.6k	1.1	14400.0
dem_1000_belg_orig_462	2192.9k	2187.7k	0.2	14400.0	2192.9k	2185.8k	0.3	14400.0
dem_1000_belg_orig_463	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_464	2518.1k	2515.6k	0.1	425.3	2518.1k	2515.7k	0.1	12438.6
dem_1000_belg_orig_465	1957.1k	1954.1k	0.2	14400.0	1957.1k	1955.2k	0.1	1887.5
dem_1000_belg_orig_466	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_467	2914.0k	2900.0k	0.5	14400.0	2914.0k	2900.0k	0.5	14400.0
dem_1000_belg_orig_468	1777.9k	1776.3k	0.1	85.0	1777.9k	1776.1k	0.1	41.9
dem_1000_belg_orig_469	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_470	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_471	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_472	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_473	1518.1k	1504.4k	0.9	14400.0	1518.1k	1503.5k	1.0	14400.0
dem_1000_belg_orig_474	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_475	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_476	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_477	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_478	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_479	2324.5k	2298.3k	1.1	14400.0	2324.5k	2300.1k	1.1	14400.0
dem_1000_belg_orig_480	2653.9k	2644.4k	0.4	14400.0	2653.9k	2641.7k	0.5	14400.0
dem_1000_belg_orig_481	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_482	1861.2k	1857.8k	0.2	14400.0	1861.2k	1855.8k	0.3	14400.0
dem_1000_belg_orig_483	2923.5k	2920.6k	0.1	298.7	2923.5k	2920.6k	0.1	273.9
dem_1000_belg_orig_484	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_485	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_486	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_487	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_488	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_489	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_490	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_491	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_492	2370.9k	2360.3k	0.5	14400.0	2371.0k	2356.2k	0.6	14400.0
dem_1000_belg_orig_493	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_494	1398.8k	1348.7k	3.7	14400.0	1398.5k	1389.0k	0.7	14400.0
dem_1000_belg_orig_495	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_496	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_497	2047.1k	2012.7k	1.7	14400.0	2047.1k	2043.4k	0.2	14400.0
dem_1000_belg_orig_498	1881.5k	1879.6k	0.1	6986.2	1881.5k	1879.6k	0.1	4862.6
dem_1000_belg_orig_499	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_belg_orig_500	2482.8k	2472.6k	0.4	14400.0	2482.8k	2472.3k	0.4	14400.0

Table A.9: Detailed results of the discrete models on *GasLib-40* with $\mathcal{B} = 50$ as summarized in Figure 3.8a and Table 3.3b. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_1	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_2	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_3	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_4	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.4	0.0	0.0	0.0	1.6
dem_50_gaslib40_5	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.6
dem_50_gaslib40_6	8218.7	8218.7	0.0	31.8	8218.7	8218.7	0.0	154.9	8218.7	8218.7	0.0	75.8
dem_50_gaslib40_7	19.2k	19.2k	0.0	14.2	19.2k	19.2k	0.0	379.1	19.2k	19.2k	0.0	100.0
dem_50_gaslib40_8	26.8k	26.8k	0.0	8.1	26.8k	26.8k	0.0	622.1	26.8k	26.8k	0.0	56.9
dem_50_gaslib40_9	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_10	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_11	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4

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Table A.9: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_12	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_13	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_14	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_15	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_16	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_17	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_18	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_19	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_20	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_21	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_22	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_23	19.2k	19.2k	0.0	41.2	19.2k	19.2k	0.0	119.2	19.2k	19.2k	0.0	122.1
dem_50_gaslib40_24	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_25	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_26	36.9k	36.9k	0.0	71.5	36.9k	36.9k	0.0	579.7	36.9k	36.9k	0.0	2252.1
dem_50_gaslib40_27	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.3
dem_50_gaslib40_28	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_29	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_30	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_31	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_32	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.6
dem_50_gaslib40_33	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_34	36.9k	36.9k	0.0	15.4	36.9k	36.9k	0.0	531.8	36.9k	36.9k	0.0	365.1
dem_50_gaslib40_35	22.8k	22.8k	0.0	23.1	22.8k	22.8k	0.0	146.7	22.8k	22.8k	0.0	136.8
dem_50_gaslib40_36	16.8k	16.8k	0.0	38.6	16.8k	16.8k	0.0	69.5	16.8k	16.8k	0.0	62.1
dem_50_gaslib40_37	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_38	23.2k	23.2k	0.0	8.0	23.2k	23.2k	0.0	119.3	23.2k	23.2k	0.0	86.1
dem_50_gaslib40_39	19.2k	19.2k	0.0	10.8	19.2k	19.2k	0.0	127.1	19.2k	19.2k	0.0	37.1
dem_50_gaslib40_40	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_41	31.7k	31.7k	0.0	16.0	31.7k	31.7k	0.0	185.4	31.7k	31.7k	0.0	482.5
dem_50_gaslib40_42	23.2k	23.2k	0.0	12.7	23.2k	23.2k	0.0	591.1	23.2k	23.2k	0.0	74.7
dem_50_gaslib40_43	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_44	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_45	8339.0	8339.0	0.0	23.2	8339.0	8339.0	0.0	131.1	8339.0	8339.0	0.0	62.6
dem_50_gaslib40_46	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_47	23.2k	23.2k	0.0	17.5	23.2k	23.2k	0.0	111.5	23.2k	23.2k	0.0	263.9
dem_50_gaslib40_48	23.2k	23.2k	0.0	12.0	23.2k	23.2k	0.0	143.2	23.2k	23.2k	0.0	101.8
dem_50_gaslib40_49	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_50	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_51	23.2k	23.2k	0.0	13.1	23.2k	23.2k	0.0	214.1	23.2k	23.2k	0.0	140.5
dem_50_gaslib40_52	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_53	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_54	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_55	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_56	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_57	36.9k	36.9k	0.0	14.0	36.9k	36.9k	0.0	176.6	36.9k	36.9k	0.0	187.1
dem_50_gaslib40_58	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_59	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_60	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_61	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_62	19.2k	19.2k	0.0	13.7	19.2k	19.2k	0.0	145.4	19.2k	19.2k	0.0	36.0
dem_50_gaslib40_63	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.5
dem_50_gaslib40_64	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_65	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_66	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_67	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_68	23.2k	23.2k	0.0	12.4	23.2k	23.2k	0.0	459.6	23.2k	23.2k	0.0	30.0
dem_50_gaslib40_69	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_70	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.3
dem_50_gaslib40_71	6955.8	6955.8	0.0	14.4	6955.8	6955.8	0.0	65.3	6955.8	6955.8	0.0	16.9
dem_50_gaslib40_72	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_73	31.7k	31.7k	0.0	21.4	31.7k	31.7k	0.0	250.2	31.7k	31.7k	0.0	334.6
dem_50_gaslib40_74	23.2k	23.2k	0.0	16.7	23.2k	23.2k	0.0	408.5	23.2k	23.2k	0.0	220.9
dem_50_gaslib40_75	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_76	23.2k	23.2k	0.0	44.3	23.2k	23.2k	0.0	314.4	23.2k	23.2k	0.0	300.6
dem_50_gaslib40_77	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_78	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.8
dem_50_gaslib40_79	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.2
dem_50_gaslib40_80	23.2k	23.2k	0.0	13.0	23.2k	23.2k	0.0	241.2	23.2k	23.2k	0.0	103.0
dem_50_gaslib40_81	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_82	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_83	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_84	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_85	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_86	23.2k	23.2k	0.0	12.0	23.2k	23.2k	0.0	159.0	23.2k	23.2k	0.0	85.0
dem_50_gaslib40_87	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.3
dem_50_gaslib40_88	0.0	0.0	0.0	0.3	0.0	0.0	0.0	81.0	0.0	0.0	0.0	0.3
dem_50_gaslib40_89	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.3

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Table A.9: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_90	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.3	0.0	0.0	0.0	0.4
dem_50_gaslib40_91	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_92	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_93	26.8k	26.8k	0.0	12.3	26.8k	26.8k	0.0	450.7	26.8k	26.8k	0.0	346.9
dem_50_gaslib40_94	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_95	9745.9	9745.9	0.0	15.6	9745.9	9745.9	0.0	144.3	9745.9	9745.9	0.0	59.6
dem_50_gaslib40_96	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_97	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_98	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_99	19.2k	19.2k	0.0	14.4	19.2k	19.2k	0.0	156.1	19.2k	19.2k	0.0	89.0
dem_50_gaslib40_100	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_101	19.2k	19.2k	0.0	23.6	19.2k	19.2k	0.0	334.5	19.2k	19.2k	0.0	112.2
dem_50_gaslib40_102	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_103	0.0	0.0	0.0	0.3	0.0	0.0	0.0	17.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_104	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_105	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_106	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_107	23.2k	23.2k	0.0	13.3	23.2k	23.2k	0.0	296.2	23.2k	23.2k	0.0	145.6
dem_50_gaslib40_108	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_109	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.5	0.0	0.0	0.0	0.5
dem_50_gaslib40_110	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_111	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_112	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_113	23.2k	23.2k	0.0	7.5	23.2k	23.2k	0.0	274.3	23.2k	23.2k	0.0	113.4
dem_50_gaslib40_114	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_115	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_116	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_117	0.0	0.0	0.0	0.2	0.0	0.0	0.0	126.3	0.0	0.0	0.0	0.4
dem_50_gaslib40_118	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_119	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_120	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.2
dem_50_gaslib40_121	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_122	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_123	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_124	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_125	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_126	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.8
dem_50_gaslib40_127	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.3
dem_50_gaslib40_128	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_129	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_130	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_131	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.6
dem_50_gaslib40_132	23.2k	23.2k	0.0	6.4	23.2k	23.2k	0.0	95.9	23.2k	23.2k	0.0	78.3
dem_50_gaslib40_133	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_134	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_135	26.8k	26.8k	0.0	12.0	26.8k	26.8k	0.0	320.9	26.8k	26.8k	0.0	77.7
dem_50_gaslib40_136	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_137	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_138	23.2k	23.2k	0.0	2.4	23.2k	23.2k	0.0	199.5	23.2k	23.2k	0.0	117.0
dem_50_gaslib40_139	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_140	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_141	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.6
dem_50_gaslib40_142	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_143	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_144	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_145	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.6
dem_50_gaslib40_146	23.2k	23.2k	0.0	55.0	23.2k	23.2k	0.0	393.2	23.2k	23.2k	0.0	312.0
dem_50_gaslib40_147	23.2k	23.2k	0.0	2.8	23.2k	23.2k	0.0	112.7	23.2k	23.2k	0.0	73.1
dem_50_gaslib40_148	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_149	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_150	23.2k	23.2k	0.0	7.8	23.2k	23.2k	0.0	443.4	23.2k	23.2k	0.0	86.9
dem_50_gaslib40_151	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_152	19.2k	19.2k	0.0	17.2	19.2k	19.2k	0.0	166.0	19.2k	19.2k	0.0	171.8
dem_50_gaslib40_153	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_154	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_155	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_156	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_157	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_158	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_159	19.2k	19.2k	0.0	45.2	19.2k	19.2k	0.0	185.7	19.2k	19.2k	0.0	52.5
dem_50_gaslib40_160	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_161	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_162	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_163	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	5.6
dem_50_gaslib40_164	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_165	0.0	0.0	0.0	0.3	0.0	0.0	0.0	18.5	0.0	0.0	0.0	0.3
dem_50_gaslib40_166	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_167	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5

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Table A.9: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_168	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_169	98.4k	98.4k	0.0	588.4	98.4k	98.4k	0.0	2061.8	111.2k	44.3k	151.1	14400.0
dem_50_gaslib40_170	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_171	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_172	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_173	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_174	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.3
dem_50_gaslib40_175	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_176	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_177	19.2k	19.2k	0.0	90.6	19.2k	19.2k	0.0	149.1	19.2k	19.2k	0.0	138.3
dem_50_gaslib40_178	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_179	17.9k	17.9k	0.0	14.3	17.9k	17.9k	0.0	71.1	17.9k	17.9k	0.0	32.6
dem_50_gaslib40_180	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_181	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_182	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.7
dem_50_gaslib40_183	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_184	26.8k	26.8k	0.0	13.0	26.8k	26.8k	0.0	118.1	26.8k	26.8k	0.0	892.4
dem_50_gaslib40_185	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_186	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_187	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.2
dem_50_gaslib40_188	31.7k	31.7k	0.0	45.4	31.7k	31.7k	0.0	549.9	31.7k	31.7k	0.0	540.6
dem_50_gaslib40_189	13.4k	13.4k	0.0	13.7	13.4k	13.4k	0.0	106.1	13.4k	13.4k	0.0	89.5
dem_50_gaslib40_190	22.8k	22.8k	0.0	47.0	22.8k	22.8k	0.0	184.4	22.8k	22.8k	0.0	124.6
dem_50_gaslib40_191	23.2k	23.2k	0.0	12.6	23.2k	23.2k	0.0	202.8	23.2k	23.2k	0.0	218.6
dem_50_gaslib40_192	19.2k	19.2k	0.0	16.5	19.2k	19.2k	0.0	162.1	19.2k	19.2k	0.0	121.2
dem_50_gaslib40_193	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_194	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.2
dem_50_gaslib40_195	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_196	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_197	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_198	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_199	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.2
dem_50_gaslib40_200	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.2
dem_50_gaslib40_201	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_202	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_203	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_204	23.2k	23.2k	0.0	14.4	23.2k	23.2k	0.0	162.2	23.2k	23.2k	0.0	98.7
dem_50_gaslib40_205	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_206	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_207	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_208	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_209	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_210	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_211	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_212	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_213	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_214	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_215	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_216	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_217	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_218	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_219	23.2k	23.2k	0.0	40.4	23.2k	23.2k	0.0	290.2	23.2k	23.2k	0.0	309.3
dem_50_gaslib40_220	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_221	23.2k	23.2k	0.0	12.7	23.2k	23.2k	0.0	203.8	23.2k	23.2k	0.0	122.1
dem_50_gaslib40_222	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_223	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.2
dem_50_gaslib40_224	19.2k	19.2k	0.0	17.9	19.2k	19.2k	0.0	133.2	19.2k	19.2k	0.0	221.6
dem_50_gaslib40_225	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.2
dem_50_gaslib40_226	19.2k	19.2k	0.0	6.8	19.2k	19.2k	0.0	247.3	19.2k	19.2k	0.0	44.7
dem_50_gaslib40_227	71.6k	71.6k	0.0	294.7	71.6k	71.6k	0.0	870.9	71.6k	71.6k	0.0	7518.3
dem_50_gaslib40_228	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_229	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_230	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_231	23.2k	23.2k	0.0	20.9	23.2k	23.2k	0.0	139.1	23.2k	23.2k	0.0	108.0
dem_50_gaslib40_232	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_233	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_234	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_235	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_236	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.3
dem_50_gaslib40_237	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_238	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_239	26.8k	26.8k	0.0	38.7	26.8k	26.8k	0.0	485.8	26.8k	26.8k	0.0	226.3
dem_50_gaslib40_240	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_241	1840.5	1840.5	0.0	15.9	1840.5	1840.5	0.0	33.7	1840.5	1840.5	0.0	12.1
dem_50_gaslib40_242	23.2k	23.2k	0.0	12.9	23.2k	23.2k	0.0	183.1	23.2k	23.2k	0.0	40.4
dem_50_gaslib40_243	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_244	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_245	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4

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Table A.9: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_246	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_247	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_248	19.2k	19.2k	0.0	19.8	19.2k	19.2k	0.0	276.1	19.2k	19.2k	0.0	159.4
dem_50_gaslib40_249	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_250	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_251	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.3	0.0	0.0	0.0	0.3
dem_50_gaslib40_252	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_253	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_254	8218.7	8218.7	0.0	8.0	8218.7	8218.7	0.0	87.3	8218.7	8218.7	0.0	20.6
dem_50_gaslib40_255	31.7k	31.7k	0.0	7.8	31.7k	31.7k	0.0	199.4	31.7k	31.7k	0.0	171.7
dem_50_gaslib40_256	23.2k	23.2k	0.0	17.1	23.2k	23.2k	0.0	215.5	23.2k	23.2k	0.0	63.4
dem_50_gaslib40_257	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_258	19.2k	19.2k	0.0	14.6	19.2k	19.2k	0.0	142.9	19.2k	19.2k	0.0	55.0
dem_50_gaslib40_259	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_260	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_261	26.8k	26.8k	0.0	2.8	26.8k	26.8k	0.0	106.1	26.8k	26.8k	0.0	20.3
dem_50_gaslib40_262	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_263	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_264	23.2k	23.2k	0.0	36.4	23.2k	23.2k	0.0	157.5	23.2k	23.2k	0.0	535.2
dem_50_gaslib40_265	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_266	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_267	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.2
dem_50_gaslib40_268	23.2k	23.2k	0.0	9.8	23.2k	23.2k	0.0	409.5	23.2k	23.2k	0.0	176.9
dem_50_gaslib40_269	69.6k	69.6k	0.0	147.3	69.6k	69.6k	0.0	927.2	69.6k	69.6k	0.0	2456.2
dem_50_gaslib40_270	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_271	41.4k	41.4k	0.0	24.7	41.4k	41.4k	0.0	617.7	41.4k	41.4k	0.0	899.8
dem_50_gaslib40_272	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_273	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.5
dem_50_gaslib40_274	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_275	23.2k	23.2k	0.0	14.9	23.2k	23.2k	0.0	396.7	23.2k	23.2k	0.0	181.4
dem_50_gaslib40_276	19.2k	19.2k	0.0	22.9	19.2k	19.2k	0.0	218.0	19.2k	19.2k	0.0	45.7
dem_50_gaslib40_277	17.3k	17.3k	0.0	9.0	17.3k	17.3k	0.0	113.8	17.3k	17.3k	0.0	51.5
dem_50_gaslib40_278	7209.5	7209.5	0.0	33.3	7209.5	7209.5	0.0	119.0	7209.5	7209.5	0.0	28.1
dem_50_gaslib40_279	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_280	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_281	19.2k	19.2k	0.0	12.4	19.2k	19.2k	0.0	114.9	19.2k	19.2k	0.0	50.7
dem_50_gaslib40_282	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_283	23.2k	23.2k	0.0	12.4	23.2k	23.2k	0.0	217.4	23.2k	23.2k	0.0	126.3
dem_50_gaslib40_284	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.8
dem_50_gaslib40_285	86.7k	86.7k	0.0	354.2	86.7k	86.7k	0.0	6199.6	87.2k	24.8k	251.9	14400.0
dem_50_gaslib40_286	19.2k	19.2k	0.0	15.3	19.2k	19.2k	0.0	85.4	19.2k	19.2k	0.0	48.7
dem_50_gaslib40_287	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.6
dem_50_gaslib40_288	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_289	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_290	19.2k	19.2k	0.0	16.6	19.2k	19.2k	0.0	139.4	19.2k	19.2k	0.0	344.9
dem_50_gaslib40_291	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_292	23.2k	23.2k	0.0	12.9	23.2k	23.2k	0.0	450.2	23.2k	23.2k	0.0	126.5
dem_50_gaslib40_293	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_294	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_295	27.0k	27.0k	0.0	24.5	27.0k	27.0k	0.0	291.7	27.0k	27.0k	0.0	232.2
dem_50_gaslib40_296	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_297	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_298	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_299	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_300	36.9k	36.9k	0.0	17.9	36.9k	36.9k	0.0	183.2	36.9k	36.9k	0.0	536.4
dem_50_gaslib40_301	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_302	23.2k	23.2k	0.0	13.3	23.2k	23.2k	0.0	157.1	23.2k	23.2k	0.0	224.7
dem_50_gaslib40_303	23.2k	23.2k	0.0	12.2	23.2k	23.2k	0.0	261.1	23.2k	23.2k	0.0	61.7
dem_50_gaslib40_304	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_305	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_306	23.2k	23.2k	0.0	13.2	23.2k	23.2k	0.0	294.6	23.2k	23.2k	0.0	122.8
dem_50_gaslib40_307	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_308	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_309	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_310	6955.8	6955.8	0.0	14.1	6955.8	6955.8	0.0	88.8	6955.8	6955.8	0.0	26.7
dem_50_gaslib40_311	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_312	21.7k	21.7k	0.0	14.0	21.7k	21.7k	0.0	347.6	21.7k	21.7k	0.0	482.4
dem_50_gaslib40_313	19.2k	19.2k	0.0	12.6	19.2k	19.2k	0.0	114.2	19.2k	19.2k	0.0	64.0
dem_50_gaslib40_314	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_315	19.2k	19.2k	0.0	11.9	19.2k	19.2k	0.0	251.5	19.2k	19.2k	0.0	35.7
dem_50_gaslib40_316	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.6
dem_50_gaslib40_317	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.7
dem_50_gaslib40_318	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_319	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_320	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_321	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_322	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_323	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.2

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Table A.9: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_324	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.6
dem_50_gaslib40_325	23.2k	23.2k	0.0	13.2	23.2k	23.2k	0.0	502.9	23.2k	23.2k	0.0	80.8
dem_50_gaslib40_326	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_327	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_328	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_329	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_330	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_331	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_332	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_333	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_334	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_335	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.3	0.0	0.0	0.0	0.4
dem_50_gaslib40_336	23.2k	23.2k	0.0	6.5	23.2k	23.2k	0.0	122.0	23.2k	23.2k	0.0	149.0
dem_50_gaslib40_337	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_338	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_339	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_340	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_341	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.3	0.0	0.0	0.0	0.4
dem_50_gaslib40_342	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.3	0.0	0.0	0.0	0.4
dem_50_gaslib40_343	26.8k	26.8k	0.0	11.3	26.8k	26.8k	0.0	248.7	26.8k	26.8k	0.0	156.2
dem_50_gaslib40_344	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_345	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_346	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_347	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_348	23.2k	23.2k	0.0	13.3	23.2k	23.2k	0.0	407.2	23.2k	23.2k	0.0	129.9
dem_50_gaslib40_349	15.8k	15.8k	0.0	30.6	15.8k	15.8k	0.0	96.8	15.8k	15.8k	0.0	106.6
dem_50_gaslib40_350	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_351	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.5
dem_50_gaslib40_352	8339.0	8339.0	0.0	17.7	8339.0	8339.0	0.0	76.8	8339.0	8339.0	0.0	59.4
dem_50_gaslib40_353	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_354	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_355	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_356	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_357	26.8k	26.8k	0.0	13.4	26.8k	26.8k	0.0	223.0	26.8k	26.8k	0.0	153.3
dem_50_gaslib40_358	9745.9	9745.9	0.0	25.0	9745.9	9745.9	0.0	73.2	9745.9	9745.9	0.0	48.1
dem_50_gaslib40_359	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_360	31.7k	31.7k	0.0	12.7	31.7k	31.7k	0.0	159.3	31.7k	31.7k	0.0	118.0
dem_50_gaslib40_361	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_362	23.2k	23.2k	0.0	7.3	23.2k	23.2k	0.0	425.3	23.2k	23.2k	0.0	182.2
dem_50_gaslib40_363	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_364	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_365	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.7
dem_50_gaslib40_366	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_367	23.2k	23.2k	0.0	7.0	23.2k	23.2k	0.0	555.3	23.2k	23.2k	0.0	72.5
dem_50_gaslib40_368	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_369	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_370	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_371	19.2k	19.2k	0.0	5.7	19.2k	19.2k	0.0	125.7	19.2k	19.2k	0.0	70.3
dem_50_gaslib40_372	23.2k	23.2k	0.0	6.4	23.2k	23.2k	0.0	561.7	23.2k	23.2k	0.0	47.1
dem_50_gaslib40_373	23.2k	23.2k	0.0	15.0	23.2k	23.2k	0.0	170.9	23.2k	23.2k	0.0	108.4
dem_50_gaslib40_374	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_375	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_376	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_377	31.7k	31.7k	0.0	13.5	31.7k	31.7k	0.0	127.1	31.7k	31.7k	0.0	223.6
dem_50_gaslib40_378	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.2
dem_50_gaslib40_379	35.6k	35.6k	0.0	13.3	35.6k	35.6k	0.0	299.7	35.6k	35.6k	0.0	567.0
dem_50_gaslib40_380	23.2k	23.2k	0.0	12.7	23.2k	23.2k	0.0	129.2	23.2k	23.2k	0.0	120.8
dem_50_gaslib40_381	19.2k	19.2k	0.0	13.8	19.2k	19.2k	0.0	194.3	19.2k	19.2k	0.0	117.4
dem_50_gaslib40_382	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_383	82.4k	82.4k	0.0	135.2	82.4k	82.4k	0.0	5451.6	82.4k	50.9k	61.7	14400.0
dem_50_gaslib40_384	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_385	6955.8	6955.8	0.0	13.3	6955.8	6955.8	0.0	94.7	6955.8	6955.8	0.0	32.3
dem_50_gaslib40_386	25.7k	25.7k	0.0	12.0	25.7k	25.7k	0.0	156.1	25.7k	25.7k	0.0	44.4
dem_50_gaslib40_387	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_388	23.2k	23.2k	0.0	12.3	23.2k	23.2k	0.0	81.8	23.2k	23.2k	0.0	128.6
dem_50_gaslib40_389	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_390	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_391	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.2
dem_50_gaslib40_392	23.2k	23.2k	0.0	11.8	23.2k	23.2k	0.0	144.1	23.2k	23.2k	0.0	24.3
dem_50_gaslib40_393	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_394	23.2k	23.2k	0.0	11.9	23.2k	23.2k	0.0	225.1	23.2k	23.2k	0.0	143.9
dem_50_gaslib40_395	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_396	19.2k	19.2k	0.0	113.9	19.2k	19.2k	0.0	108.2	19.2k	19.2k	0.0	73.3
dem_50_gaslib40_397	23.2k	23.2k	0.0	3.4	23.2k	23.2k	0.0	185.2	23.2k	23.2k	0.0	72.2
dem_50_gaslib40_398	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_399	31.7k	31.7k	0.0	12.1	31.7k	31.7k	0.0	132.4	31.7k	31.7k	0.0	6.9
dem_50_gaslib40_400	19.2k	19.2k	0.0	15.4	19.2k	19.2k	0.0	222.9	19.2k	19.2k	0.0	96.0
dem_50_gaslib40_401	19.2k	19.2k	0.0	12.3	19.2k	19.2k	0.0	152.7	19.2k	19.2k	0.0	51.0

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Table A.9: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_402	19.2k	19.2k	0.0	27.3	19.2k	19.2k	0.0	278.0	19.2k	19.2k	0.0	82.0
dem_50_gaslib40_403	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_404	31.7k	31.7k	0.0	75.4	31.7k	31.7k	0.0	893.3	31.7k	31.7k	0.0	429.6
dem_50_gaslib40_405	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_406	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_407	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_408	66.8k	66.8k	0.0	439.4	66.8k	66.8k	0.0	8350.6	78.7k	27.8k	183.0	14400.0
dem_50_gaslib40_409	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_410	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_411	19.2k	19.2k	0.0	39.2	19.2k	19.2k	0.0	278.0	19.2k	19.2k	0.0	95.2
dem_50_gaslib40_412	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_413	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_414	23.2k	23.2k	0.0	8.9	23.2k	23.2k	0.0	137.2	23.2k	23.2k	0.0	55.3
dem_50_gaslib40_415	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_416	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_417	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_418	26.8k	26.8k	0.0	16.0	26.8k	26.8k	0.0	137.8	26.8k	26.8k	0.0	263.6
dem_50_gaslib40_419	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_420	2515.4	2515.4	0.0	7.8	2515.4	2515.4	0.0	25.0	2515.4	2515.4	0.0	16.4
dem_50_gaslib40_421	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_422	23.2k	23.2k	0.0	12.3	23.2k	23.2k	0.0	524.4	23.2k	23.2k	0.0	220.1
dem_50_gaslib40_423	0.0	0.0	0.0	0.4	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_424	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_425	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_426	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_427	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_428	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_429	23.2k	23.2k	0.0	12.2	23.2k	23.2k	0.0	458.4	23.2k	23.2k	0.0	168.8
dem_50_gaslib40_430	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.6
dem_50_gaslib40_431	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_432	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_433	6955.8	6955.8	0.0	14.2	6955.8	6955.8	0.0	101.7	6955.8	6955.8	0.0	29.8
dem_50_gaslib40_434	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_435	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_436	6013.7	6013.7	0.0	6.7	6013.7	6013.7	0.0	104.4	6013.7	6013.7	0.0	46.3
dem_50_gaslib40_437	26.8k	26.8k	0.0	39.5	26.8k	26.8k	0.0	370.8	26.8k	26.8k	0.0	435.0
dem_50_gaslib40_438	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_439	19.2k	19.2k	0.0	15.7	19.2k	19.2k	0.0	125.6	19.2k	19.2k	0.0	122.5
dem_50_gaslib40_440	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.7
dem_50_gaslib40_441	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_442	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_443	2515.4	2515.4	0.0	18.5	2515.4	2515.4	0.0	122.2	2515.4	2515.4	0.0	27.7
dem_50_gaslib40_444	9745.9	9745.9	0.0	19.5	9745.9	9745.9	0.0	155.3	9745.9	9745.9	0.0	59.4
dem_50_gaslib40_445	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_446	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_447	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_448	19.2k	19.2k	0.0	31.3	19.2k	19.2k	0.0	157.3	19.2k	19.2k	0.0	172.2
dem_50_gaslib40_449	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_450	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_451	6013.7	6013.7	0.0	10.9	6013.7	6013.7	0.0	165.6	6013.7	6013.7	0.0	18.2
dem_50_gaslib40_452	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_453	23.2k	23.2k	0.0	11.7	23.2k	23.2k	0.0	154.1	23.2k	23.2k	0.0	97.3
dem_50_gaslib40_454	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_455	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_456	36.9k	36.9k	0.0	32.9	36.9k	36.9k	0.0	270.6	36.9k	36.9k	0.0	581.0
dem_50_gaslib40_457	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_458	23.2k	23.2k	0.0	12.2	23.2k	23.2k	0.0	122.8	23.2k	23.2k	0.0	40.5
dem_50_gaslib40_459	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_460	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_461	19.2k	19.2k	0.0	22.2	19.2k	19.2k	0.0	119.1	19.2k	19.2k	0.0	120.0
dem_50_gaslib40_462	31.7k	31.7k	0.0	22.7	31.7k	31.7k	0.0	336.0	31.7k	31.7k	0.0	327.0
dem_50_gaslib40_463	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_464	26.8k	26.8k	0.0	5.1	26.8k	26.8k	0.0	136.0	26.8k	26.8k	0.0	87.5
dem_50_gaslib40_465	19.2k	19.2k	0.0	18.0	19.2k	19.2k	0.0	158.5	19.2k	19.2k	0.0	190.8
dem_50_gaslib40_466	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_467	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_468	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_469	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_470	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_471	26.8k	26.8k	0.0	12.3	26.8k	26.8k	0.0	222.0	26.8k	26.8k	0.0	61.8
dem_50_gaslib40_472	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_473	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_474	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_475	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_476	23.2k	23.2k	0.0	12.0	23.2k	23.2k	0.0	253.1	23.2k	23.2k	0.0	100.7
dem_50_gaslib40_477	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_478	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_479	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4

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Table A.9: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_480	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_481	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_482	0.0	0.0	0.0	0.2	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_483	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_484	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_485	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_486	0.0	0.0	0.0	0.4	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_487	19.2k	19.2k	0.0	52.2	19.2k	19.2k	0.0	1013.4	19.2k	19.2k	0.0	112.6
dem_50_gaslib40_488	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_489	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.3	0.0	0.0	0.0	0.5
dem_50_gaslib40_490	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_491	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_492	0.0	0.0	0.0	0.3	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_493	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_494	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_495	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_496	31.7k	31.7k	0.0	17.3	31.7k	31.7k	0.0	275.4	31.7k	31.7k	0.0	588.1
dem_50_gaslib40_497	19.2k	19.2k	0.0	18.3	19.2k	19.2k	0.0	103.5	19.2k	19.2k	0.0	153.6
dem_50_gaslib40_498	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_499	23.2k	23.2k	0.0	41.7	23.2k	23.2k	0.0	758.9	23.2k	23.2k	0.0	237.4
dem_50_gaslib40_500	47.7k	47.7k	0.0	201.7	47.7k	47.7k	0.0	1994.1	47.7k	47.7k	0.0	10051.6

Table A.10: Detailed results of the split-pipe models on *GasLib-40* with $\mathcal{B} = 50$ as summarized in Figure 3.14a and Table 3.4b. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_1	0.0	0.0	0.0	6.7	0.0	0.0	0.0	1.1
dem_50_gaslib40_2	0.0	0.0	0.0	2.0	0.0	0.0	0.0	0.3
dem_50_gaslib40_3	0.0	0.0	0.0	4.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_4	0.0	0.0	0.0	4.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_5	0.0	0.0	0.0	2.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_6	1632.6	1632.6	0.0	3.9	1632.6	1632.6	0.0	1.5
dem_50_gaslib40_7	4262.3	4262.3	0.0	5.0	4262.3	4262.3	0.0	1.3
dem_50_gaslib40_8	24.3k	24.3k	0.0	2.4	24.3k	24.3k	0.0	2.4
dem_50_gaslib40_9	0.0	0.0	0.0	1.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_10	0.0	0.0	0.0	6.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_11	0.0	0.0	0.0	1.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_12	0.0	0.0	0.0	1.4	0.0	0.0	0.0	2.4
dem_50_gaslib40_13	0.0	0.0	0.0	2.0	0.0	0.0	0.0	1.0
dem_50_gaslib40_14	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.7
dem_50_gaslib40_15	0.0	0.0	0.0	3.0	0.0	0.0	0.0	1.1
dem_50_gaslib40_16	0.0	0.0	0.0	4.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_17	0.0	0.0	0.0	1.8	0.0	0.0	0.0	2.5
dem_50_gaslib40_18	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.6
dem_50_gaslib40_19	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.6
dem_50_gaslib40_20	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_21	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_22	0.0	0.0	0.0	3.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_23	5604.3	5604.3	0.0	2.0	5604.3	5604.3	0.0	1.8
dem_50_gaslib40_24	0.0	0.0	0.0	4.1	0.0	0.0	0.0	0.8
dem_50_gaslib40_25	0.0	0.0	0.0	2.4	0.0	0.0	0.0	1.4
dem_50_gaslib40_26	34.3k	34.3k	0.0	3.5	34.3k	34.3k	0.0	2.3
dem_50_gaslib40_27	0.0	0.0	0.0	8.3	0.0	0.0	0.0	0.8
dem_50_gaslib40_28	0.0	0.0	0.0	4.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_29	0.0	0.0	0.0	3.5	0.0	0.0	0.0	0.4
dem_50_gaslib40_30	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_31	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_32	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.6
dem_50_gaslib40_33	0.0	0.0	0.0	1.6	0.0	0.0	0.0	1.4
dem_50_gaslib40_34	32.6k	32.6k	0.0	3.3	32.6k	32.6k	0.0	1.8
dem_50_gaslib40_35	10.3k	10.3k	0.0	4.6	10.3k	10.3k	0.0	4.4
dem_50_gaslib40_36	4938.2	4938.2	0.0	5.8	4938.2	4938.2	0.0	1.5
dem_50_gaslib40_37	0.0	0.0	0.0	2.4	0.0	0.0	0.0	1.5
dem_50_gaslib40_38	16.6k	16.6k	0.0	2.2	16.6k	16.6k	0.1	1.5
dem_50_gaslib40_39	8349.8	8349.8	0.0	2.2	8349.8	8349.8	0.0	1.2
dem_50_gaslib40_40	0.0	0.0	0.0	1.5	0.0	0.0	0.0	0.6
dem_50_gaslib40_41	27.8k	27.8k	0.0	2.2	27.8k	27.8k	0.0	2.8
dem_50_gaslib40_42	13.0k	13.0k	0.0	2.1	13.0k	13.0k	0.0	5.0

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Table A.10: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_43	0.0	0.0	0.0	1.3	0.0	0.0	0.0	1.4
dem_50_gaslib40_44	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_45	1985.3	1985.3	0.0	1.7	1985.3	1985.3	0.0	1.3
dem_50_gaslib40_46	0.0	0.0	0.0	1.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_47	11.3k	11.3k	0.0	3.4	11.3k	11.3k	0.0	2.1
dem_50_gaslib40_48	14.6k	14.6k	0.0	2.7	14.6k	14.6k	0.0	1.4
dem_50_gaslib40_49	0.0	0.0	0.0	4.3	0.0	0.0	0.0	0.4
dem_50_gaslib40_50	0.0	0.0	0.0	6.7	0.0	0.0	0.0	0.3
dem_50_gaslib40_51	16.4k	16.4k	0.0	3.0	16.4k	16.4k	0.0	2.5
dem_50_gaslib40_52	0.0	0.0	0.0	2.6	0.0	0.0	0.0	1.6
dem_50_gaslib40_53	0.0	0.0	0.0	2.0	0.0	0.0	0.0	1.6
dem_50_gaslib40_54	0.0	0.0	0.0	2.5	0.0	0.0	0.0	1.1
dem_50_gaslib40_55	0.0	0.0	0.0	0.8	0.0	0.0	0.0	2.2
dem_50_gaslib40_56	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.4
dem_50_gaslib40_57	32.2k	32.2k	0.0	2.5	32.2k	32.2k	0.0	2.9
dem_50_gaslib40_58	0.0	0.0	0.0	1.8	0.0	0.0	0.0	2.1
dem_50_gaslib40_59	0.0	0.0	0.0	2.0	0.0	0.0	0.0	0.6
dem_50_gaslib40_60	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.4
dem_50_gaslib40_61	0.0	0.0	0.0	2.6	0.0	0.0	0.0	0.6
dem_50_gaslib40_62	5639.3	5639.1	0.0	3.7	5639.3	5639.3	0.0	1.1
dem_50_gaslib40_63	0.0	0.0	0.0	1.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_64	0.0	0.0	0.0	8.8	0.0	0.0	0.0	0.7
dem_50_gaslib40_65	0.0	0.0	0.0	1.6	0.0	0.0	0.0	1.1
dem_50_gaslib40_66	0.0	0.0	0.0	2.4	0.0	0.0	0.0	1.2
dem_50_gaslib40_67	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_68	17.1k	17.1k	0.0	3.1	17.1k	17.1k	0.0	1.2
dem_50_gaslib40_69	0.0	0.0	0.0	10.2	0.0	0.0	0.0	0.6
dem_50_gaslib40_70	0.0	0.0	0.0	2.7	0.0	0.0	0.0	2.1
dem_50_gaslib40_71	756.0	756.0	0.0	2.2	756.0	756.0	0.0	1.8
dem_50_gaslib40_72	0.0	0.0	0.0	2.4	0.0	0.0	0.0	1.5
dem_50_gaslib40_73	19.3k	19.3k	0.0	7.4	19.3k	19.3k	0.0	1.5
dem_50_gaslib40_74	12.8k	12.8k	0.0	10.3	12.8k	12.8k	0.0	2.2
dem_50_gaslib40_75	0.0	0.0	0.0	0.5	0.0	0.0	0.0	1.6
dem_50_gaslib40_76	11.4k	11.4k	0.0	3.5	11.4k	11.4k	0.0	1.3
dem_50_gaslib40_77	0.0	0.0	0.0	3.5	0.0	0.0	0.0	0.7
dem_50_gaslib40_78	0.0	0.0	0.0	9.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_79	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_80	15.1k	15.1k	0.0	3.6	15.1k	15.1k	0.0	2.6
dem_50_gaslib40_81	0.0	0.0	0.0	3.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_82	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.5
dem_50_gaslib40_83	0.0	0.0	0.0	2.0	0.0	0.0	0.0	1.6
dem_50_gaslib40_84	0.0	0.0	0.0	6.0	0.0	0.0	0.0	0.2
dem_50_gaslib40_85	0.0	0.0	0.0	3.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_86	14.0k	14.0k	0.0	5.5	14.0k	14.0k	0.1	1.1
dem_50_gaslib40_87	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.8
dem_50_gaslib40_88	0.0	0.0	0.0	3.5	0.0	0.0	0.0	1.2
dem_50_gaslib40_89	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_90	0.0	0.0	0.0	7.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_91	0.0	0.0	0.0	5.3	0.0	0.0	0.0	2.7
dem_50_gaslib40_92	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.8
dem_50_gaslib40_93	20.4k	20.4k	0.0	3.9	20.4k	20.4k	0.0	1.5
dem_50_gaslib40_94	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.3
dem_50_gaslib40_95	2380.5	2380.5	0.0	3.3	2380.5	2380.5	0.0	0.9
dem_50_gaslib40_96	0.0	0.0	0.0	2.1	0.0	0.0	0.0	4.8
dem_50_gaslib40_97	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.7
dem_50_gaslib40_98	0.0	0.0	0.0	2.4	0.0	0.0	0.0	1.0
dem_50_gaslib40_99	7093.3	7093.3	0.0	3.1	7093.3	7093.3	0.0	1.3
dem_50_gaslib40_100	0.0	0.0	0.0	2.4	0.0	0.0	0.0	3.0
dem_50_gaslib40_101	7060.7	7060.7	0.0	2.0	7060.7	7060.7	0.0	1.5
dem_50_gaslib40_102	0.0	0.0	0.0	3.5	0.0	0.0	0.0	0.4
dem_50_gaslib40_103	0.0	0.0	0.0	2.0	-0.0	-0.0	0.0	2.1
dem_50_gaslib40_104	0.0	0.0	0.0	6.4	0.0	0.0	0.0	2.0
dem_50_gaslib40_105	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_106	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.9
dem_50_gaslib40_107	16.2k	16.2k	0.0	3.1	16.2k	16.2k	0.0	1.7
dem_50_gaslib40_108	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.3
dem_50_gaslib40_109	0.0	0.0	0.0	2.3	0.0	0.0	0.0	0.5
dem_50_gaslib40_110	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_111	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.6
dem_50_gaslib40_112	0.0	0.0	0.0	4.5	0.0	0.0	0.0	0.7
dem_50_gaslib40_113	12.5k	12.5k	0.0	7.3	12.5k	12.5k	0.0	1.6
dem_50_gaslib40_114	0.0	0.0	0.0	2.1	0.0	0.0	0.0	1.0
dem_50_gaslib40_115	0.0	0.0	0.0	3.1	0.0	0.0	0.0	2.2
dem_50_gaslib40_116	0.0	0.0	0.0	8.8	0.0	0.0	0.0	3.1
dem_50_gaslib40_117	0.0	0.0	0.0	2.5	0.0	0.0	0.0	2.1
dem_50_gaslib40_118	0.0	0.0	0.0	2.3	0.0	0.0	0.0	1.1
dem_50_gaslib40_119	0.0	0.0	0.0	2.0	0.0	0.0	0.0	0.9
dem_50_gaslib40_120	0.0	0.0	0.0	4.3	0.0	0.0	0.0	2.0

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Table A.10: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_121	0.0	0.0	0.0	8.4	0.0	0.0	0.0	0.6
dem_50_gaslib40_122	0.0	0.0	0.0	7.3	0.0	0.0	0.0	2.3
dem_50_gaslib40_123	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.5
dem_50_gaslib40_124	0.0	0.0	0.0	3.6	0.0	0.0	0.0	0.7
dem_50_gaslib40_125	0.0	0.0	0.0	3.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_126	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_127	0.0	0.0	0.0	3.5	0.0	0.0	0.0	1.0
dem_50_gaslib40_128	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_129	0.0	0.0	0.0	0.6	0.0	0.0	0.0	3.2
dem_50_gaslib40_130	0.0	0.0	0.0	1.9	0.0	0.0	0.0	3.9
dem_50_gaslib40_131	0.0	0.0	0.0	4.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_132	16.2k	16.2k	0.0	3.7	16.2k	16.2k	0.0	1.4
dem_50_gaslib40_133	0.0	0.0	0.0	1.6	0.0	0.0	0.0	1.2
dem_50_gaslib40_134	0.0	0.0	0.0	2.6	0.0	0.0	0.0	3.3
dem_50_gaslib40_135	26.8k	26.8k	0.0	4.8	26.8k	26.8k	0.0	2.0
dem_50_gaslib40_136	0.0	0.0	0.0	4.8	0.0	0.0	0.0	1.3
dem_50_gaslib40_137	0.0	0.0	0.0	2.7	0.0	0.0	0.0	4.3
dem_50_gaslib40_138	14.9k	14.9k	0.0	4.8	14.9k	14.9k	0.1	1.3
dem_50_gaslib40_139	0.0	0.0	0.0	3.3	0.0	0.0	0.0	1.2
dem_50_gaslib40_140	0.0	0.0	0.0	2.6	0.0	0.0	0.0	8.3
dem_50_gaslib40_141	0.0	0.0	0.0	2.9	0.0	0.0	0.0	1.1
dem_50_gaslib40_142	0.0	0.0	0.0	4.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_143	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_144	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_145	0.0	0.0	0.0	2.7	0.0	0.0	0.0	0.8
dem_50_gaslib40_146	14.4k	14.4k	0.0	3.4	14.4k	14.4k	0.0	958.8
dem_50_gaslib40_147	17.3k	17.3k	0.0	2.0	17.3k	17.3k	0.0	1.6
dem_50_gaslib40_148	0.0	0.0	0.0	2.4	0.0	0.0	0.0	1.2
dem_50_gaslib40_149	0.0	0.0	0.0	4.3	0.0	0.0	0.0	0.5
dem_50_gaslib40_150	11.0k	11.0k	0.0	5.4	11.0k	11.0k	0.0	1.2
dem_50_gaslib40_151	0.0	0.0	0.0	1.5	0.0	0.0	0.0	0.6
dem_50_gaslib40_152	4267.9	4267.9	0.0	3.2	4267.9	4267.9	0.0	1.1
dem_50_gaslib40_153	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_154	0.0	0.0	0.0	0.6	0.0	0.0	0.0	2.3
dem_50_gaslib40_155	0.0	0.0	0.0	3.2	0.0	0.0	0.0	1.2
dem_50_gaslib40_156	0.0	0.0	0.0	2.1	0.0	0.0	0.0	2.3
dem_50_gaslib40_157	0.0	0.0	0.0	2.4	0.0	0.0	0.0	1.5
dem_50_gaslib40_158	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.5
dem_50_gaslib40_159	4903.8	4903.1	0.0	2.9	4903.8	4902.8	0.0	1.1
dem_50_gaslib40_160	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_161	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_162	0.0	0.0	0.0	4.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_163	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.6
dem_50_gaslib40_164	0.0	0.0	0.0	1.3	0.0	0.0	0.0	0.5
dem_50_gaslib40_165	0.0	0.0	0.0	6.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_166	0.0	0.0	0.0	9.2	0.0	0.0	0.0	2.1
dem_50_gaslib40_167	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.5
dem_50_gaslib40_168	0.0	0.0	0.0	3.7	0.0	0.0	0.0	1.3
dem_50_gaslib40_169	97.8k	96.9k	0.9	14400.0	97.8k	97.8k	0.0	4.5
dem_50_gaslib40_170	0.0	0.0	0.0	4.0	0.0	0.0	0.0	0.6
dem_50_gaslib40_171	0.0	0.0	0.0	3.6	0.0	0.0	0.0	0.6
dem_50_gaslib40_172	0.0	0.0	0.0	2.7	0.0	0.0	0.0	0.6
dem_50_gaslib40_173	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.6
dem_50_gaslib40_174	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.6
dem_50_gaslib40_175	0.0	0.0	0.0	3.5	0.0	0.0	0.0	0.4
dem_50_gaslib40_176	0.0	0.0	0.0	3.3	0.0	0.0	0.0	0.5
dem_50_gaslib40_177	5953.9	5953.9	0.0	2.8	5953.9	5953.9	0.0	2.5
dem_50_gaslib40_178	0.0	0.0	0.0	0.9	0.0	0.0	0.0	2.3
dem_50_gaslib40_179	3814.3	3814.3	0.0	4.9	3814.3	3814.3	0.0	1.5
dem_50_gaslib40_180	0.0	0.0	0.0	4.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_181	0.0	0.0	0.0	4.0	0.0	0.0	0.0	1.1
dem_50_gaslib40_182	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.5
dem_50_gaslib40_183	0.0	0.0	0.0	2.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_184	21.8k	21.8k	0.0	6.1	21.8k	21.8k	0.0	1.2
dem_50_gaslib40_185	0.0	0.0	0.0	4.4	0.0	0.0	0.0	0.5
dem_50_gaslib40_186	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.4
dem_50_gaslib40_187	0.0	0.0	0.0	2.6	0.0	0.0	0.0	1.3
dem_50_gaslib40_188	31.0k	31.0k	0.0	6.8	31.0k	31.0k	0.0	1.6
dem_50_gaslib40_189	5103.9	5103.9	0.0	3.0	5103.9	5103.9	0.0	1.1
dem_50_gaslib40_190	12.5k	12.5k	0.0	3.3	12.5k	12.5k	0.0	2.3
dem_50_gaslib40_191	13.5k	13.5k	0.0	2.0	13.5k	13.5k	0.0	1.8
dem_50_gaslib40_192	4027.9	4027.9	0.0	4.7	4027.9	4027.9	0.0	1.8
dem_50_gaslib40_193	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.5
dem_50_gaslib40_194	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_195	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_196	0.0	0.0	0.0	9.4	0.0	0.0	0.0	0.6
dem_50_gaslib40_197	0.0	0.0	0.0	3.6	0.0	0.0	0.0	0.4
dem_50_gaslib40_198	0.0	0.0	0.0	1.2	0.0	0.0	0.0	1.6

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Table A.10: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_199	0.0	0.0	0.0	1.5	0.0	0.0	0.0	1.1
dem_50_gaslib40_200	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.6
dem_50_gaslib40_201	0.0	0.0	0.0	3.5	0.0	0.0	0.0	2.0
dem_50_gaslib40_202	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_203	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.7
dem_50_gaslib40_204	12.7k	12.7k	0.0	2.0	12.7k	12.7k	0.0	1.5
dem_50_gaslib40_205	0.0	0.0	0.0	1.4	0.0	0.0	0.0	2.1
dem_50_gaslib40_206	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.3
dem_50_gaslib40_207	0.0	0.0	0.0	7.6	0.0	0.0	0.0	0.6
dem_50_gaslib40_208	0.0	0.0	0.0	2.8	0.0	0.0	0.0	1.3
dem_50_gaslib40_209	0.0	0.0	0.0	3.5	0.0	0.0	0.0	0.5
dem_50_gaslib40_210	0.0	0.0	0.0	8.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_211	0.0	0.0	0.0	4.6	0.0	0.0	0.0	3.4
dem_50_gaslib40_212	0.0	0.0	0.0	3.9	0.0	0.0	0.0	0.9
dem_50_gaslib40_213	0.0	0.0	0.0	2.3	0.0	0.0	0.0	0.4
dem_50_gaslib40_214	0.0	0.0	0.0	10.1	0.0	0.0	0.0	0.7
dem_50_gaslib40_215	0.0	0.0	0.0	2.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_216	0.0	0.0	0.0	1.8	0.0	0.0	0.0	1.5
dem_50_gaslib40_217	0.0	0.0	0.0	1.7	0.0	0.0	0.0	2.4
dem_50_gaslib40_218	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_219	13.9k	13.9k	0.0	114.0	13.9k	13.9k	0.0	1.2
dem_50_gaslib40_220	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.8
dem_50_gaslib40_221	19.5k	19.5k	0.0	2.9	19.5k	19.5k	0.0	1.8
dem_50_gaslib40_222	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.6
dem_50_gaslib40_223	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_224	6921.2	6921.2	0.0	4.0	6921.2	6921.2	0.0	1.4
dem_50_gaslib40_225	0.0	0.0	0.0	1.1	0.0	0.0	0.0	1.3
dem_50_gaslib40_226	8258.1	8258.1	0.0	3.3	8258.1	8258.1	0.0	1.3
dem_50_gaslib40_227	70.2k	70.2k	0.0	4.2	70.2k	70.2k	0.0	5.0
dem_50_gaslib40_228	0.0	0.0	0.0	3.0	0.0	0.0	0.0	2.4
dem_50_gaslib40_229	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.3
dem_50_gaslib40_230	0.0	0.0	0.0	3.1	0.0	0.0	0.0	1.1
dem_50_gaslib40_231	18.0k	18.0k	0.0	2.4	18.0k	18.0k	0.0	1.9
dem_50_gaslib40_232	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_233	0.0	0.0	0.0	3.7	0.0	0.0	0.0	1.1
dem_50_gaslib40_234	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_235	0.0	0.0	0.0	2.6	0.0	0.0	0.0	0.5
dem_50_gaslib40_236	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.7
dem_50_gaslib40_237	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.9
dem_50_gaslib40_238	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.6
dem_50_gaslib40_239	21.4k	21.4k	0.0	6.7	21.4k	21.4k	0.0	4.0
dem_50_gaslib40_240	0.0	0.0	0.0	2.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_241	27.4	27.4	0.0	2.9	27.4	27.4	0.0	1.6
dem_50_gaslib40_242	16.6k	16.6k	0.0	3.5	16.6k	16.6k	0.0	1.6
dem_50_gaslib40_243	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.5
dem_50_gaslib40_244	0.0	0.0	0.0	1.3	0.0	0.0	0.0	0.6
dem_50_gaslib40_245	0.0	0.0	0.0	2.5	0.0	0.0	0.0	1.7
dem_50_gaslib40_246	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_247	0.0	0.0	0.0	3.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_248	9340.9	9340.9	0.0	3.9	9340.9	9340.4	0.0	3.4
dem_50_gaslib40_249	0.0	0.0	0.0	2.6	0.0	0.0	0.0	0.4
dem_50_gaslib40_250	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.9
dem_50_gaslib40_251	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.5
dem_50_gaslib40_252	0.0	0.0	0.0	4.8	0.0	0.0	0.0	0.9
dem_50_gaslib40_253	0.0	0.0	0.0	3.4	0.0	0.0	0.0	1.0
dem_50_gaslib40_254	2067.8	2067.8	0.0	5.4	2067.8	2067.8	0.0	1.1
dem_50_gaslib40_255	27.7k	27.7k	0.0	4.3	27.7k	27.7k	0.0	1.8
dem_50_gaslib40_256	12.4k	12.4k	0.0	3.5	12.4k	12.4k	0.0	1.2
dem_50_gaslib40_257	0.0	0.0	0.0	1.5	0.0	0.0	0.0	1.0
dem_50_gaslib40_258	6320.4	6320.4	0.0	2.2	6320.4	6320.4	0.0	1.4
dem_50_gaslib40_259	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_260	0.0	0.0	0.0	6.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_261	22.8k	22.8k	0.0	5.0	22.8k	22.8k	0.0	1.5
dem_50_gaslib40_262	0.0	0.0	0.0	6.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_263	0.0	0.0	0.0	1.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_264	12.6k	12.6k	0.0	4.1	12.6k	12.6k	0.0	1.2
dem_50_gaslib40_265	0.0	0.0	0.0	4.5	0.0	0.0	0.0	0.4
dem_50_gaslib40_266	0.0	0.0	0.0	3.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_267	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_268	19.6k	19.6k	0.0	2.2	19.6k	19.6k	0.0	5.2
dem_50_gaslib40_269	64.0k	64.0k	0.0	11.2	64.0k	64.0k	0.0	4.3
dem_50_gaslib40_270	0.0	0.0	0.0	6.7	0.0	0.0	0.0	1.1
dem_50_gaslib40_271	40.7k	40.7k	0.0	4.2	40.7k	40.7k	0.1	2.5
dem_50_gaslib40_272	0.0	0.0	0.0	2.6	0.0	0.0	0.0	0.7
dem_50_gaslib40_273	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.5
dem_50_gaslib40_274	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_275	13.1k	13.0k	0.1	3.3	13.1k	13.1k	0.0	1.4
dem_50_gaslib40_276	5305.8	5305.8	0.0	3.6	5305.8	5305.8	0.0	1.2

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Table A.10: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_277	6145.8	6145.5	0.0	2.0	6145.8	6145.0	0.0	1.2
dem_50_gaslib40_278	800.8	800.8	0.0	3.2	800.8	800.8	0.0	1.1
dem_50_gaslib40_279	0.0	0.0	0.0	1.5	0.0	0.0	0.0	1.5
dem_50_gaslib40_280	0.0	0.0	0.0	8.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_281	3798.5	3798.5	0.0	3.4	3798.5	3798.5	0.0	1.1
dem_50_gaslib40_282	0.0	0.0	0.0	6.0	0.0	0.0	0.0	0.8
dem_50_gaslib40_283	11.0k	11.0k	0.0	1.7	11.0k	11.0k	0.0	1.3
dem_50_gaslib40_284	0.0	0.0	0.0	3.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_285	76.3k	76.2k	0.1	4.9	76.3k	76.3k	0.0	2.9
dem_50_gaslib40_286	8393.3	8393.3	0.0	4.2	8393.3	8393.3	0.0	3.9
dem_50_gaslib40_287	0.0	0.0	0.0	2.1	0.0	0.0	0.0	3.7
dem_50_gaslib40_288	0.0	0.0	0.0	2.5	0.0	0.0	0.0	1.1
dem_50_gaslib40_289	0.0	0.0	0.0	2.8	0.0	0.0	0.0	1.6
dem_50_gaslib40_290	4816.9	4816.9	0.0	3.0	4816.9	4816.9	0.0	1.1
dem_50_gaslib40_291	0.0	0.0	0.0	3.9	0.0	0.0	0.0	1.1
dem_50_gaslib40_292	15.8k	15.8k	0.0	2.3	15.8k	15.8k	0.0	1.5
dem_50_gaslib40_293	0.0	0.0	0.0	3.8	0.0	0.0	0.0	2.7
dem_50_gaslib40_294	0.0	0.0	0.0	4.7	0.0	0.0	0.0	0.9
dem_50_gaslib40_295	16.2k	16.2k	0.0	3.8	16.2k	16.2k	0.0	1.4
dem_50_gaslib40_296	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.8
dem_50_gaslib40_297	0.0	0.0	0.0	1.1	0.0	0.0	0.0	1.4
dem_50_gaslib40_298	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_299	0.0	0.0	0.0	4.8	0.0	0.0	0.0	1.0
dem_50_gaslib40_300	22.1k	22.1k	0.0	3.5	22.1k	22.1k	0.0	1.8
dem_50_gaslib40_301	0.0	0.0	0.0	2.8	0.0	0.0	0.0	1.7
dem_50_gaslib40_302	10.1k	10.1k	0.0	5.7	10.1k	10.1k	0.0	2.1
dem_50_gaslib40_303	13.4k	13.4k	0.0	3.4	13.4k	13.4k	0.0	2.1
dem_50_gaslib40_304	0.0	0.0	0.0	2.3	0.0	0.0	0.0	0.4
dem_50_gaslib40_305	0.0	0.0	0.0	1.6	0.0	0.0	0.0	1.2
dem_50_gaslib40_306	10.3k	10.3k	0.0	3.0	10.3k	10.3k	0.0	1.2
dem_50_gaslib40_307	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_308	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.4
dem_50_gaslib40_309	0.0	0.0	0.0	3.3	0.0	0.0	0.0	1.6
dem_50_gaslib40_310	771.6	771.6	0.0	3.4	771.6	771.6	0.0	1.1
dem_50_gaslib40_311	0.0	0.0	0.0	4.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_312	10.1k	10.1k	0.0	3.8	10.1k	10.1k	0.0	1.2
dem_50_gaslib40_313	9038.3	9035.2	0.0	3.5	9038.3	9038.0	0.0	2.4
dem_50_gaslib40_314	0.0	0.0	0.0	3.9	0.0	0.0	0.0	0.5
dem_50_gaslib40_315	9655.9	9655.6	0.0	2.7	9655.9	9655.9	0.0	1.1
dem_50_gaslib40_316	0.0	0.0	0.0	3.2	0.0	0.0	0.0	3.1
dem_50_gaslib40_317	0.0	0.0	0.0	1.6	0.0	0.0	0.0	2.2
dem_50_gaslib40_318	0.0	0.0	0.0	2.9	0.0	0.0	0.0	2.2
dem_50_gaslib40_319	0.0	0.0	0.0	2.7	0.0	0.0	0.0	0.7
dem_50_gaslib40_320	0.0	0.0	0.0	2.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_321	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.3
dem_50_gaslib40_322	0.0	0.0	0.0	3.3	0.0	0.0	0.0	0.5
dem_50_gaslib40_323	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.8
dem_50_gaslib40_324	0.0	0.0	0.0	7.4	0.0	0.0	0.0	5.6
dem_50_gaslib40_325	15.1k	15.1k	0.0	3.0	15.1k	15.1k	0.0	1.3
dem_50_gaslib40_326	0.0	0.0	0.0	2.4	0.0	0.0	0.0	1.1
dem_50_gaslib40_327	0.0	0.0	0.0	2.6	0.0	0.0	0.0	0.5
dem_50_gaslib40_328	0.0	0.0	0.0	4.8	0.0	0.0	0.0	0.8
dem_50_gaslib40_329	0.0	0.0	0.0	1.6	0.0	0.0	0.0	2.3
dem_50_gaslib40_330	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.6
dem_50_gaslib40_331	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.2
dem_50_gaslib40_332	0.0	0.0	0.0	8.4	0.0	0.0	0.0	1.7
dem_50_gaslib40_333	0.0	0.0	0.0	2.8	0.0	0.0	0.0	1.9
dem_50_gaslib40_334	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.5
dem_50_gaslib40_335	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.4
dem_50_gaslib40_336	14.5k	14.5k	0.0	2.4	14.5k	14.5k	0.0	5.3
dem_50_gaslib40_337	0.0	0.0	0.0	4.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_338	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.6
dem_50_gaslib40_339	0.0	0.0	0.0	4.0	0.0	0.0	0.0	0.2
dem_50_gaslib40_340	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_341	0.0	0.0	0.0	2.7	0.0	0.0	0.0	6.9
dem_50_gaslib40_342	0.0	0.0	0.0	1.9	0.0	0.0	0.0	1.0
dem_50_gaslib40_343	24.0k	24.0k	0.0	3.0	24.0k	24.0k	0.0	2.5
dem_50_gaslib40_344	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_345	0.0	0.0	0.0	2.2	0.0	0.0	0.0	1.0
dem_50_gaslib40_346	0.0	0.0	0.0	2.4	0.0	0.0	0.0	1.9
dem_50_gaslib40_347	0.0	0.0	0.0	3.0	0.0	0.0	0.0	1.4
dem_50_gaslib40_348	11.7k	11.7k	0.0	3.8	11.7k	11.7k	0.0	1.8
dem_50_gaslib40_349	3319.4	3319.4	0.0	4.5	3319.4	3319.4	0.0	1.2
dem_50_gaslib40_350	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.3
dem_50_gaslib40_351	0.0	0.0	0.0	4.1	0.0	0.0	0.0	0.6
dem_50_gaslib40_352	1572.1	1571.8	0.0	3.7	1572.1	1572.1	0.0	1.0
dem_50_gaslib40_353	0.0	0.0	0.0	3.4	0.0	0.0	0.0	0.8
dem_50_gaslib40_354	0.0	0.0	0.0	2.0	0.0	0.0	0.0	0.4

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Table A.10: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_355	0.0	0.0	0.0	2.8	0.0	0.0	0.0	2.0
dem_50_gaslib40_356	0.0	0.0	0.0	9.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_357	20.2k	20.2k	0.0	2.8	20.2k	20.2k	0.0	1.5
dem_50_gaslib40_358	2020.1	2020.1	0.0	3.3	2020.1	2018.9	0.1	2.2
dem_50_gaslib40_359	0.0	0.0	0.0	3.6	0.0	0.0	0.0	1.4
dem_50_gaslib40_360	28.9k	28.8k	0.1	2.7	28.9k	28.9k	0.0	1.8
dem_50_gaslib40_361	0.0	0.0	0.0	4.6	0.0	0.0	0.0	0.9
dem_50_gaslib40_362	12.4k	12.4k	0.0	5.3	12.4k	12.4k	0.0	1.9
dem_50_gaslib40_363	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_364	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_365	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.8
dem_50_gaslib40_366	0.0	0.0	0.0	9.0	0.0	0.0	0.0	2.3
dem_50_gaslib40_367	13.6k	13.6k	0.0	2.3	13.6k	13.6k	0.0	1.4
dem_50_gaslib40_368	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_369	0.0	0.0	0.0	2.6	0.0	0.0	0.0	2.3
dem_50_gaslib40_370	0.0	0.0	0.0	8.6	0.0	0.0	0.0	0.5
dem_50_gaslib40_371	4002.3	4002.3	0.0	2.4	4002.3	4002.3	0.0	1.2
dem_50_gaslib40_372	19.2k	19.2k	0.0	1.6	19.2k	19.2k	0.0	1.0
dem_50_gaslib40_373	11.7k	11.7k	0.0	4.2	11.7k	11.7k	0.0	1.0
dem_50_gaslib40_374	0.0	0.0	0.0	8.3	0.0	0.0	0.0	0.5
dem_50_gaslib40_375	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.8
dem_50_gaslib40_376	0.0	0.0	0.0	2.0	0.0	0.0	0.0	1.1
dem_50_gaslib40_377	29.1k	29.1k	0.0	6.4	29.1k	29.1k	0.0	2.6
dem_50_gaslib40_378	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.5
dem_50_gaslib40_379	31.0k	31.0k	0.0	2.3	31.0k	31.0k	0.0	1.7
dem_50_gaslib40_380	16.4k	16.3k	0.1	4.3	16.4k	16.4k	0.0	2.3
dem_50_gaslib40_381	7375.8	7375.8	0.0	2.6	7375.8	7375.8	0.0	1.2
dem_50_gaslib40_382	0.0	0.0	0.0	6.8	0.0	0.0	0.0	1.7
dem_50_gaslib40_383	74.2k	74.2k	0.0	3.3	74.2k	74.2k	0.0	7.6
dem_50_gaslib40_384	0.0	0.0	0.0	1.9	0.0	0.0	0.0	0.3
dem_50_gaslib40_385	842.9	842.9	0.0	4.3	842.9	842.9	0.0	1.1
dem_50_gaslib40_386	19.7k	19.7k	0.0	2.2	19.7k	19.7k	0.0	1.5
dem_50_gaslib40_387	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_388	12.9k	12.9k	0.0	5.9	12.9k	12.9k	0.0	2.4
dem_50_gaslib40_389	0.0	0.0	0.0	3.3	0.0	0.0	0.0	0.8
dem_50_gaslib40_390	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_391	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.4
dem_50_gaslib40_392	15.6k	15.6k	0.0	2.7	15.6k	15.6k	0.0	1.7
dem_50_gaslib40_393	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.9
dem_50_gaslib40_394	16.4k	16.4k	0.0	2.5	16.4k	16.4k	0.0	2.4
dem_50_gaslib40_395	0.0	0.0	0.0	3.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_396	6622.9	6622.9	0.0	2.9	6622.9	6622.9	0.0	1.9
dem_50_gaslib40_397	17.1k	17.1k	0.0	3.5	17.1k	17.1k	0.0	1.6
dem_50_gaslib40_398	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.8
dem_50_gaslib40_399	27.7k	27.7k	0.0	3.1	27.7k	27.7k	0.0	7.2
dem_50_gaslib40_400	9765.8	9765.8	0.0	2.5	9765.8	9765.8	0.0	1.2
dem_50_gaslib40_401	8446.9	8439.7	0.1	3.5	8446.9	8444.8	0.0	1.1
dem_50_gaslib40_402	8833.8	8833.8	0.0	3.6	8833.8	8833.8	0.0	2.1
dem_50_gaslib40_403	0.0	0.0	0.0	3.7	0.0	0.0	0.0	2.3
dem_50_gaslib40_404	28.0k	28.0k	0.0	3.7	28.0k	28.0k	0.0	2.6
dem_50_gaslib40_405	0.0	0.0	0.0	3.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_406	0.0	0.0	0.0	3.3	0.0	0.0	0.0	0.5
dem_50_gaslib40_407	0.0	0.0	0.0	3.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_408	61.7k	61.7k	0.0	4.9	61.7k	61.7k	0.0	3.9
dem_50_gaslib40_409	0.0	0.0	0.0	3.6	0.0	0.0	0.0	0.4
dem_50_gaslib40_410	0.0	0.0	0.0	1.7	0.0	0.0	0.0	1.5
dem_50_gaslib40_411	4819.0	4819.0	0.0	1.9	4819.0	4818.9	0.0	1.1
dem_50_gaslib40_412	0.0	0.0	0.0	4.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_413	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.4
dem_50_gaslib40_414	10.4k	10.4k	0.0	1.1	10.4k	10.4k	0.0	1.7
dem_50_gaslib40_415	0.0	0.0	0.0	0.9	0.0	0.0	0.0	1.3
dem_50_gaslib40_416	0.0	0.0	0.0	3.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_417	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.6
dem_50_gaslib40_418	26.6k	26.6k	0.0	2.4	26.6k	26.6k	0.0	2.9
dem_50_gaslib40_419	0.0	0.0	0.0	2.0	0.0	0.0	0.0	0.4
dem_50_gaslib40_420	141.7	141.7	0.0	2.2	141.7	141.7	0.0	1.1
dem_50_gaslib40_421	0.0	0.0	0.0	10.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_422	15.6k	15.6k	0.0	3.2	15.6k	15.6k	0.0	1.6
dem_50_gaslib40_423	0.0	0.0	0.0	3.6	0.0	0.0	0.0	0.7
dem_50_gaslib40_424	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.5
dem_50_gaslib40_425	0.0	0.0	0.0	2.7	0.0	0.0	0.0	0.9
dem_50_gaslib40_426	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_427	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.9
dem_50_gaslib40_428	0.0	0.0	0.0	8.5	0.0	0.0	0.0	0.5
dem_50_gaslib40_429	13.6k	13.6k	0.0	2.4	13.6k	13.6k	0.0	1.4
dem_50_gaslib40_430	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.6
dem_50_gaslib40_431	0.0	0.0	0.0	4.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_432	0.0	0.0	0.0	8.2	0.0	0.0	0.0	1.4

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Table A.10: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib40_433	830.2	829.9	0.0	3.7	830.2	829.6	0.1	1.8
dem_50_gaslib40_434	0.0	0.0	0.0	3.1	0.0	0.0	0.0	1.1
dem_50_gaslib40_435	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.9
dem_50_gaslib40_436	463.1	463.1	0.0	2.6	463.1	462.8	0.1	1.0
dem_50_gaslib40_437	19.9k	19.9k	0.0	4.5	19.9k	19.9k	0.0	2.6
dem_50_gaslib40_438	0.0	0.0	0.0	9.6	0.0	0.0	0.0	1.3
dem_50_gaslib40_439	5480.8	5478.6	0.0	2.1	5480.8	5480.8	0.0	1.2
dem_50_gaslib40_440	0.0	0.0	0.0	3.2	0.0	0.0	0.0	1.0
dem_50_gaslib40_441	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.8
dem_50_gaslib40_442	0.0	0.0	0.0	3.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_443	77.6	77.6	0.0	2.9	77.6	77.6	0.0	1.6
dem_50_gaslib40_444	2144.7	2144.6	0.0	2.6	2144.7	2144.6	0.0	1.8
dem_50_gaslib40_445	0.0	0.0	0.0	2.5	0.0	0.0	0.0	3.0
dem_50_gaslib40_446	0.0	0.0	0.0	1.4	0.0	0.0	0.0	1.1
dem_50_gaslib40_447	0.0	0.0	0.0	3.2	0.0	0.0	0.0	0.4
dem_50_gaslib40_448	6517.7	6517.7	0.0	1.8	6517.7	6517.6	0.0	1.9
dem_50_gaslib40_449	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.6
dem_50_gaslib40_450	0.0	0.0	0.0	8.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_451	546.8	546.8	0.0	4.3	546.8	546.8	0.0	2.0
dem_50_gaslib40_452	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.8
dem_50_gaslib40_453	13.2k	13.2k	0.0	4.4	13.2k	13.2k	0.0	4.8
dem_50_gaslib40_454	0.0	0.0	0.0	3.2	0.0	0.0	0.0	7.1
dem_50_gaslib40_455	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.3
dem_50_gaslib40_456	33.7k	33.7k	0.0	1.9	33.7k	33.7k	0.0	1.9
dem_50_gaslib40_457	0.0	0.0	0.0	10.7	0.0	0.0	0.0	1.5
dem_50_gaslib40_458	18.9k	18.9k	0.0	2.5	18.9k	18.9k	0.0	1.4
dem_50_gaslib40_459	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.8
dem_50_gaslib40_460	0.0	0.0	0.0	4.7	0.0	0.0	0.0	0.4
dem_50_gaslib40_461	8658.6	8658.6	0.0	3.0	8658.6	8658.6	0.0	3.0
dem_50_gaslib40_462	30.9k	30.9k	0.0	2.2	30.9k	30.9k	0.0	2.9
dem_50_gaslib40_463	0.0	0.0	0.0	2.8	0.0	0.0	0.0	0.7
dem_50_gaslib40_464	23.4k	23.4k	0.0	6.7	23.4k	23.4k	0.0	1.6
dem_50_gaslib40_465	6685.9	6685.9	0.0	4.4	6685.9	6685.9	0.0	1.3
dem_50_gaslib40_466	0.0	0.0	0.0	1.6	0.0	0.0	0.0	2.6
dem_50_gaslib40_467	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.4
dem_50_gaslib40_468	0.0	0.0	0.0	3.4	0.0	0.0	0.0	0.6
dem_50_gaslib40_469	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.3
dem_50_gaslib40_470	0.0	0.0	0.0	7.8	0.0	0.0	0.0	0.2
dem_50_gaslib40_471	24.5k	24.5k	0.0	2.7	24.5k	24.5k	0.0	1.5
dem_50_gaslib40_472	0.0	0.0	0.0	4.3	0.0	0.0	0.0	8.1
dem_50_gaslib40_473	0.0	0.0	0.0	1.9	0.0	0.0	0.0	1.7
dem_50_gaslib40_474	0.0	0.0	0.0	10.4	0.0	0.0	0.0	3.6
dem_50_gaslib40_475	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.8
dem_50_gaslib40_476	19.0k	19.0k	0.0	2.4	19.0k	19.0k	0.0	1.5
dem_50_gaslib40_477	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.5
dem_50_gaslib40_478	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.4
dem_50_gaslib40_479	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.5
dem_50_gaslib40_480	0.0	0.0	0.0	6.5	0.0	0.0	0.0	2.2
dem_50_gaslib40_481	0.0	0.0	0.0	2.9	0.0	0.0	0.0	1.7
dem_50_gaslib40_482	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.8
dem_50_gaslib40_483	0.0	0.0	0.0	3.5	0.0	0.0	0.0	0.4
dem_50_gaslib40_484	0.0	0.0	0.0	6.8	0.0	0.0	0.0	0.5
dem_50_gaslib40_485	0.0	0.0	0.0	2.5	0.0	0.0	0.0	3.5
dem_50_gaslib40_486	0.0	0.0	0.0	3.7	0.0	0.0	0.0	0.5
dem_50_gaslib40_487	6791.7	6791.7	0.0	2.7	6791.7	6791.0	0.0	1.3
dem_50_gaslib40_488	0.0	0.0	0.0	3.5	0.0	0.0	0.0	0.8
dem_50_gaslib40_489	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.5
dem_50_gaslib40_490	0.0	0.0	0.0	1.9	0.0	0.0	0.0	0.4
dem_50_gaslib40_491	0.0	0.0	0.0	1.5	0.0	0.0	0.0	0.6
dem_50_gaslib40_492	0.0	0.0	0.0	7.8	0.0	0.0	0.0	1.7
dem_50_gaslib40_493	0.0	0.0	0.0	3.0	0.0	0.0	0.0	1.9
dem_50_gaslib40_494	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.7
dem_50_gaslib40_495	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.3
dem_50_gaslib40_496	29.9k	29.9k	0.0	2.6	29.9k	29.9k	0.0	1.8
dem_50_gaslib40_497	9809.6	9809.5	0.0	1.9	9809.6	9808.6	0.0	1.6
dem_50_gaslib40_498	0.0	0.0	0.0	2.5	0.0	0.0	0.0	0.5
dem_50_gaslib40_499	12.9k	12.9k	0.0	2.5	12.9k	12.9k	0.0	1.6
dem_50_gaslib40_500	28.3k	28.3k	0.0	4.7	28.3k	28.3k	0.0	6.7

Table A.11: Detailed results of the discrete models on *GasLib-40* with $\mathcal{B} = 100$ as summarized in Figure 3.8b and Table 3.3b. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_1	133.0k	133.0k	0.0	151.5	133.0k	132.9k	0.1	1336.7	133.3k	105.7k	26.1	14400.0
dem_100_gaslib40_2	128.0k	128.0k	0.0	234.3	128.0k	128.0k	0.0	5053.1	129.4k	48.6k	166.1	14400.0
dem_100_gaslib40_3	178.2k	178.2k	0.0	701.9	178.2k	178.2k	0.0	5661.6	181.7k	122.2k	48.6	14400.0
dem_100_gaslib40_4	92.7k	92.7k	0.0	63.7	92.7k	92.7k	0.0	1816.5	92.7k	92.7k	0.0	4848.7
dem_100_gaslib40_5	128.1k	128.1k	0.0	1959.0	128.1k	114.1k	12.3	14400.0	163.6k	46.3k	253.0	14400.0
dem_100_gaslib40_6	285.4k	168.1k	69.8	14400.0	302.0k	179.3k	68.4	14400.0	319.1k	107.6k	196.6	14400.0
dem_100_gaslib40_7	200.0k	200.0k	0.0	3722.8	200.0k	200.0k	0.0	9074.9	210.0k	133.7k	57.1	14400.0
dem_100_gaslib40_8	220.2k	220.2k	0.0	8821.6	220.2k	202.1k	9.0	14400.0	227.5k	100.6k	126.1	14400.0
dem_100_gaslib40_9	235.9k	203.4k	16.0	14400.0	239.1k	168.4k	42.0	14400.0	236.2k	95.5k	147.3	14400.0
dem_100_gaslib40_10	332.5k	188.5k	76.4	14400.0	334.7k	182.8k	83.2	14400.0	335.3k	85.8k	290.7	14400.0
dem_100_gaslib40_11	212.9k	171.5k	24.1	14400.0	218.9k	138.6k	58.0	14400.0	230.5k	55.9k	312.3	14400.0
dem_100_gaslib40_12	209.9k	209.9k	0.0	14231.1	209.9k	180.3k	16.4	14400.0	228.5k	130.1k	75.7	14400.0
dem_100_gaslib40_13	290.2k	146.6k	97.9	14400.0	277.8k	178.3k	55.8	14400.0	343.4k	78.7k	336.2	14400.0
dem_100_gaslib40_14	172.8k	172.8k	0.0	835.5	172.8k	172.8k	0.0	10904.4	200.3k	88.5k	126.4	14400.0
dem_100_gaslib40_15	135.6k	135.6k	0.0	707.9	135.6k	135.6k	0.0	4571.3	208.5k	62.7k	232.8	14400.0
dem_100_gaslib40_16	147.0k	147.0k	0.0	2427.0	147.0k	147.0k	0.0	9423.1	151.3k	91.6k	65.2	14400.0
dem_100_gaslib40_17	301.9k	185.0k	63.2	14400.0	280.2k	203.0k	38.0	14400.0	311.7k	116.8k	166.9	14400.0
dem_100_gaslib40_18	142.1k	142.1k	0.0	311.5	142.1k	142.1k	0.0	5810.5	150.4k	76.9k	95.5	14400.0
dem_100_gaslib40_19	202.1k	202.1k	0.0	9125.6	202.1k	202.1k	0.0	14269.7	232.8k	118.4k	96.7	14400.0
dem_100_gaslib40_20	152.9k	152.9k	0.0	3242.4	152.9k	123.3k	24.0	14400.0	191.0k	41.3k	362.2	14400.0
dem_100_gaslib40_21	198.1k	198.1k	0.0	3354.8	198.1k	152.6k	29.8	14400.0	199.6k	51.4k	288.6	14400.0
dem_100_gaslib40_22	246.4k	246.4k	0.0	3380.6	246.4k	246.4k	0.0	10339.3	308.8k	163.7k	88.6	14400.0
dem_100_gaslib40_23	287.7k	178.8k	60.9	14400.0	280.2k	188.5k	48.7	14400.0	293.5k	78.9k	271.9	14400.0
dem_100_gaslib40_24	208.4k	160.3k	30.0	14400.0	200.9k	167.0k	20.3	14400.0	296.2k	64.0k	362.7	14400.0
dem_100_gaslib40_25	394.4k	176.1k	124.0	14400.0	421.1k	187.3k	124.8	14400.0	492.4k	85.8k	473.9	14400.0
dem_100_gaslib40_26	221.0k	221.0k	0.0	1483.7	221.0k	220.9k	0.0	5127.5	229.6k	159.9k	43.6	14400.0
dem_100_gaslib40_27	162.3k	162.3k	0.0	561.8	162.3k	162.3k	0.0	4798.6	165.3k	92.6k	78.5	14400.0
dem_100_gaslib40_28	138.1k	138.1k	0.0	9767.4	138.1k	138.1k	0.0	12863.2	153.9k	58.3k	164.1	14400.0
dem_100_gaslib40_29	151.6k	151.6k	0.0	1328.3	151.6k	151.6k	0.0	3740.8	172.4k	63.3k	172.4	14400.0
dem_100_gaslib40_30	297.4k	128.5k	131.5	14400.0	201.2k	141.7k	42.0	14400.0	223.9k	29.4k	662.2	14400.0
dem_100_gaslib40_31	409.1k	202.8k	101.7	14400.0	412.6k	192.7k	114.1	14400.0	417.2k	126.6k	229.5	14400.0
dem_100_gaslib40_32	70.6k	70.6k	0.0	34.5	70.6k	70.6k	0.0	1208.1	70.6k	70.6k	0.0	8261.5
dem_100_gaslib40_33	134.3k	134.3k	0.0	188.2	134.3k	134.3k	0.0	1025.4	134.3k	114.6k	17.2	14400.0
dem_100_gaslib40_34	115.3k	115.3k	0.0	25.9	115.3k	115.3k	0.0	483.7	115.3k	115.3k	0.0	1024.1
dem_100_gaslib40_35	180.5k	180.5k	0.0	1997.2	180.5k	180.5k	0.0	9606.8	208.3k	73.7k	182.7	14400.0
dem_100_gaslib40_36	182.9k	182.9k	0.0	5190.4	182.9k	182.9k	0.0	13637.4	204.2k	93.2k	119.0	14400.0
dem_100_gaslib40_37	164.5k	164.5k	0.0	944.2	165.6k	125.2k	32.3	14400.0	179.7k	62.1k	189.3	14400.0
dem_100_gaslib40_38	319.1k	199.6k	59.9	14400.0	351.5k	194.2k	81.0	14400.0	322.5k	126.1k	155.7	14400.0
dem_100_gaslib40_39	204.6k	204.6k	0.0	5408.5	205.9k	152.8k	34.8	14400.0	227.7k	84.6k	169.2	14400.0
dem_100_gaslib40_40	176.9k	176.9k	0.0	840.9	176.9k	176.9k	0.0	5317.0	207.3k	97.4k	112.9	14400.0
dem_100_gaslib40_41	279.5k	184.2k	51.8	14400.0	301.5k	190.7k	58.1	14400.0	297.9k	108.1k	175.5	14400.0
dem_100_gaslib40_42	243.9k	158.8k	53.6	14400.0	218.4k	218.4k	0.0	6858.4	314.2k	87.3k	259.8	14400.0
dem_100_gaslib40_43	193.5k	193.5k	0.0	1794.6	193.5k	181.4k	6.6	14400.0	268.6k	80.4k	234.0	14400.0
dem_100_gaslib40_44	211.3k	211.3k	0.0	10505.7	211.3k	168.6k	25.4	14400.0	217.8k	102.5k	112.5	14400.0
dem_100_gaslib40_45	198.6k	198.6k	0.0	7464.4	198.6k	198.6k	0.0	5691.5	208.5k	65.1k	220.2	14400.0
dem_100_gaslib40_46	161.6k	161.6k	0.0	930.7	161.6k	161.6k	0.0	12836.5	173.1k	67.8k	155.1	14400.0
dem_100_gaslib40_47	155.6k	155.6k	0.0	393.3	155.6k	155.6k	0.0	2981.5	161.2k	84.4k	91.1	14400.0
dem_100_gaslib40_48	291.7k	215.4k	35.4	14400.0	340.2k	194.9k	74.5	14400.0	306.2k	112.0k	173.3	14400.0
dem_100_gaslib40_49	181.7k	181.7k	0.0	567.1	181.7k	181.7k	0.0	5854.0	181.7k	128.5k	41.4	14400.0
dem_100_gaslib40_50	295.7k	203.7k	45.1	14400.0	274.7k	199.4k	37.8	14400.0	298.5k	101.3k	194.6	14400.0
dem_100_gaslib40_51	185.7k	185.7k	0.0	4709.5	185.7k	185.7k	0.0	7485.5	205.9k	100.7k	104.4	14400.0
dem_100_gaslib40_52	108.1k	108.1k	0.0	473.8	108.1k	108.1k	0.0	3981.2	125.7k	39.4k	218.7	14400.0
dem_100_gaslib40_53	210.8k	124.6k	69.2	14400.0	192.9k	164.4k	17.4	14400.0	237.5k	61.3k	287.7	14400.0
dem_100_gaslib40_54	207.2k	207.2k	0.0	1503.2	207.2k	207.2k	0.0	10428.7	208.9k	151.3k	38.1	14400.0
dem_100_gaslib40_55	238.8k	162.6k	46.9	14400.0	214.4k	157.2k	36.4	14400.0	243.4k	80.6k	201.9	14400.0
dem_100_gaslib40_56	125.9k	125.8k	0.1	2048.0	125.9k	125.9k	0.0	9016.0	131.7k	42.0k	213.5	14400.0
dem_100_gaslib40_57	169.9k	169.9k	0.0	3691.3	169.9k	128.2k	32.5	14400.0	211.1k	67.5k	212.8	14400.0
dem_100_gaslib40_58	101.5k	101.5k	0.0	1636.6	101.5k	101.5k	0.0	8714.3	112.9k	38.7k	192.1	14400.0
dem_100_gaslib40_59	245.4k	193.9k	26.6	14400.0	228.5k	228.5k	0.0	8889.4	295.1k	141.0k	109.3	14400.0
dem_100_gaslib40_60	124.3k	124.3k	0.0	261.7	124.3k	124.3k	0.0	1184.2	124.3k	97.4k	27.7	14400.0
dem_100_gaslib40_61	278.5k	187.7k	48.4	14400.0	266.7k	190.5k	40.0	14400.0	307.9k	123.0k	150.3	14400.0
dem_100_gaslib40_62	242.2k	198.4k	22.1	14400.0	1e+20	159.2k	–	14400.0	242.6k	99.9k	142.8	14400.0
dem_100_gaslib40_63	234.4k	161.2k	45.4	14400.0	218.0k	154.1k	41.5	14400.0	223.6k	67.0k	233.6	14400.0
dem_100_gaslib40_64	209.8k	209.8k	0.0	2893.7	212.5k	166.4k	27.7	14400.0	222.9k	81.5k	173.6	14400.0
dem_100_gaslib40_65	255.0k	167.9k	51.9	14400.0	251.5k	183.2k	37.3	14400.0	250.5k	88.3k	183.7	14400.0
dem_100_gaslib40_66	317.2k	184.0k	72.4	14400.0	356.4k	204.8k	74.0	14400.0	464.9k	91.8k	406.3	14400.0
dem_100_gaslib40_67	474.7k	213.1k	122.8	14400.0	452.0k	216.8k	108.4	14400.0	406.5k	167.3k	143.0	14400.0
dem_100_gaslib40_68	259.9k	173.9k	49.5	14400.0	261.4k	169.0k	54.7	14400.0	265.9k	46.5k	471.7	14400.0
dem_100_gaslib40_69	111.6k	111.6k	0.0	649.0	111.6k	111.6k	0.0	3100.6	123.0k	51.6k	138.2	14400.0
dem_100_gaslib40_70	306.2k	154.2k	98.6	14400.0	316.8k	204.9k	54.6	14400.0	322.0k	122.3k	163.3	14400.0
dem_100_gaslib40_71	204.1k	204.1k	0.0	10748.8	204.1k	164.0k	24.4	14400.0	206.0k	136.7k	50.7	14400.0
dem_100_gaslib40_72	182.7k	182.7k	0.0	815.3	182.7k	182.7k	0.0	3809.8	186.2k	97.7k	90.5	14400.0
dem_100_gaslib40_73	272.0k	199.2k	36.5	14400.0	272.9k	157.0k	73.8	14400.0	277.1k	83.6k	231.3	14400.0
dem_100_gaslib40_74	179.6k	179.6k	0.0	5032.9	179.6k	179.6k	0.0	10817.0	197.4k	80.8k	144.4	14400.0

continued on next page

Table A.11: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_75	82.0k	82.0k	0.0	96.1	82.0k	82.0k	0.0	998.4	82.0k	43.0k	90.8	14400.0
dem_100_gaslib40_76	387.9k	204.7k	89.5	14400.0	417.1k	230.1k	81.2	14400.0	568.9k	124.0k	358.9	14400.0
dem_100_gaslib40_77	181.1k	181.1k	0.0	1143.5	181.1k	181.1k	0.0	6298.4	213.2k	103.0k	106.9	14400.0
dem_100_gaslib40_78	201.0k	201.0k	0.0	11881.8	201.0k	201.0k	0.0	13083.2	244.7k	86.4k	183.3	14400.0
dem_100_gaslib40_79	202.9k	202.9k	0.0	8071.8	202.9k	202.9k	0.0	6077.0	215.8k	62.4k	245.9	14400.0
dem_100_gaslib40_80	499.4k	187.7k	166.1	14400.0	644.4k	258.7k	149.1	14400.0	756.2k	144.8k	422.2	14400.0
dem_100_gaslib40_81	225.2k	131.1k	71.8	14400.0	226.2k	156.6k	44.5	14400.0	245.2k	35.7k	587.9	14400.0
dem_100_gaslib40_82	237.0k	158.7k	49.3	14400.0	188.1k	174.8k	7.6	14400.0	237.5k	97.3k	144.1	14400.0
dem_100_gaslib40_83	207.5k	207.5k	0.0	6319.1	207.5k	207.5k	0.0	13618.5	226.6k	82.6k	174.2	14400.0
dem_100_gaslib40_84	181.6k	181.6k	0.0	716.0	181.6k	181.6k	0.0	5675.4	182.9k	116.2k	57.4	14400.0
dem_100_gaslib40_85	277.1k	167.7k	65.3	14400.0	293.0k	149.7k	95.7	14400.0	407.1k	71.4k	469.9	14400.0
dem_100_gaslib40_86	86.2k	86.2k	0.0	247.4	86.2k	86.2k	0.0	3007.3	86.8k	44.7k	94.4	14400.0
dem_100_gaslib40_87	164.6k	164.6k	0.0	207.4	164.6k	164.6k	0.0	4690.6	171.4k	95.3k	79.9	14400.0
dem_100_gaslib40_88	265.5k	172.2k	54.1	14400.0	239.7k	194.4k	23.3	14400.0	250.3k	98.3k	154.6	14400.0
dem_100_gaslib40_89	165.1k	165.1k	0.0	5045.4	165.1k	165.1k	0.0	9592.9	177.7k	70.5k	152.2	14400.0
dem_100_gaslib40_90	148.0k	148.0k	0.0	1073.1	148.0k	148.0k	0.0	7540.0	154.9k	98.5k	57.3	14400.0
dem_100_gaslib40_91	228.6k	214.2k	6.8	14400.0	225.7k	225.7k	0.0	8857.9	229.4k	120.0k	91.1	14400.0
dem_100_gaslib40_92	367.3k	228.7k	60.6	14400.0	334.0k	237.6k	40.6	14400.0	1e+20	153.7k	–	14400.0
dem_100_gaslib40_93	267.6k	177.1k	51.2	14400.0	255.4k	224.3k	13.9	14400.0	270.6k	121.7k	122.4	14400.0
dem_100_gaslib40_94	232.6k	185.8k	25.2	14400.0	214.5k	214.5k	0.0	9905.9	246.1k	116.2k	111.7	14400.0
dem_100_gaslib40_95	181.3k	181.3k	0.0	8507.0	181.3k	181.3k	0.0	14272.6	192.9k	94.3k	104.6	14400.0
dem_100_gaslib40_96	218.5k	218.5k	0.0	5660.6	218.5k	197.9k	10.4	14400.0	262.8k	88.6k	196.6	14400.0
dem_100_gaslib40_97	169.9k	169.9k	0.0	1398.5	169.9k	169.9k	0.0	1824.2	172.8k	123.2k	40.2	14400.0
dem_100_gaslib40_98	404.2k	199.4k	102.7	14400.0	409.9k	211.9k	93.4	14400.0	386.7k	135.0k	186.4	14400.0
dem_100_gaslib40_99	152.6k	152.6k	0.0	476.6	152.6k	152.6k	0.0	5007.8	157.7k	103.9k	51.7	14400.0
dem_100_gaslib40_100	128.8k	128.8k	0.0	106.5	128.8k	128.8k	0.0	2102.9	129.6k	100.7k	28.7	14400.0
dem_100_gaslib40_101	163.2k	163.2k	0.0	1134.8	163.2k	163.2k	0.0	5034.2	167.1k	117.8k	41.9	14400.0
dem_100_gaslib40_102	141.7k	141.7k	0.0	623.8	141.7k	141.7k	0.0	8053.6	156.0k	95.8k	62.9	14400.0
dem_100_gaslib40_103	315.9k	164.2k	92.4	14400.0	295.0k	196.1k	50.4	14400.0	290.6k	121.2k	139.6	14400.0
dem_100_gaslib40_104	195.7k	195.7k	0.0	7508.0	195.7k	195.7k	0.0	12388.0	283.7k	59.6k	376.0	14400.0
dem_100_gaslib40_105	275.4k	216.0k	27.5	14400.0	265.7k	192.2k	38.3	14400.0	343.7k	132.8k	158.8	14400.0
dem_100_gaslib40_106	196.3k	196.3k	0.0	5843.3	196.3k	167.9k	16.9	14400.0	246.6k	86.9k	183.8	14400.0
dem_100_gaslib40_107	219.6k	160.8k	36.6	14400.0	208.9k	147.7k	41.5	14400.0	207.8k	102.9k	102.0	14400.0
dem_100_gaslib40_108	311.2k	178.8k	74.0	14400.0	305.1k	194.9k	56.6	14400.0	391.5k	116.9k	234.8	14400.0
dem_100_gaslib40_109	138.0k	138.0k	0.0	131.5	138.0k	138.0k	0.0	1981.2	161.5k	77.6k	108.1	14400.0
dem_100_gaslib40_110	130.7k	130.7k	0.0	1239.4	130.7k	130.7k	0.0	3801.8	147.3k	71.0k	107.5	14400.0
dem_100_gaslib40_111	288.2k	172.7k	66.9	14400.0	335.8k	176.5k	90.3	14400.0	295.9k	116.6k	153.7	14400.0
dem_100_gaslib40_112	222.8k	222.8k	0.0	8328.6	222.8k	222.8k	0.0	9416.1	239.0k	122.5k	95.2	14400.0
dem_100_gaslib40_113	170.3k	170.3k	0.0	7480.3	170.3k	170.3k	0.0	5206.7	177.0k	92.3k	91.8	14400.0
dem_100_gaslib40_114	202.3k	202.3k	0.0	3514.5	202.3k	202.2k	0.1	7639.7	206.4k	131.5k	56.9	14400.0
dem_100_gaslib40_115	140.3k	140.3k	0.0	727.1	140.3k	140.3k	0.0	2164.8	143.9k	90.1k	59.7	14400.0
dem_100_gaslib40_116	469.2k	227.9k	105.9	14400.0	462.0k	266.5k	73.4	14400.0	519.6k	182.7k	184.4	14400.0
dem_100_gaslib40_117	167.7k	167.7k	0.0	765.2	167.7k	167.7k	0.0	5058.6	169.8k	104.8k	62.0	14400.0
dem_100_gaslib40_118	163.7k	163.7k	0.0	1049.3	163.7k	163.7k	0.0	4550.4	173.4k	90.5k	91.5	14400.0
dem_100_gaslib40_119	83.0k	83.0k	0.0	19.1	83.0k	83.0k	0.0	537.4	83.0k	83.0k	0.0	747.5
dem_100_gaslib40_120	470.4k	196.7k	139.1	14400.0	682.3k	234.1k	191.4	14400.0	467.1k	154.8k	201.8	14400.0
dem_100_gaslib40_121	136.9k	136.9k	0.0	12295.8	136.9k	136.9k	0.0	1629.9	169.4k	74.9k	126.2	14400.0
dem_100_gaslib40_122	166.4k	166.4k	0.0	6370.1	166.4k	166.4k	0.0	4021.0	173.5k	50.2k	245.8	14400.0
dem_100_gaslib40_123	119.6k	119.5k	0.0	142.2	119.6k	119.6k	0.0	2451.7	121.6k	75.0k	62.2	14400.0
dem_100_gaslib40_124	160.3k	160.3k	0.0	1503.6	160.3k	160.3k	0.0	2983.2	167.3k	70.8k	136.2	14400.0
dem_100_gaslib40_125	164.5k	164.5k	0.0	1257.4	164.5k	164.5k	0.0	3473.7	176.3k	94.0k	87.5	14400.0
dem_100_gaslib40_126	164.6k	164.6k	0.0	423.6	164.6k	164.6k	0.0	7390.8	166.5k	112.6k	47.9	14400.0
dem_100_gaslib40_127	216.2k	216.2k	0.0	6814.7	232.6k	172.1k	35.1	14400.0	231.4k	143.3k	61.5	14400.0
dem_100_gaslib40_128	150.1k	150.1k	0.0	2697.1	150.1k	150.1k	0.0	9052.8	162.7k	50.8k	220.4	14400.0
dem_100_gaslib40_129	179.5k	179.5k	0.0	3272.1	179.5k	179.5k	0.0	9655.9	211.4k	80.8k	161.6	14400.0
dem_100_gaslib40_130	222.2k	222.2k	0.0	13585.6	222.2k	222.2k	0.0	13085.8	307.2k	75.7k	305.8	14400.0
dem_100_gaslib40_131	158.7k	158.7k	0.0	685.9	158.7k	158.7k	0.0	9043.2	175.0k	110.8k	58.0	14400.0
dem_100_gaslib40_132	231.9k	231.9k	0.0	4345.6	231.9k	231.9k	0.0	4901.0	247.0k	170.4k	44.9	14400.0
dem_100_gaslib40_133	214.4k	214.4k	0.0	6472.6	214.4k	214.4k	0.0	9974.2	218.9k	126.2k	73.5	14400.0
dem_100_gaslib40_134	178.1k	178.1k	0.0	3337.8	178.1k	178.1k	0.0	5102.7	194.8k	91.2k	113.7	14400.0
dem_100_gaslib40_135	227.8k	187.4k	21.6	14400.0	223.9k	223.9k	0.0	12978.3	256.0k	129.5k	97.8	14400.0
dem_100_gaslib40_136	230.7k	167.4k	37.8	14400.0	243.7k	165.1k	47.6	14400.0	296.8k	105.7k	180.9	14400.0
dem_100_gaslib40_137	139.4k	139.4k	0.0	485.0	139.4k	139.4k	0.0	8604.7	141.7k	67.1k	111.1	14400.0
dem_100_gaslib40_138	320.0k	194.9k	64.2	14400.0	321.5k	187.4k	71.6	14400.0	283.9k	87.8k	223.2	14400.0
dem_100_gaslib40_139	239.0k	239.0k	0.0	2528.4	239.0k	239.0k	0.0	2882.4	239.8k	180.5k	32.9	14400.0
dem_100_gaslib40_140	215.4k	215.4k	0.0	8378.7	215.4k	215.4k	0.0	10954.5	225.1k	114.0k	97.5	14400.0
dem_100_gaslib40_141	289.8k	159.5k	81.7	14400.0	224.2k	139.5k	60.7	14400.0	246.1k	63.5k	287.4	14400.0
dem_100_gaslib40_142	301.3k	177.0k	70.2	14400.0	233.9k	190.7k	22.7	14400.0	256.7k	135.8k	89.1	14400.0
dem_100_gaslib40_143	309.6k	202.6k	52.8	14400.0	325.6k	189.6k	71.7	14400.0	318.1k	88.3k	260.3	14400.0
dem_100_gaslib40_144	213.8k	190.5k	12.2	14400.0	210.6k	210.6k	0.0	5602.6	225.5k	89.2k	152.8	14400.0
dem_100_gaslib40_145	230.8k	159.2k	45.0	14400.0	212.2k	212.2k	0.0	8074.5	228.9k	79.5k	188.0	14400.0
dem_100_gaslib40_146	190.4k	190.4k	0.0	4269.1	190.4k	190.4k	0.0	3539.7	206.8k	73.1k	182.8	14400.0
dem_100_gaslib40_147	196.0k	196.0k	0.0	3338.4	196.0k	196.0k	0.0	6285.3	249.6k	108.5k	130.1	14400.0
dem_100_gaslib40_148	153.5k	153.5k	0.0	1256.3	153.5k	153.5k	0.0	8959.1	160.4k	60.4k	165.4	14400.0
dem_100_gaslib40_149	203.6k	203.6k	0.0	13599.2	203.6k	203.6k	0.0	4589.4	211.1k	134.5k	57.0	14400.0
dem_100_gaslib40_150	369.7k	148.5k	149.0	14400.0	525.9k	213.1k	146.8	14400.0	428.3k	112.2k	281.6	14400.0
dem_100_gaslib40_151	238.4k	238.4k	0.0	4193.5	238.4k	238.4k	0.0	7679.5	240.4k	182.9k	31.4	14400.0
dem_100_gaslib40_152	183.8k	183.8k	0.0	4455.6	183.8k	183.8k	0.0	9873.7	188.9k	111.2k	69.8	14400.0

continued on next page

Table A.11: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_153	325.5k	157.0k	107.3	14400.0	305.4k	198.9k	53.5	14400.0	339.4k	124.4k	172.8	14400.0
dem_100_gaslib40_154	245.6k	157.9k	55.6	14400.0	234.4k	143.5k	63.3	14400.0	233.7k	76.0k	207.6	14400.0
dem_100_gaslib40_155	198.1k	198.1k	0.0	4907.3	198.1k	198.1k	0.0	6188.9	245.4k	102.8k	138.6	14400.0
dem_100_gaslib40_156	195.1k	195.1k	0.0	2826.7	195.1k	195.1k	0.0	5498.7	204.3k	118.9k	71.9	14400.0
dem_100_gaslib40_157	347.0k	173.7k	99.7	14400.0	306.2k	175.1k	74.9	14400.0	413.8k	112.4k	268.0	14400.0
dem_100_gaslib40_158	151.3k	151.3k	0.0	2968.6	151.3k	151.3k	0.0	5201.9	157.1k	86.3k	82.1	14400.0
dem_100_gaslib40_159	210.7k	210.7k	0.0	6820.9	210.7k	210.7k	0.0	12127.5	228.5k	100.1k	128.3	14400.0
dem_100_gaslib40_160	394.4k	202.2k	95.0	14400.0	367.2k	204.0k	80.0	14400.0	419.5k	130.9k	220.4	14400.0
dem_100_gaslib40_161	554.1k	249.4k	122.2	14400.0	511.2k	303.5k	68.4	14400.0	635.8k	192.9k	229.6	14400.0
dem_100_gaslib40_162	227.9k	227.9k	0.0	1544.8	227.9k	227.9k	0.0	3592.9	233.2k	170.8k	36.5	14400.0
dem_100_gaslib40_163	140.8k	140.8k	0.0	62.7	140.8k	140.8k	0.0	1081.9	144.3k	115.2k	25.2	14400.0
dem_100_gaslib40_164	190.9k	125.7k	51.9	14400.0	183.1k	183.1k	0.0	5297.0	197.3k	78.4k	151.6	14400.0
dem_100_gaslib40_165	183.1k	183.1k	0.0	2418.9	183.1k	183.1k	0.0	3493.0	225.9k	78.1k	189.4	14400.0
dem_100_gaslib40_166	286.3k	286.3k	0.0	7009.4	286.3k	286.3k	0.0	7261.4	304.1k	188.9k	61.0	14400.0
dem_100_gaslib40_167	186.8k	186.8k	0.0	2416.7	186.8k	186.8k	0.0	4341.9	195.8k	99.9k	96.0	14400.0
dem_100_gaslib40_168	264.9k	226.8k	16.8	14400.0	253.6k	253.6k	0.0	13915.3	271.5k	155.3k	74.8	14400.0
dem_100_gaslib40_169	101.4k	101.4k	0.0	32.8	101.4k	101.4k	0.0	1179.9	101.4k	101.4k	0.0	1627.8
dem_100_gaslib40_170	133.5k	133.5k	0.0	2816.1	133.5k	116.4k	14.6	14400.0	161.7k	30.8k	424.7	14400.0
dem_100_gaslib40_171	518.3k	192.3k	169.6	14400.0	575.4k	246.5k	133.5	14400.0	587.4k	184.8k	217.8	14400.0
dem_100_gaslib40_172	223.9k	223.9k	0.0	13675.2	224.6k	179.5k	25.1	14400.0	263.9k	84.2k	213.4	14400.0
dem_100_gaslib40_173	112.3k	112.3k	0.0	3802.4	112.3k	112.3k	0.0	995.9	153.7k	57.1k	169.2	14400.0
dem_100_gaslib40_174	194.5k	194.5k	0.0	7263.4	194.5k	194.5k	0.0	4699.9	196.2k	91.3k	114.8	14400.0
dem_100_gaslib40_175	524.7k	223.0k	135.3	14400.0	522.9k	257.2k	103.3	14400.0	511.5k	178.2k	187.1	14400.0
dem_100_gaslib40_176	215.1k	215.1k	0.0	8149.6	215.1k	215.1k	0.0	12656.7	218.4k	124.3k	75.7	14400.0
dem_100_gaslib40_177	231.1k	170.5k	35.5	14400.0	229.1k	188.8k	21.4	14400.0	253.0k	101.2k	149.9	14400.0
dem_100_gaslib40_178	511.1k	223.2k	128.9	14400.0	695.1k	249.6k	178.5	14400.0	502.3k	190.6k	163.5	14400.0
dem_100_gaslib40_179	227.3k	127.3k	78.5	14400.0	210.2k	188.2k	11.7	14400.0	266.8k	58.3k	357.3	14400.0
dem_100_gaslib40_180	150.8k	150.8k	0.0	910.6	150.8k	150.8k	0.0	2875.8	155.0k	95.1k	63.0	14400.0
dem_100_gaslib40_181	378.3k	195.9k	93.1	14400.0	402.4k	203.5k	97.8	14400.0	432.0k	99.3k	335.2	14400.0
dem_100_gaslib40_182	182.2k	182.2k	0.0	890.9	182.2k	182.2k	0.0	6963.4	192.8k	120.9k	59.5	14400.0
dem_100_gaslib40_183	659.8k	215.2k	206.7	14400.0	794.5k	357.5k	122.2	14400.0	748.3k	236.9k	215.9	14400.0
dem_100_gaslib40_184	248.4k	165.3k	50.3	14400.0	230.3k	158.1k	45.6	14400.0	253.1k	81.7k	209.9	14400.0
dem_100_gaslib40_185	160.2k	160.2k	0.0	4729.6	160.2k	160.2k	0.0	5852.5	190.5k	61.7k	208.8	14400.0
dem_100_gaslib40_186	170.0k	170.0k	0.0	1513.6	170.0k	170.0k	0.0	5351.8	176.4k	93.2k	89.2	14400.0
dem_100_gaslib40_187	263.0k	152.6k	72.3	14400.0	296.2k	169.0k	75.3	14400.0	254.0k	88.9k	185.6	14400.0
dem_100_gaslib40_188	302.2k	213.5k	41.5	14400.0	313.3k	194.7k	60.9	14400.0	312.6k	134.4k	132.6	14400.0
dem_100_gaslib40_189	196.4k	159.6k	23.1	14400.0	196.3k	196.3k	0.0	10794.3	241.0k	85.5k	181.9	14400.0
dem_100_gaslib40_190	150.0k	150.0k	0.0	1004.3	150.0k	150.0k	0.0	6679.0	152.9k	104.8k	46.0	14400.0
dem_100_gaslib40_191	238.9k	238.9k	0.0	2188.8	238.9k	238.9k	0.0	3547.2	241.8k	197.8k	22.2	14400.0
dem_100_gaslib40_192	268.1k	177.0k	51.4	14400.0	215.2k	215.2k	0.0	11731.4	232.6k	113.1k	105.7	14400.0
dem_100_gaslib40_193	64.3k	64.3k	0.0	182.8	64.3k	64.3k	0.0	1318.7	64.3k	64.3k	0.0	8766.0
dem_100_gaslib40_194	175.1k	175.1k	0.0	1406.0	175.1k	175.1k	0.0	4367.6	178.5k	97.0k	84.1	14400.0
dem_100_gaslib40_195	430.8k	182.5k	136.0	14400.0	443.4k	213.2k	108.0	14400.0	490.4k	162.4k	202.0	14400.0
dem_100_gaslib40_196	200.6k	200.6k	0.0	653.6	200.6k	200.6k	0.0	5532.4	201.5k	168.8k	19.4	14400.0
dem_100_gaslib40_197	204.1k	204.1k	0.0	6697.7	204.1k	193.7k	5.4	14400.0	217.0k	77.4k	180.5	14400.0
dem_100_gaslib40_198	132.9k	132.9k	0.0	71.8	132.9k	132.9k	0.0	2371.8	132.9k	132.9k	0.0	13569.7
dem_100_gaslib40_199	162.0k	162.0k	0.0	2250.0	162.0k	162.0k	0.0	5320.8	178.1k	66.7k	167.0	14400.0
dem_100_gaslib40_200	299.8k	144.4k	107.6	14400.0	290.8k	195.3k	48.9	14400.0	311.0k	121.7k	155.5	14400.0
dem_100_gaslib40_201	304.8k	197.5k	54.3	14400.0	292.8k	206.0k	42.2	14400.0	286.0k	86.4k	230.9	14400.0
dem_100_gaslib40_202	243.4k	243.4k	0.0	4629.4	246.9k	189.6k	30.2	14400.0	262.5k	92.8k	182.8	14400.0
dem_100_gaslib40_203	202.4k	202.4k	0.0	964.2	202.4k	202.4k	0.0	6043.8	208.3k	152.8k	36.3	14400.0
dem_100_gaslib40_204	205.2k	205.1k	0.1	9045.3	205.2k	148.5k	38.1	14400.0	270.2k	70.2k	284.6	14400.0
dem_100_gaslib40_205	206.4k	206.4k	0.0	5578.1	206.4k	206.4k	0.0	13569.7	260.3k	148.4k	75.4	14400.0
dem_100_gaslib40_206	196.6k	196.6k	0.0	718.5	196.6k	196.6k	0.0	6495.2	202.3k	126.9k	59.5	14400.0
dem_100_gaslib40_207	238.9k	175.6k	36.1	14400.0	220.9k	220.9k	0.0	8892.5	231.8k	92.9k	149.5	14400.0
dem_100_gaslib40_208	254.2k	208.9k	21.7	14400.0	243.1k	196.2k	23.9	14400.0	275.7k	88.6k	211.3	14400.0
dem_100_gaslib40_209	216.3k	216.3k	0.0	13716.5	216.3k	183.9k	17.6	14400.0	221.7k	74.1k	199.3	14400.0
dem_100_gaslib40_210	148.2k	148.2k	0.0	7957.4	148.9k	107.0k	39.2	14400.0	152.5k	38.2k	299.6	14400.0
dem_100_gaslib40_211	138.8k	138.8k	0.0	638.8	138.8k	138.7k	0.1	1287.9	142.3k	112.4k	26.6	14400.0
dem_100_gaslib40_212	196.6k	196.6k	0.0	891.2	196.6k	196.6k	0.0	3922.9	198.1k	153.9k	28.7	14400.0
dem_100_gaslib40_213	119.9k	119.9k	0.0	284.3	119.9k	119.9k	0.0	4690.1	125.1k	82.2k	52.3	14400.0
dem_100_gaslib40_214	192.4k	192.4k	0.0	6330.7	192.4k	165.9k	16.0	14400.0	200.2k	130.7k	53.2	14400.0
dem_100_gaslib40_215	248.0k	207.6k	19.4	14400.0	242.1k	196.1k	23.4	14400.0	307.3k	114.0k	169.6	14400.0
dem_100_gaslib40_216	178.0k	178.0k	0.0	6401.3	178.0k	178.0k	0.0	4046.9	193.1k	54.8k	252.5	14400.0
dem_100_gaslib40_217	439.1k	216.3k	103.0	14400.0	451.0k	231.4k	94.9	14400.0	419.6k	167.3k	150.8	14400.0
dem_100_gaslib40_218	151.5k	151.5k	0.0	259.4	151.5k	151.5k	0.0	1672.9	158.9k	79.4k	100.2	14400.0
dem_100_gaslib40_219	200.2k	200.2k	0.0	2880.7	200.2k	200.0k	0.1	11898.5	229.5k	136.1k	68.7	14400.0
dem_100_gaslib40_220	144.3k	144.3k	0.0	424.9	144.3k	144.2k	0.1	2322.7	146.8k	85.2k	72.3	14400.0
dem_100_gaslib40_221	285.6k	200.7k	42.3	14400.0	279.1k	212.4k	31.4	14400.0	289.6k	81.4k	255.5	14400.0
dem_100_gaslib40_222	219.9k	219.9k	0.0	9035.1	219.9k	186.9k	17.7	14400.0	282.8k	82.0k	245.0	14400.0
dem_100_gaslib40_223	321.6k	132.8k	142.1	14400.0	685.7k	164.2k	317.5	14400.0	293.9k	56.0k	424.4	14400.0
dem_100_gaslib40_224	248.7k	168.3k	47.8	14400.0	238.3k	238.3k	0.0	7326.4	267.1k	139.3k	91.7	14400.0
dem_100_gaslib40_225	196.2k	143.9k	36.3	14400.0	192.5k	192.5k	0.0	10472.9	220.6k	39.7k	456.4	14400.0
dem_100_gaslib40_226	273.9k	231.7k	18.2	14400.0	261.9k	246.0k	6.4	14400.0	305.7k	172.0k	77.7	14400.0
dem_100_gaslib40_227	124.8k	124.8k	0.0	2771.9	124.8k	124.8k	0.0	6349.5	214.7k	113.2k	89.6	14400.0
dem_100_gaslib40_228	151.1k	151.1k	0.0	68.4	151.1k	151.1k	0.0	965.5	152.3k	121.1k	25.7	14400.0
dem_100_gaslib40_229	354.1k	169.6k	108.8	14400.0	454.3k	183.8k	147.1	14400.0	511.7k	74.4k	587.4	14400.0
dem_100_gaslib40_230	566.2k	261.0k	116.9	14400.0	636.4k	308.8k	106.1	14400.0	776.7k	190.4k	308.0	14400.0

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Table A.11: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_231	263.3k	178.9k	47.2	14400.0	256.1k	160.7k	59.4	14400.0	287.7k	74.2k	287.9	14400.0
dem_100_gaslib40_232	178.3k	178.3k	0.0	435.9	178.3k	178.3k	0.0	1304.1	181.0k	135.5k	33.5	14400.0
dem_100_gaslib40_233	304.7k	155.9k	95.5	14400.0	379.0k	174.0k	117.8	14400.0	332.3k	79.8k	316.4	14400.0
dem_100_gaslib40_234	175.1k	175.1k	0.0	2217.4	175.1k	175.1k	0.0	5716.1	176.5k	94.9k	85.9	14400.0
dem_100_gaslib40_235	296.4k	155.1k	91.1	14400.0	283.5k	208.7k	35.8	14400.0	418.3k	65.8k	535.4	14400.0
dem_100_gaslib40_236	436.3k	187.5k	132.7	14400.0	526.9k	216.5k	143.4	14400.0	405.7k	111.4k	264.3	14400.0
dem_100_gaslib40_237	249.5k	224.0k	11.4	14400.0	246.6k	215.1k	14.6	14400.0	258.4k	77.7k	232.4	14400.0
dem_100_gaslib40_238	306.9k	143.4k	114.0	14400.0	246.7k	167.9k	46.9	14400.0	258.7k	100.8k	156.6	14400.0
dem_100_gaslib40_239	211.3k	211.3k	0.0	2395.2	211.3k	211.3k	0.0	5012.2	227.0k	116.9k	94.3	14400.0
dem_100_gaslib40_240	216.8k	216.8k	0.0	715.1	216.8k	216.8k	0.0	3675.1	220.6k	174.7k	26.2	14400.0
dem_100_gaslib40_241	212.4k	212.4k	0.0	4740.7	212.4k	212.4k	0.0	3447.9	214.1k	163.7k	30.8	14400.0
dem_100_gaslib40_242	382.4k	235.2k	62.6	14400.0	431.2k	195.1k	121.0	14400.0	398.2k	164.5k	142.1	14400.0
dem_100_gaslib40_243	162.3k	162.3k	0.0	3110.3	162.3k	162.3k	0.0	6879.1	162.3k	112.8k	43.9	14400.0
dem_100_gaslib40_244	213.3k	213.3k	0.0	9444.8	213.3k	213.3k	0.0	12133.4	217.3k	139.5k	55.7	14400.0
dem_100_gaslib40_245	470.2k	224.1k	109.8	14400.0	523.2k	274.8k	90.4	14400.0	499.9k	170.4k	193.3	14400.0
dem_100_gaslib40_246	149.6k	149.6k	0.0	3375.8	149.6k	149.6k	0.0	7164.7	154.1k	74.4k	107.0	14400.0
dem_100_gaslib40_247	459.1k	223.5k	105.4	14400.0	644.5k	208.9k	208.4	14400.0	448.7k	138.6k	223.6	14400.0
dem_100_gaslib40_248	173.1k	173.1k	0.0	1090.0	173.1k	173.1k	0.0	9208.5	197.8k	91.9k	115.4	14400.0
dem_100_gaslib40_249	147.7k	147.7k	0.0	7912.3	147.7k	147.7k	0.0	5494.2	150.7k	47.0k	220.7	14400.0
dem_100_gaslib40_250	334.3k	155.3k	115.3	14400.0	343.5k	197.1k	74.3	14400.0	343.7k	85.2k	303.3	14400.0
dem_100_gaslib40_251	216.3k	216.3k	0.0	9888.4	216.8k	165.1k	31.3	14400.0	248.5k	89.9k	176.6	14400.0
dem_100_gaslib40_252	150.0k	150.0k	0.0	827.0	150.0k	150.0k	0.0	2685.5	157.3k	97.7k	61.0	14400.0
dem_100_gaslib40_253	188.2k	188.2k	0.0	973.2	188.2k	188.2k	0.0	5631.0	203.5k	109.3k	86.2	14400.0
dem_100_gaslib40_254	256.1k	163.9k	56.3	14400.0	258.7k	154.7k	67.2	14400.0	600.9k	48.3k	1143.6	14400.0
dem_100_gaslib40_255	213.9k	213.9k	0.0	2637.5	213.9k	213.9k	0.0	6959.2	237.6k	127.2k	86.9	14400.0
dem_100_gaslib40_256	138.1k	138.1k	0.0	1665.5	138.1k	138.1k	0.0	4330.5	145.8k	68.5k	112.8	14400.0
dem_100_gaslib40_257	121.8k	121.8k	0.0	216.6	121.8k	121.8k	0.0	2089.2	122.9k	102.9k	19.4	14400.0
dem_100_gaslib40_258	492.4k	199.5k	146.8	14400.0	484.6k	274.1k	76.8	14400.0	572.4k	162.1k	253.1	14400.0
dem_100_gaslib40_259	130.8k	130.8k	0.0	75.5	130.8k	130.8k	0.0	1554.1	130.8k	115.0k	13.7	14400.0
dem_100_gaslib40_260	117.4k	117.4k	0.0	210.8	117.4k	117.4k	0.0	1855.2	129.6k	34.9k	271.9	14400.0
dem_100_gaslib40_261	244.8k	190.3k	28.6	14400.0	220.5k	203.3k	8.4	14400.0	231.0k	137.0k	68.6	14400.0
dem_100_gaslib40_262	479.8k	191.1k	151.1	14400.0	539.3k	275.4k	95.8	14400.0	787.3k	140.7k	459.7	14400.0
dem_100_gaslib40_263	322.0k	208.2k	54.7	14400.0	456.3k	207.6k	119.8	14400.0	353.6k	107.6k	228.6	14400.0
dem_100_gaslib40_264	145.6k	145.6k	0.0	389.2	145.6k	145.6k	0.0	4325.1	146.8k	101.1k	45.2	14400.0
dem_100_gaslib40_265	720.3k	172.1k	318.5	14400.0	351.2k	187.1k	87.7	14400.0	408.5k	64.2k	536.4	14400.0
dem_100_gaslib40_266	194.2k	194.2k	0.0	4810.9	194.2k	194.2k	0.0	13342.9	214.8k	70.0k	207.0	14400.0
dem_100_gaslib40_267	253.6k	147.1k	72.4	14400.0	362.4k	154.6k	134.4	14400.0	262.8k	61.6k	326.7	14400.0
dem_100_gaslib40_268	184.9k	184.9k	0.0	1444.9	184.9k	184.9k	0.0	5951.9	210.5k	115.4k	82.5	14400.0
dem_100_gaslib40_269	178.4k	178.4k	0.0	2813.2	178.4k	178.4k	0.0	2589.7	185.7k	89.0k	108.7	14400.0
dem_100_gaslib40_270	142.0k	142.0k	0.0	241.8	142.0k	142.0k	0.0	1533.0	152.3k	114.0k	33.6	14400.0
dem_100_gaslib40_271	218.7k	218.7k	0.0	1716.5	218.7k	218.7k	0.0	2723.1	232.8k	175.0k	33.0	14400.0
dem_100_gaslib40_272	259.5k	259.5k	0.0	4525.2	259.5k	259.5k	0.0	5029.6	279.6k	145.8k	91.7	14400.0
dem_100_gaslib40_273	164.0k	164.0k	0.0	1192.0	164.0k	164.0k	0.0	4754.4	164.0k	107.4k	52.7	14400.0
dem_100_gaslib40_274	598.0k	241.0k	148.1	14400.0	581.2k	290.2k	100.3	14400.0	671.9k	163.9k	309.8	14400.0
dem_100_gaslib40_275	538.7k	240.9k	123.6	14400.0	684.8k	301.0k	127.5	14400.0	583.6k	195.5k	198.5	14400.0
dem_100_gaslib40_276	428.1k	226.8k	88.8	14400.0	631.3k	224.0k	181.8	14400.0	468.7k	160.7k	191.7	14400.0
dem_100_gaslib40_277	144.7k	144.7k	0.0	563.2	144.7k	144.7k	0.0	2675.1	146.1k	74.8k	95.4	14400.0
dem_100_gaslib40_278	172.8k	172.8k	0.0	11747.4	172.8k	172.8k	0.0	14185.9	199.6k	57.7k	245.9	14400.0
dem_100_gaslib40_279	156.7k	156.7k	0.0	2667.7	156.7k	156.7k	0.0	7907.1	161.8k	54.0k	199.8	14400.0
dem_100_gaslib40_280	186.9k	186.9k	0.0	6715.9	186.9k	186.9k	0.0	9684.5	215.1k	105.5k	103.9	14400.0
dem_100_gaslib40_281	164.5k	164.5k	0.0	2986.9	164.5k	164.4k	0.1	11542.9	178.1k	56.5k	215.1	14400.0
dem_100_gaslib40_282	118.2k	118.2k	0.0	3037.5	118.2k	118.2k	0.0	7179.6	124.4k	29.4k	323.6	14400.0
dem_100_gaslib40_283	174.7k	174.7k	0.0	221.0	174.7k	174.7k	0.0	6859.4	181.6k	100.0k	81.6	14400.0
dem_100_gaslib40_284	218.3k	122.0k	78.9	14400.0	188.5k	188.5k	0.0	10508.8	226.1k	85.4k	164.9	14400.0
dem_100_gaslib40_285	209.6k	209.6k	0.0	1125.1	209.6k	209.6k	0.0	5651.1	209.6k	159.3k	31.6	14400.0
dem_100_gaslib40_286	243.5k	243.3k	0.1	8714.9	243.5k	243.5k	0.0	4283.1	247.3k	183.9k	34.4	14400.0
dem_100_gaslib40_287	145.0k	145.0k	0.0	5731.1	145.0k	145.0k	0.0	9848.7	166.1k	63.8k	160.5	14400.0
dem_100_gaslib40_288	636.0k	214.4k	196.7	14400.0	677.3k	250.5k	170.4	14400.0	758.2k	170.7k	344.2	14400.0
dem_100_gaslib40_289	288.6k	225.6k	28.0	14400.0	278.9k	223.5k	24.8	14400.0	280.0k	195.2k	43.4	14400.0
dem_100_gaslib40_290	128.2k	128.2k	0.0	2371.0	128.2k	116.5k	10.1	14400.0	140.0k	32.0k	337.5	14400.0
dem_100_gaslib40_291	483.8k	206.5k	134.3	14400.0	520.7k	224.7k	131.7	14400.0	458.2k	125.9k	263.8	14400.0
dem_100_gaslib40_292	176.0k	176.0k	0.0	289.4	176.0k	176.0k	0.0	1513.9	176.0k	123.0k	43.1	14400.0
dem_100_gaslib40_293	167.4k	167.3k	0.0	372.0	167.4k	167.4k	0.0	1506.1	168.5k	137.0k	23.0	14400.0
dem_100_gaslib40_294	200.3k	200.3k	0.0	3948.5	200.3k	200.3k	0.0	7699.6	213.2k	90.0k	137.1	14400.0
dem_100_gaslib40_295	344.6k	171.0k	101.5	14400.0	321.1k	196.1k	63.8	14400.0	460.9k	67.2k	586.4	14400.0
dem_100_gaslib40_296	233.3k	165.6k	40.8	14400.0	221.6k	164.8k	34.5	14400.0	267.0k	110.2k	142.2	14400.0
dem_100_gaslib40_297	249.5k	193.2k	29.2	14400.0	253.4k	192.5k	31.6	14400.0	247.5k	113.1k	118.9	14400.0
dem_100_gaslib40_298	171.9k	171.9k	0.0	11053.6	171.9k	171.9k	0.0	8611.6	199.5k	67.9k	193.9	14400.0
dem_100_gaslib40_299	152.9k	152.9k	0.0	4070.0	152.9k	152.9k	0.0	3026.1	172.9k	66.0k	162.2	14400.0
dem_100_gaslib40_300	148.6k	148.6k	0.0	1069.2	148.6k	148.6k	0.0	3398.7	158.2k	78.9k	100.5	14400.0
dem_100_gaslib40_301	268.2k	212.1k	26.5	14400.0	270.5k	197.6k	36.9	14400.0	266.3k	113.1k	135.4	14400.0
dem_100_gaslib40_302	218.9k	218.9k	0.0	2991.4	218.9k	218.9k	0.0	8223.9	232.7k	148.0k	57.2	14400.0
dem_100_gaslib40_303	486.8k	177.0k	175.0	14400.0	469.8k	216.8k	116.7	14400.0	431.6k	143.0k	201.8	14400.0
dem_100_gaslib40_304	124.6k	124.6k	0.0	157.7	124.6k	124.5k	0.1	3282.7	129.6k	70.4k	84.1	14400.0
dem_100_gaslib40_305	187.1k	187.1k	0.0	470.6	187.1k	187.1k	0.0	9747.1	214.0k	108.9k	96.6	14400.0
dem_100_gaslib40_306	243.5k	178.1k	36.7	14400.0	224.7k	168.0k	33.7	14400.0	293.2k	101.1k	189.9	14400.0
dem_100_gaslib40_307	199.3k	199.3k	0.0	1981.5	199.3k	199.3k	0.0	8839.3	207.5k	147.8k	40.4	14400.0
dem_100_gaslib40_308	187.3k	187.3k	0.0	1434.9	187.3k	187.3k	0.0	6613.9	188.0k	138.0k	36.2	14400.0

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Table A.11: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_309	179.1k	179.1k	0.0	224.4	179.1k	179.1k	0.0	1606.4	204.8k	125.0k	63.8	14400.0
dem_100_gaslib40_310	88.3k	88.3k	0.0	71.0	88.3k	88.3k	0.1	460.2	126.1k	40.5k	211.1	14400.0
dem_100_gaslib40_311	172.4k	172.4k	0.0	1921.7	172.4k	172.4k	0.0	9235.9	185.1k	98.2k	88.4	14400.0
dem_100_gaslib40_312	202.2k	202.2k	0.0	6404.7	202.2k	174.4k	15.9	14400.0	212.3k	103.3k	105.6	14400.0
dem_100_gaslib40_313	144.2k	144.2k	0.0	557.9	144.2k	144.2k	0.0	3004.2	160.2k	92.4k	73.4	14400.0
dem_100_gaslib40_314	267.0k	203.2k	31.4	14400.0	266.0k	213.4k	24.6	14400.0	297.8k	86.7k	243.4	14400.0
dem_100_gaslib40_315	480.7k	180.7k	166.0	14400.0	376.8k	180.5k	108.8	14400.0	504.0k	83.1k	506.3	14400.0
dem_100_gaslib40_316	242.1k	204.4k	18.5	14400.0	238.2k	196.6k	21.1	14400.0	263.3k	117.3k	124.5	14400.0
dem_100_gaslib40_317	311.9k	157.3k	98.3	14400.0	242.0k	185.7k	30.3	14400.0	288.3k	109.3k	163.7	14400.0
dem_100_gaslib40_318	176.8k	176.8k	0.0	412.4	176.8k	176.8k	0.0	5310.2	190.3k	101.3k	88.0	14400.0
dem_100_gaslib40_319	61.2k	61.2k	0.0	26.6	61.2k	61.2k	0.0	1026.0	61.2k	61.2k	0.0	4243.3
dem_100_gaslib40_320	137.3k	137.3k	0.0	1935.3	137.3k	137.3k	0.0	6001.6	150.8k	42.2k	256.9	14400.0
dem_100_gaslib40_321	481.4k	150.3k	220.2	14400.0	254.6k	191.3k	33.1	14400.0	301.8k	76.4k	295.0	14400.0
dem_100_gaslib40_322	177.2k	177.2k	0.0	7028.9	177.2k	177.2k	0.0	12282.2	200.8k	61.7k	225.3	14400.0
dem_100_gaslib40_323	295.1k	188.8k	56.3	14400.0	319.7k	186.6k	71.4	14400.0	346.8k	34.7k	899.5	14400.0
dem_100_gaslib40_324	213.3k	213.3k	0.0	2922.4	213.3k	213.3k	0.0	7229.5	231.6k	101.5k	128.1	14400.0
dem_100_gaslib40_325	246.0k	150.0k	64.1	14400.0	241.5k	155.6k	55.2	14400.0	229.8k	69.8k	229.2	14400.0
dem_100_gaslib40_326	98.1k	98.1k	0.0	230.1	98.1k	98.1k	0.0	2098.6	99.8k	71.0k	40.6	14400.0
dem_100_gaslib40_327	121.0k	121.0k	0.0	43.8	121.0k	121.0k	0.0	761.6	121.4k	105.5k	15.0	14400.0
dem_100_gaslib40_328	389.3k	276.5k	40.8	14400.0	372.8k	209.1k	78.3	14400.0	507.7k	156.4k	224.6	14400.0
dem_100_gaslib40_329	162.5k	162.5k	0.0	4652.1	162.5k	162.5k	0.0	11593.6	182.6k	75.5k	141.8	14400.0
dem_100_gaslib40_330	134.4k	134.4k	0.0	242.8	134.4k	134.4k	0.0	3218.5	142.1k	107.0k	32.8	14400.0
dem_100_gaslib40_331	263.6k	142.9k	84.5	14400.0	218.0k	176.7k	23.4	14400.0	278.9k	60.1k	364.4	14400.0
dem_100_gaslib40_332	199.4k	199.4k	0.0	2985.5	199.4k	199.4k	0.0	7196.6	355.7k	72.1k	393.3	14400.0
dem_100_gaslib40_333	203.0k	203.0k	0.0	845.3	203.0k	203.0k	0.0	4222.9	206.4k	161.1k	28.1	14400.0
dem_100_gaslib40_334	176.3k	176.3k	0.0	1372.3	176.3k	176.3k	0.0	10297.1	199.7k	51.4k	288.7	14400.0
dem_100_gaslib40_335	126.5k	126.5k	0.0	7044.8	126.5k	126.5k	0.0	8039.5	145.0k	36.8k	294.1	14400.0
dem_100_gaslib40_336	244.1k	158.8k	53.7	14400.0	207.1k	179.4k	15.5	14400.0	241.6k	79.5k	203.9	14400.0
dem_100_gaslib40_337	366.5k	190.0k	92.9	14400.0	355.7k	201.1k	76.9	14400.0	470.1k	128.7k	265.3	14400.0
dem_100_gaslib40_338	323.8k	183.0k	76.9	14400.0	297.8k	203.8k	46.1	14400.0	311.4k	132.7k	134.7	14400.0
dem_100_gaslib40_339	148.3k	148.3k	0.0	794.8	148.3k	148.3k	0.0	9777.3	168.7k	64.9k	160.0	14400.0
dem_100_gaslib40_340	200.6k	200.6k	0.0	819.7	200.6k	200.6k	0.0	6783.4	202.5k	133.6k	51.5	14400.0
dem_100_gaslib40_341	235.8k	156.7k	50.5	14400.0	240.6k	149.3k	61.1	14400.0	244.0k	68.1k	258.1	14400.0
dem_100_gaslib40_342	210.1k	210.1k	0.0	904.9	210.1k	210.1k	0.0	2407.9	211.3k	177.2k	19.3	14400.0
dem_100_gaslib40_343	416.5k	213.3k	95.2	14400.0	346.7k	210.8k	64.5	14400.0	320.9k	71.6k	348.5	14400.0
dem_100_gaslib40_344	133.3k	133.3k	0.0	83.1	133.3k	133.3k	0.0	2448.9	134.3k	88.8k	51.2	14400.0
dem_100_gaslib40_345	118.5k	118.5k	0.0	391.8	118.5k	118.5k	0.0	1751.0	119.9k	79.2k	51.4	14400.0
dem_100_gaslib40_346	191.3k	191.3k	0.0	13298.2	191.3k	158.4k	20.8	14400.0	198.3k	126.1k	57.2	14400.0
dem_100_gaslib40_347	193.4k	193.4k	0.0	630.9	193.4k	193.4k	0.0	4035.7	194.8k	151.6k	28.5	14400.0
dem_100_gaslib40_348	218.3k	218.2k	0.0	8684.4	218.3k	188.0k	16.1	14400.0	242.9k	120.0k	102.5	14400.0
dem_100_gaslib40_349	227.6k	167.9k	35.6	14400.0	226.3k	178.5k	26.8	14400.0	231.4k	81.0k	185.6	14400.0
dem_100_gaslib40_350	244.7k	171.7k	42.5	14400.0	245.2k	179.2k	36.8	14400.0	250.3k	124.2k	101.4	14400.0
dem_100_gaslib40_351	368.3k	165.1k	123.0	14400.0	316.1k	159.6k	98.1	14400.0	350.2k	79.1k	342.5	14400.0
dem_100_gaslib40_352	158.1k	158.1k	0.0	2033.5	158.1k	158.1k	0.0	7690.9	172.4k	83.9k	105.4	14400.0
dem_100_gaslib40_353	191.3k	191.3k	0.0	3680.2	191.3k	191.3k	0.0	4483.5	210.2k	72.7k	189.0	14400.0
dem_100_gaslib40_354	191.4k	191.4k	0.0	457.8	191.4k	191.4k	0.0	2016.6	193.1k	149.5k	29.1	14400.0
dem_100_gaslib40_355	237.5k	237.3k	0.1	13699.5	237.5k	237.5k	0.0	4785.1	247.5k	119.4k	107.2	14400.0
dem_100_gaslib40_356	172.7k	172.7k	0.0	1470.0	172.7k	172.7k	0.0	5358.9	179.3k	116.2k	54.4	14400.0
dem_100_gaslib40_357	141.6k	141.6k	0.0	2556.3	141.6k	141.6k	0.0	4161.3	157.0k	33.5k	368.9	14400.0
dem_100_gaslib40_358	225.3k	225.3k	0.0	3744.1	225.3k	225.3k	0.0	5007.4	226.0k	166.1k	36.1	14400.0
dem_100_gaslib40_359	173.0k	173.0k	0.0	2296.9	173.0k	173.0k	0.0	3834.1	178.3k	115.9k	53.9	14400.0
dem_100_gaslib40_360	223.2k	153.7k	45.2	14400.0	213.9k	213.9k	0.0	7154.0	219.5k	94.6k	132.1	14400.0
dem_100_gaslib40_361	179.0k	179.0k	0.0	10876.0	179.0k	179.0k	0.0	5133.8	219.8k	71.6k	207.0	14400.0
dem_100_gaslib40_362	235.1k	144.7k	62.4	14400.0	233.8k	155.5k	50.3	14400.0	305.5k	101.0k	202.5	14400.0
dem_100_gaslib40_363	162.7k	162.7k	0.0	1383.8	162.7k	162.7k	0.0	7801.0	180.0k	54.1k	232.5	14400.0
dem_100_gaslib40_364	121.6k	121.6k	0.0	2034.3	121.6k	121.6k	0.0	11185.8	127.9k	62.8k	103.6	14400.0
dem_100_gaslib40_365	320.9k	152.7k	110.2	14400.0	311.9k	169.1k	84.4	14400.0	281.4k	61.4k	358.0	14400.0
dem_100_gaslib40_366	322.6k	135.7k	137.7	14400.0	199.3k	146.9k	35.7	14400.0	260.9k	65.8k	296.8	14400.0
dem_100_gaslib40_367	238.3k	238.3k	0.0	2466.6	238.3k	238.3k	0.0	3625.9	263.2k	168.2k	56.5	14400.0
dem_100_gaslib40_368	334.4k	198.4k	68.6	14400.0	402.1k	188.7k	113.1	14400.0	410.9k	110.8k	270.9	14400.0
dem_100_gaslib40_369	212.2k	163.2k	30.0	14400.0	207.2k	141.9k	46.0	14400.0	220.8k	75.6k	191.9	14400.0
dem_100_gaslib40_370	181.0k	181.0k	0.0	13833.7	181.0k	181.0k	0.0	14031.7	189.7k	57.0k	233.0	14400.0
dem_100_gaslib40_371	264.2k	208.0k	27.1	14400.0	234.0k	192.3k	21.7	14400.0	237.4k	87.5k	171.2	14400.0
dem_100_gaslib40_372	187.0k	187.0k	0.0	1410.6	187.0k	187.0k	0.0	12522.1	188.7k	114.9k	64.2	14400.0
dem_100_gaslib40_373	316.4k	179.9k	75.9	14400.0	309.8k	224.4k	38.1	14400.0	388.2k	142.5k	172.4	14400.0
dem_100_gaslib40_374	182.2k	182.2k	0.0	5870.2	182.2k	182.2k	0.0	5229.9	188.8k	92.4k	104.2	14400.0
dem_100_gaslib40_375	202.6k	202.6k	0.0	1255.0	202.6k	202.6k	0.0	6082.3	204.1k	120.6k	69.3	14400.0
dem_100_gaslib40_376	339.9k	180.0k	88.8	14400.0	367.9k	170.5k	115.8	14400.0	390.7k	100.7k	287.9	14400.0
dem_100_gaslib40_377	293.5k	175.4k	67.3	14400.0	292.8k	181.7k	61.1	14400.0	306.0k	71.3k	329.0	14400.0
dem_100_gaslib40_378	163.5k	163.5k	0.0	1420.0	163.5k	163.5k	0.0	3328.6	193.0k	85.1k	127.0	14400.0
dem_100_gaslib40_379	205.0k	205.0k	0.0	1490.2	205.0k	205.0k	0.0	8064.5	215.4k	148.2k	45.4	14400.0
dem_100_gaslib40_380	162.0k	162.0k	0.0	1390.6	162.0k	162.0k	0.0	2604.1	191.9k	61.0k	214.5	14400.0
dem_100_gaslib40_381	321.6k	162.0k	98.5	14400.0	332.9k	174.4k	90.9	14400.0	387.8k	59.7k	550.0	14400.0
dem_100_gaslib40_382	328.5k	179.4k	83.1	14400.0	299.7k	207.5k	44.5	14400.0	301.2k	104.1k	189.3	14400.0
dem_100_gaslib40_383	164.3k	164.3k	0.0	1337.6	173.9k	119.0k	46.2	14400.0	170.5k	42.6k	299.9	14400.0
dem_100_gaslib40_384	175.2k	133.7k	31.0	14400.0	173.7k	173.7k	0.0	10625.1	174.1k	57.8k	201.3	14400.0
dem_100_gaslib40_385	205.2k	181.4k	13.1	14400.0	199.2k	171.4k	16.2	14400.0	217.3k	74.6k	191.2	14400.0
dem_100_gaslib40_386	229.0k	229.0k	0.0	3044.7	229.0k	229.0k	0.0	9118.6	240.6k	162.6k	48.0	14400.0

continued on next page

Table A.11: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_387	231.1k	173.9k	32.9	14400.0	227.8k	170.6k	33.5	14400.0	234.4k	107.5k	118.0	14400.0
dem_100_gaslib40_388	430.6k	200.0k	115.3	14400.0	502.2k	259.4k	93.6	14400.0	483.6k	160.8k	200.9	14400.0
dem_100_gaslib40_389	370.5k	145.2k	155.2	14400.0	275.4k	178.9k	53.9	14400.0	286.7k	62.2k	360.8	14400.0
dem_100_gaslib40_390	242.4k	242.4k	0.0	5446.6	242.4k	242.4k	0.0	13344.2	280.7k	138.3k	103.0	14400.0
dem_100_gaslib40_391	126.3k	126.3k	0.0	830.6	126.3k	126.3k	0.0	5148.3	144.8k	35.9k	303.9	14400.0
dem_100_gaslib40_392	337.2k	143.8k	134.4	14400.0	329.7k	157.7k	109.1	14400.0	351.3k	59.2k	493.6	14400.0
dem_100_gaslib40_393	270.3k	237.5k	13.8	14400.0	270.0k	246.0k	9.8	14400.0	277.1k	183.4k	51.1	14400.0
dem_100_gaslib40_394	178.6k	178.6k	0.0	1260.2	178.6k	178.6k	0.0	7501.9	186.4k	106.6k	74.9	14400.0
dem_100_gaslib40_395	219.3k	169.1k	29.7	14400.0	216.5k	184.5k	17.3	14400.0	219.3k	98.9k	121.8	14400.0
dem_100_gaslib40_396	92.5k	92.5k	0.0	351.6	92.5k	92.5k	0.0	2021.6	93.8k	68.3k	37.4	14400.0
dem_100_gaslib40_397	210.5k	154.7k	36.0	14400.0	205.5k	205.5k	0.0	8003.5	226.2k	105.3k	114.8	14400.0
dem_100_gaslib40_398	274.4k	175.2k	56.7	14400.0	261.3k	188.9k	38.3	14400.0	318.9k	74.2k	329.7	14400.0
dem_100_gaslib40_399	210.0k	157.7k	33.2	14400.0	210.3k	147.2k	42.9	14400.0	218.4k	43.9k	398.0	14400.0
dem_100_gaslib40_400	615.8k	201.1k	206.2	14400.0	483.0k	208.6k	131.6	14400.0	659.8k	148.7k	343.6	14400.0
dem_100_gaslib40_401	192.5k	192.5k	0.0	4445.0	192.5k	192.5k	0.0	7816.6	200.9k	78.5k	156.1	14400.0
dem_100_gaslib40_402	105.7k	105.7k	0.0	2186.4	105.7k	105.7k	0.0	786.2	112.6k	56.8k	98.3	14400.0
dem_100_gaslib40_403	131.0k	131.0k	0.0	272.3	131.0k	131.0k	0.0	1486.1	131.8k	107.2k	22.9	14400.0
dem_100_gaslib40_404	229.3k	229.3k	0.0	8602.8	229.3k	229.3k	0.0	8311.8	264.6k	115.9k	128.3	14400.0
dem_100_gaslib40_405	115.6k	115.6k	0.0	7170.2	115.6k	115.6k	0.0	1774.2	118.9k	76.2k	56.1	14400.0
dem_100_gaslib40_406	192.9k	192.9k	0.0	2728.8	192.9k	192.9k	0.0	2115.4	223.7k	84.0k	166.4	14400.0
dem_100_gaslib40_407	190.7k	190.7k	0.0	5382.6	190.7k	190.7k	0.0	8925.5	255.1k	115.5k	120.9	14400.0
dem_100_gaslib40_408	319.3k	175.2k	82.3	14400.0	356.6k	188.2k	89.5	14400.0	412.2k	115.3k	257.5	14400.0
dem_100_gaslib40_409	179.7k	179.7k	0.0	242.8	179.7k	179.7k	0.0	2103.8	187.8k	133.4k	40.8	14400.0
dem_100_gaslib40_410	179.0k	179.0k	0.0	12585.1	179.0k	179.0k	0.0	7828.1	186.2k	77.9k	139.0	14400.0
dem_100_gaslib40_411	294.2k	149.2k	97.2	14400.0	200.5k	136.5k	46.8	14400.0	269.8k	48.8k	453.0	14400.0
dem_100_gaslib40_412	228.7k	190.4k	20.1	14400.0	212.8k	212.8k	0.0	9164.2	252.5k	85.4k	195.6	14400.0
dem_100_gaslib40_413	171.9k	171.9k	0.0	524.9	171.9k	171.9k	0.0	9728.0	177.6k	128.6k	38.0	14400.0
dem_100_gaslib40_414	213.2k	213.2k	0.0	12139.1	214.3k	171.0k	25.3	14400.0	234.8k	108.1k	117.3	14400.0
dem_100_gaslib40_415	176.6k	176.6k	0.0	583.4	176.6k	176.6k	0.0	2673.3	197.2k	90.9k	117.0	14400.0
dem_100_gaslib40_416	218.0k	150.3k	45.0	14400.0	252.3k	141.2k	78.7	14400.0	311.2k	28.5k	990.3	14400.0
dem_100_gaslib40_417	295.5k	156.5k	88.8	14400.0	336.9k	160.8k	109.5	14400.0	342.1k	65.9k	419.0	14400.0
dem_100_gaslib40_418	188.0k	188.0k	0.0	1178.3	188.0k	188.0k	0.0	4380.1	196.7k	116.0k	69.5	14400.0
dem_100_gaslib40_419	183.5k	183.4k	0.1	8721.8	183.7k	145.3k	26.5	14400.0	217.1k	67.8k	219.9	14400.0
dem_100_gaslib40_420	549.2k	192.8k	184.8	14400.0	347.8k	181.2k	91.9	14400.0	427.2k	115.6k	269.5	14400.0
dem_100_gaslib40_421	427.9k	220.6k	94.0	14400.0	826.0k	233.5k	253.7	14400.0	451.3k	192.9k	133.9	14400.0
dem_100_gaslib40_422	124.6k	124.6k	0.0	68.5	124.6k	124.6k	0.0	1958.0	126.1k	77.1k	63.5	14400.0
dem_100_gaslib40_423	216.3k	165.9k	30.3	14400.0	208.1k	208.1k	0.0	6804.6	215.5k	144.6k	49.0	14400.0
dem_100_gaslib40_424	279.2k	166.6k	67.6	14400.0	248.9k	181.1k	37.5	14400.0	365.7k	96.6k	278.5	14400.0
dem_100_gaslib40_425	496.2k	208.4k	138.0	14400.0	589.8k	209.9k	181.0	14400.0	436.1k	143.7k	203.4	14400.0
dem_100_gaslib40_426	182.9k	182.9k	0.0	11107.3	184.1k	132.9k	38.6	14400.0	213.0k	44.3k	381.1	14400.0
dem_100_gaslib40_427	615.7k	184.9k	233.1	14400.0	375.6k	217.5k	72.7	14400.0	375.9k	74.7k	403.3	14400.0
dem_100_gaslib40_428	194.0k	194.0k	0.0	8370.6	194.0k	194.0k	0.0	5352.4	215.1k	118.1k	82.1	14400.0
dem_100_gaslib40_429	289.1k	201.6k	43.4	14400.0	358.4k	167.5k	114.0	14400.0	480.1k	85.8k	459.4	14400.0
dem_100_gaslib40_430	163.7k	163.7k	0.0	13895.7	163.7k	163.7k	0.0	3480.5	194.8k	57.2k	240.3	14400.0
dem_100_gaslib40_431	318.7k	220.7k	44.4	14400.0	315.7k	210.8k	49.8	14400.0	333.0k	80.8k	311.9	14400.0
dem_100_gaslib40_432	197.8k	136.3k	45.1	14400.0	159.6k	159.6k	0.0	5220.7	175.2k	78.3k	123.9	14400.0
dem_100_gaslib40_433	302.7k	162.1k	86.7	14400.0	272.2k	163.0k	66.9	14400.0	406.3k	66.7k	509.2	14400.0
dem_100_gaslib40_434	169.8k	169.8k	0.0	1170.8	169.8k	169.8k	0.0	4420.6	182.3k	85.6k	113.0	14400.0
dem_100_gaslib40_435	76.9k	76.9k	0.0	44.0	76.9k	76.9k	0.0	569.6	76.9k	76.8k	0.1	9135.9
dem_100_gaslib40_436	70.5k	70.5k	0.0	242.8	70.5k	70.5k	0.0	1102.8	109.7k	33.0k	232.5	14400.0
dem_100_gaslib40_437	179.9k	179.9k	0.0	14171.8	179.9k	140.8k	27.8	14400.0	194.5k	72.4k	168.6	14400.0
dem_100_gaslib40_438	245.1k	245.1k	0.0	1422.8	245.1k	245.1k	0.0	5107.1	255.3k	196.4k	30.0	14400.0
dem_100_gaslib40_439	214.6k	214.4k	0.1	8357.2	214.6k	214.6k	0.0	7577.5	217.1k	143.4k	51.4	14400.0
dem_100_gaslib40_440	192.0k	192.0k	0.0	2380.9	192.0k	192.0k	0.0	6835.1	211.0k	97.3k	116.9	14400.0
dem_100_gaslib40_441	205.1k	205.1k	0.0	2685.9	205.1k	205.1k	0.0	13427.7	212.1k	130.8k	62.2	14400.0
dem_100_gaslib40_442	287.7k	167.8k	71.5	14400.0	281.9k	168.2k	67.6	14400.0	419.7k	103.4k	305.9	14400.0
dem_100_gaslib40_443	242.6k	187.7k	29.2	14400.0	232.0k	232.0k	0.0	11388.6	277.1k	133.3k	108.0	14400.0
dem_100_gaslib40_444	225.2k	225.2k	0.0	12565.0	225.2k	196.4k	14.7	14400.0	305.0k	54.0k	464.7	14400.0
dem_100_gaslib40_445	147.8k	147.8k	0.0	464.0	147.8k	147.8k	0.0	5466.9	169.6k	70.1k	142.1	14400.0
dem_100_gaslib40_446	204.5k	204.5k	0.0	7465.6	204.5k	184.8k	10.7	14400.0	245.2k	103.5k	136.9	14400.0
dem_100_gaslib40_447	495.2k	236.5k	109.4	14400.0	523.5k	272.3k	92.2	14400.0	564.8k	194.4k	190.5	14400.0
dem_100_gaslib40_448	206.3k	206.3k	0.0	2642.0	206.3k	206.3k	0.0	2745.2	211.1k	127.8k	65.2	14400.0
dem_100_gaslib40_449	167.2k	167.2k	0.0	1255.6	167.2k	167.2k	0.0	3915.8	181.1k	85.9k	110.8	14400.0
dem_100_gaslib40_450	291.3k	189.0k	54.1	14400.0	278.5k	215.3k	29.3	14400.0	280.5k	128.0k	119.1	14400.0
dem_100_gaslib40_451	132.8k	132.8k	0.0	6954.4	132.8k	132.8k	0.0	2362.9	162.8k	42.5k	283.0	14400.0
dem_100_gaslib40_452	223.4k	152.5k	46.5	14400.0	223.4k	223.3k	0.0	9410.0	270.4k	102.5k	163.8	14400.0
dem_100_gaslib40_453	153.0k	153.0k	0.0	1021.8	153.0k	128.6k	18.9	14400.0	155.0k	37.7k	310.9	14400.0
dem_100_gaslib40_454	255.1k	153.9k	65.8	14400.0	231.5k	135.5k	70.9	14400.0	401.0k	32.3k	1143.1	14400.0
dem_100_gaslib40_455	351.1k	161.5k	117.3	14400.0	454.3k	172.2k	163.8	14400.0	470.5k	65.3k	620.5	14400.0
dem_100_gaslib40_456	174.0k	174.0k	0.0	664.9	174.0k	173.9k	0.0	984.5	212.2k	138.7k	53.0	14400.0
dem_100_gaslib40_457	247.3k	247.3k	0.0	3333.5	247.3k	247.3k	0.0	2970.1	253.0k	203.3k	24.4	14400.0
dem_100_gaslib40_458	314.3k	210.6k	49.3	14400.0	369.6k	200.7k	84.1	14400.0	355.3k	69.4k	412.0	14400.0
dem_100_gaslib40_459	182.9k	182.9k	0.0	4764.6	184.6k	149.0k	23.9	14400.0	195.4k	69.8k	179.7	14400.0
dem_100_gaslib40_460	180.2k	180.2k	0.0	771.1	180.2k	180.2k	0.0	1252.4	187.1k	116.5k	60.5	14400.0
dem_100_gaslib40_461	163.2k	163.2k	0.0	8396.5	163.6k	125.3k	30.5	14400.0	172.2k	40.5k	325.4	14400.0
dem_100_gaslib40_462	218.7k	193.7k	12.9	14400.0	218.7k	180.7k	21.0	14400.0	233.4k	55.2k	322.7	14400.0
dem_100_gaslib40_463	215.4k	215.4k	0.0	616.2	215.4k	215.4k	0.0	4329.8	221.1k	168.4k	31.3	14400.0
dem_100_gaslib40_464	286.1k	201.5k	42.0	14400.0	278.4k	213.2k	30.6	14400.0	309.1k	121.5k	154.4	14400.0

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Table A.11: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_465	253.1k	253.1k	0.0	13856.1	253.5k	202.6k	25.1	14400.0	258.5k	149.1k	73.4	14400.0
dem_100_gaslib40_466	184.6k	184.6k	0.0	1937.7	184.6k	184.6k	0.0	7514.8	204.1k	123.2k	65.6	14400.0
dem_100_gaslib40_467	210.3k	210.3k	0.0	3859.4	210.3k	184.4k	14.0	14400.0	212.7k	104.3k	103.9	14400.0
dem_100_gaslib40_468	132.6k	132.6k	0.0	3061.6	132.6k	132.6k	0.0	1634.4	160.7k	65.6k	145.0	14400.0
dem_100_gaslib40_469	190.7k	190.7k	0.0	2039.2	190.7k	190.7k	0.0	13280.9	192.2k	116.5k	65.0	14400.0
dem_100_gaslib40_470	220.2k	220.2k	0.0	6228.7	220.2k	192.9k	14.2	14400.0	230.7k	119.7k	92.7	14400.0
dem_100_gaslib40_471	186.4k	186.4k	0.0	1757.5	186.4k	181.0k	3.0	14400.0	189.7k	127.6k	48.7	14400.0
dem_100_gaslib40_472	340.4k	173.6k	96.0	14400.0	365.5k	220.8k	65.5	14400.0	1e+20	93.2k	–	14400.0
dem_100_gaslib40_473	151.1k	151.1k	0.0	4633.0	151.1k	151.1k	0.0	8423.1	171.2k	37.4k	357.7	14400.0
dem_100_gaslib40_474	190.7k	190.7k	0.0	12824.6	190.7k	162.4k	17.4	14400.0	209.8k	106.9k	96.3	14400.0
dem_100_gaslib40_475	243.6k	176.0k	38.4	14400.0	231.6k	231.6k	0.0	11624.0	240.7k	73.2k	228.9	14400.0
dem_100_gaslib40_476	156.5k	156.5k	0.0	919.1	156.5k	156.5k	0.0	1708.4	166.3k	70.7k	135.1	14400.0
dem_100_gaslib40_477	628.6k	216.9k	189.8	14400.0	598.1k	334.2k	79.0	14400.0	649.0k	185.6k	249.7	14400.0
dem_100_gaslib40_478	180.1k	180.1k	0.0	1903.9	180.1k	180.1k	0.0	6041.6	245.1k	106.9k	129.3	14400.0
dem_100_gaslib40_479	173.4k	173.4k	0.0	1502.0	173.4k	173.4k	0.0	4360.3	193.2k	100.8k	91.6	14400.0
dem_100_gaslib40_480	241.7k	184.7k	30.8	14400.0	256.9k	158.0k	62.6	14400.0	309.0k	74.9k	312.7	14400.0
dem_100_gaslib40_481	143.0k	143.0k	0.0	1040.9	143.0k	143.0k	0.0	3843.0	162.6k	34.7k	369.1	14400.0
dem_100_gaslib40_482	409.9k	177.5k	130.9	14400.0	403.1k	204.2k	97.4	14400.0	458.7k	109.2k	320.2	14400.0
dem_100_gaslib40_483	134.2k	134.2k	0.0	1771.7	134.2k	115.0k	16.7	14400.0	158.2k	33.7k	369.3	14400.0
dem_100_gaslib40_484	146.0k	146.0k	0.0	893.0	146.0k	145.9k	0.0	2957.1	146.0k	95.5k	52.9	14400.0
dem_100_gaslib40_485	181.8k	181.8k	0.0	1331.6	181.8k	181.8k	0.0	4371.9	200.4k	110.1k	82.0	14400.0
dem_100_gaslib40_486	167.8k	115.1k	45.8	14400.0	148.0k	148.0k	0.0	12550.4	151.7k	49.7k	205.1	14400.0
dem_100_gaslib40_487	207.6k	207.6k	0.0	792.7	207.6k	207.6k	0.0	3064.5	209.6k	147.9k	41.7	14400.0
dem_100_gaslib40_488	252.9k	207.2k	22.0	14400.0	241.8k	241.8k	0.0	8680.3	259.5k	170.5k	52.1	14400.0
dem_100_gaslib40_489	220.9k	220.9k	0.0	2951.1	220.9k	220.9k	0.0	9508.1	223.1k	122.9k	81.5	14400.0
dem_100_gaslib40_490	203.5k	203.5k	0.0	1111.5	203.5k	203.5k	0.0	10001.6	204.6k	142.7k	43.4	14400.0
dem_100_gaslib40_491	421.1k	222.2k	89.6	14400.0	312.4k	189.0k	65.3	14400.0	383.4k	130.1k	194.7	14400.0
dem_100_gaslib40_492	192.9k	192.9k	0.0	6408.3	192.9k	192.9k	0.0	5443.3	225.4k	75.0k	200.4	14400.0
dem_100_gaslib40_493	159.8k	159.8k	0.0	242.2	159.8k	159.8k	0.0	7250.6	166.8k	102.2k	63.2	14400.0
dem_100_gaslib40_494	279.6k	166.9k	67.5	14400.0	325.4k	184.9k	75.9	14400.0	760.2k	91.4k	731.4	14400.0
dem_100_gaslib40_495	321.5k	173.1k	85.8	14400.0	430.3k	165.5k	160.1	14400.0	386.6k	81.1k	376.9	14400.0
dem_100_gaslib40_496	269.2k	174.3k	54.4	14400.0	262.6k	199.1k	31.9	14400.0	289.5k	95.5k	203.2	14400.0
dem_100_gaslib40_497	156.4k	156.4k	0.0	130.3	156.4k	156.4k	0.0	1976.1	156.6k	136.2k	15.0	14400.0
dem_100_gaslib40_498	404.4k	230.2k	75.7	14400.0	402.0k	230.2k	74.7	14400.0	467.2k	109.2k	327.8	14400.0
dem_100_gaslib40_499	413.1k	232.1k	78.0	14400.0	420.7k	203.9k	106.3	14400.0	495.1k	165.3k	199.5	14400.0
dem_100_gaslib40_500	275.4k	174.0k	58.2	14400.0	211.0k	174.2k	21.1	14400.0	254.9k	121.8k	109.2	14400.0

Table A.12: Detailed results of the split-pipe models on *GasLib-40* with $\mathcal{B} = 100$ as summarized in Figure 3.14b and Table 3.4b. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_1	131.1k	130.9k	0.1	7.2	131.1k	130.9k	0.1	5.3
dem_100_gaslib40_2	127.2k	127.2k	0.0	63.4	127.2k	127.1k	0.1	6.7
dem_100_gaslib40_3	172.7k	172.6k	0.0	67.8	172.7k	172.5k	0.1	681.3
dem_100_gaslib40_4	90.5k	90.5k	0.0	11.5	90.5k	90.5k	0.0	6.4
dem_100_gaslib40_5	126.5k	110.8k	14.2	14400.0	126.5k	126.5k	0.0	128.3
dem_100_gaslib40_6	269.7k	214.7k	25.6	14400.0	269.7k	221.5k	21.8	14400.0
dem_100_gaslib40_7	197.2k	197.0k	0.1	15.8	197.2k	197.0k	0.1	34.9
dem_100_gaslib40_8	220.0k	219.8k	0.1	24.0	220.0k	219.8k	0.1	22.4
dem_100_gaslib40_9	226.3k	226.1k	0.1	142.2	226.3k	226.3k	0.0	129.7
dem_100_gaslib40_10	320.3k	244.0k	31.3	14400.0	320.3k	262.1k	22.2	14400.0
dem_100_gaslib40_11	198.7k	198.5k	0.1	539.5	198.7k	198.5k	0.1	167.3
dem_100_gaslib40_12	198.9k	198.9k	0.0	30.0	198.9k	198.8k	0.1	20.8
dem_100_gaslib40_13	247.1k	246.9k	0.1	3842.0	247.1k	246.9k	0.1	12268.0
dem_100_gaslib40_14	167.2k	167.0k	0.1	22.8	167.2k	167.0k	0.1	208.6
dem_100_gaslib40_15	132.3k	132.3k	0.0	9.3	132.3k	132.2k	0.1	6.9
dem_100_gaslib40_16	137.3k	137.3k	0.0	61.0	137.3k	137.3k	0.0	18.3
dem_100_gaslib40_17	270.1k	269.9k	0.1	25.1	270.1k	270.1k	0.0	33.2
dem_100_gaslib40_18	141.4k	141.3k	0.1	9.1	141.4k	141.3k	0.1	7.4
dem_100_gaslib40_19	189.6k	189.6k	0.0	16.4	189.6k	189.5k	0.0	14.1
dem_100_gaslib40_20	151.1k	151.1k	0.0	90.2	151.1k	151.1k	0.0	20.2
dem_100_gaslib40_21	196.7k	155.8k	26.2	14400.0	196.7k	196.5k	0.1	2104.6
dem_100_gaslib40_22	244.7k	244.5k	0.1	13.9	244.7k	244.7k	0.0	14.2
dem_100_gaslib40_23	277.6k	231.1k	20.1	14400.0	277.6k	244.8k	13.4	14400.0
dem_100_gaslib40_24	193.9k	193.7k	0.1	515.6	193.9k	193.7k	0.1	322.1
dem_100_gaslib40_25	358.3k	357.9k	0.1	7270.2	358.3k	358.3k	0.0	289.0
dem_100_gaslib40_26	219.5k	219.3k	0.1	6.0	219.5k	219.3k	0.1	24.1
dem_100_gaslib40_27	148.0k	138.3k	7.1	14400.0	148.0k	147.9k	0.1	16.4

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Table A.12: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_28	135.3k	135.3k	0.0	321.2	135.3k	135.3k	0.0	100.6
dem_100_gaslib40_29	150.5k	150.4k	0.0	5.8	150.5k	150.4k	0.1	5.2
dem_100_gaslib40_30	199.3k	199.1k	0.1	1098.8	199.3k	199.3k	0.0	995.5
dem_100_gaslib40_31	330.5k	255.7k	29.3	14400.0	330.5k	330.1k	0.1	1924.6
dem_100_gaslib40_32	63.5k	63.5k	0.0	4.4	63.5k	63.5k	0.0	2.5
dem_100_gaslib40_33	133.6k	133.5k	0.1	4.4	133.6k	133.6k	0.0	3.6
dem_100_gaslib40_34	113.1k	113.1k	0.0	3.6	113.1k	113.1k	0.0	3.0
dem_100_gaslib40_35	179.6k	179.4k	0.1	306.7	179.6k	179.6k	0.0	47.0
dem_100_gaslib40_36	180.9k	180.9k	0.0	24.4	180.9k	163.5k	10.6	14400.0
dem_100_gaslib40_37	164.5k	164.3k	0.1	895.0	164.5k	164.5k	0.0	106.0
dem_100_gaslib40_38	274.4k	274.1k	0.1	12314.6	274.4k	274.1k	0.1	2442.1
dem_100_gaslib40_39	202.6k	202.6k	0.0	46.4	202.6k	202.4k	0.1	57.3
dem_100_gaslib40_40	175.0k	174.9k	0.1	45.8	175.0k	174.8k	0.1	43.3
dem_100_gaslib40_41	240.7k	221.3k	8.8	14400.0	240.7k	240.5k	0.1	1140.6
dem_100_gaslib40_42	218.2k	218.0k	0.1	31.5	218.2k	218.0k	0.1	16.9
dem_100_gaslib40_43	192.6k	192.4k	0.1	13790.2	192.6k	192.6k	0.0	85.0
dem_100_gaslib40_44	201.6k	201.5k	0.0	13.8	201.6k	201.4k	0.1	12.6
dem_100_gaslib40_45	190.7k	190.5k	0.1	1563.9	190.7k	190.7k	0.0	70.8
dem_100_gaslib40_46	155.4k	155.3k	0.1	9.6	155.4k	155.2k	0.1	63.7
dem_100_gaslib40_47	145.6k	140.5k	3.6	14400.0	145.6k	140.4k	3.7	14400.0
dem_100_gaslib40_48	282.3k	254.7k	10.8	14400.0	282.3k	280.7k	0.5	14400.0
dem_100_gaslib40_49	175.9k	175.7k	0.1	1527.3	175.9k	175.7k	0.1	26.2
dem_100_gaslib40_50	259.5k	246.3k	5.4	14400.0	259.5k	259.5k	0.0	4718.8
dem_100_gaslib40_51	178.8k	178.7k	0.1	12.2	178.8k	178.6k	0.1	28.1
dem_100_gaslib40_52	106.0k	105.9k	0.1	26.9	106.0k	106.0k	0.0	15.0
dem_100_gaslib40_53	190.7k	190.5k	0.1	693.1	190.7k	190.6k	0.1	167.9
dem_100_gaslib40_54	202.2k	202.1k	0.1	18.0	202.3k	202.1k	0.1	13.3
dem_100_gaslib40_55	212.0k	153.7k	37.9	14400.0	212.0k	211.8k	0.1	5582.5
dem_100_gaslib40_56	125.0k	125.0k	0.0	190.3	125.0k	124.9k	0.1	14.6
dem_100_gaslib40_57	162.5k	162.4k	0.1	163.5	162.5k	162.5k	0.0	182.8
dem_100_gaslib40_58	93.1k	93.0k	0.1	46.6	93.1k	93.0k	0.1	27.3
dem_100_gaslib40_59	228.2k	228.2k	0.0	96.6	228.2k	228.0k	0.1	13.3
dem_100_gaslib40_60	123.0k	122.9k	0.1	5.3	123.0k	122.9k	0.1	4.8
dem_100_gaslib40_61	259.4k	259.4k	0.0	28.0	259.4k	259.4k	0.0	49.4
dem_100_gaslib40_62	220.9k	220.7k	0.1	26.7	220.9k	220.9k	0.0	16.1
dem_100_gaslib40_63	208.0k	187.4k	11.0	14400.0	208.0k	208.0k	0.0	171.7
dem_100_gaslib40_64	209.0k	202.6k	3.2	14400.0	209.0k	209.0k	0.0	943.9
dem_100_gaslib40_65	240.3k	240.1k	0.1	62.1	240.3k	240.1k	0.1	29.4
dem_100_gaslib40_66	312.6k	276.7k	13.0	14400.0	312.6k	251.2k	24.5	14400.0
dem_100_gaslib40_67	390.9k	309.3k	26.4	14400.0	390.9k	390.6k	0.1	828.1
dem_100_gaslib40_68	239.5k	239.3k	0.1	10308.8	239.5k	227.2k	5.4	14400.0
dem_100_gaslib40_69	102.3k	102.2k	0.1	10.8	102.3k	102.3k	0.0	5.6
dem_100_gaslib40_70	282.4k	245.1k	15.2	14400.0	282.4k	247.5k	14.1	14400.0
dem_100_gaslib40_71	202.0k	201.9k	0.0	7.7	202.0k	202.0k	0.0	22.6
dem_100_gaslib40_72	178.3k	178.1k	0.1	28.9	178.3k	178.1k	0.1	17.8
dem_100_gaslib40_73	251.1k	220.9k	13.7	14400.0	251.1k	251.1k	0.0	175.0
dem_100_gaslib40_74	174.6k	174.5k	0.1	26.8	174.7k	174.6k	0.1	8.1
dem_100_gaslib40_75	81.4k	81.4k	0.0	5.0	81.4k	81.4k	0.0	3.2
dem_100_gaslib40_76	347.2k	273.9k	26.8	14400.0	347.2k	286.5k	21.2	14400.0
dem_100_gaslib40_77	177.0k	176.9k	0.0	10.0	176.9k	176.8k	0.1	35.0
dem_100_gaslib40_78	200.0k	199.8k	0.1	16.1	200.0k	199.8k	0.1	10.1
dem_100_gaslib40_79	202.0k	201.8k	0.1	15.4	202.0k	202.0k	0.0	10.0
dem_100_gaslib40_80	456.4k	350.7k	30.1	14400.0	455.8k	351.7k	29.6	14400.0
dem_100_gaslib40_81	207.4k	207.2k	0.1	4855.6	207.4k	207.4k	0.0	1909.9
dem_100_gaslib40_82	184.7k	175.9k	5.0	14400.0	184.7k	184.7k	0.0	25.6
dem_100_gaslib40_83	206.9k	206.7k	0.1	18.9	206.9k	206.9k	0.0	13.6
dem_100_gaslib40_84	180.7k	180.7k	0.0	29.6	180.7k	180.6k	0.1	11.5
dem_100_gaslib40_85	275.4k	206.8k	33.2	14400.0	275.5k	251.8k	9.4	14400.0
dem_100_gaslib40_86	80.7k	80.6k	0.0	13.7	80.7k	80.6k	0.1	5.0
dem_100_gaslib40_87	162.3k	162.3k	0.0	8.6	162.3k	162.2k	0.1	8.6
dem_100_gaslib40_88	239.7k	239.4k	0.1	16.0	239.7k	239.5k	0.1	11.0
dem_100_gaslib40_89	161.3k	161.2k	0.0	32.1	161.3k	161.1k	0.1	11.8
dem_100_gaslib40_90	144.1k	144.0k	0.1	40.1	144.1k	144.1k	0.0	27.9
dem_100_gaslib40_91	215.8k	215.6k	0.1	37.2	215.8k	215.6k	0.1	197.9
dem_100_gaslib40_92	304.5k	251.9k	20.9	14400.0	304.6k	250.6k	21.5	14400.0
dem_100_gaslib40_93	255.3k	255.0k	0.1	22.5	255.1k	255.1k	0.0	12.5
dem_100_gaslib40_94	202.1k	202.0k	0.1	24.0	202.1k	202.0k	0.1	35.5
dem_100_gaslib40_95	180.8k	161.2k	12.2	14400.0	180.8k	180.8k	0.0	143.4
dem_100_gaslib40_96	218.0k	218.0k	0.0	27.0	218.0k	217.8k	0.1	23.2
dem_100_gaslib40_97	164.6k	164.5k	0.1	29.2	164.6k	164.5k	0.1	302.1
dem_100_gaslib40_98	365.5k	298.8k	22.3	14400.0	365.4k	365.1k	0.1	542.9
dem_100_gaslib40_99	152.3k	152.3k	0.0	15.2	152.3k	152.2k	0.1	185.4
dem_100_gaslib40_100	126.2k	126.0k	0.1	13.6	126.2k	126.1k	0.1	10.8
dem_100_gaslib40_101	162.7k	162.6k	0.1	12.0	162.7k	162.5k	0.1	7.3
dem_100_gaslib40_102	140.3k	135.4k	3.6	14400.0	140.3k	140.2k	0.0	22.9
dem_100_gaslib40_103	273.8k	229.0k	19.6	14400.0	273.8k	273.5k	0.1	6399.9
dem_100_gaslib40_104	187.8k	187.7k	0.1	111.9	187.8k	187.6k	0.1	107.9
dem_100_gaslib40_105	241.6k	241.4k	0.1	348.1	241.6k	241.6k	0.0	4006.4

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Table A.12: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_106	178.3k	178.3k	0.0	7.2	178.3k	178.2k	0.0	249.0
dem_100_gaslib40_107	199.1k	199.0k	0.0	35.6	199.0k	198.9k	0.1	31.3
dem_100_gaslib40_108	294.5k	294.2k	0.1	4127.3	294.3k	263.1k	11.9	14400.0
dem_100_gaslib40_109	136.1k	136.1k	0.0	12.9	136.1k	136.1k	0.0	10.3
dem_100_gaslib40_110	121.4k	121.3k	0.0	6.1	121.4k	121.3k	0.1	41.5
dem_100_gaslib40_111	251.6k	217.2k	15.8	14400.0	251.6k	185.1k	35.9	14400.0
dem_100_gaslib40_112	217.6k	217.4k	0.1	26.1	217.5k	217.3k	0.1	27.3
dem_100_gaslib40_113	170.0k	169.8k	0.1	28.9	169.9k	169.8k	0.1	59.9
dem_100_gaslib40_114	195.4k	195.4k	0.0	43.2	195.4k	194.0k	0.7	14400.0
dem_100_gaslib40_115	134.7k	134.7k	0.0	18.2	134.7k	134.6k	0.0	9.2
dem_100_gaslib40_116	439.5k	344.1k	27.7	14400.0	439.5k	338.6k	29.8	14400.0
dem_100_gaslib40_117	156.0k	156.0k	0.0	8.6	156.0k	155.9k	0.1	10.0
dem_100_gaslib40_118	159.3k	159.2k	0.1	10.2	159.3k	159.3k	0.0	10.2
dem_100_gaslib40_119	77.0k	77.0k	0.0	2.7	77.0k	77.0k	0.0	2.1
dem_100_gaslib40_120	431.7k	319.6k	35.1	14400.0	429.7k	413.8k	3.8	14400.0
dem_100_gaslib40_121	136.3k	136.2k	0.1	11.5	136.3k	136.2k	0.1	10.3
dem_100_gaslib40_122	161.7k	161.7k	0.0	14.0	161.7k	145.6k	11.1	14400.0
dem_100_gaslib40_123	118.0k	118.0k	0.0	60.3	118.0k	117.9k	0.1	18.4
dem_100_gaslib40_124	154.9k	154.7k	0.1	13.4	154.9k	154.8k	0.0	31.0
dem_100_gaslib40_125	152.4k	152.4k	0.0	16.1	152.4k	152.3k	0.1	5210.8
dem_100_gaslib40_126	161.7k	161.6k	0.1	8.6	161.7k	161.5k	0.1	4.7
dem_100_gaslib40_127	208.3k	208.3k	0.0	23.1	208.3k	208.2k	0.1	126.7
dem_100_gaslib40_128	146.7k	146.6k	0.1	356.9	146.7k	146.6k	0.1	69.4
dem_100_gaslib40_129	165.7k	165.6k	0.0	20.0	165.7k	165.5k	0.1	103.6
dem_100_gaslib40_130	213.0k	213.0k	0.0	67.1	213.0k	213.0k	0.0	29.3
dem_100_gaslib40_131	154.0k	153.9k	0.1	7.1	154.0k	153.9k	0.1	4.7
dem_100_gaslib40_132	230.9k	230.8k	0.0	28.6	230.8k	230.7k	0.1	7.3
dem_100_gaslib40_133	210.5k	199.3k	5.6	14400.0	205.7k	205.5k	0.1	12.3
dem_100_gaslib40_134	174.8k	174.8k	0.0	26.8	174.8k	174.7k	0.1	17.2
dem_100_gaslib40_135	220.4k	220.2k	0.1	15.1	220.4k	220.2k	0.1	13.6
dem_100_gaslib40_136	227.2k	193.3k	17.5	14400.0	227.2k	227.0k	0.1	5181.9
dem_100_gaslib40_137	134.9k	134.9k	0.0	16.7	134.9k	134.7k	0.1	4.8
dem_100_gaslib40_138	242.1k	241.9k	0.1	100.5	242.1k	242.1k	0.0	1054.6
dem_100_gaslib40_139	238.5k	238.4k	0.1	14.2	238.5k	238.3k	0.1	5.8
dem_100_gaslib40_140	203.2k	203.1k	0.0	32.0	203.2k	203.0k	0.1	16.3
dem_100_gaslib40_141	195.2k	195.0k	0.1	254.7	195.2k	195.2k	0.0	292.3
dem_100_gaslib40_142	228.6k	228.4k	0.1	4305.4	228.6k	228.4k	0.1	171.7
dem_100_gaslib40_143	303.5k	224.4k	35.3	14400.0	303.5k	233.4k	30.0	14400.0
dem_100_gaslib40_144	209.8k	209.8k	0.0	23.9	209.8k	180.7k	16.1	14400.0
dem_100_gaslib40_145	209.6k	209.6k	0.0	59.6	209.6k	209.4k	0.1	62.4
dem_100_gaslib40_146	186.5k	186.4k	0.0	31.7	186.5k	186.5k	0.0	13.7
dem_100_gaslib40_147	194.9k	194.9k	0.0	16.0	194.9k	194.8k	0.1	13.1
dem_100_gaslib40_148	147.6k	147.5k	0.0	15.3	147.6k	147.4k	0.1	16.4
dem_100_gaslib40_149	203.2k	203.0k	0.1	15.4	203.2k	203.0k	0.1	19.4
dem_100_gaslib40_150	337.8k	279.8k	20.8	14400.0	337.8k	226.5k	49.2	14400.0
dem_100_gaslib40_151	237.2k	237.1k	0.0	17.6	237.2k	236.9k	0.1	28.2
dem_100_gaslib40_152	183.7k	183.5k	0.1	2864.9	183.7k	183.7k	0.0	431.9
dem_100_gaslib40_153	281.3k	251.6k	11.8	14400.0	281.3k	250.4k	12.3	14400.0
dem_100_gaslib40_154	213.9k	213.7k	0.1	419.6	213.9k	213.9k	0.0	1104.1
dem_100_gaslib40_155	195.9k	195.9k	0.0	19.9	195.9k	195.7k	0.1	17.0
dem_100_gaslib40_156	194.0k	194.0k	0.0	103.2	194.0k	194.0k	0.0	14.0
dem_100_gaslib40_157	268.8k	230.1k	16.8	14400.0	268.7k	268.4k	0.1	29.4
dem_100_gaslib40_158	150.5k	150.5k	0.0	6.5	150.5k	150.4k	0.1	5.3
dem_100_gaslib40_159	202.3k	202.1k	0.1	18.2	202.3k	202.3k	0.0	85.1
dem_100_gaslib40_160	359.4k	280.2k	28.3	14400.0	359.4k	359.2k	0.1	876.4
dem_100_gaslib40_161	498.9k	380.6k	31.1	14400.0	498.9k	479.7k	4.0	14400.0
dem_100_gaslib40_162	226.7k	226.5k	0.1	14.5	226.7k	226.5k	0.1	11.5
dem_100_gaslib40_163	139.2k	139.1k	0.1	18.6	139.2k	139.1k	0.1	5.2
dem_100_gaslib40_164	182.3k	182.3k	0.0	12.5	182.3k	182.3k	0.0	11.4
dem_100_gaslib40_165	182.5k	174.8k	4.4	14400.0	182.5k	182.4k	0.0	8.5
dem_100_gaslib40_166	286.2k	286.0k	0.1	6.2	286.2k	286.1k	0.0	10.1
dem_100_gaslib40_167	185.5k	185.5k	0.0	19.9	185.5k	185.3k	0.1	7.8
dem_100_gaslib40_168	242.4k	242.4k	0.0	52.2	242.4k	242.4k	0.0	29.4
dem_100_gaslib40_169	99.5k	99.5k	0.0	2.4	99.5k	99.5k	0.0	2.9
dem_100_gaslib40_170	132.1k	132.0k	0.1	25.2	132.1k	132.0k	0.1	21.4
dem_100_gaslib40_171	452.9k	355.1k	27.5	14400.0	452.9k	355.7k	27.3	14400.0
dem_100_gaslib40_172	223.2k	223.1k	0.0	20.2	223.2k	223.0k	0.1	21.6
dem_100_gaslib40_173	109.9k	109.8k	0.1	41.0	109.9k	109.8k	0.2	14400.0
dem_100_gaslib40_174	192.5k	192.5k	0.0	15.9	192.5k	192.3k	0.1	10.0
dem_100_gaslib40_175	451.4k	344.9k	30.9	14400.0	451.4k	351.0k	28.6	14400.0
dem_100_gaslib40_176	208.6k	208.5k	0.1	65.0	208.6k	208.4k	0.1	32.6
dem_100_gaslib40_177	228.9k	228.9k	0.0	16.4	228.9k	228.7k	0.1	86.0
dem_100_gaslib40_178	459.4k	362.5k	26.7	14400.0	459.4k	371.8k	23.6	14400.0
dem_100_gaslib40_179	210.1k	209.9k	0.1	29.3	210.1k	209.9k	0.1	14.2
dem_100_gaslib40_180	150.2k	150.1k	0.1	9.0	150.2k	150.0k	0.1	8.1
dem_100_gaslib40_181	349.0k	252.3k	38.4	14400.0	349.0k	263.2k	32.6	14400.0
dem_100_gaslib40_182	181.4k	181.4k	0.0	16.8	181.4k	181.2k	0.1	68.7
dem_100_gaslib40_183	556.0k	436.6k	27.3	14400.0	556.0k	438.3k	26.9	14400.0

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Table A.12: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_184	224.0k	223.7k	0.1	11949.0	224.0k	204.2k	9.7	14400.0
dem_100_gaslib40_185	157.5k	157.4k	0.1	17.4	157.5k	157.4k	0.1	29.1
dem_100_gaslib40_186	168.3k	168.3k	0.0	13.9	168.3k	168.2k	0.1	37.8
dem_100_gaslib40_187	248.1k	207.8k	19.4	14400.0	248.1k	204.9k	21.1	14400.0
dem_100_gaslib40_188	287.9k	234.7k	22.7	14400.0	287.9k	197.2k	46.0	14400.0
dem_100_gaslib40_189	188.1k	187.9k	0.1	1028.5	188.1k	188.0k	0.1	273.6
dem_100_gaslib40_190	146.4k	146.3k	0.1	11.7	146.4k	146.3k	0.1	12.5
dem_100_gaslib40_191	237.8k	237.6k	0.1	7.7	237.8k	237.7k	0.0	9.8
dem_100_gaslib40_192	213.2k	213.2k	0.0	65.2	213.2k	204.9k	4.0	14400.0
dem_100_gaslib40_193	62.7k	62.7k	0.0	4.8	62.7k	62.6k	0.1	3.3
dem_100_gaslib40_194	172.9k	172.7k	0.1	107.8	172.9k	172.9k	0.0	18.0
dem_100_gaslib40_195	386.0k	316.5k	21.9	14400.0	386.0k	309.0k	24.9	14400.0
dem_100_gaslib40_196	199.2k	199.0k	0.1	7.4	199.2k	199.0k	0.1	6.2
dem_100_gaslib40_197	203.5k	203.3k	0.1	26.4	203.5k	203.5k	0.0	10.2
dem_100_gaslib40_198	131.3k	131.3k	0.0	5.3	131.3k	131.3k	0.0	3.3
dem_100_gaslib40_199	157.7k	157.7k	0.0	62.1	157.7k	156.5k	0.8	14400.0
dem_100_gaslib40_200	281.2k	205.8k	36.6	14400.0	281.2k	257.4k	9.2	14400.0
dem_100_gaslib40_201	272.9k	272.7k	0.1	122.8	272.9k	272.7k	0.1	1145.6
dem_100_gaslib40_202	243.2k	243.2k	0.0	132.6	243.2k	243.2k	0.0	1603.5
dem_100_gaslib40_203	201.0k	200.8k	0.1	6.2	201.0k	200.8k	0.1	7.4
dem_100_gaslib40_204	200.0k	172.1k	16.2	14400.0	200.0k	200.0k	0.0	1097.8
dem_100_gaslib40_205	201.9k	201.8k	0.1	14.2	201.9k	201.7k	0.1	27.1
dem_100_gaslib40_206	194.9k	194.7k	0.1	22.1	194.9k	194.7k	0.1	11.1
dem_100_gaslib40_207	219.6k	219.6k	0.0	41.5	219.7k	219.5k	0.1	14.3
dem_100_gaslib40_208	240.7k	240.7k	0.0	19.0	240.7k	240.6k	0.1	14.2
dem_100_gaslib40_209	216.0k	215.8k	0.1	18.2	215.9k	215.7k	0.1	19.8
dem_100_gaslib40_210	136.0k	136.0k	0.0	57.8	136.0k	136.0k	0.0	23.2
dem_100_gaslib40_211	138.1k	138.0k	0.1	6.7	138.1k	138.0k	0.1	10.6
dem_100_gaslib40_212	196.1k	196.0k	0.1	9.3	196.1k	196.1k	0.0	5.0
dem_100_gaslib40_213	113.0k	112.0k	0.9	14400.0	113.1k	113.0k	0.1	8.2
dem_100_gaslib40_214	188.2k	188.1k	0.1	458.9	188.2k	188.2k	0.0	94.0
dem_100_gaslib40_215	238.5k	238.5k	0.0	38.4	238.5k	238.4k	0.1	23.8
dem_100_gaslib40_216	177.2k	175.5k	1.0	14400.0	177.2k	177.1k	0.1	4.9
dem_100_gaslib40_217	385.4k	288.8k	33.5	14400.0	385.4k	324.4k	18.8	14400.0
dem_100_gaslib40_218	151.3k	151.3k	0.0	12.8	151.3k	151.1k	0.1	2.7
dem_100_gaslib40_219	191.5k	191.4k	0.1	23.5	191.5k	191.4k	0.0	27.8
dem_100_gaslib40_220	143.3k	143.2k	0.0	13.1	143.2k	143.2k	0.0	11.4
dem_100_gaslib40_221	273.2k	273.2k	0.0	145.4	273.2k	273.0k	0.1	12982.9
dem_100_gaslib40_222	205.9k	205.8k	0.1	38.2	205.9k	205.8k	0.1	25.1
dem_100_gaslib40_223	274.4k	194.0k	41.4	14400.0	274.4k	274.2k	0.1	8018.6
dem_100_gaslib40_224	237.1k	237.1k	0.0	39.9	237.1k	237.0k	0.1	11.9
dem_100_gaslib40_225	191.9k	191.9k	0.0	12.6	191.9k	191.9k	0.0	12.6
dem_100_gaslib40_226	256.0k	256.0k	0.0	27.6	256.0k	255.8k	0.1	16.9
dem_100_gaslib40_227	121.0k	120.8k	0.1	24.1	121.0k	121.0k	0.0	13.7
dem_100_gaslib40_228	149.4k	149.3k	0.1	2.7	149.4k	149.4k	0.0	3.6
dem_100_gaslib40_229	333.7k	220.9k	51.1	14400.0	333.7k	263.9k	26.4	14400.0
dem_100_gaslib40_230	504.6k	398.1k	26.7	14400.0	504.6k	504.1k	0.1	1210.4
dem_100_gaslib40_231	243.3k	208.8k	16.5	14400.0	243.3k	243.3k	0.0	3651.1
dem_100_gaslib40_232	177.0k	177.0k	0.0	5.1	177.0k	177.0k	0.0	2.9
dem_100_gaslib40_233	277.9k	211.6k	31.3	14400.0	277.9k	277.9k	0.0	3498.6
dem_100_gaslib40_234	169.8k	169.7k	0.1	11.3	169.8k	169.6k	0.1	14.8
dem_100_gaslib40_235	275.9k	265.1k	4.1	14400.0	275.9k	258.4k	6.8	14400.0
dem_100_gaslib40_236	374.4k	278.3k	34.6	14400.0	367.1k	366.8k	0.1	158.9
dem_100_gaslib40_237	246.2k	222.7k	10.6	14400.0	246.2k	215.3k	14.3	14400.0
dem_100_gaslib40_238	231.8k	231.6k	0.1	183.5	231.8k	231.8k	0.0	746.0
dem_100_gaslib40_239	209.7k	209.5k	0.1	101.7	209.7k	209.5k	0.1	594.8
dem_100_gaslib40_240	215.5k	215.3k	0.1	8.1	215.5k	215.5k	0.0	6.7
dem_100_gaslib40_241	209.9k	209.9k	0.0	12.2	209.9k	209.8k	0.0	4.9
dem_100_gaslib40_242	339.4k	219.7k	54.5	14400.0	339.4k	278.7k	21.8	14400.0
dem_100_gaslib40_243	158.6k	158.6k	0.0	283.6	158.6k	149.3k	6.3	14400.0
dem_100_gaslib40_244	212.8k	212.8k	0.0	695.7	212.8k	212.8k	0.0	28.9
dem_100_gaslib40_245	434.6k	340.5k	27.6	14400.0	428.5k	337.7k	26.9	14400.0
dem_100_gaslib40_246	149.4k	149.2k	0.1	737.6	149.4k	149.4k	0.0	27.4
dem_100_gaslib40_247	397.1k	320.4k	23.9	14400.0	397.1k	396.7k	0.1	137.5
dem_100_gaslib40_248	159.8k	159.6k	0.1	857.0	159.8k	159.8k	0.0	44.5
dem_100_gaslib40_249	143.4k	143.2k	0.1	7.6	143.4k	143.3k	0.0	5.0
dem_100_gaslib40_250	321.9k	166.3k	93.6	14400.0	317.8k	272.2k	16.8	14400.0
dem_100_gaslib40_251	214.9k	214.7k	0.1	41.8	214.9k	214.7k	0.1	10.1
dem_100_gaslib40_252	145.1k	145.1k	0.0	12.8	145.1k	145.0k	0.1	5.8
dem_100_gaslib40_253	186.4k	186.2k	0.1	545.6	186.4k	186.2k	0.1	12.3
dem_100_gaslib40_254	239.1k	178.8k	33.7	14400.0	239.1k	238.9k	0.1	3521.4
dem_100_gaslib40_255	211.8k	211.6k	0.1	15.2	211.8k	201.1k	5.3	14400.0
dem_100_gaslib40_256	137.1k	137.1k	0.0	173.1	137.1k	137.0k	0.1	24.2
dem_100_gaslib40_257	118.4k	118.3k	0.1	11.7	118.4k	118.3k	0.1	5.3
dem_100_gaslib40_258	451.6k	362.8k	24.5	14400.0	451.6k	363.2k	24.4	14400.0
dem_100_gaslib40_259	129.9k	129.9k	0.0	3.7	129.9k	129.9k	0.0	3.5
dem_100_gaslib40_260	116.0k	115.9k	0.1	32.9	116.0k	115.9k	0.1	5.4
dem_100_gaslib40_261	220.0k	219.8k	0.1	795.8	220.0k	219.8k	0.1	354.2

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Table A.12: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_262	454.9k	365.1k	24.6	14400.0	454.4k	453.9k	0.1	8567.3
dem_100_gaslib40_263	310.7k	227.3k	36.7	14400.0	310.7k	242.2k	28.3	14400.0
dem_100_gaslib40_264	142.1k	133.9k	6.1	14400.0	142.1k	139.5k	1.9	14400.0
dem_100_gaslib40_265	300.4k	222.9k	34.7	14400.0	300.4k	251.4k	19.5	14400.0
dem_100_gaslib40_266	185.2k	185.2k	0.0	19.2	185.2k	185.0k	0.1	12.5
dem_100_gaslib40_267	243.3k	228.0k	6.7	14400.0	243.3k	243.1k	0.1	1487.8
dem_100_gaslib40_268	176.7k	176.7k	0.0	21.1	176.7k	176.6k	0.1	15.5
dem_100_gaslib40_269	178.2k	178.2k	0.0	15.0	178.2k	178.1k	0.1	6.4
dem_100_gaslib40_270	141.8k	141.7k	0.1	9.0	141.8k	141.8k	0.0	4.0
dem_100_gaslib40_271	216.7k	216.7k	0.0	6.8	216.7k	216.7k	0.0	7.0
dem_100_gaslib40_272	258.5k	258.3k	0.1	10.5	258.5k	258.3k	0.1	13.0
dem_100_gaslib40_273	162.9k	162.9k	0.0	7.9	163.0k	162.8k	0.1	6.5
dem_100_gaslib40_274	500.1k	379.2k	31.9	14400.0	500.1k	485.0k	3.1	14400.0
dem_100_gaslib40_275	519.2k	402.6k	29.0	14400.0	519.2k	519.1k	0.0	2293.2
dem_100_gaslib40_276	380.4k	292.7k	29.9	14400.0	380.4k	311.4k	22.1	14400.0
dem_100_gaslib40_277	143.6k	143.5k	0.1	118.7	143.6k	143.5k	0.1	9003.5
dem_100_gaslib40_278	172.6k	172.6k	0.0	42.7	172.6k	172.6k	0.0	69.8
dem_100_gaslib40_279	156.0k	155.8k	0.1	127.7	156.0k	156.0k	0.0	433.6
dem_100_gaslib40_280	184.8k	184.8k	0.0	24.8	184.8k	184.6k	0.1	784.7
dem_100_gaslib40_281	162.8k	155.6k	4.6	14400.0	162.8k	162.8k	0.0	158.9
dem_100_gaslib40_282	117.6k	117.6k	0.0	20.5	117.6k	117.6k	0.0	57.7
dem_100_gaslib40_283	171.5k	171.4k	0.0	17.7	171.5k	171.4k	0.1	7.1
dem_100_gaslib40_284	186.0k	185.9k	0.1	17.2	186.0k	186.0k	0.0	17.4
dem_100_gaslib40_285	209.2k	209.0k	0.1	10.4	209.2k	209.1k	0.1	7.7
dem_100_gaslib40_286	242.5k	242.4k	0.0	18.8	242.5k	242.3k	0.1	26.4
dem_100_gaslib40_287	132.9k	132.8k	0.1	6.6	132.9k	132.8k	0.1	8.9
dem_100_gaslib40_288	446.4k	348.8k	28.0	14400.0	446.4k	420.0k	6.3	14400.0
dem_100_gaslib40_289	270.5k	270.2k	0.1	71.0	271.0k	268.3k	1.0	14400.0
dem_100_gaslib40_290	123.5k	123.4k	0.1	45.0	123.5k	123.5k	0.0	26.7
dem_100_gaslib40_291	378.0k	278.7k	35.6	14400.0	378.0k	269.3k	40.4	14400.0
dem_100_gaslib40_292	174.6k	174.5k	0.0	12.2	174.6k	174.4k	0.1	44.8
dem_100_gaslib40_293	164.0k	163.9k	0.0	5.5	164.0k	164.0k	0.0	3.2
dem_100_gaslib40_294	196.8k	196.7k	0.0	22.7	196.8k	183.8k	7.1	14400.0
dem_100_gaslib40_295	308.5k	217.2k	42.0	14400.0	307.3k	257.6k	19.3	14400.0
dem_100_gaslib40_296	219.4k	168.7k	30.0	14400.0	217.9k	217.9k	0.0	5724.1
dem_100_gaslib40_297	224.0k	223.9k	0.1	30.8	223.9k	223.7k	0.1	33.8
dem_100_gaslib40_298	170.5k	170.5k	0.0	12.7	170.5k	170.4k	0.1	10.8
dem_100_gaslib40_299	152.2k	152.1k	0.1	10.1	152.2k	152.2k	0.0	6.6
dem_100_gaslib40_300	148.1k	148.0k	0.1	15.4	148.0k	148.0k	0.1	17.1
dem_100_gaslib40_301	260.7k	260.7k	0.0	38.1	260.7k	260.6k	0.1	34.7
dem_100_gaslib40_302	216.1k	215.9k	0.1	19.1	216.1k	215.9k	0.1	9.0
dem_100_gaslib40_303	393.7k	303.8k	29.6	14400.0	392.7k	392.3k	0.1	364.7
dem_100_gaslib40_304	124.1k	124.1k	0.0	14.0	124.2k	124.1k	0.1	8.4
dem_100_gaslib40_305	181.9k	181.8k	0.1	9.0	181.9k	181.9k	0.0	15.4
dem_100_gaslib40_306	224.2k	204.0k	9.9	14400.0	224.2k	224.0k	0.1	4966.0
dem_100_gaslib40_307	198.7k	198.5k	0.1	36.0	198.7k	198.6k	0.1	30.9
dem_100_gaslib40_308	185.0k	184.8k	0.1	18.0	185.0k	184.9k	0.0	18.9
dem_100_gaslib40_309	178.6k	178.6k	0.0	3.7	178.6k	178.6k	0.0	3.2
dem_100_gaslib40_310	83.5k	83.5k	0.0	4.5	83.6k	83.5k	0.0	6.1
dem_100_gaslib40_311	171.0k	170.8k	0.1	25.9	171.0k	170.9k	0.1	10.6
dem_100_gaslib40_312	201.3k	201.1k	0.1	8.0	201.3k	201.1k	0.1	23.4
dem_100_gaslib40_313	134.9k	134.9k	0.0	9.6	134.9k	134.9k	0.0	4.5
dem_100_gaslib40_314	258.5k	258.3k	0.1	15.7	258.5k	258.4k	0.1	53.5
dem_100_gaslib40_315	355.5k	251.1k	41.6	14400.0	355.5k	275.5k	29.0	14400.0
dem_100_gaslib40_316	237.2k	237.1k	0.0	37.2	237.2k	237.0k	0.1	21.1
dem_100_gaslib40_317	239.1k	238.9k	0.1	403.7	239.1k	238.9k	0.1	6979.5
dem_100_gaslib40_318	168.0k	167.9k	0.1	12.1	168.0k	168.0k	0.1	12.4
dem_100_gaslib40_319	57.1k	57.1k	0.0	3.5	57.1k	57.1k	0.0	1.8
dem_100_gaslib40_320	136.3k	136.2k	0.1	12.0	136.3k	136.2k	0.1	8.6
dem_100_gaslib40_321	246.7k	246.5k	0.1	661.8	246.7k	246.7k	0.0	3845.1
dem_100_gaslib40_322	176.6k	176.6k	0.0	41.6	176.6k	176.6k	0.0	19.4
dem_100_gaslib40_323	280.9k	233.0k	20.6	14400.0	280.9k	272.5k	3.1	14400.0
dem_100_gaslib40_324	204.9k	204.7k	0.1	71.8	204.9k	204.8k	0.1	17.8
dem_100_gaslib40_325	216.7k	174.8k	23.9	14400.0	216.7k	177.5k	22.1	14400.0
dem_100_gaslib40_326	85.3k	85.2k	0.1	4.6	85.3k	85.2k	0.1	2.6
dem_100_gaslib40_327	118.5k	118.5k	0.0	2.8	118.5k	118.5k	0.0	2.5
dem_100_gaslib40_328	330.2k	267.5k	23.4	14400.0	330.2k	300.0k	10.1	14400.0
dem_100_gaslib40_329	162.1k	161.9k	0.1	129.0	162.1k	161.9k	0.1	56.8
dem_100_gaslib40_330	133.0k	133.0k	0.0	8.1	133.0k	133.0k	0.0	5.2
dem_100_gaslib40_331	204.0k	204.0k	0.0	34.3	204.0k	204.0k	0.0	560.0
dem_100_gaslib40_332	195.7k	195.5k	0.1	12.8	195.7k	195.5k	0.1	18.8
dem_100_gaslib40_333	201.8k	201.6k	0.1	24.3	201.8k	201.6k	0.1	30.4
dem_100_gaslib40_334	175.0k	175.0k	0.0	17.8	175.0k	174.9k	0.1	7.3
dem_100_gaslib40_335	124.6k	124.5k	0.1	137.1	124.6k	124.5k	0.1	83.6
dem_100_gaslib40_336	205.1k	204.9k	0.1	18.5	205.1k	204.9k	0.1	9.9
dem_100_gaslib40_337	338.2k	245.6k	37.7	14400.0	338.2k	291.3k	16.1	14400.0
dem_100_gaslib40_338	287.3k	287.3k	0.0	108.2	287.3k	287.3k	0.0	4092.3
dem_100_gaslib40_339	134.2k	126.4k	6.2	14400.0	134.2k	134.2k	0.0	13.3

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Table A.12: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_340	199.5k	199.3k	0.1	93.2	199.5k	199.3k	0.1	3773.3
dem_100_gaslib40_341	233.9k	233.9k	0.0	18.1	233.9k	233.9k	0.0	6499.4
dem_100_gaslib40_342	209.6k	209.4k	0.1	13.8	209.5k	209.3k	0.1	4.8
dem_100_gaslib40_343	313.7k	237.0k	32.3	14400.0	313.7k	266.3k	17.8	14400.0
dem_100_gaslib40_344	132.7k	132.6k	0.1	15.0	132.7k	132.7k	0.0	9.1
dem_100_gaslib40_345	113.1k	113.1k	0.0	6.3	113.2k	113.1k	0.1	3.9
dem_100_gaslib40_346	187.7k	187.5k	0.1	175.7	187.7k	187.5k	0.1	252.9
dem_100_gaslib40_347	192.8k	192.6k	0.1	8.9	192.8k	192.6k	0.1	5.7
dem_100_gaslib40_348	217.1k	216.9k	0.1	12.7	217.1k	216.9k	0.1	17.1
dem_100_gaslib40_349	223.5k	211.9k	5.5	14400.0	223.5k	223.5k	0.0	431.3
dem_100_gaslib40_350	226.1k	225.8k	0.1	9344.2	226.1k	225.9k	0.1	107.3
dem_100_gaslib40_351	300.0k	196.2k	53.0	14400.0	299.2k	252.5k	18.5	14400.0
dem_100_gaslib40_352	157.6k	157.6k	0.0	58.4	157.6k	157.5k	0.1	122.5
dem_100_gaslib40_353	190.5k	190.4k	0.1	25.8	190.5k	190.3k	0.1	11.9
dem_100_gaslib40_354	190.4k	190.2k	0.1	15.6	190.4k	190.3k	0.0	7.3
dem_100_gaslib40_355	233.5k	233.3k	0.1	17.7	233.5k	233.4k	0.1	10.4
dem_100_gaslib40_356	172.2k	172.0k	0.1	9.3	172.2k	172.1k	0.1	8.3
dem_100_gaslib40_357	140.0k	139.9k	0.1	24.7	140.0k	139.9k	0.1	4.4
dem_100_gaslib40_358	223.7k	223.5k	0.1	12.3	223.7k	223.5k	0.1	12.3
dem_100_gaslib40_359	168.2k	168.0k	0.1	22.5	168.2k	168.0k	0.1	3.9
dem_100_gaslib40_360	213.7k	213.7k	0.0	26.4	213.7k	213.5k	0.1	14.8
dem_100_gaslib40_361	178.4k	178.2k	0.1	9.4	178.4k	178.4k	0.0	14.1
dem_100_gaslib40_362	214.4k	193.3k	11.0	14400.0	214.4k	214.3k	0.1	380.6
dem_100_gaslib40_363	154.6k	154.4k	0.1	10.1	154.6k	154.5k	0.1	17.1
dem_100_gaslib40_364	119.7k	119.7k	0.0	33.3	119.7k	119.7k	0.1	9.4
dem_100_gaslib40_365	276.5k	207.2k	33.5	14400.0	276.1k	250.3k	10.3	14400.0
dem_100_gaslib40_366	195.7k	195.7k	0.0	425.8	195.7k	195.7k	0.0	706.4
dem_100_gaslib40_367	237.2k	236.9k	0.1	28.5	237.3k	237.0k	0.1	20.7
dem_100_gaslib40_368	331.1k	222.1k	49.1	14400.0	331.1k	273.5k	21.1	14400.0
dem_100_gaslib40_369	199.8k	175.2k	14.1	14400.0	199.8k	199.8k	0.0	3332.3
dem_100_gaslib40_370	176.5k	176.4k	0.1	35.4	176.5k	176.5k	0.0	28.4
dem_100_gaslib40_371	226.2k	214.3k	5.6	14400.0	226.2k	226.0k	0.1	348.1
dem_100_gaslib40_372	180.8k	180.7k	0.0	14.1	180.8k	180.6k	0.1	8.2
dem_100_gaslib40_373	301.1k	257.5k	16.9	14400.0	301.1k	252.2k	19.4	14400.0
dem_100_gaslib40_374	177.1k	177.1k	0.0	9.7	177.1k	177.0k	0.1	8.3
dem_100_gaslib40_375	199.1k	199.1k	0.0	22.2	199.1k	198.7k	0.2	14400.0
dem_100_gaslib40_376	299.9k	299.7k	0.1	5273.9	299.9k	253.6k	18.3	14400.0
dem_100_gaslib40_377	271.6k	229.3k	18.5	14400.0	271.7k	245.1k	10.9	14400.0
dem_100_gaslib40_378	155.9k	155.9k	0.0	14.5	155.9k	155.8k	0.0	10.5
dem_100_gaslib40_379	204.1k	203.9k	0.1	4.9	204.1k	203.9k	0.1	5.3
dem_100_gaslib40_380	160.4k	160.3k	0.1	8.1	160.4k	160.3k	0.1	7.4
dem_100_gaslib40_381	297.0k	228.6k	29.9	14400.0	297.0k	242.1k	22.7	14400.0
dem_100_gaslib40_382	287.3k	258.3k	11.3	14400.0	287.3k	273.7k	5.0	14400.0
dem_100_gaslib40_383	163.3k	163.1k	0.1	641.0	163.3k	163.3k	0.0	126.7
dem_100_gaslib40_384	165.6k	165.6k	0.0	47.2	165.6k	165.6k	0.0	37.5
dem_100_gaslib40_385	197.8k	197.6k	0.1	9.6	197.8k	197.7k	0.1	9.2
dem_100_gaslib40_386	224.1k	224.1k	0.0	13.8	224.1k	224.1k	0.0	20.3
dem_100_gaslib40_387	213.8k	213.8k	0.0	206.1	213.8k	213.6k	0.1	3746.8
dem_100_gaslib40_388	427.9k	329.0k	30.0	14400.0	425.6k	425.2k	0.1	4110.8
dem_100_gaslib40_389	250.2k	218.0k	14.8	14400.0	250.2k	250.2k	0.0	14236.3
dem_100_gaslib40_390	241.4k	241.2k	0.1	14.4	241.4k	241.1k	0.1	8.6
dem_100_gaslib40_391	125.1k	125.0k	0.1	257.1	125.1k	125.1k	0.0	756.0
dem_100_gaslib40_392	313.7k	241.4k	29.9	14400.0	313.7k	275.3k	14.0	14400.0
dem_100_gaslib40_393	260.4k	260.2k	0.1	12.6	260.4k	260.1k	0.1	21.5
dem_100_gaslib40_394	174.9k	174.9k	0.0	37.2	174.9k	174.8k	0.1	9.1
dem_100_gaslib40_395	216.2k	216.0k	0.1	17.1	216.1k	216.0k	0.0	23.1
dem_100_gaslib40_396	89.7k	89.7k	0.0	11.2	89.7k	89.7k	0.0	4.6
dem_100_gaslib40_397	195.5k	195.3k	0.1	69.4	195.5k	195.3k	0.1	277.8
dem_100_gaslib40_398	255.4k	255.2k	0.1	91.6	255.5k	255.2k	0.1	3651.8
dem_100_gaslib40_399	200.7k	200.5k	0.1	539.2	200.7k	200.5k	0.1	659.4
dem_100_gaslib40_400	386.1k	312.5k	23.6	14400.0	386.1k	303.5k	27.2	14400.0
dem_100_gaslib40_401	185.7k	185.7k	0.0	17.6	185.7k	185.7k	0.0	16.0
dem_100_gaslib40_402	104.6k	104.6k	0.0	5.0	104.6k	104.6k	0.0	15.8
dem_100_gaslib40_403	129.2k	129.2k	0.0	4.7	129.2k	129.1k	0.0	3.2
dem_100_gaslib40_404	227.9k	227.8k	0.0	18.0	227.9k	227.7k	0.1	11.8
dem_100_gaslib40_405	114.9k	114.9k	0.0	12.3	114.9k	114.8k	0.1	9.8
dem_100_gaslib40_406	191.7k	191.6k	0.1	16.0	191.7k	191.5k	0.1	8.3
dem_100_gaslib40_407	188.2k	188.1k	0.1	11.6	188.2k	188.1k	0.1	8.1
dem_100_gaslib40_408	282.6k	236.4k	19.5	14400.0	282.6k	282.4k	0.1	11408.0
dem_100_gaslib40_409	179.1k	179.0k	0.1	6.3	179.1k	179.1k	0.0	5.2
dem_100_gaslib40_410	177.7k	177.7k	0.0	13.6	177.7k	177.5k	0.1	1548.2
dem_100_gaslib40_411	194.3k	194.1k	0.1	118.7	194.3k	194.3k	0.0	190.6
dem_100_gaslib40_412	211.8k	211.6k	0.1	20.3	211.8k	211.7k	0.0	18.4
dem_100_gaslib40_413	169.0k	168.8k	0.1	12.6	168.9k	168.7k	0.1	11.5
dem_100_gaslib40_414	211.0k	210.8k	0.1	7632.3	211.0k	210.8k	0.1	119.3
dem_100_gaslib40_415	170.6k	170.5k	0.0	10.2	170.6k	170.6k	0.0	7.2
dem_100_gaslib40_416	217.0k	216.8k	0.1	7226.3	217.0k	216.8k	0.1	2048.0
dem_100_gaslib40_417	291.2k	210.2k	38.5	14400.0	291.2k	290.9k	0.1	9096.9

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Table A.12: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_418	185.6k	177.1k	4.7	14400.0	185.6k	185.4k	0.1	8.4
dem_100_gaslib40_419	179.8k	179.7k	0.1	33.9	179.8k	179.8k	0.0	97.2
dem_100_gaslib40_420	304.3k	226.9k	34.1	14400.0	289.9k	244.8k	18.5	14400.0
dem_100_gaslib40_421	383.8k	308.7k	24.3	14400.0	383.8k	313.0k	22.6	14400.0
dem_100_gaslib40_422	123.8k	123.8k	0.0	4.6	123.9k	123.8k	0.0	7.0
dem_100_gaslib40_423	208.1k	207.9k	0.1	20.1	208.1k	207.9k	0.1	13.4
dem_100_gaslib40_424	248.1k	209.5k	18.4	14400.0	248.1k	248.1k	0.0	7331.1
dem_100_gaslib40_425	411.6k	392.2k	4.9	14400.0	411.6k	407.4k	1.0	14400.0
dem_100_gaslib40_426	176.1k	176.0k	0.1	2063.9	176.1k	175.9k	0.1	409.9
dem_100_gaslib40_427	345.6k	280.9k	23.0	14400.0	345.6k	283.4k	22.0	14400.0
dem_100_gaslib40_428	193.4k	193.3k	0.0	166.2	193.4k	193.2k	0.1	47.5
dem_100_gaslib40_429	286.7k	210.2k	36.4	14400.0	286.7k	239.0k	20.0	14400.0
dem_100_gaslib40_430	160.4k	160.2k	0.1	29.0	160.4k	160.4k	0.0	139.8
dem_100_gaslib40_431	306.4k	219.5k	39.6	14400.0	305.0k	274.3k	11.2	14400.0
dem_100_gaslib40_432	155.8k	155.6k	0.1	9.0	155.8k	155.8k	0.0	7.4
dem_100_gaslib40_433	245.7k	245.5k	0.1	9163.4	245.7k	245.5k	0.1	1117.2
dem_100_gaslib40_434	167.8k	167.6k	0.1	18.0	167.7k	167.6k	0.1	13.2
dem_100_gaslib40_435	73.8k	73.8k	0.0	3.1	73.8k	73.8k	0.0	2.6
dem_100_gaslib40_436	69.5k	69.5k	0.1	4.5	69.5k	69.5k	0.0	5.0
dem_100_gaslib40_437	178.1k	178.1k	0.0	607.9	178.1k	178.1k	0.0	941.8
dem_100_gaslib40_438	245.0k	244.7k	0.1	25.7	244.8k	244.5k	0.1	6.8
dem_100_gaslib40_439	213.7k	213.7k	0.0	18.9	213.7k	213.7k	0.0	7.7
dem_100_gaslib40_440	190.6k	190.5k	0.1	18.3	190.5k	190.5k	0.0	12.7
dem_100_gaslib40_441	197.1k	196.9k	0.1	60.9	197.1k	196.9k	0.1	66.2
dem_100_gaslib40_442	251.3k	196.1k	28.1	14400.0	251.2k	251.0k	0.1	9453.9
dem_100_gaslib40_443	231.6k	231.3k	0.1	30.4	231.6k	231.3k	0.1	51.2
dem_100_gaslib40_444	215.4k	215.2k	0.1	47.4	215.4k	215.2k	0.1	16.4
dem_100_gaslib40_445	146.4k	146.2k	0.1	14.2	146.4k	146.2k	0.1	10.1
dem_100_gaslib40_446	191.5k	191.4k	0.1	89.7	191.5k	191.3k	0.1	57.5
dem_100_gaslib40_447	465.8k	360.0k	29.4	14400.0	466.4k	362.6k	28.6	14400.0
dem_100_gaslib40_448	204.3k	204.2k	0.1	56.2	204.3k	204.1k	0.1	14.6
dem_100_gaslib40_449	166.2k	166.0k	0.1	12.9	166.1k	166.0k	0.1	4.9
dem_100_gaslib40_450	275.3k	275.0k	0.1	82.2	275.3k	275.0k	0.1	19.5
dem_100_gaslib40_451	132.0k	132.0k	0.0	4.3	132.0k	131.9k	0.1	4.7
dem_100_gaslib40_452	222.7k	222.5k	0.1	12.8	222.7k	222.7k	0.0	20.5
dem_100_gaslib40_453	152.1k	152.0k	0.0	932.1	152.1k	151.9k	0.1	432.8
dem_100_gaslib40_454	222.5k	222.3k	0.1	9628.3	222.5k	222.3k	0.1	5689.9
dem_100_gaslib40_455	323.4k	241.2k	34.1	14400.0	316.3k	250.0k	26.5	14400.0
dem_100_gaslib40_456	172.4k	172.3k	0.1	8.2	172.4k	172.3k	0.0	5.0
dem_100_gaslib40_457	247.0k	246.9k	0.0	13.7	247.0k	247.0k	0.0	11.2
dem_100_gaslib40_458	308.2k	228.5k	34.9	14400.0	308.2k	246.5k	25.1	14400.0
dem_100_gaslib40_459	181.2k	181.1k	0.1	8.3	181.2k	181.2k	0.0	9.7
dem_100_gaslib40_460	179.8k	179.7k	0.0	4.9	179.8k	179.6k	0.1	81.7
dem_100_gaslib40_461	162.2k	162.1k	0.1	551.2	162.2k	162.2k	0.0	67.7
dem_100_gaslib40_462	218.0k	203.3k	7.2	14400.0	218.0k	217.8k	0.1	2522.6
dem_100_gaslib40_463	214.9k	214.8k	0.0	19.4	214.8k	214.6k	0.1	4.6
dem_100_gaslib40_464	273.9k	273.6k	0.1	52.8	273.9k	273.6k	0.1	33.8
dem_100_gaslib40_465	250.0k	249.8k	0.1	26.2	250.0k	249.8k	0.1	25.4
dem_100_gaslib40_466	181.9k	179.9k	1.1	14400.0	181.9k	181.9k	0.0	310.3
dem_100_gaslib40_467	209.6k	209.5k	0.1	9.0	209.5k	209.5k	0.0	12.3
dem_100_gaslib40_468	129.5k	129.4k	0.0	4.8	129.5k	129.4k	0.0	4.0
dem_100_gaslib40_469	186.5k	186.4k	0.1	18.1	186.5k	186.3k	0.1	16.5
dem_100_gaslib40_470	205.1k	204.9k	0.1	23.3	205.1k	205.0k	0.1	43.1
dem_100_gaslib40_471	182.9k	182.8k	0.1	82.3	182.9k	182.8k	0.1	36.5
dem_100_gaslib40_472	300.3k	219.1k	37.1	14400.0	300.3k	300.0k	0.1	5495.7
dem_100_gaslib40_473	145.0k	144.9k	0.1	51.8	145.0k	144.9k	0.1	4.9
dem_100_gaslib40_474	190.0k	189.8k	0.1	33.3	190.0k	189.8k	0.1	24.6
dem_100_gaslib40_475	230.2k	230.1k	0.0	12.3	230.2k	230.0k	0.1	14.3
dem_100_gaslib40_476	155.1k	155.0k	0.1	11.3	155.1k	155.1k	0.0	7.7
dem_100_gaslib40_477	539.3k	425.1k	26.9	14400.0	539.3k	425.7k	26.7	14400.0
dem_100_gaslib40_478	174.8k	174.7k	0.1	10.2	174.8k	174.7k	0.1	11.5
dem_100_gaslib40_479	172.2k	172.1k	0.1	12.0	172.2k	172.1k	0.1	115.1
dem_100_gaslib40_480	223.5k	223.3k	0.1	3458.2	223.5k	223.3k	0.1	294.8
dem_100_gaslib40_481	139.6k	139.6k	0.0	70.5	139.6k	139.5k	0.1	41.2
dem_100_gaslib40_482	376.6k	278.6k	35.2	14400.0	372.5k	287.8k	29.4	14400.0
dem_100_gaslib40_483	132.9k	132.9k	0.0	13.3	132.9k	132.7k	0.1	44.0
dem_100_gaslib40_484	138.7k	135.2k	2.6	14400.0	138.7k	138.6k	0.1	16.3
dem_100_gaslib40_485	176.5k	176.3k	0.1	39.0	176.5k	170.6k	3.4	14400.0
dem_100_gaslib40_486	147.4k	147.3k	0.1	24.5	147.4k	147.4k	0.0	49.5
dem_100_gaslib40_487	206.9k	206.7k	0.1	26.6	206.9k	206.7k	0.1	11.2
dem_100_gaslib40_488	236.6k	236.6k	0.0	57.6	236.6k	236.6k	0.0	67.0
dem_100_gaslib40_489	219.9k	219.7k	0.1	9.4	219.8k	219.7k	0.0	13.7
dem_100_gaslib40_490	202.6k	198.0k	2.3	14400.0	202.6k	202.4k	0.1	27.6
dem_100_gaslib40_491	270.0k	245.1k	10.1	14400.0	270.0k	270.0k	0.0	5368.7
dem_100_gaslib40_492	191.8k	191.6k	0.1	49.1	191.8k	191.6k	0.1	120.5
dem_100_gaslib40_493	156.3k	156.3k	0.0	15.7	156.3k	156.2k	0.1	33.0
dem_100_gaslib40_494	279.1k	279.1k	0.0	6717.4	279.1k	278.8k	0.1	4695.6
dem_100_gaslib40_495	282.5k	234.4k	20.5	14400.0	282.5k	240.3k	17.6	14400.0

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Table A.12: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib40_496	260.6k	189.2k	37.8	14400.0	260.3k	260.0k	0.1	11564.8
dem_100_gaslib40_497	152.8k	152.8k	0.0	6.0	152.8k	152.8k	0.0	4.6
dem_100_gaslib40_498	356.7k	290.7k	22.7	14400.0	356.7k	356.7k	0.0	128.6
dem_100_gaslib40_499	371.6k	275.0k	35.2	14400.0	371.6k	371.3k	0.1	301.8
dem_100_gaslib40_500	200.8k	200.7k	0.0	35.8	200.8k	200.6k	0.1	6.5

Table A.13: Detailed results of the discrete models on *GasLib-40* with $\mathcal{B} = 500$ as summarized in Figure 3.8c and Table 3.3b. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_1	2132.7k	867.5k	145.9	14400.0	2383.1k	1197.3k	99.0	14400.0	2307.4k	910.7k	153.4	14400.0
dem_500_gaslib40_2	2039.7k	831.5k	145.3	14400.0	2227.1k	1142.0k	95.0	14400.0	1935.2k	955.1k	102.6	14400.0
dem_500_gaslib40_3	2765.0k	892.3k	209.9	14400.0	2373.4k	1282.4k	85.1	14400.0	2520.5k	986.8k	155.4	14400.0
dem_500_gaslib40_4	1872.9k	568.6k	229.4	14400.0	1598.5k	849.1k	88.3	14400.0	1660.3k	688.9k	141.0	14400.0
dem_500_gaslib40_5	2264.2k	857.9k	163.9	14400.0	2643.2k	1128.1k	134.3	14400.0	2141.9k	1023.8k	109.2	14400.0
dem_500_gaslib40_6	3083.2k	908.4k	239.4	14400.0	2494.7k	1500.6k	66.2	14400.0	2693.8k	1069.3k	151.9	14400.0
dem_500_gaslib40_7	2093.9k	858.8k	143.8	14400.0	2174.9k	1280.7k	69.8	14400.0	2169.0k	889.4k	143.9	14400.0
dem_500_gaslib40_8	2470.9k	860.9k	187.0	14400.0	2709.0k	1220.4k	122.0	14400.0	2262.6k	945.8k	139.2	14400.0
dem_500_gaslib40_9	2687.8k	907.8k	196.1	14400.0	2489.0k	1465.4k	69.8	14400.0	3931.9k	962.3k	308.6	14400.0
dem_500_gaslib40_10	3001.2k	957.9k	213.3	14400.0	2416.9k	1339.5k	80.4	14400.0	2545.7k	1097.0k	132.1	14400.0
dem_500_gaslib40_11	2875.8k	1004.4k	186.3	14400.0	2767.4k	1494.3k	85.2	14400.0	2676.3k	1235.8k	116.6	14400.0
dem_500_gaslib40_12	1817.5k	853.5k	112.9	14400.0	1804.2k	1120.8k	61.0	14400.0	1e+20	859.3k	–	14400.0
dem_500_gaslib40_13	1906.9k	1060.6k	79.8	14400.0	2197.5k	1292.9k	70.0	14400.0	2020.2k	1095.5k	84.4	14400.0
dem_500_gaslib40_14	2569.4k	982.5k	161.5	14400.0	2727.5k	1285.1k	112.2	14400.0	2775.8k	1126.8k	146.3	14400.0
dem_500_gaslib40_15	2287.7k	718.3k	218.5	14400.0	1869.0k	1070.9k	74.5	14400.0	1952.1k	794.5k	145.7	14400.0
dem_500_gaslib40_16	1521.3k	872.8k	74.3	14400.0	1459.0k	965.7k	51.1	14400.0	1411.8k	917.6k	53.9	14400.0
dem_500_gaslib40_17	1882.8k	899.0k	109.4	14400.0	2135.4k	1280.2k	66.8	14400.0	2361.7k	995.3k	137.3	14400.0
dem_500_gaslib40_18	2118.0k	833.1k	154.2	14400.0	1921.2k	1270.4k	51.2	14400.0	1890.4k	928.9k	103.5	14400.0
dem_500_gaslib40_19	1730.6k	1011.6k	71.1	14400.0	1855.8k	1196.0k	55.2	14400.0	1855.1k	1288.1k	44.0	14400.0
dem_500_gaslib40_20	1757.5k	720.4k	144.0	14400.0	1629.4k	1012.2k	61.0	14400.0	1789.0k	785.6k	127.7	14400.0
dem_500_gaslib40_21	2336.6k	861.3k	171.3	14400.0	2684.8k	1106.5k	142.6	14400.0	2282.3k	983.4k	132.1	14400.0
dem_500_gaslib40_22	1e+20	2075.4k	–	14400.0	4130.9k	2773.7k	48.9	14400.0	4390.9k	2437.5k	80.1	14400.0
dem_500_gaslib40_23	2184.1k	818.0k	167.0	14400.0	2699.2k	1069.8k	152.3	14400.0	2563.5k	946.5k	170.8	14400.0
dem_500_gaslib40_24	2293.0k	902.9k	154.0	14400.0	2106.9k	1342.0k	57.0	14400.0	2236.1k	980.8k	128.0	14400.0
dem_500_gaslib40_25	1517.4k	885.9k	71.3	14400.0	1661.1k	1118.3k	48.5	14400.0	1632.3k	933.5k	74.9	14400.0
dem_500_gaslib40_26	1385.6k	991.5k	39.8	14400.0	1407.3k	1114.2k	26.3	14400.0	1353.5k	1002.2k	35.1	14400.0
dem_500_gaslib40_27	2149.8k	815.0k	163.8	14400.0	2284.5k	1198.9k	90.5	14400.0	2315.8k	949.0k	144.0	14400.0
dem_500_gaslib40_28	1914.3k	729.3k	162.5	14400.0	2012.8k	1161.3k	73.3	14400.0	2671.0k	906.9k	194.5	14400.0
dem_500_gaslib40_29	1802.1k	789.8k	128.2	14400.0	1957.0k	1197.1k	63.5	14400.0	2062.6k	913.4k	125.8	14400.0
dem_500_gaslib40_30	2600.0k	977.4k	166.0	14400.0	2696.7k	1288.8k	109.2	14400.0	2352.9k	1359.3k	73.1	14400.0
dem_500_gaslib40_31	1436.2k	599.2k	139.7	14400.0	1458.9k	927.3k	57.3	14400.0	1366.3k	682.4k	100.2	14400.0
dem_500_gaslib40_32	2845.4k	893.5k	218.4	14400.0	2121.3k	1216.0k	74.5	14400.0	3134.6k	974.9k	221.5	14400.0
dem_500_gaslib40_33	1746.2k	654.5k	166.8	14400.0	1841.0k	1107.0k	66.3	14400.0	2399.6k	749.9k	220.0	14400.0
dem_500_gaslib40_34	2415.7k	927.8k	160.4	14400.0	2596.0k	1328.2k	95.5	14400.0	2917.4k	1027.8k	183.8	14400.0
dem_500_gaslib40_35	1769.1k	710.8k	148.9	14400.0	1993.7k	1105.5k	80.3	14400.0	1e+20	769.9k	–	14400.0
dem_500_gaslib40_36	1516.9k	761.7k	99.2	14400.0	1616.0k	1076.2k	50.2	14400.0	1890.8k	804.4k	135.1	14400.0
dem_500_gaslib40_37	2290.7k	697.3k	228.5	14400.0	2068.9k	981.4k	110.8	14400.0	1864.7k	792.4k	135.3	14400.0
dem_500_gaslib40_38	2058.6k	809.5k	154.3	14400.0	1716.9k	1166.1k	47.2	14400.0	1699.7k	883.9k	92.3	14400.0
dem_500_gaslib40_39	2528.7k	891.8k	183.5	14400.0	2766.6k	1258.2k	119.9	14400.0	2467.9k	1003.7k	145.9	14400.0
dem_500_gaslib40_40	2407.9k	852.4k	182.5	14400.0	1e+20	1255.4k	–	14400.0	2726.9k	1102.0k	147.4	14400.0
dem_500_gaslib40_41	1883.1k	929.0k	102.7	14400.0	2119.1k	1235.2k	71.6	14400.0	1965.5k	1020.5k	92.6	14400.0
dem_500_gaslib40_42	2188.4k	813.8k	168.9	14400.0	2175.6k	1067.7k	103.8	14400.0	2329.6k	911.9k	155.5	14400.0
dem_500_gaslib40_43	2205.9k	873.9k	152.4	14400.0	1866.4k	1102.3k	69.3	14400.0	1834.1k	917.1k	100.0	14400.0
dem_500_gaslib40_44	2182.3k	722.0k	202.3	14400.0	1824.9k	1152.6k	58.3	14400.0	2031.5k	815.9k	149.0	14400.0
dem_500_gaslib40_45	2473.4k	901.7k	174.3	14400.0	2405.9k	1156.5k	108.0	14400.0	3174.1k	1044.4k	203.9	14400.0
dem_500_gaslib40_46	2026.0k	824.7k	145.7	14400.0	1871.0k	1273.3k	46.9	14400.0	1956.3k	1195.6k	63.6	14400.0
dem_500_gaslib40_47	2999.9k	1317.9k	127.6	14400.0	3114.8k	1782.8k	74.7	14400.0	3216.7k	1740.7k	84.8	14400.0
dem_500_gaslib40_48	2230.6k	788.4k	182.9	14400.0	2153.0k	1208.9k	78.1	14400.0	3139.8k	953.5k	229.3	14400.0
dem_500_gaslib40_49	2458.9k	939.8k	161.6	14400.0	2292.4k	1283.6k	78.6	14400.0	2094.7k	1005.6k	108.3	14400.0
dem_500_gaslib40_50	1598.9k	892.9k	79.1	14400.0	1400.0k	1164.4k	20.2	14400.0	1482.9k	871.6k	70.1	14400.0
dem_500_gaslib40_51	1e+20	952.4k	–	14400.0	2789.1k	1359.1k	105.2	14400.0	3806.4k	1186.1k	220.9	14400.0
dem_500_gaslib40_52	1891.1k	859.8k	119.9	14400.0	1883.3k	1204.9k	56.3	14400.0	2020.8k	925.7k	118.3	14400.0
dem_500_gaslib40_53	2130.1k	898.9k	137.0	14400.0	2527.6k	1178.1k	114.5	14400.0	2501.5k	980.3k	155.2	14400.0
dem_500_gaslib40_54	1792.8k	838.9k	113.7	14400.0	1615.7k	1253.0k	28.9	14400.0	1753.8k	882.0k	98.8	14400.0
dem_500_gaslib40_55	1547.1k	703.4k	120.0	14400.0	1657.3k	1116.2k	48.5	14400.0	1621.2k	918.9k	76.4	14400.0
dem_500_gaslib40_56	2893.9k	1117.8k	158.9	14400.0	3037.0k	1453.6k	108.9	14400.0	3453.9k	1270.0k	172.0	14400.0
dem_500_gaslib40_57	3115.7k	1211.4k	157.2	14400.0	3055.5k	1652.4k	84.9	14400.0	2991.8k	1615.9k	85.1	14400.0
dem_500_gaslib40_58	1668.0k	726.8k	129.5	14400.0	1636.3k	1153.3k	41.9	14400.0	1828.7k	831.0k	120.1	14400.0

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Table A.13: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_59	2000.3k	839.5k	138.3	14400.0	2568.2k	1117.4k	129.8	14400.0	2844.4k	926.5k	207.0	14400.0
dem_500_gaslib40_60	2000.0k	730.7k	173.7	14400.0	2468.8k	1125.0k	119.4	14400.0	1891.0k	808.4k	133.9	14400.0
dem_500_gaslib40_61	1e+20	1010.4k	–	14400.0	2780.8k	1446.8k	92.2	14400.0	2724.6k	1159.2k	135.0	14400.0
dem_500_gaslib40_62	2527.0k	850.1k	197.2	14400.0	2666.9k	1157.6k	130.4	14400.0	2075.4k	960.8k	116.0	14400.0
dem_500_gaslib40_63	2863.3k	1282.1k	123.3	14400.0	3256.6k	1654.8k	96.8	14400.0	3998.3k	1607.7k	148.7	14400.0
dem_500_gaslib40_64	1651.4k	991.6k	66.5	14400.0	2071.4k	1224.3k	69.2	14400.0	1682.2k	1037.6k	62.1	14400.0
dem_500_gaslib40_65	2676.5k	905.0k	195.7	14400.0	2739.1k	1225.7k	123.5	14400.0	2305.5k	1007.3k	128.9	14400.0
dem_500_gaslib40_66	2672.5k	970.7k	175.3	14400.0	2673.2k	1393.2k	91.9	14400.0	2833.7k	1301.5k	117.7	14400.0
dem_500_gaslib40_67	1568.0k	755.3k	107.6	14400.0	1522.7k	1108.7k	37.3	14400.0	1499.9k	780.0k	92.3	14400.0
dem_500_gaslib40_68	2193.5k	746.2k	194.0	14400.0	2293.9k	1051.3k	118.2	14400.0	1787.2k	750.8k	138.0	14400.0
dem_500_gaslib40_69	2251.0k	991.3k	127.1	14400.0	2348.7k	1349.5k	74.0	14400.0	2417.2k	1136.1k	112.8	14400.0
dem_500_gaslib40_70	2060.7k	942.7k	118.6	14400.0	2263.9k	1180.6k	91.7	14400.0	2357.9k	1095.1k	115.3	14400.0
dem_500_gaslib40_71	1e+20	1190.4k	–	14400.0	3234.0k	1635.3k	97.8	14400.0	2602.3k	1582.6k	64.4	14400.0
dem_500_gaslib40_72	1e+20	962.7k	–	14400.0	2778.3k	1365.1k	103.5	14400.0	2347.6k	1243.6k	88.8	14400.0
dem_500_gaslib40_73	1891.7k	607.7k	211.3	14400.0	1685.5k	1077.2k	56.5	14400.0	1895.9k	761.4k	149.0	14400.0
dem_500_gaslib40_74	2579.5k	864.7k	198.3	14400.0	2388.2k	1146.9k	108.2	14400.0	3035.5k	1001.1k	203.2	14400.0
dem_500_gaslib40_75	2817.4k	1075.1k	162.1	14400.0	1e+20	1425.1k	–	14400.0	2710.7k	1401.6k	93.4	14400.0
dem_500_gaslib40_76	2076.7k	882.9k	135.2	14400.0	2222.6k	1281.6k	73.4	14400.0	2385.7k	1050.6k	127.1	14400.0
dem_500_gaslib40_77	2233.8k	892.7k	150.2	14400.0	2399.5k	1520.8k	57.8	14400.0	2317.4k	1010.6k	129.3	14400.0
dem_500_gaslib40_78	2510.3k	951.5k	163.8	14400.0	2579.2k	1255.7k	105.4	14400.0	2870.1k	1077.0k	166.5	14400.0
dem_500_gaslib40_79	1954.3k	702.5k	178.2	14400.0	2020.9k	1048.5k	92.7	14400.0	1872.7k	733.2k	155.4	14400.0
dem_500_gaslib40_80	2420.7k	942.2k	156.9	14400.0	3390.8k	1263.3k	168.4	14400.0	2362.7k	1036.7k	127.9	14400.0
dem_500_gaslib40_81	1903.5k	909.8k	109.2	14400.0	1814.7k	1314.3k	38.1	14400.0	2050.0k	995.5k	105.9	14400.0
dem_500_gaslib40_82	2579.0k	1126.3k	129.0	14400.0	2591.9k	1510.3k	71.6	14400.0	2540.5k	1361.0k	86.7	14400.0
dem_500_gaslib40_83	2470.8k	807.3k	206.1	14400.0	2394.0k	1267.0k	89.0	14400.0	2127.0k	881.1k	141.4	14400.0
dem_500_gaslib40_84	2810.3k	1088.7k	158.1	14400.0	3061.0k	1402.9k	118.2	14400.0	3013.5k	1173.2k	156.9	14400.0
dem_500_gaslib40_85	1742.2k	956.1k	82.2	14400.0	1925.1k	1089.3k	76.7	14400.0	1784.5k	978.7k	82.3	14400.0
dem_500_gaslib40_86	1723.9k	647.5k	166.3	14400.0	2375.1k	1074.0k	121.1	14400.0	2026.3k	783.8k	158.5	14400.0
dem_500_gaslib40_87	2155.3k	1077.1k	100.1	14400.0	2574.0k	1319.8k	95.0	14400.0	2340.7k	1293.9k	80.9	14400.0
dem_500_gaslib40_88	1643.7k	950.7k	72.9	14400.0	1644.0k	1205.6k	36.4	14400.0	1560.9k	1041.3k	49.9	14400.0
dem_500_gaslib40_89	2395.2k	951.0k	151.9	14400.0	2584.9k	1503.6k	71.9	14400.0	2774.2k	1081.4k	156.5	14400.0
dem_500_gaslib40_90	1903.9k	1151.8k	65.3	14400.0	2140.4k	1289.8k	66.0	14400.0	2030.1k	1155.4k	75.7	14400.0
dem_500_gaslib40_91	3066.2k	1195.3k	156.5	14400.0	3672.1k	1638.7k	124.1	14400.0	2999.2k	1322.7k	126.7	14400.0
dem_500_gaslib40_92	1602.2k	707.2k	126.5	14400.0	1753.2k	1055.1k	66.2	14400.0	1835.5k	804.3k	128.2	14400.0
dem_500_gaslib40_93	1561.2k	852.4k	83.2	14400.0	2030.9k	1132.7k	79.3	14400.0	1619.8k	868.4k	86.5	14400.0
dem_500_gaslib40_94	1747.3k	852.1k	105.0	14400.0	1744.6k	1126.0k	54.9	14400.0	1745.0k	929.7k	87.7	14400.0
dem_500_gaslib40_95	1857.5k	777.1k	139.0	14400.0	1828.0k	1121.9k	62.9	14400.0	2132.9k	939.6k	127.0	14400.0
dem_500_gaslib40_96	2302.7k	903.0k	155.0	14400.0	2040.2k	1193.6k	70.9	14400.0	2237.5k	966.4k	131.5	14400.0
dem_500_gaslib40_97	2034.1k	790.1k	157.4	14400.0	2391.3k	1204.7k	98.5	14400.0	3540.5k	896.3k	295.0	14400.0
dem_500_gaslib40_98	2596.9k	828.6k	213.4	14400.0	3160.1k	1217.7k	159.5	14400.0	2103.2k	976.4k	115.4	14400.0
dem_500_gaslib40_99	1623.3k	805.1k	101.6	14400.0	1678.5k	1169.7k	43.5	14400.0	1922.6k	826.4k	132.6	14400.0
dem_500_gaslib40_100	2403.6k	935.9k	156.8	14400.0	2956.4k	1405.1k	110.4	14400.0	2499.3k	1087.5k	129.8	14400.0
dem_500_gaslib40_101	2665.6k	915.6k	191.1	14400.0	2914.5k	1301.5k	123.9	14400.0	3410.6k	1180.7k	188.9	14400.0
dem_500_gaslib40_102	2746.3k	1012.8k	171.2	14400.0	2730.5k	1506.8k	81.2	14400.0	2907.9k	1195.4k	143.3	14400.0
dem_500_gaslib40_103	2032.3k	784.1k	159.2	14400.0	2044.7k	1233.2k	65.8	14400.0	1999.6k	901.2k	121.9	14400.0
dem_500_gaslib40_104	2267.6k	848.8k	167.2	14400.0	2142.2k	1124.0k	90.6	14400.0	2020.4k	917.1k	120.3	14400.0
dem_500_gaslib40_105	1650.6k	1027.9k	60.6	14400.0	1988.9k	1124.7k	76.8	14400.0	1926.0k	1042.0k	84.8	14400.0
dem_500_gaslib40_106	2236.4k	831.4k	169.0	14400.0	2263.6k	1146.0k	97.5	14400.0	1858.0k	889.7k	108.8	14400.0
dem_500_gaslib40_107	2124.4k	943.1k	125.3	14400.0	2233.7k	1233.9k	81.0	14400.0	2287.8k	1052.8k	117.3	14400.0
dem_500_gaslib40_108	2362.9k	924.1k	155.7	14400.0	2319.2k	1312.1k	76.8	14400.0	2111.3k	1178.5k	79.1	14400.0
dem_500_gaslib40_109	2312.1k	976.0k	136.9	14400.0	2468.5k	1318.1k	87.3	14400.0	2345.5k	1022.4k	129.4	14400.0
dem_500_gaslib40_110	1872.9k	988.6k	89.5	14400.0	1647.2k	1160.0k	42.0	14400.0	1805.8k	997.3k	81.1	14400.0
dem_500_gaslib40_111	1675.0k	709.4k	136.1	14400.0	1861.1k	1015.2k	83.3	14400.0	1786.7k	814.7k	119.3	14400.0
dem_500_gaslib40_112	2301.6k	982.6k	134.2	14400.0	2578.3k	1287.8k	100.2	14400.0	2804.6k	1336.0k	109.9	14400.0
dem_500_gaslib40_113	2134.1k	878.6k	142.9	14400.0	2376.2k	1152.8k	106.1	14400.0	1991.3k	979.6k	103.3	14400.0
dem_500_gaslib40_114	1750.9k	709.1k	146.9	14400.0	1349.9k	1014.5k	33.1	14400.0	1339.4k	774.9k	72.9	14400.0
dem_500_gaslib40_115	1e+20	1312.8k	–	14400.0	3044.8k	1815.3k	67.7	14400.0	3416.0k	1820.8k	87.6	14400.0
dem_500_gaslib40_116	2259.2k	827.3k	173.1	14400.0	2109.2k	1090.0k	93.5	14400.0	2266.6k	943.5k	140.2	14400.0
dem_500_gaslib40_117	1764.9k	925.3k	90.7	14400.0	1667.3k	1302.4k	28.0	14400.0	1657.4k	1016.4k	63.1	14400.0
dem_500_gaslib40_118	1714.3k	843.4k	103.2	14400.0	1672.5k	1180.4k	41.7	14400.0	1581.7k	852.8k	85.5	14400.0
dem_500_gaslib40_119	2236.0k	964.0k	132.0	14400.0	2628.1k	1321.0k	99.0	14400.0	3059.9k	1217.1k	151.4	14400.0
dem_500_gaslib40_120	2003.7k	969.3k	106.7	14400.0	2358.2k	1228.0k	92.0	14400.0	1862.8k	988.8k	88.4	14400.0
dem_500_gaslib40_121	1e+20	1386.2k	–	14400.0	3367.4k	1925.2k	74.9	14400.0	3653.9k	1809.0k	102.0	14400.0
dem_500_gaslib40_122	1658.6k	800.4k	107.2	14400.0	1848.5k	1030.9k	79.3	14400.0	1653.5k	893.4k	85.1	14400.0
dem_500_gaslib40_123	1831.7k	711.9k	157.3	14400.0	2104.7k	1156.9k	81.9	14400.0	2448.4k	832.2k	194.2	14400.0
dem_500_gaslib40_124	3133.1k	1308.6k	139.4	14400.0	3091.2k	1620.0k	90.8	14400.0	3834.1k	1567.4k	144.6	14400.0
dem_500_gaslib40_125	2233.5k	911.1k	145.2	14400.0	2461.0k	1283.0k	91.8	14400.0	2679.3k	1036.3k	158.6	14400.0
dem_500_gaslib40_126	2164.3k	1253.6k	72.6	14400.0	2221.3k	1514.6k	46.7	14400.0	2147.4k	1319.2k	62.8	14400.0
dem_500_gaslib40_127	1798.2k	941.1k	91.1	14400.0	2068.1k	1124.9k	83.9	14400.0	1850.8k	1206.4k	53.4	14400.0
dem_500_gaslib40_128	2338.1k	970.2k	141.0	14400.0	2598.6k	1342.6k	93.5	14400.0	2412.9k	1087.2k	121.9	14400.0
dem_500_gaslib40_129	1650.8k	787.2k	109.7	14400.0	1744.6k	1054.2k	65.5	14400.0	1617.2k	1025.0k	57.8	14400.0
dem_500_gaslib40_130	2333.9k	902.3k	158.7	14400.0	2329.3k	1248.4k	86.6	14400.0	2383.6k	1193.0k	99.8	14400.0
dem_500_gaslib40_131	2910.6k	1043.3k	179.0	14400.0	2590.2k	1385.7k	86.9	14400.0	2691.2k	1196.5k	124.9	14400.0
dem_500_gaslib40_132	1980.2k	761.4k	160.1	14400.0	2204.6k	1090.0k	102.3	14400.0	2034.3k	933.6k	117.9	14400.0
dem_500_gaslib40_133	2078.5k	876.1k	137.3	14400.0	2894.4k	1260.4k	129.6	14400.0	3375.7k	933.5k	261.6	14400.0
dem_500_gaslib40_134	2128.3k	855.9k	148.7	14400.0	2353.1k	1108.1k	112.3	14400.0	2160.9k	983.2k	119.8	14400.0
dem_500_gaslib40_135	2930.4k	1071.8k	173.4	14400.0	2538.8k	1524.4k	66.5	14400.0	2753.6k	1272.1k	116.5	14400.0
dem_500_gaslib40_136	2156.6k	779.3k	176.7	14400.0	1888.1k	1110.3k	70.0	14400.0	2351.7k	874.5k	168.9	14400.0

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Table A.13: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_137	2650.7k	846.0k	213.3	14400.0	2350.6k	1320.2k	78.1	14400.0	2744.1k	989.0k	177.5	14400.0
dem_500_gaslib40_138	1657.4k	746.9k	121.9	14400.0	1691.6k	988.2k	71.2	14400.0	1562.4k	867.2k	80.2	14400.0
dem_500_gaslib40_139	1706.3k	638.0k	167.4	14400.0	1716.4k	1046.1k	64.1	14400.0	1617.3k	845.0k	91.4	14400.0
dem_500_gaslib40_140	1e+20	1255.3k	–	14400.0	2945.3k	1896.3k	55.3	14400.0	4586.7k	1540.5k	197.7	14400.0
dem_500_gaslib40_141	2078.8k	973.8k	113.5	14400.0	2327.6k	1385.5k	68.0	14400.0	2534.9k	1028.0k	146.6	14400.0
dem_500_gaslib40_142	1943.2k	708.3k	174.4	14400.0	2341.4k	1163.8k	101.2	14400.0	1768.9k	780.7k	126.6	14400.0
dem_500_gaslib40_143	2102.7k	711.6k	195.5	14400.0	1792.2k	1116.7k	60.5	14400.0	1701.1k	822.7k	106.8	14400.0
dem_500_gaslib40_144	1615.0k	881.5k	83.2	14400.0	1765.5k	1133.3k	55.8	14400.0	1631.0k	910.0k	79.2	14400.0
dem_500_gaslib40_145	2134.1k	809.1k	163.8	14400.0	2232.0k	1231.1k	81.3	14400.0	2200.8k	1157.6k	90.1	14400.0
dem_500_gaslib40_146	4245.2k	856.8k	395.4	14400.0	2342.6k	1219.5k	92.1	14400.0	2144.2k	972.4k	120.5	14400.0
dem_500_gaslib40_147	1746.8k	888.9k	96.5	14400.0	1833.7k	1350.5k	35.8	14400.0	1961.1k	942.5k	108.1	14400.0
dem_500_gaslib40_148	1888.4k	903.6k	109.0	14400.0	1805.9k	1172.9k	54.0	14400.0	2139.1k	872.7k	145.1	14400.0
dem_500_gaslib40_149	2619.9k	985.9k	165.8	14400.0	2876.2k	1301.6k	121.0	14400.0	2574.4k	1157.4k	122.4	14400.0
dem_500_gaslib40_150	2836.1k	882.9k	221.2	14400.0	2240.9k	1317.7k	70.1	14400.0	3051.0k	946.4k	222.4	14400.0
dem_500_gaslib40_151	2318.1k	949.0k	144.3	14400.0	2333.8k	1382.7k	68.8	14400.0	2413.5k	1053.5k	129.1	14400.0
dem_500_gaslib40_152	2210.8k	850.7k	159.9	14400.0	2968.8k	1215.7k	144.2	14400.0	2010.9k	945.8k	112.6	14400.0
dem_500_gaslib40_153	1771.1k	743.6k	138.2	14400.0	2004.2k	1013.8k	97.7	14400.0	1692.4k	802.2k	111.0	14400.0
dem_500_gaslib40_154	1e+20	2366.6k	–	14400.0	1e+20	2835.6k	–	14400.0	1e+20	2937.2k	–	14400.0
dem_500_gaslib40_155	1405.8k	955.1k	47.2	14400.0	1415.4k	1147.3k	23.4	14400.0	1591.0k	966.1k	64.7	14400.0
dem_500_gaslib40_156	2736.4k	948.9k	188.4	14400.0	2731.6k	1410.0k	93.7	14400.0	2416.9k	1083.1k	123.2	14400.0
dem_500_gaslib40_157	1e+20	2007.3k	–	14400.0	3888.6k	2742.8k	41.8	14400.0	1e+20	2332.6k	–	14400.0
dem_500_gaslib40_158	1501.4k	840.8k	78.6	14400.0	1689.1k	1019.5k	65.7	14400.0	1467.3k	881.1k	66.5	14400.0
dem_500_gaslib40_159	1989.4k	797.5k	149.5	14400.0	2299.3k	1166.9k	97.0	14400.0	3066.1k	904.2k	239.1	14400.0
dem_500_gaslib40_160	2338.6k	829.2k	182.0	14400.0	2518.9k	1208.5k	108.4	14400.0	2484.2k	1058.8k	134.6	14400.0
dem_500_gaslib40_161	2043.3k	928.8k	120.0	14400.0	2003.1k	1382.9k	44.8	14400.0	2147.3k	965.5k	122.4	14400.0
dem_500_gaslib40_162	2375.8k	890.4k	166.8	14400.0	2296.5k	1393.5k	64.8	14400.0	2324.5k	958.4k	142.5	14400.0
dem_500_gaslib40_163	2305.6k	823.0k	180.1	14400.0	2205.1k	1248.3k	76.7	14400.0	1e+20	913.7k	–	14400.0
dem_500_gaslib40_164	1785.8k	864.4k	106.6	14400.0	1789.3k	1105.5k	61.9	14400.0	1812.9k	956.3k	89.6	14400.0
dem_500_gaslib40_165	2175.5k	823.3k	164.2	14400.0	2350.8k	1251.2k	87.9	14400.0	2087.3k	1174.7k	77.7	14400.0
dem_500_gaslib40_166	2186.0k	777.2k	181.3	14400.0	2274.7k	1188.1k	91.5	14400.0	1974.8k	956.7k	106.4	14400.0
dem_500_gaslib40_167	2029.6k	715.6k	183.6	14400.0	1637.7k	1151.2k	42.3	14400.0	1698.9k	822.5k	106.6	14400.0
dem_500_gaslib40_168	4414.1k	1521.1k	190.2	14400.0	3374.4k	2125.0k	58.8	14400.0	3583.9k	2046.8k	75.1	14400.0
dem_500_gaslib40_169	1799.5k	587.3k	206.4	14400.0	2659.7k	889.0k	199.2	14400.0	1763.0k	746.5k	136.2	14400.0
dem_500_gaslib40_170	2336.9k	882.7k	164.8	14400.0	2019.8k	1270.1k	59.0	14400.0	2201.5k	964.6k	128.2	14400.0
dem_500_gaslib40_171	2512.2k	791.4k	217.4	14400.0	1956.3k	1179.1k	65.9	14400.0	2525.4k	878.3k	187.5	14400.0
dem_500_gaslib40_172	2005.1k	895.3k	124.0	14400.0	2449.0k	1189.6k	105.9	14400.0	2070.6k	1041.5k	98.8	14400.0
dem_500_gaslib40_173	3226.1k	1063.3k	203.4	14400.0	3438.6k	1305.4k	163.4	14400.0	2520.8k	1210.7k	108.2	14400.0
dem_500_gaslib40_174	1881.6k	742.6k	153.4	14400.0	2169.1k	1118.9k	93.9	14400.0	2025.7k	876.9k	131.0	14400.0
dem_500_gaslib40_175	1e+20	1223.6k	–	14400.0	3174.0k	1675.4k	89.4	14400.0	3572.9k	1703.5k	109.7	14400.0
dem_500_gaslib40_176	2698.8k	1086.7k	148.3	14400.0	2493.0k	1527.7k	63.2	14400.0	2878.8k	1418.4k	103.0	14400.0
dem_500_gaslib40_177	2172.3k	942.9k	130.4	14400.0	2207.5k	1226.4k	80.0	14400.0	2360.9k	1167.2k	102.3	14400.0
dem_500_gaslib40_178	2182.7k	962.3k	126.8	14400.0	2314.6k	1289.3k	79.5	14400.0	2720.9k	1111.6k	144.8	14400.0
dem_500_gaslib40_179	1444.7k	801.9k	80.2	14400.0	1383.2k	1097.8k	26.0	14400.0	1603.2k	847.7k	89.1	14400.0
dem_500_gaslib40_180	2190.4k	683.0k	220.7	14400.0	1864.2k	1044.3k	78.5	14400.0	1876.4k	777.8k	141.2	14400.0
dem_500_gaslib40_181	2103.6k	1022.3k	105.8	14400.0	1974.2k	1446.3k	36.5	14400.0	2015.8k	1156.7k	74.3	14400.0
dem_500_gaslib40_182	2519.4k	908.5k	177.3	14400.0	2697.7k	1371.8k	96.7	14400.0	2352.4k	999.0k	135.5	14400.0
dem_500_gaslib40_183	2076.2k	712.7k	191.3	14400.0	1682.2k	987.1k	70.4	14400.0	1768.5k	766.0k	130.9	14400.0
dem_500_gaslib40_184	1742.2k	986.2k	76.7	14400.0	2096.6k	1086.6k	93.0	14400.0	1672.7k	1009.2k	65.7	14400.0
dem_500_gaslib40_185	2471.4k	845.2k	192.4	14400.0	2428.2k	1323.4k	83.5	14400.0	2013.2k	907.2k	121.9	14400.0
dem_500_gaslib40_186	2787.1k	1234.4k	125.8	14400.0	2980.7k	1719.6k	73.3	14400.0	3342.6k	1641.8k	103.6	14400.0
dem_500_gaslib40_187	2459.4k	816.5k	201.2	14400.0	2140.5k	1164.8k	83.8	14400.0	2347.5k	1042.9k	125.1	14400.0
dem_500_gaslib40_188	1627.0k	986.2k	65.0	14400.0	1595.3k	1283.2k	24.3	14400.0	1708.8k	1036.1k	64.9	14400.0
dem_500_gaslib40_189	1670.0k	1093.7k	52.7	14400.0	2148.5k	1286.1k	67.1	14400.0	1761.0k	1124.5k	56.6	14400.0
dem_500_gaslib40_190	2119.5k	843.4k	151.3	14400.0	2243.9k	1136.1k	97.5	14400.0	2423.7k	972.9k	149.1	14400.0
dem_500_gaslib40_191	2527.9k	949.8k	166.2	14400.0	2445.6k	1312.2k	86.4	14400.0	2387.1k	1088.6k	119.3	14400.0
dem_500_gaslib40_192	1e+20	1060.2k	–	14400.0	3017.2k	1491.3k	102.3	14400.0	3734.7k	1493.9k	150.0	14400.0
dem_500_gaslib40_193	1654.9k	703.4k	135.3	14400.0	1531.4k	987.6k	55.1	14400.0	1438.7k	799.2k	80.0	14400.0
dem_500_gaslib40_194	2670.7k	1167.2k	128.8	14400.0	2940.1k	1544.0k	90.4	14400.0	2939.1k	1392.5k	111.1	14400.0
dem_500_gaslib40_195	1614.9k	975.4k	65.6	14400.0	1754.8k	1225.1k	43.2	14400.0	1698.7k	998.1k	70.2	14400.0
dem_500_gaslib40_196	2370.7k	888.3k	166.9	14400.0	2607.2k	1209.3k	115.6	14400.0	2249.1k	1162.5k	93.5	14400.0
dem_500_gaslib40_197	2537.6k	938.1k	170.5	14400.0	2703.0k	1327.2k	103.7	14400.0	2547.6k	1067.5k	138.7	14400.0
dem_500_gaslib40_198	2425.1k	968.4k	150.4	14400.0	2787.4k	1284.9k	116.9	14400.0	2478.1k	1192.3k	107.8	14400.0
dem_500_gaslib40_199	2114.9k	680.9k	210.6	14400.0	2007.7k	1010.6k	98.7	14400.0	2335.8k	741.3k	215.1	14400.0
dem_500_gaslib40_200	2486.3k	948.1k	162.2	14400.0	2484.3k	1271.4k	95.4	14400.0	2413.0k	1101.0k	119.2	14400.0
dem_500_gaslib40_201	2152.1k	934.0k	130.4	14400.0	2221.9k	1208.5k	83.9	14400.0	1924.1k	1009.3k	90.6	14400.0
dem_500_gaslib40_202	2386.8k	846.3k	182.0	14400.0	2537.4k	1244.1k	104.0	14400.0	3848.5k	1220.4k	215.3	14400.0
dem_500_gaslib40_203	1826.5k	892.1k	104.8	14400.0	1778.5k	1121.2k	58.6	14400.0	1747.1k	1114.6k	56.7	14400.0
dem_500_gaslib40_204	1895.0k	594.9k	218.6	14400.0	2166.2k	915.8k	136.5	14400.0	1806.0k	712.8k	153.4	14400.0
dem_500_gaslib40_205	1517.5k	995.8k	52.4	14400.0	1534.9k	1112.2k	38.0	14400.0	1539.5k	1015.5k	51.6	14400.0
dem_500_gaslib40_206	2430.4k	855.6k	184.1	14400.0	1756.2k	1292.7k	35.9	14400.0	2039.0k	902.3k	126.0	14400.0
dem_500_gaslib40_207	1758.0k	701.9k	150.4	14400.0	2259.1k	1114.7k	102.7	14400.0	1776.5k	814.5k	118.1	14400.0
dem_500_gaslib40_208	1782.6k	695.2k	156.4	14400.0	2468.3k	1096.1k	125.2	14400.0	1709.5k	744.5k	129.6	14400.0
dem_500_gaslib40_209	1729.1k	859.2k	101.2	14400.0	1719.0k	1119.8k	53.5	14400.0	1762.8k	873.0k	101.9	14400.0
dem_500_gaslib40_210	1e+20	1265.2k	–	14400.0	3078.7k	1960.2k	57.1	14400.0	3142.2k	1524.1k	106.2	14400.0
dem_500_gaslib40_211	1848.7k	841.8k	119.6	14400.0	1939.1k	1147.6k	69.0	14400.0	2349.0k	899.5k	161.1	14400.0
dem_500_gaslib40_212	2251.0k	910.3k	147.3	14400.0	2294.1k	1252.2k	83.2	14400.0	2271.0k	941.6k	141.2	14400.0
dem_500_gaslib40_213	1603.7k	885.1k	81.2	14400.0	1614.3k	1153.0k	40.0	14400.0	1591.0k	888.7k	79.0	14400.0
dem_500_gaslib40_214	2159.6k	854.0k	152.9	14400.0	2134.2k	1169.4k	82.5	14400.0	1994.8k	954.5k	109.0	14400.0

continued on next page

Table A.13: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_215	2631.0k	813.8k	223.3	14400.0	2105.2k	1252.0k	68.1	14400.0	1970.9k	883.0k	123.2	14400.0
dem_500_gaslib40_216	2123.4k	820.6k	158.8	14400.0	2314.1k	1199.5k	92.9	14400.0	3222.7k	1066.0k	202.3	14400.0
dem_500_gaslib40_217	2333.6k	881.4k	164.8	14400.0	2348.7k	1309.9k	79.3	14400.0	2428.8k	979.0k	148.1	14400.0
dem_500_gaslib40_218	2182.2k	830.9k	162.6	14400.0	2250.5k	1126.6k	99.8	14400.0	2768.5k	1076.2k	157.2	14400.0
dem_500_gaslib40_219	1995.9k	791.9k	152.0	14400.0	2303.6k	1196.3k	92.6	14400.0	1990.4k	1026.1k	94.0	14400.0
dem_500_gaslib40_220	2047.6k	946.8k	116.3	14400.0	2094.5k	1329.9k	57.5	14400.0	2306.7k	1016.0k	127.0	14400.0
dem_500_gaslib40_221	2211.1k	881.4k	150.9	14400.0	2842.0k	1275.9k	122.8	14400.0	2092.0k	1005.3k	108.1	14400.0
dem_500_gaslib40_222	2050.3k	801.8k	155.7	14400.0	2686.9k	1171.9k	129.3	14400.0	2394.4k	1154.1k	107.5	14400.0
dem_500_gaslib40_223	2837.5k	1197.9k	136.9	14400.0	2534.5k	1542.1k	64.4	14400.0	2388.5k	1481.4k	61.2	14400.0
dem_500_gaslib40_224	1642.3k	963.6k	70.4	14400.0	1772.3k	1020.1k	73.7	14400.0	1769.1k	971.0k	82.2	14400.0
dem_500_gaslib40_225	2363.2k	889.7k	165.6	14400.0	2873.5k	1239.0k	131.9	14400.0	2083.6k	1010.2k	106.3	14400.0
dem_500_gaslib40_226	2546.2k	747.4k	240.7	14400.0	1910.4k	1129.9k	69.1	14400.0	2242.8k	844.6k	165.5	14400.0
dem_500_gaslib40_227	2825.6k	1097.9k	157.4	14400.0	2923.9k	1488.0k	96.5	14400.0	2587.7k	1504.8k	72.0	14400.0
dem_500_gaslib40_228	2694.9k	1040.6k	159.0	14400.0	2675.1k	1430.4k	87.0	14400.0	2672.9k	1300.1k	105.6	14400.0
dem_500_gaslib40_229	2328.4k	874.0k	166.4	14400.0	2389.5k	1257.2k	90.1	14400.0	2302.0k	967.1k	138.0	14400.0
dem_500_gaslib40_230	2284.6k	896.2k	154.9	14400.0	2701.2k	1157.8k	133.3	14400.0	2182.3k	1286.7k	69.6	14400.0
dem_500_gaslib40_231	2247.0k	948.0k	137.0	14400.0	2623.3k	1361.5k	92.7	14400.0	2380.6k	1001.7k	137.7	14400.0
dem_500_gaslib40_232	1702.3k	842.1k	102.2	14400.0	1980.1k	1031.8k	91.9	14400.0	2136.3k	833.3k	156.4	14400.0
dem_500_gaslib40_233	1e+20	1459.9k	–	14400.0	3565.4k	1925.3k	85.2	14400.0	3862.2k	1817.4k	112.5	14400.0
dem_500_gaslib40_234	2048.1k	786.2k	160.5	14400.0	2456.8k	1117.6k	119.8	14400.0	1747.6k	881.3k	98.3	14400.0
dem_500_gaslib40_235	2438.8k	839.0k	190.7	14400.0	2174.0k	1197.5k	81.5	14400.0	2194.4k	1205.2k	82.1	14400.0
dem_500_gaslib40_236	2375.0k	778.4k	205.1	14400.0	1760.5k	1150.2k	53.1	14400.0	1993.3k	866.0k	130.2	14400.0
dem_500_gaslib40_237	2733.2k	898.6k	204.2	14400.0	2829.5k	1251.6k	126.1	14400.0	2934.6k	1100.1k	166.8	14400.0
dem_500_gaslib40_238	1746.7k	934.1k	87.0	14400.0	1830.7k	1201.3k	52.4	14400.0	2310.8k	997.1k	131.7	14400.0
dem_500_gaslib40_239	2257.8k	893.7k	152.6	14400.0	2187.3k	1220.9k	79.1	14400.0	2027.0k	1063.5k	90.6	14400.0
dem_500_gaslib40_240	1e+20	1107.7k	–	14400.0	3047.0k	1457.9k	109.0	14400.0	2917.1k	1239.7k	135.3	14400.0
dem_500_gaslib40_241	3312.7k	996.0k	232.6	14400.0	2766.4k	1388.9k	99.2	14400.0	2398.2k	1129.1k	112.4	14400.0
dem_500_gaslib40_242	2393.1k	1033.0k	131.7	14400.0	2597.1k	1441.7k	80.1	14400.0	2702.6k	1262.3k	114.1	14400.0
dem_500_gaslib40_243	2291.1k	777.6k	194.6	14400.0	2474.2k	1184.4k	108.9	14400.0	2162.8k	954.3k	126.6	14400.0
dem_500_gaslib40_244	2535.1k	1055.9k	140.1	14400.0	2851.3k	1413.2k	101.8	14400.0	2650.9k	1185.6k	123.6	14400.0
dem_500_gaslib40_245	1833.9k	759.1k	141.6	14400.0	1721.5k	1121.4k	53.5	14400.0	1683.2k	789.7k	113.1	14400.0
dem_500_gaslib40_246	2008.5k	878.3k	128.7	14400.0	2000.6k	1133.4k	76.5	14400.0	3020.4k	1080.0k	179.7	14400.0
dem_500_gaslib40_247	1e+20	1634.7k	–	14400.0	3861.5k	2083.7k	85.3	14400.0	4132.5k	1988.9k	107.8	14400.0
dem_500_gaslib40_248	2681.4k	1046.6k	156.2	14400.0	2904.6k	1485.8k	95.5	14400.0	3466.0k	1248.6k	177.6	14400.0
dem_500_gaslib40_249	1e+20	993.9k	–	14400.0	2569.8k	1387.2k	85.3	14400.0	2546.6k	1356.6k	87.7	14400.0
dem_500_gaslib40_250	3712.2k	1006.1k	269.0	14400.0	2525.2k	1339.1k	88.6	14400.0	2510.1k	1182.2k	112.3	14400.0
dem_500_gaslib40_251	2242.3k	628.5k	256.7	14400.0	1951.6k	984.1k	98.3	14400.0	1839.2k	733.7k	150.7	14400.0
dem_500_gaslib40_252	2385.1k	907.7k	162.8	14400.0	2638.3k	1401.1k	88.3	14400.0	2972.5k	1242.5k	139.2	14400.0
dem_500_gaslib40_253	1395.6k	692.1k	101.7	14400.0	1345.0k	875.2k	53.7	14400.0	1261.1k	791.8k	59.3	14400.0
dem_500_gaslib40_254	1e+20	1059.5k	–	14400.0	2903.2k	1573.0k	84.6	14400.0	2824.4k	1267.2k	122.9	14400.0
dem_500_gaslib40_255	2175.9k	767.2k	183.6	14400.0	2503.0k	1141.9k	119.2	14400.0	1884.3k	848.8k	122.0	14400.0
dem_500_gaslib40_256	2227.4k	743.7k	199.5	14400.0	1801.0k	1181.9k	52.4	14400.0	1879.9k	805.9k	133.3	14400.0
dem_500_gaslib40_257	2588.0k	954.3k	171.2	14400.0	2623.4k	1353.8k	93.8	14400.0	2397.9k	1129.5k	112.3	14400.0
dem_500_gaslib40_258	2259.6k	860.9k	162.5	14400.0	2942.1k	1317.6k	123.3	14400.0	2205.5k	1016.4k	117.0	14400.0
dem_500_gaslib40_259	2701.2k	802.5k	236.6	14400.0	2660.3k	1166.6k	128.0	14400.0	2267.4k	926.2k	144.8	14400.0
dem_500_gaslib40_260	1807.8k	602.6k	200.0	14400.0	1840.6k	988.6k	86.2	14400.0	2013.0k	747.0k	169.5	14400.0
dem_500_gaslib40_261	1e+20	1202.1k	–	14400.0	3264.8k	1582.1k	106.4	14400.0	2903.7k	1492.3k	94.6	14400.0
dem_500_gaslib40_262	1893.1k	743.9k	154.5	14400.0	1920.3k	1110.6k	72.9	14400.0	1747.7k	842.2k	107.5	14400.0
dem_500_gaslib40_263	2863.3k	1000.5k	186.2	14400.0	2958.3k	1372.1k	115.6	14400.0	2579.5k	1216.5k	112.0	14400.0
dem_500_gaslib40_264	2344.9k	940.5k	149.3	14400.0	2583.2k	1222.9k	111.2	14400.0	2380.5k	1135.5k	109.6	14400.0
dem_500_gaslib40_265	1566.2k	711.2k	120.2	14400.0	1846.1k	981.6k	88.1	14400.0	1961.0k	906.2k	116.4	14400.0
dem_500_gaslib40_266	1667.1k	987.9k	68.7	14400.0	1889.8k	1158.5k	63.1	14400.0	1763.2k	1135.6k	55.3	14400.0
dem_500_gaslib40_267	1979.1k	698.7k	183.3	14400.0	2010.2k	1057.7k	90.0	14400.0	2118.7k	860.8k	146.1	14400.0
dem_500_gaslib40_268	2157.5k	933.4k	131.1	14400.0	2301.8k	1225.9k	87.8	14400.0	2430.8k	940.9k	158.3	14400.0
dem_500_gaslib40_269	2297.0k	946.9k	142.6	14400.0	2565.5k	1285.9k	99.5	14400.0	2327.9k	1052.9k	121.1	14400.0
dem_500_gaslib40_270	1474.1k	898.5k	64.1	14400.0	1426.4k	1021.2k	39.7	14400.0	1346.9k	955.2k	41.0	14400.0
dem_500_gaslib40_271	2209.0k	712.7k	210.0	14400.0	2204.2k	1119.0k	97.0	14400.0	1906.6k	793.1k	140.4	14400.0
dem_500_gaslib40_272	1881.3k	836.3k	125.0	14400.0	1891.9k	1157.2k	63.5	14400.0	1917.7k	975.9k	96.5	14400.0
dem_500_gaslib40_273	1913.3k	1042.3k	83.6	14400.0	2072.9k	1265.3k	63.8	14400.0	1871.0k	1056.2k	77.1	14400.0
dem_500_gaslib40_274	1454.8k	821.2k	77.1	14400.0	1587.2k	975.8k	62.7	14400.0	1597.5k	872.8k	83.0	14400.0
dem_500_gaslib40_275	1668.9k	858.4k	94.4	14400.0	2476.9k	1091.9k	126.8	14400.0	1607.5k	949.9k	69.2	14400.0
dem_500_gaslib40_276	1705.6k	931.6k	83.1	14400.0	2132.7k	1251.9k	70.4	14400.0	1701.8k	982.9k	73.1	14400.0
dem_500_gaslib40_277	2626.5k	1032.8k	154.3	14400.0	2761.4k	1431.2k	92.9	14400.0	2814.2k	1433.1k	96.4	14400.0
dem_500_gaslib40_278	1976.1k	899.3k	119.7	14400.0	1680.0k	1085.0k	54.8	14400.0	1658.5k	979.6k	69.3	14400.0
dem_500_gaslib40_279	1961.1k	735.4k	166.7	14400.0	2000.0k	1121.8k	78.3	14400.0	1892.9k	852.3k	122.1	14400.0
dem_500_gaslib40_280	2417.1k	923.5k	161.7	14400.0	2490.5k	1321.8k	88.4	14400.0	2391.0k	1034.0k	131.2	14400.0
dem_500_gaslib40_281	2365.7k	924.7k	155.8	14400.0	1798.5k	1323.7k	35.9	14400.0	1747.7k	1029.0k	69.8	14400.0
dem_500_gaslib40_282	3412.0k	1237.5k	175.7	14400.0	3108.7k	1691.6k	83.8	14400.0	4263.3k	1629.6k	161.6	14400.0
dem_500_gaslib40_283	1906.2k	769.1k	147.8	14400.0	2212.8k	1109.3k	99.5	14400.0	1960.2k	876.2k	123.7	14400.0
dem_500_gaslib40_284	1962.0k	790.5k	148.2	14400.0	2215.1k	1192.1k	85.8	14400.0	2275.4k	855.5k	166.0	14400.0
dem_500_gaslib40_285	1718.8k	937.6k	83.3	14400.0	1902.7k	1368.8k	39.0	14400.0	1818.5k	1046.8k	73.7	14400.0
dem_500_gaslib40_286	2390.3k	833.3k	186.8	14400.0	2329.5k	1272.0k	83.1	14400.0	2570.4k	911.1k	182.1	14400.0
dem_500_gaslib40_287	2976.1k	949.6k	213.4	14400.0	2734.7k	1235.3k	121.4	14400.0	2458.1k	1008.3k	143.8	14400.0
dem_500_gaslib40_288	2648.1k	1107.0k	139.2	14400.0	2767.6k	1571.0k	76.2	14400.0	2619.1k	1313.2k	99.4	14400.0
dem_500_gaslib40_289	2527.3k	983.0k	157.1	14400.0	2325.9k	1441.7k	61.3	14400.0	2297.4k	1081.3k	112.5	14400.0
dem_500_gaslib40_290	1686.3k	899.6k	87.5	14400.0	1743.5k	1255.7k	38.8	14400.0	1906.8k	965.3k	97.5	14400.0
dem_500_gaslib40_291	2424.9k	878.4k	176.0	14400.0	2700.7k	1258.2k	114.6	14400.0	2145.8k	983.6k	118.2	14400.0
dem_500_gaslib40_292	2452.4k	888.1k	176.1	14400.0	2253.6k	1271.6k	77.2	14400.0	2093.2k	1142.4k	83.2	14400.0

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Table A.13: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_293	2520.8k	1000.9k	151.9	14400.0	2613.1k	1436.7k	81.9	14400.0	2706.4k	1300.0k	108.2	14400.0
dem_500_gaslib40_294	1706.2k	649.6k	162.7	14400.0	1642.4k	935.3k	75.6	14400.0	1576.3k	735.9k	114.2	14400.0
dem_500_gaslib40_295	3140.9k	1254.6k	150.4	14400.0	3550.1k	1705.0k	108.2	14400.0	3363.9k	1623.5k	107.2	14400.0
dem_500_gaslib40_296	1975.3k	928.8k	112.7	14400.0	2153.7k	1271.7k	69.4	14400.0	2037.2k	1088.4k	87.2	14400.0
dem_500_gaslib40_297	2808.1k	1119.0k	150.9	14400.0	3105.0k	1488.9k	108.5	14400.0	2803.0k	1502.1k	86.6	14400.0
dem_500_gaslib40_298	3158.8k	909.7k	247.2	14400.0	3019.9k	1462.7k	106.5	14400.0	3381.0k	1077.4k	213.8	14400.0
dem_500_gaslib40_299	1790.2k	999.2k	79.2	14400.0	1924.6k	1138.4k	69.1	14400.0	1854.4k	1052.6k	76.2	14400.0
dem_500_gaslib40_300	1945.6k	1007.7k	93.1	14400.0	1892.8k	1457.5k	29.9	14400.0	1985.0k	1021.5k	94.3	14400.0
dem_500_gaslib40_301	2760.3k	918.9k	200.4	14400.0	2440.2k	1243.7k	96.2	14400.0	2285.5k	1047.5k	118.2	14400.0
dem_500_gaslib40_302	1e+20	1330.8k	–	14400.0	3088.6k	1764.0k	75.1	14400.0	3740.2k	1635.0k	128.8	14400.0
dem_500_gaslib40_303	2360.5k	945.5k	149.6	14400.0	3142.6k	1352.3k	132.4	14400.0	2857.1k	1106.2k	158.3	14400.0
dem_500_gaslib40_304	2644.9k	944.9k	179.9	14400.0	2386.0k	1434.4k	66.3	14400.0	2441.4k	1128.8k	116.3	14400.0
dem_500_gaslib40_305	1481.6k	680.3k	117.8	14400.0	1427.5k	1016.0k	40.5	14400.0	1557.9k	807.7k	92.9	14400.0
dem_500_gaslib40_306	1606.3k	752.8k	113.4	14400.0	1772.2k	1135.6k	56.1	14400.0	1765.5k	803.0k	119.9	14400.0
dem_500_gaslib40_307	2844.4k	1113.9k	155.3	14400.0	3006.0k	1477.2k	103.5	14400.0	2643.6k	1346.2k	96.4	14400.0
dem_500_gaslib40_308	1691.0k	972.1k	74.0	14400.0	1737.7k	1224.1k	42.0	14400.0	1701.1k	1027.0k	65.6	14400.0
dem_500_gaslib40_309	2070.0k	743.5k	178.4	14400.0	2192.2k	1022.0k	114.5	14400.0	1961.3k	760.5k	157.9	14400.0
dem_500_gaslib40_310	1631.0k	866.7k	88.2	14400.0	1583.0k	1104.5k	43.3	14400.0	1459.7k	929.0k	57.1	14400.0
dem_500_gaslib40_311	2331.6k	896.2k	160.2	14400.0	2392.0k	1335.1k	79.2	14400.0	2168.6k	1288.6k	68.3	14400.0
dem_500_gaslib40_312	2632.6k	926.2k	184.2	14400.0	2493.4k	1305.3k	91.0	14400.0	2893.4k	1032.2k	180.3	14400.0
dem_500_gaslib40_313	2243.2k	720.1k	211.5	14400.0	2042.3k	1104.7k	84.9	14400.0	1946.5k	929.5k	109.4	14400.0
dem_500_gaslib40_314	2148.5k	713.8k	201.0	14400.0	1389.1k	912.6k	52.2	14400.0	1346.3k	782.3k	72.1	14400.0
dem_500_gaslib40_315	2330.1k	853.8k	172.9	14400.0	2590.0k	1317.0k	96.7	14400.0	3321.3k	887.9k	274.1	14400.0
dem_500_gaslib40_316	2680.9k	943.0k	184.3	14400.0	2639.1k	1319.0k	100.1	14400.0	3264.7k	1217.3k	168.2	14400.0
dem_500_gaslib40_317	1475.3k	837.2k	76.2	14400.0	1595.2k	946.7k	68.5	14400.0	1558.5k	848.2k	83.7	14400.0
dem_500_gaslib40_318	3663.3k	1037.4k	253.1	14400.0	2833.8k	1367.4k	107.2	14400.0	2945.8k	1128.4k	161.1	14400.0
dem_500_gaslib40_319	2131.5k	666.9k	219.6	14400.0	2000.2k	1111.3k	80.0	14400.0	1e+20	706.7k	–	14400.0
dem_500_gaslib40_320	2571.9k	925.6k	177.9	14400.0	2479.6k	1242.4k	99.6	14400.0	2895.9k	1144.6k	153.0	14400.0
dem_500_gaslib40_321	1932.4k	881.2k	119.3	14400.0	1816.6k	1051.3k	72.8	14400.0	1688.8k	1025.1k	64.7	14400.0
dem_500_gaslib40_322	1784.6k	826.4k	116.0	14400.0	1866.0k	1274.0k	46.5	14400.0	1714.9k	867.2k	97.7	14400.0
dem_500_gaslib40_323	2741.5k	993.2k	176.0	14400.0	2534.9k	1301.8k	94.7	14400.0	3344.3k	1119.0k	198.9	14400.0
dem_500_gaslib40_324	1568.2k	770.9k	103.4	14400.0	1487.9k	1005.4k	48.0	14400.0	1534.0k	873.2k	75.7	14400.0
dem_500_gaslib40_325	2374.9k	919.9k	158.2	14400.0	3082.2k	1360.8k	126.5	14400.0	3829.5k	1084.9k	253.0	14400.0
dem_500_gaslib40_326	1837.3k	704.3k	160.9	14400.0	2025.7k	1012.0k	100.2	14400.0	1669.8k	736.5k	126.7	14400.0
dem_500_gaslib40_327	1894.2k	1013.6k	86.9	14400.0	1863.9k	1255.3k	48.5	14400.0	2043.9k	1041.5k	96.2	14400.0
dem_500_gaslib40_328	1725.0k	672.4k	156.5	14400.0	1874.2k	1079.1k	73.7	14400.0	1837.4k	800.3k	129.6	14400.0
dem_500_gaslib40_329	1757.8k	823.6k	113.4	14400.0	1681.8k	1045.5k	60.9	14400.0	1e+20	826.9k	–	14400.0
dem_500_gaslib40_330	2512.7k	1004.4k	150.2	14400.0	2882.4k	1362.7k	111.5	14400.0	3299.9k	1103.4k	199.1	14400.0
dem_500_gaslib40_331	2364.5k	994.7k	137.7	14400.0	2338.8k	1267.4k	84.5	14400.0	2036.7k	1063.7k	91.5	14400.0
dem_500_gaslib40_332	2489.5k	1241.2k	100.6	14400.0	2557.8k	1566.8k	63.2	14400.0	2867.3k	1443.4k	98.7	14400.0
dem_500_gaslib40_333	2379.1k	888.5k	167.8	14400.0	2788.6k	1253.9k	122.4	14400.0	2311.7k	1005.1k	130.0	14400.0
dem_500_gaslib40_334	2608.3k	992.8k	162.7	14400.0	2861.3k	1509.7k	89.5	14400.0	2314.3k	1165.0k	98.7	14400.0
dem_500_gaslib40_335	1977.8k	616.9k	220.6	14400.0	1859.6k	965.0k	92.7	14400.0	2257.1k	734.2k	207.4	14400.0
dem_500_gaslib40_336	1729.5k	811.0k	113.3	14400.0	1582.9k	1165.3k	35.8	14400.0	1686.0k	843.8k	99.8	14400.0
dem_500_gaslib40_337	2844.9k	878.7k	223.8	14400.0	2876.8k	1122.8k	156.2	14400.0	2467.1k	956.0k	158.1	14400.0
dem_500_gaslib40_338	1986.1k	744.1k	166.9	14400.0	1807.7k	1194.3k	51.4	14400.0	1762.5k	844.7k	108.7	14400.0
dem_500_gaslib40_339	1668.0k	592.9k	181.3	14400.0	1459.9k	953.6k	53.1	14400.0	1478.8k	739.3k	100.0	14400.0
dem_500_gaslib40_340	2243.2k	915.6k	145.0	14400.0	2130.3k	1192.4k	78.7	14400.0	2228.8k	1027.3k	117.0	14400.0
dem_500_gaslib40_341	3142.3k	1220.6k	157.4	14400.0	4266.9k	1734.2k	146.0	14400.0	3856.5k	1431.2k	169.5	14400.0
dem_500_gaslib40_342	1835.8k	733.7k	150.2	14400.0	1937.7k	1074.4k	80.4	14400.0	1503.3k	764.8k	96.6	14400.0
dem_500_gaslib40_343	3138.8k	1153.5k	172.1	14400.0	2744.1k	1483.8k	84.9	14400.0	2803.2k	1459.8k	92.0	14400.0
dem_500_gaslib40_344	3465.2k	1465.0k	136.5	14400.0	3358.9k	1911.0k	75.8	14400.0	4745.4k	1875.0k	153.1	14400.0
dem_500_gaslib40_345	2529.3k	896.7k	182.1	14400.0	2641.0k	1249.1k	111.4	14400.0	2323.5k	1025.2k	126.6	14400.0
dem_500_gaslib40_346	2465.2k	917.4k	168.7	14400.0	2763.3k	1270.5k	117.5	14400.0	2343.6k	1133.1k	106.8	14400.0
dem_500_gaslib40_347	3120.8k	1161.0k	168.8	14400.0	3108.1k	1515.5k	105.1	14400.0	2999.0k	1480.8k	102.5	14400.0
dem_500_gaslib40_348	1831.0k	734.6k	149.2	14400.0	1841.3k	1122.1k	64.1	14400.0	1784.2k	790.4k	125.7	14400.0
dem_500_gaslib40_349	3208.5k	998.1k	221.5	14400.0	2685.9k	1403.4k	91.4	14400.0	2888.0k	1062.8k	171.7	14400.0
dem_500_gaslib40_350	2194.0k	866.4k	153.2	14400.0	2274.1k	1337.3k	70.0	14400.0	2600.6k	1080.5k	140.7	14400.0
dem_500_gaslib40_351	2349.1k	837.5k	180.5	14400.0	2251.1k	1167.8k	92.8	14400.0	2142.9k	913.5k	134.6	14400.0
dem_500_gaslib40_352	2219.6k	800.1k	177.4	14400.0	2399.7k	1165.9k	105.8	14400.0	2677.1k	866.6k	208.9	14400.0
dem_500_gaslib40_353	1458.6k	831.4k	75.4	14400.0	1672.9k	1116.0k	49.9	14400.0	1439.6k	849.0k	69.6	14400.0
dem_500_gaslib40_354	2276.0k	930.4k	144.6	14400.0	2080.9k	1254.7k	65.8	14400.0	2047.2k	1013.5k	102.0	14400.0
dem_500_gaslib40_355	2278.5k	877.5k	159.7	14400.0	2423.8k	1237.2k	95.9	14400.0	3165.6k	953.2k	232.1	14400.0
dem_500_gaslib40_356	2433.2k	1041.4k	133.7	14400.0	2535.5k	1482.6k	71.0	14400.0	3493.4k	1249.0k	179.7	14400.0
dem_500_gaslib40_357	2638.9k	920.8k	186.6	14400.0	2782.0k	1322.6k	110.4	14400.0	2372.2k	1042.9k	127.4	14400.0
dem_500_gaslib40_358	4036.8k	1461.7k	176.2	14400.0	3271.8k	1892.2k	72.9	14400.0	3112.0k	1750.5k	77.8	14400.0
dem_500_gaslib40_359	2701.8k	850.6k	217.6	14400.0	2635.9k	1207.3k	118.3	14400.0	2432.3k	988.3k	146.1	14400.0
dem_500_gaslib40_360	2409.7k	849.8k	183.6	14400.0	2880.2k	1238.5k	132.6	14400.0	2323.6k	961.2k	141.7	14400.0
dem_500_gaslib40_361	1495.8k	921.2k	62.4	14400.0	1646.5k	1047.2k	57.2	14400.0	1552.5k	933.7k	66.3	14400.0
dem_500_gaslib40_362	1305.1k	872.6k	49.6	14400.0	1398.0k	1039.8k	34.4	14400.0	1429.4k	903.9k	58.1	14400.0
dem_500_gaslib40_363	1501.3k	963.8k	55.8	14400.0	1452.1k	1119.8k	29.7	14400.0	1617.8k	974.4k	66.0	14400.0
dem_500_gaslib40_364	1966.8k	700.1k	180.9	14400.0	2068.5k	1076.2k	92.2	14400.0	2342.1k	845.5k	177.0	14400.0
dem_500_gaslib40_365	1616.6k	643.9k	151.1	14400.0	1582.3k	964.1k	64.1	14400.0	1620.6k	751.0k	115.8	14400.0
dem_500_gaslib40_366	1342.5k	939.1k	43.0	14400.0	1341.6k	1085.2k	23.6	14400.0	1399.8k	955.9k	46.4	14400.0
dem_500_gaslib40_367	2063.1k	798.8k	158.3	14400.0	2315.7k	1253.4k	84.7	14400.0	2598.6k	869.4k	198.9	14400.0
dem_500_gaslib40_368	1503.6k	712.2k	111.1	14400.0	1614.7k	1131.7k	42.7	14400.0	1504.0k	832.9k	80.6	14400.0
dem_500_gaslib40_369	2178.6k	742.5k	193.4	14400.0	1904.4k	1195.9k	59.2	14400.0	1749.7k	794.6k	120.2	14400.0
dem_500_gaslib40_370	2110.5k	627.7k	236.2	14400.0	1863.5k	1104.3k	68.8	14400.0	2026.0k	820.8k	146.8	14400.0

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Table A.13: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_371	1658.8k	901.5k	84.0	14400.0	1762.0k	1193.9k	47.6	14400.0	1772.1k	1170.5k	51.4	14400.0
dem_500_gaslib40_372	2798.5k	798.8k	250.3	14400.0	2175.5k	1245.7k	74.6	14400.0	1955.6k	923.5k	111.8	14400.0
dem_500_gaslib40_373	2879.7k	914.5k	214.9	14400.0	2503.0k	1461.4k	71.3	14400.0	2416.1k	1081.6k	123.4	14400.0
dem_500_gaslib40_374	1819.7k	825.6k	120.4	14400.0	1802.0k	1226.1k	47.0	14400.0	1966.2k	1005.4k	95.6	14400.0
dem_500_gaslib40_375	2011.8k	787.5k	155.5	14400.0	2516.7k	1209.6k	108.1	14400.0	2124.7k	833.3k	155.0	14400.0
dem_500_gaslib40_376	2840.5k	1145.0k	148.1	14400.0	3160.9k	1487.2k	112.5	14400.0	3469.1k	1468.7k	136.2	14400.0
dem_500_gaslib40_377	1651.0k	667.9k	147.2	14400.0	1651.7k	971.2k	70.1	14400.0	1893.8k	807.2k	134.6	14400.0
dem_500_gaslib40_378	2724.6k	831.6k	227.7	14400.0	2832.6k	1239.8k	128.5	14400.0	2693.8k	1061.1k	153.9	14400.0
dem_500_gaslib40_379	2165.2k	658.5k	228.8	14400.0	1682.9k	945.9k	77.9	14400.0	1662.7k	746.7k	122.7	14400.0
dem_500_gaslib40_380	1866.9k	802.0k	132.8	14400.0	1829.8k	1237.6k	47.9	14400.0	1836.9k	907.3k	102.5	14400.0
dem_500_gaslib40_381	2841.4k	1073.2k	164.8	14400.0	2716.3k	1518.0k	78.9	14400.0	2956.3k	1487.0k	98.8	14400.0
dem_500_gaslib40_382	2108.2k	844.9k	149.5	14400.0	2352.8k	1221.7k	92.6	14400.0	2178.2k	945.7k	130.3	14400.0
dem_500_gaslib40_383	1869.3k	916.4k	104.0	14400.0	2164.8k	1227.1k	76.4	14400.0	1864.4k	1125.3k	65.7	14400.0
dem_500_gaslib40_384	2645.5k	844.9k	213.1	14400.0	2237.0k	1162.3k	92.5	14400.0	2172.3k	980.1k	121.6	14400.0
dem_500_gaslib40_385	1636.5k	720.6k	127.1	14400.0	1673.3k	1073.4k	55.9	14400.0	1557.0k	980.0k	58.9	14400.0
dem_500_gaslib40_386	2055.8k	701.3k	193.1	14400.0	1749.3k	963.7k	81.5	14400.0	1682.5k	788.5k	113.4	14400.0
dem_500_gaslib40_387	2088.8k	1143.7k	82.6	14400.0	2350.8k	1424.6k	65.0	14400.0	2208.6k	1392.0k	58.7	14400.0
dem_500_gaslib40_388	2707.7k	1075.3k	151.8	14400.0	3563.9k	1371.5k	159.9	14400.0	2822.8k	1146.3k	146.3	14400.0
dem_500_gaslib40_389	3418.4k	1468.8k	132.7	14400.0	3481.1k	2035.6k	71.0	14400.0	1e+20	1811.2k	–	14400.0
dem_500_gaslib40_390	2686.8k	875.1k	207.0	14400.0	2159.9k	1244.0k	73.6	14400.0	2570.6k	981.8k	161.8	14400.0
dem_500_gaslib40_391	1999.0k	871.4k	129.4	14400.0	1962.6k	1082.1k	81.4	14400.0	1919.8k	1009.3k	90.2	14400.0
dem_500_gaslib40_392	2262.9k	730.7k	209.7	14400.0	1933.1k	1278.3k	51.2	14400.0	1846.9k	838.6k	120.2	14400.0
dem_500_gaslib40_393	2571.3k	978.8k	162.7	14400.0	2744.5k	1268.5k	116.4	14400.0	3278.1k	1045.2k	213.6	14400.0
dem_500_gaslib40_394	2584.1k	985.2k	162.3	14400.0	2429.0k	1473.0k	64.9	14400.0	2453.9k	1330.7k	84.4	14400.0
dem_500_gaslib40_395	1e+20	1232.5k	–	14400.0	3218.1k	1733.0k	85.7	14400.0	2942.2k	1515.0k	94.2	14400.0
dem_500_gaslib40_396	2773.1k	1189.3k	133.2	14400.0	3011.1k	1601.3k	88.0	14400.0	2944.8k	1439.1k	104.6	14400.0
dem_500_gaslib40_397	2146.6k	928.3k	131.2	14400.0	2103.8k	1326.2k	58.6	14400.0	2015.8k	993.7k	102.9	14400.0
dem_500_gaslib40_398	2312.0k	801.6k	188.4	14400.0	2247.4k	1169.9k	92.1	14400.0	3356.9k	1067.4k	214.5	14400.0
dem_500_gaslib40_399	1790.1k	715.6k	150.2	14400.0	2151.9k	1013.7k	112.3	14400.0	1766.2k	761.8k	131.8	14400.0
dem_500_gaslib40_400	2286.2k	925.2k	147.1	14400.0	2344.3k	1256.7k	86.5	14400.0	2373.1k	1050.8k	125.8	14400.0
dem_500_gaslib40_401	2354.4k	844.3k	178.9	14400.0	2630.3k	1173.0k	124.2	14400.0	2192.0k	991.2k	121.1	14400.0
dem_500_gaslib40_402	2327.3k	979.4k	137.6	14400.0	2834.8k	1301.5k	117.8	14400.0	2623.0k	1189.3k	120.6	14400.0
dem_500_gaslib40_403	2411.3k	820.8k	193.8	14400.0	2295.7k	1245.1k	84.4	14400.0	2421.6k	1208.7k	100.4	14400.0
dem_500_gaslib40_404	3968.7k	936.0k	324.0	14400.0	2658.7k	1312.9k	102.5	14400.0	3388.9k	1045.9k	224.0	14400.0
dem_500_gaslib40_405	2410.8k	919.0k	162.3	14400.0	2098.9k	1284.2k	63.4	14400.0	2226.2k	1020.2k	118.2	14400.0
dem_500_gaslib40_406	2346.5k	768.1k	205.5	14400.0	2365.7k	1165.5k	103.0	14400.0	2111.0k	1007.0k	109.6	14400.0
dem_500_gaslib40_407	2492.5k	859.7k	189.9	14400.0	2424.3k	1244.3k	94.8	14400.0	2260.5k	987.5k	128.9	14400.0
dem_500_gaslib40_408	2778.2k	1145.6k	142.5	14400.0	2913.5k	1452.8k	100.5	14400.0	4055.2k	1487.7k	172.6	14400.0
dem_500_gaslib40_409	2801.2k	1283.6k	118.2	14400.0	2882.3k	1723.9k	67.2	14400.0	3932.4k	1614.7k	143.5	14400.0
dem_500_gaslib40_410	2322.8k	886.1k	162.1	14400.0	3084.1k	1326.1k	132.6	14400.0	2183.1k	1020.1k	114.0	14400.0
dem_500_gaslib40_411	2753.4k	849.1k	224.3	14400.0	2814.1k	1157.7k	143.1	14400.0	2924.5k	887.0k	229.7	14400.0
dem_500_gaslib40_412	2806.0k	948.5k	195.8	14400.0	2390.1k	1357.4k	76.1	14400.0	2631.7k	1267.7k	107.6	14400.0
dem_500_gaslib40_413	2139.0k	685.0k	212.3	14400.0	2337.1k	1058.6k	120.8	14400.0	1844.3k	811.5k	127.3	14400.0
dem_500_gaslib40_414	2154.3k	1134.7k	89.9	14400.0	1964.0k	1320.0k	48.8	14400.0	1821.4k	1319.7k	38.0	14400.0
dem_500_gaslib40_415	1901.4k	660.8k	187.7	14400.0	2284.8k	986.3k	131.6	14400.0	1568.9k	730.1k	114.9	14400.0
dem_500_gaslib40_416	1979.4k	713.4k	177.5	14400.0	2284.9k	1154.9k	97.8	14400.0	2424.4k	876.6k	176.6	14400.0
dem_500_gaslib40_417	2431.0k	930.0k	161.4	14400.0	2515.2k	1290.4k	94.9	14400.0	3188.5k	1061.5k	200.4	14400.0
dem_500_gaslib40_418	2746.2k	919.1k	198.8	14400.0	2845.4k	1266.4k	124.7	14400.0	3025.8k	1105.6k	173.7	14400.0
dem_500_gaslib40_419	2085.5k	675.0k	208.9	14400.0	1948.4k	1082.6k	80.0	14400.0	1879.5k	834.6k	125.2	14400.0
dem_500_gaslib40_420	1e+20	1023.4k	–	14400.0	2681.6k	1520.8k	76.3	14400.0	3756.1k	1344.2k	179.4	14400.0
dem_500_gaslib40_421	2933.4k	1059.8k	176.8	14400.0	2919.3k	1472.2k	98.3	14400.0	2742.5k	1416.4k	93.6	14400.0
dem_500_gaslib40_422	1983.8k	773.4k	156.5	14400.0	2046.7k	1079.3k	89.6	14400.0	1884.6k	840.4k	124.2	14400.0
dem_500_gaslib40_423	2703.2k	843.8k	220.4	14400.0	2067.7k	1177.6k	75.6	14400.0	2530.4k	945.2k	167.7	14400.0
dem_500_gaslib40_424	2328.2k	797.6k	191.9	14400.0	2706.7k	1198.6k	125.8	14400.0	3041.0k	898.0k	238.6	14400.0
dem_500_gaslib40_425	1916.1k	931.6k	105.7	14400.0	1891.9k	1294.9k	46.1	14400.0	2021.1k	1166.6k	73.2	14400.0
dem_500_gaslib40_426	1782.0k	828.1k	115.2	14400.0	1706.3k	1170.8k	45.7	14400.0	1615.3k	912.7k	77.0	14400.0
dem_500_gaslib40_427	1563.0k	870.3k	79.6	14400.0	1580.6k	1073.7k	47.2	14400.0	1597.5k	929.2k	71.9	14400.0
dem_500_gaslib40_428	1771.9k	845.3k	109.6	14400.0	1566.4k	967.5k	61.9	14400.0	1600.1k	923.3k	73.3	14400.0
dem_500_gaslib40_429	1826.5k	790.1k	131.2	14400.0	1801.9k	1118.0k	61.2	14400.0	1876.5k	846.0k	121.8	14400.0
dem_500_gaslib40_430	1979.4k	732.3k	170.3	14400.0	1909.0k	1207.8k	58.1	14400.0	2214.8k	818.1k	170.7	14400.0
dem_500_gaslib40_431	1e+20	963.1k	–	14400.0	3045.7k	1326.1k	129.7	14400.0	2927.5k	1115.3k	162.5	14400.0
dem_500_gaslib40_432	1e+20	911.4k	–	14400.0	2392.8k	1205.3k	98.5	14400.0	2351.1k	1035.4k	127.1	14400.0
dem_500_gaslib40_433	1740.8k	1016.3k	71.3	14400.0	1704.3k	1473.6k	15.7	14400.0	2117.5k	1070.4k	97.8	14400.0
dem_500_gaslib40_434	2135.0k	672.2k	217.6	14400.0	1574.2k	931.8k	68.9	14400.0	1e+20	735.7k	–	14400.0
dem_500_gaslib40_435	1905.8k	916.2k	108.0	14400.0	2178.4k	1188.5k	83.3	14400.0	2069.2k	1107.8k	86.8	14400.0
dem_500_gaslib40_436	2187.4k	777.3k	181.4	14400.0	2802.2k	1157.2k	142.2	14400.0	2979.9k	949.8k	213.8	14400.0
dem_500_gaslib40_437	2372.3k	869.4k	172.9	14400.0	2281.8k	1243.5k	83.5	14400.0	2217.4k	1160.0k	91.1	14400.0
dem_500_gaslib40_438	2037.9k	981.5k	107.6	14400.0	1921.2k	1138.1k	68.8	14400.0	1966.0k	1022.9k	92.2	14400.0
dem_500_gaslib40_439	2259.3k	860.7k	162.5	14400.0	2609.3k	1181.5k	120.8	14400.0	2352.7k	1134.3k	107.4	14400.0
dem_500_gaslib40_440	2719.4k	915.4k	197.1	14400.0	2978.2k	1341.4k	122.0	14400.0	2743.0k	1096.9k	150.1	14400.0
dem_500_gaslib40_441	1852.0k	1141.1k	62.3	14400.0	1839.3k	1357.0k	35.5	14400.0	1970.4k	1149.4k	71.4	14400.0
dem_500_gaslib40_442	2084.4k	884.6k	135.6	14400.0	1879.2k	1342.9k	39.9	14400.0	2018.4k	934.8k	115.9	14400.0
dem_500_gaslib40_443	2733.2k	917.9k	197.8	14400.0	2595.2k	1269.5k	104.4	14400.0	2503.6k	1063.5k	135.4	14400.0
dem_500_gaslib40_444	1729.4k	801.5k	115.8	14400.0	2284.5k	1054.8k	116.6	14400.0	1730.7k	874.4k	97.9	14400.0
dem_500_gaslib40_445	2784.5k	1079.9k	157.9	14400.0	3372.9k	1373.9k	145.5	14400.0	3085.4k	1248.8k	147.1	14400.0
dem_500_gaslib40_446	2720.3k	980.2k	177.5	14400.0	2677.8k	1448.2k	84.9	14400.0	2808.2k	1112.4k	152.4	14400.0
dem_500_gaslib40_447	2626.2k	1023.6k	156.6	14400.0	3093.4k	1466.4k	111.0	14400.0	2542.7k	1219.5k	108.5	14400.0
dem_500_gaslib40_448	2574.9k	923.8k	178.7	14400.0	2582.1k	1372.1k	88.2	14400.0	2124.7k	1041.5k	104.0	14400.0

continued on next page

Table A.13: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_449	2009.7k	604.8k	232.3	14400.0	1790.0k	1034.5k	73.0	14400.0	1834.9k	748.6k	145.1	14400.0
dem_500_gaslib40_450	1928.4k	1159.8k	66.3	14400.0	1839.3k	1392.3k	32.1	14400.0	2004.9k	1198.9k	67.2	14400.0
dem_500_gaslib40_451	2650.5k	1062.8k	149.4	14400.0	2523.6k	1447.4k	74.3	14400.0	2470.4k	1304.2k	89.4	14400.0
dem_500_gaslib40_452	1e+20	1519.8k	–	14400.0	3460.5k	2139.5k	61.7	14400.0	3932.2k	1997.4k	96.9	14400.0
dem_500_gaslib40_453	1504.9k	966.2k	55.7	14400.0	1435.5k	1137.9k	26.2	14400.0	1527.4k	992.9k	53.8	14400.0
dem_500_gaslib40_454	2757.8k	858.9k	221.1	14400.0	2732.3k	1384.6k	97.3	14400.0	2372.3k	1218.5k	94.7	14400.0
dem_500_gaslib40_455	1595.7k	599.0k	166.4	14400.0	1976.1k	1016.3k	94.4	14400.0	1649.8k	721.4k	128.7	14400.0
dem_500_gaslib40_456	2407.7k	919.9k	161.7	14400.0	3044.4k	1198.0k	154.1	14400.0	2428.3k	1177.7k	106.2	14400.0
dem_500_gaslib40_457	1e+20	1342.9k	–	14400.0	3530.8k	1684.5k	109.6	14400.0	2995.9k	1811.4k	65.4	14400.0
dem_500_gaslib40_458	1874.0k	821.6k	128.1	14400.0	1802.1k	1276.1k	41.2	14400.0	1757.9k	922.5k	90.6	14400.0
dem_500_gaslib40_459	2176.7k	832.5k	161.5	14400.0	2259.6k	1227.4k	84.1	14400.0	2269.2k	996.5k	127.7	14400.0
dem_500_gaslib40_460	1589.9k	942.9k	68.6	14400.0	1708.3k	1121.6k	52.3	14400.0	1605.0k	948.2k	69.3	14400.0
dem_500_gaslib40_461	1678.7k	702.1k	139.1	14400.0	1586.9k	1033.8k	53.5	14400.0	1565.9k	783.7k	99.8	14400.0
dem_500_gaslib40_462	1786.8k	692.1k	158.2	14400.0	1977.6k	1032.0k	91.6	14400.0	2603.1k	779.5k	234.0	14400.0
dem_500_gaslib40_463	3128.9k	1198.8k	161.0	14400.0	3538.0k	1568.7k	125.5	14400.0	2798.6k	1506.9k	85.7	14400.0
dem_500_gaslib40_464	2763.6k	934.9k	195.6	14400.0	2768.9k	1259.1k	119.9	14400.0	2266.2k	1315.1k	72.3	14400.0
dem_500_gaslib40_465	2292.4k	937.9k	144.4	14400.0	2365.8k	1486.2k	59.2	14400.0	4302.9k	1089.4k	295.0	14400.0
dem_500_gaslib40_466	3005.6k	960.7k	212.8	14400.0	2615.8k	1382.9k	89.2	14400.0	2701.3k	1129.8k	139.1	14400.0
dem_500_gaslib40_467	2058.6k	756.4k	172.2	14400.0	2393.8k	1090.8k	119.5	14400.0	1830.8k	943.6k	94.0	14400.0
dem_500_gaslib40_468	3465.2k	1223.1k	183.3	14400.0	3003.9k	1601.8k	87.5	14400.0	2908.2k	1546.0k	88.1	14400.0
dem_500_gaslib40_469	1752.2k	1094.8k	60.0	14400.0	1857.4k	1315.7k	41.2	14400.0	1886.3k	1178.4k	60.1	14400.0
dem_500_gaslib40_470	1628.2k	704.4k	131.1	14400.0	1682.8k	995.3k	69.1	14400.0	1805.7k	767.1k	135.4	14400.0
dem_500_gaslib40_471	2240.7k	913.4k	145.3	14400.0	2479.4k	1251.9k	98.1	14400.0	2309.3k	1193.8k	93.4	14400.0
dem_500_gaslib40_472	2145.9k	917.9k	133.8	14400.0	2306.1k	1230.9k	87.4	14400.0	2277.6k	987.3k	130.7	14400.0
dem_500_gaslib40_473	2443.3k	871.3k	180.4	14400.0	2512.0k	1278.8k	96.4	14400.0	2120.9k	1081.9k	96.0	14400.0
dem_500_gaslib40_474	2895.6k	1212.3k	138.8	14400.0	3268.2k	1594.7k	104.9	14400.0	3748.0k	1317.9k	184.4	14400.0
dem_500_gaslib40_475	2523.6k	896.7k	181.4	14400.0	2384.9k	1421.3k	67.8	14400.0	2811.9k	1096.6k	156.4	14400.0
dem_500_gaslib40_476	1459.0k	816.0k	78.8	14400.0	1779.7k	1094.1k	62.7	14400.0	1515.0k	842.9k	79.7	14400.0
dem_500_gaslib40_477	2777.9k	1153.3k	140.9	14400.0	2622.6k	1474.4k	77.9	14400.0	2808.6k	1444.5k	94.4	14400.0
dem_500_gaslib40_478	2703.1k	970.1k	178.7	14400.0	2710.8k	1423.2k	90.5	14400.0	2625.1k	1147.5k	128.8	14400.0
dem_500_gaslib40_479	2256.5k	1033.3k	118.4	14400.0	2363.9k	1433.6k	64.9	14400.0	2327.0k	1085.0k	114.5	14400.0
dem_500_gaslib40_480	3658.9k	1276.8k	186.6	14400.0	3095.1k	1755.1k	76.4	14400.0	3050.4k	1656.5k	84.1	14400.0
dem_500_gaslib40_481	2354.4k	931.5k	152.8	14400.0	1825.9k	1312.6k	39.1	14400.0	1866.1k	1009.8k	84.8	14400.0
dem_500_gaslib40_482	2425.5k	805.8k	201.0	14400.0	2281.1k	1164.2k	95.9	14400.0	1812.9k	1055.4k	71.8	14400.0
dem_500_gaslib40_483	1903.9k	992.2k	91.9	14400.0	2358.0k	1289.4k	82.9	14400.0	1999.4k	1050.4k	90.4	14400.0
dem_500_gaslib40_484	2292.0k	696.1k	229.3	14400.0	1891.7k	1097.4k	72.4	14400.0	1969.0k	776.5k	153.6	14400.0
dem_500_gaslib40_485	1394.7k	627.8k	122.2	14400.0	1655.8k	899.4k	84.1	14400.0	1497.2k	671.1k	123.1	14400.0
dem_500_gaslib40_486	2061.9k	789.1k	161.3	14400.0	1855.5k	1198.3k	54.8	14400.0	2153.4k	903.5k	138.3	14400.0
dem_500_gaslib40_487	1967.8k	687.9k	186.0	14400.0	1827.7k	1053.7k	73.4	14400.0	1793.9k	747.5k	140.0	14400.0
dem_500_gaslib40_488	2253.4k	786.9k	186.4	14400.0	2118.3k	1161.1k	82.4	14400.0	2176.1k	899.1k	142.0	14400.0
dem_500_gaslib40_489	2097.6k	696.8k	201.0	14400.0	2133.3k	1075.7k	98.3	14400.0	2245.7k	853.9k	163.0	14400.0
dem_500_gaslib40_490	2183.4k	993.8k	119.7	14400.0	2113.1k	1327.0k	59.2	14400.0	2028.1k	1027.9k	97.3	14400.0
dem_500_gaslib40_491	2636.3k	1067.3k	147.0	14400.0	2659.4k	1479.9k	79.7	14400.0	2618.9k	1250.8k	109.4	14400.0
dem_500_gaslib40_492	1903.6k	918.0k	107.4	14400.0	2098.9k	1183.5k	77.4	14400.0	1807.1k	993.4k	81.9	14400.0
dem_500_gaslib40_493	1493.8k	885.2k	68.8	14400.0	1454.3k	1154.2k	26.0	14400.0	1476.7k	926.5k	59.4	14400.0
dem_500_gaslib40_494	1597.5k	940.3k	69.9	14400.0	1653.2k	1234.1k	34.0	14400.0	1725.6k	1010.2k	70.8	14400.0
dem_500_gaslib40_495	2485.0k	926.9k	168.1	14400.0	2032.9k	1148.7k	77.0	14400.0	1915.8k	1010.2k	89.6	14400.0
dem_500_gaslib40_496	2452.2k	990.0k	147.7	14400.0	2320.6k	1364.9k	70.0	14400.0	3037.7k	1137.6k	167.0	14400.0
dem_500_gaslib40_497	1302.4k	680.9k	91.3	14400.0	1347.2k	851.1k	58.3	14400.0	1251.3k	699.6k	78.9	14400.0
dem_500_gaslib40_498	2628.7k	918.2k	186.3	14400.0	2613.9k	1412.5k	85.1	14400.0	2827.4k	1050.6k	169.1	14400.0
dem_500_gaslib40_499	2035.5k	830.9k	145.0	14400.0	2195.4k	1211.3k	81.2	14400.0	2885.1k	991.5k	191.0	14400.0
dem_500_gaslib40_500	2476.4k	973.8k	154.3	14400.0	2784.6k	1332.1k	109.0	14400.0	3359.3k	1128.9k	197.6	14400.0

Table A.14: Detailed results of the split-pipe models on *GasLib-40* with $\mathcal{B} = 500$ as summarized in Figure 3.14c and Table 3.4b. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_1	1928.0k	1683.9k	14.5	14400.0	1920.7k	1487.1k	29.2	14400.0
dem_500_gaslib40_2	1744.4k	1448.2k	20.5	14400.0	1741.3k	1446.5k	20.4	14400.0
dem_500_gaslib40_3	2099.5k	1526.0k	37.6	14400.0	2090.0k	1515.7k	37.9	14400.0
dem_500_gaslib40_4	1259.2k	1119.1k	12.5	14400.0	1260.3k	1124.0k	12.1	14400.0
dem_500_gaslib40_5	1877.6k	1877.0k	0.0	11201.3	1877.6k	1591.9k	17.9	14400.0
dem_500_gaslib40_6	2306.7k	1706.0k	35.2	14400.0	2275.4k	1706.1k	33.4	14400.0
dem_500_gaslib40_7	1912.6k	1548.4k	23.5	14400.0	1915.7k	1555.0k	23.2	14400.0
dem_500_gaslib40_8	2035.3k	1705.9k	19.3	14400.0	2013.4k	1709.9k	17.7	14400.0
dem_500_gaslib40_9	2162.3k	1713.3k	26.2	14400.0	2150.7k	1709.5k	25.8	14400.0
dem_500_gaslib40_10	2260.5k	1930.3k	17.1	14400.0	2259.3k	2242.9k	0.7	14400.0
dem_500_gaslib40_11	2428.1k	2425.7k	0.1	13334.7	2428.1k	2425.7k	0.1	7750.8

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Table A.14: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_12	1645.8k	1512.1k	8.8	14400.0	1645.8k	1511.5k	8.9	14400.0
dem_500_gaslib40_13	1845.7k	1844.4k	0.1	313.8	1845.7k	1843.9k	0.1	270.9
dem_500_gaslib40_14	2202.6k	2200.4k	0.1	5597.8	2202.6k	2200.4k	0.1	2808.7
dem_500_gaslib40_15	1702.3k	1559.2k	9.2	14400.0	1679.3k	1570.2k	6.9	14400.0
dem_500_gaslib40_16	1287.0k	1285.7k	0.1	61.2	1287.0k	1285.7k	0.1	70.0
dem_500_gaslib40_17	1825.2k	1469.5k	24.2	14400.0	1820.7k	1818.8k	0.1	1552.8
dem_500_gaslib40_18	1715.9k	1319.0k	30.1	14400.0	1715.9k	1320.9k	29.9	14400.0
dem_500_gaslib40_19	1591.7k	1590.7k	0.1	136.9	1591.9k	1591.1k	0.1	139.2
dem_500_gaslib40_20	1469.7k	1467.5k	0.2	14400.0	1469.7k	1462.2k	0.5	14400.0
dem_500_gaslib40_21	1991.0k	1478.4k	34.7	14400.0	1960.1k	1921.4k	2.0	14400.0
dem_500_gaslib40_22	3942.7k	3938.8k	0.1	855.1	3942.7k	3938.8k	0.1	859.5
dem_500_gaslib40_23	2015.6k	1448.8k	39.1	14400.0	1965.8k	1441.8k	36.3	14400.0
dem_500_gaslib40_24	1937.5k	1480.5k	30.9	14400.0	1906.5k	1542.5k	23.6	14400.0
dem_500_gaslib40_25	1452.5k	1451.1k	0.1	190.4	1452.5k	1451.1k	0.1	164.5
dem_500_gaslib40_26	1313.2k	1311.9k	0.1	45.1	1313.2k	1312.0k	0.1	72.0
dem_500_gaslib40_27	1963.5k	1396.0k	40.7	14400.0	1968.1k	1402.3k	40.4	14400.0
dem_500_gaslib40_28	1729.9k	1609.2k	7.5	14400.0	1728.2k	1609.2k	7.4	14400.0
dem_500_gaslib40_29	1727.6k	1547.1k	11.7	14400.0	1688.5k	1388.8k	21.6	14400.0
dem_500_gaslib40_30	2226.3k	2171.6k	2.5	14400.0	2217.9k	2193.0k	1.1	14400.0
dem_500_gaslib40_31	1269.4k	1252.9k	1.3	14400.0	1269.0k	1267.8k	0.1	8287.5
dem_500_gaslib40_32	1965.0k	1692.0k	16.1	14400.0	1944.2k	1500.9k	29.5	14400.0
dem_500_gaslib40_33	1608.3k	1303.4k	23.4	14400.0	1556.1k	1270.9k	22.4	14400.0
dem_500_gaslib40_34	2067.8k	2065.8k	0.1	3946.1	2067.8k	2065.8k	0.1	4221.0
dem_500_gaslib40_35	1566.1k	1564.5k	0.1	4050.3	1565.9k	1564.7k	0.1	7945.8
dem_500_gaslib40_36	1417.7k	1416.3k	0.1	659.2	1417.7k	1416.3k	0.1	12456.1
dem_500_gaslib40_37	1565.6k	1237.7k	26.5	14400.0	1563.2k	1232.1k	26.9	14400.0
dem_500_gaslib40_38	1569.8k	1272.5k	23.4	14400.0	1561.1k	1267.5k	23.2	14400.0
dem_500_gaslib40_39	2070.7k	1491.4k	38.8	14400.0	2080.3k	1526.8k	36.3	14400.0
dem_500_gaslib40_40	1977.1k	1975.2k	0.1	5235.1	1977.1k	1975.2k	0.1	3727.4
dem_500_gaslib40_41	1755.7k	1753.2k	0.1	14400.0	1755.7k	1751.8k	0.2	14400.0
dem_500_gaslib40_42	1933.7k	1566.5k	23.4	14400.0	1890.6k	1888.7k	0.1	5314.4
dem_500_gaslib40_43	1626.7k	1198.3k	35.8	14400.0	1624.3k	1622.7k	0.1	189.0
dem_500_gaslib40_44	1681.6k	1450.0k	16.0	14400.0	1681.7k	1443.6k	16.5	14400.0
dem_500_gaslib40_45	1962.1k	1960.2k	0.1	3378.6	1962.1k	1960.2k	0.1	1850.6
dem_500_gaslib40_46	1778.3k	1776.5k	0.1	859.4	1778.3k	1776.5k	0.1	820.3
dem_500_gaslib40_47	2831.3k	2830.4k	0.0	2291.7	2831.3k	2828.4k	0.1	1998.2
dem_500_gaslib40_48	1892.5k	1350.5k	40.1	14400.0	1881.2k	1350.9k	39.3	14400.0
dem_500_gaslib40_49	1958.1k	1559.9k	25.5	14400.0	1956.4k	1609.5k	21.6	14400.0
dem_500_gaslib40_50	1375.4k	1374.2k	0.1	200.9	1375.4k	1374.1k	0.1	210.1
dem_500_gaslib40_51	2097.4k	1811.0k	15.8	14400.0	2073.8k	2062.1k	0.6	14400.0
dem_500_gaslib40_52	1738.1k	1358.8k	27.9	14400.0	1722.4k	1354.2k	27.2	14400.0
dem_500_gaslib40_53	1911.8k	1763.0k	8.4	14400.0	1891.2k	1889.4k	0.1	11399.6
dem_500_gaslib40_54	1506.0k	1504.5k	0.1	340.5	1505.4k	1503.9k	0.1	146.3
dem_500_gaslib40_55	1522.2k	1201.1k	26.7	14400.0	1456.2k	1226.3k	18.7	14400.0
dem_500_gaslib40_56	2469.4k	2453.7k	0.6	14400.0	2469.4k	2466.9k	0.1	8208.8
dem_500_gaslib40_57	2826.5k	2823.7k	0.1	14185.2	2825.9k	2823.1k	0.1	11800.6
dem_500_gaslib40_58	1460.5k	1459.0k	0.1	231.7	1460.5k	1459.0k	0.1	192.1
dem_500_gaslib40_59	1814.7k	1320.7k	37.4	14400.0	1805.4k	1307.1k	38.1	14400.0
dem_500_gaslib40_60	1617.1k	1457.3k	11.0	14400.0	1614.2k	1483.8k	8.8	14400.0
dem_500_gaslib40_61	2501.2k	2498.7k	0.1	11433.6	2501.2k	2498.7k	0.1	11517.3
dem_500_gaslib40_62	1894.4k	1355.6k	39.7	14400.0	1918.4k	1346.8k	42.4	14400.0
dem_500_gaslib40_63	2728.0k	2725.4k	0.1	2074.4	2728.0k	2725.3k	0.1	2011.1
dem_500_gaslib40_64	1541.6k	1540.0k	0.1	159.6	1541.1k	1539.7k	0.1	122.1
dem_500_gaslib40_65	2048.5k	2047.6k	0.0	12871.0	2048.4k	2046.4k	0.1	13908.6
dem_500_gaslib40_66	2241.6k	2239.3k	0.1	13445.0	2241.3k	2129.9k	5.2	14400.0
dem_500_gaslib40_67	1441.4k	1440.0k	0.1	298.4	1441.4k	1440.0k	0.1	4770.3
dem_500_gaslib40_68	1652.9k	1483.3k	11.4	14400.0	1642.3k	1483.3k	10.7	14400.0
dem_500_gaslib40_69	2091.2k	1609.8k	29.9	14400.0	2064.6k	1557.1k	32.6	14400.0
dem_500_gaslib40_70	1693.4k	1691.7k	0.1	1901.1	1693.4k	1691.7k	0.1	1419.2
dem_500_gaslib40_71	2533.7k	2531.1k	0.1	2113.7	2533.7k	2531.2k	0.1	1907.0
dem_500_gaslib40_72	2251.1k	1774.6k	26.8	14400.0	2242.9k	1922.9k	16.6	14400.0
dem_500_gaslib40_73	1463.9k	1292.8k	13.2	14400.0	1468.7k	1166.3k	25.9	14400.0
dem_500_gaslib40_74	2120.6k	1788.4k	18.6	14400.0	2095.3k	1802.7k	16.2	14400.0
dem_500_gaslib40_75	2461.0k	2460.0k	0.0	13704.9	2461.0k	2458.5k	0.1	7351.1
dem_500_gaslib40_76	2036.5k	1629.4k	25.0	14400.0	1974.1k	1800.0k	9.7	14400.0
dem_500_gaslib40_77	1996.8k	1650.6k	21.0	14400.0	1996.8k	1651.3k	20.9	14400.0
dem_500_gaslib40_78	2242.7k	2242.0k	0.0	9997.8	2242.6k	2240.3k	0.1	5923.5
dem_500_gaslib40_79	1508.9k	1399.1k	7.8	14400.0	1505.8k	1387.6k	8.5	14400.0
dem_500_gaslib40_80	2234.6k	1672.9k	33.6	14400.0	2214.9k	2172.7k	1.9	14400.0
dem_500_gaslib40_81	1753.9k	1431.7k	22.5	14400.0	1736.6k	1588.5k	9.3	14400.0
dem_500_gaslib40_82	2343.9k	2341.5k	0.1	5238.4	2343.9k	2339.7k	0.2	14400.0
dem_500_gaslib40_83	1914.3k	1476.1k	29.7	14400.0	1887.4k	1509.9k	25.0	14400.0
dem_500_gaslib40_84	2358.4k	2356.0k	0.1	8646.7	2358.4k	2356.1k	0.1	4572.9
dem_500_gaslib40_85	1637.8k	1636.2k	0.1	371.1	1663.4k	1169.6k	42.2	14400.0
dem_500_gaslib40_86	1613.5k	1214.6k	32.8	14400.0	1568.6k	1211.3k	29.5	14400.0
dem_500_gaslib40_87	2010.2k	2008.1k	0.1	4407.0	2010.2k	2008.1k	0.1	5549.4
dem_500_gaslib40_88	1517.3k	1515.8k	0.1	150.4	1517.0k	1515.6k	0.1	138.8
dem_500_gaslib40_89	2195.5k	1650.6k	33.0	14400.0	2179.6k	1648.3k	32.2	14400.0

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Table A.14: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_90	1761.6k	1759.9k	0.1	193.8	1761.6k	1759.9k	0.1	278.8
dem_500_gaslib40_91	2727.8k	2725.0k	0.1	12033.3	2727.8k	2725.0k	0.1	6350.0
dem_500_gaslib40_92	1508.0k	1217.3k	23.9	14400.0	1510.3k	1217.8k	24.0	14400.0
dem_500_gaslib40_93	1503.2k	1502.0k	0.1	253.2	1503.2k	1501.7k	0.1	195.6
dem_500_gaslib40_94	1577.4k	1329.2k	18.7	14400.0	1572.0k	1413.6k	11.2	14400.0
dem_500_gaslib40_95	1718.6k	1620.6k	6.0	14400.0	1718.2k	1608.4k	6.8	14400.0
dem_500_gaslib40_96	1816.3k	1330.2k	36.5	14400.0	1809.4k	1357.3k	33.3	14400.0
dem_500_gaslib40_97	1881.1k	1379.2k	36.4	14400.0	1877.3k	1329.5k	41.2	14400.0
dem_500_gaslib40_98	1877.6k	1639.9k	14.5	14400.0	1886.0k	1441.0k	30.9	14400.0
dem_500_gaslib40_99	1541.2k	1539.6k	0.1	3456.1	1541.2k	1539.6k	0.1	1642.6
dem_500_gaslib40_100	2155.0k	1618.6k	33.1	14400.0	2135.7k	1606.2k	33.0	14400.0
dem_500_gaslib40_101	2157.8k	2155.7k	0.1	2600.7	2157.8k	2155.7k	0.1	2881.1
dem_500_gaslib40_102	2262.7k	2260.5k	0.1	1810.0	2262.7k	2260.5k	0.1	1174.7
dem_500_gaslib40_103	1875.1k	1381.8k	35.7	14400.0	1778.3k	1453.6k	22.3	14400.0
dem_500_gaslib40_104	1838.2k	1484.0k	23.9	14400.0	1832.8k	1474.4k	24.3	14400.0
dem_500_gaslib40_105	1594.0k	1592.4k	0.1	142.3	1593.9k	1592.3k	0.1	110.7
dem_500_gaslib40_106	1757.6k	1463.2k	20.1	14400.0	1750.7k	1304.7k	34.2	14400.0
dem_500_gaslib40_107	1933.5k	1931.6k	0.1	2551.9	1941.8k	1729.6k	12.3	14400.0
dem_500_gaslib40_108	2046.0k	2043.9k	0.1	1361.2	2046.5k	2044.5k	0.1	883.6
dem_500_gaslib40_109	2085.2k	1677.6k	24.3	14400.0	2044.1k	1677.4k	21.9	14400.0
dem_500_gaslib40_110	1614.4k	1612.7k	0.1	333.1	1614.4k	1612.7k	0.1	143.6
dem_500_gaslib40_111	1542.2k	1068.2k	44.4	14400.0	1523.9k	1063.0k	43.4	14400.0
dem_500_gaslib40_112	2056.9k	2056.4k	0.0	4669.6	2056.9k	2054.8k	0.1	3756.7
dem_500_gaslib40_113	1804.7k	1548.2k	16.6	14400.0	1793.4k	1545.0k	16.1	14400.0
dem_500_gaslib40_114	1268.4k	1225.0k	3.5	14400.0	1268.4k	1267.2k	0.1	1309.8
dem_500_gaslib40_115	2836.0k	2833.2k	0.1	10184.8	2834.9k	2832.0k	0.1	8745.8
dem_500_gaslib40_116	1901.0k	1383.7k	37.4	14400.0	1885.2k	1386.9k	35.9	14400.0
dem_500_gaslib40_117	1541.0k	1539.4k	0.1	174.1	1541.0k	1540.4k	0.0	300.7
dem_500_gaslib40_118	1520.1k	1518.6k	0.1	287.6	1520.1k	1518.6k	0.1	178.8
dem_500_gaslib40_119	2050.2k	2048.2k	0.1	1725.4	2050.2k	2048.2k	0.1	1426.5
dem_500_gaslib40_120	1737.6k	1735.9k	0.1	178.1	1737.4k	1735.7k	0.1	177.1
dem_500_gaslib40_121	3061.0k	2970.6k	3.0	14400.0	3018.5k	2984.9k	1.1	14400.0
dem_500_gaslib40_122	1494.9k	1493.4k	0.1	813.4	1494.9k	1493.4k	0.1	197.1
dem_500_gaslib40_123	1692.1k	1525.1k	11.0	14400.0	1681.5k	1557.6k	8.0	14400.0
dem_500_gaslib40_124	2678.6k	2675.9k	0.1	1866.8	2678.6k	2675.9k	0.1	1768.7
dem_500_gaslib40_125	2068.9k	1727.6k	19.8	14400.0	2059.0k	1725.7k	19.3	14400.0
dem_500_gaslib40_126	1989.8k	1989.2k	0.0	582.8	1989.8k	1987.8k	0.1	254.3
dem_500_gaslib40_127	1615.6k	1614.0k	0.1	254.9	1615.6k	1614.0k	0.1	152.3
dem_500_gaslib40_128	2110.1k	1727.7k	22.1	14400.0	2119.7k	1727.4k	22.7	14400.0
dem_500_gaslib40_129	1466.2k	1464.7k	0.1	149.2	1466.2k	1436.8k	2.0	14400.0
dem_500_gaslib40_130	2021.3k	2019.3k	0.1	4546.5	2021.3k	2019.3k	0.1	6904.7
dem_500_gaslib40_131	2347.4k	2346.0k	0.1	7169.5	2347.4k	2345.0k	0.1	4445.8
dem_500_gaslib40_132	1825.0k	1269.5k	43.8	14400.0	1796.1k	1256.6k	42.9	14400.0
dem_500_gaslib40_133	1957.3k	1486.5k	31.7	14400.0	1944.2k	1932.4k	0.6	14400.0
dem_500_gaslib40_134	1979.7k	1977.7k	0.1	4976.4	1979.7k	1971.9k	0.4	14400.0
dem_500_gaslib40_135	2392.3k	2390.0k	0.1	7720.1	2392.3k	2389.9k	0.1	5571.2
dem_500_gaslib40_136	1602.4k	1246.6k	28.5	14400.0	1598.4k	1596.8k	0.1	3811.2
dem_500_gaslib40_137	1912.8k	1697.1k	12.7	14400.0	1912.6k	1694.2k	12.9	14400.0
dem_500_gaslib40_138	1433.2k	1431.8k	0.1	290.5	1433.2k	1431.8k	0.1	195.4
dem_500_gaslib40_139	1453.0k	1140.7k	27.4	14400.0	1453.0k	1139.4k	27.5	14400.0
dem_500_gaslib40_140	2763.8k	2761.0k	0.1	3738.0	2763.8k	2761.0k	0.1	2211.1
dem_500_gaslib40_141	1941.9k	1456.0k	33.4	14400.0	1925.1k	1453.2k	32.5	14400.0
dem_500_gaslib40_142	1630.6k	1359.1k	20.0	14400.0	1629.2k	874.7k	86.3	14400.0
dem_500_gaslib40_143	1540.1k	1219.6k	26.3	14400.0	1537.2k	1218.2k	26.2	14400.0
dem_500_gaslib40_144	1482.4k	1480.9k	0.1	289.6	1482.4k	1478.1k	0.3	14400.0
dem_500_gaslib40_145	1945.0k	1943.0k	0.1	6052.4	1944.5k	1942.5k	0.1	3681.1
dem_500_gaslib40_146	1936.5k	1402.8k	38.0	14400.0	1900.9k	1410.7k	34.7	14400.0
dem_500_gaslib40_147	1700.3k	1395.0k	21.9	14400.0	1699.0k	1390.5k	22.2	14400.0
dem_500_gaslib40_148	1598.2k	1596.6k	0.1	246.7	1598.2k	1597.2k	0.1	184.3
dem_500_gaslib40_149	2246.9k	2231.4k	0.7	14400.0	2246.9k	2244.7k	0.1	4205.3
dem_500_gaslib40_150	2013.3k	1612.8k	24.8	14400.0	2009.4k	1612.1k	24.6	14400.0
dem_500_gaslib40_151	2023.0k	1717.9k	17.8	14400.0	1997.8k	1575.2k	26.8	14400.0
dem_500_gaslib40_152	1920.2k	1366.4k	40.5	14400.0	1901.8k	1365.0k	39.3	14400.0
dem_500_gaslib40_153	1470.0k	1468.6k	0.1	200.2	1470.0k	1468.6k	0.1	107.7
dem_500_gaslib40_154	4157.6k	4154.0k	0.1	396.5	4156.3k	4152.2k	0.1	315.4
dem_500_gaslib40_155	1283.8k	1282.5k	0.1	82.3	1283.8k	1282.5k	0.1	110.5
dem_500_gaslib40_156	2260.6k	1934.7k	16.8	14400.0	2245.7k	1997.5k	12.4	14400.0
dem_500_gaslib40_157	3700.2k	3696.5k	0.1	1350.2	3700.2k	3696.5k	0.1	942.1
dem_500_gaslib40_158	1342.9k	1341.6k	0.1	231.5	1342.9k	1341.6k	0.1	106.6
dem_500_gaslib40_159	1675.0k	1229.6k	36.2	14400.0	1671.6k	1419.5k	17.8	14400.0
dem_500_gaslib40_160	2108.5k	1756.7k	20.0	14400.0	2089.2k	1759.2k	18.8	14400.0
dem_500_gaslib40_161	1929.3k	1628.5k	18.5	14400.0	1938.0k	1641.4k	18.1	14400.0
dem_500_gaslib40_162	1973.0k	1539.4k	28.2	14400.0	1978.6k	1642.5k	20.5	14400.0
dem_500_gaslib40_163	2039.3k	1503.8k	35.6	14400.0	2010.3k	1504.8k	33.6	14400.0
dem_500_gaslib40_164	1530.5k	1483.7k	3.2	14400.0	1525.4k	1525.1k	0.0	12051.5
dem_500_gaslib40_165	1948.8k	1946.8k	0.1	3070.2	1948.8k	1948.8k	0.0	1705.5
dem_500_gaslib40_166	1871.4k	1367.4k	36.9	14400.0	1822.6k	1371.8k	32.9	14400.0
dem_500_gaslib40_167	1542.5k	1240.3k	24.4	14400.0	1541.9k	1232.7k	25.1	14400.0

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Table A.14: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_168	3153.3k	3150.2k	0.1	4296.5	3153.3k	3150.2k	0.1	4720.1
dem_500_gaslib40_169	1504.2k	1179.1k	27.6	14400.0	1501.5k	1180.2k	27.2	14400.0
dem_500_gaslib40_170	1856.4k	1365.0k	36.0	14400.0	1843.8k	1366.7k	34.9	14400.0
dem_500_gaslib40_171	1822.0k	1515.4k	20.2	14400.0	1795.3k	1793.6k	0.1	3185.9
dem_500_gaslib40_172	1919.8k	1608.5k	19.4	14400.0	1904.5k	1615.1k	17.9	14400.0
dem_500_gaslib40_173	2334.0k	2331.7k	0.1	7863.3	2333.9k	2331.6k	0.1	6341.5
dem_500_gaslib40_174	1719.5k	1029.4k	67.0	14400.0	1657.6k	1269.6k	30.6	14400.0
dem_500_gaslib40_175	2654.3k	2653.1k	0.0	5434.1	2655.2k	2652.6k	0.1	4078.8
dem_500_gaslib40_176	2398.0k	2396.3k	0.1	2742.3	2397.9k	2395.5k	0.1	2394.1
dem_500_gaslib40_177	2055.1k	2053.1k	0.1	10781.9	2055.1k	2053.1k	0.1	5939.6
dem_500_gaslib40_178	2098.9k	2096.8k	0.1	3178.4	2098.9k	2096.8k	0.1	2402.5
dem_500_gaslib40_179	1344.7k	1343.4k	0.1	201.7	1344.7k	1343.3k	0.1	92.5
dem_500_gaslib40_180	1688.6k	1462.1k	15.5	14400.0	1679.9k	1444.3k	16.3	14400.0
dem_500_gaslib40_181	1724.0k	1723.2k	0.0	620.4	1724.0k	1722.4k	0.1	568.4
dem_500_gaslib40_182	2088.2k	1625.5k	28.5	14400.0	2090.9k	1617.6k	29.3	14400.0
dem_500_gaslib40_183	1362.9k	1236.4k	10.2	14400.0	1362.9k	1235.2k	10.3	14400.0
dem_500_gaslib40_184	1571.2k	1569.6k	0.1	483.0	1571.2k	1569.6k	0.1	256.2
dem_500_gaslib40_185	1912.4k	1676.2k	14.1	14400.0	1906.8k	1673.2k	14.0	14400.0
dem_500_gaslib40_186	2572.7k	2571.1k	0.1	1652.3	2572.7k	2570.2k	0.1	956.0
dem_500_gaslib40_187	1952.8k	1616.7k	20.8	14400.0	1942.8k	1668.8k	16.4	14400.0
dem_500_gaslib40_188	1537.2k	1535.8k	0.1	148.3	1537.2k	1535.7k	0.1	131.8
dem_500_gaslib40_189	1627.9k	1626.3k	0.1	101.3	1627.9k	1626.4k	0.1	112.5
dem_500_gaslib40_190	1967.1k	1628.3k	20.8	14400.0	1954.6k	1628.0k	20.1	14400.0
dem_500_gaslib40_191	2255.7k	1656.5k	36.2	14400.0	2240.5k	1654.1k	35.4	14400.0
dem_500_gaslib40_192	2467.5k	2465.0k	0.1	9295.5	2467.5k	2465.0k	0.1	5300.0
dem_500_gaslib40_193	1353.6k	1352.3k	0.1	335.9	1353.6k	1352.3k	0.1	187.4
dem_500_gaslib40_194	2318.8k	2316.4k	0.1	1839.2	2318.8k	2316.4k	0.1	1319.7
dem_500_gaslib40_195	1530.2k	1529.4k	0.1	94.5	1530.2k	1528.7k	0.1	103.8
dem_500_gaslib40_196	2010.3k	2008.2k	0.1	3970.5	2010.3k	2008.3k	0.1	2835.1
dem_500_gaslib40_197	2270.9k	1746.9k	30.0	14400.0	2261.1k	1703.0k	32.8	14400.0
dem_500_gaslib40_198	2095.4k	2093.3k	0.1	5352.3	2095.4k	2093.3k	0.1	4192.1
dem_500_gaslib40_199	1691.8k	1406.7k	20.3	14400.0	1671.4k	1400.4k	19.3	14400.0
dem_500_gaslib40_200	2147.2k	1840.8k	16.6	14400.0	2143.3k	2115.4k	1.3	14400.0
dem_500_gaslib40_201	1845.9k	1398.9k	31.9	14400.0	1825.0k	1352.8k	34.9	14400.0
dem_500_gaslib40_202	2109.2k	2096.7k	0.6	14400.0	2109.2k	2107.1k	0.1	13473.1
dem_500_gaslib40_203	1584.1k	1583.5k	0.0	400.3	1584.0k	1582.5k	0.1	214.2
dem_500_gaslib40_204	1522.9k	1186.0k	28.4	14400.0	1528.6k	1182.7k	29.2	14400.0
dem_500_gaslib40_205	1454.2k	1452.8k	0.1	159.0	1454.2k	1452.7k	0.1	167.2
dem_500_gaslib40_206	1641.1k	1639.4k	0.1	718.3	1641.0k	1639.4k	0.1	351.5
dem_500_gaslib40_207	1683.4k	1440.6k	16.9	14400.0	1696.9k	1430.1k	18.7	14400.0
dem_500_gaslib40_208	1630.0k	1377.5k	18.3	14400.0	1594.3k	1385.9k	15.0	14400.0
dem_500_gaslib40_209	1567.0k	1481.5k	5.8	14400.0	1564.0k	1473.2k	6.2	14400.0
dem_500_gaslib40_210	2835.1k	2832.3k	0.1	10224.8	2834.7k	2831.9k	0.1	10265.3
dem_500_gaslib40_211	1799.2k	1403.7k	28.2	14400.0	1798.4k	1404.9k	28.0	14400.0
dem_500_gaslib40_212	2011.3k	1627.5k	23.6	14400.0	2015.2k	1629.9k	23.6	14400.0
dem_500_gaslib40_213	1502.8k	1501.4k	0.1	368.5	1502.8k	1501.4k	0.1	337.5
dem_500_gaslib40_214	1821.0k	1328.3k	37.1	14400.0	1818.8k	1333.8k	36.4	14400.0
dem_500_gaslib40_215	1792.5k	1525.1k	17.5	14400.0	1796.1k	1513.7k	18.7	14400.0
dem_500_gaslib40_216	2079.4k	2071.0k	0.4	14400.0	2076.4k	2074.3k	0.1	13865.1
dem_500_gaslib40_217	1996.9k	1632.7k	22.3	14400.0	1990.8k	1597.0k	24.7	14400.0
dem_500_gaslib40_218	2018.8k	1444.2k	39.8	14400.0	2000.9k	1998.9k	0.1	6759.4
dem_500_gaslib40_219	1805.2k	1524.0k	18.4	14400.0	1785.9k	1566.3k	14.0	14400.0
dem_500_gaslib40_220	1822.9k	1821.1k	0.1	353.3	1822.9k	1821.1k	0.1	249.7
dem_500_gaslib40_221	1868.6k	1581.3k	18.2	14400.0	1870.1k	1401.5k	33.4	14400.0
dem_500_gaslib40_222	1890.6k	1888.7k	0.1	2902.1	1890.6k	1877.2k	0.7	14400.0
dem_500_gaslib40_223	2347.1k	2344.8k	0.1	3026.8	2347.8k	2345.5k	0.1	1754.8
dem_500_gaslib40_224	1557.4k	1555.8k	0.1	260.1	1557.4k	1555.8k	0.1	227.1
dem_500_gaslib40_225	1898.3k	1609.7k	17.9	14400.0	1889.4k	1611.0k	17.3	14400.0
dem_500_gaslib40_226	1765.2k	1475.9k	19.6	14400.0	1747.0k	1510.2k	15.7	14400.0
dem_500_gaslib40_227	2456.9k	2439.5k	0.7	14400.0	2448.7k	2446.3k	0.1	11023.9
dem_500_gaslib40_228	2320.4k	2318.0k	0.1	5984.5	2320.4k	2318.0k	0.1	4538.9
dem_500_gaslib40_229	2054.8k	1491.1k	37.8	14400.0	2033.2k	1490.6k	36.4	14400.0
dem_500_gaslib40_230	1977.2k	1975.4k	0.1	5402.5	1977.2k	1975.2k	0.1	1930.2
dem_500_gaslib40_231	1990.3k	1604.0k	24.1	14400.0	1979.8k	1603.8k	23.4	14400.0
dem_500_gaslib40_232	1465.8k	1429.1k	2.6	14400.0	1464.7k	1372.2k	6.7	14400.0
dem_500_gaslib40_233	3086.3k	3084.0k	0.1	1265.0	3086.3k	3083.2k	0.1	954.5
dem_500_gaslib40_234	1715.0k	1410.3k	21.6	14400.0	1691.7k	1690.0k	0.1	5991.8
dem_500_gaslib40_235	1959.4k	1957.4k	0.1	2620.9	1959.4k	1957.4k	0.1	1317.1
dem_500_gaslib40_236	1735.4k	1434.0k	21.0	14400.0	1679.3k	1382.3k	21.5	14400.0
dem_500_gaslib40_237	2201.7k	2199.5k	0.1	8441.0	2201.7k	2199.5k	0.1	9013.7
dem_500_gaslib40_238	1587.3k	1585.7k	0.1	291.7	1587.3k	1585.7k	0.1	178.4
dem_500_gaslib40_239	1863.7k	1368.0k	36.2	14400.0	1863.4k	1555.7k	19.8	14400.0
dem_500_gaslib40_240	2493.3k	2490.8k	0.1	14122.7	2493.3k	2475.6k	0.7	14400.0
dem_500_gaslib40_241	2322.4k	2007.9k	15.7	14400.0	2323.4k	2020.5k	15.0	14400.0
dem_500_gaslib40_242	2212.1k	2209.9k	0.1	5045.2	2212.1k	2209.9k	0.1	1894.6
dem_500_gaslib40_243	2013.6k	1366.6k	47.3	14400.0	2005.5k	1359.9k	47.5	14400.0
dem_500_gaslib40_244	2311.3k	2309.1k	0.1	9766.6	2311.3k	2309.0k	0.1	5003.4
dem_500_gaslib40_245	1512.2k	1510.7k	0.1	11639.2	1512.2k	1395.1k	8.4	14400.0

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Table A.14: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_246	1641.3k	1590.9k	3.2	14400.0	1640.2k	1591.8k	3.0	14400.0
dem_500_gaslib40_247	3298.5k	3295.3k	0.1	3391.5	3298.5k	3295.3k	0.1	3080.6
dem_500_gaslib40_248	2352.9k	2028.0k	16.0	14400.0	2339.5k	2031.0k	15.2	14400.0
dem_500_gaslib40_249	2294.9k	2292.6k	0.1	9034.0	2294.9k	2292.6k	0.1	10148.0
dem_500_gaslib40_250	2213.6k	2211.5k	0.1	2677.0	2213.3k	2211.1k	0.1	1529.2
dem_500_gaslib40_251	1479.5k	1219.2k	21.4	14400.0	1479.5k	1328.2k	11.4	14400.0
dem_500_gaslib40_252	2225.4k	2223.2k	0.1	4128.4	2225.4k	2223.2k	0.1	2194.8
dem_500_gaslib40_253	1208.8k	1207.8k	0.1	223.0	1208.8k	1207.6k	0.1	147.3
dem_500_gaslib40_254	2612.0k	2548.3k	2.5	14400.0	2592.2k	2568.5k	0.9	14400.0
dem_500_gaslib40_255	1769.5k	1375.9k	28.6	14400.0	1761.6k	1759.8k	0.1	6459.3
dem_500_gaslib40_256	1642.4k	1322.5k	24.2	14400.0	1629.4k	1348.4k	20.8	14400.0
dem_500_gaslib40_257	2301.9k	1969.5k	16.9	14400.0	2288.2k	1977.2k	15.7	14400.0
dem_500_gaslib40_258	1958.7k	1642.9k	19.2	14400.0	1954.6k	1641.5k	19.1	14400.0
dem_500_gaslib40_259	1898.8k	1187.6k	59.9	14400.0	1876.1k	1359.0k	38.1	14400.0
dem_500_gaslib40_260	1392.8k	1391.4k	0.1	11484.5	1392.8k	1271.8k	9.5	14400.0
dem_500_gaslib40_261	2703.2k	2700.6k	0.1	6504.4	2702.7k	2700.0k	0.1	7022.8
dem_500_gaslib40_262	1548.6k	1194.1k	29.7	14400.0	1535.2k	1195.2k	28.4	14400.0
dem_500_gaslib40_263	2366.7k	2364.4k	0.1	14178.8	2366.1k	2363.7k	0.1	7501.3
dem_500_gaslib40_264	2131.0k	1802.4k	18.2	14400.0	2130.6k	1801.6k	18.3	14400.0
dem_500_gaslib40_265	1457.6k	1456.5k	0.1	10418.3	1457.6k	1401.9k	4.0	14400.0
dem_500_gaslib40_266	1588.0k	1586.4k	0.1	183.4	1588.0k	1586.4k	0.1	211.5
dem_500_gaslib40_267	1636.1k	1252.2k	30.7	14400.0	1639.0k	1244.1k	31.7	14400.0
dem_500_gaslib40_268	2000.6k	1414.3k	41.5	14400.0	1978.4k	1417.8k	39.5	14400.0
dem_500_gaslib40_269	2107.4k	2105.3k	0.1	8250.6	2107.3k	2105.2k	0.1	1966.8
dem_500_gaslib40_270	1308.4k	1301.7k	0.5	14400.0	1308.4k	1307.1k	0.1	290.4
dem_500_gaslib40_271	1691.4k	1440.7k	17.4	14400.0	1683.5k	1541.1k	9.2	14400.0
dem_500_gaslib40_272	1784.6k	1370.5k	30.2	14400.0	1761.5k	1456.1k	21.0	14400.0
dem_500_gaslib40_273	1832.4k	1830.7k	0.1	356.2	1832.4k	1830.7k	0.1	247.8
dem_500_gaslib40_274	1390.4k	1389.1k	0.1	191.1	1390.4k	1389.0k	0.1	169.6
dem_500_gaslib40_275	1519.8k	1518.3k	0.1	1034.2	1519.9k	1221.5k	24.4	14400.0
dem_500_gaslib40_276	1650.6k	1649.1k	0.1	252.7	1650.5k	1648.8k	0.1	200.1
dem_500_gaslib40_277	2392.7k	2390.8k	0.1	5142.3	2392.7k	2390.3k	0.1	4511.1
dem_500_gaslib40_278	1525.6k	1524.1k	0.1	257.9	1525.6k	1524.1k	0.1	238.7
dem_500_gaslib40_279	1709.1k	1259.4k	35.7	14400.0	1696.9k	1258.6k	34.8	14400.0
dem_500_gaslib40_280	2183.9k	1860.1k	17.4	14400.0	2166.3k	1849.6k	17.1	14400.0
dem_500_gaslib40_281	1681.1k	1679.4k	0.1	185.9	1681.1k	1679.4k	0.1	91.0
dem_500_gaslib40_282	2689.5k	2687.1k	0.1	3893.7	2689.1k	2686.5k	0.1	3566.4
dem_500_gaslib40_283	1819.8k	1317.5k	38.1	14400.0	1752.3k	1320.2k	32.7	14400.0
dem_500_gaslib40_284	1899.4k	1539.6k	23.4	14400.0	1866.4k	1540.5k	21.2	14400.0
dem_500_gaslib40_285	1698.0k	1696.4k	0.1	514.7	1698.0k	1696.4k	0.1	278.6
dem_500_gaslib40_286	1968.6k	1445.9k	36.1	14400.0	1928.0k	1447.8k	33.2	14400.0
dem_500_gaslib40_287	2141.9k	1599.8k	33.9	14400.0	2140.5k	1598.6k	33.9	14400.0
dem_500_gaslib40_288	2554.2k	2551.6k	0.1	7216.4	2554.2k	2551.6k	0.1	3291.3
dem_500_gaslib40_289	2097.9k	2095.8k	0.1	7934.6	2097.9k	2075.1k	1.1	14400.0
dem_500_gaslib40_290	1558.9k	1557.4k	0.1	190.2	1558.9k	1557.5k	0.1	135.9
dem_500_gaslib40_291	2025.1k	1544.3k	31.1	14400.0	2002.0k	1975.0k	1.4	14400.0
dem_500_gaslib40_292	2043.4k	1978.2k	3.3	14400.0	2024.5k	2022.5k	0.1	9954.1
dem_500_gaslib40_293	2305.0k	2302.8k	0.1	3318.2	2305.0k	2302.7k	0.1	2944.9
dem_500_gaslib40_294	1486.1k	1260.3k	17.9	14400.0	1487.6k	1258.0k	18.3	14400.0
dem_500_gaslib40_295	2692.9k	2690.3k	0.1	2717.0	2692.6k	2689.9k	0.1	2422.8
dem_500_gaslib40_296	1704.0k	1702.3k	0.1	305.8	1704.0k	1702.3k	0.1	291.3
dem_500_gaslib40_297	2474.0k	2471.5k	0.1	2687.4	2474.0k	2471.5k	0.1	2170.0
dem_500_gaslib40_298	2313.2k	1702.5k	35.9	14400.0	2277.4k	1672.3k	36.2	14400.0
dem_500_gaslib40_299	1634.8k	1633.1k	0.1	121.9	1634.4k	1632.9k	0.1	192.1
dem_500_gaslib40_300	1791.8k	1790.0k	0.1	619.5	1791.8k	1790.0k	0.1	401.1
dem_500_gaslib40_301	1946.6k	1655.2k	17.6	14400.0	1935.7k	1658.9k	16.7	14400.0
dem_500_gaslib40_302	2892.0k	2889.1k	0.1	8563.7	2892.0k	2889.1k	0.1	4435.4
dem_500_gaslib40_303	2076.2k	1871.5k	10.9	14400.0	2075.9k	2050.2k	1.3	14400.0
dem_500_gaslib40_304	2197.5k	1859.3k	18.2	14400.0	2174.0k	1861.1k	16.8	14400.0
dem_500_gaslib40_305	1312.1k	953.1k	37.7	14400.0	1312.1k	973.7k	34.8	14400.0
dem_500_gaslib40_306	1482.5k	1481.1k	0.1	1278.9	1482.5k	1481.1k	0.1	5999.8
dem_500_gaslib40_307	2468.7k	2466.2k	0.1	12991.2	2468.7k	2466.2k	0.1	3919.9
dem_500_gaslib40_308	1631.0k	1629.4k	0.1	365.7	1631.0k	1629.4k	0.1	730.4
dem_500_gaslib40_309	1640.5k	1518.7k	8.0	14400.0	1639.6k	1439.3k	13.9	14400.0
dem_500_gaslib40_310	1416.6k	1415.5k	0.1	153.3	1416.6k	1415.2k	0.1	480.5
dem_500_gaslib40_311	2116.9k	2114.9k	0.1	1081.8	2116.9k	2114.8k	0.1	5168.6
dem_500_gaslib40_312	2169.5k	1667.3k	30.1	14400.0	2152.1k	1676.5k	28.4	14400.0
dem_500_gaslib40_313	1781.8k	1218.6k	46.2	14400.0	1748.6k	1322.6k	32.2	14400.0
dem_500_gaslib40_314	1253.6k	1252.4k	0.1	2114.0	1252.8k	1251.6k	0.1	1941.6
dem_500_gaslib40_315	1952.6k	1582.3k	23.4	14400.0	1935.9k	1580.9k	22.5	14400.0
dem_500_gaslib40_316	2086.0k	2084.3k	0.1	4939.4	2086.0k	2083.9k	0.1	4944.1
dem_500_gaslib40_317	1351.0k	1349.6k	0.1	135.3	1351.0k	1349.7k	0.1	81.2
dem_500_gaslib40_318	2319.1k	2002.2k	15.8	14400.0	2314.6k	2312.3k	0.1	9165.2
dem_500_gaslib40_319	1597.5k	1595.9k	0.1	14124.8	1597.5k	1552.7k	2.9	14400.0
dem_500_gaslib40_320	2106.7k	2104.8k	0.1	6021.5	2106.6k	2104.5k	0.1	3711.7
dem_500_gaslib40_321	1614.8k	1613.2k	0.1	461.3	1614.6k	1613.0k	0.1	188.8
dem_500_gaslib40_322	1617.4k	1615.7k	0.1	271.9	1617.4k	1615.7k	0.1	186.6
dem_500_gaslib40_323	2304.8k	1947.4k	18.4	14400.0	2291.5k	1948.0k	17.6	14400.0

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Table A.14: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_324	1390.1k	1388.9k	0.1	199.3	1390.1k	1388.7k	0.1	144.2
dem_500_gaslib40_325	2158.8k	2156.6k	0.1	8625.2	2158.8k	2143.1k	0.7	14400.0
dem_500_gaslib40_326	1484.5k	1397.6k	6.2	14400.0	1477.3k	1475.8k	0.1	11522.1
dem_500_gaslib40_327	1796.2k	1794.4k	0.1	764.3	1795.2k	1793.5k	0.1	329.4
dem_500_gaslib40_328	1553.6k	1232.4k	26.1	14400.0	1557.9k	1234.5k	26.2	14400.0
dem_500_gaslib40_329	1386.2k	1384.9k	0.1	1281.0	1386.0k	1384.6k	0.1	6085.9
dem_500_gaslib40_330	2303.9k	1780.0k	29.4	14400.0	2270.4k	1690.5k	34.3	14400.0
dem_500_gaslib40_331	1704.4k	1702.8k	0.1	263.4	1704.4k	1702.7k	0.1	176.1
dem_500_gaslib40_332	2236.6k	2234.5k	0.1	273.4	2235.6k	2233.4k	0.1	157.5
dem_500_gaslib40_333	1914.3k	1368.4k	39.9	14400.0	1860.5k	1560.7k	19.2	14400.0
dem_500_gaslib40_334	2231.0k	1983.2k	12.5	14400.0	2211.0k	1963.7k	12.6	14400.0
dem_500_gaslib40_335	1489.9k	1176.5k	26.6	14400.0	1486.6k	1173.6k	26.7	14400.0
dem_500_gaslib40_336	1530.2k	1528.7k	0.1	1141.3	1530.2k	1524.4k	0.4	14400.0
dem_500_gaslib40_337	2053.0k	1448.1k	41.8	14400.0	2027.7k	1478.2k	37.2	14400.0
dem_500_gaslib40_338	1707.2k	1371.1k	24.5	14400.0	1688.6k	1370.6k	23.2	14400.0
dem_500_gaslib40_339	1364.0k	1362.6k	0.1	4861.6	1395.7k	846.2k	64.9	14400.0
dem_500_gaslib40_340	2010.4k	1715.9k	17.2	14400.0	1974.2k	1712.6k	15.3	14400.0
dem_500_gaslib40_341	2579.4k	2576.8k	0.1	2413.6	2579.4k	2576.8k	0.1	1646.1
dem_500_gaslib40_342	1416.6k	1415.2k	0.1	9339.8	1416.6k	1415.7k	0.1	13086.5
dem_500_gaslib40_343	2561.4k	2559.2k	0.1	10998.6	2560.4k	2557.8k	0.1	13141.1
dem_500_gaslib40_344	2945.6k	2934.3k	0.4	14400.0	2945.7k	2935.4k	0.4	14400.0
dem_500_gaslib40_345	2080.8k	1501.3k	38.6	14400.0	2074.9k	1558.9k	33.1	14400.0
dem_500_gaslib40_346	2085.8k	2083.7k	0.1	2499.9	2085.8k	2083.7k	0.1	8160.2
dem_500_gaslib40_347	2522.3k	2519.8k	0.1	3377.7	2522.5k	2520.0k	0.1	1895.1
dem_500_gaslib40_348	1659.9k	1526.4k	8.7	14400.0	1659.5k	1532.6k	8.3	14400.0
dem_500_gaslib40_349	2367.2k	1818.6k	30.2	14400.0	2349.0k	1824.5k	28.7	14400.0
dem_500_gaslib40_350	1972.2k	1970.2k	0.1	9609.8	1971.6k	1969.6k	0.1	8112.7
dem_500_gaslib40_351	1880.1k	1407.9k	33.5	14400.0	1842.9k	1425.1k	29.3	14400.0
dem_500_gaslib40_352	1952.9k	1429.2k	36.6	14400.0	1917.9k	1159.1k	65.5	14400.0
dem_500_gaslib40_353	1383.2k	1381.8k	0.1	120.3	1383.1k	1381.8k	0.1	74.5
dem_500_gaslib40_354	1843.2k	1841.7k	0.1	7217.4	1843.1k	1418.4k	29.9	14400.0
dem_500_gaslib40_355	1985.5k	1492.2k	33.1	14400.0	1985.5k	1494.1k	32.9	14400.0
dem_500_gaslib40_356	2203.4k	2201.2k	0.1	1477.0	2203.4k	2201.2k	0.1	1470.5
dem_500_gaslib40_357	2168.5k	1617.6k	34.1	14400.0	2143.8k	1618.9k	32.4	14400.0
dem_500_gaslib40_358	2821.1k	2818.3k	0.1	2239.7	2821.1k	2818.3k	0.1	2564.8
dem_500_gaslib40_359	2098.3k	1454.3k	44.3	14400.0	2068.8k	1456.5k	42.0	14400.0
dem_500_gaslib40_360	2014.2k	1480.2k	36.1	14400.0	2001.5k	1480.9k	35.2	14400.0
dem_500_gaslib40_361	1416.9k	1415.4k	0.1	82.5	1416.9k	1415.5k	0.1	64.0
dem_500_gaslib40_362	1268.9k	1267.7k	0.1	62.9	1268.8k	1267.6k	0.1	65.4
dem_500_gaslib40_363	1387.4k	1386.1k	0.1	47.3	1387.4k	1386.0k	0.1	29.3
dem_500_gaslib40_364	1651.3k	1268.3k	30.2	14400.0	1647.0k	1173.5k	40.4	14400.0
dem_500_gaslib40_365	1343.4k	1207.3k	11.3	14400.0	1348.1k	915.4k	47.3	14400.0
dem_500_gaslib40_366	1265.9k	1264.7k	0.1	50.6	1265.9k	1264.7k	0.1	52.0
dem_500_gaslib40_367	1770.9k	1518.8k	16.6	14400.0	1749.4k	1521.3k	15.0	14400.0
dem_500_gaslib40_368	1431.4k	1430.0k	0.1	484.6	1431.4k	1429.9k	0.1	763.8
dem_500_gaslib40_369	1647.3k	1435.9k	14.7	14400.0	1652.6k	1434.0k	15.2	14400.0
dem_500_gaslib40_370	1620.0k	1358.1k	19.3	14400.0	1617.5k	1350.4k	19.8	14400.0
dem_500_gaslib40_371	1570.0k	1568.6k	0.1	342.0	1570.0k	1568.5k	0.1	210.4
dem_500_gaslib40_372	1801.0k	1356.4k	32.8	14400.0	1744.6k	1355.1k	28.7	14400.0
dem_500_gaslib40_373	2141.2k	1584.5k	35.1	14400.0	2128.4k	1579.2k	34.8	14400.0
dem_500_gaslib40_374	1655.6k	1654.0k	0.1	684.8	1655.6k	1654.0k	0.1	480.4
dem_500_gaslib40_375	1816.8k	1581.4k	14.9	14400.0	1809.6k	1546.1k	17.0	14400.0
dem_500_gaslib40_376	2548.3k	2545.7k	0.1	12196.8	2548.3k	2545.7k	0.1	9573.1
dem_500_gaslib40_377	1479.7k	1169.9k	26.5	14400.0	1491.6k	1165.3k	28.0	14400.0
dem_500_gaslib40_378	2141.9k	2139.8k	0.1	13857.8	2141.9k	2139.8k	0.1	13144.5
dem_500_gaslib40_379	1367.0k	1365.6k	0.1	2667.2	1367.0k	1365.6k	0.1	2595.0
dem_500_gaslib40_380	1643.2k	1307.4k	25.7	14400.0	1626.4k	1307.9k	24.4	14400.0
dem_500_gaslib40_381	2371.2k	2368.8k	0.1	4308.5	2371.2k	2368.8k	0.1	4468.9
dem_500_gaslib40_382	1866.1k	1417.2k	31.7	14400.0	1840.4k	1405.5k	30.9	14400.0
dem_500_gaslib40_383	1786.0k	1784.2k	0.1	353.7	1785.7k	1784.0k	0.1	277.2
dem_500_gaslib40_384	1972.7k	1389.2k	42.0	14400.0	1960.4k	1412.6k	38.8	14400.0
dem_500_gaslib40_385	1490.9k	1489.4k	0.1	670.7	1490.9k	1489.4k	0.1	283.9
dem_500_gaslib40_386	1576.5k	1462.1k	7.8	14400.0	1563.0k	1561.4k	0.1	13791.8
dem_500_gaslib40_387	2021.7k	2019.7k	0.1	967.4	2021.7k	2019.7k	0.1	639.7
dem_500_gaslib40_388	2331.2k	2328.9k	0.1	10779.0	2330.0k	2327.7k	0.1	6823.6
dem_500_gaslib40_389	3017.5k	3014.5k	0.1	10621.2	3017.5k	3014.5k	0.1	5454.4
dem_500_gaslib40_390	2032.6k	1488.6k	36.5	14400.0	2012.6k	1512.8k	33.0	14400.0
dem_500_gaslib40_391	1790.3k	1367.2k	30.9	14400.0	1790.3k	1378.7k	29.9	14400.0
dem_500_gaslib40_392	1722.6k	1615.1k	6.7	14400.0	1719.7k	1719.7k	0.0	7419.8
dem_500_gaslib40_393	2316.8k	1718.3k	34.8	14400.0	2284.2k	2281.9k	0.1	9676.5
dem_500_gaslib40_394	2256.4k	2213.4k	1.9	14400.0	2226.2k	2224.0k	0.1	4166.6
dem_500_gaslib40_395	2592.9k	2590.3k	0.1	3103.4	2591.5k	2589.0k	0.1	4684.6
dem_500_gaslib40_396	2547.3k	2544.7k	0.1	2464.3	2547.3k	2544.7k	0.1	2089.6
dem_500_gaslib40_397	1881.6k	1479.8k	27.1	14400.0	1879.6k	1493.7k	25.8	14400.0
dem_500_gaslib40_398	1961.6k	1672.4k	17.3	14400.0	1950.0k	1948.1k	0.1	11552.7
dem_500_gaslib40_399	1574.5k	1280.4k	23.0	14400.0	1574.5k	1280.4k	23.0	14400.0
dem_500_gaslib40_400	2173.7k	1618.9k	34.3	14400.0	2164.4k	1637.2k	32.2	14400.0
dem_500_gaslib40_401	2104.4k	1509.4k	39.4	14400.0	2101.2k	1512.3k	38.9	14400.0

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Table A.14: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_402	2132.7k	2130.5k	0.1	4472.1	2132.6k	2130.5k	0.1	4874.5
dem_500_gaslib40_403	1962.5k	1960.6k	0.1	2023.9	1962.5k	1960.6k	0.1	1290.4
dem_500_gaslib40_404	2242.7k	1584.7k	41.5	14400.0	2233.1k	1589.6k	40.5	14400.0
dem_500_gaslib40_405	1947.5k	1762.8k	10.5	14400.0	1936.9k	1473.0k	31.5	14400.0
dem_500_gaslib40_406	1957.8k	1955.9k	0.1	9207.4	1957.6k	1955.6k	0.1	6258.1
dem_500_gaslib40_407	2020.9k	1440.1k	40.3	14400.0	1997.1k	1684.2k	18.6	14400.0
dem_500_gaslib40_408	2466.0k	2463.5k	0.1	5572.9	2465.4k	2462.9k	0.1	4631.8
dem_500_gaslib40_409	2601.9k	2599.3k	0.1	2792.8	2601.9k	2599.3k	0.1	1979.7
dem_500_gaslib40_410	1964.4k	1617.4k	21.5	14400.0	1949.1k	1514.7k	28.7	14400.0
dem_500_gaslib40_411	1930.2k	1620.3k	19.1	14400.0	1910.2k	1429.3k	33.6	14400.0
dem_500_gaslib40_412	2160.2k	2158.0k	0.1	3183.6	2160.2k	2158.0k	0.1	3616.1
dem_500_gaslib40_413	1712.7k	1459.5k	17.3	14400.0	1692.4k	1451.1k	16.6	14400.0
dem_500_gaslib40_414	1699.3k	1697.7k	0.1	239.8	1699.3k	1697.7k	0.1	227.0
dem_500_gaslib40_415	1463.5k	1462.1k	0.1	1630.0	1463.4k	1461.9k	0.1	1535.6
dem_500_gaslib40_416	1773.2k	1302.5k	36.1	14400.0	1747.9k	1189.4k	47.0	14400.0
dem_500_gaslib40_417	2144.1k	1867.6k	14.8	14400.0	2127.3k	1858.9k	14.4	14400.0
dem_500_gaslib40_418	2139.1k	2136.9k	0.1	6902.0	2138.8k	2136.6k	0.1	7302.4
dem_500_gaslib40_419	1648.1k	1646.9k	0.1	5650.3	1648.1k	1219.7k	35.1	14400.0
dem_500_gaslib40_420	2461.5k	2450.7k	0.4	14400.0	2461.5k	2459.0k	0.1	7670.2
dem_500_gaslib40_421	2445.0k	2431.1k	0.6	14400.0	2439.4k	2437.0k	0.1	9281.1
dem_500_gaslib40_422	1753.3k	1255.8k	39.6	14400.0	1708.7k	1262.8k	35.3	14400.0
dem_500_gaslib40_423	1958.2k	1664.8k	17.6	14400.0	1947.8k	1689.4k	15.3	14400.0
dem_500_gaslib40_424	1827.1k	1439.1k	27.0	14400.0	1827.7k	1439.8k	26.9	14400.0
dem_500_gaslib40_425	1769.2k	1767.4k	0.1	193.0	1769.2k	1767.4k	0.1	154.5
dem_500_gaslib40_426	1571.4k	1570.1k	0.1	522.1	1571.4k	1569.8k	0.1	218.8
dem_500_gaslib40_427	1413.3k	1411.9k	0.1	253.6	1413.3k	1411.9k	0.1	147.8
dem_500_gaslib40_428	1445.5k	1444.1k	0.1	326.0	1445.5k	1444.1k	0.1	144.4
dem_500_gaslib40_429	1624.1k	1511.5k	7.5	14400.0	1611.2k	1501.1k	7.3	14400.0
dem_500_gaslib40_430	1718.0k	1473.4k	16.6	14400.0	1716.3k	1453.4k	18.1	14400.0
dem_500_gaslib40_431	2246.3k	2244.1k	0.1	4831.1	2246.3k	2244.1k	0.1	3377.0
dem_500_gaslib40_432	2155.0k	2153.1k	0.1	4167.9	2154.6k	2152.5k	0.1	3707.1
dem_500_gaslib40_433	1667.9k	1666.8k	0.1	207.3	1667.9k	1666.2k	0.1	123.1
dem_500_gaslib40_434	1480.3k	1443.4k	2.6	14400.0	1480.3k	1395.0k	6.1	14400.0
dem_500_gaslib40_435	1847.7k	1484.4k	24.5	14400.0	1847.6k	1481.4k	24.7	14400.0
dem_500_gaslib40_436	2002.6k	1705.5k	17.4	14400.0	1990.1k	1642.2k	21.2	14400.0
dem_500_gaslib40_437	2018.9k	2016.9k	0.1	7139.2	2018.9k	2016.9k	0.1	6675.4
dem_500_gaslib40_438	1666.0k	1664.3k	0.1	851.8	1666.0k	1664.3k	0.1	355.4
dem_500_gaslib40_439	1953.7k	1951.8k	0.1	7735.6	1953.7k	1951.8k	0.1	4035.7
dem_500_gaslib40_440	2238.4k	1904.3k	17.5	14400.0	2238.1k	1910.9k	17.1	14400.0
dem_500_gaslib40_441	1728.6k	1726.8k	0.1	174.2	1728.6k	1726.8k	0.1	108.0
dem_500_gaslib40_442	1778.7k	1618.8k	9.9	14400.0	1778.7k	1551.6k	14.6	14400.0
dem_500_gaslib40_443	2303.3k	1686.9k	36.5	14400.0	2242.6k	1943.2k	15.4	14400.0
dem_500_gaslib40_444	1633.6k	1439.2k	13.5	14400.0	1637.0k	1233.4k	32.7	14400.0
dem_500_gaslib40_445	2603.0k	2507.4k	3.8	14400.0	2539.3k	2536.7k	0.1	7965.5
dem_500_gaslib40_446	2452.2k	1845.2k	32.9	14400.0	2397.5k	1826.3k	31.3	14400.0
dem_500_gaslib40_447	2329.5k	2327.2k	0.1	2504.5	2329.6k	2327.2k	0.1	2212.6
dem_500_gaslib40_448	2072.3k	1672.5k	23.9	14400.0	2026.6k	1585.3k	27.8	14400.0
dem_500_gaslib40_449	1564.4k	1562.9k	0.1	5480.9	1564.3k	1492.1k	4.8	14400.0
dem_500_gaslib40_450	1797.5k	1796.5k	0.1	153.3	1797.0k	1795.3k	0.1	70.8
dem_500_gaslib40_451	2293.4k	2291.1k	0.1	5796.1	2293.4k	2291.1k	0.1	3670.5
dem_500_gaslib40_452	3256.1k	3252.8k	0.1	3619.0	3254.7k	3251.5k	0.1	4409.5
dem_500_gaslib40_453	1406.3k	1405.1k	0.1	32.4	1406.3k	1405.1k	0.1	30.0
dem_500_gaslib40_454	2185.7k	2183.5k	0.1	8626.1	2185.5k	2183.3k	0.1	9040.0
dem_500_gaslib40_455	1390.6k	1389.2k	0.1	6544.6	1390.6k	1390.6k	0.0	2314.4
dem_500_gaslib40_456	2162.3k	2160.1k	0.1	8367.8	2162.2k	2160.1k	0.1	5538.0
dem_500_gaslib40_457	2809.9k	2807.1k	0.1	3350.0	2809.9k	2807.1k	0.1	2427.1
dem_500_gaslib40_458	1669.9k	1668.2k	0.1	529.0	1669.9k	1668.2k	0.1	328.1
dem_500_gaslib40_459	1953.1k	1341.6k	45.6	14400.0	1981.5k	1332.8k	48.7	14400.0
dem_500_gaslib40_460	1373.1k	1371.7k	0.1	213.1	1373.1k	1371.8k	0.1	260.9
dem_500_gaslib40_461	1426.9k	1061.2k	34.5	14400.0	1432.1k	1062.4k	34.8	14400.0
dem_500_gaslib40_462	1531.7k	1308.7k	17.0	14400.0	1524.7k	1290.8k	18.1	14400.0
dem_500_gaslib40_463	2532.2k	2529.9k	0.1	4472.6	2531.3k	2528.8k	0.1	3655.3
dem_500_gaslib40_464	2182.5k	2180.3k	0.1	7034.1	2182.5k	2180.3k	0.1	7027.4
dem_500_gaslib40_465	2152.0k	1902.9k	13.1	14400.0	2124.2k	1855.4k	14.5	14400.0
dem_500_gaslib40_466	2328.4k	2326.1k	0.1	12482.8	2328.4k	2326.1k	0.1	7630.6
dem_500_gaslib40_467	1732.8k	1383.0k	25.3	14400.0	1702.7k	1377.9k	23.6	14400.0
dem_500_gaslib40_468	2553.9k	2551.3k	0.1	1947.0	2553.8k	2551.3k	0.1	1922.5
dem_500_gaslib40_469	1719.6k	1717.9k	0.1	278.4	1719.6k	1717.9k	0.1	239.8
dem_500_gaslib40_470	1486.4k	1484.9k	0.1	11028.5	1486.4k	1484.9k	0.1	9783.2
dem_500_gaslib40_471	2105.0k	2102.8k	0.1	9064.1	2105.0k	2102.9k	0.1	9933.5
dem_500_gaslib40_472	2109.3k	1545.5k	36.5	14400.0	2093.0k	1536.1k	36.2	14400.0
dem_500_gaslib40_473	1932.1k	1930.2k	0.1	1774.5	1932.1k	1930.2k	0.1	1936.5
dem_500_gaslib40_474	2669.9k	2605.0k	2.5	14400.0	2633.5k	2626.0k	0.3	14400.0
dem_500_gaslib40_475	2082.4k	1834.5k	13.5	14400.0	2053.5k	1831.6k	12.1	14400.0
dem_500_gaslib40_476	1433.3k	1431.9k	0.1	1122.0	1432.8k	1431.4k	0.1	883.0
dem_500_gaslib40_477	2569.9k	2517.2k	2.1	14400.0	2527.3k	2524.8k	0.1	9464.0
dem_500_gaslib40_478	2186.5k	1620.6k	34.9	14400.0	2183.5k	1636.2k	33.5	14400.0
dem_500_gaslib40_479	2097.3k	1884.5k	11.3	14400.0	2067.5k	1696.3k	21.9	14400.0

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Table A.14: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib40_480	2786.6k	2785.4k	0.0	2439.2	2787.7k	2785.1k	0.1	2138.0
dem_500_gaslib40_481	1709.3k	1581.5k	8.1	14400.0	1706.9k	1579.9k	8.0	14400.0
dem_500_gaslib40_482	1708.1k	1706.4k	0.1	7018.6	1708.1k	1706.4k	0.1	5635.1
dem_500_gaslib40_483	1765.3k	1763.5k	0.1	411.3	1765.3k	1763.5k	0.1	310.4
dem_500_gaslib40_484	1614.0k	1487.2k	8.5	14400.0	1624.4k	1416.3k	14.7	14400.0
dem_500_gaslib40_485	1245.8k	1179.6k	5.6	14400.0	1245.8k	1160.3k	7.4	14400.0
dem_500_gaslib40_486	1718.0k	1479.5k	16.1	14400.0	1720.7k	1477.2k	16.5	14400.0
dem_500_gaslib40_487	1626.6k	1514.7k	7.4	14400.0	1626.5k	1526.3k	6.6	14400.0
dem_500_gaslib40_488	1940.5k	1414.6k	37.2	14400.0	1898.6k	1436.1k	32.2	14400.0
dem_500_gaslib40_489	1697.2k	1473.5k	15.2	14400.0	1700.3k	1457.6k	16.6	14400.0
dem_500_gaslib40_490	1883.3k	1574.1k	19.6	14400.0	1878.7k	1459.2k	28.8	14400.0
dem_500_gaslib40_491	2211.4k	2209.2k	0.1	1444.3	2211.4k	2209.2k	0.1	1821.3
dem_500_gaslib40_492	1801.3k	1374.6k	31.0	14400.0	1754.3k	1480.8k	18.5	14400.0
dem_500_gaslib40_493	1307.7k	1306.9k	0.1	208.2	1307.4k	1306.1k	0.1	82.6
dem_500_gaslib40_494	1500.6k	1499.7k	0.1	305.6	1500.6k	1499.1k	0.1	189.1
dem_500_gaslib40_495	1852.8k	1512.2k	22.5	14400.0	1833.0k	1358.3k	34.9	14400.0
dem_500_gaslib40_496	2188.4k	2186.3k	0.1	951.0	2188.4k	2186.3k	0.1	548.9
dem_500_gaslib40_497	1145.2k	1144.1k	0.1	330.1	1147.9k	882.4k	30.1	14400.0
dem_500_gaslib40_498	2213.5k	1863.6k	18.8	14400.0	2193.9k	1860.8k	17.9	14400.0
dem_500_gaslib40_499	1995.0k	1727.7k	15.5	14400.0	1973.8k	1767.1k	11.7	14400.0
dem_500_gaslib40_500	2094.5k	2092.4k	0.1	2335.4	2094.4k	2092.3k	0.1	1555.9

Table A.15: Detailed results of the discrete models on *GasLib-40* with $\mathcal{B} = 1000$ as summarized in Figure 3.8d and Table 3.3b. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_1	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	274.9	1e+20	1e+20	0.0	0.7
dem_1000_gaslib40_2	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	381.8	1e+20	1e+20	0.0	7.3
dem_1000_gaslib40_3	1e+20	2481.2k	–	14400.0	4691.4k	3495.6k	34.2	14400.0	1e+20	2612.4k	–	14400.0
dem_1000_gaslib40_4	1e+20	2167.3k	–	14400.0	1e+20	2862.0k	–	14400.0	4793.7k	2773.5k	72.8	14400.0
dem_1000_gaslib40_5	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	158.3	1e+20	1e+20	0.0	5.8
dem_1000_gaslib40_6	3423.4k	1431.4k	139.2	14400.0	3152.6k	1924.9k	63.8	14400.0	4089.9k	1744.3k	134.5	14400.0
dem_1000_gaslib40_7	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	277.5	1e+20	1e+20	0.0	16.3
dem_1000_gaslib40_8	1e+20	4033.0k	–	14400.0	1e+20	4725.0k	–	14400.0	1e+20	4008.9k	–	14400.0
dem_1000_gaslib40_9	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	135.5	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_10	4724.7k	2107.3k	124.2	14400.0	1e+20	2827.8k	–	14400.0	1e+20	2619.6k	–	14400.0
dem_1000_gaslib40_11	1e+20	3058.4k	–	14400.0	1e+20	3532.3k	–	14400.0	1e+20	3235.8k	–	14400.0
dem_1000_gaslib40_12	3879.0k	2330.3k	66.5	14400.0	3862.1k	3018.2k	28.0	14400.0	1e+20	2116.4k	–	14400.0
dem_1000_gaslib40_13	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	323.4	1e+20	1e+20	0.0	20.2
dem_1000_gaslib40_14	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	97.6	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_15	1e+20	2418.0k	–	14400.0	1e+20	3145.5k	–	14400.0	1e+20	2872.3k	–	14400.0
dem_1000_gaslib40_16	4342.2k	1763.9k	146.2	14400.0	3973.1k	2619.5k	51.7	14400.0	4791.2k	1975.1k	142.6	14400.0
dem_1000_gaslib40_17	1e+20	3680.3k	–	14400.0	1e+20	3805.4k	–	14400.0	1e+20	3968.4k	–	14400.0
dem_1000_gaslib40_18	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	288.5	1e+20	1e+20	0.0	10.9
dem_1000_gaslib40_19	1e+20	2228.3k	–	14400.0	4074.6k	3063.9k	33.0	14400.0	1e+20	2603.1k	–	14400.0
dem_1000_gaslib40_20	1e+20	2081.3k	–	14400.0	1e+20	2495.9k	–	14400.0	1e+20	2500.3k	–	14400.0
dem_1000_gaslib40_21	1e+20	2364.7k	–	14400.0	1e+20	3028.7k	–	14400.0	4660.5k	2944.0k	58.3	14400.0
dem_1000_gaslib40_22	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	247.5	1e+20	1e+20	0.0	4.3
dem_1000_gaslib40_23	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_24	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	299.8	1e+20	1e+20	0.0	7.8
dem_1000_gaslib40_25	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	99.5	1e+20	1e+20	0.0	10.6
dem_1000_gaslib40_26	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	193.7	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_27	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	368.5	1e+20	1e+20	0.0	27.9
dem_1000_gaslib40_28	4682.7k	1818.9k	157.4	14400.0	4561.1k	2365.5k	92.8	14400.0	1e+20	1995.0k	–	14400.0
dem_1000_gaslib40_29	1e+20	1958.1k	–	14400.0	5260.1k	2536.5k	107.4	14400.0	1e+20	2329.7k	–	14400.0
dem_1000_gaslib40_30	1e+20	2515.4k	–	14400.0	1e+20	3466.8k	–	14400.0	1e+20	3113.7k	–	14400.0
dem_1000_gaslib40_31	1e+20	2253.0k	–	14400.0	4399.5k	3021.6k	45.6	14400.0	4112.5k	2263.8k	81.7	14400.0
dem_1000_gaslib40_32	4975.7k	2459.1k	102.3	14400.0	1e+20	2927.7k	–	14400.0	1e+20	2734.3k	–	14400.0
dem_1000_gaslib40_33	3637.9k	2136.7k	70.3	14400.0	3420.0k	2820.0k	21.3	14400.0	4039.0k	2153.6k	87.5	14400.0
dem_1000_gaslib40_34	1e+20	2220.1k	–	14400.0	4626.2k	3169.4k	46.0	14400.0	1e+20	2386.5k	–	14400.0
dem_1000_gaslib40_35	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	146.2	1e+20	1e+20	0.0	5.8
dem_1000_gaslib40_36	1e+20	2685.7k	–	14400.0	5492.9k	3186.0k	72.4	14400.0	1e+20	3134.5k	–	14400.0
dem_1000_gaslib40_37	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_38	1e+20	2099.9k	–	14400.0	3863.0k	3368.9k	14.7	14400.0	1e+20	2516.2k	–	14400.0
dem_1000_gaslib40_39	1e+20	3258.8k	–	14400.0	1e+20	3680.2k	–	14400.0	1e+20	3681.0k	–	14400.0
dem_1000_gaslib40_40	4188.1k	2769.1k	51.2	14400.0	4110.9k	3872.1k	6.2	14400.0	1e+20	2560.3k	–	14400.0
dem_1000_gaslib40_41	1e+20	2992.8k	–	14400.0	4699.2k	4265.4k	10.2	14400.0	1e+20	2631.9k	–	14400.0
dem_1000_gaslib40_42	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	107.5	1e+20	1e+20	0.0	0.2

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Table A.15: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_43	1e+20	3947.0k	–	14400.0	5628.3k	5310.0k	6.0	14400.0	1e+20	3979.0k	–	14400.0
dem_1000_gaslib40_44	5236.3k	2673.8k	95.8	14400.0	1e+20	3212.4k	–	14400.0	1e+20	3353.0k	–	14400.0
dem_1000_gaslib40_45	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	216.0	1e+20	1e+20	0.0	0.5
dem_1000_gaslib40_46	4536.1k	2076.5k	118.4	14400.0	3952.7k	2872.5k	37.6	14400.0	1e+20	2097.5k	–	14400.0
dem_1000_gaslib40_47	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	192.0	1e+20	1e+20	0.0	4.5
dem_1000_gaslib40_48	1e+20	2023.9k	–	14400.0	4015.6k	2654.4k	51.3	14400.0	4905.9k	2159.1k	127.2	14400.0
dem_1000_gaslib40_49	1e+20	2472.3k	–	14400.0	5397.3k	3684.3k	46.5	14400.0	1e+20	3179.7k	–	14400.0
dem_1000_gaslib40_50	4147.1k	1793.9k	131.2	14400.0	3967.3k	2464.7k	61.0	14400.0	1e+20	1956.3k	–	14400.0
dem_1000_gaslib40_51	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_52	1e+20	2642.5k	–	14400.0	1e+20	3263.7k	–	14400.0	1e+20	2994.1k	–	14400.0
dem_1000_gaslib40_53	1e+20	2565.9k	–	14400.0	5058.4k	3404.8k	48.6	14400.0	1e+20	2549.5k	–	14400.0
dem_1000_gaslib40_54	1e+20	1860.6k	–	14400.0	4414.9k	2456.0k	79.8	14400.0	1e+20	2038.4k	–	14400.0
dem_1000_gaslib40_55	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	9.3	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_56	3677.5k	1989.7k	84.8	14400.0	3787.3k	2336.0k	62.1	14400.0	4090.0k	2194.2k	86.4	14400.0
dem_1000_gaslib40_57	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	316.2	1e+20	1e+20	0.0	16.5
dem_1000_gaslib40_58	1e+20	2649.3k	–	14400.0	5410.4k	3273.4k	65.3	14400.0	1e+20	3172.1k	–	14400.0
dem_1000_gaslib40_59	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	458.1	1e+20	1e+20	0.0	46.7
dem_1000_gaslib40_60	3591.3k	1743.3k	106.0	14400.0	3190.2k	2162.2k	47.5	14400.0	3417.6k	1748.5k	95.5	14400.0
dem_1000_gaslib40_61	4898.4k	2087.6k	134.6	14400.0	4021.6k	2794.9k	43.9	14400.0	1e+20	2125.1k	–	14400.0
dem_1000_gaslib40_62	5643.8k	2066.4k	173.1	14400.0	4414.8k	3154.9k	39.9	14400.0	1e+20	2234.8k	–	14400.0
dem_1000_gaslib40_63	1e+20	1e+20	0.0	5867.8	1e+20	1e+20	0.0	1315.6	1e+20	4307.7k	–	14400.0
dem_1000_gaslib40_64	3987.6k	2417.6k	64.9	14400.0	4336.3k	3264.2k	32.8	14400.0	4214.7k	2170.8k	94.2	14400.0
dem_1000_gaslib40_65	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	115.3	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_66	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	127.0	1e+20	1e+20	0.0	10.6
dem_1000_gaslib40_67	1e+20	2945.8k	–	14400.0	5084.8k	3967.0k	28.2	14400.0	1e+20	3198.3k	–	14400.0
dem_1000_gaslib40_68	1e+20	2056.6k	–	14400.0	4896.1k	3136.6k	56.1	14400.0	1e+20	2376.9k	–	14400.0
dem_1000_gaslib40_69	1e+20	3120.2k	–	14400.0	1e+20	3455.5k	–	14400.0	1e+20	3360.2k	–	14400.0
dem_1000_gaslib40_70	1e+20	2312.4k	–	14400.0	5615.9k	2950.9k	90.3	14400.0	1e+20	2983.2k	–	14400.0
dem_1000_gaslib40_71	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_72	3504.3k	1918.1k	82.7	14400.0	3773.4k	2265.5k	66.6	14400.0	3558.2k	2007.1k	77.3	14400.0
dem_1000_gaslib40_73	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	121.5	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_74	1e+20	1870.9k	–	14400.0	3893.9k	2925.3k	33.1	14400.0	1e+20	2273.3k	–	14400.0
dem_1000_gaslib40_75	1e+20	2751.7k	–	14400.0	1e+20	3234.7k	–	14400.0	1e+20	3450.2k	–	14400.0
dem_1000_gaslib40_76	1e+20	2925.3k	–	14400.0	1e+20	3655.6k	–	14400.0	1e+20	3606.1k	–	14400.0
dem_1000_gaslib40_77	1e+20	2624.7k	–	14400.0	1e+20	3366.7k	–	14400.0	1e+20	3230.3k	–	14400.0
dem_1000_gaslib40_78	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_79	3940.6k	2122.9k	85.6	14400.0	4425.1k	2887.1k	53.3	14400.0	5074.4k	2109.5k	140.5	14400.0
dem_1000_gaslib40_80	1e+20	1904.9k	–	14400.0	3823.4k	2309.3k	65.6	14400.0	1e+20	2059.0k	–	14400.0
dem_1000_gaslib40_81	3310.2k	1537.3k	115.3	14400.0	3000.8k	2143.0k	40.0	14400.0	3145.9k	1681.2k	87.1	14400.0
dem_1000_gaslib40_82	5546.6k	2402.5k	130.9	14400.0	5532.8k	3316.0k	66.8	14400.0	1e+20	2990.1k	–	14400.0
dem_1000_gaslib40_83	4697.5k	2112.5k	122.4	14400.0	1e+20	2600.6k	–	14400.0	1e+20	2537.8k	–	14400.0
dem_1000_gaslib40_84	1e+20	2714.7k	–	14400.0	4159.4k	3616.0k	15.0	14400.0	1e+20	2674.1k	–	14400.0
dem_1000_gaslib40_85	1e+20	2154.2k	–	14400.0	4309.3k	2912.2k	48.0	14400.0	1e+20	2324.9k	–	14400.0
dem_1000_gaslib40_86	1e+20	2131.5k	–	14400.0	1e+20	2858.5k	–	14400.0	4800.8k	2569.2k	86.9	14400.0
dem_1000_gaslib40_87	1e+20	2935.9k	–	14400.0	1e+20	1e+20	0.0	1119.7	1e+20	2861.9k	–	14400.0
dem_1000_gaslib40_88	3754.0k	1651.5k	127.3	14400.0	3860.2k	2195.3k	75.8	14400.0	4387.7k	1886.6k	132.6	14400.0
dem_1000_gaslib40_89	3507.6k	2167.7k	61.8	14400.0	3222.4k	2714.6k	18.7	14400.0	3840.5k	2104.2k	82.5	14400.0
dem_1000_gaslib40_90	1e+20	2036.6k	–	14400.0	3771.4k	2692.8k	40.1	14400.0	3774.8k	2106.4k	79.2	14400.0
dem_1000_gaslib40_91	1e+20	1740.7k	–	14400.0	4076.7k	2411.2k	69.1	14400.0	1e+20	2006.0k	–	14400.0
dem_1000_gaslib40_92	1e+20	3195.1k	–	14400.0	1e+20	1e+20	0.0	1026.2	1e+20	2685.2k	–	14400.0
dem_1000_gaslib40_93	1e+20	3012.7k	–	14400.0	1e+20	3377.4k	–	14400.0	1e+20	3720.1k	–	14400.0
dem_1000_gaslib40_94	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	125.0	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_95	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	185.7	1e+20	1e+20	0.0	9.0
dem_1000_gaslib40_96	1e+20	3807.4k	–	14400.0	5864.6k	5864.6k	0.0	8070.0	1e+20	3977.7k	–	14400.0
dem_1000_gaslib40_97	1e+20	3172.4k	–	14400.0	1e+20	3845.3k	–	14400.0	1e+20	3612.4k	–	14400.0
dem_1000_gaslib40_98	1e+20	2605.3k	–	14400.0	5509.2k	3350.5k	64.4	14400.0	1e+20	3242.4k	–	14400.0
dem_1000_gaslib40_99	4801.1k	2409.0k	99.3	14400.0	4600.2k	3358.4k	37.0	14400.0	1e+20	2463.3k	–	14400.0
dem_1000_gaslib40_100	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	138.4	1e+20	1e+20	0.0	0.7
dem_1000_gaslib40_101	4699.5k	2197.5k	113.9	14400.0	4824.2k	2855.2k	69.0	14400.0	1e+20	2326.4k	–	14400.0
dem_1000_gaslib40_102	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	156.9	1e+20	1e+20	0.0	4.6
dem_1000_gaslib40_103	4302.7k	1839.6k	133.9	14400.0	1e+20	2543.8k	–	14400.0	4795.0k	2082.5k	130.3	14400.0
dem_1000_gaslib40_104	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	156.4	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_105	1e+20	2579.1k	–	14400.0	1e+20	3316.3k	–	14400.0	1e+20	3266.1k	–	14400.0
dem_1000_gaslib40_106	5015.8k	1908.1k	162.9	14400.0	4465.1k	2500.0k	78.6	14400.0	4441.0k	2060.2k	115.6	14400.0
dem_1000_gaslib40_107	5316.6k	2062.5k	157.8	14400.0	4418.0k	3109.0k	42.1	14400.0	1e+20	2334.9k	–	14400.0
dem_1000_gaslib40_108	1e+20	3172.6k	–	14400.0	4938.1k	3935.4k	25.5	14400.0	1e+20	2848.2k	–	14400.0
dem_1000_gaslib40_109	1e+20	3715.0k	–	14400.0	1e+20	3752.9k	–	14400.0	1e+20	3961.2k	–	14400.0
dem_1000_gaslib40_110	1e+20	3636.5k	–	14400.0	5474.9k	4663.9k	17.4	14400.0	1e+20	3886.0k	–	14400.0
dem_1000_gaslib40_111	1e+20	2751.9k	–	14400.0	1e+20	3509.8k	–	14400.0	1e+20	3702.0k	–	14400.0
dem_1000_gaslib40_112	4013.7k	1825.0k	119.9	14400.0	3703.0k	2595.7k	42.7	14400.0	4048.0k	1977.0k	104.8	14400.0
dem_1000_gaslib40_113	1e+20	2118.6k	–	14400.0	1e+20	2713.5k	–	14400.0	1e+20	2257.9k	–	14400.0
dem_1000_gaslib40_114	3890.3k	1693.8k	129.7	14400.0	3531.6k	2360.5k	49.6	14400.0	3741.1k	1868.0k	100.3	14400.0
dem_1000_gaslib40_115	3955.2k	2345.8k	68.6	14400.0	3749.8k	3283.7k	14.2	14400.0	1e+20	2222.6k	–	14400.0
dem_1000_gaslib40_116	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_117	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	99.9	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_118	1e+20	2099.3k	–	14400.0	4558.3k	2667.3k	70.9	14400.0	4647.1k	2538.0k	83.1	14400.0
dem_1000_gaslib40_119	1e+20	4557.3k	–	14400.0	1e+20	1e+20	0.0	2046.1	1e+20	4396.3k	–	14400.0
dem_1000_gaslib40_120	4386.2k	2150.5k	104.0	14400.0	4000.1k	2845.4k	40.6	14400.0	4339.5k	2197.2k	97.5	14400.0

continued on next page

Table A.15: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_121	1e+20	1907.4k	–	14400.0	1e+20	2414.5k	–	14400.0	4435.2k	2366.6k	87.4	14400.0
dem_1000_gaslib40_122	4328.0k	1966.1k	120.1	14400.0	4098.4k	2786.6k	47.1	14400.0	4148.0k	2037.8k	103.6	14400.0
dem_1000_gaslib40_123	1e+20	2014.8k	–	14400.0	4728.1k	2454.6k	92.6	14400.0	1e+20	2171.3k	–	14400.0
dem_1000_gaslib40_124	1e+20	2286.4k	–	14400.0	4691.3k	3569.8k	31.4	14400.0	1e+20	2566.9k	–	14400.0
dem_1000_gaslib40_125	1e+20	2345.7k	–	14400.0	4956.6k	3223.3k	53.8	14400.0	1e+20	2457.5k	–	14400.0
dem_1000_gaslib40_126	4166.6k	2694.4k	54.6	14400.0	3997.4k	2983.9k	34.0	14400.0	4030.7k	2885.4k	39.7	14400.0
dem_1000_gaslib40_127	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	135.0	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_128	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	110.3	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_129	5273.1k	2135.0k	147.0	14400.0	1e+20	2678.8k	–	14400.0	1e+20	2441.3k	–	14400.0
dem_1000_gaslib40_130	1e+20	2566.5k	–	14400.0	1e+20	1e+20	0.0	847.2	1e+20	2211.1k	–	14400.0
dem_1000_gaslib40_131	4406.2k	2916.8k	51.1	14400.0	4366.3k	3811.1k	14.6	14400.0	1e+20	2841.7k	–	14400.0
dem_1000_gaslib40_132	1e+20	2306.7k	–	14400.0	1e+20	2850.5k	–	14400.0	4665.4k	2936.4k	58.9	14400.0
dem_1000_gaslib40_133	4670.5k	1862.7k	150.7	14400.0	4190.3k	2461.7k	70.2	14400.0	1e+20	2068.6k	–	14400.0
dem_1000_gaslib40_134	1e+20	3108.5k	–	14400.0	1e+20	3516.8k	–	14400.0	1e+20	3614.1k	–	14400.0
dem_1000_gaslib40_135	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	125.2	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_136	1e+20	2974.7k	–	14400.0	5221.4k	3808.9k	37.1	14400.0	1e+20	3533.4k	–	14400.0
dem_1000_gaslib40_137	1e+20	2547.1k	–	14400.0	5531.3k	3265.4k	69.4	14400.0	1e+20	3214.0k	–	14400.0
dem_1000_gaslib40_138	1e+20	2747.4k	–	14400.0	1e+20	3557.2k	–	14400.0	1e+20	3168.1k	–	14400.0
dem_1000_gaslib40_139	4609.8k	1976.0k	133.3	14400.0	1e+20	2741.8k	–	14400.0	4514.2k	2312.5k	95.2	14400.0
dem_1000_gaslib40_140	3828.1k	2362.5k	62.0	14400.0	4213.3k	2969.6k	41.9	14400.0	4316.5k	2613.0k	65.2	14400.0
dem_1000_gaslib40_141	3218.7k	2021.1k	59.3	14400.0	3261.9k	2477.0k	31.7	14400.0	3628.1k	1860.3k	95.0	14400.0
dem_1000_gaslib40_142	1e+20	1739.5k	–	14400.0	4033.5k	2632.8k	53.2	14400.0	1e+20	1982.5k	–	14400.0
dem_1000_gaslib40_143	1e+20	2369.1k	–	14400.0	1e+20	3075.8k	–	14400.0	1e+20	3152.5k	–	14400.0
dem_1000_gaslib40_144	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	138.6	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_145	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	296.3	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_146	1e+20	2402.5k	–	14400.0	4777.1k	3355.4k	42.4	14400.0	1e+20	2393.8k	–	14400.0
dem_1000_gaslib40_147	1e+20	2124.2k	–	14400.0	4594.4k	2733.6k	68.1	14400.0	4656.5k	2672.5k	74.2	14400.0
dem_1000_gaslib40_148	3950.7k	1920.0k	105.8	14400.0	4029.3k	2518.5k	60.0	14400.0	3752.7k	1964.9k	91.0	14400.0
dem_1000_gaslib40_149	1e+20	2811.6k	–	14400.0	1e+20	1e+20	0.0	3401.3	1e+20	2745.9k	–	14400.0
dem_1000_gaslib40_150	1e+20	3115.4k	–	14400.0	5345.1k	3715.5k	43.9	14400.0	1e+20	3527.9k	–	14400.0
dem_1000_gaslib40_151	1e+20	2556.3k	–	14400.0	5791.5k	3988.2k	45.2	14400.0	6486.7k	2895.8k	124.0	14400.0
dem_1000_gaslib40_152	4354.0k	2783.8k	56.4	14400.0	4629.2k	3519.4k	31.5	14400.0	1e+20	2608.7k	–	14400.0
dem_1000_gaslib40_153	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	200.2	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_154	3785.2k	1889.3k	100.3	14400.0	3769.0k	2399.2k	57.1	14400.0	3894.3k	1868.9k	108.4	14400.0
dem_1000_gaslib40_155	1e+20	2568.1k	–	14400.0	1e+20	3529.9k	–	14400.0	1e+20	3387.3k	–	14400.0
dem_1000_gaslib40_156	1e+20	4790.9k	–	14400.0	1e+20	1e+20	0.0	2837.8	1e+20	4323.7k	–	14400.0
dem_1000_gaslib40_157	4523.6k	2551.8k	77.3	14400.0	4026.0k	3355.2k	20.0	14400.0	1e+20	2253.9k	–	14400.0
dem_1000_gaslib40_158	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_159	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	272.3	1e+20	1e+20	0.0	16.3
dem_1000_gaslib40_160	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	161.3	1e+20	1e+20	0.0	9.4
dem_1000_gaslib40_161	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	240.5	1e+20	1e+20	0.0	0.5
dem_1000_gaslib40_162	3536.7k	1931.6k	83.1	14400.0	3619.7k	2527.9k	43.2	14400.0	3609.1k	1842.7k	95.9	14400.0
dem_1000_gaslib40_163	4256.8k	2648.8k	60.7	14400.0	4465.2k	3839.6k	16.3	14400.0	1e+20	2148.3k	–	14400.0
dem_1000_gaslib40_164	1e+20	2217.9k	–	14400.0	4906.2k	2813.8k	74.4	14400.0	1e+20	2562.4k	–	14400.0
dem_1000_gaslib40_165	4249.6k	2077.3k	104.6	14400.0	4223.6k	3036.8k	39.1	14400.0	1e+20	2219.3k	–	14400.0
dem_1000_gaslib40_166	1e+20	2831.1k	–	14400.0	1e+20	3362.0k	–	14400.0	1e+20	3449.5k	–	14400.0
dem_1000_gaslib40_167	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	167.9	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_168	4497.3k	2997.8k	50.0	14400.0	4467.9k	3696.0k	20.9	14400.0	1e+20	2690.7k	–	14400.0
dem_1000_gaslib40_169	1e+20	2255.6k	–	14400.0	4156.3k	3417.7k	21.6	14400.0	1e+20	2059.8k	–	14400.0
dem_1000_gaslib40_170	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_171	4740.1k	1875.3k	152.8	14400.0	4210.0k	2430.0k	73.3	14400.0	4046.7k	2085.7k	94.0	14400.0
dem_1000_gaslib40_172	1e+20	2431.1k	–	14400.0	1e+20	2920.3k	–	14400.0	1e+20	2523.6k	–	14400.0
dem_1000_gaslib40_173	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	160.3	1e+20	1e+20	0.0	0.4
dem_1000_gaslib40_174	3885.6k	1781.0k	118.2	14400.0	4121.7k	2533.4k	62.7	14400.0	4371.3k	1951.3k	124.0	14400.0
dem_1000_gaslib40_175	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	121.8	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_176	1e+20	2105.7k	–	14400.0	4602.6k	2803.9k	64.2	14400.0	1e+20	2332.4k	–	14400.0
dem_1000_gaslib40_177	1e+20	2385.8k	–	14400.0	5147.1k	3256.6k	58.1	14400.0	1e+20	2672.1k	–	14400.0
dem_1000_gaslib40_178	4129.8k	2532.8k	63.1	14400.0	4158.2k	3256.4k	27.7	14400.0	4237.5k	2715.3k	56.1	14400.0
dem_1000_gaslib40_179	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	310.9	1e+20	3560.9k	–	9.1
dem_1000_gaslib40_180	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	226.2	1e+20	1e+20	0.0	15.6
dem_1000_gaslib40_181	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	187.6	1e+20	1e+20	0.0	10.7
dem_1000_gaslib40_182	1e+20	2221.9k	–	14400.0	1e+20	3018.3k	–	14400.0	1e+20	2592.3k	–	14400.0
dem_1000_gaslib40_183	1e+20	3201.6k	–	14400.0	5757.0k	3777.0k	52.4	14400.0	1e+20	3340.8k	–	14400.0
dem_1000_gaslib40_184	1e+20	3772.5k	–	14400.0	1e+20	4038.3k	–	14400.0	1e+20	4067.3k	–	14400.0
dem_1000_gaslib40_185	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	122.4	1e+20	1e+20	0.0	0.4
dem_1000_gaslib40_186	4023.9k	2371.8k	69.7	14400.0	4204.9k	2914.5k	44.3	14400.0	1e+20	2643.4k	–	14400.0
dem_1000_gaslib40_187	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	224.0	1e+20	1e+20	0.0	5.2
dem_1000_gaslib40_188	1e+20	1e+20	0.0	1.0	1e+20	1e+20	0.0	467.8	1e+20	1e+20	0.0	18.1
dem_1000_gaslib40_189	1e+20	1e+20	0.0	1.0	1e+20	1e+20	0.0	353.6	1e+20	1e+20	0.0	30.5
dem_1000_gaslib40_190	1e+20	2395.4k	–	14400.0	5164.1k	3185.3k	62.1	14400.0	1e+20	3145.6k	–	14400.0
dem_1000_gaslib40_191	1e+20	2497.3k	–	14400.0	4350.0k	3386.7k	28.4	14400.0	1e+20	2729.5k	–	14400.0
dem_1000_gaslib40_192	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	127.0	1e+20	1e+20	0.0	0.4
dem_1000_gaslib40_193	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_194	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	264.8	1e+20	1e+20	0.0	0.6
dem_1000_gaslib40_195	1e+20	2883.0k	–	14400.0	1e+20	3402.0k	–	14400.0	1e+20	3406.7k	–	14400.0
dem_1000_gaslib40_196	2890.9k	1681.9k	71.9	14400.0	3192.7k	1895.8k	68.4	14400.0	3390.8k	1718.8k	97.3	14400.0
dem_1000_gaslib40_197	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	194.2	1e+20	1e+20	0.0	8.7
dem_1000_gaslib40_198	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	0.2

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Table A.15: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_199	1e+20	1732.4k	–	14400.0	3984.0k	2538.7k	56.9	14400.0	4284.5k	1841.9k	132.6	14400.0
dem_1000_gaslib40_200	1e+20	2648.1k	–	14400.0	4623.1k	3433.3k	34.7	14400.0	1e+20	2522.2k	–	14400.0
dem_1000_gaslib40_201	1e+20	2165.1k	–	14400.0	1e+20	2465.9k	–	14400.0	1e+20	2392.2k	–	14400.0
dem_1000_gaslib40_202	1e+20	2835.4k	–	14400.0	1e+20	3022.9k	–	14400.0	1e+20	3487.6k	–	14400.0
dem_1000_gaslib40_203	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	335.1	1e+20	1e+20	0.0	71.0
dem_1000_gaslib40_204	3859.7k	1631.7k	136.5	14400.0	3918.8k	2148.7k	82.4	14400.0	3684.0k	1890.8k	94.8	14400.0
dem_1000_gaslib40_205	3757.1k	2372.2k	58.4	14400.0	3855.6k	3160.4k	22.0	14400.0	3838.6k	2134.2k	79.9	14400.0
dem_1000_gaslib40_206	1e+20	3124.5k	–	14400.0	5285.0k	4371.7k	20.9	14400.0	1e+20	2985.7k	–	14400.0
dem_1000_gaslib40_207	1e+20	2293.7k	–	14400.0	4301.6k	3359.4k	28.0	14400.0	1e+20	2539.7k	–	14400.0
dem_1000_gaslib40_208	3068.2k	1819.0k	68.7	14400.0	2996.5k	2412.0k	24.2	14400.0	3099.1k	1759.6k	76.1	14400.0
dem_1000_gaslib40_209	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	123.6	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_210	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	132.5	1e+20	1e+20	0.0	6.4
dem_1000_gaslib40_211	1e+20	2337.6k	–	14400.0	5334.7k	3175.0k	68.0	14400.0	1e+20	2952.2k	–	14400.0
dem_1000_gaslib40_212	1e+20	2119.7k	–	14400.0	4945.9k	2579.1k	91.8	14400.0	1e+20	2414.4k	–	14400.0
dem_1000_gaslib40_213	4827.2k	2155.8k	123.9	14400.0	3889.1k	3003.8k	29.5	14400.0	1e+20	2131.0k	–	14400.0
dem_1000_gaslib40_214	4084.0k	2503.9k	63.1	14400.0	4272.6k	2905.5k	47.1	14400.0	1e+20	2692.6k	–	14400.0
dem_1000_gaslib40_215	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	118.3	1e+20	1e+20	0.0	0.4
dem_1000_gaslib40_216	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	139.6	1e+20	1e+20	0.0	0.5
dem_1000_gaslib40_217	1e+20	2190.6k	–	14400.0	4760.7k	3167.2k	50.3	14400.0	1e+20	2622.9k	–	14400.0
dem_1000_gaslib40_218	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	276.2	1e+20	1e+20	0.0	8.2
dem_1000_gaslib40_219	5474.8k	2228.8k	145.6	14400.0	5002.5k	2981.0k	67.8	14400.0	1e+20	2743.3k	–	14400.0
dem_1000_gaslib40_220	5294.9k	2294.7k	130.7	14400.0	4904.7k	3234.1k	51.7	14400.0	1e+20	2447.6k	–	14400.0
dem_1000_gaslib40_221	4276.9k	2654.1k	61.1	14400.0	4171.8k	2887.4k	44.5	14400.0	4213.8k	2847.3k	48.0	14400.0
dem_1000_gaslib40_222	1e+20	2469.8k	–	14400.0	5528.9k	3497.3k	58.1	14400.0	1e+20	2830.6k	–	14400.0
dem_1000_gaslib40_223	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	436.2	1e+20	1e+20	0.0	18.2
dem_1000_gaslib40_224	1e+20	1932.0k	–	14400.0	4703.1k	2649.6k	77.5	14400.0	1e+20	2093.7k	–	14400.0
dem_1000_gaslib40_225	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	160.0	1e+20	1e+20	0.0	0.5
dem_1000_gaslib40_226	1e+20	2264.6k	–	14400.0	4724.5k	3311.6k	42.7	14400.0	1e+20	2422.9k	–	14400.0
dem_1000_gaslib40_227	1e+20	2291.1k	–	14400.0	4898.2k	2832.2k	72.9	14400.0	1e+20	2724.3k	–	14400.0
dem_1000_gaslib40_228	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_229	3910.8k	1925.8k	103.1	14400.0	4251.8k	2619.7k	62.3	14400.0	4359.3k	2051.9k	112.4	14400.0
dem_1000_gaslib40_230	1e+20	3002.9k	–	14400.0	1e+20	3843.1k	–	14400.0	1e+20	3539.0k	–	14400.0
dem_1000_gaslib40_231	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	524.2	1e+20	1e+20	0.0	25.2
dem_1000_gaslib40_232	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	259.4	1e+20	1e+20	0.0	18.4
dem_1000_gaslib40_233	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	164.2	1e+20	1e+20	0.0	7.7
dem_1000_gaslib40_234	1e+20	3387.5k	–	14400.0	5493.2k	4829.5k	13.7	14400.0	1e+20	3924.5k	–	14400.0
dem_1000_gaslib40_235	1e+20	2848.4k	–	14400.0	1e+20	1e+20	0.0	1643.2	1e+20	2416.4k	–	14400.0
dem_1000_gaslib40_236	1e+20	3224.3k	–	14400.0	1e+20	3482.2k	–	14400.0	1e+20	3419.3k	–	14400.0
dem_1000_gaslib40_237	3556.8k	2094.7k	69.8	14400.0	3671.2k	2626.7k	39.8	14400.0	3379.9k	2038.8k	65.8	14400.0
dem_1000_gaslib40_238	1e+20	2945.6k	–	14400.0	1e+20	3850.6k	–	14400.0	1e+20	3699.8k	–	14400.0
dem_1000_gaslib40_239	3739.3k	1968.9k	89.9	14400.0	3679.9k	2491.6k	47.7	14400.0	3697.9k	1923.7k	92.2	14400.0
dem_1000_gaslib40_240	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	158.1	1e+20	1e+20	0.0	0.4
dem_1000_gaslib40_241	4493.3k	1999.7k	124.7	14400.0	4660.8k	2545.0k	83.1	14400.0	1e+20	2326.3k	–	14400.0
dem_1000_gaslib40_242	1e+20	2628.6k	–	14400.0	4468.9k	4468.9k	0.0	9629.3	1e+20	2912.7k	–	14400.0
dem_1000_gaslib40_243	1e+20	3232.8k	–	14400.0	5380.8k	3830.2k	40.5	14400.0	1e+20	2895.4k	–	14400.0
dem_1000_gaslib40_244	3763.7k	2238.6k	68.1	14400.0	3936.2k	2947.1k	33.6	14400.0	3791.6k	2454.0k	54.5	14400.0
dem_1000_gaslib40_245	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	44.2	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_246	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	246.3	1e+20	1e+20	0.0	0.6
dem_1000_gaslib40_247	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_248	1e+20	3125.3k	–	14400.0	1e+20	3789.3k	–	14400.0	1e+20	3670.8k	–	14400.0
dem_1000_gaslib40_249	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	274.1	1e+20	3004.9k	–	14400.0
dem_1000_gaslib40_250	3752.8k	2140.5k	75.3	14400.0	3765.7k	2861.4k	31.6	14400.0	3904.3k	2093.0k	86.5	14400.0
dem_1000_gaslib40_251	4041.8k	2126.9k	90.0	14400.0	3662.3k	3215.3k	13.9	14400.0	3836.4k	2083.7k	84.1	14400.0
dem_1000_gaslib40_252	5146.9k	2324.9k	121.4	14400.0	5271.9k	3368.5k	56.5	14400.0	1e+20	2475.5k	–	14400.0
dem_1000_gaslib40_253	3687.1k	1917.0k	92.3	14400.0	3530.5k	2466.6k	43.1	14400.0	3835.2k	1961.2k	95.6	14400.0
dem_1000_gaslib40_254	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	194.5	1e+20	1e+20	0.0	5367.3
dem_1000_gaslib40_255	1e+20	1e+20	0.0	4.9	1e+20	1e+20	0.0	1836.2	1e+20	1e+20	0.0	2675.0
dem_1000_gaslib40_256	1e+20	1957.0k	–	14400.0	1e+20	2816.2k	–	14400.0	1e+20	2290.8k	–	14400.0
dem_1000_gaslib40_257	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	132.4	1e+20	1e+20	0.0	5.9
dem_1000_gaslib40_258	1e+20	2929.6k	–	14400.0	5206.7k	3670.4k	41.9	14400.0	1e+20	3535.2k	–	14400.0
dem_1000_gaslib40_259	3694.1k	1652.0k	123.6	14400.0	3552.9k	2070.5k	71.6	14400.0	3345.9k	1802.8k	85.6	14400.0
dem_1000_gaslib40_260	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	335.5	1e+20	1e+20	0.0	12.6
dem_1000_gaslib40_261	1e+20	2458.1k	–	14400.0	4143.8k	3759.8k	10.2	14400.0	1e+20	2709.2k	–	14400.0
dem_1000_gaslib40_262	1e+20	3291.2k	–	14400.0	1e+20	3996.5k	–	14400.0	1e+20	3804.2k	–	14400.0
dem_1000_gaslib40_263	1e+20	2093.2k	–	14400.0	4003.6k	3144.4k	27.3	14400.0	1e+20	2477.3k	–	14400.0
dem_1000_gaslib40_264	4523.7k	1948.3k	132.2	14400.0	1e+20	2546.0k	–	14400.0	1e+20	2320.5k	–	14400.0
dem_1000_gaslib40_265	5307.3k	2000.1k	165.3	14400.0	4407.7k	2452.6k	79.7	14400.0	4361.3k	2252.7k	93.6	14400.0
dem_1000_gaslib40_266	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	98.8	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_267	1e+20	3867.5k	–	14400.0	1e+20	1e+20	0.0	2016.0	1e+20	3616.8k	–	14400.0
dem_1000_gaslib40_268	3413.7k	2228.8k	53.2	14400.0	3166.9k	2728.3k	16.1	14400.0	3272.5k	1979.8k	65.3	14400.0
dem_1000_gaslib40_269	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_270	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	206.3	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_271	1e+20	3382.8k	–	14400.0	1e+20	3522.0k	–	14400.0	1e+20	3738.3k	–	14400.0
dem_1000_gaslib40_272	5062.5k	2189.7k	131.2	14400.0	1e+20	2749.6k	–	14400.0	1e+20	2445.3k	–	14400.0
dem_1000_gaslib40_273	1e+20	2472.5k	–	14400.0	5125.9k	3377.2k	51.8	14400.0	1e+20	2726.6k	–	14400.0
dem_1000_gaslib40_274	3764.5k	1913.2k	96.8	14400.0	3791.6k	2494.9k	52.0	14400.0	4207.0k	2027.5k	107.5	14400.0
dem_1000_gaslib40_275	1e+20	2007.5k	–	14400.0	4914.4k	2655.5k	85.1	14400.0	1e+20	2297.1k	–	14400.0
dem_1000_gaslib40_276	1e+20	3067.6k	–	14400.0	1e+20	3751.5k	–	14400.0	1e+20	3689.6k	–	14400.0

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Table A.15: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_277	3757.6k	2356.6k	59.5	14400.0	3784.7k	2797.2k	35.3	14400.0	3556.2k	2257.9k	57.5	14400.0
dem_1000_gaslib40_278	3850.9k	2273.2k	69.4	14400.0	3857.7k	2747.0k	40.4	14400.0	1e+20	2427.5k	–	14400.0
dem_1000_gaslib40_279	3778.7k	1941.5k	94.6	14400.0	3438.5k	2662.9k	29.1	14400.0	3752.1k	2012.5k	86.4	14400.0
dem_1000_gaslib40_280	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	13.8	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_281	1e+20	2556.7k	–	14400.0	4291.1k	3427.9k	25.2	14400.0	1e+20	2480.6k	–	14400.0
dem_1000_gaslib40_282	1e+20	2399.7k	–	14400.0	4224.5k	3382.7k	24.9	14400.0	1e+20	2307.0k	–	14400.0
dem_1000_gaslib40_283	4886.1k	2041.1k	139.4	14400.0	1e+20	2492.8k	–	14400.0	1e+20	2265.2k	–	14400.0
dem_1000_gaslib40_284	1e+20	2449.1k	–	14400.0	1e+20	2797.2k	–	14400.0	1e+20	3084.3k	–	14400.0
dem_1000_gaslib40_285	1e+20	1e+20	0.0	6.9	1e+20	1e+20	0.0	629.8	1e+20	3147.6k	–	14400.0
dem_1000_gaslib40_286	1e+20	2439.2k	–	14400.0	1e+20	2846.7k	–	14400.0	1e+20	2942.3k	–	14400.0
dem_1000_gaslib40_287	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_288	4175.6k	1946.9k	114.5	14400.0	4696.7k	2502.3k	87.7	14400.0	5162.6k	2295.1k	124.9	14400.0
dem_1000_gaslib40_289	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	106.1	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_290	5343.0k	2082.8k	156.5	14400.0	4722.1k	3236.0k	45.9	14400.0	1e+20	2459.1k	–	14400.0
dem_1000_gaslib40_291	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	161.3	1e+20	1e+20	0.0	4.8
dem_1000_gaslib40_292	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_293	1e+20	2794.3k	–	14400.0	1e+20	3324.0k	–	14400.0	1e+20	3582.0k	–	14400.0
dem_1000_gaslib40_294	4505.2k	2627.9k	71.4	14400.0	4107.4k	3459.5k	18.7	14400.0	4316.8k	2548.8k	69.4	14400.0
dem_1000_gaslib40_295	1e+20	2266.1k	–	14400.0	1e+20	2811.4k	–	14400.0	1e+20	2515.0k	–	14400.0
dem_1000_gaslib40_296	1e+20	3290.2k	–	14400.0	1e+20	3275.6k	–	14400.0	1e+20	3689.5k	–	14400.0
dem_1000_gaslib40_297	1e+20	2440.8k	–	14400.0	4349.1k	3941.6k	10.3	14400.0	1e+20	2595.9k	–	14400.0
dem_1000_gaslib40_298	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	226.9	1e+20	1e+20	0.0	0.6
dem_1000_gaslib40_299	1e+20	2515.4k	–	14400.0	1e+20	3181.9k	–	14400.0	1e+20	3149.7k	–	14400.0
dem_1000_gaslib40_300	1e+20	3582.8k	–	14400.0	1e+20	3781.5k	–	14400.0	1e+20	3730.5k	–	14400.0
dem_1000_gaslib40_301	4219.1k	2842.2k	48.4	14400.0	4168.3k	3503.2k	19.0	14400.0	1e+20	2527.4k	–	14400.0
dem_1000_gaslib40_302	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	14.1	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_303	1e+20	3480.1k	–	14400.0	1e+20	4037.9k	–	14400.0	1e+20	3858.7k	–	14400.0
dem_1000_gaslib40_304	1e+20	2273.7k	–	14400.0	1e+20	3051.7k	–	14400.0	4876.5k	2891.5k	68.7	14400.0
dem_1000_gaslib40_305	1e+20	2119.2k	–	14400.0	5439.1k	2834.7k	91.9	14400.0	1e+20	2416.3k	–	14400.0
dem_1000_gaslib40_306	1e+20	3481.1k	–	14400.0	1e+20	1e+20	0.0	514.0	1e+20	2994.5k	–	14400.0
dem_1000_gaslib40_307	3525.0k	2064.2k	70.8	14400.0	3741.6k	2755.5k	35.8	14400.0	3765.5k	2109.4k	78.5	14400.0
dem_1000_gaslib40_308	4740.0k	2114.8k	124.1	14400.0	1e+20	2692.4k	–	14400.0	1e+20	2404.9k	–	14400.0
dem_1000_gaslib40_309	4253.9k	2604.0k	63.4	14400.0	4080.1k	3430.2k	18.9	14400.0	1e+20	2743.3k	–	14400.0
dem_1000_gaslib40_310	3991.9k	2602.8k	53.4	14400.0	4290.9k	2917.1k	47.1	14400.0	4143.1k	2550.4k	62.4	14400.0
dem_1000_gaslib40_311	1e+20	2447.8k	–	14400.0	5313.5k	3217.2k	65.2	14400.0	1e+20	2961.3k	–	14400.0
dem_1000_gaslib40_312	1e+20	2769.1k	–	14400.0	5790.8k	3747.5k	54.5	14400.0	1e+20	3400.8k	–	14400.0
dem_1000_gaslib40_313	4962.0k	2188.6k	126.7	14400.0	4792.3k	2872.6k	66.8	14400.0	1e+20	2806.8k	–	14400.0
dem_1000_gaslib40_314	1e+20	2395.2k	–	14400.0	5424.0k	3243.9k	67.2	14400.0	1e+20	3066.2k	–	14400.0
dem_1000_gaslib40_315	1e+20	2873.6k	–	14400.0	4742.1k	4445.8k	6.7	14400.0	1e+20	2697.4k	–	14400.0
dem_1000_gaslib40_316	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_317	1e+20	1999.4k	–	14400.0	1e+20	2543.3k	–	14400.0	1e+20	2255.8k	–	14400.0
dem_1000_gaslib40_318	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	155.8	1e+20	1e+20	0.0	7.0
dem_1000_gaslib40_319	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	391.3	1e+20	1e+20	0.0	22.2
dem_1000_gaslib40_320	1e+20	3454.6k	–	14400.0	1e+20	4488.1k	–	14400.0	1e+20	3946.2k	–	14400.0
dem_1000_gaslib40_321	1e+20	2944.5k	–	14400.0	5709.6k	3553.1k	60.7	14400.0	1e+20	3506.2k	–	14400.0
dem_1000_gaslib40_322	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	151.6	1e+20	1e+20	0.0	1.0
dem_1000_gaslib40_323	4160.5k	1793.4k	132.0	14400.0	4705.7k	2275.3k	106.8	14400.0	4356.8k	2262.2k	92.6	14400.0
dem_1000_gaslib40_324	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_325	1e+20	2106.3k	–	14400.0	4910.0k	2917.7k	68.3	14400.0	1e+20	2613.4k	–	14400.0
dem_1000_gaslib40_326	3983.7k	1993.4k	99.8	14400.0	3560.2k	2552.4k	39.5	14400.0	3647.9k	2050.5k	77.9	14400.0
dem_1000_gaslib40_327	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	131.8	1e+20	1e+20	0.0	11.5
dem_1000_gaslib40_328	1e+20	2408.6k	–	14400.0	4194.8k	3470.7k	20.9	14400.0	1e+20	2301.5k	–	14400.0
dem_1000_gaslib40_329	4538.0k	2148.6k	111.2	14400.0	4936.7k	2700.8k	82.8	14400.0	1e+20	2368.2k	–	14400.0
dem_1000_gaslib40_330	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	9.3	1e+20	1e+20	0.0	0.4
dem_1000_gaslib40_331	1e+20	1795.9k	–	14400.0	4655.4k	2851.2k	63.3	14400.0	1e+20	2270.2k	–	14400.0
dem_1000_gaslib40_332	3401.2k	1978.5k	71.9	14400.0	3153.4k	2604.7k	21.1	14400.0	3943.5k	1950.0k	102.2	14400.0
dem_1000_gaslib40_333	1e+20	2516.7k	–	14400.0	5527.1k	3288.4k	68.1	14400.0	1e+20	2982.2k	–	14400.0
dem_1000_gaslib40_334	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	82.4	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_335	3439.6k	1860.3k	84.9	14400.0	3621.9k	2313.9k	56.5	14400.0	3564.9k	1881.3k	89.5	14400.0
dem_1000_gaslib40_336	1e+20	2243.9k	–	14400.0	4911.8k	3306.7k	48.5	14400.0	1e+20	2474.8k	–	14400.0
dem_1000_gaslib40_337	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_338	1e+20	2807.3k	–	14400.0	4619.3k	3697.1k	24.9	14400.0	1e+20	2776.3k	–	14400.0
dem_1000_gaslib40_339	1e+20	2685.4k	–	14400.0	4285.3k	3825.1k	12.0	14400.0	1e+20	2628.1k	–	14400.0
dem_1000_gaslib40_340	1e+20	2959.1k	–	14400.0	5344.5k	3645.7k	46.6	14400.0	1e+20	3523.3k	–	14400.0
dem_1000_gaslib40_341	4737.7k	2195.8k	115.8	14400.0	4974.0k	2839.9k	75.1	14400.0	1e+20	2349.9k	–	14400.0
dem_1000_gaslib40_342	4309.0k	1942.9k	121.8	14400.0	4127.9k	2705.9k	52.5	14400.0	4797.4k	2084.2k	130.2	14400.0
dem_1000_gaslib40_343	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.4
dem_1000_gaslib40_344	1e+20	1e+20	0.0	10.5	1e+20	1e+20	0.0	492.7	1e+20	2942.6k	–	14400.0
dem_1000_gaslib40_345	3523.8k	1865.7k	88.9	14400.0	3744.0k	2276.6k	64.5	14400.0	4346.9k	2185.6k	98.9	14400.0
dem_1000_gaslib40_346	1e+20	1958.7k	–	14400.0	4797.4k	2583.7k	85.7	14400.0	4454.7k	2067.0k	115.5	14400.0
dem_1000_gaslib40_347	1e+20	2865.9k	–	14400.0	5076.9k	4214.7k	20.5	14400.0	5549.5k	2633.2k	110.8	14400.0
dem_1000_gaslib40_348	4341.6k	2048.2k	112.0	14400.0	4114.3k	2979.9k	38.1	14400.0	1e+20	2361.6k	–	14400.0
dem_1000_gaslib40_349	1e+20	2446.0k	–	14400.0	5203.6k	3112.4k	67.2	14400.0	1e+20	3223.4k	–	14400.0
dem_1000_gaslib40_350	4323.5k	2017.9k	114.3	14400.0	1e+20	2792.7k	–	14400.0	4349.0k	2471.7k	76.0	14400.0
dem_1000_gaslib40_351	3875.6k	2452.7k	58.0	14400.0	3990.3k	2686.6k	48.5	14400.0	3935.7k	2429.7k	62.0	14400.0
dem_1000_gaslib40_352	1e+20	4649.8k	–	14400.0	1e+20	1e+20	0.0	5115.4	1e+20	4276.2k	–	14400.0
dem_1000_gaslib40_353	1e+20	3391.1k	–	14400.0	1e+20	4326.0k	–	14400.0	1e+20	3878.0k	–	14400.0
dem_1000_gaslib40_354	5098.9k	2157.2k	136.4	14400.0	1e+20	2697.6k	–	14400.0	4536.9k	2717.9k	66.9	14400.0

continued on next page

Table A.15: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_355	3683.1k	1884.7k	95.4	14400.0	3518.5k	2376.7k	48.0	14400.0	3626.6k	1851.2k	95.9	14400.0
dem_1000_gaslib40_356	1e+20	2474.3k	–	14400.0	1e+20	2889.1k	–	14400.0	1e+20	2779.8k	–	14400.0
dem_1000_gaslib40_357	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	19.2	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_358	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	187.5	1e+20	1e+20	0.0	0.7
dem_1000_gaslib40_359	1e+20	2532.6k	–	14400.0	1e+20	3184.3k	–	14400.0	5236.9k	3084.8k	69.8	14400.0
dem_1000_gaslib40_360	1e+20	2600.6k	–	14400.0	5195.0k	3433.7k	51.3	14400.0	1e+20	3100.8k	–	14400.0
dem_1000_gaslib40_361	3840.2k	1807.9k	112.4	14400.0	3646.1k	2391.0k	52.5	14400.0	3863.7k	1926.1k	100.6	14400.0
dem_1000_gaslib40_362	4172.7k	2841.1k	46.9	14400.0	4331.6k	3416.9k	26.8	14400.0	4168.8k	2880.2k	44.7	14400.0
dem_1000_gaslib40_363	1e+20	2900.5k	–	14400.0	1e+20	3550.2k	–	14400.0	1e+20	3575.3k	–	14400.0
dem_1000_gaslib40_364	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_365	1e+20	2211.5k	–	14400.0	4694.9k	2923.0k	60.6	14400.0	1e+20	2330.6k	–	14400.0
dem_1000_gaslib40_366	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	143.8	1e+20	1e+20	0.0	0.4
dem_1000_gaslib40_367	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	221.6	1e+20	1e+20	0.0	0.7
dem_1000_gaslib40_368	1e+20	2140.1k	–	14400.0	1e+20	2872.3k	–	14400.0	4888.1k	2860.2k	70.9	14400.0
dem_1000_gaslib40_369	4111.0k	2539.5k	61.9	14400.0	3969.2k	3411.0k	16.4	14400.0	4259.2k	2695.8k	58.0	14400.0
dem_1000_gaslib40_370	4294.9k	1799.8k	138.6	14400.0	4081.6k	2676.4k	52.5	14400.0	1e+20	1974.8k	–	14400.0
dem_1000_gaslib40_371	1e+20	2875.3k	–	14400.0	1e+20	3403.3k	–	14400.0	1e+20	3302.5k	–	14400.0
dem_1000_gaslib40_372	5511.4k	2021.7k	172.6	14400.0	4875.9k	3096.2k	57.5	14400.0	1e+20	2390.3k	–	14400.0
dem_1000_gaslib40_373	5806.6k	2207.0k	163.1	14400.0	1e+20	2913.9k	–	14400.0	1e+20	2368.6k	–	14400.0
dem_1000_gaslib40_374	5124.9k	1880.5k	172.5	14400.0	1e+20	2673.7k	–	14400.0	1e+20	2232.0k	–	14400.0
dem_1000_gaslib40_375	1e+20	2375.3k	–	14400.0	1e+20	3218.8k	–	14400.0	1e+20	3093.4k	–	14400.0
dem_1000_gaslib40_376	1e+20	2622.2k	–	14400.0	4310.6k	3750.1k	14.9	14400.0	1e+20	2405.9k	–	14400.0
dem_1000_gaslib40_377	1e+20	1957.3k	–	14400.0	3940.2k	2802.8k	40.6	14400.0	4013.1k	2327.7k	72.4	14400.0
dem_1000_gaslib40_378	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_379	1e+20	2179.3k	–	14400.0	4818.2k	2927.2k	64.6	14400.0	1e+20	2698.4k	–	14400.0
dem_1000_gaslib40_380	1e+20	2306.9k	–	14400.0	1e+20	2975.7k	–	14400.0	1e+20	2398.2k	–	14400.0
dem_1000_gaslib40_381	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_382	3978.0k	1737.7k	128.9	14400.0	4111.0k	2567.8k	60.1	14400.0	4739.8k	2016.3k	135.1	14400.0
dem_1000_gaslib40_383	1e+20	2195.7k	–	14400.0	1e+20	2859.2k	–	14400.0	1e+20	2508.3k	–	14400.0
dem_1000_gaslib40_384	3837.1k	2552.2k	50.3	14400.0	3797.9k	3435.1k	10.6	14400.0	1e+20	2385.8k	–	14400.0
dem_1000_gaslib40_385	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	158.2	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_386	1e+20	3275.9k	–	14400.0	1e+20	3608.8k	–	14400.0	1e+20	3738.2k	–	14400.0
dem_1000_gaslib40_387	5508.3k	2366.1k	132.8	14400.0	1e+20	3093.2k	–	14400.0	1e+20	2731.1k	–	14400.0
dem_1000_gaslib40_388	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	163.2	1e+20	1e+20	0.0	6.4
dem_1000_gaslib40_389	1e+20	3457.5k	–	14400.0	5732.1k	4313.9k	32.9	14400.0	1e+20	3912.2k	–	14400.0
dem_1000_gaslib40_390	4124.8k	1969.2k	109.5	14400.0	4556.1k	2838.8k	60.5	14400.0	1e+20	2316.0k	–	14400.0
dem_1000_gaslib40_391	1e+20	2064.8k	–	14400.0	4921.2k	2487.7k	97.8	14400.0	5045.2k	2458.7k	105.2	14400.0
dem_1000_gaslib40_392	1e+20	2446.3k	–	14400.0	4137.4k	3197.4k	29.4	14400.0	1e+20	2339.7k	–	14400.0
dem_1000_gaslib40_393	1e+20	2646.0k	–	14400.0	1e+20	3043.0k	–	14400.0	5021.9k	2681.0k	87.3	14400.0
dem_1000_gaslib40_394	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	336.7	1e+20	1e+20	0.0	3106.1
dem_1000_gaslib40_395	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	108.4	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_396	1e+20	2374.6k	–	14400.0	4241.7k	3411.0k	24.4	14400.0	1e+20	2256.1k	–	14400.0
dem_1000_gaslib40_397	3897.3k	1818.5k	114.3	14400.0	3569.6k	2538.0k	40.6	14400.0	4739.0k	1844.1k	157.0	14400.0
dem_1000_gaslib40_398	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	6.5	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_399	1e+20	2587.1k	–	14400.0	4869.0k	3227.8k	50.8	14400.0	1e+20	2563.2k	–	14400.0
dem_1000_gaslib40_400	1e+20	2738.9k	–	14400.0	1e+20	3298.3k	–	14400.0	1e+20	3492.0k	–	14400.0
dem_1000_gaslib40_401	1e+20	2333.6k	–	14400.0	5236.9k	2949.3k	77.6	14400.0	1e+20	2748.1k	–	14400.0
dem_1000_gaslib40_402	1e+20	3298.9k	–	14400.0	5261.2k	4615.6k	14.0	14400.0	1e+20	2881.1k	–	14400.0
dem_1000_gaslib40_403	1e+20	2323.6k	–	14400.0	4717.1k	2816.5k	67.5	14400.0	1e+20	2407.7k	–	14400.0
dem_1000_gaslib40_404	1e+20	2245.8k	–	14400.0	1e+20	2607.5k	–	14400.0	1e+20	2307.0k	–	14400.0
dem_1000_gaslib40_405	4996.1k	2146.2k	132.8	14400.0	4467.6k	3112.0k	43.6	14400.0	1e+20	2544.4k	–	14400.0
dem_1000_gaslib40_406	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_407	5265.2k	2141.2k	145.9	14400.0	4883.2k	2860.8k	70.7	14400.0	1e+20	2257.3k	–	14400.0
dem_1000_gaslib40_408	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	142.6	1e+20	1e+20	0.0	1.2
dem_1000_gaslib40_409	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_410	1e+20	2987.0k	–	14400.0	1e+20	3496.9k	–	14400.0	1e+20	3525.0k	–	14400.0
dem_1000_gaslib40_411	1e+20	3062.2k	–	14400.0	1e+20	1e+20	0.0	2265.6	1e+20	2605.9k	–	14400.0
dem_1000_gaslib40_412	1e+20	2559.2k	–	14400.0	4339.0k	3066.4k	41.5	14400.0	1e+20	2341.0k	–	14400.0
dem_1000_gaslib40_413	1e+20	3169.1k	–	14400.0	1e+20	1e+20	0.0	2777.7	1e+20	3186.1k	–	14400.0
dem_1000_gaslib40_414	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_415	1e+20	3084.1k	–	14400.0	5566.6k	3442.2k	61.7	14400.0	1e+20	3527.7k	–	14400.0
dem_1000_gaslib40_416	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_417	3907.9k	2118.0k	84.5	14400.0	1e+20	2731.1k	–	14400.0	1e+20	2054.9k	–	14400.0
dem_1000_gaslib40_418	3458.7k	1836.8k	88.3	14400.0	3369.8k	2541.5k	32.6	14400.0	3529.9k	1928.8k	83.0	14400.0
dem_1000_gaslib40_419	1e+20	2612.5k	–	14400.0	5406.9k	3025.0k	78.7	14400.0	1e+20	2988.3k	–	14400.0
dem_1000_gaslib40_420	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	5.1	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_421	3625.3k	1792.1k	102.3	14400.0	3513.4k	2510.9k	39.9	14400.0	3452.0k	1900.9k	81.6	14400.0
dem_1000_gaslib40_422	3513.6k	1822.5k	92.8	14400.0	3503.1k	2359.2k	48.5	14400.0	3573.0k	1882.8k	89.8	14400.0
dem_1000_gaslib40_423	1e+20	2148.9k	–	14400.0	4058.2k	2722.9k	49.0	14400.0	4065.0k	2241.4k	81.4	14400.0
dem_1000_gaslib40_424	1e+20	2294.2k	–	14400.0	4954.2k	2981.2k	66.2	14400.0	5274.6k	2642.2k	99.6	14400.0
dem_1000_gaslib40_425	1e+20	2061.2k	–	14400.0	4200.1k	3007.5k	39.7	14400.0	1e+20	2172.1k	–	14400.0
dem_1000_gaslib40_426	3796.7k	2187.5k	73.6	14400.0	1e+20	2548.8k	–	14400.0	3612.9k	2299.7k	57.1	14400.0
dem_1000_gaslib40_427	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	274.5	1e+20	1e+20	0.0	0.6
dem_1000_gaslib40_428	4817.0k	2340.8k	105.8	14400.0	5256.1k	3134.2k	67.7	14400.0	1e+20	2617.7k	–	14400.0
dem_1000_gaslib40_429	4207.6k	2667.0k	57.8	14400.0	4141.8k	3516.3k	17.8	14400.0	4375.7k	2369.6k	84.7	14400.0
dem_1000_gaslib40_430	3756.3k	1824.5k	105.9	14400.0	3606.2k	2467.9k	46.1	14400.0	3792.8k	1888.7k	100.8	14400.0
dem_1000_gaslib40_431	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	6.3	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_432	1e+20	2048.5k	–	14400.0	3895.3k	2560.7k	52.1	14400.0	3943.9k	2097.3k	88.0	14400.0

continued on next page

Table A.15: Comparison of discrete models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_433	1e+20	2265.5k	–	14400.0	1e+20	3097.7k	–	14400.0	1e+20	3039.9k	–	14400.0
dem_1000_gaslib40_434	4185.4k	1866.1k	124.3	14400.0	1e+20	2364.3k	–	14400.0	4636.0k	2269.9k	104.2	14400.0
dem_1000_gaslib40_435	3628.2k	2138.5k	69.7	14400.0	3620.3k	3092.9k	17.1	14400.0	3904.0k	2137.9k	82.6	14400.0
dem_1000_gaslib40_436	4266.6k	2182.5k	95.5	14400.0	1e+20	2637.8k	–	14400.0	1e+20	2525.7k	–	14400.0
dem_1000_gaslib40_437	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	324.6	1e+20	1e+20	0.0	8.3
dem_1000_gaslib40_438	1e+20	2850.9k	–	14400.0	5309.3k	3425.5k	55.0	14400.0	6303.8k	2767.2k	127.8	14400.0
dem_1000_gaslib40_439	1e+20	1932.2k	–	14400.0	4064.8k	2507.1k	62.1	14400.0	4015.7k	1953.5k	105.6	14400.0
dem_1000_gaslib40_440	1e+20	2625.7k	–	14400.0	1e+20	3048.1k	–	14400.0	1e+20	3167.8k	–	14400.0
dem_1000_gaslib40_441	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	126.6	1e+20	1e+20	0.0	5.0
dem_1000_gaslib40_442	1e+20	3588.2k	–	14400.0	6164.4k	5050.1k	22.1	14400.0	1e+20	3869.3k	–	14400.0
dem_1000_gaslib40_443	4457.9k	1958.3k	127.6	14400.0	4012.5k	2639.0k	52.0	14400.0	1e+20	2237.7k	–	14400.0
dem_1000_gaslib40_444	4808.7k	2208.5k	117.7	14400.0	4467.1k	2765.6k	61.5	14400.0	1e+20	2465.4k	–	14400.0
dem_1000_gaslib40_445	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_446	1e+20	2330.0k	–	14400.0	1e+20	3002.0k	–	14400.0	1e+20	2822.3k	–	14400.0
dem_1000_gaslib40_447	3312.8k	1710.8k	93.6	14400.0	3410.6k	2367.8k	44.0	14400.0	3533.4k	2056.0k	71.9	14400.0
dem_1000_gaslib40_448	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	165.4	1e+20	1e+20	0.0	0.7
dem_1000_gaslib40_449	1e+20	2761.4k	–	14400.0	4978.5k	3779.6k	31.7	14400.0	1e+20	2822.9k	–	14400.0
dem_1000_gaslib40_450	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	135.2	1e+20	1e+20	0.0	6.9
dem_1000_gaslib40_451	1e+20	3079.6k	–	14400.0	1e+20	4085.8k	–	14400.0	1e+20	3496.2k	–	14400.0
dem_1000_gaslib40_452	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	147.2	1e+20	1e+20	0.0	0.4
dem_1000_gaslib40_453	3931.1k	1943.4k	102.3	14400.0	3722.6k	2576.4k	44.5	14400.0	3607.8k	2216.7k	62.8	14400.0
dem_1000_gaslib40_454	1e+20	1942.5k	–	14400.0	4647.2k	2622.1k	77.2	14400.0	1e+20	2268.7k	–	14400.0
dem_1000_gaslib40_455	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	137.6	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_456	4371.9k	1859.6k	135.1	14400.0	4046.5k	2456.9k	64.7	14400.0	4005.7k	1981.7k	102.1	14400.0
dem_1000_gaslib40_457	3813.0k	1787.1k	113.4	14400.0	4196.3k	2223.0k	88.8	14400.0	4216.0k	1810.0k	132.9	14400.0
dem_1000_gaslib40_458	3668.1k	2533.0k	44.8	14400.0	3568.2k	3263.0k	9.4	14400.0	3849.0k	2326.0k	65.5	14400.0
dem_1000_gaslib40_459	3423.2k	1727.6k	98.2	14400.0	3247.7k	2456.9k	32.2	14400.0	3624.7k	1789.0k	102.6	14400.0
dem_1000_gaslib40_460	1e+20	2164.0k	–	14400.0	4918.8k	3134.6k	56.9	14400.0	1e+20	3040.7k	–	14400.0
dem_1000_gaslib40_461	1e+20	2288.2k	–	14400.0	4578.7k	3228.1k	41.8	14400.0	1e+20	2434.0k	–	14400.0
dem_1000_gaslib40_462	4557.9k	1938.2k	135.2	14400.0	1e+20	2434.8k	–	14400.0	1e+20	2400.4k	–	14400.0
dem_1000_gaslib40_463	4234.9k	1536.5k	175.6	14400.0	3488.5k	2153.9k	62.0	14400.0	3380.3k	1624.5k	108.1	14400.0
dem_1000_gaslib40_464	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	0.1
dem_1000_gaslib40_465	4376.1k	2119.4k	106.5	14400.0	4754.0k	2808.4k	69.3	14400.0	1e+20	2352.5k	–	14400.0
dem_1000_gaslib40_466	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	12.2	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_467	1e+20	2309.8k	–	14400.0	1e+20	3214.0k	–	14400.0	1e+20	3047.4k	–	14400.0
dem_1000_gaslib40_468	1e+20	3270.1k	–	14400.0	1e+20	1e+20	0.0	1644.2	1e+20	3220.2k	–	14400.0
dem_1000_gaslib40_469	1e+20	3454.4k	–	14400.0	1e+20	3967.9k	–	14400.0	1e+20	3679.3k	–	14400.0
dem_1000_gaslib40_470	3743.0k	2610.4k	43.4	14400.0	3724.9k	3260.3k	14.3	14400.0	3944.3k	2471.5k	59.6	14400.0
dem_1000_gaslib40_471	1e+20	2129.5k	–	14400.0	4668.0k	2630.2k	77.5	14400.0	1e+20	2148.3k	–	14400.0
dem_1000_gaslib40_472	3756.2k	1542.2k	143.6	14400.0	3519.7k	2194.6k	60.4	14400.0	3819.8k	1687.1k	126.4	14400.0
dem_1000_gaslib40_473	3506.8k	1883.0k	86.2	14400.0	3533.3k	2516.4k	40.4	14400.0	3904.8k	1846.3k	111.5	14400.0
dem_1000_gaslib40_474	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	345.7	1e+20	1e+20	0.0	8.8
dem_1000_gaslib40_475	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_476	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	465.4	1e+20	1e+20	0.0	5.1
dem_1000_gaslib40_477	4705.7k	2416.9k	94.7	14400.0	1e+20	2862.3k	–	14400.0	1e+20	2760.8k	–	14400.0
dem_1000_gaslib40_478	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	37.9	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_479	1e+20	2983.9k	–	14400.0	1e+20	3608.6k	–	14400.0	1e+20	3601.2k	–	14400.0
dem_1000_gaslib40_480	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	127.6	1e+20	1e+20	0.0	0.5
dem_1000_gaslib40_481	1e+20	2671.5k	–	14400.0	4545.3k	4209.2k	8.0	14400.0	1e+20	2889.5k	–	14400.0
dem_1000_gaslib40_482	1e+20	3093.3k	–	14400.0	1e+20	1e+20	0.0	597.0	1e+20	2439.4k	–	14400.0
dem_1000_gaslib40_483	5291.7k	2312.5k	128.8	14400.0	4469.9k	3031.9k	47.4	14400.0	4988.9k	2309.0k	116.1	14400.0
dem_1000_gaslib40_484	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_485	1e+20	1858.4k	–	14400.0	4394.1k	2698.4k	62.8	14400.0	1e+20	2183.5k	–	14400.0
dem_1000_gaslib40_486	3507.1k	1750.6k	100.3	14400.0	3248.6k	2118.0k	53.4	14400.0	3737.9k	1748.8k	113.7	14400.0
dem_1000_gaslib40_487	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_488	1e+20	2364.1k	–	14400.0	1e+20	3436.5k	–	14400.0	1e+20	3434.7k	–	14400.0
dem_1000_gaslib40_489	4953.5k	1988.3k	149.1	14400.0	4747.0k	2602.7k	82.4	14400.0	4825.2k	2028.7k	137.8	14400.0
dem_1000_gaslib40_490	4524.9k	1829.5k	147.3	14400.0	4453.8k	2540.4k	75.3	14400.0	1e+20	1971.4k	–	14400.0
dem_1000_gaslib40_491	2950.3k	1775.0k	66.2	14400.0	2857.3k	2169.9k	31.7	14400.0	1e+20	1710.3k	–	14400.0
dem_1000_gaslib40_492	1e+20	2186.3k	–	14400.0	4872.3k	3106.2k	56.9	14400.0	1e+20	2306.4k	–	14400.0
dem_1000_gaslib40_493	4982.1k	3025.3k	64.7	14400.0	4502.4k	3905.5k	15.3	14400.0	1e+20	2919.0k	–	14400.0
dem_1000_gaslib40_494	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	133.8	1e+20	1e+20	0.0	0.3
dem_1000_gaslib40_495	1e+20	2280.1k	–	14400.0	1e+20	3161.6k	–	14400.0	1e+20	2798.6k	–	14400.0
dem_1000_gaslib40_496	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	0.2
dem_1000_gaslib40_497	1e+20	2039.5k	–	14400.0	3955.1k	2875.7k	37.5	14400.0	4183.5k	2339.0k	78.9	14400.0
dem_1000_gaslib40_498	1e+20	2613.4k	–	14400.0	1e+20	2781.8k	–	14400.0	4990.0k	3334.5k	49.6	14400.0
dem_1000_gaslib40_499	1e+20	3009.4k	–	14400.0	1e+20	3948.9k	–	14400.0	1e+20	3659.7k	–	14400.0
dem_1000_gaslib40_500	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	102.1	1e+20	1e+20	0.0	0.4

Table A.16: Detailed results of the split-pipe models on *GasLib-40* with $\mathcal{B} = 1000$ as summarized in Figure 3.14d and Table 3.4b. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_1	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_2	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_3	4522.6k	4518.1k	0.1	4523.4	4522.6k	4518.1k	0.1	5296.1
dem_1000_gaslib40_4	4491.5k	4444.0k	1.1	14400.0	4474.1k	4469.6k	0.1	13409.0
dem_1000_gaslib40_5	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_6	2974.1k	2934.2k	1.4	14400.0	2942.0k	2939.0k	0.1	8197.8
dem_1000_gaslib40_7	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_8	5646.2k	5644.1k	0.0	40.6	5646.9k	5643.1k	0.1	31.0
dem_1000_gaslib40_9	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_10	4323.4k	4230.7k	2.2	14400.0	4307.2k	4285.5k	0.5	14400.0
dem_1000_gaslib40_11	5128.9k	5123.8k	0.1	5247.7	5128.9k	5123.8k	0.1	3066.1
dem_1000_gaslib40_12	3712.6k	3709.8k	0.1	8626.8	3728.7k	3333.3k	11.9	14400.0
dem_1000_gaslib40_13	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	19.5
dem_1000_gaslib40_14	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_15	4727.7k	4723.0k	0.1	13809.5	4726.2k	4721.5k	0.1	13268.9
dem_1000_gaslib40_16	3605.4k	3601.8k	0.1	12820.3	3603.2k	3599.6k	0.1	5760.0
dem_1000_gaslib40_17	1e+20	1e+20	0.0	8.1	1e+20	1e+20	0.0	11.4
dem_1000_gaslib40_18	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_19	3843.8k	3840.0k	0.1	3641.1	3843.6k	3733.8k	2.9	14400.0
dem_1000_gaslib40_20	4219.0k	4178.8k	1.0	14400.0	4212.0k	4142.2k	1.7	14400.0
dem_1000_gaslib40_21	4294.5k	4284.9k	0.2	14400.0	4290.6k	4286.3k	0.1	9703.9
dem_1000_gaslib40_22	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_23	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_24	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_25	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_26	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_27	1e+20	1e+20	0.0	5.7	1e+20	1e+20	0.0	12.6
dem_1000_gaslib40_28	4003.6k	3927.8k	1.9	14400.0	3985.3k	3937.1k	1.2	14400.0
dem_1000_gaslib40_29	4145.0k	4065.0k	2.0	14400.0	4131.7k	4072.7k	1.4	14400.0
dem_1000_gaslib40_30	5145.7k	5140.5k	0.1	9029.9	5141.8k	5136.6k	0.1	3899.1
dem_1000_gaslib40_31	3822.5k	3819.1k	0.1	8385.2	3822.5k	3818.7k	0.1	7239.7
dem_1000_gaslib40_32	4289.9k	4285.7k	0.1	6797.5	4287.6k	4283.3k	0.1	5748.3
dem_1000_gaslib40_33	3406.7k	3266.1k	4.3	14400.0	3384.5k	3254.1k	4.0	14400.0
dem_1000_gaslib40_34	4345.2k	4340.9k	0.1	6616.2	4345.2k	4340.9k	0.1	5854.3
dem_1000_gaslib40_35	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_36	5014.1k	5012.8k	0.0	3109.8	5013.8k	5008.8k	0.1	2457.5
dem_1000_gaslib40_37	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_38	3808.2k	3804.4k	0.1	1381.3	3808.2k	3768.5k	1.1	14400.0
dem_1000_gaslib40_39	5112.1k	5107.0k	0.1	862.9	5112.8k	5107.7k	0.1	756.2
dem_1000_gaslib40_40	4093.4k	4090.5k	0.1	47.9	4091.7k	4087.8k	0.1	39.8
dem_1000_gaslib40_41	4624.4k	4619.9k	0.1	190.0	4621.8k	4617.2k	0.1	95.9
dem_1000_gaslib40_42	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_43	5611.9k	5607.3k	0.1	17.6	5613.3k	5607.9k	0.1	16.0
dem_1000_gaslib40_44	4883.0k	4878.1k	0.1	1168.0	4883.0k	4878.1k	0.1	1161.9
dem_1000_gaslib40_45	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_46	3758.4k	3248.7k	15.7	14400.0	3745.7k	3268.8k	14.6	14400.0
dem_1000_gaslib40_47	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_48	3911.3k	3885.7k	0.7	14400.0	3908.4k	3782.5k	3.3	14400.0
dem_1000_gaslib40_49	5029.1k	5023.1k	0.1	14400.0	5028.9k	5023.9k	0.1	10628.0
dem_1000_gaslib40_50	3734.3k	3706.3k	0.8	14400.0	3715.9k	3712.1k	0.1	10231.7
dem_1000_gaslib40_51	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_52	4757.4k	4752.0k	0.1	14400.0	4757.6k	4752.8k	0.1	7016.9
dem_1000_gaslib40_53	4630.8k	4629.6k	0.0	8304.3	4633.1k	4628.5k	0.1	5814.3
dem_1000_gaslib40_54	3972.7k	3919.7k	1.4	14400.0	3943.7k	3939.7k	0.1	9347.4
dem_1000_gaslib40_55	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_56	3397.9k	3271.5k	3.9	14400.0	3393.3k	3207.7k	5.8	14400.0
dem_1000_gaslib40_57	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_58	5013.1k	5012.3k	0.0	8416.7	5013.1k	5008.1k	0.1	5833.0
dem_1000_gaslib40_59	1e+20	1e+20	0.0	10.5	1e+20	1e+20	0.0	5.4
dem_1000_gaslib40_60	3013.8k	3012.0k	0.1	13028.5	3013.5k	3010.4k	0.1	9137.7
dem_1000_gaslib40_61	3700.0k	3696.6k	0.1	5620.9	3700.0k	3681.2k	0.5	14400.0
dem_1000_gaslib40_62	4207.6k	4205.7k	0.0	13050.6	4207.6k	4203.4k	0.1	5546.1
dem_1000_gaslib40_63	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	6.2
dem_1000_gaslib40_64	3788.4k	3785.4k	0.1	8374.5	3850.6k	3387.4k	13.7	14400.0
dem_1000_gaslib40_65	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_66	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_67	4801.8k	4797.0k	0.1	5454.2	4799.9k	4795.1k	0.1	3550.3
dem_1000_gaslib40_68	4228.5k	4177.9k	1.2	14400.0	4200.9k	4196.7k	0.1	9128.7
dem_1000_gaslib40_69	5527.3k	5521.8k	0.1	1797.0	5528.0k	5522.5k	0.1	1579.5
dem_1000_gaslib40_70	4583.0k	4558.1k	0.5	14400.0	4578.6k	4566.6k	0.3	14400.0
dem_1000_gaslib40_71	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_72	3265.2k	3205.6k	1.9	14400.0	3259.3k	2994.0k	8.9	14400.0
dem_1000_gaslib40_73	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_74	3712.9k	3630.6k	2.3	14400.0	3685.6k	3632.1k	1.5	14400.0

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Table A.16: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_75	4824.0k	4819.1k	0.1	5624.6	4824.0k	4819.1k	0.1	6005.6
dem_1000_gaslib40_76	5269.9k	5266.0k	0.1	1505.6	5269.9k	5264.7k	0.1	1494.5
dem_1000_gaslib40_77	4957.7k	4956.1k	0.0	7333.2	4957.7k	4952.8k	0.1	5136.7
dem_1000_gaslib40_78	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_79	3843.8k	3798.3k	1.2	14400.0	3844.9k	3822.3k	0.6	14400.0
dem_1000_gaslib40_80	3426.5k	3414.2k	0.4	14400.0	3424.5k	3109.7k	10.1	14400.0
dem_1000_gaslib40_81	2819.0k	2816.2k	0.1	10140.2	2818.1k	2815.3k	0.1	4078.2
dem_1000_gaslib40_82	4911.4k	4910.2k	0.0	10004.0	4911.5k	4906.6k	0.1	8481.6
dem_1000_gaslib40_83	4270.6k	4246.3k	0.6	14400.0	4264.1k	4259.9k	0.1	13291.4
dem_1000_gaslib40_84	4086.9k	4082.8k	0.1	439.3	4086.9k	3885.7k	5.2	14400.0
dem_1000_gaslib40_85	4100.9k	4098.6k	0.1	12974.5	4100.9k	4096.8k	0.1	11328.2
dem_1000_gaslib40_86	4250.6k	4208.4k	1.0	14400.0	4246.7k	4208.6k	0.9	14400.0
dem_1000_gaslib40_87	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	19.1
dem_1000_gaslib40_88	3403.9k	3380.5k	0.7	14400.0	3397.0k	3393.6k	0.1	9137.6
dem_1000_gaslib40_89	3117.7k	2979.3k	4.6	14400.0	3110.2k	3107.1k	0.1	4264.7
dem_1000_gaslib40_90	3674.8k	3622.9k	1.4	14400.0	3635.1k	3631.5k	0.1	8935.2
dem_1000_gaslib40_91	3680.9k	3638.1k	1.2	14400.0	3676.9k	3643.2k	0.9	14400.0
dem_1000_gaslib40_92	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	3.1
dem_1000_gaslib40_93	5291.8k	5288.4k	0.1	2587.5	5291.8k	5286.5k	0.1	4354.1
dem_1000_gaslib40_94	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_95	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_96	5809.6k	5804.0k	0.1	26.7	5811.0k	5807.1k	0.1	32.5
dem_1000_gaslib40_97	5268.9k	5263.7k	0.1	8955.4	5265.2k	5260.0k	0.1	5361.4
dem_1000_gaslib40_98	5079.5k	5074.4k	0.1	5137.0	5079.5k	5074.4k	0.1	4250.7
dem_1000_gaslib40_99	4465.1k	4460.7k	0.1	5120.9	4465.1k	4460.7k	0.1	4682.6
dem_1000_gaslib40_100	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_101	4282.1k	4277.8k	0.1	6317.6	4279.2k	4274.9k	0.1	5304.1
dem_1000_gaslib40_102	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_103	3740.7k	3737.0k	0.1	11463.2	3739.1k	3735.4k	0.1	5820.7
dem_1000_gaslib40_104	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_105	4863.3k	4858.4k	0.1	14294.0	4862.8k	4857.9k	0.1	10936.9
dem_1000_gaslib40_106	3695.0k	3691.3k	0.1	13005.1	3693.2k	3689.5k	0.1	13734.0
dem_1000_gaslib40_107	4081.2k	4077.1k	0.1	6972.3	4081.2k	4077.1k	0.1	5191.8
dem_1000_gaslib40_108	4811.5k	4806.7k	0.1	651.8	4811.5k	4806.7k	0.1	486.2
dem_1000_gaslib40_109	1e+20	1e+20	0.0	8.3	1e+20	1e+20	0.0	18.3
dem_1000_gaslib40_110	5400.8k	5396.3k	0.1	698.1	5399.6k	5394.2k	0.1	334.6
dem_1000_gaslib40_111	5409.2k	5403.8k	0.1	1297.4	5409.2k	5403.8k	0.1	1099.6
dem_1000_gaslib40_112	3520.8k	3517.6k	0.1	4570.1	3519.9k	3516.4k	0.1	3896.3
dem_1000_gaslib40_113	4336.4k	4281.9k	1.3	14400.0	4315.5k	4292.0k	0.5	14400.0
dem_1000_gaslib40_114	3328.8k	3213.3k	3.6	14400.0	3311.9k	2796.2k	18.4	14400.0
dem_1000_gaslib40_115	3695.1k	3691.4k	0.1	5347.8	3695.1k	3691.5k	0.1	7882.1
dem_1000_gaslib40_116	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_117	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_118	4066.1k	4047.0k	0.5	14400.0	4058.2k	4054.2k	0.1	9626.4
dem_1000_gaslib40_119	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	16.6
dem_1000_gaslib40_120	3885.8k	3850.7k	0.9	14400.0	3880.0k	3854.1k	0.7	14400.0
dem_1000_gaslib40_121	4074.0k	4069.9k	0.1	11184.9	4072.8k	4068.8k	0.1	6674.2
dem_1000_gaslib40_122	3916.8k	3915.0k	0.0	12660.5	3916.8k	3912.9k	0.1	7083.2
dem_1000_gaslib40_123	3989.5k	3924.9k	1.6	14400.0	3956.4k	3952.5k	0.1	9693.5
dem_1000_gaslib40_124	4461.4k	4457.0k	0.1	6862.7	4458.8k	4454.3k	0.1	5647.3
dem_1000_gaslib40_125	4389.3k	4250.6k	3.3	14400.0	4373.2k	4340.3k	0.8	14400.0
dem_1000_gaslib40_126	3830.4k	3826.6k	0.1	134.7	3828.4k	3824.6k	0.1	141.0
dem_1000_gaslib40_127	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_128	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_129	4238.4k	4234.2k	0.1	11909.8	4236.4k	4232.2k	0.1	12562.7
dem_1000_gaslib40_130	1e+20	1e+20	0.0	10.9	1e+20	1e+20	0.0	5.5
dem_1000_gaslib40_131	4303.0k	4298.8k	0.1	1184.0	4302.4k	4302.4k	0.0	635.4
dem_1000_gaslib40_132	4225.8k	4221.6k	0.1	3131.0	4225.8k	4221.6k	0.1	5446.9
dem_1000_gaslib40_133	3909.8k	3905.9k	0.1	12940.2	3908.7k	3904.8k	0.1	8124.2
dem_1000_gaslib40_134	5603.1k	5601.7k	0.0	4725.4	5603.2k	5600.7k	0.0	4486.7
dem_1000_gaslib40_135	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_136	5042.0k	5036.9k	0.1	3083.6	5042.0k	5036.9k	0.1	2289.7
dem_1000_gaslib40_137	4986.2k	4985.0k	0.0	10343.7	4985.7k	4980.8k	0.1	5086.9
dem_1000_gaslib40_138	5055.5k	5052.6k	0.1	8353.2	5055.3k	5050.3k	0.1	9310.8
dem_1000_gaslib40_139	4191.8k	4172.0k	0.5	14400.0	4182.4k	4178.2k	0.1	11728.2
dem_1000_gaslib40_140	3714.9k	3711.3k	0.1	177.3	3714.9k	3711.3k	0.1	133.8
dem_1000_gaslib40_141	3091.6k	3072.3k	0.6	14400.0	3078.9k	2915.2k	5.6	14400.0
dem_1000_gaslib40_142	3754.1k	3750.3k	0.1	8004.6	3753.6k	3749.8k	0.1	4208.7
dem_1000_gaslib40_143	4487.8k	4466.5k	0.5	14400.0	4479.0k	4474.5k	0.1	10074.9
dem_1000_gaslib40_144	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_145	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_146	4386.7k	4382.3k	0.1	3043.7	4386.7k	4382.3k	0.1	2656.2
dem_1000_gaslib40_147	4330.5k	4294.1k	0.8	14400.0	4320.8k	4292.9k	0.7	14400.0
dem_1000_gaslib40_148	3488.6k	3485.1k	0.1	13584.2	3488.5k	3485.1k	0.1	8457.7
dem_1000_gaslib40_149	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	19.1
dem_1000_gaslib40_150	5060.9k	5055.8k	0.1	1506.0	5060.9k	5055.8k	0.1	1420.1
dem_1000_gaslib40_151	5212.1k	5165.9k	0.9	14400.0	5173.6k	5168.4k	0.1	14351.8
dem_1000_gaslib40_152	4179.0k	4037.5k	3.5	14400.0	4179.0k	4038.8k	3.5	14400.0

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Table A.16: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_153	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_154	3395.1k	3391.7k	0.1	12047.8	3395.1k	3392.1k	0.1	10457.6
dem_1000_gaslib40_155	4976.0k	4971.0k	0.1	5844.3	4976.0k	4971.0k	0.1	10745.8
dem_1000_gaslib40_156	1e+20	1e+20	0.0	7.5	1e+20	1e+20	0.0	18.5
dem_1000_gaslib40_157	3861.6k	3857.8k	0.1	4993.9	3867.4k	3470.5k	11.4	14400.0
dem_1000_gaslib40_158	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_159	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_160	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_161	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_162	3234.4k	3231.2k	0.1	8309.0	3234.4k	3231.2k	0.1	7836.4
dem_1000_gaslib40_163	3994.0k	3990.1k	0.1	95.6	3994.0k	3990.0k	0.1	167.1
dem_1000_gaslib40_164	4583.2k	4508.1k	1.7	14400.0	4583.2k	4574.7k	0.2	14400.0
dem_1000_gaslib40_165	3995.8k	3990.6k	0.1	14400.0	3995.8k	3991.8k	0.1	9937.5
dem_1000_gaslib40_166	5266.0k	5257.4k	0.2	14400.0	5266.0k	5260.8k	0.1	2480.9
dem_1000_gaslib40_167	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_168	4343.2k	4338.9k	0.1	7170.1	4343.2k	4338.9k	0.1	13271.4
dem_1000_gaslib40_169	3894.9k	3891.0k	0.1	1789.3	3894.0k	3890.1k	0.1	1861.3
dem_1000_gaslib40_170	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_171	3785.0k	3782.3k	0.1	10171.5	3783.7k	3779.9k	0.1	4672.7
dem_1000_gaslib40_172	4602.0k	4600.6k	0.0	12200.1	4602.2k	4585.5k	0.4	14400.0
dem_1000_gaslib40_173	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_174	3490.2k	3486.7k	0.1	6752.5	3490.1k	3486.6k	0.1	4255.7
dem_1000_gaslib40_175	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_176	4029.6k	4026.4k	0.1	7534.9	4027.9k	4023.8k	0.1	6236.3
dem_1000_gaslib40_177	4630.0k	4628.7k	0.0	10420.3	4629.8k	4625.2k	0.1	11087.3
dem_1000_gaslib40_178	3962.5k	3793.5k	4.5	14400.0	3958.8k	3841.9k	3.0	14400.0
dem_1000_gaslib40_179	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_180	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	5.7
dem_1000_gaslib40_181	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_182	4670.5k	4619.4k	1.1	14400.0	4649.7k	4620.8k	0.6	14400.0
dem_1000_gaslib40_183	5624.5k	5620.5k	0.1	2479.7	5624.5k	5618.9k	0.1	1579.2
dem_1000_gaslib40_184	5632.6k	5627.1k	0.1	58.2	5632.6k	5627.1k	0.1	39.2
dem_1000_gaslib40_185	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_186	3854.9k	3708.8k	3.9	14400.0	3854.4k	3711.4k	3.9	14400.0
dem_1000_gaslib40_187	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_188	1e+20	1e+20	0.0	12.3	1e+20	1e+20	0.0	8.1
dem_1000_gaslib40_189	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	9.7
dem_1000_gaslib40_190	4858.9k	4856.7k	0.0	7980.8	4858.9k	4854.0k	0.1	5480.3
dem_1000_gaslib40_191	4290.8k	4286.8k	0.1	1644.1	4290.8k	4286.5k	0.1	1067.7
dem_1000_gaslib40_192	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_193	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_194	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_195	4953.6k	4948.6k	0.1	3561.4	4951.3k	4946.3k	0.1	2384.9
dem_1000_gaslib40_196	2665.2k	2387.7k	11.6	14400.0	2608.5k	2385.5k	9.3	14400.0
dem_1000_gaslib40_197	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_198	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_199	3571.4k	3514.9k	1.6	14400.0	3546.0k	3542.4k	0.1	9881.3
dem_1000_gaslib40_200	4363.4k	4359.1k	0.1	3632.5	4362.3k	4358.0k	0.1	2181.5
dem_1000_gaslib40_201	4003.3k	3999.3k	0.1	6988.3	4002.8k	3998.8k	0.1	13636.6
dem_1000_gaslib40_202	5362.1k	5356.8k	0.1	6354.3	5357.8k	5352.5k	0.1	3659.6
dem_1000_gaslib40_203	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	15.1
dem_1000_gaslib40_204	3360.7k	3357.4k	0.1	12501.6	3360.6k	3357.2k	0.1	10772.6
dem_1000_gaslib40_205	3609.2k	3605.6k	0.1	3822.6	3608.5k	3267.6k	10.4	14400.0
dem_1000_gaslib40_206	4986.4k	4981.4k	0.1	949.4	4984.0k	4979.0k	0.1	615.5
dem_1000_gaslib40_207	4114.6k	3714.1k	10.8	14400.0	4110.6k	3721.4k	10.5	14400.0
dem_1000_gaslib40_208	2925.9k	2923.0k	0.1	8116.7	2925.9k	2731.6k	7.1	14400.0
dem_1000_gaslib40_209	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_210	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_211	4824.5k	4820.7k	0.1	11318.6	4824.5k	4819.7k	0.1	5733.8
dem_1000_gaslib40_212	4232.2k	4197.6k	0.8	14400.0	4227.4k	4208.3k	0.5	14400.0
dem_1000_gaslib40_213	3793.6k	3772.8k	0.5	14400.0	3778.8k	3775.0k	0.1	12372.8
dem_1000_gaslib40_214	3980.4k	3799.7k	4.8	14400.0	3983.1k	3809.1k	4.6	14400.0
dem_1000_gaslib40_215	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_216	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_217	4524.8k	4468.0k	1.3	14400.0	4502.8k	4476.2k	0.6	14400.0
dem_1000_gaslib40_218	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_219	4572.6k	4499.6k	1.6	14400.0	4551.9k	4508.1k	1.0	14400.0
dem_1000_gaslib40_220	4772.6k	4697.1k	1.6	14400.0	4749.0k	4674.0k	1.6	14400.0
dem_1000_gaslib40_221	3972.9k	3968.9k	0.1	7200.8	3972.9k	3952.6k	0.5	14400.0
dem_1000_gaslib40_222	4925.4k	4820.4k	2.2	14400.0	4925.4k	4920.5k	0.1	8471.4
dem_1000_gaslib40_223	1e+20	1e+20	0.0	6.6	1e+20	1e+20	0.0	21.0
dem_1000_gaslib40_224	3922.8k	3912.7k	0.3	14400.0	3922.8k	3918.8k	0.1	5999.0
dem_1000_gaslib40_225	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_226	4219.7k	4215.5k	0.1	10381.4	4219.5k	4216.1k	0.1	8187.7
dem_1000_gaslib40_227	4451.4k	4402.9k	1.1	14400.0	4427.4k	4423.0k	0.1	13970.6
dem_1000_gaslib40_228	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_229	3736.0k	3732.2k	0.1	8371.3	3736.0k	3732.2k	0.1	6020.8
dem_1000_gaslib40_230	5166.2k	5165.1k	0.0	553.2	5165.7k	5160.5k	0.1	305.0

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Table A.16: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_231	1e+20	1e+20	0.0	27.7	1e+20	1e+20	0.0	7.9
dem_1000_gaslib40_232	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_233	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_234	5411.4k	5408.0k	0.1	126.9	5411.3k	5405.9k	0.1	68.3
dem_1000_gaslib40_235	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	3.4
dem_1000_gaslib40_236	5114.1k	5109.0k	0.1	818.2	5114.4k	5109.3k	0.1	692.5
dem_1000_gaslib40_237	3112.3k	3110.1k	0.1	2232.5	3114.4k	2898.7k	7.4	14400.0
dem_1000_gaslib40_238	5205.5k	5203.8k	0.0	4637.7	5204.7k	5199.5k	0.1	3551.4
dem_1000_gaslib40_239	3301.7k	3298.8k	0.1	10964.2	3301.1k	3297.8k	0.1	4747.4
dem_1000_gaslib40_240	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_241	4222.8k	4180.8k	1.0	14400.0	4204.3k	4200.1k	0.1	10003.9
dem_1000_gaslib40_242	4448.4k	4446.4k	0.0	17.1	4449.7k	4445.4k	0.1	16.5
dem_1000_gaslib40_243	5032.0k	5027.0k	0.1	6186.8	5032.0k	5027.0k	0.1	5270.7
dem_1000_gaslib40_244	3583.3k	3579.8k	0.1	289.4	3583.3k	3579.8k	0.1	226.7
dem_1000_gaslib40_245	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_246	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_247	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_248	4922.6k	4917.7k	0.1	1460.7	4922.6k	4917.7k	0.1	2453.5
dem_1000_gaslib40_249	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	16.7
dem_1000_gaslib40_250	3577.0k	3573.4k	0.1	8617.6	3577.0k	3573.4k	0.1	6380.0
dem_1000_gaslib40_251	3578.4k	3576.3k	0.1	7371.5	3578.1k	3387.7k	5.6	14400.0
dem_1000_gaslib40_252	4567.6k	4564.5k	0.1	9619.8	4565.9k	4550.9k	0.3	14400.0
dem_1000_gaslib40_253	3433.5k	3373.2k	1.8	14400.0	3393.6k	3390.2k	0.1	13644.0
dem_1000_gaslib40_254	1e+20	1e+20	0.0	14.5	1e+20	1e+20	0.0	18.1
dem_1000_gaslib40_255	1e+20	1e+20	0.0	22.7	1e+20	1e+20	0.0	7.4
dem_1000_gaslib40_256	3888.1k	3439.5k	13.0	14400.0	3884.7k	3434.9k	13.1	14400.0
dem_1000_gaslib40_257	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_258	5025.4k	5020.4k	0.1	542.7	5025.4k	5020.4k	0.1	282.1
dem_1000_gaslib40_259	3179.3k	3143.7k	1.1	14400.0	3159.4k	3156.2k	0.1	10398.6
dem_1000_gaslib40_260	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_261	4130.3k	4126.3k	0.1	107.0	4127.3k	4125.8k	0.0	92.0
dem_1000_gaslib40_262	5571.1k	5566.3k	0.1	5180.7	5570.4k	5564.8k	0.1	3314.7
dem_1000_gaslib40_263	3897.6k	3754.6k	3.8	14400.0	3897.6k	3754.6k	3.8	14400.0
dem_1000_gaslib40_264	4044.9k	3961.1k	2.1	14400.0	4022.9k	3979.0k	1.1	14400.0
dem_1000_gaslib40_265	3997.8k	3993.8k	0.1	5845.9	3995.2k	3991.2k	0.1	3927.0
dem_1000_gaslib40_266	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_267	1e+20	1e+20	0.0	4.6	1e+20	1e+20	0.0	20.1
dem_1000_gaslib40_268	3107.3k	3106.4k	0.0	271.0	3106.7k	3103.6k	0.1	143.4
dem_1000_gaslib40_269	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_270	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_271	5218.2k	5213.4k	0.1	2117.9	5217.0k	5211.8k	0.1	1407.6
dem_1000_gaslib40_272	4271.2k	4266.9k	0.1	5827.5	4271.2k	4266.9k	0.1	4722.2
dem_1000_gaslib40_273	4563.1k	4560.2k	0.1	3194.1	4562.6k	4558.0k	0.1	1757.8
dem_1000_gaslib40_274	3531.5k	3530.1k	0.0	9289.9	3531.5k	3527.9k	0.1	3573.4
dem_1000_gaslib40_275	4106.8k	3939.3k	4.3	14400.0	4057.7k	4053.7k	0.1	13015.1
dem_1000_gaslib40_276	5422.3k	5419.5k	0.1	3187.8	5422.3k	5416.8k	0.1	1393.0
dem_1000_gaslib40_277	3417.6k	3414.2k	0.1	3067.2	3417.5k	3414.1k	0.1	2885.3
dem_1000_gaslib40_278	3567.7k	3425.9k	4.1	14400.0	3567.7k	3426.7k	4.1	14400.0
dem_1000_gaslib40_279	3299.3k	3040.2k	8.5	14400.0	3285.7k	3140.3k	4.6	14400.0
dem_1000_gaslib40_280	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_281	4193.4k	4191.5k	0.0	696.8	4193.3k	4189.1k	0.1	611.0
dem_1000_gaslib40_282	4079.7k	4075.7k	0.1	2877.8	4077.8k	4073.7k	0.1	1679.5
dem_1000_gaslib40_283	4087.9k	4021.4k	1.7	14400.0	4036.3k	4032.3k	0.1	6721.2
dem_1000_gaslib40_284	4559.5k	4557.9k	0.0	8666.0	4559.1k	4554.5k	0.1	6509.5
dem_1000_gaslib40_285	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	20.6
dem_1000_gaslib40_286	4559.6k	4555.0k	0.1	7880.0	4558.0k	4553.4k	0.1	4514.8
dem_1000_gaslib40_287	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_288	3888.9k	3887.4k	0.0	9849.8	3888.6k	3884.7k	0.1	6042.6
dem_1000_gaslib40_289	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_290	4365.1k	4360.7k	0.1	7528.8	4365.1k	4360.7k	0.1	6426.2
dem_1000_gaslib40_291	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_292	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_293	5291.9k	5289.2k	0.1	2517.0	5291.9k	5286.6k	0.1	1714.0
dem_1000_gaslib40_294	3982.3k	3979.9k	0.1	346.3	3980.7k	3976.7k	0.1	190.1
dem_1000_gaslib40_295	4394.0k	4360.2k	0.8	14400.0	4384.4k	4365.7k	0.4	14400.0
dem_1000_gaslib40_296	5137.9k	5132.7k	0.1	1726.0	5137.3k	5132.2k	0.1	1275.7
dem_1000_gaslib40_297	4223.5k	4219.3k	0.1	611.8	4224.9k	4221.4k	0.1	151.6
dem_1000_gaslib40_298	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_299	4876.6k	4875.3k	0.0	6893.4	4876.1k	4871.2k	0.1	5288.1
dem_1000_gaslib40_300	5344.9k	5339.5k	0.1	607.7	5344.9k	5339.5k	0.1	429.8
dem_1000_gaslib40_301	4019.9k	3805.4k	5.6	14400.0	4008.8k	3808.3k	5.3	14400.0
dem_1000_gaslib40_302	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_303	5133.6k	5128.4k	0.1	604.3	5131.7k	5126.6k	0.1	850.7
dem_1000_gaslib40_304	4440.7k	4436.3k	0.1	13660.2	4438.8k	4434.4k	0.1	12002.0
dem_1000_gaslib40_305	4582.5k	4554.7k	0.6	14400.0	4573.1k	4568.6k	0.1	13796.4
dem_1000_gaslib40_306	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	4.4
dem_1000_gaslib40_307	3285.1k	3281.8k	0.1	7143.7	3285.1k	3281.8k	0.1	7360.0
dem_1000_gaslib40_308	4164.9k	4160.8k	0.1	9280.5	4164.9k	4160.8k	0.1	9664.1

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Table A.16: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_309	3966.8k	3962.9k	0.1	1903.0	3967.7k	3963.8k	0.1	1609.1
dem_1000_gaslib40_310	3798.5k	3797.4k	0.0	298.9	3800.4k	3796.6k	0.1	262.1
dem_1000_gaslib40_311	4823.9k	4819.1k	0.1	13041.2	4824.2k	4813.8k	0.2	14400.0
dem_1000_gaslib40_312	5013.2k	5008.2k	0.1	3324.7	5013.2k	5008.2k	0.1	3938.0
dem_1000_gaslib40_313	4486.9k	4484.4k	0.1	13237.1	4486.1k	4481.7k	0.1	9596.4
dem_1000_gaslib40_314	4911.0k	4896.2k	0.3	14400.0	4905.2k	4900.3k	0.1	10642.0
dem_1000_gaslib40_315	4691.7k	4687.0k	0.1	81.4	4691.7k	4687.0k	0.1	79.2
dem_1000_gaslib40_316	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_317	4375.8k	4284.8k	2.1	14400.0	4352.4k	4308.3k	1.0	14400.0
dem_1000_gaslib40_318	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_319	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	6.8
dem_1000_gaslib40_320	5447.0k	5441.5k	0.1	193.2	5450.8k	5445.3k	0.1	315.8
dem_1000_gaslib40_321	5292.4k	5288.8k	0.1	4142.9	5290.4k	5285.1k	0.1	5875.6
dem_1000_gaslib40_322	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_323	3765.2k	3672.6k	2.5	14400.0	3734.2k	3672.0k	1.7	14400.0
dem_1000_gaslib40_324	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_325	4524.2k	4460.0k	1.4	14400.0	4511.1k	4447.0k	1.4	14400.0
dem_1000_gaslib40_326	3451.9k	3443.9k	0.2	14400.0	3449.4k	3263.3k	5.7	14400.0
dem_1000_gaslib40_327	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_328	3917.6k	3913.7k	0.1	1231.8	3917.5k	3913.6k	0.1	1341.3
dem_1000_gaslib40_329	4315.7k	4222.7k	2.2	14400.0	4279.3k	4243.3k	0.8	14400.0
dem_1000_gaslib40_330	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_331	4038.1k	4034.1k	0.1	8334.0	4038.1k	4034.1k	0.1	7082.9
dem_1000_gaslib40_332	3048.2k	3045.2k	0.1	5676.5	3048.2k	2979.3k	2.3	14400.0
dem_1000_gaslib40_333	4528.2k	4524.9k	0.1	2941.1	4528.2k	4523.6k	0.1	3112.3
dem_1000_gaslib40_334	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_335	3144.3k	3142.2k	0.1	9460.9	3144.1k	3141.0k	0.1	8521.3
dem_1000_gaslib40_336	4611.8k	4607.4k	0.1	7624.0	4611.9k	4607.3k	0.1	8497.5
dem_1000_gaslib40_337	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_338	4446.2k	4442.5k	0.1	1011.1	4446.2k	4441.7k	0.1	517.6
dem_1000_gaslib40_339	4253.2k	4249.1k	0.1	133.1	4253.2k	4249.1k	0.1	167.0
dem_1000_gaslib40_340	5072.0k	5067.0k	0.1	2875.2	5072.0k	5067.0k	0.1	2170.1
dem_1000_gaslib40_341	4285.5k	4282.9k	0.1	7062.9	4284.6k	4280.3k	0.1	8886.3
dem_1000_gaslib40_342	3899.2k	3895.3k	0.1	8819.9	3897.8k	3893.9k	0.1	4744.2
dem_1000_gaslib40_343	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_344	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	6.2
dem_1000_gaslib40_345	3149.7k	2979.1k	5.7	14400.0	3148.0k	3004.9k	4.8	14400.0
dem_1000_gaslib40_346	4004.0k	3956.8k	1.2	14400.0	3978.6k	3974.7k	0.1	11697.1
dem_1000_gaslib40_347	4816.6k	4811.8k	0.1	3835.8	4814.5k	4809.7k	0.1	2075.4
dem_1000_gaslib40_348	3847.7k	3843.8k	0.1	10234.8	3847.7k	3843.8k	0.1	8198.6
dem_1000_gaslib40_349	4776.7k	4762.1k	0.3	14400.0	4774.7k	4766.0k	0.2	14400.0
dem_1000_gaslib40_350	4014.1k	3931.7k	2.1	14400.0	3993.2k	3937.5k	1.4	14400.0
dem_1000_gaslib40_351	3564.3k	3437.7k	3.7	14400.0	3564.4k	3435.6k	3.7	14400.0
dem_1000_gaslib40_352	1e+20	1e+20	0.0	9.5	1e+20	1e+20	0.0	11.1
dem_1000_gaslib40_353	5471.6k	5468.0k	0.1	388.3	5472.5k	5467.1k	0.1	323.9
dem_1000_gaslib40_354	4255.5k	4251.3k	0.1	10288.4	4255.5k	4243.7k	0.3	14400.0
dem_1000_gaslib40_355	3184.2k	3156.1k	0.9	14400.0	3174.9k	3158.9k	0.5	14400.0
dem_1000_gaslib40_356	4691.1k	4689.3k	0.0	9081.6	4690.2k	4571.9k	2.6	14400.0
dem_1000_gaslib40_357	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_358	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_359	4781.5k	4776.7k	0.1	6673.2	4781.0k	4776.2k	0.1	6205.0
dem_1000_gaslib40_360	4646.7k	4636.1k	0.2	14400.0	4639.9k	4635.2k	0.1	9224.4
dem_1000_gaslib40_361	3486.2k	3482.7k	0.1	11434.0	3483.5k	3480.0k	0.1	6634.8
dem_1000_gaslib40_362	3993.3k	3989.5k	0.1	159.1	3991.5k	3987.5k	0.1	154.8
dem_1000_gaslib40_363	5144.6k	5139.5k	0.1	4263.5	5141.1k	5136.0k	0.1	4100.9
dem_1000_gaslib40_364	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_365	4252.9k	4248.6k	0.1	8873.9	4250.7k	4246.4k	0.1	6860.1
dem_1000_gaslib40_366	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_367	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_368	4285.0k	4251.3k	0.8	14400.0	4281.1k	4260.2k	0.5	14400.0
dem_1000_gaslib40_369	3951.2k	3947.3k	0.1	148.4	3948.2k	3944.3k	0.1	112.5
dem_1000_gaslib40_370	3748.2k	3673.4k	2.0	14400.0	3724.6k	3675.5k	1.3	14400.0
dem_1000_gaslib40_371	5061.8k	5056.7k	0.1	7319.9	5058.4k	5053.4k	0.1	5157.5
dem_1000_gaslib40_372	4482.4k	4417.3k	1.5	14400.0	4442.2k	4437.8k	0.1	10152.7
dem_1000_gaslib40_373	4510.6k	4479.6k	0.7	14400.0	4509.4k	4483.3k	0.6	14400.0
dem_1000_gaslib40_374	4054.9k	3962.6k	2.3	14400.0	4028.3k	3981.0k	1.2	14400.0
dem_1000_gaslib40_375	4825.4k	4805.7k	0.4	14400.0	4809.0k	4804.2k	0.1	9038.6
dem_1000_gaslib40_376	4238.0k	4233.8k	0.1	3036.7	4238.0k	4233.8k	0.1	1704.5
dem_1000_gaslib40_377	3665.3k	3521.2k	4.1	14400.0	3657.6k	3541.9k	3.3	14400.0
dem_1000_gaslib40_378	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_379	4535.9k	4487.8k	1.1	14400.0	4527.4k	4502.0k	0.6	14400.0
dem_1000_gaslib40_380	4557.8k	4554.1k	0.1	7554.0	4556.5k	4552.0k	0.1	3888.6
dem_1000_gaslib40_381	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_382	3754.1k	3636.6k	3.2	14400.0	3711.7k	3656.9k	1.5	14400.0
dem_1000_gaslib40_383	4311.4k	4310.1k	0.0	6733.5	4311.4k	4307.1k	0.1	4531.4
dem_1000_gaslib40_384	3764.7k	3574.9k	5.3	14400.0	3754.3k	3750.6k	0.1	1737.5
dem_1000_gaslib40_385	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_386	5223.9k	5218.7k	0.1	599.6	5223.3k	5218.1k	0.1	622.0

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Table A.16: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_387	4663.4k	4592.8k	1.5	14400.0	4658.5k	4606.2k	1.1	14400.0
dem_1000_gaslib40_388	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_389	5376.3k	5371.0k	0.1	115.9	5376.2k	5370.8k	0.1	197.2
dem_1000_gaslib40_390	3921.4k	3917.5k	0.1	11581.0	3919.3k	3915.4k	0.1	9096.5
dem_1000_gaslib40_391	4186.0k	4089.1k	2.4	14400.0	4146.9k	4065.9k	2.0	14400.0
dem_1000_gaslib40_392	3949.4k	3945.2k	0.1	14400.0	3949.2k	3945.2k	0.1	2740.7
dem_1000_gaslib40_393	4371.3k	4366.9k	0.1	2778.0	4371.3k	4366.9k	0.1	2283.3
dem_1000_gaslib40_394	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	3.9
dem_1000_gaslib40_395	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_396	3963.9k	3962.4k	0.0	4425.8	3963.8k	3959.9k	0.1	6538.1
dem_1000_gaslib40_397	3486.6k	3483.1k	0.1	9023.8	3486.6k	3483.1k	0.1	7536.8
dem_1000_gaslib40_398	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_399	4341.3k	4328.2k	0.3	14400.0	4341.3k	4318.8k	0.5	14400.0
dem_1000_gaslib40_400	4970.6k	4969.4k	0.0	1431.8	4970.2k	4965.2k	0.1	1073.6
dem_1000_gaslib40_401	4433.5k	4406.0k	0.6	14400.0	4427.8k	4423.4k	0.1	8138.6
dem_1000_gaslib40_402	5073.4k	5068.3k	0.1	294.9	5073.5k	5068.5k	0.1	473.5
dem_1000_gaslib40_403	4400.0k	4395.6k	0.1	3617.3	4397.5k	4393.1k	0.1	2366.9
dem_1000_gaslib40_404	4122.4k	4118.3k	0.1	6146.4	4122.4k	4118.3k	0.1	7196.4
dem_1000_gaslib40_405	4364.3k	4360.0k	0.1	9263.1	4363.5k	4359.1k	0.1	5434.7
dem_1000_gaslib40_406	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_407	4219.7k	4215.5k	0.1	10240.1	4219.2k	4215.0k	0.1	8348.2
dem_1000_gaslib40_408	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_409	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_410	5007.8k	5006.5k	0.0	3250.2	5007.5k	5002.5k	0.1	2696.8
dem_1000_gaslib40_411	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	2.5
dem_1000_gaslib40_412	4118.2k	4114.1k	0.1	2059.0	4118.2k	4114.1k	0.1	1720.8
dem_1000_gaslib40_413	1e+20	1e+20	0.0	6.1	1e+20	1e+20	0.0	20.3
dem_1000_gaslib40_414	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_415	5066.2k	5061.1k	0.1	3392.4	5066.1k	5061.1k	0.1	3270.0
dem_1000_gaslib40_416	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_417	3598.1k	3335.3k	7.9	14400.0	3595.4k	3335.0k	7.8	14400.0
dem_1000_gaslib40_418	3233.1k	3228.8k	0.1	14400.0	3233.1k	3229.9k	0.1	8199.1
dem_1000_gaslib40_419	4793.2k	4788.4k	0.1	7172.9	4791.4k	4775.8k	0.3	14400.0
dem_1000_gaslib40_420	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_421	3160.9k	2928.7k	7.9	14400.0	3160.9k	2969.4k	6.4	14400.0
dem_1000_gaslib40_422	3258.2k	3225.2k	1.0	14400.0	3248.4k	3245.2k	0.1	14334.3
dem_1000_gaslib40_423	3825.2k	3760.1k	1.7	14400.0	3816.0k	3812.1k	0.1	8097.6
dem_1000_gaslib40_424	4534.0k	4519.7k	0.3	14400.0	4525.6k	4521.1k	0.1	10470.3
dem_1000_gaslib40_425	4010.0k	4006.0k	0.1	5295.8	4010.0k	4006.0k	0.1	3724.5
dem_1000_gaslib40_426	3346.6k	3295.9k	1.5	14400.0	3346.7k	3285.5k	1.9	14400.0
dem_1000_gaslib40_427	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_428	4589.6k	4533.4k	1.2	14400.0	4573.4k	4548.4k	0.5	14400.0
dem_1000_gaslib40_429	3967.1k	3965.9k	0.0	1415.6	3967.5k	3966.1k	0.0	1227.0
dem_1000_gaslib40_430	3393.6k	3224.9k	5.2	14400.0	3288.9k	3259.3k	0.9	14400.0
dem_1000_gaslib40_431	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_432	3591.8k	3589.5k	0.1	7568.4	3591.5k	3587.9k	0.1	4324.6
dem_1000_gaslib40_433	4813.2k	4762.0k	1.1	14400.0	4788.3k	4783.5k	0.1	13517.6
dem_1000_gaslib40_434	3847.0k	3843.2k	0.1	11245.7	3846.4k	3845.1k	0.0	10661.9
dem_1000_gaslib40_435	3500.9k	3499.7k	0.0	303.6	3500.6k	3497.2k	0.1	212.9
dem_1000_gaslib40_436	4022.0k	4017.9k	0.1	7716.9	4022.0k	4009.5k	0.3	14400.0
dem_1000_gaslib40_437	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_438	5002.5k	4997.2k	0.1	14400.0	4999.2k	4994.2k	0.1	6812.7
dem_1000_gaslib40_439	3627.0k	3625.2k	0.0	11946.0	3626.4k	3622.8k	0.1	6956.8
dem_1000_gaslib40_440	5125.5k	5078.2k	0.9	14400.0	5101.2k	5096.1k	0.1	11714.5
dem_1000_gaslib40_441	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_442	5834.9k	5829.1k	0.1	235.4	5832.2k	5826.4k	0.1	199.6
dem_1000_gaslib40_443	3574.1k	3540.6k	0.9	14400.0	3560.4k	3556.8k	0.1	10914.4
dem_1000_gaslib40_444	4226.0k	4186.1k	1.0	14400.0	4195.0k	4193.8k	0.0	11413.4
dem_1000_gaslib40_445	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_446	4937.0k	4876.4k	1.2	14400.0	4919.7k	4885.3k	0.7	14400.0
dem_1000_gaslib40_447	3042.9k	2997.2k	1.5	14400.0	3037.2k	2986.0k	1.7	14400.0
dem_1000_gaslib40_448	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_449	4682.7k	4644.8k	0.8	14400.0	4663.6k	4658.9k	0.1	8053.3
dem_1000_gaslib40_450	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_451	5513.4k	5507.9k	0.1	2040.9	5511.4k	5505.9k	0.1	2443.2
dem_1000_gaslib40_452	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_453	3489.5k	3472.1k	0.5	14400.0	3479.0k	3475.5k	0.1	12549.4
dem_1000_gaslib40_454	4342.7k	4333.8k	0.2	14400.0	4340.5k	4336.1k	0.1	11453.0
dem_1000_gaslib40_455	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_456	3616.9k	3615.9k	0.0	9861.9	3616.9k	3613.3k	0.1	6096.5
dem_1000_gaslib40_457	3418.4k	3369.6k	1.4	14400.0	3405.0k	3375.6k	0.9	14400.0
dem_1000_gaslib40_458	3556.5k	3553.0k	0.1	207.9	3553.5k	3550.0k	0.1	107.0
dem_1000_gaslib40_459	3128.7k	3125.6k	0.1	8258.8	3127.9k	3126.4k	0.1	7631.5
dem_1000_gaslib40_460	4545.1k	4501.1k	1.0	14400.0	4542.3k	4509.7k	0.7	14400.0
dem_1000_gaslib40_461	4377.4k	4375.3k	0.0	9463.1	4377.4k	4373.0k	0.1	5240.9
dem_1000_gaslib40_462	3988.8k	3927.0k	1.6	14400.0	3957.7k	3894.7k	1.6	14400.0
dem_1000_gaslib40_463	3140.0k	3103.4k	1.2	14400.0	3119.1k	3116.0k	0.1	9477.6
dem_1000_gaslib40_464	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.16: Comparison of split-pipe models on *GasLib-40* with $\mathcal{B} = 1000$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_1000_gaslib40_465	4124.2k	4053.2k	1.8	14400.0	4085.7k	4065.4k	0.5	14400.0
dem_1000_gaslib40_466	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_467	4715.8k	4653.1k	1.3	14400.0	4693.8k	4664.8k	0.6	14400.0
dem_1000_gaslib40_468	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	6.9
dem_1000_gaslib40_469	5705.6k	5700.5k	0.1	492.6	5701.8k	5696.1k	0.1	342.4
dem_1000_gaslib40_470	3669.2k	3637.1k	0.9	14400.0	3669.3k	3623.3k	1.3	14400.0
dem_1000_gaslib40_471	3792.2k	3790.6k	0.0	4577.9	3792.1k	3788.3k	0.1	3225.9
dem_1000_gaslib40_472	2996.5k	2994.4k	0.1	8338.9	2995.8k	2992.8k	0.1	8121.2
dem_1000_gaslib40_473	3125.7k	3029.8k	3.2	14400.0	3105.6k	2890.7k	7.4	14400.0
dem_1000_gaslib40_474	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_475	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_476	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_477	4380.3k	4332.6k	1.1	14400.0	4356.5k	4352.1k	0.1	10571.8
dem_1000_gaslib40_478	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_479	5295.6k	5290.3k	0.1	2947.4	5292.8k	5287.5k	0.1	2385.9
dem_1000_gaslib40_480	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_481	4523.9k	4399.7k	2.8	14400.0	4539.7k	4409.2k	3.0	14400.0
dem_1000_gaslib40_482	1e+20	1e+20	0.0	7.5	1e+20	1e+20	0.0	3.1
dem_1000_gaslib40_483	4342.0k	4339.5k	0.1	11451.3	4342.0k	4337.6k	0.1	6520.2
dem_1000_gaslib40_484	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_485	3843.0k	3801.1k	1.1	14400.0	3830.9k	3811.3k	0.5	14400.0
dem_1000_gaslib40_486	3044.3k	3041.3k	0.1	8381.0	3044.3k	3041.3k	0.1	3771.6
dem_1000_gaslib40_487	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_488	4898.5k	4896.2k	0.0	6206.1	4897.5k	4892.6k	0.1	4873.5
dem_1000_gaslib40_489	3770.7k	3760.9k	0.3	14400.0	3770.7k	3766.9k	0.1	10723.8
dem_1000_gaslib40_490	3885.0k	3830.6k	1.4	14400.0	3857.4k	3853.5k	0.1	11730.7
dem_1000_gaslib40_491	2777.4k	2675.5k	3.8	14400.0	2775.8k	2621.1k	5.9	14400.0
dem_1000_gaslib40_492	4288.3k	4285.6k	0.1	6625.5	4287.9k	4283.7k	0.1	5281.2
dem_1000_gaslib40_493	4399.3k	4271.3k	3.0	14400.0	4399.1k	4394.7k	0.1	1782.9
dem_1000_gaslib40_494	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_495	4540.9k	4503.7k	0.8	14400.0	4519.6k	4515.1k	0.1	11778.9
dem_1000_gaslib40_496	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_1000_gaslib40_497	3752.3k	3306.7k	13.5	14400.0	3715.4k	3711.7k	0.1	5288.0
dem_1000_gaslib40_498	4706.6k	4702.4k	0.1	11660.7	4706.6k	4701.9k	0.1	6078.5
dem_1000_gaslib40_499	5137.8k	5136.5k	0.0	1644.1	5137.5k	5132.4k	0.1	839.0
dem_1000_gaslib40_500	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

Table A.17: Detailed results of the split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ as summarized in Figure 3.9a and Table 3.3c. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_1	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_2	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_3	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_4	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_5	0.0	0.0	0.0	0.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.4
dem_50_gaslib135_6	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_7	0.0	0.0	0.0	0.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.0
dem_50_gaslib135_8	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_9	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_10	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_11	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_12	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_50_gaslib135_13	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	6951.3k	35.8k	Large	14400.0
dem_50_gaslib135_14	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_15	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_16	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_17	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9
dem_50_gaslib135_18	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.9
dem_50_gaslib135_19	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_20	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_50_gaslib135_21	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_50_gaslib135_22	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_23	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.2
dem_50_gaslib135_24	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_25	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.2
dem_50_gaslib135_26	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_27	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4

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Table A.17: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_28	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_29	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.8
dem_50_gaslib135_30	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_31	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_32	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.2
dem_50_gaslib135_33	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.1
dem_50_gaslib135_34	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	12.3
dem_50_gaslib135_35	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_36	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_37	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	12.3
dem_50_gaslib135_38	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_50_gaslib135_39	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.4
dem_50_gaslib135_40	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_41	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_42	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_43	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_44	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_45	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.4
dem_50_gaslib135_46	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.2
dem_50_gaslib135_47	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_48	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_49	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	91.2
dem_50_gaslib135_50	1e+20	50.0k	–	14400.0	1e+20	50.3k	–	14400.0	1e+20	50.0k	–	14400.0
dem_50_gaslib135_51	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.6
dem_50_gaslib135_52	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.1
dem_50_gaslib135_53	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_54	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_55	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.9
dem_50_gaslib135_56	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_57	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_50_gaslib135_58	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_59	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.8
dem_50_gaslib135_60	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_61	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_62	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_63	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.1
dem_50_gaslib135_64	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	65.7
dem_50_gaslib135_65	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_66	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_50_gaslib135_67	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_68	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_69	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.1
dem_50_gaslib135_70	0.0	0.0	0.0	0.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_71	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_72	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_73	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.3
dem_50_gaslib135_74	0.0	0.0	0.0	0.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	12.5
dem_50_gaslib135_75	0.0	0.0	0.0	0.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.5
dem_50_gaslib135_76	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.9
dem_50_gaslib135_77	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	72.0
dem_50_gaslib135_78	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_79	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	12.1
dem_50_gaslib135_80	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.2
dem_50_gaslib135_81	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_50_gaslib135_82	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	12.1
dem_50_gaslib135_83	0.0	0.0	0.0	0.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.8
dem_50_gaslib135_84	12.3k	1624.3	657.2	14400.0	1e+20	490.6	–	14400.0	12.3k	12.3k	0.0	4496.2
dem_50_gaslib135_85	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.2
dem_50_gaslib135_86	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_87	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_88	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_89	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	68.3
dem_50_gaslib135_90	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.2
dem_50_gaslib135_91	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_92	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.9
dem_50_gaslib135_93	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.5
dem_50_gaslib135_94	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_50_gaslib135_95	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	91.2
dem_50_gaslib135_96	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_97	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_50_gaslib135_98	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.8
dem_50_gaslib135_99	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_100	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_50_gaslib135_101	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.0
dem_50_gaslib135_102	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_103	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	100.7
dem_50_gaslib135_104	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.6
dem_50_gaslib135_105	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1

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Table A.17: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_106	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	1.6
dem_50_gaslib135_107	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_108	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_50_gaslib135_109	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_110	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_111	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_112	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_113	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_50_gaslib135_114	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9
dem_50_gaslib135_115	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	87.7
dem_50_gaslib135_116	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.1
dem_50_gaslib135_117	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	105.4
dem_50_gaslib135_118	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.7
dem_50_gaslib135_119	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	12.1
dem_50_gaslib135_120	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_121	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_122	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_123	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	82.3
dem_50_gaslib135_124	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_125	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.4
dem_50_gaslib135_126	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.6
dem_50_gaslib135_127	0.0	0.0	0.0	5.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.8
dem_50_gaslib135_128	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_129	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_130	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_131	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_132	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_133	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	79.2
dem_50_gaslib135_134	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	86.8
dem_50_gaslib135_135	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.1
dem_50_gaslib135_136	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	95.4
dem_50_gaslib135_137	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.4
dem_50_gaslib135_138	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.9
dem_50_gaslib135_139	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_50_gaslib135_140	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	76.1
dem_50_gaslib135_141	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.2
dem_50_gaslib135_142	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.9
dem_50_gaslib135_143	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.3
dem_50_gaslib135_144	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_50_gaslib135_145	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	6.5
dem_50_gaslib135_146	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	89.7
dem_50_gaslib135_147	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_148	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.2
dem_50_gaslib135_149	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.5
dem_50_gaslib135_150	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_151	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.2
dem_50_gaslib135_152	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	76.7
dem_50_gaslib135_153	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_154	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	76.7
dem_50_gaslib135_155	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.9
dem_50_gaslib135_156	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.7
dem_50_gaslib135_157	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_158	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.6
dem_50_gaslib135_159	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_160	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_161	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_162	1e+20	1488.0	–	14400.0	1e+20	9711.3	–	14400.0	440.4k	1488.0	Large	14400.0
dem_50_gaslib135_163	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_164	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_165	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_166	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.7
dem_50_gaslib135_167	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	84.4
dem_50_gaslib135_168	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	83.3
dem_50_gaslib135_169	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.4
dem_50_gaslib135_170	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_171	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_172	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_173	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.0
dem_50_gaslib135_174	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.6
dem_50_gaslib135_175	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_176	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.7
dem_50_gaslib135_177	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_178	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_179	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_180	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_181	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_182	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_183	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9

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Table A.17: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_184	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_185	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_186	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.6
dem_50_gaslib135_187	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_50_gaslib135_188	0.0	0.0	0.0	3.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_189	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.4
dem_50_gaslib135_190	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_191	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.8
dem_50_gaslib135_192	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9
dem_50_gaslib135_193	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_194	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.5
dem_50_gaslib135_195	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	61.5
dem_50_gaslib135_196	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_197	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_198	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_199	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_200	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.5
dem_50_gaslib135_201	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.9
dem_50_gaslib135_202	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.9
dem_50_gaslib135_203	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_204	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_205	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_206	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.6
dem_50_gaslib135_207	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_208	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_209	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_210	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	73.4
dem_50_gaslib135_211	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_212	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_213	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_214	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_50_gaslib135_215	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	83.5
dem_50_gaslib135_216	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_217	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_50_gaslib135_218	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_219	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_220	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_221	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.6
dem_50_gaslib135_222	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	49.9
dem_50_gaslib135_223	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.4
dem_50_gaslib135_224	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_225	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.8
dem_50_gaslib135_226	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_227	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_228	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_229	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_230	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_231	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.3
dem_50_gaslib135_232	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.6
dem_50_gaslib135_233	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_234	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.5
dem_50_gaslib135_235	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_236	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.8
dem_50_gaslib135_237	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_238	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.2
dem_50_gaslib135_239	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_240	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.8
dem_50_gaslib135_241	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_242	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	91.8
dem_50_gaslib135_243	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_244	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	86.6
dem_50_gaslib135_245	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_50_gaslib135_246	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.8
dem_50_gaslib135_247	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_50_gaslib135_248	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_249	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_250	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_251	292.4k	0.0	–	14400.0	1e+20	37.6	–	14400.0	10.6k	10.6k	0.0	5274.0
dem_50_gaslib135_252	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_253	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_254	10.6k	8790.1	21.0	14400.0	1e+20	0.0	–	14400.0	10.6k	10.6k	0.0	14251.2
dem_50_gaslib135_255	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_256	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.2
dem_50_gaslib135_257	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_258	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_259	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	80.7
dem_50_gaslib135_260	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.9
dem_50_gaslib135_261	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8

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Table A.17: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_262	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.3
dem_50_gaslib135_263	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.2
dem_50_gaslib135_264	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_265	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_266	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.2
dem_50_gaslib135_267	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_268	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9
dem_50_gaslib135_269	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	63.3
dem_50_gaslib135_270	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_271	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_50_gaslib135_272	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_273	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_274	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_275	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_276	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_277	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_50_gaslib135_278	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.3
dem_50_gaslib135_279	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_280	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_281	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_282	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_283	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_284	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_50_gaslib135_285	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.3
dem_50_gaslib135_286	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_287	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_288	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.4
dem_50_gaslib135_289	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_290	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_291	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.4
dem_50_gaslib135_292	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_293	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_294	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_295	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_50_gaslib135_296	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.3
dem_50_gaslib135_297	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_298	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_50_gaslib135_299	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_300	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_301	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.9
dem_50_gaslib135_302	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.6
dem_50_gaslib135_303	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_304	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.4
dem_50_gaslib135_305	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_306	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_307	12.3k	1624.3	657.2	14400.0	1e+20	321.7	–	14400.0	13.8k	12.1k	13.7	14400.0
dem_50_gaslib135_308	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.4
dem_50_gaslib135_309	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_50_gaslib135_310	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_311	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_50_gaslib135_312	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.8
dem_50_gaslib135_313	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_314	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.2
dem_50_gaslib135_315	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	96.0
dem_50_gaslib135_316	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_317	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	53.1
dem_50_gaslib135_318	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_319	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_320	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_321	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_322	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_323	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_324	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_325	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_326	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_50_gaslib135_327	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_328	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_329	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_330	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_331	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_50_gaslib135_332	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_333	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_334	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_335	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.2
dem_50_gaslib135_336	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	80.5
dem_50_gaslib135_337	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_338	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_339	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	69.5

continued on next page

Table A.17: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_340	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_341	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_342	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_50_gaslib135_343	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_344	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	58.8
dem_50_gaslib135_345	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.6
dem_50_gaslib135_346	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	81.1
dem_50_gaslib135_347	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_348	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.9
dem_50_gaslib135_349	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.7
dem_50_gaslib135_350	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	12.2
dem_50_gaslib135_351	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.3
dem_50_gaslib135_352	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.8
dem_50_gaslib135_353	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_354	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.9
dem_50_gaslib135_355	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_356	21.8k	1624.3	1242.0	14400.0	1e+20	324.8	–	14400.0	12.3k	12.3k	0.0	3303.1
dem_50_gaslib135_357	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_358	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.0
dem_50_gaslib135_359	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_360	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_361	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.8
dem_50_gaslib135_362	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_363	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_364	0.0	0.0	0.0	3.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.7
dem_50_gaslib135_365	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_366	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.4
dem_50_gaslib135_367	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_50_gaslib135_368	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.4
dem_50_gaslib135_369	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_370	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_50_gaslib135_371	0.0	0.0	0.0	3.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_372	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	88.3
dem_50_gaslib135_373	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_374	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.1
dem_50_gaslib135_375	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_376	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.7
dem_50_gaslib135_377	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.4
dem_50_gaslib135_378	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_379	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_50_gaslib135_380	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_381	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_382	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	99.4
dem_50_gaslib135_383	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_384	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_385	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.2
dem_50_gaslib135_386	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.4
dem_50_gaslib135_387	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_388	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_389	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_390	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_391	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_392	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.3
dem_50_gaslib135_393	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	95.5
dem_50_gaslib135_394	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_395	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_396	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_397	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_398	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_399	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.7
dem_50_gaslib135_400	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.1
dem_50_gaslib135_401	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_402	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_50_gaslib135_403	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_404	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_50_gaslib135_405	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_406	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_407	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	75.2
dem_50_gaslib135_408	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_409	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_410	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_411	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_412	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_413	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_414	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_415	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.9
dem_50_gaslib135_416	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.1
dem_50_gaslib135_417	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5

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Table A.17: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_418	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_419	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_420	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_421	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_422	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.9
dem_50_gaslib135_423	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_424	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_50_gaslib135_425	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.9
dem_50_gaslib135_426	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_427	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.4
dem_50_gaslib135_428	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.2
dem_50_gaslib135_429	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.6
dem_50_gaslib135_430	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_431	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_432	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_50_gaslib135_433	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_50_gaslib135_434	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_435	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9
dem_50_gaslib135_436	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_437	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.6
dem_50_gaslib135_438	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.5
dem_50_gaslib135_439	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	79.5
dem_50_gaslib135_440	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_441	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.1
dem_50_gaslib135_442	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_50_gaslib135_443	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_444	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	65.4
dem_50_gaslib135_445	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.6
dem_50_gaslib135_446	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_447	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	85.1
dem_50_gaslib135_448	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_50_gaslib135_449	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.1
dem_50_gaslib135_450	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_451	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.2
dem_50_gaslib135_452	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_50_gaslib135_453	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_50_gaslib135_454	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_455	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_456	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	101.3
dem_50_gaslib135_457	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	1.9
dem_50_gaslib135_458	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	98.3
dem_50_gaslib135_459	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_460	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_461	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.5
dem_50_gaslib135_462	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.2
dem_50_gaslib135_463	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_464	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_465	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_466	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_50_gaslib135_467	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	70.1
dem_50_gaslib135_468	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_469	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_470	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	62.1
dem_50_gaslib135_471	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_50_gaslib135_472	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_50_gaslib135_473	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_50_gaslib135_474	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.6
dem_50_gaslib135_475	0.0	0.0	0.0	3.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_476	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.8
dem_50_gaslib135_477	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_50_gaslib135_478	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_50_gaslib135_479	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_50_gaslib135_480	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_481	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.3
dem_50_gaslib135_482	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	70.2
dem_50_gaslib135_483	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_50_gaslib135_484	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_485	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_50_gaslib135_486	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.6
dem_50_gaslib135_487	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_50_gaslib135_488	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.2
dem_50_gaslib135_489	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_50_gaslib135_490	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_50_gaslib135_491	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_50_gaslib135_492	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.7
dem_50_gaslib135_493	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_494	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_50_gaslib135_495	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0

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Table A.17: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_496	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.8
dem_50_gaslib135_497	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_50_gaslib135_498	12.3k	1488.0	726.5	14400.0	1e+20	454.1	–	14400.0	1e+20	1488.0	–	14400.0
dem_50_gaslib135_499	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_50_gaslib135_500	12.3k	1624.3	657.2	14400.0	1e+20	2071.6	–	14400.0	12.3k	1624.3	657.2	14400.0

Table A.18: Detailed results of the split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ as summarized in Figure 3.15a and Table 3.4c. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_1	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_2	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_3	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_4	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_5	0.0	0.0	0.0	0.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_6	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_7	0.0	0.0	0.0	0.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_8	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_9	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_10	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_11	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_12	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_13	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_14	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_15	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_16	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_17	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_18	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_19	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_20	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_21	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_22	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_23	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_24	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_25	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_26	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_27	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_28	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_29	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_30	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_31	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_32	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_33	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_34	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_35	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_36	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_37	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_38	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_39	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_40	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_41	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_42	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_43	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_44	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_45	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_46	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_47	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_48	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_49	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_50	1e+20	50.0k	–	14400.0	1e+20	50.3k	–	14400.0
dem_50_gaslib135_51	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_52	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_53	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_54	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_55	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_56	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_57	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_58	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0

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Table A.18: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_59	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_60	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_61	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_62	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_63	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_64	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_65	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_66	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_67	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_68	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_69	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_70	0.0	0.0	0.0	0.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_71	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_72	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_73	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_74	0.0	0.0	0.0	0.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_75	0.0	0.0	0.0	0.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_76	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_77	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_78	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_79	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_80	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_81	0.0	0.0	0.0	0.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_82	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_83	0.0	0.0	0.0	0.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_84	12.3k	1624.3	657.2	14400.0	1e+20	490.6	–	14400.0
dem_50_gaslib135_85	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_86	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_87	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_88	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_89	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_90	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_91	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_92	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_93	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_94	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_95	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_96	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_97	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_98	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_99	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_100	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_101	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_102	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_103	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_104	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_105	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_106	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_107	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_108	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_109	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_110	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_111	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_112	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_113	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_114	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_115	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_116	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_117	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_118	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_119	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_120	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_121	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_122	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_123	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_124	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_125	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_126	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_127	0.0	0.0	0.0	5.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_128	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_129	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_130	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_131	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_132	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_133	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_134	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_135	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_136	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0

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Table A.18: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_137	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_138	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_139	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_140	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_141	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_142	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_143	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_144	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_145	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_146	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_147	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_148	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_149	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_150	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_151	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_152	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_153	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_154	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_155	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_156	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_157	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_158	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_159	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_160	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_161	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_162	1e+20	1488.0	–	14400.0	1e+20	9711.3	–	14400.0
dem_50_gaslib135_163	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_164	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_165	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_166	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_167	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_168	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_169	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_170	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_171	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_172	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_173	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_174	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_175	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_176	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_177	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_178	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_179	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_180	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_181	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_182	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_183	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_184	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_185	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_186	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_187	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_188	0.0	0.0	0.0	3.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_189	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_190	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_191	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_192	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_193	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_194	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_195	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_196	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_197	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_198	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_199	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_200	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_201	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_202	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_203	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_204	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_205	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_206	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_207	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_208	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_209	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_210	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_211	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_212	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_213	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_214	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0

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Table A.18: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_215	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_216	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_217	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_218	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_219	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_220	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_221	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_222	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_223	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_224	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_225	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_226	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_227	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_228	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_229	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_230	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_231	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_232	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_233	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_234	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_235	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_236	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_237	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_238	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_239	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_240	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_241	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_242	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_243	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_244	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_245	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_246	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_247	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_248	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_249	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_250	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_251	292.4k	0.0	–	14400.0	1e+20	37.6	–	14400.0
dem_50_gaslib135_252	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_253	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_254	10.6k	8790.1	21.0	14400.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_255	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_256	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_257	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_258	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_259	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_260	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_261	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_262	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_263	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_264	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_265	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_266	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_267	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_268	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_269	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_270	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_271	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_272	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_273	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_274	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_275	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_276	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_277	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_278	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_279	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_280	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_281	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_282	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_283	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_284	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_285	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_286	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_287	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_288	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_289	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_290	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_291	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_292	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0

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Table A.18: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_293	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_294	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_295	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_296	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_297	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_298	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_299	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_300	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_301	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_302	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_303	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_304	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_305	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_306	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_307	12.3k	1624.3	657.2	14400.0	1e+20	321.7	–	14400.0
dem_50_gaslib135_308	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_309	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_310	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_311	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_312	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_313	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_314	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_315	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_316	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_317	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_318	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_319	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_320	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_321	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_322	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_323	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_324	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_325	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_326	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_327	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_328	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_329	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_330	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_331	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_332	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_333	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_334	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_335	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_336	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_337	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_338	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_339	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_340	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_341	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_342	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_343	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_344	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_345	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_346	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_347	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_348	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_349	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_350	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_351	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_352	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_353	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_354	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_355	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_356	21.8k	1624.3	1242.0	14400.0	1e+20	324.8	–	14400.0
dem_50_gaslib135_357	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_358	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_359	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_360	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_361	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_362	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_363	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_364	0.0	0.0	0.0	3.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_365	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_366	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_367	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_368	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_369	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_370	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0

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Table A.18: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_371	0.0	0.0	0.0	3.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_372	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_373	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_374	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_375	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_376	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_377	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_378	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_379	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_380	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_381	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_382	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_383	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_384	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_385	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_386	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_387	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_388	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_389	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_390	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_391	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_392	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_393	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_394	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_395	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_396	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_397	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_398	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_399	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_400	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_401	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_402	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_403	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_404	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_405	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_406	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_407	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_408	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_409	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_410	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_411	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_412	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_413	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_414	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_415	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_416	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_417	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_418	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_419	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_420	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_421	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_422	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_423	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_424	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_425	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_426	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_427	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_428	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_429	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_430	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_431	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_432	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_433	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_434	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_435	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_436	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_437	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_438	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_439	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_440	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_441	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_442	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_443	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_444	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_445	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_446	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_447	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_448	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0

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Table A.18: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 50$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_50_gaslib135_449	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_450	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_451	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_452	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_453	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_454	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_455	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_456	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_457	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_458	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_50_gaslib135_459	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_460	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_461	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_462	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_463	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_464	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_465	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_466	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_467	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_468	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_469	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_470	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_471	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_472	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_473	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_474	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_475	0.0	0.0	0.0	3.3	1e+20	0.0	–	14400.0
dem_50_gaslib135_476	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_50_gaslib135_477	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_478	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_479	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_480	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_481	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_482	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_483	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_484	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_485	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_50_gaslib135_486	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_487	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_488	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_50_gaslib135_489	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_490	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_491	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_50_gaslib135_492	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_493	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_50_gaslib135_494	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_495	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_496	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_50_gaslib135_497	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_498	12.3k	1488.0	726.5	14400.0	1e+20	454.1	–	14400.0
dem_50_gaslib135_499	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_50_gaslib135_500	12.3k	1624.3	657.2	14400.0	1e+20	2071.6	–	14400.0

Table A.19: Detailed results of the split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ as summarized in Figure 3.9b and Table 3.3c. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_1	1e+20	0.0	–	14400.0	1e+20	12.8	–	14400.0	24.3k	0.0	–	14400.0
dem_100_gaslib135_2	0.0	0.0	0.0	52.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_100_gaslib135_3	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	1.9
dem_100_gaslib135_4	48.0k	1624.3	2854.5	14400.0	1e+20	8653.7	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_5	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.3
dem_100_gaslib135_6	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_100_gaslib135_7	1e+20	28.2k	–	14400.0	1e+20	36.7k	–	14400.0	625.5k	27.3k	2193.7	14400.0
dem_100_gaslib135_8	74.2k	31.7k	134.2	14400.0	1e+20	44.5k	–	14400.0	66.4k	31.7k	109.6	14400.0
dem_100_gaslib135_9	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.3
dem_100_gaslib135_10	0.0	0.0	0.0	6.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	54.3
dem_100_gaslib135_11	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	55.0

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Table A.19: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_12	103.1k	36.5k	182.1	14400.0	1e+20	54.3k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_13	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_100_gaslib135_14	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_100_gaslib135_15	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	6.5
dem_100_gaslib135_16	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_100_gaslib135_17	10.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0	830.3k	0.0	–	14400.0
dem_100_gaslib135_18	1e+20	58.6k	–	14400.0	1e+20	70.0k	–	14400.0	101.9k	58.6k	73.8	14400.0
dem_100_gaslib135_19	0.0	0.0	0.0	5.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_100_gaslib135_20	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_100_gaslib135_21	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_100_gaslib135_22	81.0k	63.0k	28.6	14400.0	1e+20	61.0k	–	14400.0	1e+20	50.0k	–	14400.0
dem_100_gaslib135_23	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_100_gaslib135_24	1e+20	1488.0	–	14400.0	1e+20	4561.8	–	14400.0	65.7k	1488.0	4313.2	14400.0
dem_100_gaslib135_25	1e+20	1488.0	–	14400.0	1e+20	12.2k	–	14400.0	48.0k	1488.0	3125.2	14400.0
dem_100_gaslib135_26	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_100_gaslib135_27	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.7
dem_100_gaslib135_28	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_29	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	72.3
dem_100_gaslib135_30	1e+20	1488.0	–	14400.0	1e+20	19.9k	–	14400.0	1e+20	19.9k	–	14400.0
dem_100_gaslib135_31	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_100_gaslib135_32	383.8k	1624.3	Large	14400.0	1e+20	27.8k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_33	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9
dem_100_gaslib135_34	1e+20	36.5k	–	14400.0	1e+20	55.2k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_35	10.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0	16.2k	0.0	–	14400.0
dem_100_gaslib135_36	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_37	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	14.5k	14.5k	0.0	10680.5
dem_100_gaslib135_38	1e+20	58.6k	–	14400.0	1e+20	66.5k	–	14400.0	101.9k	101.9k	0.0	8865.6
dem_100_gaslib135_39	249.1k	64.9k	283.8	14400.0	1e+20	71.4k	–	14400.0	1e+20	60.3k	–	14400.0
dem_100_gaslib135_40	48.0k	1624.3	2854.5	14400.0	1e+20	1055.5	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_41	1e+20	1624.3	–	14400.0	1e+20	1176.6	–	14400.0	41.2k	6845.5	502.6	14400.0
dem_100_gaslib135_42	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.6
dem_100_gaslib135_43	41.2k	1488.0	2672.2	14400.0	1e+20	975.9	–	14400.0	253.4k	1488.0	Large	14400.0
dem_100_gaslib135_44	251.5k	1488.0	Large	14400.0	1e+20	9441.1	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_45	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_100_gaslib135_46	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.3
dem_100_gaslib135_47	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	63.9
dem_100_gaslib135_48	1e+20	26.4k	–	14400.0	1e+20	36.6k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_49	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_100_gaslib135_50	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_100_gaslib135_51	1e+20	1624.3	–	14400.0	1e+20	21.1k	–	14400.0	54.3k	1488.0	3551.1	14400.0
dem_100_gaslib135_52	1e+20	0.0	–	14400.0	1e+20	2319.8	–	14400.0	34.9k	0.0	–	14400.0
dem_100_gaslib135_53	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9
dem_100_gaslib135_54	221.5k	1488.0	Large	14400.0	1e+20	30.1k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_55	403.0k	31.7k	1172.4	14400.0	1e+20	39.8k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_56	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.3
dem_100_gaslib135_57	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.6
dem_100_gaslib135_58	0.0	0.0	0.0	6.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_100_gaslib135_59	89.9k	36.5k	146.0	14400.0	1e+20	46.7k	–	14400.0	71.2k	36.4k	95.7	14400.0
dem_100_gaslib135_60	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_100_gaslib135_61	207.6k	0.0	–	14400.0	1e+20	6277.5	–	14400.0	1e+20	6885.8	–	14400.0
dem_100_gaslib135_62	1e+20	31.7k	–	14400.0	1e+20	38.3k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_63	1e+20	1488.0	–	14400.0	1e+20	4273.2	–	14400.0	41.2k	1488.0	2672.2	14400.0
dem_100_gaslib135_64	108.8k	42.5k	156.2	14400.0	1e+20	50.8k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_65	16.9k	14.5k	16.3	14400.0	1e+20	426.9	–	14400.0	18.4k	1488.0	1136.2	14400.0
dem_100_gaslib135_66	1e+20	1624.3	–	14400.0	1e+20	30.4k	–	14400.0	69.0k	1488.0	4538.3	14400.0
dem_100_gaslib135_67	1e+20	26.4k	–	14400.0	1e+20	35.6k	–	14400.0	1e+20	35.6k	–	14400.0
dem_100_gaslib135_68	61.2k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_69	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_100_gaslib135_70	41.2k	3229.4	1177.3	14400.0	1e+20	868.2	–	14400.0	48.0k	1488.0	3125.2	14400.0
dem_100_gaslib135_71	1e+20	26.4k	–	14400.0	1e+20	29.7k	–	14400.0	1e+20	26.5k	–	14400.0
dem_100_gaslib135_72	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	54.4
dem_100_gaslib135_73	85.9k	36.4k	136.1	14400.0	1e+20	59.0k	–	14400.0	78.8k	43.7k	80.3	14400.0
dem_100_gaslib135_74	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.0
dem_100_gaslib135_75	1e+20	31.7k	–	14400.0	1e+20	44.4k	–	14400.0	98.5k	31.7k	210.9	14400.0
dem_100_gaslib135_76	1e+20	36.5k	–	14400.0	1e+20	41.7k	–	14400.0	103.7k	36.4k	184.8	14400.0
dem_100_gaslib135_77	221.5k	26.4k	737.6	14400.0	1e+20	30.3k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_78	1e+20	1488.0	–	14400.0	1e+20	14.3k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_79	67.1k	36.4k	84.4	14400.0	1e+20	44.5k	–	14400.0	1e+20	36.5k	–	14400.0
dem_100_gaslib135_80	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_100_gaslib135_81	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.3
dem_100_gaslib135_82	1e+20	26.4k	–	14400.0	1e+20	33.4k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_83	1e+20	23.1k	–	14400.0	1e+20	6063.2	–	14400.0	55.8k	1488.0	3648.8	14400.0
dem_100_gaslib135_84	0.0	0.0	0.0	7.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.5
dem_100_gaslib135_85	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	99.2
dem_100_gaslib135_86	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	46.5
dem_100_gaslib135_87	1e+20	36.5k	–	14400.0	1e+20	46.1k	–	14400.0	1110.5k	36.4k	2950.9	14400.0
dem_100_gaslib135_88	3095.1k	26.4k	Large	14400.0	1e+20	32.8k	–	14400.0	135.6k	26.4k	413.0	14400.0
dem_100_gaslib135_89	327.0k	26.4k	1136.7	14400.0	1e+20	39.0k	–	14400.0	1e+20	26.4k	–	14400.0

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Table A.19: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_90	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_100_gaslib135_91	0.0	0.0	0.0	7.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	73.5
dem_100_gaslib135_92	1e+20	1488.0	–	14400.0	1e+20	1108.5	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_93	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.1
dem_100_gaslib135_94	1e+20	31.7k	–	14400.0	1e+20	36.7k	–	14400.0	60.3k	31.7k	90.1	14400.0
dem_100_gaslib135_95	1e+20	1488.0	–	14400.0	1e+20	1083.7	–	14400.0	48.0k	1488.0	3125.2	14400.0
dem_100_gaslib135_96	1e+20	1488.0	–	14400.0	1e+20	13.2k	–	14400.0	236.7k	1488.0	Large	14400.0
dem_100_gaslib135_97	1e+20	26.4k	–	14400.0	1e+20	36.1k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_98	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.2
dem_100_gaslib135_99	108.8k	36.5k	197.9	14400.0	1e+20	44.4k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_100	70.5k	1488.0	4638.3	14400.0	1e+20	1959.2	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_101	1e+20	25.0k	–	14400.0	1e+20	42.5k	–	14400.0	1053.7k	1488.0	Large	14400.0
dem_100_gaslib135_102	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_100_gaslib135_103	211.2k	1488.0	Large	14400.0	1e+20	4579.9	–	14400.0	375.0k	1488.0	Large	14400.0
dem_100_gaslib135_104	0.0	0.0	0.0	6.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	61.6
dem_100_gaslib135_105	0.0	0.0	0.0	7.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.6
dem_100_gaslib135_106	1e+20	3229.4	–	14400.0	1e+20	2730.9	–	14400.0	1e+20	2730.9	–	14400.0
dem_100_gaslib135_107	1e+20	31.7k	–	14400.0	1e+20	42.5k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_108	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	51.2
dem_100_gaslib135_109	44.5k	0.0	–	14400.0	1e+20	0.0	–	14400.0	113.1k	41.3k	173.8	14400.0
dem_100_gaslib135_110	51.8k	1488.0	3382.4	14400.0	1e+20	15.3k	–	14400.0	700.4k	1488.0	Large	14400.0
dem_100_gaslib135_111	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.7
dem_100_gaslib135_112	53.5k	1624.5	3196.3	14400.0	1e+20	13.5k	–	14400.0	54.3k	1488.0	3551.1	14400.0
dem_100_gaslib135_113	14.0k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1155.8k	0.0	Large	14400.0
dem_100_gaslib135_114	1e+20	36.4k	–	14400.0	1e+20	51.8k	–	14400.0	155.0k	45.4k	241.3	14400.0
dem_100_gaslib135_115	1e+20	31.7k	–	14400.0	1e+20	49.2k	–	14400.0	70.1k	31.7k	121.2	14400.0
dem_100_gaslib135_116	71.2k	42.9k	66.2	14400.0	1e+20	59.7k	–	14400.0	3635.1k	36.4k	9886.8	14400.0
dem_100_gaslib135_117	14.5k	0.0	–	14400.0	1e+20	232.9	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_118	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_100_gaslib135_119	48.0k	1624.3	2854.5	14400.0	1e+20	2719.7	–	14400.0	53.5k	1488.0	3493.3	14400.0
dem_100_gaslib135_120	1e+20	0.0	–	14400.0	1e+20	156.7	–	14400.0	19.1k	14.2k	35.0	14400.0
dem_100_gaslib135_121	66.6k	1488.0	4377.7	14400.0	1e+20	38.9k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_122	1e+20	26.4k	–	14400.0	1e+20	38.0k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_123	0.0	0.0	0.0	55.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_100_gaslib135_124	0.0	0.0	0.0	7.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	66.2
dem_100_gaslib135_125	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	184.6k	0.0	–	14400.0
dem_100_gaslib135_126	201.4k	36.4k	453.3	14400.0	1e+20	45.5k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_127	1e+20	1624.3	–	14400.0	1e+20	32.1k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_128	11.6k	11.6k	0.0	12336.5	1e+20	0.0	–	14400.0	446.6k	0.0	Large	14400.0
dem_100_gaslib135_129	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_100_gaslib135_130	0.0	0.0	0.0	7.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	74.3
dem_100_gaslib135_131	54.3k	1624.3	3244.7	14400.0	1e+20	28.2k	–	14400.0	56.8k	1488.0	3715.9	14400.0
dem_100_gaslib135_132	1e+20	1488.0	–	14400.0	1e+20	20.0k	–	14400.0	54.3k	1488.0	3551.1	14400.0
dem_100_gaslib135_133	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	59.0
dem_100_gaslib135_134	72.9k	0.0	–	14400.0	1e+20	0.0	–	14400.0	924.8k	0.0	Large	14400.0
dem_100_gaslib135_135	0.0	0.0	0.0	5.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_100_gaslib135_136	0.0	0.0	0.0	7.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	98.7
dem_100_gaslib135_137	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.4
dem_100_gaslib135_138	77.6k	0.0	–	14400.0	1e+20	1096.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_139	1e+20	1624.3	–	14400.0	1e+20	14.9k	–	14400.0	54.3k	23.9k	127.6	14400.0
dem_100_gaslib135_140	64.9k	58.2k	11.6	14400.0	1e+20	41.9k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_141	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_142	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	59.9
dem_100_gaslib135_143	66.6k	34.9k	90.8	14400.0	1e+20	39.4k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_144	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	50.5
dem_100_gaslib135_145	1474.8k	0.0	–	14400.0	1e+20	21.0	–	14400.0	1e+20	8576.8	–	14400.0
dem_100_gaslib135_146	73.5k	71.2k	3.1	14400.0	1e+20	47.6k	–	14400.0	73.5k	58.9k	24.7	14400.0
dem_100_gaslib135_147	246.4k	4758.5	5078.2	14400.0	1e+20	1721.5	–	14400.0	49.5k	1501.5	3195.3	14400.0
dem_100_gaslib135_148	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.9
dem_100_gaslib135_149	1e+20	33.4k	–	14400.0	1e+20	38.6k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_150	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	77.2
dem_100_gaslib135_151	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.1
dem_100_gaslib135_152	47.2k	1624.3	2806.5	14400.0	1e+20	5830.1	–	14400.0	61.9k	1488.0	4058.3	14400.0
dem_100_gaslib135_153	1e+20	50.9k	–	14400.0	1e+20	42.0k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_154	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.7
dem_100_gaslib135_155	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.8
dem_100_gaslib135_156	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	66.0
dem_100_gaslib135_157	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.8
dem_100_gaslib135_158	0.0	0.0	0.0	5.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.9
dem_100_gaslib135_159	1e+20	26.4k	–	14400.0	1e+20	46.9k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_160	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	41.8
dem_100_gaslib135_161	1e+20	1488.0	–	14400.0	1e+20	29.5k	–	14400.0	60.4k	1488.0	3958.4	14400.0
dem_100_gaslib135_162	54.3k	0.0	–	14400.0	1e+20	273.4	–	14400.0	795.1k	0.0	Large	14400.0
dem_100_gaslib135_163	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_100_gaslib135_164	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.7
dem_100_gaslib135_165	74.2k	31.7k	134.2	14400.0	1e+20	46.0k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_166	48.0k	1488.0	3125.2	14400.0	1e+20	9554.1	–	14400.0	49.5k	1488.0	3225.2	14400.0
dem_100_gaslib135_167	99.4k	86.2k	15.3	14400.0	1e+20	69.3k	–	14400.0	609.0k	51.9k	1073.8	14400.0

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Table A.19: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_168	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	78.3
dem_100_gaslib135_169	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.2
dem_100_gaslib135_170	1e+20	26.4k	–	14400.0	1e+20	44.4k	–	14400.0	58.2k	31.3k	86.1	14400.0
dem_100_gaslib135_171	48.0k	41.2k	16.3	14400.0	1e+20	15.3k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_172	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.6
dem_100_gaslib135_173	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	77.7
dem_100_gaslib135_174	114.7k	50.1k	129.1	14400.0	1e+20	61.1k	–	14400.0	1e+20	50.0k	–	14400.0
dem_100_gaslib135_175	54.3k	1488.0	3551.1	14400.0	1e+20	20.4k	–	14400.0	41.2k	20.8k	98.6	14400.0
dem_100_gaslib135_176	1e+20	1488.0	–	14400.0	1e+20	10.1k	–	14400.0	1e+20	10.1k	–	14400.0
dem_100_gaslib135_177	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	74.5
dem_100_gaslib135_178	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_100_gaslib135_179	1e+20	0.0	–	14400.0	1e+20	1804.6	–	14400.0	269.8k	0.0	Large	14400.0
dem_100_gaslib135_180	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_100_gaslib135_181	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.5
dem_100_gaslib135_182	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	65.2
dem_100_gaslib135_183	391.9k	52.2k	650.9	14400.0	1e+20	66.7k	–	14400.0	1e+20	51.9k	–	14400.0
dem_100_gaslib135_184	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.8
dem_100_gaslib135_185	34.9k	3229.4	981.1	14400.0	1e+20	331.2	–	14400.0	33.1k	1488.0	2125.5	14400.0
dem_100_gaslib135_186	365.9k	25.0k	1366.4	14400.0	1e+20	19.3k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_187	0.0	0.0	0.0	3.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	63.2
dem_100_gaslib135_188	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_100_gaslib135_189	48.0k	1624.3	2854.5	14400.0	1e+20	2245.3	–	14400.0	687.0k	1488.0	Large	14400.0
dem_100_gaslib135_190	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	59.8
dem_100_gaslib135_191	1e+20	29.3k	–	14400.0	1e+20	36.8k	–	14400.0	353.7k	26.4k	1237.6	14400.0
dem_100_gaslib135_192	1e+20	31.8k	–	14400.0	1e+20	47.2k	–	14400.0	69.8k	31.7k	120.4	14400.0
dem_100_gaslib135_193	1e+20	0.0	–	14400.0	1e+20	414.3	–	14400.0	21.8k	21.8k	0.0	8597.0
dem_100_gaslib135_194	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	52.4
dem_100_gaslib135_195	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	58.1
dem_100_gaslib135_196	92.5k	50.0k	84.8	14400.0	1e+20	55.7k	–	14400.0	238.4k	52.0k	358.7	14400.0
dem_100_gaslib135_197	375.0k	29.5k	1170.4	14400.0	1e+20	39.6k	–	14400.0	77.8k	26.4k	194.3	14400.0
dem_100_gaslib135_198	0.0	0.0	0.0	3.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.1
dem_100_gaslib135_199	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	39.2
dem_100_gaslib135_200	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.9
dem_100_gaslib135_201	1e+20	51.9k	–	14400.0	1e+20	62.9k	–	14400.0	1e+20	53.5k	–	14400.0
dem_100_gaslib135_202	103.1k	51.9k	98.7	14400.0	1e+20	57.8k	–	14400.0	1e+20	51.9k	–	14400.0
dem_100_gaslib135_203	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_100_gaslib135_204	41.2k	0.0	–	14400.0	1e+20	906.1	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_205	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.3
dem_100_gaslib135_206	56.8k	1488.0	3715.9	14400.0	1e+20	10.4k	–	14400.0	58.3k	1488.0	3815.9	14400.0
dem_100_gaslib135_207	55.8k	55.8k	0.0	2759.2	1e+20	21.7k	–	14400.0	96.5k	1488.0	6387.5	14400.0
dem_100_gaslib135_208	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	58.8
dem_100_gaslib135_209	123.0k	0.0	–	14400.0	1e+20	0.0	–	14400.0	264.2k	0.0	–	14400.0
dem_100_gaslib135_210	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	74.1
dem_100_gaslib135_211	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.5
dem_100_gaslib135_212	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.1
dem_100_gaslib135_213	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	48.3
dem_100_gaslib135_214	96.5k	1624.3	5843.1	14400.0	1e+20	17.6k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_215	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_100_gaslib135_216	69.0k	26.4k	161.0	14400.0	1e+20	32.3k	–	14400.0	61.9k	44.8k	38.1	14400.0
dem_100_gaslib135_217	1e+20	1488.0	–	14400.0	1e+20	28.9k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_218	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.2
dem_100_gaslib135_219	239.6k	0.0	–	14400.0	1e+20	202.2	–	14400.0	1e+20	24.0k	–	14400.0
dem_100_gaslib135_220	71.2k	40.8k	74.5	14400.0	1e+20	52.1k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_221	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	1.5
dem_100_gaslib135_222	1e+20	1488.0	–	14400.0	1e+20	862.2	–	14400.0	41.2k	1488.0	2672.2	14400.0
dem_100_gaslib135_223	1e+20	36.4k	–	14400.0	1e+20	39.8k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_224	0.0	0.0	0.0	3.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.5
dem_100_gaslib135_225	0.0	0.0	0.0	48.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_100_gaslib135_226	388.8k	26.4k	1370.6	14400.0	1e+20	36.3k	–	14400.0	1e+20	32.4k	–	14400.0
dem_100_gaslib135_227	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.8
dem_100_gaslib135_228	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	62.3
dem_100_gaslib135_229	41.2k	1624.3	2439.5	14400.0	1e+20	562.7	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_230	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_100_gaslib135_231	60.3k	1624.3	3611.7	14400.0	1e+20	21.4k	–	14400.0	65.3k	1488.0	4286.4	14400.0
dem_100_gaslib135_232	1e+20	36.4k	–	14400.0	1e+20	42.0k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_233	0.0	0.0	0.0	49.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.4
dem_100_gaslib135_234	56.7k	11.8k	381.2	14400.0	1e+20	30.7k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_235	0.0	0.0	0.0	6.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_100_gaslib135_236	1e+20	0.0	–	14400.0	1e+20	292.7	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_237	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	83.7
dem_100_gaslib135_238	92.0k	31.7k	190.6	14400.0	1e+20	40.3k	–	14400.0	421.7k	31.7k	1231.6	14400.0
dem_100_gaslib135_239	1e+20	30.2k	–	14400.0	1e+20	41.8k	–	14400.0	334.7k	30.2k	1008.9	14400.0
dem_100_gaslib135_240	1e+20	1624.3	–	14400.0	1e+20	9550.7	–	14400.0	69.0k	1488.0	4538.3	14400.0
dem_100_gaslib135_241	34.0k	1624.3	1993.7	14400.0	1e+20	322.9	–	14400.0	35.5k	1488.0	2285.5	14400.0
dem_100_gaslib135_242	433.6k	83.7k	418.2	14400.0	1e+20	71.4k	–	14400.0	105.3k	76.7k	37.2	14400.0
dem_100_gaslib135_243	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_100_gaslib135_244	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_100_gaslib135_245	211.2k	26.4k	698.7	14400.0	1e+20	31.4k	–	14400.0	54.3k	26.5k	105.4	14400.0

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Table A.19: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_246	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_247	1e+20	1488.0	–	14400.0	1e+20	13.7k	–	14400.0	48.0k	3971.7	1108.3	14400.0
dem_100_gaslib135_248	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	77.6
dem_100_gaslib135_249	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	50.8
dem_100_gaslib135_250	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	69.5
dem_100_gaslib135_251	0.0	0.0	0.0	6.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	76.7
dem_100_gaslib135_252	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_100_gaslib135_253	55.8k	28.9k	93.3	14400.0	1e+20	37.2k	–	14400.0	375.0k	1488.0	Large	14400.0
dem_100_gaslib135_254	0.0	0.0	0.0	49.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_100_gaslib135_255	0.0	0.0	0.0	5.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	65.5
dem_100_gaslib135_256	1e+20	1488.0	–	14400.0	1e+20	16.6k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_257	69.0k	1624.3	4149.0	14400.0	1e+20	19.1k	–	14400.0	82.0k	1488.0	5410.7	14400.0
dem_100_gaslib135_258	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	347.4k	0.0	–	14400.0
dem_100_gaslib135_259	346.5k	10.9k	3075.5	14400.0	1e+20	37.0k	–	14400.0	123.0k	1488.0	8165.4	14400.0
dem_100_gaslib135_260	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.0
dem_100_gaslib135_261	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	72.9
dem_100_gaslib135_262	54.3k	1624.3	3244.7	14400.0	1e+20	9200.0	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_263	529.3k	0.0	–	14400.0	1e+20	361.7	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_264	0.0	0.0	0.0	4.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.7
dem_100_gaslib135_265	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	96.9
dem_100_gaslib135_266	1e+20	0.0	–	14400.0	1e+20	1.8	–	14400.0	19.1k	0.0	Large	14400.0
dem_100_gaslib135_267	1e+20	1624.3	–	14400.0	1e+20	31.7k	–	14400.0	51.9k	1488.0	3386.8	14400.0
dem_100_gaslib135_268	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_100_gaslib135_269	1e+20	58.6k	–	14400.0	1e+20	64.0k	–	14400.0	96.7k	62.8k	54.1	14400.0
dem_100_gaslib135_270	62.5k	62.5k	0.0	10979.7	1e+20	36.5k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_271	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	49.2
dem_100_gaslib135_272	1e+20	30.2k	–	14400.0	1e+20	36.5k	–	14400.0	66.6k	26.5k	151.3	14400.0
dem_100_gaslib135_273	76.1k	50.0k	52.1	14400.0	1e+20	59.8k	–	14400.0	1e+20	50.0k	–	14400.0
dem_100_gaslib135_274	1e+20	1488.0	–	14400.0	1e+20	24.6k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_275	139.5k	45.5k	206.4	14400.0	1e+20	56.7k	–	14400.0	1e+20	56.7k	–	14400.0
dem_100_gaslib135_276	520.4k	58.6k	787.6	14400.0	1e+20	78.4k	–	14400.0	541.5k	61.0k	787.2	14400.0
dem_100_gaslib135_277	1e+20	31.7k	–	14400.0	1e+20	48.0k	–	14400.0	1097.7k	31.7k	3366.1	14400.0
dem_100_gaslib135_278	0.0	0.0	0.0	51.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	83.1
dem_100_gaslib135_279	10.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0	142.4k	0.0	–	14400.0
dem_100_gaslib135_280	640.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0	132.1k	0.0	Large	14400.0
dem_100_gaslib135_281	0.0	0.0	0.0	48.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_100_gaslib135_282	0.0	0.0	0.0	6.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_100_gaslib135_283	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	92.5
dem_100_gaslib135_284	48.0k	25.0k	92.3	14400.0	1e+20	29.2k	–	14400.0	48.0k	25.0k	92.3	14400.0
dem_100_gaslib135_285	1e+20	36.5k	–	14400.0	1e+20	54.5k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_286	1e+20	31.7k	–	14400.0	1e+20	38.9k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_287	210.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0	132.1k	0.0	Large	14400.0
dem_100_gaslib135_288	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	53.1
dem_100_gaslib135_289	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	91.9
dem_100_gaslib135_290	0.0	0.0	0.0	3.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_100_gaslib135_291	0.0	0.0	0.0	6.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_100_gaslib135_292	1e+20	36.4k	–	14400.0	1e+20	39.0k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_293	55.8k	30.2k	84.8	14400.0	1e+20	33.0k	–	14400.0	55.8k	26.9k	107.5	14400.0
dem_100_gaslib135_294	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.2
dem_100_gaslib135_295	1e+20	0.0	–	14400.0	1e+20	986.3	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_296	65.6k	42.3k	55.0	14400.0	1e+20	39.2k	–	14400.0	1e+20	39.2k	–	14400.0
dem_100_gaslib135_297	1e+20	0.0	–	14400.0	1e+20	346.2	–	14400.0	48.0k	48.0k	0.0	3676.7
dem_100_gaslib135_298	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	424.7k	0.0	–	14400.0
dem_100_gaslib135_299	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	45.1
dem_100_gaslib135_300	0.0	0.0	0.0	6.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.7
dem_100_gaslib135_301	60.4k	60.4k	0.0	11978.8	1e+20	25.4k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_302	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	62.4
dem_100_gaslib135_303	54.3k	1488.0	3551.1	14400.0	1e+20	12.2k	–	14400.0	48.0k	1488.0	3125.2	14400.0
dem_100_gaslib135_304	48.0k	1488.0	3125.2	14400.0	1e+20	5888.9	–	14400.0	48.0k	1490.3	3120.2	14400.0
dem_100_gaslib135_305	0.0	0.0	0.0	6.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.4
dem_100_gaslib135_306	74.2k	31.7k	134.2	14400.0	1e+20	40.7k	–	14400.0	75.3k	33.3k	126.3	14400.0
dem_100_gaslib135_307	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	68.9
dem_100_gaslib135_308	125.0k	4542.9	2651.7	14400.0	1e+20	7678.5	–	14400.0	61.9k	1488.0	4058.2	14400.0
dem_100_gaslib135_309	0.0	0.0	0.0	6.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	66.4
dem_100_gaslib135_310	0.0	0.0	0.0	5.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	84.4
dem_100_gaslib135_311	96.8k	31.7k	205.7	14400.0	1e+20	45.2k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_312	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6
dem_100_gaslib135_313	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.8
dem_100_gaslib135_314	1e+20	1488.0	–	14400.0	1e+20	12.3k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_315	1e+20	1488.0	–	14400.0	1e+20	16.4k	–	14400.0	211.2k	1488.0	Large	14400.0
dem_100_gaslib135_316	64.9k	34.9k	85.9	14400.0	1e+20	43.8k	–	14400.0	221.1k	31.7k	598.2	14400.0
dem_100_gaslib135_317	41.2k	1624.3	2439.5	14400.0	1e+20	1469.0	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_318	0.0	0.0	0.0	7.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.7
dem_100_gaslib135_319	97.0k	1488.0	6420.5	14400.0	1e+20	564.6	–	14400.0	34.9k	1488.0	2246.2	14400.0
dem_100_gaslib135_320	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0	0.0	0.0	0.0	70.1
dem_100_gaslib135_321	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.8
dem_100_gaslib135_322	754.1k	1488.0	Large	14400.0	1e+20	7534.1	–	14400.0	48.0k	1488.0	3125.2	14400.0
dem_100_gaslib135_323	94.5k	3315.4	2749.6	14400.0	1e+20	24.4k	–	14400.0	58.2k	1488.0	3808.4	14400.0

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Table A.19: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_324	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.1
dem_100_gaslib135_325	162.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0	66.9k	0.0	Large	14400.0
dem_100_gaslib135_326	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.2
dem_100_gaslib135_327	335.6k	83.1k	303.8	14400.0	1e+20	104.6k	–	14400.0	259.1k	83.1k	211.8	14400.0
dem_100_gaslib135_328	21.8k	0.0	–	14400.0	1e+20	238.2	–	14400.0	27.9k	0.4	Large	14400.0
dem_100_gaslib135_329	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_100_gaslib135_330	1480.0k	1624.3	Large	14400.0	1e+20	9776.9	–	14400.0	55.8k	1488.0	3651.1	14400.0
dem_100_gaslib135_331	100.7k	38.1k	164.0	14400.0	1e+20	51.6k	–	14400.0	1e+20	43.7k	–	14400.0
dem_100_gaslib135_332	1495.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0	14.5k	0.0	Large	14400.0
dem_100_gaslib135_333	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	421.5
dem_100_gaslib135_334	48.0k	25.0k	92.3	14400.0	1e+20	29.4k	–	14400.0	48.0k	22.6k	112.4	14400.0
dem_100_gaslib135_335	1e+20	31.7k	–	14400.0	1e+20	38.6k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_336	0.0	0.0	0.0	7.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.5
dem_100_gaslib135_337	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.7
dem_100_gaslib135_338	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.3
dem_100_gaslib135_339	376.5k	26.4k	1324.1	14400.0	1e+20	37.3k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_340	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	80.4
dem_100_gaslib135_341	51.8k	47.2k	9.8	14400.0	1e+20	29.7k	–	14400.0	77.6k	1488.0	5113.0	14400.0
dem_100_gaslib135_342	0.0	0.0	0.0	50.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.1
dem_100_gaslib135_343	99.4k	42.7k	132.5	14400.0	1e+20	55.2k	–	14400.0	139.9k	42.7k	227.4	14400.0
dem_100_gaslib135_344	0.0	0.0	0.0	5.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.8
dem_100_gaslib135_345	0.0	0.0	0.0	48.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	69.2
dem_100_gaslib135_346	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_100_gaslib135_347	1e+20	26.4k	–	14400.0	1e+20	33.4k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_348	387.3k	31.7k	1123.0	14400.0	1e+20	42.8k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_349	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	51.0
dem_100_gaslib135_350	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	56.0
dem_100_gaslib135_351	1e+20	58.6k	–	14400.0	1e+20	93.0k	–	14400.0	1e+20	60.3k	–	14400.0
dem_100_gaslib135_352	237.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0	2934.5k	0.0	Large	14400.0
dem_100_gaslib135_353	1e+20	0.0	–	14400.0	1e+20	401.4	–	14400.0	1232.7k	0.0	–	14400.0
dem_100_gaslib135_354	86.4k	31.7k	172.7	14400.0	1e+20	45.8k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_355	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_100_gaslib135_356	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	74.7
dem_100_gaslib135_357	1e+20	1488.0	–	14400.0	1e+20	2734.4	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_358	263.8k	31.7k	732.9	14400.0	1e+20	39.9k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_359	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	91.5
dem_100_gaslib135_360	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_100_gaslib135_361	1e+20	36.4k	–	14400.0	1e+20	43.6k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_362	1e+20	1624.3	–	14400.0	1e+20	34.7k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_363	861.1k	0.0	–	14400.0	1e+20	98.4	–	14400.0	446.5k	0.0	Large	14400.0
dem_100_gaslib135_364	1e+20	31.7k	–	14400.0	1e+20	37.8k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_365	54.3k	1624.3	3244.7	14400.0	1e+20	6020.8	–	14400.0	49.5k	1488.1	3225.0	14400.0
dem_100_gaslib135_366	1e+20	0.0	–	14400.0	1e+20	493.4	–	14400.0	364.7k	0.0	–	14400.0
dem_100_gaslib135_367	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	49.0
dem_100_gaslib135_368	0.0	0.0	0.0	7.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_100_gaslib135_369	1113.3k	1488.0	Large	14400.0	1e+20	24.9k	–	14400.0	58.2k	1489.8	3803.8	14400.0
dem_100_gaslib135_370	79.7k	26.4k	201.3	14400.0	1e+20	36.9k	–	14400.0	458.5k	26.4k	1634.1	14400.0
dem_100_gaslib135_371	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_372	60.4k	58.2k	3.8	14400.0	1e+20	35.8k	–	14400.0	65.6k	26.4k	148.0	14400.0
dem_100_gaslib135_373	258.8k	1624.3	Large	14400.0	1e+20	33.5k	–	14400.0	426.5k	1488.0	Large	14400.0
dem_100_gaslib135_374	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_100_gaslib135_375	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	50.2
dem_100_gaslib135_376	54.3k	3588.0	1414.2	14400.0	1e+20	18.7k	–	14400.0	58.3k	1488.0	3815.9	14400.0
dem_100_gaslib135_377	61.9k	3229.4	1816.0	14400.0	1e+20	1335.9	–	14400.0	122.1k	1488.0	8104.7	14400.0
dem_100_gaslib135_378	1e+20	31.7k	–	14400.0	1e+20	37.5k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_379	0.0	0.0	0.0	7.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.0
dem_100_gaslib135_380	1e+20	53.5k	–	14400.0	1e+20	62.1k	–	14400.0	1e+20	52.3k	–	14400.0
dem_100_gaslib135_381	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9
dem_100_gaslib135_382	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_100_gaslib135_383	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.9
dem_100_gaslib135_384	1e+20	1488.0	–	14400.0	1e+20	5234.1	–	14400.0	54.3k	6482.0	738.2	14400.0
dem_100_gaslib135_385	495.8k	1624.3	Large	14400.0	1e+20	807.2	–	14400.0	41.2k	3719.7	1008.9	14400.0
dem_100_gaslib135_386	1e+20	26.4k	–	14400.0	1e+20	37.3k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_387	288.0k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_388	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.0
dem_100_gaslib135_389	250.0k	1624.3	Large	14400.0	1e+20	2785.2	–	14400.0	43.7k	10.7k	309.9	14400.0
dem_100_gaslib135_390	1e+20	1488.0	–	14400.0	1e+20	2451.4	–	14400.0	840.6k	1488.0	Large	14400.0
dem_100_gaslib135_391	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.4
dem_100_gaslib135_392	1e+20	1488.0	–	14400.0	1e+20	2144.6	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_393	44.7k	1488.0	2905.0	14400.0	1e+20	3330.6	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_394	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.9
dem_100_gaslib135_395	0.0	0.0	0.0	49.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_100_gaslib135_396	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.4
dem_100_gaslib135_397	1e+20	1624.3	–	14400.0	1e+20	32.2k	–	14400.0	221.5k	1488.0	Large	14400.0
dem_100_gaslib135_398	687.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_399	1e+20	23.5k	–	14400.0	1e+20	24.9k	–	14400.0	180.1k	1488.0	Large	14400.0
dem_100_gaslib135_400	1e+20	36.5k	–	14400.0	1e+20	47.4k	–	14400.0	233.8k	36.4k	542.4	14400.0
dem_100_gaslib135_401	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.6

continued on next page

Table A.19: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_402	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_100_gaslib135_403	1e+20	31.7k	–	14400.0	1e+20	35.9k	–	14400.0	67.1k	32.8k	104.8	14400.0
dem_100_gaslib135_404	1e+20	26.4k	–	14400.0	1e+20	45.2k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_405	1e+20	36.4k	–	14400.0	1e+20	52.5k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_406	166.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0	155.9k	0.0	–	14400.0
dem_100_gaslib135_407	69.0k	1488.0	4538.3	14400.0	1e+20	11.3k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_408	1e+20	0.0	–	14400.0	1e+20	212.7	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_409	1e+20	1624.3	–	14400.0	1e+20	33.5k	–	14400.0	86.6k	1488.0	5719.7	14400.0
dem_100_gaslib135_410	1e+20	1488.0	–	14400.0	1e+20	15.1k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_411	1e+20	0.0	–	14400.0	1e+20	9.2	–	14400.0	19.1k	0.0	Large	14400.0
dem_100_gaslib135_412	42.5k	1798.6	2261.9	14400.0	1e+20	910.3	–	14400.0	43.7k	1488.0	2836.9	14400.0
dem_100_gaslib135_413	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	1e+20	0.0	8793.1
dem_100_gaslib135_414	1e+20	1488.0	–	14400.0	1e+20	1409.9	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_415	2118.0k	31.7k	6587.4	14400.0	1e+20	41.4k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_416	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_100_gaslib135_417	0.0	0.0	0.0	5.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	8.6
dem_100_gaslib135_418	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.2
dem_100_gaslib135_419	0.0	0.0	0.0	6.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	66.0
dem_100_gaslib135_420	1e+20	1488.0	–	14400.0	1e+20	13.2k	–	14400.0	61.9k	1488.0	4058.2	14400.0
dem_100_gaslib135_421	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	74.5
dem_100_gaslib135_422	48.0k	1488.0	3125.2	14400.0	1e+20	9428.7	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_423	1e+20	1624.3	–	14400.0	1e+20	37.8k	–	14400.0	79.7k	8910.0	794.0	14400.0
dem_100_gaslib135_424	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.9
dem_100_gaslib135_425	1e+20	43.7k	–	14400.0	1e+20	52.9k	–	14400.0	75.3k	45.5k	65.4	14400.0
dem_100_gaslib135_426	1e+20	51.9k	–	14400.0	1e+20	62.0k	–	14400.0	1e+20	51.9k	–	14400.0
dem_100_gaslib135_427	334.7k	31.7k	956.7	14400.0	1e+20	36.7k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_428	49.4k	4542.9	988.4	14400.0	1e+20	25.3k	–	14400.0	49.4k	1488.0	3222.8	14400.0
dem_100_gaslib135_429	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	30.2k	0.0	Large	14400.0
dem_100_gaslib135_430	1e+20	26.4k	–	14400.0	1e+20	35.8k	–	14400.0	60.4k	26.6k	127.3	14400.0
dem_100_gaslib135_431	0.0	0.0	0.0	53.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.2
dem_100_gaslib135_432	0.0	0.0	0.0	3.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	76.9
dem_100_gaslib135_433	1e+20	36.4k	–	14400.0	1e+20	41.3k	–	14400.0	68.9k	46.1k	49.4	14400.0
dem_100_gaslib135_434	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.6
dem_100_gaslib135_435	1e+20	60.3k	–	14400.0	1e+20	70.7k	–	14400.0	101.9k	101.9k	0.0	1747.4
dem_100_gaslib135_436	1e+20	31.7k	–	14400.0	1e+20	35.5k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_437	1e+20	1488.0	–	14400.0	1e+20	33.9k	–	14400.0	60.3k	8586.9	602.1	14400.0
dem_100_gaslib135_438	74.2k	34.9k	112.5	14400.0	1e+20	43.3k	–	14400.0	1e+20	34.7k	–	14400.0
dem_100_gaslib135_439	73.5k	42.9k	71.4	14400.0	1e+20	57.8k	–	14400.0	263.8k	42.7k	517.3	14400.0
dem_100_gaslib135_440	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	69.5
dem_100_gaslib135_441	48.0k	47.1k	1.9	14400.0	1e+20	10.3k	–	14400.0	1e+20	1488.0	–	14400.0
dem_100_gaslib135_442	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.7
dem_100_gaslib135_443	48.0k	1624.3	2854.5	14400.0	1e+20	475.4	–	14400.0	48.0k	1490.0	3120.9	14400.0
dem_100_gaslib135_444	8790.1	0.0	–	14400.0	1e+20	0.0	–	14400.0	553.3k	0.0	–	14400.0
dem_100_gaslib135_445	0.0	0.0	0.0	7.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	65.3
dem_100_gaslib135_446	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.4
dem_100_gaslib135_447	1e+20	1488.0	–	14400.0	1e+20	26.6k	–	14400.0	434.2k	1488.0	Large	14400.0
dem_100_gaslib135_448	16.9k	12.3k	37.5	14400.0	1e+20	158.5	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_449	48.0k	1488.0	3125.2	14400.0	1e+20	15.9k	–	14400.0	48.0k	1514.8	3068.1	14400.0
dem_100_gaslib135_450	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.6
dem_100_gaslib135_451	1e+20	3229.4	–	14400.0	1e+20	6225.8	–	14400.0	517.6k	1488.0	Large	14400.0
dem_100_gaslib135_452	61.9k	1488.0	4058.2	14400.0	1e+20	9918.2	–	14400.0	48.0k	1488.0	3125.2	14400.0
dem_100_gaslib135_453	142.4k	36.4k	291.2	14400.0	1e+20	61.9k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_454	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_100_gaslib135_455	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.5
dem_100_gaslib135_456	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.7
dem_100_gaslib135_457	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.0
dem_100_gaslib135_458	12.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_459	18.5k	0.0	–	14400.0	1e+20	0.0	–	14400.0	692.5k	0.0	Large	14400.0
dem_100_gaslib135_460	1e+20	36.5k	–	14400.0	1e+20	57.8k	–	14400.0	1e+20	37.9k	–	14400.0
dem_100_gaslib135_461	62.2k	1624.3	3731.7	14400.0	1e+20	42.0k	–	14400.0	77.6k	1488.0	5113.0	14400.0
dem_100_gaslib135_462	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	55.6
dem_100_gaslib135_463	573.1k	42.9k	1236.7	14400.0	1e+20	63.4k	–	14400.0	102.2k	45.2k	125.9	14400.0
dem_100_gaslib135_464	1e+20	36.5k	–	14400.0	1e+20	48.9k	–	14400.0	1e+20	36.4k	–	14400.0
dem_100_gaslib135_465	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	5.1
dem_100_gaslib135_466	14.5k	14.5k	0.0	4403.3	1e+20	0.0	–	14400.0	2903.9k	0.0	Large	14400.0
dem_100_gaslib135_467	1e+20	1488.0	–	14400.0	1e+20	13.1k	–	14400.0	253.9k	1488.0	Large	14400.0
dem_100_gaslib135_468	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	97.5
dem_100_gaslib135_469	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.8
dem_100_gaslib135_470	58.2k	27.0k	115.1	14400.0	1e+20	37.7k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_471	1110.0k	50.0k	2118.1	14400.0	1e+20	57.2k	–	14400.0	1e+20	51.9k	–	14400.0
dem_100_gaslib135_472	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.6
dem_100_gaslib135_473	41.2k	5742.2	618.4	14400.0	1e+20	2971.0	–	14400.0	784.5k	1488.0	Large	14400.0
dem_100_gaslib135_474	1e+20	0.0	–	14400.0	1e+20	4270.3	–	14400.0	125.0k	0.0	Large	14400.0
dem_100_gaslib135_475	1e+20	31.7k	–	14400.0	1e+20	37.9k	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_476	48.0k	1624.3	2854.5	14400.0	1e+20	2290.9	–	14400.0	41.2k	1610.1	2462.0	14400.0
dem_100_gaslib135_477	122.9k	1624.3	7466.4	14400.0	1e+20	582.6	–	14400.0	34.9k	1488.0	2246.2	14400.0
dem_100_gaslib135_478	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	67.9
dem_100_gaslib135_479	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	41.2

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Table A.19: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_480	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_100_gaslib135_481	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0	0.0	0.0	0.0	10.8
dem_100_gaslib135_482	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	57.0
dem_100_gaslib135_483	0.0	0.0	0.0	46.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	2.5
dem_100_gaslib135_484	1e+20	26.4k	–	14400.0	1e+20	45.9k	–	14400.0	68.9k	26.4k	160.5	14400.0
dem_100_gaslib135_485	51.8k	51.8k	0.0	7241.3	1e+20	32.5k	–	14400.0	1055.3k	1566.7	Large	14400.0
dem_100_gaslib135_486	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	11.6
dem_100_gaslib135_487	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	48.7
dem_100_gaslib135_488	1e+20	26.4k	–	14400.0	1e+20	49.3k	–	14400.0	1e+20	26.4k	–	14400.0
dem_100_gaslib135_489	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.0
dem_100_gaslib135_490	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_100_gaslib135_491	1e+20	26.4k	–	14400.0	1e+20	38.3k	–	14400.0	65.6k	65.6k	0.0	10809.1
dem_100_gaslib135_492	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0	0.0	0.0	0.0	90.7
dem_100_gaslib135_493	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.3
dem_100_gaslib135_494	10.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0	48.0k	0.0	Large	14400.0
dem_100_gaslib135_495	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.1
dem_100_gaslib135_496	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.5
dem_100_gaslib135_497	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1140.9k	0.0	–	14400.0
dem_100_gaslib135_498	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.9
dem_100_gaslib135_499	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0	0.0	0.0	0.0	3.9
dem_100_gaslib135_500	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5

Table A.20: Detailed results of the split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ as summarized in Figure 3.15b and Table 3.4c. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_1	1e+20	0.0	–	14400.0	1e+20	12.8	–	14400.0
dem_100_gaslib135_2	0.0	0.0	0.0	52.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_3	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_4	48.0k	1624.3	2854.5	14400.0	1e+20	8653.7	–	14400.0
dem_100_gaslib135_5	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_6	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_7	1e+20	28.2k	–	14400.0	1e+20	36.7k	–	14400.0
dem_100_gaslib135_8	74.2k	31.7k	134.2	14400.0	1e+20	44.5k	–	14400.0
dem_100_gaslib135_9	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_10	0.0	0.0	0.0	6.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_11	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_12	103.1k	36.5k	182.1	14400.0	1e+20	54.3k	–	14400.0
dem_100_gaslib135_13	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_14	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_15	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_16	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_17	10.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_18	1e+20	58.6k	–	14400.0	1e+20	70.0k	–	14400.0
dem_100_gaslib135_19	0.0	0.0	0.0	5.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_20	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_21	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_22	81.0k	63.0k	28.6	14400.0	1e+20	61.0k	–	14400.0
dem_100_gaslib135_23	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_24	1e+20	1488.0	–	14400.0	1e+20	4561.8	–	14400.0
dem_100_gaslib135_25	1e+20	1488.0	–	14400.0	1e+20	12.2k	–	14400.0
dem_100_gaslib135_26	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_27	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_28	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_29	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_30	1e+20	1488.0	–	14400.0	1e+20	19.9k	–	14400.0
dem_100_gaslib135_31	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_32	383.8k	1624.3	Large	14400.0	1e+20	27.8k	–	14400.0
dem_100_gaslib135_33	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_34	1e+20	36.5k	–	14400.0	1e+20	55.2k	–	14400.0
dem_100_gaslib135_35	10.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_36	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_37	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_38	1e+20	58.6k	–	14400.0	1e+20	66.5k	–	14400.0
dem_100_gaslib135_39	249.1k	64.9k	283.8	14400.0	1e+20	71.4k	–	14400.0
dem_100_gaslib135_40	48.0k	1624.3	2854.5	14400.0	1e+20	1055.5	–	14400.0
dem_100_gaslib135_41	1e+20	1624.3	–	14400.0	1e+20	1176.6	–	14400.0
dem_100_gaslib135_42	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0

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Table A.20: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_43	41.2k	1488.0	2672.2	14400.0	1e+20	975.9	–	14400.0
dem_100_gaslib135_44	251.5k	1488.0	Large	14400.0	1e+20	9441.1	–	14400.0
dem_100_gaslib135_45	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_46	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_47	0.0	0.0	0.0	0.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_48	1e+20	26.4k	–	14400.0	1e+20	36.6k	–	14400.0
dem_100_gaslib135_49	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_50	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_51	1e+20	1624.3	–	14400.0	1e+20	21.1k	–	14400.0
dem_100_gaslib135_52	1e+20	0.0	–	14400.0	1e+20	2319.8	–	14400.0
dem_100_gaslib135_53	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_54	221.5k	1488.0	Large	14400.0	1e+20	30.1k	–	14400.0
dem_100_gaslib135_55	403.0k	31.7k	1172.4	14400.0	1e+20	39.8k	–	14400.0
dem_100_gaslib135_56	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_57	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_58	0.0	0.0	0.0	6.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_59	89.9k	36.5k	146.0	14400.0	1e+20	46.7k	–	14400.0
dem_100_gaslib135_60	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_61	207.6k	0.0	–	14400.0	1e+20	6277.5	–	14400.0
dem_100_gaslib135_62	1e+20	31.7k	–	14400.0	1e+20	38.3k	–	14400.0
dem_100_gaslib135_63	1e+20	1488.0	–	14400.0	1e+20	4273.2	–	14400.0
dem_100_gaslib135_64	108.8k	42.5k	156.2	14400.0	1e+20	50.8k	–	14400.0
dem_100_gaslib135_65	16.9k	14.5k	16.3	14400.0	1e+20	426.9	–	14400.0
dem_100_gaslib135_66	1e+20	1624.3	–	14400.0	1e+20	30.4k	–	14400.0
dem_100_gaslib135_67	1e+20	26.4k	–	14400.0	1e+20	35.6k	–	14400.0
dem_100_gaslib135_68	61.2k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_69	0.0	0.0	0.0	3.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_70	41.2k	3229.4	1177.3	14400.0	1e+20	868.2	–	14400.0
dem_100_gaslib135_71	1e+20	26.4k	–	14400.0	1e+20	29.7k	–	14400.0
dem_100_gaslib135_72	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_73	85.9k	36.4k	136.1	14400.0	1e+20	59.0k	–	14400.0
dem_100_gaslib135_74	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_75	1e+20	31.7k	–	14400.0	1e+20	44.4k	–	14400.0
dem_100_gaslib135_76	1e+20	36.5k	–	14400.0	1e+20	41.7k	–	14400.0
dem_100_gaslib135_77	221.5k	26.4k	737.6	14400.0	1e+20	30.3k	–	14400.0
dem_100_gaslib135_78	1e+20	1488.0	–	14400.0	1e+20	14.3k	–	14400.0
dem_100_gaslib135_79	67.1k	36.4k	84.4	14400.0	1e+20	44.5k	–	14400.0
dem_100_gaslib135_80	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_81	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_82	1e+20	26.4k	–	14400.0	1e+20	33.4k	–	14400.0
dem_100_gaslib135_83	1e+20	23.1k	–	14400.0	1e+20	6063.2	–	14400.0
dem_100_gaslib135_84	0.0	0.0	0.0	7.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_85	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_86	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_87	1e+20	36.5k	–	14400.0	1e+20	46.1k	–	14400.0
dem_100_gaslib135_88	3095.1k	26.4k	Large	14400.0	1e+20	32.8k	–	14400.0
dem_100_gaslib135_89	327.0k	26.4k	1136.7	14400.0	1e+20	39.0k	–	14400.0
dem_100_gaslib135_90	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_91	0.0	0.0	0.0	7.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_92	1e+20	1488.0	–	14400.0	1e+20	1108.5	–	14400.0
dem_100_gaslib135_93	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_94	1e+20	31.7k	–	14400.0	1e+20	36.7k	–	14400.0
dem_100_gaslib135_95	1e+20	1488.0	–	14400.0	1e+20	1083.7	–	14400.0
dem_100_gaslib135_96	1e+20	1488.0	–	14400.0	1e+20	13.2k	–	14400.0
dem_100_gaslib135_97	1e+20	26.4k	–	14400.0	1e+20	36.1k	–	14400.0
dem_100_gaslib135_98	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_99	108.8k	36.5k	197.9	14400.0	1e+20	44.4k	–	14400.0
dem_100_gaslib135_100	70.5k	1488.0	4638.3	14400.0	1e+20	1959.2	–	14400.0
dem_100_gaslib135_101	1e+20	25.0k	–	14400.0	1e+20	42.5k	–	14400.0
dem_100_gaslib135_102	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_103	211.2k	1488.0	Large	14400.0	1e+20	4579.9	–	14400.0
dem_100_gaslib135_104	0.0	0.0	0.0	6.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_105	0.0	0.0	0.0	7.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_106	1e+20	3229.4	–	14400.0	1e+20	2730.9	–	14400.0
dem_100_gaslib135_107	1e+20	31.7k	–	14400.0	1e+20	42.5k	–	14400.0
dem_100_gaslib135_108	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_109	44.5k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_110	51.8k	1488.0	3382.4	14400.0	1e+20	15.3k	–	14400.0
dem_100_gaslib135_111	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_112	53.5k	1624.5	3196.3	14400.0	1e+20	13.5k	–	14400.0
dem_100_gaslib135_113	14.0k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_114	1e+20	36.4k	–	14400.0	1e+20	51.8k	–	14400.0
dem_100_gaslib135_115	1e+20	31.7k	–	14400.0	1e+20	49.2k	–	14400.0
dem_100_gaslib135_116	71.2k	42.9k	66.2	14400.0	1e+20	59.7k	–	14400.0
dem_100_gaslib135_117	14.5k	0.0	–	14400.0	1e+20	232.9	–	14400.0
dem_100_gaslib135_118	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_119	48.0k	1624.3	2854.5	14400.0	1e+20	2719.7	–	14400.0
dem_100_gaslib135_120	1e+20	0.0	–	14400.0	1e+20	156.7	–	14400.0

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Table A.20: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_121	66.6k	1488.0	4377.7	14400.0	1e+20	38.9k	–	14400.0
dem_100_gaslib135_122	1e+20	26.4k	–	14400.0	1e+20	38.0k	–	14400.0
dem_100_gaslib135_123	0.0	0.0	0.0	55.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_124	0.0	0.0	0.0	7.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_125	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_126	201.4k	36.4k	453.3	14400.0	1e+20	45.5k	–	14400.0
dem_100_gaslib135_127	1e+20	1624.3	–	14400.0	1e+20	32.1k	–	14400.0
dem_100_gaslib135_128	11.6k	11.6k	0.0	12336.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_129	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_130	0.0	0.0	0.0	7.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_131	54.3k	1624.3	3244.7	14400.0	1e+20	28.2k	–	14400.0
dem_100_gaslib135_132	1e+20	1488.0	–	14400.0	1e+20	20.0k	–	14400.0
dem_100_gaslib135_133	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_134	72.9k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_135	0.0	0.0	0.0	5.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_136	0.0	0.0	0.0	7.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_137	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_138	77.6k	0.0	–	14400.0	1e+20	1096.0	–	14400.0
dem_100_gaslib135_139	1e+20	1624.3	–	14400.0	1e+20	14.9k	–	14400.0
dem_100_gaslib135_140	64.9k	58.2k	11.6	14400.0	1e+20	41.9k	–	14400.0
dem_100_gaslib135_141	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_142	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_143	66.6k	34.9k	90.8	14400.0	1e+20	39.4k	–	14400.0
dem_100_gaslib135_144	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_145	1474.8k	0.0	–	14400.0	1e+20	21.0	–	14400.0
dem_100_gaslib135_146	73.5k	71.2k	3.1	14400.0	1e+20	47.6k	–	14400.0
dem_100_gaslib135_147	246.4k	4758.5	5078.2	14400.0	1e+20	1721.5	–	14400.0
dem_100_gaslib135_148	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_149	1e+20	33.4k	–	14400.0	1e+20	38.6k	–	14400.0
dem_100_gaslib135_150	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_151	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_152	47.2k	1624.3	2806.5	14400.0	1e+20	5830.1	–	14400.0
dem_100_gaslib135_153	1e+20	50.9k	–	14400.0	1e+20	42.0k	–	14400.0
dem_100_gaslib135_154	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_155	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_156	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_157	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_158	0.0	0.0	0.0	5.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_159	1e+20	26.4k	–	14400.0	1e+20	46.9k	–	14400.0
dem_100_gaslib135_160	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_161	1e+20	1488.0	–	14400.0	1e+20	29.5k	–	14400.0
dem_100_gaslib135_162	54.3k	0.0	–	14400.0	1e+20	273.4	–	14400.0
dem_100_gaslib135_163	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_164	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_165	74.2k	31.7k	134.2	14400.0	1e+20	46.0k	–	14400.0
dem_100_gaslib135_166	48.0k	1488.0	3125.2	14400.0	1e+20	9554.1	–	14400.0
dem_100_gaslib135_167	99.4k	86.2k	15.3	14400.0	1e+20	69.3k	–	14400.0
dem_100_gaslib135_168	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_169	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_170	1e+20	26.4k	–	14400.0	1e+20	44.4k	–	14400.0
dem_100_gaslib135_171	48.0k	41.2k	16.3	14400.0	1e+20	15.3k	–	14400.0
dem_100_gaslib135_172	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_173	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_174	114.7k	50.1k	129.1	14400.0	1e+20	61.1k	–	14400.0
dem_100_gaslib135_175	54.3k	1488.0	3551.1	14400.0	1e+20	20.4k	–	14400.0
dem_100_gaslib135_176	1e+20	1488.0	–	14400.0	1e+20	10.1k	–	14400.0
dem_100_gaslib135_177	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_178	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_179	1e+20	0.0	–	14400.0	1e+20	1804.6	–	14400.0
dem_100_gaslib135_180	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_181	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_182	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_183	391.9k	52.2k	650.9	14400.0	1e+20	66.7k	–	14400.0
dem_100_gaslib135_184	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_185	34.9k	3229.4	981.1	14400.0	1e+20	331.2	–	14400.0
dem_100_gaslib135_186	365.9k	25.0k	1366.4	14400.0	1e+20	19.3k	–	14400.0
dem_100_gaslib135_187	0.0	0.0	0.0	3.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_188	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_189	48.0k	1624.3	2854.5	14400.0	1e+20	2245.3	–	14400.0
dem_100_gaslib135_190	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_191	1e+20	29.3k	–	14400.0	1e+20	36.8k	–	14400.0
dem_100_gaslib135_192	1e+20	31.8k	–	14400.0	1e+20	47.2k	–	14400.0
dem_100_gaslib135_193	1e+20	0.0	–	14400.0	1e+20	414.3	–	14400.0
dem_100_gaslib135_194	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_195	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_196	92.5k	50.0k	84.8	14400.0	1e+20	55.7k	–	14400.0
dem_100_gaslib135_197	375.0k	29.5k	1170.4	14400.0	1e+20	39.6k	–	14400.0
dem_100_gaslib135_198	0.0	0.0	0.0	3.6	1e+20	0.0	–	14400.0

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Table A.20: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_199	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_200	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_201	1e+20	51.9k	–	14400.0	1e+20	62.9k	–	14400.0
dem_100_gaslib135_202	103.1k	51.9k	98.7	14400.0	1e+20	57.8k	–	14400.0
dem_100_gaslib135_203	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_204	41.2k	0.0	–	14400.0	1e+20	906.1	–	14400.0
dem_100_gaslib135_205	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_206	56.8k	1488.0	3715.9	14400.0	1e+20	10.4k	–	14400.0
dem_100_gaslib135_207	55.8k	55.8k	0.0	2759.2	1e+20	21.7k	–	14400.0
dem_100_gaslib135_208	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_209	123.0k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_210	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_211	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_212	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_213	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_214	96.5k	1624.3	5843.1	14400.0	1e+20	17.6k	–	14400.0
dem_100_gaslib135_215	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_216	69.0k	26.4k	161.0	14400.0	1e+20	32.3k	–	14400.0
dem_100_gaslib135_217	1e+20	1488.0	–	14400.0	1e+20	28.9k	–	14400.0
dem_100_gaslib135_218	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_219	239.6k	0.0	–	14400.0	1e+20	202.2	–	14400.0
dem_100_gaslib135_220	71.2k	40.8k	74.5	14400.0	1e+20	52.1k	–	14400.0
dem_100_gaslib135_221	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_222	1e+20	1488.0	–	14400.0	1e+20	862.2	–	14400.0
dem_100_gaslib135_223	1e+20	36.4k	–	14400.0	1e+20	39.8k	–	14400.0
dem_100_gaslib135_224	0.0	0.0	0.0	3.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_225	0.0	0.0	0.0	48.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_226	388.8k	26.4k	1370.6	14400.0	1e+20	36.3k	–	14400.0
dem_100_gaslib135_227	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_228	0.0	0.0	0.0	3.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_229	41.2k	1624.3	2439.5	14400.0	1e+20	562.7	–	14400.0
dem_100_gaslib135_230	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_231	60.3k	1624.3	3611.7	14400.0	1e+20	21.4k	–	14400.0
dem_100_gaslib135_232	1e+20	36.4k	–	14400.0	1e+20	42.0k	–	14400.0
dem_100_gaslib135_233	0.0	0.0	0.0	49.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_234	56.7k	11.8k	381.2	14400.0	1e+20	30.7k	–	14400.0
dem_100_gaslib135_235	0.0	0.0	0.0	6.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_236	1e+20	0.0	–	14400.0	1e+20	292.7	–	14400.0
dem_100_gaslib135_237	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_238	92.0k	31.7k	190.6	14400.0	1e+20	40.3k	–	14400.0
dem_100_gaslib135_239	1e+20	30.2k	–	14400.0	1e+20	41.8k	–	14400.0
dem_100_gaslib135_240	1e+20	1624.3	–	14400.0	1e+20	9550.7	–	14400.0
dem_100_gaslib135_241	34.0k	1624.3	1993.7	14400.0	1e+20	322.9	–	14400.0
dem_100_gaslib135_242	433.6k	83.7k	418.2	14400.0	1e+20	71.4k	–	14400.0
dem_100_gaslib135_243	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_244	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_245	211.2k	26.4k	698.7	14400.0	1e+20	31.4k	–	14400.0
dem_100_gaslib135_246	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_247	1e+20	1488.0	–	14400.0	1e+20	13.7k	–	14400.0
dem_100_gaslib135_248	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_249	0.0	0.0	0.0	1.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_250	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_251	0.0	0.0	0.0	6.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_252	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_253	55.8k	28.9k	93.3	14400.0	1e+20	37.2k	–	14400.0
dem_100_gaslib135_254	0.0	0.0	0.0	49.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_255	0.0	0.0	0.0	5.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_256	1e+20	1488.0	–	14400.0	1e+20	16.6k	–	14400.0
dem_100_gaslib135_257	69.0k	1624.3	4149.0	14400.0	1e+20	19.1k	–	14400.0
dem_100_gaslib135_258	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_259	346.5k	10.9k	3075.5	14400.0	1e+20	37.0k	–	14400.0
dem_100_gaslib135_260	0.0	0.0	0.0	1.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_261	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_262	54.3k	1624.3	3244.7	14400.0	1e+20	9200.0	–	14400.0
dem_100_gaslib135_263	529.3k	0.0	–	14400.0	1e+20	361.7	–	14400.0
dem_100_gaslib135_264	0.0	0.0	0.0	4.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_265	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_266	1e+20	0.0	–	14400.0	1e+20	1.8	–	14400.0
dem_100_gaslib135_267	1e+20	1624.3	–	14400.0	1e+20	31.7k	–	14400.0
dem_100_gaslib135_268	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_269	1e+20	58.6k	–	14400.0	1e+20	64.0k	–	14400.0
dem_100_gaslib135_270	62.5k	62.5k	0.0	10979.7	1e+20	36.5k	–	14400.0
dem_100_gaslib135_271	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_272	1e+20	30.2k	–	14400.0	1e+20	36.5k	–	14400.0
dem_100_gaslib135_273	76.1k	50.0k	52.1	14400.0	1e+20	59.8k	–	14400.0
dem_100_gaslib135_274	1e+20	1488.0	–	14400.0	1e+20	24.6k	–	14400.0
dem_100_gaslib135_275	139.5k	45.5k	206.4	14400.0	1e+20	56.7k	–	14400.0
dem_100_gaslib135_276	520.4k	58.6k	787.6	14400.0	1e+20	78.4k	–	14400.0

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Table A.20: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_277	1e+20	31.7k	–	14400.0	1e+20	48.0k	–	14400.0
dem_100_gaslib135_278	0.0	0.0	0.0	51.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_279	10.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_280	640.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_281	0.0	0.0	0.0	48.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_282	0.0	0.0	0.0	6.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_283	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_284	48.0k	25.0k	92.3	14400.0	1e+20	29.2k	–	14400.0
dem_100_gaslib135_285	1e+20	36.5k	–	14400.0	1e+20	54.5k	–	14400.0
dem_100_gaslib135_286	1e+20	31.7k	–	14400.0	1e+20	38.9k	–	14400.0
dem_100_gaslib135_287	210.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_288	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_289	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_290	0.0	0.0	0.0	3.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_291	0.0	0.0	0.0	6.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_292	1e+20	36.4k	–	14400.0	1e+20	39.0k	–	14400.0
dem_100_gaslib135_293	55.8k	30.2k	84.8	14400.0	1e+20	33.0k	–	14400.0
dem_100_gaslib135_294	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_295	1e+20	0.0	–	14400.0	1e+20	986.3	–	14400.0
dem_100_gaslib135_296	65.6k	42.3k	55.0	14400.0	1e+20	39.2k	–	14400.0
dem_100_gaslib135_297	1e+20	0.0	–	14400.0	1e+20	346.2	–	14400.0
dem_100_gaslib135_298	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_299	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_300	0.0	0.0	0.0	6.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_301	60.4k	60.4k	0.0	11978.8	1e+20	25.4k	–	14400.0
dem_100_gaslib135_302	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_303	54.3k	1488.0	3551.1	14400.0	1e+20	12.2k	–	14400.0
dem_100_gaslib135_304	48.0k	1488.0	3125.2	14400.0	1e+20	5888.9	–	14400.0
dem_100_gaslib135_305	0.0	0.0	0.0	6.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_306	74.2k	31.7k	134.2	14400.0	1e+20	40.7k	–	14400.0
dem_100_gaslib135_307	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_308	125.0k	4542.9	2651.7	14400.0	1e+20	7678.5	–	14400.0
dem_100_gaslib135_309	0.0	0.0	0.0	6.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_310	0.0	0.0	0.0	5.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_311	96.8k	31.7k	205.7	14400.0	1e+20	45.2k	–	14400.0
dem_100_gaslib135_312	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_313	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_314	1e+20	1488.0	–	14400.0	1e+20	12.3k	–	14400.0
dem_100_gaslib135_315	1e+20	1488.0	–	14400.0	1e+20	16.4k	–	14400.0
dem_100_gaslib135_316	64.9k	34.9k	85.9	14400.0	1e+20	43.8k	–	14400.0
dem_100_gaslib135_317	41.2k	1624.3	2439.5	14400.0	1e+20	1469.0	–	14400.0
dem_100_gaslib135_318	0.0	0.0	0.0	7.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_319	97.0k	1488.0	6420.5	14400.0	1e+20	564.6	–	14400.0
dem_100_gaslib135_320	0.0	0.0	0.0	2.8	1e+20	0.0	–	14400.0
dem_100_gaslib135_321	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_322	754.1k	1488.0	Large	14400.0	1e+20	7534.1	–	14400.0
dem_100_gaslib135_323	94.5k	3315.4	2749.6	14400.0	1e+20	24.4k	–	14400.0
dem_100_gaslib135_324	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_325	162.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_326	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_327	335.6k	83.1k	303.8	14400.0	1e+20	104.6k	–	14400.0
dem_100_gaslib135_328	21.8k	0.0	–	14400.0	1e+20	238.2	–	14400.0
dem_100_gaslib135_329	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_330	1480.0k	1624.3	Large	14400.0	1e+20	9776.9	–	14400.0
dem_100_gaslib135_331	100.7k	38.1k	164.0	14400.0	1e+20	51.6k	–	14400.0
dem_100_gaslib135_332	1495.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_333	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_334	48.0k	25.0k	92.3	14400.0	1e+20	29.4k	–	14400.0
dem_100_gaslib135_335	1e+20	31.7k	–	14400.0	1e+20	38.6k	–	14400.0
dem_100_gaslib135_336	0.0	0.0	0.0	7.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_337	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_338	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_339	376.5k	26.4k	1324.1	14400.0	1e+20	37.3k	–	14400.0
dem_100_gaslib135_340	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_341	51.8k	47.2k	9.8	14400.0	1e+20	29.7k	–	14400.0
dem_100_gaslib135_342	0.0	0.0	0.0	50.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_343	99.4k	42.7k	132.5	14400.0	1e+20	55.2k	–	14400.0
dem_100_gaslib135_344	0.0	0.0	0.0	5.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_345	0.0	0.0	0.0	48.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_346	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_347	1e+20	26.4k	–	14400.0	1e+20	33.4k	–	14400.0
dem_100_gaslib135_348	387.3k	31.7k	1123.0	14400.0	1e+20	42.8k	–	14400.0
dem_100_gaslib135_349	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_350	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_351	1e+20	58.6k	–	14400.0	1e+20	93.0k	–	14400.0
dem_100_gaslib135_352	237.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_353	1e+20	0.0	–	14400.0	1e+20	401.4	–	14400.0
dem_100_gaslib135_354	86.4k	31.7k	172.7	14400.0	1e+20	45.8k	–	14400.0

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Table A.20: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_355	0.0	0.0	0.0	1.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_356	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_357	1e+20	1488.0	–	14400.0	1e+20	2734.4	–	14400.0
dem_100_gaslib135_358	263.8k	31.7k	732.9	14400.0	1e+20	39.9k	–	14400.0
dem_100_gaslib135_359	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_360	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_361	1e+20	36.4k	–	14400.0	1e+20	43.6k	–	14400.0
dem_100_gaslib135_362	1e+20	1624.3	–	14400.0	1e+20	34.7k	–	14400.0
dem_100_gaslib135_363	861.1k	0.0	–	14400.0	1e+20	98.4	–	14400.0
dem_100_gaslib135_364	1e+20	31.7k	–	14400.0	1e+20	37.8k	–	14400.0
dem_100_gaslib135_365	54.3k	1624.3	3244.7	14400.0	1e+20	6020.8	–	14400.0
dem_100_gaslib135_366	1e+20	0.0	–	14400.0	1e+20	493.4	–	14400.0
dem_100_gaslib135_367	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_368	0.0	0.0	0.0	7.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_369	1113.3k	1488.0	Large	14400.0	1e+20	24.9k	–	14400.0
dem_100_gaslib135_370	79.7k	26.4k	201.3	14400.0	1e+20	36.9k	–	14400.0
dem_100_gaslib135_371	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_372	60.4k	58.2k	3.8	14400.0	1e+20	35.8k	–	14400.0
dem_100_gaslib135_373	258.8k	1624.3	Large	14400.0	1e+20	33.5k	–	14400.0
dem_100_gaslib135_374	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_375	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_376	54.3k	3588.0	1414.2	14400.0	1e+20	18.7k	–	14400.0
dem_100_gaslib135_377	61.9k	3229.4	1816.0	14400.0	1e+20	1335.9	–	14400.0
dem_100_gaslib135_378	1e+20	31.7k	–	14400.0	1e+20	37.5k	–	14400.0
dem_100_gaslib135_379	0.0	0.0	0.0	7.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_380	1e+20	53.5k	–	14400.0	1e+20	62.1k	–	14400.0
dem_100_gaslib135_381	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_382	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_383	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_384	1e+20	1488.0	–	14400.0	1e+20	5234.1	–	14400.0
dem_100_gaslib135_385	495.8k	1624.3	Large	14400.0	1e+20	807.2	–	14400.0
dem_100_gaslib135_386	1e+20	26.4k	–	14400.0	1e+20	37.3k	–	14400.0
dem_100_gaslib135_387	288.0k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_388	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_389	250.0k	1624.3	Large	14400.0	1e+20	2785.2	–	14400.0
dem_100_gaslib135_390	1e+20	1488.0	–	14400.0	1e+20	2451.4	–	14400.0
dem_100_gaslib135_391	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_392	1e+20	1488.0	–	14400.0	1e+20	2144.6	–	14400.0
dem_100_gaslib135_393	44.7k	1488.0	2905.0	14400.0	1e+20	3330.6	–	14400.0
dem_100_gaslib135_394	0.0	0.0	0.0	1.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_395	0.0	0.0	0.0	49.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_396	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_397	1e+20	1624.3	–	14400.0	1e+20	32.2k	–	14400.0
dem_100_gaslib135_398	687.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_399	1e+20	23.5k	–	14400.0	1e+20	24.9k	–	14400.0
dem_100_gaslib135_400	1e+20	36.5k	–	14400.0	1e+20	47.4k	–	14400.0
dem_100_gaslib135_401	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_402	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_403	1e+20	31.7k	–	14400.0	1e+20	35.9k	–	14400.0
dem_100_gaslib135_404	1e+20	26.4k	–	14400.0	1e+20	45.2k	–	14400.0
dem_100_gaslib135_405	1e+20	36.4k	–	14400.0	1e+20	52.5k	–	14400.0
dem_100_gaslib135_406	166.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_407	69.0k	1488.0	4538.3	14400.0	1e+20	11.3k	–	14400.0
dem_100_gaslib135_408	1e+20	0.0	–	14400.0	1e+20	212.7	–	14400.0
dem_100_gaslib135_409	1e+20	1624.3	–	14400.0	1e+20	33.5k	–	14400.0
dem_100_gaslib135_410	1e+20	1488.0	–	14400.0	1e+20	15.1k	–	14400.0
dem_100_gaslib135_411	1e+20	0.0	–	14400.0	1e+20	9.2	–	14400.0
dem_100_gaslib135_412	42.5k	1798.6	2261.9	14400.0	1e+20	910.3	–	14400.0
dem_100_gaslib135_413	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_414	1e+20	1488.0	–	14400.0	1e+20	1409.9	–	14400.0
dem_100_gaslib135_415	2118.0k	31.7k	6587.4	14400.0	1e+20	41.4k	–	14400.0
dem_100_gaslib135_416	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_417	0.0	0.0	0.0	5.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_418	0.0	0.0	0.0	2.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_419	0.0	0.0	0.0	6.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_420	1e+20	1488.0	–	14400.0	1e+20	13.2k	–	14400.0
dem_100_gaslib135_421	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_422	48.0k	1488.0	3125.2	14400.0	1e+20	9428.7	–	14400.0
dem_100_gaslib135_423	1e+20	1624.3	–	14400.0	1e+20	37.8k	–	14400.0
dem_100_gaslib135_424	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_425	1e+20	43.7k	–	14400.0	1e+20	52.9k	–	14400.0
dem_100_gaslib135_426	1e+20	51.9k	–	14400.0	1e+20	62.0k	–	14400.0
dem_100_gaslib135_427	334.7k	31.7k	956.7	14400.0	1e+20	36.7k	–	14400.0
dem_100_gaslib135_428	49.4k	4542.9	988.4	14400.0	1e+20	25.3k	–	14400.0
dem_100_gaslib135_429	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_430	1e+20	26.4k	–	14400.0	1e+20	35.8k	–	14400.0
dem_100_gaslib135_431	0.0	0.0	0.0	53.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_432	0.0	0.0	0.0	3.1	1e+20	0.0	–	14400.0

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Table A.20: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_gaslib135_433	1e+20	36.4k	–	14400.0	1e+20	41.3k	–	14400.0
dem_100_gaslib135_434	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_435	1e+20	60.3k	–	14400.0	1e+20	70.7k	–	14400.0
dem_100_gaslib135_436	1e+20	31.7k	–	14400.0	1e+20	35.5k	–	14400.0
dem_100_gaslib135_437	1e+20	1488.0	–	14400.0	1e+20	33.9k	–	14400.0
dem_100_gaslib135_438	74.2k	34.9k	112.5	14400.0	1e+20	43.3k	–	14400.0
dem_100_gaslib135_439	73.5k	42.9k	71.4	14400.0	1e+20	57.8k	–	14400.0
dem_100_gaslib135_440	0.0	0.0	0.0	1.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_441	48.0k	47.1k	1.9	14400.0	1e+20	10.3k	–	14400.0
dem_100_gaslib135_442	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_443	48.0k	1624.3	2854.5	14400.0	1e+20	475.4	–	14400.0
dem_100_gaslib135_444	8790.1	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_445	0.0	0.0	0.0	7.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_446	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_447	1e+20	1488.0	–	14400.0	1e+20	26.6k	–	14400.0
dem_100_gaslib135_448	16.9k	12.3k	37.5	14400.0	1e+20	158.5	–	14400.0
dem_100_gaslib135_449	48.0k	1488.0	3125.2	14400.0	1e+20	15.9k	–	14400.0
dem_100_gaslib135_450	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_451	1e+20	3229.4	–	14400.0	1e+20	6225.8	–	14400.0
dem_100_gaslib135_452	61.9k	1488.0	4058.2	14400.0	1e+20	9918.2	–	14400.0
dem_100_gaslib135_453	142.4k	36.4k	291.2	14400.0	1e+20	61.9k	–	14400.0
dem_100_gaslib135_454	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_455	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_456	0.0	0.0	0.0	1.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_457	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_458	12.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_459	18.5k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_460	1e+20	36.5k	–	14400.0	1e+20	57.8k	–	14400.0
dem_100_gaslib135_461	62.2k	1624.3	3731.7	14400.0	1e+20	42.0k	–	14400.0
dem_100_gaslib135_462	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_463	573.1k	42.9k	1236.7	14400.0	1e+20	63.4k	–	14400.0
dem_100_gaslib135_464	1e+20	36.5k	–	14400.0	1e+20	48.9k	–	14400.0
dem_100_gaslib135_465	0.0	0.0	0.0	2.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_466	14.5k	14.5k	0.0	4403.3	1e+20	0.0	–	14400.0
dem_100_gaslib135_467	1e+20	1488.0	–	14400.0	1e+20	13.1k	–	14400.0
dem_100_gaslib135_468	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_469	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_470	58.2k	27.0k	115.1	14400.0	1e+20	37.7k	–	14400.0
dem_100_gaslib135_471	1110.0k	50.0k	2118.1	14400.0	1e+20	57.2k	–	14400.0
dem_100_gaslib135_472	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_473	41.2k	5742.2	618.4	14400.0	1e+20	2971.0	–	14400.0
dem_100_gaslib135_474	1e+20	0.0	–	14400.0	1e+20	4270.3	–	14400.0
dem_100_gaslib135_475	1e+20	31.7k	–	14400.0	1e+20	37.9k	–	14400.0
dem_100_gaslib135_476	48.0k	1624.3	2854.5	14400.0	1e+20	2290.9	–	14400.0
dem_100_gaslib135_477	122.9k	1624.3	7466.4	14400.0	1e+20	582.6	–	14400.0
dem_100_gaslib135_478	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_479	0.0	0.0	0.0	1.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_480	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_481	0.0	0.0	0.0	2.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_482	0.0	0.0	0.0	0.9	1e+20	0.0	–	14400.0
dem_100_gaslib135_483	0.0	0.0	0.0	46.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_484	1e+20	26.4k	–	14400.0	1e+20	45.9k	–	14400.0
dem_100_gaslib135_485	51.8k	51.8k	0.0	7241.3	1e+20	32.5k	–	14400.0
dem_100_gaslib135_486	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_487	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_488	1e+20	26.4k	–	14400.0	1e+20	49.3k	–	14400.0
dem_100_gaslib135_489	0.0	0.0	0.0	2.6	1e+20	0.0	–	14400.0
dem_100_gaslib135_490	0.0	0.0	0.0	2.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_491	1e+20	26.4k	–	14400.0	1e+20	38.3k	–	14400.0
dem_100_gaslib135_492	0.0	0.0	0.0	2.4	1e+20	0.0	–	14400.0
dem_100_gaslib135_493	0.0	0.0	0.0	1.2	1e+20	0.0	–	14400.0
dem_100_gaslib135_494	10.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_495	0.0	0.0	0.0	2.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_496	0.0	0.0	0.0	2.1	1e+20	0.0	–	14400.0
dem_100_gaslib135_497	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_100_gaslib135_498	0.0	0.0	0.0	1.7	1e+20	0.0	–	14400.0
dem_100_gaslib135_499	0.0	0.0	0.0	1.5	1e+20	0.0	–	14400.0
dem_100_gaslib135_500	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0

Table A.21: Detailed results of the split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ as summarized in Figure 3.9c and Table 3.3c. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_1	82.4k	0.0	–	14400.0	1e+20	0.0	–	14400.0	101.8k	0.0	–	14400.0
dem_150_gaslib135_2	146.8k	32.1k	356.6	14400.0	1e+20	60.3k	–	14400.0	99.4k	35.2k	182.6	14400.0
dem_150_gaslib135_3	1e+20	58.2k	–	14400.0	1e+20	95.3k	–	14400.0	703.7k	51.9k	1256.3	14400.0
dem_150_gaslib135_4	0.0	0.0	0.0	47.1	1e+20	0.0	–	14400.0	0.0	0.0	0.0	9.7
dem_150_gaslib135_5	1e+20	0.0	–	14400.0	1e+20	48.9k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_6	1e+20	5200.7	–	14400.0	1e+20	2203.5	–	14400.0	1269.9k	0.0	–	14400.0
dem_150_gaslib135_7	273.6k	42.9k	538.1	14400.0	1e+20	84.1k	–	14400.0	722.3k	42.8k	1588.9	14400.0
dem_150_gaslib135_8	1e+20	58.6k	–	14400.0	1e+20	122.1k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_9	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_10	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	45.6
dem_150_gaslib135_11	1e+20	36.4k	–	14400.0	1e+20	60.4k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_12	375.0k	1488.0	Large	14400.0	1e+20	2736.1	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_13	1e+20	0.0	–	14400.0	1e+20	35.6k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_14	405.4k	60.9k	565.5	14400.0	1e+20	82.8k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_15	1e+20	58.6k	–	14400.0	1e+20	102.3k	–	14400.0	1e+20	83.0k	–	14400.0
dem_150_gaslib135_16	1e+20	24.0k	–	14400.0	1e+20	27.7k	–	14400.0	1e+20	1254.5	–	14400.0
dem_150_gaslib135_17	1e+20	50.0k	–	14400.0	1e+20	75.0k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_18	339.8k	0.0	–	14400.0	1e+20	1126.1	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_19	1e+20	120.9k	–	14400.0	1e+20	152.9k	–	14400.0	1e+20	76.4k	–	14400.0
dem_150_gaslib135_20	1e+20	1488.0	–	14400.0	1e+20	49.4k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_21	740.8k	40.0k	1753.3	14400.0	1e+20	24.8k	–	14400.0	1e+20	5148.5	–	14400.0
dem_150_gaslib135_22	311.1k	0.0	–	14400.0	1e+20	189.6	–	14400.0	146.2k	0.0	Large	14400.0
dem_150_gaslib135_23	1e+20	42.9k	–	14400.0	1e+20	64.8k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_24	1e+20	36.5k	–	14400.0	1e+20	73.2k	–	14400.0	1e+20	38.0k	–	14400.0
dem_150_gaslib135_25	1e+20	55.5k	–	14400.0	1e+20	29.1k	–	14400.0	1e+20	0.5	–	14400.0
dem_150_gaslib135_26	1e+20	0.0	–	14400.0	1e+20	15.1k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_27	99.3k	16.2k	512.7	14400.0	1e+20	283.9	–	14400.0	707.0k	0.0	–	14400.0
dem_150_gaslib135_28	1e+20	39.3k	–	14400.0	1e+20	76.0k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_29	1e+20	63.0k	–	14400.0	1e+20	97.8k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_30	82.7k	16.2k	410.3	14400.0	1e+20	16.6k	–	14400.0	120.0k	0.0	Large	14400.0
dem_150_gaslib135_31	1e+20	36.5k	–	14400.0	1e+20	83.3k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_32	393.4k	36.4k	980.5	14400.0	1e+20	51.8k	–	14400.0	1e+20	36.5k	–	14400.0
dem_150_gaslib135_33	1e+20	98.1k	–	14400.0	1e+20	144.8k	–	14400.0	1e+20	60.4k	–	14400.0
dem_150_gaslib135_34	197.1k	0.0	–	14400.0	1e+20	601.8	–	14400.0	622.9k	0.0	–	14400.0
dem_150_gaslib135_35	213.4k	36.4k	486.3	14400.0	1e+20	76.5k	–	14400.0	554.4k	40.8k	1258.2	14400.0
dem_150_gaslib135_36	1e+20	0.0	–	14400.0	1e+20	34.6k	–	14400.0	1e+20	472.7	–	14400.0
dem_150_gaslib135_37	2142.4k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_38	1e+20	44.5k	–	14400.0	1e+20	85.1k	–	14400.0	2885.4k	36.4k	7827.0	14400.0
dem_150_gaslib135_39	1e+20	19.3k	–	14400.0	1e+20	71.1k	–	14400.0	1e+20	1491.7	–	14400.0
dem_150_gaslib135_40	1e+20	0.0	–	14400.0	1e+20	3504.3	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_41	375.0k	1488.0	Large	14400.0	1e+20	18.6k	–	14400.0	375.0k	1488.0	Large	14400.0
dem_150_gaslib135_42	1e+20	14.0k	–	14400.0	1e+20	28.9k	–	14400.0	7927.4k	0.0	Large	14400.0
dem_150_gaslib135_43	70.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0	763.2k	0.0	Large	14400.0
dem_150_gaslib135_44	1e+20	50.1k	–	14400.0	1e+20	83.2k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_45	1e+20	7.1	–	14400.0	1e+20	26.3k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_46	455.0k	46.7k	874.3	14400.0	1e+20	65.2k	–	14400.0	1e+20	1510.0	–	14400.0
dem_150_gaslib135_47	1e+20	0.0	–	14400.0	1e+20	29.5k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_48	1e+20	1488.0	–	14400.0	1e+20	50.3k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_49	1e+20	1488.0	–	14400.0	1e+20	39.8k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_50	637.9k	78.1k	716.6	14400.0	1e+20	107.3k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_51	75.9k	14.7k	415.3	14400.0	1e+20	20.6k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_52	1e+20	26.4k	–	14400.0	1e+20	43.0k	–	14400.0	1e+20	26.4k	–	14400.0
dem_150_gaslib135_53	1e+20	2194.8	–	14400.0	1e+20	31.2k	–	14400.0	1e+20	4.7	–	14400.0
dem_150_gaslib135_54	1e+20	36.5k	–	14400.0	1e+20	69.7k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_55	1e+20	36.5k	–	14400.0	1e+20	61.1k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_56	244.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0	931.6k	0.0	–	14400.0
dem_150_gaslib135_57	1e+20	31.7k	–	14400.0	1e+20	48.8k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_58	76.8k	31.7k	142.5	14400.0	1e+20	54.1k	–	14400.0	181.6k	31.7k	473.4	14400.0
dem_150_gaslib135_59	1e+20	60.4k	–	14400.0	1e+20	83.3k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_60	499.7k	0.0	–	14400.0	1e+20	2090.1	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_61	160.5k	0.0	–	14400.0	1e+20	0.0	–	14400.0	363.9k	0.0	–	14400.0
dem_150_gaslib135_62	1e+20	0.0	–	14400.0	1e+20	37.9k	–	14400.0	1e+20	14.2k	–	14400.0
dem_150_gaslib135_63	341.6k	5793.7	5796.4	14400.0	1e+20	210.6	–	14400.0	414.2k	0.0	Large	14400.0
dem_150_gaslib135_64	1e+20	1488.0	–	14400.0	1e+20	61.6k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_65	1145.1k	98.1k	1067.0	14400.0	1e+20	118.4k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_66	1e+20	36.5k	–	14400.0	1e+20	91.3k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_67	260.7k	38.9k	569.3	14400.0	1e+20	72.7k	–	14400.0	1e+20	49.4k	–	14400.0
dem_150_gaslib135_68	1e+20	0.0	–	14400.0	1e+20	11.1k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_69	1e+20	42.9k	–	14400.0	1e+20	67.1k	–	14400.0	1e+20	42.9k	–	14400.0
dem_150_gaslib135_70	1e+20	31.8k	–	14400.0	1e+20	61.5k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_71	1e+20	34.9k	–	14400.0	1e+20	76.2k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_72	1e+20	58.6k	–	14400.0	1e+20	88.6k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_73	966.1k	25.0k	3772.1	14400.0	1e+20	58.0k	–	14400.0	207.4k	8720.6	2278.0	14400.0
dem_150_gaslib135_74	1e+20	0.0	–	14400.0	1e+20	1134.0	–	14400.0	3452.8k	0.0	Large	14400.0

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Table A.21: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_75	1e+20	0.3	–	14400.0	1e+20	38.5k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_76	1e+20	38.4k	–	14400.0	1e+20	60.5k	–	14400.0	1829.8k	36.4k	4927.0	14400.0
dem_150_gaslib135_77	664.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0	336.1k	45.9k	631.8	14400.0
dem_150_gaslib135_78	2441.1k	105.3	Large	14400.0	1e+20	18.7k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_79	1e+20	50.0k	–	14400.0	1e+20	82.9k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_80	1e+20	31.7k	–	14400.0	1e+20	51.6k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_81	1e+20	5599.4	–	14400.0	1e+20	61.3k	–	14400.0	2522.0k	2802.3	Large	14400.0
dem_150_gaslib135_82	1e+20	0.0	–	14400.0	1e+20	36.1k	–	14400.0	4743.0k	0.0	Large	14400.0
dem_150_gaslib135_83	591.6k	0.0	–	14400.0	1e+20	1169.1	–	14400.0	70.0k	0.0	–	14400.0
dem_150_gaslib135_84	1e+20	4542.9	–	14400.0	1e+20	57.0k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_85	591.1k	0.0	–	14400.0	1e+20	3076.7	–	14400.0	726.5k	0.0	Large	14400.0
dem_150_gaslib135_86	3480.1k	0.0	–	14400.0	1e+20	21.9k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_87	1e+20	0.0	–	14400.0	1e+20	34.1k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_88	1e+20	3229.4	–	14400.0	1e+20	9197.2	–	14400.0	343.8k	1488.0	Large	14400.0
dem_150_gaslib135_89	1e+20	26.4k	–	14400.0	1e+20	41.4k	–	14400.0	82.0k	26.4k	210.1	14400.0
dem_150_gaslib135_90	403.0k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_91	159.6k	0.0	–	14400.0	1e+20	125.7	–	14400.0	162.5k	0.0	–	14400.0
dem_150_gaslib135_92	1e+20	52.6k	–	14400.0	1e+20	83.1k	–	14400.0	1e+20	43.7k	–	14400.0
dem_150_gaslib135_93	648.0k	53.9k	1102.1	14400.0	1e+20	18.9k	–	14400.0	942.4k	1315.8	Large	14400.0
dem_150_gaslib135_94	1e+20	133.4k	–	14400.0	1e+20	97.4k	–	14400.0	1e+20	62.7k	–	14400.0
dem_150_gaslib135_95	919.2k	45.9k	1901.2	14400.0	1e+20	33.1k	–	14400.0	496.2k	0.0	Large	14400.0
dem_150_gaslib135_96	1e+20	11.6k	–	14400.0	1e+20	2132.6	–	14400.0	1875.7k	0.0	Large	14400.0
dem_150_gaslib135_97	1e+20	31.7k	–	14400.0	1e+20	70.9k	–	14400.0	1e+20	32.6k	–	14400.0
dem_150_gaslib135_98	1e+20	42.9k	–	14400.0	1e+20	87.8k	–	14400.0	504.4k	42.7k	1080.2	14400.0
dem_150_gaslib135_99	1e+20	0.0	–	14400.0	1e+20	10.2k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_100	1e+20	29.3k	–	14400.0	1e+20	86.2k	–	14400.0	1e+20	26.4k	–	14400.0
dem_150_gaslib135_101	1e+20	0.0	–	14400.0	1e+20	37.3k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_102	1e+20	79.7k	–	14400.0	1e+20	138.7k	–	14400.0	1e+20	68.9k	–	14400.0
dem_150_gaslib135_103	1e+20	1488.0	–	14400.0	1e+20	47.3k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_104	264.5k	44.5k	494.8	14400.0	1e+20	56.3k	–	14400.0	171.7k	42.7k	301.8	14400.0
dem_150_gaslib135_105	1492.2k	0.0	–	14400.0	1e+20	0.0	–	14400.0	969.8k	0.0	Large	14400.0
dem_150_gaslib135_106	1e+20	36.4k	–	14400.0	1e+20	65.0k	–	14400.0	758.7k	36.4k	1984.3	14400.0
dem_150_gaslib135_107	1e+20	0.0	–	14400.0	1e+20	10.9k	–	14400.0	3772.9k	0.0	Large	14400.0
dem_150_gaslib135_108	1062.7k	0.0	–	14400.0	1e+20	1717.1	–	14400.0	919.3k	0.5	Large	14400.0
dem_150_gaslib135_109	1e+20	1488.0	–	14400.0	1e+20	90.8k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_110	1e+20	0.0	–	14400.0	1e+20	29.4k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_111	1e+20	0.0	–	14400.0	1e+20	29.4k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_112	1076.1k	73.8k	1359.0	14400.0	1e+20	137.4k	–	14400.0	1e+20	68.9k	–	14400.0
dem_150_gaslib135_113	1e+20	0.0	–	14400.0	1e+20	38.7k	–	14400.0	3370.4k	2.1	Large	14400.0
dem_150_gaslib135_114	570.7k	0.0	–	14400.0	1e+20	4661.3	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_115	1e+20	66.5k	–	14400.0	1e+20	127.8k	–	14400.0	1e+20	68.9k	–	14400.0
dem_150_gaslib135_116	141.9k	86.2k	64.7	14400.0	1e+20	63.4k	–	14400.0	234.6k	42.7k	448.8	14400.0
dem_150_gaslib135_117	1e+20	96.6k	–	14400.0	1e+20	138.3k	–	14400.0	1e+20	69.3k	–	14400.0
dem_150_gaslib135_118	1e+20	10.5k	–	14400.0	1e+20	63.3k	–	14400.0	1e+20	13.5k	–	14400.0
dem_150_gaslib135_119	1e+20	36.4k	–	14400.0	1e+20	72.6k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_120	1e+20	0.0	–	14400.0	1e+20	25.7k	–	14400.0	1e+20	0.2	–	14400.0
dem_150_gaslib135_121	1e+20	65.0k	–	14400.0	1e+20	124.5k	–	14400.0	1e+20	72.8k	–	14400.0
dem_150_gaslib135_122	1e+20	0.0	–	14400.0	1e+20	33.8k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_123	1e+20	44.5k	–	14400.0	1e+20	79.4k	–	14400.0	785.2k	50.1k	1466.4	14400.0
dem_150_gaslib135_124	1e+20	31.7k	–	14400.0	1e+20	90.6k	–	14400.0	1e+20	31.8k	–	14400.0
dem_150_gaslib135_125	1592.6k	42.7k	3626.4	14400.0	1e+20	76.8k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_126	1e+20	36.7k	–	14400.0	1e+20	63.7k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_127	1e+20	42.9k	–	14400.0	1e+20	57.9k	–	14400.0	479.9k	36.4k	1218.6	14400.0
dem_150_gaslib135_128	1e+20	16.2k	–	14400.0	1e+20	37.3k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_129	180.1k	45.8k	293.2	14400.0	1e+20	80.4k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_130	1e+20	65.0k	–	14400.0	1e+20	115.3k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_131	1e+20	36.4k	–	14400.0	1e+20	57.7k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_132	71.2k	26.5k	168.8	14400.0	1e+20	55.3k	–	14400.0	546.6k	26.4k	1967.4	14400.0
dem_150_gaslib135_133	1e+20	26.4k	–	14400.0	1e+20	52.3k	–	14400.0	1e+20	26.4k	–	14400.0
dem_150_gaslib135_134	478.7k	36.4k	1215.0	14400.0	1e+20	68.6k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_135	233.0k	53.5k	335.1	14400.0	1e+20	76.8k	–	14400.0	3462.2k	49.1k	6955.7	14400.0
dem_150_gaslib135_136	1e+20	36.4k	–	14400.0	1e+20	76.2k	–	14400.0	1e+20	40.5k	–	14400.0
dem_150_gaslib135_137	1155.4k	4399.3	Large	14400.0	1e+20	0.0	–	14400.0	1e+20	30.3	–	14400.0
dem_150_gaslib135_138	1e+20	32.1	–	14400.0	1e+20	0.0	–	14400.0	1377.6k	0.0	–	14400.0
dem_150_gaslib135_139	1e+20	36.5k	–	14400.0	1e+20	95.1k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_140	1e+20	0.0	–	14400.0	1e+20	74.8k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_141	1e+20	0.0	–	14400.0	1e+20	19.7k	–	14400.0	5526.8k	0.0	Large	14400.0
dem_150_gaslib135_142	1e+20	0.2	–	14400.0	1e+20	6972.3	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_143	1e+20	17.5k	–	14400.0	1e+20	22.3k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_144	1e+20	42.9k	–	14400.0	1e+20	67.8k	–	14400.0	1e+20	48.4k	–	14400.0
dem_150_gaslib135_145	1e+20	62.5k	–	14400.0	1e+20	80.0k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_146	81.7k	75.1k	8.8	14400.0	1e+20	52.8k	–	14400.0	1e+20	1624.3	–	14400.0
dem_150_gaslib135_147	206.9k	58.6k	252.9	14400.0	1e+20	85.7k	–	14400.0	1e+20	58.9k	–	14400.0
dem_150_gaslib135_148	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_149	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1945.8k	0.0	–	14400.0
dem_150_gaslib135_150	1e+20	58.6k	–	14400.0	1e+20	106.3k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_151	1e+20	68.6k	–	14400.0	1e+20	102.1k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_152	1e+20	1488.0	–	14400.0	1e+20	60.5k	–	14400.0	1e+20	1488.0	–	14400.0

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Table A.21: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_153	1e+20	97.9k	–	14400.0	1e+20	132.1k	–	14400.0	1e+20	68.9k	–	14400.0
dem_150_gaslib135_154	1e+20	31.7k	–	14400.0	1e+20	49.2k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_155	1e+20	1624.3	–	14400.0	1e+20	36.5k	–	14400.0	95.0k	1488.0	6281.9	14400.0
dem_150_gaslib135_156	1e+20	28.8k	–	14400.0	1e+20	68.0k	–	14400.0	762.2k	30.2k	2425.4	14400.0
dem_150_gaslib135_157	1e+20	0.0	–	14400.0	1e+20	14.7k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_158	1e+20	1488.0	–	14400.0	1e+20	50.5k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_159	1e+20	52.8k	–	14400.0	1e+20	127.6k	–	14400.0	1e+20	42.9k	–	14400.0
dem_150_gaslib135_160	1e+20	1488.0	–	14400.0	1e+20	56.6k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_161	1e+20	1488.0	–	14400.0	1e+20	54.4k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_162	750.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_163	127.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_164	1347.4k	58.6k	2198.4	14400.0	1e+20	81.6k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_165	303.2k	106.6k	184.4	14400.0	1e+20	96.8k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_166	434.9k	1488.4	Large	14400.0	1e+20	16.0k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_167	1e+20	53.5k	–	14400.0	1e+20	45.6k	–	14400.0	1683.2k	14.5	Large	14400.0
dem_150_gaslib135_168	1e+20	31.7k	–	14400.0	1e+20	51.0k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_169	2068.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0	101.8k	0.0	–	14400.0
dem_150_gaslib135_170	218.3k	42.9k	409.1	14400.0	1e+20	61.1k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_171	840.8k	55.2k	1423.5	14400.0	1e+20	26.3k	–	14400.0	1302.0k	5720.7	Large	14400.0
dem_150_gaslib135_172	345.4k	60.4k	472.0	14400.0	1e+20	70.6k	–	14400.0	4162.5k	50.0k	8218.4	14400.0
dem_150_gaslib135_173	1e+20	68.1k	–	14400.0	1e+20	99.7k	–	14400.0	1108.1k	50.0k	2114.5	14400.0
dem_150_gaslib135_174	1e+20	0.0	–	14400.0	1e+20	1656.5	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_175	1e+20	1624.3	–	14400.0	1e+20	38.8k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_176	1e+20	1488.0	–	14400.0	1e+20	56.8k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_177	1e+20	31.7k	–	14400.0	1e+20	48.8k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_178	1e+20	50.0k	–	14400.0	1e+20	86.5k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_179	1e+20	42.9k	–	14400.0	1e+20	79.5k	–	14400.0	1867.6k	36.5k	5020.3	14400.0
dem_150_gaslib135_180	1e+20	30.2k	–	14400.0	1e+20	60.8k	–	14400.0	1e+20	29.5k	–	14400.0
dem_150_gaslib135_181	1e+20	14.0k	–	14400.0	1e+20	34.8k	–	14400.0	1e+20	95.1	–	14400.0
dem_150_gaslib135_182	1e+20	36.4k	–	14400.0	1e+20	79.6k	–	14400.0	1e+20	36.5k	–	14400.0
dem_150_gaslib135_183	1e+20	36.4k	–	14400.0	1e+20	85.3k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_184	1e+20	6142.5	–	14400.0	1e+20	44.6k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_185	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_186	80.2k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1149.2k	0.0	Large	14400.0
dem_150_gaslib135_187	71.2k	31.7k	124.9	14400.0	1e+20	52.9k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_188	2557.2k	56.8k	4403.7	14400.0	1e+20	96.6k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_189	1e+20	54.3k	–	14400.0	1e+20	36.1k	–	14400.0	1e+20	2516.5	–	14400.0
dem_150_gaslib135_190	1055.7k	16.2k	6413.4	14400.0	1e+20	731.8	–	14400.0	7197.8k	239.3k	2908.3	14400.0
dem_150_gaslib135_191	1e+20	45.0	–	14400.0	1e+20	11.5k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_192	817.6k	42.7k	1813.0	14400.0	1e+20	92.2k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_193	1e+20	0.0	–	14400.0	1e+20	6874.0	–	14400.0	3825.2k	0.0	Large	14400.0
dem_150_gaslib135_194	1e+20	37.2	–	14400.0	1e+20	127.1	–	14400.0	2873.0k	0.0	Large	14400.0
dem_150_gaslib135_195	1e+20	64.9k	–	14400.0	1e+20	94.4k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_196	1e+20	76.3k	–	14400.0	1e+20	104.2k	–	14400.0	1051.5k	58.6k	1693.7	14400.0
dem_150_gaslib135_197	1e+20	1624.3	–	14400.0	1e+20	60.5k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_198	1e+20	31.7k	–	14400.0	1e+20	54.5k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_199	1e+20	58.6k	–	14400.0	1e+20	100.2k	–	14400.0	1e+20	59.4k	–	14400.0
dem_150_gaslib135_200	1e+20	36.4k	–	14400.0	1e+20	59.1k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_201	185.1k	0.0	–	14400.0	1e+20	980.0	–	14400.0	612.6k	0.0	Large	14400.0
dem_150_gaslib135_202	1e+20	0.0	–	14400.0	1e+20	28.9k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_203	1e+20	138.2k	–	14400.0	1e+20	197.6k	–	14400.0	1e+20	136.5k	–	14400.0
dem_150_gaslib135_204	1e+20	14.8k	–	14400.0	1e+20	29.0k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_205	1e+20	31.7k	–	14400.0	1e+20	96.4k	–	14400.0	7044.7k	31.7k	Large	14400.0
dem_150_gaslib135_206	1e+20	40.7k	–	14400.0	1e+20	72.9k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_207	1e+20	81.8k	–	14400.0	1e+20	94.7k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_208	1e+20	0.0	–	14400.0	1e+20	19.6k	–	14400.0	1e+20	26.9k	–	14400.0
dem_150_gaslib135_209	1e+20	72.7k	–	14400.0	1e+20	81.2k	–	14400.0	1e+20	177.8k	–	14400.0
dem_150_gaslib135_210	1e+20	97.6k	–	14400.0	1e+20	102.0k	–	14400.0	1e+20	6180.1	–	14400.0
dem_150_gaslib135_211	1e+20	88.5k	–	14400.0	1e+20	99.8k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_212	1e+20	0.0	–	14400.0	1e+20	6818.9	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_213	1e+20	42.7k	–	14400.0	1e+20	54.7k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_214	1e+20	43.7k	–	14400.0	1e+20	91.8k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_215	1e+20	36.5k	–	14400.0	1e+20	66.1k	–	14400.0	243.6k	36.4k	569.3	14400.0
dem_150_gaslib135_216	1e+20	0.0	–	14400.0	1e+20	1501.1	–	14400.0	1e+20	5.2	–	14400.0
dem_150_gaslib135_217	305.2k	18.9k	1511.8	14400.0	1e+20	851.0	–	14400.0	973.8k	0.0	–	14400.0
dem_150_gaslib135_218	1e+20	36.4k	–	14400.0	1e+20	56.8k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_219	2205.2k	65.4k	3273.3	14400.0	1e+20	135.7k	–	14400.0	1e+20	77.6k	–	14400.0
dem_150_gaslib135_220	1e+20	4.2	–	14400.0	1e+20	449.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_221	1e+20	31.7k	–	14400.0	1e+20	42.5k	–	14400.0	71.3k	32.4k	120.0	14400.0
dem_150_gaslib135_222	1e+20	36.5k	–	14400.0	1e+20	88.3k	–	14400.0	1357.2k	40.3k	3269.7	14400.0
dem_150_gaslib135_223	252.0k	60.3k	317.9	14400.0	1e+20	81.4k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_224	1e+20	162.0k	–	14400.0	1e+20	215.2k	–	14400.0	1e+20	147.1k	–	14400.0
dem_150_gaslib135_225	432.0k	47.5k	810.0	14400.0	1e+20	57.0k	–	14400.0	1337.5k	36.4k	3574.5	14400.0
dem_150_gaslib135_226	1e+20	149.0k	–	14400.0	1e+20	199.6k	–	14400.0	1877.9k	141.8k	1224.7	14400.0
dem_150_gaslib135_227	1e+20	1488.0	–	14400.0	1e+20	4855.5	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_228	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1224.7k	0.0	–	14400.0
dem_150_gaslib135_229	1e+20	48.9k	–	14400.0	1e+20	86.1k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_230	205.6k	59.1k	248.0	14400.0	1e+20	82.1k	–	14400.0	819.2k	47.5k	1623.2	14400.0

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Table A.21: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_231	1e+20	1624.3	–	14400.0	1e+20	20.8k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_232	1e+20	10.3k	–	14400.0	1e+20	48.4k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_233	1e+20	34.2k	–	14400.0	1e+20	81.3k	–	14400.0	1e+20	32.4k	–	14400.0
dem_150_gaslib135_234	245.9k	44.5k	452.7	14400.0	1e+20	67.9k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_235	1e+20	31.7k	–	14400.0	1e+20	55.3k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_236	812.4k	31.7k	2465.1	14400.0	1e+20	57.7k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_237	738.0k	83.1k	788.2	14400.0	1e+20	133.7k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_238	1e+20	54.9k	–	14400.0	1e+20	117.6k	–	14400.0	856.5k	51.9k	1550.8	14400.0
dem_150_gaslib135_239	495.5k	80.0k	519.2	14400.0	1e+20	83.9k	–	14400.0	1e+20	50.1k	–	14400.0
dem_150_gaslib135_240	208.2k	0.0	–	14400.0	1e+20	0.0	–	14400.0	2294.2k	0.0	Large	14400.0
dem_150_gaslib135_241	1e+20	0.0	–	14400.0	1e+20	11.9k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_242	2411.5k	0.0	–	14400.0	1e+20	596.8	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_243	1e+20	1488.0	–	14400.0	1e+20	43.4k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_244	1e+20	1488.0	–	14400.0	1e+20	8514.0	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_245	694.7k	61.9k	1021.6	14400.0	1e+20	51.6k	–	14400.0	1e+20	20.2k	–	14400.0
dem_150_gaslib135_246	1e+20	26.6k	–	14400.0	1e+20	58.2k	–	14400.0	1e+20	26.4k	–	14400.0
dem_150_gaslib135_247	454.5k	65.0k	599.6	14400.0	1e+20	96.2k	–	14400.0	1e+20	66.6k	–	14400.0
dem_150_gaslib135_248	1e+20	42.7k	–	14400.0	1e+20	91.1k	–	14400.0	1e+20	51.3k	–	14400.0
dem_150_gaslib135_249	1e+20	42.9k	–	14400.0	1e+20	87.5k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_250	1e+20	31.7k	–	14400.0	1e+20	50.4k	–	14400.0	388.8k	31.7k	1127.7	14400.0
dem_150_gaslib135_251	249.5k	50.0k	398.6	14400.0	1e+20	73.8k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_252	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0	0.0	0.0	0.0	4.5
dem_150_gaslib135_253	1e+20	50.0k	–	14400.0	1e+20	96.9k	–	14400.0	5020.7k	50.0k	9933.3	14400.0
dem_150_gaslib135_254	1e+20	50.0k	–	14400.0	1e+20	92.3k	–	14400.0	1e+20	43.7k	–	14400.0
dem_150_gaslib135_255	70.8k	0.0	–	14399.6	1e+20	0.0	–	14400.0	158.5k	0.0	Large	14400.0
dem_150_gaslib135_256	1335.0k	1488.0	Large	14400.0	1e+20	41.0k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_257	270.2k	103.0k	162.4	14400.0	1e+20	86.2k	–	14400.0	1e+20	58.8k	–	14400.0
dem_150_gaslib135_258	1e+20	83.7k	–	14400.0	1e+20	103.2k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_259	1e+20	36.8k	–	14400.0	1e+20	59.7k	–	14400.0	1e+20	8.3	–	14400.0
dem_150_gaslib135_260	1e+20	36.4k	–	14400.0	1e+20	69.5k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_261	339.4k	70.2k	383.4	14400.0	1e+20	127.0k	–	14400.0	1e+20	61.1k	–	14400.0
dem_150_gaslib135_262	611.6k	36.4k	1580.2	14400.0	1e+20	72.5k	–	14400.0	689.5k	36.4k	1794.2	14400.0
dem_150_gaslib135_263	1e+20	0.0	–	14400.0	1e+20	11.5k	–	14400.0	1e+20	5148.5	–	14400.0
dem_150_gaslib135_264	1e+20	36.4k	–	14400.0	1e+20	69.0k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_265	1e+20	58.6k	–	14400.0	1e+20	98.7k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_266	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_267	190.0k	79.7k	138.3	14400.0	1e+20	74.0k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_268	793.3k	51.9k	1429.0	14400.0	1e+20	90.1k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_269	338.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_270	1e+20	160.8k	–	14400.0	1e+20	162.7k	–	14400.0	1e+20	77.6k	–	14400.0
dem_150_gaslib135_271	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	4235.2k	0.0	Large	14400.0
dem_150_gaslib135_272	245.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0	248.2k	0.0	Large	14400.0
dem_150_gaslib135_273	1e+20	42.8k	–	14400.0	1e+20	35.6k	–	14400.0	1e+20	1700.9	–	14400.0
dem_150_gaslib135_274	1e+20	72.6k	–	14400.0	1e+20	108.8k	–	14400.0	1e+20	60.4k	–	14400.0
dem_150_gaslib135_275	475.9k	118.2k	302.6	14400.0	1e+20	125.9k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_276	223.5k	51.9k	330.7	14400.0	1e+20	75.4k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_277	534.1k	92.9k	475.0	14400.0	1e+20	105.5k	–	14400.0	1e+20	66.6k	–	14400.0
dem_150_gaslib135_278	1e+20	144.8k	–	14400.0	1e+20	183.5k	–	14400.0	1e+20	84.2k	–	14400.0
dem_150_gaslib135_279	1e+20	38.1k	–	14400.0	1e+20	83.9k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_280	373.6k	0.0	–	14400.0	1e+20	727.8	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_281	1e+20	0.0	–	14400.0	1e+20	6202.0	–	14400.0	1e+20	4542.9	–	14400.0
dem_150_gaslib135_282	1e+20	36.4k	–	14400.0	1e+20	60.9k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_283	231.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_284	1e+20	1488.0	–	14400.0	1e+20	41.1k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_285	1e+20	14.0k	–	14400.0	1e+20	16.8k	–	14400.0	1242.3k	19.8	Large	14400.0
dem_150_gaslib135_286	1e+20	58.6k	–	14400.0	1e+20	127.5k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_287	472.7k	78.5k	502.2	14400.0	1e+20	112.2k	–	14400.0	1e+20	68.9k	–	14400.0
dem_150_gaslib135_288	1e+20	58.6k	–	14400.0	1e+20	104.6k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_289	1e+20	51.9k	–	14400.0	1e+20	79.5k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_290	1e+20	68.9k	–	14400.0	1e+20	79.8k	–	14400.0	1e+20	58.7k	–	14400.0
dem_150_gaslib135_291	1e+20	11.6k	–	14400.0	1e+20	18.5k	–	14400.0	1e+20	2.2	–	14400.0
dem_150_gaslib135_292	1e+20	36.5k	–	14400.0	1e+20	83.7k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_293	1e+20	31.7k	–	14400.0	1e+20	62.7k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_294	1e+20	115.7k	–	14400.0	1e+20	174.5k	–	14400.0	1e+20	73.1k	–	14400.0
dem_150_gaslib135_295	1e+20	36.4k	–	14400.0	1e+20	50.1k	–	14400.0	85.0k	45.0k	89.1	14400.0
dem_150_gaslib135_296	1e+20	50.0k	–	14400.0	1e+20	91.9k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_297	83.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0	687.2k	0.0	Large	14400.0
dem_150_gaslib135_298	7931.4	0.0	–	14400.0	1e+20	0.0	–	14400.0	22.2k	0.0	Large	14400.0
dem_150_gaslib135_299	1e+20	31.7k	–	14400.0	1e+20	49.1k	–	14400.0	142.4k	31.7k	349.6	14400.0
dem_150_gaslib135_300	1e+20	0.0	–	14400.0	1e+20	14.8k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_301	1e+20	5132.3	–	14400.0	1e+20	32.5k	–	14400.0	1e+20	1488.1	–	14400.0
dem_150_gaslib135_302	1e+20	76.1k	–	14400.0	1e+20	122.1k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_303	1e+20	5920.3	–	14400.0	1e+20	52.8k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_304	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_305	1e+20	92.2k	–	14400.0	1e+20	78.2k	–	14400.0	1e+20	4042.7	–	14400.0
dem_150_gaslib135_306	282.6k	91.6k	208.4	14400.0	1e+20	105.3k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_307	1e+20	36.4k	–	14400.0	1e+20	72.9k	–	14400.0	952.5k	36.4k	2514.3	14400.0
dem_150_gaslib135_308	384.4k	73.6k	422.5	14400.0	1e+20	113.0k	–	14400.0	1e+20	60.3k	–	14400.0

continued on next page

Table A.21: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_309	478.7k	1488.0	Large	14400.0	1e+20	49.0k	–	14400.0	126.0k	1488.0	8370.3	14400.0
dem_150_gaslib135_310	1089.6k	0.0	–	14400.0	1e+20	16.4k	–	14400.0	1e+20	209.9	–	14400.0
dem_150_gaslib135_311	509.4k	42.7k	1091.9	14400.0	1e+20	87.4k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_312	1053.0k	42.9k	2356.1	14400.0	1e+20	69.9k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_313	1e+20	52.8k	–	14400.0	1e+20	97.0k	–	14400.0	1021.1k	43.3k	2258.5	14400.0
dem_150_gaslib135_314	1e+20	47.5k	–	14400.0	1e+20	69.6k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_315	1e+20	65.0k	–	14400.0	1e+20	108.3k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_316	509.9k	0.0	–	14400.0	1e+20	7414.9	–	14400.0	392.4k	0.0	–	14400.0
dem_150_gaslib135_317	1e+20	19.2k	–	14400.0	1e+20	17.1k	–	14400.0	1e+20	159.0	–	14400.0
dem_150_gaslib135_318	263.8k	31.7k	732.9	14400.0	1e+20	49.8k	–	14400.0	71.3k	33.7k	111.8	14400.0
dem_150_gaslib135_319	1677.4k	42.9k	3812.4	14400.0	1e+20	63.1k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_320	1431.9k	0.0	–	14399.9	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_321	1e+20	36.5k	–	14400.0	1e+20	90.0k	–	14400.0	1e+20	36.5k	–	14400.0
dem_150_gaslib135_322	437.7k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1535.9k	70.4k	2080.9	14400.0
dem_150_gaslib135_323	1e+20	37.4k	–	14400.0	1e+20	141.5k	–	14400.0	1e+20	37.4k	–	14400.0
dem_150_gaslib135_324	1e+20	73.3k	–	14400.0	1e+20	104.2k	–	14400.0	1e+20	64.9k	–	14400.0
dem_150_gaslib135_325	1e+20	0.0	–	14400.0	1e+20	47.1k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_326	1e+20	50.0k	–	14400.0	1e+20	76.7k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_327	1573.2k	65.4k	2304.2	14400.0	1e+20	75.4k	–	14400.0	1e+20	38.5k	–	14400.0
dem_150_gaslib135_328	84.1k	20.7k	305.8	14400.0	1e+20	45.2k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_329	1e+20	1624.3	–	14400.0	1e+20	49.9k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_330	1e+20	31.7k	–	14400.0	1e+20	71.6k	–	14400.0	1710.8k	30.2k	5567.9	14400.0
dem_150_gaslib135_331	1e+20	0.0	–	14400.0	1e+20	603.4	–	14400.0	48.0k	1556.6	2983.0	14400.0
dem_150_gaslib135_332	609.4k	0.0	–	14400.0	1e+20	1699.3	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_333	185.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0	269.9k	0.0	Large	14400.0
dem_150_gaslib135_334	84.1k	10.2k	724.1	14400.0	1e+20	0.0	–	14400.0	230.1k	45.7k	403.4	14400.0
dem_150_gaslib135_335	139.5k	26.4k	427.8	14400.0	1e+20	50.1k	–	14400.0	1e+20	29.4k	–	14400.0
dem_150_gaslib135_336	1e+20	50.0k	–	14400.0	1e+20	91.6k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_337	69.0k	22.5k	207.3	14400.0	1e+20	44.2k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_338	1e+20	168.2k	–	14400.0	1e+20	190.0k	–	14400.0	1e+20	91.5k	–	14400.0
dem_150_gaslib135_339	1e+20	31.7k	–	14400.0	1e+20	61.7k	–	14400.0	225.7k	32.3k	598.5	14400.0
dem_150_gaslib135_340	1e+20	37.9k	–	14400.0	1e+20	64.4k	–	14400.0	1e+20	1495.7	–	14400.0
dem_150_gaslib135_341	1e+20	150.0k	–	14400.0	1e+20	208.7k	–	14400.0	1e+20	150.0k	–	14400.0
dem_150_gaslib135_342	1e+20	26.4k	–	14400.0	1e+20	54.5k	–	14400.0	1e+20	26.4k	–	14400.0
dem_150_gaslib135_343	1e+20	0.0	–	14400.0	1e+20	12.5k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_344	1e+20	36.5k	–	14400.0	1e+20	78.6k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_345	1e+20	11.6k	–	14400.0	1e+20	22.2k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_346	74.2k	34.0k	118.0	14400.0	1e+20	44.2k	–	14400.0	75.4k	26.4k	185.3	14400.0
dem_150_gaslib135_347	1e+20	14.0k	–	14400.0	1e+20	487.2	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_348	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_349	842.0k	126.1k	567.8	14400.0	1e+20	167.3k	–	14400.0	1525.2k	84.2k	1710.6	14400.0
dem_150_gaslib135_350	1e+20	42.9k	–	14400.0	1e+20	72.8k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_351	917.6k	0.0	–	14400.0	1e+20	13.3k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_352	1e+20	0.0	–	14400.0	1e+20	1021.9	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_353	1e+20	1488.0	–	14400.0	1e+20	52.7k	–	14400.0	114.5k	2082.0	5399.6	14400.0
dem_150_gaslib135_354	1e+20	0.4	–	14400.0	1e+20	4242.3	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_355	1e+20	83.5k	–	14400.0	1e+20	113.1k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_356	1e+20	1488.0	–	14400.0	1e+20	53.7k	–	14400.0	1e+20	16.0k	–	14400.0
dem_150_gaslib135_357	1e+20	74.9k	–	14400.0	1e+20	102.8k	–	14400.0	1e+20	78.8k	–	14400.0
dem_150_gaslib135_358	1e+20	31.7k	–	14400.0	1e+20	60.7k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_359	1e+20	51.9k	–	14400.0	1e+20	68.5k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_360	1e+20	1111.5	–	14400.0	1e+20	37.8k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_361	1e+20	0.0	–	14400.0	1e+20	64.7k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_362	1154.5k	0.0	–	14400.0	1e+20	3924.6	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_363	1049.4k	50.1k	1995.8	14400.0	1e+20	89.2k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_364	1e+20	211.4k	–	14400.0	1e+20	274.6k	–	14400.0	1e+20	187.1k	–	14400.0
dem_150_gaslib135_365	169.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0	673.5k	6885.8	9681.3	14400.0
dem_150_gaslib135_366	1e+20	0.0	–	14400.0	1e+20	25.7k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_367	1e+20	44.5k	–	14400.0	1e+20	94.4k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_368	1e+20	3229.4	–	14400.0	1e+20	36.8k	–	14400.0	351.7k	1488.0	Large	14400.0
dem_150_gaslib135_369	1e+20	0.0	–	14400.0	1e+20	53.0	–	14400.0	1e+20	0.0	–	14400.0
dem_50_gaslib135_370	1e+20	110.7k	–	14400.0	1e+20	189.4k	–	14400.0	5913.3k	110.7k	5240.3	14400.0
dem_150_gaslib135_371	99.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0	618.0k	0.0	Large	14400.0
dem_150_gaslib135_372	1e+20	0.0	–	14400.0	1e+20	9005.6	–	14400.0	41.2k	0.0	–	14400.0
dem_150_gaslib135_373	1e+20	5088.0	–	14400.0	1e+20	12.8k	–	14400.0	1e+20	0.8	–	14400.0
dem_150_gaslib135_374	1e+20	73.0k	–	14400.0	1e+20	85.1k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_375	1e+20	48.5k	–	14400.0	1e+20	47.8k	–	14400.0	2523.2k	1488.0	Large	14400.0
dem_150_gaslib135_376	1e+20	58.6k	–	14400.0	1e+20	75.2k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_377	1e+20	1488.0	–	14400.0	1e+20	48.6k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_378	402.8k	50.0k	705.0	14400.0	1e+20	76.7k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_379	405.9k	64.9k	525.4	14400.0	1e+20	111.5k	–	14400.0	860.1k	52.0k	1553.1	14400.0
dem_150_gaslib135_380	711.4k	0.0	–	14400.0	1e+20	764.5	–	14400.0	1796.2k	0.0	–	14400.0
dem_150_gaslib135_381	1e+20	3251.7	–	14400.0	1e+20	68.9k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_382	1e+20	36.5k	–	14400.0	1e+20	58.6k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_383	359.1k	16.2k	2115.4	14400.0	1e+20	13.7k	–	14400.0	2207.2k	0.0	Large	14400.0
dem_150_gaslib135_384	435.4k	0.0	–	14400.0	1e+20	154.7	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_385	614.7k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_386	1e+20	31.7k	–	14400.0	1e+20	54.9k	–	14400.0	1e+20	31.7k	–	14400.0

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Table A.21: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_387	1e+20	0.0	–	14400.0	1e+20	24.2k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_388	1e+20	0.0	–	14400.0	1e+20	745.7	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_389	1e+20	31.7k	–	14400.0	1e+20	56.9k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_390	1e+20	51.1k	–	14400.0	1e+20	117.7k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_391	1e+20	1624.3	–	14400.0	1e+20	63.0k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_392	1e+20	0.0	–	14400.0	1e+20	166.2	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_393	820.3k	58.6k	1299.3	14400.0	1e+20	121.6k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_394	1e+20	41.8k	–	14400.0	1e+20	37.0k	–	14400.0	1301.9k	1488.0	Large	14400.0
dem_150_gaslib135_395	1e+20	863.1	–	14400.0	1e+20	0.0	–	14400.0	1127.7k	0.0	Large	14400.0
dem_150_gaslib135_396	1e+20	65.2k	–	14400.0	1e+20	84.4k	–	14400.0	1e+20	43.7k	–	14400.0
dem_150_gaslib135_397	1e+20	5740.3	–	14400.0	1e+20	89.5k	–	14400.0	4053.5k	4626.0	Large	14400.0
dem_150_gaslib135_398	26.3k	7773.6	238.5	14400.0	1e+20	3.9	–	14400.0	143.6k	0.0	–	14400.0
dem_150_gaslib135_399	1e+20	38.5k	–	14400.0	1e+20	28.4k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_400	1e+20	24.7k	–	14400.0	1e+20	47.4k	–	14400.0	675.9k	1488.0	Large	14400.0
dem_150_gaslib135_401	1123.6k	91.7k	1125.0	14400.0	1e+20	105.4k	–	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_402	1e+20	0.0	–	14400.0	1e+20	33.4k	–	14400.0	1e+20	32.8k	–	14400.0
dem_150_gaslib135_403	1e+20	0.0	–	14400.0	1e+20	863.4	–	14400.0	1e+20	1e+20	0.0	13899.4
dem_150_gaslib135_404	1e+20	34.9k	–	14400.0	1e+20	67.8k	–	14400.0	81.0k	36.9k	119.8	14400.0
dem_150_gaslib135_405	93.3k	34.9k	167.3	14400.0	1e+20	56.2k	–	14400.0	1e+20	32.3k	–	14400.0
dem_150_gaslib135_406	1e+20	26.6k	–	14400.0	1e+20	54.3k	–	14400.0	1e+20	26.4k	–	14400.0
dem_150_gaslib135_407	1e+20	1488.0	–	14400.0	1e+20	24.7k	–	14400.0	1e+20	1535.1	–	14400.0
dem_150_gaslib135_408	1e+20	93.0k	–	14400.0	1e+20	114.7k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_409	205.8k	0.0	–	14400.0	1e+20	664.4	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_410	1239.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1627.2k	0.0	–	14400.0
dem_150_gaslib135_411	77.6k	0.5	Large	14400.0	1e+20	15.9k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_412	4594.8k	128.1k	3486.7	14400.0	1e+20	208.3k	–	14400.0	1e+20	82.1k	–	14400.0
dem_150_gaslib135_413	1e+20	31.7k	–	14400.0	1e+20	52.0k	–	14400.0	1e+20	1e+20	0.0	13637.7
dem_150_gaslib135_414	1e+20	71.0k	–	14400.0	1e+20	135.9k	–	14400.0	1e+20	69.5k	–	14400.0
dem_150_gaslib135_415	140.9k	16.2k	769.1	14400.0	1e+20	48.7k	–	14400.0	1e+20	1500.0	–	14400.0
dem_150_gaslib135_416	1e+20	65.0k	–	14400.0	1e+20	102.1k	–	14400.0	1e+20	66.6k	–	14400.0
dem_150_gaslib135_417	1e+20	10.9k	–	14400.0	1e+20	10.0k	–	14400.0	5048.3k	0.0	Large	14400.0
dem_150_gaslib135_418	1e+20	26.4k	–	14400.0	1e+20	52.2k	–	14400.0	1e+20	26.4k	–	14400.0
dem_150_gaslib135_419	506.6k	6233.3	8027.4	14400.0	1e+20	12.9k	–	14400.0	774.2k	257.0	Large	14400.0
dem_150_gaslib135_420	1265.1k	0.0	–	14400.0	1e+20	781.2	–	14400.0	1009.4k	0.0	–	14400.0
dem_150_gaslib135_421	525.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_422	1e+20	83.2k	–	14400.0	1e+20	120.1k	–	14400.0	3149.6k	66.8k	4616.2	14400.0
dem_150_gaslib135_423	1e+20	115.6k	–	14400.0	1e+20	148.7k	–	14400.0	2483.0k	82.6k	2904.5	14400.0
dem_150_gaslib135_424	1562.3k	74.1k	2007.4	14400.0	1e+20	145.4k	–	14400.0	1e+20	82.6k	–	14400.0
dem_150_gaslib135_425	1e+20	120.5k	–	14400.0	1e+20	132.8k	–	14400.0	1e+20	53.2k	–	14400.0
dem_150_gaslib135_426	137.1k	53.0k	158.5	14400.0	1e+20	78.3k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_427	198.5k	0.0	–	14400.0	1e+20	0.0	–	14400.0	726.5k	0.0	Large	14400.0
dem_150_gaslib135_428	1e+20	67.3k	–	14400.0	1e+20	104.8k	–	14400.0	1e+20	33.7k	–	14400.0
dem_150_gaslib135_429	1e+20	34.9k	–	14400.0	1e+20	49.3k	–	14400.0	141.4k	36.5k	287.7	14400.0
dem_150_gaslib135_430	1e+20	75.6k	–	14400.0	1e+20	69.1k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_431	1e+20	1488.0	–	14400.0	1e+20	26.0k	–	14400.0	1262.4k	1488.0	Large	14400.0
dem_150_gaslib135_432	1e+20	0.0	–	14400.0	1e+20	16.0k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_433	120.2k	51.9k	131.5	14400.0	1e+20	62.7k	–	14400.0	1e+20	57.8k	–	14400.0
dem_150_gaslib135_434	1e+20	15.9k	–	14400.0	1e+20	93.4k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_435	537.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0	2071.7k	0.0	Large	14400.0
dem_150_gaslib135_436	1e+20	0.0	–	14400.0	1e+20	1250.8	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_437	1e+20	3229.4	–	14400.0	1e+20	47.7k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_438	1e+20	60.4k	–	14400.0	1e+20	104.3k	–	14400.0	1e+20	44.8k	–	14400.0
dem_150_gaslib135_439	1e+20	38.1k	–	14400.0	1e+20	55.1k	–	14400.0	415.2k	36.4k	1040.6	14400.0
dem_150_gaslib135_440	1e+20	65.0k	–	14400.0	1e+20	93.9k	–	14400.0	1e+20	66.6k	–	14400.0
dem_150_gaslib135_441	1e+20	36.4k	–	14400.0	1e+20	54.2k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_442	1e+20	128.7k	–	14400.0	1e+20	151.4k	–	14400.0	1e+20	69.2k	–	14400.0
dem_150_gaslib135_443	1e+20	1624.3	–	14400.0	1e+20	74.1k	–	14400.0	1e+20	4316.7	–	14400.0
dem_150_gaslib135_444	1e+20	37.5k	–	14400.0	1e+20	74.4k	–	14400.0	1e+20	1500.0	–	14400.0
dem_150_gaslib135_445	1e+20	115.9k	–	14400.0	1e+20	117.8k	–	14400.0	1560.7k	69.0k	2160.3	14400.0
dem_150_gaslib135_446	1230.6k	83.8k	1368.3	14400.0	1e+20	123.5k	–	14400.0	1e+20	68.9k	–	14400.0
dem_150_gaslib135_447	1e+20	42.9k	–	14400.0	1e+20	67.3k	–	14400.0	243.8k	38.5k	533.4	14400.0
dem_150_gaslib135_448	141.9k	56.4k	151.5	14400.0	1e+20	74.8k	–	14400.0	141.9k	58.6k	142.1	14400.0
dem_150_gaslib135_449	1603.4k	66.1k	2323.9	14400.0	1e+20	121.4k	–	14400.0	1e+20	52.3k	–	14400.0
dem_150_gaslib135_450	1e+20	26.4k	–	14400.0	1e+20	57.7k	–	14400.0	1e+20	26.4k	–	14400.0
dem_150_gaslib135_451	175.4k	66.4k	164.1	14400.0	1e+20	70.1k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_452	1e+20	24.5k	–	14400.0	1e+20	13.2k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_453	682.4k	65.0k	950.4	14400.0	1e+20	97.8k	–	14400.0	1e+20	60.9k	–	14400.0
dem_150_gaslib135_454	1e+20	20.8k	–	14400.0	1e+20	69.6k	–	14400.0	1e+20	14.3k	–	14400.0
dem_150_gaslib135_455	4671.4k	3.8	Large	14400.0	1e+20	74.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_456	420.5k	42.7k	884.0	14400.0	1e+20	73.3k	–	14400.0	1e+20	42.7k	–	14400.0
dem_150_gaslib135_457	1e+20	0.0	–	14400.0	1e+20	32.4k	–	14400.0	1e+20	32.7	–	14400.0
dem_150_gaslib135_458	1e+20	0.0	–	14400.0	1e+20	17.6k	–	14400.0	5613.3k	0.0	Large	14400.0
dem_150_gaslib135_459	251.5k	1488.0	Large	14400.0	1e+20	40.2k	–	14400.0	174.9k	1508.0	Large	14400.0
dem_150_gaslib135_460	1e+20	37.7k	–	14400.0	1e+20	65.0k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_461	107.9k	49.7k	117.2	14400.0	1e+20	76.9k	–	14400.0	107.9k	55.1k	95.9	14400.0
dem_150_gaslib135_462	447.2k	66.1k	576.9	14400.0	1e+20	82.0k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_463	1e+20	51.9k	–	14400.0	1e+20	80.8k	–	14400.0	1e+20	51.9k	–	14400.0
dem_150_gaslib135_464	1e+20	0.0	–	14400.0	1e+20	93.0k	–	14400.0	1e+20	0.0	–	14400.0

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Table A.21: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_465	1790.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0	1052.5k	0.0	Large	14400.0
dem_150_gaslib135_466	1e+20	36.5k	–	14400.0	1e+20	88.9k	–	14400.0	263.5k	38.6k	582.9	14400.0
dem_150_gaslib135_467	1e+20	55.8k	–	14400.0	1e+20	73.3k	–	14400.0	584.6k	51.9k	1026.7	14400.0
dem_150_gaslib135_468	1e+20	58.0k	–	14400.0	1e+20	62.5k	–	14400.0	819.3k	12.6k	6392.3	14400.0
dem_150_gaslib135_469	1e+20	31.7k	–	14400.0	1e+20	52.5k	–	14400.0	1e+20	31.7k	–	14400.0
dem_150_gaslib135_470	375.8k	0.0	–	14400.0	1e+20	23.1k	–	14400.0	693.5k	0.0	Large	14400.0
dem_150_gaslib135_471	1e+20	6892.8	–	14400.0	1e+20	52.1k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_472	1e+20	0.0	–	14400.0	1e+20	64.7k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_473	1e+20	36.5k	–	14400.0	1e+20	76.1k	–	14400.0	1e+20	36.4k	–	14400.0
dem_150_gaslib135_474	1e+20	50.0k	–	14400.0	1e+20	89.1k	–	14400.0	1e+20	50.0k	–	14400.0
dem_150_gaslib135_475	1e+20	65.0k	–	14400.0	1e+20	118.3k	–	14400.0	1e+20	68.9k	–	14400.0
dem_150_gaslib135_476	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_477	1665.8k	69.8k	2284.9	14400.0	1e+20	114.4k	–	14400.0	1748.7k	60.3k	2800.5	14400.0
dem_150_gaslib135_478	1e+20	98.3k	–	14400.0	1e+20	139.7k	–	14400.0	1e+20	95.2k	–	14400.0
dem_150_gaslib135_479	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	861.1k	0.0	–	14400.0
dem_150_gaslib135_480	1e+20	33.2k	–	14400.0	1e+20	15.8k	–	14400.0	1060.4k	0.0	–	14400.0
dem_150_gaslib135_481	1e+20	58.6k	–	14400.0	1e+20	75.5k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_482	1779.6k	11.6k	Large	14400.0	1e+20	2472.5	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_483	920.2k	65.0k	1316.5	14400.0	1e+20	123.7k	–	14400.0	1790.8k	69.5k	2478.1	14400.0
dem_150_gaslib135_484	56.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0	487.9k	0.0	Large	14400.0
dem_150_gaslib135_485	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_486	1e+20	82.2k	–	14400.0	1e+20	156.6k	–	14400.0	1e+20	72.5k	–	14400.0
dem_150_gaslib135_487	1e+20	0.0	–	14400.0	1e+20	31.4k	–	14400.0	2204.1k	0.0	Large	14400.0
dem_150_gaslib135_488	1e+20	81.2k	–	14400.0	1e+20	151.8k	–	14400.0	1e+20	65.5k	–	14400.0
dem_150_gaslib135_489	1936.1k	55.8k	3370.9	14400.0	1e+20	73.8k	–	14400.0	1e+20	68.8k	–	14400.0
dem_150_gaslib135_490	1e+20	0.0	–	14400.0	1e+20	3359.0	–	14400.0	223.1k	507.7	Large	14400.0
dem_150_gaslib135_491	60.9k	0.0	–	14400.0	1e+20	0.0	–	14400.0	93.4k	0.0	Large	14400.0
dem_150_gaslib135_492	1e+20	1488.0	–	14400.0	1e+20	43.4k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_493	518.6k	11.6k	4376.7	14400.0	1e+20	875.2	–	14400.0	1592.5k	263.1k	505.2	14400.0
dem_150_gaslib135_494	1e+20	27.0k	–	14400.0	1e+20	44.8k	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_495	19.2k	11.8	Large	14400.0	1e+20	0.0	–	14400.0	19.2k	13.1k	45.9	14400.0
dem_150_gaslib135_496	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0	5395.6k	0.0	Large	14400.0
dem_150_gaslib135_497	1e+20	1964.9	–	14400.0	1e+20	69.7k	–	14400.0	1e+20	2179.3	–	14400.0
dem_150_gaslib135_498	1e+20	1488.0	–	14400.0	1e+20	59.4k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_499	1e+20	13.4k	–	14400.0	1e+20	15.0k	–	14400.0	1e+20	1488.0	–	14400.0
dem_150_gaslib135_500	523.5k	53.9k	871.5	14400.0	1e+20	99.9k	–	14400.0	1e+20	50.0k	–	14400.0

Table A.22: Detailed results of the split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ as summarized in Figure 3.15c and Table 3.4c. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_1	82.4k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_2	146.8k	32.1k	356.6	14400.0	1e+20	60.3k	–	14400.0
dem_150_gaslib135_3	1e+20	58.2k	–	14400.0	1e+20	95.3k	–	14400.0
dem_150_gaslib135_4	0.0	0.0	0.0	47.1	1e+20	0.0	–	14400.0
dem_150_gaslib135_5	1e+20	0.0	–	14400.0	1e+20	48.9k	–	14400.0
dem_150_gaslib135_6	1e+20	5200.7	–	14400.0	1e+20	2203.5	–	14400.0
dem_150_gaslib135_7	273.6k	42.9k	538.1	14400.0	1e+20	84.1k	–	14400.0
dem_150_gaslib135_8	1e+20	58.6k	–	14400.0	1e+20	122.1k	–	14400.0
dem_150_gaslib135_9	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_10	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_150_gaslib135_11	1e+20	36.4k	–	14400.0	1e+20	60.4k	–	14400.0
dem_150_gaslib135_12	375.0k	1488.0	Large	14400.0	1e+20	2736.1	–	14400.0
dem_150_gaslib135_13	1e+20	0.0	–	14400.0	1e+20	35.6k	–	14400.0
dem_150_gaslib135_14	405.4k	60.9k	565.5	14400.0	1e+20	82.8k	–	14400.0
dem_150_gaslib135_15	1e+20	58.6k	–	14400.0	1e+20	102.3k	–	14400.0
dem_150_gaslib135_16	1e+20	24.0k	–	14400.0	1e+20	27.7k	–	14400.0
dem_150_gaslib135_17	1e+20	50.0k	–	14400.0	1e+20	75.0k	–	14400.0
dem_150_gaslib135_18	339.8k	0.0	–	14400.0	1e+20	1126.1	–	14400.0
dem_150_gaslib135_19	1e+20	120.9k	–	14400.0	1e+20	152.9k	–	14400.0
dem_150_gaslib135_20	1e+20	1488.0	–	14400.0	1e+20	49.4k	–	14400.0
dem_150_gaslib135_21	740.8k	40.0k	1753.3	14400.0	1e+20	24.8k	–	14400.0
dem_150_gaslib135_22	311.1k	0.0	–	14400.0	1e+20	189.6	–	14400.0
dem_150_gaslib135_23	1e+20	42.9k	–	14400.0	1e+20	64.8k	–	14400.0
dem_150_gaslib135_24	1e+20	36.5k	–	14400.0	1e+20	73.2k	–	14400.0
dem_150_gaslib135_25	1e+20	55.5k	–	14400.0	1e+20	29.1k	–	14400.0
dem_150_gaslib135_26	1e+20	0.0	–	14400.0	1e+20	15.1k	–	14400.0
dem_150_gaslib135_27	99.3k	16.2k	512.7	14400.0	1e+20	283.9	–	14400.0

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Table A.22: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_28	1e+20	39.3k	–	14400.0	1e+20	76.0k	–	14400.0
dem_150_gaslib135_29	1e+20	63.0k	–	14400.0	1e+20	97.8k	–	14400.0
dem_150_gaslib135_30	82.7k	16.2k	410.3	14400.0	1e+20	16.6k	–	14400.0
dem_150_gaslib135_31	1e+20	36.5k	–	14400.0	1e+20	83.3k	–	14400.0
dem_150_gaslib135_32	393.4k	36.4k	980.5	14400.0	1e+20	51.8k	–	14400.0
dem_150_gaslib135_33	1e+20	98.1k	–	14400.0	1e+20	144.8k	–	14400.0
dem_150_gaslib135_34	197.1k	0.0	–	14400.0	1e+20	601.8	–	14400.0
dem_150_gaslib135_35	213.4k	36.4k	486.3	14400.0	1e+20	76.5k	–	14400.0
dem_150_gaslib135_36	1e+20	0.0	–	14400.0	1e+20	34.6k	–	14400.0
dem_150_gaslib135_37	2142.4k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_38	1e+20	44.5k	–	14400.0	1e+20	85.1k	–	14400.0
dem_150_gaslib135_39	1e+20	19.3k	–	14400.0	1e+20	71.1k	–	14400.0
dem_150_gaslib135_40	1e+20	0.0	–	14400.0	1e+20	3504.3	–	14400.0
dem_150_gaslib135_41	375.0k	1488.0	Large	14400.0	1e+20	18.6k	–	14400.0
dem_150_gaslib135_42	1e+20	14.0k	–	14400.0	1e+20	28.9k	–	14400.0
dem_150_gaslib135_43	70.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_44	1e+20	50.1k	–	14400.0	1e+20	83.2k	–	14400.0
dem_150_gaslib135_45	1e+20	7.1	–	14400.0	1e+20	26.3k	–	14400.0
dem_150_gaslib135_46	455.0k	46.7k	874.3	14400.0	1e+20	65.2k	–	14400.0
dem_150_gaslib135_47	1e+20	0.0	–	14400.0	1e+20	29.5k	–	14400.0
dem_150_gaslib135_48	1e+20	1488.0	–	14400.0	1e+20	50.3k	–	14400.0
dem_150_gaslib135_49	1e+20	1488.0	–	14400.0	1e+20	39.8k	–	14400.0
dem_150_gaslib135_50	637.9k	78.1k	716.6	14400.0	1e+20	107.3k	–	14400.0
dem_150_gaslib135_51	75.9k	14.7k	415.3	14400.0	1e+20	20.6k	–	14400.0
dem_150_gaslib135_52	1e+20	26.4k	–	14400.0	1e+20	43.0k	–	14400.0
dem_150_gaslib135_53	1e+20	2194.8	–	14400.0	1e+20	31.2k	–	14400.0
dem_150_gaslib135_54	1e+20	36.5k	–	14400.0	1e+20	69.7k	–	14400.0
dem_150_gaslib135_55	1e+20	36.5k	–	14400.0	1e+20	61.1k	–	14400.0
dem_150_gaslib135_56	244.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_57	1e+20	31.7k	–	14400.0	1e+20	48.8k	–	14400.0
dem_150_gaslib135_58	76.8k	31.7k	142.5	14400.0	1e+20	54.1k	–	14400.0
dem_150_gaslib135_59	1e+20	60.4k	–	14400.0	1e+20	83.3k	–	14400.0
dem_150_gaslib135_60	499.7k	0.0	–	14400.0	1e+20	2090.1	–	14400.0
dem_150_gaslib135_61	160.5k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_62	1e+20	0.0	–	14400.0	1e+20	37.9k	–	14400.0
dem_150_gaslib135_63	341.6k	5793.7	5796.4	14400.0	1e+20	210.6	–	14400.0
dem_150_gaslib135_64	1e+20	1488.0	–	14400.0	1e+20	61.6k	–	14400.0
dem_150_gaslib135_65	1145.1k	98.1k	1067.0	14400.0	1e+20	118.4k	–	14400.0
dem_150_gaslib135_66	1e+20	36.5k	–	14400.0	1e+20	91.3k	–	14400.0
dem_150_gaslib135_67	260.7k	38.9k	569.3	14400.0	1e+20	72.7k	–	14400.0
dem_150_gaslib135_68	1e+20	0.0	–	14400.0	1e+20	11.1k	–	14400.0
dem_150_gaslib135_69	1e+20	42.9k	–	14400.0	1e+20	67.1k	–	14400.0
dem_150_gaslib135_70	1e+20	31.8k	–	14400.0	1e+20	61.5k	–	14400.0
dem_150_gaslib135_71	1e+20	34.9k	–	14400.0	1e+20	76.2k	–	14400.0
dem_150_gaslib135_72	1e+20	58.6k	–	14400.0	1e+20	88.6k	–	14400.0
dem_150_gaslib135_73	966.1k	25.0k	3772.1	14400.0	1e+20	58.0k	–	14400.0
dem_150_gaslib135_74	1e+20	0.0	–	14400.0	1e+20	1134.0	–	14400.0
dem_150_gaslib135_75	1e+20	0.3	–	14400.0	1e+20	38.5k	–	14400.0
dem_150_gaslib135_76	1e+20	38.4k	–	14400.0	1e+20	60.5k	–	14400.0
dem_150_gaslib135_77	664.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_78	2441.1k	105.3	Large	14400.0	1e+20	18.7k	–	14400.0
dem_150_gaslib135_79	1e+20	50.0k	–	14400.0	1e+20	82.9k	–	14400.0
dem_150_gaslib135_80	1e+20	31.7k	–	14400.0	1e+20	51.6k	–	14400.0
dem_150_gaslib135_81	1e+20	5599.4	–	14400.0	1e+20	61.3k	–	14400.0
dem_150_gaslib135_82	1e+20	0.0	–	14400.0	1e+20	36.1k	–	14400.0
dem_150_gaslib135_83	591.6k	0.0	–	14400.0	1e+20	1169.1	–	14400.0
dem_150_gaslib135_84	1e+20	4542.9	–	14400.0	1e+20	57.0k	–	14400.0
dem_150_gaslib135_85	591.1k	0.0	–	14400.0	1e+20	3076.7	–	14400.0
dem_150_gaslib135_86	3480.1k	0.0	–	14400.0	1e+20	21.9k	–	14400.0
dem_150_gaslib135_87	1e+20	0.0	–	14400.0	1e+20	34.1k	–	14400.0
dem_150_gaslib135_88	1e+20	3229.4	–	14400.0	1e+20	9197.2	–	14400.0
dem_150_gaslib135_89	1e+20	26.4k	–	14400.0	1e+20	41.4k	–	14400.0
dem_150_gaslib135_90	403.0k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_91	159.6k	0.0	–	14400.0	1e+20	125.7	–	14400.0
dem_150_gaslib135_92	1e+20	52.6k	–	14400.0	1e+20	83.1k	–	14400.0
dem_150_gaslib135_93	648.0k	53.9k	1102.1	14400.0	1e+20	18.9k	–	14400.0
dem_150_gaslib135_94	1e+20	133.4k	–	14400.0	1e+20	97.4k	–	14400.0
dem_150_gaslib135_95	919.2k	45.9k	1901.2	14400.0	1e+20	33.1k	–	14400.0
dem_150_gaslib135_96	1e+20	11.6k	–	14400.0	1e+20	2132.6	–	14400.0
dem_150_gaslib135_97	1e+20	31.7k	–	14400.0	1e+20	70.9k	–	14400.0
dem_150_gaslib135_98	1e+20	42.9k	–	14400.0	1e+20	87.8k	–	14400.0
dem_150_gaslib135_99	1e+20	0.0	–	14400.0	1e+20	10.2k	–	14400.0
dem_150_gaslib135_100	1e+20	29.3k	–	14400.0	1e+20	86.2k	–	14400.0
dem_150_gaslib135_101	1e+20	0.0	–	14400.0	1e+20	37.3k	–	14400.0
dem_150_gaslib135_102	1e+20	79.7k	–	14400.0	1e+20	138.7k	–	14400.0
dem_150_gaslib135_103	1e+20	1488.0	–	14400.0	1e+20	47.3k	–	14400.0
dem_150_gaslib135_104	264.5k	44.5k	494.8	14400.0	1e+20	56.3k	–	14400.0
dem_150_gaslib135_105	1492.2k	0.0	–	14400.0	1e+20	0.0	–	14400.0

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Table A.22: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_106	1e+20	36.4k	–	14400.0	1e+20	65.0k	–	14400.0
dem_150_gaslib135_107	1e+20	0.0	–	14400.0	1e+20	10.9k	–	14400.0
dem_150_gaslib135_108	1062.7k	0.0	–	14400.0	1e+20	1717.1	–	14400.0
dem_150_gaslib135_109	1e+20	1488.0	–	14400.0	1e+20	90.8k	–	14400.0
dem_150_gaslib135_110	1e+20	0.0	–	14400.0	1e+20	29.4k	–	14400.0
dem_150_gaslib135_111	1e+20	0.0	–	14400.0	1e+20	29.4k	–	14400.0
dem_150_gaslib135_112	1076.1k	73.8k	1359.0	14400.0	1e+20	137.4k	–	14400.0
dem_150_gaslib135_113	1e+20	0.0	–	14400.0	1e+20	38.7k	–	14400.0
dem_150_gaslib135_114	570.7k	0.0	–	14400.0	1e+20	4661.3	–	14400.0
dem_150_gaslib135_115	1e+20	66.5k	–	14400.0	1e+20	127.8k	–	14400.0
dem_150_gaslib135_116	141.9k	86.2k	64.7	14400.0	1e+20	63.4k	–	14400.0
dem_150_gaslib135_117	1e+20	96.6k	–	14400.0	1e+20	138.3k	–	14400.0
dem_150_gaslib135_118	1e+20	10.5k	–	14400.0	1e+20	63.3k	–	14400.0
dem_150_gaslib135_119	1e+20	36.4k	–	14400.0	1e+20	72.6k	–	14400.0
dem_150_gaslib135_120	1e+20	0.0	–	14400.0	1e+20	25.7k	–	14400.0
dem_150_gaslib135_121	1e+20	65.0k	–	14400.0	1e+20	124.5k	–	14400.0
dem_150_gaslib135_122	1e+20	0.0	–	14400.0	1e+20	33.8k	–	14400.0
dem_150_gaslib135_123	1e+20	44.5k	–	14400.0	1e+20	79.4k	–	14400.0
dem_150_gaslib135_124	1e+20	31.7k	–	14400.0	1e+20	90.6k	–	14400.0
dem_150_gaslib135_125	1592.6k	42.7k	3626.4	14400.0	1e+20	76.8k	–	14400.0
dem_150_gaslib135_126	1e+20	36.7k	–	14400.0	1e+20	63.7k	–	14400.0
dem_150_gaslib135_127	1e+20	42.9k	–	14400.0	1e+20	57.9k	–	14400.0
dem_150_gaslib135_128	1e+20	16.2k	–	14400.0	1e+20	37.3k	–	14400.0
dem_150_gaslib135_129	180.1k	45.8k	293.2	14400.0	1e+20	80.4k	–	14400.0
dem_150_gaslib135_130	1e+20	65.0k	–	14400.0	1e+20	115.3k	–	14400.0
dem_150_gaslib135_131	1e+20	36.4k	–	14400.0	1e+20	57.7k	–	14400.0
dem_150_gaslib135_132	71.2k	26.5k	168.8	14400.0	1e+20	55.3k	–	14400.0
dem_150_gaslib135_133	1e+20	26.4k	–	14400.0	1e+20	52.3k	–	14400.0
dem_150_gaslib135_134	478.7k	36.4k	1215.0	14400.0	1e+20	68.6k	–	14400.0
dem_150_gaslib135_135	233.0k	53.5k	335.1	14400.0	1e+20	76.8k	–	14400.0
dem_150_gaslib135_136	1e+20	36.4k	–	14400.0	1e+20	76.2k	–	14400.0
dem_150_gaslib135_137	1155.4k	4399.3	Large	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_138	1e+20	32.1	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_139	1e+20	36.5k	–	14400.0	1e+20	95.1k	–	14400.0
dem_150_gaslib135_140	1e+20	0.0	–	14400.0	1e+20	74.8k	–	14400.0
dem_150_gaslib135_141	1e+20	0.0	–	14400.0	1e+20	19.7k	–	14400.0
dem_150_gaslib135_142	1e+20	0.2	–	14400.0	1e+20	6972.3	–	14400.0
dem_150_gaslib135_143	1e+20	17.5k	–	14400.0	1e+20	22.3k	–	14400.0
dem_150_gaslib135_144	1e+20	42.9k	–	14400.0	1e+20	67.8k	–	14400.0
dem_150_gaslib135_145	1e+20	62.5k	–	14400.0	1e+20	80.0k	–	14400.0
dem_150_gaslib135_146	81.7k	75.1k	8.8	14400.0	1e+20	52.8k	–	14400.0
dem_150_gaslib135_147	206.9k	58.6k	252.9	14400.0	1e+20	85.7k	–	14400.0
dem_150_gaslib135_148	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_149	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_150	1e+20	58.6k	–	14400.0	1e+20	106.3k	–	14400.0
dem_150_gaslib135_151	1e+20	68.6k	–	14400.0	1e+20	102.1k	–	14400.0
dem_150_gaslib135_152	1e+20	1488.0	–	14400.0	1e+20	60.5k	–	14400.0
dem_150_gaslib135_153	1e+20	97.9k	–	14400.0	1e+20	132.1k	–	14400.0
dem_150_gaslib135_154	1e+20	31.7k	–	14400.0	1e+20	49.2k	–	14400.0
dem_150_gaslib135_155	1e+20	1624.3	–	14400.0	1e+20	36.5k	–	14400.0
dem_150_gaslib135_156	1e+20	28.8k	–	14400.0	1e+20	68.0k	–	14400.0
dem_150_gaslib135_157	1e+20	0.0	–	14400.0	1e+20	14.7k	–	14400.0
dem_150_gaslib135_158	1e+20	1488.0	–	14400.0	1e+20	50.5k	–	14400.0
dem_150_gaslib135_159	1e+20	52.8k	–	14400.0	1e+20	127.6k	–	14400.0
dem_150_gaslib135_160	1e+20	1488.0	–	14400.0	1e+20	56.6k	–	14400.0
dem_150_gaslib135_161	1e+20	1488.0	–	14400.0	1e+20	54.4k	–	14400.0
dem_150_gaslib135_162	750.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_163	127.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_164	1347.4k	58.6k	2198.4	14400.0	1e+20	81.6k	–	14400.0
dem_150_gaslib135_165	303.2k	106.6k	184.4	14400.0	1e+20	96.8k	–	14400.0
dem_150_gaslib135_166	434.9k	1488.4	Large	14400.0	1e+20	16.0k	–	14400.0
dem_150_gaslib135_167	1e+20	53.5k	–	14400.0	1e+20	45.6k	–	14400.0
dem_150_gaslib135_168	1e+20	31.7k	–	14400.0	1e+20	51.0k	–	14400.0
dem_150_gaslib135_169	2068.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_170	218.3k	42.9k	409.1	14400.0	1e+20	61.1k	–	14400.0
dem_150_gaslib135_171	840.8k	55.2k	1423.5	14400.0	1e+20	26.3k	–	14400.0
dem_150_gaslib135_172	345.4k	60.4k	472.0	14400.0	1e+20	70.6k	–	14400.0
dem_150_gaslib135_173	1e+20	68.1k	–	14400.0	1e+20	99.7k	–	14400.0
dem_150_gaslib135_174	1e+20	0.0	–	14400.0	1e+20	1656.5	–	14400.0
dem_150_gaslib135_175	1e+20	1624.3	–	14400.0	1e+20	38.8k	–	14400.0
dem_150_gaslib135_176	1e+20	1488.0	–	14400.0	1e+20	56.8k	–	14400.0
dem_150_gaslib135_177	1e+20	31.7k	–	14400.0	1e+20	48.8k	–	14400.0
dem_150_gaslib135_178	1e+20	50.0k	–	14400.0	1e+20	86.5k	–	14400.0
dem_150_gaslib135_179	1e+20	42.9k	–	14400.0	1e+20	79.5k	–	14400.0
dem_150_gaslib135_180	1e+20	30.2k	–	14400.0	1e+20	60.8k	–	14400.0
dem_150_gaslib135_181	1e+20	14.0k	–	14400.0	1e+20	34.8k	–	14400.0
dem_150_gaslib135_182	1e+20	36.4k	–	14400.0	1e+20	79.6k	–	14400.0
dem_150_gaslib135_183	1e+20	36.4k	–	14400.0	1e+20	85.3k	–	14400.0

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Table A.22: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_184	1e+20	6142.5	–	14400.0	1e+20	44.6k	–	14400.0
dem_150_gaslib135_185	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_186	80.2k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_187	71.2k	31.7k	124.9	14400.0	1e+20	52.9k	–	14400.0
dem_150_gaslib135_188	2557.2k	56.8k	4403.7	14400.0	1e+20	96.6k	–	14400.0
dem_150_gaslib135_189	1e+20	54.3k	–	14400.0	1e+20	36.1k	–	14400.0
dem_150_gaslib135_190	1055.7k	16.2k	6413.4	14400.0	1e+20	731.8	–	14400.0
dem_150_gaslib135_191	1e+20	45.0	–	14400.0	1e+20	11.5k	–	14400.0
dem_150_gaslib135_192	817.6k	42.7k	1813.0	14400.0	1e+20	92.2k	–	14400.0
dem_150_gaslib135_193	1e+20	0.0	–	14400.0	1e+20	6874.0	–	14400.0
dem_150_gaslib135_194	1e+20	37.2	–	14400.0	1e+20	127.1	–	14400.0
dem_150_gaslib135_195	1e+20	64.9k	–	14400.0	1e+20	94.4k	–	14400.0
dem_150_gaslib135_196	1e+20	76.3k	–	14400.0	1e+20	104.2k	–	14400.0
dem_150_gaslib135_197	1e+20	1624.3	–	14400.0	1e+20	60.5k	–	14400.0
dem_150_gaslib135_198	1e+20	31.7k	–	14400.0	1e+20	54.5k	–	14400.0
dem_150_gaslib135_199	1e+20	58.6k	–	14400.0	1e+20	100.2k	–	14400.0
dem_150_gaslib135_200	1e+20	36.4k	–	14400.0	1e+20	59.1k	–	14400.0
dem_150_gaslib135_201	185.1k	0.0	–	14400.0	1e+20	980.0	–	14400.0
dem_150_gaslib135_202	1e+20	0.0	–	14400.0	1e+20	28.9k	–	14400.0
dem_150_gaslib135_203	1e+20	138.2k	–	14400.0	1e+20	197.6k	–	14400.0
dem_150_gaslib135_204	1e+20	14.8k	–	14400.0	1e+20	29.0k	–	14400.0
dem_150_gaslib135_205	1e+20	31.7k	–	14400.0	1e+20	96.4k	–	14400.0
dem_150_gaslib135_206	1e+20	40.7k	–	14400.0	1e+20	72.9k	–	14400.0
dem_150_gaslib135_207	1e+20	81.8k	–	14400.0	1e+20	94.7k	–	14400.0
dem_150_gaslib135_208	1e+20	0.0	–	14400.0	1e+20	19.6k	–	14400.0
dem_150_gaslib135_209	1e+20	72.7k	–	14400.0	1e+20	81.2k	–	14400.0
dem_150_gaslib135_210	1e+20	97.6k	–	14400.0	1e+20	102.0k	–	14400.0
dem_150_gaslib135_211	1e+20	88.5k	–	14400.0	1e+20	99.8k	–	14400.0
dem_150_gaslib135_212	1e+20	0.0	–	14400.0	1e+20	6818.9	–	14400.0
dem_150_gaslib135_213	1e+20	42.7k	–	14400.0	1e+20	54.7k	–	14400.0
dem_150_gaslib135_214	1e+20	43.7k	–	14400.0	1e+20	91.8k	–	14400.0
dem_150_gaslib135_215	1e+20	36.5k	–	14400.0	1e+20	66.1k	–	14400.0
dem_150_gaslib135_216	1e+20	0.0	–	14400.0	1e+20	1501.1	–	14400.0
dem_150_gaslib135_217	305.2k	18.9k	1511.8	14400.0	1e+20	851.0	–	14400.0
dem_150_gaslib135_218	1e+20	36.4k	–	14400.0	1e+20	56.8k	–	14400.0
dem_150_gaslib135_219	2205.2k	65.4k	3273.3	14400.0	1e+20	135.7k	–	14400.0
dem_150_gaslib135_220	1e+20	4.2	–	14400.0	1e+20	449.0	–	14400.0
dem_150_gaslib135_221	1e+20	31.7k	–	14400.0	1e+20	42.5k	–	14400.0
dem_150_gaslib135_222	1e+20	36.5k	–	14400.0	1e+20	88.3k	–	14400.0
dem_150_gaslib135_223	252.0k	60.3k	317.9	14400.0	1e+20	81.4k	–	14400.0
dem_150_gaslib135_224	1e+20	162.0k	–	14400.0	1e+20	215.2k	–	14400.0
dem_150_gaslib135_225	432.0k	47.5k	810.0	14400.0	1e+20	57.0k	–	14400.0
dem_150_gaslib135_226	1e+20	149.0k	–	14400.0	1e+20	199.6k	–	14400.0
dem_150_gaslib135_227	1e+20	1488.0	–	14400.0	1e+20	4855.5	–	14400.0
dem_150_gaslib135_228	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_229	1e+20	48.9k	–	14400.0	1e+20	86.1k	–	14400.0
dem_150_gaslib135_230	205.6k	59.1k	248.0	14400.0	1e+20	82.1k	–	14400.0
dem_150_gaslib135_231	1e+20	1624.3	–	14400.0	1e+20	20.8k	–	14400.0
dem_150_gaslib135_232	1e+20	10.3k	–	14400.0	1e+20	48.4k	–	14400.0
dem_150_gaslib135_233	1e+20	34.2k	–	14400.0	1e+20	81.3k	–	14400.0
dem_150_gaslib135_234	245.9k	44.5k	452.7	14400.0	1e+20	67.9k	–	14400.0
dem_150_gaslib135_235	1e+20	31.7k	–	14400.0	1e+20	55.3k	–	14400.0
dem_150_gaslib135_236	812.4k	31.7k	2465.1	14400.0	1e+20	57.7k	–	14400.0
dem_150_gaslib135_237	738.0k	83.1k	788.2	14400.0	1e+20	133.7k	–	14400.0
dem_150_gaslib135_238	1e+20	54.9k	–	14400.0	1e+20	117.6k	–	14400.0
dem_150_gaslib135_239	495.5k	80.0k	519.2	14400.0	1e+20	83.9k	–	14400.0
dem_150_gaslib135_240	208.2k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_241	1e+20	0.0	–	14400.0	1e+20	11.9k	–	14400.0
dem_150_gaslib135_242	2411.5k	0.0	–	14400.0	1e+20	596.8	–	14400.0
dem_150_gaslib135_243	1e+20	1488.0	–	14400.0	1e+20	43.4k	–	14400.0
dem_150_gaslib135_244	1e+20	1488.0	–	14400.0	1e+20	8514.0	–	14400.0
dem_150_gaslib135_245	694.7k	61.9k	1021.6	14400.0	1e+20	51.6k	–	14400.0
dem_150_gaslib135_246	1e+20	26.6k	–	14400.0	1e+20	58.2k	–	14400.0
dem_150_gaslib135_247	454.5k	65.0k	599.6	14400.0	1e+20	96.2k	–	14400.0
dem_150_gaslib135_248	1e+20	42.7k	–	14400.0	1e+20	91.1k	–	14400.0
dem_150_gaslib135_249	1e+20	42.9k	–	14400.0	1e+20	87.5k	–	14400.0
dem_150_gaslib135_250	1e+20	31.7k	–	14400.0	1e+20	50.4k	–	14400.0
dem_150_gaslib135_251	249.5k	50.0k	398.6	14400.0	1e+20	73.8k	–	14400.0
dem_150_gaslib135_252	0.0	0.0	0.0	2.9	1e+20	0.0	–	14400.0
dem_150_gaslib135_253	1e+20	50.0k	–	14400.0	1e+20	96.9k	–	14400.0
dem_150_gaslib135_254	1e+20	50.0k	–	14400.0	1e+20	92.3k	–	14400.0
dem_150_gaslib135_255	70.8k	0.0	–	14399.6	1e+20	0.0	–	14400.0
dem_150_gaslib135_256	1335.0k	1488.0	Large	14400.0	1e+20	41.0k	–	14400.0
dem_150_gaslib135_257	270.2k	103.0k	162.4	14400.0	1e+20	86.2k	–	14400.0
dem_150_gaslib135_258	1e+20	83.7k	–	14400.0	1e+20	103.2k	–	14400.0
dem_150_gaslib135_259	1e+20	36.8k	–	14400.0	1e+20	59.7k	–	14400.0
dem_150_gaslib135_260	1e+20	36.4k	–	14400.0	1e+20	69.5k	–	14400.0
dem_150_gaslib135_261	339.4k	70.2k	383.4	14400.0	1e+20	127.0k	–	14400.0

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Table A.22: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_262	611.6k	36.4k	1580.2	14400.0	1e+20	72.5k	–	14400.0
dem_150_gaslib135_263	1e+20	0.0	–	14400.0	1e+20	11.5k	–	14400.0
dem_150_gaslib135_264	1e+20	36.4k	–	14400.0	1e+20	69.0k	–	14400.0
dem_150_gaslib135_265	1e+20	58.6k	–	14400.0	1e+20	98.7k	–	14400.0
dem_150_gaslib135_266	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_267	190.0k	79.7k	138.3	14400.0	1e+20	74.0k	–	14400.0
dem_150_gaslib135_268	793.3k	51.9k	1429.0	14400.0	1e+20	90.1k	–	14400.0
dem_150_gaslib135_269	338.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_270	1e+20	160.8k	–	14400.0	1e+20	162.7k	–	14400.0
dem_150_gaslib135_271	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_272	245.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_273	1e+20	42.8k	–	14400.0	1e+20	35.6k	–	14400.0
dem_150_gaslib135_274	1e+20	72.6k	–	14400.0	1e+20	108.8k	–	14400.0
dem_150_gaslib135_275	475.9k	118.2k	302.6	14400.0	1e+20	125.9k	–	14400.0
dem_150_gaslib135_276	223.5k	51.9k	330.7	14400.0	1e+20	75.4k	–	14400.0
dem_150_gaslib135_277	534.1k	92.9k	475.0	14400.0	1e+20	105.5k	–	14400.0
dem_150_gaslib135_278	1e+20	144.8k	–	14400.0	1e+20	183.5k	–	14400.0
dem_150_gaslib135_279	1e+20	38.1k	–	14400.0	1e+20	83.9k	–	14400.0
dem_150_gaslib135_280	373.6k	0.0	–	14400.0	1e+20	727.8	–	14400.0
dem_150_gaslib135_281	1e+20	0.0	–	14400.0	1e+20	6202.0	–	14400.0
dem_150_gaslib135_282	1e+20	36.4k	–	14400.0	1e+20	60.9k	–	14400.0
dem_150_gaslib135_283	231.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_284	1e+20	1488.0	–	14400.0	1e+20	41.1k	–	14400.0
dem_150_gaslib135_285	1e+20	14.0k	–	14400.0	1e+20	16.8k	–	14400.0
dem_150_gaslib135_286	1e+20	58.6k	–	14400.0	1e+20	127.5k	–	14400.0
dem_150_gaslib135_287	472.7k	78.5k	502.2	14400.0	1e+20	112.2k	–	14400.0
dem_150_gaslib135_288	1e+20	58.6k	–	14400.0	1e+20	104.6k	–	14400.0
dem_150_gaslib135_289	1e+20	51.9k	–	14400.0	1e+20	79.5k	–	14400.0
dem_150_gaslib135_290	1e+20	68.9k	–	14400.0	1e+20	79.8k	–	14400.0
dem_150_gaslib135_291	1e+20	11.6k	–	14400.0	1e+20	18.5k	–	14400.0
dem_150_gaslib135_292	1e+20	36.5k	–	14400.0	1e+20	83.7k	–	14400.0
dem_150_gaslib135_293	1e+20	31.7k	–	14400.0	1e+20	62.7k	–	14400.0
dem_150_gaslib135_294	1e+20	115.7k	–	14400.0	1e+20	174.5k	–	14400.0
dem_150_gaslib135_295	1e+20	36.4k	–	14400.0	1e+20	50.1k	–	14400.0
dem_150_gaslib135_296	1e+20	50.0k	–	14400.0	1e+20	91.9k	–	14400.0
dem_150_gaslib135_297	83.1k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_298	7931.4	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_299	1e+20	31.7k	–	14400.0	1e+20	49.1k	–	14400.0
dem_150_gaslib135_300	1e+20	0.0	–	14400.0	1e+20	14.8k	–	14400.0
dem_150_gaslib135_301	1e+20	5132.3	–	14400.0	1e+20	32.5k	–	14400.0
dem_150_gaslib135_302	1e+20	76.1k	–	14400.0	1e+20	122.1k	–	14400.0
dem_150_gaslib135_303	1e+20	5920.3	–	14400.0	1e+20	52.8k	–	14400.0
dem_150_gaslib135_304	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_305	1e+20	92.2k	–	14400.0	1e+20	78.2k	–	14400.0
dem_150_gaslib135_306	282.6k	91.6k	208.4	14400.0	1e+20	105.3k	–	14400.0
dem_150_gaslib135_307	1e+20	36.4k	–	14400.0	1e+20	72.9k	–	14400.0
dem_150_gaslib135_308	384.4k	73.6k	422.5	14400.0	1e+20	113.0k	–	14400.0
dem_150_gaslib135_309	478.7k	1488.0	Large	14400.0	1e+20	49.0k	–	14400.0
dem_150_gaslib135_310	1089.6k	0.0	–	14400.0	1e+20	16.4k	–	14400.0
dem_150_gaslib135_311	509.4k	42.7k	1091.9	14400.0	1e+20	87.4k	–	14400.0
dem_150_gaslib135_312	1053.0k	42.9k	2356.1	14400.0	1e+20	69.9k	–	14400.0
dem_150_gaslib135_313	1e+20	52.8k	–	14400.0	1e+20	97.0k	–	14400.0
dem_150_gaslib135_314	1e+20	47.5k	–	14400.0	1e+20	69.6k	–	14400.0
dem_150_gaslib135_315	1e+20	65.0k	–	14400.0	1e+20	108.3k	–	14400.0
dem_150_gaslib135_316	509.9k	0.0	–	14400.0	1e+20	7414.9	–	14400.0
dem_150_gaslib135_317	1e+20	19.2k	–	14400.0	1e+20	17.1k	–	14400.0
dem_150_gaslib135_318	263.8k	31.7k	732.9	14400.0	1e+20	49.8k	–	14400.0
dem_150_gaslib135_319	1677.4k	42.9k	3812.4	14400.0	1e+20	63.1k	–	14400.0
dem_150_gaslib135_320	1431.9k	0.0	–	14399.9	1e+20	0.0	–	14400.0
dem_150_gaslib135_321	1e+20	36.5k	–	14400.0	1e+20	90.0k	–	14400.0
dem_150_gaslib135_322	437.7k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_323	1e+20	37.4k	–	14400.0	1e+20	141.5k	–	14400.0
dem_150_gaslib135_324	1e+20	73.3k	–	14400.0	1e+20	104.2k	–	14400.0
dem_150_gaslib135_325	1e+20	0.0	–	14400.0	1e+20	47.1k	–	14400.0
dem_150_gaslib135_326	1e+20	50.0k	–	14400.0	1e+20	76.7k	–	14400.0
dem_150_gaslib135_327	1573.2k	65.4k	2304.2	14400.0	1e+20	75.4k	–	14400.0
dem_150_gaslib135_328	84.1k	20.7k	305.8	14400.0	1e+20	45.2k	–	14400.0
dem_150_gaslib135_329	1e+20	1624.3	–	14400.0	1e+20	49.9k	–	14400.0
dem_150_gaslib135_330	1e+20	31.7k	–	14400.0	1e+20	71.6k	–	14400.0
dem_150_gaslib135_331	1e+20	0.0	–	14400.0	1e+20	603.4	–	14400.0
dem_150_gaslib135_332	609.4k	0.0	–	14400.0	1e+20	1699.3	–	14400.0
dem_150_gaslib135_333	185.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_334	84.1k	10.2k	724.1	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_335	139.5k	26.4k	427.8	14400.0	1e+20	50.1k	–	14400.0
dem_150_gaslib135_336	1e+20	50.0k	–	14400.0	1e+20	91.6k	–	14400.0
dem_150_gaslib135_337	69.0k	22.5k	207.3	14400.0	1e+20	44.2k	–	14400.0
dem_150_gaslib135_338	1e+20	168.2k	–	14400.0	1e+20	190.0k	–	14400.0
dem_150_gaslib135_339	1e+20	31.7k	–	14400.0	1e+20	61.7k	–	14400.0

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Table A.22: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_340	1e+20	37.9k	–	14400.0	1e+20	64.4k	–	14400.0
dem_150_gaslib135_341	1e+20	150.0k	–	14400.0	1e+20	208.7k	–	14400.0
dem_150_gaslib135_342	1e+20	26.4k	–	14400.0	1e+20	54.5k	–	14400.0
dem_150_gaslib135_343	1e+20	0.0	–	14400.0	1e+20	12.5k	–	14400.0
dem_150_gaslib135_344	1e+20	36.5k	–	14400.0	1e+20	78.6k	–	14400.0
dem_150_gaslib135_345	1e+20	11.6k	–	14400.0	1e+20	22.2k	–	14400.0
dem_150_gaslib135_346	74.2k	34.0k	118.0	14400.0	1e+20	44.2k	–	14400.0
dem_150_gaslib135_347	1e+20	14.0k	–	14400.0	1e+20	487.2	–	14400.0
dem_150_gaslib135_348	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_349	842.0k	126.1k	567.8	14400.0	1e+20	167.3k	–	14400.0
dem_150_gaslib135_350	1e+20	42.9k	–	14400.0	1e+20	72.8k	–	14400.0
dem_150_gaslib135_351	917.6k	0.0	–	14400.0	1e+20	13.3k	–	14400.0
dem_150_gaslib135_352	1e+20	0.0	–	14400.0	1e+20	1021.9	–	14400.0
dem_150_gaslib135_353	1e+20	1488.0	–	14400.0	1e+20	52.7k	–	14400.0
dem_150_gaslib135_354	1e+20	0.4	–	14400.0	1e+20	4242.3	–	14400.0
dem_150_gaslib135_355	1e+20	83.5k	–	14400.0	1e+20	113.1k	–	14400.0
dem_150_gaslib135_356	1e+20	1488.0	–	14400.0	1e+20	53.7k	–	14400.0
dem_150_gaslib135_357	1e+20	74.9k	–	14400.0	1e+20	102.8k	–	14400.0
dem_150_gaslib135_358	1e+20	31.7k	–	14400.0	1e+20	60.7k	–	14400.0
dem_150_gaslib135_359	1e+20	51.9k	–	14400.0	1e+20	68.5k	–	14400.0
dem_150_gaslib135_360	1e+20	1111.5	–	14400.0	1e+20	37.8k	–	14400.0
dem_150_gaslib135_361	1e+20	0.0	–	14400.0	1e+20	64.7k	–	14400.0
dem_150_gaslib135_362	1154.5k	0.0	–	14400.0	1e+20	3924.6	–	14400.0
dem_150_gaslib135_363	1049.4k	50.1k	1995.8	14400.0	1e+20	89.2k	–	14400.0
dem_150_gaslib135_364	1e+20	211.4k	–	14400.0	1e+20	274.6k	–	14400.0
dem_150_gaslib135_365	169.6k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_366	1e+20	0.0	–	14400.0	1e+20	25.7k	–	14400.0
dem_150_gaslib135_367	1e+20	44.5k	–	14400.0	1e+20	94.4k	–	14400.0
dem_150_gaslib135_368	1e+20	3229.4	–	14400.0	1e+20	36.8k	–	14400.0
dem_150_gaslib135_369	1e+20	0.0	–	14400.0	1e+20	53.0	–	14400.0
dem_50_gaslib135_370	1e+20	110.7k	–	14400.0	1e+20	189.4k	–	14400.0
dem_150_gaslib135_371	99.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_372	1e+20	0.0	–	14400.0	1e+20	9005.6	–	14400.0
dem_150_gaslib135_373	1e+20	5088.0	–	14400.0	1e+20	12.8k	–	14400.0
dem_150_gaslib135_374	1e+20	73.0k	–	14400.0	1e+20	85.1k	–	14400.0
dem_150_gaslib135_375	1e+20	48.5k	–	14400.0	1e+20	47.8k	–	14400.0
dem_150_gaslib135_376	1e+20	58.6k	–	14400.0	1e+20	75.2k	–	14400.0
dem_150_gaslib135_377	1e+20	1488.0	–	14400.0	1e+20	48.6k	–	14400.0
dem_150_gaslib135_378	402.8k	50.0k	705.0	14400.0	1e+20	76.7k	–	14400.0
dem_150_gaslib135_379	405.9k	64.9k	525.4	14400.0	1e+20	111.5k	–	14400.0
dem_150_gaslib135_380	711.4k	0.0	–	14400.0	1e+20	764.5	–	14400.0
dem_150_gaslib135_381	1e+20	3251.7	–	14400.0	1e+20	68.9k	–	14400.0
dem_150_gaslib135_382	1e+20	36.5k	–	14400.0	1e+20	58.6k	–	14400.0
dem_150_gaslib135_383	359.1k	16.2k	2115.4	14400.0	1e+20	13.7k	–	14400.0
dem_150_gaslib135_384	435.4k	0.0	–	14400.0	1e+20	154.7	–	14400.0
dem_150_gaslib135_385	614.7k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_386	1e+20	31.7k	–	14400.0	1e+20	54.9k	–	14400.0
dem_150_gaslib135_387	1e+20	0.0	–	14400.0	1e+20	24.2k	–	14400.0
dem_150_gaslib135_388	1e+20	0.0	–	14400.0	1e+20	745.7	–	14400.0
dem_150_gaslib135_389	1e+20	31.7k	–	14400.0	1e+20	56.9k	–	14400.0
dem_150_gaslib135_390	1e+20	51.1k	–	14400.0	1e+20	117.7k	–	14400.0
dem_150_gaslib135_391	1e+20	1624.3	–	14400.0	1e+20	63.0k	–	14400.0
dem_150_gaslib135_392	1e+20	0.0	–	14400.0	1e+20	166.2	–	14400.0
dem_150_gaslib135_393	820.3k	58.6k	1299.3	14400.0	1e+20	121.6k	–	14400.0
dem_150_gaslib135_394	1e+20	41.8k	–	14400.0	1e+20	37.0k	–	14400.0
dem_150_gaslib135_395	1e+20	863.1	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_396	1e+20	65.2k	–	14400.0	1e+20	84.4k	–	14400.0
dem_150_gaslib135_397	1e+20	5740.3	–	14400.0	1e+20	89.5k	–	14400.0
dem_150_gaslib135_398	26.3k	7773.6	238.5	14400.0	1e+20	3.9	–	14400.0
dem_150_gaslib135_399	1e+20	38.5k	–	14400.0	1e+20	28.4k	–	14400.0
dem_150_gaslib135_400	1e+20	24.7k	–	14400.0	1e+20	47.4k	–	14400.0
dem_150_gaslib135_401	1123.6k	91.7k	1125.0	14400.0	1e+20	105.4k	–	14400.0
dem_150_gaslib135_402	1e+20	0.0	–	14400.0	1e+20	33.4k	–	14400.0
dem_150_gaslib135_403	1e+20	0.0	–	14400.0	1e+20	863.4	–	14400.0
dem_150_gaslib135_404	1e+20	34.9k	–	14400.0	1e+20	67.8k	–	14400.0
dem_150_gaslib135_405	93.3k	34.9k	167.3	14400.0	1e+20	56.2k	–	14400.0
dem_150_gaslib135_406	1e+20	26.6k	–	14400.0	1e+20	54.3k	–	14400.0
dem_150_gaslib135_407	1e+20	1488.0	–	14400.0	1e+20	24.7k	–	14400.0
dem_150_gaslib135_408	1e+20	93.0k	–	14400.0	1e+20	114.7k	–	14400.0
dem_150_gaslib135_409	205.8k	0.0	–	14400.0	1e+20	664.4	–	14400.0
dem_150_gaslib135_410	1239.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_411	77.6k	0.5	Large	14400.0	1e+20	15.9k	–	14400.0
dem_150_gaslib135_412	4594.8k	128.1k	3486.7	14400.0	1e+20	208.3k	–	14400.0
dem_150_gaslib135_413	1e+20	31.7k	–	14400.0	1e+20	52.0k	–	14400.0
dem_150_gaslib135_414	1e+20	71.0k	–	14400.0	1e+20	135.9k	–	14400.0
dem_150_gaslib135_415	140.9k	16.2k	769.1	14400.0	1e+20	48.7k	–	14400.0
dem_150_gaslib135_416	1e+20	65.0k	–	14400.0	1e+20	102.1k	–	14400.0
dem_150_gaslib135_417	1e+20	10.9k	–	14400.0	1e+20	10.0k	–	14400.0

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Table A.22: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_418	1e+20	26.4k	–	14400.0	1e+20	52.2k	–	14400.0
dem_150_gaslib135_419	506.6k	6233.3	8027.4	14400.0	1e+20	12.9k	–	14400.0
dem_150_gaslib135_420	1265.1k	0.0	–	14400.0	1e+20	781.2	–	14400.0
dem_150_gaslib135_421	525.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_422	1e+20	83.2k	–	14400.0	1e+20	120.1k	–	14400.0
dem_150_gaslib135_423	1e+20	115.6k	–	14400.0	1e+20	148.7k	–	14400.0
dem_150_gaslib135_424	1562.3k	74.1k	2007.4	14400.0	1e+20	145.4k	–	14400.0
dem_150_gaslib135_425	1e+20	120.5k	–	14400.0	1e+20	132.8k	–	14400.0
dem_150_gaslib135_426	137.1k	53.0k	158.5	14400.0	1e+20	78.3k	–	14400.0
dem_150_gaslib135_427	198.5k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_428	1e+20	67.3k	–	14400.0	1e+20	104.8k	–	14400.0
dem_150_gaslib135_429	1e+20	34.9k	–	14400.0	1e+20	49.3k	–	14400.0
dem_150_gaslib135_430	1e+20	75.6k	–	14400.0	1e+20	69.1k	–	14400.0
dem_150_gaslib135_431	1e+20	1488.0	–	14400.0	1e+20	26.0k	–	14400.0
dem_150_gaslib135_432	1e+20	0.0	–	14400.0	1e+20	16.0k	–	14400.0
dem_150_gaslib135_433	120.2k	51.9k	131.5	14400.0	1e+20	62.7k	–	14400.0
dem_150_gaslib135_434	1e+20	15.9k	–	14400.0	1e+20	93.4k	–	14400.0
dem_150_gaslib135_435	537.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_436	1e+20	0.0	–	14400.0	1e+20	1250.8	–	14400.0
dem_150_gaslib135_437	1e+20	3229.4	–	14400.0	1e+20	47.7k	–	14400.0
dem_150_gaslib135_438	1e+20	60.4k	–	14400.0	1e+20	104.3k	–	14400.0
dem_150_gaslib135_439	1e+20	38.1k	–	14400.0	1e+20	55.1k	–	14400.0
dem_150_gaslib135_440	1e+20	65.0k	–	14400.0	1e+20	93.9k	–	14400.0
dem_150_gaslib135_441	1e+20	36.4k	–	14400.0	1e+20	54.2k	–	14400.0
dem_150_gaslib135_442	1e+20	128.7k	–	14400.0	1e+20	151.4k	–	14400.0
dem_150_gaslib135_443	1e+20	1624.3	–	14400.0	1e+20	74.1k	–	14400.0
dem_150_gaslib135_444	1e+20	37.5k	–	14400.0	1e+20	74.4k	–	14400.0
dem_150_gaslib135_445	1e+20	115.9k	–	14400.0	1e+20	117.8k	–	14400.0
dem_150_gaslib135_446	1230.6k	83.8k	1368.3	14400.0	1e+20	123.5k	–	14400.0
dem_150_gaslib135_447	1e+20	42.9k	–	14400.0	1e+20	67.3k	–	14400.0
dem_150_gaslib135_448	141.9k	56.4k	151.5	14400.0	1e+20	74.8k	–	14400.0
dem_150_gaslib135_449	1603.4k	66.1k	2323.9	14400.0	1e+20	121.4k	–	14400.0
dem_150_gaslib135_450	1e+20	26.4k	–	14400.0	1e+20	57.7k	–	14400.0
dem_150_gaslib135_451	175.4k	66.4k	164.1	14400.0	1e+20	70.1k	–	14400.0
dem_150_gaslib135_452	1e+20	24.5k	–	14400.0	1e+20	13.2k	–	14400.0
dem_150_gaslib135_453	682.4k	65.0k	950.4	14400.0	1e+20	97.8k	–	14400.0
dem_150_gaslib135_454	1e+20	20.8k	–	14400.0	1e+20	69.6k	–	14400.0
dem_150_gaslib135_455	4671.4k	3.8	Large	14400.0	1e+20	74.0	–	14400.0
dem_150_gaslib135_456	420.5k	42.7k	884.0	14400.0	1e+20	73.3k	–	14400.0
dem_150_gaslib135_457	1e+20	0.0	–	14400.0	1e+20	32.4k	–	14400.0
dem_150_gaslib135_458	1e+20	0.0	–	14400.0	1e+20	17.6k	–	14400.0
dem_150_gaslib135_459	251.5k	1488.0	Large	14400.0	1e+20	40.2k	–	14400.0
dem_150_gaslib135_460	1e+20	37.7k	–	14400.0	1e+20	65.0k	–	14400.0
dem_150_gaslib135_461	107.9k	49.7k	117.2	14400.0	1e+20	76.9k	–	14400.0
dem_150_gaslib135_462	447.2k	66.1k	576.9	14400.0	1e+20	82.0k	–	14400.0
dem_150_gaslib135_463	1e+20	51.9k	–	14400.0	1e+20	80.8k	–	14400.0
dem_150_gaslib135_464	1e+20	0.0	–	14400.0	1e+20	93.0k	–	14400.0
dem_150_gaslib135_465	1790.3k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_466	1e+20	36.5k	–	14400.0	1e+20	88.9k	–	14400.0
dem_150_gaslib135_467	1e+20	55.8k	–	14400.0	1e+20	73.3k	–	14400.0
dem_150_gaslib135_468	1e+20	58.0k	–	14400.0	1e+20	62.5k	–	14400.0
dem_150_gaslib135_469	1e+20	31.7k	–	14400.0	1e+20	52.5k	–	14400.0
dem_150_gaslib135_470	375.8k	0.0	–	14400.0	1e+20	23.1k	–	14400.0
dem_150_gaslib135_471	1e+20	6892.8	–	14400.0	1e+20	52.1k	–	14400.0
dem_150_gaslib135_472	1e+20	0.0	–	14400.0	1e+20	64.7k	–	14400.0
dem_150_gaslib135_473	1e+20	36.5k	–	14400.0	1e+20	76.1k	–	14400.0
dem_150_gaslib135_474	1e+20	50.0k	–	14400.0	1e+20	89.1k	–	14400.0
dem_150_gaslib135_475	1e+20	65.0k	–	14400.0	1e+20	118.3k	–	14400.0
dem_150_gaslib135_476	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_477	1665.8k	69.8k	2284.9	14400.0	1e+20	114.4k	–	14400.0
dem_150_gaslib135_478	1e+20	98.3k	–	14400.0	1e+20	139.7k	–	14400.0
dem_150_gaslib135_479	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_480	1e+20	33.2k	–	14400.0	1e+20	15.8k	–	14400.0
dem_150_gaslib135_481	1e+20	58.6k	–	14400.0	1e+20	75.5k	–	14400.0
dem_150_gaslib135_482	1779.6k	11.6k	Large	14400.0	1e+20	2472.5	–	14400.0
dem_150_gaslib135_483	920.2k	65.0k	1316.5	14400.0	1e+20	123.7k	–	14400.0
dem_150_gaslib135_484	56.8k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_485	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_486	1e+20	82.2k	–	14400.0	1e+20	156.6k	–	14400.0
dem_150_gaslib135_487	1e+20	0.0	–	14400.0	1e+20	31.4k	–	14400.0
dem_150_gaslib135_488	1e+20	81.2k	–	14400.0	1e+20	151.8k	–	14400.0
dem_150_gaslib135_489	1936.1k	55.8k	3370.9	14400.0	1e+20	73.8k	–	14400.0
dem_150_gaslib135_490	1e+20	0.0	–	14400.0	1e+20	3359.0	–	14400.0
dem_150_gaslib135_491	60.9k	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_492	1e+20	1488.0	–	14400.0	1e+20	43.4k	–	14400.0
dem_150_gaslib135_493	518.6k	11.6k	4376.7	14400.0	1e+20	875.2	–	14400.0
dem_150_gaslib135_494	1e+20	27.0k	–	14400.0	1e+20	44.8k	–	14400.0
dem_150_gaslib135_495	19.2k	11.8	Large	14400.0	1e+20	0.0	–	14400.0

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Table A.22: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 150$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_150_gaslib135_496	1e+20	0.0	–	14400.0	1e+20	0.0	–	14400.0
dem_150_gaslib135_497	1e+20	1964.9	–	14400.0	1e+20	69.7k	–	14400.0
dem_150_gaslib135_498	1e+20	1488.0	–	14400.0	1e+20	59.4k	–	14400.0
dem_150_gaslib135_499	1e+20	13.4k	–	14400.0	1e+20	15.0k	–	14400.0
dem_150_gaslib135_500	523.5k	53.9k	871.5	14400.0	1e+20	99.9k	–	14400.0

Table A.23: Detailed results of the split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ as summarized in Figure 3.9d and Table 3.3c. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_1	1e+20	1050.1k	–	14400.0	1e+20	1319.9k	–	14400.0	1e+20	877.5k	–	14400.0
dem_500_gaslib135_2	1e+20	1041.8k	–	14400.0	1e+20	1253.1k	–	14400.0	1e+20	810.2k	–	14400.0
dem_500_gaslib135_3	1e+20	1107.2k	–	14400.0	1e+20	1450.0k	–	14400.0	1e+20	985.1k	–	14400.0
dem_500_gaslib135_4	1e+20	534.6k	–	14400.0	1e+20	1209.2k	–	14400.0	1e+20	759.3k	–	14400.0
dem_500_gaslib135_5	1e+20	735.9k	–	14400.0	1e+20	1264.7k	–	14400.0	1e+20	829.0k	–	14400.0
dem_500_gaslib135_6	1e+20	845.7k	–	14400.0	1e+20	1775.0k	–	14400.0	1e+20	989.2k	–	14400.0
dem_500_gaslib135_7	1e+20	1047.3k	–	14400.0	1e+20	1470.6k	–	14400.0	1e+20	1027.8k	–	14400.0
dem_500_gaslib135_8	1e+20	626.5k	–	14400.0	1e+20	1055.8k	–	14400.0	1e+20	768.3k	–	14400.0
dem_500_gaslib135_9	1e+20	427.2k	–	14400.0	1e+20	1290.8k	–	14400.0	1e+20	709.4k	–	14400.0
dem_500_gaslib135_10	1e+20	617.0k	–	14400.0	1e+20	1068.3k	–	14400.0	1e+20	744.2k	–	14400.0
dem_500_gaslib135_11	1e+20	976.6k	–	14400.0	1e+20	1338.3k	–	14400.0	1e+20	891.9k	–	14400.0
dem_500_gaslib135_12	1e+20	979.9k	–	14400.0	1e+20	1214.8k	–	14400.0	1e+20	753.5k	–	14400.0
dem_500_gaslib135_13	1e+20	861.6k	–	14400.0	1e+20	1380.1k	–	14400.0	1e+20	826.0k	–	14400.0
dem_500_gaslib135_14	1e+20	930.9k	–	14400.0	1e+20	1628.0k	–	14400.0	1e+20	881.0k	–	14400.0
dem_500_gaslib135_15	1e+20	1010.1k	–	14400.0	1e+20	1e+20	0.0	427.6	1e+20	1148.1k	–	14400.0
dem_500_gaslib135_16	1e+20	933.9k	–	14400.0	1e+20	1592.5k	–	14400.0	1e+20	1005.5k	–	14400.0
dem_500_gaslib135_17	1e+20	1214.5k	–	14400.0	1e+20	2007.2k	–	14400.0	1e+20	1018.1k	–	14400.0
dem_500_gaslib135_18	1e+20	578.2k	–	14400.0	1e+20	927.7k	–	14400.0	1e+20	683.8k	–	14400.0
dem_500_gaslib135_19	1e+20	833.9k	–	14400.0	1e+20	1249.7k	–	14400.0	1e+20	658.0k	–	14400.0
dem_500_gaslib135_20	1e+20	727.0k	–	14400.0	1e+20	1127.3k	–	14400.0	1e+20	774.1k	–	14400.0
dem_500_gaslib135_21	1e+20	1039.0k	–	14400.0	1e+20	2032.6k	–	14400.0	1e+20	1361.3k	–	14400.0
dem_500_gaslib135_22	1e+20	741.4k	–	14400.0	1e+20	1455.2k	–	14400.0	1e+20	992.9k	–	14400.0
dem_500_gaslib135_23	1e+20	834.5k	–	14400.0	1e+20	1406.0k	–	14400.0	1e+20	695.6k	–	14400.0
dem_500_gaslib135_24	1e+20	963.6k	–	14400.0	1e+20	1958.0k	–	14400.0	1e+20	772.8k	–	14400.0
dem_500_gaslib135_25	1e+20	1035.5k	–	14400.0	1e+20	1602.9k	–	14400.0	1e+20	1070.8k	–	14400.0
dem_500_gaslib135_26	1e+20	1140.9k	–	14400.0	1e+20	1515.9k	–	14400.0	1e+20	971.9k	–	14400.0
dem_500_gaslib135_27	1e+20	608.5k	–	14400.0	1e+20	1522.4k	–	14400.0	1e+20	790.7k	–	14400.0
dem_500_gaslib135_28	1e+20	393.0k	–	14400.0	1e+20	1026.1k	–	14400.0	1e+20	615.3k	–	14400.0
dem_500_gaslib135_29	1e+20	868.3k	–	14400.0	1e+20	1301.2k	–	14400.0	1e+20	869.0k	–	14400.0
dem_500_gaslib135_30	1e+20	801.4k	–	14400.0	1e+20	1389.4k	–	14400.0	1e+20	796.3k	–	14400.0
dem_500_gaslib135_31	1e+20	962.7k	–	14400.0	1e+20	1373.5k	–	14400.0	1e+20	911.7k	–	14400.0
dem_500_gaslib135_32	1e+20	1179.9k	–	14400.0	1e+20	1529.4k	–	14400.0	1e+20	1089.2k	–	14400.0
dem_500_gaslib135_33	1e+20	785.9k	–	14400.0	1e+20	1273.7k	–	14400.0	1e+20	798.1k	–	14400.0
dem_500_gaslib135_34	1e+20	1050.7k	–	14400.0	1e+20	1372.4k	–	14400.0	1e+20	889.9k	–	14400.0
dem_500_gaslib135_35	1e+20	1008.4k	–	14400.0	1e+20	1637.5k	–	14400.0	1e+20	1190.8k	–	14400.0
dem_500_gaslib135_36	1e+20	1111.5k	–	14400.0	1e+20	1475.2k	–	14400.0	1e+20	858.2k	–	14400.0
dem_500_gaslib135_37	1e+20	632.6k	–	14400.0	1e+20	1010.9k	–	14400.0	1e+20	691.1k	–	14400.0
dem_500_gaslib135_38	1e+20	648.4k	–	14400.0	1e+20	1316.9k	–	14400.0	1e+20	829.2k	–	14400.0
dem_500_gaslib135_39	1e+20	646.9k	–	14400.0	1e+20	1417.3k	–	14400.0	1e+20	802.5k	–	14400.0
dem_500_gaslib135_40	1e+20	992.8k	–	14400.0	1e+20	1288.9k	–	14400.0	1e+20	819.0k	–	14400.0
dem_500_gaslib135_41	1e+20	972.6k	–	14400.0	1e+20	1335.1k	–	14400.0	1e+20	846.0k	–	14400.0
dem_500_gaslib135_42	1e+20	917.9k	–	14400.0	1e+20	1355.0k	–	14400.0	1e+20	718.3k	–	14400.0
dem_500_gaslib135_43	1e+20	750.6k	–	14400.0	1e+20	1357.2k	–	14400.0	1e+20	974.5k	–	14400.0
dem_500_gaslib135_44	1e+20	647.3k	–	14400.0	1e+20	1338.0k	–	14400.0	1e+20	855.6k	–	14400.0
dem_500_gaslib135_45	1e+20	936.3k	–	14400.0	1e+20	1077.9k	–	14400.0	1e+20	747.6k	–	14400.0
dem_500_gaslib135_46	1e+20	727.0k	–	14400.0	1e+20	1385.7k	–	14400.0	1e+20	885.4k	–	14400.0
dem_500_gaslib135_47	1e+20	676.1k	–	14400.0	1e+20	1377.8k	–	14400.0	1e+20	907.7k	–	14400.0
dem_500_gaslib135_48	1e+20	1095.4k	–	14400.0	1e+20	1663.7k	–	14400.0	1e+20	1040.8k	–	14400.0
dem_500_gaslib135_49	1e+20	837.5k	–	14400.0	1e+20	1523.1k	–	14400.0	1e+20	936.3k	–	14400.0
dem_500_gaslib135_50	1e+20	879.3k	–	14400.0	1e+20	1279.4k	–	14400.0	1e+20	789.8k	–	14400.0
dem_500_gaslib135_51	1e+20	792.1k	–	14400.0	1e+20	1558.5k	–	14400.0	1e+20	972.4k	–	14400.0
dem_500_gaslib135_52	1e+20	507.4k	–	14400.0	1e+20	1081.8k	–	14400.0	1e+20	789.3k	–	14400.0
dem_500_gaslib135_53	1e+20	965.5k	–	14400.0	1e+20	1648.9k	–	14400.0	1e+20	1205.2k	–	14400.0
dem_500_gaslib135_54	1e+20	802.2k	–	14400.0	1e+20	1218.1k	–	14400.0	1e+20	611.3k	–	14400.0
dem_500_gaslib135_55	1e+20	1510.6k	–	14400.0	1e+20	2002.1k	–	14400.0	1e+20	1312.9k	–	14400.0
dem_500_gaslib135_56	1e+20	692.8k	–	14400.0	1e+20	1165.4k	–	14400.0	1e+20	767.4k	–	14400.0
dem_500_gaslib135_57	1e+20	892.1k	–	14400.0	1e+20	1288.4k	–	14400.0	1e+20	730.1k	–	14400.0
dem_500_gaslib135_58	1e+20	667.3k	–	14400.0	1e+20	1271.2k	–	14400.0	1e+20	771.9k	–	14400.0

continued on next page

Table A.23: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_59	1e+20	834.2k	–	14400.0	1e+20	1313.0k	–	14400.0	1e+20	707.3k	–	14400.0
dem_500_gaslib135_60	1e+20	931.2k	–	14400.0	1e+20	1518.7k	–	14400.0	1e+20	841.3k	–	14400.0
dem_500_gaslib135_61	1e+20	649.4k	–	14400.0	1e+20	1188.8k	–	14400.0	1e+20	762.2k	–	14400.0
dem_500_gaslib135_62	1e+20	1018.3k	–	14400.0	1e+20	1892.8k	–	14400.0	1e+20	747.2k	–	14400.0
dem_500_gaslib135_63	1e+20	1152.4k	–	14400.0	1e+20	1734.9k	–	14400.0	1e+20	986.2k	–	14400.0
dem_500_gaslib135_64	1e+20	1030.1k	–	14400.0	1e+20	1264.9k	–	14400.0	1e+20	944.3k	–	14400.0
dem_500_gaslib135_65	1e+20	911.4k	–	14400.0	1e+20	1225.9k	–	14400.0	1e+20	865.7k	–	14400.0
dem_500_gaslib135_66	1e+20	1102.0k	–	14400.0	1e+20	1423.6k	–	14400.0	1e+20	982.2k	–	14400.0
dem_500_gaslib135_67	1e+20	1141.8k	–	14400.0	1e+20	1481.7k	–	14400.0	1e+20	985.5k	–	14400.0
dem_500_gaslib135_68	1e+20	1090.0k	–	14400.0	1e+20	1929.2k	–	14400.0	1e+20	1177.4k	–	14400.0
dem_500_gaslib135_69	1e+20	420.9k	–	14400.0	1e+20	1164.2k	–	14400.0	1e+20	722.4k	–	14400.0
dem_500_gaslib135_70	1e+20	651.4k	–	14400.0	1e+20	1944.8k	–	14400.0	1e+20	882.9k	–	14400.0
dem_500_gaslib135_71	1e+20	369.0k	–	14400.0	1e+20	1298.6k	–	14400.0	1e+20	861.1k	–	14400.0
dem_500_gaslib135_72	1e+20	728.2k	–	14400.0	1e+20	1632.5k	–	14400.0	1e+20	781.2k	–	14400.0
dem_500_gaslib135_73	1e+20	789.1k	–	14400.0	1e+20	1074.0k	–	14400.0	1e+20	581.0k	–	14400.0
dem_500_gaslib135_74	1e+20	832.0k	–	14400.0	1e+20	1322.0k	–	14400.0	1e+20	729.9k	–	14400.0
dem_500_gaslib135_75	1e+20	385.0k	–	14400.0	1e+20	1146.4k	–	14400.0	1e+20	502.8k	–	14400.0
dem_500_gaslib135_76	1e+20	672.4k	–	14400.0	1e+20	1127.8k	–	14400.0	1e+20	674.9k	–	14400.0
dem_500_gaslib135_77	1e+20	423.1k	–	14400.0	1e+20	1129.1k	–	14400.0	1e+20	674.0k	–	14400.0
dem_500_gaslib135_78	1e+20	781.1k	–	14400.0	1e+20	1615.5k	–	14400.0	1e+20	1067.8k	–	14400.0
dem_500_gaslib135_79	1e+20	820.4k	–	14400.0	1e+20	1155.6k	–	14400.0	1e+20	635.8k	–	14400.0
dem_500_gaslib135_80	1e+20	928.9k	–	14400.0	1e+20	1414.5k	–	14400.0	1e+20	869.0k	–	14400.0
dem_500_gaslib135_81	1e+20	886.7k	–	14400.0	1e+20	1751.7k	–	14400.0	1e+20	1179.7k	–	14400.0
dem_500_gaslib135_82	1e+20	1007.2k	–	14400.0	1e+20	1535.8k	–	14400.0	1e+20	962.6k	–	14400.0
dem_500_gaslib135_83	1e+20	1008.9k	–	14400.0	1e+20	1370.4k	–	14400.0	1e+20	916.6k	–	14400.0
dem_500_gaslib135_84	1e+20	854.3k	–	14400.0	1e+20	1497.9k	–	14400.0	1e+20	840.7k	–	14400.0
dem_500_gaslib135_85	1e+20	699.8k	–	14400.0	1e+20	1379.1k	–	14400.0	1e+20	907.8k	–	14400.0
dem_500_gaslib135_86	1e+20	917.8k	–	14400.0	1e+20	1602.8k	–	14400.0	1e+20	814.2k	–	14400.0
dem_500_gaslib135_87	1e+20	737.7k	–	14400.0	1e+20	1289.9k	–	14400.0	1e+20	832.0k	–	14400.0
dem_500_gaslib135_88	1e+20	1155.9k	–	14400.0	1e+20	1657.1k	–	14400.0	1e+20	987.9k	–	14400.0
dem_500_gaslib135_89	1e+20	981.3k	–	14400.0	1e+20	1425.5k	–	14400.0	1e+20	759.7k	–	14400.0
dem_500_gaslib135_90	1e+20	1094.0k	–	14400.0	1e+20	1921.8k	–	14400.0	1e+20	1025.5k	–	14400.0
dem_500_gaslib135_91	1e+20	889.6k	–	14400.0	1e+20	1364.9k	–	14400.0	1e+20	896.5k	–	14400.0
dem_500_gaslib135_92	1e+20	743.2k	–	14400.0	1e+20	1320.6k	–	14400.0	1e+20	827.4k	–	14400.0
dem_500_gaslib135_93	1e+20	1053.8k	–	14400.0	1e+20	1300.9k	–	14400.0	1e+20	900.3k	–	14400.0
dem_500_gaslib135_94	1e+20	765.5k	–	14400.0	1e+20	1209.2k	–	14400.0	1e+20	650.3k	–	14400.0
dem_500_gaslib135_95	1e+20	843.9k	–	14400.0	1e+20	1272.0k	–	14400.0	1e+20	822.3k	–	14400.0
dem_500_gaslib135_96	1e+20	1009.8k	–	14400.0	1e+20	1606.4k	–	14400.0	1e+20	1002.8k	–	14400.0
dem_500_gaslib135_97	1e+20	894.9k	–	14400.0	1e+20	1241.6k	–	14400.0	1e+20	784.8k	–	14400.0
dem_500_gaslib135_98	1e+20	1091.9k	–	14400.0	1e+20	1473.5k	–	14400.0	1e+20	940.8k	–	14400.0
dem_500_gaslib135_99	1e+20	968.7k	–	14400.0	1e+20	1399.7k	–	14400.0	1e+20	758.5k	–	14400.0
dem_500_gaslib135_100	1e+20	1478.3k	–	14400.0	1e+20	1889.5k	–	14400.0	1e+20	1338.1k	–	14400.0
dem_500_gaslib135_101	1e+20	986.0k	–	14400.0	1e+20	1383.3k	–	14400.0	1e+20	876.2k	–	14400.0
dem_500_gaslib135_102	1e+20	1211.2k	–	14400.0	1e+20	1548.1k	–	14400.0	1e+20	1e+20	0.0	9086.4
dem_500_gaslib135_103	1e+20	1038.1k	–	14400.0	1e+20	1395.6k	–	14400.0	1e+20	782.2k	–	14400.0
dem_500_gaslib135_104	1e+20	921.6k	–	14400.0	1e+20	1448.9k	–	14400.0	1e+20	867.5k	–	14400.0
dem_500_gaslib135_105	1e+20	892.3k	–	14400.0	1e+20	1185.5k	–	14400.0	1e+20	712.7k	–	14400.0
dem_500_gaslib135_106	1e+20	1086.9k	–	14400.0	1e+20	1480.6k	–	14400.0	1e+20	971.0k	–	14400.0
dem_500_gaslib135_107	1e+20	705.9k	–	14400.0	1e+20	1275.9k	–	14400.0	1e+20	834.3k	–	14400.0
dem_500_gaslib135_108	1e+20	854.6k	–	14400.0	1e+20	1319.2k	–	14400.0	1e+20	780.1k	–	14400.0
dem_500_gaslib135_109	1e+20	548.5k	–	14400.0	1e+20	1250.2k	–	14400.0	1e+20	815.0k	–	14400.0
dem_500_gaslib135_110	1e+20	847.2k	–	14400.0	1e+20	1213.2k	–	14400.0	1e+20	671.3k	–	14400.0
dem_500_gaslib135_111	1e+20	1082.5k	–	14400.0	1e+20	1477.3k	–	14400.0	1e+20	755.1k	–	14400.0
dem_500_gaslib135_112	1e+20	922.3k	–	14400.0	1e+20	1310.7k	–	14400.0	1e+20	838.6k	–	14400.0
dem_500_gaslib135_113	1e+20	381.7k	–	14400.0	1e+20	1069.3k	–	14400.0	1e+20	496.3k	–	14400.0
dem_500_gaslib135_114	1e+20	770.0k	–	14400.0	1e+20	1354.3k	–	14400.0	1e+20	851.3k	–	14400.0
dem_500_gaslib135_115	1e+20	1098.2k	–	14400.0	1e+20	1671.3k	–	14400.0	1e+20	1114.6k	–	14400.0
dem_500_gaslib135_116	1e+20	718.1k	–	14400.0	1e+20	1305.7k	–	14400.0	1e+20	769.7k	–	14400.0
dem_500_gaslib135_117	1e+20	1239.7k	–	14400.0	1e+20	1687.5k	–	14400.0	1e+20	1044.9k	–	14400.0
dem_500_gaslib135_118	1e+20	816.6k	–	14400.0	1e+20	1223.9k	–	14400.0	1e+20	830.0k	–	14400.0
dem_500_gaslib135_119	1e+20	786.2k	–	14400.0	1e+20	1273.3k	–	14400.0	1e+20	831.1k	–	14400.0
dem_500_gaslib135_120	1e+20	761.7k	–	14400.0	1e+20	1656.6k	–	14400.0	1e+20	903.6k	–	14400.0
dem_500_gaslib135_121	1e+20	979.9k	–	14400.0	1e+20	1538.1k	–	14400.0	1e+20	900.2k	–	14400.0
dem_500_gaslib135_122	1e+20	852.3k	–	14400.0	1e+20	1130.4k	–	14400.0	1e+20	725.8k	–	14400.0
dem_500_gaslib135_123	1e+20	1149.2k	–	14400.0	1e+20	1712.7k	–	14400.0	1e+20	886.6k	–	14400.0
dem_500_gaslib135_124	1e+20	935.2k	–	14400.0	1e+20	1539.0k	–	14400.0	1e+20	1018.3k	–	14400.0
dem_500_gaslib135_125	1e+20	1096.7k	–	14400.0	1e+20	1467.3k	–	14400.0	1e+20	959.5k	–	14400.0
dem_500_gaslib135_126	1e+20	651.0k	–	14400.0	1e+20	1114.0k	–	14400.0	1e+20	596.5k	–	14400.0
dem_500_gaslib135_127	1e+20	717.2k	–	14400.0	1e+20	1333.6k	–	14400.0	1e+20	908.1k	–	14400.0
dem_500_gaslib135_128	1e+20	695.4k	–	14400.0	1e+20	1193.8k	–	14400.0	1e+20	823.1k	–	14400.0
dem_500_gaslib135_129	1e+20	566.7k	–	14400.0	1e+20	1340.4k	–	14400.0	1e+20	824.6k	–	14400.0
dem_500_gaslib135_130	1e+20	923.0k	–	14400.0	1e+20	1452.0k	–	14400.0	1e+20	1033.0k	–	14400.0
dem_500_gaslib135_131	1e+20	663.4k	–	14400.0	1e+20	1287.5k	–	14400.0	1e+20	806.2k	–	14400.0
dem_500_gaslib135_132	1e+20	1227.3k	–	14400.0	1e+20	1681.2k	–	14400.0	1e+20	1121.1k	–	14400.0
dem_500_gaslib135_133	1e+20	875.8k	–	14400.0	1e+20	1676.7k	–	14400.0	1e+20	1209.1k	–	14400.0
dem_500_gaslib135_134	1e+20	1195.7k	–	14400.0	1e+20	1707.2k	–	14400.0	1e+20	1041.0k	–	14400.0
dem_500_gaslib135_135	1e+20	629.6k	–	14400.0	1e+20	1127.5k	–	14400.0	1e+20	782.7k	–	14400.0
dem_500_gaslib135_136	1e+20	621.4k	–	14400.0	1e+20	1367.2k	–	14400.0	1e+20	778.6k	–	14400.0

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Table A.23: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_137	1e+20	764.3k	–	14400.0	1e+20	1777.4k	–	14400.0	1e+20	859.9k	–	14400.0
dem_500_gaslib135_138	1e+20	994.7k	–	14400.0	1e+20	1739.0k	–	14400.0	1e+20	911.3k	–	14400.0
dem_500_gaslib135_139	1e+20	877.8k	–	14400.0	1e+20	1344.7k	–	14400.0	1e+20	679.3k	–	14400.0
dem_500_gaslib135_140	1e+20	827.5k	–	14400.0	1e+20	1537.2k	–	14400.0	1e+20	1203.6k	–	14400.0
dem_500_gaslib135_141	1e+20	715.9k	–	14400.0	1e+20	1438.2k	–	14400.0	1e+20	987.1k	–	14400.0
dem_500_gaslib135_142	1e+20	1148.3k	–	14400.0	1e+20	1746.6k	–	14400.0	1e+20	984.7k	–	14400.0
dem_500_gaslib135_143	1e+20	1071.8k	–	14400.0	1e+20	1329.9k	–	14400.0	1e+20	844.8k	–	14400.0
dem_500_gaslib135_144	1e+20	636.9k	–	14400.0	1e+20	1127.2k	–	14400.0	1e+20	779.2k	–	14400.0
dem_500_gaslib135_145	1e+20	853.0k	–	14400.0	1e+20	1408.0k	–	14400.0	1e+20	732.1k	–	14400.0
dem_500_gaslib135_146	1e+20	1071.9k	–	14400.0	1e+20	1609.4k	–	14400.0	1e+20	972.1k	–	14400.0
dem_500_gaslib135_147	1e+20	967.4k	–	14400.0	1e+20	1291.2k	–	14400.0	1e+20	964.8k	–	14400.0
dem_500_gaslib135_148	1e+20	1027.5k	–	14400.0	1e+20	1549.3k	–	14400.0	1e+20	960.4k	–	14400.0
dem_500_gaslib135_149	1e+20	1275.8k	–	14400.0	1e+20	1542.5k	–	14400.0	1e+20	1160.3k	–	14400.0
dem_500_gaslib135_150	1e+20	804.2k	–	14400.0	1e+20	1097.4k	–	14400.0	1e+20	770.3k	–	14400.0
dem_500_gaslib135_151	1e+20	1006.9k	–	14400.0	1e+20	1551.9k	–	14400.0	1e+20	1104.7k	–	14400.0
dem_500_gaslib135_152	1e+20	1116.1k	–	14400.0	1e+20	1886.7k	–	14400.0	1e+20	1077.4k	–	14400.0
dem_500_gaslib135_153	1e+20	579.4k	–	14400.0	1e+20	1230.9k	–	14400.0	1e+20	882.2k	–	14400.0
dem_500_gaslib135_154	1e+20	388.4k	–	14400.0	1e+20	1108.3k	–	14400.0	1e+20	560.4k	–	14400.0
dem_500_gaslib135_155	1e+20	974.4k	–	14400.0	1e+20	1347.0k	–	14400.0	1e+20	880.3k	–	14400.0
dem_500_gaslib135_156	1e+20	1106.1k	–	14400.0	1e+20	1852.9k	–	14400.0	1e+20	1094.8k	–	14400.0
dem_500_gaslib135_157	1e+20	1333.9k	–	14400.0	1e+20	2307.3k	–	14400.0	1e+20	1273.4k	–	14400.0
dem_500_gaslib135_158	1e+20	1592.6k	–	14400.0	1e+20	2399.0k	–	14400.0	1e+20	1461.2k	–	14400.0
dem_500_gaslib135_159	1e+20	819.7k	–	14400.0	1e+20	1242.4k	–	14400.0	1e+20	868.5k	–	14400.0
dem_500_gaslib135_160	1e+20	769.8k	–	14400.0	1e+20	1455.3k	–	14400.0	1e+20	937.5k	–	14400.0
dem_500_gaslib135_161	1e+20	1220.0k	–	14400.0	1e+20	1649.7k	–	14400.0	1e+20	1137.5k	–	14400.0
dem_500_gaslib135_162	1e+20	556.1k	–	14400.0	1e+20	1440.2k	–	14400.0	1e+20	908.1k	–	14400.0
dem_500_gaslib135_163	1e+20	667.4k	–	14400.0	1e+20	1580.6k	–	14400.0	1e+20	790.3k	–	14400.0
dem_500_gaslib135_164	1e+20	966.4k	–	14400.0	1e+20	1277.3k	–	14400.0	1e+20	892.9k	–	14400.0
dem_500_gaslib135_165	1e+20	1086.7k	–	14400.0	1e+20	2043.7k	–	14400.0	1e+20	1020.8k	–	14400.0
dem_500_gaslib135_166	1e+20	536.0k	–	14400.0	1e+20	1099.5k	–	14400.0	1e+20	707.2k	–	14400.0
dem_500_gaslib135_167	1e+20	924.9k	–	14400.0	1e+20	1393.9k	–	14400.0	1e+20	848.3k	–	14400.0
dem_500_gaslib135_168	1e+20	649.3k	–	14400.0	1e+20	1471.0k	–	14400.0	1e+20	823.7k	–	14400.0
dem_500_gaslib135_169	1e+20	952.1k	–	14400.0	1e+20	1264.5k	–	14400.0	1e+20	799.0k	–	14400.0
dem_500_gaslib135_170	1e+20	870.5k	–	14400.0	1e+20	1078.8k	–	14400.0	1e+20	784.8k	–	14400.0
dem_500_gaslib135_171	1e+20	881.0k	–	14400.0	1e+20	1372.8k	–	14400.0	1e+20	723.1k	–	14400.0
dem_500_gaslib135_172	1e+20	992.7k	–	14400.0	1e+20	1440.5k	–	14400.0	1e+20	818.7k	–	14400.0
dem_500_gaslib135_173	1e+20	374.4k	–	14400.0	1e+20	959.9k	–	14400.0	1e+20	587.2k	–	14400.0
dem_500_gaslib135_174	1e+20	556.9k	–	14400.0	1e+20	1166.0k	–	14400.0	1e+20	867.7k	–	14400.0
dem_500_gaslib135_175	1e+20	483.4k	–	14400.0	1e+20	1157.4k	–	14400.0	1e+20	596.8k	–	14400.0
dem_500_gaslib135_176	1e+20	1052.3k	–	14400.0	1e+20	1414.5k	–	14400.0	1e+20	873.1k	–	14400.0
dem_500_gaslib135_177	1e+20	823.1k	–	14400.0	1e+20	1251.8k	–	14400.0	1e+20	716.5k	–	14400.0
dem_500_gaslib135_178	1e+20	378.4k	–	14400.0	1e+20	1184.2k	–	14400.0	1e+20	570.7k	–	14400.0
dem_500_gaslib135_179	1e+20	821.6k	–	14400.0	1e+20	1202.3k	–	14400.0	1e+20	873.1k	–	14400.0
dem_500_gaslib135_180	1e+20	902.7k	–	14400.0	1e+20	1579.4k	–	14400.0	1e+20	863.8k	–	14400.0
dem_500_gaslib135_181	1e+20	707.9k	–	14400.0	1e+20	1451.9k	–	14400.0	1e+20	878.3k	–	14400.0
dem_500_gaslib135_182	1e+20	408.7k	–	14400.0	1e+20	991.4k	–	14400.0	1e+20	479.1k	–	14400.0
dem_500_gaslib135_183	1e+20	618.1k	–	14400.0	1e+20	1233.0k	–	14400.0	1e+20	797.2k	–	14400.0
dem_500_gaslib135_184	1e+20	322.3k	–	14400.0	1e+20	782.7k	–	14400.0	1e+20	569.7k	–	14400.0
dem_500_gaslib135_185	1e+20	844.8k	–	14400.0	1e+20	1324.6k	–	14400.0	1e+20	897.7k	–	14400.0
dem_500_gaslib135_186	1e+20	828.7k	–	14400.0	1e+20	1182.7k	–	14400.0	1e+20	707.5k	–	14400.0
dem_500_gaslib135_187	1e+20	1025.8k	–	14400.0	1e+20	1505.8k	–	14400.0	1e+20	922.1k	–	14400.0
dem_500_gaslib135_188	1e+20	913.3k	–	14400.0	1e+20	1473.2k	–	14400.0	1e+20	737.2k	–	14400.0
dem_500_gaslib135_189	1e+20	938.5k	–	14400.0	1e+20	1284.0k	–	14400.0	1e+20	739.5k	–	14400.0
dem_500_gaslib135_190	1e+20	986.9k	–	14400.0	1e+20	1355.9k	–	14400.0	1e+20	876.1k	–	14400.0
dem_500_gaslib135_191	1e+20	1114.5k	–	14400.0	1e+20	1555.9k	–	14400.0	1e+20	978.6k	–	14400.0
dem_500_gaslib135_192	1e+20	490.1k	–	14400.0	1e+20	1179.7k	–	14400.0	1e+20	611.5k	–	14400.0
dem_500_gaslib135_193	1e+20	811.3k	–	14400.0	1e+20	1627.1k	–	14400.0	1e+20	1102.0k	–	14400.0
dem_500_gaslib135_194	1e+20	971.2k	–	14400.0	1e+20	1568.6k	–	14400.0	1e+20	960.0k	–	14400.0
dem_500_gaslib135_195	1e+20	861.1k	–	14400.0	1e+20	1110.7k	–	14400.0	1e+20	678.6k	–	14400.0
dem_500_gaslib135_196	1e+20	942.7k	–	14400.0	1e+20	1336.4k	–	14400.0	1e+20	926.7k	–	14400.0
dem_500_gaslib135_197	1e+20	1191.1k	–	14400.0	1e+20	1619.8k	–	14400.0	1e+20	1095.0k	–	14400.0
dem_500_gaslib135_198	1e+20	626.7k	–	14400.0	1e+20	1388.4k	–	14400.0	1e+20	839.6k	–	14400.0
dem_500_gaslib135_199	1e+20	826.5k	–	14400.0	1e+20	1568.3k	–	14400.0	1e+20	1053.2k	–	14400.0
dem_500_gaslib135_200	1e+20	773.4k	–	14400.0	1e+20	1271.5k	–	14400.0	1e+20	537.6k	–	14400.0
dem_500_gaslib135_201	1e+20	699.5k	–	14400.0	1e+20	1268.7k	–	14400.0	1e+20	790.1k	–	14400.0
dem_500_gaslib135_202	1e+20	1059.3k	–	14400.0	1e+20	1442.3k	–	14400.0	1e+20	992.9k	–	14400.0
dem_500_gaslib135_203	1e+20	1260.7k	–	14400.0	1e+20	1705.2k	–	14400.0	1e+20	1041.2k	–	14400.0
dem_500_gaslib135_204	1e+20	1511.4k	–	14400.0	1e+20	1929.9k	–	14400.0	1e+20	1405.9k	–	14400.0
dem_500_gaslib135_205	1e+20	1082.5k	–	14400.0	1e+20	1486.0k	–	14400.0	1e+20	960.4k	–	14400.0
dem_500_gaslib135_206	1e+20	567.3k	–	14400.0	1e+20	1243.3k	–	14400.0	1e+20	658.5k	–	14400.0
dem_500_gaslib135_207	1e+20	365.4k	–	14400.0	1e+20	1013.9k	–	14400.0	1e+20	509.5k	–	14400.0
dem_500_gaslib135_208	1e+20	712.2k	–	14400.0	1e+20	1100.9k	–	14400.0	1e+20	697.6k	–	14400.0
dem_500_gaslib135_209	1e+20	587.8k	–	14400.0	1e+20	1195.6k	–	14400.0	1e+20	752.5k	–	14400.0
dem_500_gaslib135_210	1e+20	1044.8k	–	14400.0	1e+20	1313.7k	–	14400.0	1e+20	917.5k	–	14400.0
dem_500_gaslib135_211	1e+20	552.2k	–	14400.0	1e+20	1153.8k	–	14400.0	1e+20	780.1k	–	14400.0
dem_500_gaslib135_212	1e+20	865.7k	–	14400.0	1e+20	1535.3k	–	14400.0	1e+20	1011.4k	–	14400.0
dem_500_gaslib135_213	1e+20	902.1k	–	14400.0	1e+20	1382.1k	–	14400.0	1e+20	891.2k	–	14400.0
dem_500_gaslib135_214	1e+20	904.8k	–	14400.0	1e+20	1161.2k	–	14400.0	1e+20	823.2k	–	14400.0

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Table A.23: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_215	1e+20	986.2k	–	14400.0	1e+20	1308.9k	–	14400.0	1e+20	768.6k	–	14400.0
dem_500_gaslib135_216	1e+20	816.8k	–	14400.0	1e+20	1361.4k	–	14400.0	1e+20	858.5k	–	14400.0
dem_500_gaslib135_217	1e+20	1158.6k	–	14400.0	1e+20	1457.1k	–	14400.0	1e+20	1018.6k	–	14400.0
dem_500_gaslib135_218	1e+20	1091.2k	–	14400.0	1e+20	1474.6k	–	14400.0	1e+20	954.6k	–	14400.0
dem_500_gaslib135_219	1e+20	583.9k	–	14400.0	1e+20	1224.6k	–	14400.0	1e+20	857.4k	–	14400.0
dem_500_gaslib135_220	1e+20	456.9k	–	14400.0	1e+20	1014.8k	–	14400.0	1e+20	652.0k	–	14400.0
dem_500_gaslib135_221	1e+20	880.2k	–	14400.0	1e+20	1753.2k	–	14400.0	1e+20	1144.6k	–	14400.0
dem_500_gaslib135_222	1e+20	437.1k	–	14400.0	1e+20	1006.8k	–	14400.0	1e+20	562.3k	–	14400.0
dem_500_gaslib135_223	1e+20	809.3k	–	14400.0	1e+20	1209.2k	–	14400.0	1e+20	673.6k	–	14400.0
dem_500_gaslib135_224	1e+20	720.0k	–	14400.0	1e+20	1668.0k	–	14400.0	1e+20	1071.1k	–	14400.0
dem_500_gaslib135_225	1e+20	1118.4k	–	14400.0	1e+20	1750.4k	–	14400.0	1e+20	928.9k	–	14400.0
dem_500_gaslib135_226	1e+20	479.1k	–	14400.0	1e+20	1210.8k	–	14400.0	1e+20	770.1k	–	14400.0
dem_500_gaslib135_227	1e+20	864.8k	–	14400.0	1e+20	1448.2k	–	14400.0	1e+20	934.4k	–	14400.0
dem_500_gaslib135_228	1e+20	660.2k	–	14400.0	1e+20	1224.7k	–	14400.0	1e+20	762.5k	–	14400.0
dem_500_gaslib135_229	1e+20	1051.7k	–	14400.0	1e+20	1403.8k	–	14400.0	1e+20	962.7k	–	14400.0
dem_500_gaslib135_230	1e+20	805.4k	–	14400.0	1e+20	1116.3k	–	14400.0	1e+20	790.1k	–	14400.0
dem_500_gaslib135_231	1e+20	558.1k	–	14400.0	1e+20	1228.5k	–	14400.0	1e+20	758.6k	–	14400.0
dem_500_gaslib135_232	1e+20	756.3k	–	14400.0	1e+20	1060.9k	–	14400.0	1e+20	747.3k	–	14400.0
dem_500_gaslib135_233	1e+20	642.0k	–	14400.0	1e+20	1248.9k	–	14400.0	1e+20	711.6k	–	14400.0
dem_500_gaslib135_234	1e+20	1265.5k	–	14400.0	1e+20	1623.1k	–	14400.0	1e+20	1097.9k	–	14400.0
dem_500_gaslib135_235	1e+20	669.6k	–	14400.0	1e+20	1088.0k	–	14400.0	1e+20	540.5k	–	14400.0
dem_500_gaslib135_236	1e+20	1052.2k	–	14400.0	1e+20	1504.7k	–	14400.0	1e+20	1015.7k	–	14400.0
dem_500_gaslib135_237	1e+20	798.0k	–	14400.0	1e+20	1230.5k	–	14400.0	1e+20	812.2k	–	14400.0
dem_500_gaslib135_238	1e+20	903.7k	–	14400.0	1e+20	1375.8k	–	14400.0	1e+20	940.3k	–	14400.0
dem_500_gaslib135_239	1e+20	1040.5k	–	14400.0	1e+20	1456.7k	–	14400.0	1e+20	840.7k	–	14400.0
dem_500_gaslib135_240	1e+20	1008.5k	–	14400.0	1e+20	1711.0k	–	14400.0	1e+20	916.0k	–	14400.0
dem_500_gaslib135_241	1e+20	763.1k	–	14400.0	1e+20	1481.1k	–	14400.0	1e+20	597.1k	–	14400.0
dem_500_gaslib135_242	1e+20	635.0k	–	14400.0	1e+20	1177.3k	–	14400.0	1e+20	580.4k	–	14400.0
dem_500_gaslib135_243	1e+20	679.3k	–	14400.0	1e+20	1305.4k	–	14400.0	1e+20	897.2k	–	14400.0
dem_500_gaslib135_244	1e+20	934.7k	–	14400.0	1e+20	1380.9k	–	14400.0	1e+20	826.4k	–	14400.0
dem_500_gaslib135_245	1e+20	922.3k	–	14400.0	1e+20	1356.9k	–	14400.0	1e+20	913.5k	–	14400.0
dem_500_gaslib135_246	1e+20	663.8k	–	14400.0	1e+20	1215.9k	–	14400.0	1e+20	812.1k	–	14400.0
dem_500_gaslib135_247	1e+20	1004.9k	–	14400.0	1e+20	1454.6k	–	14400.0	1e+20	948.5k	–	14400.0
dem_500_gaslib135_248	1e+20	1025.7k	–	14400.0	1e+20	1533.8k	–	14400.0	1e+20	918.3k	–	14400.0
dem_500_gaslib135_249	1e+20	686.4k	–	14400.0	1e+20	1263.8k	–	14400.0	1e+20	761.3k	–	14400.0
dem_500_gaslib135_250	1e+20	785.7k	–	14400.0	1e+20	1081.6k	–	14400.0	1e+20	560.3k	–	14400.0
dem_500_gaslib135_251	1e+20	819.3k	–	14400.0	1e+20	1650.4k	–	14400.0	1e+20	1057.4k	–	14400.0
dem_500_gaslib135_252	1e+20	1158.4k	–	14400.0	1e+20	2202.7k	–	14400.0	1e+20	1006.0k	–	14400.0
dem_500_gaslib135_253	1e+20	778.7k	–	14400.0	1e+20	1303.9k	–	14400.0	1e+20	848.8k	–	14400.0
dem_500_gaslib135_254	1e+20	687.2k	–	14400.0	1e+20	1344.7k	–	14400.0	24.1M	969.1k	2390.4	14400.0
dem_500_gaslib135_255	1e+20	906.2k	–	14400.0	1e+20	1446.4k	–	14400.0	1e+20	946.7k	–	14400.0
dem_500_gaslib135_256	1e+20	788.0k	–	14400.0	1e+20	911.3k	–	14400.0	1e+20	637.2k	–	14400.0
dem_500_gaslib135_257	1e+20	626.6k	–	14400.0	1e+20	1290.8k	–	14400.0	1e+20	687.2k	–	14400.0
dem_500_gaslib135_258	1e+20	1019.2k	–	14400.0	1e+20	1431.0k	–	14400.0	1e+20	810.7k	–	14400.0
dem_500_gaslib135_259	1e+20	909.3k	–	14400.0	1e+20	1459.2k	–	14400.0	1e+20	702.6k	–	14400.0
dem_500_gaslib135_260	1e+20	682.4k	–	14400.0	1e+20	1269.9k	–	14400.0	1e+20	1013.0k	–	14400.0
dem_500_gaslib135_261	1e+20	1035.1k	–	14400.0	1e+20	1433.4k	–	14400.0	1e+20	1010.4k	–	14400.0
dem_500_gaslib135_262	1e+20	735.6k	–	14400.0	1e+20	1233.4k	–	14400.0	1e+20	835.9k	–	14400.0
dem_500_gaslib135_263	1e+20	893.6k	–	14400.0	1e+20	1458.5k	–	14400.0	1e+20	860.6k	–	14400.0
dem_500_gaslib135_264	1e+20	927.4k	–	14400.0	1e+20	1283.6k	–	14400.0	1e+20	801.4k	–	14400.0
dem_500_gaslib135_265	1e+20	791.0k	–	14400.0	1e+20	1112.9k	–	14400.0	1e+20	674.7k	–	14400.0
dem_500_gaslib135_266	1e+20	1027.0k	–	14400.0	1e+20	1305.7k	–	14400.0	1e+20	894.3k	–	14400.0
dem_500_gaslib135_267	1e+20	680.8k	–	14400.0	1e+20	1207.9k	–	14400.0	1e+20	833.9k	–	14400.0
dem_500_gaslib135_268	1e+20	1187.4k	–	14400.0	1e+20	1538.2k	–	14400.0	1e+20	985.8k	–	14400.0
dem_500_gaslib135_269	1e+20	751.1k	–	14400.0	1e+20	1124.9k	–	14400.0	1e+20	629.3k	–	14400.0
dem_500_gaslib135_270	1e+20	762.3k	–	14400.0	1e+20	1417.7k	–	14400.0	1e+20	846.7k	–	14400.0
dem_500_gaslib135_271	1e+20	899.4k	–	14400.0	1e+20	1670.6k	–	14400.0	1e+20	1176.7k	–	14400.0
dem_500_gaslib135_272	1e+20	1280.4k	–	14400.0	1e+20	1799.7k	–	14400.0	1e+20	1288.8k	–	14400.0
dem_500_gaslib135_273	1e+20	970.6k	–	14400.0	1e+20	1293.7k	–	14400.0	1e+20	899.5k	–	14400.0
dem_500_gaslib135_274	1e+20	851.0k	–	14400.0	1e+20	1323.5k	–	14400.0	1e+20	972.9k	–	14400.0
dem_500_gaslib135_275	1e+20	1177.0k	–	14400.0	1e+20	1554.2k	–	14400.0	1e+20	1031.9k	–	14400.0
dem_500_gaslib135_276	1e+20	1006.4k	–	14400.0	1e+20	1362.0k	–	14400.0	1e+20	819.8k	–	14400.0
dem_500_gaslib135_277	1e+20	1013.0k	–	14400.0	1e+20	1512.6k	–	14400.0	1e+20	930.3k	–	14400.0
dem_500_gaslib135_278	1e+20	1120.9k	–	14400.0	1e+20	1696.5k	–	14400.0	1e+20	1178.9k	–	14400.0
dem_500_gaslib135_279	1e+20	653.3k	–	14400.0	1e+20	1316.7k	–	14400.0	1e+20	867.1k	–	14400.0
dem_500_gaslib135_280	1e+20	1007.7k	–	14400.0	1e+20	1643.7k	–	14400.0	1e+20	1038.8k	–	14400.0
dem_500_gaslib135_281	1e+20	722.3k	–	14400.0	1e+20	1344.6k	–	14400.0	1e+20	775.2k	–	14400.0
dem_500_gaslib135_282	1e+20	685.6k	–	14400.0	1e+20	1144.4k	–	14400.0	1e+20	769.8k	–	14400.0
dem_500_gaslib135_283	1e+20	1271.3k	–	14400.0	1e+20	1644.6k	–	14400.0	1e+20	1137.7k	–	14400.0
dem_500_gaslib135_284	1e+20	981.0k	–	14400.0	1e+20	1308.5k	–	14400.0	1e+20	824.0k	–	14400.0
dem_500_gaslib135_285	1e+20	810.3k	–	14400.0	1e+20	1216.8k	–	14400.0	1e+20	653.6k	–	14400.0
dem_500_gaslib135_286	1e+20	770.3k	–	14400.0	1e+20	1494.5k	–	14400.0	1e+20	874.0k	–	14400.0
dem_500_gaslib135_287	1e+20	331.4k	–	14400.0	1e+20	977.6k	–	14400.0	1e+20	482.1k	–	14400.0
dem_500_gaslib135_288	1e+20	777.1k	–	14400.0	1e+20	1064.2k	–	14400.0	1e+20	714.9k	–	14400.0
dem_500_gaslib135_289	1e+20	327.3k	–	14400.0	1e+20	970.9k	–	14400.0	1e+20	591.0k	–	14400.0
dem_500_gaslib135_290	1e+20	1234.1k	–	14400.0	1e+20	1494.1k	–	14400.0	1e+20	1012.4k	–	14400.0
dem_500_gaslib135_291	1e+20	935.5k	–	14400.0	1e+20	1481.7k	–	14400.0	1e+20	829.9k	–	14400.0
dem_500_gaslib135_292	1e+20	623.5k	–	14400.0	1e+20	1268.2k	–	14400.0	1e+20	790.5k	–	14400.0

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Table A.23: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_293	1e+20	486.9k	–	14400.0	1e+20	1113.7k	–	14400.0	1e+20	466.0k	–	14400.0
dem_500_gaslib135_294	1e+20	948.2k	–	14400.0	1e+20	1327.8k	–	14400.0	1e+20	858.0k	–	14400.0
dem_500_gaslib135_295	1e+20	903.9k	–	14400.0	1e+20	1178.4k	–	14400.0	1e+20	571.8k	–	14400.0
dem_500_gaslib135_296	1e+20	835.7k	–	14400.0	1e+20	1457.0k	–	14400.0	1e+20	995.7k	–	14400.0
dem_500_gaslib135_297	1e+20	959.5k	–	14400.0	1e+20	1434.3k	–	14400.0	1e+20	782.8k	–	14400.0
dem_500_gaslib135_298	1e+20	974.0k	–	14400.0	1e+20	1339.8k	–	14400.0	1e+20	955.4k	–	14400.0
dem_500_gaslib135_299	1e+20	676.7k	–	14400.0	1e+20	1109.4k	–	14400.0	1e+20	702.1k	–	14400.0
dem_500_gaslib135_300	1e+20	604.2k	–	14400.0	1e+20	1109.9k	–	14400.0	1e+20	668.0k	–	14400.0
dem_500_gaslib135_301	1e+20	431.1k	–	14400.0	1e+20	1132.2k	–	14400.0	1e+20	721.1k	–	14400.0
dem_500_gaslib135_302	1e+20	1087.1k	–	14400.0	1e+20	1341.1k	–	14400.0	1e+20	924.1k	–	14400.0
dem_500_gaslib135_303	1e+20	575.8k	–	14400.0	1e+20	1254.0k	–	14400.0	1e+20	707.7k	–	14400.0
dem_500_gaslib135_304	1e+20	816.0k	–	14400.0	1e+20	1638.5k	–	14400.0	1e+20	971.5k	–	14400.0
dem_500_gaslib135_305	1e+20	722.1k	–	14400.0	1e+20	1611.1k	–	14400.0	1e+20	951.4k	–	14400.0
dem_500_gaslib135_306	1e+20	942.0k	–	14400.0	1e+20	1399.6k	–	14400.0	1e+20	829.3k	–	14400.0
dem_500_gaslib135_307	1e+20	609.6k	–	14400.0	1e+20	1122.7k	–	14400.0	1e+20	796.3k	–	14400.0
dem_500_gaslib135_308	1e+20	1043.1k	–	14400.0	1e+20	1454.9k	–	14400.0	1e+20	892.3k	–	14400.0
dem_500_gaslib135_309	1e+20	823.5k	–	14400.0	1e+20	1659.0k	–	14400.0	1e+20	868.9k	–	14400.0
dem_500_gaslib135_310	1e+20	913.7k	–	14400.0	1e+20	1511.8k	–	14400.0	1e+20	998.3k	–	14400.0
dem_500_gaslib135_311	1e+20	911.9k	–	14400.0	1e+20	1936.6k	–	14400.0	1e+20	1175.0k	–	14400.0
dem_500_gaslib135_312	1e+20	1193.9k	–	14400.0	1e+20	1742.2k	–	14400.0	1e+20	1240.9k	–	14400.0
dem_500_gaslib135_313	1e+20	885.3k	–	14400.0	1e+20	1296.8k	–	14400.0	1e+20	830.3k	–	14400.0
dem_500_gaslib135_314	1e+20	1000.4k	–	14400.0	1e+20	1662.3k	–	14400.0	1e+20	810.0k	–	14400.0
dem_500_gaslib135_315	1e+20	714.5k	–	14400.0	1e+20	1194.2k	–	14400.0	1e+20	715.4k	–	14400.0
dem_500_gaslib135_316	1e+20	710.8k	–	14400.0	1e+20	1439.8k	–	14400.0	1e+20	926.8k	–	14400.0
dem_500_gaslib135_317	1e+20	782.7k	–	14400.0	1e+20	1457.6k	–	14400.0	1e+20	1029.5k	–	14400.0
dem_500_gaslib135_318	1e+20	943.0k	–	14400.0	1e+20	1362.6k	–	14400.0	1e+20	1e+20	0.0	2670.6
dem_500_gaslib135_319	1e+20	970.8k	–	14400.0	1e+20	1348.8k	–	14400.0	1e+20	863.6k	–	14400.0
dem_500_gaslib135_320	1e+20	753.0k	–	14400.0	1e+20	1150.1k	–	14400.0	1e+20	822.2k	–	14400.0
dem_500_gaslib135_321	1e+20	928.8k	–	14400.0	1e+20	1577.0k	–	14400.0	1e+20	1035.7k	–	14400.0
dem_500_gaslib135_322	1e+20	624.2k	–	14400.0	1e+20	1456.7k	–	14400.0	1e+20	843.4k	–	14400.0
dem_500_gaslib135_323	1e+20	1197.9k	–	14400.0	1e+20	1599.3k	–	14400.0	1e+20	934.5k	–	14400.0
dem_500_gaslib135_324	1e+20	1059.8k	–	14400.0	1e+20	1558.6k	–	14400.0	1e+20	1022.5k	–	14400.0
dem_500_gaslib135_325	1e+20	1181.5k	–	14400.0	1e+20	1777.0k	–	14400.0	1e+20	1056.5k	–	14400.0
dem_500_gaslib135_326	1e+20	869.1k	–	14400.0	1e+20	1162.3k	–	14400.0	1e+20	748.4k	–	14400.0
dem_500_gaslib135_327	1e+20	528.8k	–	14400.0	1e+20	1143.5k	–	14400.0	1e+20	805.1k	–	14400.0
dem_500_gaslib135_328	1e+20	588.3k	–	14400.0	1e+20	1270.2k	–	14400.0	1e+20	787.4k	–	14400.0
dem_500_gaslib135_329	1e+20	713.1k	–	14400.0	1e+20	1572.8k	–	14400.0	1e+20	1055.6k	–	14400.0
dem_500_gaslib135_330	1e+20	1149.6k	–	14400.0	1e+20	1402.6k	–	14400.0	1e+20	870.3k	–	14400.0
dem_500_gaslib135_331	1e+20	865.9k	–	14400.0	1e+20	1276.8k	–	14400.0	1e+20	800.8k	–	14400.0
dem_500_gaslib135_332	1e+20	842.9k	–	14400.0	1e+20	1490.5k	–	14400.0	1e+20	889.6k	–	14400.0
dem_500_gaslib135_333	1e+20	930.5k	–	14400.0	1e+20	1343.5k	–	14400.0	1e+20	686.8k	–	14400.0
dem_500_gaslib135_334	1e+20	496.8k	–	14400.0	1e+20	1186.7k	–	14400.0	1e+20	697.2k	–	14400.0
dem_500_gaslib135_335	1e+20	523.6k	–	14400.0	1e+20	1291.7k	–	14400.0	1e+20	904.0k	–	14400.0
dem_500_gaslib135_336	1e+20	688.8k	–	14400.0	1e+20	1429.5k	–	14400.0	1e+20	987.4k	–	14400.0
dem_500_gaslib135_337	1e+20	547.0k	–	14400.0	1e+20	960.0k	–	14400.0	1e+20	602.7k	–	14400.0
dem_500_gaslib135_338	1e+20	649.6k	–	14400.0	1e+20	1378.7k	–	14400.0	1e+20	780.6k	–	14400.0
dem_500_gaslib135_339	1e+20	1145.5k	–	14400.0	1e+20	1516.7k	–	14400.0	1e+20	972.6k	–	14400.0
dem_500_gaslib135_340	1e+20	906.5k	–	14400.0	1e+20	1700.1k	–	14400.0	1e+20	1019.6k	–	14400.0
dem_500_gaslib135_341	1e+20	820.8k	–	14400.0	1e+20	1842.1k	–	14400.0	1e+20	946.4k	–	14400.0
dem_500_gaslib135_342	1e+20	760.0k	–	14400.0	1e+20	1423.8k	–	14400.0	1e+20	831.1k	–	14400.0
dem_500_gaslib135_343	1e+20	822.2k	–	14400.0	1e+20	968.3k	–	14400.0	1e+20	638.5k	–	14400.0
dem_500_gaslib135_344	1e+20	376.9k	–	14400.0	1e+20	1036.7k	–	14400.0	1e+20	530.9k	–	14400.0
dem_500_gaslib135_345	1e+20	1076.3k	–	14400.0	1e+20	1561.1k	–	14400.0	1e+20	986.7k	–	14400.0
dem_500_gaslib135_346	1e+20	619.0k	–	14400.0	1e+20	1304.7k	–	14400.0	1e+20	885.1k	–	14400.0
dem_500_gaslib135_347	1e+20	1049.6k	–	14400.0	1e+20	1730.0k	–	14400.0	1e+20	906.0k	–	14400.0
dem_500_gaslib135_348	1e+20	686.6k	–	14400.0	1e+20	1382.7k	–	14400.0	1e+20	749.4k	–	14400.0
dem_500_gaslib135_349	1e+20	495.6k	–	14400.0	1e+20	1194.3k	–	14400.0	1e+20	619.1k	–	14400.0
dem_500_gaslib135_350	1e+20	1056.2k	–	14400.0	1e+20	1642.0k	–	14400.0	1e+20	901.5k	–	14400.0
dem_500_gaslib135_351	1e+20	608.1k	–	14400.0	1e+20	1349.1k	–	14400.0	1e+20	633.8k	–	14400.0
dem_500_gaslib135_352	1e+20	752.1k	–	14400.0	1e+20	1308.3k	–	14400.0	1e+20	816.7k	–	14400.0
dem_500_gaslib135_353	1e+20	553.0k	–	14400.0	1e+20	1029.7k	–	14400.0	1e+20	604.7k	–	14400.0
dem_500_gaslib135_354	1e+20	926.0k	–	14400.0	1e+20	1295.1k	–	14400.0	1e+20	830.7k	–	14400.0
dem_500_gaslib135_355	1e+20	317.9k	–	14400.0	1e+20	1088.2k	–	14400.0	1e+20	540.5k	–	14400.0
dem_500_gaslib135_356	1e+20	1196.0k	–	14400.0	1e+20	1499.8k	–	14400.0	1e+20	951.7k	–	14400.0
dem_500_gaslib135_357	1e+20	768.6k	–	14400.0	1e+20	1391.8k	–	14400.0	1e+20	841.6k	–	14400.0
dem_500_gaslib135_358	1e+20	974.5k	–	14400.0	1e+20	1304.8k	–	14400.0	1e+20	847.9k	–	14400.0
dem_500_gaslib135_359	1e+20	994.7k	–	14400.0	1e+20	1258.5k	–	14400.0	1e+20	761.3k	–	14400.0
dem_500_gaslib135_360	1e+20	860.7k	–	14400.0	1e+20	1160.8k	–	14400.0	1e+20	728.3k	–	14400.0
dem_500_gaslib135_361	1e+20	681.7k	–	14400.0	1e+20	1194.1k	–	14400.0	1e+20	738.8k	–	14400.0
dem_500_gaslib135_362	1e+20	912.7k	–	14400.0	1e+20	1260.0k	–	14400.0	1e+20	765.3k	–	14400.0
dem_500_gaslib135_363	1e+20	760.3k	–	14400.0	1e+20	1456.7k	–	14400.0	1e+20	993.7k	–	14400.0
dem_500_gaslib135_364	1e+20	865.1k	–	14400.0	1e+20	1592.7k	–	14400.0	1e+20	998.1k	–	14400.0
dem_500_gaslib135_365	1e+20	974.7k	–	14400.0	1e+20	1729.2k	–	14400.0	1e+20	886.1k	–	14400.0
dem_500_gaslib135_366	1e+20	771.2k	–	14400.0	1e+20	1136.2k	–	14400.0	1e+20	672.0k	–	14400.0
dem_500_gaslib135_367	1e+20	737.5k	–	14400.0	1e+20	1534.4k	–	14400.0	1e+20	940.6k	–	14400.0
dem_500_gaslib135_368	1e+20	918.1k	–	14400.0	1e+20	1332.2k	–	14400.0	1e+20	825.2k	–	14400.0
dem_500_gaslib135_369	1e+20	1039.1k	–	14400.0	1e+20	1310.1k	–	14400.0	1e+20	904.4k	–	14400.0
dem_500_gaslib135_370	1e+20	710.5k	–	14400.0	1e+20	1277.8k	–	14400.0	1e+20	922.9k	–	14400.0

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Table A.23: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_371	1e+20	581.6k	–	14400.0	1e+20	1171.5k	–	14400.0	1e+20	655.3k	–	14400.0
dem_500_gaslib135_372	1e+20	805.1k	–	14400.0	1e+20	1609.3k	–	14400.0	1e+20	882.3k	–	14400.0
dem_500_gaslib135_373	1e+20	1106.4k	–	14400.0	1e+20	1674.1k	–	14400.0	1e+20	990.4k	–	14400.0
dem_500_gaslib135_374	1e+20	675.1k	–	14400.0	1e+20	1307.1k	–	14400.0	1e+20	811.5k	–	14400.0
dem_500_gaslib135_375	1e+20	1360.5k	–	14400.0	1e+20	1770.7k	–	14400.0	1e+20	1e+20	0.0	13283.0
dem_500_gaslib135_376	1e+20	867.8k	–	14400.0	1e+20	1616.7k	–	14400.0	1e+20	890.3k	–	14400.0
dem_500_gaslib135_377	1e+20	1156.6k	–	14400.0	1e+20	1568.6k	–	14400.0	1e+20	993.9k	–	14400.0
dem_500_gaslib135_378	1e+20	926.4k	–	14400.0	1e+20	1346.1k	–	14400.0	1e+20	870.2k	–	14400.0
dem_500_gaslib135_379	1e+20	999.6k	–	14400.0	1e+20	1140.7k	–	14400.0	1e+20	784.6k	–	14400.0
dem_500_gaslib135_380	1e+20	1212.7k	–	14400.0	1e+20	1814.4k	–	14400.0	1e+20	983.2k	–	14400.0
dem_500_gaslib135_381	1e+20	663.7k	–	14400.0	1e+20	1104.6k	–	14400.0	1e+20	772.3k	–	14400.0
dem_500_gaslib135_382	1e+20	697.7k	–	14400.0	1e+20	1268.5k	–	14400.0	1e+20	807.5k	–	14400.0
dem_500_gaslib135_383	1e+20	1192.3k	–	14400.0	1e+20	1470.4k	–	14400.0	1e+20	989.6k	–	14400.0
dem_500_gaslib135_384	1e+20	813.1k	–	14400.0	1e+20	1558.8k	–	14400.0	1e+20	997.4k	–	14400.0
dem_500_gaslib135_385	1e+20	725.1k	–	14400.0	1e+20	1326.6k	–	14400.0	1e+20	823.7k	–	14400.0
dem_500_gaslib135_386	1e+20	996.8k	–	14400.0	1e+20	1405.2k	–	14400.0	1e+20	936.4k	–	14400.0
dem_500_gaslib135_387	1e+20	847.8k	–	14400.0	1e+20	1608.1k	–	14400.0	1e+20	999.9k	–	14400.0
dem_500_gaslib135_388	1e+20	726.8k	–	14400.0	1e+20	1093.6k	–	14400.0	1e+20	672.2k	–	14400.0
dem_500_gaslib135_389	1e+20	812.7k	–	14400.0	1e+20	1660.5k	–	14400.0	1e+20	1108.1k	–	14400.0
dem_500_gaslib135_390	1e+20	755.7k	–	14400.0	1e+20	1802.7k	–	14400.0	1e+20	1040.3k	–	14400.0
dem_500_gaslib135_391	1e+20	700.9k	–	14400.0	1e+20	1247.4k	–	14400.0	1e+20	717.8k	–	14400.0
dem_500_gaslib135_392	1e+20	547.7k	–	14400.0	1e+20	1354.2k	–	14400.0	1e+20	891.2k	–	14400.0
dem_500_gaslib135_393	1e+20	702.5k	–	14400.0	1e+20	1964.2k	–	14400.0	1e+20	1087.2k	–	14400.0
dem_500_gaslib135_394	1e+20	494.1k	–	14400.0	1e+20	1257.7k	–	14400.0	1e+20	731.9k	–	14400.0
dem_500_gaslib135_395	1e+20	1105.4k	–	14400.0	1e+20	1565.5k	–	14400.0	1e+20	960.3k	–	14400.0
dem_500_gaslib135_396	1e+20	928.5k	–	14400.0	1e+20	1454.8k	–	14400.0	1e+20	1008.7k	–	14400.0
dem_500_gaslib135_397	1e+20	861.5k	–	14400.0	1e+20	1388.2k	–	14400.0	1e+20	800.1k	–	14400.0
dem_500_gaslib135_398	1e+20	787.2k	–	14400.0	1e+20	1411.9k	–	14400.0	1e+20	839.8k	–	14400.0
dem_500_gaslib135_399	1e+20	613.6k	–	14400.0	1e+20	1085.1k	–	14400.0	1e+20	625.4k	–	14400.0
dem_500_gaslib135_400	1e+20	981.6k	–	14400.0	1e+20	1344.9k	–	14400.0	1e+20	807.3k	–	14400.0
dem_500_gaslib135_401	1e+20	1175.0k	–	14400.0	1e+20	1392.0k	–	14400.0	1e+20	1022.5k	–	14400.0
dem_500_gaslib135_402	1e+20	321.3k	–	14400.0	1e+20	959.1k	–	14400.0	1e+20	572.8k	–	14400.0
dem_500_gaslib135_403	1e+20	748.6k	–	14400.0	1e+20	1693.0k	–	14400.0	1e+20	954.3k	–	14400.0
dem_500_gaslib135_404	1e+20	1069.2k	–	14400.0	1e+20	1464.3k	–	14400.0	1e+20	985.4k	–	14400.0
dem_500_gaslib135_405	1e+20	678.2k	–	14400.0	1e+20	972.5k	–	14400.0	1e+20	670.4k	–	14400.0
dem_500_gaslib135_406	1e+20	702.0k	–	14400.0	1e+20	1431.1k	–	14400.0	1e+20	683.7k	–	14400.0
dem_500_gaslib135_407	1e+20	701.9k	–	14400.0	1e+20	1435.1k	–	14400.0	1e+20	802.4k	–	14400.0
dem_500_gaslib135_408	1e+20	949.8k	–	14400.0	1e+20	1456.3k	–	14400.0	1e+20	877.3k	–	14400.0
dem_500_gaslib135_409	1e+20	956.1k	–	14400.0	1e+20	1815.9k	–	14400.0	1e+20	1116.4k	–	14400.0
dem_500_gaslib135_410	1e+20	1130.3k	–	14400.0	1e+20	1502.2k	–	14400.0	1e+20	1029.2k	–	14400.0
dem_500_gaslib135_411	1e+20	1051.2k	–	14400.0	1e+20	1386.0k	–	14400.0	1e+20	879.5k	–	14400.0
dem_500_gaslib135_412	1e+20	956.2k	–	14400.0	1e+20	1884.0k	–	14400.0	1e+20	1190.9k	–	14400.0
dem_500_gaslib135_413	1e+20	891.6k	–	14400.0	1e+20	1313.1k	–	14400.0	1e+20	794.0k	–	14400.0
dem_500_gaslib135_414	1e+20	1001.7k	–	14400.0	1e+20	1465.9k	–	14400.0	1e+20	859.3k	–	14400.0
dem_500_gaslib135_415	1e+20	1041.7k	–	14400.0	1e+20	1538.6k	–	14400.0	1e+20	880.7k	–	14400.0
dem_500_gaslib135_416	1e+20	688.9k	–	14400.0	1e+20	1356.5k	–	14400.0	1e+20	839.0k	–	14400.0
dem_500_gaslib135_417	1e+20	481.6k	–	14400.0	1e+20	1180.5k	–	14400.0	1e+20	628.7k	–	14400.0
dem_500_gaslib135_418	1e+20	1059.2k	–	14400.0	1e+20	1533.3k	–	14400.0	1e+20	951.1k	–	14400.0
dem_500_gaslib135_419	1e+20	749.1k	–	14400.0	1e+20	1329.3k	–	14400.0	1e+20	933.3k	–	14400.0
dem_500_gaslib135_420	1e+20	680.5k	–	14400.0	1e+20	1e+20	0.0	10119.9	1e+20	1096.8k	–	14400.0
dem_500_gaslib135_421	1e+20	1033.3k	–	14400.0	1e+20	1294.1k	–	14400.0	1e+20	904.8k	–	14400.0
dem_500_gaslib135_422	1e+20	499.2k	–	14400.0	1e+20	1324.8k	–	14400.0	1e+20	895.7k	–	14400.0
dem_500_gaslib135_423	1e+20	449.8k	–	14400.0	1e+20	942.3k	–	14400.0	1e+20	506.8k	–	14400.0
dem_500_gaslib135_424	1e+20	1085.7k	–	14400.0	1e+20	1589.8k	–	14400.0	1e+20	904.4k	–	14400.0
dem_500_gaslib135_425	1e+20	885.0k	–	14400.0	1e+20	1385.8k	–	14400.0	1e+20	816.6k	–	14400.0
dem_500_gaslib135_426	1e+20	1042.9k	–	14400.0	1e+20	1963.3k	–	14400.0	1e+20	898.1k	–	14400.0
dem_500_gaslib135_427	1e+20	921.3k	–	14400.0	1e+20	1285.1k	–	14400.0	1e+20	873.3k	–	14400.0
dem_500_gaslib135_428	1e+20	718.2k	–	14400.0	1e+20	1321.0k	–	14400.0	1e+20	827.9k	–	14400.0
dem_500_gaslib135_429	1e+20	1038.6k	–	14400.0	1e+20	1356.1k	–	14400.0	1e+20	817.7k	–	14400.0
dem_500_gaslib135_430	1e+20	888.9k	–	14400.0	1e+20	1347.0k	–	14400.0	1e+20	846.4k	–	14400.0
dem_500_gaslib135_431	1e+20	991.6k	–	14400.0	1e+20	1435.7k	–	14400.0	1e+20	892.7k	–	14400.0
dem_500_gaslib135_432	1e+20	1030.4k	–	14400.0	1e+20	1849.1k	–	14400.0	1e+20	888.8k	–	14400.0
dem_500_gaslib135_433	1e+20	617.7k	–	14400.0	1e+20	1320.5k	–	14400.0	1e+20	744.1k	–	14400.0
dem_500_gaslib135_434	1e+20	851.3k	–	14400.0	1e+20	1330.4k	–	14400.0	1e+20	784.1k	–	14400.0
dem_500_gaslib135_435	1e+20	1019.8k	–	14400.0	1e+20	1743.8k	–	14400.0	1e+20	1221.1k	–	14400.0
dem_500_gaslib135_436	1e+20	1041.2k	–	14400.0	1e+20	1790.0k	–	14400.0	1e+20	901.5k	–	14400.0
dem_500_gaslib135_437	1e+20	657.5k	–	14400.0	1e+20	1196.2k	–	14400.0	1e+20	797.0k	–	14400.0
dem_500_gaslib135_438	1e+20	939.8k	–	14400.0	1e+20	1686.0k	–	14400.0	1e+20	970.2k	–	14400.0
dem_500_gaslib135_439	1e+20	991.1k	–	14400.0	1e+20	1755.3k	–	14400.0	1e+20	1095.2k	–	14400.0
dem_500_gaslib135_440	1e+20	677.4k	–	14400.0	1e+20	1140.1k	–	14400.0	1e+20	717.8k	–	14400.0
dem_500_gaslib135_441	1e+20	1185.7k	–	14400.0	1e+20	1725.3k	–	14400.0	1e+20	1056.9k	–	14400.0
dem_500_gaslib135_442	1e+20	1068.2k	–	14400.0	1e+20	1425.2k	–	14400.0	1e+20	936.9k	–	14400.0
dem_500_gaslib135_443	1e+20	817.8k	–	14400.0	1e+20	1434.1k	–	14400.0	1e+20	927.5k	–	14400.0
dem_500_gaslib135_444	1e+20	664.5k	–	14400.0	1e+20	1194.0k	–	14400.0	1e+20	653.2k	–	14400.0
dem_500_gaslib135_445	1e+20	912.5k	–	14400.0	1e+20	1418.2k	–	14400.0	1e+20	943.7k	–	14400.0
dem_500_gaslib135_446	1e+20	970.8k	–	14400.0	1e+20	1615.6k	–	14400.0	1e+20	960.3k	–	14400.0
dem_500_gaslib135_447	1e+20	1081.8k	–	14400.0	1e+20	1501.8k	–	14400.0	1e+20	995.2k	–	14400.0
dem_500_gaslib135_448	1e+20	549.6k	–	14400.0	1e+20	1274.7k	–	14400.0	1e+20	737.0k	–	14400.0

continued on next page

Table A.23: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_449	1e+20	958.0k	–	14400.0	1e+20	1661.4k	–	14400.0	1e+20	1028.6k	–	14400.0
dem_500_gaslib135_450	1e+20	1028.8k	–	14400.0	1e+20	1378.9k	–	14400.0	1e+20	906.7k	–	14400.0
dem_500_gaslib135_451	1e+20	964.0k	–	14400.0	1e+20	1786.2k	–	14400.0	1e+20	685.2k	–	14400.0
dem_500_gaslib135_452	1e+20	477.5k	–	14400.0	1e+20	1166.4k	–	14400.0	1e+20	788.6k	–	14400.0
dem_500_gaslib135_453	1e+20	430.2k	–	14400.0	1e+20	954.2k	–	14400.0	1e+20	502.0k	–	14400.0
dem_500_gaslib135_454	1e+20	854.6k	–	14400.0	1e+20	1679.8k	–	14400.0	1e+20	1055.3k	–	14400.0
dem_500_gaslib135_455	1e+20	963.2k	–	14400.0	1e+20	1284.2k	–	14400.0	1e+20	813.0k	–	14400.0
dem_500_gaslib135_456	1e+20	851.6k	–	14400.0	1e+20	1601.5k	–	14400.0	1e+20	1059.6k	–	14400.0
dem_500_gaslib135_457	1e+20	462.6k	–	14400.0	1e+20	1207.7k	–	14400.0	1e+20	758.8k	–	14400.0
dem_500_gaslib135_458	1e+20	1086.0k	–	14400.0	1e+20	1586.4k	–	14400.0	1e+20	897.6k	–	14400.0
dem_500_gaslib135_459	1e+20	531.8k	–	14400.0	1e+20	1274.9k	–	14400.0	1e+20	766.2k	–	14400.0
dem_500_gaslib135_460	1e+20	809.7k	–	14400.0	1e+20	1316.5k	–	14400.0	1e+20	557.3k	–	14400.0
dem_500_gaslib135_461	1e+20	739.6k	–	14400.0	1e+20	1509.9k	–	14400.0	1e+20	695.5k	–	14400.0
dem_500_gaslib135_462	1e+20	589.6k	–	14400.0	1e+20	1197.0k	–	14400.0	1e+20	659.4k	–	14400.0
dem_500_gaslib135_463	1e+20	1035.6k	–	14400.0	1e+20	1404.4k	–	14400.0	1e+20	998.7k	–	14400.0
dem_500_gaslib135_464	1e+20	417.8k	–	14400.0	1e+20	1177.6k	–	14400.0	1e+20	542.3k	–	14400.0
dem_500_gaslib135_465	1e+20	688.0k	–	14400.0	1e+20	1251.9k	–	14400.0	1e+20	916.6k	–	14400.0
dem_500_gaslib135_466	1e+20	638.7k	–	14400.0	1e+20	1299.9k	–	14400.0	1e+20	763.7k	–	14400.0
dem_500_gaslib135_467	1e+20	779.9k	–	14400.0	1e+20	1189.1k	–	14400.0	1e+20	807.9k	–	14400.0
dem_500_gaslib135_468	1e+20	854.9k	–	14400.0	1e+20	1186.4k	–	14400.0	1e+20	754.4k	–	14400.0
dem_500_gaslib135_469	1e+20	735.8k	–	14400.0	1e+20	1392.5k	–	14400.0	1e+20	581.7k	–	14400.0
dem_500_gaslib135_470	1e+20	985.4k	–	14400.0	1e+20	1332.6k	–	14400.0	1e+20	717.7k	–	14400.0
dem_500_gaslib135_471	1e+20	851.2k	–	14400.0	1e+20	1336.6k	–	14400.0	1e+20	866.0k	–	14400.0
dem_500_gaslib135_472	1e+20	760.1k	–	14400.0	1e+20	1561.0k	–	14400.0	1e+20	997.8k	–	14400.0
dem_500_gaslib135_473	1e+20	708.5k	–	14400.0	1e+20	1268.5k	–	14400.0	1e+20	942.7k	–	14400.0
dem_500_gaslib135_474	1e+20	748.3k	–	14400.0	1e+20	1208.4k	–	14400.0	1e+20	710.9k	–	14400.0
dem_500_gaslib135_475	1e+20	1006.8k	–	14400.0	1e+20	1457.9k	–	14400.0	1e+20	791.2k	–	14400.0
dem_500_gaslib135_476	1e+20	944.0k	–	14400.0	1e+20	1337.9k	–	14400.0	1e+20	825.9k	–	14400.0
dem_500_gaslib135_477	1e+20	1006.7k	–	14400.0	1e+20	1740.9k	–	14400.0	1e+20	1109.5k	–	14400.0
dem_500_gaslib135_478	1e+20	903.7k	–	14400.0	1e+20	1277.5k	–	14400.0	1e+20	932.8k	–	14400.0
dem_500_gaslib135_479	1e+20	982.5k	–	14400.0	1e+20	1407.5k	–	14400.0	1e+20	826.4k	–	14400.0
dem_500_gaslib135_480	1e+20	778.4k	–	14400.0	1e+20	1073.3k	–	14400.0	1e+20	764.5k	–	14400.0
dem_500_gaslib135_481	1e+20	881.3k	–	14400.0	1e+20	1246.8k	–	14400.0	1e+20	818.3k	–	14400.0
dem_500_gaslib135_482	1e+20	1043.6k	–	14400.0	1e+20	1402.5k	–	14400.0	1e+20	905.2k	–	14400.0
dem_500_gaslib135_483	1e+20	921.3k	–	14400.0	1e+20	1455.5k	–	14400.0	1e+20	772.0k	–	14400.0
dem_500_gaslib135_484	1e+20	570.3k	–	14400.0	1e+20	1303.2k	–	14400.0	1e+20	840.2k	–	14400.0
dem_500_gaslib135_485	1e+20	664.4k	–	14400.0	1e+20	1121.4k	–	14400.0	1e+20	648.1k	–	14400.0
dem_500_gaslib135_486	1e+20	876.4k	–	14400.0	1e+20	1371.9k	–	14400.0	1e+20	949.5k	–	14400.0
dem_500_gaslib135_487	1e+20	988.9k	–	14400.0	1e+20	1231.8k	–	14400.0	1e+20	953.1k	–	14400.0
dem_500_gaslib135_488	1e+20	916.7k	–	14400.0	1e+20	1243.5k	–	14400.0	1e+20	848.7k	–	14400.0
dem_500_gaslib135_489	1e+20	1090.5k	–	14400.0	1e+20	1539.7k	–	14400.0	1e+20	1074.6k	–	14400.0
dem_500_gaslib135_490	1e+20	360.9k	–	14400.0	1e+20	1001.0k	–	14400.0	1e+20	665.6k	–	14400.0
dem_500_gaslib135_491	1e+20	716.4k	–	14400.0	1e+20	1261.2k	–	14400.0	1e+20	765.7k	–	14400.0
dem_500_gaslib135_492	1e+20	964.7k	–	14400.0	1e+20	1518.7k	–	14400.0	1e+20	1076.0k	–	14400.0
dem_500_gaslib135_493	1e+20	939.3k	–	14400.0	1e+20	1384.5k	–	14400.0	1e+20	714.0k	–	14400.0
dem_500_gaslib135_494	1e+20	762.2k	–	14400.0	1e+20	1086.0k	–	14400.0	1e+20	589.4k	–	14400.0
dem_500_gaslib135_495	1e+20	668.6k	–	14400.0	1e+20	989.3k	–	14400.0	1e+20	717.0k	–	14400.0
dem_500_gaslib135_496	1e+20	1135.5k	–	14400.0	1e+20	1453.9k	–	14400.0	1e+20	937.9k	–	14400.0
dem_500_gaslib135_497	1e+20	1102.7k	–	14400.0	1e+20	1614.1k	–	14400.0	1e+20	859.8k	–	14400.0
dem_500_gaslib135_498	1e+20	361.4k	–	14400.0	1e+20	805.0k	–	14400.0	1e+20	443.8k	–	14400.0
dem_500_gaslib135_499	1e+20	805.7k	–	14400.0	1e+20	1665.8k	–	14400.0	1e+20	891.4k	–	14400.0
dem_500_gaslib135_500	1e+20	915.4k	–	14400.0	1e+20	1691.0k	–	14400.0	1e+20	965.7k	–	14400.0

Table A.24: Detailed results of the split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ as summarized in Figure 3.15d and Table 3.4c. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_1	1e+20	1050.1k	–	14400.0	1e+20	1319.9k	–	14400.0
dem_500_gaslib135_2	1e+20	1041.8k	–	14400.0	1e+20	1253.1k	–	14400.0
dem_500_gaslib135_3	1e+20	1107.2k	–	14400.0	1e+20	1450.0k	–	14400.0
dem_500_gaslib135_4	1e+20	534.6k	–	14400.0	1e+20	1209.2k	–	14400.0
dem_500_gaslib135_5	1e+20	735.9k	–	14400.0	1e+20	1264.7k	–	14400.0
dem_500_gaslib135_6	1e+20	845.7k	–	14400.0	1e+20	1775.0k	–	14400.0
dem_500_gaslib135_7	1e+20	1047.3k	–	14400.0	1e+20	1470.6k	–	14400.0
dem_500_gaslib135_8	1e+20	626.5k	–	14400.0	1e+20	1055.8k	–	14400.0
dem_500_gaslib135_9	1e+20	427.2k	–	14400.0	1e+20	1290.8k	–	14400.0
dem_500_gaslib135_10	1e+20	617.0k	–	14400.0	1e+20	1068.3k	–	14400.0
dem_500_gaslib135_11	1e+20	976.6k	–	14400.0	1e+20	1338.3k	–	14400.0

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Table A.24: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_12	1e+20	979.9k	–	14400.0	1e+20	1214.8k	–	14400.0
dem_500_gaslib135_13	1e+20	861.6k	–	14400.0	1e+20	1380.1k	–	14400.0
dem_500_gaslib135_14	1e+20	930.9k	–	14400.0	1e+20	1628.0k	–	14400.0
dem_500_gaslib135_15	1e+20	1010.1k	–	14400.0	1e+20	1e+20	0.0	427.6
dem_500_gaslib135_16	1e+20	933.9k	–	14400.0	1e+20	1592.5k	–	14400.0
dem_500_gaslib135_17	1e+20	1214.5k	–	14400.0	1e+20	2007.2k	–	14400.0
dem_500_gaslib135_18	1e+20	578.2k	–	14400.0	1e+20	927.7k	–	14400.0
dem_500_gaslib135_19	1e+20	833.9k	–	14400.0	1e+20	1249.7k	–	14400.0
dem_500_gaslib135_20	1e+20	727.0k	–	14400.0	1e+20	1127.3k	–	14400.0
dem_500_gaslib135_21	1e+20	1039.0k	–	14400.0	1e+20	2032.6k	–	14400.0
dem_500_gaslib135_22	1e+20	741.4k	–	14400.0	1e+20	1455.2k	–	14400.0
dem_500_gaslib135_23	1e+20	834.5k	–	14400.0	1e+20	1406.0k	–	14400.0
dem_500_gaslib135_24	1e+20	963.6k	–	14400.0	1e+20	1958.0k	–	14400.0
dem_500_gaslib135_25	1e+20	1035.5k	–	14400.0	1e+20	1602.9k	–	14400.0
dem_500_gaslib135_26	1e+20	1140.9k	–	14400.0	1e+20	1515.9k	–	14400.0
dem_500_gaslib135_27	1e+20	608.5k	–	14400.0	1e+20	1522.4k	–	14400.0
dem_500_gaslib135_28	1e+20	393.0k	–	14400.0	1e+20	1026.1k	–	14400.0
dem_500_gaslib135_29	1e+20	868.3k	–	14400.0	1e+20	1301.2k	–	14400.0
dem_500_gaslib135_30	1e+20	801.4k	–	14400.0	1e+20	1389.4k	–	14400.0
dem_500_gaslib135_31	1e+20	962.7k	–	14400.0	1e+20	1373.5k	–	14400.0
dem_500_gaslib135_32	1e+20	1179.9k	–	14400.0	1e+20	1529.4k	–	14400.0
dem_500_gaslib135_33	1e+20	785.9k	–	14400.0	1e+20	1273.7k	–	14400.0
dem_500_gaslib135_34	1e+20	1050.7k	–	14400.0	1e+20	1372.4k	–	14400.0
dem_500_gaslib135_35	1e+20	1008.4k	–	14400.0	1e+20	1637.5k	–	14400.0
dem_500_gaslib135_36	1e+20	1111.5k	–	14400.0	1e+20	1475.2k	–	14400.0
dem_500_gaslib135_37	1e+20	632.6k	–	14400.0	1e+20	1010.9k	–	14400.0
dem_500_gaslib135_38	1e+20	648.4k	–	14400.0	1e+20	1316.9k	–	14400.0
dem_500_gaslib135_39	1e+20	646.9k	–	14400.0	1e+20	1417.3k	–	14400.0
dem_500_gaslib135_40	1e+20	992.8k	–	14400.0	1e+20	1288.9k	–	14400.0
dem_500_gaslib135_41	1e+20	972.6k	–	14400.0	1e+20	1335.1k	–	14400.0
dem_500_gaslib135_42	1e+20	917.9k	–	14400.0	1e+20	1355.0k	–	14400.0
dem_500_gaslib135_43	1e+20	750.6k	–	14400.0	1e+20	1357.2k	–	14400.0
dem_500_gaslib135_44	1e+20	647.3k	–	14400.0	1e+20	1338.0k	–	14400.0
dem_500_gaslib135_45	1e+20	936.3k	–	14400.0	1e+20	1077.9k	–	14400.0
dem_500_gaslib135_46	1e+20	727.0k	–	14400.0	1e+20	1385.7k	–	14400.0
dem_500_gaslib135_47	1e+20	676.1k	–	14400.0	1e+20	1377.8k	–	14400.0
dem_500_gaslib135_48	1e+20	1095.4k	–	14400.0	1e+20	1663.7k	–	14400.0
dem_500_gaslib135_49	1e+20	837.5k	–	14400.0	1e+20	1523.1k	–	14400.0
dem_500_gaslib135_50	1e+20	879.3k	–	14400.0	1e+20	1279.4k	–	14400.0
dem_500_gaslib135_51	1e+20	792.1k	–	14400.0	1e+20	1558.5k	–	14400.0
dem_500_gaslib135_52	1e+20	507.4k	–	14400.0	1e+20	1081.8k	–	14400.0
dem_500_gaslib135_53	1e+20	965.5k	–	14400.0	1e+20	1648.9k	–	14400.0
dem_500_gaslib135_54	1e+20	802.2k	–	14400.0	1e+20	1218.1k	–	14400.0
dem_500_gaslib135_55	1e+20	1510.6k	–	14400.0	1e+20	2002.1k	–	14400.0
dem_500_gaslib135_56	1e+20	692.8k	–	14400.0	1e+20	1165.4k	–	14400.0
dem_500_gaslib135_57	1e+20	892.1k	–	14400.0	1e+20	1288.4k	–	14400.0
dem_500_gaslib135_58	1e+20	667.3k	–	14400.0	1e+20	1271.2k	–	14400.0
dem_500_gaslib135_59	1e+20	834.2k	–	14400.0	1e+20	1313.0k	–	14400.0
dem_500_gaslib135_60	1e+20	931.2k	–	14400.0	1e+20	1518.7k	–	14400.0
dem_500_gaslib135_61	1e+20	649.4k	–	14400.0	1e+20	1188.8k	–	14400.0
dem_500_gaslib135_62	1e+20	1018.3k	–	14400.0	1e+20	1892.8k	–	14400.0
dem_500_gaslib135_63	1e+20	1152.4k	–	14400.0	1e+20	1734.9k	–	14400.0
dem_500_gaslib135_64	1e+20	1030.1k	–	14400.0	1e+20	1264.9k	–	14400.0
dem_500_gaslib135_65	1e+20	911.4k	–	14400.0	1e+20	1225.9k	–	14400.0
dem_500_gaslib135_66	1e+20	1102.0k	–	14400.0	1e+20	1423.6k	–	14400.0
dem_500_gaslib135_67	1e+20	1141.8k	–	14400.0	1e+20	1481.7k	–	14400.0
dem_500_gaslib135_68	1e+20	1090.0k	–	14400.0	1e+20	1929.2k	–	14400.0
dem_500_gaslib135_69	1e+20	420.9k	–	14400.0	1e+20	1164.2k	–	14400.0
dem_500_gaslib135_70	1e+20	651.4k	–	14400.0	1e+20	1944.8k	–	14400.0
dem_500_gaslib135_71	1e+20	369.0k	–	14400.0	1e+20	1298.6k	–	14400.0
dem_500_gaslib135_72	1e+20	728.2k	–	14400.0	1e+20	1632.5k	–	14400.0
dem_500_gaslib135_73	1e+20	789.1k	–	14400.0	1e+20	1074.0k	–	14400.0
dem_500_gaslib135_74	1e+20	832.0k	–	14400.0	1e+20	1322.0k	–	14400.0
dem_500_gaslib135_75	1e+20	385.0k	–	14400.0	1e+20	1146.4k	–	14400.0
dem_500_gaslib135_76	1e+20	672.4k	–	14400.0	1e+20	1127.8k	–	14400.0
dem_500_gaslib135_77	1e+20	423.1k	–	14400.0	1e+20	1129.1k	–	14400.0
dem_500_gaslib135_78	1e+20	781.1k	–	14400.0	1e+20	1615.5k	–	14400.0
dem_500_gaslib135_79	1e+20	820.4k	–	14400.0	1e+20	1155.6k	–	14400.0
dem_500_gaslib135_80	1e+20	928.9k	–	14400.0	1e+20	1414.5k	–	14400.0
dem_500_gaslib135_81	1e+20	886.7k	–	14400.0	1e+20	1751.7k	–	14400.0
dem_500_gaslib135_82	1e+20	1007.2k	–	14400.0	1e+20	1535.8k	–	14400.0
dem_500_gaslib135_83	1e+20	1008.9k	–	14400.0	1e+20	1370.4k	–	14400.0
dem_500_gaslib135_84	1e+20	854.3k	–	14400.0	1e+20	1497.9k	–	14400.0
dem_500_gaslib135_85	1e+20	699.8k	–	14400.0	1e+20	1379.1k	–	14400.0
dem_500_gaslib135_86	1e+20	917.8k	–	14400.0	1e+20	1602.8k	–	14400.0
dem_500_gaslib135_87	1e+20	737.7k	–	14400.0	1e+20	1289.9k	–	14400.0
dem_500_gaslib135_88	1e+20	1155.9k	–	14400.0	1e+20	1657.1k	–	14400.0
dem_500_gaslib135_89	1e+20	981.3k	–	14400.0	1e+20	1425.5k	–	14400.0

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Table A.24: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_90	1e+20	1094.0k	–	14400.0	1e+20	1921.8k	–	14400.0
dem_500_gaslib135_91	1e+20	889.6k	–	14400.0	1e+20	1364.9k	–	14400.0
dem_500_gaslib135_92	1e+20	743.2k	–	14400.0	1e+20	1320.6k	–	14400.0
dem_500_gaslib135_93	1e+20	1053.8k	–	14400.0	1e+20	1300.9k	–	14400.0
dem_500_gaslib135_94	1e+20	765.5k	–	14400.0	1e+20	1209.2k	–	14400.0
dem_500_gaslib135_95	1e+20	843.9k	–	14400.0	1e+20	1272.0k	–	14400.0
dem_500_gaslib135_96	1e+20	1009.8k	–	14400.0	1e+20	1606.4k	–	14400.0
dem_500_gaslib135_97	1e+20	894.9k	–	14400.0	1e+20	1241.6k	–	14400.0
dem_500_gaslib135_98	1e+20	1091.9k	–	14400.0	1e+20	1473.5k	–	14400.0
dem_500_gaslib135_99	1e+20	968.7k	–	14400.0	1e+20	1399.7k	–	14400.0
dem_500_gaslib135_100	1e+20	1478.3k	–	14400.0	1e+20	1889.5k	–	14400.0
dem_500_gaslib135_101	1e+20	986.0k	–	14400.0	1e+20	1383.3k	–	14400.0
dem_500_gaslib135_102	1e+20	1211.2k	–	14400.0	1e+20	1548.1k	–	14400.0
dem_500_gaslib135_103	1e+20	1038.1k	–	14400.0	1e+20	1395.6k	–	14400.0
dem_500_gaslib135_104	1e+20	921.6k	–	14400.0	1e+20	1448.9k	–	14400.0
dem_500_gaslib135_105	1e+20	892.3k	–	14400.0	1e+20	1185.5k	–	14400.0
dem_500_gaslib135_106	1e+20	1086.9k	–	14400.0	1e+20	1480.6k	–	14400.0
dem_500_gaslib135_107	1e+20	705.9k	–	14400.0	1e+20	1275.9k	–	14400.0
dem_500_gaslib135_108	1e+20	854.6k	–	14400.0	1e+20	1319.2k	–	14400.0
dem_500_gaslib135_109	1e+20	548.5k	–	14400.0	1e+20	1250.2k	–	14400.0
dem_500_gaslib135_110	1e+20	847.2k	–	14400.0	1e+20	1213.2k	–	14400.0
dem_500_gaslib135_111	1e+20	1082.5k	–	14400.0	1e+20	1477.3k	–	14400.0
dem_500_gaslib135_112	1e+20	922.3k	–	14400.0	1e+20	1310.7k	–	14400.0
dem_500_gaslib135_113	1e+20	381.7k	–	14400.0	1e+20	1069.3k	–	14400.0
dem_500_gaslib135_114	1e+20	770.0k	–	14400.0	1e+20	1354.3k	–	14400.0
dem_500_gaslib135_115	1e+20	1098.2k	–	14400.0	1e+20	1671.3k	–	14400.0
dem_500_gaslib135_116	1e+20	718.1k	–	14400.0	1e+20	1305.7k	–	14400.0
dem_500_gaslib135_117	1e+20	1239.7k	–	14400.0	1e+20	1687.5k	–	14400.0
dem_500_gaslib135_118	1e+20	816.6k	–	14400.0	1e+20	1223.9k	–	14400.0
dem_500_gaslib135_119	1e+20	786.2k	–	14400.0	1e+20	1273.3k	–	14400.0
dem_500_gaslib135_120	1e+20	761.7k	–	14400.0	1e+20	1656.6k	–	14400.0
dem_500_gaslib135_121	1e+20	979.9k	–	14400.0	1e+20	1538.1k	–	14400.0
dem_500_gaslib135_122	1e+20	852.3k	–	14400.0	1e+20	1130.4k	–	14400.0
dem_500_gaslib135_123	1e+20	1149.2k	–	14400.0	1e+20	1712.7k	–	14400.0
dem_500_gaslib135_124	1e+20	935.2k	–	14400.0	1e+20	1539.0k	–	14400.0
dem_500_gaslib135_125	1e+20	1096.7k	–	14400.0	1e+20	1467.3k	–	14400.0
dem_500_gaslib135_126	1e+20	651.0k	–	14400.0	1e+20	1114.0k	–	14400.0
dem_500_gaslib135_127	1e+20	717.2k	–	14400.0	1e+20	1333.6k	–	14400.0
dem_500_gaslib135_128	1e+20	695.4k	–	14400.0	1e+20	1193.8k	–	14400.0
dem_500_gaslib135_129	1e+20	566.7k	–	14400.0	1e+20	1340.4k	–	14400.0
dem_500_gaslib135_130	1e+20	923.0k	–	14400.0	1e+20	1452.0k	–	14400.0
dem_500_gaslib135_131	1e+20	663.4k	–	14400.0	1e+20	1287.5k	–	14400.0
dem_500_gaslib135_132	1e+20	1227.3k	–	14400.0	1e+20	1681.2k	–	14400.0
dem_500_gaslib135_133	1e+20	875.8k	–	14400.0	1e+20	1676.7k	–	14400.0
dem_500_gaslib135_134	1e+20	1195.7k	–	14400.0	1e+20	1707.2k	–	14400.0
dem_500_gaslib135_135	1e+20	629.6k	–	14400.0	1e+20	1127.5k	–	14400.0
dem_500_gaslib135_136	1e+20	621.4k	–	14400.0	1e+20	1367.2k	–	14400.0
dem_500_gaslib135_137	1e+20	764.3k	–	14400.0	1e+20	1777.4k	–	14400.0
dem_500_gaslib135_138	1e+20	994.7k	–	14400.0	1e+20	1739.0k	–	14400.0
dem_500_gaslib135_139	1e+20	877.8k	–	14400.0	1e+20	1344.7k	–	14400.0
dem_500_gaslib135_140	1e+20	827.5k	–	14400.0	1e+20	1537.2k	–	14400.0
dem_500_gaslib135_141	1e+20	715.9k	–	14400.0	1e+20	1438.2k	–	14400.0
dem_500_gaslib135_142	1e+20	1148.3k	–	14400.0	1e+20	1746.6k	–	14400.0
dem_500_gaslib135_143	1e+20	1071.8k	–	14400.0	1e+20	1329.9k	–	14400.0
dem_500_gaslib135_144	1e+20	636.9k	–	14400.0	1e+20	1127.2k	–	14400.0
dem_500_gaslib135_145	1e+20	853.0k	–	14400.0	1e+20	1408.0k	–	14400.0
dem_500_gaslib135_146	1e+20	1071.9k	–	14400.0	1e+20	1609.4k	–	14400.0
dem_500_gaslib135_147	1e+20	967.4k	–	14400.0	1e+20	1291.2k	–	14400.0
dem_500_gaslib135_148	1e+20	1027.5k	–	14400.0	1e+20	1549.3k	–	14400.0
dem_500_gaslib135_149	1e+20	1275.8k	–	14400.0	1e+20	1542.5k	–	14400.0
dem_500_gaslib135_150	1e+20	804.2k	–	14400.0	1e+20	1097.4k	–	14400.0
dem_500_gaslib135_151	1e+20	1006.9k	–	14400.0	1e+20	1551.9k	–	14400.0
dem_500_gaslib135_152	1e+20	1116.1k	–	14400.0	1e+20	1886.7k	–	14400.0
dem_500_gaslib135_153	1e+20	579.4k	–	14400.0	1e+20	1230.9k	–	14400.0
dem_500_gaslib135_154	1e+20	388.4k	–	14400.0	1e+20	1108.3k	–	14400.0
dem_500_gaslib135_155	1e+20	974.4k	–	14400.0	1e+20	1347.0k	–	14400.0
dem_500_gaslib135_156	1e+20	1106.1k	–	14400.0	1e+20	1852.9k	–	14400.0
dem_500_gaslib135_157	1e+20	1333.9k	–	14400.0	1e+20	2307.3k	–	14400.0
dem_500_gaslib135_158	1e+20	1592.6k	–	14400.0	1e+20	2399.0k	–	14400.0
dem_500_gaslib135_159	1e+20	819.7k	–	14400.0	1e+20	1242.4k	–	14400.0
dem_500_gaslib135_160	1e+20	769.8k	–	14400.0	1e+20	1455.3k	–	14400.0
dem_500_gaslib135_161	1e+20	1220.0k	–	14400.0	1e+20	1649.7k	–	14400.0
dem_500_gaslib135_162	1e+20	556.1k	–	14400.0	1e+20	1440.2k	–	14400.0
dem_500_gaslib135_163	1e+20	667.4k	–	14400.0	1e+20	1580.6k	–	14400.0
dem_500_gaslib135_164	1e+20	966.4k	–	14400.0	1e+20	1277.3k	–	14400.0
dem_500_gaslib135_165	1e+20	1086.7k	–	14400.0	1e+20	2043.7k	–	14400.0
dem_500_gaslib135_166	1e+20	536.0k	–	14400.0	1e+20	1099.5k	–	14400.0
dem_500_gaslib135_167	1e+20	924.9k	–	14400.0	1e+20	1393.9k	–	14400.0

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Table A.24: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_168	1e+20	649.3k	–	14400.0	1e+20	1471.0k	–	14400.0
dem_500_gaslib135_169	1e+20	952.1k	–	14400.0	1e+20	1264.5k	–	14400.0
dem_500_gaslib135_170	1e+20	870.5k	–	14400.0	1e+20	1078.8k	–	14400.0
dem_500_gaslib135_171	1e+20	881.0k	–	14400.0	1e+20	1372.8k	–	14400.0
dem_500_gaslib135_172	1e+20	992.7k	–	14400.0	1e+20	1440.5k	–	14400.0
dem_500_gaslib135_173	1e+20	374.4k	–	14400.0	1e+20	959.9k	–	14400.0
dem_500_gaslib135_174	1e+20	556.9k	–	14400.0	1e+20	1166.0k	–	14400.0
dem_500_gaslib135_175	1e+20	483.4k	–	14400.0	1e+20	1157.4k	–	14400.0
dem_500_gaslib135_176	1e+20	1052.3k	–	14400.0	1e+20	1414.5k	–	14400.0
dem_500_gaslib135_177	1e+20	823.1k	–	14400.0	1e+20	1251.8k	–	14400.0
dem_500_gaslib135_178	1e+20	378.4k	–	14400.0	1e+20	1184.2k	–	14400.0
dem_500_gaslib135_179	1e+20	821.6k	–	14400.0	1e+20	1202.3k	–	14400.0
dem_500_gaslib135_180	1e+20	902.7k	–	14400.0	1e+20	1579.4k	–	14400.0
dem_500_gaslib135_181	1e+20	707.9k	–	14400.0	1e+20	1451.9k	–	14400.0
dem_500_gaslib135_182	1e+20	408.7k	–	14400.0	1e+20	991.4k	–	14400.0
dem_500_gaslib135_183	1e+20	618.1k	–	14400.0	1e+20	1233.0k	–	14400.0
dem_500_gaslib135_184	1e+20	322.3k	–	14400.0	1e+20	782.7k	–	14400.0
dem_500_gaslib135_185	1e+20	844.8k	–	14400.0	1e+20	1324.6k	–	14400.0
dem_500_gaslib135_186	1e+20	828.7k	–	14400.0	1e+20	1182.7k	–	14400.0
dem_500_gaslib135_187	1e+20	1025.8k	–	14400.0	1e+20	1505.8k	–	14400.0
dem_500_gaslib135_188	1e+20	913.3k	–	14400.0	1e+20	1473.2k	–	14400.0
dem_500_gaslib135_189	1e+20	938.5k	–	14400.0	1e+20	1284.0k	–	14400.0
dem_500_gaslib135_190	1e+20	986.9k	–	14400.0	1e+20	1355.9k	–	14400.0
dem_500_gaslib135_191	1e+20	1114.5k	–	14400.0	1e+20	1555.9k	–	14400.0
dem_500_gaslib135_192	1e+20	490.1k	–	14400.0	1e+20	1179.7k	–	14400.0
dem_500_gaslib135_193	1e+20	811.3k	–	14400.0	1e+20	1627.1k	–	14400.0
dem_500_gaslib135_194	1e+20	971.2k	–	14400.0	1e+20	1568.6k	–	14400.0
dem_500_gaslib135_195	1e+20	861.1k	–	14400.0	1e+20	1110.7k	–	14400.0
dem_500_gaslib135_196	1e+20	942.7k	–	14400.0	1e+20	1336.4k	–	14400.0
dem_500_gaslib135_197	1e+20	1191.1k	–	14400.0	1e+20	1619.8k	–	14400.0
dem_500_gaslib135_198	1e+20	626.7k	–	14400.0	1e+20	1388.4k	–	14400.0
dem_500_gaslib135_199	1e+20	826.5k	–	14400.0	1e+20	1568.3k	–	14400.0
dem_500_gaslib135_200	1e+20	773.4k	–	14400.0	1e+20	1271.5k	–	14400.0
dem_500_gaslib135_201	1e+20	699.5k	–	14400.0	1e+20	1268.7k	–	14400.0
dem_500_gaslib135_202	1e+20	1059.3k	–	14400.0	1e+20	1442.3k	–	14400.0
dem_500_gaslib135_203	1e+20	1260.7k	–	14400.0	1e+20	1705.2k	–	14400.0
dem_500_gaslib135_204	1e+20	1511.4k	–	14400.0	1e+20	1929.9k	–	14400.0
dem_500_gaslib135_205	1e+20	1082.5k	–	14400.0	1e+20	1486.0k	–	14400.0
dem_500_gaslib135_206	1e+20	567.3k	–	14400.0	1e+20	1243.3k	–	14400.0
dem_500_gaslib135_207	1e+20	365.4k	–	14400.0	1e+20	1013.9k	–	14400.0
dem_500_gaslib135_208	1e+20	712.2k	–	14400.0	1e+20	1100.9k	–	14400.0
dem_500_gaslib135_209	1e+20	587.8k	–	14400.0	1e+20	1195.6k	–	14400.0
dem_500_gaslib135_210	1e+20	1044.8k	–	14400.0	1e+20	1313.7k	–	14400.0
dem_500_gaslib135_211	1e+20	552.2k	–	14400.0	1e+20	1153.8k	–	14400.0
dem_500_gaslib135_212	1e+20	865.7k	–	14400.0	1e+20	1535.3k	–	14400.0
dem_500_gaslib135_213	1e+20	902.1k	–	14400.0	1e+20	1382.1k	–	14400.0
dem_500_gaslib135_214	1e+20	904.8k	–	14400.0	1e+20	1161.2k	–	14400.0
dem_500_gaslib135_215	1e+20	986.2k	–	14400.0	1e+20	1308.9k	–	14400.0
dem_500_gaslib135_216	1e+20	816.8k	–	14400.0	1e+20	1361.4k	–	14400.0
dem_500_gaslib135_217	1e+20	1158.6k	–	14400.0	1e+20	1457.1k	–	14400.0
dem_500_gaslib135_218	1e+20	1091.2k	–	14400.0	1e+20	1474.6k	–	14400.0
dem_500_gaslib135_219	1e+20	583.9k	–	14400.0	1e+20	1224.6k	–	14400.0
dem_500_gaslib135_220	1e+20	456.9k	–	14400.0	1e+20	1014.8k	–	14400.0
dem_500_gaslib135_221	1e+20	880.2k	–	14400.0	1e+20	1753.2k	–	14400.0
dem_500_gaslib135_222	1e+20	437.1k	–	14400.0	1e+20	1006.8k	–	14400.0
dem_500_gaslib135_223	1e+20	809.3k	–	14400.0	1e+20	1209.2k	–	14400.0
dem_500_gaslib135_224	1e+20	720.0k	–	14400.0	1e+20	1668.0k	–	14400.0
dem_500_gaslib135_225	1e+20	1118.4k	–	14400.0	1e+20	1750.4k	–	14400.0
dem_500_gaslib135_226	1e+20	479.1k	–	14400.0	1e+20	1210.8k	–	14400.0
dem_500_gaslib135_227	1e+20	864.8k	–	14400.0	1e+20	1448.2k	–	14400.0
dem_500_gaslib135_228	1e+20	660.2k	–	14400.0	1e+20	1224.7k	–	14400.0
dem_500_gaslib135_229	1e+20	1051.7k	–	14400.0	1e+20	1403.8k	–	14400.0
dem_500_gaslib135_230	1e+20	805.4k	–	14400.0	1e+20	1116.3k	–	14400.0
dem_500_gaslib135_231	1e+20	558.1k	–	14400.0	1e+20	1228.5k	–	14400.0
dem_500_gaslib135_232	1e+20	756.3k	–	14400.0	1e+20	1060.9k	–	14400.0
dem_500_gaslib135_233	1e+20	642.0k	–	14400.0	1e+20	1248.9k	–	14400.0
dem_500_gaslib135_234	1e+20	1265.5k	–	14400.0	1e+20	1623.1k	–	14400.0
dem_500_gaslib135_235	1e+20	669.6k	–	14400.0	1e+20	1088.0k	–	14400.0
dem_500_gaslib135_236	1e+20	1052.2k	–	14400.0	1e+20	1504.7k	–	14400.0
dem_500_gaslib135_237	1e+20	798.0k	–	14400.0	1e+20	1230.5k	–	14400.0
dem_500_gaslib135_238	1e+20	903.7k	–	14400.0	1e+20	1375.8k	–	14400.0
dem_500_gaslib135_239	1e+20	1040.5k	–	14400.0	1e+20	1456.7k	–	14400.0
dem_500_gaslib135_240	1e+20	1008.5k	–	14400.0	1e+20	1711.0k	–	14400.0
dem_500_gaslib135_241	1e+20	763.1k	–	14400.0	1e+20	1481.1k	–	14400.0
dem_500_gaslib135_242	1e+20	635.0k	–	14400.0	1e+20	1177.3k	–	14400.0
dem_500_gaslib135_243	1e+20	679.3k	–	14400.0	1e+20	1305.4k	–	14400.0
dem_500_gaslib135_244	1e+20	934.7k	–	14400.0	1e+20	1380.9k	–	14400.0
dem_500_gaslib135_245	1e+20	922.3k	–	14400.0	1e+20	1356.9k	–	14400.0

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Table A.24: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_246	1e+20	663.8k	–	14400.0	1e+20	1215.9k	–	14400.0
dem_500_gaslib135_247	1e+20	1004.9k	–	14400.0	1e+20	1454.6k	–	14400.0
dem_500_gaslib135_248	1e+20	1025.7k	–	14400.0	1e+20	1533.8k	–	14400.0
dem_500_gaslib135_249	1e+20	686.4k	–	14400.0	1e+20	1263.8k	–	14400.0
dem_500_gaslib135_250	1e+20	785.7k	–	14400.0	1e+20	1081.6k	–	14400.0
dem_500_gaslib135_251	1e+20	819.3k	–	14400.0	1e+20	1650.4k	–	14400.0
dem_500_gaslib135_252	1e+20	1158.4k	–	14400.0	1e+20	2202.7k	–	14400.0
dem_500_gaslib135_253	1e+20	778.7k	–	14400.0	1e+20	1303.9k	–	14400.0
dem_500_gaslib135_254	1e+20	687.2k	–	14400.0	1e+20	1344.7k	–	14400.0
dem_500_gaslib135_255	1e+20	906.2k	–	14400.0	1e+20	1446.4k	–	14400.0
dem_500_gaslib135_256	1e+20	788.0k	–	14400.0	1e+20	911.3k	–	14400.0
dem_500_gaslib135_257	1e+20	626.6k	–	14400.0	1e+20	1290.8k	–	14400.0
dem_500_gaslib135_258	1e+20	1019.2k	–	14400.0	1e+20	1431.0k	–	14400.0
dem_500_gaslib135_259	1e+20	909.3k	–	14400.0	1e+20	1459.2k	–	14400.0
dem_500_gaslib135_260	1e+20	682.4k	–	14400.0	1e+20	1269.9k	–	14400.0
dem_500_gaslib135_261	1e+20	1035.1k	–	14400.0	1e+20	1433.4k	–	14400.0
dem_500_gaslib135_262	1e+20	735.6k	–	14400.0	1e+20	1233.4k	–	14400.0
dem_500_gaslib135_263	1e+20	893.6k	–	14400.0	1e+20	1458.5k	–	14400.0
dem_500_gaslib135_264	1e+20	927.4k	–	14400.0	1e+20	1283.6k	–	14400.0
dem_500_gaslib135_265	1e+20	791.0k	–	14400.0	1e+20	1112.9k	–	14400.0
dem_500_gaslib135_266	1e+20	1027.0k	–	14400.0	1e+20	1305.7k	–	14400.0
dem_500_gaslib135_267	1e+20	680.8k	–	14400.0	1e+20	1207.9k	–	14400.0
dem_500_gaslib135_268	1e+20	1187.4k	–	14400.0	1e+20	1538.2k	–	14400.0
dem_500_gaslib135_269	1e+20	751.1k	–	14400.0	1e+20	1124.9k	–	14400.0
dem_500_gaslib135_270	1e+20	762.3k	–	14400.0	1e+20	1417.7k	–	14400.0
dem_500_gaslib135_271	1e+20	899.4k	–	14400.0	1e+20	1670.6k	–	14400.0
dem_500_gaslib135_272	1e+20	1280.4k	–	14400.0	1e+20	1799.7k	–	14400.0
dem_500_gaslib135_273	1e+20	970.6k	–	14400.0	1e+20	1293.7k	–	14400.0
dem_500_gaslib135_274	1e+20	851.0k	–	14400.0	1e+20	1323.5k	–	14400.0
dem_500_gaslib135_275	1e+20	1177.0k	–	14400.0	1e+20	1554.2k	–	14400.0
dem_500_gaslib135_276	1e+20	1006.4k	–	14400.0	1e+20	1362.0k	–	14400.0
dem_500_gaslib135_277	1e+20	1013.0k	–	14400.0	1e+20	1512.6k	–	14400.0
dem_500_gaslib135_278	1e+20	1120.9k	–	14400.0	1e+20	1696.5k	–	14400.0
dem_500_gaslib135_279	1e+20	653.3k	–	14400.0	1e+20	1316.7k	–	14400.0
dem_500_gaslib135_280	1e+20	1007.7k	–	14400.0	1e+20	1643.7k	–	14400.0
dem_500_gaslib135_281	1e+20	722.3k	–	14400.0	1e+20	1344.6k	–	14400.0
dem_500_gaslib135_282	1e+20	685.6k	–	14400.0	1e+20	1144.4k	–	14400.0
dem_500_gaslib135_283	1e+20	1271.3k	–	14400.0	1e+20	1644.6k	–	14400.0
dem_500_gaslib135_284	1e+20	981.0k	–	14400.0	1e+20	1308.5k	–	14400.0
dem_500_gaslib135_285	1e+20	810.3k	–	14400.0	1e+20	1216.8k	–	14400.0
dem_500_gaslib135_286	1e+20	770.3k	–	14400.0	1e+20	1494.5k	–	14400.0
dem_500_gaslib135_287	1e+20	331.4k	–	14400.0	1e+20	977.6k	–	14400.0
dem_500_gaslib135_288	1e+20	777.1k	–	14400.0	1e+20	1064.2k	–	14400.0
dem_500_gaslib135_289	1e+20	327.3k	–	14400.0	1e+20	970.9k	–	14400.0
dem_500_gaslib135_290	1e+20	1234.1k	–	14400.0	1e+20	1494.1k	–	14400.0
dem_500_gaslib135_291	1e+20	935.5k	–	14400.0	1e+20	1481.7k	–	14400.0
dem_500_gaslib135_292	1e+20	623.5k	–	14400.0	1e+20	1268.2k	–	14400.0
dem_500_gaslib135_293	1e+20	486.9k	–	14400.0	1e+20	1113.7k	–	14400.0
dem_500_gaslib135_294	1e+20	948.2k	–	14400.0	1e+20	1327.8k	–	14400.0
dem_500_gaslib135_295	1e+20	903.9k	–	14400.0	1e+20	1178.4k	–	14400.0
dem_500_gaslib135_296	1e+20	835.7k	–	14400.0	1e+20	1457.0k	–	14400.0
dem_500_gaslib135_297	1e+20	959.5k	–	14400.0	1e+20	1434.3k	–	14400.0
dem_500_gaslib135_298	1e+20	974.0k	–	14400.0	1e+20	1339.8k	–	14400.0
dem_500_gaslib135_299	1e+20	676.7k	–	14400.0	1e+20	1109.4k	–	14400.0
dem_500_gaslib135_300	1e+20	604.2k	–	14400.0	1e+20	1109.9k	–	14400.0
dem_500_gaslib135_301	1e+20	431.1k	–	14400.0	1e+20	1132.2k	–	14400.0
dem_500_gaslib135_302	1e+20	1087.1k	–	14400.0	1e+20	1341.1k	–	14400.0
dem_500_gaslib135_303	1e+20	575.8k	–	14400.0	1e+20	1254.0k	–	14400.0
dem_500_gaslib135_304	1e+20	816.0k	–	14400.0	1e+20	1638.5k	–	14400.0
dem_500_gaslib135_305	1e+20	722.1k	–	14400.0	1e+20	1611.1k	–	14400.0
dem_500_gaslib135_306	1e+20	942.0k	–	14400.0	1e+20	1399.6k	–	14400.0
dem_500_gaslib135_307	1e+20	609.6k	–	14400.0	1e+20	1122.7k	–	14400.0
dem_500_gaslib135_308	1e+20	1043.1k	–	14400.0	1e+20	1454.9k	–	14400.0
dem_500_gaslib135_309	1e+20	823.5k	–	14400.0	1e+20	1659.0k	–	14400.0
dem_500_gaslib135_310	1e+20	913.7k	–	14400.0	1e+20	1511.8k	–	14400.0
dem_500_gaslib135_311	1e+20	911.9k	–	14400.0	1e+20	1936.6k	–	14400.0
dem_500_gaslib135_312	1e+20	1193.9k	–	14400.0	1e+20	1742.2k	–	14400.0
dem_500_gaslib135_313	1e+20	885.3k	–	14400.0	1e+20	1296.8k	–	14400.0
dem_500_gaslib135_314	1e+20	1000.4k	–	14400.0	1e+20	1662.3k	–	14400.0
dem_500_gaslib135_315	1e+20	714.5k	–	14400.0	1e+20	1194.2k	–	14400.0
dem_500_gaslib135_316	1e+20	710.8k	–	14400.0	1e+20	1439.8k	–	14400.0
dem_500_gaslib135_317	1e+20	782.7k	–	14400.0	1e+20	1457.6k	–	14400.0
dem_500_gaslib135_318	1e+20	943.0k	–	14400.0	1e+20	1362.6k	–	14400.0
dem_500_gaslib135_319	1e+20	970.8k	–	14400.0	1e+20	1348.8k	–	14400.0
dem_500_gaslib135_320	1e+20	753.0k	–	14400.0	1e+20	1150.1k	–	14400.0
dem_500_gaslib135_321	1e+20	928.8k	–	14400.0	1e+20	1577.0k	–	14400.0
dem_500_gaslib135_322	1e+20	624.2k	–	14400.0	1e+20	1456.7k	–	14400.0
dem_500_gaslib135_323	1e+20	1197.9k	–	14400.0	1e+20	1599.3k	–	14400.0

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Table A.24: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_324	1e+20	1059.8k	–	14400.0	1e+20	1558.6k	–	14400.0
dem_500_gaslib135_325	1e+20	1181.5k	–	14400.0	1e+20	1777.0k	–	14400.0
dem_500_gaslib135_326	1e+20	869.1k	–	14400.0	1e+20	1162.3k	–	14400.0
dem_500_gaslib135_327	1e+20	528.8k	–	14400.0	1e+20	1143.5k	–	14400.0
dem_500_gaslib135_328	1e+20	588.3k	–	14400.0	1e+20	1270.2k	–	14400.0
dem_500_gaslib135_329	1e+20	713.1k	–	14400.0	1e+20	1572.8k	–	14400.0
dem_500_gaslib135_330	1e+20	1149.6k	–	14400.0	1e+20	1402.6k	–	14400.0
dem_500_gaslib135_331	1e+20	865.9k	–	14400.0	1e+20	1276.8k	–	14400.0
dem_500_gaslib135_332	1e+20	842.9k	–	14400.0	1e+20	1490.5k	–	14400.0
dem_500_gaslib135_333	1e+20	930.5k	–	14400.0	1e+20	1343.5k	–	14400.0
dem_500_gaslib135_334	1e+20	496.8k	–	14400.0	1e+20	1186.7k	–	14400.0
dem_500_gaslib135_335	1e+20	523.6k	–	14400.0	1e+20	1291.7k	–	14400.0
dem_500_gaslib135_336	1e+20	688.8k	–	14400.0	1e+20	1429.5k	–	14400.0
dem_500_gaslib135_337	1e+20	547.0k	–	14400.0	1e+20	960.0k	–	14400.0
dem_500_gaslib135_338	1e+20	649.6k	–	14400.0	1e+20	1378.7k	–	14400.0
dem_500_gaslib135_339	1e+20	1145.5k	–	14400.0	1e+20	1516.7k	–	14400.0
dem_500_gaslib135_340	1e+20	906.5k	–	14400.0	1e+20	1700.1k	–	14400.0
dem_500_gaslib135_341	1e+20	820.8k	–	14400.0	1e+20	1842.1k	–	14400.0
dem_500_gaslib135_342	1e+20	760.0k	–	14400.0	1e+20	1423.8k	–	14400.0
dem_500_gaslib135_343	1e+20	822.2k	–	14400.0	1e+20	968.3k	–	14400.0
dem_500_gaslib135_344	1e+20	376.9k	–	14400.0	1e+20	1036.7k	–	14400.0
dem_500_gaslib135_345	1e+20	1076.3k	–	14400.0	1e+20	1561.1k	–	14400.0
dem_500_gaslib135_346	1e+20	619.0k	–	14400.0	1e+20	1304.7k	–	14400.0
dem_500_gaslib135_347	1e+20	1049.6k	–	14400.0	1e+20	1730.0k	–	14400.0
dem_500_gaslib135_348	1e+20	686.6k	–	14400.0	1e+20	1382.7k	–	14400.0
dem_500_gaslib135_349	1e+20	495.6k	–	14400.0	1e+20	1194.3k	–	14400.0
dem_500_gaslib135_350	1e+20	1056.2k	–	14400.0	1e+20	1642.0k	–	14400.0
dem_500_gaslib135_351	1e+20	608.1k	–	14400.0	1e+20	1349.1k	–	14400.0
dem_500_gaslib135_352	1e+20	752.1k	–	14400.0	1e+20	1308.3k	–	14400.0
dem_500_gaslib135_353	1e+20	553.0k	–	14400.0	1e+20	1029.7k	–	14400.0
dem_500_gaslib135_354	1e+20	926.0k	–	14400.0	1e+20	1295.1k	–	14400.0
dem_500_gaslib135_355	1e+20	317.9k	–	14400.0	1e+20	1088.2k	–	14400.0
dem_500_gaslib135_356	1e+20	1196.0k	–	14400.0	1e+20	1499.8k	–	14400.0
dem_500_gaslib135_357	1e+20	768.6k	–	14400.0	1e+20	1391.8k	–	14400.0
dem_500_gaslib135_358	1e+20	974.5k	–	14400.0	1e+20	1304.8k	–	14400.0
dem_500_gaslib135_359	1e+20	994.7k	–	14400.0	1e+20	1258.5k	–	14400.0
dem_500_gaslib135_360	1e+20	860.7k	–	14400.0	1e+20	1160.8k	–	14400.0
dem_500_gaslib135_361	1e+20	681.7k	–	14400.0	1e+20	1194.1k	–	14400.0
dem_500_gaslib135_362	1e+20	912.7k	–	14400.0	1e+20	1260.0k	–	14400.0
dem_500_gaslib135_363	1e+20	760.3k	–	14400.0	1e+20	1456.7k	–	14400.0
dem_500_gaslib135_364	1e+20	865.1k	–	14400.0	1e+20	1592.7k	–	14400.0
dem_500_gaslib135_365	1e+20	974.7k	–	14400.0	1e+20	1729.2k	–	14400.0
dem_500_gaslib135_366	1e+20	771.2k	–	14400.0	1e+20	1136.2k	–	14400.0
dem_500_gaslib135_367	1e+20	737.5k	–	14400.0	1e+20	1534.4k	–	14400.0
dem_500_gaslib135_368	1e+20	918.1k	–	14400.0	1e+20	1332.2k	–	14400.0
dem_500_gaslib135_369	1e+20	1039.1k	–	14400.0	1e+20	1310.1k	–	14400.0
dem_500_gaslib135_370	1e+20	710.5k	–	14400.0	1e+20	1277.8k	–	14400.0
dem_500_gaslib135_371	1e+20	581.6k	–	14400.0	1e+20	1171.5k	–	14400.0
dem_500_gaslib135_372	1e+20	805.1k	–	14400.0	1e+20	1609.3k	–	14400.0
dem_500_gaslib135_373	1e+20	1106.4k	–	14400.0	1e+20	1674.1k	–	14400.0
dem_500_gaslib135_374	1e+20	675.1k	–	14400.0	1e+20	1307.1k	–	14400.0
dem_500_gaslib135_375	1e+20	1360.5k	–	14400.0	1e+20	1770.7k	–	14400.0
dem_500_gaslib135_376	1e+20	867.8k	–	14400.0	1e+20	1616.7k	–	14400.0
dem_500_gaslib135_377	1e+20	1156.6k	–	14400.0	1e+20	1568.6k	–	14400.0
dem_500_gaslib135_378	1e+20	926.4k	–	14400.0	1e+20	1346.1k	–	14400.0
dem_500_gaslib135_379	1e+20	999.6k	–	14400.0	1e+20	1140.7k	–	14400.0
dem_500_gaslib135_380	1e+20	1212.7k	–	14400.0	1e+20	1814.4k	–	14400.0
dem_500_gaslib135_381	1e+20	663.7k	–	14400.0	1e+20	1104.6k	–	14400.0
dem_500_gaslib135_382	1e+20	697.7k	–	14400.0	1e+20	1268.5k	–	14400.0
dem_500_gaslib135_383	1e+20	1192.3k	–	14400.0	1e+20	1470.4k	–	14400.0
dem_500_gaslib135_384	1e+20	813.1k	–	14400.0	1e+20	1558.8k	–	14400.0
dem_500_gaslib135_385	1e+20	725.1k	–	14400.0	1e+20	1326.6k	–	14400.0
dem_500_gaslib135_386	1e+20	996.8k	–	14400.0	1e+20	1405.2k	–	14400.0
dem_500_gaslib135_387	1e+20	847.8k	–	14400.0	1e+20	1608.1k	–	14400.0
dem_500_gaslib135_388	1e+20	726.8k	–	14400.0	1e+20	1093.6k	–	14400.0
dem_500_gaslib135_389	1e+20	812.7k	–	14400.0	1e+20	1660.5k	–	14400.0
dem_500_gaslib135_390	1e+20	755.7k	–	14400.0	1e+20	1802.7k	–	14400.0
dem_500_gaslib135_391	1e+20	700.9k	–	14400.0	1e+20	1247.4k	–	14400.0
dem_500_gaslib135_392	1e+20	547.7k	–	14400.0	1e+20	1354.2k	–	14400.0
dem_500_gaslib135_393	1e+20	702.5k	–	14400.0	1e+20	1964.2k	–	14400.0
dem_500_gaslib135_394	1e+20	494.1k	–	14400.0	1e+20	1257.7k	–	14400.0
dem_500_gaslib135_395	1e+20	1105.4k	–	14400.0	1e+20	1565.5k	–	14400.0
dem_500_gaslib135_396	1e+20	928.5k	–	14400.0	1e+20	1454.8k	–	14400.0
dem_500_gaslib135_397	1e+20	861.5k	–	14400.0	1e+20	1388.2k	–	14400.0
dem_500_gaslib135_398	1e+20	787.2k	–	14400.0	1e+20	1411.9k	–	14400.0
dem_500_gaslib135_399	1e+20	613.6k	–	14400.0	1e+20	1085.1k	–	14400.0
dem_500_gaslib135_400	1e+20	981.6k	–	14400.0	1e+20	1344.9k	–	14400.0
dem_500_gaslib135_401	1e+20	1175.0k	–	14400.0	1e+20	1392.0k	–	14400.0

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Table A.24: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_402	1e+20	321.3k	–	14400.0	1e+20	959.1k	–	14400.0
dem_500_gaslib135_403	1e+20	748.6k	–	14400.0	1e+20	1693.0k	–	14400.0
dem_500_gaslib135_404	1e+20	1069.2k	–	14400.0	1e+20	1464.3k	–	14400.0
dem_500_gaslib135_405	1e+20	678.2k	–	14400.0	1e+20	972.5k	–	14400.0
dem_500_gaslib135_406	1e+20	702.0k	–	14400.0	1e+20	1431.1k	–	14400.0
dem_500_gaslib135_407	1e+20	701.9k	–	14400.0	1e+20	1435.1k	–	14400.0
dem_500_gaslib135_408	1e+20	949.8k	–	14400.0	1e+20	1456.3k	–	14400.0
dem_500_gaslib135_409	1e+20	956.1k	–	14400.0	1e+20	1815.9k	–	14400.0
dem_500_gaslib135_410	1e+20	1130.3k	–	14400.0	1e+20	1502.2k	–	14400.0
dem_500_gaslib135_411	1e+20	1051.2k	–	14400.0	1e+20	1386.0k	–	14400.0
dem_500_gaslib135_412	1e+20	956.2k	–	14400.0	1e+20	1884.0k	–	14400.0
dem_500_gaslib135_413	1e+20	891.6k	–	14400.0	1e+20	1313.1k	–	14400.0
dem_500_gaslib135_414	1e+20	1001.7k	–	14400.0	1e+20	1465.9k	–	14400.0
dem_500_gaslib135_415	1e+20	1041.7k	–	14400.0	1e+20	1538.6k	–	14400.0
dem_500_gaslib135_416	1e+20	688.9k	–	14400.0	1e+20	1356.5k	–	14400.0
dem_500_gaslib135_417	1e+20	481.6k	–	14400.0	1e+20	1180.5k	–	14400.0
dem_500_gaslib135_418	1e+20	1059.2k	–	14400.0	1e+20	1533.3k	–	14400.0
dem_500_gaslib135_419	1e+20	749.1k	–	14400.0	1e+20	1329.3k	–	14400.0
dem_500_gaslib135_420	1e+20	680.5k	–	14400.0	1e+20	1e+20	0.0	10119.9
dem_500_gaslib135_421	1e+20	1033.3k	–	14400.0	1e+20	1294.1k	–	14400.0
dem_500_gaslib135_422	1e+20	499.2k	–	14400.0	1e+20	1324.8k	–	14400.0
dem_500_gaslib135_423	1e+20	449.8k	–	14400.0	1e+20	942.3k	–	14400.0
dem_500_gaslib135_424	1e+20	1085.7k	–	14400.0	1e+20	1589.8k	–	14400.0
dem_500_gaslib135_425	1e+20	885.0k	–	14400.0	1e+20	1385.8k	–	14400.0
dem_500_gaslib135_426	1e+20	1042.9k	–	14400.0	1e+20	1963.3k	–	14400.0
dem_500_gaslib135_427	1e+20	921.3k	–	14400.0	1e+20	1285.1k	–	14400.0
dem_500_gaslib135_428	1e+20	718.2k	–	14400.0	1e+20	1321.0k	–	14400.0
dem_500_gaslib135_429	1e+20	1038.6k	–	14400.0	1e+20	1356.1k	–	14400.0
dem_500_gaslib135_430	1e+20	888.9k	–	14400.0	1e+20	1347.0k	–	14400.0
dem_500_gaslib135_431	1e+20	991.6k	–	14400.0	1e+20	1435.7k	–	14400.0
dem_500_gaslib135_432	1e+20	1030.4k	–	14400.0	1e+20	1849.1k	–	14400.0
dem_500_gaslib135_433	1e+20	617.7k	–	14400.0	1e+20	1320.5k	–	14400.0
dem_500_gaslib135_434	1e+20	851.3k	–	14400.0	1e+20	1330.4k	–	14400.0
dem_500_gaslib135_435	1e+20	1019.8k	–	14400.0	1e+20	1743.8k	–	14400.0
dem_500_gaslib135_436	1e+20	1041.2k	–	14400.0	1e+20	1790.0k	–	14400.0
dem_500_gaslib135_437	1e+20	657.5k	–	14400.0	1e+20	1196.2k	–	14400.0
dem_500_gaslib135_438	1e+20	939.8k	–	14400.0	1e+20	1686.0k	–	14400.0
dem_500_gaslib135_439	1e+20	991.1k	–	14400.0	1e+20	1755.3k	–	14400.0
dem_500_gaslib135_440	1e+20	677.4k	–	14400.0	1e+20	1140.1k	–	14400.0
dem_500_gaslib135_441	1e+20	1185.7k	–	14400.0	1e+20	1725.3k	–	14400.0
dem_500_gaslib135_442	1e+20	1068.2k	–	14400.0	1e+20	1425.2k	–	14400.0
dem_500_gaslib135_443	1e+20	817.8k	–	14400.0	1e+20	1434.1k	–	14400.0
dem_500_gaslib135_444	1e+20	664.5k	–	14400.0	1e+20	1194.0k	–	14400.0
dem_500_gaslib135_445	1e+20	912.5k	–	14400.0	1e+20	1418.2k	–	14400.0
dem_500_gaslib135_446	1e+20	970.8k	–	14400.0	1e+20	1615.6k	–	14400.0
dem_500_gaslib135_447	1e+20	1081.8k	–	14400.0	1e+20	1501.8k	–	14400.0
dem_500_gaslib135_448	1e+20	549.6k	–	14400.0	1e+20	1274.7k	–	14400.0
dem_500_gaslib135_449	1e+20	958.0k	–	14400.0	1e+20	1661.4k	–	14400.0
dem_500_gaslib135_450	1e+20	1028.8k	–	14400.0	1e+20	1378.9k	–	14400.0
dem_500_gaslib135_451	1e+20	964.0k	–	14400.0	1e+20	1786.2k	–	14400.0
dem_500_gaslib135_452	1e+20	477.5k	–	14400.0	1e+20	1166.4k	–	14400.0
dem_500_gaslib135_453	1e+20	430.2k	–	14400.0	1e+20	954.2k	–	14400.0
dem_500_gaslib135_454	1e+20	854.6k	–	14400.0	1e+20	1679.8k	–	14400.0
dem_500_gaslib135_455	1e+20	963.2k	–	14400.0	1e+20	1284.2k	–	14400.0
dem_500_gaslib135_456	1e+20	851.6k	–	14400.0	1e+20	1601.5k	–	14400.0
dem_500_gaslib135_457	1e+20	462.6k	–	14400.0	1e+20	1207.7k	–	14400.0
dem_500_gaslib135_458	1e+20	1086.0k	–	14400.0	1e+20	1586.4k	–	14400.0
dem_500_gaslib135_459	1e+20	531.8k	–	14400.0	1e+20	1274.9k	–	14400.0
dem_500_gaslib135_460	1e+20	809.7k	–	14400.0	1e+20	1316.5k	–	14400.0
dem_500_gaslib135_461	1e+20	739.6k	–	14400.0	1e+20	1509.9k	–	14400.0
dem_500_gaslib135_462	1e+20	589.6k	–	14400.0	1e+20	1197.0k	–	14400.0
dem_500_gaslib135_463	1e+20	1035.6k	–	14400.0	1e+20	1404.4k	–	14400.0
dem_500_gaslib135_464	1e+20	417.8k	–	14400.0	1e+20	1177.6k	–	14400.0
dem_500_gaslib135_465	1e+20	688.0k	–	14400.0	1e+20	1251.9k	–	14400.0
dem_500_gaslib135_466	1e+20	638.7k	–	14400.0	1e+20	1299.9k	–	14400.0
dem_500_gaslib135_467	1e+20	779.9k	–	14400.0	1e+20	1189.1k	–	14400.0
dem_500_gaslib135_468	1e+20	854.9k	–	14400.0	1e+20	1186.4k	–	14400.0
dem_500_gaslib135_469	1e+20	735.8k	–	14400.0	1e+20	1392.5k	–	14400.0
dem_500_gaslib135_470	1e+20	985.4k	–	14400.0	1e+20	1332.6k	–	14400.0
dem_500_gaslib135_471	1e+20	851.2k	–	14400.0	1e+20	1336.6k	–	14400.0
dem_500_gaslib135_472	1e+20	760.1k	–	14400.0	1e+20	1561.0k	–	14400.0
dem_500_gaslib135_473	1e+20	708.5k	–	14400.0	1e+20	1268.5k	–	14400.0
dem_500_gaslib135_474	1e+20	748.3k	–	14400.0	1e+20	1208.4k	–	14400.0
dem_500_gaslib135_475	1e+20	1006.8k	–	14400.0	1e+20	1457.9k	–	14400.0
dem_500_gaslib135_476	1e+20	944.0k	–	14400.0	1e+20	1337.9k	–	14400.0
dem_500_gaslib135_477	1e+20	1006.7k	–	14400.0	1e+20	1740.9k	–	14400.0
dem_500_gaslib135_478	1e+20	903.7k	–	14400.0	1e+20	1277.5k	–	14400.0
dem_500_gaslib135_479	1e+20	982.5k	–	14400.0	1e+20	1407.5k	–	14400.0

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Table A.24: Comparison of split-pipe models on *GasLib-135* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_gaslib135_480	1e+20	778.4k	–	14400.0	1e+20	1073.3k	–	14400.0
dem_500_gaslib135_481	1e+20	881.3k	–	14400.0	1e+20	1246.8k	–	14400.0
dem_500_gaslib135_482	1e+20	1043.6k	–	14400.0	1e+20	1402.5k	–	14400.0
dem_500_gaslib135_483	1e+20	921.3k	–	14400.0	1e+20	1455.5k	–	14400.0
dem_500_gaslib135_484	1e+20	570.3k	–	14400.0	1e+20	1303.2k	–	14400.0
dem_500_gaslib135_485	1e+20	664.4k	–	14400.0	1e+20	1121.4k	–	14400.0
dem_500_gaslib135_486	1e+20	876.4k	–	14400.0	1e+20	1371.9k	–	14400.0
dem_500_gaslib135_487	1e+20	988.9k	–	14400.0	1e+20	1231.8k	–	14400.0
dem_500_gaslib135_488	1e+20	916.7k	–	14400.0	1e+20	1243.5k	–	14400.0
dem_500_gaslib135_489	1e+20	1090.5k	–	14400.0	1e+20	1539.7k	–	14400.0
dem_500_gaslib135_490	1e+20	360.9k	–	14400.0	1e+20	1001.0k	–	14400.0
dem_500_gaslib135_491	1e+20	716.4k	–	14400.0	1e+20	1261.2k	–	14400.0
dem_500_gaslib135_492	1e+20	964.7k	–	14400.0	1e+20	1518.7k	–	14400.0
dem_500_gaslib135_493	1e+20	939.3k	–	14400.0	1e+20	1384.5k	–	14400.0
dem_500_gaslib135_494	1e+20	762.2k	–	14400.0	1e+20	1086.0k	–	14400.0
dem_500_gaslib135_495	1e+20	668.6k	–	14400.0	1e+20	989.3k	–	14400.0
dem_500_gaslib135_496	1e+20	1135.5k	–	14400.0	1e+20	1453.9k	–	14400.0
dem_500_gaslib135_497	1e+20	1102.7k	–	14400.0	1e+20	1614.1k	–	14400.0
dem_500_gaslib135_498	1e+20	361.4k	–	14400.0	1e+20	805.0k	–	14400.0
dem_500_gaslib135_499	1e+20	805.7k	–	14400.0	1e+20	1665.8k	–	14400.0
dem_500_gaslib135_500	1e+20	915.4k	–	14400.0	1e+20	1691.0k	–	14400.0

Table A.25: Detailed results of the discrete models on *Circuit rank* + 2 arcs, as summarized in Figure 3.10b and Table 3.3d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_1	569.7k	569.7k	0.0	262.7	569.7k	569.7k	0.0	568.5	569.7k	569.7k	0.0	4374.9
belg+2arcs_2	478.3k	443.8k	7.8	14400.0	478.3k	466.5k	2.5	14400.0	519.4k	363.0k	43.1	14400.0
belg+2arcs_3	435.8k	359.0k	21.4	14400.0	458.7k	406.3k	12.9	14400.0	537.3k	285.2k	88.4	14400.0
belg+2arcs_4	664.4k	566.9k	17.2	14400.0	635.1k	567.5k	11.9	14400.0	923.9k	433.4k	113.2	14400.0
belg+2arcs_5	658.9k	658.9k	0.0	7203.7	658.9k	658.9k	0.0	7808.9	666.3k	584.2k	14.1	14400.0
belg+2arcs_6	324.0k	324.0k	0.0	794.9	324.0k	324.0k	0.0	527.0	324.0k	324.0k	0.0	1326.8
belg+2arcs_7	776.3k	581.9k	33.4	14400.0	723.6k	660.3k	9.6	14400.0	1027.3k	521.2k	97.1	14400.0
belg+2arcs_8	134.3k	134.3k	0.0	137.8	134.3k	134.3k	0.0	210.4	134.3k	134.3k	0.0	733.5
belg+2arcs_9	498.7k	498.7k	0.0	5587.9	498.7k	498.7k	0.0	4126.7	516.3k	393.9k	31.1	14400.0
belg+2arcs_10	527.5k	455.3k	15.9	14400.0	517.7k	517.7k	0.0	6059.2	909.6k	331.3k	174.6	14400.0
belg+2arcs_11	460.0k	460.0k	0.0	639.6	460.0k	460.0k	0.0	1078.6	460.0k	460.0k	0.0	557.5
belg+2arcs_12	281.5k	281.5k	0.0	2803.5	281.5k	281.5k	0.0	4228.8	313.9k	233.8k	34.2	14400.0
belg+2arcs_13	477.2k	477.2k	0.0	485.4	477.2k	477.2k	0.0	3401.1	477.2k	477.2k	0.0	6383.7
belg+2arcs_14	691.4k	691.0k	0.1	76.7	691.4k	691.4k	0.0	549.9	691.4k	691.4k	0.0	493.1
belg+2arcs_15	197.5k	197.5k	0.0	16.7	197.5k	197.3k	0.1	230.7	197.5k	197.5k	0.0	91.0
belg+2arcs_16	661.1k	661.1k	0.0	6391.5	661.1k	661.0k	0.0	8118.5	687.7k	504.0k	36.4	14400.0
belg+2arcs_17	388.1k	388.1k	0.0	341.4	388.1k	388.1k	0.0	562.4	388.1k	388.1k	0.0	1540.2
belg+2arcs_18	590.8k	542.8k	8.8	14400.0	588.4k	561.3k	4.8	14400.0	589.4k	461.1k	27.8	14400.0
belg+2arcs_19	299.9k	299.9k	0.0	59.9	299.9k	299.9k	0.0	183.4	299.9k	299.9k	0.0	138.6
belg+2arcs_20	552.8k	552.8k	0.0	4835.8	552.8k	552.8k	0.0	14150.6	652.9k	424.2k	53.9	14400.0
belg+2arcs_21	193.0k	193.0k	0.0	205.2	193.0k	193.0k	0.0	374.4	193.0k	193.0k	0.0	218.0
belg+2arcs_22	477.1k	351.6k	35.7	14400.0	449.9k	426.8k	5.4	14400.0	483.4k	279.2k	73.1	14400.0
belg+2arcs_23	348.8k	348.5k	0.1	10387.6	348.8k	348.8k	0.0	3502.3	499.4k	275.3k	81.4	14400.0
belg+2arcs_24	437.4k	437.4k	0.0	853.9	437.4k	437.4k	0.0	1172.2	437.4k	437.4k	0.0	8711.0
belg+2arcs_25	387.5k	387.5k	0.0	1649.1	387.5k	387.5k	0.0	8563.5	387.5k	374.5k	3.5	14400.0
belg+2arcs_26	516.0k	516.0k	0.0	5490.0	516.0k	516.0k	0.0	5661.5	534.3k	426.0k	25.4	14400.0
belg+2arcs_27	272.1k	272.1k	0.0	87.7	272.1k	272.1k	0.0	148.9	272.1k	272.1k	0.0	135.8
belg+2arcs_28	528.7k	528.3k	0.1	1998.2	528.7k	528.7k	0.0	4001.0	652.0k	456.3k	42.9	14400.0
belg+2arcs_29	382.3k	382.3k	0.0	4189.9	382.3k	382.3k	0.0	3631.7	392.3k	343.7k	14.1	14400.0
belg+2arcs_30	354.3k	354.3k	0.0	3121.8	354.3k	354.3k	0.0	1856.3	444.8k	252.5k	76.2	14400.0
belg+2arcs_31	339.9k	339.9k	0.0	156.3	339.9k	339.9k	0.0	613.5	339.9k	339.9k	0.0	469.7
belg+2arcs_32	628.4k	467.5k	34.4	14400.0	523.2k	523.2k	0.0	1414.2	523.2k	523.1k	0.0	4598.5
belg+2arcs_33	278.5k	278.5k	0.0	2012.2	278.5k	278.4k	0.0	6970.3	281.5k	224.7k	25.3	14400.0
belg+2arcs_34	199.8k	199.6k	0.1	198.3	199.8k	199.8k	0.0	242.5	199.8k	199.8k	0.0	143.8
belg+2arcs_35	393.4k	393.4k	0.0	46.5	393.4k	393.4k	0.0	402.6	393.4k	393.4k	0.0	204.0
belg+2arcs_36	520.9k	520.9k	0.0	3056.6	520.9k	520.9k	0.0	3535.2	530.5k	446.5k	18.8	14400.0
belg+2arcs_37	320.1k	261.3k	22.5	14400.0	303.0k	280.2k	8.1	14400.0	305.7k	234.2k	30.6	14400.0
belg+2arcs_38	701.6k	653.6k	7.3	14400.0	701.6k	669.5k	4.8	14400.0	739.9k	474.4k	56.0	14400.0
belg+2arcs_39	265.3k	265.3k	0.0	563.0	265.3k	265.3k	0.0	1406.7	265.3k	265.3k	0.0	8765.0
belg+2arcs_40	392.5k	317.5k	23.6	14400.0	385.8k	359.4k	7.3	14400.0	644.3k	252.9k	154.8	14400.0
belg+2arcs_41	585.2k	584.9k	0.1	2078.9	585.2k	585.2k	0.0	3481.3	587.1k	542.3k	8.3	14400.0
belg+2arcs_42	481.3k	481.0k	0.1	8940.6	481.3k	481.3k	0.0	7676.2	604.0k	425.8k	41.9	14400.0

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Table A.25: Comparison of discrete models on *Circuit rank* + 2 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_43	376.4k	376.4k	0.0	405.4	376.4k	376.4k	0.0	725.3	376.4k	376.4k	0.0	11335.9
belg+2arcs_44	370.2k	370.2k	0.0	3191.4	370.2k	370.2k	0.0	2376.4	381.7k	273.9k	39.3	14400.0
belg+2arcs_45	157.2k	157.2k	0.0	204.9	157.2k	157.2k	0.0	1159.7	157.2k	157.2k	0.0	960.4
belg+2arcs_46	505.4k	504.9k	0.1	12056.6	505.4k	505.4k	0.0	4178.6	557.7k	422.6k	32.0	14400.0
belg+2arcs_47	411.4k	411.3k	0.0	1241.7	411.4k	411.4k	0.0	1582.5	413.3k	375.6k	10.0	14400.0
belg+2arcs_48	394.2k	394.2k	0.0	1022.6	394.2k	394.2k	0.0	1056.0	394.2k	394.2k	0.0	14076.8
belg+2arcs_49	274.8k	274.8k	0.0	1076.2	274.8k	274.8k	0.0	1398.0	274.8k	274.8k	0.0	8808.9
belg+2arcs_50	540.2k	540.2k	0.0	2152.8	540.2k	540.2k	0.0	6258.6	544.9k	471.2k	15.6	14400.0
belg+2arcs_51	244.6k	244.6k	0.0	1611.0	244.6k	244.6k	0.0	2424.5	244.6k	244.6k	0.0	2933.7
belg+2arcs_52	445.9k	445.9k	0.0	679.7	445.9k	445.9k	0.0	2744.4	445.9k	432.3k	3.2	14400.0
belg+2arcs_53	314.1k	314.1k	0.0	305.3	314.1k	314.0k	0.0	911.3	314.1k	314.1k	0.0	12568.4
belg+2arcs_54	457.6k	457.6k	0.0	13480.2	457.6k	457.6k	0.0	7730.1	462.0k	374.6k	23.3	14400.0
belg+2arcs_55	396.6k	396.6k	0.0	1629.6	396.6k	396.6k	0.0	4847.3	428.1k	298.0k	43.7	14400.0
belg+2arcs_56	448.6k	448.6k	0.0	7790.1	448.6k	448.6k	0.0	2601.0	474.6k	372.2k	27.5	14400.0
belg+2arcs_57	453.9k	397.9k	14.1	14400.0	449.1k	449.1k	0.0	8049.6	637.5k	308.4k	106.7	14400.0
belg+2arcs_58	395.0k	395.0k	0.0	1747.3	395.0k	390.6k	1.1	14400.0	395.0k	359.7k	9.8	14400.0
belg+2arcs_59	239.1k	239.1k	0.0	54.0	239.1k	239.1k	0.0	440.0	239.1k	239.1k	0.0	7474.9
belg+2arcs_60	390.0k	390.0k	0.0	674.9	390.0k	390.0k	0.0	1600.1	391.6k	335.6k	16.7	14400.0
belg+2arcs_61	415.3k	415.3k	0.0	1253.0	415.3k	415.3k	0.0	822.6	415.3k	415.3k	0.0	4827.2
belg+2arcs_62	621.8k	621.8k	0.0	2500.3	621.8k	621.8k	0.0	3638.8	625.4k	522.5k	19.7	14400.0
belg+2arcs_63	457.7k	457.7k	0.0	10036.3	457.7k	457.7k	0.0	14169.3	460.1k	417.5k	10.2	14400.0
belg+2arcs_64	264.5k	264.5k	0.0	297.6	264.5k	264.4k	0.1	155.8	264.5k	264.5k	0.0	362.3
belg+2arcs_65	485.6k	485.6k	0.0	12608.4	485.6k	473.9k	2.5	14400.0	485.8k	408.1k	19.0	14400.0
belg+2arcs_66	468.9k	468.9k	0.0	16.5	468.9k	468.9k	0.0	192.6	468.9k	468.5k	0.1	81.7
belg+2arcs_67	385.5k	385.1k	0.1	2531.6	385.5k	385.5k	0.0	3980.1	392.7k	337.1k	16.5	14400.0
belg+2arcs_68	507.0k	482.8k	5.0	14400.0	506.9k	498.2k	1.8	14400.0	516.2k	372.8k	38.5	14400.0
belg+2arcs_69	583.1k	502.3k	16.1	14400.0	573.7k	523.2k	9.7	14400.0	743.4k	381.9k	94.7	14400.0
belg+2arcs_70	437.4k	437.4k	0.0	214.1	437.4k	437.4k	0.0	1163.6	437.4k	437.4k	0.0	1262.6
belg+2arcs_71	446.0k	446.0k	0.0	1445.6	446.0k	446.0k	0.0	750.7	446.0k	445.7k	0.1	4266.0
belg+2arcs_72	269.9k	269.9k	0.0	4454.4	269.9k	269.9k	0.0	3295.2	343.9k	206.1k	66.8	14400.0
belg+2arcs_73	199.6k	199.6k	0.0	449.3	199.6k	199.6k	0.0	1920.1	199.6k	178.8k	11.6	14400.0
belg+2arcs_74	615.4k	615.4k	0.0	114.4	615.4k	615.4k	0.0	378.9	615.4k	615.4k	0.0	969.3
belg+2arcs_75	480.7k	480.7k	0.0	1773.3	480.7k	480.7k	0.0	2253.3	484.1k	454.3k	6.6	14400.0
belg+2arcs_76	244.1k	244.1k	0.0	301.1	244.1k	243.9k	0.1	680.5	244.1k	244.1k	0.0	2643.3
belg+2arcs_77	358.4k	343.6k	4.3	14400.0	356.1k	356.1k	0.0	2694.3	376.2k	287.0k	31.1	14400.0
belg+2arcs_78	261.0k	261.0k	0.0	390.7	261.0k	261.0k	0.0	220.4	261.0k	261.0k	0.0	494.7
belg+2arcs_79	279.8k	279.8k	0.0	4506.3	281.2k	259.6k	8.3	14400.0	386.9k	221.7k	74.5	14400.0
belg+2arcs_80	413.6k	413.6k	0.0	1156.8	413.6k	413.6k	0.0	2164.2	413.6k	413.6k	0.0	11639.5
belg+2arcs_81	458.7k	458.7k	0.0	23.2	458.7k	458.7k	0.0	262.7	458.7k	458.7k	0.0	526.8
belg+2arcs_82	113.3k	113.3k	0.0	147.6	113.3k	113.3k	0.0	236.1	113.3k	113.3k	0.0	361.2
belg+2arcs_83	570.0k	570.0k	0.0	418.9	570.0k	569.6k	0.1	617.4	570.0k	570.0k	0.0	3107.0
belg+2arcs_84	559.1k	559.1k	0.0	213.7	559.1k	559.1k	0.0	345.4	559.1k	559.1k	0.0	162.3
belg+2arcs_85	445.6k	445.6k	0.0	916.7	445.6k	445.6k	0.0	5138.5	449.1k	422.2k	6.4	14400.0
belg+2arcs_86	481.0k	481.0k	0.0	339.1	481.0k	481.0k	0.0	1083.3	481.0k	481.0k	0.0	355.5
belg+2arcs_87	394.8k	355.1k	11.2	14400.0	383.9k	383.9k	0.0	1099.5	383.9k	383.9k	0.0	2478.8
belg+2arcs_88	461.4k	461.4k	0.0	124.4	461.4k	461.4k	0.0	459.9	461.4k	461.4k	0.0	896.5
belg+2arcs_89	981.2k	877.0k	11.9	14400.0	973.3k	899.4k	8.2	14400.0	1026.3k	792.7k	29.5	14400.0
belg+2arcs_90	526.3k	526.2k	0.0	274.3	526.3k	526.3k	0.0	630.9	526.3k	526.3k	0.0	376.4
belg+2arcs_91	588.3k	588.3k	0.0	11869.0	588.3k	588.3k	0.0	13973.6	595.8k	501.1k	18.9	14400.0
belg+2arcs_92	569.6k	536.6k	6.2	14400.0	564.2k	564.2k	0.0	6081.1	564.2k	476.5k	18.4	14400.0
belg+2arcs_93	753.9k	707.8k	6.5	14400.0	739.0k	739.0k	0.0	8812.8	806.6k	633.1k	27.4	14400.0
belg+2arcs_94	439.4k	439.4k	0.0	8156.8	439.4k	432.9k	1.5	14400.0	445.1k	366.3k	21.5	14400.0
belg+2arcs_95	250.7k	250.7k	0.0	367.4	250.7k	250.7k	0.0	1145.0	250.7k	250.7k	0.0	2358.5
belg+2arcs_96	407.4k	407.4k	0.0	5950.8	407.4k	407.4k	0.0	2734.2	407.4k	407.4k	0.0	4853.4
belg+2arcs_97	294.5k	294.5k	0.0	48.9	294.5k	294.3k	0.1	650.9	294.5k	294.5k	0.0	643.1
belg+2arcs_98	465.4k	465.4k	0.0	659.6	465.4k	465.4k	0.0	2227.2	465.4k	465.0k	0.1	7861.7
belg+2arcs_99	566.0k	544.4k	4.0	14400.0	563.6k	531.9k	6.0	14400.0	572.5k	490.7k	16.7	14400.0
belg+2arcs_100	474.2k	445.9k	6.3	14400.0	472.6k	472.6k	0.0	10106.5	473.6k	369.0k	28.3	14400.0
belg+2arcs_101	386.8k	386.8k	0.0	47.7	386.8k	386.8k	0.0	688.4	386.8k	386.8k	0.0	487.1
belg+2arcs_102	533.0k	533.0k	0.0	3913.0	533.0k	533.0k	0.0	11081.2	563.1k	482.2k	16.8	14400.0
belg+2arcs_103	288.7k	288.7k	0.0	5078.1	311.8k	272.2k	14.6	14400.0	294.4k	228.1k	29.1	14400.0
belg+2arcs_104	365.0k	365.0k	0.0	7892.8	365.0k	356.5k	2.4	14400.0	377.2k	256.3k	47.2	14400.0
belg+2arcs_105	253.2k	253.2k	0.0	122.2	253.2k	253.2k	0.0	533.5	253.2k	253.2k	0.0	460.9
belg+2arcs_106	418.9k	368.6k	13.6	14400.0	408.2k	398.7k	2.4	14400.0	549.9k	297.5k	84.9	14400.0
belg+2arcs_107	562.3k	509.0k	10.5	14400.0	548.4k	511.8k	7.1	14400.0	628.7k	426.0k	47.6	14400.0
belg+2arcs_108	209.5k	209.5k	0.0	47.5	209.5k	209.5k	0.0	535.4	209.5k	209.5k	0.0	1154.0
belg+2arcs_109	496.5k	379.8k	30.7	14400.0	481.5k	481.5k	0.0	5127.4	485.7k	288.7k	68.2	14400.0
belg+2arcs_110	524.4k	524.4k	0.0	1813.7	524.4k	524.4k	0.0	6906.6	535.8k	433.8k	23.5	14400.0
belg+2arcs_111	508.3k	508.3k	0.0	714.9	508.3k	508.3k	0.0	2232.4	508.3k	508.3k	0.0	1345.5
belg+2arcs_112	290.3k	290.3k	0.0	23.0	290.3k	290.3k	0.0	493.8	290.3k	290.3k	0.0	1469.8
belg+2arcs_113	226.8k	226.8k	0.0	111.4	226.8k	226.8k	0.0	283.9	226.8k	226.8k	0.0	183.7
belg+2arcs_114	215.5k	215.5k	0.0	745.0	215.5k	215.5k	0.0	1652.8	215.5k	215.5k	0.0	1928.0
belg+2arcs_115	593.9k	543.3k	9.3	14400.0	584.7k	565.7k	3.4	14400.0	788.5k	366.2k	115.3	14400.0
belg+2arcs_116	343.3k	343.3k	0.0	175.0	343.3k	343.3k	0.0	1378.8	343.3k	343.1k	0.1	741.6
belg+2arcs_117	329.7k	189.4k	74.1	14400.0	302.9k	271.8k	11.4	14400.0	320.6k	174.4k	83.9	14400.0
belg+2arcs_118	359.9k	359.9k	0.0	3955.8	359.9k	359.9k	0.0	1042.7	365.8k	316.1k	15.7	14400.0
belg+2arcs_119	417.6k	417.6k	0.0	525.7	417.6k	417.6k	0.0	1806.0	417.6k	417.6k	0.0	917.1
belg+2arcs_120	343.6k	343.6k	0.0	353.6	343.6k	343.6k	0.0	1500.7	343.6k	343.6k	0.0	7792.6

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Table A.25: Comparison of discrete models on *Circuit rank* + 2 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_121	289.8k	289.8k	0.0	117.1	289.8k	289.8k	0.0	758.0	289.8k	289.8k	0.0	726.1
belg+2arcs_122	445.4k	445.2k	0.1	1828.7	445.4k	445.4k	0.0	3492.4	446.6k	418.8k	6.6	14400.0
belg+2arcs_123	297.5k	297.5k	0.0	43.2	297.5k	297.5k	0.0	254.9	297.5k	297.5k	0.0	348.8
belg+2arcs_124	479.3k	479.3k	0.0	3299.6	479.3k	479.3k	0.0	5034.1	486.7k	446.1k	9.1	14400.0
belg+2arcs_125	256.1k	256.1k	0.0	235.1	256.1k	256.1k	0.0	538.9	256.1k	256.1k	0.0	580.4
belg+2arcs_126	383.6k	383.6k	0.0	1652.1	383.6k	383.6k	0.0	1516.3	383.6k	383.6k	0.0	2291.6
belg+2arcs_127	383.0k	383.0k	0.0	1220.5	383.0k	383.0k	0.0	1782.5	383.0k	383.0k	0.0	8503.4
belg+2arcs_128	446.9k	446.9k	0.0	1530.1	446.9k	446.9k	0.0	1157.8	446.9k	446.9k	0.0	5160.2
belg+2arcs_129	255.4k	255.4k	0.0	3647.6	255.4k	255.4k	0.0	9567.7	279.8k	221.7k	26.2	14400.0
belg+2arcs_130	509.3k	472.1k	7.9	14400.0	509.3k	509.3k	0.0	4978.8	525.6k	408.5k	28.7	14400.0
belg+2arcs_131	409.2k	409.2k	0.0	143.9	409.2k	409.2k	0.0	566.8	409.2k	409.2k	0.0	1144.8
belg+2arcs_132	293.8k	293.8k	0.0	147.1	293.8k	293.8k	0.0	299.9	293.8k	293.8k	0.0	296.9
belg+2arcs_133	538.9k	485.0k	11.1	14400.0	524.6k	524.6k	0.0	8177.9	534.4k	360.3k	48.3	14400.0
belg+2arcs_134	719.7k	719.7k	0.0	1935.8	719.7k	719.3k	0.1	4811.9	801.9k	627.0k	27.9	14400.0
belg+2arcs_135	335.9k	335.9k	0.0	331.1	335.9k	335.9k	0.0	669.5	335.9k	335.9k	0.0	1838.2
belg+2arcs_136	973.6k	819.6k	18.8	14400.0	941.9k	917.0k	2.7	14400.0	1122.1k	648.9k	72.9	14400.0
belg+2arcs_137	664.5k	664.5k	0.0	1832.2	664.5k	664.5k	0.0	4088.3	669.2k	549.6k	21.8	14400.0
belg+2arcs_138	542.0k	497.8k	8.9	14400.0	540.8k	540.8k	0.0	6774.6	542.2k	461.3k	17.5	14400.0
belg+2arcs_139	386.4k	386.4k	0.0	216.0	386.4k	386.4k	0.0	1088.4	386.4k	386.4k	0.0	1159.2
belg+2arcs_140	331.3k	331.3k	0.0	2587.3	331.3k	331.1k	0.1	2082.1	339.1k	302.7k	12.0	14400.0
belg+2arcs_141	453.1k	406.7k	11.4	14400.0	439.8k	439.8k	0.0	4334.3	443.9k	359.8k	23.4	14400.0
belg+2arcs_142	524.0k	498.6k	5.1	14400.0	521.5k	521.5k	0.0	11713.3	608.2k	442.7k	37.4	14400.0
belg+2arcs_143	272.1k	272.0k	0.0	514.8	272.1k	272.1k	0.0	747.6	272.1k	272.1k	0.0	1778.8
belg+2arcs_144	325.4k	263.8k	23.4	14400.0	323.8k	281.8k	14.9	14400.0	525.2k	233.2k	125.2	14400.0
belg+2arcs_145	409.1k	409.1k	0.0	4677.1	409.1k	409.1k	0.0	4818.0	474.2k	318.9k	48.7	14400.0
belg+2arcs_146	363.9k	363.9k	0.0	146.8	363.9k	363.9k	0.0	686.6	363.9k	363.9k	0.0	1566.2
belg+2arcs_147	520.2k	458.8k	13.4	14400.0	494.4k	494.4k	0.0	2174.1	516.2k	429.7k	20.1	14400.0
belg+2arcs_148	595.1k	435.4k	36.7	14400.0	511.3k	511.3k	0.0	7722.0	569.5k	356.1k	59.9	14400.0
belg+2arcs_149	433.7k	300.0k	44.6	14400.0	412.8k	369.4k	11.7	14400.0	543.4k	279.6k	94.3	14400.0
belg+2arcs_150	649.5k	649.3k	0.0	893.4	649.5k	649.5k	0.0	2767.4	649.5k	629.3k	3.2	14400.0
belg+2arcs_151	285.6k	285.6k	0.0	141.9	285.6k	285.6k	0.0	470.9	285.6k	285.6k	0.0	494.5
belg+2arcs_152	369.9k	369.9k	0.0	4095.8	369.9k	369.9k	0.0	449.9	369.9k	369.9k	0.0	1617.4
belg+2arcs_153	445.2k	445.2k	0.0	209.3	445.2k	445.2k	0.0	1495.8	445.2k	427.8k	4.1	14400.0
belg+2arcs_154	351.6k	351.6k	0.0	513.4	351.6k	351.6k	0.0	2101.1	351.6k	351.6k	0.0	4554.6
belg+2arcs_155	565.2k	565.2k	0.0	835.0	565.2k	565.2k	0.0	2162.8	577.3k	532.1k	8.5	14400.0
belg+2arcs_156	368.6k	342.0k	7.8	14400.0	363.0k	362.6k	0.1	3260.9	373.4k	336.8k	10.9	14400.0
belg+2arcs_157	372.3k	372.3k	0.0	101.6	372.3k	372.3k	0.0	468.1	372.3k	372.3k	0.0	1394.9
belg+2arcs_158	413.2k	413.2k	0.0	4377.8	413.2k	412.7k	0.1	6164.9	431.8k	340.8k	26.7	14400.0
belg+2arcs_159	342.2k	342.2k	0.0	270.4	342.2k	342.2k	0.0	957.7	346.8k	334.5k	3.7	14400.0
belg+2arcs_160	333.3k	333.3k	0.0	251.7	333.3k	333.3k	0.0	734.4	333.3k	333.3k	0.0	200.6
belg+2arcs_161	455.9k	455.9k	0.0	369.2	455.9k	455.9k	0.0	570.1	455.9k	455.9k	0.0	1116.2
belg+2arcs_162	551.8k	551.8k	0.0	4480.1	551.8k	551.6k	0.0	1954.4	553.3k	428.9k	29.0	14400.0
belg+2arcs_163	638.2k	600.1k	6.4	14400.0	636.8k	619.0k	2.9	14400.0	671.5k	405.7k	65.5	14400.0
belg+2arcs_164	258.8k	258.8k	0.0	38.4	258.8k	258.7k	0.1	172.3	258.8k	258.8k	0.0	206.2
belg+2arcs_165	300.2k	300.2k	0.0	245.2	300.2k	300.2k	0.0	1129.9	300.2k	300.2k	0.0	7793.0
belg+2arcs_166	255.9k	255.9k	0.0	1578.5	255.9k	255.9k	0.0	6378.5	255.9k	201.7k	26.9	14400.0
belg+2arcs_167	470.2k	446.9k	5.2	14400.0	470.2k	470.2k	0.0	6971.7	476.7k	351.6k	35.6	14400.0
belg+2arcs_168	492.0k	491.6k	0.1	22.3	492.0k	492.0k	0.0	432.3	492.0k	492.0k	0.0	439.7
belg+2arcs_169	281.6k	281.4k	0.1	2474.4	281.6k	281.6k	0.0	2281.9	290.7k	256.3k	13.5	14400.0
belg+2arcs_170	369.3k	369.3k	0.0	6117.2	369.3k	369.3k	0.0	438.8	369.3k	369.3k	0.0	3145.0
belg+2arcs_171	401.7k	401.3k	0.1	333.3	401.7k	401.7k	0.0	735.0	401.7k	401.7k	0.0	549.9
belg+2arcs_172	377.0k	360.4k	4.6	14400.0	376.5k	376.5k	0.0	2702.4	377.9k	323.3k	16.9	14400.0
belg+2arcs_173	580.5k	580.2k	0.0	4141.5	580.5k	580.5k	0.0	3338.3	616.5k	467.0k	32.0	14400.0
belg+2arcs_174	382.7k	382.7k	0.0	1425.9	382.7k	382.7k	0.0	2771.3	392.8k	332.7k	18.1	14400.0
belg+2arcs_175	220.6k	220.6k	0.0	53.0	220.6k	220.4k	0.1	108.7	220.6k	220.6k	0.0	74.8
belg+2arcs_176	424.3k	424.3k	0.0	767.5	424.3k	424.3k	0.0	607.0	424.3k	424.3k	0.0	4631.0
belg+2arcs_177	527.4k	527.4k	0.0	5567.3	527.4k	527.4k	0.0	13453.3	559.2k	363.5k	53.8	14400.0
belg+2arcs_178	361.3k	361.3k	0.0	363.1	361.3k	361.3k	0.0	606.6	361.3k	361.3k	0.0	6559.1
belg+2arcs_179	418.8k	418.8k	0.0	171.4	418.8k	418.8k	0.0	449.8	418.8k	418.8k	0.0	1117.1
belg+2arcs_180	241.0k	241.0k	0.0	236.0	241.0k	241.0k	0.0	751.3	241.0k	241.0k	0.0	2782.4
belg+2arcs_181	294.8k	294.8k	0.0	1447.8	294.8k	294.8k	0.0	3275.0	321.3k	267.2k	20.2	14400.0
belg+2arcs_182	370.4k	370.4k	0.0	1070.7	370.4k	370.4k	0.0	1566.7	392.0k	340.6k	15.1	14400.0
belg+2arcs_183	292.6k	243.1k	20.4	14400.0	294.4k	286.2k	2.9	14400.0	297.2k	243.3k	22.2	14400.0
belg+2arcs_184	324.2k	206.2k	57.2	14400.0	317.1k	270.1k	17.4	14400.0	351.1k	206.1k	70.4	14400.0
belg+2arcs_185	267.8k	267.8k	0.0	740.1	267.8k	267.8k	0.0	564.8	267.8k	267.8k	0.0	4765.9
belg+2arcs_186	424.5k	424.5k	0.0	1747.1	424.5k	424.5k	0.0	2380.6	510.6k	348.5k	46.5	14400.0
belg+2arcs_187	709.3k	708.8k	0.1	5917.8	709.3k	709.1k	0.0	3298.0	941.0k	511.5k	84.0	14400.0
belg+2arcs_188	228.0k	228.0k	0.0	19.4	228.0k	228.0k	0.0	103.3	228.0k	228.0k	0.0	80.1
belg+2arcs_189	516.6k	375.1k	37.7	14400.0	470.9k	441.0k	6.8	14400.0	521.8k	265.1k	96.8	14400.0
belg+2arcs_190	380.5k	282.8k	34.6	14400.0	370.5k	338.5k	9.5	14400.0	430.3k	208.7k	106.2	14400.0
belg+2arcs_191	588.4k	588.4k	0.0	6524.0	588.4k	573.6k	2.6	14400.0	590.1k	541.1k	9.0	14400.0
belg+2arcs_192	405.5k	405.5k	0.0	923.8	405.5k	405.5k	0.0	951.0	405.5k	405.5k	0.0	4659.5
belg+2arcs_193	297.3k	297.3k	0.0	324.0	297.3k	297.1k	0.1	509.3	297.3k	297.3k	0.0	1566.6
belg+2arcs_194	462.3k	462.2k	0.0	59.7	462.3k	462.3k	0.0	231.6	462.3k	462.3k	0.0	111.5
belg+2arcs_195	413.4k	413.4k	0.0	7637.4	413.4k	413.2k	0.1	13890.7	415.6k	335.8k	23.8	14400.0
belg+2arcs_196	557.0k	557.0k	0.0	2968.5	557.0k	557.0k	0.0	4863.4	568.1k	534.8k	6.2	14400.0
belg+2arcs_197	729.0k	612.5k	19.0	14400.0	716.8k	661.4k	8.4	14400.0	799.6k	517.8k	54.4	14400.0
belg+2arcs_198	371.4k	371.4k	0.0	2189.0	371.4k	371.2k	0.1	10951.3	371.4k	316.2k	17.5	14400.0

continued on next page

Table A.25: Comparison of discrete models on *Circuit rank* + 2 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_199	206.8k	206.8k	0.0	341.3	206.8k	206.8k	0.0	410.8	206.8k	206.8k	0.0	5081.8
belg+2arcs_200	481.8k	481.8k	0.0	133.2	481.8k	481.7k	0.0	488.4	481.8k	481.8k	0.0	405.5
belg+2arcs_201	243.9k	243.9k	0.0	3902.9	243.9k	243.9k	0.0	3202.2	245.0k	180.1k	36.1	14400.0
belg+2arcs_202	336.0k	336.0k	0.0	988.1	336.0k	336.0k	0.0	2133.0	336.0k	336.0k	0.0	9815.6
belg+2arcs_203	435.4k	367.8k	18.4	14400.0	434.9k	385.5k	12.8	14400.0	515.5k	284.3k	81.3	14400.0
belg+2arcs_204	381.1k	381.1k	0.0	2855.5	381.1k	381.1k	0.0	2065.0	381.1k	370.6k	2.8	14400.0
belg+2arcs_205	589.1k	545.2k	8.1	14400.0	570.1k	555.1k	2.7	14400.0	617.1k	475.3k	29.8	14400.0
belg+2arcs_206	279.5k	279.5k	0.0	598.9	279.5k	279.5k	0.0	982.0	279.5k	266.4k	4.9	14400.0
belg+2arcs_207	454.6k	454.3k	0.1	54.5	454.6k	454.2k	0.1	233.3	454.6k	454.6k	0.0	335.3
belg+2arcs_208	341.6k	341.6k	0.0	1291.9	341.6k	341.6k	0.0	2697.4	345.4k	320.2k	7.9	14400.0
belg+2arcs_209	268.5k	268.5k	0.0	4630.8	268.5k	265.6k	1.1	14400.0	273.9k	202.3k	35.4	14400.0
belg+2arcs_210	454.8k	454.8k	0.0	4443.8	454.8k	454.8k	0.0	2256.1	481.2k	414.9k	16.0	14400.0
belg+2arcs_211	471.3k	471.3k	0.0	2313.6	471.3k	471.3k	0.0	3798.3	471.3k	436.9k	7.9	14400.0
belg+2arcs_212	268.6k	268.6k	0.0	8674.6	268.6k	268.5k	0.1	444.4	268.6k	268.6k	0.0	515.3
belg+2arcs_213	651.8k	553.0k	17.9	14400.0	653.7k	626.1k	4.4	14400.0	665.9k	461.8k	44.2	14400.0
belg+2arcs_214	402.0k	402.0k	0.0	2281.6	402.0k	402.0k	0.0	7161.5	405.1k	318.7k	27.1	14400.0
belg+2arcs_215	305.6k	305.6k	0.0	365.0	305.6k	305.5k	0.0	645.9	305.6k	305.6k	0.0	2034.0
belg+2arcs_216	519.6k	519.6k	0.0	12737.7	519.6k	519.6k	0.0	2012.2	527.7k	486.8k	8.4	14400.0
belg+2arcs_217	351.0k	351.0k	0.0	918.1	351.0k	351.0k	0.0	1827.3	352.5k	332.4k	6.1	14400.0
belg+2arcs_218	578.1k	578.1k	0.0	4407.6	578.1k	578.1k	0.0	10757.1	579.2k	521.8k	11.0	14400.0
belg+2arcs_219	440.3k	440.3k	0.0	781.2	440.3k	440.3k	0.0	2819.2	443.7k	413.7k	7.3	14400.0
belg+2arcs_220	453.1k	453.1k	0.0	13243.9	453.1k	445.6k	1.7	14400.0	459.8k	439.7k	4.6	14400.0
belg+2arcs_221	196.5k	196.5k	0.0	13.7	196.5k	196.5k	0.0	138.7	196.5k	196.5k	0.0	249.3
belg+2arcs_222	449.0k	449.0k	0.0	368.4	449.0k	449.0k	0.0	1024.5	449.0k	449.0k	0.0	1098.0
belg+2arcs_223	383.5k	383.5k	0.0	268.9	383.5k	383.5k	0.0	398.4	383.5k	383.5k	0.0	530.4
belg+2arcs_224	532.7k	504.1k	5.7	14400.0	531.2k	498.3k	6.6	14400.0	562.8k	374.1k	50.4	14400.0
belg+2arcs_225	303.2k	303.2k	0.0	1217.9	303.2k	303.2k	0.0	2598.0	304.0k	267.6k	13.6	14400.0
belg+2arcs_226	378.5k	378.5k	0.0	245.8	378.5k	378.5k	0.0	888.3	378.5k	378.5k	0.0	8850.6
belg+2arcs_227	216.5k	216.5k	0.0	126.1	216.5k	216.5k	0.0	314.7	216.5k	216.5k	0.0	559.4
belg+2arcs_228	448.1k	448.1k	0.0	336.6	448.1k	448.1k	0.0	265.2	448.1k	448.1k	0.0	659.4
belg+2arcs_229	535.2k	504.1k	6.2	14400.0	533.3k	507.5k	5.1	14400.0	546.2k	426.3k	28.1	14400.0
belg+2arcs_230	522.7k	435.0k	20.2	14400.0	508.3k	442.6k	14.8	14400.0	746.6k	334.5k	123.2	14400.0
belg+2arcs_231	274.4k	274.4k	0.0	24.3	274.4k	274.4k	0.0	217.6	274.4k	274.4k	0.0	116.2
belg+2arcs_232	307.9k	307.9k	0.0	257.4	307.9k	307.9k	0.0	535.2	307.9k	307.9k	0.0	970.8
belg+2arcs_233	494.1k	484.4k	2.0	14400.0	494.0k	493.7k	0.1	9787.8	516.3k	414.7k	24.5	14400.0
belg+2arcs_234	263.5k	263.5k	0.0	335.8	263.5k	263.2k	0.1	770.9	263.5k	263.5k	0.0	1565.5
belg+2arcs_235	320.9k	320.9k	0.0	150.5	320.9k	320.9k	0.0	305.1	320.9k	320.9k	0.0	1446.6
belg+2arcs_236	399.0k	399.0k	0.0	6377.8	399.0k	399.0k	0.0	10257.0	409.0k	362.6k	12.8	14400.0
belg+2arcs_237	329.5k	329.5k	0.0	2066.8	329.5k	329.2k	0.1	2736.2	330.9k	269.7k	22.7	14400.0
belg+2arcs_238	484.8k	484.8k	0.0	302.5	484.8k	484.8k	0.0	470.4	484.8k	484.8k	0.0	602.2
belg+2arcs_239	542.7k	542.7k	0.0	501.5	542.7k	542.7k	0.0	1635.0	542.7k	542.7k	0.0	7096.1
belg+2arcs_240	566.0k	549.9k	2.9	14400.0	566.0k	566.0k	0.0	7986.0	664.5k	378.3k	75.6	14400.0
belg+2arcs_241	509.8k	509.8k	0.0	2625.6	509.8k	509.8k	0.0	1936.2	509.9k	470.9k	8.3	14400.0
belg+2arcs_242	374.3k	374.0k	0.1	234.6	374.3k	374.0k	0.1	349.0	374.3k	374.3k	0.0	385.5
belg+2arcs_243	455.5k	455.5k	0.0	3782.9	455.5k	448.6k	1.5	14400.0	463.9k	376.9k	23.1	14400.0
belg+2arcs_244	461.6k	461.6k	0.0	9355.5	461.6k	461.6k	0.0	8689.4	641.3k	383.4k	67.3	14400.0
belg+2arcs_245	200.3k	200.3k	0.0	128.1	200.3k	200.3k	0.0	251.1	200.3k	200.3k	0.0	621.9
belg+2arcs_246	469.8k	469.8k	0.0	3093.4	469.8k	469.8k	0.0	5102.4	483.1k	381.2k	26.7	14400.0
belg+2arcs_247	357.9k	357.9k	0.0	1907.5	357.9k	357.9k	0.0	1892.5	360.1k	290.7k	23.9	14400.0
belg+2arcs_248	728.9k	514.2k	41.8	14400.0	624.6k	624.6k	0.0	13440.8	729.9k	500.3k	45.9	14400.0
belg+2arcs_249	348.2k	348.0k	0.1	847.9	348.2k	348.2k	0.0	514.9	348.2k	348.0k	0.1	4331.8
belg+2arcs_250	416.9k	416.9k	0.0	45.8	416.9k	416.5k	0.1	447.8	416.9k	416.9k	0.0	439.8
belg+2arcs_251	453.9k	453.9k	0.0	2016.0	453.9k	453.9k	0.0	9868.5	455.8k	429.7k	6.1	14400.0
belg+2arcs_252	317.0k	317.0k	0.0	835.5	317.0k	317.0k	0.0	1414.1	325.4k	308.5k	5.5	14400.0
belg+2arcs_253	758.6k	613.4k	23.7	14400.0	672.1k	638.7k	5.2	14400.0	766.0k	541.6k	41.4	14400.0
belg+2arcs_254	617.6k	492.6k	25.4	14400.0	603.2k	564.2k	6.9	14400.0	779.4k	429.2k	81.6	14400.0
belg+2arcs_255	170.2k	170.2k	0.0	144.9	170.2k	170.2k	0.0	153.5	170.2k	170.2k	0.0	235.3
belg+2arcs_256	486.3k	486.3k	0.0	3842.2	486.3k	486.3k	0.0	2855.0	685.0k	373.6k	83.4	14400.0
belg+2arcs_257	379.3k	379.3k	0.0	13471.4	379.3k	366.4k	3.5	14400.0	382.4k	347.0k	10.2	14400.0
belg+2arcs_258	306.5k	306.5k	0.0	246.7	306.5k	306.5k	0.0	326.5	306.5k	306.4k	0.0	1796.8
belg+2arcs_259	394.6k	394.6k	0.0	3299.4	394.6k	394.6k	0.0	7201.9	403.9k	339.2k	19.1	14400.0
belg+2arcs_260	411.8k	411.8k	0.0	3160.1	411.8k	411.8k	0.0	1173.0	432.2k	355.3k	21.7	14400.0
belg+2arcs_261	145.0k	145.0k	0.0	14.6	145.0k	145.0k	0.0	25.1	145.0k	145.0k	0.0	79.5
belg+2arcs_262	296.8k	296.8k	0.0	88.1	296.8k	296.7k	0.0	409.4	296.8k	296.8k	0.0	134.1
belg+2arcs_263	518.7k	518.7k	0.0	6870.6	518.7k	507.6k	2.2	14400.0	528.5k	468.9k	12.7	14400.0
belg+2arcs_264	421.4k	421.4k	0.0	487.1	421.4k	421.4k	0.0	252.8	421.4k	421.4k	0.0	514.2
belg+2arcs_265	940.5k	583.5k	61.2	14400.0	812.5k	672.2k	20.9	14400.0	1378.2k	526.3k	161.9	14400.0
belg+2arcs_266	347.9k	347.9k	0.0	1279.8	347.9k	347.9k	0.0	1072.6	354.4k	318.8k	11.2	14400.0
belg+2arcs_267	615.4k	615.4k	0.0	421.3	615.4k	615.4k	0.0	1664.9	615.4k	615.4k	0.0	1580.7
belg+2arcs_268	523.3k	483.0k	8.3	14400.0	515.8k	495.1k	4.2	14400.0	553.1k	436.4k	26.7	14400.0
belg+2arcs_269	499.9k	418.8k	19.4	14400.0	484.6k	484.2k	0.1	10926.2	534.7k	321.0k	66.6	14400.0
belg+2arcs_270	326.0k	326.0k	0.0	1578.3	326.0k	326.0k	0.0	3569.4	326.0k	303.3k	7.5	14400.0
belg+2arcs_271	511.9k	511.9k	0.0	5334.8	511.9k	511.9k	0.0	7777.9	529.9k	437.7k	21.0	14400.0
belg+2arcs_272	588.3k	588.3k	0.0	10478.1	588.3k	588.3k	0.0	7169.6	592.0k	478.9k	23.6	14400.0
belg+2arcs_273	635.6k	580.8k	9.4	14400.0	635.6k	635.6k	0.0	11395.1	642.8k	480.9k	33.7	14400.0
belg+2arcs_274	286.9k	286.9k	0.0	24.5	286.9k	286.9k	0.0	369.8	286.9k	286.9k	0.0	2745.5
belg+2arcs_275	247.4k	247.4k	0.0	1609.8	247.4k	247.4k	0.0	596.1	257.3k	209.0k	23.1	14400.0
belg+2arcs_276	232.7k	232.7k	0.0	718.5	232.7k	232.7k	0.0	587.2	252.1k	227.5k	10.8	14400.0

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Table A.25: Comparison of discrete models on *Circuit rank* + 2 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_277	324.2k	324.2k	0.0	4579.6	324.2k	324.2k	0.0	7365.4	334.0k	261.2k	27.9	14400.0
belg+2arcs_278	252.2k	252.2k	0.0	174.9	252.2k	252.2k	0.0	281.1	252.2k	252.2k	0.0	516.6
belg+2arcs_279	310.0k	309.8k	0.1	621.9	310.0k	310.0k	0.0	3775.9	310.0k	310.0k	0.0	5685.9
belg+2arcs_280	540.2k	539.7k	0.1	664.5	540.2k	540.2k	0.0	1632.3	561.2k	505.4k	11.0	14400.0
belg+2arcs_281	323.0k	323.0k	0.0	375.6	323.0k	323.0k	0.0	243.5	323.0k	323.0k	0.0	437.5
belg+2arcs_282	548.5k	511.5k	7.2	14400.0	541.4k	530.5k	2.1	14400.0	881.7k	414.5k	112.7	14400.0
belg+2arcs_283	435.5k	435.5k	0.0	5053.1	435.5k	435.5k	0.0	8030.2	436.4k	376.1k	16.0	14400.0
belg+2arcs_284	420.2k	420.2k	0.0	1377.3	420.2k	420.2k	0.0	8640.0	420.2k	357.8k	17.4	14400.0
belg+2arcs_285	267.9k	267.9k	0.0	109.5	267.9k	267.9k	0.0	607.2	267.9k	267.9k	0.0	1032.4
belg+2arcs_286	424.9k	424.9k	0.0	179.9	424.9k	424.9k	0.0	665.5	424.9k	424.9k	0.0	1447.2
belg+2arcs_287	411.8k	411.8k	0.0	3881.2	411.8k	411.8k	0.0	3701.4	541.2k	334.9k	61.6	14400.0
belg+2arcs_288	344.3k	344.3k	0.0	6675.8	344.3k	344.1k	0.1	5369.7	345.1k	321.8k	7.2	14400.0
belg+2arcs_289	414.5k	360.3k	15.0	14400.0	409.9k	409.9k	0.0	9734.7	693.4k	288.7k	140.2	14400.0
belg+2arcs_290	625.3k	432.1k	44.7	14400.0	605.8k	545.6k	11.0	14400.0	564.4k	382.1k	47.7	14400.0
belg+2arcs_291	518.2k	518.2k	0.0	1739.7	518.2k	518.2k	0.0	2716.8	518.7k	481.3k	7.8	14400.0
belg+2arcs_292	313.2k	313.2k	0.0	196.8	313.2k	313.2k	0.0	787.8	313.2k	313.2k	0.0	6606.4
belg+2arcs_293	237.3k	237.3k	0.0	1729.5	237.3k	237.3k	0.0	3379.2	240.8k	205.6k	17.1	14400.0
belg+2arcs_294	327.5k	327.5k	0.0	89.9	327.5k	327.5k	0.0	774.5	327.5k	327.5k	0.0	1923.0
belg+2arcs_295	466.9k	456.9k	2.2	14400.0	462.9k	462.9k	0.0	2394.3	462.9k	462.9k	0.0	7961.3
belg+2arcs_296	463.7k	463.7k	0.0	9941.8	463.7k	463.5k	0.0	5965.1	466.5k	403.8k	15.5	14400.0
belg+2arcs_297	289.7k	289.7k	0.0	1093.4	289.7k	289.7k	0.0	964.6	289.7k	273.7k	5.9	14400.0
belg+2arcs_298	325.3k	325.3k	0.0	5415.1	325.3k	306.0k	6.3	14400.0	436.4k	241.1k	81.0	14400.0
belg+2arcs_299	668.2k	573.9k	16.4	14400.0	661.1k	643.2k	2.8	14400.0	665.9k	529.6k	25.7	14400.0
belg+2arcs_300	307.4k	307.4k	0.0	5225.5	307.4k	299.6k	2.6	14400.0	311.7k	243.8k	27.9	14400.0
belg+2arcs_301	317.3k	317.1k	0.1	114.9	317.3k	317.3k	0.0	438.2	317.3k	317.3k	0.0	406.7
belg+2arcs_302	457.6k	457.6k	0.0	610.2	457.6k	457.6k	0.0	3326.6	457.6k	457.6k	0.0	3288.7
belg+2arcs_303	601.9k	535.4k	12.4	14400.0	600.0k	560.0k	7.1	14400.0	621.9k	497.1k	25.1	14400.0
belg+2arcs_304	495.6k	482.3k	2.8	14400.0	495.3k	495.0k	0.1	8836.8	498.2k	439.8k	13.3	14400.0
belg+2arcs_305	444.8k	444.4k	0.1	76.3	444.8k	444.7k	0.0	606.3	444.8k	444.8k	0.0	1478.6
belg+2arcs_306	442.7k	442.7k	0.0	5216.9	442.7k	435.8k	1.6	14400.0	451.1k	385.9k	16.9	14400.0
belg+2arcs_307	291.0k	291.0k	0.0	945.0	291.0k	291.0k	0.0	662.5	291.0k	269.1k	8.1	14400.0
belg+2arcs_308	252.4k	252.4k	0.0	2171.0	259.5k	242.7k	6.9	14400.0	254.9k	234.1k	8.9	14400.0
belg+2arcs_309	493.7k	370.2k	33.3	14400.0	480.8k	446.8k	7.6	14400.0	706.0k	332.9k	112.1	14400.0
belg+2arcs_310	286.3k	286.3k	0.0	833.7	286.3k	286.3k	0.0	4833.5	286.3k	286.3k	0.0	7617.1
belg+2arcs_311	303.0k	302.8k	0.1	192.6	303.0k	303.0k	0.0	163.3	303.0k	303.0k	0.0	473.8
belg+2arcs_312	267.2k	267.2k	0.0	83.2	267.2k	267.2k	0.0	161.9	267.2k	267.2k	0.0	438.9
belg+2arcs_313	517.7k	517.7k	0.0	536.0	517.7k	517.7k	0.0	671.3	517.7k	517.7k	0.0	2935.4
belg+2arcs_314	713.3k	713.3k	0.0	1340.1	713.3k	713.3k	0.0	2599.9	721.0k	695.4k	3.7	14400.0
belg+2arcs_315	491.6k	491.6k	0.0	460.4	491.6k	491.6k	0.0	1689.2	491.6k	491.2k	0.1	7797.9
belg+2arcs_316	274.1k	274.1k	0.0	336.2	274.1k	274.1k	0.0	161.7	274.1k	274.1k	0.0	644.4
belg+2arcs_317	463.5k	463.5k	0.0	5559.0	463.5k	463.5k	0.0	10031.1	478.4k	404.6k	18.2	14400.0
belg+2arcs_318	267.2k	267.2k	0.0	9265.9	267.2k	267.2k	0.0	7406.7	304.2k	231.8k	31.2	14400.0
belg+2arcs_319	313.3k	290.7k	7.8	14400.0	300.3k	300.3k	0.0	507.4	300.3k	300.3k	0.0	955.8
belg+2arcs_320	367.0k	367.0k	0.0	933.0	367.0k	366.8k	0.1	2774.9	367.0k	325.0k	12.9	14400.0
belg+2arcs_321	250.3k	250.3k	0.0	5897.8	250.3k	250.3k	0.0	2402.2	254.7k	221.4k	15.1	14400.0
belg+2arcs_322	323.5k	323.5k	0.0	13765.7	323.9k	309.0k	4.8	14400.0	358.4k	256.0k	40.0	14400.0
belg+2arcs_323	295.8k	295.8k	0.0	374.4	295.8k	295.8k	0.0	360.6	295.8k	295.8k	0.0	2837.1
belg+2arcs_324	324.4k	324.4k	0.0	414.4	324.4k	324.4k	0.0	578.0	324.4k	324.4k	0.0	1181.5
belg+2arcs_325	551.1k	551.1k	0.0	4066.7	551.1k	551.1k	0.0	1794.0	557.6k	495.8k	12.5	14400.0
belg+2arcs_326	394.5k	394.5k	0.0	50.5	394.5k	394.5k	0.0	239.6	394.5k	394.5k	0.0	782.6
belg+2arcs_327	227.0k	227.0k	0.0	187.3	227.0k	227.0k	0.0	769.5	227.0k	227.0k	0.0	2588.8
belg+2arcs_328	275.7k	275.7k	0.0	1365.9	275.7k	275.5k	0.0	1448.3	276.7k	247.6k	11.8	14400.0
belg+2arcs_329	338.0k	338.0k	0.0	87.6	338.0k	338.0k	0.0	578.9	338.0k	338.0k	0.0	1053.0
belg+2arcs_330	395.6k	395.6k	0.0	1262.8	395.6k	395.6k	0.0	1116.3	395.6k	352.3k	12.3	14400.0
belg+2arcs_331	308.3k	308.3k	0.0	52.9	308.3k	308.3k	0.0	204.3	308.3k	308.3k	0.0	141.3
belg+2arcs_332	920.1k	566.7k	62.4	14400.0	740.8k	668.8k	10.8	14400.0	844.4k	490.2k	72.3	14400.0
belg+2arcs_333	321.8k	321.8k	0.0	12486.5	321.8k	321.8k	0.0	1388.3	321.8k	321.8k	0.0	12446.4
belg+2arcs_334	516.5k	468.5k	10.3	14400.0	506.1k	505.6k	0.1	7881.9	518.5k	364.3k	42.3	14400.0
belg+2arcs_335	465.0k	465.0k	0.0	957.0	465.0k	465.0k	0.0	1098.5	465.0k	465.0k	0.0	1475.2
belg+2arcs_336	322.2k	322.1k	0.0	757.5	322.2k	322.2k	0.0	1210.1	322.2k	322.2k	0.0	6166.4
belg+2arcs_337	337.7k	337.7k	0.0	453.7	337.7k	337.7k	0.0	2074.4	337.7k	337.7k	0.0	1593.3
belg+2arcs_338	320.5k	320.5k	0.0	21.2	320.5k	320.5k	0.0	235.1	320.5k	320.5k	0.0	293.4
belg+2arcs_339	648.6k	648.6k	0.0	5418.4	648.6k	648.6k	0.0	4722.3	648.6k	612.2k	5.9	14400.0
belg+2arcs_340	649.8k	627.2k	3.6	14400.0	647.3k	647.3k	0.0	13305.7	652.7k	520.0k	25.5	14400.0
belg+2arcs_341	290.8k	290.8k	0.0	656.6	290.8k	290.8k	0.0	9161.8	290.8k	290.8k	0.0	3070.4
belg+2arcs_342	461.9k	332.4k	39.0	14400.0	455.1k	381.2k	19.4	14400.0	742.5k	279.9k	165.2	14400.0
belg+2arcs_343	345.3k	345.2k	0.0	154.7	345.3k	345.3k	0.0	346.0	345.3k	345.3k	0.0	620.7
belg+2arcs_344	314.7k	314.7k	0.0	254.3	314.7k	314.7k	0.0	1516.0	323.4k	273.0k	18.5	14400.0
belg+2arcs_345	519.4k	519.4k	0.0	11673.6	519.4k	503.0k	3.3	14400.0	529.2k	447.7k	18.2	14400.0
belg+2arcs_346	377.7k	377.7k	0.0	27.1	377.7k	377.7k	0.0	686.5	377.7k	377.7k	0.0	152.8
belg+2arcs_347	567.6k	567.6k	0.0	566.4	567.6k	567.6k	0.0	827.6	567.6k	567.6k	0.0	1609.4
belg+2arcs_348	270.9k	270.9k	0.0	60.3	270.9k	270.7k	0.1	126.6	270.9k	270.9k	0.0	226.5
belg+2arcs_349	312.7k	253.6k	23.3	14400.0	292.1k	272.5k	7.2	14400.0	461.8k	201.4k	129.3	14400.0
belg+2arcs_350	406.6k	406.6k	0.0	1219.1	406.6k	406.6k	0.0	1187.2	408.0k	361.7k	12.8	14400.0
belg+2arcs_351	482.4k	482.4k	0.0	178.9	482.4k	482.4k	0.0	462.8	482.4k	482.4k	0.0	276.3
belg+2arcs_352	541.3k	466.2k	16.1	14400.0	541.3k	541.3k	0.0	8781.3	557.9k	419.5k	33.0	14400.0
belg+2arcs_353	396.4k	396.4k	0.0	374.2	396.4k	396.4k	0.0	681.9	396.4k	396.4k	0.0	1294.7
belg+2arcs_354	314.2k	314.2k	0.0	388.3	314.2k	314.0k	0.1	386.2	314.2k	314.2k	0.0	842.6

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Table A.25: Comparison of discrete models on *Circuit rank* + 2 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_355	207.4k	207.4k	0.0	280.7	207.4k	207.4k	0.0	502.1	207.4k	207.4k	0.0	943.1
belg+2arcs_356	471.7k	471.7k	0.0	225.1	471.7k	471.7k	0.0	1350.7	471.7k	471.7k	0.0	848.9
belg+2arcs_357	406.6k	406.6k	0.0	142.7	406.6k	406.6k	0.0	571.2	406.6k	406.6k	0.0	1231.1
belg+2arcs_358	708.4k	649.9k	9.0	14400.0	698.9k	664.9k	5.1	14400.0	773.0k	601.1k	28.6	14400.0
belg+2arcs_359	496.3k	399.1k	24.4	14400.0	480.2k	447.4k	7.3	14400.0	593.4k	313.0k	89.6	14400.0
belg+2arcs_360	264.0k	264.0k	0.0	117.0	264.0k	264.0k	0.0	244.7	264.0k	264.0k	0.0	144.3
belg+2arcs_361	619.9k	619.3k	0.1	905.1	619.9k	619.5k	0.1	1349.1	624.5k	538.8k	15.9	14400.0
belg+2arcs_362	328.8k	328.8k	0.0	56.3	328.8k	328.8k	0.0	670.7	328.8k	328.8k	0.0	362.4
belg+2arcs_363	423.1k	423.1k	0.0	2119.8	423.1k	423.1k	0.0	2285.1	423.1k	386.5k	9.5	14400.0
belg+2arcs_364	400.0k	400.0k	0.0	109.8	400.0k	400.0k	0.0	726.6	400.0k	400.0k	0.0	5499.9
belg+2arcs_365	602.3k	500.3k	20.4	14400.0	564.1k	548.5k	2.9	14400.0	688.2k	414.1k	66.2	14400.0
belg+2arcs_366	624.4k	624.4k	0.0	4096.2	624.4k	624.4k	0.0	4234.8	694.3k	461.8k	50.3	14400.0
belg+2arcs_367	363.8k	363.8k	0.0	678.6	363.8k	363.8k	0.0	1885.8	364.4k	333.9k	9.1	14400.0
belg+2arcs_368	342.4k	342.4k	0.0	1728.2	342.4k	339.3k	0.9	14400.0	345.6k	314.7k	9.8	14400.0
belg+2arcs_369	228.2k	228.2k	0.0	170.5	228.2k	228.2k	0.0	201.0	228.2k	228.2k	0.0	450.1
belg+2arcs_370	298.2k	298.1k	0.0	154.5	298.2k	298.2k	0.0	187.9	298.2k	298.2k	0.0	6248.1
belg+2arcs_371	593.6k	458.3k	29.5	14400.0	589.7k	589.7k	0.0	12720.7	681.9k	369.7k	84.4	14400.0
belg+2arcs_372	457.1k	403.2k	13.3	14400.0	431.5k	431.5k	0.0	1216.8	734.8k	339.6k	116.4	14400.0
belg+2arcs_373	487.4k	487.0k	0.1	2151.2	487.4k	487.4k	0.0	1380.7	495.0k	444.7k	11.3	14400.0
belg+2arcs_374	311.9k	296.7k	5.1	14400.0	310.8k	296.7k	4.8	14400.0	310.8k	287.3k	8.2	14400.0
belg+2arcs_375	266.9k	266.9k	0.0	152.9	266.9k	266.9k	0.0	121.5	266.9k	266.9k	0.0	1326.0
belg+2arcs_376	687.0k	687.0k	0.0	4663.2	687.0k	686.9k	0.0	3209.8	693.7k	577.6k	20.1	14400.0
belg+2arcs_377	1331.4k	291.3k	357.1	14400.0	416.0k	383.7k	8.4	14400.0	1010.9k	211.7k	377.4	14400.0
belg+2arcs_378	353.6k	353.6k	0.0	1088.5	353.6k	353.6k	0.0	580.9	353.6k	324.3k	9.0	14400.0
belg+2arcs_379	512.5k	512.5k	0.0	557.9	512.5k	512.5k	0.0	2840.0	516.4k	413.7k	24.8	14400.0
belg+2arcs_380	389.7k	389.7k	0.0	247.6	389.7k	389.7k	0.0	1480.1	389.7k	389.7k	0.0	5114.2
belg+2arcs_381	565.1k	565.1k	0.0	10059.7	565.1k	565.1k	0.0	9822.8	578.9k	431.9k	34.0	14400.0
belg+2arcs_382	570.3k	570.3k	0.0	2660.0	570.3k	570.3k	0.0	7771.3	570.8k	532.0k	7.3	14400.0
belg+2arcs_383	250.1k	218.4k	14.5	14400.0	248.5k	248.5k	0.0	4163.1	290.5k	173.5k	67.4	14400.0
belg+2arcs_384	384.9k	384.9k	0.0	214.2	384.9k	384.9k	0.0	597.4	384.9k	384.9k	0.0	1733.3
belg+2arcs_385	440.1k	440.1k	0.0	948.8	440.1k	440.1k	0.0	1650.7	466.0k	416.2k	12.0	14400.0
belg+2arcs_386	746.9k	585.5k	27.6	14400.0	667.8k	647.9k	3.1	14400.0	704.7k	517.7k	36.1	14400.0
belg+2arcs_387	359.2k	310.2k	15.8	14400.0	348.1k	317.5k	9.6	14400.0	435.9k	261.2k	66.9	14400.0
belg+2arcs_388	257.7k	257.7k	0.0	493.3	257.7k	257.5k	0.1	593.5	257.7k	257.7k	0.0	5937.6
belg+2arcs_389	325.5k	325.5k	0.0	385.0	325.5k	325.2k	0.1	624.9	325.5k	325.5k	0.0	1215.4
belg+2arcs_390	455.9k	455.9k	0.0	354.0	455.9k	455.9k	0.0	908.4	455.9k	455.9k	0.0	3972.3
belg+2arcs_391	373.4k	373.4k	0.0	514.0	373.4k	373.4k	0.0	1211.9	373.4k	373.4k	0.0	7213.5
belg+2arcs_392	271.1k	271.1k	0.0	758.8	271.1k	271.1k	0.0	1190.4	271.1k	254.5k	6.5	14400.0
belg+2arcs_393	435.3k	435.3k	0.0	5474.4	435.3k	435.3k	0.0	3622.2	438.7k	386.5k	13.5	14400.0
belg+2arcs_394	315.5k	315.5k	0.0	195.0	315.5k	315.5k	0.0	515.3	315.5k	315.5k	0.0	216.9
belg+2arcs_395	402.8k	402.8k	0.0	150.4	402.8k	402.8k	0.0	544.1	402.8k	402.8k	0.0	143.8
belg+2arcs_396	284.1k	284.1k	0.0	8715.6	284.1k	268.6k	5.8	14400.0	312.4k	224.8k	39.0	14400.0
belg+2arcs_397	649.0k	571.7k	13.5	14400.0	654.3k	588.3k	11.2	14400.0	863.7k	445.6k	93.8	14400.0
belg+2arcs_398	331.2k	331.2k	0.0	185.4	331.2k	331.2k	0.0	834.7	331.2k	331.2k	0.0	789.0
belg+2arcs_399	320.0k	320.0k	0.0	1214.7	320.0k	320.0k	0.0	6834.0	344.2k	283.0k	21.6	14400.0
belg+2arcs_400	298.4k	298.2k	0.1	303.3	298.4k	298.4k	0.0	285.4	298.4k	298.4k	0.0	200.3
belg+2arcs_401	460.2k	396.5k	16.1	14400.0	432.1k	432.1k	0.0	8361.5	458.7k	349.8k	31.1	14400.0
belg+2arcs_402	374.3k	374.3k	0.0	1128.9	374.3k	374.3k	0.0	1001.1	374.3k	340.6k	9.9	14400.0
belg+2arcs_403	467.9k	429.5k	8.9	14400.0	461.3k	424.3k	8.7	14400.0	538.1k	326.0k	65.1	14400.0
belg+2arcs_404	482.1k	482.1k	0.0	1941.9	482.1k	482.1k	0.0	2136.5	487.3k	463.1k	5.2	14400.0
belg+2arcs_405	595.5k	553.6k	7.6	14400.0	589.0k	589.0k	0.0	10919.6	600.2k	444.1k	35.1	14400.0
belg+2arcs_406	263.8k	263.8k	0.0	1637.2	263.8k	263.8k	0.0	755.7	263.8k	263.8k	0.0	2584.7
belg+2arcs_407	478.0k	478.0k	0.0	785.0	478.0k	478.0k	0.0	2687.7	478.0k	478.0k	0.0	14385.5
belg+2arcs_408	173.1k	173.1k	0.0	17.7	173.1k	173.1k	0.0	100.3	173.1k	173.1k	0.0	98.5
belg+2arcs_409	305.4k	269.9k	13.2	14400.0	305.4k	305.4k	0.0	6049.8	305.4k	305.4k	0.0	11310.4
belg+2arcs_410	349.3k	349.2k	0.0	7185.0	349.3k	349.3k	0.0	5321.1	442.0k	284.3k	55.5	14400.0
belg+2arcs_411	418.7k	418.7k	0.0	200.7	418.7k	418.7k	0.0	374.9	418.7k	418.7k	0.0	1446.1
belg+2arcs_412	230.5k	230.5k	0.0	242.6	230.5k	230.5k	0.0	192.1	230.5k	230.5k	0.0	178.4
belg+2arcs_413	354.5k	354.5k	0.0	2562.4	354.5k	354.5k	0.0	5072.9	354.9k	334.0k	6.2	14400.0
belg+2arcs_414	304.0k	304.0k	0.0	647.9	304.0k	304.0k	0.0	242.0	304.0k	261.4k	16.3	14400.0
belg+2arcs_415	227.6k	227.6k	0.0	138.6	227.6k	227.6k	0.0	1003.3	227.6k	227.6k	0.0	7155.8
belg+2arcs_416	513.4k	513.4k	0.0	9306.9	513.4k	501.6k	2.4	14400.0	542.1k	432.5k	25.4	14400.0
belg+2arcs_417	370.4k	370.4k	0.0	793.8	370.4k	370.4k	0.0	1359.6	370.4k	359.6k	3.0	14400.0
belg+2arcs_418	599.5k	599.5k	0.0	4948.5	599.5k	599.0k	0.1	2443.7	601.6k	504.7k	19.2	14400.0
belg+2arcs_419	357.9k	357.9k	0.0	1742.7	357.9k	357.9k	0.0	1475.7	357.9k	357.9k	0.0	5311.5
belg+2arcs_420	335.5k	335.5k	0.0	162.2	335.5k	335.5k	0.0	243.9	335.5k	335.5k	0.0	269.2
belg+2arcs_421	339.1k	339.1k	0.0	90.7	339.1k	339.1k	0.0	1497.0	339.1k	339.1k	0.0	3319.1
belg+2arcs_422	389.2k	389.2k	0.0	128.5	389.2k	389.2k	0.0	1033.4	389.2k	389.2k	0.0	5869.8
belg+2arcs_423	296.7k	296.7k	0.0	244.6	296.7k	296.7k	0.0	1125.3	296.7k	296.7k	0.0	11524.9
belg+2arcs_424	574.8k	509.8k	12.8	14400.0	570.4k	570.4k	0.0	4196.8	584.1k	473.3k	23.4	14400.0
belg+2arcs_425	386.2k	386.2k	0.0	1085.0	386.2k	386.1k	0.0	1413.6	386.2k	386.2k	0.0	6425.2
belg+2arcs_426	408.4k	306.4k	33.3	14400.0	396.7k	343.0k	15.6	14400.0	574.5k	222.2k	158.6	14400.0
belg+2arcs_427	332.2k	332.2k	0.0	392.8	332.2k	332.2k	0.0	573.4	332.2k	332.2k	0.0	4005.8
belg+2arcs_428	285.8k	285.8k	0.0	196.0	285.8k	285.8k	0.0	637.9	285.8k	285.8k	0.0	645.3
belg+2arcs_429	569.1k	569.1k	0.0	951.7	569.1k	569.1k	0.0	2166.9	569.1k	569.1k	0.0	5177.6
belg+2arcs_430	485.7k	485.7k	0.0	500.1	485.7k	485.7k	0.0	1335.3	485.7k	485.7k	0.0	6114.2
belg+2arcs_431	755.3k	695.5k	8.6	14400.0	754.6k	754.6k	0.0	10457.6	982.7k	545.2k	80.2	14400.0
belg+2arcs_432	236.5k	236.5k	0.0	67.2	236.5k	236.5k	0.0	295.2	236.5k	236.5k	0.0	79.4

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Table A.25: Comparison of discrete models on *Circuit rank* + 2 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_433	234.3k	234.3k	0.0	41.0	234.3k	234.3k	0.0	190.2	234.3k	234.3k	0.0	295.0
belg+2arcs_434	510.2k	492.8k	3.5	14400.0	510.2k	510.2k	0.0	910.1	510.2k	510.2k	0.0	6998.4
belg+2arcs_435	511.4k	511.4k	0.0	1403.2	511.4k	511.4k	0.0	4208.7	517.7k	455.5k	13.7	14400.0
belg+2arcs_436	383.7k	383.7k	0.0	1882.5	383.7k	383.7k	0.0	2824.1	723.4k	276.8k	161.3	14400.0
belg+2arcs_437	355.4k	355.4k	0.0	3204.5	355.4k	355.4k	0.0	3327.1	384.5k	299.0k	28.6	14400.0
belg+2arcs_438	421.3k	421.3k	0.0	487.6	421.3k	421.3k	0.0	466.2	421.3k	421.3k	0.0	1441.2
belg+2arcs_439	554.1k	554.1k	0.0	7176.3	554.1k	553.9k	0.0	5880.6	562.8k	332.7k	69.2	14400.0
belg+2arcs_440	603.1k	603.1k	0.0	1860.2	603.1k	603.1k	0.0	6569.2	627.6k	490.2k	28.0	14400.0
belg+2arcs_441	339.7k	339.7k	0.0	151.7	339.7k	339.4k	0.1	202.6	339.7k	339.7k	0.0	356.2
belg+2arcs_442	234.3k	234.3k	0.0	178.0	234.3k	234.2k	0.1	951.1	238.1k	223.4k	6.6	14400.0
belg+2arcs_443	420.8k	420.8k	0.0	3528.6	420.8k	420.7k	0.0	4614.3	427.6k	369.9k	15.6	14400.0
belg+2arcs_444	276.7k	276.7k	0.0	302.2	276.7k	276.7k	0.0	349.1	276.7k	276.7k	0.0	1074.0
belg+2arcs_445	657.0k	598.3k	9.8	14400.0	681.0k	591.8k	15.1	14400.0	676.8k	490.4k	38.0	14400.0
belg+2arcs_446	340.8k	340.8k	0.0	304.7	340.8k	340.8k	0.0	700.5	340.8k	340.8k	0.0	3171.6
belg+2arcs_447	392.0k	392.0k	0.0	202.7	392.0k	392.0k	0.0	887.1	392.0k	392.0k	0.0	364.0
belg+2arcs_448	407.6k	319.5k	27.6	14400.0	398.1k	382.8k	4.0	14400.0	597.7k	255.0k	134.4	14400.0
belg+2arcs_449	384.0k	384.0k	0.0	2123.0	384.0k	384.0k	0.0	2079.2	389.8k	344.7k	13.1	14400.0
belg+2arcs_450	355.5k	355.5k	0.0	9242.3	355.5k	355.2k	0.1	6695.2	360.1k	283.9k	26.8	14400.0
belg+2arcs_451	165.2k	165.2k	0.0	50.9	165.2k	165.2k	0.0	131.9	165.2k	165.2k	0.0	86.8
belg+2arcs_452	424.6k	424.6k	0.0	882.9	424.6k	424.6k	0.0	1099.2	424.6k	424.6k	0.0	3990.6
belg+2arcs_453	686.4k	610.4k	12.4	14400.0	671.0k	650.5k	3.1	14400.0	693.5k	515.7k	34.5	14400.0
belg+2arcs_454	257.9k	257.9k	0.0	302.0	257.9k	257.9k	0.0	744.9	257.9k	257.6k	0.1	584.6
belg+2arcs_455	207.7k	207.7k	0.0	82.9	207.7k	207.7k	0.0	146.3	207.7k	207.7k	0.0	724.9
belg+2arcs_456	566.2k	512.7k	10.4	14400.0	550.8k	550.8k	0.0	6681.4	614.1k	314.8k	95.0	14400.0
belg+2arcs_457	339.2k	317.8k	6.7	14400.0	338.2k	338.2k	0.0	2097.6	341.2k	306.6k	11.3	14400.0
belg+2arcs_458	720.8k	608.6k	18.4	14400.0	715.9k	642.8k	11.4	14400.0	855.7k	459.7k	86.1	14400.0
belg+2arcs_459	167.3k	167.3k	0.0	17.2	167.3k	167.3k	0.0	62.9	167.3k	167.3k	0.0	36.8
belg+2arcs_460	236.8k	236.8k	0.0	71.5	236.8k	236.8k	0.0	140.7	236.8k	236.8k	0.0	149.2
belg+2arcs_461	383.8k	383.4k	0.1	271.5	383.8k	383.8k	0.0	779.1	383.8k	383.8k	0.0	696.1
belg+2arcs_462	313.6k	313.5k	0.0	1924.3	313.6k	313.6k	0.0	9580.2	321.0k	273.2k	17.5	14400.0
belg+2arcs_463	336.8k	336.8k	0.0	187.8	336.8k	336.8k	0.0	1082.3	336.8k	336.8k	0.0	7598.7
belg+2arcs_464	398.8k	278.0k	43.4	14400.0	310.1k	310.1k	0.0	615.2	310.3k	286.2k	8.4	14400.0
belg+2arcs_465	364.1k	364.1k	0.0	1191.1	364.1k	364.1k	0.0	2353.6	364.1k	340.8k	6.8	14400.0
belg+2arcs_466	290.7k	290.7k	0.0	2097.1	290.7k	290.7k	0.0	211.5	290.7k	290.7k	0.0	621.9
belg+2arcs_467	537.5k	537.5k	0.0	10176.8	537.5k	537.5k	0.0	8456.7	557.4k	414.3k	34.6	14400.0
belg+2arcs_468	447.8k	301.7k	48.4	14400.0	463.8k	374.2k	24.0	14400.0	725.6k	205.6k	253.0	14400.0
belg+2arcs_469	335.2k	335.2k	0.0	208.5	335.2k	335.2k	0.0	2054.8	335.2k	335.2k	0.0	2503.9
belg+2arcs_470	327.7k	327.7k	0.0	1766.2	327.7k	327.4k	0.1	2449.1	327.7k	327.7k	0.0	6460.2
belg+2arcs_471	424.0k	424.0k	0.0	243.2	424.0k	424.0k	0.0	852.7	424.0k	424.0k	0.0	789.9
belg+2arcs_472	286.4k	286.4k	0.0	13.3	286.4k	286.4k	0.0	257.9	286.4k	286.4k	0.0	192.0
belg+2arcs_473	612.3k	480.7k	27.4	14400.0	570.5k	529.4k	7.8	14400.0	622.0k	382.8k	62.5	14400.0
belg+2arcs_474	254.0k	254.0k	0.0	353.7	254.0k	254.0k	0.0	344.7	254.0k	254.0k	0.0	690.6
belg+2arcs_475	273.5k	273.5k	0.0	1112.6	273.5k	273.5k	0.0	304.8	273.5k	273.5k	0.0	4295.0
belg+2arcs_476	461.9k	461.9k	0.0	263.5	461.9k	461.9k	0.0	3187.4	462.5k	429.7k	7.6	14400.0
belg+2arcs_477	439.6k	439.6k	0.0	2497.7	439.6k	439.6k	0.0	6224.6	439.6k	439.6k	0.0	11018.3
belg+2arcs_478	471.8k	471.8k	0.0	3189.7	471.8k	471.8k	0.0	5123.1	473.6k	449.9k	5.3	14400.0
belg+2arcs_479	480.5k	403.1k	19.2	14400.0	474.4k	458.6k	3.4	14400.0	635.2k	329.3k	92.9	14400.0
belg+2arcs_480	278.2k	278.2k	0.0	765.9	278.2k	278.2k	0.0	496.3	278.2k	278.0k	0.1	6641.0
belg+2arcs_481	482.5k	482.5k	0.0	1140.1	482.5k	482.5k	0.0	2379.4	482.5k	482.5k	0.0	2978.9
belg+2arcs_482	415.6k	415.6k	0.0	5103.9	415.6k	415.6k	0.0	12364.1	416.4k	393.8k	5.8	14400.0
belg+2arcs_483	238.0k	238.0k	0.0	17.0	238.0k	238.0k	0.0	297.9	238.0k	238.0k	0.0	251.7
belg+2arcs_484	257.4k	257.4k	0.0	373.4	257.4k	257.4k	0.0	307.5	257.4k	257.4k	0.0	1292.8
belg+2arcs_485	269.7k	269.7k	0.0	236.5	269.7k	269.7k	0.0	685.0	269.7k	269.7k	0.0	1016.8
belg+2arcs_486	552.4k	538.5k	2.6	14400.0	548.5k	548.5k	0.0	2947.8	566.6k	502.8k	12.7	14400.0
belg+2arcs_487	327.0k	327.0k	0.0	2034.2	327.0k	327.0k	0.0	2138.1	327.0k	327.0k	0.0	3804.3
belg+2arcs_488	451.5k	451.5k	0.0	1976.6	451.5k	451.5k	0.0	1192.0	451.6k	420.2k	7.5	14400.0
belg+2arcs_489	489.7k	489.7k	0.0	730.0	489.7k	489.7k	0.0	2198.0	489.7k	489.7k	0.0	5765.3
belg+2arcs_490	279.6k	279.6k	0.0	1078.6	279.6k	279.6k	0.0	640.4	279.6k	279.6k	0.0	1281.2
belg+2arcs_491	343.5k	343.5k	0.0	13708.7	343.7k	331.1k	3.8	14400.0	351.2k	272.1k	29.1	14400.0
belg+2arcs_492	270.7k	270.7k	0.0	365.0	270.7k	270.7k	0.0	491.9	270.7k	270.7k	0.0	3939.6
belg+2arcs_493	456.1k	456.1k	0.0	803.6	456.1k	456.1k	0.0	2263.3	456.1k	456.1k	0.0	8785.7
belg+2arcs_494	407.6k	407.6k	0.0	4173.1	407.6k	407.6k	0.0	7767.5	420.7k	365.5k	15.1	14400.0
belg+2arcs_495	519.2k	476.9k	8.9	14400.0	512.0k	460.7k	11.1	14400.0	587.5k	390.5k	50.5	14400.0
belg+2arcs_496	323.3k	323.3k	0.0	146.8	323.3k	323.3k	0.0	1249.0	323.3k	323.3k	0.0	3183.8
belg+2arcs_497	386.4k	386.4k	0.0	248.4	386.4k	386.4k	0.0	636.2	386.4k	386.4k	0.0	9774.4
belg+2arcs_498	516.1k	516.1k	0.0	163.7	516.1k	516.1k	0.0	256.1	516.1k	516.1k	0.0	1276.4
belg+2arcs_499	432.8k	401.6k	7.8	14400.0	432.8k	432.8k	0.0	5609.4	438.0k	336.3k	30.2	14400.0
belg+2arcs_500	316.4k	316.4k	0.0	4879.3	316.4k	316.4k	0.0	11969.2	337.4k	252.1k	33.9	14400.0

Table A.26: Detailed results of the split-pipe models on *Circuit rank* + 2 arcs, as summarized in Figure 3.16b and Table 3.4d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_1	554.5k	554.0k	0.1	14400.0	554.5k	554.0k	0.1	440.1
belg+2arcs_2	472.2k	450.0k	4.9	14400.0	472.2k	425.4k	11.0	14400.0
belg+2arcs_3	421.9k	345.0k	22.3	14400.0	421.9k	369.9k	14.1	14400.0
belg+2arcs_4	627.5k	560.7k	11.9	14400.0	627.5k	558.7k	12.3	14400.0
belg+2arcs_5	657.8k	649.1k	1.3	14400.0	657.8k	646.3k	1.8	14400.0
belg+2arcs_6	319.1k	318.8k	0.1	14.6	319.1k	318.8k	0.1	465.6
belg+2arcs_7	722.6k	686.7k	5.2	14400.0	722.6k	716.4k	0.9	14400.0
belg+2arcs_8	133.8k	124.5k	7.5	14400.0	133.8k	133.7k	0.0	3.8
belg+2arcs_9	492.5k	463.0k	6.4	14400.0	492.5k	472.7k	4.2	14400.0
belg+2arcs_10	513.0k	488.6k	5.0	14400.0	513.0k	512.5k	0.1	10792.2
belg+2arcs_11	454.8k	454.4k	0.1	17.8	454.8k	453.7k	0.2	14400.0
belg+2arcs_12	280.5k	279.8k	0.2	14400.0	280.5k	280.2k	0.1	97.8
belg+2arcs_13	469.5k	448.9k	4.6	14400.0	469.5k	453.0k	3.6	14400.0
belg+2arcs_14	672.3k	669.5k	0.4	14400.0	672.3k	671.7k	0.1	55.1
belg+2arcs_15	192.4k	192.2k	0.1	1658.5	192.4k	192.2k	0.1	1.9
belg+2arcs_16	654.2k	652.5k	0.3	14400.0	654.2k	653.5k	0.1	2918.5
belg+2arcs_17	386.3k	385.3k	0.3	14400.0	386.3k	385.9k	0.1	350.6
belg+2arcs_18	582.6k	582.0k	0.1	476.3	582.6k	581.2k	0.2	14400.0
belg+2arcs_19	297.6k	297.3k	0.1	1280.3	297.6k	297.6k	0.0	1.4
belg+2arcs_20	536.1k	497.3k	7.8	14400.0	535.9k	471.3k	13.7	14400.0
belg+2arcs_21	191.0k	190.8k	0.1	11.5	191.0k	190.8k	0.1	217.5
belg+2arcs_22	439.1k	431.8k	1.7	14400.0	439.1k	438.6k	0.1	128.3
belg+2arcs_23	347.0k	317.8k	9.2	14400.0	347.0k	346.7k	0.1	40.2
belg+2arcs_24	435.1k	432.1k	0.7	14400.0	435.1k	434.8k	0.1	2363.9
belg+2arcs_25	376.5k	374.7k	0.5	14400.0	376.5k	374.7k	0.5	14400.0
belg+2arcs_26	513.3k	512.8k	0.1	766.9	513.3k	512.8k	0.1	146.7
belg+2arcs_27	264.6k	252.6k	4.7	14400.0	264.6k	258.1k	2.5	14400.0
belg+2arcs_28	525.4k	446.8k	17.6	14400.0	525.4k	524.5k	0.2	14400.0
belg+2arcs_29	380.2k	326.4k	16.5	14400.0	380.2k	379.8k	0.1	12505.0
belg+2arcs_30	351.0k	328.2k	7.0	14400.0	351.0k	350.7k	0.1	11078.8
belg+2arcs_31	339.5k	337.7k	0.5	14400.0	339.5k	338.2k	0.4	14400.0
belg+2arcs_32	521.1k	520.3k	0.2	14400.0	521.1k	497.6k	4.7	14400.0
belg+2arcs_33	269.9k	207.1k	30.3	14400.0	269.9k	269.9k	0.0	41.4
belg+2arcs_34	193.5k	193.3k	0.1	2.3	193.5k	193.4k	0.1	1.3
belg+2arcs_35	392.7k	390.7k	0.5	14400.0	392.7k	389.2k	0.9	14400.0
belg+2arcs_36	514.1k	475.1k	8.2	14400.0	514.1k	480.5k	7.0	14400.0
belg+2arcs_37	300.3k	297.3k	1.0	14400.0	300.3k	300.0k	0.1	270.1
belg+2arcs_38	679.1k	678.4k	0.1	196.4	679.1k	677.9k	0.2	14400.0
belg+2arcs_39	261.0k	261.0k	0.0	15.8	261.0k	260.7k	0.1	31.7
belg+2arcs_40	382.9k	362.6k	5.6	14400.0	382.9k	382.5k	0.1	406.4
belg+2arcs_41	583.1k	533.0k	9.4	14400.0	582.8k	529.6k	10.1	14400.0
belg+2arcs_42	472.9k	448.7k	5.4	14400.0	473.3k	427.5k	10.7	14400.0
belg+2arcs_43	373.9k	360.5k	3.7	14400.0	370.6k	369.6k	0.3	14400.0
belg+2arcs_44	366.1k	355.1k	3.1	14400.0	366.1k	365.8k	0.1	24.0
belg+2arcs_45	149.2k	145.7k	2.4	14400.0	149.2k	149.1k	0.1	4.0
belg+2arcs_46	496.5k	424.9k	16.8	14400.0	496.5k	496.0k	0.1	74.1
belg+2arcs_47	407.9k	357.0k	14.3	14400.0	407.9k	363.3k	12.3	14400.0
belg+2arcs_48	387.4k	379.8k	2.0	14400.0	387.4k	384.2k	0.8	14400.0
belg+2arcs_49	271.2k	255.2k	6.3	14400.0	271.2k	271.0k	0.1	6.7
belg+2arcs_50	522.4k	522.0k	0.1	69.2	522.3k	479.0k	9.0	14400.0
belg+2arcs_51	231.0k	230.7k	0.1	269.3	231.0k	230.7k	0.1	2028.1
belg+2arcs_52	441.7k	441.2k	0.1	2222.0	441.7k	436.8k	1.1	14400.0
belg+2arcs_53	313.7k	291.9k	7.5	14400.0	313.8k	289.4k	8.4	14400.0
belg+2arcs_54	451.5k	416.3k	8.5	14400.0	451.5k	451.0k	0.1	184.9
belg+2arcs_55	386.4k	325.6k	18.7	14400.0	386.4k	325.3k	18.8	14400.0
belg+2arcs_56	447.5k	444.3k	0.7	14400.0	447.5k	444.3k	0.7	14400.0
belg+2arcs_57	449.4k	442.9k	1.5	14400.0	448.4k	431.1k	4.0	14400.0
belg+2arcs_58	393.7k	393.3k	0.1	34.0	393.7k	393.3k	0.1	325.5
belg+2arcs_59	231.6k	228.5k	1.4	14400.0	231.6k	227.1k	2.0	14400.0
belg+2arcs_60	379.7k	334.8k	13.4	14400.0	379.7k	365.5k	3.9	14400.0
belg+2arcs_61	408.6k	408.0k	0.2	14400.0	408.6k	387.2k	5.5	14400.0
belg+2arcs_62	620.8k	612.7k	1.3	14400.0	620.8k	611.6k	1.5	14400.0
belg+2arcs_63	457.5k	444.1k	3.0	14400.0	457.5k	450.6k	1.5	14400.0
belg+2arcs_64	263.6k	263.6k	0.0	45.7	263.6k	263.6k	0.0	1.7
belg+2arcs_65	480.9k	458.2k	5.0	14400.0	480.9k	450.4k	6.8	14400.0
belg+2arcs_66	465.9k	465.9k	0.0	13.8	465.9k	465.4k	0.1	967.0
belg+2arcs_67	370.7k	352.8k	5.1	14400.0	370.7k	342.1k	8.3	14400.0
belg+2arcs_68	504.4k	502.7k	0.3	14400.0	504.4k	494.8k	1.9	14400.0
belg+2arcs_69	564.7k	533.6k	5.8	14400.0	564.6k	510.4k	10.6	14400.0
belg+2arcs_70	432.8k	430.4k	0.6	14400.0	432.8k	427.9k	1.1	14400.0
belg+2arcs_71	439.9k	439.8k	0.0	22.4	439.9k	434.4k	1.3	14400.0
belg+2arcs_72	269.0k	261.5k	2.9	14400.0	269.0k	266.5k	1.0	14400.0
belg+2arcs_73	195.2k	190.3k	2.6	14400.0	195.2k	195.1k	0.0	2.4
belg+2arcs_74	612.4k	611.8k	0.1	5.8	612.4k	611.4k	0.2	14400.0

continued on next page

Table A.26: Comparison of split-pipe models on *Circuit rank* + 2 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_75	462.6k	461.3k	0.3	14400.0	462.6k	456.1k	1.4	14400.0
belg+2arcs_76	239.0k	230.9k	3.5	14400.0	239.0k	238.8k	0.1	3.0
belg+2arcs_77	352.1k	345.5k	1.9	14400.0	352.1k	321.7k	9.4	14400.0
belg+2arcs_78	253.2k	252.9k	0.1	30.7	253.2k	253.1k	0.0	0.9
belg+2arcs_79	277.4k	265.1k	4.6	14400.0	277.4k	230.5k	20.4	14400.0
belg+2arcs_80	406.7k	393.2k	3.4	14400.0	406.7k	395.7k	2.8	14400.0
belg+2arcs_81	457.2k	455.6k	0.4	14400.0	457.2k	455.8k	0.3	14400.0
belg+2arcs_82	101.2k	101.0k	0.2	14400.0	101.2k	101.1k	0.1	1.2
belg+2arcs_83	567.1k	555.6k	2.1	14400.0	567.1k	553.6k	2.4	14400.0
belg+2arcs_84	558.9k	558.4k	0.1	17.7	558.9k	558.4k	0.1	5906.8
belg+2arcs_85	442.6k	415.6k	6.5	14400.0	442.6k	433.7k	2.0	14400.0
belg+2arcs_86	479.2k	479.2k	0.0	126.3	479.2k	478.7k	0.1	15.1
belg+2arcs_87	372.6k	366.2k	1.7	14400.0	372.6k	346.9k	7.4	14400.0
belg+2arcs_88	454.0k	453.6k	0.1	4430.1	454.0k	453.6k	0.1	150.0
belg+2arcs_89	967.3k	966.3k	0.1	587.9	967.3k	866.9k	11.6	14400.0
belg+2arcs_90	524.7k	524.2k	0.1	11257.1	524.7k	524.2k	0.1	44.7
belg+2arcs_91	586.0k	585.4k	0.1	4895.0	586.0k	557.3k	5.1	14400.0
belg+2arcs_92	549.1k	508.9k	7.9	14400.0	549.1k	548.6k	0.1	103.1
belg+2arcs_93	737.5k	648.6k	13.7	14400.0	737.6k	714.8k	3.2	14400.0
belg+2arcs_94	438.3k	417.5k	5.0	14400.0	438.3k	424.0k	3.4	14400.0
belg+2arcs_95	232.1k	220.3k	5.4	14400.0	232.1k	229.9k	1.0	14400.0
belg+2arcs_96	403.6k	382.2k	5.6	14400.0	403.6k	393.8k	2.5	14400.0
belg+2arcs_97	293.7k	290.5k	1.1	14400.0	293.7k	293.4k	0.1	175.9
belg+2arcs_98	462.7k	461.6k	0.2	14400.0	462.7k	459.7k	0.7	14400.0
belg+2arcs_99	559.8k	556.3k	0.6	14400.0	559.8k	558.6k	0.2	14400.0
belg+2arcs_100	470.6k	467.1k	0.7	14400.0	470.6k	470.1k	0.1	488.9
belg+2arcs_101	385.7k	385.7k	0.0	10.8	385.7k	385.3k	0.1	3325.4
belg+2arcs_102	520.4k	519.8k	0.1	14400.0	520.6k	497.9k	4.6	14400.0
belg+2arcs_103	286.4k	286.1k	0.1	95.3	286.4k	286.1k	0.1	54.2
belg+2arcs_104	359.3k	339.0k	6.0	14400.0	359.3k	359.3k	0.0	196.1
belg+2arcs_105	249.3k	249.1k	0.1	20.9	249.3k	249.0k	0.1	0.8
belg+2arcs_106	406.9k	372.5k	9.3	14400.0	406.9k	406.5k	0.1	587.7
belg+2arcs_107	544.6k	540.7k	0.7	14400.0	544.6k	506.0k	7.6	14400.0
belg+2arcs_108	206.1k	205.7k	0.2	14400.0	206.1k	205.9k	0.1	29.5
belg+2arcs_109	473.9k	473.4k	0.1	11199.5	473.9k	456.4k	3.8	14400.0
belg+2arcs_110	523.6k	516.9k	1.3	14400.0	523.6k	433.8k	20.7	14400.0
belg+2arcs_111	497.7k	492.2k	1.1	14400.0	497.7k	497.3k	0.1	37.3
belg+2arcs_112	288.6k	287.9k	0.2	14400.0	288.6k	287.7k	0.3	14400.0
belg+2arcs_113	223.6k	215.1k	3.9	14400.0	223.6k	223.3k	0.1	1.2
belg+2arcs_114	206.1k	203.3k	1.4	14400.0	206.1k	206.0k	0.1	2.5
belg+2arcs_115	583.7k	577.9k	1.0	14400.0	583.3k	579.6k	0.6	14400.0
belg+2arcs_116	341.4k	330.4k	3.3	14400.0	341.4k	327.5k	4.2	14400.0
belg+2arcs_117	286.9k	286.6k	0.1	459.2	286.9k	286.6k	0.1	2366.7
belg+2arcs_118	359.4k	338.3k	6.2	14400.0	359.4k	325.0k	10.6	14400.0
belg+2arcs_119	409.3k	404.4k	1.2	14400.0	409.3k	396.5k	3.2	14400.0
belg+2arcs_120	342.3k	341.9k	0.1	59.8	342.3k	341.9k	0.1	3.7
belg+2arcs_121	288.9k	288.6k	0.1	28.1	288.9k	288.6k	0.1	4.3
belg+2arcs_122	444.6k	393.1k	13.1	14400.0	444.6k	391.1k	13.7	14400.0
belg+2arcs_123	278.0k	277.7k	0.1	14400.0	278.2k	277.9k	0.1	1.9
belg+2arcs_124	461.9k	459.1k	0.6	14400.0	462.0k	461.6k	0.1	54.6
belg+2arcs_125	253.3k	250.0k	1.3	14400.0	253.3k	253.0k	0.1	14400.0
belg+2arcs_126	383.0k	382.7k	0.1	331.0	383.0k	382.7k	0.1	19.5
belg+2arcs_127	377.0k	377.0k	0.0	963.4	377.0k	376.6k	0.1	3.0
belg+2arcs_128	444.9k	444.5k	0.1	20.5	444.9k	429.2k	3.7	14400.0
belg+2arcs_129	252.6k	225.8k	11.9	14400.0	252.5k	252.3k	0.1	13.5
belg+2arcs_130	502.1k	501.6k	0.1	293.6	502.1k	470.6k	6.7	14400.0
belg+2arcs_131	408.4k	408.0k	0.1	2.7	408.4k	408.0k	0.1	5.5
belg+2arcs_132	286.8k	286.8k	0.0	9.3	286.8k	286.5k	0.1	5.3
belg+2arcs_133	523.6k	513.7k	1.9	14400.0	523.5k	516.5k	1.4	14400.0
belg+2arcs_134	716.8k	715.3k	0.2	14400.0	716.8k	683.5k	4.9	14400.0
belg+2arcs_135	333.9k	333.5k	0.1	14400.0	333.9k	333.7k	0.1	2.0
belg+2arcs_136	925.0k	906.2k	2.1	14400.0	925.0k	902.3k	2.5	14400.0
belg+2arcs_137	658.1k	655.1k	0.5	14400.0	658.1k	655.6k	0.4	14400.0
belg+2arcs_138	537.9k	533.1k	0.9	14400.0	537.9k	489.1k	10.0	14400.0
belg+2arcs_139	384.6k	384.2k	0.1	206.7	384.6k	369.4k	4.1	14400.0
belg+2arcs_140	330.8k	328.9k	0.6	14400.0	330.8k	328.8k	0.6	14400.0
belg+2arcs_141	431.3k	430.9k	0.1	4085.5	431.3k	400.0k	7.8	14400.0
belg+2arcs_142	520.6k	520.1k	0.1	103.7	520.6k	484.4k	7.5	14400.0
belg+2arcs_143	270.6k	270.4k	0.1	40.0	270.6k	265.4k	2.0	14400.0
belg+2arcs_144	302.9k	285.1k	6.2	14400.0	302.9k	302.9k	0.0	542.2
belg+2arcs_145	406.9k	406.5k	0.1	124.4	406.9k	406.5k	0.1	149.0
belg+2arcs_146	360.3k	360.0k	0.1	87.2	360.3k	360.0k	0.1	1.9
belg+2arcs_147	511.1k	437.6k	16.8	14400.0	511.1k	428.7k	19.2	14400.0
belg+2arcs_148	504.2k	497.3k	1.4	14400.0	504.2k	503.6k	0.1	14400.0
belg+2arcs_149	405.3k	404.9k	0.1	2704.9	405.3k	335.7k	20.7	14400.0
belg+2arcs_150	639.3k	617.9k	3.5	14400.0	639.3k	637.9k	0.2	14400.0
belg+2arcs_151	270.9k	262.4k	3.2	14400.0	270.9k	270.9k	0.0	1.2
belg+2arcs_152	359.3k	356.4k	0.8	14400.0	359.3k	359.3k	0.0	4.6

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Table A.26: Comparison of split-pipe models on *Circuit rank* + 2 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_153	439.7k	432.8k	1.6	14400.0	439.7k	432.0k	1.8	14400.0
belg+2arcs_154	349.6k	348.8k	0.2	14400.0	349.6k	349.2k	0.1	26.1
belg+2arcs_155	553.3k	529.3k	4.5	14400.0	553.3k	541.2k	2.2	14400.0
belg+2arcs_156	354.8k	351.4k	1.0	14400.0	354.8k	340.7k	4.1	14400.0
belg+2arcs_157	368.6k	368.6k	0.0	809.4	368.6k	368.3k	0.1	20.5
belg+2arcs_158	409.6k	372.4k	10.0	14400.0	409.6k	390.9k	4.8	14400.0
belg+2arcs_159	336.4k	323.8k	3.9	14400.0	336.4k	334.8k	0.5	14400.0
belg+2arcs_160	332.6k	325.5k	2.2	14400.0	332.6k	330.3k	0.7	14400.0
belg+2arcs_161	449.6k	448.5k	0.2	14400.0	449.6k	449.1k	0.1	543.9
belg+2arcs_162	548.5k	524.5k	4.6	14400.0	548.5k	547.9k	0.1	853.1
belg+2arcs_163	631.5k	629.7k	0.3	14400.0	631.5k	595.5k	6.0	14400.0
belg+2arcs_164	257.6k	257.3k	0.1	11.3	257.6k	257.3k	0.1	5190.5
belg+2arcs_165	295.7k	295.4k	0.1	8.7	295.7k	274.0k	7.9	14400.0
belg+2arcs_166	253.7k	202.6k	25.2	14400.0	253.7k	253.7k	0.0	89.5
belg+2arcs_167	468.9k	435.5k	7.7	14400.0	468.7k	410.2k	14.3	14400.0
belg+2arcs_168	488.5k	488.0k	0.1	13108.3	488.5k	488.0k	0.1	4.1
belg+2arcs_169	273.6k	229.8k	19.1	14400.0	273.6k	273.3k	0.1	182.8
belg+2arcs_170	360.1k	359.7k	0.1	2497.5	360.1k	359.9k	0.1	2.9
belg+2arcs_171	399.0k	395.5k	0.9	14400.0	399.0k	398.6k	0.1	10097.3
belg+2arcs_172	366.3k	365.9k	0.1	4.0	366.3k	365.9k	0.1	7.0
belg+2arcs_173	578.5k	577.9k	0.1	200.8	578.5k	577.9k	0.1	238.6
belg+2arcs_174	380.1k	379.4k	0.2	14400.0	380.1k	353.1k	7.7	14400.0
belg+2arcs_175	217.3k	217.1k	0.1	4.0	217.3k	217.0k	0.1	2.9
belg+2arcs_176	416.7k	414.8k	0.5	14400.0	416.7k	415.2k	0.4	14400.0
belg+2arcs_177	524.8k	456.6k	14.9	14400.0	524.5k	458.2k	14.5	14400.0
belg+2arcs_178	359.8k	350.4k	2.7	14400.0	359.8k	352.4k	2.1	14400.0
belg+2arcs_179	417.5k	410.2k	1.8	14400.0	417.5k	402.7k	3.7	14400.0
belg+2arcs_180	235.0k	234.7k	0.1	14400.0	235.0k	234.8k	0.1	3.2
belg+2arcs_181	290.9k	286.4k	1.6	14400.0	290.9k	290.6k	0.1	59.5
belg+2arcs_182	363.8k	363.4k	0.1	18.9	363.8k	346.3k	5.0	14400.0
belg+2arcs_183	288.9k	288.6k	0.1	533.0	288.9k	288.6k	0.1	2003.9
belg+2arcs_184	295.4k	247.6k	19.3	14400.0	295.4k	295.2k	0.1	59.3
belg+2arcs_185	248.1k	247.9k	0.1	2.5	248.1k	248.0k	0.1	1.0
belg+2arcs_186	423.8k	421.5k	0.5	14400.0	423.8k	391.2k	8.3	14400.0
belg+2arcs_187	700.7k	699.2k	0.2	14400.0	700.7k	689.4k	1.6	14400.0
belg+2arcs_188	223.2k	223.0k	0.1	12.6	223.2k	223.0k	0.1	38.3
belg+2arcs_189	464.3k	348.1k	33.4	14400.0	464.3k	457.7k	1.4	14400.0
belg+2arcs_190	363.3k	324.9k	11.8	14400.0	363.3k	363.0k	0.1	421.4
belg+2arcs_191	580.9k	553.4k	5.0	14400.0	580.9k	543.1k	7.0	14400.0
belg+2arcs_192	397.2k	396.8k	0.1	40.6	397.2k	380.2k	4.5	14400.0
belg+2arcs_193	297.0k	268.5k	10.6	14400.0	297.0k	296.7k	0.1	23.1
belg+2arcs_194	456.5k	455.7k	0.2	14400.0	456.5k	456.0k	0.1	67.7
belg+2arcs_195	411.0k	366.6k	12.1	14400.0	411.0k	368.5k	11.5	14400.0
belg+2arcs_196	547.7k	547.0k	0.1	14400.0	547.7k	547.2k	0.1	163.4
belg+2arcs_197	712.4k	671.1k	6.1	14400.0	712.4k	616.2k	15.6	14400.0
belg+2arcs_198	368.5k	343.9k	7.2	14400.0	368.5k	366.9k	0.4	14400.0
belg+2arcs_199	206.0k	206.0k	0.0	202.1	206.0k	206.0k	0.0	4.1
belg+2arcs_200	477.4k	475.4k	0.4	14400.0	477.4k	467.0k	2.2	14400.0
belg+2arcs_201	242.2k	239.9k	1.0	14400.0	242.2k	242.0k	0.1	975.0
belg+2arcs_202	333.1k	332.8k	0.1	25.2	333.1k	332.1k	0.3	14400.0
belg+2arcs_203	419.3k	341.7k	22.7	14400.0	419.3k	397.6k	5.4	14400.0
belg+2arcs_204	372.4k	365.1k	2.0	14400.0	372.4k	370.0k	0.6	14400.0
belg+2arcs_205	566.9k	550.9k	2.9	14400.0	566.9k	566.2k	0.1	14400.0
belg+2arcs_206	275.2k	275.0k	0.1	4.6	275.2k	274.4k	0.3	14400.0
belg+2arcs_207	451.8k	450.8k	0.2	14400.0	451.8k	451.3k	0.1	2.1
belg+2arcs_208	334.0k	333.7k	0.1	6.2	334.0k	333.9k	0.0	4.1
belg+2arcs_209	264.7k	263.1k	0.6	14400.0	264.7k	264.4k	0.1	21.9
belg+2arcs_210	449.3k	448.8k	0.1	47.8	449.3k	448.8k	0.1	73.7
belg+2arcs_211	464.9k	449.9k	3.3	14400.0	464.9k	450.4k	3.2	14400.0
belg+2arcs_212	267.8k	267.5k	0.1	29.8	267.8k	267.5k	0.1	5.9
belg+2arcs_213	646.6k	635.9k	1.7	14400.0	646.6k	646.0k	0.1	136.1
belg+2arcs_214	397.8k	397.4k	0.1	4921.6	397.8k	386.7k	2.9	14400.0
belg+2arcs_215	301.2k	275.4k	9.3	14400.0	301.2k	285.4k	5.5	14400.0
belg+2arcs_216	513.7k	513.2k	0.1	48.7	513.7k	506.8k	1.4	14400.0
belg+2arcs_217	348.3k	337.5k	3.2	14400.0	348.3k	348.0k	0.1	1786.5
belg+2arcs_218	571.9k	525.2k	8.9	14400.0	571.9k	562.6k	1.7	14400.0
belg+2arcs_219	431.7k	403.8k	6.9	14400.0	431.7k	409.6k	5.4	14400.0
belg+2arcs_220	451.6k	444.5k	1.6	14400.0	451.6k	432.9k	4.3	14400.0
belg+2arcs_221	191.3k	191.3k	0.0	1.5	191.3k	191.2k	0.1	1.4
belg+2arcs_222	434.7k	425.7k	2.1	14400.0	434.7k	425.8k	2.1	14400.0
belg+2arcs_223	382.6k	382.2k	0.1	8.3	382.6k	381.6k	0.3	14400.0
belg+2arcs_224	529.5k	497.6k	6.4	14400.0	529.5k	524.6k	0.9	14400.0
belg+2arcs_225	298.1k	295.3k	0.9	14400.0	298.1k	297.8k	0.1	515.4
belg+2arcs_226	372.9k	352.6k	5.8	14400.0	372.9k	372.9k	0.0	16.9
belg+2arcs_227	215.0k	214.8k	0.1	16.2	215.0k	214.9k	0.0	1.4
belg+2arcs_228	447.5k	443.1k	1.0	14400.0	447.5k	447.1k	0.1	226.4
belg+2arcs_229	526.3k	489.6k	7.5	14400.0	526.3k	525.5k	0.2	14400.0
belg+2arcs_230	501.8k	501.3k	0.1	2661.2	501.8k	500.3k	0.3	14400.0

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Table A.26: Comparison of split-pipe models on *Circuit rank* + 2 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_231	269.8k	269.5k	0.1	7.2	269.8k	269.5k	0.1	2.5
belg+2arcs_232	306.3k	277.8k	10.3	14400.0	306.3k	306.0k	0.1	13.9
belg+2arcs_233	491.0k	467.1k	5.1	14400.0	491.0k	465.2k	5.5	14400.0
belg+2arcs_234	258.0k	255.9k	0.8	14400.0	258.0k	257.8k	0.1	4.4
belg+2arcs_235	318.7k	318.4k	0.1	15.8	318.6k	311.2k	2.4	14400.0
belg+2arcs_236	395.3k	394.9k	0.1	1508.2	395.3k	389.8k	1.4	14400.0
belg+2arcs_237	325.5k	302.5k	7.6	14400.0	325.5k	307.4k	5.9	14400.0
belg+2arcs_238	481.8k	481.3k	0.1	602.1	481.8k	481.4k	0.1	9.1
belg+2arcs_239	531.2k	530.7k	0.1	100.8	531.2k	531.2k	0.0	11.1
belg+2arcs_240	563.1k	542.5k	3.8	14400.0	563.2k	540.7k	4.2	14400.0
belg+2arcs_241	508.7k	502.6k	1.2	14400.0	508.7k	504.9k	0.8	14400.0
belg+2arcs_242	372.5k	371.5k	0.2	14400.0	372.5k	371.7k	0.2	14400.0
belg+2arcs_243	452.2k	451.8k	0.1	96.7	453.5k	419.7k	8.1	14400.0
belg+2arcs_244	457.1k	434.2k	5.3	14400.0	457.1k	456.6k	0.1	43.9
belg+2arcs_245	199.2k	199.2k	0.0	15.4	199.2k	199.0k	0.1	4.1
belg+2arcs_246	468.2k	388.7k	20.5	14400.0	468.2k	465.2k	0.7	14400.0
belg+2arcs_247	356.1k	355.0k	0.3	14400.0	356.1k	355.8k	0.1	9460.5
belg+2arcs_248	620.6k	557.3k	11.3	14400.0	620.6k	540.6k	14.8	14400.0
belg+2arcs_249	345.6k	345.3k	0.1	33.6	345.6k	345.2k	0.1	21.3
belg+2arcs_250	415.6k	415.2k	0.1	286.0	415.6k	415.2k	0.1	7048.3
belg+2arcs_251	452.3k	428.2k	5.6	14400.0	452.3k	428.9k	5.5	14400.0
belg+2arcs_252	315.5k	314.5k	0.3	14400.0	315.5k	315.2k	0.1	93.0
belg+2arcs_253	670.0k	610.0k	9.8	14400.0	670.0k	600.2k	11.6	14400.0
belg+2arcs_254	601.2k	569.7k	5.5	14400.0	606.8k	588.5k	3.1	14400.0
belg+2arcs_255	167.9k	167.8k	0.1	18.5	167.9k	167.7k	0.1	1.0
belg+2arcs_256	486.0k	466.2k	4.2	14400.0	486.0k	485.5k	0.1	464.1
belg+2arcs_257	370.8k	370.4k	0.1	111.3	370.8k	359.5k	3.1	14400.0
belg+2arcs_258	306.2k	306.0k	0.1	5.5	306.2k	295.6k	3.6	14400.0
belg+2arcs_259	385.6k	374.7k	2.9	14400.0	385.6k	341.8k	12.8	14400.0
belg+2arcs_260	408.3k	384.9k	6.1	14400.0	408.3k	407.9k	0.1	65.1
belg+2arcs_261	144.2k	144.1k	0.1	1.2	144.2k	144.2k	0.0	2.1
belg+2arcs_262	295.5k	295.5k	0.0	7.2	295.5k	295.3k	0.1	4.6
belg+2arcs_263	517.8k	478.2k	8.3	14400.0	517.5k	488.0k	6.1	14400.0
belg+2arcs_264	419.3k	416.5k	0.7	14400.0	419.3k	417.2k	0.5	14400.0
belg+2arcs_265	775.5k	770.8k	0.6	14400.0	775.5k	695.9k	11.4	14400.0
belg+2arcs_266	347.1k	341.7k	1.6	14400.0	347.1k	346.1k	0.3	14400.0
belg+2arcs_267	608.1k	607.8k	0.0	41.7	608.1k	599.1k	1.5	14400.0
belg+2arcs_268	504.9k	478.9k	5.4	14400.0	504.9k	457.9k	10.3	14400.0
belg+2arcs_269	482.4k	437.9k	10.2	14400.0	482.4k	461.5k	4.5	14400.0
belg+2arcs_270	323.6k	317.8k	1.8	14400.0	323.6k	323.6k	0.0	33.8
belg+2arcs_271	510.8k	464.7k	9.9	14400.0	510.8k	421.8k	21.1	14400.0
belg+2arcs_272	588.1k	587.5k	0.1	265.9	588.3k	587.7k	0.1	258.4
belg+2arcs_273	631.3k	614.7k	2.7	14400.0	631.3k	603.2k	4.7	14400.0
belg+2arcs_274	279.0k	278.8k	0.1	21.2	278.8k	278.7k	0.0	1.2
belg+2arcs_275	232.7k	232.6k	0.1	9.8	232.7k	223.2k	4.3	14400.0
belg+2arcs_276	230.5k	217.3k	6.1	14400.0	230.5k	217.8k	5.8	14400.0
belg+2arcs_277	315.7k	310.3k	1.7	14400.0	315.7k	315.4k	0.1	359.9
belg+2arcs_278	250.5k	250.2k	0.1	5.1	250.5k	250.3k	0.0	2.1
belg+2arcs_279	299.1k	298.8k	0.1	161.9	299.1k	298.8k	0.1	139.1
belg+2arcs_280	538.9k	536.4k	0.5	14400.0	538.9k	516.6k	4.3	14400.0
belg+2arcs_281	320.2k	319.9k	0.1	14.3	320.2k	319.9k	0.1	2.3
belg+2arcs_282	530.8k	466.0k	13.9	14400.0	530.8k	480.7k	10.4	14400.0
belg+2arcs_283	424.0k	422.3k	0.4	14400.0	424.0k	374.4k	13.2	14400.0
belg+2arcs_284	419.1k	407.4k	2.9	14400.0	419.1k	395.9k	5.9	14400.0
belg+2arcs_285	264.2k	263.9k	0.1	14400.0	264.2k	262.8k	0.5	14400.0
belg+2arcs_286	421.1k	413.1k	2.0	14400.0	421.1k	417.5k	0.9	14400.0
belg+2arcs_287	405.4k	380.0k	6.7	14400.0	405.5k	378.9k	7.0	14400.0
belg+2arcs_288	341.3k	340.9k	0.1	1918.8	341.3k	340.9k	0.1	89.9
belg+2arcs_289	406.3k	331.8k	22.4	14400.0	406.0k	403.3k	0.7	14400.0
belg+2arcs_290	532.3k	506.1k	5.2	14400.0	532.4k	519.4k	2.5	14400.0
belg+2arcs_291	513.4k	508.3k	1.0	14400.0	513.4k	509.9k	0.7	14400.0
belg+2arcs_292	312.5k	311.8k	0.2	14400.0	312.5k	312.0k	0.2	14400.0
belg+2arcs_293	236.4k	236.2k	0.1	8.8	236.4k	236.2k	0.1	8368.2
belg+2arcs_294	321.7k	321.4k	0.1	3.3	321.9k	321.6k	0.1	4.3
belg+2arcs_295	457.9k	457.5k	0.1	2812.1	457.9k	457.2k	0.2	14400.0
belg+2arcs_296	461.0k	443.2k	4.0	14400.0	461.0k	460.6k	0.1	590.4
belg+2arcs_297	286.5k	285.9k	0.2	14400.0	286.5k	286.2k	0.1	9.3
belg+2arcs_298	320.8k	258.1k	24.3	14400.0	320.8k	320.5k	0.1	143.7
belg+2arcs_299	660.2k	650.8k	1.4	14400.0	660.2k	658.8k	0.2	14400.0
belg+2arcs_300	302.8k	296.9k	2.0	14400.0	302.8k	252.6k	19.8	14400.0
belg+2arcs_301	316.6k	315.8k	0.3	14400.0	316.6k	316.3k	0.1	1900.3
belg+2arcs_302	441.8k	428.1k	3.2	14400.0	441.8k	432.4k	2.2	14400.0
belg+2arcs_303	591.2k	579.1k	2.1	14400.0	591.2k	562.6k	5.1	14400.0
belg+2arcs_304	493.4k	485.4k	1.7	14400.0	493.4k	481.6k	2.5	14400.0
belg+2arcs_305	436.9k	436.6k	0.1	3.7	436.9k	435.3k	0.4	14400.0
belg+2arcs_306	436.5k	400.7k	8.9	14400.0	436.5k	400.6k	9.0	14400.0
belg+2arcs_307	296.4k	246.2k	20.4	14400.0	284.8k	284.8k	0.0	11.3
belg+2arcs_308	220.5k	220.3k	0.1	344.1	220.5k	220.3k	0.1	20.6

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Table A.26: Comparison of split-pipe models on *Circuit rank* + 2 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_309	479.8k	415.6k	15.4	14400.0	479.8k	479.3k	0.1	1145.3
belg+2arcs_310	283.5k	283.4k	0.1	8.8	283.5k	283.5k	0.0	3.1
belg+2arcs_311	300.1k	299.4k	0.2	14400.0	300.1k	299.8k	0.1	242.7
belg+2arcs_312	263.1k	255.6k	2.9	14400.0	263.1k	262.8k	0.1	9.3
belg+2arcs_313	512.4k	511.9k	0.1	508.0	512.4k	494.2k	3.7	14400.0
belg+2arcs_314	710.5k	680.7k	4.4	14400.0	710.5k	708.5k	0.3	14400.0
belg+2arcs_315	486.5k	485.9k	0.1	14400.0	486.5k	467.9k	4.0	14400.0
belg+2arcs_316	267.6k	267.3k	0.1	576.2	267.6k	267.4k	0.1	1.5
belg+2arcs_317	459.1k	453.7k	1.2	14400.0	459.0k	455.9k	0.7	14400.0
belg+2arcs_318	266.0k	246.7k	7.8	14400.0	266.0k	266.0k	0.0	779.0
belg+2arcs_319	297.9k	297.7k	0.1	45.3	297.8k	297.6k	0.1	8.3
belg+2arcs_320	365.3k	364.9k	0.1	16.0	365.3k	364.9k	0.1	38.6
belg+2arcs_321	242.1k	242.1k	0.0	33.5	242.1k	234.6k	3.2	14400.0
belg+2arcs_322	320.1k	274.8k	16.5	14400.0	320.1k	319.8k	0.1	130.6
belg+2arcs_323	293.7k	287.4k	2.2	14400.0	293.7k	293.4k	0.1	4.6
belg+2arcs_324	323.1k	311.9k	3.6	14400.0	323.1k	322.7k	0.1	17.6
belg+2arcs_325	549.5k	512.1k	7.3	14400.0	549.5k	483.4k	13.7	14400.0
belg+2arcs_326	394.1k	392.5k	0.4	14400.0	394.1k	393.7k	0.1	479.9
belg+2arcs_327	212.9k	201.0k	5.9	14400.0	212.9k	212.9k	0.0	2.3
belg+2arcs_328	254.6k	251.8k	1.1	14400.0	254.6k	254.4k	0.1	13.6
belg+2arcs_329	321.6k	321.3k	0.1	1852.3	321.6k	305.2k	5.4	14400.0
belg+2arcs_330	389.6k	388.9k	0.2	14400.0	389.6k	389.6k	0.0	85.9
belg+2arcs_331	306.5k	306.2k	0.1	21.8	306.5k	306.2k	0.1	5.8
belg+2arcs_332	729.8k	720.9k	1.2	14400.0	729.8k	676.6k	7.9	14400.0
belg+2arcs_333	319.9k	314.7k	1.7	14400.0	319.9k	318.6k	0.4	14400.0
belg+2arcs_334	507.2k	460.5k	10.1	14400.0	502.1k	499.5k	0.5	14400.0
belg+2arcs_335	446.7k	446.7k	0.0	23.3	446.7k	427.6k	4.5	14400.0
belg+2arcs_336	319.9k	319.6k	0.1	29.6	319.9k	319.6k	0.1	47.0
belg+2arcs_337	326.8k	326.5k	0.1	2792.9	326.8k	325.8k	0.3	14400.0
belg+2arcs_338	318.7k	318.4k	0.1	327.3	318.7k	318.4k	0.1	1.8
belg+2arcs_339	648.0k	627.7k	3.2	14400.0	648.0k	647.3k	0.1	4809.4
belg+2arcs_340	642.8k	639.0k	0.6	14400.0	642.8k	641.5k	0.2	14400.0
belg+2arcs_341	282.8k	255.1k	10.9	14400.0	282.8k	282.8k	0.0	8.8
belg+2arcs_342	439.1k	412.0k	6.6	14400.0	439.1k	384.3k	14.3	14400.0
belg+2arcs_343	343.5k	340.5k	0.9	14400.0	343.5k	343.2k	0.1	1.3
belg+2arcs_344	306.7k	302.2k	1.5	14400.0	306.7k	306.4k	0.1	9.2
belg+2arcs_345	521.2k	481.1k	8.3	14400.0	519.1k	517.8k	0.2	14400.0
belg+2arcs_346	376.7k	376.3k	0.1	44.0	376.7k	373.9k	0.7	14400.0
belg+2arcs_347	560.3k	554.5k	1.1	14400.0	560.3k	559.8k	0.1	18.3
belg+2arcs_348	263.1k	262.8k	0.1	1650.9	263.1k	262.9k	0.1	1.6
belg+2arcs_349	289.8k	273.2k	6.1	14400.0	289.8k	273.3k	6.1	14400.0
belg+2arcs_350	405.6k	405.2k	0.1	6620.3	405.6k	405.2k	0.1	19.9
belg+2arcs_351	478.6k	478.2k	0.1	10.3	478.6k	478.2k	0.1	6691.8
belg+2arcs_352	539.6k	536.0k	0.7	14400.0	539.6k	538.2k	0.3	14400.0
belg+2arcs_353	393.7k	381.5k	3.2	14400.0	393.7k	379.0k	3.9	14400.0
belg+2arcs_354	293.2k	293.2k	0.0	2.8	293.2k	293.2k	0.0	2.2
belg+2arcs_355	203.2k	195.0k	4.2	14400.0	203.2k	203.2k	0.0	2.5
belg+2arcs_356	462.3k	461.4k	0.2	14400.0	462.3k	461.8k	0.1	13.9
belg+2arcs_357	403.6k	403.6k	0.0	10.8	403.6k	403.2k	0.1	2.5
belg+2arcs_358	696.2k	675.0k	3.1	14400.0	696.2k	661.2k	5.3	14400.0
belg+2arcs_359	478.8k	478.3k	0.1	399.1	478.8k	478.3k	0.1	207.6
belg+2arcs_360	263.6k	263.4k	0.1	2.3	263.6k	263.6k	0.0	0.9
belg+2arcs_361	618.4k	599.3k	3.2	14400.0	618.4k	615.3k	0.5	14400.0
belg+2arcs_362	328.1k	325.8k	0.7	14400.0	328.1k	327.8k	0.1	4.4
belg+2arcs_363	416.7k	382.5k	8.9	14400.0	416.7k	393.7k	5.8	14400.0
belg+2arcs_364	389.1k	388.8k	0.1	8.3	389.1k	388.8k	0.1	1.8
belg+2arcs_365	551.3k	523.3k	5.3	14400.0	551.3k	527.2k	4.6	14400.0
belg+2arcs_366	619.6k	552.1k	12.2	14400.0	619.6k	619.0k	0.1	47.9
belg+2arcs_367	353.7k	328.9k	7.5	14400.0	353.7k	353.5k	0.1	4.9
belg+2arcs_368	337.9k	336.0k	0.6	14400.0	337.9k	326.4k	3.5	14400.0
belg+2arcs_369	227.4k	218.8k	3.9	14400.0	226.4k	226.2k	0.1	1.7
belg+2arcs_370	295.4k	295.4k	0.0	34.8	295.4k	295.4k	0.0	22.2
belg+2arcs_371	584.4k	568.5k	2.8	14400.0	584.4k	583.8k	0.1	195.7
belg+2arcs_372	429.8k	425.9k	0.9	14400.0	429.8k	409.6k	4.9	14400.0
belg+2arcs_373	481.6k	458.0k	5.1	14400.0	481.6k	456.3k	5.6	14400.0
belg+2arcs_374	303.4k	303.0k	0.1	61.5	303.4k	289.4k	4.8	14400.0
belg+2arcs_375	265.7k	265.5k	0.1	46.3	265.7k	265.5k	0.1	1.3
belg+2arcs_376	680.8k	667.0k	2.1	14400.0	680.8k	680.1k	0.1	98.3
belg+2arcs_377	414.6k	376.4k	10.1	14400.0	414.8k	376.6k	10.1	14400.0
belg+2arcs_378	349.7k	337.9k	3.5	14400.0	349.7k	339.9k	2.9	14400.0
belg+2arcs_379	510.6k	505.3k	1.0	14400.0	510.6k	510.1k	0.1	2210.3
belg+2arcs_380	373.0k	372.6k	0.1	68.4	373.0k	356.4k	4.6	14400.0
belg+2arcs_381	563.3k	561.1k	0.4	14400.0	563.3k	555.2k	1.5	14400.0
belg+2arcs_382	568.0k	534.5k	6.3	14400.0	567.9k	515.5k	10.2	14400.0
belg+2arcs_383	240.1k	235.6k	1.9	14400.0	240.1k	239.8k	0.1	4282.9
belg+2arcs_384	383.0k	382.6k	0.1	3897.6	383.0k	377.7k	1.4	14400.0
belg+2arcs_385	436.2k	424.4k	2.8	14400.0	436.2k	434.9k	0.3	14400.0
belg+2arcs_386	665.5k	649.5k	2.5	14400.0	665.5k	635.0k	4.8	14400.0

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Table A.26: Comparison of split-pipe models on *Circuit rank* + 2 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_387	345.9k	310.6k	11.4	14400.0	345.9k	330.5k	4.7	14400.0
belg+2arcs_388	254.8k	254.6k	0.1	17.6	254.8k	252.9k	0.7	14400.0
belg+2arcs_389	323.1k	321.5k	0.5	14400.0	323.1k	322.9k	0.1	15.1
belg+2arcs_390	451.3k	445.1k	1.4	14400.0	451.3k	448.0k	0.7	14400.0
belg+2arcs_391	365.7k	348.5k	4.9	14400.0	365.8k	364.3k	0.4	14400.0
belg+2arcs_392	268.5k	265.9k	1.0	14400.0	268.5k	268.5k	0.0	7.3
belg+2arcs_393	432.2k	419.2k	3.1	14400.0	432.2k	416.1k	3.9	14400.0
belg+2arcs_394	314.5k	314.2k	0.1	9.9	314.5k	314.1k	0.1	59.2
belg+2arcs_395	401.5k	400.9k	0.2	14400.0	401.5k	401.1k	0.1	3286.3
belg+2arcs_396	276.9k	268.1k	3.3	14400.0	276.9k	263.6k	5.0	14400.0
belg+2arcs_397	634.3k	633.6k	0.1	14400.0	634.3k	600.2k	5.7	14400.0
belg+2arcs_398	329.7k	322.8k	2.2	14400.0	329.7k	329.4k	0.1	307.4
belg+2arcs_399	309.5k	284.2k	8.9	14400.0	309.5k	292.0k	6.0	14400.0
belg+2arcs_400	296.0k	292.8k	1.1	14400.0	296.0k	295.7k	0.1	3171.1
belg+2arcs_401	430.0k	429.6k	0.1	1233.9	430.0k	429.6k	0.1	145.5
belg+2arcs_402	371.4k	336.4k	10.4	14400.0	371.4k	371.1k	0.1	77.4
belg+2arcs_403	460.0k	371.8k	23.7	14400.0	459.2k	442.5k	3.8	14400.0
belg+2arcs_404	471.4k	471.0k	0.1	54.7	471.4k	435.6k	8.2	14400.0
belg+2arcs_405	587.4k	586.9k	0.1	142.1	587.4k	586.8k	0.1	141.8
belg+2arcs_406	262.6k	262.6k	0.0	24.7	262.6k	262.3k	0.1	32.8
belg+2arcs_407	470.0k	463.1k	1.5	14400.0	470.0k	469.9k	0.0	12.6
belg+2arcs_408	166.6k	165.1k	0.9	14400.0	166.6k	166.6k	0.0	1.4
belg+2arcs_409	297.6k	266.1k	11.8	14400.0	297.6k	294.9k	0.9	14400.0
belg+2arcs_410	347.2k	347.2k	0.0	30.9	347.2k	318.5k	9.0	14400.0
belg+2arcs_411	416.7k	408.6k	2.0	14400.0	416.7k	416.3k	0.1	12.8
belg+2arcs_412	227.7k	210.1k	8.4	14400.0	227.7k	227.7k	0.0	7.0
belg+2arcs_413	353.1k	320.9k	10.0	14400.0	352.6k	345.9k	1.9	14400.0
belg+2arcs_414	303.4k	303.2k	0.1	25.9	303.4k	303.4k	0.0	116.1
belg+2arcs_415	219.5k	218.2k	0.6	14400.0	219.5k	219.0k	0.3	14400.0
belg+2arcs_416	509.1k	508.6k	0.1	251.2	509.1k	508.4k	0.1	14400.0
belg+2arcs_417	366.6k	349.3k	4.9	14400.0	366.6k	360.2k	1.8	14400.0
belg+2arcs_418	597.0k	595.7k	0.2	14400.0	597.0k	595.0k	0.3	14400.0
belg+2arcs_419	355.2k	345.6k	2.8	14400.0	355.2k	354.8k	0.1	9674.2
belg+2arcs_420	319.9k	319.6k	0.1	1166.5	319.9k	319.6k	0.1	4.0
belg+2arcs_421	332.9k	319.4k	4.2	14400.0	332.9k	332.5k	0.1	14400.0
belg+2arcs_422	370.4k	370.0k	0.1	12472.0	370.4k	370.0k	0.1	45.0
belg+2arcs_423	296.3k	287.0k	3.2	14400.0	293.7k	293.4k	0.1	607.1
belg+2arcs_424	567.3k	532.9k	6.5	14400.0	567.3k	566.7k	0.1	912.0
belg+2arcs_425	374.9k	370.7k	1.1	14400.0	375.1k	356.0k	5.4	14400.0
belg+2arcs_426	393.3k	373.7k	5.2	14400.0	393.3k	328.6k	19.7	14400.0
belg+2arcs_427	306.7k	306.4k	0.1	39.4	306.7k	306.6k	0.0	4.8
belg+2arcs_428	279.1k	277.2k	0.7	14400.0	279.1k	278.8k	0.1	14400.0
belg+2arcs_429	566.4k	566.1k	0.0	20.3	566.4k	565.4k	0.2	14400.0
belg+2arcs_430	484.8k	483.7k	0.2	14400.0	484.8k	483.7k	0.2	14400.0
belg+2arcs_431	745.2k	729.2k	2.2	14400.0	745.3k	737.1k	1.1	14400.0
belg+2arcs_432	233.6k	233.3k	0.1	14400.0	233.6k	233.6k	0.0	2.0
belg+2arcs_433	218.8k	218.8k	0.0	8.2	218.8k	218.7k	0.0	1.0
belg+2arcs_434	508.9k	489.4k	4.0	14400.0	508.9k	482.0k	5.6	14400.0
belg+2arcs_435	509.5k	509.3k	0.0	96.8	509.5k	498.9k	2.1	14400.0
belg+2arcs_436	382.7k	376.3k	1.7	14400.0	382.7k	333.4k	14.8	14400.0
belg+2arcs_437	354.5k	307.3k	15.4	14400.0	354.4k	354.0k	0.1	593.3
belg+2arcs_438	418.9k	414.3k	1.1	14400.0	418.9k	406.8k	3.0	14400.0
belg+2arcs_439	559.2k	518.2k	7.9	14400.0	553.3k	547.3k	1.1	14400.0
belg+2arcs_440	594.6k	589.9k	0.8	14400.0	594.6k	593.4k	0.2	14400.0
belg+2arcs_441	331.0k	329.5k	0.4	14400.0	331.0k	330.5k	0.1	14400.0
belg+2arcs_442	227.9k	227.9k	0.0	26.4	227.9k	227.8k	0.1	7.9
belg+2arcs_443	420.1k	419.6k	0.1	14400.0	420.1k	409.1k	2.7	14400.0
belg+2arcs_444	271.9k	261.4k	4.0	14400.0	272.0k	261.3k	4.1	14400.0
belg+2arcs_445	648.3k	596.6k	8.7	14400.0	648.3k	620.8k	4.4	14400.0
belg+2arcs_446	334.0k	333.2k	0.2	14400.0	334.0k	333.9k	0.0	2.8
belg+2arcs_447	383.6k	382.0k	0.4	14400.0	383.6k	383.2k	0.1	10133.0
belg+2arcs_448	396.7k	368.1k	7.8	14400.0	394.7k	394.4k	0.1	106.8
belg+2arcs_449	376.2k	375.8k	0.1	1710.0	376.2k	375.8k	0.1	23.2
belg+2arcs_450	354.7k	343.0k	3.4	14400.0	354.7k	351.9k	0.8	14400.0
belg+2arcs_451	161.5k	161.3k	0.1	865.5	161.5k	161.4k	0.1	1.0
belg+2arcs_452	422.3k	421.3k	0.2	14400.0	422.3k	418.8k	0.8	14400.0
belg+2arcs_453	666.3k	666.3k	0.0	156.0	666.3k	523.2k	27.4	14400.0
belg+2arcs_454	252.3k	252.0k	0.1	220.2	252.3k	252.3k	0.0	5.9
belg+2arcs_455	202.9k	202.8k	0.0	6.3	202.9k	202.8k	0.0	1.1
belg+2arcs_456	550.0k	543.3k	1.2	14400.0	550.0k	543.6k	1.2	14400.0
belg+2arcs_457	337.0k	332.1k	1.5	14400.0	337.0k	332.3k	1.4	14400.0
belg+2arcs_458	707.7k	606.8k	16.6	14400.0	707.7k	698.1k	1.4	14400.0
belg+2arcs_459	166.8k	160.6k	3.8	14400.0	166.8k	166.7k	0.1	1.2
belg+2arcs_460	236.6k	236.4k	0.1	6.3	236.6k	236.5k	0.0	1.0
belg+2arcs_461	374.5k	372.3k	0.6	14400.0	374.5k	363.0k	3.2	14400.0
belg+2arcs_462	308.6k	308.0k	0.2	14400.0	308.6k	303.5k	1.7	14400.0
belg+2arcs_463	334.0k	323.9k	3.1	14400.0	334.0k	327.6k	2.0	14400.0
belg+2arcs_464	292.4k	292.2k	0.1	1.9	292.4k	292.4k	0.0	1.4

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Table A.26: Comparison of split-pipe models on *Circuit rank* + 2 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+2arcs_465	363.3k	356.0k	2.0	14400.0	363.3k	343.9k	5.7	14400.0
belg+2arcs_466	289.9k	286.4k	1.2	14400.0	289.9k	289.7k	0.1	3.6
belg+2arcs_467	534.0k	533.5k	0.1	401.4	534.0k	533.5k	0.1	137.6
belg+2arcs_468	444.8k	444.4k	0.1	14400.0	444.8k	443.1k	0.4	14400.0
belg+2arcs_469	320.5k	320.2k	0.1	2581.1	320.5k	320.4k	0.0	2.1
belg+2arcs_470	323.6k	323.3k	0.1	74.4	323.6k	323.3k	0.1	80.6
belg+2arcs_471	405.9k	402.3k	0.9	14400.0	405.9k	405.5k	0.1	15.9
belg+2arcs_472	286.3k	286.0k	0.1	2.5	286.3k	285.9k	0.1	14400.0
belg+2arcs_473	566.4k	559.4k	1.2	14400.0	566.6k	500.0k	13.3	14400.0
belg+2arcs_474	252.6k	252.4k	0.1	10.7	252.6k	233.3k	8.3	14400.0
belg+2arcs_475	270.4k	264.9k	2.1	14400.0	270.4k	270.1k	0.1	51.0
belg+2arcs_476	459.3k	451.6k	1.7	14400.0	459.3k	458.9k	0.1	100.7
belg+2arcs_477	433.1k	421.3k	2.8	14400.0	433.1k	432.7k	0.1	38.0
belg+2arcs_478	471.0k	459.5k	2.5	14400.0	471.0k	464.5k	1.4	14400.0
belg+2arcs_479	472.0k	423.9k	11.3	14400.0	469.0k	462.8k	1.3	14400.0
belg+2arcs_480	278.0k	259.7k	7.0	14400.0	278.0k	278.0k	0.0	11.4
belg+2arcs_481	475.1k	474.6k	0.1	1757.1	475.1k	474.6k	0.1	471.9
belg+2arcs_482	410.0k	396.4k	3.4	14400.0	410.0k	402.3k	1.9	14400.0
belg+2arcs_483	232.1k	229.4k	1.2	14400.0	232.1k	231.9k	0.1	3.5
belg+2arcs_484	256.8k	254.9k	0.7	14400.0	256.8k	256.8k	0.0	2.3
belg+2arcs_485	259.3k	248.9k	4.2	14400.0	259.3k	259.1k	0.1	12.8
belg+2arcs_486	546.0k	545.4k	0.1	26.9	546.0k	523.8k	4.2	14400.0
belg+2arcs_487	316.4k	294.4k	7.5	14400.0	316.4k	316.1k	0.1	69.9
belg+2arcs_488	450.9k	449.7k	0.3	14400.0	450.9k	430.9k	4.6	14400.0
belg+2arcs_489	486.8k	486.3k	0.1	266.3	486.8k	486.4k	0.1	14.2
belg+2arcs_490	274.8k	274.5k	0.1	616.1	274.8k	274.6k	0.1	6.7
belg+2arcs_491	342.9k	330.9k	3.6	14400.0	342.9k	326.4k	5.0	14400.0
belg+2arcs_492	263.3k	260.2k	1.2	14400.0	263.3k	263.3k	0.0	14.1
belg+2arcs_493	456.1k	454.4k	0.4	14400.0	456.1k	455.6k	0.1	23.5
belg+2arcs_494	388.8k	375.1k	3.6	14400.0	388.8k	377.5k	3.0	14400.0
belg+2arcs_495	505.5k	477.1k	5.9	14400.0	505.5k	505.0k	0.1	310.6
belg+2arcs_496	321.1k	320.1k	0.3	14400.0	321.1k	320.8k	0.1	3.7
belg+2arcs_497	377.9k	375.0k	0.8	14400.0	403.3k	352.8k	14.3	14400.0
belg+2arcs_498	509.4k	504.6k	1.0	14400.0	509.4k	509.0k	0.1	16.6
belg+2arcs_499	432.4k	373.0k	15.9	14400.0	432.2k	383.3k	12.7	14400.0
belg+2arcs_500	315.6k	296.4k	6.5	14400.0	315.6k	315.0k	0.2	14400.0

Table A.27: Detailed results of the discrete models on *Circuit rank* + 4 arcs, as summarized in Figure 3.10c and Table 3.3d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_1	645.5k	384.1k	68.1	14400.0	496.4k	496.4k	0.0	7877.0	497.0k	381.7k	30.2	14400.0
belg+4arcs_2	324.4k	324.4k	0.0	116.2	324.4k	324.4k	0.0	4028.4	324.4k	222.4k	45.9	14400.0
belg+4arcs_3	354.4k	288.3k	22.9	14400.0	302.9k	277.4k	9.2	14400.0	348.0k	198.6k	75.3	14400.0
belg+4arcs_4	453.8k	434.7k	4.4	14400.0	452.1k	452.1k	0.0	9393.5	452.1k	301.7k	49.9	14400.0
belg+4arcs_5	742.7k	362.5k	104.9	14400.0	753.9k	439.4k	71.6	14400.0	580.0k	309.6k	87.3	14400.0
belg+4arcs_6	263.2k	263.2k	0.0	8229.6	263.2k	263.2k	0.0	5470.6	280.2k	224.1k	25.0	14400.0
belg+4arcs_7	524.0k	524.0k	0.0	8752.9	620.9k	405.3k	53.2	14400.0	542.7k	296.4k	83.1	14400.0
belg+4arcs_8	11.4k	11.4k	0.0	197.9	1e+20	3979.4	–	14400.0	11.4k	11.4k	0.0	927.8
belg+4arcs_9	389.6k	389.6k	0.0	5412.2	389.6k	389.6k	0.0	3975.7	568.9k	242.2k	134.8	14400.0
belg+4arcs_10	363.2k	363.2k	0.0	3362.4	363.2k	363.2k	0.0	10826.2	363.2k	236.9k	53.3	14400.0
belg+4arcs_11	390.1k	333.4k	17.0	14400.0	388.5k	319.3k	21.7	14400.0	545.9k	244.9k	123.0	14400.0
belg+4arcs_12	163.4k	163.4k	0.0	731.0	163.4k	163.4k	0.0	5947.2	196.5k	128.1k	53.4	14400.0
belg+4arcs_13	397.2k	346.1k	14.8	14400.0	1e+20	301.6k	–	14400.0	458.9k	238.2k	92.6	14400.0
belg+4arcs_14	552.7k	552.7k	0.0	5082.9	667.1k	512.6k	30.1	14400.0	585.1k	470.0k	24.5	14400.0
belg+4arcs_15	65.8k	65.8k	0.0	435.9	65.8k	65.8k	0.0	2002.2	65.8k	21.9k	200.9	14400.0
belg+4arcs_16	896.0k	423.2k	111.7	14400.0	611.4k	505.8k	20.9	14400.0	614.8k	308.1k	99.5	14400.0
belg+4arcs_17	363.8k	258.1k	40.9	14400.0	301.4k	301.4k	0.0	6844.1	303.2k	230.3k	31.7	14400.0
belg+4arcs_18	394.4k	394.4k	0.0	357.8	394.4k	394.3k	0.0	4698.7	418.9k	252.9k	65.7	14400.0
belg+4arcs_19	222.9k	222.9k	0.0	366.9	222.9k	222.9k	0.0	595.8	222.9k	222.9k	0.0	2764.9
belg+4arcs_20	409.0k	409.0k	0.0	1406.5	409.0k	409.0k	0.0	4318.0	416.2k	281.4k	47.9	14400.0
belg+4arcs_21	46.4k	46.4k	0.0	45.0	46.4k	46.4k	0.0	2272.8	46.4k	46.4k	0.0	119.2
belg+4arcs_22	303.5k	262.2k	15.7	14400.0	300.1k	300.1k	0.0	12245.7	303.5k	202.2k	50.1	14400.0
belg+4arcs_23	190.6k	190.6k	0.0	3778.5	190.6k	190.6k	0.0	2326.9	190.6k	145.2k	31.3	14400.0
belg+4arcs_24	274.6k	274.6k	0.0	1774.8	274.6k	274.6k	0.0	2287.2	274.6k	274.6k	0.0	7944.9
belg+4arcs_25	313.4k	313.4k	0.0	11636.0	313.4k	286.2k	9.5	14400.0	489.5k	209.6k	133.6	14400.0
belg+4arcs_26	336.5k	336.5k	0.0	503.9	336.5k	336.5k	0.0	4208.6	337.4k	227.1k	48.5	14400.0
belg+4arcs_27	191.0k	191.0k	0.0	431.8	191.0k	191.0k	0.0	299.9	191.0k	191.0k	0.0	111.7

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Table A.27: Comparison of discrete models on *Circuit rank* + 4 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_28	426.8k	316.2k	34.9	14400.0	419.8k	419.8k	0.0	5323.9	516.4k	271.2k	90.4	14400.0
belg+4arcs_29	240.0k	224.7k	6.8	14400.0	236.7k	236.7k	0.0	2715.3	262.4k	142.1k	84.6	14400.0
belg+4arcs_30	189.2k	189.2k	0.0	12687.2	216.6k	161.1k	34.5	14400.0	192.1k	100.8k	90.6	14400.0
belg+4arcs_31	308.8k	210.2k	46.9	14400.0	260.5k	260.5k	0.0	5506.3	265.8k	207.9k	27.8	14400.0
belg+4arcs_32	412.0k	412.0k	0.0	1221.6	412.0k	412.0k	0.0	2164.0	415.8k	372.5k	11.6	14400.0
belg+4arcs_33	152.1k	152.1k	0.0	1356.4	152.1k	152.0k	0.1	322.8	152.1k	152.1k	0.0	27.0
belg+4arcs_34	28.3k	28.3k	0.0	37.2	28.3k	28.3k	0.0	196.2	28.3k	28.3k	0.0	285.4
belg+4arcs_35	311.6k	311.6k	0.0	1096.2	311.6k	311.6k	0.0	12444.3	311.9k	224.6k	38.9	14400.0
belg+4arcs_36	379.1k	341.1k	11.1	14400.0	372.4k	372.4k	0.0	2719.8	372.4k	300.7k	23.8	14400.0
belg+4arcs_37	168.1k	168.1k	0.0	104.4	168.1k	168.1k	0.0	1917.1	168.1k	168.1k	0.0	1934.1
belg+4arcs_38	516.9k	440.8k	17.3	14400.0	489.5k	489.5k	0.0	11846.0	552.0k	310.9k	77.6	14400.0
belg+4arcs_39	165.7k	165.7k	0.0	19.3	165.7k	165.7k	0.0	224.4	165.7k	165.7k	0.0	203.7
belg+4arcs_40	309.9k	204.4k	51.6	14400.0	239.0k	239.0k	0.0	12088.3	309.9k	144.3k	114.8	14400.0
belg+4arcs_41	543.2k	361.4k	50.3	14400.0	448.2k	448.2k	0.0	3672.3	525.9k	313.7k	67.6	14400.0
belg+4arcs_42	339.6k	339.6k	0.0	3477.2	339.6k	339.6k	0.0	3962.2	339.6k	261.5k	29.8	14400.0
belg+4arcs_43	308.7k	308.7k	0.0	9833.9	308.7k	308.7k	0.0	10718.9	316.9k	184.0k	72.2	14400.0
belg+4arcs_44	185.8k	185.8k	0.0	16.0	185.8k	185.8k	0.0	292.8	185.8k	185.8k	0.0	2.4
belg+4arcs_45	53.7k	53.7k	0.0	802.8	53.7k	53.7k	0.0	11019.3	53.7k	24.1k	122.6	14400.0
belg+4arcs_46	378.9k	312.4k	21.3	14400.0	380.6k	319.0k	19.3	14400.0	380.6k	219.5k	73.4	14400.0
belg+4arcs_47	275.0k	275.0k	0.0	73.4	275.0k	275.0k	0.0	1098.2	275.0k	275.0k	0.0	879.5
belg+4arcs_48	293.3k	293.3k	0.0	1691.8	293.3k	293.3k	0.0	6202.5	338.6k	239.3k	41.5	14400.0
belg+4arcs_49	163.1k	163.1k	0.0	403.0	163.1k	163.1k	0.0	4570.0	163.1k	163.1k	0.0	1733.9
belg+4arcs_50	452.3k	362.4k	24.8	14400.0	448.6k	448.6k	0.0	5859.3	616.2k	318.6k	93.5	14400.0
belg+4arcs_51	131.2k	131.2k	0.0	935.4	131.2k	131.2k	0.0	419.4	131.2k	131.2k	0.0	1826.5
belg+4arcs_52	436.8k	267.6k	63.2	14400.0	346.9k	346.9k	0.0	4370.8	346.9k	265.4k	30.7	14400.0
belg+4arcs_53	195.4k	195.4k	0.0	21.0	195.4k	195.2k	0.1	244.2	195.4k	195.4k	0.0	341.4
belg+4arcs_54	337.4k	253.4k	33.1	14400.0	312.4k	312.4k	0.0	5955.6	337.4k	232.6k	45.0	14400.0
belg+4arcs_55	236.6k	236.6k	0.0	37.9	236.6k	236.6k	0.0	1521.8	236.6k	236.6k	0.0	991.9
belg+4arcs_56	320.8k	300.0k	6.9	14400.0	310.9k	310.9k	0.0	8994.6	320.8k	258.4k	24.2	14400.0
belg+4arcs_57	356.2k	283.3k	25.7	14400.0	334.5k	334.5k	0.0	9351.6	535.2k	194.7k	174.8	14400.0
belg+4arcs_58	420.2k	256.4k	63.9	14400.0	336.4k	277.4k	21.3	14400.0	446.0k	202.3k	120.5	14400.0
belg+4arcs_59	144.8k	144.8k	0.0	22.4	144.8k	144.8k	0.0	154.0	144.8k	144.8k	0.0	10.7
belg+4arcs_60	259.7k	218.7k	18.7	14400.0	252.7k	252.7k	0.0	5416.0	265.4k	197.5k	34.4	14400.0
belg+4arcs_61	383.1k	272.2k	40.7	14400.0	353.8k	268.8k	31.6	14400.0	366.3k	219.0k	67.2	14400.0
belg+4arcs_62	579.8k	426.2k	36.0	14400.0	543.9k	543.9k	0.0	11957.7	649.0k	344.2k	88.6	14400.0
belg+4arcs_63	279.6k	279.6k	0.0	165.7	279.6k	279.4k	0.1	2288.6	279.6k	279.6k	0.0	799.8
belg+4arcs_64	209.7k	209.7k	0.0	1239.3	209.7k	209.7k	0.0	2667.7	209.7k	170.8k	22.8	14400.0
belg+4arcs_65	358.7k	337.9k	6.2	14400.0	356.6k	356.6k	0.0	3636.8	356.6k	268.8k	32.7	14400.0
belg+4arcs_66	405.7k	405.3k	0.1	528.1	405.7k	405.7k	0.0	5565.8	407.9k	341.8k	19.3	14400.0
belg+4arcs_67	262.6k	262.6k	0.0	518.2	262.6k	262.6k	0.0	640.5	271.5k	230.8k	17.6	14400.0
belg+4arcs_68	420.2k	310.9k	35.2	14400.0	410.0k	326.6k	25.5	14400.0	702.5k	230.2k	205.2	14400.0
belg+4arcs_69	423.8k	423.8k	0.0	7223.7	423.8k	423.8k	0.0	13125.0	468.0k	291.4k	60.6	14400.0
belg+4arcs_70	377.4k	312.5k	20.8	14400.0	375.1k	375.1k	0.0	11769.2	443.5k	234.0k	89.5	14400.0
belg+4arcs_71	402.7k	369.9k	8.9	14400.0	400.1k	337.6k	18.5	14400.0	411.5k	246.0k	67.3	14400.0
belg+4arcs_72	160.0k	91.5k	74.7	14400.0	161.2k	65.1k	147.5	14400.0	162.4k	23.6k	586.8	14400.0
belg+4arcs_73	46.0k	46.0k	0.0	54.5	46.0k	46.0k	0.0	2642.7	46.0k	46.0k	0.0	36.6
belg+4arcs_74	500.8k	466.4k	7.4	14400.0	500.8k	500.8k	0.0	9003.8	611.5k	358.1k	70.7	14400.0
belg+4arcs_75	435.1k	346.1k	25.7	14400.0	403.7k	403.7k	0.0	8642.9	489.7k	235.4k	108.0	14400.0
belg+4arcs_76	135.5k	135.5k	0.0	26.0	135.5k	135.5k	0.0	4725.0	135.5k	135.5k	0.0	1750.6
belg+4arcs_77	215.5k	215.5k	0.0	439.4	215.5k	215.5k	0.0	2352.5	246.6k	156.6k	57.5	14400.0
belg+4arcs_78	183.3k	183.3k	0.0	2393.8	183.3k	183.3k	0.0	2716.9	183.3k	183.3k	0.0	1087.4
belg+4arcs_79	55.3k	55.3k	0.0	7151.1	55.3k	55.3k	0.0	537.9	55.3k	55.3k	0.0	10075.0
belg+4arcs_80	328.2k	233.5k	40.5	14400.0	303.2k	274.1k	10.6	14400.0	303.2k	227.9k	33.0	14400.0
belg+4arcs_81	392.4k	392.4k	0.0	4548.5	392.4k	392.4k	0.0	5761.9	416.6k	279.8k	48.9	14400.0
belg+4arcs_82	20.5k	20.5k	0.0	13.0	20.5k	20.5k	0.0	99.5	20.5k	20.5k	0.0	9.3
belg+4arcs_83	596.5k	375.4k	58.9	14400.0	487.4k	487.4k	0.0	11950.6	512.2k	346.3k	47.9	14400.0
belg+4arcs_84	494.9k	378.1k	30.9	14400.0	492.4k	492.4k	0.0	2993.1	495.9k	381.7k	29.9	14400.0
belg+4arcs_85	361.8k	272.2k	32.9	14400.0	357.1k	357.1k	0.0	6547.7	361.8k	265.1k	36.5	14400.0
belg+4arcs_86	414.9k	347.4k	19.4	14400.0	414.9k	414.9k	0.0	6053.0	423.3k	299.9k	41.2	14400.0
belg+4arcs_87	280.1k	280.1k	0.0	2807.6	280.1k	280.1k	0.0	2875.9	280.1k	271.7k	3.1	14400.0
belg+4arcs_88	439.9k	349.3k	25.9	14400.0	406.6k	406.6k	0.0	12345.2	427.3k	269.6k	58.5	14400.0
belg+4arcs_89	761.4k	645.8k	17.9	14400.0	748.7k	703.2k	6.5	14400.0	755.8k	623.7k	21.2	14400.0
belg+4arcs_90	493.9k	357.0k	38.4	14400.0	454.2k	454.2k	0.0	6570.5	455.2k	337.6k	34.9	14400.0
belg+4arcs_91	500.4k	386.2k	29.6	14400.0	622.6k	405.3k	53.6	14400.0	696.0k	284.0k	145.1	14400.0
belg+4arcs_92	428.2k	427.9k	0.1	1136.8	428.2k	428.2k	0.0	5830.7	434.0k	336.0k	29.2	14400.0
belg+4arcs_93	682.0k	483.5k	41.0	14400.0	611.8k	516.5k	18.5	14400.0	625.6k	412.8k	51.5	14400.0
belg+4arcs_94	347.9k	229.0k	51.9	14400.0	334.2k	284.8k	17.4	14400.0	320.9k	213.2k	50.5	14400.0
belg+4arcs_95	169.1k	58.3k	190.0	14400.0	126.9k	126.9k	0.0	8364.6	170.1k	57.7k	194.9	14400.0
belg+4arcs_96	312.1k	237.3k	31.5	14400.0	290.3k	290.3k	0.0	8636.5	290.6k	215.2k	35.0	14400.0
belg+4arcs_97	540.5k	161.8k	234.0	14400.0	218.2k	218.2k	0.0	7412.1	262.7k	147.2k	78.4	14400.0
belg+4arcs_98	383.7k	277.5k	38.2	14400.0	380.9k	368.1k	3.5	14400.0	425.4k	222.7k	91.0	14400.0
belg+4arcs_99	479.7k	357.8k	34.1	14400.0	467.7k	391.6k	19.4	14400.0	775.5k	260.4k	197.8	14400.0
belg+4arcs_100	321.3k	287.8k	11.7	14400.0	313.3k	313.3k	0.0	8902.5	322.1k	214.6k	50.0	14400.0
belg+4arcs_101	340.6k	283.9k	20.0	14400.0	1e+20	264.0k	–	14400.0	397.9k	206.3k	92.8	14400.0
belg+4arcs_102	425.4k	342.7k	24.2	14400.0	423.8k	372.8k	13.7	14400.0	417.3k	284.6k	46.6	14400.0
belg+4arcs_103	169.6k	169.6k	0.0	2381.3	171.3k	154.8k	10.7	14400.0	170.1k	139.9k	21.6	14400.0
belg+4arcs_104	186.4k	186.4k	0.0	13.1	186.4k	186.3k	0.0	107.3	186.4k	186.4k	0.0	12.6
belg+4arcs_105	195.9k	195.9k	0.0	3631.8	195.9k	195.9k	0.0	2953.8	195.9k	195.9k	0.0	484.4

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Table A.27: Comparison of discrete models on *Circuit rank* + 4 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_106	225.8k	225.8k	0.0	449.2	225.8k	225.8k	0.0	818.5	225.8k	225.8k	0.0	4405.4
belg+4arcs_107	363.8k	313.0k	16.2	14400.0	356.0k	356.0k	0.0	3400.9	363.8k	258.3k	40.9	14400.0
belg+4arcs_108	34.0k	34.0k	0.0	1031.4	34.0k	34.0k	0.0	116.0	34.0k	34.0k	0.0	497.6
belg+4arcs_109	354.4k	310.1k	14.3	14400.0	352.1k	319.7k	10.1	14400.0	366.3k	203.1k	80.4	14400.0
belg+4arcs_110	452.5k	343.4k	31.8	14400.0	366.1k	366.1k	0.0	9891.6	366.1k	259.5k	41.1	14400.0
belg+4arcs_111	608.6k	333.2k	82.7	14400.0	422.3k	390.9k	8.0	14400.0	442.2k	314.0k	40.8	14400.0
belg+4arcs_112	222.5k	222.5k	0.0	7269.3	222.5k	222.5k	0.0	9770.2	230.7k	164.1k	40.5	14400.0
belg+4arcs_113	130.8k	130.8k	0.0	73.1	130.8k	130.8k	0.0	446.3	130.8k	130.8k	0.0	258.3
belg+4arcs_114	115.3k	115.3k	0.0	97.1	115.3k	115.3k	0.0	331.5	115.3k	115.3k	0.0	85.3
belg+4arcs_115	500.1k	383.0k	30.6	14400.0	491.4k	382.0k	28.6	14400.0	824.3k	248.0k	232.3	14400.0
belg+4arcs_116	297.4k	225.3k	32.0	14400.0	292.7k	292.7k	0.0	4559.7	297.3k	222.5k	33.6	14400.0
belg+4arcs_117	174.1k	89.3k	94.9	14400.0	199.4k	125.7k	58.7	14400.0	252.7k	24.4k	936.6	14400.0
belg+4arcs_118	238.2k	238.2k	0.0	241.1	238.2k	238.2k	0.0	1108.0	238.2k	203.8k	16.9	14400.0
belg+4arcs_119	346.6k	298.7k	16.0	14400.0	337.4k	337.4k	0.0	4229.4	337.4k	337.4k	0.0	8116.4
belg+4arcs_120	369.1k	201.9k	82.8	14400.0	420.9k	211.0k	99.5	14400.0	365.0k	165.2k	120.9	14400.0
belg+4arcs_121	199.0k	199.0k	0.0	237.5	199.0k	199.0k	0.0	1378.5	199.0k	199.0k	0.0	1620.9
belg+4arcs_122	298.3k	253.7k	17.6	14400.0	298.3k	298.3k	0.0	1706.3	314.7k	226.2k	39.1	14400.0
belg+4arcs_123	214.4k	214.4k	0.0	729.4	214.4k	214.4k	0.0	6421.5	217.9k	169.5k	28.6	14400.0
belg+4arcs_124	398.1k	349.1k	14.0	14400.0	396.5k	396.5k	0.0	9772.4	402.8k	283.4k	42.1	14400.0
belg+4arcs_125	164.9k	119.1k	38.5	14400.0	158.6k	158.6k	0.0	8039.8	190.6k	78.8k	141.8	14400.0
belg+4arcs_126	367.1k	291.1k	26.1	14400.0	381.8k	284.2k	34.3	14400.0	353.8k	201.7k	75.4	14400.0
belg+4arcs_127	379.7k	228.5k	66.2	14400.0	308.9k	308.9k	0.0	10974.6	430.0k	206.4k	108.3	14400.0
belg+4arcs_128	386.9k	307.0k	26.0	14400.0	366.8k	366.8k	0.0	9534.1	441.2k	219.6k	100.9	14400.0
belg+4arcs_129	130.6k	130.6k	0.0	255.1	130.6k	130.6k	0.0	2115.7	130.6k	121.6k	7.4	14400.0
belg+4arcs_130	344.6k	344.6k	0.0	697.5	344.6k	344.6k	0.0	2389.8	345.3k	246.7k	40.0	14400.0
belg+4arcs_131	431.7k	290.9k	48.4	14400.0	360.4k	360.4k	0.0	5874.7	384.6k	236.2k	62.8	14400.0
belg+4arcs_132	200.8k	200.8k	0.0	10632.0	200.8k	200.8k	0.0	242.6	200.8k	200.8k	0.0	89.5
belg+4arcs_133	518.7k	360.9k	43.7	14400.0	485.7k	355.5k	36.6	14400.0	602.5k	195.7k	207.9	14400.0
belg+4arcs_134	625.7k	479.6k	30.5	14400.0	655.3k	497.7k	31.7	14400.0	702.2k	372.2k	88.7	14400.0
belg+4arcs_135	231.7k	231.6k	0.0	438.6	231.7k	231.7k	0.0	2956.7	231.7k	231.7k	0.0	3150.4
belg+4arcs_136	865.8k	598.2k	44.7	14400.0	844.6k	631.9k	33.7	14400.0	853.3k	522.7k	63.2	14400.0
belg+4arcs_137	636.7k	439.8k	44.8	14400.0	578.0k	513.5k	12.6	14400.0	719.9k	338.9k	112.4	14400.0
belg+4arcs_138	405.6k	369.8k	9.7	14400.0	393.2k	393.2k	0.0	7792.2	399.7k	270.4k	47.8	14400.0
belg+4arcs_139	304.2k	241.0k	26.2	14400.0	276.7k	276.7k	0.0	10312.8	289.1k	207.8k	39.2	14400.0
belg+4arcs_140	265.5k	173.7k	52.9	14400.0	243.2k	200.4k	21.4	14400.0	257.5k	133.9k	92.4	14400.0
belg+4arcs_141	286.5k	242.5k	18.2	14400.0	286.5k	286.5k	0.0	1190.3	287.3k	231.9k	23.9	14400.0
belg+4arcs_142	485.7k	329.8k	47.3	14400.0	452.4k	364.1k	24.3	14400.0	482.3k	280.3k	72.0	14400.0
belg+4arcs_143	186.2k	186.2k	0.0	130.5	186.2k	186.1k	0.0	701.3	186.2k	186.2k	0.0	312.5
belg+4arcs_144	163.8k	110.9k	47.6	14400.0	138.8k	138.8k	0.0	5127.2	154.5k	114.2k	35.3	14400.0
belg+4arcs_145	343.4k	280.8k	22.3	14400.0	1e+20	261.1k	–	14400.0	564.8k	164.5k	243.3	14400.0
belg+4arcs_146	423.1k	206.9k	104.5	14400.0	376.3k	242.7k	55.1	14400.0	297.0k	176.2k	68.6	14400.0
belg+4arcs_147	429.0k	313.4k	36.9	14400.0	384.5k	384.5k	0.0	9115.3	412.1k	269.2k	53.1	14400.0
belg+4arcs_148	690.9k	318.7k	116.8	14400.0	557.8k	302.6k	84.4	14400.0	719.3k	188.7k	281.2	14400.0
belg+4arcs_149	221.1k	221.0k	0.1	3548.2	221.1k	220.9k	0.1	838.2	221.1k	221.1k	0.0	8012.8
belg+4arcs_150	636.3k	475.7k	33.8	14400.0	552.9k	552.9k	0.0	7501.2	555.4k	344.7k	61.1	14400.0
belg+4arcs_151	197.1k	197.1k	0.0	51.2	197.1k	197.1k	0.0	2205.6	197.1k	197.1k	0.0	29.0
belg+4arcs_152	277.1k	277.1k	0.0	1003.8	277.1k	277.1k	0.0	7583.4	290.0k	219.1k	32.4	14400.0
belg+4arcs_153	369.1k	305.0k	21.0	14400.0	365.6k	365.6k	0.0	2853.2	382.8k	248.1k	54.3	14400.0
belg+4arcs_154	249.1k	235.2k	5.9	14400.0	249.1k	249.1k	0.0	1641.2	274.7k	213.3k	28.8	14400.0
belg+4arcs_155	589.0k	400.6k	47.0	14400.0	494.2k	494.2k	0.0	9505.2	508.5k	320.6k	58.6	14400.0
belg+4arcs_156	232.6k	232.6k	0.0	1592.7	232.6k	232.6k	0.0	947.3	232.6k	232.4k	0.1	4554.4
belg+4arcs_157	280.8k	280.8k	0.0	1344.2	280.8k	280.8k	0.0	2695.9	307.6k	199.7k	54.0	14400.0
belg+4arcs_158	279.0k	279.0k	0.0	2051.8	279.0k	279.0k	0.0	4118.4	279.0k	197.3k	41.4	14400.0
belg+4arcs_159	337.6k	249.5k	35.3	14400.0	398.3k	244.6k	62.8	14400.0	613.0k	175.9k	248.4	14400.0
belg+4arcs_160	317.6k	228.5k	39.0	14400.0	342.1k	204.6k	67.2	14400.0	287.0k	174.8k	64.2	14400.0
belg+4arcs_161	384.3k	384.3k	0.0	12975.1	384.3k	384.3k	0.0	4517.4	387.7k	253.7k	52.8	14400.0
belg+4arcs_162	539.7k	370.9k	45.5	14400.0	519.6k	377.3k	37.7	14400.0	541.2k	249.5k	116.9	14400.0
belg+4arcs_163	596.7k	432.8k	37.9	14400.0	495.5k	403.5k	22.8	14400.0	495.5k	303.4k	63.3	14400.0
belg+4arcs_164	181.6k	181.6k	0.0	256.1	181.6k	181.4k	0.1	7072.7	191.1k	130.3k	46.6	14400.0
belg+4arcs_165	205.1k	205.1k	0.0	177.7	205.1k	205.0k	0.1	659.1	205.1k	205.1k	0.0	22.8
belg+4arcs_166	125.3k	125.3k	0.0	407.5	125.3k	125.3k	0.0	257.7	125.3k	125.3k	0.0	799.6
belg+4arcs_167	344.8k	344.7k	0.0	979.2	344.8k	344.8k	0.0	7916.4	361.1k	211.4k	70.8	14400.0
belg+4arcs_168	463.4k	349.1k	32.7	14400.0	459.9k	459.9k	0.0	13273.9	646.0k	285.8k	126.0	14400.0
belg+4arcs_169	181.4k	181.4k	0.0	68.8	181.4k	181.4k	0.0	1773.7	181.4k	181.4k	0.0	1844.3
belg+4arcs_170	273.4k	273.4k	0.0	1768.3	273.4k	273.4k	0.0	2669.8	273.4k	241.0k	13.4	14400.0
belg+4arcs_171	331.5k	331.5k	0.0	3908.1	1e+20	261.9k	–	14400.0	332.7k	220.9k	50.6	14400.0
belg+4arcs_172	237.4k	185.2k	28.2	14400.0	228.9k	193.6k	18.2	14400.0	253.6k	83.2k	204.8	14400.0
belg+4arcs_173	398.9k	398.9k	0.0	614.0	398.9k	398.9k	0.0	1913.5	398.9k	398.9k	0.0	11559.2
belg+4arcs_174	232.4k	232.4k	0.0	1076.1	232.4k	232.4k	0.0	1745.0	232.4k	221.1k	5.1	14400.0
belg+4arcs_175	115.9k	115.9k	0.0	693.6	115.9k	115.9k	0.0	1069.6	115.9k	115.9k	0.0	3491.5
belg+4arcs_176	366.0k	366.0k	0.0	1451.8	373.5k	333.6k	12.0	14400.0	385.1k	260.7k	47.7	14400.0
belg+4arcs_177	337.4k	337.4k	0.0	10681.0	337.4k	337.4k	0.0	583.4	337.4k	313.3k	7.7	14400.0
belg+4arcs_178	429.2k	182.2k	135.5	14400.0	264.1k	264.1k	0.0	3807.1	288.8k	196.1k	47.3	14400.0
belg+4arcs_179	333.9k	259.2k	28.8	14400.0	333.9k	333.9k	0.0	4300.6	386.0k	256.9k	50.3	14400.0
belg+4arcs_180	142.7k	130.4k	9.4	14400.0	157.9k	97.9k	61.2	14400.0	149.4k	39.0k	283.4	14400.0
belg+4arcs_181	161.4k	161.4k	0.0	118.4	161.4k	161.4k	0.0	923.3	161.4k	161.4k	0.0	2808.6
belg+4arcs_182	264.8k	264.8k	0.0	7632.3	264.8k	264.8k	0.0	9133.6	288.9k	169.8k	70.1	14400.0
belg+4arcs_183	164.7k	164.7k	0.0	147.0	164.7k	164.7k	0.0	314.7	164.7k	164.7k	0.0	31.0

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Table A.27: Comparison of discrete models on *Circuit rank* + 4 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_184	147.1k	147.1k	0.0	4273.3	147.1k	147.1k	0.0	2604.4	162.2k	130.1k	24.7	14400.0
belg+4arcs_185	197.4k	197.4k	0.0	811.2	197.4k	197.4k	0.0	4164.2	197.4k	197.4k	0.0	10280.0
belg+4arcs_186	268.5k	268.5k	0.0	1063.1	268.5k	268.5k	0.0	2250.7	268.5k	238.5k	12.5	14400.0
belg+4arcs_187	1142.3k	489.3k	133.5	14400.0	627.9k	536.1k	17.1	14400.0	634.2k	339.6k	86.8	14400.0
belg+4arcs_188	160.8k	160.8k	0.0	259.3	160.8k	160.8k	0.0	10200.6	160.8k	160.7k	0.0	1154.9
belg+4arcs_189	307.9k	287.1k	7.2	14400.0	309.4k	270.0k	14.6	14400.0	319.7k	209.1k	52.9	14400.0
belg+4arcs_190	255.8k	176.9k	44.6	14400.0	245.8k	186.9k	31.6	14400.0	504.9k	137.1k	268.2	14400.0
belg+4arcs_191	457.1k	371.1k	23.2	14400.0	457.1k	457.1k	0.0	6202.4	457.1k	307.0k	48.9	14400.0
belg+4arcs_192	335.3k	335.3k	0.0	2056.0	367.3k	299.1k	22.8	14400.0	353.6k	216.0k	63.7	14400.0
belg+4arcs_193	182.2k	182.2k	0.0	19.0	182.2k	182.2k	0.0	1058.5	182.2k	182.2k	0.0	109.4
belg+4arcs_194	696.5k	330.4k	110.8	14400.0	405.0k	405.0k	0.0	6538.8	410.0k	318.3k	28.8	14400.0
belg+4arcs_195	267.7k	267.7k	0.0	1734.0	267.7k	260.2k	2.9	14400.0	267.7k	239.9k	11.6	14400.0
belg+4arcs_196	489.2k	411.1k	19.0	14400.0	493.0k	406.5k	21.3	14400.0	565.8k	301.8k	87.4	14400.0
belg+4arcs_197	580.4k	511.4k	13.5	14400.0	575.5k	489.9k	17.5	14400.0	640.9k	320.9k	99.7	14400.0
belg+4arcs_198	232.2k	232.2k	0.0	399.5	232.2k	232.2k	0.0	2959.2	232.2k	232.2k	0.0	1878.4
belg+4arcs_199	45.2k	45.2k	0.0	12.1	45.2k	45.2k	0.0	3077.7	45.2k	45.2k	0.0	481.6
belg+4arcs_200	445.1k	308.2k	44.4	14400.0	400.4k	383.5k	4.4	14400.0	418.2k	274.5k	52.3	14400.0
belg+4arcs_201	75.5k	75.5k	0.0	1926.5	75.5k	75.5k	0.0	6747.5	106.0k	29.5k	258.8	14400.0
belg+4arcs_202	241.8k	241.8k	0.0	566.8	241.8k	241.8k	0.0	2753.4	279.4k	183.4k	52.3	14400.0
belg+4arcs_203	295.1k	230.6k	28.0	14400.0	275.8k	275.8k	0.0	9331.4	305.5k	152.8k	100.0	14400.0
belg+4arcs_204	296.3k	296.3k	0.0	6711.5	454.0k	226.8k	100.2	14400.0	314.7k	162.9k	93.2	14400.0
belg+4arcs_205	456.8k	311.5k	46.6	14400.0	436.8k	436.7k	0.0	8117.9	554.9k	280.0k	98.2	14400.0
belg+4arcs_206	195.8k	189.1k	3.6	14400.0	195.3k	195.3k	0.0	1574.4	195.3k	195.3k	0.0	899.0
belg+4arcs_207	385.4k	330.5k	16.6	14400.0	385.4k	385.4k	0.0	4289.7	388.3k	287.3k	35.2	14400.0
belg+4arcs_208	267.8k	240.3k	11.4	14400.0	252.0k	252.0k	0.0	2451.9	252.0k	252.0k	0.0	4823.3
belg+4arcs_209	156.5k	156.5k	0.0	765.1	156.5k	156.5k	0.0	663.6	156.5k	156.5k	0.0	4072.5
belg+4arcs_210	344.8k	262.5k	31.3	14400.0	335.2k	335.2k	0.0	8423.1	336.8k	249.4k	35.0	14400.0
belg+4arcs_211	384.5k	384.5k	0.0	3492.0	384.5k	384.5k	0.0	7856.2	391.9k	248.1k	58.0	14400.0
belg+4arcs_212	169.1k	169.1k	0.0	79.5	169.1k	169.1k	0.0	118.5	169.1k	169.1k	0.0	21.6
belg+4arcs_213	700.3k	421.5k	66.1	14400.0	719.1k	430.3k	67.1	14400.0	646.2k	294.4k	119.5	14400.0
belg+4arcs_214	252.3k	252.3k	0.0	2281.4	252.3k	252.3k	0.0	988.6	252.3k	252.3k	0.0	770.7
belg+4arcs_215	230.8k	230.8k	0.0	889.9	230.8k	230.8k	0.0	1181.3	230.8k	230.8k	0.0	9079.1
belg+4arcs_216	489.9k	322.3k	52.0	14400.0	414.6k	414.6k	0.0	8270.9	531.4k	268.3k	98.1	14400.0
belg+4arcs_217	240.9k	240.9k	0.0	294.8	240.9k	240.9k	0.0	2242.9	240.9k	210.0k	14.7	14400.0
belg+4arcs_218	484.2k	399.7k	21.2	14400.0	445.4k	445.4k	0.0	2879.5	469.1k	355.3k	32.0	14400.0
belg+4arcs_219	351.5k	351.5k	0.0	1951.6	351.5k	351.5k	0.0	3775.8	388.5k	251.0k	54.8	14400.0
belg+4arcs_220	543.0k	336.8k	61.2	14400.0	534.2k	319.2k	67.4	14400.0	682.0k	234.0k	191.4	14400.0
belg+4arcs_221	124.5k	124.5k	0.0	19.1	124.5k	124.5k	0.0	4652.7	124.5k	124.5k	0.0	2455.5
belg+4arcs_222	369.6k	369.6k	0.0	12095.8	403.7k	281.6k	43.4	14400.0	411.2k	254.7k	61.4	14400.0
belg+4arcs_223	343.1k	257.5k	33.2	14400.0	334.4k	334.4k	0.0	9079.4	359.2k	200.4k	79.3	14400.0
belg+4arcs_224	979.6k	343.2k	185.5	14400.0	501.9k	324.3k	54.8	14400.0	730.2k	213.5k	242.1	14400.0
belg+4arcs_225	217.6k	217.6k	0.0	3571.4	226.1k	179.4k	26.0	14400.0	236.3k	122.4k	93.2	14400.0
belg+4arcs_226	276.8k	276.8k	0.0	2458.9	276.8k	276.8k	0.0	4407.5	278.8k	241.7k	15.3	14400.0
belg+4arcs_227	172.5k	59.4k	190.5	14400.0	1e+20	92.2k	–	14400.0	178.5k	28.8k	520.8	14400.0
belg+4arcs_228	378.0k	377.8k	0.1	9615.3	378.0k	378.0k	0.0	8307.9	387.9k	211.5k	83.4	14400.0
belg+4arcs_229	418.5k	327.0k	28.0	14400.0	416.2k	355.9k	16.9	14400.0	418.5k	268.1k	56.1	14400.0
belg+4arcs_230	336.5k	298.7k	12.6	14400.0	336.5k	314.6k	6.9	14400.0	349.4k	228.0k	53.3	14400.0
belg+4arcs_231	218.0k	191.0k	14.1	14400.0	206.9k	206.9k	0.0	4094.6	260.9k	162.8k	60.3	14400.0
belg+4arcs_232	186.7k	186.7k	0.0	62.3	186.7k	186.7k	0.0	181.5	186.7k	186.7k	0.0	11.9
belg+4arcs_233	413.4k	303.5k	36.2	14400.0	384.6k	299.0k	28.7	14400.0	625.5k	221.7k	182.2	14400.0
belg+4arcs_234	192.3k	192.3k	0.0	1004.8	192.3k	192.2k	0.0	6039.8	192.7k	167.3k	15.2	14400.0
belg+4arcs_235	260.0k	260.0k	0.0	2525.2	260.0k	260.0k	0.0	3340.2	260.8k	232.3k	12.2	14400.0
belg+4arcs_236	333.1k	332.9k	0.1	7674.9	333.1k	333.1k	0.0	12694.8	501.4k	204.5k	145.1	14400.0
belg+4arcs_237	241.6k	176.4k	37.0	14400.0	228.6k	228.6k	0.0	6228.7	270.0k	120.8k	123.6	14400.0
belg+4arcs_238	432.0k	432.0k	0.0	5742.1	432.0k	432.0k	0.0	10102.1	521.8k	269.8k	93.4	14400.0
belg+4arcs_239	461.9k	369.9k	24.9	14400.0	449.7k	381.2k	18.0	14400.0	461.7k	290.6k	58.9	14400.0
belg+4arcs_240	468.7k	413.3k	13.4	14400.0	470.5k	393.8k	19.5	14400.0	549.6k	238.5k	130.4	14400.0
belg+4arcs_241	464.0k	367.1k	26.4	14400.0	439.2k	439.2k	0.0	8849.3	460.3k	289.3k	59.1	14400.0
belg+4arcs_242	308.5k	223.9k	37.8	14400.0	313.7k	282.8k	10.9	14400.0	313.7k	236.6k	32.6	14400.0
belg+4arcs_243	253.7k	231.6k	9.5	14400.0	253.7k	253.7k	0.0	1828.1	253.7k	253.5k	0.1	8381.5
belg+4arcs_244	232.4k	232.4k	0.0	184.6	232.4k	232.4k	0.0	319.0	232.4k	232.4k	0.0	30.3
belg+4arcs_245	46.4k	46.4k	0.0	3577.7	46.4k	46.4k	0.0	2010.0	46.4k	46.4k	0.0	1993.6
belg+4arcs_246	327.3k	327.3k	0.0	4481.3	327.3k	327.3k	0.0	6077.2	327.3k	201.9k	62.1	14400.0
belg+4arcs_247	248.8k	194.4k	27.9	14400.0	1e+20	194.9k	–	14400.0	318.6k	115.0k	177.1	14400.0
belg+4arcs_248	454.0k	454.0k	0.0	8650.1	454.0k	454.0k	0.0	2511.0	459.1k	310.0k	48.1	14400.0
belg+4arcs_249	352.2k	210.8k	67.1	14400.0	1e+20	181.8k	–	14400.0	474.8k	124.1k	282.5	14400.0
belg+4arcs_250	360.5k	360.5k	0.0	14291.7	360.5k	360.5k	0.0	11219.8	404.5k	206.8k	95.6	14400.0
belg+4arcs_251	355.9k	229.6k	55.0	14400.0	333.5k	333.5k	0.0	7128.5	334.7k	237.1k	41.2	14400.0
belg+4arcs_252	194.9k	194.9k	0.0	2229.7	194.9k	194.9k	0.0	3957.8	194.9k	178.6k	9.1	14400.0
belg+4arcs_253	545.4k	472.9k	15.3	14400.0	544.6k	473.5k	15.0	14400.0	569.0k	331.0k	71.9	14400.0
belg+4arcs_254	508.9k	373.6k	36.2	14400.0	428.4k	428.4k	0.0	11428.8	477.0k	266.2k	79.2	14400.0
belg+4arcs_255	23.4k	23.4k	0.0	144.8	23.4k	23.4k	0.0	1450.8	23.4k	23.4k	0.0	4.0
belg+4arcs_256	333.6k	333.6k	0.0	3200.0	333.6k	333.6k	0.0	6250.4	335.7k	236.5k	42.0	14400.0
belg+4arcs_257	291.2k	291.2k	0.0	11368.4	1e+20	220.9k	–	14400.0	293.8k	156.7k	87.5	14400.0
belg+4arcs_258	228.6k	228.6k	0.0	1193.9	228.6k	228.6k	0.0	565.6	228.6k	228.6k	0.0	5695.5
belg+4arcs_259	258.8k	258.8k	0.0	684.1	258.8k	258.8k	0.0	1425.9	258.8k	228.1k	13.4	14400.0
belg+4arcs_260	312.4k	312.4k	0.0	5566.9	312.4k	312.4k	0.0	9423.7	321.6k	183.7k	75.1	14400.0
belg+4arcs_261	18.1k	18.1k	0.0	16.9	18.1k	18.1k	0.0	1746.7	18.1k	18.1k	0.0	1.3

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Table A.27: Comparison of discrete models on *Circuit rank* + 4 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_262	249.6k	213.9k	16.7	14400.0	248.2k	248.2k	0.0	8070.0	263.3k	130.3k	102.1	14400.0
belg+4arcs_263	471.0k	299.5k	57.3	14400.0	421.1k	421.1k	0.0	12515.5	435.9k	305.8k	42.6	14400.0
belg+4arcs_264	392.3k	328.1k	19.6	14400.0	362.3k	362.3k	0.0	4785.6	399.0k	225.5k	77.0	14400.0
belg+4arcs_265	657.2k	444.4k	47.9	14400.0	602.7k	471.7k	27.8	14400.0	672.9k	360.9k	86.5	14400.0
belg+4arcs_266	239.5k	239.5k	0.0	7394.8	239.5k	239.5k	0.0	2371.7	263.8k	141.7k	86.1	14400.0
belg+4arcs_267	546.5k	444.8k	22.9	14400.0	713.5k	469.6k	51.9	14400.0	546.7k	368.2k	48.5	14400.0
belg+4arcs_268	395.2k	278.4k	42.0	14400.0	382.0k	382.0k	0.0	9384.5	587.1k	264.9k	121.7	14400.0
belg+4arcs_269	367.0k	284.9k	28.8	14400.0	1e+20	272.0k	–	14400.0	879.5k	187.3k	369.6	14400.0
belg+4arcs_270	233.6k	233.6k	0.0	775.9	233.6k	233.6k	0.0	972.2	233.6k	233.6k	0.0	197.1
belg+4arcs_271	375.2k	375.2k	0.0	3956.4	375.2k	366.9k	2.3	14400.0	382.3k	286.2k	33.6	14400.0
belg+4arcs_272	415.9k	415.9k	0.0	1005.5	415.9k	415.9k	0.0	4814.8	420.4k	316.6k	32.8	14400.0
belg+4arcs_273	559.1k	437.1k	27.9	14400.0	773.4k	407.1k	90.0	14400.0	633.3k	265.1k	138.9	14400.0
belg+4arcs_274	200.3k	200.3k	0.0	134.9	200.3k	200.3k	0.0	327.4	200.3k	200.3k	0.0	1608.0
belg+4arcs_275	156.3k	139.6k	11.9	14400.0	147.1k	147.1k	0.0	6730.9	153.4k	113.7k	34.9	14400.0
belg+4arcs_276	85.0k	56.1k	51.6	14400.0	85.0k	85.0k	0.0	2651.3	85.0k	85.0k	0.0	4152.6
belg+4arcs_277	233.5k	233.5k	0.0	5230.3	233.5k	233.5k	0.0	8086.4	245.5k	157.3k	56.1	14400.0
belg+4arcs_278	154.8k	154.8k	0.0	343.3	154.8k	154.8k	0.0	563.7	154.8k	154.8k	0.0	1961.8
belg+4arcs_279	231.9k	202.3k	14.6	14400.0	229.7k	229.7k	0.0	6650.4	294.0k	151.9k	93.5	14400.0
belg+4arcs_280	493.0k	328.8k	49.9	14400.0	428.5k	428.5k	0.0	7875.2	462.8k	319.8k	44.7	14400.0
belg+4arcs_281	339.3k	216.7k	56.6	14400.0	298.3k	276.9k	7.7	14400.0	310.3k	189.8k	63.5	14400.0
belg+4arcs_282	363.8k	363.8k	0.0	2212.8	363.8k	363.8k	0.0	1335.7	371.1k	305.6k	21.4	14400.0
belg+4arcs_283	263.8k	263.8k	0.0	363.9	263.8k	263.8k	0.0	169.6	263.8k	263.8k	0.0	16.4
belg+4arcs_284	288.1k	288.1k	0.0	828.2	288.1k	288.1k	0.0	1838.5	288.1k	228.4k	26.1	14400.0
belg+4arcs_285	204.3k	204.1k	0.1	4753.1	204.5k	191.4k	6.9	14400.0	204.5k	177.8k	15.0	14400.0
belg+4arcs_286	335.9k	231.0k	45.4	14400.0	333.4k	333.4k	0.0	4226.7	362.7k	241.3k	50.3	14400.0
belg+4arcs_287	251.1k	251.1k	0.0	365.3	251.1k	251.1k	0.0	2802.7	251.1k	251.1k	0.0	1412.7
belg+4arcs_288	224.2k	224.2k	0.0	3235.6	224.2k	224.2k	0.0	7399.9	224.2k	224.2k	0.0	280.8
belg+4arcs_289	255.9k	255.9k	0.0	1495.9	255.9k	255.9k	0.0	6217.9	337.5k	155.8k	116.7	14400.0
belg+4arcs_290	415.4k	415.4k	0.0	12533.4	415.4k	385.0k	7.9	14400.0	457.6k	219.1k	108.9	14400.0
belg+4arcs_291	458.1k	325.7k	40.7	14400.0	1e+20	336.7k	–	14400.0	537.8k	274.0k	96.2	14400.0
belg+4arcs_292	236.6k	169.7k	39.4	14400.0	223.3k	223.3k	0.0	3468.7	231.7k	162.5k	42.6	14400.0
belg+4arcs_293	116.5k	116.5k	0.0	1006.4	116.5k	116.5k	0.0	2599.7	116.5k	116.5k	0.0	3479.3
belg+4arcs_294	235.1k	235.1k	0.0	4369.6	235.1k	235.1k	0.0	2991.1	235.1k	211.8k	11.0	14400.0
belg+4arcs_295	535.0k	348.2k	53.6	14400.0	391.2k	363.1k	7.7	14400.0	391.2k	270.4k	44.6	14400.0
belg+4arcs_296	310.8k	310.8k	0.0	409.2	310.8k	310.8k	0.0	2922.5	310.8k	310.8k	0.0	8995.4
belg+4arcs_297	177.8k	177.8k	0.0	1351.6	177.8k	177.8k	0.0	209.6	177.8k	177.8k	0.0	25.4
belg+4arcs_298	149.1k	149.1k	0.0	3033.4	149.1k	149.1k	0.0	137.5	149.1k	149.1k	0.0	19.1
belg+4arcs_299	590.6k	431.1k	37.0	14400.0	722.1k	456.0k	58.4	14400.0	825.2k	291.7k	182.9	14400.0
belg+4arcs_300	187.9k	187.9k	0.0	24.1	187.9k	187.9k	0.0	1700.5	187.9k	187.9k	0.0	20.8
belg+4arcs_301	261.3k	192.0k	36.1	14400.0	250.6k	250.6k	0.0	4119.1	254.2k	199.5k	27.4	14400.0
belg+4arcs_302	364.4k	364.4k	0.0	10748.3	364.4k	364.4k	0.0	5118.2	370.1k	254.8k	45.3	14400.0
belg+4arcs_303	417.3k	417.3k	0.0	427.9	417.3k	417.3k	0.0	2553.3	417.3k	351.5k	18.7	14400.0
belg+4arcs_304	417.8k	398.5k	4.9	14400.0	414.6k	384.3k	7.9	14400.0	435.8k	275.4k	58.3	14400.0
belg+4arcs_305	405.2k	300.5k	34.8	14400.0	378.5k	378.2k	0.1	7726.6	403.2k	240.1k	67.9	14400.0
belg+4arcs_306	364.0k	265.5k	37.1	14400.0	323.1k	323.0k	0.0	4732.7	368.4k	239.7k	53.7	14400.0
belg+4arcs_307	189.4k	175.5k	7.9	14400.0	189.4k	189.4k	0.0	3426.3	189.4k	189.4k	0.0	11641.4
belg+4arcs_308	95.8k	95.8k	0.0	1049.1	95.8k	67.3k	42.4	14400.0	100.2k	52.4k	91.2	14400.0
belg+4arcs_309	319.5k	319.5k	0.0	9932.5	319.5k	303.5k	5.3	14400.0	425.1k	218.1k	94.9	14400.0
belg+4arcs_310	162.1k	162.1k	0.0	1527.2	162.1k	162.1k	0.0	2603.7	162.1k	162.1k	0.0	6593.9
belg+4arcs_311	241.2k	241.2k	0.0	3702.5	241.2k	241.2k	0.0	2310.0	247.1k	189.3k	30.5	14400.0
belg+4arcs_312	207.1k	207.1k	0.0	2839.7	207.1k	207.1k	0.0	3795.0	207.1k	207.1k	0.0	14237.3
belg+4arcs_313	466.5k	385.5k	21.0	14400.0	445.6k	389.1k	14.5	14400.0	449.1k	296.1k	51.7	14400.0
belg+4arcs_314	724.4k	478.5k	51.4	14400.0	1e+20	513.0k	–	14400.0	644.7k	370.0k	74.3	14400.0
belg+4arcs_315	394.1k	324.7k	21.4	14400.0	384.2k	384.2k	0.0	9622.7	395.9k	271.3k	45.9	14400.0
belg+4arcs_316	217.0k	217.0k	0.0	9313.3	352.9k	199.0k	77.4	14400.0	235.9k	169.5k	39.1	14400.0
belg+4arcs_317	411.2k	318.0k	29.3	14400.0	420.2k	335.9k	25.1	14400.0	566.0k	204.0k	177.5	14400.0
belg+4arcs_318	101.1k	101.1k	0.0	224.4	101.1k	101.1k	0.0	305.2	101.1k	101.1k	0.0	1256.2
belg+4arcs_319	209.1k	209.1k	0.0	506.3	209.1k	209.1k	0.0	4864.5	209.1k	209.1k	0.0	2697.0
belg+4arcs_320	245.6k	245.6k	0.0	668.3	245.6k	245.6k	0.0	922.0	245.6k	245.6k	0.0	4843.3
belg+4arcs_321	167.2k	167.1k	0.1	1439.2	167.2k	167.2k	0.0	1058.4	167.2k	167.2k	0.0	1789.1
belg+4arcs_322	193.5k	193.5k	0.0	6819.8	193.5k	193.4k	0.1	3131.8	198.2k	156.8k	26.4	14400.0
belg+4arcs_323	199.7k	199.7k	0.0	122.5	199.7k	199.7k	0.0	927.9	199.7k	199.7k	0.0	2202.1
belg+4arcs_324	286.8k	286.8k	0.0	7127.6	302.9k	255.1k	18.7	14400.0	329.4k	180.7k	82.4	14400.0
belg+4arcs_325	468.6k	295.0k	58.9	14400.0	474.0k	401.2k	18.2	14400.0	471.3k	275.4k	71.1	14400.0
belg+4arcs_326	344.3k	344.3k	0.0	768.5	344.3k	344.0k	0.1	2927.3	352.8k	227.0k	55.5	14400.0
belg+4arcs_327	51.7k	51.7k	0.0	184.6	51.7k	51.7k	0.0	178.5	51.7k	51.7k	0.0	1864.9
belg+4arcs_328	174.7k	174.7k	0.0	450.1	174.7k	174.7k	0.0	1144.9	174.7k	174.7k	0.0	2649.0
belg+4arcs_329	293.8k	197.3k	48.9	14400.0	248.5k	248.5k	0.0	5829.8	294.0k	191.0k	53.9	14400.0
belg+4arcs_330	253.7k	253.7k	0.0	682.8	253.7k	253.7k	0.0	775.3	253.7k	253.7k	0.0	40.2
belg+4arcs_331	237.1k	237.1k	0.0	281.8	237.1k	237.1k	0.0	2252.5	237.1k	190.6k	24.4	14400.0
belg+4arcs_332	629.4k	449.7k	40.0	14400.0	631.4k	511.4k	23.5	14400.0	776.1k	323.0k	140.2	14400.0
belg+4arcs_333	208.8k	188.8k	10.6	14400.0	204.1k	204.1k	0.0	924.3	204.1k	194.9k	4.8	14400.0
belg+4arcs_334	383.3k	383.3k	0.0	7069.9	383.3k	383.3k	0.0	11924.4	478.8k	216.9k	120.7	14400.0
belg+4arcs_335	419.7k	292.0k	43.7	14400.0	386.0k	331.8k	16.4	14400.0	471.7k	250.6k	88.2	14400.0
belg+4arcs_336	219.8k	219.8k	0.0	157.8	219.8k	219.8k	0.0	440.6	219.8k	219.8k	0.0	198.2
belg+4arcs_337	303.4k	264.2k	14.9	14400.0	1e+20	209.8k	–	14400.0	627.8k	171.9k	265.2	14400.0
belg+4arcs_338	274.0k	225.3k	21.6	14400.0	268.6k	268.6k	0.0	5878.9	275.7k	196.9k	40.0	14400.0
belg+4arcs_339	724.3k	406.7k	78.1	14400.0	658.3k	448.4k	46.8	14400.0	599.9k	338.2k	77.4	14400.0

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Table A.27: Comparison of discrete models on *Circuit rank* + 4 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_340	602.1k	459.3k	31.1	14400.0	652.1k	430.7k	51.4	14400.0	584.4k	337.2k	73.3	14400.0
belg+4arcs_341	187.2k	187.2k	0.0	220.6	187.2k	187.2k	0.0	183.9	187.2k	187.2k	0.0	10.0
belg+4arcs_342	368.7k	246.2k	49.7	14400.0	297.6k	261.8k	13.7	14400.0	411.2k	199.8k	105.8	14400.0
belg+4arcs_343	328.3k	283.6k	15.8	14400.0	304.5k	304.5k	0.0	8941.3	312.7k	225.0k	39.0	14400.0
belg+4arcs_344	211.2k	197.3k	7.0	14400.0	208.4k	208.4k	0.0	783.9	210.4k	189.4k	11.1	14400.0
belg+4arcs_345	368.4k	368.4k	0.0	3482.5	368.4k	368.4k	0.0	4620.8	391.9k	234.3k	67.3	14400.0
belg+4arcs_346	349.1k	219.5k	59.1	14400.0	1e+20	264.1k	–	14400.0	317.2k	214.0k	48.2	14400.0
belg+4arcs_347	611.3k	435.9k	40.2	14400.0	1e+20	416.8k	–	14400.0	568.4k	281.7k	101.8	14400.0
belg+4arcs_348	183.6k	183.6k	0.0	47.2	183.6k	183.6k	0.0	386.1	183.6k	183.6k	0.0	102.7
belg+4arcs_349	121.3k	121.3k	0.0	11550.7	121.3k	121.3k	0.0	5216.6	121.3k	91.5k	32.5	14400.0
belg+4arcs_350	280.5k	280.5k	0.0	563.4	280.5k	280.5k	0.0	2989.3	295.9k	190.5k	55.4	14400.0
belg+4arcs_351	422.8k	422.8k	0.0	10046.2	1e+20	350.1k	–	14400.0	463.6k	301.3k	53.9	14400.0
belg+4arcs_352	1e+20	324.3k	–	14400.0	459.5k	381.6k	20.4	14400.0	602.8k	258.1k	133.5	14400.0
belg+4arcs_353	342.7k	264.7k	29.5	14400.0	538.1k	266.1k	102.2	14400.0	342.7k	229.5k	49.3	14400.0
belg+4arcs_354	243.5k	243.5k	0.0	2500.5	243.5k	243.5k	0.0	4740.5	264.1k	181.9k	45.2	14400.0
belg+4arcs_355	38.7k	38.7k	0.0	372.5	38.7k	38.7k	0.0	253.3	38.7k	38.7k	0.0	547.8
belg+4arcs_356	436.8k	331.9k	31.6	14400.0	1e+20	342.8k	–	14400.0	436.5k	327.3k	33.4	14400.0
belg+4arcs_357	338.0k	338.0k	0.0	10745.0	338.0k	338.0k	0.0	5932.8	346.2k	221.7k	56.2	14400.0
belg+4arcs_358	582.2k	459.2k	26.8	14400.0	566.7k	566.7k	0.0	13450.3	578.2k	337.7k	71.2	14400.0
belg+4arcs_359	391.7k	299.8k	30.6	14400.0	353.7k	310.6k	13.9	14400.0	392.1k	180.1k	117.7	14400.0
belg+4arcs_360	212.1k	190.3k	11.4	14400.0	207.8k	207.8k	0.0	1138.8	207.8k	207.8k	0.0	9525.4
belg+4arcs_361	575.4k	486.8k	18.2	14400.0	662.5k	460.6k	43.8	14400.0	677.0k	310.4k	118.1	14400.0
belg+4arcs_362	276.2k	276.2k	0.0	9429.6	276.2k	276.2k	0.0	13776.4	307.1k	163.3k	88.1	14400.0
belg+4arcs_363	340.5k	245.3k	38.8	14400.0	323.6k	323.6k	0.0	8221.6	340.5k	236.9k	43.7	14400.0
belg+4arcs_364	320.5k	320.5k	0.0	487.1	320.5k	320.5k	0.0	3494.8	343.6k	235.0k	46.2	14400.0
belg+4arcs_365	392.0k	392.0k	0.0	4165.6	392.0k	392.0k	0.0	7334.4	393.5k	250.0k	57.4	14400.0
belg+4arcs_366	392.7k	392.6k	0.0	283.9	392.7k	392.7k	0.0	4420.2	392.7k	306.9k	28.0	14400.0
belg+4arcs_367	251.3k	251.3k	0.0	1500.0	251.3k	251.2k	0.0	1099.1	271.6k	226.1k	20.1	14400.0
belg+4arcs_368	268.0k	127.6k	110.1	14400.0	1e+20	132.3k	–	14400.0	544.4k	46.3k	1074.7	14400.0
belg+4arcs_369	139.3k	139.3k	0.0	127.9	139.3k	139.3k	0.0	396.6	139.3k	139.3k	0.0	421.5
belg+4arcs_370	208.9k	208.9k	0.0	978.3	208.9k	208.9k	0.0	713.3	208.9k	208.9k	0.0	2811.4
belg+4arcs_371	440.2k	321.3k	37.0	14400.0	433.6k	346.7k	25.1	14400.0	418.0k	260.0k	60.8	14400.0
belg+4arcs_372	301.0k	301.0k	0.0	871.0	301.0k	301.0k	0.0	3291.4	301.0k	300.9k	0.0	11078.3
belg+4arcs_373	393.2k	393.2k	0.0	1851.6	393.2k	393.2k	0.0	8807.8	417.3k	249.7k	67.1	14400.0
belg+4arcs_374	205.7k	205.7k	0.0	328.9	205.7k	205.7k	0.0	2003.7	205.7k	205.7k	0.0	4314.8
belg+4arcs_375	189.9k	189.9k	0.0	53.5	189.9k	189.9k	0.0	863.7	189.9k	189.9k	0.0	1428.2
belg+4arcs_376	467.5k	467.5k	0.0	926.5	467.5k	467.5k	0.0	2755.0	467.5k	467.5k	0.0	4242.5
belg+4arcs_377	615.6k	112.6k	446.6	14400.0	205.3k	104.9k	95.7	14400.0	331.6k	25.4k	1205.1	14400.0
belg+4arcs_378	292.3k	292.3k	0.0	14047.6	292.3k	292.3k	0.0	6032.3	417.1k	166.6k	150.3	14400.0
belg+4arcs_379	533.6k	350.7k	52.1	14400.0	449.1k	449.1k	0.0	11665.6	778.5k	232.2k	235.3	14400.0
belg+4arcs_380	331.3k	256.4k	29.2	14400.0	1e+20	267.9k	–	14400.0	320.6k	215.4k	48.9	14400.0
belg+4arcs_381	650.8k	403.9k	61.1	14400.0	484.3k	410.2k	18.1	14400.0	678.1k	269.6k	151.5	14400.0
belg+4arcs_382	562.9k	360.7k	56.1	14400.0	535.5k	401.6k	33.3	14400.0	502.4k	295.9k	69.8	14400.0
belg+4arcs_383	87.6k	87.6k	0.0	1951.3	87.6k	87.6k	0.0	3581.9	144.0k	39.2k	267.1	14400.0
belg+4arcs_384	408.3k	239.9k	70.2	14400.0	312.1k	312.1k	0.0	8299.5	367.1k	170.1k	115.8	14400.0
belg+4arcs_385	335.9k	268.5k	25.1	14400.0	520.0k	299.6k	73.6	14400.0	338.2k	250.3k	35.1	14400.0
belg+4arcs_386	530.7k	402.4k	31.9	14400.0	530.3k	432.7k	22.5	14400.0	864.9k	302.1k	186.3	14400.0
belg+4arcs_387	203.3k	134.3k	51.5	14400.0	186.0k	145.1k	28.2	14400.0	190.7k	98.6k	93.4	14400.0
belg+4arcs_388	143.4k	143.4k	0.0	34.2	143.4k	143.4k	0.0	184.2	143.4k	143.4k	0.0	17.3
belg+4arcs_389	282.1k	196.0k	44.0	14400.0	223.7k	223.7k	0.0	1173.8	223.7k	197.5k	13.3	14400.0
belg+4arcs_390	454.0k	279.6k	62.4	14400.0	354.0k	354.0k	0.0	10546.8	447.6k	226.2k	97.9	14400.0
belg+4arcs_391	254.5k	254.5k	0.0	12610.7	254.5k	254.5k	0.0	2618.1	254.5k	229.8k	10.7	14400.0
belg+4arcs_392	182.2k	168.3k	8.2	14400.0	182.2k	182.2k	0.0	1498.2	182.2k	182.2k	0.0	2738.8
belg+4arcs_393	270.0k	270.0k	0.0	392.4	270.0k	270.0k	0.0	856.1	270.0k	244.8k	10.3	14400.0
belg+4arcs_394	293.4k	208.8k	40.5	14400.0	262.8k	262.8k	0.0	10258.7	263.1k	192.2k	36.9	14400.0
belg+4arcs_395	386.7k	288.2k	34.2	14400.0	360.9k	360.9k	0.0	3921.7	362.3k	305.4k	18.6	14400.0
belg+4arcs_396	181.1k	181.1k	0.0	597.6	181.1k	181.1k	0.0	1780.0	181.1k	181.1k	0.0	2615.1
belg+4arcs_397	437.3k	437.3k	0.0	4833.1	437.3k	437.3k	0.0	8322.2	437.3k	345.2k	26.7	14400.0
belg+4arcs_398	273.2k	273.2k	0.0	12529.6	273.2k	249.3k	9.6	14400.0	292.6k	154.7k	89.1	14400.0
belg+4arcs_399	229.2k	200.2k	14.5	14400.0	221.0k	221.0k	0.0	9982.3	231.5k	155.0k	49.3	14400.0
belg+4arcs_400	233.0k	197.8k	17.8	14400.0	229.0k	229.0k	0.0	8298.9	230.0k	163.7k	40.6	14400.0
belg+4arcs_401	309.4k	309.4k	0.0	2199.7	309.4k	309.4k	0.0	5660.0	312.2k	206.8k	51.0	14400.0
belg+4arcs_402	253.2k	253.2k	0.0	2957.8	253.2k	253.2k	0.0	6860.0	260.0k	201.9k	28.8	14400.0
belg+4arcs_403	277.7k	277.7k	0.0	301.3	277.7k	277.7k	0.0	943.4	277.7k	277.7k	0.0	1934.0
belg+4arcs_404	387.5k	387.5k	0.0	913.5	387.5k	387.5k	0.0	3161.0	403.3k	293.3k	37.5	14400.0
belg+4arcs_405	403.5k	403.5k	0.0	630.2	403.5k	403.5k	0.0	837.7	416.7k	349.6k	19.2	14400.0
belg+4arcs_406	175.3k	175.3k	0.0	740.5	175.3k	175.3k	0.0	3494.1	175.3k	175.3k	0.0	3784.8
belg+4arcs_407	421.2k	383.5k	9.8	14400.0	421.2k	420.8k	0.1	10775.4	540.9k	280.5k	92.9	14400.0
belg+4arcs_408	23.4k	23.4k	0.0	29.4	23.4k	23.4k	0.0	103.5	23.4k	23.4k	0.0	59.2
belg+4arcs_409	191.4k	191.4k	0.0	629.1	191.4k	191.4k	0.0	3893.4	191.4k	191.4k	0.0	4072.6
belg+4arcs_410	242.1k	242.1k	0.0	432.8	242.1k	242.1k	0.0	2065.6	242.1k	242.1k	0.0	1741.8
belg+4arcs_411	376.5k	273.3k	37.7	14400.0	350.9k	350.9k	0.0	6557.2	366.6k	245.9k	49.1	14400.0
belg+4arcs_412	127.0k	127.0k	0.0	182.7	127.0k	127.0k	0.0	3517.7	156.9k	93.2k	68.4	14400.0
belg+4arcs_413	1e+20	70.8k	–	14400.0	228.5k	228.5k	0.0	570.8	228.5k	228.5k	0.0	2327.3
belg+4arcs_414	188.7k	188.7k	0.0	151.6	188.7k	188.7k	0.0	493.7	188.7k	188.7k	0.0	18.4
belg+4arcs_415	142.6k	142.6k	0.0	189.7	142.6k	142.6k	0.0	2853.3	193.8k	113.0k	71.4	14400.0
belg+4arcs_416	394.6k	282.4k	39.7	14400.0	349.3k	349.3k	0.0	3114.8	376.9k	239.1k	57.7	14400.0
belg+4arcs_417	283.6k	197.3k	43.7	14400.0	283.6k	283.6k	0.0	2679.1	286.2k	197.3k	45.1	14400.0

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Table A.27: Comparison of discrete models on *Circuit rank* + 4 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_418	568.5k	409.2k	38.9	14400.0	705.1k	416.5k	69.3	14400.0	699.7k	294.2k	137.8	14400.0
belg+4arcs_419	302.3k	214.1k	41.2	14400.0	504.4k	226.0k	123.1	14400.0	294.5k	176.2k	67.2	14400.0
belg+4arcs_420	246.6k	246.6k	0.0	10617.4	246.6k	246.6k	0.0	3177.3	254.4k	190.1k	33.9	14400.0
belg+4arcs_421	239.5k	183.2k	30.7	14400.0	239.5k	239.4k	0.0	1607.5	240.1k	181.2k	32.5	14400.0
belg+4arcs_422	355.7k	255.7k	39.1	14400.0	328.2k	328.2k	0.0	12262.1	349.0k	213.2k	63.7	14400.0
belg+4arcs_423	185.6k	185.6k	0.0	19.0	185.6k	185.6k	0.0	189.7	185.6k	185.6k	0.0	24.5
belg+4arcs_424	431.9k	431.9k	0.0	5047.2	431.9k	431.9k	0.0	10219.7	456.2k	270.6k	68.6	14400.0
belg+4arcs_425	304.8k	251.1k	21.4	14400.0	301.5k	301.5k	0.0	13794.0	319.3k	241.3k	32.3	14400.0
belg+4arcs_426	330.6k	200.4k	65.0	14400.0	489.0k	211.1k	131.6	14400.0	531.6k	122.6k	333.6	14400.0
belg+4arcs_427	225.3k	225.3k	0.0	356.9	225.3k	225.3k	0.0	889.2	225.3k	225.3k	0.0	294.8
belg+4arcs_428	205.2k	161.5k	27.1	14400.0	195.9k	195.9k	0.0	3928.3	205.2k	167.8k	22.3	14400.0
belg+4arcs_429	461.0k	369.6k	24.7	14400.0	458.4k	458.4k	0.0	9844.6	484.8k	334.9k	44.8	14400.0
belg+4arcs_430	437.1k	361.0k	21.1	14400.0	1219.2k	324.4k	275.8	14400.0	1294.3k	251.6k	414.5	14400.0
belg+4arcs_431	778.0k	504.4k	54.2	14400.0	657.6k	529.7k	24.1	14400.0	784.9k	358.1k	119.2	14400.0
belg+4arcs_432	146.8k	146.8k	0.0	11205.2	146.8k	146.8k	0.0	1069.9	165.7k	105.8k	56.5	14400.0
belg+4arcs_433	163.9k	163.9k	0.0	497.1	163.9k	163.9k	0.0	6562.7	163.9k	133.6k	22.7	14400.0
belg+4arcs_434	438.6k	348.4k	25.9	14400.0	425.4k	425.4k	0.0	11252.9	425.4k	279.5k	52.2	14400.0
belg+4arcs_435	418.6k	277.2k	51.0	14400.0	396.4k	396.4k	0.0	13164.0	439.2k	235.6k	86.4	14400.0
belg+4arcs_436	218.0k	218.0k	0.0	243.3	218.0k	218.0k	0.0	343.8	218.0k	218.0k	0.0	247.3
belg+4arcs_437	274.9k	274.9k	0.0	9062.0	274.9k	274.9k	0.0	7997.4	297.7k	167.8k	77.5	14400.0
belg+4arcs_438	353.2k	278.1k	27.0	14400.0	341.5k	341.5k	0.0	2638.7	359.1k	253.4k	41.7	14400.0
belg+4arcs_439	486.7k	401.4k	21.3	14400.0	1e+20	356.2k	–	14400.0	645.3k	218.9k	194.8	14400.0
belg+4arcs_440	553.1k	439.1k	26.0	14400.0	524.1k	419.1k	25.0	14400.0	535.3k	307.2k	74.2	14400.0
belg+4arcs_441	272.2k	219.4k	24.1	14400.0	268.5k	268.5k	0.0	3186.6	272.2k	193.9k	40.3	14400.0
belg+4arcs_442	126.9k	126.9k	0.0	561.2	126.9k	126.9k	0.0	496.8	126.9k	126.9k	0.0	2902.9
belg+4arcs_443	304.7k	304.7k	0.0	2144.1	304.7k	304.7k	0.0	8481.0	316.6k	190.6k	66.2	14400.0
belg+4arcs_444	172.7k	172.7k	0.0	27.0	172.7k	172.7k	0.0	825.6	172.7k	172.7k	0.0	23.5
belg+4arcs_445	520.8k	449.8k	15.8	14400.0	494.0k	447.7k	10.3	14400.0	548.9k	289.1k	89.9	14400.0
belg+4arcs_446	256.8k	256.8k	0.0	501.3	256.8k	256.8k	0.0	5227.1	256.8k	205.0k	25.2	14400.0
belg+4arcs_447	369.3k	231.9k	59.3	14400.0	330.4k	330.4k	0.0	13905.7	372.6k	205.4k	81.4	14400.0
belg+4arcs_448	285.9k	219.3k	30.3	14400.0	260.9k	226.3k	15.3	14400.0	423.8k	172.6k	145.6	14400.0
belg+4arcs_449	251.6k	210.8k	19.3	14400.0	251.6k	251.6k	0.0	6803.3	272.8k	190.2k	43.4	14400.0
belg+4arcs_450	250.2k	181.5k	37.8	14400.0	289.2k	174.0k	66.2	14400.0	465.0k	108.5k	328.5	14400.0
belg+4arcs_451	29.3k	29.3k	0.0	515.7	29.3k	29.3k	0.0	53.9	29.3k	29.3k	0.0	52.3
belg+4arcs_452	344.8k	259.7k	32.8	14400.0	343.9k	300.2k	14.6	14400.0	361.1k	236.4k	52.7	14400.0
belg+4arcs_453	355.5k	355.5k	0.0	3656.9	355.5k	355.5k	0.0	6575.3	355.5k	355.5k	0.0	11005.3
belg+4arcs_454	174.7k	174.7k	0.0	729.5	174.7k	174.7k	0.0	5412.8	174.7k	174.7k	0.0	10306.5
belg+4arcs_455	81.8k	81.8k	0.0	4847.2	81.8k	81.8k	0.0	5717.5	81.8k	48.8k	67.8	14400.0
belg+4arcs_456	452.3k	346.6k	30.5	14400.0	478.2k	340.6k	40.4	14400.0	614.6k	204.2k	200.9	14400.0
belg+4arcs_457	182.2k	182.2k	0.0	37.3	182.2k	182.2k	0.0	573.7	182.2k	182.2k	0.0	670.3
belg+4arcs_458	452.5k	422.7k	7.1	14400.0	452.5k	452.5k	0.0	12735.6	452.5k	303.2k	49.3	14400.0
belg+4arcs_459	0.0	0.0	0.0	0.1	0.0	0.0	0.0	41.6	0.0	0.0	0.0	13.5
belg+4arcs_460	161.5k	161.5k	0.0	593.5	161.5k	161.5k	0.0	640.0	161.5k	161.5k	0.0	9807.2
belg+4arcs_461	309.4k	274.5k	12.7	14400.0	305.8k	305.8k	0.0	3115.5	311.5k	250.6k	24.3	14400.0
belg+4arcs_462	224.3k	224.3k	0.0	1010.7	224.3k	224.3k	0.0	709.3	224.3k	204.7k	9.6	14400.0
belg+4arcs_463	182.2k	182.2k	0.0	19.3	182.2k	182.2k	0.0	177.0	182.2k	182.2k	0.0	38.3
belg+4arcs_464	248.0k	200.4k	23.8	14400.0	246.6k	246.6k	0.0	7619.0	267.2k	189.6k	40.9	14400.0
belg+4arcs_465	247.4k	247.4k	0.0	9047.6	1e+20	195.2k	–	14400.0	259.5k	138.5k	87.4	14400.0
belg+4arcs_466	185.8k	185.8k	0.0	7138.0	185.8k	185.8k	0.0	669.3	185.8k	166.6k	11.5	14400.0
belg+4arcs_467	343.4k	343.4k	0.0	4592.8	343.4k	343.4k	0.0	6257.7	343.4k	258.6k	32.8	14400.0
belg+4arcs_468	289.8k	191.7k	51.2	14400.0	335.7k	193.5k	73.5	14400.0	365.0k	70.9k	415.1	14400.0
belg+4arcs_469	262.6k	221.3k	18.6	14400.0	262.6k	262.6k	0.0	2333.9	333.2k	168.6k	97.6	14400.0
belg+4arcs_470	244.8k	180.7k	35.5	14400.0	306.0k	196.4k	55.8	14400.0	246.6k	117.0k	110.8	14400.0
belg+4arcs_471	352.4k	294.0k	19.9	14400.0	351.9k	351.9k	0.0	4751.8	397.9k	242.1k	64.3	14400.0
belg+4arcs_472	217.5k	172.7k	25.9	14400.0	214.6k	214.6k	0.0	12806.8	256.7k	151.9k	69.0	14400.0
belg+4arcs_473	382.9k	382.9k	0.0	1264.5	382.9k	382.9k	0.0	5363.4	390.5k	253.7k	53.9	14400.0
belg+4arcs_474	166.6k	166.6k	0.0	60.9	166.6k	166.6k	0.0	477.3	166.6k	166.6k	0.0	1500.3
belg+4arcs_475	168.3k	168.3k	0.0	56.4	168.3k	168.3k	0.0	360.4	168.3k	168.3k	0.0	14.2
belg+4arcs_476	370.1k	272.2k	36.0	14400.0	369.6k	369.6k	0.0	7284.7	387.3k	249.9k	55.0	14400.0
belg+4arcs_477	348.7k	254.9k	36.8	14400.0	342.5k	342.5k	0.0	10341.2	563.0k	226.7k	148.3	14400.0
belg+4arcs_478	359.5k	359.5k	0.0	11009.2	359.7k	312.6k	15.1	14400.0	465.7k	217.5k	114.2	14400.0
belg+4arcs_479	311.5k	311.5k	0.0	2654.5	311.5k	311.5k	0.0	8837.7	347.5k	191.3k	81.6	14400.0
belg+4arcs_480	167.5k	167.5k	0.0	21.7	167.5k	167.5k	0.0	96.4	167.5k	167.5k	0.0	10.2
belg+4arcs_481	632.5k	310.7k	103.6	14400.0	387.3k	387.3k	0.0	9585.6	389.4k	290.5k	34.1	14400.0
belg+4arcs_482	494.6k	225.3k	119.5	14400.0	291.0k	291.0k	0.0	9581.3	308.0k	217.9k	41.4	14400.0
belg+4arcs_483	186.2k	186.2k	0.0	4135.6	186.2k	186.2k	0.0	9112.0	193.6k	162.3k	19.3	14400.0
belg+4arcs_484	165.4k	165.4k	0.0	972.2	165.4k	165.4k	0.0	1359.5	165.4k	165.4k	0.0	673.9
belg+4arcs_485	169.7k	169.7k	0.0	1711.0	169.7k	169.7k	0.0	528.9	169.7k	169.7k	0.0	32.7
belg+4arcs_486	450.7k	351.8k	28.1	14400.0	444.9k	444.9k	0.0	6948.5	451.5k	304.7k	48.2	14400.0
belg+4arcs_487	213.7k	213.7k	0.0	407.3	213.7k	213.7k	0.0	635.6	213.7k	213.7k	0.0	1370.7
belg+4arcs_488	346.4k	306.3k	13.1	14400.0	333.5k	333.4k	0.0	3797.4	336.3k	277.8k	21.1	14400.0
belg+4arcs_489	648.8k	350.7k	85.0	14400.0	407.8k	407.8k	0.0	12772.2	408.7k	276.3k	47.9	14400.0
belg+4arcs_490	192.9k	192.9k	0.0	840.7	192.9k	192.9k	0.0	1691.9	192.9k	192.9k	0.0	1573.3
belg+4arcs_491	262.4k	200.2k	31.1	14400.0	239.2k	239.2k	0.0	11172.0	268.1k	168.6k	59.0	14400.0
belg+4arcs_492	167.3k	167.3k	0.0	37.3	167.3k	167.3k	0.0	232.9	167.3k	167.3k	0.0	15.6
belg+4arcs_493	378.0k	334.0k	13.2	14400.0	356.5k	356.5k	0.0	13860.5	761.6k	226.2k	236.6	14400.0
belg+4arcs_494	353.8k	250.0k	41.5	14400.0	303.5k	303.5k	0.0	1519.0	311.5k	234.4k	32.9	14400.0
belg+4arcs_495	412.7k	320.7k	28.7	14400.0	395.1k	326.2k	21.1	14400.0	399.8k	209.8k	90.6	14400.0

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Table A.27: Comparison of discrete models on *Circuit rank* + 4 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_496	322.2k	205.7k	56.7	14400.0	264.4k	239.6k	10.4	14400.0	272.5k	165.0k	65.1	14400.0
belg+4arcs_497	315.0k	238.7k	32.0	14400.0	290.6k	290.6k	0.0	5710.1	298.3k	187.0k	59.6	14400.0
belg+4arcs_498	485.4k	375.2k	29.4	14400.0	469.7k	469.7k	0.0	8440.9	476.2k	306.6k	55.3	14400.0
belg+4arcs_499	451.9k	251.5k	79.7	14400.0	264.9k	264.9k	0.0	1965.9	281.7k	236.6k	19.1	14400.0
belg+4arcs_500	186.8k	186.8k	0.0	226.4	186.8k	186.8k	0.0	600.5	186.8k	186.8k	0.0	151.8

Table A.28: Detailed results of the split-pipe models on *Circuit rank* + 4 arcs, as summarized in Figure 3.16c and Table 3.4d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_1	487.4k	485.9k	0.3	14400.0	487.4k	430.9k	13.1	14400.0
belg+4arcs_2	317.6k	317.3k	0.1	8088.8	317.6k	316.1k	0.5	14400.0
belg+4arcs_3	297.4k	296.4k	0.3	14400.0	297.4k	286.7k	3.7	14400.0
belg+4arcs_4	446.2k	438.0k	1.9	14400.0	446.2k	444.9k	0.3	14400.0
belg+4arcs_5	573.8k	569.0k	0.8	14400.0	573.8k	473.5k	21.2	14400.0
belg+4arcs_6	252.3k	232.0k	8.8	14400.0	252.3k	228.3k	10.5	14400.0
belg+4arcs_7	508.1k	505.5k	0.5	14400.0	508.1k	433.1k	17.3	14400.0
belg+4arcs_8	3012.8	2975.6	1.2	14400.0	3012.8	3012.8	0.0	13.6
belg+4arcs_9	385.1k	384.7k	0.1	1154.3	385.1k	384.7k	0.1	5849.8
belg+4arcs_10	358.5k	319.5k	12.2	14400.0	358.5k	353.7k	1.4	14400.0
belg+4arcs_11	407.3k	299.0k	36.2	14400.0	384.2k	355.3k	8.1	14400.0
belg+4arcs_12	150.8k	126.9k	18.8	14400.0	150.8k	150.7k	0.1	2831.7
belg+4arcs_13	390.1k	296.5k	31.6	14400.0	382.5k	318.5k	20.1	14400.0
belg+4arcs_14	613.5k	481.2k	27.5	14400.0	551.6k	470.9k	17.1	14400.0
belg+4arcs_15	62.9k	61.4k	2.6	14400.0	62.9k	3787.5	1561.9	14400.0
belg+4arcs_16	601.0k	573.0k	4.9	14400.0	666.0k	470.7k	41.5	14400.0
belg+4arcs_17	307.0k	295.1k	4.0	14400.0	299.8k	256.0k	17.1	14400.0
belg+4arcs_18	391.7k	366.6k	6.8	14400.0	391.7k	391.4k	0.1	1565.0
belg+4arcs_19	211.5k	208.8k	1.3	14400.0	211.5k	211.4k	0.1	5.2
belg+4arcs_20	395.7k	315.6k	25.4	14400.0	395.8k	332.3k	19.1	14400.0
belg+4arcs_21	45.4k	45.4k	0.0	68.9	45.4k	45.4k	0.0	9.2
belg+4arcs_22	292.7k	214.2k	36.6	14400.0	292.7k	258.6k	13.2	14400.0
belg+4arcs_23	177.1k	176.9k	0.1	5573.1	177.1k	176.9k	0.1	15.6
belg+4arcs_24	261.3k	241.9k	8.0	14400.0	261.3k	261.1k	0.1	34.8
belg+4arcs_25	300.5k	224.5k	33.8	14400.0	300.5k	237.1k	26.8	14400.0
belg+4arcs_26	329.8k	324.3k	1.7	14400.0	329.8k	329.5k	0.1	9889.0
belg+4arcs_27	185.8k	182.7k	1.7	14400.0	185.8k	185.2k	0.3	14400.0
belg+4arcs_28	432.5k	269.8k	60.3	14400.0	409.4k	406.7k	0.7	14400.0
belg+4arcs_29	222.9k	222.7k	0.1	14374.2	222.9k	190.7k	16.9	14400.0
belg+4arcs_30	188.2k	170.3k	10.6	14400.0	188.2k	167.9k	12.1	14400.0
belg+4arcs_31	256.3k	197.9k	29.5	14400.0	256.3k	228.0k	12.4	14400.0
belg+4arcs_32	457.3k	375.7k	21.7	14400.0	457.3k	376.3k	21.5	14400.0
belg+4arcs_33	147.1k	145.9k	0.8	14400.0	147.1k	147.0k	0.1	4.6
belg+4arcs_34	23.5k	19.6k	19.7	14400.0	23.5k	23.5k	0.0	0.9
belg+4arcs_35	309.7k	218.7k	41.6	14400.0	309.7k	307.2k	0.8	14400.0
belg+4arcs_36	363.6k	338.2k	7.5	14400.0	363.6k	363.3k	0.1	266.8
belg+4arcs_37	159.9k	157.0k	1.9	14400.0	159.9k	159.8k	0.1	7.5
belg+4arcs_38	484.5k	481.3k	0.7	14400.0	484.5k	392.7k	23.4	14400.0
belg+4arcs_39	158.1k	158.1k	0.0	50.7	158.1k	157.9k	0.1	33.4
belg+4arcs_40	224.6k	199.7k	12.4	14400.0	224.6k	205.3k	9.4	14400.0
belg+4arcs_41	487.6k	303.0k	60.9	14400.0	447.5k	442.6k	1.1	14400.0
belg+4arcs_42	337.0k	256.2k	31.6	14400.0	337.0k	336.7k	0.1	84.6
belg+4arcs_43	300.5k	255.1k	17.8	14400.0	299.1k	277.3k	7.8	14400.0
belg+4arcs_44	183.9k	183.8k	0.1	1.6	183.9k	183.8k	0.1	2.0
belg+4arcs_45	46.9k	32.4k	44.5	14400.0	46.9k	19.6k	138.8	14400.0
belg+4arcs_46	364.7k	307.2k	18.7	14400.0	364.7k	301.4k	21.0	14400.0
belg+4arcs_47	264.8k	252.6k	4.8	14400.0	264.8k	264.6k	0.1	387.3
belg+4arcs_48	280.2k	230.9k	21.4	14400.0	280.2k	231.6k	21.0	14400.0
belg+4arcs_49	156.1k	146.2k	6.8	14400.0	155.2k	154.3k	0.6	14400.0
belg+4arcs_50	470.2k	433.7k	8.4	14400.0	438.1k	343.6k	27.5	14400.0
belg+4arcs_51	112.1k	111.2k	0.7	14400.0	112.1k	109.5k	2.3	14400.0
belg+4arcs_52	350.4k	256.6k	36.6	14400.0	342.6k	251.4k	36.3	14400.0
belg+4arcs_53	186.0k	185.8k	0.1	257.0	186.0k	185.9k	0.1	15.0
belg+4arcs_54	310.7k	288.9k	7.6	14400.0	310.8k	303.6k	2.4	14400.0
belg+4arcs_55	228.6k	228.4k	0.1	1128.4	228.6k	228.4k	0.1	10.5
belg+4arcs_56	302.3k	263.7k	14.6	14400.0	302.3k	269.3k	12.3	14400.0
belg+4arcs_57	330.9k	297.9k	11.1	14400.0	330.9k	285.1k	16.1	14400.0
belg+4arcs_58	369.5k	228.2k	61.9	14400.0	332.4k	292.5k	13.6	14400.0

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Table A.28: Comparison of split-pipe models on *Circuit rank* + 4 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_59	130.3k	130.1k	0.1	1689.8	130.3k	130.1k	0.1	9102.8
belg+4arcs_60	243.3k	208.9k	16.4	14400.0	243.3k	243.0k	0.1	218.6
belg+4arcs_61	341.1k	311.4k	9.5	14400.0	345.0k	268.2k	28.6	14400.0
belg+4arcs_62	543.6k	522.2k	4.1	14400.0	602.8k	393.9k	53.0	14400.0
belg+4arcs_63	263.3k	263.0k	0.1	215.3	263.3k	263.1k	0.1	8.8
belg+4arcs_64	196.8k	176.6k	11.5	14400.0	196.8k	196.2k	0.3	14400.0
belg+4arcs_65	356.3k	333.0k	7.0	14400.0	356.1k	335.1k	6.3	14400.0
belg+4arcs_66	471.3k	306.9k	53.6	14400.0	404.6k	358.1k	13.0	14400.0
belg+4arcs_67	269.0k	211.9k	26.9	14400.0	260.7k	211.6k	23.2	14400.0
belg+4arcs_68	394.6k	381.5k	3.5	14400.0	394.6k	321.3k	22.8	14400.0
belg+4arcs_69	414.7k	412.9k	0.4	14400.0	414.7k	414.3k	0.1	3730.1
belg+4arcs_70	362.7k	361.9k	0.2	14400.0	362.7k	333.8k	8.7	14400.0
belg+4arcs_71	380.4k	306.4k	24.1	14400.0	380.4k	298.8k	27.3	14400.0
belg+4arcs_72	125.3k	63.9k	96.2	14400.0	125.3k	63.9k	96.1	14400.0
belg+4arcs_73	41.3k	41.3k	0.1	227.2	41.3k	41.3k	0.1	1.4
belg+4arcs_74	579.6k	404.1k	43.4	14400.0	579.6k	408.7k	41.8	14400.0
belg+4arcs_75	393.5k	375.9k	4.7	14400.0	393.5k	341.3k	15.3	14400.0
belg+4arcs_76	94.1k	94.0k	0.1	11392.9	94.1k	94.0k	0.1	20.5
belg+4arcs_77	209.6k	155.9k	34.4	14400.0	209.6k	209.4k	0.1	127.9
belg+4arcs_78	176.3k	148.0k	19.1	14400.0	176.3k	176.3k	0.0	10921.0
belg+4arcs_79	51.6k	34.9k	47.9	14400.0	51.6k	51.6k	0.0	47.2
belg+4arcs_80	302.9k	285.4k	6.1	14400.0	302.2k	283.3k	6.7	14400.0
belg+4arcs_81	406.4k	290.6k	39.9	14400.0	389.9k	324.9k	20.0	14400.0
belg+4arcs_82	18.6k	18.6k	0.1	425.3	18.6k	18.6k	0.1	0.9
belg+4arcs_83	563.5k	361.9k	55.7	14400.0	481.2k	481.2k	0.0	7378.4
belg+4arcs_84	576.7k	407.2k	41.6	14400.0	492.0k	423.9k	16.1	14400.0
belg+4arcs_85	364.4k	269.5k	35.2	14400.0	351.3k	292.8k	20.0	14400.0
belg+4arcs_86	411.7k	409.3k	0.6	14400.0	411.7k	350.6k	17.4	14400.0
belg+4arcs_87	272.4k	271.4k	0.4	14400.0	272.4k	247.1k	10.2	14400.0
belg+4arcs_88	459.3k	305.1k	50.6	14400.0	402.5k	364.8k	10.3	14400.0
belg+4arcs_89	738.1k	729.5k	1.2	14400.0	738.1k	615.3k	20.0	14400.0
belg+4arcs_90	504.0k	385.2k	30.8	14400.0	454.2k	404.9k	12.2	14400.0
belg+4arcs_91	538.7k	390.7k	37.9	14400.0	488.9k	405.2k	20.7	14400.0
belg+4arcs_92	411.5k	410.7k	0.2	14400.0	411.5k	411.1k	0.1	282.2
belg+4arcs_93	614.0k	570.8k	7.6	14400.0	614.0k	488.3k	25.7	14400.0
belg+4arcs_94	303.3k	299.4k	1.3	14400.0	303.3k	233.8k	29.7	14400.0
belg+4arcs_95	116.6k	44.2k	163.6	14400.0	116.6k	49.3k	136.6	14400.0
belg+4arcs_96	303.4k	200.7k	51.2	14400.0	287.4k	287.1k	0.1	216.0
belg+4arcs_97	207.7k	163.2k	27.3	14400.0	207.7k	144.5k	43.7	14400.0
belg+4arcs_98	374.0k	371.1k	0.8	14400.0	374.0k	318.1k	17.6	14400.0
belg+4arcs_99	461.6k	454.3k	1.6	14400.0	529.9k	376.8k	40.6	14400.0
belg+4arcs_100	299.7k	279.5k	7.2	14400.0	299.7k	271.5k	10.4	14400.0
belg+4arcs_101	311.9k	214.0k	45.7	14400.0	311.9k	300.8k	3.7	14400.0
belg+4arcs_102	405.1k	338.7k	19.6	14400.0	405.1k	354.2k	14.4	14400.0
belg+4arcs_103	157.3k	123.8k	27.1	14400.0	157.3k	134.6k	16.8	14400.0
belg+4arcs_104	180.0k	179.8k	0.1	91.3	180.0k	179.8k	0.1	28.1
belg+4arcs_105	191.4k	185.4k	3.2	14400.0	191.4k	185.3k	3.3	14400.0
belg+4arcs_106	221.1k	220.8k	0.1	250.7	221.1k	220.9k	0.1	13.8
belg+4arcs_107	352.6k	352.6k	0.0	116.2	352.8k	302.7k	16.5	14400.0
belg+4arcs_108	30.7k	30.6k	0.1	3.6	30.7k	30.6k	0.1	3.1
belg+4arcs_109	348.4k	345.6k	0.8	14400.0	348.4k	299.9k	16.2	14400.0
belg+4arcs_110	361.9k	361.5k	0.1	146.5	361.9k	361.6k	0.1	44.9
belg+4arcs_111	499.7k	383.4k	30.3	14400.0	421.0k	380.1k	10.8	14400.0
belg+4arcs_112	216.8k	207.8k	4.4	14400.0	216.8k	186.4k	16.3	14400.0
belg+4arcs_113	126.0k	115.4k	9.2	14400.0	126.0k	125.9k	0.1	30.7
belg+4arcs_114	102.6k	102.6k	0.0	240.7	102.6k	102.5k	0.1	121.1
belg+4arcs_115	526.2k	385.1k	36.6	14400.0	476.0k	392.6k	21.3	14400.0
belg+4arcs_116	292.2k	276.8k	5.6	14400.0	292.2k	236.3k	23.7	14400.0
belg+4arcs_117	151.0k	91.7k	64.8	14400.0	151.0k	84.0k	79.9	14400.0
belg+4arcs_118	224.8k	218.1k	3.1	14400.0	224.8k	209.3k	7.4	14400.0
belg+4arcs_119	391.9k	295.1k	32.8	14400.0	334.9k	317.8k	5.4	14400.0
belg+4arcs_120	277.5k	198.6k	39.7	14400.0	262.9k	239.7k	9.7	14400.0
belg+4arcs_121	185.4k	173.3k	7.0	14400.0	185.4k	185.2k	0.1	15.7
belg+4arcs_122	295.0k	294.2k	0.3	14400.0	295.0k	294.7k	0.1	1057.5
belg+4arcs_123	214.4k	160.3k	33.8	14400.0	210.4k	161.4k	30.3	14400.0
belg+4arcs_124	386.7k	325.0k	19.0	14400.0	386.7k	326.7k	18.4	14400.0
belg+4arcs_125	131.5k	69.3k	89.7	14400.0	131.5k	75.9k	73.1	14400.0
belg+4arcs_126	351.0k	217.3k	61.5	14400.0	322.9k	266.0k	21.4	14400.0
belg+4arcs_127	311.5k	299.5k	4.0	14400.0	311.5k	215.4k	44.6	14400.0
belg+4arcs_128	360.9k	356.7k	1.2	14400.0	360.9k	346.8k	4.1	14400.0
belg+4arcs_129	105.0k	91.1k	15.2	14400.0	105.0k	104.5k	0.5	14400.0
belg+4arcs_130	339.7k	339.1k	0.2	14400.0	339.7k	339.4k	0.1	8203.9
belg+4arcs_131	355.7k	321.7k	10.6	14400.0	355.7k	320.9k	10.8	14400.0
belg+4arcs_132	198.7k	193.8k	2.5	14400.0	198.1k	197.4k	0.3	14400.0
belg+4arcs_133	444.4k	434.6k	2.3	14400.0	469.8k	336.5k	39.6	14400.0
belg+4arcs_134	600.7k	422.8k	42.1	14400.0	600.7k	496.2k	21.0	14400.0
belg+4arcs_135	222.3k	222.3k	0.0	95.6	222.3k	194.5k	14.3	14400.0
belg+4arcs_136	872.4k	648.0k	34.6	14400.0	770.9k	657.7k	17.2	14400.0

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Table A.28: Comparison of split-pipe models on *Circuit rank* + 4 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_137	659.5k	465.5k	41.7	14400.0	575.7k	454.8k	26.6	14400.0
belg+4arcs_138	384.0k	315.3k	21.8	14400.0	384.0k	335.3k	14.5	14400.0
belg+4arcs_139	270.1k	258.6k	4.5	14400.0	270.1k	206.2k	31.0	14400.0
belg+4arcs_140	231.8k	144.0k	61.0	14400.0	231.6k	185.2k	25.1	14400.0
belg+4arcs_141	282.4k	202.6k	39.4	14400.0	282.4k	282.1k	0.1	105.5
belg+4arcs_142	443.9k	283.9k	56.4	14400.0	439.5k	418.8k	4.9	14400.0
belg+4arcs_143	185.0k	173.5k	6.6	14400.0	183.4k	173.4k	5.8	14400.0
belg+4arcs_144	107.2k	107.1k	0.1	478.6	107.2k	107.1k	0.1	16.9
belg+4arcs_145	328.4k	265.8k	23.5	14400.0	328.4k	269.4k	21.9	14400.0
belg+4arcs_146	293.3k	293.1k	0.1	897.3	293.3k	259.2k	13.2	14400.0
belg+4arcs_147	377.6k	297.2k	27.1	14400.0	377.6k	306.3k	23.3	14400.0
belg+4arcs_148	474.0k	351.8k	34.7	14400.0	423.4k	342.4k	23.6	14400.0
belg+4arcs_149	216.4k	205.8k	5.1	14400.0	216.4k	216.2k	0.1	7.9
belg+4arcs_150	548.4k	547.8k	0.1	6713.8	548.4k	488.4k	12.3	14400.0
belg+4arcs_151	195.5k	177.0k	10.4	14400.0	195.5k	195.3k	0.1	4047.2
belg+4arcs_152	274.2k	212.9k	28.8	14400.0	274.2k	268.1k	2.3	14400.0
belg+4arcs_153	351.0k	276.3k	27.0	14400.0	351.0k	268.6k	30.7	14400.0
belg+4arcs_154	241.0k	235.6k	2.3	14400.0	241.0k	213.3k	13.0	14400.0
belg+4arcs_155	550.3k	382.2k	44.0	14400.0	487.8k	470.5k	3.7	14400.0
belg+4arcs_156	235.2k	191.7k	22.7	14400.0	231.3k	195.2k	18.5	14400.0
belg+4arcs_157	278.7k	278.4k	0.1	2738.0	291.4k	207.7k	40.3	14400.0
belg+4arcs_158	272.0k	264.3k	2.9	14400.0	272.0k	268.0k	1.5	14400.0
belg+4arcs_159	326.7k	235.7k	38.6	14400.0	326.7k	234.2k	39.5	14400.0
belg+4arcs_160	272.6k	202.3k	34.7	14400.0	275.0k	201.9k	36.2	14400.0
belg+4arcs_161	438.5k	336.4k	30.3	14400.0	382.8k	337.0k	13.6	14400.0
belg+4arcs_162	469.8k	469.8k	0.0	389.9	469.8k	464.2k	1.2	14400.0
belg+4arcs_163	491.9k	414.7k	18.6	14400.0	491.9k	454.7k	8.2	14400.0
belg+4arcs_164	173.1k	172.4k	0.4	14400.0	173.1k	172.9k	0.1	4958.4
belg+4arcs_165	201.9k	201.7k	0.1	1127.5	201.9k	201.7k	0.1	245.4
belg+4arcs_166	111.6k	111.6k	0.0	67.5	111.6k	111.5k	0.1	18.7
belg+4arcs_167	337.4k	337.1k	0.1	435.0	337.4k	326.8k	3.2	14400.0
belg+4arcs_168	446.3k	438.3k	1.8	14400.0	446.3k	438.4k	1.8	14400.0
belg+4arcs_169	164.7k	163.6k	0.7	14400.0	164.7k	164.5k	0.1	9.6
belg+4arcs_170	259.2k	224.0k	15.7	14400.0	259.2k	259.2k	0.0	51.4
belg+4arcs_171	343.4k	276.4k	24.2	14400.0	330.8k	257.1k	28.6	14400.0
belg+4arcs_172	224.5k	218.9k	2.6	14400.0	224.5k	185.9k	20.8	14400.0
belg+4arcs_173	389.2k	388.9k	0.1	111.7	389.2k	381.0k	2.2	14400.0
belg+4arcs_174	213.5k	213.3k	0.1	1590.5	213.5k	212.5k	0.5	14400.0
belg+4arcs_175	79.6k	79.5k	0.1	8818.8	79.6k	74.7k	6.5	14400.0
belg+4arcs_176	355.6k	338.7k	5.0	14400.0	355.6k	333.7k	6.6	14400.0
belg+4arcs_177	322.5k	250.2k	28.9	14400.0	322.5k	259.9k	24.1	14400.0
belg+4arcs_178	277.3k	192.7k	43.9	14400.0	277.3k	208.3k	33.1	14400.0
belg+4arcs_179	326.8k	249.0k	31.3	14400.0	326.8k	323.6k	1.0	14400.0
belg+4arcs_180	124.3k	63.1k	97.0	14400.0	124.3k	79.5k	56.3	14400.0
belg+4arcs_181	141.1k	128.1k	10.1	14400.0	141.1k	140.9k	0.1	205.8
belg+4arcs_182	249.4k	184.0k	35.5	14400.0	249.4k	228.4k	9.2	14400.0
belg+4arcs_183	149.3k	147.4k	1.3	14400.0	149.3k	149.3k	0.0	3.1
belg+4arcs_184	122.1k	119.1k	2.5	14400.0	122.1k	122.0k	0.1	9.2
belg+4arcs_185	187.2k	155.4k	20.5	14400.0	187.2k	187.0k	0.1	487.0
belg+4arcs_186	266.0k	261.0k	1.9	14400.0	266.0k	248.4k	7.1	14400.0
belg+4arcs_187	716.7k	575.3k	24.6	14400.0	710.1k	581.7k	22.1	14400.0
belg+4arcs_188	155.5k	135.5k	14.7	14400.0	155.5k	139.8k	11.2	14400.0
belg+4arcs_189	283.9k	251.0k	13.1	14400.0	283.9k	251.5k	12.9	14400.0
belg+4arcs_190	216.4k	176.6k	22.6	14400.0	216.4k	165.7k	30.6	14400.0
belg+4arcs_191	438.6k	321.6k	36.4	14400.0	438.5k	350.8k	25.0	14400.0
belg+4arcs_192	332.4k	237.6k	39.9	14400.0	332.4k	311.7k	6.6	14400.0
belg+4arcs_193	179.2k	172.5k	3.9	14400.0	179.2k	179.1k	0.1	164.0
belg+4arcs_194	470.8k	329.7k	42.8	14400.0	404.6k	365.4k	10.7	14400.0
belg+4arcs_195	256.5k	215.6k	19.0	14400.0	256.5k	256.3k	0.1	104.5
belg+4arcs_196	559.9k	405.3k	38.1	14400.0	486.2k	427.2k	13.8	14400.0
belg+4arcs_197	550.9k	462.7k	19.1	14400.0	545.8k	471.5k	15.8	14400.0
belg+4arcs_198	219.5k	215.4k	1.9	14400.0	219.5k	219.3k	0.1	12.6
belg+4arcs_199	39.1k	39.1k	0.1	37.9	39.1k	39.1k	0.1	7.0
belg+4arcs_200	438.0k	346.7k	26.3	14400.0	398.9k	329.8k	20.9	14400.0
belg+4arcs_201	75.1k	67.8k	10.8	14400.0	75.1k	59.8k	25.7	14400.0
belg+4arcs_202	242.6k	218.9k	10.8	14400.0	230.3k	191.4k	20.3	14400.0
belg+4arcs_203	257.5k	257.3k	0.1	771.1	257.5k	257.3k	0.1	9059.3
belg+4arcs_204	288.3k	241.9k	19.2	14400.0	288.3k	226.1k	27.5	14400.0
belg+4arcs_205	419.3k	316.3k	32.6	14400.0	419.3k	286.0k	46.6	14400.0
belg+4arcs_206	187.7k	179.5k	4.5	14400.0	187.7k	184.0k	2.0	14400.0
belg+4arcs_207	402.5k	367.3k	9.6	14400.0	384.3k	309.6k	24.1	14400.0
belg+4arcs_208	247.5k	233.5k	6.0	14400.0	247.5k	246.7k	0.3	14400.0
belg+4arcs_209	137.0k	132.0k	3.8	14400.0	137.0k	136.8k	0.1	97.6
belg+4arcs_210	309.8k	228.6k	35.6	14400.0	309.8k	299.0k	3.6	14400.0
belg+4arcs_211	378.8k	344.2k	10.0	14400.0	378.8k	349.7k	8.3	14400.0
belg+4arcs_212	165.1k	163.6k	0.9	14400.0	165.1k	164.9k	0.1	61.8
belg+4arcs_213	560.1k	556.6k	0.6	14400.0	578.4k	420.0k	37.7	14400.0
belg+4arcs_214	236.6k	236.4k	0.1	405.8	236.6k	236.4k	0.1	143.3

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Table A.28: Comparison of split-pipe models on *Circuit rank* + 4 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_215	228.6k	194.7k	17.4	14400.0	228.6k	193.0k	18.5	14400.0
belg+4arcs_216	404.5k	279.7k	44.6	14400.0	404.5k	385.0k	5.1	14400.0
belg+4arcs_217	242.8k	210.3k	15.5	14400.0	239.6k	214.5k	11.7	14400.0
belg+4arcs_218	433.2k	393.5k	10.1	14400.0	433.2k	394.4k	9.8	14400.0
belg+4arcs_219	352.7k	224.8k	56.9	14400.0	347.6k	235.6k	47.5	14400.0
belg+4arcs_220	404.0k	290.4k	39.1	14400.0	432.0k	290.8k	48.6	14400.0
belg+4arcs_221	108.4k	103.3k	4.9	14400.0	108.4k	92.6k	17.1	14400.0
belg+4arcs_222	408.8k	269.1k	51.9	14400.0	371.0k	316.2k	17.3	14400.0
belg+4arcs_223	347.9k	242.2k	43.6	14400.0	330.3k	319.8k	3.3	14400.0
belg+4arcs_224	418.8k	413.2k	1.4	14400.0	418.8k	369.2k	13.4	14400.0
belg+4arcs_225	194.0k	175.6k	10.5	14400.0	194.0k	157.8k	22.9	14400.0
belg+4arcs_226	276.6k	253.8k	9.0	14400.0	275.9k	269.9k	2.3	14400.0
belg+4arcs_227	134.3k	32.2k	316.9	14400.0	134.3k	25.8k	420.1	14400.0
belg+4arcs_228	374.3k	373.3k	0.3	14400.0	374.3k	332.6k	12.5	14400.0
belg+4arcs_229	405.8k	330.8k	22.7	14400.0	405.8k	402.1k	0.9	14400.0
belg+4arcs_230	327.3k	303.3k	7.9	14400.0	327.3k	302.8k	8.1	14400.0
belg+4arcs_231	196.2k	162.1k	21.0	14400.0	192.7k	171.4k	12.5	14400.0
belg+4arcs_232	177.0k	176.8k	0.1	78.3	177.0k	176.9k	0.0	17.6
belg+4arcs_233	370.7k	276.7k	34.0	14400.0	370.7k	310.2k	19.5	14400.0
belg+4arcs_234	181.7k	170.0k	6.9	14400.0	181.7k	170.2k	6.8	14400.0
belg+4arcs_235	246.0k	216.1k	13.8	14400.0	246.0k	245.8k	0.1	3432.9
belg+4arcs_236	320.2k	255.2k	25.5	14400.0	320.2k	266.0k	20.4	14400.0
belg+4arcs_237	214.5k	182.6k	17.5	14400.0	214.5k	183.6k	16.8	14400.0
belg+4arcs_238	430.1k	428.8k	0.3	14400.0	455.1k	360.7k	26.2	14400.0
belg+4arcs_239	478.1k	404.1k	18.3	14400.0	476.3k	391.5k	21.7	14400.0
belg+4arcs_240	469.4k	424.2k	10.7	14400.0	462.2k	409.1k	13.0	14400.0
belg+4arcs_241	461.1k	312.4k	47.6	14400.0	437.6k	374.4k	16.9	14400.0
belg+4arcs_242	290.4k	283.0k	2.6	14400.0	331.1k	238.9k	38.6	14400.0
belg+4arcs_243	233.2k	228.2k	2.2	14400.0	233.2k	233.0k	0.1	20.0
belg+4arcs_244	224.8k	222.4k	1.1	14400.0	224.8k	224.5k	0.1	15.5
belg+4arcs_245	39.4k	35.6k	10.5	14400.0	39.4k	39.3k	0.1	47.1
belg+4arcs_246	325.4k	316.0k	3.0	14400.0	325.4k	322.9k	0.8	14400.0
belg+4arcs_247	245.0k	194.6k	25.9	14400.0	245.0k	182.8k	34.0	14400.0
belg+4arcs_248	449.3k	442.5k	1.5	14400.0	449.3k	443.1k	1.4	14400.0
belg+4arcs_249	281.7k	267.4k	5.3	14400.0	281.7k	171.4k	64.4	14400.0
belg+4arcs_250	358.5k	358.2k	0.1	374.8	358.5k	317.3k	13.0	14400.0
belg+4arcs_251	347.7k	275.8k	26.0	14400.0	332.6k	326.9k	1.7	14400.0
belg+4arcs_252	169.0k	168.8k	0.1	1375.5	169.0k	168.8k	0.1	10673.3
belg+4arcs_253	513.1k	445.8k	15.1	14400.0	513.1k	512.6k	0.1	5166.7
belg+4arcs_254	426.8k	397.4k	7.4	14400.0	426.8k	385.5k	10.7	14400.0
belg+4arcs_255	22.2k	21.5k	3.3	14400.0	22.2k	22.2k	0.0	22.2
belg+4arcs_256	322.1k	298.4k	7.9	14400.0	322.1k	302.8k	6.4	14400.0
belg+4arcs_257	287.4k	205.2k	40.1	14400.0	287.4k	205.4k	39.9	14400.0
belg+4arcs_258	225.2k	194.9k	15.5	14400.0	225.2k	225.2k	0.0	1366.0
belg+4arcs_259	253.3k	198.3k	27.7	14400.0	248.1k	247.8k	0.1	91.9
belg+4arcs_260	292.9k	286.1k	2.4	14400.0	292.9k	291.6k	0.4	14400.0
belg+4arcs_261	17.7k	17.7k	0.1	3.8	17.7k	17.7k	0.0	1.3
belg+4arcs_262	233.1k	190.0k	22.7	14400.0	243.2k	176.4k	37.9	14400.0
belg+4arcs_263	443.1k	296.2k	49.6	14400.0	420.1k	287.2k	46.3	14400.0
belg+4arcs_264	352.6k	341.3k	3.3	14400.0	352.6k	328.0k	7.5	14400.0
belg+4arcs_265	534.3k	470.6k	13.5	14400.0	534.3k	501.3k	6.6	14400.0
belg+4arcs_266	241.1k	205.5k	17.3	14400.0	237.1k	237.1k	0.0	2372.8
belg+4arcs_267	612.1k	404.0k	51.5	14400.0	550.5k	456.7k	20.5	14400.0
belg+4arcs_268	358.8k	247.2k	45.2	14400.0	358.8k	289.6k	23.9	14400.0
belg+4arcs_269	334.4k	276.3k	21.0	14400.0	334.4k	282.5k	18.4	14400.0
belg+4arcs_270	226.0k	225.8k	0.1	1852.9	226.0k	225.8k	0.1	10.6
belg+4arcs_271	367.4k	345.3k	6.4	14400.0	367.4k	325.9k	12.7	14400.0
belg+4arcs_272	399.8k	394.6k	1.3	14400.0	399.8k	395.4k	1.1	14400.0
belg+4arcs_273	518.1k	413.6k	25.3	14400.0	518.1k	459.1k	12.8	14400.0
belg+4arcs_274	194.0k	193.4k	0.3	14400.0	194.0k	193.8k	0.1	463.3
belg+4arcs_275	117.3k	87.4k	34.2	14400.0	117.3k	117.1k	0.2	14400.0
belg+4arcs_276	80.5k	77.4k	4.0	14400.0	80.5k	60.5k	33.2	14400.0
belg+4arcs_277	218.3k	197.1k	10.7	14400.0	218.3k	218.0k	0.1	14400.0
belg+4arcs_278	148.7k	146.8k	1.3	14400.0	148.7k	141.0k	5.5	14400.0
belg+4arcs_279	224.0k	162.1k	38.1	14400.0	224.0k	191.9k	16.7	14400.0
belg+4arcs_280	420.1k	416.1k	0.9	14400.0	441.5k	359.7k	22.7	14400.0
belg+4arcs_281	311.4k	227.2k	37.0	14400.0	295.7k	289.7k	2.1	14400.0
belg+4arcs_282	353.6k	353.2k	0.1	3484.3	353.6k	351.4k	0.6	14400.0
belg+4arcs_283	262.5k	251.1k	4.5	14400.0	262.5k	262.3k	0.1	213.7
belg+4arcs_284	283.9k	283.1k	0.3	14400.0	283.9k	226.9k	25.1	14400.0
belg+4arcs_285	200.2k	164.3k	21.8	14400.0	200.2k	182.8k	9.5	14400.0
belg+4arcs_286	325.0k	256.5k	26.7	14400.0	325.0k	256.3k	26.8	14400.0
belg+4arcs_287	255.1k	226.8k	12.5	14400.0	248.6k	248.0k	0.2	14400.0
belg+4arcs_288	221.0k	199.0k	11.1	14400.0	221.0k	218.6k	1.1	14400.0
belg+4arcs_289	251.9k	237.1k	6.2	14400.0	251.9k	251.1k	0.3	14400.0
belg+4arcs_290	414.6k	390.5k	6.2	14400.0	414.6k	356.3k	16.3	14400.0
belg+4arcs_291	489.9k	324.2k	51.1	14400.0	414.6k	378.2k	9.6	14400.0
belg+4arcs_292	214.7k	198.5k	8.2	14400.0	214.5k	171.2k	25.3	14400.0

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Table A.28: Comparison of split-pipe models on *Circuit rank* + 4 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_293	88.0k	82.7k	6.4	14400.0	88.0k	88.0k	0.0	12.3
belg+4arcs_294	220.9k	187.0k	18.1	14400.0	220.9k	189.2k	16.8	14400.0
belg+4arcs_295	433.9k	362.7k	19.6	14400.0	387.8k	352.5k	10.0	14400.0
belg+4arcs_296	302.9k	298.1k	1.6	14400.0	302.9k	302.6k	0.1	214.6
belg+4arcs_297	176.6k	171.0k	3.3	14400.0	175.9k	175.7k	0.1	13.5
belg+4arcs_298	134.9k	131.9k	2.3	14400.0	134.9k	134.8k	0.1	51.5
belg+4arcs_299	625.7k	480.2k	30.3	14400.0	573.2k	474.6k	20.8	14400.0
belg+4arcs_300	179.7k	179.5k	0.1	49.5	179.7k	179.5k	0.1	3.2
belg+4arcs_301	247.3k	240.2k	3.0	14400.0	247.3k	198.6k	24.5	14400.0
belg+4arcs_302	362.2k	355.3k	2.0	14400.0	362.2k	355.3k	2.0	14400.0
belg+4arcs_303	414.1k	396.9k	4.3	14400.0	414.1k	413.7k	0.1	37.5
belg+4arcs_304	411.3k	334.4k	23.0	14400.0	412.0k	319.0k	29.2	14400.0
belg+4arcs_305	398.8k	307.8k	29.6	14400.0	371.2k	367.8k	0.9	14400.0
belg+4arcs_306	302.6k	244.7k	23.7	14400.0	302.6k	302.1k	0.2	14400.0
belg+4arcs_307	176.6k	176.3k	0.2	14400.0	176.6k	174.0k	1.5	14400.0
belg+4arcs_308	83.2k	52.4k	58.8	14400.0	83.1k	82.4k	0.9	14400.0
belg+4arcs_309	304.4k	273.1k	11.5	14400.0	304.3k	262.6k	15.9	14400.0
belg+4arcs_310	160.6k	130.1k	23.5	14400.0	160.4k	160.3k	0.1	596.6
belg+4arcs_311	235.4k	180.9k	30.2	14400.0	235.4k	232.8k	1.1	14400.0
belg+4arcs_312	205.4k	189.9k	8.2	14400.0	203.1k	202.9k	0.1	1281.2
belg+4arcs_313	442.4k	365.8k	21.0	14400.0	442.4k	386.4k	14.5	14400.0
belg+4arcs_314	785.9k	496.9k	58.2	14400.0	632.0k	472.3k	33.8	14400.0
belg+4arcs_315	384.5k	292.7k	31.4	14400.0	379.9k	309.9k	22.6	14400.0
belg+4arcs_316	212.7k	183.0k	16.2	14400.0	210.8k	179.7k	17.3	14400.0
belg+4arcs_317	399.2k	396.1k	0.8	14400.0	399.2k	326.6k	22.2	14400.0
belg+4arcs_318	97.6k	87.1k	12.1	14400.0	97.6k	97.6k	0.0	31.7
belg+4arcs_319	197.5k	197.3k	0.1	183.7	197.5k	177.5k	11.3	14400.0
belg+4arcs_320	229.3k	228.6k	0.3	14400.0	229.3k	229.0k	0.1	30.3
belg+4arcs_321	166.0k	154.8k	7.3	14400.0	166.0k	161.2k	3.0	14400.0
belg+4arcs_322	180.0k	169.1k	6.4	14400.0	180.0k	165.6k	8.7	14400.0
belg+4arcs_323	187.3k	185.1k	1.2	14400.0	187.3k	187.2k	0.1	20.6
belg+4arcs_324	275.4k	219.9k	25.3	14400.0	275.5k	215.7k	27.7	14400.0
belg+4arcs_325	425.0k	324.7k	30.9	14400.0	425.0k	312.7k	35.9	14400.0
belg+4arcs_326	343.4k	343.0k	0.1	301.0	382.7k	288.9k	32.4	14400.0
belg+4arcs_327	46.8k	40.8k	14.9	14400.0	46.8k	46.8k	0.1	2475.4
belg+4arcs_328	165.9k	164.9k	0.6	14400.0	165.9k	165.9k	0.0	5952.5
belg+4arcs_329	241.5k	241.2k	0.1	5979.3	241.5k	231.3k	4.4	14400.0
belg+4arcs_330	251.6k	244.8k	2.8	14400.0	251.6k	234.8k	7.2	14400.0
belg+4arcs_331	225.8k	212.0k	6.5	14400.0	225.8k	169.0k	33.7	14400.0
belg+4arcs_332	619.5k	608.8k	1.8	14400.0	619.8k	450.6k	37.5	14400.0
belg+4arcs_333	191.4k	183.6k	4.3	14400.0	191.4k	186.1k	2.9	14400.0
belg+4arcs_334	428.8k	316.9k	35.3	14400.0	384.5k	281.0k	36.9	14400.0
belg+4arcs_335	402.6k	258.9k	55.5	14400.0	374.9k	365.2k	2.6	14400.0
belg+4arcs_336	211.9k	203.5k	4.1	14400.0	211.9k	211.6k	0.1	6620.1
belg+4arcs_337	285.6k	189.9k	50.4	14400.0	273.0k	196.5k	38.9	14400.0
belg+4arcs_338	259.3k	188.3k	37.7	14400.0	259.3k	210.6k	23.2	14400.0
belg+4arcs_339	600.7k	481.0k	24.9	14400.0	581.3k	502.9k	15.6	14400.0
belg+4arcs_340	550.2k	451.0k	22.0	14400.0	550.2k	453.8k	21.3	14400.0
belg+4arcs_341	183.7k	181.1k	1.5	14400.0	183.7k	183.6k	0.1	4.3
belg+4arcs_342	276.8k	241.7k	14.5	14400.0	276.8k	248.5k	11.4	14400.0
belg+4arcs_343	323.4k	214.2k	51.0	14400.0	303.5k	247.6k	22.6	14400.0
belg+4arcs_344	199.4k	168.4k	18.4	14400.0	199.4k	199.3k	0.0	289.7
belg+4arcs_345	361.2k	354.9k	1.8	14400.0	361.2k	360.8k	0.1	14400.0
belg+4arcs_346	306.7k	217.4k	41.0	14400.0	306.7k	248.8k	23.2	14400.0
belg+4arcs_347	579.2k	464.1k	24.8	14400.0	483.2k	415.1k	16.4	14400.0
belg+4arcs_348	181.5k	181.3k	0.1	90.6	181.5k	181.0k	0.2	14400.0
belg+4arcs_349	86.8k	46.6k	86.2	14400.0	78.4k	78.3k	0.1	175.7
belg+4arcs_350	278.6k	278.1k	0.2	14400.0	278.6k	272.3k	2.3	14400.0
belg+4arcs_351	493.6k	327.2k	50.8	14400.0	493.2k	336.3k	46.7	14400.0
belg+4arcs_352	453.8k	446.8k	1.6	14400.0	497.2k	382.9k	29.9	14400.0
belg+4arcs_353	329.4k	309.2k	6.5	14400.0	318.5k	310.4k	2.6	14400.0
belg+4arcs_354	237.1k	228.7k	3.7	14400.0	237.1k	235.5k	0.7	14400.0
belg+4arcs_355	30.7k	29.6k	3.8	14400.0	30.7k	30.7k	0.0	15.1
belg+4arcs_356	471.8k	337.9k	39.6	14400.0	417.5k	368.4k	13.3	14400.0
belg+4arcs_357	350.7k	211.6k	65.7	14400.0	350.7k	221.3k	58.5	14400.0
belg+4arcs_358	553.7k	462.8k	19.6	14400.0	553.7k	455.1k	21.7	14400.0
belg+4arcs_359	340.4k	312.1k	9.1	14400.0	340.4k	303.2k	12.2	14400.0
belg+4arcs_360	215.4k	201.5k	6.9	14400.0	206.2k	205.4k	0.4	14400.0
belg+4arcs_361	569.9k	569.3k	0.1	1253.7	600.0k	486.3k	23.4	14400.0
belg+4arcs_362	274.6k	195.1k	40.8	14400.0	274.6k	268.0k	2.5	14400.0
belg+4arcs_363	317.8k	252.2k	26.0	14400.0	317.8k	317.5k	0.1	525.5
belg+4arcs_364	318.7k	302.5k	5.4	14400.0	316.0k	313.4k	0.8	14400.0
belg+4arcs_365	388.8k	368.8k	5.4	14400.0	379.4k	334.4k	13.5	14400.0
belg+4arcs_366	387.7k	371.3k	4.4	14400.0	387.7k	387.4k	0.1	99.2
belg+4arcs_367	237.1k	221.4k	7.1	14400.0	237.1k	236.9k	0.1	39.5
belg+4arcs_368	236.4k	79.3k	198.1	14400.0	236.5k	98.8k	139.4	14400.0
belg+4arcs_369	138.6k	134.6k	3.0	14400.0	138.0k	137.9k	0.1	13404.4
belg+4arcs_370	199.1k	182.8k	8.9	14400.0	199.1k	198.9k	0.1	41.7

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Table A.28: Comparison of split-pipe models on *Circuit rank* + 4 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_371	407.6k	406.3k	0.3	14400.0	407.6k	391.5k	4.1	14400.0
belg+4arcs_372	297.5k	297.1k	0.1	14400.0	297.5k	297.2k	0.1	1683.6
belg+4arcs_373	385.7k	324.4k	18.9	14400.0	385.7k	348.3k	10.7	14400.0
belg+4arcs_374	197.7k	193.3k	2.2	14400.0	197.7k	182.4k	8.3	14400.0
belg+4arcs_375	179.7k	156.1k	15.1	14400.0	179.7k	179.5k	0.1	3516.1
belg+4arcs_376	457.2k	454.8k	0.5	14400.0	457.2k	454.4k	0.6	14400.0
belg+4arcs_377	194.8k	154.0k	26.5	14400.0	194.8k	163.6k	19.0	14400.0
belg+4arcs_378	285.4k	228.2k	25.1	14400.0	285.4k	228.4k	24.9	14400.0
belg+4arcs_379	489.9k	351.2k	39.5	14400.0	442.2k	412.0k	7.3	14400.0
belg+4arcs_380	306.7k	201.8k	52.0	14400.0	306.7k	208.2k	47.3	14400.0
belg+4arcs_381	522.1k	477.0k	9.5	14400.0	482.9k	431.5k	11.9	14400.0
belg+4arcs_382	458.7k	458.0k	0.1	14400.0	458.7k	393.8k	16.5	14400.0
belg+4arcs_383	84.1k	64.0k	31.5	14400.0	84.1k	66.0k	27.4	14400.0
belg+4arcs_384	355.9k	256.1k	39.0	14400.0	310.2k	271.6k	14.2	14400.0
belg+4arcs_385	364.6k	261.9k	39.2	14400.0	327.2k	288.8k	13.3	14400.0
belg+4arcs_386	548.5k	372.5k	47.3	14400.0	526.5k	414.2k	27.1	14400.0
belg+4arcs_387	131.1k	98.5k	33.1	14400.0	125.9k	125.8k	0.1	310.4
belg+4arcs_388	132.9k	132.4k	0.4	14400.0	132.9k	132.9k	0.0	4.0
belg+4arcs_389	219.1k	174.4k	25.6	14400.0	219.1k	173.9k	26.0	14400.0
belg+4arcs_390	359.0k	347.3k	3.4	14400.0	353.8k	349.1k	1.3	14400.0
belg+4arcs_391	245.9k	245.9k	0.0	1561.2	245.9k	219.1k	12.3	14400.0
belg+4arcs_392	173.4k	146.6k	18.3	14400.0	173.4k	167.8k	3.4	14400.0
belg+4arcs_393	239.1k	227.6k	5.1	14400.0	239.1k	238.9k	0.1	46.6
belg+4arcs_394	258.2k	251.3k	2.7	14400.0	258.2k	206.7k	24.9	14400.0
belg+4arcs_395	358.9k	289.9k	23.8	14400.0	358.9k	358.6k	0.1	2966.9
belg+4arcs_396	168.4k	163.3k	3.1	14400.0	168.4k	168.3k	0.1	53.8
belg+4arcs_397	429.1k	415.3k	3.3	14400.0	429.1k	428.7k	0.1	215.7
belg+4arcs_398	269.2k	262.3k	2.6	14400.0	269.2k	254.1k	6.0	14400.0
belg+4arcs_399	213.5k	186.3k	14.6	14400.0	213.5k	209.0k	2.2	14400.0
belg+4arcs_400	227.7k	220.3k	3.4	14400.0	227.7k	188.8k	20.6	14400.0
belg+4arcs_401	305.1k	260.4k	17.2	14400.0	305.1k	268.5k	13.6	14400.0
belg+4arcs_402	236.3k	236.0k	0.1	14400.0	236.3k	191.1k	23.7	14400.0
belg+4arcs_403	255.8k	255.5k	0.1	125.1	255.8k	255.5k	0.1	6.1
belg+4arcs_404	390.1k	272.3k	43.2	14400.0	386.0k	379.5k	1.7	14400.0
belg+4arcs_405	393.9k	387.7k	1.6	14400.0	393.9k	393.3k	0.1	14400.0
belg+4arcs_406	160.0k	152.9k	4.7	14400.0	160.0k	159.8k	0.1	53.9
belg+4arcs_407	463.2k	355.2k	30.4	14400.0	463.2k	369.9k	25.2	14400.0
belg+4arcs_408	21.6k	20.9k	3.7	14400.0	21.6k	21.6k	0.1	1.7
belg+4arcs_409	169.7k	169.5k	0.1	122.0	169.7k	169.5k	0.1	2.4
belg+4arcs_410	229.0k	219.1k	4.5	14400.0	229.0k	224.6k	1.9	14400.0
belg+4arcs_411	369.3k	301.5k	22.5	14400.0	343.8k	305.2k	12.7	14400.0
belg+4arcs_412	101.6k	94.2k	7.9	14400.0	101.6k	52.2k	94.6	14400.0
belg+4arcs_413	214.0k	213.5k	0.2	14400.0	214.0k	213.8k	0.1	8.6
belg+4arcs_414	182.4k	180.7k	1.0	14400.0	182.4k	182.3k	0.1	4.8
belg+4arcs_415	124.3k	98.3k	26.5	14400.0	124.3k	98.5k	26.2	14400.0
belg+4arcs_416	343.9k	343.5k	0.1	967.6	343.9k	288.3k	19.3	14400.0
belg+4arcs_417	271.2k	189.8k	42.9	14400.0	271.2k	188.6k	43.8	14400.0
belg+4arcs_418	573.3k	431.7k	32.8	14400.0	533.6k	418.4k	27.5	14400.0
belg+4arcs_419	270.1k	269.9k	0.1	2527.1	286.7k	221.2k	29.6	14400.0
belg+4arcs_420	227.6k	218.6k	4.1	14400.0	227.6k	195.4k	16.5	14400.0
belg+4arcs_421	235.3k	181.9k	29.4	14400.0	235.3k	235.1k	0.1	283.9
belg+4arcs_422	333.2k	235.5k	41.5	14400.0	333.2k	217.6k	53.1	14400.0
belg+4arcs_423	180.1k	179.9k	0.1	38.2	180.1k	180.1k	0.0	3.5
belg+4arcs_424	423.6k	375.8k	12.7	14400.0	423.6k	408.1k	3.8	14400.0
belg+4arcs_425	280.4k	245.4k	14.2	14400.0	280.4k	244.0k	14.9	14400.0
belg+4arcs_426	280.4k	231.3k	21.2	14400.0	280.4k	206.0k	36.1	14400.0
belg+4arcs_427	218.4k	185.7k	17.6	14400.0	218.4k	210.8k	3.6	14400.0
belg+4arcs_428	193.3k	156.7k	23.4	14400.0	192.8k	183.6k	5.0	14400.0
belg+4arcs_429	524.5k	359.5k	45.9	14400.0	455.8k	408.3k	11.6	14400.0
belg+4arcs_430	470.9k	292.4k	61.0	14400.0	401.6k	367.0k	9.4	14400.0
belg+4arcs_431	720.7k	569.8k	26.5	14400.0	720.7k	575.5k	25.2	14400.0
belg+4arcs_432	135.9k	135.5k	0.3	14400.0	135.9k	103.4k	31.4	14400.0
belg+4arcs_433	158.2k	109.2k	44.9	14400.0	158.2k	156.8k	0.9	14400.0
belg+4arcs_434	411.6k	392.4k	4.9	14400.0	411.6k	353.0k	16.6	14400.0
belg+4arcs_435	391.8k	326.9k	19.9	14400.0	391.8k	311.5k	25.8	14400.0
belg+4arcs_436	213.8k	210.1k	1.7	14400.0	213.8k	213.6k	0.1	11.9
belg+4arcs_437	278.8k	232.4k	20.0	14400.0	278.8k	256.0k	8.9	14400.0
belg+4arcs_438	337.0k	261.2k	29.0	14400.0	337.0k	335.3k	0.5	14400.0
belg+4arcs_439	470.3k	460.3k	2.2	14400.0	470.3k	421.0k	11.7	14400.0
belg+4arcs_440	563.9k	469.0k	20.2	14400.0	514.4k	478.7k	7.5	14400.0
belg+4arcs_441	268.8k	184.4k	45.7	14400.0	268.8k	203.6k	32.0	14400.0
belg+4arcs_442	115.6k	83.8k	38.0	14400.0	115.6k	94.6k	22.2	14400.0
belg+4arcs_443	297.4k	292.1k	1.8	14400.0	297.4k	293.0k	1.5	14400.0
belg+4arcs_444	165.2k	160.6k	2.9	14400.0	165.2k	165.1k	0.1	13.3
belg+4arcs_445	490.9k	423.6k	15.9	14400.0	490.9k	451.4k	8.7	14400.0
belg+4arcs_446	248.7k	191.5k	29.9	14400.0	248.7k	248.4k	0.1	2447.7
belg+4arcs_447	348.2k	265.4k	31.2	14400.0	318.3k	255.8k	24.4	14400.0
belg+4arcs_448	254.0k	231.3k	9.9	14400.0	254.0k	237.7k	6.9	14400.0

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Table A.28: Comparison of split-pipe models on *Circuit rank* + 4 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+4arcs_449	237.6k	180.2k	31.9	14400.0	237.6k	237.3k	0.1	2958.3
belg+4arcs_450	225.0k	151.8k	48.3	14400.0	225.0k	142.5k	57.9	14400.0
belg+4arcs_451	25.7k	25.1k	2.3	14400.0	25.7k	25.7k	0.1	105.2
belg+4arcs_452	339.8k	337.9k	0.6	14400.0	360.2k	235.8k	52.7	14400.0
belg+4arcs_453	340.4k	332.5k	2.4	14400.0	340.4k	299.2k	13.8	14400.0
belg+4arcs_454	154.4k	147.1k	4.9	14400.0	154.4k	154.2k	0.1	12.5
belg+4arcs_455	78.4k	73.9k	6.0	14400.0	78.4k	46.2k	69.7	14400.0
belg+4arcs_456	443.6k	331.9k	33.7	14400.0	443.6k	349.3k	27.0	14400.0
belg+4arcs_457	178.8k	164.3k	8.8	14400.0	178.8k	178.8k	0.0	20.3
belg+4arcs_458	451.3k	388.0k	16.3	14400.0	451.2k	450.8k	0.1	9082.8
belg+4arcs_459	0.0	0.0	0.0	15.8	0.0	0.0	0.0	0.3
belg+4arcs_460	154.4k	122.8k	25.8	14400.0	154.4k	127.6k	21.0	14400.0
belg+4arcs_461	313.3k	236.9k	32.3	14400.0	313.3k	255.4k	22.7	14400.0
belg+4arcs_462	222.3k	220.7k	0.7	14400.0	222.3k	211.0k	5.3	14400.0
belg+4arcs_463	181.8k	176.9k	2.8	14400.0	181.8k	181.8k	0.0	2.2
belg+4arcs_464	243.8k	181.3k	34.5	14400.0	245.6k	227.9k	7.8	14400.0
belg+4arcs_465	239.0k	198.2k	20.6	14400.0	238.5k	213.9k	11.5	14400.0
belg+4arcs_466	178.8k	163.9k	9.1	14400.0	178.8k	177.6k	0.7	14400.0
belg+4arcs_467	342.8k	306.1k	12.0	14400.0	342.8k	329.8k	4.0	14400.0
belg+4arcs_468	242.7k	239.7k	1.3	14400.0	242.7k	170.9k	42.1	14400.0
belg+4arcs_469	261.3k	258.4k	1.1	14400.0	261.3k	260.4k	0.3	14400.0
belg+4arcs_470	229.2k	191.3k	19.8	14400.0	229.2k	168.7k	35.8	14400.0
belg+4arcs_471	347.0k	243.7k	42.4	14400.0	347.0k	297.7k	16.6	14400.0
belg+4arcs_472	210.0k	201.2k	4.4	14400.0	210.0k	162.3k	29.4	14400.0
belg+4arcs_473	380.2k	362.2k	5.0	14400.0	394.9k	338.6k	16.6	14400.0
belg+4arcs_474	163.2k	162.7k	0.3	14400.0	163.2k	163.0k	0.1	1636.9
belg+4arcs_475	165.7k	165.2k	0.3	14400.0	165.7k	165.6k	0.1	5.5
belg+4arcs_476	376.9k	266.9k	41.2	14400.0	361.8k	279.2k	29.5	14400.0
belg+4arcs_477	342.1k	253.3k	35.1	14400.0	339.9k	275.9k	23.2	14400.0
belg+4arcs_478	350.2k	303.1k	15.6	14400.0	350.2k	286.0k	22.5	14400.0
belg+4arcs_479	329.4k	284.8k	15.7	14400.0	307.5k	307.5k	0.0	178.7
belg+4arcs_480	159.5k	159.4k	0.1	470.6	159.5k	159.5k	0.0	2.4
belg+4arcs_481	466.3k	345.5k	35.0	14400.0	386.6k	354.4k	9.1	14400.0
belg+4arcs_482	285.6k	275.0k	3.8	14400.0	285.6k	231.4k	23.4	14400.0
belg+4arcs_483	179.2k	144.1k	24.3	14400.0	177.8k	146.7k	21.2	14400.0
belg+4arcs_484	164.7k	164.4k	0.1	14400.0	164.7k	142.9k	15.3	14400.0
belg+4arcs_485	158.5k	155.5k	2.0	14400.0	158.5k	158.4k	0.1	14.7
belg+4arcs_486	440.4k	421.8k	4.4	14400.0	440.4k	415.1k	6.1	14400.0
belg+4arcs_487	198.6k	198.4k	0.1	1700.6	198.6k	198.4k	0.1	26.9
belg+4arcs_488	378.4k	256.6k	47.5	14400.0	331.1k	328.0k	1.0	14400.0
belg+4arcs_489	426.9k	400.2k	6.7	14400.0	402.6k	399.3k	0.8	14400.0
belg+4arcs_490	187.1k	169.7k	10.3	14400.0	186.3k	170.6k	9.2	14400.0
belg+4arcs_491	233.8k	229.3k	2.0	14400.0	233.8k	189.6k	23.3	14400.0
belg+4arcs_492	161.6k	161.4k	0.1	6424.2	161.6k	161.5k	0.0	6.8
belg+4arcs_493	409.0k	293.8k	39.2	14400.0	351.4k	281.5k	24.9	14400.0
belg+4arcs_494	285.3k	229.9k	24.1	14400.0	285.3k	285.0k	0.1	22.0
belg+4arcs_495	376.7k	317.1k	18.8	14400.0	376.7k	376.3k	0.1	3922.0
belg+4arcs_496	256.5k	227.3k	12.8	14400.0	256.5k	185.4k	38.4	14400.0
belg+4arcs_497	281.0k	281.0k	0.0	145.2	281.0k	233.1k	20.5	14400.0
belg+4arcs_498	503.7k	335.9k	50.0	14400.0	456.3k	383.9k	18.9	14400.0
belg+4arcs_499	258.8k	256.8k	0.8	14400.0	258.8k	258.4k	0.1	14400.0
belg+4arcs_500	173.3k	172.0k	0.7	14400.0	173.3k	173.1k	0.1	4.7

Table A.29: Detailed results of the discrete models on *Circuit rank* + 6 arcs, as summarized in Figure 3.10d and Table 3.3d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_1	425.8k	264.4k	61.0	14400.0	1e+20	290.6k	–	14400.0	377.5k	242.8k	55.4	14400.0
belg+6arcs_2	180.7k	180.7k	0.0	546.2	180.7k	180.7k	0.0	2650.7	180.7k	156.0k	15.8	14400.0
belg+6arcs_3	124.3k	124.3k	0.0	5867.9	1e+20	82.1k	–	14400.0	216.8k	41.6k	421.8	14400.0
belg+6arcs_4	302.5k	220.3k	37.3	14400.0	288.2k	288.2k	0.0	9793.2	302.1k	187.9k	60.8	14400.0
belg+6arcs_5	566.9k	271.9k	108.5	14400.0	659.2k	284.2k	132.0	14400.0	490.8k	188.1k	161.0	14400.0
belg+6arcs_6	206.7k	206.7k	0.0	1607.1	206.7k	184.5k	12.0	14400.0	206.7k	206.7k	0.0	13916.5
belg+6arcs_7	352.7k	352.7k	0.0	8659.1	352.7k	352.7k	0.0	10703.4	390.0k	229.1k	70.3	14400.0
belg+6arcs_8	11.4k	11.4k	0.0	268.3	11.4k	11.4k	0.0	10831.3	11.4k	11.4k	0.0	1480.3
belg+6arcs_9	359.7k	160.6k	123.9	14400.0	1e+20	156.9k	–	14400.0	363.9k	54.0k	573.5	14400.0
belg+6arcs_10	234.1k	233.9k	0.1	2439.5	234.1k	226.0k	3.6	14400.0	242.7k	207.1k	17.2	14400.0
belg+6arcs_11	304.9k	204.7k	49.0	14400.0	291.0k	208.6k	39.5	14400.0	285.8k	198.0k	44.3	14400.0

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Table A.29: Comparison of discrete models on *Circuit rank* + 6 (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_12	61.5k	61.5k	0.0	2698.2	61.5k	61.5k	0.0	1169.3	61.5k	61.5k	0.0	14079.0
belg+6arcs_13	214.9k	214.9k	0.0	1310.9	1e+20	188.2k	–	14400.0	214.9k	214.9k	0.0	3893.7
belg+6arcs_14	474.0k	353.8k	34.0	14400.0	520.0k	374.2k	39.0	14400.0	474.0k	279.4k	69.6	14400.0
belg+6arcs_15	2976.0	2976.0	0.0	22.2	2976.0	2976.0	0.0	517.7	2976.0	2976.0	0.0	43.8
belg+6arcs_16	601.0k	307.5k	95.4	14400.0	1e+20	308.4k	–	14400.0	727.7k	182.4k	299.0	14400.0
belg+6arcs_17	262.1k	186.0k	40.9	14400.0	335.8k	187.3k	79.2	14400.0	262.4k	162.2k	61.7	14400.0
belg+6arcs_18	234.8k	234.8k	0.0	401.2	234.8k	234.8k	0.0	1337.7	234.8k	234.8k	0.0	1938.2
belg+6arcs_19	167.5k	167.5k	0.0	174.6	167.5k	167.5k	0.0	878.4	167.5k	167.5k	0.0	190.2
belg+6arcs_20	279.4k	279.4k	0.0	970.3	279.4k	279.2k	0.1	3334.4	297.1k	252.1k	17.9	14400.0
belg+6arcs_21	46.4k	46.4k	0.0	25.8	46.4k	46.4k	0.0	3765.8	46.4k	46.4k	0.0	158.3
belg+6arcs_22	213.9k	213.9k	0.0	3848.9	239.2k	206.7k	15.7	14400.0	239.2k	191.1k	25.2	14400.0
belg+6arcs_23	185.8k	64.6k	187.7	14400.0	1e+20	43.5k	–	14400.0	120.0k	13.2k	806.4	14400.0
belg+6arcs_24	222.7k	222.7k	0.0	3749.7	222.7k	222.7k	0.0	12684.1	222.7k	222.7k	0.0	13635.2
belg+6arcs_25	214.8k	80.8k	166.0	14400.0	183.3k	71.4k	156.7	14400.0	338.6k	14.0k	2313.1	14400.0
belg+6arcs_26	179.4k	179.4k	0.0	175.7	179.4k	179.4k	0.0	1487.1	179.4k	179.4k	0.0	22.9
belg+6arcs_27	143.9k	143.9k	0.0	185.8	143.9k	143.9k	0.0	6652.0	143.9k	143.9k	0.0	869.0
belg+6arcs_28	261.3k	211.2k	23.7	14400.0	254.5k	254.5k	0.0	8214.3	261.3k	226.7k	15.3	14400.0
belg+6arcs_29	3600.0	3600.0	0.0	20.6	3600.0	3600.0	0.0	553.1	3600.0	3600.0	0.0	107.6
belg+6arcs_30	71.5k	71.5k	0.0	389.9	71.5k	71.5k	0.0	4324.6	71.9k	49.3k	46.0	14400.0
belg+6arcs_31	196.8k	133.2k	47.8	14400.0	168.1k	156.6k	7.4	14400.0	196.8k	118.3k	66.3	14400.0
belg+6arcs_32	310.3k	282.7k	9.8	14400.0	1e+20	236.2k	–	14400.0	320.4k	211.1k	51.8	14400.0
belg+6arcs_33	168.1k	54.5k	208.4	14400.0	106.8k	75.4k	41.6	14400.0	168.1k	30.5k	451.6	14400.0
belg+6arcs_34	20.5k	20.5k	0.0	17.5	20.5k	20.5k	0.0	2266.6	20.5k	20.5k	0.0	47.2
belg+6arcs_35	190.7k	190.7k	0.0	549.1	190.7k	190.7k	0.0	1887.3	190.7k	190.7k	0.0	264.7
belg+6arcs_36	216.6k	216.6k	0.0	1580.8	216.6k	216.6k	0.0	7774.7	216.7k	171.8k	26.1	14400.0
belg+6arcs_37	87.1k	87.1k	0.0	10999.4	87.1k	87.1k	0.0	7449.8	117.5k	29.0k	305.3	14400.0
belg+6arcs_38	359.8k	311.3k	15.6	14400.0	347.0k	347.0k	0.0	4115.2	359.8k	280.7k	28.2	14400.0
belg+6arcs_39	133.6k	133.6k	0.0	6929.0	1e+20	87.9k	–	14400.0	143.5k	64.5k	122.3	14400.0
belg+6arcs_40	222.5k	124.2k	79.2	14400.0	1e+20	110.0k	–	14400.0	217.1k	19.9k	989.1	14400.0
belg+6arcs_41	361.6k	235.7k	53.4	14400.0	323.7k	292.9k	10.5	14400.0	334.2k	241.6k	38.4	14400.0
belg+6arcs_42	271.8k	222.6k	22.1	14400.0	247.1k	247.1k	0.0	4973.6	249.3k	229.6k	8.6	14400.0
belg+6arcs_43	146.2k	79.1k	84.8	14400.0	1e+20	59.4k	–	14400.0	154.3k	32.4k	376.5	14400.0
belg+6arcs_44	173.9k	173.9k	0.0	978.1	173.9k	173.9k	0.0	2854.2	173.9k	173.9k	0.0	819.7
belg+6arcs_45	18.1k	18.1k	0.0	85.1	18.1k	18.1k	0.0	3880.9	18.1k	18.1k	0.0	228.4
belg+6arcs_46	194.3k	194.3k	0.0	982.0	194.3k	194.3k	0.0	9988.9	194.3k	194.3k	0.0	3051.6
belg+6arcs_47	229.7k	229.7k	0.0	265.3	229.7k	229.7k	0.0	4947.6	229.7k	229.7k	0.0	458.6
belg+6arcs_48	238.1k	238.1k	0.0	13254.1	238.1k	238.1k	0.0	14315.9	241.4k	195.4k	23.6	14400.0
belg+6arcs_49	40.1k	17.3k	132.1	14400.0	40.1k	40.1k	0.0	2911.1	40.1k	40.1k	0.0	1957.5
belg+6arcs_50	285.6k	245.3k	16.4	14400.0	278.6k	278.6k	0.0	7263.9	284.5k	229.3k	24.0	14400.0
belg+6arcs_51	38.4k	38.4k	0.0	2054.4	60.5k	28.0k	115.8	14400.0	38.4k	38.4k	0.0	11136.1
belg+6arcs_52	229.5k	229.5k	0.0	2844.3	229.5k	229.5k	0.0	9652.2	238.7k	182.0k	31.1	14400.0
belg+6arcs_53	145.9k	145.9k	0.0	93.2	145.9k	145.9k	0.0	1102.8	145.9k	145.9k	0.0	505.9
belg+6arcs_54	188.7k	188.7k	0.0	48.0	188.7k	188.5k	0.1	420.6	188.7k	188.7k	0.0	16.8
belg+6arcs_55	221.0k	38.7k	471.3	14400.0	221.0k	221.0k	0.0	3930.0	221.0k	221.0k	0.0	9649.8
belg+6arcs_56	169.8k	169.8k	0.0	405.1	169.8k	169.7k	0.1	5933.0	169.8k	169.8k	0.0	3311.7
belg+6arcs_57	263.5k	263.5k	0.0	7023.6	1e+20	187.5k	–	14400.0	276.1k	119.3k	131.5	14400.0
belg+6arcs_58	186.5k	151.4k	23.2	14400.0	1e+20	120.7k	–	14400.0	186.5k	107.2k	74.1	14400.0
belg+6arcs_59	29.9k	29.9k	0.0	281.0	29.9k	29.9k	0.0	4180.6	29.9k	29.9k	0.0	938.6
belg+6arcs_60	136.5k	136.5k	0.0	2642.1	136.5k	136.5k	0.0	7951.1	136.5k	136.5k	0.0	103.8
belg+6arcs_61	154.2k	154.2k	0.0	1897.7	154.2k	154.2k	0.0	10494.2	154.2k	122.4k	26.0	14400.0
belg+6arcs_62	400.5k	276.3k	44.9	14400.0	1e+20	303.8k	–	14400.0	431.0k	217.5k	98.1	14400.0
belg+6arcs_63	241.6k	241.6k	0.0	203.3	241.6k	241.6k	0.0	1612.6	241.6k	241.6k	0.0	1659.2
belg+6arcs_64	136.4k	106.8k	27.7	14400.0	1e+20	55.7k	–	14400.0	136.4k	63.2k	115.9	14400.0
belg+6arcs_65	257.3k	206.5k	24.6	14400.0	1e+20	171.3k	–	14400.0	263.6k	141.8k	85.9	14400.0
belg+6arcs_66	339.6k	210.5k	61.3	14400.0	1e+20	241.6k	–	14400.0	344.4k	178.6k	92.8	14400.0
belg+6arcs_67	182.6k	182.6k	0.0	138.0	182.6k	182.6k	0.0	407.6	182.6k	182.6k	0.0	1233.7
belg+6arcs_68	267.1k	247.7k	7.8	14400.0	253.7k	253.7k	0.0	10296.8	267.1k	206.0k	29.6	14400.0
belg+6arcs_69	339.0k	222.7k	52.2	14400.0	477.5k	216.4k	120.7	14400.0	365.1k	126.9k	187.6	14400.0
belg+6arcs_70	208.5k	208.5k	0.0	4799.1	1e+20	170.6k	–	14400.0	240.9k	167.8k	43.5	14400.0
belg+6arcs_71	223.1k	136.4k	63.5	14400.0	206.2k	169.3k	21.8	14400.0	221.2k	138.7k	59.4	14400.0
belg+6arcs_72	43.1k	43.1k	0.0	352.8	43.1k	43.1k	0.0	1080.2	43.1k	43.1k	0.0	2132.2
belg+6arcs_73	35.3k	35.3k	0.0	71.5	35.3k	35.3k	0.0	2457.2	35.3k	35.2k	0.1	316.9
belg+6arcs_74	463.8k	318.9k	45.5	14400.0	1e+20	313.8k	–	14400.0	451.4k	231.5k	95.0	14400.0
belg+6arcs_75	201.2k	201.2k	0.0	3901.6	201.2k	201.2k	0.0	2634.9	201.2k	201.2k	0.0	4146.4
belg+6arcs_76	49.4k	49.4k	0.0	83.1	49.4k	49.4k	0.0	2306.5	49.4k	49.4k	0.0	1131.8
belg+6arcs_77	113.3k	113.3k	0.0	2187.8	113.3k	113.3k	0.0	8766.8	113.3k	44.4k	155.2	14400.0
belg+6arcs_78	55.8k	55.8k	0.0	64.5	55.8k	55.8k	0.0	13950.0	55.8k	55.8k	0.0	191.3
belg+6arcs_79	195.6k	38.9k	403.3	14400.0	168.6k	44.3k	280.7	14400.0	105.2k	22.7k	363.9	14400.0
belg+6arcs_80	157.8k	157.8k	0.0	2282.8	157.8k	157.8k	0.0	3079.8	157.8k	157.7k	0.0	5402.2
belg+6arcs_81	264.2k	264.2k	0.0	1646.2	264.2k	264.2k	0.0	7454.1	290.4k	195.8k	48.3	14400.0
belg+6arcs_82	20.5k	20.5k	0.0	21.5	20.5k	20.5k	0.0	26.4	20.5k	20.5k	0.0	7.0
belg+6arcs_83	386.2k	311.6k	24.0	14400.0	1e+20	292.6k	–	14400.0	450.9k	246.4k	83.0	14400.0
belg+6arcs_84	367.1k	325.6k	12.7	14400.0	1e+20	300.4k	–	14400.0	368.7k	257.6k	43.1	14400.0
belg+6arcs_85	221.8k	221.8k	0.0	1853.0	221.8k	221.8k	0.0	5682.4	221.8k	165.3k	34.2	14400.0
belg+6arcs_86	316.0k	316.0k	0.0	12931.9	1e+20	233.7k	–	14400.0	327.7k	182.2k	79.9	14400.0
belg+6arcs_87	221.7k	221.7k	0.0	2511.0	221.7k	221.7k	0.0	8147.4	221.7k	221.7k	0.0	3915.9
belg+6arcs_88	302.5k	215.7k	40.3	14400.0	1e+20	214.3k	–	14400.0	289.1k	199.7k	44.8	14400.0
belg+6arcs_89	725.1k	410.8k	76.5	14400.0	1e+20	430.3k	–	14400.0	632.4k	348.1k	81.7	14400.0

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Table A.29: Comparison of discrete models on *Circuit rank* + 6 (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_90	320.3k	320.3k	0.0	1339.7	320.3k	283.7k	12.9	14400.0	320.3k	320.2k	0.0	12148.6
belg+6arcs_91	319.9k	319.9k	0.0	11422.0	319.9k	265.5k	20.5	14400.0	374.8k	161.0k	132.9	14400.0
belg+6arcs_92	337.5k	337.5k	0.0	5993.1	1e+20	270.7k	–	14400.0	355.3k	190.5k	86.5	14400.0
belg+6arcs_93	504.3k	348.7k	44.6	14400.0	1e+20	326.4k	–	14400.0	509.5k	244.5k	108.4	14400.0
belg+6arcs_94	191.2k	191.1k	0.1	3387.1	191.2k	191.2k	0.0	2572.9	191.2k	191.2k	0.0	773.3
belg+6arcs_95	46.2k	46.2k	0.0	60.5	46.2k	46.2k	0.0	133.8	46.2k	46.2k	0.0	252.6
belg+6arcs_96	211.2k	170.7k	23.7	14400.0	195.7k	195.7k	0.0	4152.3	195.7k	195.7k	0.0	7698.8
belg+6arcs_97	108.9k	108.9k	0.0	8260.6	1e+20	74.6k	–	14400.0	108.9k	101.8k	7.0	14400.0
belg+6arcs_98	225.5k	225.5k	0.0	597.7	225.5k	225.5k	0.0	6877.7	234.6k	197.2k	19.0	14400.0
belg+6arcs_99	638.3k	247.7k	157.6	14400.0	1e+20	231.2k	–	14400.0	337.7k	170.5k	98.0	14400.0
belg+6arcs_100	187.1k	187.1k	0.0	175.9	187.1k	187.1k	0.0	533.4	187.1k	187.1k	0.0	764.8
belg+6arcs_101	223.0k	223.0k	0.0	4004.4	1e+20	187.1k	–	14400.0	226.1k	196.0k	15.3	14400.0
belg+6arcs_102	208.7k	208.7k	0.0	1493.7	208.7k	208.7k	0.0	3194.7	209.7k	180.9k	15.9	14400.0
belg+6arcs_103	190.4k	48.0k	296.6	14400.0	1e+20	48.7k	–	14400.0	204.8k	16.1k	1170.2	14400.0
belg+6arcs_104	218.5k	218.5k	0.0	13864.5	227.3k	188.8k	20.4	14400.0	219.2k	152.7k	43.6	14400.0
belg+6arcs_105	112.4k	112.4k	0.0	91.5	112.4k	112.4k	0.0	1156.4	112.4k	112.4k	0.0	1970.8
belg+6arcs_106	196.9k	196.9k	0.0	8788.9	196.9k	196.9k	0.0	12891.0	201.4k	129.0k	56.1	14400.0
belg+6arcs_107	211.2k	211.2k	0.0	146.2	211.2k	211.2k	0.0	5940.0	211.2k	211.2k	0.0	21.7
belg+6arcs_108	50.0k	50.0k	0.0	2472.0	50.0k	50.0k	0.0	8105.9	50.0k	50.0k	0.0	7852.9
belg+6arcs_109	187.3k	187.3k	0.0	8026.0	1e+20	125.5k	–	14400.0	188.0k	105.2k	78.6	14400.0
belg+6arcs_110	216.5k	203.0k	6.6	14398.6	207.4k	207.4k	0.0	3440.6	207.4k	207.4k	0.0	2861.9
belg+6arcs_111	373.6k	278.4k	34.2	14400.0	1e+20	248.0k	–	14400.0	451.1k	182.9k	146.6	14400.0
belg+6arcs_112	154.7k	98.6k	56.9	14400.0	1e+20	74.9k	–	14400.0	140.1k	62.0k	126.0	14400.0
belg+6arcs_113	3600.0	3600.0	0.0	62.4	3600.0	3600.0	0.0	98.0	3600.0	3600.0	0.0	47.9
belg+6arcs_114	4164.0	4164.0	0.0	15.0	4164.0	4164.0	0.0	2043.7	4164.0	4164.0	0.0	2.7
belg+6arcs_115	378.2k	166.2k	127.5	14400.0	1e+20	229.8k	–	14400.0	400.0k	150.7k	165.4	14400.0
belg+6arcs_116	177.8k	177.8k	0.0	745.5	1e+20	149.6k	–	14400.0	181.3k	142.8k	26.9	14400.0
belg+6arcs_117	133.4k	58.6k	127.5	14400.0	1e+20	49.4k	–	14400.0	215.5k	17.7k	1115.5	14400.0
belg+6arcs_118	168.3k	168.3k	0.0	783.0	168.3k	168.3k	0.0	6915.1	168.3k	168.3k	0.0	820.6
belg+6arcs_119	213.3k	213.3k	0.0	7512.2	213.3k	213.3k	0.0	14235.8	213.3k	178.3k	19.7	14400.0
belg+6arcs_120	186.8k	8560.1	2082.4	14400.0	1e+20	36.0k	–	14400.0	186.8k	5453.9	3325.3	14400.0
belg+6arcs_121	143.4k	143.4k	0.0	251.2	143.4k	143.4k	0.0	2112.8	143.4k	143.4k	0.0	128.0
belg+6arcs_122	177.8k	177.8k	0.0	329.4	177.8k	177.8k	0.0	10334.5	177.8k	177.8k	0.0	1316.2
belg+6arcs_123	23.0k	23.0k	0.0	277.3	23.0k	23.0k	0.0	1959.3	23.0k	23.0k	0.0	233.0
belg+6arcs_124	248.3k	157.2k	57.9	14400.0	406.5k	156.3k	160.2	14400.0	274.1k	113.5k	141.4	14400.0
belg+6arcs_125	18.1k	18.1k	0.0	281.9	18.1k	18.1k	0.0	7271.3	18.1k	18.1k	0.0	143.8
belg+6arcs_126	113.1k	113.1k	0.0	7873.7	113.1k	113.1k	0.0	11613.0	124.1k	27.4k	353.3	14400.0
belg+6arcs_127	194.5k	194.5k	0.0	1187.2	194.5k	194.5k	0.0	3018.0	194.5k	161.6k	20.4	14400.0
belg+6arcs_128	195.1k	195.1k	0.0	1036.5	199.5k	183.5k	8.8	14400.0	199.5k	170.5k	17.0	14400.0
belg+6arcs_129	32.2k	21.4k	50.8	14400.0	32.2k	32.2k	0.0	5278.8	32.2k	32.2k	0.0	1440.0
belg+6arcs_130	220.0k	147.1k	49.6	14400.0	185.2k	185.2k	0.0	1228.4	185.2k	155.0k	19.5	14400.0
belg+6arcs_131	290.2k	224.6k	29.2	14400.0	359.1k	198.0k	81.4	14400.0	354.8k	162.7k	118.1	14400.0
belg+6arcs_132	140.5k	140.5k	0.0	838.0	140.5k	140.5k	0.0	2065.5	140.5k	140.5k	0.0	4282.5
belg+6arcs_133	340.4k	224.0k	52.0	14400.0	1e+20	204.6k	–	14400.0	398.1k	141.5k	181.4	14400.0
belg+6arcs_134	433.9k	289.4k	49.9	14400.0	375.9k	375.9k	0.0	4826.3	422.5k	239.0k	76.8	14400.0
belg+6arcs_135	158.6k	158.6k	0.0	66.6	158.6k	158.6k	0.0	2563.4	158.6k	158.6k	0.0	488.5
belg+6arcs_136	659.0k	344.8k	91.1	14400.0	778.8k	428.8k	81.6	14400.0	641.3k	263.5k	143.4	14400.0
belg+6arcs_137	481.8k	283.1k	70.2	14400.0	524.8k	322.8k	62.6	14400.0	431.3k	241.6k	78.5	14400.0
belg+6arcs_138	184.5k	184.4k	0.1	154.2	184.5k	184.4k	0.0	734.6	184.5k	184.5k	0.0	24.7
belg+6arcs_139	182.2k	182.2k	0.0	614.4	182.2k	182.2k	0.0	2478.6	182.2k	182.2k	0.0	1803.5
belg+6arcs_140	349.0k	20.1k	1638.2	14400.0	1e+20	33.1k	–	14400.0	73.0k	36.2k	101.9	14400.0
belg+6arcs_141	183.5k	183.5k	0.0	237.8	183.5k	183.5k	0.0	4166.7	183.5k	183.5k	0.0	597.1
belg+6arcs_142	255.3k	255.3k	0.0	11460.6	255.3k	255.3k	0.0	4380.1	259.0k	195.1k	32.8	14400.0
belg+6arcs_143	127.9k	127.9k	0.0	322.7	127.9k	127.9k	0.0	3361.2	127.9k	127.9k	0.0	1924.8
belg+6arcs_144	57.3k	57.3k	0.0	11326.2	1e+20	28.3k	–	14400.0	76.3k	11.2k	580.2	14400.0
belg+6arcs_145	255.8k	124.1k	106.1	14400.0	1e+20	85.5k	–	14400.0	283.9k	13.4k	2021.1	14400.0
belg+6arcs_146	156.2k	156.2k	0.0	5195.1	156.2k	156.1k	0.1	8205.9	264.6k	111.2k	137.9	14400.0
belg+6arcs_147	184.4k	184.4k	0.0	195.2	184.4k	184.4k	0.0	1133.2	184.4k	184.4k	0.0	1259.0
belg+6arcs_148	369.0k	185.8k	98.6	14400.0	1e+20	174.7k	–	14400.0	390.2k	126.4k	208.6	14400.0
belg+6arcs_149	272.7k	203.7k	33.9	14400.0	259.6k	194.2k	33.7	14400.0	272.7k	137.3k	98.6	14400.0
belg+6arcs_150	360.9k	340.3k	6.1	14400.0	356.2k	356.2k	0.0	10886.0	356.2k	227.9k	56.3	14400.0
belg+6arcs_151	57.0k	57.0k	0.0	101.6	57.0k	57.0k	0.0	2877.8	57.0k	57.0k	0.0	21.7
belg+6arcs_152	198.2k	198.2k	0.0	8875.6	198.2k	198.2k	0.0	4144.6	198.2k	198.2k	0.0	14374.1
belg+6arcs_153	228.1k	228.1k	0.0	1021.8	228.1k	228.1k	0.0	3552.4	233.2k	195.7k	19.1	14400.0
belg+6arcs_154	198.1k	198.1k	0.0	2981.7	198.1k	198.1k	0.0	3441.9	201.1k	148.7k	35.2	14400.0
belg+6arcs_155	297.0k	297.0k	0.0	1671.6	297.0k	297.0k	0.0	3427.1	297.0k	297.0k	0.0	7621.1
belg+6arcs_156	162.3k	162.3k	0.0	1046.8	162.3k	162.3k	0.0	2188.5	162.3k	162.3k	0.0	6990.5
belg+6arcs_157	172.0k	140.4k	22.5	14400.0	1e+20	140.7k	–	14400.0	170.0k	144.9k	17.3	14400.0
belg+6arcs_158	79.8k	79.8k	0.0	2965.9	97.3k	66.6k	46.1	14400.0	95.6k	18.8k	409.2	14400.0
belg+6arcs_159	220.3k	143.1k	53.9	14400.0	1e+20	76.1k	–	14400.0	412.1k	34.9k	1082.1	14400.0
belg+6arcs_160	91.7k	2976.0	2981.9	14400.0	34.4k	34.4k	0.0	2821.7	34.4k	12.9k	166.0	14400.0
belg+6arcs_161	280.4k	198.7k	41.1	14400.0	257.8k	226.7k	13.7	14400.0	286.3k	193.1k	48.3	14400.0
belg+6arcs_162	327.9k	297.7k	10.1	14400.0	1e+20	239.4k	–	14400.0	421.0k	165.5k	154.4	14400.0
belg+6arcs_163	338.5k	255.0k	32.7	14400.0	329.2k	250.6k	31.4	14400.0	420.0k	177.2k	137.0	14400.0
belg+6arcs_164	0.0	0.0	0.0	0.2	0.0	0.0	0.0	211.6	0.0	0.0	0.0	0.3
belg+6arcs_165	152.2k	152.2k	0.0	105.4	152.2k	152.2k	0.0	3045.2	152.2k	152.2k	0.0	401.6
belg+6arcs_166	4164.0	4164.0	0.0	14.9	4164.0	4164.0	0.0	51.3	4164.0	4164.0	0.0	8.3
belg+6arcs_167	233.4k	201.0k	16.1	14400.0	225.2k	209.7k	7.4	14400.0	278.2k	79.0k	252.1	14400.0

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Table A.29: Comparison of discrete models on *Circuit rank* + 6 (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_168	447.6k	255.4k	75.2	14400.0	1e+20	262.1k	–	14400.0	359.3k	195.9k	83.4	14400.0
belg+6arcs_169	165.6k	165.6k	0.0	5792.7	165.8k	125.1k	32.6	14400.0	201.9k	107.1k	88.5	14400.0
belg+6arcs_170	185.6k	185.6k	0.0	95.4	185.6k	185.6k	0.0	1204.5	185.6k	185.6k	0.0	620.6
belg+6arcs_171	235.6k	205.9k	14.4	14400.0	253.9k	178.9k	41.9	14400.0	235.6k	152.9k	54.0	14400.0
belg+6arcs_172	119.7k	4219.3	2736.0	14400.0	65.3k	65.3k	0.0	10017.4	65.3k	3600.4	1713.1	14400.0
belg+6arcs_173	309.9k	280.8k	10.3	14400.0	308.4k	282.1k	9.3	14400.0	312.8k	196.9k	58.9	14400.0
belg+6arcs_174	148.3k	148.3k	0.0	176.3	148.3k	148.3k	0.0	307.6	148.3k	148.3k	0.0	90.4
belg+6arcs_175	54.6k	54.6k	0.0	626.7	1e+20	34.1k	–	14400.0	54.6k	54.6k	0.0	7172.8
belg+6arcs_176	172.9k	97.0k	78.2	14400.0	1e+20	90.8k	–	14400.0	172.5k	21.1k	718.9	14400.0
belg+6arcs_177	234.8k	234.8k	0.0	32.5	234.8k	234.8k	0.0	1054.4	234.8k	234.8k	0.0	17.3
belg+6arcs_178	183.5k	183.5k	0.0	1467.5	183.5k	183.5k	0.0	9689.1	183.5k	159.9k	14.8	14400.0
belg+6arcs_179	252.4k	252.4k	0.0	2887.7	252.4k	252.4k	0.0	13441.0	252.4k	252.4k	0.0	12580.0
belg+6arcs_180	45.5k	45.5k	0.0	297.2	45.5k	45.5k	0.0	2218.7	45.5k	45.5k	0.0	7113.9
belg+6arcs_181	35.0k	35.0k	0.0	694.4	1e+20	24.0k	–	14400.0	35.0k	35.0k	0.0	4349.8
belg+6arcs_182	137.8k	137.8k	0.0	967.6	137.8k	137.8k	0.0	14288.3	137.8k	137.8k	0.0	4931.1
belg+6arcs_183	189.8k	139.4k	36.1	14400.0	462.1k	117.0k	295.0	14400.0	186.4k	117.2k	59.0	14400.0
belg+6arcs_184	64.3k	64.3k	0.0	5209.5	1e+20	34.3k	–	14400.0	178.9k	7781.7	2198.9	14400.0
belg+6arcs_185	114.6k	114.6k	0.0	205.7	114.6k	114.6k	0.0	3450.1	114.6k	114.6k	0.0	1740.4
belg+6arcs_186	169.1k	169.1k	0.0	545.8	169.1k	169.1k	0.0	243.5	169.1k	169.1k	0.0	132.8
belg+6arcs_187	537.3k	343.4k	56.5	14400.0	1e+20	319.5k	–	14400.0	551.2k	234.7k	134.9	14400.0
belg+6arcs_188	17.4k	17.4k	0.0	29.3	17.4k	17.4k	0.0	986.5	17.4k	17.4k	0.0	264.3
belg+6arcs_189	169.8k	169.8k	0.0	32.7	169.8k	169.8k	0.0	3258.0	169.8k	169.8k	0.0	219.7
belg+6arcs_190	220.2k	54.2k	306.3	14400.0	1e+20	82.9k	–	14400.0	231.6k	18.6k	1146.2	14400.0
belg+6arcs_191	272.6k	272.6k	0.0	542.5	272.6k	272.6k	0.0	837.4	272.6k	272.5k	0.1	493.7
belg+6arcs_192	149.1k	124.1k	20.2	14400.0	135.8k	135.8k	0.0	6675.4	135.8k	135.8k	0.0	7329.8
belg+6arcs_193	153.0k	153.0k	0.0	172.5	153.0k	153.0k	0.0	2802.2	153.0k	153.0k	0.0	530.1
belg+6arcs_194	314.2k	286.4k	9.7	14400.0	1e+20	214.7k	–	14400.0	335.4k	184.9k	81.5	14400.0
belg+6arcs_195	187.2k	187.2k	0.0	126.1	187.2k	187.2k	0.0	5656.0	187.2k	187.1k	0.1	62.9
belg+6arcs_196	885.4k	255.4k	246.6	14400.0	460.3k	245.9k	87.2	14400.0	464.9k	131.4k	253.9	14400.0
belg+6arcs_197	498.6k	257.0k	94.0	14400.0	870.7k	303.0k	187.3	14400.0	426.3k	208.8k	104.1	14400.0
belg+6arcs_198	149.1k	149.1k	0.0	48.7	149.1k	149.1k	0.0	519.1	149.1k	149.1k	0.0	28.7
belg+6arcs_199	41.3k	41.3k	0.0	80.2	41.3k	41.3k	0.0	3600.6	41.3k	41.3k	0.0	465.9
belg+6arcs_200	222.4k	222.4k	0.0	11709.6	1e+20	203.0k	–	14400.0	222.4k	222.4k	0.0	6783.6
belg+6arcs_201	157.7k	39.5k	299.5	14400.0	108.6k	52.2k	108.2	14400.0	226.2k	22.1k	922.4	14400.0
belg+6arcs_202	151.8k	151.8k	0.0	2056.6	151.8k	151.8k	0.0	6290.6	151.8k	151.8k	0.0	9651.1
belg+6arcs_203	139.2k	45.4k	206.3	14400.0	1e+20	57.0k	–	14400.0	183.5k	13.6k	1248.4	14400.0
belg+6arcs_204	25.4k	25.4k	0.0	227.8	25.4k	25.4k	0.0	405.4	25.4k	25.4k	0.0	191.7
belg+6arcs_205	276.9k	276.9k	0.0	1988.8	276.9k	276.9k	0.0	7151.2	276.9k	233.6k	18.5	14400.0
belg+6arcs_206	140.8k	140.8k	0.0	1544.1	140.8k	140.8k	0.0	8744.8	153.8k	105.3k	46.0	14400.0
belg+6arcs_207	292.5k	196.4k	48.9	14400.0	267.5k	267.5k	0.0	14375.8	292.5k	198.1k	47.7	14400.0
belg+6arcs_208	211.4k	211.4k	0.0	444.2	211.4k	211.4k	0.0	4179.6	211.4k	211.4k	0.0	1158.2
belg+6arcs_209	231.6k	46.0k	403.3	14400.0	208.9k	54.8k	281.1	14400.0	187.9k	17.4k	981.9	14400.0
belg+6arcs_210	205.1k	205.1k	0.0	1560.3	205.1k	205.1k	0.0	8172.8	205.1k	205.1k	0.0	8431.7
belg+6arcs_211	238.4k	114.6k	108.0	14400.0	1e+20	124.2k	–	14400.0	298.7k	77.4k	286.0	14400.0
belg+6arcs_212	190.6k	104.1k	83.0	14400.0	127.8k	127.8k	0.0	4064.1	127.8k	127.8k	0.0	11321.6
belg+6arcs_213	392.9k	270.2k	45.4	14400.0	1e+20	257.3k	–	14400.0	453.8k	196.6k	130.8	14400.0
belg+6arcs_214	235.1k	234.9k	0.1	3305.5	1e+20	203.4k	–	14400.0	251.5k	197.4k	27.4	14400.0
belg+6arcs_215	185.2k	185.2k	0.0	12410.6	185.2k	185.2k	0.0	6210.3	185.2k	185.2k	0.0	10194.7
belg+6arcs_216	225.3k	225.3k	0.0	59.6	225.3k	225.3k	0.0	1202.9	225.3k	225.3k	0.0	770.9
belg+6arcs_217	154.5k	154.5k	0.0	233.5	154.5k	154.5k	0.0	4761.5	154.5k	154.5k	0.0	1591.4
belg+6arcs_218	344.3k	283.4k	21.5	14400.0	311.8k	311.8k	0.0	13113.0	344.3k	174.4k	97.4	14400.0
belg+6arcs_219	209.8k	209.8k	0.0	76.1	209.8k	209.8k	0.0	885.9	209.8k	209.8k	0.0	42.8
belg+6arcs_220	845.3k	193.1k	337.8	14400.0	1e+20	170.7k	–	14400.0	868.0k	87.6k	890.9	14400.0
belg+6arcs_221	25.1k	25.1k	0.0	895.7	25.1k	25.1k	0.0	3580.2	25.1k	25.1k	0.0	860.7
belg+6arcs_222	231.4k	231.4k	0.0	1637.2	231.4k	231.4k	0.0	7126.4	231.4k	231.4k	0.0	7466.9
belg+6arcs_223	199.7k	199.7k	0.0	6891.5	1e+20	162.1k	–	14400.0	235.4k	155.7k	51.2	14400.0
belg+6arcs_224	293.0k	261.4k	12.1	14400.0	1e+20	210.1k	–	14400.0	392.8k	175.5k	123.8	14400.0
belg+6arcs_225	11.4k	11.4k	0.0	30.6	11.4k	11.4k	0.0	262.8	11.4k	11.4k	0.0	141.4
belg+6arcs_226	189.1k	189.1k	0.0	1455.4	189.1k	189.1k	0.0	4938.9	189.1k	189.1k	0.0	5298.9
belg+6arcs_227	29.7k	29.7k	0.0	263.7	29.7k	29.7k	0.0	1872.9	29.7k	29.7k	0.0	408.3
belg+6arcs_228	361.8k	217.0k	66.8	14400.0	1e+20	202.5k	–	14400.0	338.7k	152.3k	122.4	14400.0
belg+6arcs_229	207.1k	188.7k	9.8	14400.0	207.1k	207.1k	0.0	2692.9	207.1k	207.1k	0.0	6269.0
belg+6arcs_230	188.7k	188.7k	0.0	968.2	188.7k	188.7k	0.0	512.9	188.7k	188.7k	0.0	711.3
belg+6arcs_231	28.3k	28.3k	0.0	56.3	28.3k	28.3k	0.0	58.9	28.3k	28.3k	0.0	53.0
belg+6arcs_232	161.8k	161.8k	0.0	5321.8	161.8k	161.8k	0.0	10405.9	161.8k	161.8k	0.0	12215.2
belg+6arcs_233	261.0k	213.7k	22.1	14400.0	1e+20	176.6k	–	14400.0	391.8k	130.5k	200.3	14400.0
belg+6arcs_234	57.6k	57.6k	0.0	584.6	57.6k	57.6k	0.0	12361.2	57.6k	57.6k	0.0	1942.4
belg+6arcs_235	176.6k	176.6k	0.0	95.9	176.6k	176.6k	0.0	8542.7	176.6k	176.6k	0.0	6095.1
belg+6arcs_236	200.3k	200.3k	0.0	988.0	200.3k	200.3k	0.0	7152.4	207.1k	166.5k	24.4	14400.0
belg+6arcs_237	3600.0	3600.0	0.0	16.7	3600.0	3600.0	0.0	31.0	3600.0	3600.0	0.0	51.1
belg+6arcs_238	375.9k	236.4k	59.0	14400.0	1e+20	237.8k	–	14400.0	524.4k	179.8k	191.6	14400.0
belg+6arcs_239	262.0k	262.0k	0.0	9591.6	262.0k	262.0k	0.0	14195.4	269.3k	180.3k	49.4	14400.0
belg+6arcs_240	341.8k	260.3k	31.3	14400.0	308.1k	240.5k	28.1	14400.0	368.9k	204.4k	80.5	14400.0
belg+6arcs_241	273.4k	191.6k	42.7	14400.0	270.5k	270.5k	0.0	7292.1	277.6k	210.6k	31.9	14400.0
belg+6arcs_242	198.1k	198.1k	0.0	8092.9	198.1k	198.1k	0.0	9585.9	223.6k	125.4k	78.3	14400.0
belg+6arcs_243	186.4k	186.4k	0.0	18.1	186.4k	186.4k	0.0	1267.9	186.4k	186.4k	0.0	152.0
belg+6arcs_244	202.3k	202.3k	0.0	2005.3	202.3k	202.3k	0.0	6570.8	202.3k	202.3k	0.0	1166.6
belg+6arcs_245	41.0k	41.0k	0.0	312.8	41.0k	41.0k	0.0	1129.9	41.0k	41.0k	0.0	3690.8

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Table A.29: Comparison of discrete models on *Circuit rank* + 6 (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_246	178.8k	178.8k	0.0	2309.9	178.8k	178.8k	0.0	3231.1	178.8k	178.8k	0.0	2350.6
belg+6arcs_247	56.4k	56.4k	0.0	2287.5	56.4k	56.4k	0.0	6777.0	153.2k	10.6k	1343.3	14400.0
belg+6arcs_248	407.9k	328.0k	24.3	14400.0	398.2k	301.1k	32.2	14400.0	437.5k	224.2k	95.2	14400.0
belg+6arcs_249	199.7k	51.8k	285.4	14400.0	98.2k	98.2k	0.0	12471.9	100.9k	48.1k	109.6	14400.0
belg+6arcs_250	366.2k	209.3k	75.0	14400.0	266.5k	208.5k	27.8	14400.0	380.8k	153.5k	148.0	14400.0
belg+6arcs_251	192.8k	192.8k	0.0	9151.9	192.8k	192.8k	0.0	9221.9	229.2k	149.6k	53.2	14400.0
belg+6arcs_252	190.2k	139.1k	36.7	14400.0	1e+20	123.3k	–	14400.0	186.1k	109.5k	69.9	14400.0
belg+6arcs_253	317.1k	210.5k	50.7	14400.0	313.4k	313.4k	0.0	4569.0	322.2k	218.7k	47.3	14400.0
belg+6arcs_254	301.9k	245.4k	23.0	14400.0	278.9k	278.9k	0.0	14170.9	284.7k	214.8k	32.5	14400.0
belg+6arcs_255	23.4k	23.4k	0.0	202.8	23.4k	23.4k	0.0	1989.3	23.4k	23.4k	0.0	27.7
belg+6arcs_256	187.9k	187.9k	0.0	185.3	187.9k	187.9k	0.0	4735.5	187.9k	187.9k	0.0	379.3
belg+6arcs_257	220.4k	38.5k	472.1	14400.0	1e+20	63.2k	–	14400.0	258.6k	23.2k	1014.4	14400.0
belg+6arcs_258	177.9k	177.9k	0.0	719.8	177.9k	177.9k	0.0	4948.4	177.9k	177.9k	0.0	6522.5
belg+6arcs_259	181.2k	181.2k	0.0	420.4	181.2k	181.2k	0.0	6215.9	181.2k	181.2k	0.0	2545.1
belg+6arcs_260	41.0k	41.0k	0.0	52.4	41.0k	41.0k	0.0	244.6	41.0k	41.0k	0.0	662.4
belg+6arcs_261	18.1k	18.1k	0.0	34.5	18.1k	18.1k	0.0	83.3	18.1k	18.1k	0.0	39.8
belg+6arcs_262	20.5k	20.5k	0.0	180.2	20.5k	20.5k	0.0	2477.8	20.5k	20.5k	0.0	445.6
belg+6arcs_263	294.1k	294.0k	0.0	2786.1	307.1k	257.7k	19.2	14400.0	305.4k	214.7k	42.3	14400.0
belg+6arcs_264	194.1k	194.1k	0.0	1991.4	194.1k	194.1k	0.0	10073.0	197.8k	159.9k	23.7	14400.0
belg+6arcs_265	458.1k	332.0k	38.0	14400.0	443.8k	339.9k	30.5	14400.0	466.8k	226.6k	106.0	14400.0
belg+6arcs_266	4920.0	4920.0	0.0	22.7	4920.0	4920.0	0.0	21.3	4920.0	4920.0	0.0	49.0
belg+6arcs_267	332.4k	332.3k	0.0	1628.7	332.4k	332.4k	0.0	7784.4	387.3k	247.6k	56.4	14400.0
belg+6arcs_268	345.6k	227.7k	51.7	14400.0	250.4k	250.4k	0.0	2558.6	250.4k	250.4k	0.0	9200.7
belg+6arcs_269	350.5k	236.9k	48.0	14400.0	276.6k	230.0k	20.3	14400.0	348.2k	130.0k	167.8	14400.0
belg+6arcs_270	202.6k	202.6k	0.0	1247.0	202.6k	202.6k	0.0	4817.5	202.6k	202.6k	0.0	2989.3
belg+6arcs_271	189.5k	189.5k	0.0	2070.3	189.5k	189.5k	0.0	1751.3	189.5k	189.5k	0.0	1525.1
belg+6arcs_272	306.9k	247.5k	24.0	14400.0	293.2k	293.2k	0.0	9103.3	306.0k	198.7k	54.0	14400.0
belg+6arcs_273	385.8k	263.0k	46.7	14400.0	389.1k	248.2k	56.8	14400.0	358.8k	198.6k	80.7	14400.0
belg+6arcs_274	143.9k	143.9k	0.0	320.5	143.9k	143.9k	0.0	6065.7	143.9k	143.9k	0.0	1243.8
belg+6arcs_275	19.2k	19.2k	0.0	87.2	19.2k	19.2k	0.0	288.5	19.2k	19.2k	0.0	90.5
belg+6arcs_276	72.6k	67.5k	7.6	14400.0	81.7k	41.2k	98.5	14400.0	72.6k	45.6k	59.3	14400.0
belg+6arcs_277	97.8k	97.8k	0.0	12425.3	97.8k	97.8k	0.0	7713.0	102.6k	50.0k	105.4	14400.0
belg+6arcs_278	13.2k	13.2k	0.0	24.9	13.2k	13.2k	0.0	949.7	13.2k	13.2k	0.0	62.8
belg+6arcs_279	110.3k	77.4k	42.4	14400.0	1e+20	36.7k	–	14400.0	123.2k	35.0k	251.7	14400.0
belg+6arcs_280	289.7k	228.6k	26.7	14400.0	276.2k	276.2k	0.0	5853.4	276.2k	211.5k	30.6	14400.0
belg+6arcs_281	223.2k	123.4k	81.0	14400.0	1e+20	122.9k	–	14400.0	328.9k	74.2k	343.1	14400.0
belg+6arcs_282	295.7k	295.7k	0.0	1324.3	295.7k	295.7k	0.0	4144.2	295.7k	221.0k	33.8	14400.0
belg+6arcs_283	225.1k	225.1k	0.0	177.2	225.1k	225.1k	0.0	1657.0	225.1k	225.1k	0.0	497.5
belg+6arcs_284	210.5k	187.1k	12.5	14400.0	206.0k	206.0k	0.0	5315.4	207.1k	177.6k	16.6	14400.0
belg+6arcs_285	161.4k	161.4k	0.0	7446.6	162.6k	141.3k	15.1	14400.0	163.2k	139.6k	16.9	14400.0
belg+6arcs_286	206.4k	206.4k	0.0	601.6	206.4k	206.4k	0.0	1733.7	206.4k	206.4k	0.0	3905.7
belg+6arcs_287	230.5k	230.5k	0.0	452.2	230.5k	230.5k	0.0	5453.8	230.5k	230.5k	0.0	6909.3
belg+6arcs_288	176.9k	176.7k	0.1	330.7	176.9k	176.9k	0.0	4669.3	176.9k	176.9k	0.0	1351.3
belg+6arcs_289	227.5k	127.9k	77.9	14400.0	212.5k	153.3k	38.6	14400.0	211.7k	35.0k	504.3	14400.0
belg+6arcs_290	329.8k	201.5k	63.7	14400.0	1e+20	173.2k	–	14400.0	349.3k	139.7k	150.0	14400.0
belg+6arcs_291	303.4k	303.4k	0.0	9585.5	1e+20	240.0k	–	14400.0	369.8k	183.5k	101.6	14400.0
belg+6arcs_292	129.5k	129.5k	0.0	11741.4	1e+20	42.2k	–	14400.0	130.2k	29.5k	341.3	14400.0
belg+6arcs_293	159.3k	23.3k	583.9	14400.0	1e+20	27.2k	–	14400.0	146.8k	17.5k	738.2	14400.0
belg+6arcs_294	166.0k	166.0k	0.0	237.2	166.0k	166.0k	0.0	1632.2	166.0k	166.0k	0.0	1974.9
belg+6arcs_295	222.6k	179.2k	24.2	14400.0	212.8k	212.8k	0.0	13921.0	261.0k	128.3k	103.4	14400.0
belg+6arcs_296	166.1k	166.1k	0.0	1339.3	1e+20	120.3k	–	14400.0	220.6k	72.0k	206.2	14400.0
belg+6arcs_297	132.4k	132.4k	0.0	1935.9	132.4k	132.4k	0.0	507.3	132.4k	132.4k	0.0	1538.2
belg+6arcs_298	154.5k	23.7k	552.9	14400.0	1e+20	38.5k	–	14400.0	168.3k	13.2k	1173.6	14400.0
belg+6arcs_299	486.1k	333.9k	45.6	14400.0	1e+20	295.5k	–	14400.0	486.1k	170.4k	185.4	14400.0
belg+6arcs_300	162.1k	162.1k	0.0	1935.3	162.1k	162.1k	0.0	14079.8	162.1k	162.1k	0.0	11205.0
belg+6arcs_301	173.0k	173.0k	0.0	11324.6	255.9k	148.5k	72.3	14400.0	177.3k	146.9k	20.7	14400.0
belg+6arcs_302	240.2k	240.2k	0.0	4277.4	240.2k	227.1k	5.8	14400.0	245.5k	205.1k	19.7	14400.0
belg+6arcs_303	311.6k	296.3k	5.2	14400.0	1e+20	244.0k	–	14400.0	329.3k	159.8k	106.1	14400.0
belg+6arcs_304	250.6k	188.7k	32.8	14400.0	242.8k	242.7k	0.1	12315.1	243.9k	198.6k	22.8	14400.0
belg+6arcs_305	330.9k	199.3k	66.0	14400.0	243.1k	243.1k	0.0	7611.5	246.2k	202.2k	21.8	14400.0
belg+6arcs_306	203.5k	203.5k	0.0	351.7	203.5k	203.5k	0.0	851.7	203.5k	203.5k	0.0	2621.0
belg+6arcs_307	142.5k	112.9k	26.2	14400.0	1e+20	92.0k	–	14400.0	137.0k	94.8k	44.5	14400.0
belg+6arcs_308	46.2k	46.2k	0.0	728.1	46.2k	46.2k	0.0	2197.0	46.2k	46.2k	0.0	818.5
belg+6arcs_309	417.8k	232.9k	79.4	14400.0	741.6k	230.0k	222.4	14400.0	467.1k	129.0k	262.2	14400.0
belg+6arcs_310	31.2k	31.2k	0.0	484.6	31.2k	31.2k	0.0	2260.1	31.2k	31.2k	0.0	236.5
belg+6arcs_311	145.3k	145.3k	0.0	192.8	145.3k	145.3k	0.0	9953.4	145.3k	145.3k	0.0	2306.8
belg+6arcs_312	137.2k	137.2k	0.0	1212.8	1e+20	112.5k	–	14400.0	137.2k	137.2k	0.0	9616.6
belg+6arcs_313	249.8k	249.8k	0.0	3615.1	249.8k	249.8k	0.0	3911.7	254.8k	171.4k	48.7	14400.0
belg+6arcs_314	547.4k	345.5k	58.5	14400.0	1e+20	309.9k	–	14400.0	627.5k	239.5k	162.0	14400.0
belg+6arcs_315	275.6k	187.1k	47.3	14400.0	419.0k	202.2k	107.2	14400.0	252.6k	187.2k	35.0	14400.0
belg+6arcs_316	178.0k	130.3k	36.6	14400.0	175.1k	132.8k	31.8	14400.0	253.7k	97.3k	160.7	14400.0
belg+6arcs_317	215.1k	193.5k	11.2	14400.0	215.1k	215.1k	0.0	14108.4	223.9k	147.3k	52.0	14400.0
belg+6arcs_318	94.0k	94.0k	0.0	477.4	94.0k	94.0k	0.0	276.8	94.0k	94.0k	0.0	3132.1
belg+6arcs_319	146.9k	146.9k	0.0	1274.2	146.9k	146.9k	0.0	3507.4	146.9k	146.9k	0.0	2531.1
belg+6arcs_320	181.4k	181.4k	0.0	352.5	181.4k	181.4k	0.0	9979.6	181.4k	181.4k	0.0	471.0
belg+6arcs_321	155.9k	146.0k	6.8	14400.0	151.7k	151.7k	0.0	10789.8	151.7k	151.7k	0.0	2991.1
belg+6arcs_322	244.2k	99.8k	144.7	14400.0	232.8k	85.7k	171.6	14400.0	244.2k	31.0k	686.8	14400.0
belg+6arcs_323	140.7k	140.7k	0.0	620.6	140.7k	140.7k	0.0	3176.4	157.5k	122.8k	28.2	14400.0

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Table A.29: Comparison of discrete models on *Circuit rank* + 6 (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_324	95.2k	95.2k	0.0	1202.2	1e+20	69.3k	–	14400.0	101.2k	63.3k	59.9	14400.0
belg+6arcs_325	242.1k	242.1k	0.0	602.9	242.1k	242.1k	0.0	7790.5	242.1k	241.9k	0.1	2640.0
belg+6arcs_326	244.7k	220.5k	11.0	14400.0	1e+20	195.9k	–	14400.0	263.6k	167.7k	57.2	14400.0
belg+6arcs_327	31.3k	31.3k	0.0	79.4	31.3k	31.3k	0.0	474.6	31.3k	31.3k	0.0	260.0
belg+6arcs_328	11.4k	11.4k	0.0	19.0	11.4k	11.4k	0.0	61.2	11.4k	11.4k	0.0	314.8
belg+6arcs_329	187.1k	171.3k	9.2	14400.0	177.1k	177.1k	0.0	10221.5	178.2k	162.7k	9.6	14400.0
belg+6arcs_330	227.8k	227.8k	0.0	111.3	227.8k	227.8k	0.0	1895.3	227.8k	227.7k	0.0	1222.0
belg+6arcs_331	143.4k	128.7k	11.5	14400.0	143.4k	143.4k	0.0	8388.6	143.4k	143.4k	0.0	482.2
belg+6arcs_332	496.6k	305.3k	62.7	14400.0	528.2k	328.7k	60.7	14400.0	496.6k	232.8k	113.4	14400.0
belg+6arcs_333	118.2k	118.2k	0.0	272.1	118.2k	118.2k	0.0	6627.9	118.2k	118.2k	0.0	1255.0
belg+6arcs_334	275.9k	275.9k	0.0	8838.0	275.9k	244.8k	12.7	14400.0	389.2k	189.1k	105.8	14400.0
belg+6arcs_335	272.9k	219.5k	24.3	14400.0	1e+20	202.6k	–	14400.0	274.7k	191.3k	43.6	14400.0
belg+6arcs_336	182.8k	182.8k	0.0	1639.5	1e+20	156.4k	–	14400.0	189.3k	156.0k	21.3	14400.0
belg+6arcs_337	85.0k	85.0k	0.0	11637.4	85.0k	85.0k	0.0	14258.4	85.0k	27.0k	214.4	14400.0
belg+6arcs_338	188.7k	188.7k	0.0	7962.6	1e+20	145.1k	–	14400.0	192.8k	147.6k	30.6	14400.0
belg+6arcs_339	327.6k	327.6k	0.0	546.5	327.6k	327.6k	0.0	5074.1	327.6k	327.6k	0.0	10106.3
belg+6arcs_340	474.2k	235.6k	101.3	14400.0	452.1k	268.3k	68.5	14400.0	545.5k	148.0k	268.5	14400.0
belg+6arcs_341	172.9k	172.9k	0.0	6987.4	172.9k	172.9k	0.0	2689.5	172.9k	172.8k	0.0	10157.5
belg+6arcs_342	326.8k	215.4k	51.7	14400.0	1e+20	218.6k	–	14400.0	371.4k	160.9k	130.8	14400.0
belg+6arcs_343	175.4k	175.4k	0.0	728.8	175.4k	175.4k	0.0	2268.7	189.6k	166.7k	13.7	14400.0
belg+6arcs_344	137.3k	137.3k	0.0	354.9	137.3k	137.3k	0.0	2459.9	137.3k	137.3k	0.0	930.8
belg+6arcs_345	187.9k	187.9k	0.0	235.6	187.9k	187.9k	0.0	210.1	187.9k	187.9k	0.0	2466.9
belg+6arcs_346	198.5k	198.5k	0.0	1298.6	1e+20	173.8k	–	14400.0	199.7k	193.1k	3.4	14400.0
belg+6arcs_347	412.4k	325.2k	26.8	14400.0	1e+20	287.4k	–	14400.0	420.1k	190.3k	120.7	14400.0
belg+6arcs_348	141.3k	141.3k	0.0	796.9	141.3k	141.3k	0.0	3260.6	141.3k	141.3k	0.0	1866.2
belg+6arcs_349	82.4k	82.4k	0.0	6588.1	1e+20	38.5k	–	14400.0	105.7k	23.7k	346.9	14400.0
belg+6arcs_350	57.1k	57.1k	0.0	308.3	57.1k	57.1k	0.0	10673.8	57.1k	21.6k	163.8	14400.0
belg+6arcs_351	1103.1k	233.0k	373.4	14400.0	344.4k	265.5k	29.7	14400.0	460.6k	179.1k	157.2	14400.0
belg+6arcs_352	306.0k	242.9k	26.0	14400.0	296.1k	238.7k	24.0	14400.0	303.2k	166.4k	82.2	14400.0
belg+6arcs_353	144.2k	144.2k	0.0	1580.6	144.2k	144.2k	0.0	4306.2	144.2k	144.2k	0.0	1524.4
belg+6arcs_354	124.5k	124.5k	0.0	1406.5	124.5k	124.5k	0.0	7940.7	124.5k	112.8k	10.3	14400.0
belg+6arcs_355	35.3k	35.3k	0.0	32.7	35.3k	35.3k	0.0	8157.5	35.3k	35.3k	0.0	406.5
belg+6arcs_356	384.0k	258.2k	48.7	14400.0	1e+20	233.6k	–	14400.0	354.3k	167.5k	111.5	14400.0
belg+6arcs_357	242.6k	200.5k	21.0	14400.0	229.6k	193.7k	18.5	14400.0	283.9k	169.0k	68.0	14400.0
belg+6arcs_358	349.6k	349.6k	0.0	9159.7	349.6k	349.6k	0.0	6818.6	430.8k	237.0k	81.8	14400.0
belg+6arcs_359	277.2k	183.5k	51.0	14400.0	244.6k	177.7k	37.7	14400.0	245.9k	129.7k	89.7	14400.0
belg+6arcs_360	148.3k	148.3k	0.0	6532.7	148.3k	148.3k	0.0	9696.8	148.3k	148.3k	0.0	6575.4
belg+6arcs_361	505.1k	291.0k	73.6	14400.0	537.7k	286.9k	87.4	14400.0	510.1k	222.8k	129.0	14400.0
belg+6arcs_362	71.5k	71.5k	0.0	3048.8	71.5k	71.5k	0.0	9051.7	71.5k	5097.6	1302.5	14400.0
belg+6arcs_363	180.6k	180.6k	0.0	780.9	180.6k	180.6k	0.0	1504.9	180.6k	180.6k	0.0	1231.0
belg+6arcs_364	189.1k	189.1k	0.0	11447.9	295.3k	157.4k	87.6	14400.0	202.7k	126.5k	60.2	14400.0
belg+6arcs_365	206.6k	206.6k	0.0	5807.8	206.6k	206.6k	0.0	2591.1	206.6k	206.6k	0.0	1710.2
belg+6arcs_366	254.5k	233.2k	9.1	14400.0	254.5k	254.5k	0.0	2099.5	259.7k	220.5k	17.8	14400.0
belg+6arcs_367	185.4k	185.4k	0.0	818.1	185.4k	185.4k	0.0	1592.2	185.4k	185.4k	0.0	580.0
belg+6arcs_368	94.9k	65.9k	43.9	14400.0	104.3k	49.2k	112.1	14400.0	89.0k	51.8k	72.0	14400.0
belg+6arcs_369	4920.0	4920.0	0.0	50.2	4920.0	4920.0	0.0	686.8	4920.0	4920.0	0.0	111.4
belg+6arcs_370	163.3k	163.3k	0.0	144.9	163.3k	163.3k	0.0	5018.8	163.3k	163.3k	0.0	312.6
belg+6arcs_371	276.6k	276.6k	0.0	11099.0	276.6k	276.6k	0.0	10215.3	305.5k	195.1k	56.6	14400.0
belg+6arcs_372	219.4k	219.4k	0.0	4249.8	219.4k	219.4k	0.0	6810.1	278.0k	133.8k	107.8	14400.0
belg+6arcs_373	184.5k	184.5k	0.0	524.1	184.5k	184.4k	0.1	4082.6	184.5k	184.5k	0.0	111.4
belg+6arcs_374	126.9k	126.9k	0.0	5439.7	126.9k	126.9k	0.0	7752.4	126.9k	110.5k	14.8	14400.0
belg+6arcs_375	46.4k	46.4k	0.0	22.0	46.4k	46.4k	0.0	343.9	46.4k	46.4k	0.0	193.4
belg+6arcs_376	390.7k	386.9k	1.0	14400.0	389.0k	389.0k	0.0	6513.7	393.1k	267.3k	47.1	14400.0
belg+6arcs_377	396.7k	70.6k	462.0	14400.0	628.2k	83.7k	650.1	14400.0	501.4k	12.3k	3983.4	14400.0
belg+6arcs_378	119.3k	119.3k	0.0	1875.7	119.3k	119.3k	0.0	11105.1	133.8k	49.8k	168.4	14400.0
belg+6arcs_379	380.9k	233.4k	63.2	14400.0	1e+20	217.4k	–	14400.0	417.0k	160.0k	160.6	14400.0
belg+6arcs_380	176.8k	176.8k	0.0	1035.1	176.8k	176.8k	0.0	13310.2	176.8k	176.8k	0.0	6473.7
belg+6arcs_381	527.9k	218.5k	141.6	14400.0	1e+20	211.5k	–	14400.0	451.8k	156.0k	189.6	14400.0
belg+6arcs_382	253.0k	253.0k	0.0	1461.3	253.0k	253.0k	0.0	13338.9	254.2k	192.7k	31.9	14400.0
belg+6arcs_383	63.6k	63.6k	0.0	475.4	1e+20	30.7k	–	14400.0	63.6k	63.6k	0.0	4316.4
belg+6arcs_384	362.7k	117.2k	209.5	14400.0	328.4k	129.0k	154.6	14400.0	334.7k	25.1k	1236.2	14400.0
belg+6arcs_385	219.0k	219.0k	0.0	1967.0	219.0k	219.0k	0.0	11800.3	242.3k	172.4k	40.5	14400.0
belg+6arcs_386	484.5k	306.5k	58.1	14400.0	376.5k	376.5k	0.0	12049.1	456.5k	233.2k	95.7	14400.0
belg+6arcs_387	236.3k	60.6k	289.8	14400.0	1e+20	58.6k	–	14400.0	303.0k	11.2k	2611.7	14400.0
belg+6arcs_388	134.7k	63.3k	112.6	14400.0	1e+20	29.7k	–	14400.0	120.3k	12.5k	860.1	14400.0
belg+6arcs_389	158.3k	158.3k	0.0	291.1	158.3k	158.3k	0.0	10977.6	158.3k	158.3k	0.0	7906.3
belg+6arcs_390	260.9k	232.3k	12.3	14400.0	251.2k	251.2k	0.0	8613.4	268.5k	197.3k	36.1	14400.0
belg+6arcs_391	221.2k	186.0k	18.9	14400.0	192.5k	192.5k	0.0	7056.2	192.5k	192.5k	0.0	6732.5
belg+6arcs_392	108.9k	108.9k	0.0	27.7	1e+20	98.4k	–	14400.0	108.9k	108.9k	0.0	469.9
belg+6arcs_393	221.9k	221.9k	0.0	575.2	221.9k	221.9k	0.0	12075.9	221.9k	221.9k	0.0	6227.3
belg+6arcs_394	137.1k	25.4k	439.1	14400.0	124.5k	57.1k	118.0	14400.0	122.2k	26.8k	355.9	14400.0
belg+6arcs_395	374.0k	154.7k	141.8	14400.0	1e+20	181.3k	–	14400.0	447.1k	135.4k	230.2	14400.0
belg+6arcs_396	79.4k	79.4k	0.0	839.9	79.4k	79.4k	0.0	2875.4	107.7k	51.0k	111.2	14400.0
belg+6arcs_397	381.5k	208.7k	82.8	14400.0	435.4k	259.2k	68.0	14400.0	529.4k	164.7k	221.4	14400.0
belg+6arcs_398	43.5k	43.5k	0.0	6309.4	43.5k	43.5k	0.0	7754.5	49.2k	23.5k	109.6	14400.0
belg+6arcs_399	4920.0	4920.0	0.0	16.4	4920.0	4920.0	0.0	685.8	4920.0	4920.0	0.0	9.3
belg+6arcs_400	102.2k	28.3k	261.3	14400.0	1e+20	34.9k	–	14400.0	102.2k	24.6k	314.6	14400.0
belg+6arcs_401	140.7k	140.7k	0.0	83.2	140.7k	140.5k	0.1	5090.6	140.7k	140.7k	0.0	645.3

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Table A.29: Comparison of discrete models on *Circuit rank* + 6 (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_402	161.3k	161.3k	0.0	178.1	161.3k	161.1k	0.1	2815.5	161.3k	161.3k	0.0	365.8
belg+6arcs_403	270.4k	270.4k	0.0	4876.8	528.9k	238.7k	121.6	14400.0	285.2k	218.8k	30.4	14400.0
belg+6arcs_404	259.2k	259.2k	0.0	1600.9	259.2k	259.2k	0.0	11619.8	260.1k	213.4k	21.9	14400.0
belg+6arcs_405	331.3k	331.3k	0.0	5725.9	331.3k	300.4k	10.3	14400.0	342.0k	187.7k	82.2	14400.0
belg+6arcs_406	68.6k	68.6k	0.0	293.7	68.6k	68.5k	0.0	2706.0	68.6k	68.6k	0.0	1164.8
belg+6arcs_407	249.4k	201.1k	24.0	14400.0	304.5k	209.4k	45.4	14400.0	238.7k	180.8k	32.0	14400.0
belg+6arcs_408	36.5k	36.5k	0.0	756.4	1e+20	25.5k	–	14400.0	36.5k	36.5k	0.0	115.2
belg+6arcs_409	161.0k	161.0k	0.0	3987.2	1e+20	133.5k	–	14400.0	161.0k	161.0k	0.0	5368.6
belg+6arcs_410	196.3k	196.3k	0.0	2734.0	196.3k	196.3k	0.0	5912.9	202.8k	179.4k	13.0	14400.0
belg+6arcs_411	255.1k	182.0k	40.2	14400.0	201.2k	201.2k	0.0	13203.5	255.9k	160.1k	59.8	14400.0
belg+6arcs_412	16.8k	16.8k	0.0	21.6	16.8k	16.8k	0.0	3426.0	16.8k	16.8k	0.0	765.9
belg+6arcs_413	191.2k	191.2k	0.0	6305.7	1e+20	143.5k	–	14400.0	209.9k	120.4k	74.4	14400.0
belg+6arcs_414	159.4k	159.4k	0.0	2786.9	159.4k	159.4k	0.0	1146.7	159.4k	159.4k	0.0	507.2
belg+6arcs_415	24.3k	24.3k	0.0	365.9	24.3k	24.3k	0.0	3817.3	24.3k	24.3k	0.0	941.0
belg+6arcs_416	190.7k	190.7k	0.0	689.8	190.7k	190.7k	0.0	8697.7	190.7k	190.7k	0.0	1073.7
belg+6arcs_417	161.3k	161.3k	0.0	2024.3	161.3k	161.3k	0.0	3425.1	161.3k	161.3k	0.0	1164.8
belg+6arcs_418	476.6k	236.3k	101.7	14400.0	1e+20	250.5k	–	14400.0	624.3k	138.7k	350.2	14400.0
belg+6arcs_419	150.2k	112.7k	33.3	14400.0	228.8k	115.4k	98.2	14400.0	150.1k	99.4k	51.0	14400.0
belg+6arcs_420	143.4k	143.4k	0.0	741.2	208.7k	137.3k	52.0	14400.0	143.4k	143.4k	0.0	3019.1
belg+6arcs_421	127.7k	127.7k	0.0	3184.8	148.8k	73.7k	101.7	14400.0	148.9k	66.2k	124.9	14400.0
belg+6arcs_422	268.0k	39.4k	579.7	14400.0	1e+20	51.7k	–	14400.0	186.5k	19.9k	834.9	14400.0
belg+6arcs_423	183.0k	183.0k	0.0	2369.1	183.0k	183.0k	0.0	9069.1	192.1k	154.2k	24.6	14400.0
belg+6arcs_424	344.6k	260.4k	32.3	14400.0	1e+20	258.4k	–	14400.0	342.5k	206.6k	65.8	14400.0
belg+6arcs_425	223.7k	223.7k	0.0	1351.0	223.7k	223.7k	0.0	10181.8	223.7k	223.7k	0.0	10690.0
belg+6arcs_426	304.7k	94.2k	223.5	14400.0	1e+20	96.2k	–	14400.0	779.4k	11.1k	6947.3	14400.0
belg+6arcs_427	147.2k	147.2k	0.0	132.8	147.2k	147.1k	0.0	3148.7	147.2k	147.2k	0.0	492.6
belg+6arcs_428	112.8k	71.9k	56.9	14400.0	264.5k	44.8k	490.1	14400.0	116.0k	22.6k	413.3	14400.0
belg+6arcs_429	330.8k	330.8k	0.0	13330.5	1e+20	242.6k	–	14400.0	344.1k	223.3k	54.1	14400.0
belg+6arcs_430	367.6k	200.7k	83.2	14400.0	285.7k	256.4k	11.4	14400.0	435.4k	163.7k	166.0	14400.0
belg+6arcs_431	751.0k	313.0k	140.0	14400.0	1e+20	354.2k	–	14400.0	532.8k	217.4k	145.0	14400.0
belg+6arcs_432	4164.0	4164.0	0.0	18.2	4164.0	4164.0	0.0	64.8	4164.0	4164.0	0.0	8.1
belg+6arcs_433	32.2k	32.2k	0.0	171.3	32.2k	32.2k	0.0	2178.0	32.2k	32.2k	0.0	1259.1
belg+6arcs_434	219.2k	219.2k	0.0	842.8	219.2k	219.2k	0.0	7184.6	219.2k	219.2k	0.0	8100.4
belg+6arcs_435	370.0k	192.0k	92.7	14400.0	252.5k	252.5k	0.0	9757.8	257.6k	188.9k	36.4	14400.0
belg+6arcs_436	332.9k	190.9k	74.4	14400.0	215.5k	215.5k	0.0	4863.6	215.5k	187.0k	15.2	14400.0
belg+6arcs_437	143.2k	143.2k	0.0	1425.7	143.2k	143.2k	0.0	6218.7	202.9k	50.3k	303.4	14400.0
belg+6arcs_438	258.1k	193.3k	33.5	14400.0	236.8k	236.8k	0.0	8949.2	251.7k	186.4k	35.1	14400.0
belg+6arcs_439	399.7k	150.2k	166.1	14400.0	471.1k	248.2k	89.8	14400.0	399.7k	159.0k	151.4	14400.0
belg+6arcs_440	413.5k	202.3k	104.4	14400.0	419.5k	237.0k	77.0	14400.0	503.1k	158.3k	217.8	14400.0
belg+6arcs_441	214.4k	163.1k	31.4	14400.0	1e+20	137.7k	–	14400.0	298.0k	119.2k	150.1	14400.0
belg+6arcs_442	3600.0	3600.0	0.0	25.5	3600.0	3600.0	0.0	32.5	3600.0	3600.0	0.0	23.6
belg+6arcs_443	129.1k	129.1k	0.0	35.7	129.1k	129.1k	0.0	177.4	129.1k	129.1k	0.0	78.6
belg+6arcs_444	58.3k	58.3k	0.0	2715.9	1e+20	43.3k	–	14400.0	58.3k	58.3k	0.0	10281.2
belg+6arcs_445	405.0k	295.2k	37.2	14400.0	1e+20	255.1k	–	14400.0	477.9k	191.1k	150.1	14400.0
belg+6arcs_446	157.4k	157.4k	0.0	824.7	157.4k	157.4k	0.0	4183.4	157.4k	157.4k	0.0	2947.3
belg+6arcs_447	246.8k	191.6k	28.8	14400.0	201.9k	201.9k	0.0	8266.3	233.8k	159.9k	46.2	14400.0
belg+6arcs_448	340.3k	181.7k	87.2	14400.0	1e+20	180.0k	–	14400.0	341.2k	135.6k	151.6	14400.0
belg+6arcs_449	152.4k	152.4k	0.0	425.3	152.4k	152.4k	0.0	3759.7	152.4k	152.4k	0.0	615.8
belg+6arcs_450	86.9k	56.4k	54.2	14400.0	65.9k	65.9k	0.0	12747.2	65.9k	65.9k	0.0	2053.2
belg+6arcs_451	38.3k	38.3k	0.0	41.9	38.3k	38.3k	0.0	253.2	38.3k	38.3k	0.0	657.6
belg+6arcs_452	213.5k	213.5k	0.0	7147.9	213.5k	195.1k	9.4	14400.0	230.7k	171.2k	34.7	14400.0
belg+6arcs_453	269.9k	269.9k	0.0	2175.2	269.9k	269.9k	0.0	1879.1	269.9k	269.9k	0.0	990.5
belg+6arcs_454	99.2k	99.2k	0.0	4401.1	1e+20	45.0k	–	14400.0	107.7k	60.9k	77.0	14400.0
belg+6arcs_455	52.1k	26.1k	100.0	14400.0	1e+20	24.7k	–	14400.0	43.6k	43.6k	0.0	10901.1
belg+6arcs_456	632.1k	208.0k	203.8	14400.0	1e+20	211.5k	–	14400.0	490.9k	121.0k	305.6	14400.0
belg+6arcs_457	108.9k	108.9k	0.0	563.2	108.9k	108.9k	0.0	2055.7	108.9k	108.9k	0.0	235.9
belg+6arcs_458	261.3k	261.3k	0.0	181.7	261.3k	261.3k	0.0	5286.9	261.3k	261.3k	0.0	2423.5
belg+6arcs_459	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.3
belg+6arcs_460	4164.0	4164.0	0.0	21.4	4164.0	4164.0	0.0	249.9	4164.0	4164.0	0.0	27.4
belg+6arcs_461	221.7k	221.7k	0.0	3262.7	221.7k	195.5k	13.4	14400.0	277.9k	183.2k	51.7	14400.0
belg+6arcs_462	175.9k	175.9k	0.0	352.7	175.9k	175.9k	0.0	10672.0	178.4k	157.4k	13.3	14400.0
belg+6arcs_463	143.4k	143.4k	0.0	202.2	143.4k	143.4k	0.0	753.6	143.4k	143.4k	0.0	212.7
belg+6arcs_464	194.2k	142.6k	36.2	14400.0	152.5k	152.5k	0.0	12723.5	164.9k	131.9k	25.0	14400.0
belg+6arcs_465	5724.0	5724.0	0.0	40.1	5724.0	5724.0	0.0	27.6	5724.0	5724.0	0.0	9.8
belg+6arcs_466	89.0k	89.0k	0.0	58.7	89.0k	89.0k	0.0	1266.9	89.0k	89.0k	0.0	330.1
belg+6arcs_467	188.7k	188.6k	0.0	1479.1	188.7k	188.7k	0.0	691.3	188.7k	188.7k	0.0	634.9
belg+6arcs_468	297.6k	107.7k	176.3	14400.0	1e+20	105.1k	–	14400.0	616.8k	16.3k	3694.7	14400.0
belg+6arcs_469	122.1k	122.1k	0.0	552.5	122.1k	122.1k	0.0	5706.9	122.1k	122.1k	0.0	4859.2
belg+6arcs_470	42.4k	21.1k	101.0	14400.0	39.8k	39.8k	0.0	10930.0	39.8k	39.8k	0.0	12233.9
belg+6arcs_471	181.0k	181.0k	0.0	631.6	181.0k	181.0k	0.0	4047.2	181.0k	181.0k	0.0	43.6
belg+6arcs_472	113.8k	51.1k	122.7	14400.0	1e+20	30.8k	–	14400.0	113.8k	25.6k	344.0	14400.0
belg+6arcs_473	259.3k	259.3k	0.0	1569.4	259.3k	259.2k	0.0	7935.9	370.7k	145.1k	155.5	14400.0
belg+6arcs_474	127.0k	104.9k	21.1	14400.0	1e+20	68.6k	–	14400.0	116.0k	92.9k	24.8	14400.0
belg+6arcs_475	131.6k	131.6k	0.0	550.2	131.6k	131.6k	0.0	3023.8	131.6k	131.6k	0.0	392.9
belg+6arcs_476	274.7k	218.3k	25.8	14400.0	255.0k	220.2k	15.8	14400.0	274.7k	191.7k	43.3	14400.0
belg+6arcs_477	259.9k	226.1k	14.9	14400.0	252.4k	230.7k	9.4	14400.0	343.1k	181.7k	88.8	14400.0
belg+6arcs_478	256.8k	186.4k	37.8	14400.0	273.0k	217.8k	25.3	14400.0	331.9k	167.1k	98.6	14400.0
belg+6arcs_479	174.7k	174.7k	0.0	3529.4	174.7k	174.7k	0.0	6673.3	186.2k	145.4k	28.1	14400.0

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Table A.29: Comparison of discrete models on *Circuit rank* + 6 (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_480	164.7k	164.7k	0.0	9212.1	1e+20	126.2k	–	14400.0	166.8k	130.0k	28.2	14400.0
belg+6arcs_481	331.1k	225.9k	46.5	14400.0	1e+20	224.1k	–	14400.0	549.1k	160.3k	242.5	14400.0
belg+6arcs_482	194.4k	188.5k	3.1	14400.0	188.7k	188.7k	0.0	12893.6	188.7k	188.7k	0.0	7827.3
belg+6arcs_483	38.3k	38.3k	0.0	138.0	38.3k	38.3k	0.0	2901.4	38.3k	38.3k	0.0	542.2
belg+6arcs_484	84.8k	84.8k	0.0	9820.5	90.9k	48.7k	86.4	14400.0	94.9k	40.1k	136.9	14400.0
belg+6arcs_485	59.4k	59.4k	0.0	288.0	59.4k	59.4k	0.0	3292.6	59.4k	59.4k	0.0	13776.6
belg+6arcs_486	283.5k	283.5k	0.0	10348.7	285.0k	254.2k	12.1	14400.0	283.5k	227.7k	24.5	14400.0
belg+6arcs_487	154.3k	154.3k	0.0	965.6	154.3k	154.3k	0.0	11771.8	154.3k	154.3k	0.0	8934.2
belg+6arcs_488	206.9k	206.9k	0.0	623.0	206.9k	206.9k	0.0	10954.2	244.9k	182.2k	34.5	14400.0
belg+6arcs_489	220.6k	220.6k	0.0	4549.2	220.6k	220.6k	0.0	12333.2	238.3k	149.9k	59.0	14400.0
belg+6arcs_490	133.5k	133.5k	0.0	193.6	133.5k	133.5k	0.0	5058.7	133.5k	133.4k	0.1	5306.2
belg+6arcs_491	128.6k	128.6k	0.0	894.4	128.6k	128.6k	0.0	2527.6	132.6k	114.3k	16.0	14400.0
belg+6arcs_492	137.6k	137.6k	0.0	7890.4	137.6k	137.6k	0.0	9942.9	156.9k	118.1k	32.8	14400.0
belg+6arcs_493	272.2k	272.2k	0.0	9405.6	1e+20	206.4k	–	14400.0	296.9k	160.9k	84.5	14400.0
belg+6arcs_494	197.5k	197.5k	0.0	472.5	197.5k	197.5k	0.0	629.5	197.5k	197.5k	0.0	21.6
belg+6arcs_495	213.3k	134.1k	59.1	14400.0	165.7k	165.7k	0.0	1250.0	165.7k	165.7k	0.0	3155.1
belg+6arcs_496	147.0k	126.0k	16.7	14400.0	143.9k	120.1k	19.9	14400.0	147.0k	110.6k	32.9	14400.0
belg+6arcs_497	69.6k	69.6k	0.0	2129.9	1e+20	42.6k	–	14400.0	78.4k	22.3k	251.8	14400.0
belg+6arcs_498	284.2k	232.7k	22.2	14400.0	282.6k	282.6k	0.0	11065.5	284.2k	241.3k	17.8	14400.0
belg+6arcs_499	202.7k	202.7k	0.0	483.9	202.7k	202.7k	0.0	4379.5	202.7k	202.7k	0.0	5145.7
belg+6arcs_500	169.3k	169.3k	0.0	4404.0	169.3k	169.3k	0.0	7097.4	170.3k	136.9k	24.4	14400.0

Table A.30: Detailed results of the split-pipe models on *Circuit rank* + 6 arcs, as summarized in Figure 3.16d and Table 3.4d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_1	366.0k	261.9k	39.7	14400.0	366.0k	276.1k	32.6	14400.0
belg+6arcs_2	174.8k	153.5k	13.9	14400.0	174.8k	174.1k	0.4	14400.0
belg+6arcs_3	117.3k	105.2k	11.5	14400.0	117.3k	110.1k	6.5	14400.0
belg+6arcs_4	284.5k	213.1k	33.5	14400.0	287.3k	229.6k	25.1	14400.0
belg+6arcs_5	417.1k	311.9k	33.7	14400.0	417.3k	320.5k	30.2	14400.0
belg+6arcs_6	200.1k	193.5k	3.4	14400.0	200.1k	198.4k	0.8	14400.0
belg+6arcs_7	343.6k	309.5k	11.0	14400.0	343.6k	294.2k	16.8	14400.0
belg+6arcs_8	4519.6	3737.1	20.9	14400.0	4519.6	4515.2	0.1	3.6
belg+6arcs_9	253.2k	155.9k	62.4	14400.0	253.2k	197.1k	28.4	14400.0
belg+6arcs_10	227.6k	217.9k	4.4	14400.0	227.6k	214.8k	6.0	14400.0
belg+6arcs_11	279.7k	206.0k	35.8	14400.0	262.3k	186.1k	41.0	14400.0
belg+6arcs_12	55.4k	48.5k	14.2	14400.0	55.4k	55.3k	0.2	14400.0
belg+6arcs_13	205.5k	203.1k	1.2	14400.0	205.5k	200.0k	2.8	14400.0
belg+6arcs_14	522.4k	396.9k	31.6	14400.0	456.4k	308.7k	47.8	14400.0
belg+6arcs_15	1595.0	1548.5	3.0	14400.0	1595.0	1594.4	0.0	1.6
belg+6arcs_16	475.1k	408.7k	16.2	14400.0	475.1k	391.4k	21.4	14400.0
belg+6arcs_17	253.0k	169.7k	49.1	14400.0	230.5k	189.8k	21.5	14400.0
belg+6arcs_18	232.4k	208.4k	11.6	14400.0	232.4k	232.3k	0.1	188.4
belg+6arcs_19	161.0k	160.4k	0.4	14400.0	161.0k	160.9k	0.1	6015.5
belg+6arcs_20	273.7k	226.4k	20.9	14400.0	273.7k	228.6k	19.7	14400.0
belg+6arcs_21	44.9k	40.4k	11.2	14400.0	44.9k	44.9k	0.1	9.2
belg+6arcs_22	207.3k	161.9k	28.0	14400.0	207.3k	198.8k	4.2	14400.0
belg+6arcs_23	104.7k	35.0k	199.5	14400.0	104.7k	103.5k	1.2	14400.0
belg+6arcs_24	205.3k	203.8k	0.7	14400.0	205.3k	205.1k	0.1	116.8
belg+6arcs_25	140.0k	34.8k	302.0	14400.0	140.0k	44.5k	214.7	14400.0
belg+6arcs_26	176.0k	173.8k	1.3	14400.0	176.0k	175.9k	0.1	40.2
belg+6arcs_27	127.4k	124.4k	2.4	14400.0	127.4k	127.3k	0.1	19.9
belg+6arcs_28	249.3k	223.3k	11.6	14400.0	249.3k	211.8k	17.7	14400.0
belg+6arcs_29	2420.4	778.0	211.1	14400.0	2420.4	2418.4	0.1	1.2
belg+6arcs_30	70.4k	68.8k	2.3	14400.0	70.4k	70.2k	0.3	14400.0
belg+6arcs_31	163.6k	148.2k	10.4	14400.0	163.6k	162.8k	0.5	14400.0
belg+6arcs_32	301.7k	275.4k	9.5	14400.0	301.7k	265.1k	13.8	14400.0
belg+6arcs_33	83.3k	55.1k	51.1	14400.0	83.3k	74.5k	11.8	14400.0
belg+6arcs_34	19.4k	19.4k	0.1	898.9	19.4k	19.4k	0.1	4.4
belg+6arcs_35	199.2k	176.6k	12.8	14400.0	188.5k	188.3k	0.1	1223.4
belg+6arcs_36	220.7k	163.9k	34.6	14400.0	215.8k	199.7k	8.1	14400.0
belg+6arcs_37	82.0k	56.1k	46.1	14400.0	82.0k	79.7k	2.8	14400.0
belg+6arcs_38	336.6k	269.3k	25.0	14400.0	336.6k	254.1k	32.5	14400.0
belg+6arcs_39	95.6k	83.6k	14.4	14400.0	95.6k	92.3k	3.6	14400.0
belg+6arcs_40	194.8k	101.9k	91.1	14400.0	194.8k	119.1k	63.5	14400.0
belg+6arcs_41	323.4k	233.5k	38.5	14400.0	321.6k	239.1k	34.5	14400.0
belg+6arcs_42	239.8k	228.4k	5.0	14400.0	239.8k	211.2k	13.5	14400.0

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Table A.30: Comparison of split-pipe models on *Circuit rank* + 6 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_43	109.1k	75.2k	45.1	14400.0	109.1k	75.0k	45.6	14400.0
belg+6arcs_44	165.0k	158.4k	4.2	14400.0	165.0k	164.8k	0.1	69.8
belg+6arcs_45	17.7k	13.6k	30.5	14400.0	17.7k	17.7k	0.1	5.6
belg+6arcs_46	190.3k	170.1k	11.9	14400.0	190.3k	186.4k	2.1	14400.0
belg+6arcs_47	223.6k	223.3k	0.1	3070.6	223.6k	223.3k	0.1	36.8
belg+6arcs_48	226.0k	188.1k	20.2	14400.0	226.0k	198.5k	13.8	14400.0
belg+6arcs_49	29.4k	29.1k	1.2	14400.0	29.4k	28.6k	3.0	14400.0
belg+6arcs_50	284.7k	256.0k	11.2	14400.0	277.5k	225.6k	23.0	14400.0
belg+6arcs_51	32.1k	13.0k	146.5	14400.0	32.1k	32.1k	0.0	92.4
belg+6arcs_52	226.2k	167.4k	35.1	14400.0	226.2k	166.6k	35.8	14400.0
belg+6arcs_53	134.8k	134.0k	0.6	14400.0	134.8k	134.8k	0.0	27.5
belg+6arcs_54	176.7k	174.8k	1.1	14400.0	176.7k	176.6k	0.0	3.5
belg+6arcs_55	199.6k	196.6k	1.5	14400.0	199.6k	199.5k	0.0	24.4
belg+6arcs_56	161.8k	161.5k	0.1	14400.0	161.8k	161.6k	0.1	403.2
belg+6arcs_57	259.5k	231.4k	12.1	14400.0	259.4k	236.6k	9.6	14400.0
belg+6arcs_58	164.5k	79.0k	108.3	14400.0	164.5k	84.1k	95.6	14400.0
belg+6arcs_59	23.3k	19.5k	19.4	14400.0	23.3k	23.3k	0.1	12.0
belg+6arcs_60	133.8k	131.8k	1.5	14400.0	133.8k	133.6k	0.1	394.4
belg+6arcs_61	153.2k	125.4k	22.1	14400.0	153.2k	153.2k	0.0	6820.9
belg+6arcs_62	471.1k	280.5k	67.9	14400.0	391.7k	271.5k	44.3	14400.0
belg+6arcs_63	234.0k	233.6k	0.2	14400.0	234.0k	233.7k	0.1	168.1
belg+6arcs_64	115.7k	81.9k	41.3	14400.0	115.7k	85.0k	36.1	14400.0
belg+6arcs_65	247.1k	188.6k	31.0	14400.0	230.6k	193.3k	19.3	14400.0
belg+6arcs_66	326.1k	313.5k	4.0	14400.0	338.5k	249.4k	35.7	14400.0
belg+6arcs_67	176.6k	170.8k	3.4	14400.0	175.0k	174.8k	0.1	4091.2
belg+6arcs_68	250.2k	216.6k	15.5	14400.0	250.2k	216.7k	15.4	14400.0
belg+6arcs_69	309.9k	277.4k	11.7	14400.0	309.9k	278.5k	11.2	14400.0
belg+6arcs_70	213.8k	171.1k	25.0	14400.0	205.2k	180.9k	13.4	14400.0
belg+6arcs_71	204.3k	122.7k	66.5	14400.0	204.3k	144.9k	41.0	14400.0
belg+6arcs_72	40.2k	24.8k	62.0	14400.0	40.2k	40.2k	0.1	72.5
belg+6arcs_73	30.3k	27.6k	9.7	14400.0	30.3k	30.3k	0.1	10.7
belg+6arcs_74	414.2k	412.7k	0.4	14400.0	437.5k	294.3k	48.6	14400.0
belg+6arcs_75	206.7k	173.2k	19.4	14400.0	200.0k	173.8k	15.0	14400.0
belg+6arcs_76	44.9k	44.9k	0.1	704.7	44.9k	44.9k	0.1	772.5
belg+6arcs_77	82.5k	77.0k	7.2	14400.0	82.5k	59.8k	37.9	14400.0
belg+6arcs_78	49.8k	47.3k	5.3	14400.0	49.8k	49.8k	0.1	9.8
belg+6arcs_79	97.9k	19.6k	398.3	14400.0	97.9k	63.8k	53.5	14400.0
belg+6arcs_80	144.5k	144.3k	0.2	14400.0	144.5k	144.4k	0.1	439.9
belg+6arcs_81	260.8k	228.8k	14.0	14400.0	260.8k	232.9k	12.0	14400.0
belg+6arcs_82	19.3k	17.9k	7.5	14400.0	19.3k	19.3k	0.1	2.1
belg+6arcs_83	457.6k	299.7k	52.7	14400.0	395.0k	249.8k	58.1	14400.0
belg+6arcs_84	356.8k	314.1k	13.6	14400.0	356.8k	295.1k	20.9	14400.0
belg+6arcs_85	234.7k	170.7k	37.5	14400.0	219.7k	163.0k	34.8	14400.0
belg+6arcs_86	321.1k	301.2k	6.6	14400.0	315.1k	260.5k	21.0	14400.0
belg+6arcs_87	215.8k	213.6k	1.0	14400.0	215.8k	213.8k	1.0	14400.0
belg+6arcs_88	323.0k	206.3k	56.6	14400.0	264.8k	214.0k	23.7	14400.0
belg+6arcs_89	585.7k	434.8k	34.7	14400.0	585.7k	577.3k	1.5	14400.0
belg+6arcs_90	314.7k	283.4k	11.1	14400.0	362.2k	257.9k	40.4	14400.0
belg+6arcs_91	318.0k	272.4k	16.7	14400.0	318.0k	246.9k	28.8	14400.0
belg+6arcs_92	332.1k	327.7k	1.3	14400.0	341.0k	276.0k	23.6	14400.0
belg+6arcs_93	440.6k	335.1k	31.5	14400.0	440.6k	336.8k	30.8	14400.0
belg+6arcs_94	188.2k	186.0k	1.1	14400.0	188.2k	175.3k	7.4	14400.0
belg+6arcs_95	42.1k	40.9k	3.1	14400.0	42.1k	42.1k	0.1	53.1
belg+6arcs_96	191.6k	146.4k	30.8	14400.0	191.6k	165.0k	16.1	14400.0
belg+6arcs_97	92.8k	43.2k	114.7	14400.0	92.8k	43.9k	111.3	14400.0
belg+6arcs_98	266.2k	181.6k	46.6	14400.0	225.2k	192.1k	17.2	14400.0
belg+6arcs_99	314.3k	244.2k	28.7	14400.0	314.3k	248.5k	26.5	14400.0
belg+6arcs_100	172.7k	168.7k	2.4	14400.0	172.7k	172.6k	0.1	685.9
belg+6arcs_101	213.9k	194.6k	9.9	14400.0	213.9k	179.2k	19.4	14400.0
belg+6arcs_102	214.4k	168.3k	27.4	14400.0	207.9k	207.6k	0.1	14400.0
belg+6arcs_103	140.1k	87.0k	61.0	14400.0	140.1k	65.9k	112.5	14400.0
belg+6arcs_104	207.6k	159.1k	30.5	14400.0	207.6k	190.8k	8.8	14400.0
belg+6arcs_105	87.5k	85.0k	3.0	14400.0	87.5k	87.4k	0.1	1809.4
belg+6arcs_106	196.3k	160.4k	22.4	14400.0	195.2k	178.6k	9.3	14400.0
belg+6arcs_107	205.6k	192.1k	7.0	14400.0	205.6k	205.4k	0.1	54.8
belg+6arcs_108	41.3k	23.6k	75.4	14400.0	41.3k	41.3k	0.1	101.5
belg+6arcs_109	183.0k	154.2k	18.6	14400.0	183.0k	163.6k	11.9	14400.0
belg+6arcs_110	206.6k	183.6k	12.5	14400.0	206.6k	203.0k	1.8	14400.0
belg+6arcs_111	350.0k	249.5k	40.3	14400.0	334.5k	265.9k	25.8	14400.0
belg+6arcs_112	131.0k	64.9k	101.9	14400.0	131.0k	54.4k	140.9	14400.0
belg+6arcs_113	2821.6	0.0	–	14400.0	2821.6	2819.9	0.1	1.5
belg+6arcs_114	3689.5	3685.8	0.1	2.4	3689.5	3688.3	0.0	2.1
belg+6arcs_115	360.5k	336.4k	7.2	14400.0	359.7k	345.9k	4.0	14400.0
belg+6arcs_116	173.9k	153.2k	13.5	14400.0	174.6k	132.0k	32.3	14400.0
belg+6arcs_117	108.3k	59.7k	81.4	14400.0	108.3k	49.9k	117.1	14400.0
belg+6arcs_118	153.8k	150.2k	2.4	14400.0	153.8k	153.7k	0.1	7.2
belg+6arcs_119	210.7k	189.1k	11.4	14400.0	210.7k	165.2k	27.5	14400.0
belg+6arcs_120	104.0k	88.3k	17.9	14400.0	104.0k	89.7k	16.0	14400.0

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Table A.30: Comparison of split-pipe models on *Circuit rank* + 6 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_121	129.7k	42.1k	208.1	14400.0	129.7k	129.6k	0.1	5.4
belg+6arcs_122	176.4k	169.1k	4.3	14400.0	176.4k	176.2k	0.1	1768.4
belg+6arcs_123	20.3k	18.9k	7.4	14400.0	20.3k	20.0k	1.3	14400.0
belg+6arcs_124	239.0k	191.3k	24.9	14400.0	239.0k	150.3k	59.0	14400.0
belg+6arcs_125	17.3k	15.8k	9.0	14400.0	17.3k	17.3k	0.1	2.3
belg+6arcs_126	110.0k	59.2k	86.0	14400.0	110.0k	107.2k	2.7	14400.0
belg+6arcs_127	187.6k	164.5k	14.0	14400.0	187.6k	168.1k	11.5	14400.0
belg+6arcs_128	184.0k	152.3k	20.8	14400.0	184.0k	155.9k	18.0	14400.0
belg+6arcs_129	30.4k	29.3k	3.8	14400.0	30.4k	29.5k	3.0	14400.0
belg+6arcs_130	176.0k	145.8k	20.7	14400.0	176.0k	176.0k	0.0	188.5
belg+6arcs_131	277.9k	184.0k	51.0	14400.0	257.7k	207.5k	24.2	14400.0
belg+6arcs_132	122.6k	122.2k	0.3	14400.0	122.6k	122.5k	0.1	8.4
belg+6arcs_133	319.6k	291.9k	9.5	14400.0	320.4k	245.2k	30.7	14400.0
belg+6arcs_134	374.1k	254.1k	47.2	14400.0	428.8k	245.6k	74.6	14400.0
belg+6arcs_135	161.6k	150.8k	7.2	14400.0	156.6k	155.0k	1.0	14400.0
belg+6arcs_136	680.7k	365.9k	86.0	14400.0	618.7k	509.7k	21.4	14400.0
belg+6arcs_137	490.7k	287.4k	70.7	14400.0	413.8k	266.0k	55.6	14400.0
belg+6arcs_138	190.6k	169.9k	12.1	14400.0	183.5k	183.3k	0.1	723.1
belg+6arcs_139	181.5k	173.0k	4.9	14400.0	181.5k	181.4k	0.1	1958.8
belg+6arcs_140	72.8k	56.6k	28.6	14400.0	72.8k	38.9k	87.2	14400.0
belg+6arcs_141	178.2k	171.9k	3.7	14400.0	178.2k	174.1k	2.4	14400.0
belg+6arcs_142	250.1k	239.5k	4.4	14400.0	250.1k	245.7k	1.8	14400.0
belg+6arcs_143	99.4k	99.3k	0.1	14400.0	99.4k	99.3k	0.1	43.9
belg+6arcs_144	50.1k	33.0k	51.6	14400.0	50.1k	50.1k	0.0	206.8
belg+6arcs_145	209.5k	103.6k	102.2	14400.0	209.5k	173.5k	20.8	14400.0
belg+6arcs_146	153.7k	104.2k	47.5	14400.0	153.7k	104.1k	47.6	14400.0
belg+6arcs_147	183.5k	183.3k	0.1	2166.7	183.5k	183.3k	0.1	4501.5
belg+6arcs_148	289.6k	237.4k	22.0	14400.0	289.6k	212.2k	36.5	14400.0
belg+6arcs_149	249.7k	208.4k	19.8	14400.0	249.0k	245.7k	1.3	14400.0
belg+6arcs_150	350.3k	279.5k	25.3	14400.0	350.3k	350.0k	0.1	285.8
belg+6arcs_151	54.9k	54.9k	0.1	645.3	54.9k	54.9k	0.1	7.2
belg+6arcs_152	200.2k	170.0k	17.8	14400.0	193.3k	184.1k	5.0	14400.0
belg+6arcs_153	224.1k	184.5k	21.5	14400.0	224.1k	184.7k	21.3	14400.0
belg+6arcs_154	189.8k	180.1k	5.4	14400.0	189.8k	189.6k	0.1	2615.0
belg+6arcs_155	292.9k	286.1k	2.4	14400.0	292.9k	262.8k	11.5	14400.0
belg+6arcs_156	158.6k	100.6k	57.7	14400.0	158.6k	154.0k	2.9	14400.0
belg+6arcs_157	159.6k	135.2k	18.0	14400.0	159.6k	138.7k	15.1	14400.0
belg+6arcs_158	78.0k	68.0k	14.6	14400.0	78.0k	68.8k	13.3	14400.0
belg+6arcs_159	194.3k	162.5k	19.5	14400.0	194.3k	153.3k	26.7	14400.0
belg+6arcs_160	30.2k	2501.2	1108.1	14400.0	30.2k	30.2k	0.0	12183.3
belg+6arcs_161	242.6k	204.5k	18.6	14400.0	242.5k	187.4k	29.4	14400.0
belg+6arcs_162	338.3k	183.1k	84.7	14400.0	322.2k	223.6k	44.1	14400.0
belg+6arcs_163	332.5k	241.8k	37.5	14400.0	322.1k	208.6k	54.4	14400.0
belg+6arcs_164	0.0	0.0	0.0	16.5	0.0	0.0	0.0	0.5
belg+6arcs_165	144.6k	144.5k	0.1	1135.4	144.6k	144.6k	0.0	27.1
belg+6arcs_166	4077.7	4075.6	0.1	3.0	4077.7	4073.9	0.1	2.2
belg+6arcs_167	223.0k	210.4k	6.0	14400.0	223.0k	196.5k	13.5	14400.0
belg+6arcs_168	395.4k	246.1k	60.6	14400.0	343.9k	235.7k	45.9	14400.0
belg+6arcs_169	142.4k	125.9k	13.1	14400.0	142.4k	141.3k	0.8	14400.0
belg+6arcs_170	171.2k	171.0k	0.1	781.9	171.2k	171.0k	0.1	17.9
belg+6arcs_171	231.2k	152.5k	51.6	14400.0	231.2k	151.4k	52.7	14400.0
belg+6arcs_172	61.6k	34.3k	79.7	14400.0	61.6k	7211.4	754.4	14400.0
belg+6arcs_173	311.3k	238.1k	30.7	14400.0	309.1k	286.8k	7.8	14400.0
belg+6arcs_174	141.1k	139.2k	1.4	14400.0	141.1k	141.0k	0.1	14.0
belg+6arcs_175	46.9k	43.1k	8.8	14400.0	46.9k	43.2k	8.4	14400.0
belg+6arcs_176	128.6k	126.5k	1.7	14400.0	128.6k	128.5k	0.1	2074.7
belg+6arcs_177	223.1k	218.9k	1.9	14400.0	223.1k	222.9k	0.1	7.9
belg+6arcs_178	186.9k	154.9k	20.7	14400.0	180.0k	169.9k	5.9	14400.0
belg+6arcs_179	241.9k	240.0k	0.8	14400.0	241.9k	219.2k	10.3	14400.0
belg+6arcs_180	40.5k	36.7k	10.4	14400.0	40.5k	40.3k	0.5	14400.0
belg+6arcs_181	30.5k	28.8k	6.0	14400.0	30.5k	24.6k	24.0	14400.0
belg+6arcs_182	129.2k	115.6k	11.8	14400.0	129.2k	119.3k	8.4	14400.0
belg+6arcs_183	145.2k	122.8k	18.3	14400.0	145.2k	126.5k	14.8	14400.0
belg+6arcs_184	61.1k	25.3k	141.3	14400.0	61.1k	58.2k	4.8	14400.0
belg+6arcs_185	97.0k	85.4k	13.6	14400.0	97.0k	96.9k	0.1	522.9
belg+6arcs_186	163.7k	163.4k	0.2	14400.0	163.7k	163.5k	0.1	3.9
belg+6arcs_187	442.7k	366.0k	21.0	14400.0	442.7k	368.4k	20.2	14400.0
belg+6arcs_188	13.3k	12.9k	2.8	14400.0	13.3k	13.2k	0.1	2.7
belg+6arcs_189	162.4k	160.3k	1.3	14400.0	162.4k	162.2k	0.1	17.7
belg+6arcs_190	190.9k	87.7k	117.7	14400.0	190.9k	138.2k	38.1	14400.0
belg+6arcs_191	253.5k	245.1k	3.5	14400.0	253.5k	253.3k	0.1	13.3
belg+6arcs_192	133.8k	99.0k	35.2	14400.0	133.8k	98.9k	35.3	14400.0
belg+6arcs_193	149.0k	147.0k	1.3	14400.0	149.0k	148.8k	0.1	145.1
belg+6arcs_194	289.3k	222.0k	30.4	14400.0	289.3k	222.5k	30.1	14400.0
belg+6arcs_195	183.5k	179.9k	2.0	14400.0	183.5k	183.3k	0.1	77.0
belg+6arcs_196	349.1k	318.7k	9.5	14400.0	349.1k	341.6k	2.2	14400.0
belg+6arcs_197	419.4k	273.3k	53.5	14400.0	390.3k	315.0k	23.9	14400.0
belg+6arcs_198	142.2k	139.7k	1.8	14400.0	142.2k	142.1k	0.1	5.6

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Table A.30: Comparison of split-pipe models on *Circuit rank* + 6 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_199	38.2k	38.1k	0.3	14400.0	38.2k	38.2k	0.1	11.6
belg+6arcs_200	232.9k	188.3k	23.7	14400.0	220.4k	201.5k	9.4	14400.0
belg+6arcs_201	97.3k	33.3k	192.6	14400.0	97.3k	43.2k	125.2	14400.0
belg+6arcs_202	146.0k	123.7k	18.1	14400.0	146.0k	146.0k	0.0	435.9
belg+6arcs_203	104.5k	13.6k	667.3	14400.0	104.5k	104.4k	0.1	14400.0
belg+6arcs_204	23.2k	23.1k	0.1	1.8	23.2k	23.1k	0.1	1.5
belg+6arcs_205	271.9k	216.5k	25.6	14400.0	271.9k	216.6k	25.5	14400.0
belg+6arcs_206	105.5k	94.9k	11.2	14400.0	101.2k	101.1k	0.1	2553.0
belg+6arcs_207	269.2k	203.0k	32.6	14400.0	266.0k	182.9k	45.4	14400.0
belg+6arcs_208	202.3k	200.9k	0.7	14400.0	202.3k	202.0k	0.1	14400.0
belg+6arcs_209	117.5k	101.8k	15.4	14400.0	117.5k	65.1k	80.6	14400.0
belg+6arcs_210	192.2k	184.8k	4.0	14400.0	192.2k	192.0k	0.1	1490.9
belg+6arcs_211	199.1k	96.1k	107.1	14400.0	199.1k	198.9k	0.1	2111.1
belg+6arcs_212	104.3k	76.6k	36.1	14400.0	104.3k	104.2k	0.1	341.3
belg+6arcs_213	392.0k	288.5k	35.9	14400.0	346.6k	280.2k	23.7	14400.0
belg+6arcs_214	229.3k	214.9k	6.7	14400.0	228.8k	225.4k	1.5	14400.0
belg+6arcs_215	177.6k	167.4k	6.1	14400.0	177.6k	174.9k	1.5	14400.0
belg+6arcs_216	224.2k	223.3k	0.4	14400.0	224.2k	221.1k	1.4	14400.0
belg+6arcs_217	133.4k	130.7k	2.1	14400.0	133.4k	133.2k	0.1	14.1
belg+6arcs_218	334.8k	258.5k	29.5	14400.0	308.5k	293.1k	5.2	14400.0
belg+6arcs_219	206.6k	202.6k	2.0	14400.0	206.6k	206.5k	0.1	19.0
belg+6arcs_220	310.2k	218.8k	41.8	14400.0	310.2k	174.0k	78.3	14400.0
belg+6arcs_221	20.6k	18.1k	13.7	14400.0	20.6k	20.4k	1.1	14400.0
belg+6arcs_222	226.4k	210.1k	7.8	14400.0	238.0k	210.6k	13.0	14400.0
belg+6arcs_223	222.6k	148.9k	49.4	14400.0	197.6k	151.5k	30.4	14400.0
belg+6arcs_224	273.9k	247.0k	10.9	14400.0	273.9k	207.4k	32.1	14400.0
belg+6arcs_225	3267.4	2862.4	14.1	14400.0	3267.4	3264.6	0.1	1.7
belg+6arcs_226	177.2k	158.5k	11.8	14400.0	177.2k	171.4k	3.4	14400.0
belg+6arcs_227	28.1k	14.0k	100.3	14400.0	28.1k	28.1k	0.0	118.2
belg+6arcs_228	297.5k	239.9k	24.0	14400.0	297.5k	204.1k	45.7	14400.0
belg+6arcs_229	204.0k	176.9k	15.3	14400.0	204.0k	203.8k	0.1	2005.1
belg+6arcs_230	184.2k	182.8k	0.8	14400.0	184.2k	184.0k	0.1	75.7
belg+6arcs_231	26.2k	23.6k	11.1	14400.0	26.2k	26.1k	0.1	4.4
belg+6arcs_232	150.5k	137.0k	9.8	14400.0	146.2k	146.0k	0.1	93.7
belg+6arcs_233	252.9k	250.0k	1.2	14400.0	260.4k	222.0k	17.3	14400.0
belg+6arcs_234	53.7k	40.5k	32.3	14400.0	53.7k	53.6k	0.1	98.0
belg+6arcs_235	174.8k	169.6k	3.1	14400.0	174.8k	169.6k	3.1	14400.0
belg+6arcs_236	194.9k	190.2k	2.5	14400.0	194.9k	168.3k	15.8	14400.0
belg+6arcs_237	2771.1	2565.1	8.0	14400.0	2771.1	2768.6	0.1	2.3
belg+6arcs_238	343.0k	309.5k	10.8	14400.0	382.8k	260.2k	47.1	14400.0
belg+6arcs_239	252.9k	179.7k	40.7	14400.0	252.9k	177.3k	42.6	14400.0
belg+6arcs_240	322.2k	262.3k	22.8	14400.0	289.0k	259.1k	11.5	14400.0
belg+6arcs_241	268.0k	183.6k	46.0	14400.0	268.0k	208.5k	28.5	14400.0
belg+6arcs_242	196.2k	133.7k	46.7	14400.0	196.2k	154.1k	27.3	14400.0
belg+6arcs_243	182.9k	182.6k	0.2	14400.0	182.9k	182.9k	0.0	10.8
belg+6arcs_244	197.4k	192.6k	2.5	14400.0	197.4k	197.3k	0.1	339.6
belg+6arcs_245	39.6k	19.9k	99.2	14400.0	39.5k	39.5k	0.0	58.5
belg+6arcs_246	177.5k	168.1k	5.6	14400.0	176.4k	175.7k	0.4	14400.0
belg+6arcs_247	48.6k	15.9k	205.6	14400.0	48.6k	48.5k	0.3	14400.0
belg+6arcs_248	378.1k	322.7k	17.2	14400.0	378.1k	345.3k	9.5	14400.0
belg+6arcs_249	95.3k	35.8k	166.4	14400.0	95.3k	62.3k	53.0	14400.0
belg+6arcs_250	278.3k	189.1k	47.2	14400.0	278.3k	170.3k	63.5	14400.0
belg+6arcs_251	196.3k	158.1k	24.2	14400.0	192.2k	179.7k	7.0	14400.0
belg+6arcs_252	162.0k	129.6k	25.0	14400.0	162.0k	162.0k	0.0	1856.5
belg+6arcs_253	314.5k	286.3k	9.8	14400.0	312.3k	229.0k	36.4	14400.0
belg+6arcs_254	271.2k	268.5k	1.0	14400.0	271.2k	249.9k	8.5	14400.0
belg+6arcs_255	22.6k	22.5k	0.5	14400.0	22.6k	22.6k	0.1	3.6
belg+6arcs_256	182.6k	177.4k	2.9	14400.0	182.6k	182.4k	0.1	28.7
belg+6arcs_257	135.5k	87.6k	54.7	14400.0	132.5k	34.7k	281.2	14400.0
belg+6arcs_258	171.4k	162.5k	5.5	14400.0	171.4k	171.2k	0.1	656.3
belg+6arcs_259	171.6k	165.1k	3.9	14400.0	171.6k	171.4k	0.1	1017.1
belg+6arcs_260	35.5k	28.9k	22.7	14400.0	35.5k	35.5k	0.1	1.3
belg+6arcs_261	17.9k	17.0k	5.0	14400.0	17.9k	17.9k	0.1	3.9
belg+6arcs_262	18.0k	18.0k	0.1	482.5	18.0k	18.0k	0.1	2.6
belg+6arcs_263	284.1k	260.5k	9.1	14400.0	284.1k	224.2k	26.7	14400.0
belg+6arcs_264	186.9k	150.7k	24.0	14400.0	186.4k	172.1k	8.3	14400.0
belg+6arcs_265	438.9k	299.8k	46.4	14400.0	430.9k	320.1k	34.6	14400.0
belg+6arcs_266	4695.9	4691.2	0.1	476.1	4695.9	4693.0	0.1	1.3
belg+6arcs_267	330.4k	249.9k	32.2	14400.0	330.4k	319.2k	3.5	14400.0
belg+6arcs_268	245.3k	217.0k	13.0	14400.0	245.3k	218.9k	12.1	14400.0
belg+6arcs_269	270.9k	236.8k	14.4	14400.0	270.9k	252.4k	7.3	14400.0
belg+6arcs_270	197.5k	197.3k	0.1	14400.0	197.5k	197.4k	0.1	14.2
belg+6arcs_271	198.3k	163.1k	21.5	14400.0	188.7k	175.9k	7.3	14400.0
belg+6arcs_272	285.4k	242.9k	17.5	14400.0	285.4k	243.8k	17.1	14400.0
belg+6arcs_273	374.4k	296.8k	26.1	14400.0	339.5k	242.7k	39.9	14400.0
belg+6arcs_274	136.0k	133.5k	1.9	14400.0	136.0k	135.6k	0.3	14400.0
belg+6arcs_275	18.3k	18.3k	0.1	2.0	18.3k	18.3k	0.1	17.6
belg+6arcs_276	70.9k	44.4k	59.9	14400.0	70.9k	48.5k	46.3	14400.0

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Table A.30: Comparison of split-pipe models on *Circuit rank* + 6 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_277	93.1k	69.1k	34.8	14400.0	93.1k	92.3k	0.9	14400.0
belg+6arcs_278	10.3k	10.3k	0.0	6.6	10.3k	10.3k	0.1	2.9
belg+6arcs_279	69.3k	64.8k	7.0	14400.0	69.3k	65.0k	6.6	14400.0
belg+6arcs_280	252.9k	222.0k	13.9	14400.0	252.9k	246.0k	2.8	14400.0
belg+6arcs_281	204.8k	161.4k	26.9	14400.0	204.8k	142.7k	43.5	14400.0
belg+6arcs_282	285.9k	284.4k	0.5	14400.0	285.9k	285.4k	0.2	14400.0
belg+6arcs_283	221.4k	217.8k	1.7	14400.0	221.4k	220.2k	0.6	14400.0
belg+6arcs_284	195.6k	194.1k	0.8	14400.0	195.6k	172.8k	13.2	14400.0
belg+6arcs_285	147.7k	128.9k	14.5	14400.0	147.7k	147.5k	0.1	3684.8
belg+6arcs_286	203.9k	203.1k	0.4	14400.0	203.9k	203.6k	0.1	14400.0
belg+6arcs_287	215.8k	207.0k	4.3	14400.0	215.8k	211.2k	2.2	14400.0
belg+6arcs_288	171.8k	164.3k	4.5	14400.0	171.8k	171.6k	0.1	92.2
belg+6arcs_289	207.2k	190.3k	8.9	14400.0	206.2k	137.2k	50.3	14400.0
belg+6arcs_290	259.7k	157.0k	65.4	14400.0	243.6k	207.7k	17.3	14400.0
belg+6arcs_291	310.5k	219.8k	41.3	14400.0	302.5k	224.9k	34.5	14400.0
belg+6arcs_292	111.0k	44.1k	151.7	14400.0	111.0k	40.9k	171.2	14400.0
belg+6arcs_293	62.7k	16.7k	274.8	14400.0	62.7k	58.6k	7.0	14400.0
belg+6arcs_294	151.0k	150.9k	0.1	9043.3	151.0k	150.9k	0.1	12.9
belg+6arcs_295	208.1k	196.8k	5.7	14400.0	208.1k	204.4k	1.8	14400.0
belg+6arcs_296	153.8k	51.7k	197.6	14400.0	153.8k	153.6k	0.1	2167.8
belg+6arcs_297	113.9k	107.4k	6.1	14400.0	113.9k	113.9k	0.1	11.9
belg+6arcs_298	85.1k	25.5k	233.9	14400.0	85.1k	77.9k	9.2	14400.0
belg+6arcs_299	450.9k	368.0k	22.6	14400.0	479.7k	374.3k	28.1	14400.0
belg+6arcs_300	153.6k	145.9k	5.3	14400.0	153.6k	153.4k	0.1	5422.1
belg+6arcs_301	159.6k	134.5k	18.6	14400.0	159.6k	140.3k	13.8	14400.0
belg+6arcs_302	237.4k	207.3k	14.5	14400.0	237.4k	237.1k	0.1	11452.4
belg+6arcs_303	309.4k	304.0k	1.8	14400.0	321.1k	243.8k	31.7	14400.0
belg+6arcs_304	236.3k	236.1k	0.1	4164.3	236.3k	184.4k	28.1	14400.0
belg+6arcs_305	246.8k	189.2k	30.4	14400.0	240.1k	229.9k	4.4	14400.0
belg+6arcs_306	198.8k	192.9k	3.0	14400.0	198.8k	198.6k	0.1	26.6
belg+6arcs_307	89.6k	74.3k	20.6	14400.0	89.6k	79.8k	12.3	14400.0
belg+6arcs_308	40.0k	28.7k	39.2	14400.0	40.0k	40.0k	0.1	54.8
belg+6arcs_309	383.3k	117.1k	227.4	14400.0	334.7k	316.5k	5.8	14400.0
belg+6arcs_310	25.7k	25.4k	1.2	14400.0	25.7k	25.7k	0.1	236.4
belg+6arcs_311	143.1k	123.0k	16.3	14400.0	143.1k	132.5k	8.0	14400.0
belg+6arcs_312	101.6k	101.5k	0.1	14400.0	101.6k	101.5k	0.1	1343.4
belg+6arcs_313	273.5k	162.7k	68.1	14400.0	273.5k	185.2k	47.6	14400.0
belg+6arcs_314	499.4k	378.3k	32.0	14400.0	523.6k	372.5k	40.6	14400.0
belg+6arcs_315	244.5k	211.2k	15.8	14400.0	251.7k	175.7k	43.2	14400.0
belg+6arcs_316	147.3k	115.0k	28.0	14400.0	147.3k	118.1k	24.7	14400.0
belg+6arcs_317	219.4k	205.1k	7.0	14400.0	219.4k	182.3k	20.3	14400.0
belg+6arcs_318	93.0k	84.3k	10.3	14400.0	93.0k	84.0k	10.7	14400.0
belg+6arcs_319	144.3k	140.6k	2.6	14400.0	144.3k	143.9k	0.3	14400.0
belg+6arcs_320	171.8k	171.3k	0.3	14400.0	171.8k	171.6k	0.1	14400.0
belg+6arcs_321	153.3k	144.0k	6.5	14400.0	149.2k	141.9k	5.2	14400.0
belg+6arcs_322	161.7k	87.3k	85.1	14400.0	161.7k	136.3k	18.6	14400.0
belg+6arcs_323	118.2k	99.8k	18.4	14400.0	118.2k	118.2k	0.0	135.9
belg+6arcs_324	83.6k	73.2k	14.2	14400.0	83.6k	73.2k	14.3	14400.0
belg+6arcs_325	240.1k	221.0k	8.6	14400.0	240.1k	228.8k	5.0	14400.0
belg+6arcs_326	279.1k	168.4k	65.7	14400.0	244.3k	168.2k	45.2	14400.0
belg+6arcs_327	26.0k	18.2k	42.4	14400.0	26.0k	25.9k	0.1	7.4
belg+6arcs_328	5372.2	5368.5	0.1	3.1	5372.2	5366.9	0.1	1.6
belg+6arcs_329	170.7k	158.2k	7.9	14400.0	170.7k	167.7k	1.7	14400.0
belg+6arcs_330	211.2k	207.9k	1.6	14400.0	211.2k	211.0k	0.1	32.2
belg+6arcs_331	134.3k	119.7k	12.2	14400.0	134.3k	134.2k	0.1	839.8
belg+6arcs_332	445.4k	405.5k	9.8	14400.0	439.1k	349.7k	25.5	14400.0
belg+6arcs_333	100.9k	93.9k	7.5	14400.0	100.9k	98.7k	2.2	14400.0
belg+6arcs_334	272.3k	267.2k	1.9	14400.0	278.4k	255.6k	8.9	14400.0
belg+6arcs_335	260.9k	205.5k	27.0	14400.0	250.1k	238.6k	4.8	14400.0
belg+6arcs_336	166.9k	154.5k	8.0	14400.0	166.9k	165.5k	0.8	14400.0
belg+6arcs_337	80.0k	25.0k	219.5	14400.0	80.0k	57.0k	40.5	14400.0
belg+6arcs_338	180.7k	139.7k	29.4	14400.0	181.1k	141.6k	27.9	14400.0
belg+6arcs_339	321.4k	320.5k	0.3	14400.0	345.6k	259.3k	33.3	14400.0
belg+6arcs_340	353.9k	326.4k	8.4	14400.0	385.2k	243.7k	58.1	14400.0
belg+6arcs_341	162.4k	155.4k	4.5	14400.0	162.4k	162.2k	0.1	76.8
belg+6arcs_342	309.7k	232.6k	33.2	14400.0	309.7k	288.5k	7.4	14400.0
belg+6arcs_343	179.7k	146.4k	22.8	14400.0	171.8k	162.1k	6.0	14400.0
belg+6arcs_344	130.3k	130.2k	0.1	7016.3	130.3k	127.9k	1.9	14400.0
belg+6arcs_345	187.0k	174.2k	7.3	14400.0	187.0k	186.8k	0.1	48.2
belg+6arcs_346	193.9k	192.7k	0.6	14400.0	193.9k	182.1k	6.5	14400.0
belg+6arcs_347	365.7k	256.6k	42.5	14400.0	397.4k	298.8k	33.0	14400.0
belg+6arcs_348	124.7k	124.5k	0.2	14400.0	124.7k	124.6k	0.1	561.8
belg+6arcs_349	74.8k	52.1k	43.7	14400.0	74.8k	55.9k	33.7	14400.0
belg+6arcs_350	51.8k	51.8k	0.1	367.8	51.8k	48.8k	6.1	14400.0
belg+6arcs_351	341.4k	313.8k	8.8	14400.0	341.4k	301.3k	13.3	14400.0
belg+6arcs_352	292.2k	282.7k	3.3	14400.0	292.2k	235.9k	23.9	14400.0
belg+6arcs_353	140.3k	135.8k	3.4	14400.0	140.3k	123.5k	13.7	14400.0
belg+6arcs_354	108.2k	79.6k	35.9	14400.0	108.2k	96.3k	12.3	14400.0

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Table A.30: Comparison of split-pipe models on *Circuit rank* + 6 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_355	35.0k	27.3k	28.5	14400.0	30.9k	30.9k	0.0	12.1
belg+6arcs_356	310.7k	214.0k	45.2	14400.0	310.7k	211.4k	47.0	14400.0
belg+6arcs_357	223.5k	182.5k	22.5	14400.0	216.8k	164.9k	31.5	14400.0
belg+6arcs_358	401.9k	236.3k	70.1	14400.0	348.6k	270.1k	29.1	14400.0
belg+6arcs_359	220.6k	188.7k	16.9	14400.0	220.6k	207.9k	6.1	14400.0
belg+6arcs_360	138.3k	103.7k	33.4	14400.0	138.3k	138.3k	0.0	2533.5
belg+6arcs_361	470.4k	386.6k	21.7	14400.0	424.0k	284.3k	49.2	14400.0
belg+6arcs_362	68.1k	20.7k	229.6	14400.0	68.1k	68.0k	0.1	5980.1
belg+6arcs_363	174.7k	172.0k	1.6	14400.0	174.7k	174.5k	0.1	3107.0
belg+6arcs_364	181.7k	58.5k	210.7	14400.0	181.7k	113.0k	60.7	14400.0
belg+6arcs_365	205.8k	137.0k	50.2	14400.0	204.1k	194.8k	4.8	14400.0
belg+6arcs_366	235.1k	209.0k	12.5	14400.0	235.1k	227.4k	3.4	14400.0
belg+6arcs_367	180.5k	180.4k	0.1	10214.7	180.5k	180.4k	0.1	10.1
belg+6arcs_368	87.6k	22.2k	295.3	14400.0	87.6k	57.1k	53.4	14400.0
belg+6arcs_369	4302.8	4119.8	4.4	14400.0	4302.8	4299.6	0.1	2.8
belg+6arcs_370	155.9k	150.6k	3.5	14400.0	155.9k	155.4k	0.3	14400.0
belg+6arcs_371	265.7k	250.3k	6.1	14400.0	265.7k	235.9k	12.6	14400.0
belg+6arcs_372	212.3k	143.9k	47.5	14400.0	212.3k	212.1k	0.1	4755.0
belg+6arcs_373	202.8k	159.5k	27.1	14400.0	184.0k	159.6k	15.3	14400.0
belg+6arcs_374	77.9k	69.0k	12.8	14400.0	77.9k	72.8k	6.9	14400.0
belg+6arcs_375	45.1k	41.1k	9.6	14400.0	45.1k	45.1k	0.1	6.4
belg+6arcs_376	389.1k	383.2k	1.5	14400.0	387.4k	385.6k	0.5	14400.0
belg+6arcs_377	219.8k	142.2k	54.5	14400.0	219.8k	149.9k	46.7	14400.0
belg+6arcs_378	106.4k	64.0k	66.4	14400.0	106.4k	77.0k	38.2	14400.0
belg+6arcs_379	330.3k	308.0k	7.2	14400.0	330.3k	262.3k	25.9	14400.0
belg+6arcs_380	180.2k	156.8k	14.9	14400.0	170.1k	169.9k	0.1	4241.5
belg+6arcs_381	350.6k	189.9k	84.7	14400.0	350.6k	250.1k	40.2	14400.0
belg+6arcs_382	242.7k	231.8k	4.7	14400.0	242.7k	210.4k	15.3	14400.0
belg+6arcs_383	58.8k	52.0k	13.0	14400.0	58.8k	58.8k	0.1	277.5
belg+6arcs_384	232.5k	105.7k	119.9	14400.0	232.5k	92.7k	150.7	14400.0
belg+6arcs_385	237.6k	173.1k	37.3	14400.0	237.5k	176.4k	34.6	14400.0
belg+6arcs_386	374.6k	292.4k	28.1	14400.0	374.6k	283.9k	32.0	14400.0
belg+6arcs_387	124.3k	78.6k	58.1	14400.0	124.3k	65.3k	90.4	14400.0
belg+6arcs_388	70.9k	40.7k	74.3	14400.0	70.9k	15.5k	358.4	14400.0
belg+6arcs_389	151.9k	133.0k	14.2	14400.0	149.0k	147.4k	1.1	14400.0
belg+6arcs_390	246.6k	186.8k	32.0	14400.0	246.6k	204.8k	20.4	14400.0
belg+6arcs_391	181.2k	176.6k	2.6	14400.0	181.2k	177.5k	2.1	14400.0
belg+6arcs_392	97.1k	81.8k	18.8	14400.0	97.1k	92.5k	5.1	14400.0
belg+6arcs_393	197.2k	195.5k	0.9	14400.0	197.2k	197.0k	0.1	42.0
belg+6arcs_394	112.1k	83.8k	33.8	14400.0	112.1k	23.9k	368.8	14400.0
belg+6arcs_395	262.6k	219.7k	19.5	14400.0	262.6k	199.4k	31.7	14400.0
belg+6arcs_396	75.7k	65.7k	15.3	14400.0	75.7k	75.7k	0.0	3333.2
belg+6arcs_397	379.2k	298.7k	26.9	14400.0	364.9k	341.0k	7.0	14400.0
belg+6arcs_398	25.4k	24.3k	4.7	14400.0	25.4k	25.4k	0.1	2388.0
belg+6arcs_399	4857.5	4855.5	0.0	1.2	4857.5	4853.2	0.1	1.3
belg+6arcs_400	86.9k	29.8k	192.0	14400.0	86.9k	26.0k	233.9	14400.0
belg+6arcs_401	137.1k	128.6k	6.6	14400.0	137.1k	136.9k	0.1	1316.0
belg+6arcs_402	152.7k	145.7k	4.9	14400.0	152.7k	152.6k	0.1	1158.9
belg+6arcs_403	262.9k	256.8k	2.4	14400.0	262.9k	262.6k	0.1	1038.7
belg+6arcs_404	249.5k	209.1k	19.3	14400.0	249.5k	232.9k	7.1	14400.0
belg+6arcs_405	337.0k	320.6k	5.1	14400.0	329.2k	327.1k	0.7	14400.0
belg+6arcs_406	68.0k	58.3k	16.6	14400.0	68.0k	68.0k	0.0	96.8
belg+6arcs_407	232.3k	219.4k	5.9	14400.0	232.3k	230.2k	0.9	14400.0
belg+6arcs_408	29.6k	28.9k	2.2	14400.0	29.6k	29.5k	0.1	4816.2
belg+6arcs_409	143.3k	132.9k	7.8	14400.0	143.3k	143.1k	0.1	80.9
belg+6arcs_410	176.3k	170.9k	3.2	14400.0	176.3k	176.2k	0.1	100.5
belg+6arcs_411	220.1k	163.6k	34.5	14400.0	200.9k	164.1k	22.5	14400.0
belg+6arcs_412	11.8k	11.4k	4.0	14400.0	11.8k	11.8k	0.1	2.5
belg+6arcs_413	176.8k	159.5k	10.9	14400.0	171.1k	171.0k	0.1	404.1
belg+6arcs_414	152.0k	151.2k	0.5	14400.0	152.0k	151.8k	0.1	45.1
belg+6arcs_415	21.9k	0.0	–	14400.0	21.9k	21.9k	0.1	17.2
belg+6arcs_416	188.7k	174.7k	8.0	14400.0	188.7k	179.5k	5.1	14400.0
belg+6arcs_417	155.5k	141.3k	10.1	14400.0	155.5k	152.1k	2.2	14400.0
belg+6arcs_418	432.9k	276.7k	56.5	14400.0	426.4k	328.7k	29.7	14400.0
belg+6arcs_419	143.5k	79.1k	81.3	14400.0	143.5k	94.9k	51.2	14400.0
belg+6arcs_420	135.7k	121.1k	12.1	14400.0	135.7k	120.9k	12.3	14400.0
belg+6arcs_421	109.5k	43.8k	150.0	14400.0	109.5k	52.5k	108.7	14400.0
belg+6arcs_422	126.5k	106.5k	18.8	14400.0	126.5k	101.8k	24.2	14400.0
belg+6arcs_423	173.0k	163.1k	6.1	14400.0	173.0k	171.2k	1.0	14400.0
belg+6arcs_424	316.8k	234.3k	35.2	14400.0	316.8k	237.5k	33.4	14400.0
belg+6arcs_425	220.3k	208.6k	5.6	14400.0	219.8k	219.6k	0.1	769.3
belg+6arcs_426	230.4k	105.2k	119.0	14400.0	230.4k	141.7k	62.6	14400.0
belg+6arcs_427	134.5k	131.0k	2.7	14400.0	134.5k	134.4k	0.1	74.1
belg+6arcs_428	78.0k	40.3k	93.3	14400.0	78.0k	36.4k	114.1	14400.0
belg+6arcs_429	322.8k	297.4k	8.5	14400.0	322.8k	264.8k	21.9	14400.0
belg+6arcs_430	291.5k	216.4k	34.7	14400.0	276.8k	219.1k	26.3	14400.0
belg+6arcs_431	514.4k	412.8k	24.6	14400.0	514.4k	419.6k	22.6	14400.0
belg+6arcs_432	4005.1	4001.1	0.1	11409.1	4005.1	4003.0	0.1	1.7

continued on next page

Table A.30: Comparison of split-pipe models on *Circuit rank* + 6 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+6arcs_433	27.1k	25.3k	7.2	14400.0	27.1k	27.1k	0.1	44.9
belg+6arcs_434	230.9k	181.3k	27.3	14400.0	218.4k	177.7k	22.9	14400.0
belg+6arcs_435	246.1k	185.2k	32.9	14400.0	246.1k	184.8k	33.1	14400.0
belg+6arcs_436	206.2k	202.7k	1.7	14400.0	206.2k	206.1k	0.1	1484.2
belg+6arcs_437	126.3k	44.9k	181.1	14400.0	126.1k	115.3k	9.4	14400.0
belg+6arcs_438	228.6k	182.6k	25.2	14400.0	228.6k	182.8k	25.1	14400.0
belg+6arcs_439	341.3k	253.3k	34.7	14400.0	338.5k	262.1k	29.2	14400.0
belg+6arcs_440	340.2k	280.5k	21.3	14400.0	343.1k	275.5k	24.5	14400.0
belg+6arcs_441	195.3k	147.4k	32.5	14400.0	195.6k	146.4k	33.6	14400.0
belg+6arcs_442	3013.6	2831.9	6.4	14400.0	3013.6	3012.0	0.1	1.9
belg+6arcs_443	120.6k	106.4k	13.3	14400.0	120.6k	119.7k	0.8	14400.0
belg+6arcs_444	49.9k	46.3k	7.9	14400.0	49.9k	49.9k	0.0	110.3
belg+6arcs_445	395.4k	259.4k	52.4	14400.0	394.8k	241.6k	63.4	14400.0
belg+6arcs_446	152.8k	141.3k	8.1	14400.0	152.8k	144.8k	5.5	14400.0
belg+6arcs_447	222.9k	156.3k	42.6	14400.0	196.1k	157.2k	24.7	14400.0
belg+6arcs_448	291.2k	221.1k	31.7	14400.0	273.7k	248.1k	10.3	14400.0
belg+6arcs_449	142.9k	134.8k	5.9	14400.0	142.9k	142.9k	0.0	807.1
belg+6arcs_450	64.9k	47.6k	36.4	14400.0	64.9k	53.7k	20.9	14400.0
belg+6arcs_451	33.1k	31.2k	5.9	14400.0	33.1k	33.0k	0.1	2459.4
belg+6arcs_452	213.0k	204.4k	4.2	14400.0	213.0k	165.6k	28.6	14400.0
belg+6arcs_453	264.8k	259.9k	1.9	14400.0	264.8k	264.5k	0.1	48.6
belg+6arcs_454	52.8k	22.4k	135.3	14400.0	52.8k	46.3k	14.0	14400.0
belg+6arcs_455	33.5k	24.1k	39.1	14400.0	33.5k	33.4k	0.1	2287.8
belg+6arcs_456	345.5k	253.7k	36.2	14400.0	345.5k	293.4k	17.7	14400.0
belg+6arcs_457	77.6k	66.7k	16.3	14400.0	77.6k	77.6k	0.0	7.7
belg+6arcs_458	257.4k	233.3k	10.3	14400.0	257.4k	252.1k	2.1	14400.0
belg+6arcs_459	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.3
belg+6arcs_460	3552.7	3552.7	0.0	3979.4	3552.7	3551.7	0.0	2.5
belg+6arcs_461	217.1k	186.7k	16.3	14400.0	214.6k	193.4k	11.0	14400.0
belg+6arcs_462	169.3k	153.6k	10.2	14400.0	169.3k	162.9k	3.9	14400.0
belg+6arcs_463	131.1k	129.9k	0.9	14400.0	131.1k	131.1k	0.0	4.7
belg+6arcs_464	148.9k	126.9k	17.4	14400.0	148.9k	148.8k	0.1	1589.5
belg+6arcs_465	5014.0	4740.9	5.8	14400.0	5014.0	5013.6	0.0	2.1
belg+6arcs_466	62.7k	61.8k	1.5	14400.0	62.7k	62.7k	0.1	79.6
belg+6arcs_467	185.1k	169.1k	9.4	14400.0	185.1k	184.9k	0.1	134.0
belg+6arcs_468	264.1k	197.3k	33.9	14400.0	264.1k	189.2k	39.6	14400.0
belg+6arcs_469	117.4k	81.8k	43.6	14400.0	117.4k	103.4k	13.5	14400.0
belg+6arcs_470	30.4k	19.1k	58.8	14400.0	30.4k	30.4k	0.1	13.6
belg+6arcs_471	178.0k	174.4k	2.1	14400.0	178.0k	176.8k	0.7	14400.0
belg+6arcs_472	98.7k	4483.2	2101.1	14400.0	98.7k	33.5k	194.8	14400.0
belg+6arcs_473	254.5k	250.9k	1.4	14400.0	254.5k	250.7k	1.5	14400.0
belg+6arcs_474	83.6k	64.5k	29.5	14400.0	83.6k	79.0k	5.7	14400.0
belg+6arcs_475	122.7k	119.0k	3.1	14400.0	122.7k	122.6k	0.1	10.7
belg+6arcs_476	250.3k	171.6k	45.8	14400.0	250.3k	244.1k	2.5	14400.0
belg+6arcs_477	249.0k	222.1k	12.1	14400.0	246.3k	221.9k	11.0	14400.0
belg+6arcs_478	247.7k	172.6k	43.6	14400.0	247.7k	193.0k	28.4	14400.0
belg+6arcs_479	165.5k	155.2k	6.7	14400.0	165.5k	165.2k	0.2	14400.0
belg+6arcs_480	154.9k	123.3k	25.6	14400.0	154.7k	121.3k	27.5	14400.0
belg+6arcs_481	321.7k	265.4k	21.2	14400.0	317.4k	213.7k	48.5	14400.0
belg+6arcs_482	187.7k	168.0k	11.7	14400.0	182.2k	175.8k	3.7	14400.0
belg+6arcs_483	30.6k	30.5k	0.2	14400.0	30.6k	30.6k	0.1	133.7
belg+6arcs_484	55.0k	55.0k	0.1	992.9	55.0k	32.2k	70.9	14400.0
belg+6arcs_485	57.4k	51.0k	12.6	14400.0	57.0k	57.0k	0.1	64.1
belg+6arcs_486	281.6k	243.2k	15.8	14400.0	281.6k	224.9k	25.2	14400.0
belg+6arcs_487	122.5k	118.9k	3.0	14400.0	122.5k	122.4k	0.1	14.6
belg+6arcs_488	224.0k	181.7k	23.3	14400.0	205.2k	205.0k	0.1	242.1
belg+6arcs_489	216.2k	195.1k	10.8	14400.0	216.2k	140.2k	54.2	14400.0
belg+6arcs_490	114.5k	86.6k	32.1	14400.0	114.5k	114.5k	0.0	25.5
belg+6arcs_491	89.3k	85.0k	5.0	14400.0	89.3k	89.0k	0.3	14400.0
belg+6arcs_492	109.1k	102.7k	6.3	14400.0	109.1k	108.8k	0.3	14400.0
belg+6arcs_493	277.4k	199.3k	39.1	14400.0	262.3k	198.7k	32.0	14400.0
belg+6arcs_494	194.7k	188.0k	3.5	14400.0	194.7k	192.3k	1.2	14400.0
belg+6arcs_495	181.2k	128.4k	41.2	14400.0	163.9k	127.7k	28.3	14400.0
belg+6arcs_496	141.8k	132.9k	6.7	14400.0	141.8k	94.4k	50.2	14400.0
belg+6arcs_497	1e+20	0.0	–	14400.0	66.9k	52.6k	27.3	14400.0
belg+6arcs_498	316.1k	230.6k	37.0	14400.0	281.8k	230.9k	22.0	14400.0
belg+6arcs_499	200.5k	192.1k	4.4	14400.0	199.1k	192.9k	3.2	14400.0
belg+6arcs_500	151.4k	146.8k	3.1	14400.0	151.4k	151.2k	0.1	1742.2

Table A.31: Detailed results of the discrete models on *Circuit rank* + 8 arcs, as summarized in Figure 3.10e and Table 3.3d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_1	436.3k	254.0k	71.8	14400.0	1e+20	265.1k	–	14400.0	375.1k	212.7k	76.4	14400.0
belg+8arcs_2	151.7k	151.7k	0.0	287.6	151.7k	151.7k	0.0	5320.2	151.7k	151.7k	0.0	2139.0
belg+8arcs_3	118.9k	98.4k	20.8	14400.0	1e+20	61.7k	–	14400.0	166.3k	30.6k	443.9	14400.0
belg+8arcs_4	213.1k	213.1k	0.0	1120.3	213.1k	213.1k	0.0	6414.3	213.1k	213.1k	0.0	5341.9
belg+8arcs_5	475.5k	201.6k	135.8	14400.0	748.2k	247.0k	202.9	14400.0	350.8k	190.3k	84.4	14400.0
belg+8arcs_6	203.5k	203.5k	0.0	2000.2	203.5k	183.6k	10.8	14400.0	203.5k	203.5k	0.0	13304.0
belg+8arcs_7	286.1k	257.6k	11.1	14400.0	286.1k	286.1k	0.0	9144.5	325.2k	239.5k	35.8	14400.0
belg+8arcs_8	11.4k	11.4k	0.0	174.4	11.4k	11.4k	0.0	13019.7	11.4k	11.4k	0.0	346.2
belg+8arcs_9	306.5k	149.2k	105.5	14400.0	1e+20	123.9k	–	14400.0	291.0k	51.6k	464.4	14400.0
belg+8arcs_10	195.4k	195.4k	0.0	956.1	195.4k	195.4k	0.0	2617.9	195.4k	195.4k	0.0	3192.0
belg+8arcs_11	235.2k	202.0k	16.5	14400.0	1e+20	194.8k	–	14400.0	240.3k	202.5k	18.7	14400.0
belg+8arcs_12	44.9k	44.9k	0.0	348.9	44.9k	44.9k	0.0	2237.2	44.9k	44.9k	0.0	3875.3
belg+8arcs_13	214.6k	207.6k	3.4	14400.0	1e+20	180.8k	–	14400.0	212.2k	212.2k	0.0	390.3
belg+8arcs_14	478.9k	305.8k	56.6	14400.0	571.3k	327.8k	74.3	14400.0	469.4k	263.1k	78.4	14400.0
belg+8arcs_15	2976.0	2976.0	0.0	152.6	2976.0	2976.0	0.0	1878.9	2976.0	2976.0	0.0	50.3
belg+8arcs_16	543.9k	246.7k	120.5	14400.0	1e+20	257.0k	–	14400.0	633.0k	172.5k	267.0	14400.0
belg+8arcs_17	215.0k	192.4k	11.7	14400.0	217.3k	172.9k	25.7	14400.0	313.2k	153.4k	104.2	14400.0
belg+8arcs_18	228.6k	228.6k	0.0	999.4	228.6k	228.6k	0.0	7173.4	228.6k	228.6k	0.0	2638.3
belg+8arcs_19	165.7k	165.7k	0.0	235.3	165.7k	165.7k	0.0	479.0	165.7k	165.7k	0.0	99.8
belg+8arcs_20	271.7k	255.0k	6.6	14400.0	268.5k	268.5k	0.0	10434.0	268.5k	255.0k	5.3	14400.0
belg+8arcs_21	46.4k	46.4k	0.0	44.5	46.4k	46.4k	0.0	3583.7	46.4k	46.4k	0.0	90.7
belg+8arcs_22	194.7k	194.7k	0.0	1248.4	194.7k	194.5k	0.1	7686.6	210.5k	162.1k	29.9	14400.0
belg+8arcs_23	89.3k	47.2k	89.3	14400.0	94.4k	37.9k	149.0	14400.0	176.8k	5304.3	3232.5	14400.0
belg+8arcs_24	224.5k	224.5k	0.0	3895.3	224.5k	224.5k	0.0	7370.8	224.5k	213.9k	5.0	14400.0
belg+8arcs_25	159.5k	36.3k	338.7	14400.0	772.5k	34.0k	2174.2	14400.0	177.1k	11.1k	1492.3	14400.0
belg+8arcs_26	176.8k	176.8k	0.0	187.7	176.8k	176.8k	0.0	914.8	176.8k	176.8k	0.0	45.6
belg+8arcs_27	142.6k	142.6k	0.0	652.6	148.6k	131.5k	13.0	14400.0	142.6k	142.6k	0.0	802.3
belg+8arcs_28	236.1k	236.1k	0.0	2068.1	236.1k	236.1k	0.0	6868.8	236.1k	236.1k	0.0	7272.0
belg+8arcs_29	3600.0	3600.0	0.0	195.8	3600.0	3600.0	0.0	2831.4	3600.0	3600.0	0.0	679.4
belg+8arcs_30	43.9k	43.9k	0.0	1001.0	1e+20	20.1k	–	14400.0	43.9k	43.9k	0.0	3900.3
belg+8arcs_31	145.0k	145.0k	0.0	2154.6	145.0k	145.0k	0.0	5750.6	145.0k	145.0k	0.0	10656.7
belg+8arcs_32	352.4k	200.1k	76.1	14400.0	372.8k	221.6k	68.2	14400.0	301.6k	190.7k	58.2	14400.0
belg+8arcs_33	63.9k	63.9k	0.0	10110.7	79.4k	40.4k	96.4	14400.0	71.0k	30.3k	134.4	14400.0
belg+8arcs_34	20.5k	20.5k	0.0	23.6	20.5k	20.5k	0.0	1686.0	20.5k	20.5k	0.0	29.9
belg+8arcs_35	184.5k	184.5k	0.0	534.6	184.5k	184.5k	0.0	2376.2	184.5k	184.4k	0.1	47.7
belg+8arcs_36	208.9k	182.1k	14.7	14400.0	207.7k	207.7k	0.0	11507.2	208.9k	176.6k	18.3	14400.0
belg+8arcs_37	50.6k	50.6k	0.0	2395.2	50.6k	50.6k	0.0	4316.2	50.6k	50.6k	0.0	4048.3
belg+8arcs_38	331.1k	278.6k	18.8	14400.0	320.8k	320.8k	0.0	4880.0	328.8k	274.5k	19.8	14400.0
belg+8arcs_39	105.3k	105.3k	0.0	1231.3	105.3k	93.1k	13.1	14400.0	114.3k	50.3k	127.4	14400.0
belg+8arcs_40	182.2k	82.5k	120.8	14400.0	1e+20	67.4k	–	14400.0	182.2k	16.1k	1032.8	14400.0
belg+8arcs_41	277.7k	277.7k	0.0	567.4	277.7k	277.7k	0.0	6673.1	277.7k	277.7k	0.0	11260.7
belg+8arcs_42	222.6k	222.5k	0.0	724.2	222.6k	222.6k	0.0	9939.1	222.6k	222.6k	0.0	1577.0
belg+8arcs_43	77.0k	77.0k	0.0	12684.7	1e+20	39.6k	–	14400.0	128.5k	30.9k	315.4	14400.0
belg+8arcs_44	166.3k	166.3k	0.0	69.0	166.3k	166.3k	0.0	1931.6	166.3k	166.3k	0.0	295.4
belg+8arcs_45	18.1k	18.1k	0.0	42.6	18.1k	18.1k	0.0	4322.2	18.1k	18.1k	0.0	270.7
belg+8arcs_46	183.0k	183.0k	0.0	827.4	183.0k	183.0k	0.0	2412.0	183.0k	182.9k	0.1	774.1
belg+8arcs_47	229.7k	229.7k	0.0	1511.1	229.7k	229.7k	0.0	6954.0	229.7k	229.7k	0.0	1517.7
belg+8arcs_48	229.9k	229.9k	0.0	11387.7	1e+20	193.5k	–	14400.0	233.2k	198.4k	17.5	14400.0
belg+8arcs_49	30.2k	30.2k	0.0	987.4	30.2k	30.2k	0.0	8210.4	30.2k	30.2k	0.0	3828.7
belg+8arcs_50	262.0k	262.0k	0.0	3185.3	262.0k	262.0k	0.0	3713.2	262.0k	262.0k	0.0	1969.6
belg+8arcs_51	19.2k	19.2k	0.0	184.5	19.2k	19.2k	0.0	1476.5	19.2k	19.2k	0.0	305.6
belg+8arcs_52	191.0k	191.0k	0.0	337.0	191.0k	191.0k	0.0	8688.8	191.0k	191.0k	0.0	2028.8
belg+8arcs_53	144.8k	144.7k	0.0	119.3	144.8k	144.8k	0.0	6217.1	144.8k	144.8k	0.0	948.0
belg+8arcs_54	188.7k	188.7k	0.0	370.6	188.7k	188.7k	0.0	3591.1	188.7k	188.7k	0.0	367.4
belg+8arcs_55	215.6k	215.6k	0.0	11959.8	215.6k	215.6k	0.0	11816.3	215.6k	215.6k	0.0	4852.6
belg+8arcs_56	168.3k	168.3k	0.0	659.9	168.3k	168.1k	0.1	5336.5	168.3k	168.3k	0.0	11829.3
belg+8arcs_57	261.7k	134.6k	94.5	14400.0	1e+20	162.4k	–	14400.0	313.0k	104.9k	198.5	14400.0
belg+8arcs_58	129.7k	129.7k	0.0	2213.3	129.7k	108.1k	20.0	14400.0	149.1k	93.4k	59.7	14400.0
belg+8arcs_59	20.5k	20.5k	0.0	56.6	20.5k	20.5k	0.0	35.5	20.5k	20.5k	0.0	43.5
belg+8arcs_60	131.8k	131.8k	0.0	159.7	131.8k	131.8k	0.0	8683.3	131.8k	131.8k	0.0	35.0
belg+8arcs_61	143.4k	143.4k	0.0	8134.6	180.3k	117.3k	53.7	14400.0	143.4k	120.9k	18.6	14400.0
belg+8arcs_62	431.9k	248.5k	73.8	14400.0	312.5k	283.4k	10.3	14400.0	358.6k	196.0k	83.0	14400.0
belg+8arcs_63	235.1k	235.1k	0.0	750.2	235.1k	235.1k	0.0	10034.3	235.1k	235.1k	0.0	2570.7
belg+8arcs_64	153.5k	86.1k	78.3	14400.0	1e+20	56.4k	–	14400.0	137.2k	31.7k	332.4	14400.0
belg+8arcs_65	202.4k	186.6k	8.4	14400.0	1e+20	155.0k	–	14400.0	212.6k	149.5k	42.2	14400.0
belg+8arcs_66	330.1k	207.1k	59.4	14400.0	1e+20	223.4k	–	14400.0	379.0k	189.0k	100.6	14400.0
belg+8arcs_67	182.1k	182.1k	0.0	186.5	182.1k	182.1k	0.0	1619.9	182.1k	182.1k	0.0	393.9
belg+8arcs_68	241.3k	219.1k	10.1	14400.0	238.8k	238.8k	0.0	12136.6	391.9k	183.0k	114.2	14400.0
belg+8arcs_69	341.0k	192.9k	76.8	14400.0	1e+20	196.2k	–	14400.0	324.8k	129.1k	151.7	14400.0
belg+8arcs_70	196.8k	196.8k	0.0	2322.3	349.6k	169.1k	106.7	14400.0	200.8k	179.1k	12.1	14400.0
belg+8arcs_71	163.6k	163.6k	0.0	3974.3	1e+20	128.7k	–	14400.0	166.3k	133.5k	24.6	14400.0
belg+8arcs_72	31.1k	31.1k	0.0	161.8	31.1k	31.1k	0.0	11035.2	31.1k	31.1k	0.0	1451.7
belg+8arcs_73	23.4k	23.4k	0.0	84.9	23.4k	23.4k	0.0	3139.8	23.4k	23.4k	0.0	18.5
belg+8arcs_74	556.3k	269.7k	106.2	14400.0	1e+20	301.6k	–	14400.0	602.5k	170.7k	252.9	14400.0

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Table A.31: Comparison of discrete models on *Circuit rank* + 8 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_75	187.3k	187.3k	0.0	289.9	187.3k	187.3k	0.0	3369.0	187.3k	187.3k	0.0	2543.6
belg+8arcs_76	49.4k	49.4k	0.0	653.2	49.4k	49.4k	0.0	4751.1	49.4k	49.4k	0.0	994.5
belg+8arcs_77	101.4k	101.4k	0.0	4928.5	110.9k	57.6k	92.5	14400.0	103.9k	14.7k	608.8	14400.0
belg+8arcs_78	53.2k	53.2k	0.0	464.0	56.3k	47.6k	18.3	14400.0	53.2k	53.2k	0.0	927.4
belg+8arcs_79	55.3k	55.3k	0.0	7241.3	133.5k	22.6k	491.8	14400.0	71.1k	20.3k	251.1	14400.0
belg+8arcs_80	156.3k	156.3k	0.0	2914.4	156.3k	156.3k	0.0	2661.1	156.3k	156.3k	0.0	11638.1
belg+8arcs_81	229.8k	207.7k	10.6	14400.0	229.8k	229.8k	0.0	9430.8	231.1k	196.3k	17.7	14400.0
belg+8arcs_82	20.5k	20.5k	0.0	247.4	1e+20	18.4k	–	14400.0	20.5k	20.5k	0.0	10.1
belg+8arcs_83	1e+20	155.8k	–	14400.0	719.8k	253.1k	184.4	14400.0	437.1k	222.3k	96.6	14400.0
belg+8arcs_84	364.5k	254.0k	43.5	14400.0	1e+20	272.0k	–	14400.0	475.0k	213.8k	122.1	14400.0
belg+8arcs_85	184.5k	184.5k	0.0	2563.4	184.5k	184.5k	0.0	6861.4	184.5k	164.0k	12.5	14400.0
belg+8arcs_86	407.6k	199.8k	104.0	14400.0	486.3k	226.4k	114.8	14400.0	347.0k	181.9k	90.8	14400.0
belg+8arcs_87	220.4k	220.4k	0.0	7855.0	220.4k	220.4k	0.0	13250.4	220.4k	220.4k	0.0	6052.4
belg+8arcs_88	245.7k	245.7k	0.0	13909.6	1e+20	197.4k	–	14400.0	326.4k	193.2k	69.0	14400.0
belg+8arcs_89	596.4k	440.7k	35.3	14400.0	777.4k	449.9k	72.8	14400.0	596.4k	343.9k	73.4	14400.0
belg+8arcs_90	304.1k	225.8k	34.6	14400.0	375.8k	240.2k	56.5	14400.0	330.5k	224.1k	47.5	14400.0
belg+8arcs_91	291.2k	188.2k	54.7	14400.0	1e+20	196.9k	–	14400.0	376.8k	162.5k	131.8	14400.0
belg+8arcs_92	340.1k	235.3k	44.5	14400.0	1e+20	237.0k	–	14400.0	349.5k	182.3k	91.8	14400.0
belg+8arcs_93	644.1k	241.4k	166.8	14400.0	1e+20	291.2k	–	14400.0	556.7k	232.3k	139.6	14400.0
belg+8arcs_94	182.7k	182.7k	0.0	460.8	182.7k	182.7k	0.0	1963.2	182.7k	182.7k	0.0	483.7
belg+8arcs_95	48.6k	48.6k	0.0	949.5	48.6k	48.6k	0.0	5126.0	48.6k	48.6k	0.0	465.0
belg+8arcs_96	189.6k	170.7k	11.0	14400.0	184.3k	184.3k	0.0	6177.4	184.3k	184.3k	0.0	6820.2
belg+8arcs_97	108.9k	98.2k	10.8	14400.0	1e+20	60.9k	–	14400.0	108.9k	59.9k	81.8	14400.0
belg+8arcs_98	211.8k	211.8k	0.0	5286.5	211.8k	211.8k	0.0	3881.5	211.8k	211.8k	0.0	9283.2
belg+8arcs_99	311.7k	206.7k	50.8	14400.0	1e+20	205.3k	–	14400.0	339.6k	157.6k	115.4	14400.0
belg+8arcs_100	179.9k	179.9k	0.0	504.2	179.9k	179.9k	0.0	1057.7	179.9k	179.8k	0.0	1645.6
belg+8arcs_101	219.3k	219.3k	0.0	3657.8	1e+20	182.1k	–	14400.0	245.1k	177.9k	37.8	14400.0
belg+8arcs_102	193.9k	193.9k	0.0	2509.0	193.9k	193.9k	0.0	1542.4	193.9k	193.9k	0.0	4100.6
belg+8arcs_103	168.7k	18.9k	791.3	14400.0	139.2k	40.4k	244.8	14400.0	182.2k	4091.7	4352.6	14400.0
belg+8arcs_104	231.1k	148.3k	55.8	14400.0	179.8k	179.8k	0.0	3475.8	179.8k	179.7k	0.0	11395.5
belg+8arcs_105	109.1k	109.1k	0.0	223.4	109.1k	109.1k	0.0	3547.6	109.1k	109.1k	0.0	1498.0
belg+8arcs_106	204.8k	136.2k	50.3	14400.0	160.5k	160.5k	0.0	7969.6	160.5k	117.8k	36.3	14400.0
belg+8arcs_107	211.2k	211.2k	0.0	289.5	211.2k	211.2k	0.0	7055.1	211.2k	211.2k	0.0	741.5
belg+8arcs_108	29.7k	29.7k	0.0	148.1	29.7k	29.7k	0.0	1409.0	29.7k	29.7k	0.0	368.4
belg+8arcs_109	151.5k	151.5k	0.0	7906.7	152.8k	116.9k	30.7	14400.0	158.8k	89.5k	77.5	14400.0
belg+8arcs_110	199.7k	199.7k	0.0	142.7	199.7k	199.7k	0.0	6985.7	199.7k	199.7k	0.0	321.9
belg+8arcs_111	552.7k	180.8k	205.6	14400.0	1e+20	205.1k	–	14400.0	333.2k	148.2k	124.9	14400.0
belg+8arcs_112	133.2k	116.0k	14.8	14400.0	1e+20	57.2k	–	14400.0	133.2k	45.8k	190.8	14400.0
belg+8arcs_113	3600.0	3600.0	0.0	65.1	3600.0	3600.0	0.0	677.5	3600.0	3600.0	0.0	745.9
belg+8arcs_114	4164.0	4164.0	0.0	30.0	4164.0	4164.0	0.0	4805.2	4164.0	4164.0	0.0	13.0
belg+8arcs_115	336.0k	208.2k	61.4	14400.0	1e+20	167.8k	–	14400.0	445.4k	155.2k	187.0	14400.0
belg+8arcs_116	161.5k	161.5k	0.0	1238.3	161.5k	161.5k	0.0	11323.5	161.5k	149.0k	8.4	14400.0
belg+8arcs_117	503.0k	39.6k	1169.6	14400.0	290.4k	40.3k	620.7	14400.0	194.3k	7225.5	2589.5	14400.0
belg+8arcs_118	149.1k	149.1k	0.0	66.3	149.1k	149.1k	0.0	2149.4	149.1k	149.1k	0.0	776.5
belg+8arcs_119	165.7k	165.7k	0.0	1582.4	165.7k	165.7k	0.0	4727.6	165.7k	165.7k	0.0	3362.5
belg+8arcs_120	54.4k	54.4k	0.0	7545.4	1e+20	11.8k	–	14400.0	58.6k	3736.9	1467.5	14400.0
belg+8arcs_121	143.4k	143.4k	0.0	52.4	1e+20	131.8k	–	14400.0	143.4k	143.4k	0.0	210.0
belg+8arcs_122	172.0k	168.3k	2.2	14399.7	169.1k	169.1k	0.0	4249.1	169.1k	169.1k	0.0	895.8
belg+8arcs_123	22.3k	22.3k	0.0	342.0	22.3k	22.3k	0.0	3153.4	22.3k	22.3k	0.0	181.3
belg+8arcs_124	187.5k	187.5k	0.0	13638.4	1e+20	135.1k	–	14400.0	301.3k	105.7k	185.0	14400.0
belg+8arcs_125	18.1k	18.1k	0.0	23.9	18.1k	18.1k	0.0	2657.8	18.1k	18.1k	0.0	57.8
belg+8arcs_126	89.2k	41.4k	115.7	14400.0	1e+20	30.4k	–	14400.0	249.4k	14.1k	1673.3	14400.0
belg+8arcs_127	186.9k	186.9k	0.0	5765.4	186.9k	186.9k	0.0	7250.7	186.9k	186.9k	0.0	11123.5
belg+8arcs_128	179.6k	179.6k	0.0	1806.3	179.6k	179.6k	0.0	6594.8	179.6k	179.6k	0.0	9979.5
belg+8arcs_129	25.5k	25.5k	0.0	635.4	25.5k	25.5k	0.0	2815.7	25.5k	25.5k	0.0	6340.8
belg+8arcs_130	166.9k	166.9k	0.0	281.4	202.9k	150.0k	35.2	14400.0	166.9k	166.9k	0.0	3919.5
belg+8arcs_131	324.1k	202.8k	59.8	14400.0	1e+20	188.4k	–	14400.0	354.1k	168.6k	110.0	14400.0
belg+8arcs_132	142.0k	135.1k	5.1	14400.0	139.2k	139.2k	0.0	2183.4	139.2k	139.2k	0.0	824.8
belg+8arcs_133	324.6k	158.2k	105.2	14400.0	1e+20	169.0k	–	14400.0	257.3k	135.2k	90.2	14400.0
belg+8arcs_134	446.0k	282.5k	57.9	14400.0	1e+20	257.2k	–	14400.0	395.6k	238.5k	65.9	14400.0
belg+8arcs_135	157.0k	156.9k	0.1	168.7	157.0k	157.0k	0.0	4375.4	157.0k	157.0k	0.0	719.8
belg+8arcs_136	627.5k	340.8k	84.1	14400.0	1e+20	356.6k	–	14400.0	627.5k	256.9k	144.2	14400.0
belg+8arcs_137	353.8k	236.3k	49.7	14400.0	346.2k	299.2k	15.7	14400.0	438.7k	235.5k	86.3	14400.0
belg+8arcs_138	183.0k	183.0k	0.0	790.5	183.0k	182.9k	0.1	1221.7	183.0k	183.0k	0.0	36.0
belg+8arcs_139	182.2k	182.2k	0.0	529.7	182.2k	182.2k	0.0	10531.9	182.2k	182.2k	0.0	7734.5
belg+8arcs_140	36.2k	36.2k	0.0	1745.8	36.6k	23.5k	55.6	14400.0	84.1k	15.2k	454.4	14400.0
belg+8arcs_141	187.1k	168.3k	11.2	14400.0	179.4k	179.2k	0.1	3203.2	179.4k	179.4k	0.0	2387.0
belg+8arcs_142	209.5k	209.5k	0.0	1987.3	209.5k	209.5k	0.0	2555.7	209.5k	209.5k	0.0	1282.4
belg+8arcs_143	125.4k	125.4k	0.0	331.2	125.4k	125.4k	0.0	3093.9	125.4k	125.4k	0.0	6441.1
belg+8arcs_144	35.9k	28.8k	24.8	14400.0	37.5k	24.1k	55.4	14400.0	50.0k	5532.7	804.4	14400.0
belg+8arcs_145	186.4k	68.5k	172.2	14400.0	219.0k	55.0k	298.0	14400.0	144.6k	7667.8	1785.3	14400.0
belg+8arcs_146	150.4k	129.3k	16.3	14400.0	199.5k	117.0k	70.6	14400.0	163.4k	117.6k	39.0	14400.0
belg+8arcs_147	184.4k	184.4k	0.0	463.6	184.4k	184.4k	0.0	5375.2	184.4k	184.4k	0.0	1117.5
belg+8arcs_148	251.3k	176.5k	42.4	14400.0	214.9k	160.9k	33.6	14400.0	327.2k	110.1k	197.0	14400.0
belg+8arcs_149	281.8k	176.8k	59.4	14400.0	212.0k	176.0k	20.5	14400.0	227.4k	146.2k	55.5	14400.0
belg+8arcs_150	344.9k	263.8k	30.7	14400.0	1e+20	278.4k	–	14400.0	344.1k	208.3k	65.2	14400.0
belg+8arcs_151	54.7k	54.7k	0.0	619.5	54.7k	54.7k	0.0	1607.2	54.7k	54.7k	0.0	15.6
belg+8arcs_152	191.9k	191.9k	0.0	1381.2	191.9k	191.8k	0.0	10103.3	191.9k	191.9k	0.0	6313.9

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Table A.31: Comparison of discrete models on *Circuit rank* + 8 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_153	210.5k	210.5k	0.0	1168.7	210.5k	210.5k	0.0	948.8	210.5k	210.5k	0.0	958.6
belg+8arcs_154	187.4k	187.4k	0.0	3652.9	187.4k	187.4k	0.0	3683.1	187.4k	187.4k	0.0	10601.0
belg+8arcs_155	289.6k	275.9k	5.0	14400.0	289.6k	289.6k	0.0	5672.3	289.6k	289.6k	0.0	11391.8
belg+8arcs_156	183.4k	153.7k	19.4	14400.0	159.5k	159.5k	0.0	13040.7	159.5k	159.5k	0.0	9298.6
belg+8arcs_157	162.4k	162.4k	0.0	3383.7	162.4k	162.4k	0.0	10129.8	162.4k	135.8k	19.5	14400.0
belg+8arcs_158	66.1k	66.1k	0.0	3532.7	1e+20	29.4k	–	14400.0	67.8k	15.2k	345.6	14400.0
belg+8arcs_159	241.5k	79.2k	204.7	14400.0	1e+20	57.5k	–	14400.0	251.4k	26.3k	856.8	14400.0
belg+8arcs_160	3600.0	3600.0	0.0	219.0	3600.0	3600.0	0.0	53.3	3600.0	3600.0	0.0	264.5
belg+8arcs_161	213.1k	213.1k	0.0	5182.4	213.1k	213.1k	0.0	11238.5	259.2k	196.5k	31.9	14400.0
belg+8arcs_162	245.9k	245.9k	0.0	10556.2	245.9k	245.9k	0.0	8766.4	325.1k	174.8k	86.0	14400.0
belg+8arcs_163	236.4k	198.0k	19.4	14400.0	228.3k	228.3k	0.0	8695.9	228.3k	207.9k	9.8	14400.0
belg+8arcs_164	0.0	0.0	0.0	0.2	0.0	0.0	0.0	7440.4	0.0	0.0	0.0	0.6
belg+8arcs_165	152.2k	152.2k	0.0	320.0	1e+20	142.3k	–	14400.0	152.2k	152.2k	0.0	846.0
belg+8arcs_166	4164.0	4164.0	0.0	30.7	4164.0	4164.0	0.0	1659.6	4164.0	4164.0	0.0	14.9
belg+8arcs_167	250.6k	123.1k	103.5	14400.0	1e+20	123.3k	–	14400.0	247.2k	67.6k	265.6	14400.0
belg+8arcs_168	410.2k	261.9k	56.6	14400.0	1e+20	243.7k	–	14400.0	424.7k	192.6k	120.5	14400.0
belg+8arcs_169	139.7k	139.7k	0.0	10515.0	139.7k	139.7k	0.0	14149.6	149.9k	110.3k	35.9	14400.0
belg+8arcs_170	185.6k	185.6k	0.0	158.1	185.6k	185.6k	0.0	2384.6	185.6k	185.5k	0.1	1668.7
belg+8arcs_171	244.1k	167.5k	45.7	14400.0	200.1k	200.1k	0.0	9693.5	219.9k	151.5k	45.2	14400.0
belg+8arcs_172	3600.0	3600.0	0.0	143.2	3600.0	3600.0	0.0	518.6	3600.0	3600.0	0.0	149.1
belg+8arcs_173	313.3k	262.2k	19.5	14400.0	294.0k	243.5k	20.8	14400.0	312.0k	186.6k	67.2	14400.0
belg+8arcs_174	146.8k	146.8k	0.0	86.7	146.8k	146.8k	0.0	902.8	146.8k	146.8k	0.0	290.8
belg+8arcs_175	40.9k	40.9k	0.0	1280.2	1e+20	24.4k	–	14400.0	40.9k	40.9k	0.0	10972.3
belg+8arcs_176	144.1k	110.6k	30.3	14400.0	1e+20	45.9k	–	14400.0	168.0k	23.2k	623.0	14400.0
belg+8arcs_177	234.8k	234.8k	0.0	47.4	234.8k	234.8k	0.0	3578.3	234.8k	234.8k	0.0	184.2
belg+8arcs_178	174.0k	174.0k	0.0	1914.8	174.0k	174.0k	0.0	11159.0	188.7k	151.9k	24.2	14400.0
belg+8arcs_179	252.4k	252.4k	0.0	6849.2	1e+20	211.9k	–	14400.0	256.6k	239.9k	7.0	14400.0
belg+8arcs_180	44.6k	44.6k	0.0	754.4	1e+20	30.5k	–	14400.0	44.6k	44.6k	0.0	1147.8
belg+8arcs_181	21.3k	21.3k	0.0	768.9	21.3k	21.3k	0.0	2563.3	21.3k	21.3k	0.0	613.0
belg+8arcs_182	124.8k	124.7k	0.1	1847.5	146.5k	108.3k	35.2	14400.0	124.8k	124.8k	0.0	4418.0
belg+8arcs_183	134.2k	134.2k	0.0	4016.6	140.6k	118.1k	19.0	14400.0	135.6k	101.0k	34.2	14400.0
belg+8arcs_184	26.3k	14.5k	81.6	14400.0	21.7k	21.7k	0.0	10262.8	21.7k	4164.4	421.6	14400.0
belg+8arcs_185	114.6k	114.6k	0.0	2845.2	1e+20	80.0k	–	14400.0	114.6k	114.6k	0.0	3685.0
belg+8arcs_186	169.1k	169.1k	0.0	374.6	169.1k	168.9k	0.1	8868.3	169.1k	169.1k	0.0	424.8
belg+8arcs_187	513.2k	241.3k	112.7	14400.0	1e+20	273.7k	–	14400.0	547.8k	191.5k	186.0	14400.0
belg+8arcs_188	17.4k	17.4k	0.0	69.8	17.4k	17.4k	0.0	4243.0	17.4k	17.4k	0.0	124.1
belg+8arcs_189	169.8k	169.8k	0.0	1978.2	169.8k	169.8k	0.0	4629.0	169.8k	169.8k	0.0	522.1
belg+8arcs_190	204.8k	45.8k	347.3	14400.0	209.7k	74.2k	182.6	14400.0	204.8k	7983.2	2464.9	14400.0
belg+8arcs_191	272.6k	252.1k	8.2	14400.0	272.5k	272.5k	0.0	3329.8	272.5k	272.5k	0.0	4237.9
belg+8arcs_192	127.6k	127.6k	0.0	1927.5	127.6k	127.6k	0.0	9310.8	127.6k	127.6k	0.0	8829.7
belg+8arcs_193	143.4k	143.4k	0.0	19.2	143.4k	143.4k	0.0	856.1	143.4k	143.4k	0.0	109.6
belg+8arcs_194	440.4k	177.7k	147.8	14400.0	1e+20	188.9k	–	14400.0	311.5k	161.6k	92.8	14400.0
belg+8arcs_195	184.6k	184.6k	0.0	248.5	184.6k	184.6k	0.0	3556.1	184.6k	184.6k	0.0	460.6
belg+8arcs_196	366.4k	207.3k	76.8	14400.0	1e+20	204.7k	–	14400.0	455.3k	154.7k	194.4	14400.0
belg+8arcs_197	296.5k	296.5k	0.0	10025.4	296.5k	296.5k	0.0	8191.4	350.5k	177.0k	98.1	14400.0
belg+8arcs_198	149.1k	149.1k	0.0	558.5	149.1k	149.1k	0.0	3469.7	149.1k	149.1k	0.0	83.7
belg+8arcs_199	40.4k	40.4k	0.0	1216.1	40.4k	40.4k	0.0	6528.9	40.4k	40.4k	0.0	1726.4
belg+8arcs_200	207.4k	207.4k	0.0	2414.7	418.7k	192.8k	117.1	14400.0	207.4k	207.4k	0.0	2435.7
belg+8arcs_201	216.8k	26.1k	729.1	14400.0	1e+20	37.1k	–	14400.0	114.8k	25.9k	343.0	14400.0
belg+8arcs_202	145.8k	145.8k	0.0	974.4	1e+20	128.0k	–	14400.0	145.8k	145.8k	0.0	2978.6
belg+8arcs_203	210.3k	68.0k	209.3	14400.0	1e+20	40.9k	–	14400.0	268.9k	11.2k	2308.5	14400.0
belg+8arcs_204	25.4k	25.4k	0.0	1638.8	25.4k	25.4k	0.0	69.0	25.4k	25.4k	0.0	2395.8
belg+8arcs_205	253.5k	233.9k	8.4	14400.0	247.7k	247.7k	0.0	2680.9	247.7k	247.7k	0.0	1764.5
belg+8arcs_206	134.7k	134.7k	0.0	7508.6	189.9k	98.6k	92.5	14400.0	157.2k	101.0k	55.6	14400.0
belg+8arcs_207	221.0k	221.0k	0.0	4765.2	221.0k	221.0k	0.0	3491.3	224.4k	196.0k	14.5	14400.0
belg+8arcs_208	197.8k	197.8k	0.0	1087.9	197.8k	197.8k	0.0	10598.9	197.8k	197.8k	0.0	4135.7
belg+8arcs_209	168.3k	30.4k	452.8	14400.0	168.3k	39.0k	330.9	14400.0	168.3k	8555.1	1866.7	14400.0
belg+8arcs_210	200.1k	200.1k	0.0	838.1	200.1k	200.1k	0.0	8713.9	200.1k	200.1k	0.0	2409.0
belg+8arcs_211	141.0k	141.0k	0.0	3264.3	141.0k	141.0k	0.0	2693.5	144.8k	94.4k	53.4	14400.0
belg+8arcs_212	127.8k	127.8k	0.0	5325.6	127.8k	127.8k	0.0	7956.7	127.8k	127.8k	0.0	7090.9
belg+8arcs_213	256.7k	256.7k	0.0	12528.5	256.7k	256.7k	0.0	5639.8	256.7k	219.0k	17.2	14400.0
belg+8arcs_214	235.1k	235.1k	0.0	3247.9	1e+20	188.9k	–	14400.0	235.1k	199.1k	18.1	14400.0
belg+8arcs_215	237.9k	169.3k	40.6	14400.0	191.5k	172.0k	11.3	14400.0	201.2k	143.9k	39.8	14400.0
belg+8arcs_216	227.1k	227.1k	0.0	1098.3	227.1k	227.1k	0.0	2584.6	227.1k	227.1k	0.0	770.0
belg+8arcs_217	144.6k	144.6k	0.0	964.7	144.6k	144.6k	0.0	1513.5	144.6k	144.6k	0.0	682.0
belg+8arcs_218	307.3k	218.3k	40.8	14400.0	1e+20	217.9k	–	14400.0	301.4k	187.6k	60.6	14400.0
belg+8arcs_219	206.1k	206.1k	0.0	170.9	206.1k	206.1k	0.0	5688.7	206.1k	206.1k	0.0	43.8
belg+8arcs_220	574.3k	149.7k	283.7	14400.0	1e+20	149.8k	–	14400.0	777.7k	82.4k	843.7	14400.0
belg+8arcs_221	15.6k	15.6k	0.0	23.1	1e+20	14.2k	–	14400.0	15.6k	15.6k	0.0	31.0
belg+8arcs_222	229.8k	229.7k	0.1	3265.2	1e+20	191.1k	–	14400.0	229.8k	229.6k	0.1	9853.5
belg+8arcs_223	181.5k	151.8k	19.5	14400.0	278.4k	156.0k	78.5	14400.0	181.5k	157.9k	14.9	14400.0
belg+8arcs_224	389.1k	193.0k	101.6	14400.0	233.7k	233.7k	0.0	12148.0	385.0k	160.4k	139.9	14400.0
belg+8arcs_225	11.4k	11.4k	0.0	124.6	11.4k	11.4k	0.0	146.3	11.4k	11.4k	0.0	677.2
belg+8arcs_226	172.0k	172.0k	0.0	1079.8	172.0k	172.0k	0.0	2720.9	172.0k	172.0k	0.0	2412.5
belg+8arcs_227	29.7k	29.7k	0.0	2558.7	29.7k	29.7k	0.0	9102.4	29.7k	29.7k	0.0	430.7
belg+8arcs_228	624.7k	184.4k	238.7	14400.0	1e+20	183.3k	–	14400.0	323.8k	150.0k	115.8	14400.0
belg+8arcs_229	189.5k	189.3k	0.1	1279.3	189.5k	189.5k	0.0	6423.3	189.5k	189.5k	0.0	1559.6
belg+8arcs_230	187.9k	187.9k	0.0	668.6	187.9k	187.9k	0.0	6930.5	187.9k	187.9k	0.0	522.5

continued on next page

Table A.31: Comparison of discrete models on *Circuit rank* + 8 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_231	25.4k	25.4k	0.0	33.3	25.4k	25.4k	0.0	1084.3	25.4k	25.4k	0.0	66.6
belg+8arcs_232	152.7k	152.7k	0.0	11825.7	152.7k	152.7k	0.0	6886.5	152.7k	152.7k	0.0	8021.1
belg+8arcs_233	209.4k	170.9k	22.5	14400.0	192.8k	192.8k	0.0	13304.1	415.1k	130.7k	217.7	14400.0
belg+8arcs_234	54.3k	54.3k	0.0	1676.8	54.3k	54.3k	0.0	8454.4	54.3k	54.3k	0.0	2829.3
belg+8arcs_235	177.9k	177.9k	0.0	1605.3	1e+20	153.0k	–	14400.0	177.9k	177.9k	0.0	2979.6
belg+8arcs_236	192.8k	167.3k	15.3	14400.0	189.2k	154.8k	22.3	14400.0	194.4k	153.7k	26.5	14400.0
belg+8arcs_237	3600.0	3600.0	0.0	3659.4	3600.0	3600.0	0.0	1285.5	3600.0	3600.0	0.0	54.0
belg+8arcs_238	559.4k	222.7k	151.2	14400.0	1e+20	226.5k	–	14400.0	881.7k	163.8k	438.2	14400.0
belg+8arcs_239	303.9k	199.3k	52.5	14400.0	1e+20	192.1k	–	14400.0	261.4k	176.6k	48.0	14400.0
belg+8arcs_240	238.9k	224.8k	6.2	14400.0	238.6k	238.6k	0.0	4027.9	241.1k	195.5k	23.3	14400.0
belg+8arcs_241	263.1k	187.1k	40.6	14400.0	219.2k	219.2k	0.0	4645.5	219.2k	206.3k	6.2	14400.0
belg+8arcs_242	235.4k	132.4k	77.8	14400.0	1e+20	104.2k	–	14400.0	191.0k	118.6k	61.1	14400.0
belg+8arcs_243	186.4k	186.4k	0.0	15.4	186.4k	186.4k	0.0	5654.4	186.4k	186.4k	0.0	138.6
belg+8arcs_244	187.1k	187.1k	0.0	38.4	187.1k	186.9k	0.1	1414.0	187.1k	187.1k	0.0	212.2
belg+8arcs_245	26.1k	26.1k	0.0	559.3	26.1k	26.1k	0.0	499.4	26.1k	26.1k	0.0	793.4
belg+8arcs_246	167.2k	167.2k	0.0	424.9	167.2k	167.2k	0.0	2416.3	167.2k	167.2k	0.0	1143.5
belg+8arcs_247	33.6k	15.6k	115.8	14400.0	97.8k	14.3k	582.1	14400.0	33.6k	12.2k	175.6	14400.0
belg+8arcs_248	377.3k	244.0k	54.6	14400.0	352.3k	271.8k	29.6	14400.0	418.0k	226.5k	84.5	14400.0
belg+8arcs_249	234.5k	33.3k	604.9	14400.0	1e+20	31.1k	–	14400.0	92.0k	28.8k	219.0	14400.0
belg+8arcs_250	294.3k	174.3k	68.8	14400.0	234.2k	180.7k	29.6	14400.0	236.9k	165.5k	43.1	14400.0
belg+8arcs_251	181.5k	181.5k	0.0	2473.8	187.8k	161.8k	16.1	14400.0	186.4k	152.2k	22.5	14400.0
belg+8arcs_252	163.0k	163.0k	0.0	5987.1	1e+20	116.8k	–	14400.0	218.4k	100.3k	117.8	14400.0
belg+8arcs_253	354.0k	210.5k	68.2	14400.0	258.0k	258.0k	0.0	3634.8	261.7k	212.5k	23.2	14400.0
belg+8arcs_254	242.5k	242.5k	0.0	934.0	242.5k	242.5k	0.0	9934.4	242.5k	214.1k	13.3	14400.0
belg+8arcs_255	23.4k	23.4k	0.0	86.8	1e+20	20.9k	–	14400.0	23.4k	23.4k	0.0	545.9
belg+8arcs_256	187.9k	187.9k	0.0	389.5	187.9k	187.9k	0.0	4109.8	187.9k	187.9k	0.0	280.9
belg+8arcs_257	216.1k	28.8k	649.9	14400.0	1e+20	31.2k	–	14400.0	107.0k	4923.1	2073.3	14400.0
belg+8arcs_258	174.6k	174.6k	0.0	2873.3	174.6k	174.6k	0.0	7090.6	174.6k	174.6k	0.0	8062.1
belg+8arcs_259	181.2k	181.2k	0.0	1885.4	181.2k	181.2k	0.0	3997.0	181.2k	181.2k	0.0	3605.1
belg+8arcs_260	41.0k	41.0k	0.0	65.6	41.0k	41.0k	0.0	2442.7	41.0k	41.0k	0.0	185.8
belg+8arcs_261	18.1k	18.1k	0.0	137.8	18.1k	18.1k	0.0	3547.2	18.1k	18.1k	0.0	502.8
belg+8arcs_262	20.5k	20.5k	0.0	63.2	20.5k	20.5k	0.0	2724.2	20.5k	20.5k	0.0	253.0
belg+8arcs_263	255.3k	255.1k	0.1	2241.4	1e+20	224.7k	–	14400.0	261.3k	216.3k	20.8	14400.0
belg+8arcs_264	181.8k	181.8k	0.0	908.1	1e+20	152.2k	–	14400.0	191.2k	144.6k	32.2	14400.0
belg+8arcs_265	488.3k	267.7k	82.4	14400.0	1e+20	263.9k	–	14400.0	618.6k	171.1k	261.5	14400.0
belg+8arcs_266	4920.0	4920.0	0.0	98.7	4920.0	4920.0	0.0	26.3	4920.0	4920.0	0.0	55.4
belg+8arcs_267	325.4k	211.2k	54.1	14400.0	372.0k	277.6k	34.0	14400.0	325.4k	217.7k	49.5	14400.0
belg+8arcs_268	237.3k	237.3k	0.0	1513.9	237.3k	237.3k	0.0	2594.8	237.3k	237.3k	0.0	2115.3
belg+8arcs_269	251.6k	178.9k	40.6	14400.0	1e+20	162.3k	–	14400.0	334.3k	122.3k	173.3	14400.0
belg+8arcs_270	197.2k	197.2k	0.0	443.5	197.2k	197.2k	0.0	9557.9	197.2k	197.2k	0.0	2797.3
belg+8arcs_271	185.8k	185.8k	0.0	1451.2	185.8k	185.8k	0.0	2912.1	185.8k	185.8k	0.0	1977.1
belg+8arcs_272	268.7k	240.0k	11.9	14400.0	1e+20	212.7k	–	14400.0	279.8k	186.2k	50.3	14400.0
belg+8arcs_273	263.2k	263.2k	0.0	4937.4	263.2k	263.2k	0.0	12489.5	274.8k	207.3k	32.6	14400.0
belg+8arcs_274	142.5k	142.5k	0.0	2179.7	142.5k	142.5k	0.0	5051.3	142.5k	142.5k	0.0	1406.8
belg+8arcs_275	19.2k	19.2k	0.0	19.2	19.2k	19.2k	0.0	648.5	19.2k	19.2k	0.0	64.5
belg+8arcs_276	63.7k	63.7k	0.0	5352.3	1e+20	17.9k	–	14400.0	69.0k	26.1k	164.5	14400.0
belg+8arcs_277	61.2k	61.2k	0.0	757.9	1e+20	28.7k	–	14400.0	61.2k	61.2k	0.0	1876.2
belg+8arcs_278	13.2k	13.2k	0.0	255.8	1e+20	7137.9	–	14400.0	13.2k	13.2k	0.0	145.3
belg+8arcs_279	86.6k	86.6k	0.0	7468.0	1e+20	19.4k	–	14400.0	108.1k	11.6k	831.1	14400.0
belg+8arcs_280	251.5k	251.5k	0.0	834.6	251.5k	251.5k	0.0	4882.3	273.4k	227.4k	20.2	14400.0
belg+8arcs_281	250.3k	121.1k	106.7	14400.0	1e+20	81.6k	–	14400.0	237.0k	59.6k	297.6	14400.0
belg+8arcs_282	288.3k	288.3k	0.0	4814.4	288.3k	288.3k	0.0	2589.7	291.3k	217.9k	33.7	14400.0
belg+8arcs_283	225.1k	225.1k	0.0	367.7	225.1k	224.9k	0.1	2692.7	225.1k	225.1k	0.0	1360.0
belg+8arcs_284	184.5k	184.5k	0.0	451.3	184.5k	184.5k	0.0	4665.0	184.5k	184.5k	0.0	1977.0
belg+8arcs_285	147.7k	124.2k	18.9	14400.0	136.7k	136.7k	0.0	4271.4	137.2k	116.8k	17.5	14400.0
belg+8arcs_286	204.2k	204.2k	0.0	727.8	204.2k	204.2k	0.0	1862.7	204.2k	204.2k	0.0	1483.8
belg+8arcs_287	219.3k	219.3k	0.0	639.6	219.3k	219.3k	0.0	3210.2	219.3k	219.3k	0.0	853.9
belg+8arcs_288	172.5k	172.5k	0.0	554.2	172.5k	172.5k	0.0	3125.8	172.5k	172.5k	0.0	766.5
belg+8arcs_289	200.3k	85.5k	134.2	14400.0	198.6k	102.0k	94.8	14400.0	204.8k	23.5k	771.2	14400.0
belg+8arcs_290	200.6k	200.6k	0.0	6615.1	200.6k	164.0k	22.3	14400.0	221.4k	133.0k	66.5	14400.0
belg+8arcs_291	300.0k	204.5k	46.7	14400.0	1e+20	199.3k	–	14400.0	351.0k	169.0k	107.7	14400.0
belg+8arcs_292	120.8k	79.1k	52.8	14400.0	1e+20	33.3k	–	14400.0	154.4k	14.8k	946.3	14400.0
belg+8arcs_293	47.4k	47.4k	0.0	4903.2	50.2k	33.0k	52.3	14400.0	59.2k	12.7k	365.9	14400.0
belg+8arcs_294	162.5k	162.5k	0.0	647.6	162.5k	162.5k	0.0	9196.2	162.5k	162.5k	0.0	1002.5
belg+8arcs_295	224.1k	133.2k	68.3	14400.0	1e+20	133.8k	–	14400.0	208.5k	122.8k	69.8	14400.0
belg+8arcs_296	161.8k	161.8k	0.0	13432.2	1e+20	93.1k	–	14400.0	287.2k	54.9k	422.7	14400.0
belg+8arcs_297	141.8k	141.8k	0.0	7831.0	141.8k	141.8k	0.0	5440.2	141.8k	122.5k	15.7	14400.0
belg+8arcs_298	55.4k	55.4k	0.0	9073.9	480.5k	23.7k	1926.6	14400.0	69.9k	14.2k	391.0	14400.0
belg+8arcs_299	579.6k	239.4k	142.1	14400.0	1e+20	261.7k	–	14400.0	658.2k	160.5k	310.1	14400.0
belg+8arcs_300	151.3k	151.3k	0.0	749.7	151.3k	151.3k	0.0	3771.6	151.3k	151.3k	0.0	4654.2
belg+8arcs_301	149.7k	149.7k	0.0	260.8	182.7k	133.8k	36.6	14400.0	149.7k	149.7k	0.0	2545.6
belg+8arcs_302	243.9k	197.5k	23.5	14400.0	228.9k	228.9k	0.0	10801.9	228.9k	212.4k	7.7	14400.0
belg+8arcs_303	297.5k	213.8k	39.2	14400.0	1e+20	184.5k	–	14400.0	295.4k	161.2k	83.3	14400.0
belg+8arcs_304	207.4k	207.4k	0.0	4444.6	207.4k	207.4k	0.0	8276.8	207.4k	207.4k	0.0	3614.8
belg+8arcs_305	233.3k	201.1k	16.0	14400.0	233.3k	233.3k	0.0	7344.9	246.4k	203.6k	21.0	14400.0
belg+8arcs_306	202.7k	202.7k	0.0	210.8	202.7k	202.7k	0.0	3211.5	202.7k	202.7k	0.0	811.8
belg+8arcs_307	124.2k	124.2k	0.0	11211.6	1e+20	89.7k	–	14400.0	173.3k	81.0k	113.9	14400.0
belg+8arcs_308	41.8k	41.8k	0.0	277.1	41.8k	41.8k	0.0	2681.9	41.8k	41.8k	0.0	2663.3

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Table A.31: Comparison of discrete models on *Circuit rank* + 8 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_309	382.8k	171.0k	123.9	14400.0	668.3k	190.2k	251.5	14400.0	557.9k	120.4k	363.3	14400.0
belg+8arcs_310	23.0k	23.0k	0.0	66.6	23.0k	23.0k	0.0	1158.9	23.0k	23.0k	0.0	510.3
belg+8arcs_311	139.3k	139.3k	0.0	1065.8	139.3k	139.3k	0.0	9253.5	139.3k	139.3k	0.0	3561.8
belg+8arcs_312	137.2k	137.2k	0.0	3553.3	1e+20	98.5k	–	14400.0	195.1k	94.2k	107.2	14400.0
belg+8arcs_313	217.1k	217.1k	0.0	12433.9	297.6k	191.3k	55.6	14400.0	236.1k	161.3k	46.4	14400.0
belg+8arcs_314	586.0k	261.2k	124.4	14400.0	675.4k	309.4k	118.3	14400.0	511.3k	224.4k	127.9	14400.0
belg+8arcs_315	212.1k	211.9k	0.1	2865.4	1e+20	187.9k	–	14400.0	214.1k	190.8k	12.2	14400.0
belg+8arcs_316	164.1k	119.1k	37.9	14400.0	1e+20	89.9k	–	14400.0	225.3k	74.1k	203.9	14400.0
belg+8arcs_317	192.8k	192.8k	0.0	4244.2	192.8k	192.8k	0.0	7154.3	204.5k	145.3k	40.8	14400.0
belg+8arcs_318	74.6k	74.6k	0.0	53.7	74.6k	74.6k	0.0	1220.1	74.6k	74.6k	0.0	476.1
belg+8arcs_319	144.2k	144.2k	0.0	227.9	144.2k	144.2k	0.0	8729.8	144.2k	144.2k	0.0	633.5
belg+8arcs_320	181.4k	181.4k	0.0	377.8	181.4k	181.4k	0.0	13969.6	181.4k	181.4k	0.0	297.2
belg+8arcs_321	140.6k	140.6k	0.0	1717.8	1e+20	89.0k	–	14400.0	143.7k	90.2k	59.4	14400.0
belg+8arcs_322	237.3k	42.3k	461.1	14400.0	213.1k	52.1k	309.1	14400.0	248.8k	24.2k	929.4	14400.0
belg+8arcs_323	138.0k	138.0k	0.0	4492.9	138.0k	138.0k	0.0	7814.4	138.0k	115.0k	20.0	14400.0
belg+8arcs_324	78.2k	78.2k	0.0	2518.2	78.2k	78.2k	0.0	6202.3	125.3k	54.4k	130.2	14400.0
belg+8arcs_325	230.4k	230.3k	0.0	99.4	230.4k	230.4k	0.0	225.4	230.4k	230.4k	0.0	332.5
belg+8arcs_326	214.2k	177.4k	20.8	14400.0	209.7k	181.9k	15.3	14400.0	224.9k	158.9k	41.5	14400.0
belg+8arcs_327	28.7k	28.7k	0.0	1583.1	28.7k	28.7k	0.0	1332.7	28.7k	28.7k	0.0	3344.8
belg+8arcs_328	11.4k	11.4k	0.0	25.2	11.4k	11.4k	0.0	116.0	11.4k	11.4k	0.0	61.5
belg+8arcs_329	173.4k	173.4k	0.0	9217.7	173.4k	158.7k	9.2	14400.0	202.1k	145.3k	39.1	14400.0
belg+8arcs_330	227.8k	227.8k	0.0	5226.3	227.8k	227.8k	0.0	13558.5	227.8k	227.8k	0.0	9869.9
belg+8arcs_331	143.4k	126.7k	13.2	14400.0	1e+20	126.6k	–	14400.0	137.5k	137.5k	0.0	566.9
belg+8arcs_332	530.3k	257.1k	106.3	14400.0	1e+20	287.4k	–	14400.0	452.6k	197.9k	128.7	14400.0
belg+8arcs_333	118.4k	118.4k	0.0	4514.5	118.4k	118.4k	0.0	11667.6	118.4k	118.4k	0.0	9849.2
belg+8arcs_334	244.5k	244.5k	0.0	8065.0	244.5k	244.5k	0.0	11021.3	245.7k	205.1k	19.8	14400.0
belg+8arcs_335	224.7k	224.7k	0.0	7945.8	229.1k	200.5k	14.3	14400.0	227.4k	189.5k	20.0	14400.0
belg+8arcs_336	171.5k	171.5k	0.0	844.9	171.5k	171.5k	0.0	7693.1	171.5k	171.5k	0.0	10828.1
belg+8arcs_337	78.2k	78.2k	0.0	11221.8	1e+20	30.0k	–	14400.0	78.2k	27.9k	180.7	14400.0
belg+8arcs_338	210.5k	159.7k	31.8	14400.0	1e+20	131.8k	–	14400.0	210.6k	141.6k	48.8	14400.0
belg+8arcs_339	327.8k	245.0k	33.8	14400.0	291.0k	291.0k	0.0	12953.1	330.5k	211.3k	56.4	14400.0
belg+8arcs_340	296.1k	229.7k	28.9	14400.0	1e+20	195.5k	–	14400.0	453.7k	153.2k	196.2	14400.0
belg+8arcs_341	152.9k	152.9k	0.0	5233.7	152.9k	152.9k	0.0	5158.0	152.9k	152.9k	0.0	2397.9
belg+8arcs_342	303.5k	186.0k	63.1	14400.0	270.0k	192.8k	40.1	14400.0	382.1k	151.0k	153.0	14400.0
belg+8arcs_343	169.9k	169.9k	0.0	7491.4	169.9k	169.9k	0.0	3863.9	169.9k	169.9k	0.0	3624.5
belg+8arcs_344	132.6k	132.6k	0.0	402.6	132.6k	132.6k	0.0	4554.9	132.6k	132.6k	0.0	57.1
belg+8arcs_345	187.1k	187.1k	0.0	102.2	187.1k	187.1k	0.0	421.4	187.1k	187.1k	0.0	975.4
belg+8arcs_346	189.0k	189.0k	0.0	805.7	189.0k	189.0k	0.0	14354.3	189.0k	189.0k	0.0	499.7
belg+8arcs_347	365.7k	274.8k	33.1	14400.0	1e+20	257.8k	–	14400.0	380.6k	165.7k	129.7	14400.0
belg+8arcs_348	135.5k	135.5k	0.0	640.9	135.5k	135.5k	0.0	8466.8	135.5k	135.5k	0.0	940.1
belg+8arcs_349	48.0k	48.0k	0.0	7322.2	138.2k	36.0k	284.4	14400.0	59.0k	22.6k	160.8	14400.0
belg+8arcs_350	45.2k	45.2k	0.0	994.2	45.2k	45.2k	0.0	2296.2	45.2k	17.7k	154.7	14400.0
belg+8arcs_351	387.0k	228.8k	69.1	14400.0	1e+20	223.9k	–	14400.0	1401.5k	166.1k	744.0	14400.0
belg+8arcs_352	236.5k	218.1k	8.4	14400.0	231.5k	183.9k	25.9	14400.0	270.1k	139.4k	93.9	14400.0
belg+8arcs_353	131.8k	131.8k	0.0	1103.8	1e+20	119.9k	–	14400.0	131.8k	131.8k	0.0	187.3
belg+8arcs_354	124.5k	101.6k	22.5	14400.0	1e+20	92.4k	–	14400.0	124.5k	97.5k	27.7	14400.0
belg+8arcs_355	35.3k	35.3k	0.0	197.8	1e+20	9807.4	–	14400.0	35.3k	35.3k	0.0	774.1
belg+8arcs_356	427.9k	185.5k	130.7	14400.0	1e+20	203.7k	–	14400.0	333.0k	176.4k	88.8	14400.0
belg+8arcs_357	216.3k	175.9k	23.0	14400.0	230.7k	169.7k	35.9	14400.0	257.6k	159.1k	61.9	14400.0
belg+8arcs_358	288.7k	288.7k	0.0	5537.0	288.7k	288.7k	0.0	5700.9	311.8k	231.9k	34.5	14400.0
belg+8arcs_359	277.0k	166.9k	65.9	14400.0	236.2k	131.9k	79.1	14400.0	229.6k	116.5k	97.0	14400.0
belg+8arcs_360	150.2k	150.2k	0.0	12282.5	150.2k	150.2k	0.0	10847.2	159.2k	138.2k	15.2	14400.0
belg+8arcs_361	333.1k	267.7k	24.4	14400.0	1e+20	272.1k	–	14400.0	357.2k	210.7k	69.6	14400.0
belg+8arcs_362	57.9k	45.4k	27.4	14400.0	59.9k	12.5k	380.5	14400.0	76.8k	3011.6	2448.8	14400.0
belg+8arcs_363	178.1k	178.1k	0.0	61.8	178.1k	178.1k	0.0	3996.1	178.1k	178.1k	0.0	624.2
belg+8arcs_364	149.7k	149.7k	0.0	3864.9	149.7k	149.7k	0.0	8642.8	172.3k	133.0k	29.5	14400.0
belg+8arcs_365	201.9k	201.9k	0.0	3172.5	201.9k	201.9k	0.0	8738.8	201.9k	201.9k	0.0	3433.3
belg+8arcs_366	254.5k	254.5k	0.0	2660.1	254.5k	254.5k	0.0	3649.9	254.5k	224.6k	13.3	14400.0
belg+8arcs_367	185.4k	185.4k	0.0	444.2	185.4k	185.4k	0.0	2953.1	185.4k	185.4k	0.0	3251.3
belg+8arcs_368	56.3k	20.5k	174.3	14400.0	132.0k	26.3k	401.3	14400.0	92.3k	14.5k	534.7	14400.0
belg+8arcs_369	4164.0	4164.0	0.0	119.3	4164.0	4164.0	0.0	1412.5	4164.0	4164.0	0.0	32.0
belg+8arcs_370	163.3k	163.3k	0.0	90.2	1e+20	145.5k	–	14400.0	163.3k	163.3k	0.0	41.3
belg+8arcs_371	237.6k	213.5k	11.3	14400.0	236.0k	236.0k	0.0	4261.3	251.5k	190.7k	31.9	14400.0
belg+8arcs_372	214.9k	182.8k	17.6	14400.0	1e+20	156.1k	–	14400.0	266.6k	127.7k	108.8	14400.0
belg+8arcs_373	177.8k	177.8k	0.0	2088.9	1e+20	163.1k	–	14400.0	177.8k	177.8k	0.0	93.8
belg+8arcs_374	125.3k	125.3k	0.0	1117.2	1e+20	94.1k	–	14400.0	127.1k	106.0k	19.9	14400.0
belg+8arcs_375	46.4k	46.4k	0.0	26.5	46.4k	46.4k	0.0	1070.5	46.4k	46.4k	0.0	101.7
belg+8arcs_376	382.0k	382.0k	0.0	5493.1	1e+20	303.4k	–	14400.0	385.6k	247.7k	55.7	14400.0
belg+8arcs_377	413.3k	24.0k	1623.3	14400.0	341.8k	36.9k	825.8	14400.0	238.5k	4843.9	4824.0	14400.0
belg+8arcs_378	113.3k	113.3k	0.0	7414.9	113.3k	113.3k	0.0	14074.0	132.6k	51.9k	155.3	14400.0
belg+8arcs_379	243.9k	199.2k	22.4	14400.0	1e+20	183.0k	–	14400.0	355.3k	133.1k	167.0	14400.0
belg+8arcs_380	175.3k	175.3k	0.0	555.2	175.3k	175.3k	0.0	6659.3	175.3k	175.3k	0.0	1854.1
belg+8arcs_381	646.7k	157.9k	309.6	14400.0	436.1k	191.6k	127.6	14400.0	492.0k	138.7k	254.8	14400.0
belg+8arcs_382	203.9k	203.9k	0.0	6274.1	203.9k	203.9k	0.0	1056.8	203.9k	203.9k	0.0	5113.8
belg+8arcs_383	51.8k	51.8k	0.0	6578.1	1e+20	16.7k	–	14400.0	74.2k	26.3k	181.9	14400.0
belg+8arcs_384	294.8k	89.4k	229.9	14400.0	1e+20	64.4k	–	14400.0	239.0k	14.5k	1544.7	14400.0
belg+8arcs_385	188.1k	188.1k	0.0	1002.7	188.1k	188.1k	0.0	1806.3	204.2k	178.6k	14.3	14400.0
belg+8arcs_386	358.8k	254.8k	40.8	14400.0	323.7k	279.5k	15.8	14400.0	390.3k	213.3k	83.0	14400.0

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Table A.31: Comparison of discrete models on *Circuit rank* + 8 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_387	148.6k	35.2k	322.0	14400.0	128.7k	34.6k	271.5	14400.0	127.2k	7771.1	1537.3	14400.0
belg+8arcs_388	114.6k	27.2k	321.5	14400.0	113.3k	22.1k	413.5	14400.0	163.3k	4261.9	3731.1	14400.0
belg+8arcs_389	155.3k	155.3k	0.0	4298.2	1e+20	128.6k	–	14400.0	155.3k	155.3k	0.0	5658.6
belg+8arcs_390	254.4k	205.2k	24.0	14400.0	222.7k	199.3k	11.8	14400.0	222.7k	222.6k	0.1	13760.9
belg+8arcs_391	194.9k	194.8k	0.1	868.6	205.1k	176.7k	16.1	14400.0	194.9k	176.5k	10.5	14400.0
belg+8arcs_392	108.9k	108.9k	0.0	165.1	186.6k	89.7k	108.0	14400.0	108.9k	108.9k	0.0	189.9
belg+8arcs_393	213.7k	213.7k	0.0	3027.3	213.7k	213.7k	0.0	6236.8	213.7k	213.7k	0.0	8294.0
belg+8arcs_394	137.2k	40.9k	235.4	14400.0	1e+20	36.2k	–	14400.0	125.9k	27.2k	363.3	14400.0
belg+8arcs_395	416.3k	172.9k	140.8	14400.0	1e+20	164.5k	–	14400.0	381.4k	126.2k	202.1	14400.0
belg+8arcs_396	49.0k	49.0k	0.0	1895.8	49.0k	49.0k	0.0	8298.2	49.0k	49.0k	0.0	1823.6
belg+8arcs_397	327.6k	227.1k	44.3	14400.0	1e+20	207.7k	–	14400.0	367.3k	162.5k	126.0	14400.0
belg+8arcs_398	24.4k	24.4k	0.0	703.6	24.4k	24.4k	0.0	5143.1	24.4k	24.4k	0.0	2826.5
belg+8arcs_399	4920.0	4920.0	0.0	24.6	4920.0	4920.0	0.0	48.0	4920.0	4920.0	0.0	11.6
belg+8arcs_400	182.8k	28.3k	546.2	14400.0	1e+20	31.4k	–	14400.0	90.0k	31.5k	185.4	14400.0
belg+8arcs_401	139.3k	139.3k	0.0	415.2	139.3k	139.3k	0.0	6281.4	139.3k	139.3k	0.0	561.1
belg+8arcs_402	161.3k	161.2k	0.1	1736.7	1e+20	149.2k	–	14400.0	161.3k	161.3k	0.0	400.3
belg+8arcs_403	236.6k	236.6k	0.0	3355.3	236.6k	236.6k	0.0	13734.6	236.6k	236.5k	0.1	2350.8
belg+8arcs_404	220.0k	220.0k	0.0	779.6	220.0k	220.0k	0.0	2238.7	220.0k	220.0k	0.0	2055.2
belg+8arcs_405	319.0k	319.0k	0.0	12376.7	319.0k	319.0k	0.0	13615.5	331.9k	182.6k	81.8	14400.0
belg+8arcs_406	68.6k	68.6k	0.0	2044.2	68.6k	68.6k	0.0	7691.9	68.6k	68.6k	0.0	5779.7
belg+8arcs_407	192.9k	163.9k	17.7	14400.0	192.7k	192.7k	0.0	4816.8	202.3k	171.3k	18.1	14400.0
belg+8arcs_408	36.5k	36.5k	0.0	1763.2	1e+20	16.6k	–	14400.0	36.5k	36.5k	0.0	1516.6
belg+8arcs_409	146.3k	146.3k	0.0	430.3	146.3k	146.3k	0.0	5645.2	146.3k	146.3k	0.0	2682.2
belg+8arcs_410	182.3k	182.3k	0.0	2345.8	182.3k	182.3k	0.0	5789.3	182.3k	182.3k	0.0	3940.2
belg+8arcs_411	199.9k	199.9k	0.0	4055.5	1e+20	161.3k	–	14400.0	252.7k	163.3k	54.8	14400.0
belg+8arcs_412	16.8k	16.8k	0.0	226.8	16.8k	16.8k	0.0	2631.4	16.8k	16.8k	0.0	227.5
belg+8arcs_413	169.0k	169.0k	0.0	13801.2	185.4k	155.5k	19.2	14400.0	259.7k	108.2k	140.1	14400.0
belg+8arcs_414	149.9k	149.9k	0.0	648.3	149.9k	149.9k	0.0	8725.5	149.9k	149.9k	0.0	199.7
belg+8arcs_415	20.5k	20.5k	0.0	2933.9	20.5k	20.5k	0.0	6638.6	20.5k	20.5k	0.0	719.4
belg+8arcs_416	184.5k	184.5k	0.0	520.4	184.5k	184.5k	0.0	7204.6	184.5k	184.5k	0.0	1392.4
belg+8arcs_417	156.2k	156.2k	0.0	627.8	156.2k	156.2k	0.0	6993.1	156.2k	156.2k	0.0	1879.0
belg+8arcs_418	441.5k	164.1k	169.1	14400.0	1e+20	213.6k	–	14400.0	538.8k	127.8k	321.7	14400.0
belg+8arcs_419	147.2k	147.2k	0.0	7139.5	1e+20	79.2k	–	14400.0	147.2k	80.1k	83.8	14400.0
belg+8arcs_420	142.4k	142.4k	0.0	287.4	1e+20	129.7k	–	14400.0	142.4k	142.4k	0.0	3617.9
belg+8arcs_421	121.8k	102.3k	19.0	14400.0	1e+20	54.1k	–	14400.0	136.6k	36.7k	272.1	14400.0
belg+8arcs_422	297.6k	38.5k	672.2	14400.0	1e+20	30.5k	–	14400.0	338.4k	15.2k	2124.7	14400.0
belg+8arcs_423	151.4k	151.4k	0.0	1264.9	151.4k	151.4k	0.0	8606.8	151.4k	151.4k	0.0	637.9
belg+8arcs_424	357.1k	235.7k	51.5	14400.0	319.6k	266.5k	20.0	14400.0	345.9k	236.0k	46.6	14400.0
belg+8arcs_425	223.7k	223.7k	0.0	4859.4	1e+20	189.8k	–	14400.0	225.1k	196.9k	14.4	14400.0
belg+8arcs_426	284.0k	14.1k	1917.5	14400.0	1e+20	67.7k	–	14400.0	600.8k	4494.3	Large	14400.0
belg+8arcs_427	148.0k	148.0k	0.0	380.8	567.0k	139.5k	306.5	14400.0	148.0k	148.0k	0.0	504.0
belg+8arcs_428	110.0k	57.6k	90.9	14400.0	1e+20	24.3k	–	14400.0	114.6k	30.8k	272.6	14400.0
belg+8arcs_429	408.1k	210.5k	93.9	14400.0	309.8k	234.6k	32.0	14400.0	308.7k	211.6k	45.9	14400.0
belg+8arcs_430	340.2k	202.8k	67.7	14400.0	1e+20	183.2k	–	14400.0	295.2k	160.8k	83.6	14400.0
belg+8arcs_431	512.4k	279.9k	83.1	14400.0	1e+20	308.6k	–	14400.0	842.3k	140.8k	498.2	14400.0
belg+8arcs_432	4164.0	4164.0	0.0	48.2	4164.0	4164.0	0.0	1296.9	4164.0	4164.0	0.0	18.4
belg+8arcs_433	35.3k	35.3k	0.0	783.4	164.1k	14.8k	1006.4	14400.0	35.3k	35.3k	0.0	6198.4
belg+8arcs_434	206.9k	206.9k	0.0	7967.7	206.9k	206.9k	0.0	9400.7	206.9k	206.9k	0.0	1994.2
belg+8arcs_435	216.5k	216.5k	0.0	2364.6	216.5k	197.8k	9.5	14400.0	222.9k	187.9k	18.6	14400.0
belg+8arcs_436	192.4k	192.4k	0.0	3451.6	192.4k	192.4k	0.0	10209.2	192.4k	192.4k	0.0	1894.3
belg+8arcs_437	91.2k	91.2k	0.0	1876.6	91.2k	91.2k	0.0	2408.4	173.3k	39.9k	334.9	14400.0
belg+8arcs_438	209.7k	209.7k	0.0	399.0	217.4k	202.6k	7.3	14400.0	209.7k	195.6k	7.2	14400.0
belg+8arcs_439	256.0k	180.9k	41.5	14400.0	249.9k	216.7k	15.3	14400.0	264.3k	164.3k	60.8	14400.0
belg+8arcs_440	394.7k	183.0k	115.7	14400.0	1e+20	202.6k	–	14400.0	341.9k	170.1k	101.0	14400.0
belg+8arcs_441	258.2k	150.1k	72.0	14400.0	1e+20	124.3k	–	14400.0	304.3k	107.9k	182.1	14400.0
belg+8arcs_442	13.0k	3600.0	261.8	14400.0	1e+20	3805.6	–	14400.0	13.0k	13.0k	0.0	3321.2
belg+8arcs_443	126.8k	126.8k	0.0	510.9	126.8k	126.8k	0.0	8873.4	126.8k	126.8k	0.0	771.9
belg+8arcs_444	66.3k	66.3k	0.0	6790.6	1e+20	35.2k	–	14400.0	121.9k	26.4k	362.5	14400.0
belg+8arcs_445	358.7k	242.8k	47.7	14400.0	281.6k	281.6k	0.0	10810.5	397.1k	177.3k	123.9	14400.0
belg+8arcs_446	156.1k	156.1k	0.0	2339.7	157.4k	142.1k	10.7	14400.0	156.1k	156.1k	0.0	4468.6
belg+8arcs_447	195.1k	195.1k	0.0	5756.7	195.1k	195.1k	0.0	11232.3	204.2k	160.8k	27.1	14400.0
belg+8arcs_448	259.7k	153.1k	69.6	14400.0	253.0k	173.0k	46.2	14400.0	259.7k	134.2k	93.5	14400.0
belg+8arcs_449	144.8k	144.8k	0.0	14076.7	144.8k	144.8k	0.0	8736.8	144.8k	144.7k	0.1	957.7
belg+8arcs_450	52.3k	52.3k	0.0	182.3	52.3k	52.3k	0.0	2867.0	52.3k	52.3k	0.0	495.4
belg+8arcs_451	38.3k	38.3k	0.0	107.9	38.3k	38.3k	0.0	6218.7	38.3k	38.3k	0.0	631.4
belg+8arcs_452	183.0k	183.0k	0.0	5650.9	183.0k	183.0k	0.0	5169.3	183.0k	183.0k	0.0	496.4
belg+8arcs_453	265.7k	265.7k	0.0	759.0	265.7k	265.7k	0.0	5476.1	265.7k	265.6k	0.0	835.9
belg+8arcs_454	99.2k	95.8k	3.5	14400.0	1e+20	46.5k	–	14400.0	107.7k	28.1k	284.0	14400.0
belg+8arcs_455	94.4k	31.0k	204.2	14400.0	1e+20	24.4k	–	14400.0	46.4k	46.4k	0.0	13895.7
belg+8arcs_456	352.1k	164.0k	114.6	14400.0	1e+20	156.7k	–	14400.0	388.5k	118.4k	228.2	14400.0
belg+8arcs_457	108.9k	108.9k	0.0	892.2	108.9k	108.9k	0.0	7052.1	108.9k	108.8k	0.1	343.9
belg+8arcs_458	260.5k	239.9k	8.6	14400.0	260.5k	260.3k	0.1	5553.2	260.5k	238.8k	9.1	14400.0
belg+8arcs_459	0.0	0.0	0.0	0.1	0.0	0.0	0.0	5699.6	0.0	0.0	0.0	0.2
belg+8arcs_460	3600.0	3600.0	0.0	12.3	3600.0	3600.0	0.0	372.9	3600.0	3600.0	0.0	27.3
belg+8arcs_461	212.3k	212.3k	0.0	4981.1	502.6k	178.7k	181.3	14400.0	219.0k	187.1k	17.1	14400.0
belg+8arcs_462	155.9k	155.9k	0.0	11892.7	155.9k	155.9k	0.0	10848.2	155.9k	155.9k	0.0	11608.7
belg+8arcs_463	137.5k	137.5k	0.0	68.6	137.5k	137.5k	0.0	1942.5	137.5k	137.5k	0.0	622.4
belg+8arcs_464	161.1k	138.8k	16.1	14400.0	154.4k	154.4k	0.0	12546.3	161.8k	135.0k	19.9	14400.0

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Table A.31: Comparison of discrete models on *Circuit rank* + 8 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_465	4920.0	4920.0	0.0	22.3	4920.0	4920.0	0.0	38.6	4920.0	4920.0	0.0	9.4
belg+8arcs_466	86.6k	86.6k	0.0	128.4	86.6k	86.6k	0.0	4132.9	86.6k	86.6k	0.0	580.6
belg+8arcs_467	188.7k	188.7k	0.0	520.5	188.7k	188.7k	0.0	4215.8	188.7k	188.7k	0.0	2669.2
belg+8arcs_468	269.0k	69.9k	284.7	14400.0	181.9k	115.5k	57.5	14400.0	289.3k	4516.0	6306.5	14400.0
belg+8arcs_469	110.3k	110.3k	0.0	1614.9	110.3k	110.3k	0.0	13146.1	110.3k	110.3k	0.0	1616.0
belg+8arcs_470	24.1k	24.1k	0.0	157.8	1e+20	17.6k	–	14400.0	24.1k	24.1k	0.0	368.2
belg+8arcs_471	177.8k	177.6k	0.1	373.6	177.8k	177.7k	0.1	11672.3	177.8k	177.8k	0.0	5185.4
belg+8arcs_472	102.6k	102.6k	0.0	5424.7	667.5k	22.2k	2909.1	14400.0	106.6k	25.7k	315.0	14400.0
belg+8arcs_473	254.9k	254.9k	0.0	11339.1	1e+20	201.0k	–	14400.0	264.7k	161.2k	64.2	14400.0
belg+8arcs_474	109.9k	109.9k	0.0	980.7	109.9k	89.9k	22.2	14400.0	113.5k	30.4k	273.2	14400.0
belg+8arcs_475	128.3k	128.3k	0.0	1001.2	128.3k	128.3k	0.0	3917.0	128.3k	128.3k	0.0	535.4
belg+8arcs_476	208.7k	208.7k	0.0	1428.4	221.7k	201.5k	10.0	14400.0	208.7k	208.5k	0.1	3241.6
belg+8arcs_477	319.7k	189.6k	68.6	14400.0	231.1k	202.5k	14.1	14400.0	385.1k	151.1k	154.9	14400.0
belg+8arcs_478	203.5k	203.5k	0.0	2400.3	203.5k	194.7k	4.5	14400.0	215.5k	158.7k	35.8	14400.0
belg+8arcs_479	174.4k	150.9k	15.6	14400.0	156.5k	156.5k	0.0	9435.3	156.5k	156.5k	0.0	13517.9
belg+8arcs_480	166.9k	122.1k	36.7	14400.0	149.3k	129.0k	15.8	14400.0	166.9k	118.7k	40.6	14400.0
belg+8arcs_481	340.9k	180.2k	89.2	14400.0	1e+20	199.7k	–	14400.0	288.9k	147.7k	95.6	14400.0
belg+8arcs_482	178.9k	178.9k	0.0	1714.7	1e+20	156.7k	–	14400.0	178.9k	178.9k	0.0	4874.7
belg+8arcs_483	41.0k	41.0k	0.0	1017.9	41.0k	41.0k	0.0	8219.2	41.0k	41.0k	0.0	3938.3
belg+8arcs_484	64.2k	64.2k	0.0	4417.5	1e+20	21.2k	–	14400.0	93.6k	21.9k	327.0	14400.0
belg+8arcs_485	57.3k	57.3k	0.0	4671.2	88.7k	44.3k	100.2	14400.0	59.4k	40.8k	45.7	14400.0
belg+8arcs_486	244.0k	244.0k	0.0	7706.7	264.0k	230.5k	14.6	14400.0	248.8k	218.1k	14.0	14400.0
belg+8arcs_487	157.5k	157.5k	0.0	393.3	157.5k	157.5k	0.0	10037.2	157.5k	157.5k	0.0	7093.2
belg+8arcs_488	185.1k	185.1k	0.0	1620.9	185.1k	185.1k	0.0	1012.2	185.1k	185.0k	0.0	760.6
belg+8arcs_489	179.1k	179.1k	0.0	5312.0	191.8k	150.6k	27.3	14400.0	236.2k	152.4k	55.0	14400.0
belg+8arcs_490	133.5k	133.5k	0.0	3981.0	1e+20	114.2k	–	14400.0	133.5k	133.5k	0.0	3585.5
belg+8arcs_491	123.0k	123.0k	0.0	1296.9	123.0k	123.0k	0.0	4351.6	132.7k	75.1k	76.7	14400.0
belg+8arcs_492	116.0k	116.0k	0.0	1652.3	116.0k	116.0k	0.0	2668.4	116.0k	116.0k	0.0	1490.7
belg+8arcs_493	228.5k	228.5k	0.0	3221.5	1e+20	177.8k	–	14400.0	287.1k	180.4k	59.1	14400.0
belg+8arcs_494	197.5k	197.5k	0.0	127.0	197.5k	197.5k	0.0	7641.7	197.5k	197.5k	0.0	21.2
belg+8arcs_495	146.4k	146.4k	0.0	8841.0	146.4k	146.4k	0.0	6669.0	146.4k	146.4k	0.0	2939.1
belg+8arcs_496	128.3k	128.3k	0.0	5457.0	1e+20	89.0k	–	14400.0	130.3k	100.9k	29.1	14400.0
belg+8arcs_497	54.8k	54.8k	0.0	5550.3	54.8k	54.8k	0.0	5672.4	56.9k	38.1k	49.3	14400.0
belg+8arcs_498	366.5k	234.6k	56.2	14400.0	278.6k	278.6k	0.0	8161.9	278.6k	237.6k	17.3	14400.0
belg+8arcs_499	207.6k	192.1k	8.1	14400.0	1e+20	176.4k	–	14400.0	205.4k	205.4k	0.0	4200.8
belg+8arcs_500	147.7k	147.7k	0.0	1406.3	147.7k	147.7k	0.0	6711.1	183.9k	120.9k	52.0	14400.0

Table A.32: Detailed results of the split-pipe models on *Circuit rank* + 8 arcs, as summarized in Figure 3.16e and Table 3.4d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_1	357.1k	232.0k	53.9	14400.0	353.4k	233.3k	51.5	14400.0
belg+8arcs_2	146.9k	142.6k	3.1	14400.0	146.9k	146.8k	0.1	75.1
belg+8arcs_3	108.8k	76.0k	43.1	14400.0	109.1k	63.3k	72.4	14400.0
belg+8arcs_4	212.9k	204.9k	3.9	14400.0	212.9k	212.5k	0.2	14400.0
belg+8arcs_5	350.5k	267.4k	31.1	14400.0	341.8k	281.0k	21.6	14400.0
belg+8arcs_6	199.7k	187.8k	6.3	14400.0	199.7k	188.4k	6.0	14400.0
belg+8arcs_7	278.2k	223.2k	24.7	14400.0	278.2k	275.2k	1.1	14400.0
belg+8arcs_8	4164.0	3119.8	33.5	14400.0	4164.0	4160.2	0.1	7.7
belg+8arcs_9	226.5k	122.0k	85.7	14400.0	221.4k	172.8k	28.1	14400.0
belg+8arcs_10	191.9k	191.4k	0.2	14400.0	191.9k	191.7k	0.1	621.1
belg+8arcs_11	239.2k	186.6k	28.2	14400.0	229.6k	223.2k	2.9	14400.0
belg+8arcs_12	38.2k	37.8k	1.2	14400.0	38.2k	38.2k	0.1	961.4
belg+8arcs_13	198.9k	193.7k	2.7	14400.0	198.9k	198.1k	0.4	14400.0
belg+8arcs_14	506.1k	279.9k	80.9	14400.0	495.6k	256.3k	93.4	14400.0
belg+8arcs_15	1135.9	572.9	98.3	14400.0	1135.9	1135.9	0.0	4.9
belg+8arcs_16	415.2k	273.8k	51.6	14400.0	407.4k	317.9k	28.1	14400.0
belg+8arcs_17	208.2k	197.6k	5.3	14400.0	208.2k	200.6k	3.8	14400.0
belg+8arcs_18	221.8k	200.8k	10.5	14400.0	221.8k	207.5k	6.9	14400.0
belg+8arcs_19	159.0k	152.8k	4.1	14400.0	159.0k	158.9k	0.1	65.3
belg+8arcs_20	255.4k	244.8k	4.4	14400.0	255.4k	255.2k	0.1	330.5
belg+8arcs_21	45.0k	45.0k	0.0	6527.5	45.0k	44.9k	0.1	33.3
belg+8arcs_22	190.3k	167.1k	13.8	14400.0	190.3k	189.8k	0.3	14400.0
belg+8arcs_23	72.8k	4584.4	1488.1	14400.0	72.8k	72.7k	0.1	9158.5
belg+8arcs_24	207.7k	205.8k	0.9	14400.0	207.7k	203.5k	2.1	14400.0
belg+8arcs_25	112.8k	48.6k	132.0	14400.0	112.8k	90.0k	25.3	14400.0
belg+8arcs_26	170.5k	165.4k	3.1	14400.0	170.5k	170.4k	0.1	166.1
belg+8arcs_27	125.9k	121.8k	3.4	14400.0	125.9k	125.3k	0.5	14400.0

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Table A.32: Comparison of split-pipe models on *Circuit rank* + 8 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_28	239.3k	229.2k	4.4	14400.0	235.5k	231.5k	1.8	14400.0
belg+8arcs_29	2749.0	2749.0	0.0	479.7	2749.0	2746.3	0.1	4.2
belg+8arcs_30	41.9k	16.9k	148.5	14400.0	41.9k	41.8k	0.1	7321.7
belg+8arcs_31	142.5k	106.4k	34.0	14400.0	142.5k	111.5k	27.9	14400.0
belg+8arcs_32	272.1k	212.7k	27.9	14400.0	272.1k	210.3k	29.4	14400.0
belg+8arcs_33	43.1k	42.0k	2.6	14400.0	43.1k	43.1k	0.0	768.6
belg+8arcs_34	19.1k	18.8k	1.2	14400.0	19.1k	19.0k	0.1	7.3
belg+8arcs_35	191.5k	175.3k	9.3	14400.0	183.8k	183.6k	0.1	1712.8
belg+8arcs_36	197.3k	164.8k	19.7	14400.0	197.3k	166.1k	18.8	14400.0
belg+8arcs_37	46.4k	35.2k	31.9	14400.0	46.4k	43.6k	6.4	14400.0
belg+8arcs_38	312.5k	253.1k	23.5	14400.0	312.5k	252.8k	23.6	14400.0
belg+8arcs_39	68.9k	19.3k	256.5	14400.0	68.9k	66.4k	3.8	14400.0
belg+8arcs_40	163.8k	73.8k	122.2	14400.0	164.0k	70.0k	134.1	14400.0
belg+8arcs_41	293.5k	238.8k	22.9	14400.0	276.8k	269.8k	2.6	14400.0
belg+8arcs_42	214.0k	213.8k	0.1	14400.0	214.0k	212.7k	0.6	14400.0
belg+8arcs_43	71.3k	30.7k	132.6	14400.0	71.3k	52.3k	36.5	14400.0
belg+8arcs_44	155.5k	45.0k	245.5	14400.0	155.5k	155.4k	0.1	8.7
belg+8arcs_45	18.0k	11.1k	63.0	14400.0	18.0k	18.0k	0.0	13.4
belg+8arcs_46	185.0k	179.0k	3.3	14400.0	182.6k	169.3k	7.9	14400.0
belg+8arcs_47	222.9k	222.2k	0.3	14400.0	222.9k	220.9k	0.9	14400.0
belg+8arcs_48	221.5k	207.5k	6.7	14400.0	221.5k	213.9k	3.5	14400.0
belg+8arcs_49	18.5k	14.5k	27.6	14400.0	18.5k	18.5k	0.1	64.9
belg+8arcs_50	261.6k	226.8k	15.3	14400.0	253.1k	245.6k	3.1	14400.0
belg+8arcs_51	15.6k	15.4k	1.3	14400.0	15.6k	15.6k	0.1	16.6
belg+8arcs_52	187.6k	165.1k	13.7	14400.0	187.6k	165.9k	13.1	14400.0
belg+8arcs_53	135.1k	127.7k	5.8	14400.0	135.1k	135.0k	0.1	75.6
belg+8arcs_54	173.7k	172.6k	0.6	14400.0	173.7k	173.5k	0.1	5.9
belg+8arcs_55	194.3k	193.0k	0.6	14400.0	194.3k	194.3k	0.0	454.0
belg+8arcs_56	158.4k	4848.2	3168.0	14400.0	158.4k	154.1k	2.8	14400.0
belg+8arcs_57	220.6k	138.2k	59.6	14400.0	220.6k	216.3k	2.0	14400.0
belg+8arcs_58	127.1k	75.3k	68.8	14400.0	127.1k	90.0k	41.3	14400.0
belg+8arcs_59	19.4k	17.9k	8.3	14400.0	19.4k	19.4k	0.1	76.2
belg+8arcs_60	129.3k	121.5k	6.4	14400.0	129.3k	128.8k	0.4	14400.0
belg+8arcs_61	140.4k	122.4k	14.7	14400.0	140.4k	126.1k	11.4	14400.0
belg+8arcs_62	349.3k	248.0k	40.8	14400.0	306.3k	235.3k	30.2	14400.0
belg+8arcs_63	224.0k	208.3k	7.6	14400.0	224.0k	223.8k	0.1	198.7
belg+8arcs_64	112.3k	68.2k	64.7	14400.0	112.3k	73.3k	53.2	14400.0
belg+8arcs_65	189.2k	133.0k	42.3	14400.0	189.2k	158.6k	19.3	14400.0
belg+8arcs_66	320.4k	190.4k	68.3	14400.0	304.1k	189.9k	60.2	14400.0
belg+8arcs_67	173.7k	172.4k	0.8	14400.0	173.7k	173.5k	0.1	782.9
belg+8arcs_68	234.7k	214.0k	9.7	14400.0	234.7k	214.1k	9.6	14400.0
belg+8arcs_69	269.4k	219.2k	22.9	14400.0	266.9k	228.0k	17.1	14400.0
belg+8arcs_70	199.9k	173.6k	15.1	14400.0	193.8k	176.6k	9.7	14400.0
belg+8arcs_71	160.5k	147.6k	8.8	14400.0	160.5k	140.9k	13.9	14400.0
belg+8arcs_72	25.1k	25.1k	0.1	625.6	25.1k	25.1k	0.1	40.5
belg+8arcs_73	22.5k	19.5k	15.3	14400.0	22.5k	22.5k	0.1	6.4
belg+8arcs_74	482.5k	289.4k	66.7	14400.0	408.9k	244.4k	67.3	14400.0
belg+8arcs_75	184.0k	172.2k	6.9	14400.0	184.0k	183.6k	0.2	14400.0
belg+8arcs_76	43.8k	40.0k	9.5	14400.0	43.8k	43.8k	0.0	99.9
belg+8arcs_77	68.7k	21.1k	225.5	14400.0	68.7k	68.6k	0.1	228.2
belg+8arcs_78	47.1k	43.5k	8.2	14400.0	47.1k	47.0k	0.1	19.3
belg+8arcs_79	48.9k	21.8k	123.9	14400.0	48.9k	40.8k	20.0	14400.0
belg+8arcs_80	142.7k	136.1k	4.8	14400.0	142.7k	142.5k	0.1	6862.0
belg+8arcs_81	221.3k	186.5k	18.7	14400.0	221.3k	187.8k	17.8	14400.0
belg+8arcs_82	19.3k	17.4k	11.4	14400.0	19.3k	19.3k	0.0	24.4
belg+8arcs_83	445.1k	238.7k	86.5	14400.0	371.2k	257.4k	44.2	14400.0
belg+8arcs_84	354.0k	268.8k	31.7	14400.0	356.2k	222.7k	59.9	14400.0
belg+8arcs_85	184.2k	180.6k	2.0	14400.0	184.2k	170.1k	8.3	14400.0
belg+8arcs_86	340.8k	242.1k	40.7	14400.0	291.5k	206.6k	41.1	14400.0
belg+8arcs_87	217.2k	209.4k	3.8	14400.0	215.8k	211.0k	2.3	14400.0
belg+8arcs_88	280.7k	211.9k	32.4	14400.0	241.0k	207.1k	16.4	14400.0
belg+8arcs_89	577.5k	395.4k	46.1	14400.0	562.3k	334.7k	68.0	14400.0
belg+8arcs_90	300.5k	196.6k	52.8	14400.0	300.5k	249.6k	20.4	14400.0
belg+8arcs_91	287.7k	176.2k	63.3	14400.0	275.3k	180.2k	52.8	14400.0
belg+8arcs_92	360.5k	250.3k	44.0	14400.0	311.2k	272.5k	14.2	14400.0
belg+8arcs_93	407.5k	360.3k	13.1	14400.0	407.5k	279.3k	45.9	14400.0
belg+8arcs_94	180.9k	179.0k	1.1	14400.0	180.9k	180.4k	0.3	14400.0
belg+8arcs_95	42.1k	39.9k	5.7	14400.0	42.1k	42.1k	0.1	2142.8
belg+8arcs_96	187.0k	164.9k	13.4	14400.0	182.2k	181.9k	0.2	14400.0
belg+8arcs_97	81.4k	43.7k	86.4	14400.0	81.4k	48.7k	67.1	14400.0
belg+8arcs_98	204.1k	202.0k	1.0	14400.0	204.1k	184.7k	10.5	14400.0
belg+8arcs_99	288.1k	200.3k	43.8	14400.0	288.1k	236.6k	21.7	14400.0
belg+8arcs_100	169.9k	165.9k	2.4	14400.0	169.9k	169.8k	0.1	64.5
belg+8arcs_101	211.8k	176.8k	19.8	14400.0	211.8k	179.7k	17.8	14400.0
belg+8arcs_102	192.8k	162.4k	18.7	14400.0	192.7k	192.5k	0.1	4259.4
belg+8arcs_103	101.8k	3398.3	2894.7	14400.0	101.8k	68.3k	49.1	14400.0
belg+8arcs_104	176.3k	147.7k	19.3	14400.0	176.3k	168.7k	4.5	14400.0
belg+8arcs_105	83.3k	60.8k	37.0	14400.0	83.3k	76.3k	9.2	14400.0

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Table A.32: Comparison of split-pipe models on *Circuit rank* + 8 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_106	158.0k	13.1	Large	14400.0	157.1k	148.2k	6.0	14400.0
belg+8arcs_107	203.7k	193.7k	5.1	14400.0	203.7k	199.0k	2.3	14400.0
belg+8arcs_108	28.9k	23.5k	23.1	14400.0	28.9k	28.9k	0.1	16.1
belg+8arcs_109	138.7k	104.8k	32.3	14400.0	138.7k	112.9k	22.9	14400.0
belg+8arcs_110	198.1k	162.3k	22.1	14400.0	198.1k	190.5k	4.0	14400.0
belg+8arcs_111	308.7k	216.3k	42.7	14400.0	316.5k	180.6k	75.2	14400.0
belg+8arcs_112	114.7k	5557.3	1964.7	14400.0	114.7k	60.1k	90.8	14400.0
belg+8arcs_113	2420.3	1827.0	32.5	14400.0	2420.3	2419.2	0.0	2.6
belg+8arcs_114	3564.0	3561.0	0.1	2.3	3564.0	3560.9	0.1	2.0
belg+8arcs_115	282.5k	222.5k	26.9	14400.0	259.9k	211.8k	22.7	14400.0
belg+8arcs_116	163.0k	134.5k	21.1	14400.0	160.7k	159.8k	0.6	14400.0
belg+8arcs_117	99.8k	33.9k	194.0	14400.0	99.8k	54.6k	82.6	14400.0
belg+8arcs_118	148.6k	148.4k	0.1	4516.2	148.6k	148.5k	0.1	30.2
belg+8arcs_119	163.8k	158.9k	3.1	14400.0	163.8k	158.3k	3.5	14400.0
belg+8arcs_120	51.6k	45.0k	14.8	14400.0	51.6k	40.6k	27.0	14400.0
belg+8arcs_121	128.8k	127.3k	1.2	14400.0	128.8k	128.7k	0.1	48.4
belg+8arcs_122	169.9k	160.0k	6.2	14400.0	168.2k	168.0k	0.1	446.2
belg+8arcs_123	20.3k	17.7k	14.4	14400.0	20.3k	20.3k	0.1	10.5
belg+8arcs_124	183.4k	173.4k	5.8	14400.0	183.4k	181.3k	1.2	14400.0
belg+8arcs_125	17.5k	16.6k	5.3	14400.0	17.5k	17.4k	0.1	3.6
belg+8arcs_126	87.1k	14.1k	518.9	14400.0	87.1k	87.0k	0.1	1806.0
belg+8arcs_127	182.5k	161.6k	12.9	14400.0	182.5k	182.2k	0.2	14400.0
belg+8arcs_128	172.4k	146.6k	17.6	14400.0	172.4k	157.2k	9.7	14400.0
belg+8arcs_129	15.6k	3152.1	393.8	14400.0	15.6k	15.6k	0.1	5.0
belg+8arcs_130	165.5k	135.3k	22.3	14400.0	165.5k	162.8k	1.6	14400.0
belg+8arcs_131	272.3k	187.1k	45.5	14400.0	243.3k	203.5k	19.6	14400.0
belg+8arcs_132	124.5k	118.0k	5.5	14400.0	124.5k	124.4k	0.1	333.2
belg+8arcs_133	250.0k	184.8k	35.3	14400.0	239.3k	206.0k	16.2	14400.0
belg+8arcs_134	404.0k	318.2k	27.0	14400.0	354.8k	226.1k	56.9	14400.0
belg+8arcs_135	157.1k	147.9k	6.2	14400.0	153.3k	153.1k	0.1	751.1
belg+8arcs_136	668.9k	371.4k	80.1	14400.0	572.1k	394.8k	44.9	14400.0
belg+8arcs_137	339.6k	313.6k	8.3	14400.0	339.6k	235.5k	44.2	14400.0
belg+8arcs_138	178.2k	177.6k	0.4	14400.0	178.2k	178.1k	0.1	200.1
belg+8arcs_139	174.3k	154.1k	13.1	14400.0	174.3k	163.7k	6.5	14400.0
belg+8arcs_140	25.1k	10.1k	148.8	14400.0	25.1k	25.1k	0.1	212.6
belg+8arcs_141	173.8k	160.4k	8.3	14400.0	173.8k	161.8k	7.4	14400.0
belg+8arcs_142	208.0k	182.0k	14.3	14400.0	208.0k	206.5k	0.8	14400.0
belg+8arcs_143	99.4k	91.1k	9.1	14400.0	99.4k	97.0k	2.5	14400.0
belg+8arcs_144	31.1k	2554.4	1117.3	14400.0	31.1k	27.2k	14.2	14400.0
belg+8arcs_145	122.0k	83.3k	46.5	14400.0	122.0k	111.8k	9.1	14400.0
belg+8arcs_146	143.0k	105.6k	35.4	14400.0	143.0k	103.1k	38.7	14400.0
belg+8arcs_147	193.2k	171.7k	12.5	14400.0	183.2k	183.0k	0.1	726.2
belg+8arcs_148	208.5k	142.2k	46.6	14400.0	208.5k	172.8k	20.6	14400.0
belg+8arcs_149	210.2k	178.9k	17.5	14400.0	210.2k	192.1k	9.4	14400.0
belg+8arcs_150	341.5k	339.3k	0.6	14400.0	365.9k	286.1k	27.9	14400.0
belg+8arcs_151	54.0k	53.8k	0.4	14400.0	54.0k	54.0k	0.1	38.1
belg+8arcs_152	187.9k	175.3k	7.2	14400.0	187.9k	180.7k	4.0	14400.0
belg+8arcs_153	209.7k	187.9k	11.6	14400.0	209.7k	183.4k	14.3	14400.0
belg+8arcs_154	183.4k	159.5k	15.0	14400.0	183.4k	168.7k	8.7	14400.0
belg+8arcs_155	302.9k	261.9k	15.7	14400.0	280.9k	265.8k	5.7	14400.0
belg+8arcs_156	155.9k	153.3k	1.7	14400.0	155.9k	155.7k	0.1	3858.3
belg+8arcs_157	149.7k	146.7k	2.1	14400.0	149.7k	149.7k	0.0	722.9
belg+8arcs_158	57.3k	30.0k	90.9	14400.0	57.3k	57.3k	0.0	937.9
belg+8arcs_159	199.2k	46.8k	326.0	14400.0	196.7k	104.1k	88.9	14400.0
belg+8arcs_160	2629.9	1978.7	32.9	14400.0	2629.9	2627.7	0.1	4.4
belg+8arcs_161	264.8k	196.2k	34.9	14400.0	209.3k	196.7k	6.4	14400.0
belg+8arcs_162	268.4k	200.7k	33.7	14400.0	244.7k	244.5k	0.1	8459.5
belg+8arcs_163	247.7k	200.0k	23.8	14400.0	228.2k	191.5k	19.2	14400.0
belg+8arcs_164	0.0	0.0	0.0	19.2	0.0	0.0	0.0	0.7
belg+8arcs_165	141.3k	135.4k	4.4	14400.0	141.3k	141.2k	0.1	47.8
belg+8arcs_166	3997.4	3808.1	5.0	14400.0	3997.4	3994.0	0.1	7.2
belg+8arcs_167	195.3k	80.1k	144.0	14400.0	195.3k	186.4k	4.8	14400.0
belg+8arcs_168	360.6k	270.1k	33.5	14400.0	296.3k	246.7k	20.1	14400.0
belg+8arcs_169	117.9k	104.9k	12.4	14400.0	117.9k	117.9k	0.0	156.8
belg+8arcs_170	171.0k	169.6k	0.8	14400.0	171.0k	170.8k	0.1	216.2
belg+8arcs_171	197.6k	170.4k	16.0	14400.0	197.6k	155.7k	26.9	14400.0
belg+8arcs_172	2874.4	2436.5	18.0	14400.0	2874.4	2872.0	0.1	2.6
belg+8arcs_173	293.8k	222.4k	32.1	14400.0	290.8k	270.2k	7.6	14400.0
belg+8arcs_174	138.8k	138.3k	0.3	14400.0	138.8k	138.6k	0.1	21.2
belg+8arcs_175	36.2k	26.4k	37.3	14400.0	36.2k	36.2k	0.0	515.4
belg+8arcs_176	111.4k	89.4k	24.6	14400.0	111.4k	53.9k	106.6	14400.0
belg+8arcs_177	221.5k	219.3k	1.0	14400.0	221.5k	221.3k	0.1	112.0
belg+8arcs_178	170.1k	148.8k	14.3	14400.0	170.1k	167.1k	1.8	14400.0
belg+8arcs_179	243.7k	218.9k	11.3	14400.0	243.7k	242.8k	0.3	14400.0
belg+8arcs_180	41.7k	36.6k	13.8	14400.0	41.7k	41.6k	0.1	155.4
belg+8arcs_181	17.0k	13.1k	29.0	14400.0	17.0k	16.9k	0.1	9.7
belg+8arcs_182	115.5k	107.7k	7.3	14400.0	115.5k	100.3k	15.1	14400.0
belg+8arcs_183	107.1k	84.7k	26.5	14400.0	107.1k	107.0k	0.1	6687.9

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Table A.32: Comparison of split-pipe models on *Circuit rank* + 8 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_184	14.8k	14.8k	0.1	13.1	14.8k	14.8k	0.1	12.7
belg+8arcs_185	90.4k	76.1k	18.7	14400.0	90.4k	67.2k	34.6	14400.0
belg+8arcs_186	161.7k	161.4k	0.2	14400.0	161.7k	161.6k	0.1	9.7
belg+8arcs_187	374.2k	328.1k	14.0	14400.0	374.2k	298.0k	25.6	14400.0
belg+8arcs_188	13.4k	11.3k	18.4	14400.0	13.4k	13.4k	0.1	14.0
belg+8arcs_189	159.2k	156.4k	1.8	14400.0	159.2k	159.0k	0.1	8.9
belg+8arcs_190	150.9k	51.2k	194.6	14400.0	151.0k	35.4k	326.9	14400.0
belg+8arcs_191	252.5k	239.9k	5.2	14400.0	252.5k	252.3k	0.1	13.4
belg+8arcs_192	126.6k	98.7k	28.3	14400.0	126.6k	114.6k	10.4	14400.0
belg+8arcs_193	138.5k	135.8k	2.0	14400.0	138.5k	138.4k	0.1	21.9
belg+8arcs_194	282.0k	182.6k	54.5	14400.0	294.7k	179.1k	64.6	14400.0
belg+8arcs_195	180.5k	179.8k	0.4	14400.0	180.5k	180.4k	0.1	38.7
belg+8arcs_196	299.6k	172.4k	73.8	14400.0	307.0k	237.8k	29.1	14400.0
belg+8arcs_197	335.6k	213.7k	57.1	14400.0	294.9k	247.3k	19.3	14400.0
belg+8arcs_198	139.6k	83.4k	67.4	14400.0	139.6k	139.5k	0.1	29.2
belg+8arcs_199	30.7k	29.0k	5.9	14400.0	30.7k	30.7k	0.1	25.1
belg+8arcs_200	222.0k	186.5k	19.0	14400.0	206.6k	190.8k	8.3	14400.0
belg+8arcs_201	67.8k	17.9k	279.7	14400.0	67.8k	67.8k	0.0	7649.2
belg+8arcs_202	137.6k	136.4k	0.9	14400.0	137.6k	137.3k	0.2	14400.0
belg+8arcs_203	105.7k	27.2k	289.2	14400.0	105.7k	47.9k	120.6	14400.0
belg+8arcs_204	23.1k	23.1k	0.1	2.4	23.1k	23.1k	0.1	3.2
belg+8arcs_205	241.0k	238.7k	0.9	14400.0	241.0k	216.8k	11.1	14400.0
belg+8arcs_206	89.7k	76.5k	17.2	14400.0	89.7k	89.7k	0.0	11819.6
belg+8arcs_207	223.8k	181.8k	23.1	14400.0	220.1k	219.9k	0.1	5528.0
belg+8arcs_208	187.8k	184.7k	1.6	14400.0	187.8k	183.9k	2.1	14400.0
belg+8arcs_209	88.2k	22.8k	287.0	14400.0	88.2k	68.7k	28.4	14400.0
belg+8arcs_210	190.4k	186.3k	2.2	14400.0	190.4k	190.2k	0.1	4852.1
belg+8arcs_211	136.7k	94.9k	44.0	14400.0	136.7k	121.5k	12.5	14400.0
belg+8arcs_212	107.1k	70.4k	52.0	14400.0	107.1k	107.0k	0.1	559.0
belg+8arcs_213	277.9k	229.4k	21.1	14400.0	252.7k	218.6k	15.6	14400.0
belg+8arcs_214	216.8k	193.2k	12.2	14400.0	216.8k	216.6k	0.1	6205.9
belg+8arcs_215	178.1k	175.7k	1.4	14400.0	178.1k	162.6k	9.6	14400.0
belg+8arcs_216	223.7k	219.7k	1.9	14400.0	223.7k	218.1k	2.6	14400.0
belg+8arcs_217	129.3k	125.9k	2.7	14400.0	129.3k	129.2k	0.1	19.1
belg+8arcs_218	322.5k	180.5k	78.6	14400.0	290.4k	265.7k	9.3	14400.0
belg+8arcs_219	202.9k	196.9k	3.0	14400.0	202.9k	198.9k	2.0	14400.0
belg+8arcs_220	304.3k	237.4k	28.2	14400.0	304.3k	174.2k	74.7	14400.0
belg+8arcs_221	13.0k	12.9k	0.1	16.9	13.0k	13.0k	0.0	58.5
belg+8arcs_222	234.5k	211.2k	11.0	14400.0	224.4k	210.3k	6.7	14400.0
belg+8arcs_223	180.7k	175.5k	3.0	14400.0	180.7k	176.3k	2.5	14400.0
belg+8arcs_224	228.1k	216.3k	5.5	14400.0	228.1k	203.2k	12.3	14400.0
belg+8arcs_225	3805.6	3254.9	16.9	14400.0	3805.6	3803.6	0.1	4.8
belg+8arcs_226	169.6k	156.9k	8.1	14400.0	168.6k	153.1k	10.1	14400.0
belg+8arcs_227	28.7k	25.2k	14.2	14400.0	28.7k	28.7k	0.1	326.6
belg+8arcs_228	296.0k	195.1k	51.7	14400.0	296.0k	197.6k	49.8	14400.0
belg+8arcs_229	187.1k	183.5k	1.9	14400.0	187.1k	175.3k	6.7	14400.0
belg+8arcs_230	177.4k	176.4k	0.5	14400.0	177.4k	177.2k	0.1	339.6
belg+8arcs_231	24.8k	23.7k	4.5	14400.0	24.8k	24.7k	0.1	7.7
belg+8arcs_232	127.9k	123.9k	3.2	14400.0	127.9k	127.8k	0.1	21.2
belg+8arcs_233	187.1k	185.2k	1.0	14400.0	187.1k	145.7k	28.4	14400.0
belg+8arcs_234	47.8k	37.8k	26.5	14400.0	47.8k	47.8k	0.1	7370.5
belg+8arcs_235	175.7k	172.5k	1.8	14400.0	175.7k	175.5k	0.1	13392.6
belg+8arcs_236	183.2k	173.8k	5.4	14400.0	183.2k	166.5k	10.0	14400.0
belg+8arcs_237	2120.5	2037.5	4.1	14400.0	2120.5	2119.0	0.1	3.8
belg+8arcs_238	344.9k	229.3k	50.4	14400.0	344.9k	282.4k	22.1	14400.0
belg+8arcs_239	275.6k	175.3k	57.2	14400.0	231.7k	176.9k	31.0	14400.0
belg+8arcs_240	234.5k	225.5k	4.0	14400.0	234.5k	214.1k	9.5	14400.0
belg+8arcs_241	231.6k	205.0k	12.9	14400.0	218.4k	179.5k	21.7	14400.0
belg+8arcs_242	161.1k	137.0k	17.6	14400.0	161.1k	115.0k	40.0	14400.0
belg+8arcs_243	182.6k	133.6k	36.7	14400.0	182.6k	182.5k	0.1	9.9
belg+8arcs_244	182.0k	168.6k	7.9	14400.0	182.0k	181.8k	0.1	21.9
belg+8arcs_245	25.5k	15.9k	60.1	14400.0	25.5k	25.5k	0.0	22.9
belg+8arcs_246	164.8k	156.1k	5.6	14400.0	164.8k	158.6k	3.9	14400.0
belg+8arcs_247	24.5k	7042.3	248.1	14400.0	24.5k	24.5k	0.1	474.3
belg+8arcs_248	310.4k	277.5k	11.9	14400.0	310.4k	298.5k	4.0	14400.0
belg+8arcs_249	82.4k	32.3k	155.4	14400.0	82.4k	44.4k	85.6	14400.0
belg+8arcs_250	211.6k	193.1k	9.6	14400.0	211.6k	182.3k	16.1	14400.0
belg+8arcs_251	176.7k	167.9k	5.2	14400.0	176.7k	176.7k	0.0	2300.8
belg+8arcs_252	142.4k	96.5k	47.7	14400.0	142.4k	114.2k	24.7	14400.0
belg+8arcs_253	273.6k	209.6k	30.6	14400.0	255.7k	253.1k	1.0	14400.0
belg+8arcs_254	241.5k	199.9k	20.8	14400.0	241.5k	216.3k	11.6	14400.0
belg+8arcs_255	22.8k	20.2k	12.8	14400.0	22.8k	22.8k	0.1	11.5
belg+8arcs_256	180.4k	174.9k	3.1	14400.0	180.4k	180.2k	0.1	653.2
belg+8arcs_257	94.2k	48.2k	95.5	14400.0	94.1k	86.8k	8.3	14400.0
belg+8arcs_258	170.2k	168.8k	0.8	14400.0	170.2k	165.4k	2.9	14400.0
belg+8arcs_259	167.6k	166.9k	0.4	14400.0	167.6k	167.5k	0.1	93.5
belg+8arcs_260	36.3k	33.2k	9.4	14400.0	36.3k	36.3k	0.1	3.6
belg+8arcs_261	17.6k	16.0k	10.2	14400.0	17.6k	17.6k	0.1	7.5

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Table A.32: Comparison of split-pipe models on *Circuit rank* + 8 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_262	18.2k	18.1k	1.1	14400.0	18.2k	18.2k	0.1	9.1
belg+8arcs_263	252.1k	241.5k	4.4	14400.0	252.1k	210.0k	20.0	14400.0
belg+8arcs_264	177.4k	146.3k	21.2	14400.0	175.7k	151.6k	15.9	14400.0
belg+8arcs_265	379.7k	316.8k	19.9	14400.0	367.7k	284.0k	29.5	14400.0
belg+8arcs_266	4554.9	3669.3	24.1	14400.0	4554.9	4551.7	0.1	2.4
belg+8arcs_267	314.1k	203.2k	54.6	14400.0	394.7k	205.1k	92.4	14400.0
belg+8arcs_268	233.8k	216.5k	8.0	14400.0	231.9k	231.6k	0.1	496.3
belg+8arcs_269	239.3k	181.9k	31.6	14400.0	238.4k	208.6k	14.3	14400.0
belg+8arcs_270	191.6k	177.6k	7.9	14400.0	191.6k	191.4k	0.1	115.5
belg+8arcs_271	185.0k	174.9k	5.8	14400.0	185.0k	171.8k	7.7	14400.0
belg+8arcs_272	262.5k	230.8k	13.7	14400.0	262.5k	262.3k	0.1	2200.6
belg+8arcs_273	275.7k	195.2k	41.3	14400.0	261.1k	208.0k	25.6	14400.0
belg+8arcs_274	129.9k	125.8k	3.2	14400.0	129.9k	129.8k	0.1	517.2
belg+8arcs_275	18.3k	11.2k	62.9	14400.0	18.3k	18.3k	0.1	49.1
belg+8arcs_276	49.8k	24.9k	100.0	14400.0	49.8k	39.5k	26.0	14400.0
belg+8arcs_277	55.4k	54.7k	1.3	14400.0	55.4k	55.3k	0.1	2493.7
belg+8arcs_278	10.1k	9341.1	8.3	14400.0	10.1k	10.1k	0.1	11.7
belg+8arcs_279	50.1k	38.4k	30.2	14400.0	50.1k	20.3k	146.1	14400.0
belg+8arcs_280	240.0k	214.8k	11.7	14400.0	240.0k	222.4k	7.9	14400.0
belg+8arcs_281	205.7k	122.6k	67.7	14400.0	205.7k	127.4k	61.4	14400.0
belg+8arcs_282	281.0k	275.2k	2.1	14400.0	281.0k	280.7k	0.1	705.6
belg+8arcs_283	220.0k	126.5k	73.9	14400.0	220.0k	219.8k	0.1	1285.1
belg+8arcs_284	181.5k	168.3k	7.8	14400.0	181.5k	169.2k	7.3	14400.0
belg+8arcs_285	130.0k	113.4k	14.7	14400.0	130.0k	108.7k	19.7	14400.0
belg+8arcs_286	198.8k	189.8k	4.7	14400.0	198.8k	192.1k	3.5	14400.0
belg+8arcs_287	209.8k	205.7k	2.0	14400.0	209.8k	209.6k	0.1	6667.9
belg+8arcs_288	169.6k	160.3k	5.8	14400.0	169.6k	169.4k	0.1	2684.1
belg+8arcs_289	173.5k	138.5k	25.2	14400.0	173.5k	93.7k	85.2	14400.0
belg+8arcs_290	199.1k	179.9k	10.7	14400.0	199.1k	197.1k	1.0	14400.0
belg+8arcs_291	292.9k	174.4k	68.0	14400.0	276.8k	175.0k	58.2	14400.0
belg+8arcs_292	89.7k	25.6k	249.9	14400.0	89.7k	48.3k	85.8	14400.0
belg+8arcs_293	45.6k	2014.5	2162.6	14400.0	45.3k	45.3k	0.1	878.1
belg+8arcs_294	150.9k	150.4k	0.3	14400.0	150.9k	150.7k	0.1	194.6
belg+8arcs_295	187.9k	143.7k	30.7	14400.0	187.9k	128.6k	46.1	14400.0
belg+8arcs_296	148.6k	116.9k	27.1	14400.0	148.6k	134.9k	10.2	14400.0
belg+8arcs_297	112.9k	107.5k	5.0	14400.0	112.9k	112.7k	0.1	24.6
belg+8arcs_298	39.5k	6469.7	510.1	14400.0	39.5k	5919.2	566.8	14400.0
belg+8arcs_299	421.1k	246.1k	71.1	14400.0	371.8k	279.6k	33.0	14400.0
belg+8arcs_300	137.7k	126.5k	8.9	14400.0	137.7k	137.6k	0.1	43.0
belg+8arcs_301	150.7k	130.4k	15.5	14400.0	147.5k	139.2k	6.0	14400.0
belg+8arcs_302	233.9k	188.1k	24.4	14400.0	226.9k	188.2k	20.6	14400.0
belg+8arcs_303	278.0k	180.7k	53.8	14400.0	278.0k	239.6k	16.0	14400.0
belg+8arcs_304	214.9k	183.4k	17.2	14400.0	206.7k	198.8k	4.0	14400.0
belg+8arcs_305	234.6k	207.8k	12.9	14400.0	229.8k	193.6k	18.7	14400.0
belg+8arcs_306	196.8k	193.2k	1.9	14400.0	196.8k	191.2k	2.9	14400.0
belg+8arcs_307	78.8k	58.8k	34.0	14400.0	78.8k	71.5k	10.3	14400.0
belg+8arcs_308	30.6k	20.5k	49.1	14400.0	30.6k	30.6k	0.1	22.8
belg+8arcs_309	315.7k	201.8k	56.4	14400.0	298.7k	265.9k	12.3	14400.0
belg+8arcs_310	21.1k	17.6k	19.9	14400.0	21.1k	21.1k	0.1	5.0
belg+8arcs_311	134.9k	122.3k	10.3	14400.0	134.9k	134.7k	0.1	2275.6
belg+8arcs_312	102.8k	77.4k	32.8	14400.0	102.8k	100.4k	2.4	14400.0
belg+8arcs_313	248.6k	160.1k	55.2	14400.0	215.7k	160.2k	34.6	14400.0
belg+8arcs_314	540.2k	272.9k	98.0	14400.0	443.1k	330.4k	34.1	14400.0
belg+8arcs_315	211.6k	191.8k	10.4	14400.0	211.6k	174.4k	21.3	14400.0
belg+8arcs_316	143.3k	97.6k	46.8	14400.0	143.3k	112.2k	27.7	14400.0
belg+8arcs_317	195.3k	153.8k	26.9	14400.0	187.6k	170.8k	9.8	14400.0
belg+8arcs_318	70.9k	69.9k	1.4	14400.0	70.9k	70.8k	0.1	20.9
belg+8arcs_319	139.2k	135.1k	3.1	14400.0	139.2k	138.8k	0.3	14400.0
belg+8arcs_320	170.3k	165.7k	2.7	14400.0	170.3k	170.1k	0.1	15.0
belg+8arcs_321	138.9k	130.6k	6.4	14400.0	138.9k	126.7k	9.6	14400.0
belg+8arcs_322	129.6k	45.2k	186.5	14400.0	129.6k	114.0k	13.7	14400.0
belg+8arcs_323	106.2k	42.9k	147.7	14400.0	106.2k	106.1k	0.1	3857.3
belg+8arcs_324	62.7k	56.5k	10.9	14400.0	62.7k	58.9k	6.4	14400.0
belg+8arcs_325	232.4k	226.6k	2.5	14400.0	230.2k	214.1k	7.5	14400.0
belg+8arcs_326	237.0k	166.6k	42.2	14400.0	209.4k	162.9k	28.6	14400.0
belg+8arcs_327	21.2k	19.0k	11.5	14400.0	21.2k	21.2k	0.1	29.3
belg+8arcs_328	6205.7	4945.8	25.5	14400.0	6205.7	6200.5	0.1	6.4
belg+8arcs_329	172.7k	149.1k	15.8	14400.0	167.2k	149.1k	12.1	14400.0
belg+8arcs_330	212.6k	205.7k	3.4	14400.0	212.6k	208.1k	2.1	14400.0
belg+8arcs_331	127.7k	119.0k	7.3	14400.0	127.7k	127.5k	0.1	108.5
belg+8arcs_332	382.8k	321.2k	19.2	14400.0	378.4k	362.1k	4.5	14400.0
belg+8arcs_333	103.8k	87.7k	18.4	14400.0	103.8k	86.1k	20.5	14400.0
belg+8arcs_334	239.8k	233.2k	2.8	14400.0	239.8k	222.2k	7.9	14400.0
belg+8arcs_335	231.1k	182.3k	26.8	14400.0	222.5k	182.1k	22.2	14400.0
belg+8arcs_336	150.3k	137.7k	9.1	14400.0	150.3k	150.1k	0.1	54.4
belg+8arcs_337	76.2k	18.9k	302.7	14400.0	76.2k	40.1k	90.1	14400.0
belg+8arcs_338	174.5k	144.0k	21.2	14400.0	173.2k	161.0k	7.6	14400.0
belg+8arcs_339	303.4k	197.4k	53.7	14400.0	290.4k	226.4k	28.3	14400.0

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Table A.32: Comparison of split-pipe models on *Circuit rank* + 8 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_340	288.6k	210.1k	37.4	14400.0	288.6k	208.4k	38.5	14400.0
belg+8arcs_341	145.0k	141.8k	2.3	14400.0	145.0k	144.9k	0.1	25.2
belg+8arcs_342	255.5k	180.0k	42.0	14400.0	255.5k	191.8k	33.2	14400.0
belg+8arcs_343	176.3k	147.0k	20.0	14400.0	168.5k	168.4k	0.1	4579.6
belg+8arcs_344	122.1k	116.1k	5.1	14400.0	122.1k	122.1k	0.0	168.2
belg+8arcs_345	184.5k	183.0k	0.8	14400.0	184.5k	184.0k	0.2	14400.0
belg+8arcs_346	201.2k	168.1k	19.7	14400.0	188.6k	183.8k	2.6	14400.0
belg+8arcs_347	360.5k	272.9k	32.1	14400.0	363.7k	286.5k	26.9	14400.0
belg+8arcs_348	122.3k	111.5k	9.6	14400.0	122.3k	122.2k	0.1	6086.6
belg+8arcs_349	43.5k	12.0k	262.5	14400.0	43.5k	43.5k	0.0	322.1
belg+8arcs_350	41.8k	8009.8	422.2	14400.0	41.8k	41.8k	0.0	1550.1
belg+8arcs_351	338.8k	180.3k	87.9	14400.0	338.8k	268.2k	26.3	14400.0
belg+8arcs_352	244.7k	175.6k	39.4	14400.0	226.6k	174.9k	29.6	14400.0
belg+8arcs_353	133.6k	121.3k	10.1	14400.0	131.6k	131.5k	0.1	4996.1
belg+8arcs_354	109.7k	65.7k	67.0	14400.0	109.7k	83.7k	31.1	14400.0
belg+8arcs_355	31.0k	28.2k	9.8	14400.0	31.0k	30.9k	0.1	51.3
belg+8arcs_356	308.9k	203.1k	52.1	14400.0	322.7k	205.4k	57.1	14400.0
belg+8arcs_357	198.8k	187.0k	6.3	14400.0	198.8k	166.0k	19.7	14400.0
belg+8arcs_358	342.0k	258.2k	32.4	14400.0	342.0k	217.8k	57.0	14400.0
belg+8arcs_359	205.4k	174.0k	18.0	14400.0	205.4k	165.1k	24.5	14400.0
belg+8arcs_360	138.7k	116.9k	18.7	14400.0	138.7k	117.0k	18.5	14400.0
belg+8arcs_361	316.2k	312.4k	1.2	14400.0	316.2k	315.9k	0.1	8506.6
belg+8arcs_362	51.4k	23.3k	120.4	14400.0	51.4k	36.7k	40.1	14400.0
belg+8arcs_363	173.4k	168.1k	3.2	14400.0	173.4k	169.0k	2.6	14400.0
belg+8arcs_364	160.3k	116.7k	37.3	14400.0	146.8k	133.4k	10.1	14400.0
belg+8arcs_365	196.2k	191.2k	2.6	14400.0	196.2k	192.5k	1.9	14400.0
belg+8arcs_366	237.8k	223.9k	6.2	14400.0	237.8k	227.2k	4.7	14400.0
belg+8arcs_367	181.0k	180.1k	0.5	14400.0	181.0k	180.8k	0.1	233.1
belg+8arcs_368	51.3k	16.0k	221.1	14400.0	51.3k	16.4k	213.7	14400.0
belg+8arcs_369	4046.9	3916.3	3.3	14400.0	4046.9	4043.0	0.1	9.8
belg+8arcs_370	159.1k	158.4k	0.4	14400.0	159.1k	158.5k	0.4	14400.0
belg+8arcs_371	228.4k	197.1k	15.9	14400.0	228.4k	226.6k	0.8	14400.0
belg+8arcs_372	201.0k	130.0k	54.6	14400.0	201.0k	182.0k	10.4	14400.0
belg+8arcs_373	186.4k	157.3k	18.5	14400.0	175.6k	174.4k	0.7	14400.0
belg+8arcs_374	78.3k	69.5k	12.7	14400.0	78.3k	67.8k	15.5	14400.0
belg+8arcs_375	44.3k	37.9k	16.7	14400.0	44.3k	44.2k	0.1	23.7
belg+8arcs_376	391.3k	315.7k	23.9	14400.0	381.6k	354.6k	7.6	14400.0
belg+8arcs_377	153.0k	19.9k	667.9	14400.0	153.0k	38.7k	295.6	14400.0
belg+8arcs_378	105.1k	86.4k	21.7	14400.0	105.1k	69.3k	51.7	14400.0
belg+8arcs_379	274.1k	200.1k	37.0	14400.0	228.7k	207.9k	10.0	14400.0
belg+8arcs_380	169.1k	163.6k	3.3	14400.0	169.1k	167.8k	0.8	14400.0
belg+8arcs_381	278.8k	200.7k	38.9	14400.0	262.1k	194.8k	34.5	14400.0
belg+8arcs_382	219.4k	182.5k	20.2	14400.0	201.0k	200.8k	0.1	14400.0
belg+8arcs_383	41.0k	35.3k	16.1	14400.0	41.0k	41.0k	0.0	97.2
belg+8arcs_384	193.1k	75.8k	154.7	14400.0	193.1k	92.5k	108.7	14400.0
belg+8arcs_385	184.9k	162.8k	13.6	14400.0	184.9k	177.7k	4.1	14400.0
belg+8arcs_386	319.1k	229.9k	38.8	14400.0	319.2k	225.2k	41.8	14400.0
belg+8arcs_387	101.1k	61.3k	64.9	14400.0	101.1k	17.9k	463.8	14400.0
belg+8arcs_388	56.6k	0.0	–	14400.0	56.3k	32.6k	73.0	14400.0
belg+8arcs_389	146.4k	136.7k	7.1	14400.0	145.2k	145.1k	0.1	13479.0
belg+8arcs_390	215.2k	186.6k	15.3	14400.0	215.2k	195.0k	10.4	14400.0
belg+8arcs_391	180.6k	173.1k	4.3	14400.0	180.6k	180.4k	0.1	10391.2
belg+8arcs_392	89.9k	78.1k	15.2	14400.0	89.9k	78.2k	15.0	14400.0
belg+8arcs_393	196.5k	190.0k	3.4	14400.0	196.5k	196.3k	0.1	534.1
belg+8arcs_394	108.5k	23.9k	354.8	14400.0	108.5k	25.0k	334.5	14400.0
belg+8arcs_395	264.8k	156.4k	69.3	14400.0	232.7k	168.6k	38.0	14400.0
belg+8arcs_396	43.3k	29.5k	47.2	14400.0	43.3k	43.3k	0.1	181.2
belg+8arcs_397	309.5k	248.8k	24.4	14400.0	309.5k	267.9k	15.5	14400.0
belg+8arcs_398	10.7k	9535.4	11.8	14400.0	10.7k	10.7k	0.1	10.5
belg+8arcs_399	4726.3	4656.1	1.5	14400.0	4726.3	4722.3	0.1	2.3
belg+8arcs_400	78.5k	31.0k	153.3	14400.0	78.5k	31.2k	151.5	14400.0
belg+8arcs_401	133.1k	133.0k	0.1	259.1	133.1k	133.0k	0.1	760.2
belg+8arcs_402	148.4k	143.7k	3.3	14400.0	148.4k	148.2k	0.1	175.0
belg+8arcs_403	228.9k	225.5k	1.5	14400.0	228.9k	228.7k	0.1	71.1
belg+8arcs_404	216.7k	195.8k	10.6	14400.0	216.7k	216.5k	0.1	14347.4
belg+8arcs_405	315.0k	310.5k	1.5	14400.0	315.0k	314.7k	0.1	2373.9
belg+8arcs_406	66.2k	54.1k	22.4	14400.0	66.2k	65.7k	0.7	14400.0
belg+8arcs_407	189.7k	169.9k	11.7	14400.0	189.7k	158.5k	19.7	14400.0
belg+8arcs_408	29.7k	20.5k	44.8	14400.0	29.7k	28.7k	3.3	14400.0
belg+8arcs_409	138.6k	124.9k	10.9	14400.0	138.6k	138.6k	0.0	1696.2
belg+8arcs_410	162.3k	83.4k	94.7	14400.0	162.3k	162.1k	0.1	129.6
belg+8arcs_411	196.9k	188.2k	4.6	14400.0	196.9k	167.0k	17.9	14400.0
belg+8arcs_412	12.0k	10.6k	13.4	14400.0	12.0k	12.0k	0.0	6.3
belg+8arcs_413	143.2k	114.5k	25.0	14400.0	143.2k	143.1k	0.1	525.8
belg+8arcs_414	149.4k	146.9k	1.7	14400.0	149.4k	149.3k	0.1	94.7
belg+8arcs_415	19.2k	14.3k	34.6	14400.0	19.2k	19.2k	0.1	81.0
belg+8arcs_416	184.4k	184.1k	0.2	14400.0	184.4k	177.1k	4.1	14400.0
belg+8arcs_417	151.7k	143.8k	5.5	14400.0	149.7k	149.4k	0.2	14400.0

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Table A.32: Comparison of split-pipe models on *Circuit rank* + 8 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_418	380.9k	284.2k	34.0	14400.0	358.9k	288.7k	24.3	14400.0
belg+8arcs_419	138.0k	19.5k	606.3	14400.0	137.8k	111.4k	23.7	14400.0
belg+8arcs_420	130.3k	115.5k	12.8	14400.0	130.3k	120.8k	7.9	14400.0
belg+8arcs_421	91.1k	68.9k	32.1	14400.0	91.1k	58.9k	54.5	14400.0
belg+8arcs_422	131.0k	45.7k	186.5	14399.7	122.8k	29.8k	312.2	14400.0
belg+8arcs_423	141.8k	137.7k	3.0	14400.0	141.8k	141.7k	0.1	33.2
belg+8arcs_424	313.2k	297.9k	5.1	14400.0	313.2k	292.4k	7.1	14400.0
belg+8arcs_425	221.2k	214.3k	3.2	14400.0	220.4k	211.8k	4.1	14400.0
belg+8arcs_426	168.8k	93.5k	80.5	14400.0	168.8k	130.2k	29.6	14400.0
belg+8arcs_427	132.3k	129.9k	1.8	14400.0	132.3k	132.2k	0.1	16.8
belg+8arcs_428	71.9k	25.2k	185.2	14400.0	71.9k	35.6k	101.8	14400.0
belg+8arcs_429	369.2k	216.9k	70.2	14400.0	287.7k	195.7k	47.0	14400.0
belg+8arcs_430	274.8k	172.8k	59.0	14400.0	237.3k	171.9k	38.0	14400.0
belg+8arcs_431	507.9k	322.9k	57.3	14400.0	487.5k	345.8k	41.0	14400.0
belg+8arcs_432	3912.1	3825.8	2.3	14400.0	3912.1	3909.3	0.1	3.7
belg+8arcs_433	28.1k	24.4k	15.2	14400.0	28.1k	28.1k	0.1	117.1
belg+8arcs_434	205.2k	179.9k	14.0	14400.0	205.2k	197.1k	4.1	14400.0
belg+8arcs_435	212.8k	197.6k	7.7	14400.0	212.8k	182.2k	16.8	14400.0
belg+8arcs_436	184.1k	169.8k	8.5	14400.0	184.1k	184.0k	0.1	1174.1
belg+8arcs_437	84.7k	59.6k	42.2	14400.0	84.7k	80.9k	4.7	14400.0
belg+8arcs_438	208.4k	182.1k	14.4	14400.0	208.4k	208.2k	0.1	3710.9
belg+8arcs_439	270.9k	210.8k	28.5	14400.0	241.9k	221.5k	9.2	14400.0
belg+8arcs_440	243.3k	209.0k	16.4	14400.0	243.3k	172.2k	41.3	14400.0
belg+8arcs_441	190.8k	142.8k	33.6	14400.0	190.8k	142.8k	33.6	14400.0
belg+8arcs_442	5811.3	2621.1	121.7	14400.0	5811.3	5811.3	0.0	30.4
belg+8arcs_443	115.6k	104.4k	10.7	14400.0	115.6k	115.5k	0.1	1266.4
belg+8arcs_444	63.4k	25.8k	145.8	14400.0	60.7k	60.3k	0.8	14400.0
belg+8arcs_445	281.3k	265.6k	5.9	14400.0	281.3k	232.9k	20.8	14400.0
belg+8arcs_446	148.6k	138.5k	7.3	14400.0	148.6k	148.4k	0.1	1424.6
belg+8arcs_447	200.1k	155.4k	28.8	14400.0	186.4k	168.6k	10.6	14400.0
belg+8arcs_448	263.2k	134.7k	95.3	14400.0	250.1k	213.8k	17.0	14400.0
belg+8arcs_449	136.7k	121.6k	12.4	14400.0	136.7k	128.1k	6.7	14400.0
belg+8arcs_450	48.2k	45.8k	5.2	14400.0	48.2k	48.1k	0.1	14400.0
belg+8arcs_451	32.7k	31.7k	3.0	14400.0	32.7k	32.6k	0.2	14400.0
belg+8arcs_452	200.9k	167.9k	19.6	14400.0	182.6k	179.1k	1.9	14400.0
belg+8arcs_453	262.8k	257.9k	1.9	14400.0	262.8k	262.6k	0.1	6420.1
belg+8arcs_454	50.5k	19.3k	161.2	14400.0	50.5k	50.5k	0.0	7201.0
belg+8arcs_455	34.6k	29.7k	16.5	14400.0	34.6k	34.6k	0.1	5487.0
belg+8arcs_456	271.9k	199.4k	36.4	14400.0	271.9k	211.6k	28.5	14400.0
belg+8arcs_457	77.5k	65.7k	17.9	14400.0	77.5k	77.5k	0.1	27.2
belg+8arcs_458	240.3k	240.1k	0.1	3904.8	240.3k	240.1k	0.1	1119.4
belg+8arcs_459	0.0	0.0	0.0	12.2	0.0	0.0	0.0	0.5
belg+8arcs_460	3317.3	2829.8	17.2	14400.0	3317.3	3317.3	0.0	1120.1
belg+8arcs_461	210.9k	177.6k	18.7	14400.0	208.5k	195.6k	6.6	14400.0
belg+8arcs_462	148.0k	134.0k	10.4	14400.0	148.0k	147.8k	0.1	779.1
belg+8arcs_463	129.2k	124.6k	3.7	14400.0	129.2k	129.1k	0.1	199.8
belg+8arcs_464	150.9k	125.1k	20.7	14400.0	152.5k	125.3k	21.7	14400.0
belg+8arcs_465	4882.3	4572.3	6.8	14400.0	4882.3	4878.2	0.1	4.2
belg+8arcs_466	53.5k	51.4k	4.0	14400.0	53.5k	53.4k	0.1	5520.7
belg+8arcs_467	182.5k	173.0k	5.5	14400.0	182.5k	176.1k	3.6	14400.0
belg+8arcs_468	180.9k	64.5k	180.3	14400.0	180.9k	119.5k	51.4	14400.0
belg+8arcs_469	103.4k	38.6k	167.6	14400.0	103.4k	73.9k	39.9	14400.0
belg+8arcs_470	20.9k	16.3k	27.9	14400.0	20.9k	20.9k	0.1	10.8
belg+8arcs_471	172.8k	159.7k	8.2	14400.0	172.8k	167.3k	3.3	14400.0
belg+8arcs_472	80.1k	45.2k	77.2	14400.0	80.1k	26.9k	198.0	14400.0
belg+8arcs_473	251.9k	209.8k	20.1	14400.0	251.9k	251.5k	0.2	14400.0
belg+8arcs_474	68.9k	55.4k	24.3	14400.0	68.9k	68.8k	0.1	353.3
belg+8arcs_475	114.7k	96.2k	19.2	14400.0	114.7k	114.7k	0.1	37.5
belg+8arcs_476	206.4k	191.6k	7.7	14400.0	206.4k	206.4k	0.0	1181.2
belg+8arcs_477	227.5k	197.8k	15.0	14400.0	222.2k	202.1k	9.9	14400.0
belg+8arcs_478	197.2k	164.2k	20.1	14400.0	197.2k	192.6k	2.4	14400.0
belg+8arcs_479	145.9k	119.1k	22.5	14400.0	145.9k	145.8k	0.1	3388.0
belg+8arcs_480	139.5k	119.8k	16.4	14400.0	139.4k	123.1k	13.2	14400.0
belg+8arcs_481	284.1k	165.8k	71.4	14400.0	296.4k	236.3k	25.4	14400.0
belg+8arcs_482	175.8k	159.1k	10.5	14400.0	171.6k	171.4k	0.1	2463.7
belg+8arcs_483	32.3k	27.3k	18.6	14400.0	32.2k	32.2k	0.1	175.1
belg+8arcs_484	43.4k	42.2k	2.8	14400.0	43.4k	43.3k	0.2	14400.0
belg+8arcs_485	38.9k	28.6k	36.0	14400.0	38.9k	38.9k	0.1	25.1
belg+8arcs_486	243.0k	201.1k	20.8	14400.0	243.0k	218.1k	11.4	14400.0
belg+8arcs_487	132.0k	129.4k	2.1	14400.0	132.0k	131.9k	0.1	226.4
belg+8arcs_488	190.3k	171.3k	11.1	14400.0	183.0k	181.4k	0.9	14400.0
belg+8arcs_489	173.5k	150.5k	15.3	14400.0	173.5k	146.7k	18.3	14400.0
belg+8arcs_490	111.1k	96.1k	15.6	14400.0	111.1k	111.0k	0.1	72.6
belg+8arcs_491	77.2k	70.0k	10.3	14400.0	77.2k	77.2k	0.0	1534.7
belg+8arcs_492	86.7k	76.1k	14.0	14400.0	86.7k	86.6k	0.1	85.1
belg+8arcs_493	230.6k	190.0k	21.4	14400.0	218.1k	189.3k	15.2	14400.0
belg+8arcs_494	193.3k	191.1k	1.2	14400.0	193.3k	190.6k	1.4	14400.0
belg+8arcs_495	145.0k	142.8k	1.5	14400.0	145.0k	121.6k	19.2	14400.0

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Table A.32: Comparison of split-pipe models on *Circuit rank* + 8 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+8arcs_496	126.1k	116.0k	8.7	14400.0	126.4k	93.6k	35.0	14400.0
belg+8arcs_497	52.1k	40.9k	27.4	14400.0	52.1k	42.7k	22.0	14400.0
belg+8arcs_498	312.9k	261.7k	19.6	14400.0	276.5k	230.0k	20.2	14400.0
belg+8arcs_499	194.7k	189.8k	2.6	14400.0	194.7k	194.0k	0.4	14400.0
belg+8arcs_500	129.6k	100.0k	29.6	14400.0	129.6k	129.6k	0.0	161.4

Table A.33: Detailed results of the discrete models on *Circuit rank* + 10 arcs, as summarized in Figure 3.10f and Table 3.3d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_1	211.4k	211.4k	0.0	503.8	211.4k	211.4k	0.0	2629.1	211.4k	211.4k	0.0	2380.2
belg+10arcs_2	4164.0	4164.0	0.0	2269.5	4164.0	4164.0	0.0	47.0	4164.0	4164.0	0.0	238.3
belg+10arcs_3	150.4k	35.2k	327.6	14400.0	1e+20	41.5k	–	14400.0	152.6k	22.1k	590.9	14400.0
belg+10arcs_4	150.8k	150.8k	0.0	291.9	150.8k	150.8k	0.0	10038.3	150.8k	150.8k	0.0	3412.5
belg+10arcs_5	252.4k	164.7k	53.3	14400.0	1e+20	169.0k	–	14400.0	271.1k	125.4k	116.2	14400.0
belg+10arcs_6	61.9k	61.9k	0.0	949.7	61.9k	61.9k	0.0	1956.3	61.9k	61.9k	0.0	1666.3
belg+10arcs_7	224.3k	224.3k	0.0	3529.5	317.4k	197.3k	60.9	14400.0	233.2k	204.9k	13.8	14400.0
belg+10arcs_8	11.4k	11.4k	0.0	680.5	1e+20	315.7	–	14400.0	11.4k	11.4k	0.0	376.3
belg+10arcs_9	76.7k	76.7k	0.0	772.7	76.7k	76.7k	0.0	8904.9	76.7k	76.7k	0.0	3405.3
belg+10arcs_10	134.1k	134.1k	0.0	499.7	134.1k	134.1k	0.0	1993.4	134.1k	134.1k	0.0	733.6
belg+10arcs_11	154.8k	154.8k	0.0	818.9	154.8k	154.8k	0.0	11665.4	154.8k	154.8k	0.0	13835.9
belg+10arcs_12	40.5k	40.5k	0.0	2224.3	40.5k	40.5k	0.0	7948.9	40.5k	40.5k	0.0	3182.8
belg+10arcs_13	159.6k	159.6k	0.0	510.5	163.6k	115.0k	42.2	14400.0	160.0k	144.1k	11.1	14400.0
belg+10arcs_14	276.1k	276.1k	0.0	2021.5	1e+20	225.6k	–	14400.0	317.2k	239.0k	32.7	14400.0
belg+10arcs_15	2976.0	2976.0	0.0	284.3	2976.0	2976.0	0.0	752.8	2976.0	2976.0	0.0	930.0
belg+10arcs_16	396.0k	108.2k	266.1	14400.0	1e+20	125.2k	–	14400.0	494.5k	41.9k	1079.8	14400.0
belg+10arcs_17	142.9k	112.8k	26.7	14400.0	200.8k	47.2k	325.2	14400.0	157.2k	17.3k	809.2	14400.0
belg+10arcs_18	188.7k	188.7k	0.0	2368.0	192.4k	171.0k	12.5	14400.0	188.7k	173.5k	8.8	14400.0
belg+10arcs_19	84.6k	84.6k	0.0	419.9	87.8k	54.5k	60.9	14400.0	84.6k	26.8k	216.2	14400.0
belg+10arcs_20	222.6k	201.8k	10.3	14400.0	209.0k	183.8k	13.7	14400.0	208.1k	208.1k	0.0	4328.9
belg+10arcs_21	46.4k	46.4k	0.0	446.5	1e+20	23.2k	–	14400.0	46.4k	46.4k	0.0	218.4
belg+10arcs_22	232.4k	27.5k	745.0	14400.0	1e+20	47.6k	–	14400.0	187.9k	11.8k	1493.8	14400.0
belg+10arcs_23	86.4k	24.3k	256.1	14400.0	357.3k	16.9k	2014.0	14400.0	138.8k	5218.9	2559.8	14400.0
belg+10arcs_24	163.2k	163.2k	0.0	452.5	163.2k	163.1k	0.1	14014.4	163.2k	163.2k	0.0	2448.9
belg+10arcs_25	17.4k	17.4k	0.0	551.3	17.4k	17.4k	0.0	8695.3	17.4k	17.4k	0.0	491.6
belg+10arcs_26	132.4k	132.4k	0.0	1712.0	1e+20	106.3k	–	14400.0	132.4k	132.4k	0.0	365.9
belg+10arcs_27	20.5k	20.5k	0.0	149.6	1e+20	16.3k	–	14400.0	20.5k	20.5k	0.0	885.7
belg+10arcs_28	177.0k	177.0k	0.0	645.7	1e+20	166.9k	–	14400.0	177.0k	177.0k	0.0	25.5
belg+10arcs_29	2976.0	2976.0	0.0	133.2	2976.0	2976.0	0.0	7096.2	2976.0	2976.0	0.0	520.3
belg+10arcs_30	27.8k	27.8k	0.0	643.7	27.8k	27.8k	0.0	9284.7	27.8k	27.8k	0.0	2770.6
belg+10arcs_31	6480.0	6480.0	0.0	281.6	6480.0	6480.0	0.0	971.1	6480.0	6480.0	0.0	489.2
belg+10arcs_32	188.4k	188.4k	0.0	1151.1	1e+20	137.2k	–	14400.0	188.4k	152.3k	23.7	14400.0
belg+10arcs_33	43.9k	25.2k	74.3	14400.0	43.9k	35.9k	22.3	14400.0	55.5k	22.7k	144.1	14400.0
belg+10arcs_34	20.5k	20.5k	0.0	39.9	20.5k	20.5k	0.0	7531.2	20.5k	20.5k	0.0	179.1
belg+10arcs_35	125.1k	125.1k	0.0	64.4	1e+20	104.7k	–	14400.0	125.1k	125.1k	0.0	19.0
belg+10arcs_36	140.8k	140.8k	0.0	3589.7	140.8k	140.8k	0.0	8044.5	147.6k	65.8k	124.3	14400.0
belg+10arcs_37	41.0k	41.0k	0.0	76.6	41.0k	41.0k	0.0	3579.6	41.0k	41.0k	0.0	336.2
belg+10arcs_38	247.1k	225.3k	9.7	14400.0	237.6k	237.6k	0.0	6438.9	237.6k	237.4k	0.1	3294.9
belg+10arcs_39	21.7k	21.7k	0.0	153.1	21.7k	21.7k	0.0	3944.0	21.7k	21.7k	0.0	465.6
belg+10arcs_40	163.3k	3600.0	4437.0	14400.0	431.3k	23.1k	1764.2	14400.0	138.6k	2998.8	4522.8	14400.0
belg+10arcs_41	214.1k	214.1k	0.0	6407.2	214.1k	214.1k	0.0	7037.6	214.1k	214.1k	0.0	3290.3
belg+10arcs_42	176.3k	176.3k	0.0	1708.3	176.3k	176.3k	0.0	5759.4	176.3k	176.3k	0.0	3532.4
belg+10arcs_43	39.7k	39.7k	0.0	361.1	39.7k	39.7k	0.0	14386.1	39.7k	39.7k	0.0	1814.5
belg+10arcs_44	117.9k	41.4k	185.0	14400.0	163.3k	73.5k	122.2	14400.0	129.7k	15.3k	747.9	14400.0
belg+10arcs_45	18.1k	18.1k	0.0	2916.7	1e+20	11.9k	–	14400.0	18.1k	18.1k	0.0	414.7
belg+10arcs_46	124.3k	124.3k	0.0	880.4	131.0k	108.2k	21.0	14400.0	124.3k	124.3k	0.0	1922.4
belg+10arcs_47	201.1k	187.2k	7.5	14400.0	1e+20	153.2k	–	14400.0	196.1k	196.1k	0.0	6731.4
belg+10arcs_48	166.8k	166.8k	0.0	2607.7	1e+20	117.1k	–	14400.0	176.3k	150.3k	17.3	14400.0
belg+10arcs_49	16.3k	16.3k	0.0	1950.0	16.3k	16.3k	0.0	953.7	16.3k	16.3k	0.0	2548.5
belg+10arcs_50	301.5k	79.5k	279.3	14400.0	205.7k	196.8k	4.5	14400.0	203.7k	203.7k	0.0	11663.5
belg+10arcs_51	18.6k	18.6k	0.0	50.6	18.6k	18.6k	0.0	147.2	18.6k	18.6k	0.0	107.3
belg+10arcs_52	120.0k	120.0k	0.0	521.0	194.4k	83.6k	132.5	14400.0	120.0k	120.0k	0.0	3351.8
belg+10arcs_53	13.2k	13.2k	0.0	110.2	13.2k	13.2k	0.0	2576.8	13.2k	13.2k	0.0	74.6
belg+10arcs_54	134.1k	134.1k	0.0	978.6	134.1k	133.9k	0.1	12273.5	134.1k	134.1k	0.0	1191.5
belg+10arcs_55	166.1k	166.1k	0.0	3200.6	1e+20	118.2k	–	14400.0	213.4k	105.2k	102.9	14400.0
belg+10arcs_56	64.2k	64.2k	0.0	3265.9	1e+20	16.4k	–	14400.0	92.9k	9378.6	890.7	14400.0
belg+10arcs_57	44.5k	44.5k	0.0	575.7	44.5k	44.5k	0.0	860.6	44.5k	44.5k	0.0	935.6
belg+10arcs_58	4920.0	4920.0	0.0	76.8	4920.0	4920.0	0.0	32.1	4920.0	4920.0	0.0	340.7

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Table A.33: Comparison of discrete models on *Circuit rank* + 10 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_59	20.5k	20.5k	0.0	168.3	20.5k	20.5k	0.0	1582.8	20.5k	20.5k	0.0	83.1
belg+10arcs_60	5724.0	5724.0	0.0	20.7	1e+20	5515.5	–	14400.0	5724.0	5724.0	0.0	24.3
belg+10arcs_61	0.0	0.0	0.0	0.4	0.0	0.0	0.0	2752.3	0.0	0.0	0.0	0.5
belg+10arcs_62	230.8k	200.1k	15.4	14400.0	208.3k	208.3k	0.0	2639.8	210.1k	196.5k	6.9	14400.0
belg+10arcs_63	202.8k	196.3k	3.3	14400.0	1e+20	163.6k	–	14400.0	215.5k	157.1k	37.2	14400.0
belg+10arcs_64	27.5k	27.5k	0.0	5249.3	27.5k	27.5k	0.0	5173.8	27.5k	27.5k	0.0	2157.9
belg+10arcs_65	122.5k	42.1k	191.0	14400.0	1e+20	31.4k	–	14400.0	112.6k	15.5k	626.0	14400.0
belg+10arcs_66	175.5k	175.5k	0.0	1463.3	197.4k	138.8k	42.2	14400.0	175.5k	175.5k	0.0	13320.1
belg+10arcs_67	129.5k	129.5k	0.0	4707.7	129.5k	129.4k	0.0	4683.7	129.5k	129.5k	0.0	8600.2
belg+10arcs_68	161.8k	161.8k	0.0	9874.6	161.8k	161.8k	0.0	9945.3	230.1k	112.1k	105.3	14400.0
belg+10arcs_69	88.5k	88.5k	0.0	2865.9	1e+20	61.9k	–	14400.0	88.5k	88.5k	0.0	12046.0
belg+10arcs_70	44.3k	44.3k	0.0	830.4	44.3k	44.3k	0.0	9735.8	44.3k	44.3k	0.0	10996.1
belg+10arcs_71	2976.0	2976.0	0.0	247.2	2976.0	2976.0	0.0	427.6	2976.0	2976.0	0.0	293.5
belg+10arcs_72	31.1k	31.1k	0.0	1130.5	31.1k	31.1k	0.0	3137.0	31.1k	31.1k	0.0	2737.8
belg+10arcs_73	23.4k	23.4k	0.0	482.3	23.4k	23.4k	0.0	6043.8	23.4k	23.4k	0.0	24.2
belg+10arcs_74	270.2k	270.2k	0.0	3518.1	270.2k	270.1k	0.0	11982.8	274.3k	140.2k	95.6	14400.0
belg+10arcs_75	110.1k	110.1k	0.0	345.5	110.1k	110.1k	0.0	7153.1	110.1k	110.1k	0.0	3822.6
belg+10arcs_76	49.4k	49.4k	0.0	536.0	1e+20	32.2k	–	14400.0	49.4k	49.4k	0.0	3884.1
belg+10arcs_77	8390.0	8390.0	0.0	381.2	8390.0	8390.0	0.0	1093.1	8390.0	8390.0	0.0	665.0
belg+10arcs_78	55.8k	55.8k	0.0	611.0	1e+20	43.4k	–	14400.0	55.8k	55.8k	0.0	638.0
belg+10arcs_79	43.8k	19.2k	128.3	14400.0	392.5k	17.3k	2171.0	14400.0	43.8k	28.3k	54.8	14400.0
belg+10arcs_80	13.6k	13.6k	0.0	172.6	13.6k	13.6k	0.0	2363.6	13.6k	13.6k	0.0	3470.0
belg+10arcs_81	156.5k	156.5k	0.0	599.6	156.5k	156.5k	0.0	528.5	156.5k	156.5k	0.0	170.1
belg+10arcs_82	20.5k	20.5k	0.0	46.0	20.5k	20.5k	0.0	7424.5	20.5k	20.5k	0.0	17.6
belg+10arcs_83	232.6k	232.6k	0.0	8285.2	933.0k	180.8k	416.0	14400.0	232.6k	201.5k	15.4	14400.0
belg+10arcs_84	238.7k	238.7k	0.0	2568.8	238.7k	238.7k	0.0	12669.5	238.7k	182.3k	31.0	14400.0
belg+10arcs_85	82.5k	82.5k	0.0	676.8	82.5k	82.5k	0.0	7733.2	82.5k	82.5k	0.0	2444.3
belg+10arcs_86	191.9k	191.9k	0.0	2389.9	1e+20	141.9k	–	14400.0	191.9k	152.6k	25.7	14400.0
belg+10arcs_87	161.8k	161.8k	0.0	1530.0	1e+20	118.5k	–	14400.0	162.5k	129.4k	25.6	14400.0
belg+10arcs_88	179.9k	179.9k	0.0	1676.2	1e+20	136.1k	–	14400.0	179.9k	168.3k	6.9	14400.0
belg+10arcs_89	454.8k	422.1k	7.7	14400.0	1e+20	345.2k	–	14400.0	460.6k	320.3k	43.8	14400.0
belg+10arcs_90	208.6k	208.6k	0.0	3592.7	311.2k	181.0k	71.9	14400.0	221.1k	153.3k	44.2	14400.0
belg+10arcs_91	190.8k	173.9k	9.8	14400.0	1e+20	113.0k	–	14400.0	263.1k	85.9k	206.3	14400.0
belg+10arcs_92	216.4k	166.9k	29.6	14400.0	1e+20	112.3k	–	14400.0	292.7k	50.4k	481.3	14400.0
belg+10arcs_93	353.6k	174.4k	102.8	14400.0	1e+20	201.0k	–	14400.0	366.8k	153.2k	139.5	14400.0
belg+10arcs_94	122.2k	109.3k	11.8	14400.0	120.3k	111.3k	8.0	14400.0	116.3k	116.3k	0.0	1297.3
belg+10arcs_95	49.0k	49.0k	0.0	520.3	1e+20	37.6k	–	14400.0	49.0k	49.0k	0.0	480.8
belg+10arcs_96	121.6k	121.6k	0.0	1200.2	1e+20	77.3k	–	14400.0	121.6k	121.6k	0.0	303.8
belg+10arcs_97	0.0	0.0	0.0	0.2	0.0	0.0	0.0	58.6	0.0	0.0	0.0	2.7
belg+10arcs_98	148.5k	138.3k	7.4	14400.0	148.5k	148.5k	0.0	13976.4	148.5k	148.5k	0.0	14287.5
belg+10arcs_99	187.0k	57.3k	226.5	14400.0	1e+20	58.4k	–	14400.0	340.7k	18.0k	1791.2	14400.0
belg+10arcs_100	132.4k	132.4k	0.0	232.4	132.4k	132.4k	0.0	7540.9	132.4k	132.4k	0.0	3359.2
belg+10arcs_101	153.2k	153.2k	0.0	6328.7	1e+20	98.4k	–	14400.0	158.1k	98.7k	60.2	14400.0
belg+10arcs_102	124.3k	124.3k	0.0	471.5	124.3k	124.3k	0.0	4851.0	124.3k	124.3k	0.0	1425.1
belg+10arcs_103	163.3k	23.8k	587.2	14400.0	164.5k	18.6k	782.4	14400.0	163.3k	2977.1	5386.2	14400.0
belg+10arcs_104	186.4k	84.5k	120.5	14400.0	1e+20	48.0k	–	14400.0	187.1k	8365.2	2136.7	14400.0
belg+10arcs_105	24.7k	22.3k	10.7	14400.0	24.7k	24.7k	0.0	3579.8	24.7k	24.7k	0.0	399.7
belg+10arcs_106	89.0k	7331.4	1113.8	14400.0	1e+20	19.6k	–	14400.0	107.4k	1737.0	6081.0	14400.0
belg+10arcs_107	169.8k	169.8k	0.0	1584.8	169.8k	169.6k	0.1	8379.0	169.8k	169.8k	0.0	1851.2
belg+10arcs_108	32.9k	32.9k	0.0	675.4	32.9k	32.9k	0.0	910.5	32.9k	32.9k	0.0	933.7
belg+10arcs_109	0.0	0.0	0.0	1.5	0.0	0.0	0.0	919.2	0.0	0.0	0.0	3.0
belg+10arcs_110	140.7k	140.7k	0.0	172.7	140.7k	140.7k	0.0	6781.1	140.7k	140.6k	0.1	1877.1
belg+10arcs_111	344.6k	104.2k	230.8	14400.0	1e+20	124.1k	–	14400.0	448.3k	71.5k	526.8	14400.0
belg+10arcs_112	5724.0	5724.0	0.0	26.9	5724.0	5724.0	0.0	5551.0	5724.0	5724.0	0.0	109.0
belg+10arcs_113	13.5k	13.5k	0.0	4528.6	13.5k	13.5k	0.0	5157.9	13.5k	13.5k	0.0	7352.7
belg+10arcs_114	3600.0	3600.0	0.0	6.6	3600.0	3600.0	0.0	10964.8	3600.0	3600.0	0.0	1.3
belg+10arcs_115	102.2k	102.2k	0.0	2431.2	102.2k	102.1k	0.1	12118.3	110.5k	41.1k	168.7	14400.0
belg+10arcs_116	22.1k	22.1k	0.0	13915.7	22.1k	17.3k	27.7	14400.0	22.1k	22.1k	0.0	9817.1
belg+10arcs_117	61.7k	37.7k	63.9	14400.0	1e+20	14.3k	–	14400.0	195.1k	8342.2	2239.3	14400.0
belg+10arcs_118	92.0k	5794.1	1487.9	14400.0	74.8k	29.6k	152.6	14400.0	92.0k	10.7k	757.3	14400.0
belg+10arcs_119	19.7k	19.7k	0.0	2423.8	19.7k	19.7k	0.0	3294.1	19.7k	6487.9	203.3	14400.0
belg+10arcs_120	0.0	0.0	0.0	0.7	0.0	0.0	0.0	56.7	0.0	0.0	0.0	0.4
belg+10arcs_121	2976.0	2976.0	0.0	95.2	1e+20	180.0	–	14400.0	2976.0	2976.0	0.0	460.1
belg+10arcs_122	97.7k	97.7k	0.1	231.4	97.7k	97.7k	0.0	2832.3	97.7k	97.7k	0.0	1530.8
belg+10arcs_123	21.7k	21.7k	0.0	1166.3	21.7k	21.7k	0.0	6864.1	21.7k	21.7k	0.0	166.6
belg+10arcs_124	23.3k	23.3k	0.0	4026.2	23.3k	23.3k	0.0	8186.9	29.9k	7145.6	318.3	14400.0
belg+10arcs_125	18.1k	18.1k	0.0	49.0	18.1k	18.1k	0.0	1520.2	18.1k	18.1k	0.0	31.2
belg+10arcs_126	60.0k	19.2k	212.8	14400.0	60.0k	23.2k	158.8	14400.0	344.8k	9174.1	3658.6	14400.0
belg+10arcs_127	42.7k	42.7k	0.0	5753.7	42.7k	42.7k	0.0	6099.0	42.7k	42.7k	0.0	10586.8
belg+10arcs_128	52.4k	52.4k	0.0	1752.2	54.3k	40.7k	33.3	14400.0	54.3k	39.6k	36.9	14400.0
belg+10arcs_129	4164.0	4164.0	0.0	12686.5	4164.0	4164.0	0.0	226.8	4164.0	4164.0	0.0	70.1
belg+10arcs_130	44.6k	44.6k	0.0	2072.5	44.6k	44.6k	0.0	3277.1	44.6k	23.9k	86.6	14400.0
belg+10arcs_131	175.4k	77.7k	125.8	14400.0	1e+20	53.4k	–	14400.0	238.5k	30.2k	689.7	14400.0
belg+10arcs_132	25.4k	25.4k	0.0	127.5	25.4k	25.4k	0.0	2479.9	25.4k	25.4k	0.0	517.0
belg+10arcs_133	98.8k	46.5k	112.4	14400.0	78.7k	31.1k	153.2	14400.0	80.2k	24.7k	225.0	14400.0
belg+10arcs_134	465.9k	65.3k	613.9	14400.0	1e+20	198.3k	–	14400.0	217.9k	217.9k	0.0	1163.8
belg+10arcs_135	11.7k	11.7k	0.0	479.1	11.7k	11.7k	0.0	3424.2	11.7k	11.7k	0.0	1402.7
belg+10arcs_136	604.8k	204.3k	196.0	14400.0	1e+20	245.7k	–	14400.0	435.3k	175.0k	148.7	14400.0

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Table A.33: Comparison of discrete models on *Circuit rank* + 10 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_137	219.4k	219.4k	0.0	783.9	219.4k	219.4k	0.0	7404.9	219.4k	219.4k	0.0	51.9
belg+10arcs_138	133.2k	133.2k	0.0	583.7	1e+20	115.6k	–	14400.0	133.2k	133.2k	0.0	87.4
belg+10arcs_139	101.7k	101.7k	0.0	386.5	1e+20	26.5k	–	14400.0	101.9k	65.6k	55.5	14400.0
belg+10arcs_140	23.4k	23.4k	0.0	1588.9	1e+20	14.8k	–	14400.0	23.4k	23.4k	0.0	1598.3
belg+10arcs_141	103.3k	103.3k	0.0	1757.3	103.3k	103.3k	0.0	5974.7	103.3k	103.3k	0.0	917.3
belg+10arcs_142	140.0k	140.0k	0.0	148.7	140.0k	140.0k	0.0	919.6	140.0k	140.0k	0.0	881.3
belg+10arcs_143	26.2k	26.2k	0.0	105.4	26.2k	26.2k	0.0	3225.2	26.2k	26.2k	0.0	223.0
belg+10arcs_144	38.9k	14.4k	170.3	14400.0	35.9k	26.2k	36.8	14400.0	38.9k	3273.6	1087.1	14400.0
belg+10arcs_145	9540.0	9540.0	0.0	484.1	9540.0	9540.0	0.0	7418.7	9540.0	9540.0	0.0	6163.3
belg+10arcs_146	29.7k	29.7k	0.0	285.2	29.7k	29.7k	0.0	4336.6	29.7k	29.7k	0.0	2346.7
belg+10arcs_147	137.3k	137.3k	0.0	2916.7	137.3k	104.9k	30.9	14400.0	138.4k	107.2k	29.2	14400.0
belg+10arcs_148	38.7k	38.7k	0.0	763.2	1e+20	18.5k	–	14400.0	39.5k	7047.4	460.7	14400.0
belg+10arcs_149	607.1k	53.4k	1037.0	14400.0	1e+20	63.9k	–	14400.0	275.1k	21.2k	1196.5	14400.0
belg+10arcs_150	265.5k	188.7k	40.7	14400.0	1e+20	186.0k	–	14400.0	227.1k	193.2k	17.6	14400.0
belg+10arcs_151	54.7k	54.7k	0.0	192.6	1e+20	44.7k	–	14400.0	54.7k	54.7k	0.0	21.1
belg+10arcs_152	137.4k	137.4k	0.0	3497.9	1e+20	100.0k	–	14400.0	137.4k	137.3k	0.0	1425.2
belg+10arcs_153	140.0k	140.0k	0.0	114.7	140.0k	140.0k	0.0	5583.7	140.0k	140.0k	0.0	564.1
belg+10arcs_154	41.1k	41.1k	0.0	855.0	41.1k	41.1k	0.0	8700.5	41.1k	41.1k	0.0	3982.7
belg+10arcs_155	228.3k	228.3k	0.0	325.9	228.3k	228.3k	0.0	8783.8	228.3k	228.3k	0.0	3330.1
belg+10arcs_156	26.2k	26.2k	0.0	309.1	26.2k	26.2k	0.0	4487.8	26.2k	26.2k	0.0	10.4
belg+10arcs_157	9424.0	9424.0	0.0	136.2	1e+20	2082.5	–	14400.0	9424.0	9424.0	0.0	497.0
belg+10arcs_158	29.0k	29.0k	0.0	2850.4	1e+20	7392.2	–	14400.0	29.0k	12.1k	139.9	14400.0
belg+10arcs_159	56.7k	56.7k	0.0	575.2	1e+20	29.8k	–	14400.0	339.6k	23.0k	1376.7	14400.0
belg+10arcs_160	2976.0	2976.0	0.0	10.7	1e+20	2976.0	–	14400.0	2976.0	2976.0	0.0	45.1
belg+10arcs_161	147.3k	147.3k	0.0	718.9	1e+20	116.5k	–	14400.0	147.3k	147.3k	0.0	3218.0
belg+10arcs_162	157.8k	157.8k	0.0	4803.6	157.8k	157.8k	0.0	3697.4	157.8k	157.8k	0.0	6364.4
belg+10arcs_163	158.6k	158.6k	0.0	1150.0	158.6k	158.6k	0.0	14192.9	158.6k	158.6k	0.0	484.9
belg+10arcs_164	0.0	0.0	0.0	0.3	0.0	0.0	0.0	3866.9	0.0	0.0	0.0	0.7
belg+10arcs_165	25.5k	25.5k	0.0	1679.5	25.5k	25.5k	0.0	12150.4	25.5k	25.5k	0.0	464.0
belg+10arcs_166	4164.0	4164.0	0.0	197.3	4164.0	4164.0	0.0	5591.9	4164.0	4164.0	0.0	25.8
belg+10arcs_167	16.5k	16.5k	0.0	270.6	16.5k	16.5k	0.0	1620.3	16.5k	16.5k	0.0	2779.4
belg+10arcs_168	223.8k	196.0k	14.2	14400.0	517.1k	169.9k	204.3	14400.0	234.3k	142.5k	64.3	14400.0
belg+10arcs_169	23.0k	23.0k	0.0	35.0	23.0k	23.0k	0.0	348.2	23.0k	23.0k	0.0	694.2
belg+10arcs_170	111.6k	111.6k	0.0	1034.3	1e+20	38.6k	–	14400.0	114.4k	39.3k	191.5	14400.0
belg+10arcs_171	103.6k	72.7k	42.5	14400.0	1e+20	16.0k	–	14400.0	88.1k	17.9k	392.5	14400.0
belg+10arcs_172	3600.0	3600.0	0.0	305.2	3600.0	3600.0	0.0	4029.9	3600.0	3600.0	0.0	246.3
belg+10arcs_173	236.9k	176.5k	34.2	14400.0	1e+20	142.6k	–	14400.0	296.0k	108.8k	172.2	14400.0
belg+10arcs_174	24.8k	24.8k	0.0	3107.3	24.8k	24.8k	0.0	13088.5	33.2k	4920.1	574.0	14400.0
belg+10arcs_175	37.9k	37.9k	0.0	3975.2	37.9k	37.9k	0.0	7173.5	37.9k	37.9k	0.0	1124.8
belg+10arcs_176	15.6k	15.6k	0.0	43.9	15.6k	15.6k	0.0	658.2	15.6k	15.6k	0.0	141.6
belg+10arcs_177	189.6k	189.6k	0.0	25.5	189.6k	189.6k	0.0	6868.4	189.6k	189.6k	0.0	317.4
belg+10arcs_178	61.4k	41.0k	49.8	14400.0	147.9k	23.8k	520.1	14400.0	58.8k	22.2k	164.4	14400.0
belg+10arcs_179	188.1k	188.0k	0.1	2905.3	188.1k	188.1k	0.0	12658.0	188.1k	188.1k	0.0	5017.5
belg+10arcs_180	44.6k	44.6k	0.0	626.7	44.6k	44.6k	0.0	7797.9	44.6k	44.6k	0.0	2528.4
belg+10arcs_181	20.5k	20.5k	0.0	158.1	20.5k	20.5k	0.0	6430.3	20.5k	20.5k	0.0	1341.3
belg+10arcs_182	2976.0	2976.0	0.0	65.6	2976.0	2976.0	0.0	7400.9	2976.0	2976.0	0.0	240.1
belg+10arcs_183	34.3k	34.3k	0.0	8599.9	42.0k	24.2k	73.3	14400.0	188.8k	6163.9	2963.4	14400.0
belg+10arcs_184	17.4k	17.4k	0.0	342.6	17.4k	17.4k	0.0	1992.7	17.4k	17.4k	0.0	847.6
belg+10arcs_185	16.8k	16.8k	0.0	141.5	1e+20	16.3k	–	14400.0	16.8k	16.8k	0.0	311.7
belg+10arcs_186	135.6k	135.6k	0.0	5446.8	135.6k	135.6k	0.0	10393.1	143.0k	95.2k	50.2	14400.0
belg+10arcs_187	271.2k	271.2k	0.0	14377.4	1e+20	186.1k	–	14400.0	342.8k	146.6k	133.9	14400.0
belg+10arcs_188	16.8k	16.8k	0.0	61.8	16.8k	16.8k	0.0	807.9	16.8k	16.8k	0.0	1791.1
belg+10arcs_189	133.2k	133.2k	0.0	3034.8	1e+20	71.6k	–	14400.0	133.2k	22.1k	501.7	14400.0
belg+10arcs_190	208.4k	1995.0	Large	14400.0	326.6k	37.1k	781.2	14400.0	166.5k	2454.2	6682.5	14400.0
belg+10arcs_191	228.6k	206.8k	10.6	14400.0	1e+20	200.5k	–	14400.0	226.1k	214.6k	5.4	14400.0
belg+10arcs_192	4920.0	4920.0	0.0	402.2	4920.0	4920.0	0.0	3332.7	4920.0	4920.0	0.0	14.3
belg+10arcs_193	92.2k	0.0	–	14400.0	109.1k	12.3k	790.2	14400.0	62.1k	4747.9	1208.5	14400.0
belg+10arcs_194	195.3k	174.4k	12.0	14400.0	363.6k	119.6k	204.0	14400.0	244.8k	90.3k	171.0	14400.0
belg+10arcs_195	150.5k	150.5k	0.0	1401.5	1e+20	110.9k	–	14400.0	150.5k	150.5k	0.0	13855.9
belg+10arcs_196	380.0k	124.8k	204.5	14400.0	1e+20	103.5k	–	14400.0	393.1k	68.5k	473.6	14400.0
belg+10arcs_197	218.7k	218.7k	0.0	885.4	218.7k	218.7k	0.0	4100.0	218.7k	218.7k	0.0	7804.5
belg+10arcs_198	6480.0	6480.0	0.0	8.5	6480.0	6480.0	0.0	1919.2	6480.0	6480.0	0.0	25.5
belg+10arcs_199	33.6k	33.6k	0.0	5654.3	1244.0k	20.1k	6074.6	14400.0	33.6k	33.6k	0.0	1396.1
belg+10arcs_200	140.7k	140.7k	0.0	78.7	140.7k	140.7k	0.0	3145.6	140.7k	140.7k	0.0	14.1
belg+10arcs_201	181.7k	19.7k	820.4	14400.0	54.8k	44.2k	24.1	14400.0	62.6k	24.8k	152.7	14400.0
belg+10arcs_202	18.9k	18.9k	0.0	159.8	18.9k	18.9k	0.0	3966.6	18.9k	18.9k	0.0	1406.8
belg+10arcs_203	195.4k	9424.0	1973.2	14400.0	1e+20	16.7k	–	14400.0	204.5k	11.4k	1693.9	14400.0
belg+10arcs_204	24.7k	24.7k	0.0	97.4	1e+20	24.6k	–	14400.0	24.7k	24.7k	0.0	304.6
belg+10arcs_205	193.4k	193.4k	0.0	577.6	193.4k	193.4k	0.0	8557.2	193.4k	193.4k	0.0	145.1
belg+10arcs_206	50.4k	50.4k	0.0	786.5	50.4k	50.4k	0.0	10917.8	50.4k	50.4k	0.0	2390.1
belg+10arcs_207	157.8k	157.8k	0.0	803.6	157.8k	157.8k	0.0	2084.3	157.8k	157.8k	0.0	2535.6
belg+10arcs_208	102.6k	102.6k	0.0	2579.1	1e+20	48.6k	–	14400.0	106.8k	60.5k	76.5	14400.0
belg+10arcs_209	168.3k	4920.0	3319.8	14400.0	45.4k	26.0k	74.5	14400.0	71.1k	5179.2	1272.9	14400.0
belg+10arcs_210	161.3k	161.3k	0.0	2781.4	161.3k	141.5k	14.0	14400.0	185.2k	135.4k	36.8	14400.0
belg+10arcs_211	4920.0	4920.0	0.0	46.5	4920.0	4920.0	0.0	39.8	4920.0	4920.0	0.0	117.6
belg+10arcs_212	20.5k	20.5k	0.0	386.9	20.5k	20.5k	0.0	10819.9	20.5k	20.5k	0.0	2018.6
belg+10arcs_213	202.8k	202.8k	0.0	1487.5	202.8k	202.7k	0.0	13969.8	202.8k	202.6k	0.1	8028.4
belg+10arcs_214	300.8k	123.4k	143.7	14400.0	1e+20	127.5k	–	14400.0	256.4k	115.4k	122.2	14400.0

continued on next page

Table A.33: Comparison of discrete models on *Circuit rank* + 10 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_215	29.5k	29.5k	0.0	1642.9	29.5k	29.5k	0.0	6851.0	29.5k	29.5k	0.0	11618.3
belg+10arcs_216	183.7k	183.7k	0.0	606.3	183.7k	183.7k	0.0	2231.4	183.7k	183.7k	0.0	2990.7
belg+10arcs_217	17.1k	17.1k	0.0	125.4	17.1k	17.1k	0.0	5458.6	17.1k	17.1k	0.0	107.0
belg+10arcs_218	219.0k	219.0k	0.0	12903.8	1e+20	132.6k	–	14400.0	253.2k	109.0k	132.2	14400.0
belg+10arcs_219	148.1k	148.1k	0.0	355.9	148.1k	148.1k	0.0	2415.0	148.1k	148.1k	0.0	71.4
belg+10arcs_220	214.8k	53.3k	303.1	14400.0	1e+20	48.5k	–	14400.0	437.8k	24.2k	1707.4	14400.0
belg+10arcs_221	15.6k	15.6k	0.0	33.1	15.6k	15.6k	0.0	4852.2	15.6k	15.6k	0.0	68.0
belg+10arcs_222	163.4k	163.3k	0.0	9380.1	1e+20	113.7k	–	14400.0	172.2k	133.6k	28.9	14400.0
belg+10arcs_223	67.4k	53.8k	25.2	14400.0	766.4k	12.6k	5983.0	14400.0	67.4k	21.2k	217.7	14400.0
belg+10arcs_224	168.0k	168.0k	0.0	4933.3	1e+20	102.6k	–	14400.0	220.8k	78.2k	182.3	14400.0
belg+10arcs_225	11.4k	11.4k	0.0	189.7	11.4k	11.4k	0.0	162.6	11.4k	11.4k	0.0	217.3
belg+10arcs_226	46.7k	46.7k	0.0	499.5	46.7k	46.7k	0.0	3046.4	46.7k	46.7k	0.0	8718.2
belg+10arcs_227	34.7k	27.0k	28.6	14400.0	29.7k	29.7k	0.0	12721.1	29.7k	29.7k	0.0	3417.5
belg+10arcs_228	233.5k	67.7k	244.9	14400.0	1e+20	43.2k	–	14400.0	270.3k	14.9k	1716.4	14400.0
belg+10arcs_229	130.1k	130.0k	0.1	952.1	130.1k	130.1k	0.0	7059.1	130.1k	130.1k	0.0	148.3
belg+10arcs_230	149.1k	133.2k	12.0	14400.0	1e+20	116.6k	–	14400.0	149.1k	149.1k	0.0	5124.1
belg+10arcs_231	24.7k	24.7k	0.0	614.5	24.7k	24.7k	0.0	88.4	24.7k	24.7k	0.0	100.1
belg+10arcs_232	32.5k	32.5k	0.0	1576.0	32.5k	32.5k	0.0	112.2	32.5k	32.5k	0.0	59.0
belg+10arcs_233	69.4k	69.4k	0.0	4193.4	1e+20	23.9k	–	14400.0	79.5k	6946.2	1044.3	14400.0
belg+10arcs_234	50.4k	50.4k	0.0	576.0	50.4k	50.4k	0.0	12742.0	50.4k	50.4k	0.0	1550.7
belg+10arcs_235	55.2k	55.2k	0.0	260.1	1e+20	43.7k	–	14400.0	55.2k	55.2k	0.0	815.3
belg+10arcs_236	46.1k	46.1k	0.0	557.8	46.1k	46.1k	0.0	5724.2	46.1k	46.1k	0.0	9768.1
belg+10arcs_237	2976.0	2976.0	0.0	16.2	2976.0	2976.0	0.0	1289.4	2976.0	2976.0	0.0	916.9
belg+10arcs_238	237.6k	196.6k	20.9	14400.0	1e+20	146.8k	–	14400.0	621.6k	123.5k	403.3	14400.0
belg+10arcs_239	172.6k	172.6k	0.0	4546.6	1e+20	103.2k	–	14400.0	172.6k	108.2k	59.6	14400.0
belg+10arcs_240	170.6k	170.6k	0.0	1377.7	170.6k	170.6k	0.0	1346.7	170.6k	170.6k	0.0	1282.9
belg+10arcs_241	144.1k	144.1k	0.0	3987.2	161.5k	126.0k	28.1	14400.0	144.1k	144.1k	0.0	3069.2
belg+10arcs_242	20.2k	20.2k	0.0	1373.0	20.2k	20.2k	0.0	4133.0	43.9k	5724.1	666.7	14400.0
belg+10arcs_243	157.9k	157.9k	0.0	1369.2	157.9k	157.9k	0.0	13393.9	157.9k	157.9k	0.0	13742.8
belg+10arcs_244	231.6k	141.5k	63.7	14400.0	1e+20	133.4k	–	14400.0	231.6k	129.7k	78.6	14400.0
belg+10arcs_245	29.3k	29.3k	0.0	6163.3	29.3k	29.3k	0.0	3430.1	29.3k	29.3k	0.0	1135.5
belg+10arcs_246	46.7k	46.7k	0.0	128.2	46.7k	46.7k	0.0	10705.7	46.7k	18.4k	153.7	14400.0
belg+10arcs_247	18.1k	18.1k	0.0	406.5	1e+20	12.8k	–	14400.0	18.1k	18.1k	0.0	119.0
belg+10arcs_248	236.0k	236.0k	0.0	7967.9	236.0k	203.2k	16.2	14400.0	249.9k	154.5k	61.8	14400.0
belg+10arcs_249	75.4k	39.8k	89.3	14400.0	1e+20	25.4k	–	14400.0	72.6k	21.9k	231.0	14400.0
belg+10arcs_250	144.8k	81.4k	77.9	14400.0	1e+20	37.5k	–	14400.0	144.8k	15.5k	836.4	14400.0
belg+10arcs_251	65.8k	65.8k	0.0	7101.1	74.0k	18.9k	290.9	14400.0	65.8k	31.1k	111.9	14400.0
belg+10arcs_252	179.7k	12.9k	1293.3	14400.0	1e+20	24.1k	–	14400.0	179.0k	5928.5	2919.0	14400.0
belg+10arcs_253	197.4k	197.4k	0.0	1614.1	197.4k	197.3k	0.1	13345.9	197.4k	197.4k	0.0	1784.1
belg+10arcs_254	181.1k	181.1k	0.0	1591.7	181.1k	181.1k	0.0	1183.4	181.1k	181.1k	0.0	11504.1
belg+10arcs_255	23.4k	23.4k	0.0	6192.3	23.4k	23.4k	0.0	10992.3	23.4k	23.4k	0.0	8.0
belg+10arcs_256	149.1k	149.1k	0.0	217.8	1e+20	113.1k	–	14400.0	149.1k	124.3k	20.0	14400.0
belg+10arcs_257	65.8k	65.8k	0.0	6442.6	1e+20	20.4k	–	14400.0	65.8k	6955.1	846.7	14400.0
belg+10arcs_258	32.0k	32.0k	0.0	1130.4	85.5k	22.3k	284.1	14400.0	32.0k	32.0k	0.0	7328.3
belg+10arcs_259	162.3k	105.9k	53.3	14400.0	159.8k	80.8k	97.6	14400.0	146.3k	91.9k	59.2	14400.0
belg+10arcs_260	41.0k	41.0k	0.0	211.4	41.0k	41.0k	0.0	1722.5	41.0k	41.0k	0.0	97.7
belg+10arcs_261	18.1k	18.1k	0.0	34.7	18.1k	18.1k	0.0	9591.0	18.1k	18.1k	0.0	112.0
belg+10arcs_262	19.7k	19.7k	0.0	138.7	19.7k	19.7k	0.0	1762.4	19.7k	19.7k	0.0	72.8
belg+10arcs_263	187.8k	187.8k	0.0	2044.4	187.8k	187.8k	0.0	4861.4	187.8k	187.8k	0.0	4052.6
belg+10arcs_264	58.8k	15.6k	277.5	14400.0	31.0k	31.0k	0.0	9315.6	31.0k	31.0k	0.0	12343.2
belg+10arcs_265	265.2k	225.0k	17.9	14400.0	283.3k	202.3k	40.1	14400.0	283.3k	138.8k	104.1	14400.0
belg+10arcs_266	4164.0	4164.0	0.0	72.2	4164.0	4164.0	0.0	4606.9	4164.0	4164.0	0.0	19.9
belg+10arcs_267	196.6k	196.6k	0.0	1532.2	196.6k	196.6k	0.0	2169.0	196.6k	196.6k	0.0	1427.8
belg+10arcs_268	195.9k	195.9k	0.0	1410.1	195.9k	195.9k	0.0	11266.0	204.2k	136.7k	49.4	14400.0
belg+10arcs_269	53.3k	53.3k	0.0	938.8	1e+20	30.2k	–	14400.0	53.3k	53.3k	0.0	2727.1
belg+10arcs_270	72.6k	53.8k	34.8	14400.0	69.8k	69.8k	0.0	6996.9	69.8k	69.8k	0.0	9935.4
belg+10arcs_271	137.7k	137.7k	0.0	3920.1	1e+20	76.6k	–	14400.0	146.5k	32.1k	357.0	14400.0
belg+10arcs_272	199.0k	199.0k	0.0	1988.0	199.0k	163.4k	21.8	14400.0	207.3k	134.9k	53.7	14400.0
belg+10arcs_273	176.3k	176.3k	0.0	461.9	176.3k	176.3k	0.0	10866.1	176.3k	176.3k	0.0	119.2
belg+10arcs_274	22.3k	22.3k	0.0	531.2	22.3k	22.3k	0.0	414.6	22.3k	22.3k	0.0	479.4
belg+10arcs_275	21.7k	21.7k	0.0	256.1	21.7k	21.7k	0.0	4623.4	21.7k	21.7k	0.0	354.6
belg+10arcs_276	69.0k	58.5k	18.0	14400.0	1e+20	18.9k	–	14400.0	70.0k	22.3k	214.3	14400.0
belg+10arcs_277	41.3k	41.3k	0.0	506.5	41.3k	41.3k	0.0	2750.2	41.3k	41.3k	0.0	2252.5
belg+10arcs_278	13.2k	13.2k	0.0	118.8	13.2k	13.2k	0.0	10160.0	13.2k	13.2k	0.0	591.7
belg+10arcs_279	13.6k	13.6k	0.0	267.6	1e+20	4274.1	–	14400.0	13.6k	13.6k	0.0	197.1
belg+10arcs_280	190.8k	190.8k	0.0	988.3	190.8k	190.8k	0.0	2882.5	190.8k	190.8k	0.0	7889.6
belg+10arcs_281	61.3k	31.8k	92.6	14400.0	1e+20	18.2k	–	14400.0	81.7k	24.4k	234.3	14400.0
belg+10arcs_282	249.6k	249.6k	0.0	6913.3	249.6k	202.6k	23.2	14400.0	294.2k	134.0k	119.5	14400.0
belg+10arcs_283	180.0k	180.0k	0.0	8874.8	180.0k	179.9k	0.0	7457.2	180.0k	153.5k	17.3	14400.0
belg+10arcs_284	128.3k	128.3k	0.0	1149.8	128.3k	128.3k	0.0	10338.8	128.3k	128.3k	0.0	4595.5
belg+10arcs_285	4920.0	4920.0	0.0	87.8	4920.0	4920.0	0.0	35.9	4920.0	4920.0	0.0	97.1
belg+10arcs_286	147.7k	147.7k	0.0	6710.9	147.7k	147.7k	0.0	10365.8	147.7k	147.7k	0.0	1635.8
belg+10arcs_287	163.0k	163.0k	0.0	1813.0	163.0k	163.0k	0.0	4579.3	163.0k	163.0k	0.0	514.5
belg+10arcs_288	115.4k	115.4k	0.0	642.1	1e+20	85.3k	–	14400.0	115.4k	115.4k	0.0	509.4
belg+10arcs_289	182.2k	39.8k	357.4	14400.0	541.8k	26.8k	1924.6	14400.0	182.2k	3109.4	5759.3	14400.0
belg+10arcs_290	34.6k	34.6k	0.0	579.9	34.6k	34.6k	0.0	640.5	34.6k	34.6k	0.0	6622.4
belg+10arcs_291	189.5k	138.0k	37.3	14400.0	208.8k	134.3k	55.4	14400.0	201.1k	106.4k	89.0	14400.0
belg+10arcs_292	16.3k	16.3k	0.0	651.3	16.3k	16.3k	0.0	7132.8	16.3k	16.3k	0.0	259.0

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Table A.33: Comparison of discrete models on *Circuit rank* + 10 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_293	130.3k	2976.0	4277.7	14400.0	16.7k	16.7k	0.0	3563.0	16.7k	16.7k	0.0	8400.8
belg+10arcs_294	103.4k	17.4k	495.6	14400.0	1e+20	9335.8	–	14400.0	190.7k	4756.6	3909.0	14400.0
belg+10arcs_295	40.2k	40.2k	0.0	5888.8	40.2k	40.2k	0.0	8154.2	40.2k	6501.0	518.1	14400.0
belg+10arcs_296	29.8k	29.8k	0.0	287.9	1e+20	7167.9	–	14400.0	29.8k	29.8k	0.0	4172.9
belg+10arcs_297	17.1k	17.1k	0.0	830.9	1e+20	8203.0	–	14400.0	17.1k	17.1k	0.0	2454.1
belg+10arcs_298	29.5k	29.5k	0.0	5729.3	32.4k	12.0k	169.6	14400.0	29.5k	13.7k	115.5	14400.0
belg+10arcs_299	925.1k	145.5k	535.9	14400.0	1e+20	161.9k	–	14400.0	785.7k	113.1k	594.7	14400.0
belg+10arcs_300	27.0k	27.0k	0.0	6213.8	27.0k	27.0k	0.0	432.2	27.0k	27.0k	0.0	1708.1
belg+10arcs_301	6480.0	6480.0	0.0	48.2	6480.0	6480.0	0.0	10393.5	6480.0	6480.0	0.0	42.7
belg+10arcs_302	158.6k	158.6k	0.0	2777.4	1e+20	141.1k	–	14400.0	158.6k	158.6k	0.0	1796.7
belg+10arcs_303	218.1k	160.2k	36.1	14400.0	1e+20	88.1k	–	14400.0	249.4k	17.5k	1325.7	14400.0
belg+10arcs_304	151.5k	151.5k	0.0	1513.6	151.5k	151.5k	0.0	6711.6	151.5k	151.5k	0.0	713.4
belg+10arcs_305	153.2k	153.2k	0.0	1076.8	153.2k	153.2k	0.0	8332.5	189.0k	122.1k	54.8	14400.0
belg+10arcs_306	148.0k	148.0k	0.0	2040.2	148.0k	147.9k	0.1	2775.7	148.0k	148.0k	0.0	3583.7
belg+10arcs_307	46.2k	46.2k	0.0	1828.5	1e+20	26.2k	–	14400.0	46.2k	46.2k	0.0	9816.2
belg+10arcs_308	38.6k	38.6k	0.0	1776.6	38.6k	38.6k	0.0	4383.9	38.6k	38.6k	0.0	8008.7
belg+10arcs_309	480.5k	66.4k	624.1	14400.0	267.9k	107.4k	149.4	14400.0	399.1k	8311.0	4702.2	14400.0
belg+10arcs_310	22.3k	22.3k	0.0	56.4	22.3k	22.3k	0.0	1200.6	22.3k	22.3k	0.0	12.7
belg+10arcs_311	4164.0	3600.0	15.7	14400.0	4164.0	4164.0	0.0	193.4	4164.0	4164.0	0.0	237.5
belg+10arcs_312	47.7k	47.7k	0.0	526.0	1e+20	23.9k	–	14400.0	47.7k	47.7k	0.0	1738.9
belg+10arcs_313	135.1k	135.1k	0.0	13029.6	1e+20	85.9k	–	14400.0	154.9k	12.5k	1135.4	14400.0
belg+10arcs_314	262.0k	262.0k	0.0	13447.3	1e+20	214.2k	–	14400.0	433.6k	126.2k	243.6	14400.0
belg+10arcs_315	140.0k	140.0k	0.0	1165.4	1e+20	123.6k	–	14400.0	140.0k	140.0k	0.0	2393.4
belg+10arcs_316	20.5k	20.5k	0.0	676.6	20.5k	20.5k	0.0	5115.0	25.0k	12.7k	96.2	14400.0
belg+10arcs_317	45.3k	45.3k	0.0	1799.7	45.3k	45.3k	0.0	10006.3	45.3k	45.3k	0.0	7393.1
belg+10arcs_318	74.6k	74.6k	0.0	1066.2	74.6k	74.6k	0.0	7689.6	74.6k	74.6k	0.0	444.7
belg+10arcs_319	15.9k	15.9k	0.0	645.8	15.9k	15.9k	0.0	1228.6	15.9k	15.9k	0.0	2042.9
belg+10arcs_320	129.4k	129.4k	0.0	8376.9	1e+20	89.4k	–	14400.0	129.4k	129.4k	0.0	9742.3
belg+10arcs_321	111.4k	111.4k	0.0	6842.8	1e+20	61.4k	–	14400.0	111.4k	111.4k	0.0	4900.5
belg+10arcs_322	202.8k	37.4k	442.3	14400.0	1e+20	35.7k	–	14400.0	204.5k	20.6k	893.5	14400.0
belg+10arcs_323	36.7k	36.7k	0.0	5422.8	36.7k	36.7k	0.0	1968.7	36.7k	36.7k	0.0	4807.1
belg+10arcs_324	46.4k	46.4k	0.0	47.9	1e+20	34.8k	–	14400.0	46.4k	46.4k	0.0	154.6
belg+10arcs_325	185.0k	185.0k	0.0	544.8	1e+20	170.5k	–	14400.0	185.0k	185.0k	0.0	146.4
belg+10arcs_326	123.5k	123.5k	0.0	2564.7	1e+20	43.7k	–	14400.0	123.5k	12.2k	911.9	14400.0
belg+10arcs_327	21.7k	21.7k	0.0	23.8	21.7k	21.7k	0.0	1703.9	21.7k	21.7k	0.0	85.0
belg+10arcs_328	13.2k	13.2k	0.0	256.4	1e+20	3310.1	–	14400.0	13.2k	13.2k	0.0	475.2
belg+10arcs_329	22.1k	22.1k	0.0	358.5	22.1k	22.1k	0.0	9332.2	22.1k	22.1k	0.0	880.7
belg+10arcs_330	167.3k	167.3k	0.0	416.4	167.3k	167.3k	0.0	10373.7	167.3k	167.1k	0.1	9543.6
belg+10arcs_331	0.0	0.0	0.0	0.2	0.0	0.0	0.0	8668.8	0.0	0.0	0.0	0.4
belg+10arcs_332	380.4k	182.8k	108.1	14400.0	1e+20	183.5k	–	14400.0	394.2k	135.7k	190.5	14400.0
belg+10arcs_333	4920.0	4920.0	0.0	17.6	4920.0	4920.0	0.0	26.9	4920.0	4920.0	0.0	151.6
belg+10arcs_334	177.2k	177.2k	0.0	5588.2	1e+20	129.8k	–	14400.0	194.0k	89.5k	116.8	14400.0
belg+10arcs_335	153.9k	153.9k	0.0	2462.6	1e+20	105.6k	–	14400.0	166.1k	114.9k	44.6	14400.0
belg+10arcs_336	69.7k	69.7k	0.0	218.6	69.7k	69.7k	0.0	2169.1	69.7k	69.7k	0.0	4767.9
belg+10arcs_337	83.9k	37.8k	121.9	14400.0	1e+20	25.8k	–	14400.0	69.3k	17.9k	287.6	14400.0
belg+10arcs_338	66.6k	37.6k	77.0	14400.0	1e+20	9393.3	–	14400.0	63.5k	8729.8	627.9	14400.0
belg+10arcs_339	205.1k	205.1k	0.0	3856.3	1e+20	156.0k	–	14400.0	205.1k	205.1k	0.0	11382.4
belg+10arcs_340	271.4k	90.5k	200.0	14400.0	1e+20	127.0k	–	14400.0	419.6k	67.5k	521.7	14400.0
belg+10arcs_341	29.1k	29.1k	0.0	884.9	29.1k	29.1k	0.0	870.9	29.1k	29.1k	0.0	864.5
belg+10arcs_342	837.7k	79.9k	948.4	14400.0	270.3k	90.6k	198.3	14400.0	625.0k	12.8k	4779.3	14400.0
belg+10arcs_343	26.2k	26.2k	0.0	114.9	26.2k	26.2k	0.0	2506.7	26.2k	26.2k	0.0	177.2
belg+10arcs_344	6480.0	6480.0	0.0	389.4	6480.0	6480.0	0.0	275.5	6480.0	6480.0	0.0	320.2
belg+10arcs_345	139.2k	123.5k	12.7	14400.0	135.2k	135.2k	0.0	9210.0	139.2k	104.7k	33.0	14400.0
belg+10arcs_346	140.7k	140.7k	0.0	2665.4	1e+20	88.0k	–	14400.0	146.4k	126.5k	15.7	14400.0
belg+10arcs_347	257.0k	224.9k	14.2	14400.0	1e+20	173.4k	–	14400.0	251.7k	174.7k	44.1	14400.0
belg+10arcs_348	25.4k	25.4k	0.1	1122.4	25.4k	25.4k	0.0	124.1	25.4k	25.4k	0.0	443.9
belg+10arcs_349	41.0k	41.0k	0.0	13774.2	41.0k	41.0k	0.0	6620.3	41.0k	28.9k	42.1	14400.0
belg+10arcs_350	29.4k	29.4k	0.0	353.2	29.4k	29.4k	0.0	3811.9	29.4k	29.4k	0.0	9034.3
belg+10arcs_351	292.3k	128.3k	127.7	14400.0	1e+20	122.2k	–	14400.0	302.7k	103.4k	192.8	14400.0
belg+10arcs_352	164.0k	79.8k	105.5	14400.0	1e+20	46.8k	–	14400.0	140.0k	13.8k	911.5	14400.0
belg+10arcs_353	5724.0	5724.0	0.0	268.9	5724.0	5724.0	0.0	4115.2	5724.0	5724.0	0.0	207.5
belg+10arcs_354	15.6k	15.6k	0.0	78.5	125.6k	12.1k	938.9	14400.0	15.6k	15.6k	0.0	93.0
belg+10arcs_355	35.3k	35.3k	0.0	1483.5	1e+20	6160.9	–	14400.0	35.3k	35.3k	0.0	1086.5
belg+10arcs_356	333.3k	106.8k	212.1	14400.0	1e+20	113.3k	–	14400.0	295.1k	76.4k	286.4	14400.0
belg+10arcs_357	187.7k	112.3k	67.2	14400.0	1e+20	102.1k	–	14400.0	163.5k	92.6k	76.6	14400.0
belg+10arcs_358	219.4k	219.4k	0.0	1563.0	346.1k	193.1k	79.2	14400.0	219.4k	219.4k	0.0	553.4
belg+10arcs_359	93.5k	54.1k	73.0	14400.0	1e+20	18.1k	–	14400.0	85.7k	11.4k	648.3	14400.0
belg+10arcs_360	31.8k	29.1k	9.3	14400.0	31.8k	31.8k	0.0	6866.4	31.8k	31.8k	0.0	1731.5
belg+10arcs_361	225.5k	225.5k	0.0	9073.8	225.5k	225.5k	0.0	13902.5	239.2k	179.4k	33.3	14400.0
belg+10arcs_362	43.2k	20.5k	111.2	14400.0	43.2k	4439.6	873.1	14400.0	43.2k	0.0	–	14400.0
belg+10arcs_363	112.7k	87.7k	28.6	14400.0	1e+20	35.7k	–	14400.0	121.6k	21.2k	474.0	14400.0
belg+10arcs_364	6480.0	6480.0	0.0	61.1	6480.0	6480.0	0.0	332.3	6480.0	6480.0	0.0	116.2
belg+10arcs_365	150.8k	150.8k	0.0	380.3	150.8k	150.8k	0.0	6186.3	150.8k	150.8k	0.0	431.1
belg+10arcs_366	223.2k	223.2k	0.0	1185.2	223.2k	223.2k	0.0	7368.2	223.9k	185.5k	20.7	14400.0
belg+10arcs_367	148.0k	148.0k	0.0	6291.2	786.6k	106.4k	639.5	14400.0	148.0k	107.9k	37.2	14400.0
belg+10arcs_368	49.4k	23.3k	112.2	14400.0	1e+20	15.7k	–	14400.0	49.4k	18.1k	172.8	14400.0
belg+10arcs_369	3600.0	3600.0	0.0	18.9	1e+20	3600.0	–	14400.0	3600.0	3600.0	0.0	12.8
belg+10arcs_370	74.2k	74.2k	0.0	13434.7	1e+20	9392.8	–	14400.0	74.2k	12.5k	492.9	14400.0

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Table A.33: Comparison of discrete models on *Circuit rank* + 10 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_371	178.0k	178.0k	0.0	2493.9	1e+20	143.5k	–	14400.0	178.0k	145.1k	22.7	14400.0
belg+10arcs_372	77.7k	77.7k	0.0	3404.6	77.7k	77.7k	0.0	11609.2	100.3k	5682.5	1664.6	14400.0
belg+10arcs_373	108.4k	108.4k	0.0	5078.1	108.4k	108.4k	0.0	5867.0	108.4k	29.3k	269.7	14400.0
belg+10arcs_374	52.1k	52.1k	0.0	588.4	1e+20	27.7k	–	14400.0	52.1k	52.1k	0.0	3446.7
belg+10arcs_375	46.4k	46.4k	0.0	390.8	46.4k	46.4k	0.0	7734.0	46.4k	46.4k	0.0	585.0
belg+10arcs_376	352.2k	193.5k	82.0	14400.0	1e+20	240.0k	–	14400.0	355.1k	201.4k	76.3	14400.0
belg+10arcs_377	226.0k	21.1k	970.3	14400.0	1e+20	20.9k	–	14400.0	331.8k	617.3	Large	14400.0
belg+10arcs_378	100.6k	68.2k	47.4	14400.0	1e+20	46.5k	–	14400.0	148.5k	36.4k	307.7	14400.0
belg+10arcs_379	142.5k	90.5k	57.4	14400.0	1e+20	58.5k	–	14400.0	109.2k	14.6k	647.5	14400.0
belg+10arcs_380	103.4k	69.4k	48.9	14400.0	1e+20	19.6k	–	14400.0	108.5k	7639.5	1320.5	14400.0
belg+10arcs_381	189.2k	88.7k	113.2	14400.0	1e+20	49.9k	–	14400.0	140.8k	7597.5	1753.2	14400.0
belg+10arcs_382	159.7k	159.7k	0.0	1613.3	159.7k	159.7k	0.0	5717.5	159.7k	159.7k	0.0	212.3
belg+10arcs_383	36.9k	36.9k	0.0	263.6	36.9k	36.9k	0.0	6271.4	36.9k	36.9k	0.0	2250.7
belg+10arcs_384	84.7k	35.8k	136.9	14400.0	195.2k	35.2k	453.9	14400.0	86.4k	20.6k	320.0	14400.0
belg+10arcs_385	89.1k	89.1k	0.0	1140.4	904.9k	44.0k	1954.7	14400.0	89.1k	43.1k	106.7	14400.0
belg+10arcs_386	308.9k	210.1k	47.0	14400.0	247.5k	214.9k	15.2	14400.0	327.8k	142.2k	130.4	14400.0
belg+10arcs_387	171.3k	40.1k	326.7	14400.0	1e+20	22.5k	–	14400.0	251.0k	9.2	Large	14400.0
belg+10arcs_388	29.8k	0.0	–	14400.0	18.6k	18.6k	0.0	4829.9	18.6k	18.6k	0.0	5726.4
belg+10arcs_389	14.3k	14.3k	0.0	667.1	14.3k	14.3k	0.0	9176.6	14.3k	14.3k	0.0	1091.4
belg+10arcs_390	157.3k	157.3k	0.0	5313.9	1e+20	111.0k	–	14400.0	180.5k	126.7k	42.5	14400.0
belg+10arcs_391	140.7k	140.7k	0.0	2766.3	1e+20	98.9k	–	14400.0	181.4k	101.6k	78.6	14400.0
belg+10arcs_392	0.0	0.0	0.0	0.3	0.0	0.0	0.0	2910.3	0.0	0.0	0.0	0.4
belg+10arcs_393	182.2k	182.2k	0.0	12417.7	1e+20	152.5k	–	14400.0	200.3k	150.0k	33.6	14400.0
belg+10arcs_394	27.5k	27.5k	0.0	663.6	27.5k	27.5k	0.0	3968.9	27.5k	27.5k	0.0	5108.6
belg+10arcs_395	83.2k	83.2k	0.0	5747.0	1e+20	39.9k	–	14400.0	83.2k	35.3k	135.6	14400.0
belg+10arcs_396	35.3k	35.3k	0.0	37.5	35.3k	35.3k	0.0	1298.0	35.3k	35.3k	0.0	605.4
belg+10arcs_397	244.0k	140.4k	73.7	14400.0	575.1k	101.7k	465.4	14400.0	198.9k	54.7k	263.3	14400.0
belg+10arcs_398	16.3k	16.3k	0.0	35.9	16.3k	16.3k	0.0	4672.6	16.3k	16.3k	0.0	298.4
belg+10arcs_399	4164.0	4164.0	0.0	280.2	4164.0	4164.0	0.0	3493.8	4164.0	4164.0	0.0	22.4
belg+10arcs_400	47.7k	47.7k	0.0	2983.1	1e+20	18.5k	–	14400.0	47.7k	33.1k	44.0	14400.0
belg+10arcs_401	4920.0	4920.0	0.0	146.8	4920.0	4920.0	0.0	52.2	4920.0	4920.0	0.0	31.3
belg+10arcs_402	53.9k	53.9k	0.0	9210.6	142.8k	19.8k	620.1	14400.0	153.4k	8464.0	1712.1	14400.0
belg+10arcs_403	215.0k	215.0k	0.0	3167.9	1e+20	167.9k	–	14400.0	215.5k	163.1k	32.1	14400.0
belg+10arcs_404	175.8k	159.4k	10.3	14400.0	165.7k	165.7k	0.0	8730.3	165.7k	165.7k	0.0	517.3
belg+10arcs_405	281.5k	62.3k	351.6	14400.0	1e+20	127.9k	–	14400.0	297.2k	37.2k	699.1	14400.0
belg+10arcs_406	67.5k	67.5k	0.0	423.1	67.5k	67.5k	0.0	2228.5	67.5k	67.5k	0.0	1650.5
belg+10arcs_407	103.4k	57.8k	78.8	14400.0	1e+20	23.6k	–	14400.0	106.8k	32.0k	233.7	14400.0
belg+10arcs_408	40.6k	40.6k	0.0	1399.7	1e+20	14.9k	–	14400.0	40.6k	40.6k	0.0	1545.4
belg+10arcs_409	24.6k	24.6k	0.0	10482.2	24.6k	24.6k	0.0	14277.1	24.6k	24.6k	0.0	8923.9
belg+10arcs_410	72.8k	72.8k	0.0	3545.5	72.8k	72.8k	0.0	5992.1	72.8k	72.8k	0.0	1117.5
belg+10arcs_411	97.3k	58.3k	66.9	14400.0	1e+20	31.8k	–	14400.0	91.6k	20.6k	345.4	14400.0
belg+10arcs_412	16.8k	16.8k	0.0	40.0	16.8k	16.8k	0.0	6792.1	16.8k	16.8k	0.0	137.2
belg+10arcs_413	233.8k	25.8k	805.5	14400.0	1e+20	46.3k	–	14400.0	80.7k	36.0k	124.2	14400.0
belg+10arcs_414	149.9k	19.0k	687.5	14400.0	1e+20	20.0k	–	14400.0	168.3k	7708.4	2082.7	14400.0
belg+10arcs_415	20.5k	20.5k	0.0	205.9	420.0k	8962.4	4586.1	14400.0	20.5k	20.5k	0.0	837.5
belg+10arcs_416	140.0k	125.1k	11.9	14400.0	1e+20	84.0k	–	14400.0	140.0k	112.1k	24.9	14400.0
belg+10arcs_417	17.1k	17.1k	0.0	106.8	17.1k	17.1k	0.0	1286.9	17.1k	17.1k	0.0	104.5
belg+10arcs_418	458.6k	17.8k	2473.5	14400.0	1e+20	76.0k	–	14400.0	304.1k	5014.1	5964.3	14400.0
belg+10arcs_419	20.5k	20.5k	0.0	838.1	1e+20	16.0k	–	14400.0	20.5k	20.5k	0.0	217.8
belg+10arcs_420	20.5k	20.5k	0.0	59.3	20.5k	20.5k	0.0	4398.1	20.5k	20.5k	0.0	181.0
belg+10arcs_421	26.1k	26.1k	0.0	154.9	26.1k	26.1k	0.0	92.2	26.1k	26.1k	0.0	34.5
belg+10arcs_422	41.3k	41.3k	0.0	2415.9	1e+20	17.8k	–	14400.0	41.3k	41.3k	0.0	12139.3
belg+10arcs_423	27.5k	27.5k	0.0	1192.1	27.5k	27.5k	0.0	412.4	27.5k	27.5k	0.0	1406.0
belg+10arcs_424	224.4k	224.4k	0.0	3479.6	224.4k	224.4k	0.0	12668.2	245.1k	214.6k	14.2	14400.0
belg+10arcs_425	162.4k	162.4k	0.0	4927.6	168.2k	121.3k	38.6	14400.0	213.6k	123.8k	72.5	14400.0
belg+10arcs_426	251.2k	27.6k	809.8	14400.0	1e+20	40.7k	–	14400.0	257.7k	13.4k	1825.1	14400.0
belg+10arcs_427	23.0k	23.0k	0.0	166.1	1e+20	22.1k	–	14400.0	23.0k	23.0k	0.0	267.8
belg+10arcs_428	22.3k	22.3k	0.0	1450.5	22.3k	16.8k	32.6	14400.0	22.3k	22.3k	0.0	567.9
belg+10arcs_429	177.0k	177.0k	0.0	1735.3	177.0k	140.3k	26.2	14400.0	177.0k	177.0k	0.0	4120.1
belg+10arcs_430	207.1k	84.8k	144.3	14400.0	1e+20	52.0k	–	14400.0	255.1k	22.1k	1054.3	14400.0
belg+10arcs_431	309.0k	223.4k	38.3	14400.0	275.2k	275.2k	0.0	12160.0	286.0k	149.2k	91.7	14400.0
belg+10arcs_432	4164.0	4164.0	0.0	64.0	4164.0	4164.0	0.0	8548.1	4164.0	4164.0	0.0	36.0
belg+10arcs_433	35.3k	35.3k	0.0	1162.8	1e+20	13.5k	–	14400.0	35.3k	35.3k	0.0	6308.3
belg+10arcs_434	139.2k	139.2k	0.0	253.2	139.2k	139.2k	0.0	8297.5	139.2k	139.2k	0.0	8935.9
belg+10arcs_435	140.0k	140.0k	0.0	184.3	1e+20	117.3k	–	14400.0	140.0k	140.0k	0.0	1875.4
belg+10arcs_436	205.0k	53.3k	284.4	14400.0	1e+20	75.6k	–	14400.0	210.5k	38.1k	452.9	14400.0
belg+10arcs_437	42.4k	42.4k	0.0	499.4	42.4k	42.4k	0.0	5524.6	42.4k	42.4k	0.0	1051.9
belg+10arcs_438	146.1k	146.1k	0.0	1549.9	146.1k	146.1k	0.0	4156.3	146.1k	146.1k	0.0	1590.1
belg+10arcs_439	206.3k	97.9k	110.8	14400.0	1e+20	57.5k	–	14400.0	184.5k	12.0k	1442.8	14400.0
belg+10arcs_440	141.6k	141.6k	0.0	5096.5	1e+20	110.2k	–	14400.0	141.6k	141.6k	0.0	286.4
belg+10arcs_441	46.5k	46.5k	0.0	10168.0	1e+20	25.9k	–	14400.0	62.3k	31.5k	97.7	14400.0
belg+10arcs_442	15.0k	15.0k	0.0	1693.5	1e+20	3611.0	–	14400.0	15.0k	15.0k	0.0	3999.4
belg+10arcs_443	5724.0	5724.0	0.0	62.9	5724.0	5724.0	0.0	9642.8	5724.0	5724.0	0.0	19.0
belg+10arcs_444	46.2k	46.2k	0.0	1119.0	46.2k	46.2k	0.0	13574.9	47.1k	37.4k	26.2	14400.0
belg+10arcs_445	203.9k	203.9k	0.0	3831.0	203.9k	203.7k	0.1	8835.1	212.0k	142.7k	48.6	14400.0
belg+10arcs_446	21.7k	21.7k	0.0	430.5	21.7k	21.7k	0.0	3550.0	21.7k	21.7k	0.0	961.3
belg+10arcs_447	138.0k	59.6k	131.3	14400.0	1e+20	23.8k	–	14400.0	109.3k	11.5k	851.6	14400.0
belg+10arcs_448	232.4k	21.9k	958.9	14400.0	1e+20	67.9k	–	14400.0	232.4k	11.2k	1982.4	14400.0

continued on next page

Table A.33: Comparison of discrete models on *Circuit rank* + 10 arcs (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_449	2976.0	2976.0	0.0	94.3	2976.0	2976.0	0.0	1462.7	2976.0	2976.0	0.0	798.6
belg+10arcs_450	52.3k	52.3k	0.1	492.1	1e+20	37.1k	–	14400.0	52.3k	52.3k	0.0	1430.9
belg+10arcs_451	40.6k	40.6k	0.0	897.8	40.6k	40.6k	0.0	13359.5	40.6k	40.6k	0.0	1063.6
belg+10arcs_452	125.1k	125.0k	0.1	461.9	1e+20	86.6k	–	14400.0	125.1k	106.0k	18.0	14400.0
belg+10arcs_453	234.8k	234.8k	0.0	118.3	234.8k	234.8k	0.0	5342.1	234.8k	234.8k	0.0	333.2
belg+10arcs_454	15.6k	15.6k	0.0	5067.8	1e+20	9430.4	–	14400.0	15.6k	15.6k	0.0	230.0
belg+10arcs_455	40.6k	40.6k	0.0	4703.1	1e+20	19.3k	–	14400.0	40.6k	40.6k	0.0	3201.5
belg+10arcs_456	77.5k	77.5k	0.0	7979.0	1e+20	38.6k	–	14400.0	77.5k	38.8k	99.6	14400.0
belg+10arcs_457	0.0	0.0	0.0	0.5	0.0	0.0	0.0	4767.1	0.0	0.0	0.0	0.6
belg+10arcs_458	210.5k	210.5k	0.0	90.3	1e+20	203.3k	–	14400.0	210.5k	210.5k	0.0	459.6
belg+10arcs_459	0.0	0.0	0.0	0.2	1e+20	0.0	–	14400.0	0.0	0.0	0.0	0.2
belg+10arcs_460	2976.0	2976.0	0.0	24.8	2976.0	2976.0	0.0	2961.0	2976.0	2976.0	0.0	160.1
belg+10arcs_461	69.0k	69.0k	0.0	1881.9	80.1k	51.2k	56.3	14400.0	69.0k	69.0k	0.0	9159.7
belg+10arcs_462	5724.0	5724.0	0.0	29.1	5724.0	5724.0	0.0	20.8	5724.0	5724.0	0.0	19.9
belg+10arcs_463	0.0	0.0	0.0	0.2	0.0	0.0	0.0	25.2	0.0	0.0	0.0	0.4
belg+10arcs_464	27.5k	27.5k	0.0	351.7	27.5k	27.5k	0.0	2424.7	27.5k	27.5k	0.0	343.6
belg+10arcs_465	4920.0	4164.0	18.2	14400.0	1e+20	4164.0	–	14400.0	4920.0	4920.0	0.0	613.7
belg+10arcs_466	2976.0	2976.0	0.0	56.0	2976.0	2976.0	0.0	7564.7	2976.0	2976.0	0.0	34.7
belg+10arcs_467	149.9k	149.9k	0.0	754.7	1e+20	101.6k	–	14400.0	149.9k	110.0k	36.2	14400.0
belg+10arcs_468	535.9k	48.3k	1009.7	14400.0	162.7k	80.4k	102.5	14400.0	853.4k	17.7	Large	14400.0
belg+10arcs_469	2976.0	2976.0	0.0	49.7	2976.0	2976.0	0.0	3495.4	2976.0	2976.0	0.0	502.3
belg+10arcs_470	20.5k	18.1k	13.2	14400.0	1e+20	15.2k	–	14400.0	20.5k	20.5k	0.0	129.4
belg+10arcs_471	76.2k	39.3k	93.6	14400.0	80.1k	34.5k	132.0	14400.0	69.3k	31.3k	121.3	14400.0
belg+10arcs_472	4164.0	4164.0	0.0	169.3	4164.0	4164.0	0.0	4980.3	4164.0	4164.0	0.0	68.8
belg+10arcs_473	227.7k	9497.2	2297.0	14400.0	1e+20	119.1k	–	14400.0	231.3k	24.6k	841.0	14400.0
belg+10arcs_474	23.0k	23.0k	0.0	5128.9	23.0k	23.0k	0.0	3440.9	23.0k	23.0k	0.0	321.1
belg+10arcs_475	4164.0	4164.0	0.0	27.6	4164.0	4164.0	0.0	509.8	4164.0	4164.0	0.0	38.1
belg+10arcs_476	139.2k	139.2k	0.0	147.2	1e+20	108.9k	–	14400.0	139.2k	139.2k	0.0	1256.2
belg+10arcs_477	162.0k	162.0k	0.0	3181.1	309.5k	98.6k	214.1	14400.0	187.2k	33.0k	467.6	14400.0
belg+10arcs_478	136.7k	136.7k	0.0	1568.5	1e+20	72.4k	–	14400.0	144.8k	94.8k	52.8	14400.0
belg+10arcs_479	11.7k	11.7k	0.0	1514.2	11.7k	11.7k	0.0	4387.9	11.7k	11.7k	0.0	791.5
belg+10arcs_480	147.0k	3600.0	3983.7	14400.0	1e+20	10.4k	–	14400.0	33.5k	3696.7	807.1	14400.0
belg+10arcs_481	173.5k	132.4k	31.0	14400.0	1e+20	85.3k	–	14400.0	285.8k	11.3k	2436.3	14400.0
belg+10arcs_482	101.5k	101.5k	0.0	1384.2	1e+20	42.3k	–	14400.0	104.2k	41.1k	153.6	14400.0
belg+10arcs_483	40.2k	40.2k	0.0	110.2	40.2k	40.2k	0.0	5210.6	40.2k	40.2k	0.0	7021.9
belg+10arcs_484	4920.0	4920.0	0.0	473.6	4920.0	4920.0	0.0	1092.3	4920.0	4920.0	0.0	108.3
belg+10arcs_485	28.3k	28.3k	0.0	128.0	28.3k	28.3k	0.0	7400.2	28.3k	28.3k	0.0	1644.4
belg+10arcs_486	177.9k	177.9k	0.0	583.5	177.9k	177.9k	0.0	9929.9	177.9k	177.9k	0.0	64.8
belg+10arcs_487	56.5k	56.5k	0.0	137.6	56.5k	56.5k	0.0	7426.5	56.5k	56.5k	0.0	1756.1
belg+10arcs_488	126.0k	126.0k	0.0	240.8	126.0k	126.0k	0.0	2253.0	126.0k	126.0k	0.0	10.7
belg+10arcs_489	50.3k	50.3k	0.0	3803.6	50.3k	50.3k	0.0	10195.7	50.6k	22.2k	128.1	14400.0
belg+10arcs_490	23.9k	23.9k	0.0	40.9	23.9k	23.9k	0.0	1015.5	23.9k	23.9k	0.0	2021.6
belg+10arcs_491	20.5k	20.5k	0.0	176.3	20.5k	20.5k	0.0	3687.0	20.5k	20.5k	0.0	194.3
belg+10arcs_492	25.4k	25.4k	0.0	2890.0	25.4k	25.4k	0.0	719.5	25.4k	25.4k	0.0	420.6
belg+10arcs_493	113.7k	113.7k	0.0	5985.0	1e+20	54.1k	–	14400.0	275.8k	28.3k	873.9	14400.0
belg+10arcs_494	150.3k	150.2k	0.1	941.0	150.3k	150.3k	0.0	13259.2	150.3k	150.3k	0.0	747.2
belg+10arcs_495	6480.0	6480.0	0.0	90.8	6480.0	6480.0	0.0	26.9	6480.0	6480.0	0.0	13.3
belg+10arcs_496	2976.0	2976.0	0.0	466.2	2976.0	2976.0	0.0	1925.2	2976.0	2976.0	0.0	164.8
belg+10arcs_497	41.0k	2776.0	1377.7	14400.0	32.8k	32.8k	0.0	7580.4	32.8k	32.8k	0.0	6473.7
belg+10arcs_498	197.6k	197.6k	0.0	212.1	1e+20	187.3k	–	14400.0	197.6k	197.6k	0.0	2138.5
belg+10arcs_499	145.2k	145.2k	0.0	1887.6	1e+20	90.4k	–	14400.0	191.6k	100.4k	90.8	14400.0
belg+10arcs_500	44.0k	38.2k	15.1	14400.0	99.7k	30.9k	222.2	14400.0	44.0k	27.4k	60.7	14400.0

Table A.34: Detailed results of the split-pipe models on *Circuit rank* + 10 arcs, as summarized in Figure 3.16f and Table 3.4d. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_1	202.4k	70.2k	188.1	14400.0	202.4k	202.2k	0.1	663.9
belg+10arcs_2	3585.5	3432.2	4.5	14400.0	3585.5	3584.3	0.0	2.2
belg+10arcs_3	65.6k	17.5k	273.6	14400.0	65.5k	40.4k	61.9	14400.0
belg+10arcs_4	144.8k	133.4k	8.5	14400.0	144.8k	141.5k	2.3	14400.0
belg+10arcs_5	241.9k	201.0k	20.4	14400.0	242.4k	199.5k	21.5	14400.0
belg+10arcs_6	60.0k	57.2k	5.0	14400.0	60.0k	60.0k	0.1	165.8
belg+10arcs_7	215.4k	198.7k	8.4	14400.0	215.4k	207.7k	3.7	14400.0
belg+10arcs_8	4106.5	3165.8	29.7	14400.0	4106.5	4104.4	0.1	28.1
belg+10arcs_9	73.4k	65.3k	12.3	14400.0	71.3k	71.3k	0.1	228.2
belg+10arcs_10	145.9k	93.2k	56.5	14400.0	124.6k	124.6k	0.0	13.3
belg+10arcs_11	146.7k	11.1k	1219.1	14400.0	146.7k	144.5k	1.5	14400.0

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Table A.34: Comparison of split-pipe models on *Circuit rank* + 10 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_12	28.8k	24.9k	15.7	14400.0	28.8k	28.8k	0.1	470.5
belg+10arcs_13	130.3k	36.9k	253.4	14400.0	130.3k	130.1k	0.1	1325.6
belg+10arcs_14	274.6k	176.2k	55.8	14400.0	274.6k	271.3k	1.2	14400.0
belg+10arcs_15	95.9	95.9	0.0	71.3	95.9	95.8	0.1	289.1
belg+10arcs_16	284.2k	67.1k	323.2	14400.0	278.1k	240.2k	15.7	14400.0
belg+10arcs_17	124.3k	60.0k	107.3	14400.0	124.3k	105.3k	18.1	14400.0
belg+10arcs_18	173.6k	165.4k	5.0	14400.0	173.6k	169.3k	2.6	14400.0
belg+10arcs_19	40.5k	29.9k	35.5	14400.0	40.5k	40.5k	0.0	14010.1
belg+10arcs_20	196.9k	189.3k	4.0	14400.0	196.9k	196.7k	0.1	9811.1
belg+10arcs_21	45.1k	42.5k	6.2	14400.0	45.1k	45.1k	0.1	163.6
belg+10arcs_22	161.7k	18.1k	793.0	14400.0	161.7k	79.7k	102.8	14400.0
belg+10arcs_23	62.3k	2936.1	2023.4	14400.0	62.3k	62.3k	0.1	11591.3
belg+10arcs_24	155.8k	148.8k	4.7	14400.0	155.8k	155.6k	0.1	82.2
belg+10arcs_25	13.2k	8044.4	63.6	14400.0	13.2k	13.1k	0.1	12.9
belg+10arcs_26	116.3k	95.4k	21.9	14400.0	116.3k	116.3k	0.0	149.2
belg+10arcs_27	17.4k	13.5k	28.9	14400.0	17.4k	17.4k	0.1	18.3
belg+10arcs_28	173.5k	5948.8	2816.9	14400.0	173.5k	173.4k	0.1	651.9
belg+10arcs_29	61.0	0.0	–	14400.0	61.0	61.0	0.0	112.1
belg+10arcs_30	22.3k	8953.0	148.8	14400.0	22.3k	22.3k	0.1	19.0
belg+10arcs_31	5792.1	4468.9	29.6	14400.0	5792.1	5790.5	0.0	4.6
belg+10arcs_32	176.9k	9806.3	1703.9	14400.0	176.9k	166.7k	6.1	14400.0
belg+10arcs_33	33.7k	20.6k	63.3	14400.0	33.7k	33.6k	0.1	181.5
belg+10arcs_34	19.1k	18.5k	3.3	14400.0	19.1k	19.0k	0.1	17.9
belg+10arcs_35	120.2k	4629.9	2495.6	14400.0	120.2k	120.1k	0.1	730.6
belg+10arcs_36	144.1k	46.5k	210.1	14400.0	138.1k	129.0k	7.1	14400.0
belg+10arcs_37	38.1k	35.0k	9.1	14400.0	38.1k	38.1k	0.1	22.4
belg+10arcs_38	229.2k	209.0k	9.7	14400.0	229.2k	229.2k	0.0	2704.2
belg+10arcs_39	19.9k	13.3k	50.3	14400.0	19.9k	19.9k	0.1	28.8
belg+10arcs_40	105.6k	0.0	–	14400.0	105.6k	48.8k	116.4	14400.0
belg+10arcs_41	203.9k	187.2k	8.9	14400.0	203.9k	203.7k	0.1	1905.9
belg+10arcs_42	167.3k	8143.9	1954.4	14400.0	166.5k	166.3k	0.1	5737.9
belg+10arcs_43	30.3k	28.2k	7.3	14400.0	30.3k	30.3k	0.1	13.0
belg+10arcs_44	100.3k	86.9k	15.3	14400.0	100.3k	68.8k	45.7	14400.0
belg+10arcs_45	17.6k	13.9k	26.7	14400.0	17.6k	17.6k	0.1	88.5
belg+10arcs_46	109.0k	76.4k	42.7	14400.0	109.0k	106.9k	2.0	14400.0
belg+10arcs_47	178.4k	89.8k	98.6	14400.0	178.4k	178.2k	0.1	151.4
belg+10arcs_48	160.6k	149.9k	7.1	14400.0	160.6k	160.5k	0.1	1185.7
belg+10arcs_49	10.8k	10.8k	0.1	44.9	10.8k	10.8k	0.1	61.0
belg+10arcs_50	185.9k	179.5k	3.5	14400.0	185.9k	185.7k	0.1	255.8
belg+10arcs_51	15.7k	15.6k	0.1	21.4	15.7k	15.6k	0.1	29.4
belg+10arcs_52	98.6k	25.2k	291.4	14400.0	98.6k	84.6k	16.5	14400.0
belg+10arcs_53	11.3k	1660.4	577.8	14400.0	11.3k	11.2k	0.1	7.1
belg+10arcs_54	127.9k	120.1k	6.5	14400.0	127.9k	127.8k	0.1	36.4
belg+10arcs_55	152.3k	122.0k	24.8	14400.0	152.3k	152.1k	0.1	715.7
belg+10arcs_56	58.2k	5459.2	966.3	14400.0	58.3k	17.9k	226.5	14400.0
belg+10arcs_57	41.3k	22.6k	82.9	14400.0	41.3k	41.3k	0.0	24.3
belg+10arcs_58	4631.1	4441.6	4.3	14400.0	4631.1	4626.6	0.1	6.0
belg+10arcs_59	20.4k	16.4k	24.5	14400.0	20.4k	20.4k	0.0	10.3
belg+10arcs_60	5119.5	5052.7	1.3	14400.0	5119.5	5115.8	0.1	3.5
belg+10arcs_61	0.0	0.0	0.0	110.7	0.0	0.0	0.0	0.9
belg+10arcs_62	200.0k	185.8k	7.7	14400.0	200.0k	198.6k	0.7	14400.0
belg+10arcs_63	197.5k	153.2k	28.9	14400.0	197.5k	197.3k	0.1	1111.5
belg+10arcs_64	24.1k	21.4k	12.7	14400.0	24.1k	24.1k	0.1	37.5
belg+10arcs_65	95.2k	5606.4	1597.7	14400.0	94.5k	76.8k	23.0	14400.0
belg+10arcs_66	169.9k	152.7k	11.2	14400.0	169.9k	169.7k	0.1	850.4
belg+10arcs_67	102.0k	88.2k	15.6	14400.0	102.0k	102.0k	0.0	199.1
belg+10arcs_68	137.9k	100.1k	37.7	14400.0	137.9k	132.4k	4.1	14400.0
belg+10arcs_69	84.8k	20.9k	306.0	14400.0	84.8k	56.2k	51.1	14400.0
belg+10arcs_70	41.2k	30.7k	34.1	14400.0	41.2k	41.2k	0.0	1011.3
belg+10arcs_71	816.9	0.0	–	14400.0	816.9	816.1	0.1	24.9
belg+10arcs_72	18.1k	9444.1	91.5	14400.0	18.1k	18.1k	0.0	7.9
belg+10arcs_73	22.7k	16.4k	37.9	14400.0	22.7k	22.7k	0.0	81.6
belg+10arcs_74	266.5k	175.0k	52.3	14400.0	266.5k	236.8k	12.5	14400.0
belg+10arcs_75	81.0k	25.2k	221.8	14400.0	81.0k	81.0k	0.0	1251.5
belg+10arcs_76	45.1k	34.1k	32.3	14400.0	45.1k	43.4k	4.0	14400.0
belg+10arcs_77	6288.0	4488.9	40.1	14400.0	6288.0	6282.5	0.1	27.1
belg+10arcs_78	47.9k	46.4k	3.2	14400.0	47.9k	47.9k	0.1	29.1
belg+10arcs_79	39.3k	11.0k	256.5	14400.0	39.1k	39.1k	0.0	743.0
belg+10arcs_80	6750.5	3749.2	80.1	14400.0	6750.5	6745.7	0.1	7.1
belg+10arcs_81	148.9k	1596.7	9227.3	14400.0	148.9k	148.0k	0.7	14400.0
belg+10arcs_82	19.4k	18.8k	3.3	14400.0	19.4k	19.4k	0.1	15.0
belg+10arcs_83	216.5k	66.3k	226.8	14400.0	216.5k	213.1k	1.6	14400.0
belg+10arcs_84	234.3k	233.2k	0.5	14400.0	234.3k	222.5k	5.3	14400.0
belg+10arcs_85	80.2k	79.7k	0.7	14400.0	80.2k	80.2k	0.1	437.5
belg+10arcs_86	186.7k	155.9k	19.7	14400.0	186.7k	186.5k	0.1	5628.5
belg+10arcs_87	139.0k	138.1k	0.7	14400.0	139.0k	138.9k	0.1	1435.7
belg+10arcs_88	169.7k	155.4k	9.2	14400.0	169.7k	169.7k	0.0	1599.9
belg+10arcs_89	442.7k	301.3k	46.9	14400.0	443.9k	310.5k	43.0	14400.0

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Table A.34: Comparison of split-pipe models on *Circuit rank* + 10 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_90	202.8k	14.0k	1347.7	14400.0	202.8k	188.1k	7.8	14400.0
belg+10arcs_91	175.1k	4064.5	4209.1	14400.0	175.1k	165.5k	5.8	14400.0
belg+10arcs_92	200.1k	87.6k	128.4	14400.0	200.1k	193.6k	3.4	14400.0
belg+10arcs_93	289.5k	172.5k	67.8	14400.0	289.5k	222.0k	30.4	14400.0
belg+10arcs_94	99.6k	7285.1	1267.7	14400.0	99.6k	97.7k	2.0	14400.0
belg+10arcs_95	42.2k	39.7k	6.3	14400.0	42.2k	42.1k	0.1	13715.1
belg+10arcs_96	107.9k	95.5k	12.9	14400.0	107.9k	97.6k	10.5	14400.0
belg+10arcs_97	0.0	0.0	0.0	175.6	0.0	0.0	0.0	0.8
belg+10arcs_98	136.1k	132.1k	3.0	14400.0	136.1k	123.9k	9.8	14400.0
belg+10arcs_99	157.3k	45.3k	247.5	14400.0	157.5k	105.6k	49.2	14400.0
belg+10arcs_100	122.5k	96.7k	26.6	14400.0	122.5k	122.4k	0.1	70.7
belg+10arcs_101	141.3k	74.8k	88.8	14400.0	141.3k	135.5k	4.3	14400.0
belg+10arcs_102	119.0k	108.8k	9.3	14400.0	119.0k	111.3k	6.9	14400.0
belg+10arcs_103	76.1k	1175.5	6373.0	14400.0	76.1k	4022.7	1791.4	14400.0
belg+10arcs_104	173.5k	21.9k	690.4	14400.0	173.5k	59.2k	193.1	14400.0
belg+10arcs_105	23.1k	21.0k	9.8	14400.0	23.1k	23.1k	0.1	30.6
belg+10arcs_106	80.9k	0.0	–	14400.0	80.9k	54.2k	49.3	14400.0
belg+10arcs_107	161.3k	5663.1	2748.3	14400.0	161.3k	161.1k	0.1	342.2
belg+10arcs_108	30.5k	22.9k	33.3	14400.0	30.5k	30.5k	0.0	47.3
belg+10arcs_109	0.0	0.0	0.0	149.3	0.0	0.0	0.0	12.6
belg+10arcs_110	139.0k	113.5k	22.4	14400.0	139.0k	131.6k	5.6	14400.0
belg+10arcs_111	190.3k	156.3k	21.8	14400.0	190.3k	175.0k	8.8	14400.0
belg+10arcs_112	5429.8	4454.5	21.9	14400.0	5429.8	5427.7	0.0	8.6
belg+10arcs_113	3255.3	0.0	–	14400.0	3255.3	3255.3	0.0	67.5
belg+10arcs_114	2963.2	2923.9	1.3	14400.0	2963.2	2960.5	0.1	2.7
belg+10arcs_115	101.0k	74.0k	36.5	14400.0	101.0k	86.0k	17.4	14400.0
belg+10arcs_116	20.3k	6675.7	204.3	14400.0	20.3k	20.0k	1.6	14400.0
belg+10arcs_117	48.2k	2063.4	2234.2	14400.0	48.2k	15.9k	203.7	14400.0
belg+10arcs_118	48.5k	3837.2	1164.2	14400.0	48.5k	5463.1	787.9	14400.0
belg+10arcs_119	17.5k	6073.5	188.0	14400.0	17.5k	17.4k	0.3	14400.0
belg+10arcs_120	0.0	0.0	0.0	473.5	0.0	0.0	0.0	0.6
belg+10arcs_121	1052.7	1051.7	0.1	3184.4	1052.7	1052.2	0.0	4.2
belg+10arcs_122	70.4k	4749.5	1382.4	14400.0	70.1k	70.1k	0.0	6281.5
belg+10arcs_123	19.4k	15.7k	23.6	14400.0	19.4k	19.3k	0.1	274.2
belg+10arcs_124	12.7k	12.2k	4.2	14400.0	12.7k	12.7k	0.1	57.3
belg+10arcs_125	17.7k	16.9k	4.8	14400.0	17.7k	17.6k	0.1	17.6
belg+10arcs_126	47.4k	13.2k	258.8	14400.0	47.4k	16.9k	180.6	14400.0
belg+10arcs_127	31.4k	28.3k	11.0	14400.0	31.4k	31.3k	0.1	109.3
belg+10arcs_128	31.3k	11.8k	165.6	14400.0	31.3k	23.5k	33.4	14400.0
belg+10arcs_129	3439.0	2978.0	15.5	14400.0	3439.0	3437.7	0.0	2.2
belg+10arcs_130	42.0k	0.0	–	14400.0	42.0k	41.9k	0.1	246.2
belg+10arcs_131	132.5k	43.9k	202.0	14400.0	132.5k	89.4k	48.2	14400.0
belg+10arcs_132	24.3k	21.1k	15.0	14400.0	24.3k	24.3k	0.1	71.5
belg+10arcs_133	70.7k	59.8k	18.3	14400.0	70.7k	60.6k	16.5	14400.0
belg+10arcs_134	214.4k	205.4k	4.4	14400.0	214.4k	206.0k	4.1	14400.0
belg+10arcs_135	6233.8	4646.9	34.1	14400.0	6233.8	6228.2	0.1	10.2
belg+10arcs_136	387.7k	250.5k	54.8	14400.0	389.6k	294.0k	32.5	14400.0
belg+10arcs_137	218.0k	166.8k	30.7	14400.0	218.0k	217.6k	0.2	14400.0
belg+10arcs_138	124.0k	121.8k	1.8	14400.0	124.0k	123.9k	0.1	135.7
belg+10arcs_139	78.6k	0.0	–	14400.0	78.6k	78.6k	0.1	7097.9
belg+10arcs_140	20.4k	10.1k	102.1	14400.0	20.4k	20.4k	0.1	337.8
belg+10arcs_141	80.1k	3654.6	2091.7	14400.0	80.1k	80.0k	0.1	2350.6
belg+10arcs_142	137.9k	4060.6	3295.3	14400.0	137.9k	137.7k	0.1	313.8
belg+10arcs_143	24.6k	20.1k	22.3	14400.0	24.6k	24.6k	0.1	27.5
belg+10arcs_144	27.8k	1719.1	1514.3	14400.0	27.5k	27.5k	0.0	682.9
belg+10arcs_145	3554.0	3550.4	0.1	6281.8	3554.0	3550.5	0.1	12.2
belg+10arcs_146	27.7k	20.9k	32.3	14400.0	27.7k	26.4k	4.9	14400.0
belg+10arcs_147	128.0k	113.2k	13.1	14400.0	128.0k	118.4k	8.1	14400.0
belg+10arcs_148	29.5k	13.5k	118.8	14400.0	29.5k	29.5k	0.1	8.4
belg+10arcs_149	155.2k	15.2k	918.0	14400.0	155.2k	106.8k	45.3	14400.0
belg+10arcs_150	212.4k	200.8k	5.8	14400.0	212.4k	4823.1	4303.3	14400.0
belg+10arcs_151	53.0k	50.2k	5.6	14400.0	53.0k	53.0k	0.1	13.0
belg+10arcs_152	131.8k	122.9k	7.3	14400.0	131.8k	122.8k	7.3	14400.0
belg+10arcs_153	136.7k	135.8k	0.7	14400.0	136.7k	136.5k	0.1	8082.9
belg+10arcs_154	32.4k	28.3k	14.2	14400.0	32.4k	31.3k	3.5	14400.0
belg+10arcs_155	219.1k	216.3k	1.3	14400.0	219.1k	218.9k	0.1	592.8
belg+10arcs_156	25.5k	24.2k	5.4	14400.0	25.5k	25.5k	0.1	8.2
belg+10arcs_157	2373.5	830.3	185.9	14400.0	2373.5	2371.1	0.1	97.3
belg+10arcs_158	19.2k	4482.5	328.9	14400.0	19.2k	19.2k	0.0	87.0
belg+10arcs_159	55.3k	47.0k	17.7	14400.0	55.3k	53.8k	2.7	14400.0
belg+10arcs_160	1576.2	1574.6	0.1	7365.1	1576.2	1574.7	0.1	5.3
belg+10arcs_161	140.7k	118.0k	19.3	14400.0	140.7k	129.4k	8.8	14400.0
belg+10arcs_162	152.1k	137.3k	10.8	14400.0	152.1k	147.5k	3.1	14400.0
belg+10arcs_163	150.3k	109.0k	37.9	14400.0	150.3k	150.1k	0.1	600.4
belg+10arcs_164	0.0	0.0	0.0	116.5	0.0	0.0	0.0	0.8
belg+10arcs_165	24.6k	24.2k	1.8	14400.0	24.6k	24.6k	0.1	22.3
belg+10arcs_166	3526.5	3523.2	0.1	7.7	3526.5	3523.3	0.1	6.0
belg+10arcs_167	15.5k	6774.1	128.5	14400.0	15.5k	15.5k	0.1	33.5

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Table A.34: Comparison of split-pipe models on *Circuit rank* + 10 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_168	199.5k	178.8k	11.6	14400.0	199.5k	195.2k	2.2	14400.0
belg+10arcs_169	21.0k	3922.8	435.9	14400.0	21.0k	21.0k	0.0	17.8
belg+10arcs_170	75.0k	47.3k	58.7	14400.0	75.0k	72.7k	3.2	14400.0
belg+10arcs_171	83.7k	25.9k	223.7	14400.0	83.7k	76.7k	9.1	14400.0
belg+10arcs_172	2176.1	2021.1	7.7	14400.0	2176.1	2174.2	0.1	2.8
belg+10arcs_173	224.0k	206.2k	8.6	14400.0	224.4k	203.1k	10.5	14400.0
belg+10arcs_174	20.8k	4838.6	329.9	14400.0	20.8k	20.8k	0.0	52.3
belg+10arcs_175	35.0k	27.3k	28.3	14400.0	35.0k	34.9k	0.1	153.4
belg+10arcs_176	11.8k	7132.8	64.8	14400.0	11.8k	11.7k	0.1	34.9
belg+10arcs_177	180.3k	177.6k	1.5	14400.0	180.3k	180.1k	0.1	15.3
belg+10arcs_178	54.7k	4503.9	1114.8	14400.0	54.7k	34.4k	59.1	14400.0
belg+10arcs_179	180.8k	172.4k	4.9	14400.0	180.8k	177.0k	2.2	14400.0
belg+10arcs_180	42.2k	40.7k	3.8	14400.0	42.2k	42.2k	0.0	535.2
belg+10arcs_181	17.4k	17.4k	0.1	62.5	17.4k	17.4k	0.1	17.2
belg+10arcs_182	1540.4	0.0	–	14400.0	1540.4	1539.0	0.1	41.6
belg+10arcs_183	29.3k	13.0k	125.0	14400.0	29.3k	17.9k	63.3	14400.0
belg+10arcs_184	12.8k	6747.9	89.0	14400.0	12.8k	12.7k	0.0	9.9
belg+10arcs_185	13.4k	12.8k	4.8	14400.0	13.4k	13.4k	0.1	20.3
belg+10arcs_186	111.4k	46.0k	141.9	14400.0	111.4k	111.4k	0.0	3169.9
belg+10arcs_187	265.9k	248.5k	7.0	14400.0	265.9k	253.4k	4.9	14400.0
belg+10arcs_188	12.4k	9487.9	30.5	14400.0	12.4k	12.4k	0.1	20.4
belg+10arcs_189	120.0k	74.0k	62.1	14400.0	120.0k	116.9k	2.7	14400.0
belg+10arcs_190	115.7k	17.3k	569.6	14400.0	115.7k	54.9k	111.0	14400.0
belg+10arcs_191	209.6k	97.6k	114.8	14400.0	209.6k	206.0k	1.8	14400.0
belg+10arcs_192	4401.2	3964.4	11.0	14400.0	4401.2	4397.3	0.1	10.0
belg+10arcs_193	40.4k	25.2k	60.6	14400.0	40.1k	11.8k	240.6	14400.0
belg+10arcs_194	180.0k	54.5k	230.6	14400.0	180.0k	149.1k	20.8	14400.0
belg+10arcs_195	137.2k	21.2k	548.1	14400.0	137.2k	137.1k	0.1	936.9
belg+10arcs_196	201.5k	149.4k	34.9	14400.0	201.5k	170.9k	18.0	14400.0
belg+10arcs_197	213.5k	163.9k	30.3	14400.0	213.5k	187.7k	13.7	14400.0
belg+10arcs_198	5896.6	5333.1	10.6	14400.0	5896.6	5891.0	0.1	53.4
belg+10arcs_199	25.3k	21.0k	20.6	14400.0	25.3k	25.3k	0.1	10.2
belg+10arcs_200	139.8k	137.0k	2.0	14400.0	139.8k	4855.4	2780.0	14400.0
belg+10arcs_201	66.9k	26.9k	148.9	14400.0	52.8k	52.7k	0.1	10562.6
belg+10arcs_202	13.9k	12.0k	16.0	14400.0	13.9k	13.9k	0.1	100.2
belg+10arcs_203	64.0k	6135.7	943.0	14400.0	64.0k	35.3k	81.4	14400.0
belg+10arcs_204	22.6k	22.3k	1.5	14400.0	22.6k	22.6k	0.1	7.5
belg+10arcs_205	186.0k	176.3k	5.5	14400.0	186.0k	185.9k	0.1	590.0
belg+10arcs_206	45.5k	31.0k	46.8	14400.0	45.5k	45.3k	0.4	14400.0
belg+10arcs_207	146.2k	142.0k	3.0	14400.0	146.2k	146.1k	0.1	1292.6
belg+10arcs_208	70.8k	59.7k	18.5	14400.0	70.8k	61.0k	16.0	14400.0
belg+10arcs_209	44.3k	39.2k	13.0	14400.0	44.3k	42.9k	3.3	14400.0
belg+10arcs_210	139.0k	4574.3	2938.3	14399.5	139.0k	138.8k	0.1	824.0
belg+10arcs_211	4852.1	4847.3	0.1	6578.8	4852.1	4847.9	0.1	2.0
belg+10arcs_212	17.4k	16.4k	5.6	14400.0	17.4k	17.4k	0.1	48.1
belg+10arcs_213	198.4k	183.3k	8.2	14400.0	198.4k	198.2k	0.1	2669.5
belg+10arcs_214	211.7k	100.7k	110.2	14400.0	211.7k	141.5k	49.6	14400.0
belg+10arcs_215	27.5k	18.0k	52.5	14400.0	27.5k	27.5k	0.1	2374.9
belg+10arcs_216	179.2k	163.2k	9.8	14400.0	179.2k	166.9k	7.4	14400.0
belg+10arcs_217	12.3k	8978.3	37.4	14400.0	12.3k	12.3k	0.1	5.1
belg+10arcs_218	217.3k	142.7k	52.3	14400.0	217.3k	170.8k	27.2	14400.0
belg+10arcs_219	144.9k	4482.7	3132.5	14400.0	144.9k	144.8k	0.1	75.3
belg+10arcs_220	152.2k	50.1k	203.8	14400.0	152.2k	82.9k	83.6	14400.0
belg+10arcs_221	13.4k	13.4k	0.2	14400.0	13.4k	13.4k	0.1	19.3
belg+10arcs_222	142.9k	90.4k	58.1	14400.0	142.9k	131.0k	9.1	14400.0
belg+10arcs_223	63.4k	7387.8	758.2	14400.0	63.4k	48.7k	30.3	14400.0
belg+10arcs_224	153.0k	146.6k	4.4	14400.0	153.0k	135.4k	13.0	14400.0
belg+10arcs_225	4373.8	0.0	–	14400.0	4373.8	4370.3	0.1	42.2
belg+10arcs_226	25.6k	25.1k	2.0	14400.0	25.6k	25.6k	0.1	3.0
belg+10arcs_227	27.2k	22.0k	23.3	14400.0	27.2k	27.2k	0.0	150.9
belg+10arcs_228	177.5k	130.7k	35.8	14400.0	171.7k	119.2k	44.1	14400.0
belg+10arcs_229	122.9k	69.3k	77.4	14400.0	122.9k	122.7k	0.1	2596.1
belg+10arcs_230	133.3k	4891.9	2624.6	14400.0	133.3k	133.2k	0.1	112.7
belg+10arcs_231	22.9k	19.2k	19.1	14400.0	22.9k	22.9k	0.1	6.9
belg+10arcs_232	31.7k	12.7k	150.6	14400.0	31.7k	31.7k	0.0	24.6
belg+10arcs_233	55.3k	33.5k	65.2	14400.0	55.3k	33.0k	67.8	14400.0
belg+10arcs_234	44.0k	40.3k	9.2	14400.0	44.0k	43.9k	0.1	603.1
belg+10arcs_235	53.8k	52.1k	3.3	14400.0	53.8k	53.7k	0.1	119.6
belg+10arcs_236	36.7k	22.0k	66.4	14400.0	36.7k	36.6k	0.1	781.2
belg+10arcs_237	126.1	125.9	0.1	20.3	126.1	126.1	0.0	17.0
belg+10arcs_238	226.3k	217.4k	4.1	14400.0	226.3k	218.3k	3.7	14400.0
belg+10arcs_239	164.2k	133.0k	23.4	14400.0	172.6k	142.0k	21.6	14400.0
belg+10arcs_240	166.2k	131.5k	26.4	14400.0	166.2k	166.0k	0.1	13414.7
belg+10arcs_241	138.9k	63.3k	119.3	14400.0	138.9k	138.8k	0.1	618.0
belg+10arcs_242	13.6k	4907.9	176.9	14400.0	13.6k	13.6k	0.1	1875.3
belg+10arcs_243	149.1k	99.0k	50.6	14400.0	149.1k	148.9k	0.1	126.2
belg+10arcs_244	166.1k	139.1k	19.4	14400.0	166.1k	166.1k	0.0	14387.8
belg+10arcs_245	26.9k	11.6k	131.7	14400.0	26.9k	26.9k	0.1	3038.7

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Table A.34: Comparison of split-pipe models on *Circuit rank* + 10 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_246	40.7k	12.1k	236.6	14400.0	40.7k	40.6k	0.1	13295.3
belg+10arcs_247	16.8k	12.6k	33.1	14400.0	16.8k	16.7k	0.1	54.8
belg+10arcs_248	226.4k	166.0k	36.4	14400.0	226.4k	221.7k	2.1	14400.0
belg+10arcs_249	70.8k	22.2k	218.8	14400.0	70.8k	34.6k	104.7	14400.0
belg+10arcs_250	121.5k	32.9k	268.7	14400.0	118.5k	79.4k	49.2	14400.0
belg+10arcs_251	68.2k	4006.8	1602.4	14400.0	61.6k	61.6k	0.0	2735.9
belg+10arcs_252	54.3k	35.5k	53.1	14400.0	54.3k	54.3k	0.0	3315.4
belg+10arcs_253	179.9k	169.0k	6.4	14400.0	179.9k	178.9k	0.6	14400.0
belg+10arcs_254	180.4k	45.8k	294.0	14400.0	178.6k	168.0k	6.3	14400.0
belg+10arcs_255	23.3k	23.0k	1.0	14400.0	23.3k	23.2k	0.1	37.8
belg+10arcs_256	134.1k	86.4k	55.3	14400.0	134.1k	134.1k	0.0	99.1
belg+10arcs_257	60.3k	3818.4	1478.7	14400.0	60.3k	56.9k	6.0	14400.0
belg+10arcs_258	27.8k	21.2k	30.9	14400.0	27.8k	27.7k	0.1	3599.6
belg+10arcs_259	121.3k	71.6k	69.3	14400.0	121.3k	115.0k	5.4	14400.0
belg+10arcs_260	36.7k	36.6k	0.1	195.6	36.7k	36.6k	0.1	10.4
belg+10arcs_261	17.4k	16.7k	3.7	14400.0	17.4k	17.3k	0.1	12.9
belg+10arcs_262	18.2k	13.9k	31.3	14400.0	18.2k	18.2k	0.1	15.7
belg+10arcs_263	180.6k	65.0k	177.7	14400.0	180.6k	180.4k	0.1	405.6
belg+10arcs_264	22.5k	6936.0	224.4	14400.0	22.5k	22.5k	0.1	423.6
belg+10arcs_265	256.5k	211.6k	21.2	14400.0	256.5k	213.4k	20.2	14400.0
belg+10arcs_266	3855.4	3851.6	0.1	1540.9	3855.4	3852.0	0.1	2.1
belg+10arcs_267	193.6k	166.4k	16.3	14400.0	193.6k	187.1k	3.5	14400.0
belg+10arcs_268	185.4k	4107.0	4413.9	14400.0	185.4k	178.7k	3.7	14400.0
belg+10arcs_269	48.1k	33.0k	45.5	14400.0	48.1k	47.8k	0.5	14400.0
belg+10arcs_270	64.6k	51.8k	24.7	14400.0	64.6k	63.7k	1.4	14400.0
belg+10arcs_271	107.8k	4931.1	2086.5	14400.0	107.8k	106.5k	1.3	14400.0
belg+10arcs_272	195.9k	89.7k	118.5	14400.0	195.9k	195.7k	0.1	1379.7
belg+10arcs_273	170.9k	158.4k	7.9	14400.0	170.9k	170.7k	0.1	3584.3
belg+10arcs_274	20.2k	16.1k	25.6	14400.0	20.2k	20.2k	0.1	59.3
belg+10arcs_275	18.6k	17.8k	4.3	14400.0	18.6k	18.6k	0.1	18.9
belg+10arcs_276	49.9k	27.0k	84.9	14400.0	49.9k	26.4k	88.9	14400.0
belg+10arcs_277	36.2k	15.7k	131.1	14400.0	36.2k	36.2k	0.1	121.9
belg+10arcs_278	10.4k	4761.8	119.1	14400.0	10.4k	10.4k	0.1	28.9
belg+10arcs_279	5960.8	4058.9	46.9	14400.0	5960.8	5955.8	0.1	6.6
belg+10arcs_280	182.5k	167.2k	9.1	14400.0	182.3k	182.1k	0.1	1218.2
belg+10arcs_281	58.7k	38.3k	53.5	14400.0	58.7k	38.2k	53.6	14400.0
belg+10arcs_282	239.9k	97.6k	145.8	14400.0	239.9k	235.1k	2.0	14400.0
belg+10arcs_283	162.9k	61.1k	166.6	14400.0	162.9k	162.7k	0.1	261.1
belg+10arcs_284	108.8k	105.6k	3.0	14399.7	108.8k	108.7k	0.1	524.2
belg+10arcs_285	4654.4	0.0	–	14400.0	4654.4	4649.8	0.1	10.3
belg+10arcs_286	125.8k	125.1k	0.5	14400.0	125.8k	125.7k	0.1	118.3
belg+10arcs_287	154.7k	135.1k	14.5	14400.0	154.7k	154.5k	0.1	807.5
belg+10arcs_288	91.0k	82.5k	10.3	14400.0	91.0k	91.0k	0.0	466.6
belg+10arcs_289	93.3k	10.8k	767.0	14400.0	93.2k	93.2k	0.0	1727.4
belg+10arcs_290	31.4k	11.9k	162.7	14400.0	31.4k	31.3k	0.1	212.0
belg+10arcs_291	178.8k	135.9k	31.6	14400.0	178.8k	144.2k	24.0	14400.0
belg+10arcs_292	10.6k	10.6k	0.1	57.5	10.6k	10.6k	0.1	10.7
belg+10arcs_293	13.9k	380.4	3541.1	14400.0	13.9k	13.8k	0.1	60.6
belg+10arcs_294	56.3k	9939.8	466.3	14400.0	56.3k	35.6k	58.0	14400.0
belg+10arcs_295	30.2k	5562.3	443.7	14400.0	30.2k	30.2k	0.1	7733.3
belg+10arcs_296	19.0k	0.0	–	14400.0	19.0k	18.9k	0.0	132.8
belg+10arcs_297	10.5k	4759.7	120.9	14400.0	10.5k	10.5k	0.1	13.0
belg+10arcs_298	28.2k	26.8k	5.5	14400.0	28.2k	28.2k	0.0	1187.4
belg+10arcs_299	241.9k	200.1k	20.9	14400.0	241.9k	218.5k	10.7	14400.0
belg+10arcs_300	25.7k	20.7k	24.0	14400.0	25.7k	25.7k	0.1	21.5
belg+10arcs_301	6208.7	6208.7	0.0	4037.7	6208.7	6203.5	0.1	4.7
belg+10arcs_302	151.3k	82.7k	83.0	14400.0	151.3k	151.2k	0.1	1113.0
belg+10arcs_303	186.0k	6651.9	2695.6	14400.0	186.0k	4403.1	4123.4	14398.6
belg+10arcs_304	142.6k	126.1k	13.1	14400.0	142.6k	142.6k	0.0	465.6
belg+10arcs_305	147.6k	122.5k	20.5	14400.0	147.6k	147.5k	0.1	8398.8
belg+10arcs_306	135.1k	128.8k	4.9	14400.0	135.1k	135.0k	0.1	379.4
belg+10arcs_307	37.7k	25.5k	47.6	14400.0	37.7k	37.7k	0.0	383.9
belg+10arcs_308	24.9k	21.8k	14.0	14400.0	24.9k	24.8k	0.1	24.3
belg+10arcs_309	203.0k	106.9k	89.9	14400.0	203.0k	128.7k	57.7	14400.0
belg+10arcs_310	20.4k	20.2k	1.0	14400.0	20.4k	20.3k	0.1	11.7
belg+10arcs_311	3499.3	3466.2	1.0	14400.0	3499.3	3495.9	0.1	8.0
belg+10arcs_312	41.0k	40.8k	0.3	14400.0	41.0k	40.9k	0.1	1075.8
belg+10arcs_313	121.8k	4001.4	2943.2	14400.0	121.8k	100.6k	21.1	14400.0
belg+10arcs_314	255.8k	233.7k	9.5	14400.0	255.8k	252.1k	1.5	14400.0
belg+10arcs_315	135.4k	126.4k	7.1	14400.0	135.4k	135.3k	0.1	3255.1
belg+10arcs_316	18.0k	13.7k	31.1	14400.0	18.0k	17.9k	0.1	86.3
belg+10arcs_317	38.8k	14.2k	173.6	14400.0	38.8k	38.8k	0.1	344.1
belg+10arcs_318	70.8k	60.8k	16.4	14400.0	70.8k	70.8k	0.1	97.7
belg+10arcs_319	7706.0	5429.3	41.9	14400.0	7705.4	7698.1	0.1	87.2
belg+10arcs_320	109.9k	11.3k	869.5	14400.0	109.9k	109.9k	0.0	10475.8
belg+10arcs_321	113.3k	92.8k	22.1	14400.0	110.3k	89.5k	23.2	14400.0
belg+10arcs_322	119.5k	19.2k	522.9	14400.0	118.5k	17.2k	588.6	14400.0
belg+10arcs_323	34.1k	21.9k	55.7	14400.0	34.1k	34.0k	0.1	227.0

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Table A.34: Comparison of split-pipe models on *Circuit rank* + 10 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_324	46.2k	42.1k	9.9	14400.0	45.3k	45.2k	0.1	20.3
belg+10arcs_325	182.5k	124.4k	46.7	14400.0	182.5k	181.0k	0.8	14400.0
belg+10arcs_326	115.1k	72.6k	58.5	14400.0	115.1k	91.5k	25.8	14400.0
belg+10arcs_327	20.3k	19.4k	4.6	14400.0	20.3k	20.2k	0.1	14.0
belg+10arcs_328	7872.4	4761.8	65.3	14400.0	7872.4	7866.1	0.1	43.7
belg+10arcs_329	20.4k	19.6k	3.7	14400.0	20.4k	20.3k	0.1	516.9
belg+10arcs_330	161.4k	18.3k	782.6	14400.0	161.4k	161.2k	0.1	1279.8
belg+10arcs_331	0.0	0.0	0.0	448.6	0.0	0.0	0.0	0.6
belg+10arcs_332	293.7k	191.2k	53.6	14400.0	293.7k	253.2k	16.0	14400.0
belg+10arcs_333	4826.6	4821.8	0.1	24.1	4826.6	4821.8	0.1	7.3
belg+10arcs_334	161.9k	152.1k	6.5	14400.0	161.9k	161.8k	0.1	2948.7
belg+10arcs_335	149.5k	132.5k	12.8	14400.0	149.5k	142.5k	4.9	14400.0
belg+10arcs_336	64.7k	54.0k	19.8	14400.0	64.7k	64.6k	0.1	113.9
belg+10arcs_337	64.2k	26.5k	142.4	14400.0	64.2k	29.1k	120.3	14400.0
belg+10arcs_338	48.2k	4719.7	922.0	14400.0	48.2k	17.2k	180.0	14400.0
belg+10arcs_339	198.4k	167.4k	18.5	14400.0	198.4k	193.1k	2.7	14400.0
belg+10arcs_340	180.8k	135.6k	33.4	14400.0	180.8k	153.6k	17.8	14400.0
belg+10arcs_341	26.0k	23.4k	11.2	14400.0	26.0k	26.0k	0.1	5.9
belg+10arcs_342	246.1k	12.1k	1939.5	14400.0	246.1k	123.6k	99.0	14400.0
belg+10arcs_343	25.5k	23.8k	7.0	14400.0	25.5k	25.4k	0.1	10.5
belg+10arcs_344	5884.7	4574.9	28.6	14400.0	5884.7	5884.0	0.0	5.7
belg+10arcs_345	128.6k	70.0k	83.8	14400.0	128.6k	109.9k	17.0	14400.0
belg+10arcs_346	131.8k	87.6k	50.5	14400.0	131.8k	131.7k	0.1	3192.6
belg+10arcs_347	238.1k	19.0k	1156.2	14400.0	238.1k	235.5k	1.1	14400.0
belg+10arcs_348	22.9k	20.8k	10.4	14400.0	22.9k	22.9k	0.1	31.3
belg+10arcs_349	35.4k	8877.3	298.3	14400.0	35.4k	35.4k	0.0	137.4
belg+10arcs_350	13.4k	2923.0	359.2	14400.0	13.4k	13.4k	0.1	78.2
belg+10arcs_351	221.9k	190.3k	16.6	14400.0	221.9k	198.2k	11.9	14400.0
belg+10arcs_352	121.5k	74.6k	63.0	14400.0	121.5k	88.9k	36.8	14400.0
belg+10arcs_353	4988.0	4911.8	1.6	14400.0	4988.0	4983.2	0.1	5.1
belg+10arcs_354	13.8k	12.0k	14.5	14400.0	13.8k	13.8k	0.1	68.6
belg+10arcs_355	31.9k	31.9k	0.1	396.1	31.9k	31.9k	0.0	82.1
belg+10arcs_356	200.5k	44.1k	354.9	14400.0	200.3k	137.4k	45.8	14400.0
belg+10arcs_357	117.8k	3913.1	2909.4	14400.0	117.6k	103.8k	13.3	14400.0
belg+10arcs_358	205.5k	186.1k	10.5	14400.0	225.1k	185.2k	21.6	14400.0
belg+10arcs_359	83.8k	0.0	–	14400.0	83.8k	71.0k	18.1	14400.0
belg+10arcs_360	28.7k	27.3k	5.2	14400.0	28.7k	28.7k	0.1	56.5
belg+10arcs_361	219.7k	187.6k	17.1	14400.0	219.7k	199.3k	10.2	14400.0
belg+10arcs_362	36.0k	14.9k	141.0	14400.0	36.0k	36.0k	0.0	1107.6
belg+10arcs_363	69.4k	6972.8	894.7	14400.0	69.4k	63.7k	8.8	14400.0
belg+10arcs_364	5987.3	4855.8	23.3	14400.0	5987.3	5982.1	0.1	3.2
belg+10arcs_365	149.1k	145.3k	2.6	14400.0	149.1k	149.0k	0.1	33.7
belg+10arcs_366	209.4k	36.5k	473.7	14400.0	209.4k	205.1k	2.1	14400.0
belg+10arcs_367	135.8k	123.3k	10.1	14400.0	135.8k	135.7k	0.1	3484.6
belg+10arcs_368	33.4k	15.7k	112.8	14400.0	33.4k	33.4k	0.1	1006.1
belg+10arcs_369	3010.6	2961.8	1.6	14400.0	3010.6	3007.6	0.1	3071.5
belg+10arcs_370	34.2k	1104.2	2992.8	14400.0	34.2k	34.1k	0.1	2098.9
belg+10arcs_371	170.4k	153.0k	11.4	14400.0	170.4k	170.2k	0.1	4348.3
belg+10arcs_372	76.2k	2850.5	2573.4	14400.0	75.6k	71.0k	6.6	14400.0
belg+10arcs_373	84.9k	4694.5	1708.2	14400.0	84.9k	82.0k	3.5	14400.0
belg+10arcs_374	48.3k	42.6k	13.3	14400.0	48.3k	48.2k	0.1	2153.0
belg+10arcs_375	44.8k	31.4k	42.6	14400.0	44.8k	40.5k	10.4	14400.0
belg+10arcs_376	350.2k	179.8k	94.8	14400.0	332.5k	327.8k	1.5	14400.0
belg+10arcs_377	122.9k	0.0	–	14400.0	122.9k	56.2k	118.6	14400.0
belg+10arcs_378	87.1k	51.7k	68.6	14400.0	87.1k	83.7k	4.1	14400.0
belg+10arcs_379	106.3k	90.8k	17.0	14400.0	106.3k	103.7k	2.5	14400.0
belg+10arcs_380	64.3k	10.4k	519.6	14400.0	64.3k	43.5k	47.9	14400.0
belg+10arcs_381	129.3k	54.2k	138.3	14400.0	129.3k	94.3k	37.1	14400.0
belg+10arcs_382	150.2k	5485.4	2637.3	14400.0	150.2k	149.4k	0.5	14400.0
belg+10arcs_383	29.1k	23.0k	26.5	14400.0	29.1k	29.1k	0.1	443.1
belg+10arcs_384	65.6k	26.7k	145.2	14400.0	65.6k	65.6k	0.0	2160.8
belg+10arcs_385	84.6k	19.3k	338.8	14400.0	84.6k	75.4k	12.2	14400.0
belg+10arcs_386	243.0k	222.2k	9.4	14400.0	243.0k	206.7k	17.5	14400.0
belg+10arcs_387	97.5k	23.5k	315.0	14400.0	97.5k	15.2k	539.4	14400.0
belg+10arcs_388	14.1k	4761.3	195.3	14400.0	14.1k	14.0k	0.1	225.5
belg+10arcs_389	7198.3	4463.1	61.3	14400.0	7198.3	7192.8	0.1	70.7
belg+10arcs_390	145.3k	143.2k	1.5	14400.0	145.3k	141.8k	2.5	14400.0
belg+10arcs_391	120.4k	104.3k	15.4	14400.0	120.4k	114.8k	4.8	14400.0
belg+10arcs_392	0.0	0.0	0.0	86.5	0.0	0.0	0.0	38.0
belg+10arcs_393	163.1k	157.2k	3.7	14400.0	163.1k	162.9k	0.1	171.2
belg+10arcs_394	26.8k	22.8k	17.2	14400.0	26.8k	26.7k	0.1	124.7
belg+10arcs_395	80.1k	51.3k	56.2	14400.0	80.1k	61.3k	30.7	14400.0
belg+10arcs_396	32.2k	28.0k	15.2	14400.0	32.2k	32.2k	0.1	13.6
belg+10arcs_397	193.1k	3770.7	5022.3	14400.0	193.1k	164.1k	17.7	14400.0
belg+10arcs_398	10.0k	8077.7	23.9	14400.0	10.0k	9996.9	0.1	8.6
belg+10arcs_399	3819.0	3213.0	18.9	14400.0	3819.0	3815.4	0.1	2.8
belg+10arcs_400	42.2k	20.4k	107.5	14400.0	42.2k	42.2k	0.1	1588.8
belg+10arcs_401	4821.0	3409.0	41.4	14400.0	4821.0	4820.1	0.0	3.5

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Table A.34: Comparison of split-pipe models on *Circuit rank* + 10 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_402	46.1k	8406.1	448.8	14400.0	46.1k	46.1k	0.1	10499.4
belg+10arcs_403	202.0k	48.0k	320.5	14400.0	202.0k	201.8k	0.1	9217.9
belg+10arcs_404	163.0k	155.5k	4.8	14400.0	163.0k	159.4k	2.3	14400.0
belg+10arcs_405	249.8k	40.6k	514.8	14400.0	235.6k	227.7k	3.5	14400.0
belg+10arcs_406	65.6k	54.9k	19.4	14400.0	65.6k	65.6k	0.0	249.6
belg+10arcs_407	63.6k	40.4k	57.5	14400.0	63.6k	45.8k	39.0	14400.0
belg+10arcs_408	31.3k	19.5k	60.6	14400.0	31.3k	29.9k	4.7	14400.0
belg+10arcs_409	21.2k	12.4k	70.4	14400.0	21.2k	21.2k	0.0	19.3
belg+10arcs_410	71.4k	63.2k	13.0	14400.0	71.4k	71.3k	0.1	420.9
belg+10arcs_411	80.5k	28.1k	186.1	14400.0	80.5k	29.8k	170.5	14400.0
belg+10arcs_412	12.2k	9155.0	33.0	14400.0	12.2k	12.2k	0.0	20.1
belg+10arcs_413	69.2k	28.8k	140.0	14400.0	69.2k	69.2k	0.0	447.2
belg+10arcs_414	69.1k	6970.5	891.6	14400.0	69.1k	18.8k	268.0	14400.0
belg+10arcs_415	20.2k	14.6k	38.3	14400.0	20.2k	20.2k	0.0	421.0
belg+10arcs_416	123.1k	119.5k	3.0	14400.0	123.1k	119.4k	3.0	14400.0
belg+10arcs_417	11.2k	9040.4	24.0	14400.0	11.2k	11.2k	0.1	8.2
belg+10arcs_418	185.6k	60.6k	206.1	14400.0	183.3k	141.8k	29.2	14400.0
belg+10arcs_419	18.6k	18.1k	3.2	14400.0	18.6k	18.6k	0.1	22.9
belg+10arcs_420	18.2k	17.5k	4.0	14400.0	18.2k	18.2k	0.1	20.3
belg+10arcs_421	23.7k	19.6k	21.1	14400.0	23.7k	23.7k	0.1	26.1
belg+10arcs_422	36.7k	28.7k	28.0	14400.0	36.7k	36.7k	0.1	368.3
belg+10arcs_423	25.0k	21.1k	18.1	14400.0	25.0k	24.9k	0.1	9.3
belg+10arcs_424	222.4k	206.8k	7.6	14400.0	222.4k	216.7k	2.6	14400.0
belg+10arcs_425	143.5k	26.7k	438.0	14400.0	143.5k	142.6k	0.7	14400.0
belg+10arcs_426	146.9k	1759.0	8250.5	14400.0	146.9k	67.5k	117.6	14400.0
belg+10arcs_427	21.6k	15.3k	41.2	14400.0	21.6k	21.1k	2.4	14400.0
belg+10arcs_428	19.6k	12.4k	57.7	14400.0	19.6k	19.6k	0.1	28.3
belg+10arcs_429	173.9k	155.2k	12.1	14400.0	173.9k	173.8k	0.1	3775.6
belg+10arcs_430	138.6k	103.8k	33.6	14400.0	138.6k	104.1k	33.1	14400.0
belg+10arcs_431	273.6k	133.1k	105.6	14400.0	273.6k	264.0k	3.6	14400.0
belg+10arcs_432	3536.4	3500.9	1.0	14400.0	3536.4	3532.9	0.1	4.0
belg+10arcs_433	29.9k	22.6k	32.4	14400.0	29.9k	29.9k	0.1	13529.5
belg+10arcs_434	134.1k	75.4k	77.9	14400.0	134.1k	134.0k	0.1	3987.1
belg+10arcs_435	138.5k	103.1k	34.3	14400.0	138.5k	138.3k	0.1	791.4
belg+10arcs_436	162.4k	97.1k	67.3	14400.0	162.4k	81.2k	99.9	14400.0
belg+10arcs_437	34.7k	23.2k	49.5	14400.0	34.7k	34.7k	0.1	82.2
belg+10arcs_438	140.6k	140.4k	0.1	12417.1	140.6k	137.3k	2.4	14400.0
belg+10arcs_439	142.2k	22.4k	535.0	14400.0	142.2k	88.9k	60.1	14400.0
belg+10arcs_440	139.5k	88.1k	58.3	14400.0	139.5k	126.3k	10.4	14400.0
belg+10arcs_441	43.4k	34.1k	27.4	14400.0	43.4k	39.1k	11.1	14400.0
belg+10arcs_442	7735.9	1110.7	596.5	14400.0	7735.9	7734.3	0.0	1170.6
belg+10arcs_443	5382.9	5251.8	2.5	14400.0	5382.9	5379.0	0.1	1.8
belg+10arcs_444	41.3k	25.0k	65.3	14400.0	41.3k	39.3k	5.3	14400.0
belg+10arcs_445	199.4k	147.0k	35.6	14400.0	199.3k	189.1k	5.4	14400.0
belg+10arcs_446	20.2k	14.2k	42.6	14400.0	20.2k	20.2k	0.1	63.4
belg+10arcs_447	91.3k	10.5k	766.2	14400.0	91.3k	73.2k	24.8	14400.0
belg+10arcs_448	146.6k	20.2k	625.6	14400.0	146.6k	6009.1	2339.5	14400.0
belg+10arcs_449	113.9	70.4	61.9	14400.0	113.9	113.8	0.1	41.0
belg+10arcs_450	46.2k	36.8k	25.5	14400.0	46.2k	46.2k	0.1	232.6
belg+10arcs_451	33.6k	31.9k	5.1	14400.0	33.6k	33.5k	0.1	363.7
belg+10arcs_452	111.5k	93.6k	19.1	14400.0	111.5k	95.1k	17.2	14400.0
belg+10arcs_453	228.5k	226.0k	1.1	14400.0	228.5k	228.3k	0.1	428.1
belg+10arcs_454	12.9k	8571.2	50.1	14400.0	12.9k	12.9k	0.1	34.4
belg+10arcs_455	32.0k	18.0k	78.0	14400.0	32.0k	32.0k	0.0	502.0
belg+10arcs_456	77.2k	33.4k	131.4	14400.0	72.2k	55.7k	29.7	14400.0
belg+10arcs_457	0.0	0.0	0.0	121.7	0.0	0.0	0.0	0.4
belg+10arcs_458	200.0k	190.1k	5.2	14400.0	200.0k	199.8k	0.1	77.9
belg+10arcs_459	0.0	0.0	0.0	2.4	0.0	0.0	0.0	0.4
belg+10arcs_460	1294.4	0.0	–	14400.0	1294.4	1293.1	0.1	4.2
belg+10arcs_461	64.0k	56.9k	12.4	14400.0	64.0k	62.7k	2.0	14400.0
belg+10arcs_462	5711.9	5138.1	11.2	14400.0	5711.9	5706.9	0.1	3.0
belg+10arcs_463	0.0	0.0	0.0	1.9	0.0	0.0	0.0	1.5
belg+10arcs_464	26.5k	22.2k	19.1	14400.0	26.5k	26.4k	0.1	55.3
belg+10arcs_465	4165.2	4150.8	0.3	14400.0	4165.2	4162.3	0.1	4.5
belg+10arcs_466	797.5	151.6	426.2	14400.0	797.5	796.8	0.1	1.4
belg+10arcs_467	137.9k	16.4k	739.5	14400.0	137.9k	137.7k	0.1	961.6
belg+10arcs_468	162.4k	40.8k	297.7	14400.0	162.3k	86.9k	86.8	14400.0
belg+10arcs_469	907.7	782.6	16.0	14400.0	907.7	906.8	0.1	3.9
belg+10arcs_470	18.4k	15.5k	18.8	14400.0	18.4k	18.4k	0.1	39.0
belg+10arcs_471	40.0k	20.7k	93.4	14400.0	40.0k	36.0k	11.1	14400.0
belg+10arcs_472	3917.7	3753.5	4.4	14400.0	3917.7	3914.8	0.1	6.5
belg+10arcs_473	194.8k	96.7k	101.4	14400.0	194.8k	191.4k	1.7	14400.0
belg+10arcs_474	22.3k	13.8k	61.1	14400.0	22.3k	22.2k	0.1	40.2
belg+10arcs_475	4093.1	3757.4	8.9	14400.0	4093.1	4093.1	0.0	2.1
belg+10arcs_476	131.7k	3790.9	3374.5	14400.0	131.7k	131.6k	0.1	12394.2
belg+10arcs_477	146.5k	117.3k	24.8	14400.0	146.5k	131.7k	11.2	14400.0
belg+10arcs_478	122.3k	21.6k	466.7	14400.0	122.3k	122.1k	0.1	14400.0
belg+10arcs_479	7374.6	5212.5	41.5	14400.0	7374.6	7369.6	0.1	6.6

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Table A.34: Comparison of split-pipe models on *Circuit rank* + 10 arcs (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
belg+10arcs_480	31.3k	30.0k	4.3	14400.0	31.3k	31.3k	0.0	581.8
belg+10arcs_481	162.5k	94.8k	71.4	14400.0	162.5k	137.4k	18.3	14400.0
belg+10arcs_482	61.6k	11.4k	442.9	14400.0	61.6k	50.1k	23.1	14400.0
belg+10arcs_483	33.8k	29.9k	12.9	14400.0	31.4k	31.4k	0.0	82.6
belg+10arcs_484	4221.4	4221.4	0.0	5362.1	4221.4	4217.2	0.1	7.9
belg+10arcs_485	26.7k	23.5k	13.3	14400.0	26.7k	26.7k	0.1	12.0
belg+10arcs_486	173.5k	148.0k	17.2	14400.0	173.5k	167.0k	3.9	14400.0
belg+10arcs_487	55.3k	53.3k	3.7	14400.0	55.3k	55.2k	0.1	205.9
belg+10arcs_488	113.8k	107.8k	5.5	14400.0	113.8k	113.6k	0.1	205.7
belg+10arcs_489	40.4k	6440.0	527.9	14400.0	40.4k	6896.4	486.4	14400.0
belg+10arcs_490	23.0k	19.7k	16.8	14400.0	23.0k	23.0k	0.1	17.4
belg+10arcs_491	17.9k	9945.3	79.5	14400.0	17.9k	17.8k	0.1	37.3
belg+10arcs_492	23.5k	22.7k	3.8	14400.0	23.5k	23.5k	0.1	7.1
belg+10arcs_493	105.3k	78.8k	33.7	14400.0	105.3k	95.3k	10.4	14400.0
belg+10arcs_494	126.4k	50.7k	149.4	14400.0	126.4k	126.3k	0.1	78.2
belg+10arcs_495	6074.1	6074.1	0.0	20.9	6074.1	6068.7	0.1	2.7
belg+10arcs_496	556.7	233.3	138.6	14400.0	556.7	556.2	0.1	5.5
belg+10arcs_497	18.7k	18.6k	0.1	37.7	18.7k	18.6k	0.1	50.7
belg+10arcs_498	195.7k	189.9k	3.0	14400.0	195.7k	195.5k	0.1	7644.8
belg+10arcs_499	124.4k	105.1k	18.3	14400.0	124.4k	124.4k	0.0	2854.1
belg+10arcs_500	36.6k	18.0k	103.7	14400.0	36.6k	36.6k	0.0	2532.4

Table A.35: Detailed results of the discrete models on *New York* with $\mathcal{B} = 100$ as summarized in Figure 3.11a and Table 3.3e. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_1	156.6M	72.8M	115.0	14459.6	146.9M	131.8M	11.4	14402.4	172.5M	83.2M	107.3	14400.0
dem_100_newyork_2	166.4M	75.6M	120.3	14400.0	143.1M	143.1M	0.0	7194.0	165.3M	86.6M	90.8	14400.0
dem_100_newyork_3	208.7M	69.1M	202.1	14400.3	172.7M	157.3M	9.8	14400.0	225.9M	83.2M	171.5	14400.0
dem_100_newyork_4	163.4M	75.6M	116.0	14400.4	159.2M	150.3M	5.9	14400.0	182.0M	92.9M	95.9	14400.0
dem_100_newyork_5	174.2M	83.6M	108.4	14400.1	150.4M	135.3M	11.1	14401.7	181.7M	93.4M	94.6	14400.0
dem_100_newyork_6	166.5M	77.5M	114.8	14400.1	160.7M	142.8M	12.5	14403.3	167.8M	93.2M	79.9	14400.0
dem_100_newyork_7	142.4M	73.7M	93.3	14400.0	130.8M	130.8M	0.0	8313.3	170.3M	77.6M	119.5	14400.0
dem_100_newyork_8	192.3M	77.3M	148.8	14441.9	171.2M	150.2M	14.0	14400.0	194.1M	92.4M	109.9	14400.0
dem_100_newyork_9	115.5M	50.1M	130.4	14400.0	105.3M	105.3M	0.0	8179.7	160.3M	56.1M	185.6	14400.0
dem_100_newyork_10	178.0M	95.1M	87.2	14400.0	177.5M	177.5M	0.0	12521.9	207.4M	114.3M	81.5	14400.0
dem_100_newyork_11	158.1M	76.8M	105.8	14400.0	150.2M	150.2M	0.0	8652.9	192.8M	89.6M	115.2	14400.0
dem_100_newyork_12	192.1M	81.7M	135.0	14400.0	185.9M	170.5M	9.0	14400.0	206.1M	96.6M	113.3	14400.0
dem_100_newyork_13	190.5M	74.2M	156.6	14400.0	177.6M	157.2M	12.9	14400.0	211.8M	97.1M	118.0	14400.0
dem_100_newyork_14	131.4M	53.6M	145.1	14400.0	121.8M	98.7M	23.5	14400.0	146.4M	59.8M	144.8	14400.0
dem_100_newyork_15	163.8M	62.3M	163.1	14454.6	141.0M	141.0M	0.0	11540.2	166.3M	78.1M	113.0	14400.0
dem_100_newyork_16	155.6M	60.9M	155.7	14400.0	146.0M	146.0M	0.0	11788.6	172.6M	77.7M	122.0	14400.0
dem_100_newyork_17	142.2M	71.0M	100.4	14400.1	140.8M	130.1M	8.2	14400.0	161.0M	88.4M	82.1	14400.0
dem_100_newyork_18	138.1M	66.0M	109.2	14400.0	134.7M	134.7M	0.0	6092.1	168.1M	80.0M	110.0	14400.0
dem_100_newyork_19	179.2M	82.5M	117.1	14400.0	177.0M	162.0M	9.3	14403.1	198.4M	104.2M	90.5	14400.0
dem_100_newyork_20	178.2M	91.9M	93.9	14400.0	172.6M	152.1M	13.5	14400.0	175.1M	106.2M	64.9	14400.0
dem_100_newyork_21	142.4M	68.7M	107.2	14400.0	137.9M	127.2M	8.4	14400.0	146.1M	82.2M	77.7	14400.0
dem_100_newyork_22	141.7M	68.6M	106.7	14400.0	140.0M	140.0M	0.0	14226.3	148.8M	84.9M	75.3	14400.0
dem_100_newyork_23	221.0M	80.0M	176.4	14400.0	183.9M	169.0M	8.8	14402.5	215.9M	98.8M	118.6	14400.0
dem_100_newyork_24	102.5M	44.4M	130.8	14400.0	77.4M	77.4M	0.0	5345.6	89.2M	48.4M	84.5	14400.0
dem_100_newyork_25	191.9M	80.8M	137.4	14400.0	172.3M	150.8M	14.2	14404.0	183.5M	99.0M	85.4	14400.0
dem_100_newyork_26	181.2M	71.4M	153.6	14400.4	161.0M	139.0M	15.8	14402.4	178.6M	87.4M	104.3	14400.0
dem_100_newyork_27	129.0M	73.1M	76.4	14400.3	126.1M	126.1M	0.0	6583.5	138.5M	87.6M	58.2	14400.7
dem_100_newyork_28	157.9M	73.7M	114.2	14400.0	152.2M	133.7M	13.8	14400.0	157.8M	89.5M	76.3	14400.0
dem_100_newyork_29	191.2M	84.0M	127.6	14400.0	178.1M	155.5M	14.5	14400.0	200.8M	99.6M	101.6	14400.0
dem_100_newyork_30	146.6M	71.3M	105.6	14400.0	137.1M	137.1M	0.0	7020.1	166.5M	78.6M	111.9	14400.0
dem_100_newyork_31	175.1M	67.3M	160.2	14400.0	154.7M	134.5M	15.0	14402.6	187.0M	76.8M	143.5	14400.0
dem_100_newyork_32	148.3M	62.0M	139.4	14400.0	133.1M	126.2M	5.5	14400.0	165.8M	74.4M	122.8	14400.0
dem_100_newyork_33	192.3M	88.7M	116.7	14454.6	179.8M	159.9M	12.4	14402.7	265.9M	103.1M	157.9	14400.0
dem_100_newyork_34	143.7M	67.6M	112.5	14400.3	132.1M	132.1M	0.0	7558.7	161.2M	75.3M	113.9	14400.0
dem_100_newyork_35	190.6M	80.1M	138.1	14400.0	169.5M	148.6M	14.1	14400.0	199.3M	98.3M	102.8	14400.0
dem_100_newyork_36	165.8M	73.0M	127.2	14400.0	147.0M	133.9M	9.8	14402.0	150.9M	89.3M	68.9	14400.0
dem_100_newyork_37	160.6M	61.0M	163.3	14400.0	142.3M	128.7M	10.5	14400.0	169.3M	69.4M	143.8	14400.5
dem_100_newyork_38	168.1M	76.5M	119.9	14400.0	162.8M	162.8M	0.0	11586.2	194.5M	95.8M	103.1	14400.0
dem_100_newyork_39	169.6M	62.1M	173.0	14400.0	143.8M	122.3M	17.5	14400.0	169.5M	76.7M	120.9	14400.0
dem_100_newyork_40	122.2M	66.3M	84.4	14400.0	122.2M	122.2M	0.0	10661.8	129.7M	75.5M	71.9	14400.0
dem_100_newyork_41	158.9M	75.7M	110.0	14400.0	157.0M	145.4M	7.9	14403.7	165.4M	90.0M	83.7	14400.0
dem_100_newyork_42	163.2M	79.8M	104.5	14400.0	158.8M	144.1M	10.2	14402.0	211.2M	89.3M	136.6	14400.7

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Table A.35: Comparison of discrete models on *New York* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_43	144.8M	63.9M	126.4	14400.0	129.5M	121.1M	6.9	14400.0	149.3M	76.0M	96.4	14400.0
dem_100_newyork_44	134.9M	54.6M	147.0	14400.0	129.2M	129.2M	0.0	8145.2	161.7M	64.0M	152.8	14400.0
dem_100_newyork_45	146.8M	73.6M	99.4	14400.0	130.6M	130.3M	0.2	12081.2	173.5M	82.6M	110.1	14400.0
dem_100_newyork_46	127.9M	59.3M	115.7	14400.3	110.6M	110.6M	0.0	8459.4	117.5M	67.9M	72.9	14400.0
dem_100_newyork_47	201.3M	90.4M	122.7	14400.0	188.4M	167.4M	12.6	14400.0	199.1M	111.4M	78.7	14400.0
dem_100_newyork_48	124.6M	61.0M	104.2	14464.5	114.0M	114.0M	0.0	4678.5	129.5M	72.3M	79.2	14400.0
dem_100_newyork_49	159.9M	80.5M	98.7	14400.0	159.1M	151.3M	5.2	14400.0	176.0M	90.6M	94.2	14400.0
dem_100_newyork_50	146.8M	77.1M	90.5	14400.0	140.2M	128.5M	9.1	14400.0	169.5M	85.3M	98.7	14400.0
dem_100_newyork_51	165.0M	71.4M	131.0	14400.0	162.8M	143.3M	13.6	14402.5	176.7M	84.7M	108.6	14400.0
dem_100_newyork_52	128.3M	66.3M	93.6	14400.0	122.0M	122.0M	0.0	11528.7	148.6M	77.3M	92.2	14400.0
dem_100_newyork_53	217.8M	85.5M	154.7	14400.2	188.7M	171.9M	9.7	14401.8	208.7M	111.5M	87.1	14400.0
dem_100_newyork_54	158.8M	74.4M	113.5	14400.4	148.2M	129.2M	14.7	14402.3	173.0M	86.8M	99.2	14400.0
dem_100_newyork_55	158.7M	78.2M	102.9	14400.0	153.3M	140.0M	9.5	14402.6	186.7M	91.9M	103.1	14400.0
dem_100_newyork_56	169.0M	72.3M	133.7	14400.0	135.0M	135.0M	0.0	10577.0	158.5M	79.2M	100.2	14400.0
dem_100_newyork_57	214.4M	86.0M	149.2	14400.0	196.3M	180.4M	8.8	14401.6	217.8M	114.3M	90.5	14400.0
dem_100_newyork_58	159.5M	64.0M	149.0	14400.0	150.7M	150.7M	0.0	13695.9	170.4M	76.1M	124.0	14400.0
dem_100_newyork_59	134.1M	62.3M	115.4	14400.0	125.9M	125.9M	0.0	14319.2	160.0M	75.7M	111.5	14400.0
dem_100_newyork_60	230.3M	92.1M	150.1	14400.0	207.1M	189.5M	9.3	14401.5	234.5M	120.3M	94.9	14400.0
dem_100_newyork_61	165.7M	75.1M	120.7	14400.1	155.3M	142.0M	9.4	14401.8	180.5M	88.0M	105.1	14400.0
dem_100_newyork_62	157.1M	75.5M	108.2	14400.0	148.5M	135.0M	10.0	14400.0	185.3M	76.6M	141.9	14400.0
dem_100_newyork_63	177.2M	75.8M	133.8	14400.0	154.8M	141.9M	9.1	14400.0	158.9M	92.8M	71.2	14400.0
dem_100_newyork_64	146.3M	48.5M	201.6	14400.1	103.2M	94.1M	9.7	14400.0	114.8M	52.2M	119.8	14400.2
dem_100_newyork_65	214.2M	89.4M	139.5	14400.0	198.2M	177.9M	11.4	14400.0	213.8M	111.8M	91.3	14400.0
dem_100_newyork_66	154.1M	75.0M	105.4	14400.0	148.6M	134.8M	10.2	14400.0	164.1M	91.7M	79.0	14400.0
dem_100_newyork_67	158.7M	71.7M	121.3	14400.0	128.1M	128.1M	0.0	5536.2	149.3M	86.4M	72.7	14400.0
dem_100_newyork_68	167.6M	69.1M	142.6	14400.0	160.5M	143.6M	11.8	14400.0	195.8M	87.2M	124.7	14400.0
dem_100_newyork_69	172.4M	80.5M	114.3	14400.8	167.1M	154.9M	7.9	14400.0	174.9M	103.7M	68.7	14400.0
dem_100_newyork_70	117.3M	61.0M	92.3	14400.1	109.8M	109.8M	0.0	7452.1	135.3M	69.3M	95.2	14400.0
dem_100_newyork_71	164.4M	76.4M	115.2	14400.0	151.5M	128.6M	17.8	14400.0	163.7M	87.6M	86.9	14400.0
dem_100_newyork_72	143.4M	68.1M	110.5	14400.0	135.0M	135.0M	0.0	5771.0	174.7M	76.1M	129.5	14400.0
dem_100_newyork_73	214.8M	83.7M	156.6	14400.0	190.5M	178.5M	6.7	14400.0	206.3M	111.6M	84.8	14400.0
dem_100_newyork_74	199.8M	84.3M	137.1	14459.6	181.6M	159.2M	14.1	14400.0	193.1M	105.0M	84.0	14400.0
dem_100_newyork_75	125.3M	64.1M	95.6	14400.4	118.9M	99.3M	19.8	14403.0	126.7M	70.3M	80.1	14400.0
dem_100_newyork_76	144.9M	69.6M	108.2	14400.0	143.3M	124.9M	14.8	14402.5	156.9M	80.3M	95.5	14400.0
dem_100_newyork_77	136.0M	70.3M	93.3	14400.0	129.7M	114.5M	13.3	14403.0	152.3M	76.0M	100.4	14400.0
dem_100_newyork_78	178.6M	70.2M	154.6	14400.0	162.9M	152.8M	6.6	14401.5	185.7M	85.8M	116.4	14400.0
dem_100_newyork_79	170.9M	65.4M	161.3	14400.4	145.3M	126.4M	14.9	14402.2	154.1M	78.6M	96.2	14400.0
dem_100_newyork_80	131.8M	63.1M	108.8	14400.0	123.2M	113.5M	8.6	14401.9	145.8M	70.5M	107.0	14400.0
dem_100_newyork_81	117.5M	54.2M	117.0	14400.0	111.9M	111.9M	0.0	6764.3	146.0M	63.9M	128.5	14400.0
dem_100_newyork_82	184.7M	88.3M	109.2	14400.0	170.7M	137.8M	23.8	14400.0	177.5M	101.0M	75.7	14400.0
dem_100_newyork_83	115.5M	64.2M	79.9	14400.0	109.7M	109.7M	0.0	4554.4	126.3M	75.9M	66.4	14400.0
dem_100_newyork_84	119.5M	51.3M	133.1	14400.9	112.5M	112.5M	0.0	5064.3	139.0M	60.3M	130.6	14400.0
dem_100_newyork_85	170.9M	58.6M	191.6	14400.0	150.9M	134.8M	12.0	14402.0	188.1M	72.7M	158.7	14400.0
dem_100_newyork_86	216.0M	86.9M	148.5	14400.0	177.9M	162.8M	9.3	14400.0	196.9M	102.1M	92.8	14400.0
dem_100_newyork_87	203.6M	83.1M	144.9	14400.0	186.9M	164.1M	13.9	14401.9	219.2M	105.4M	108.0	14400.0
dem_100_newyork_88	157.7M	83.5M	88.9	14400.2	145.9M	145.9M	0.0	5529.8	173.4M	96.9M	78.9	14400.0
dem_100_newyork_89	134.1M	66.5M	101.5	14448.0	125.6M	125.6M	0.0	3269.2	147.2M	77.6M	89.7	14400.0
dem_100_newyork_90	142.7M	83.7M	70.5	14400.0	141.2M	141.2M	0.0	5405.4	150.9M	100.4M	50.2	14400.0
dem_100_newyork_91	176.7M	78.6M	124.9	14400.0	163.9M	144.6M	13.3	14402.8	170.1M	94.3M	80.4	14400.0
dem_100_newyork_92	185.6M	79.9M	132.3	14400.0	171.4M	147.7M	16.0	14400.0	199.8M	92.6M	115.8	14400.0
dem_100_newyork_93	182.2M	89.8M	102.9	14400.0	178.9M	161.3M	10.9	14400.0	208.6M	109.4M	90.7	14400.0
dem_100_newyork_94	154.8M	59.1M	162.0	14400.0	142.5M	122.7M	16.1	14400.0	176.0M	66.0M	166.6	14400.0
dem_100_newyork_95	172.4M	77.4M	122.7	14400.0	164.2M	149.6M	9.8	14404.7	195.6M	85.3M	129.3	14400.0
dem_100_newyork_96	160.2M	75.4M	112.5	14400.0	158.5M	142.9M	11.0	14403.0	175.5M	91.3M	92.1	14400.0
dem_100_newyork_97	186.2M	72.5M	157.0	14400.0	159.0M	140.6M	13.1	14403.8	173.0M	91.1M	90.0	14400.0
dem_100_newyork_98	91.2M	46.1M	98.0	14400.3	90.9M	90.9M	0.0	12331.7	95.5M	52.1M	83.2	14400.0
dem_100_newyork_99	217.8M	86.1M	152.9	14455.2	181.1M	173.3M	4.5	14400.0	218.1M	106.1M	105.6	14400.0
dem_100_newyork_100	153.0M	70.7M	116.5	14400.2	150.2M	130.2M	15.4	14400.0	203.3M	80.0M	153.9	14400.0
dem_100_newyork_101	173.1M	76.0M	127.8	14400.0	169.4M	154.9M	9.3	14403.9	187.1M	101.7M	84.0	14400.0
dem_100_newyork_102	149.7M	73.4M	103.9	14400.1	140.8M	121.8M	15.5	14400.0	153.6M	82.4M	86.4	14400.0
dem_100_newyork_103	139.6M	68.3M	104.4	14400.0	130.9M	130.9M	0.0	5031.6	163.9M	77.1M	112.7	14400.0
dem_100_newyork_104	171.2M	78.1M	119.2	14400.0	155.1M	154.4M	0.5	7661.6	176.6M	92.0M	92.1	14400.0
dem_100_newyork_105	152.5M	72.4M	110.7	14400.2	146.6M	133.1M	10.1	14400.0	160.9M	90.4M	78.0	14400.0
dem_100_newyork_106	135.2M	59.1M	128.7	14400.0	110.8M	110.7M	0.1	3079.3	121.1M	66.8M	81.2	14400.0
dem_100_newyork_107	156.1M	73.5M	112.2	14400.0	146.1M	127.5M	14.6	14402.4	159.9M	80.9M	97.7	14400.3
dem_100_newyork_108	129.9M	62.1M	109.3	14400.0	113.5M	113.4M	0.1	6215.4	137.4M	69.8M	96.8	14400.0
dem_100_newyork_109	203.8M	85.3M	138.9	14400.1	181.3M	164.8M	10.0	14400.0	227.4M	95.1M	139.1	14400.0
dem_100_newyork_110	143.5M	63.3M	126.9	14400.0	127.9M	127.9M	0.0	10410.4	151.0M	71.1M	112.5	14400.0
dem_100_newyork_111	199.7M	81.0M	146.4	14400.0	175.2M	149.2M	17.4	14402.5	216.4M	98.1M	120.6	14400.0
dem_100_newyork_112	172.3M	75.0M	129.6	14400.0	159.1M	138.1M	15.2	14402.1	184.6M	94.6M	95.3	14400.0
dem_100_newyork_113	165.0M	68.6M	140.7	14400.0	155.9M	141.9M	9.9	14400.0	184.9M	81.3M	127.4	14400.0
dem_100_newyork_114	158.7M	75.4M	110.4	14400.0	145.6M	145.6M	0.0	5907.0	171.7M	87.0M	97.2	14400.0
dem_100_newyork_115	127.8M	70.5M	81.3	14400.0	121.6M	121.6M	0.0	7549.4	136.0M	79.8M	70.3	14400.0
dem_100_newyork_116	179.5M	77.6M	131.3	14400.0	168.9M	150.0M	12.6	14402.7	206.0M	94.7M	117.5	14400.0
dem_100_newyork_117	169.1M	73.4M	130.5	14400.0	149.5M	137.1M	9.0	14400.9	192.4M	87.0M	121.0	14400.0
dem_100_newyork_118	151.1M	67.8M	122.8	14400.2	134.5M	117.6M	14.4	14403.6	159.3M	71.4M	123.3	14400.5
dem_100_newyork_119	132.4M	51.7M	156.0	14400.0	125.7M	109.6M	14.7	14400.0	147.3M	61.5M	139.6	14400.4
dem_100_newyork_120	181.7M	81.2M	123.7	14400.0	170.0M	156.8M	8.4	14400.0	178.2M	107.5M	65.8	14400.0

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Table A.35: Comparison of discrete models on *New York* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_121	164.0M	66.6M	146.2	14400.0	148.9M	131.1M	13.6	14400.0	171.9M	68.6M	150.6	14400.0
dem_100_newyork_122	163.5M	67.3M	143.0	14400.2	144.0M	124.0M	16.1	14406.8	161.3M	79.7M	102.2	14400.4
dem_100_newyork_123	171.4M	81.5M	110.2	14400.5	163.5M	144.5M	13.2	14400.0	184.1M	91.4M	101.3	14400.5
dem_100_newyork_124	159.1M	74.9M	112.5	14400.0	151.5M	130.7M	15.9	14400.0	178.5M	81.7M	118.6	14400.0
dem_100_newyork_125	139.5M	71.8M	94.2	14400.0	135.9M	135.9M	0.0	5160.9	148.5M	85.0M	74.8	14400.0
dem_100_newyork_126	206.3M	89.2M	131.4	14400.0	188.2M	172.0M	9.4	14402.1	192.1M	109.5M	75.4	14400.0
dem_100_newyork_127	214.9M	99.3M	116.3	14400.2	193.7M	176.2M	9.9	14400.0	201.5M	121.8M	65.5	14400.0
dem_100_newyork_128	175.8M	77.1M	128.0	14400.0	156.2M	156.2M	0.0	12024.3	172.1M	83.1M	107.2	14400.0
dem_100_newyork_129	130.1M	72.6M	79.1	14400.0	130.1M	130.1M	0.0	8075.9	145.8M	80.4M	81.3	14400.0
dem_100_newyork_130	205.0M	94.2M	117.6	14400.0	184.4M	167.2M	10.3	14402.4	220.6M	108.5M	103.4	14400.0
dem_100_newyork_131	126.1M	55.8M	125.8	14400.3	120.0M	120.0M	0.0	5384.4	157.0M	65.1M	141.3	14400.0
dem_100_newyork_132	196.2M	83.2M	135.7	14400.0	179.9M	160.4M	12.1	14400.0	248.1M	95.7M	159.2	14400.0
dem_100_newyork_133	147.9M	67.4M	119.4	14455.5	130.3M	130.3M	0.0	8892.4	154.6M	79.5M	94.5	14400.0
dem_100_newyork_134	204.7M	75.5M	171.1	14401.0	175.2M	158.4M	10.6	14402.5	184.2M	91.4M	101.4	14400.0
dem_100_newyork_135	173.4M	69.3M	150.3	14400.0	149.4M	141.5M	5.6	14400.0	165.3M	84.2M	96.3	14400.0
dem_100_newyork_136	194.5M	82.3M	136.4	14400.0	167.9M	155.8M	7.8	14400.0	238.2M	101.9M	133.8	14400.0
dem_100_newyork_137	147.5M	60.9M	142.3	14400.0	135.2M	110.8M	22.0	14402.7	136.5M	72.9M	87.3	14400.0
dem_100_newyork_138	175.4M	73.9M	137.3	14400.0	160.7M	144.5M	11.2	14403.2	177.8M	84.8M	109.7	14400.0
dem_100_newyork_139	159.1M	65.3M	143.6	14400.0	150.1M	121.5M	23.6	14400.0	161.3M	77.7M	107.6	14400.0
dem_100_newyork_140	152.4M	63.8M	138.8	14400.0	145.0M	123.3M	17.6	14400.0	164.9M	69.4M	137.5	14400.0
dem_100_newyork_141	152.9M	61.7M	148.0	14400.0	136.2M	120.6M	12.9	14400.0	155.5M	77.5M	100.7	14400.0
dem_100_newyork_142	129.7M	62.3M	108.3	14400.0	120.7M	106.6M	13.2	14400.0	133.2M	66.4M	100.6	14400.0
dem_100_newyork_143	186.5M	85.9M	117.2	14400.0	174.2M	174.2M	0.0	13034.4	208.1M	100.7M	106.7	14400.0
dem_100_newyork_144	201.8M	76.9M	162.3	14400.0	177.0M	155.4M	13.9	14400.0	200.5M	96.8M	107.0	14400.0
dem_100_newyork_145	153.2M	73.1M	109.5	14400.2	146.4M	131.8M	11.1	14401.5	160.4M	87.8M	82.7	14400.0
dem_100_newyork_146	125.6M	63.7M	97.1	14400.0	123.1M	111.0M	10.9	14401.8	153.6M	70.1M	119.2	14400.4
dem_100_newyork_147	183.8M	81.6M	125.2	14400.0	179.4M	156.0M	15.0	14402.9	211.5M	95.2M	122.3	14400.0
dem_100_newyork_148	154.4M	68.6M	125.1	14400.4	145.9M	135.0M	8.1	14400.0	188.3M	83.2M	126.4	14400.0
dem_100_newyork_149	177.9M	70.4M	152.9	14400.2	155.5M	136.4M	14.0	14400.0	159.9M	86.5M	85.0	14400.0
dem_100_newyork_150	177.2M	74.7M	137.3	14400.0	156.9M	143.7M	9.2	14404.1	188.6M	89.0M	112.0	14400.0
dem_100_newyork_151	159.6M	79.6M	100.6	14400.0	154.8M	138.1M	12.1	14401.7	176.3M	92.6M	90.3	14400.0
dem_100_newyork_152	112.4M	46.6M	141.2	14400.6	106.6M	106.6M	0.0	10342.0	121.0M	58.9M	105.3	14400.0
dem_100_newyork_153	133.8M	58.0M	130.9	14400.0	117.9M	106.9M	10.3	14400.0	131.0M	63.9M	104.9	14400.0
dem_100_newyork_154	140.5M	70.3M	99.9	14400.0	132.1M	132.1M	0.0	3211.6	163.9M	78.8M	108.1	14400.0
dem_100_newyork_155	193.4M	81.0M	138.8	14400.0	176.3M	151.5M	16.4	14403.0	203.4M	97.1M	109.4	14400.0
dem_100_newyork_156	185.8M	78.3M	137.4	14400.5	174.8M	154.7M	13.0	14400.0	184.5M	98.0M	88.2	14400.5
dem_100_newyork_157	177.6M	81.2M	118.8	14400.1	167.8M	152.7M	9.9	14402.0	196.2M	101.8M	92.6	14400.0
dem_100_newyork_158	164.6M	69.7M	136.3	14400.0	149.0M	132.3M	12.6	14402.2	167.4M	84.0M	99.3	14400.0
dem_100_newyork_159	167.6M	70.8M	136.7	14400.0	162.8M	142.4M	14.3	14400.0	187.5M	84.9M	121.0	14400.0
dem_100_newyork_160	127.2M	59.9M	112.2	14400.0	122.8M	113.2M	8.5	14400.0	130.9M	63.2M	107.1	14400.0
dem_100_newyork_161	132.0M	57.6M	129.2	14454.5	131.5M	118.8M	10.7	14400.0	145.6M	67.9M	114.5	14400.0
dem_100_newyork_162	187.5M	81.8M	129.3	14400.0	171.8M	159.0M	8.0	14402.0	200.0M	96.0M	108.3	14400.0
dem_100_newyork_163	151.5M	70.3M	115.4	14400.0	140.3M	127.0M	10.5	14401.6	152.8M	80.3M	90.4	14400.0
dem_100_newyork_164	197.4M	86.4M	128.3	14400.0	188.2M	162.6M	15.8	14400.0	194.5M	110.8M	75.6	14400.0
dem_100_newyork_165	155.9M	71.7M	117.4	14400.0	153.9M	132.7M	16.0	14400.0	170.3M	88.3M	92.8	14400.0
dem_100_newyork_166	172.2M	79.5M	116.7	14400.0	166.2M	142.1M	16.9	14402.6	182.5M	98.0M	86.3	14400.0
dem_100_newyork_167	200.6M	87.6M	129.0	14451.4	180.9M	154.1M	17.4	14400.0	204.0M	98.1M	108.0	14400.0
dem_100_newyork_168	163.2M	66.8M	144.3	14400.0	151.3M	129.9M	16.4	14400.0	190.0M	77.1M	146.5	14400.4
dem_100_newyork_169	151.7M	53.2M	185.0	14400.5	132.4M	116.0M	14.1	14400.0	147.8M	66.8M	121.3	14400.0
dem_100_newyork_170	183.1M	73.7M	148.3	14468.1	157.4M	143.0M	10.0	14402.1	176.8M	95.2M	85.7	14400.0
dem_100_newyork_171	150.7M	67.7M	122.7	14400.4	144.9M	127.8M	13.4	14402.6	157.2M	84.3M	86.5	14400.0
dem_100_newyork_172	157.3M	64.3M	144.7	14400.0	133.5M	120.1M	11.2	14402.4	139.5M	76.8M	81.6	14400.0
dem_100_newyork_173	190.4M	85.8M	121.9	14400.1	183.2M	166.4M	10.1	14400.0	189.2M	99.1M	90.8	14400.0
dem_100_newyork_174	143.9M	69.0M	108.5	14400.0	133.9M	133.9M	0.0	6414.8	166.4M	80.8M	106.0	14400.2
dem_100_newyork_175	203.9M	83.1M	145.4	14400.0	166.1M	145.7M	14.0	14402.9	177.4M	90.1M	96.8	14400.0
dem_100_newyork_176	188.5M	76.1M	147.7	14400.1	175.9M	175.8M	0.0	12201.3	188.4M	110.0M	71.2	14400.0
dem_100_newyork_177	181.6M	71.7M	153.3	14400.0	155.3M	140.5M	10.6	14402.3	188.3M	82.2M	129.0	14400.0
dem_100_newyork_178	193.4M	86.0M	125.0	14443.5	182.3M	169.5M	7.6	14400.0	231.3M	101.6M	127.7	14400.0
dem_100_newyork_179	152.1M	75.1M	102.5	14400.3	141.7M	141.7M	0.0	9384.6	158.6M	83.2M	90.7	14400.0
dem_100_newyork_180	153.7M	67.3M	128.4	14400.0	142.1M	142.1M	0.0	8167.6	161.7M	80.6M	100.5	14400.0
dem_100_newyork_181	215.8M	81.1M	166.2	14479.3	164.4M	147.6M	11.4	14400.0	192.6M	98.1M	96.4	14400.0
dem_100_newyork_182	123.5M	61.3M	101.6	14400.0	122.4M	113.3M	8.1	14400.0	126.2M	69.2M	82.4	14400.0
dem_100_newyork_183	208.4M	79.6M	161.8	14400.0	188.3M	166.3M	13.2	14404.4	201.9M	107.6M	87.7	14400.0
dem_100_newyork_184	141.3M	69.5M	103.2	14400.0	136.2M	124.5M	9.4	14402.2	147.4M	73.5M	100.6	14400.0
dem_100_newyork_185	159.0M	63.1M	152.2	14450.5	144.4M	131.7M	9.6	14400.0	198.8M	71.1M	179.8	14400.0
dem_100_newyork_186	160.4M	72.2M	122.1	14400.0	156.3M	152.1M	2.8	14400.2	180.6M	91.4M	97.7	14400.0
dem_100_newyork_187	162.7M	70.4M	131.0	14400.0	158.5M	152.4M	4.0	14400.3	205.1M	83.5M	145.6	14400.0
dem_100_newyork_188	198.7M	82.6M	140.5	14400.5	173.6M	151.3M	14.7	14400.0	195.5M	101.2M	93.2	14400.0
dem_100_newyork_189	159.3M	78.3M	103.5	14400.0	150.9M	133.9M	12.7	14402.4	172.3M	84.9M	103.0	14400.5
dem_100_newyork_190	196.0M	83.7M	134.1	14465.2	173.3M	154.0M	12.5	14400.0	185.8M	101.7M	82.8	14400.0
dem_100_newyork_191	123.1M	73.6M	67.4	14400.0	123.1M	114.1M	7.9	14400.0	131.7M	83.1M	58.6	14400.0
dem_100_newyork_192	169.1M	74.9M	125.8	14400.2	153.0M	141.0M	8.5	14400.0	194.2M	81.3M	138.7	14400.3
dem_100_newyork_193	167.9M	78.3M	114.6	14400.0	164.2M	148.3M	10.7	14400.0	187.9M	92.1M	104.1	14400.0
dem_100_newyork_194	151.7M	81.0M	87.4	14400.0	144.7M	144.7M	0.0	10319.6	144.9M	98.3M	47.5	14400.0
dem_100_newyork_195	195.4M	83.2M	134.9	14400.0	176.3M	151.3M	16.5	14400.0	183.0M	106.1M	72.5	14400.0
dem_100_newyork_196	130.3M	66.4M	96.3	14400.0	113.8M	113.8M	0.0	5513.6	129.6M	75.0M	72.8	14400.0
dem_100_newyork_197	181.0M	85.1M	112.6	14400.0	173.4M	156.7M	10.7	14400.0	213.3M	95.7M	122.8	14400.0
dem_100_newyork_198	144.9M	63.6M	127.9	14400.0	128.3M	118.7M	8.1	14400.0	165.9M	74.0M	124.1	14400.0

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Table A.35: Comparison of discrete models on *New York* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_199	138.1M	67.8M	103.6	14400.0	130.7M	125.4M	4.2	14400.9	156.4M	78.6M	99.2	14400.0
dem_100_newyork_200	183.3M	83.1M	120.5	14400.0	166.8M	150.7M	10.7	14400.0	173.2M	92.5M	87.4	14400.0
dem_100_newyork_201	150.7M	65.0M	131.8	14400.5	132.5M	118.3M	12.1	14403.0	160.2M	74.8M	114.3	14400.3
dem_100_newyork_202	159.6M	81.2M	96.5	14400.0	157.8M	143.4M	10.0	14400.0	185.7M	90.5M	105.2	14400.0
dem_100_newyork_203	132.8M	62.1M	113.9	14400.0	125.6M	115.9M	8.4	14400.0	136.6M	70.7M	93.4	14400.0
dem_100_newyork_204	173.4M	81.0M	114.1	14400.0	163.8M	163.8M	0.0	13536.9	217.7M	93.6M	132.7	14400.0
dem_100_newyork_205	177.9M	81.1M	119.3	14400.0	169.6M	154.8M	9.6	14400.0	182.5M	95.6M	91.0	14400.0
dem_100_newyork_206	149.9M	65.6M	128.6	14400.0	132.9M	119.5M	11.2	14402.0	150.2M	72.4M	107.5	14400.0
dem_100_newyork_207	153.9M	68.3M	125.5	14400.2	146.5M	126.9M	15.5	14400.0	171.9M	80.2M	114.4	14400.0
dem_100_newyork_208	119.4M	49.5M	141.3	14443.7	103.1M	103.0M	0.1	4153.0	139.6M	56.4M	147.5	14400.2
dem_100_newyork_209	199.8M	79.1M	152.7	14456.1	178.6M	162.0M	10.2	14402.1	229.7M	89.0M	158.2	14400.0
dem_100_newyork_210	180.9M	77.7M	133.0	14454.2	164.1M	151.1M	8.6	14402.0	170.9M	103.4M	65.3	14400.0
dem_100_newyork_211	239.8M	74.2M	223.0	14400.0	189.7M	165.8M	14.5	14402.9	203.2M	97.9M	107.4	14400.0
dem_100_newyork_212	165.7M	62.7M	164.2	14400.2	148.9M	137.6M	8.2	14402.2	178.7M	76.5M	133.5	14400.0
dem_100_newyork_213	163.6M	68.9M	137.6	14400.3	150.4M	150.4M	0.0	13853.9	188.5M	80.4M	134.5	14400.4
dem_100_newyork_214	122.2M	50.2M	143.4	14400.4	108.0M	95.3M	13.4	14400.0	122.9M	54.6M	124.9	14400.0
dem_100_newyork_215	171.0M	76.7M	123.1	14400.8	154.1M	139.2M	10.8	14400.0	180.0M	87.8M	105.0	14400.0
dem_100_newyork_216	161.7M	82.4M	96.2	14400.0	155.3M	155.3M	0.0	11082.9	185.7M	94.6M	96.4	14400.0
dem_100_newyork_217	184.8M	83.6M	120.9	14400.0	174.1M	157.1M	10.8	14400.0	189.1M	95.0M	99.1	14400.0
dem_100_newyork_218	156.7M	73.8M	112.3	14400.0	150.2M	130.1M	15.4	14400.0	164.0M	81.5M	101.1	14400.0
dem_100_newyork_219	172.8M	60.3M	186.4	14400.6	154.3M	154.3M	0.0	12470.9	169.8M	81.3M	108.9	14400.0
dem_100_newyork_220	148.0M	68.8M	115.3	14400.0	143.5M	139.1M	3.2	14400.0	168.6M	87.5M	92.6	14400.0
dem_100_newyork_221	168.7M	81.2M	107.7	14451.7	155.0M	150.0M	3.3	14400.2	180.9M	93.8M	92.8	14400.0
dem_100_newyork_222	177.3M	77.8M	128.0	14400.2	165.3M	142.1M	16.3	14400.0	173.0M	98.5M	75.6	14400.0
dem_100_newyork_223	115.6M	51.8M	123.2	14400.0	95.8M	95.8M	0.0	11999.9	112.5M	58.2M	93.4	14400.0
dem_100_newyork_224	164.5M	67.8M	142.6	14400.0	155.8M	130.1M	19.7	14402.8	168.4M	72.5M	132.2	14400.0
dem_100_newyork_225	117.1M	57.4M	104.1	14400.0	102.4M	102.4M	0.0	5809.1	132.4M	63.5M	108.6	14400.0
dem_100_newyork_226	144.7M	67.0M	116.1	14400.0	124.0M	112.9M	9.8	14401.6	141.2M	73.5M	92.0	14400.0
dem_100_newyork_227	157.6M	74.3M	112.1	14400.2	138.0M	125.9M	9.6	14400.0	158.6M	77.6M	104.2	14400.0
dem_100_newyork_228	148.0M	60.1M	146.1	14446.9	139.1M	139.1M	0.0	13439.8	167.2M	69.7M	140.0	14400.0
dem_100_newyork_229	134.4M	73.8M	82.2	14400.0	132.4M	119.0M	11.3	14400.0	154.1M	88.0M	75.0	14400.0
dem_100_newyork_230	204.7M	73.0M	180.5	14400.0	168.0M	145.7M	15.3	14403.5	173.0M	96.9M	78.5	14400.0
dem_100_newyork_231	140.5M	63.3M	121.9	14400.0	121.3M	105.0M	15.5	14402.5	129.7M	68.5M	89.5	14400.0
dem_100_newyork_232	161.1M	58.8M	174.2	14400.2	147.9M	131.8M	12.3	14400.0	165.6M	73.4M	125.8	14400.2
dem_100_newyork_233	137.6M	59.8M	130.0	14400.5	126.8M	126.7M	0.0	14036.8	170.8M	66.7M	156.0	14400.0
dem_100_newyork_234	162.0M	67.8M	139.0	14400.0	156.5M	143.0M	9.4	14400.0	169.5M	86.3M	96.4	14400.0
dem_100_newyork_235	157.6M	64.6M	143.9	14400.4	140.2M	132.3M	6.0	14402.8	153.7M	73.9M	108.0	14400.0
dem_100_newyork_236	169.0M	77.4M	118.5	14400.0	163.1M	147.3M	10.8	14402.6	180.0M	95.6M	88.3	14400.0
dem_100_newyork_237	168.8M	76.9M	119.5	14400.0	154.6M	132.9M	16.3	14400.0	173.5M	89.8M	93.3	14400.0
dem_100_newyork_238	172.7M	77.9M	121.7	14400.0	163.5M	163.5M	0.0	13991.9	185.1M	92.7M	99.7	14400.0
dem_100_newyork_239	191.5M	85.1M	125.1	14457.0	186.8M	164.9M	13.3	14400.0	209.0M	111.2M	87.8	14400.0
dem_100_newyork_240	158.2M	63.0M	151.0	14400.0	148.8M	125.3M	18.7	14400.0	158.7M	78.9M	101.1	14400.0
dem_100_newyork_241	150.9M	64.4M	134.4	14400.0	136.2M	120.7M	12.8	14400.0	165.5M	74.3M	122.7	14400.0
dem_100_newyork_242	136.8M	68.7M	99.3	14400.0	135.9M	122.1M	11.3	14400.0	150.5M	79.4M	89.5	14400.0
dem_100_newyork_243	171.9M	75.0M	129.3	14400.0	157.4M	143.3M	9.9	14401.9	180.9M	93.2M	94.2	14400.0
dem_100_newyork_244	162.6M	68.0M	139.0	14400.0	146.8M	141.8M	3.5	14400.0	152.7M	88.0M	73.5	14400.0
dem_100_newyork_245	160.5M	74.2M	116.3	14400.0	145.8M	130.6M	11.6	14400.0	173.8M	87.3M	99.0	14400.0
dem_100_newyork_246	164.6M	88.2M	86.5	14457.6	155.0M	150.2M	3.1	14402.0	155.7M	109.1M	42.7	14400.0
dem_100_newyork_247	177.4M	77.5M	128.9	14400.0	172.4M	151.3M	14.0	14400.0	199.0M	101.5M	96.0	14411.9
dem_100_newyork_248	135.5M	58.6M	131.0	14400.0	125.0M	125.0M	0.0	12591.7	159.5M	67.1M	137.6	14400.0
dem_100_newyork_249	195.3M	82.1M	137.7	14400.0	174.5M	160.6M	8.7	14401.9	188.5M	102.8M	83.3	14400.0
dem_100_newyork_250	153.0M	78.0M	96.1	14400.5	148.5M	139.6M	6.4	14400.0	188.7M	83.2M	126.8	14400.0
dem_100_newyork_251	139.4M	44.4M	214.3	14400.0	123.6M	109.3M	13.1	14402.1	154.8M	60.6M	155.3	14400.0
dem_100_newyork_252	143.6M	55.3M	159.6	14400.0	130.2M	120.1M	8.4	14402.2	187.0M	68.4M	173.5	14400.0
dem_100_newyork_253	169.8M	77.1M	120.3	14400.2	157.8M	157.8M	0.0	9979.6	186.0M	94.2M	97.5	14400.0
dem_100_newyork_254	184.3M	88.9M	107.3	14449.5	159.5M	159.5M	0.0	9867.1	190.4M	98.6M	93.2	14400.1
dem_100_newyork_255	194.8M	83.7M	132.8	14458.2	174.7M	161.0M	8.5	14402.7	217.6M	98.3M	121.3	14400.3
dem_100_newyork_256	197.0M	77.9M	152.8	14442.7	172.7M	156.3M	10.5	14400.0	195.8M	91.9M	113.0	14400.0
dem_100_newyork_257	129.9M	40.4M	221.4	14443.5	111.5M	96.2M	15.9	14400.0	136.9M	47.1M	191.0	14400.0
dem_100_newyork_258	142.5M	63.3M	125.3	14400.0	126.3M	126.3M	0.0	13389.2	154.8M	73.8M	109.6	14400.0
dem_100_newyork_259	103.1M	58.3M	76.8	14400.0	100.6M	100.6M	0.0	4900.5	117.4M	64.6M	81.5	14400.0
dem_100_newyork_260	201.3M	96.2M	109.1	14400.0	198.7M	178.3M	11.4	14403.5	237.8M	118.8M	100.2	14400.1
dem_100_newyork_261	177.1M	72.0M	145.8	14400.0	160.1M	135.1M	18.5	14400.0	207.3M	85.0M	144.0	14400.0
dem_100_newyork_262	126.0M	61.4M	105.1	14400.0	123.6M	110.7M	11.7	14402.6	145.5M	72.5M	100.8	14400.0
dem_100_newyork_263	171.8M	77.9M	120.6	14400.1	166.1M	149.0M	11.5	14403.0	183.3M	94.8M	93.3	14400.0
dem_100_newyork_264	135.0M	68.8M	96.2	14400.0	132.3M	107.4M	23.2	14402.4	140.0M	80.2M	74.5	14400.0
dem_100_newyork_265	168.1M	61.4M	173.8	14400.0	151.0M	138.5M	9.0	14401.9	168.6M	70.6M	139.0	14400.0
dem_100_newyork_266	149.8M	67.3M	122.6	14400.0	133.7M	128.7M	3.9	14400.5	179.9M	69.9M	157.4	14400.1
dem_100_newyork_267	148.0M	61.0M	142.5	14400.0	130.9M	114.9M	13.9	14404.4	158.2M	69.6M	127.4	14400.0
dem_100_newyork_268	152.2M	81.9M	85.7	14400.0	144.8M	144.8M	0.0	11184.8	176.9M	90.3M	95.8	14400.0
dem_100_newyork_269	173.9M	78.3M	122.0	14400.0	157.6M	146.8M	7.4	14401.9	175.4M	91.9M	90.9	14400.0
dem_100_newyork_270	169.9M	82.3M	106.5	14400.0	165.4M	149.3M	10.8	14402.7	174.4M	98.8M	76.5	14400.0
dem_100_newyork_271	152.9M	76.9M	99.0	14400.0	152.9M	135.2M	13.1	14400.0	180.4M	85.6M	110.8	14400.0
dem_100_newyork_272	158.6M	79.7M	99.0	14458.2	148.6M	135.9M	9.3	14403.0	157.6M	92.0M	71.3	14400.0
dem_100_newyork_273	150.9M	70.5M	113.9	14400.0	142.7M	142.6M	0.0	9644.6	170.9M	83.8M	104.0	14400.0
dem_100_newyork_274	171.5M	77.5M	121.2	14400.0	154.7M	141.9M	9.0	14401.5	204.5M	86.7M	135.8	14400.0
dem_100_newyork_275	175.0M	73.8M	137.1	14400.0	163.9M	153.7M	6.7	14401.5	197.2M	87.9M	124.4	14400.0
dem_100_newyork_276	200.8M	80.2M	150.2	14400.1	188.8M	161.6M	16.9	14400.0	193.5M	104.9M	84.4	14400.0

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Table A.35: Comparison of discrete models on *New York* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_277	176.7M	71.3M	147.7	14400.0	165.0M	135.2M	22.0	14400.0	175.0M	80.3M	117.9	14400.0
dem_100_newyork_278	186.4M	78.6M	137.3	14447.6	178.9M	157.6M	13.5	14400.0	267.9M	91.9M	191.6	14400.4
dem_100_newyork_279	150.2M	64.6M	132.6	14400.0	141.6M	131.7M	7.5	14400.7	165.6M	78.1M	112.1	14400.0
dem_100_newyork_280	153.9M	80.9M	90.3	14400.0	150.7M	150.7M	0.0	6809.3	159.6M	99.3M	60.7	14400.4
dem_100_newyork_281	154.3M	66.4M	132.5	14400.0	152.1M	136.7M	11.3	14400.0	164.8M	82.8M	99.1	14400.0
dem_100_newyork_282	182.4M	76.4M	138.7	14450.4	163.2M	163.2M	0.0	12916.2	174.7M	91.6M	90.7	14400.0
dem_100_newyork_283	136.6M	68.3M	100.1	14400.0	134.1M	125.7M	6.7	14401.8	156.7M	76.8M	104.1	14400.0
dem_100_newyork_284	159.0M	61.3M	159.3	14400.0	150.2M	136.4M	10.1	14402.3	153.0M	84.3M	81.5	14400.0
dem_100_newyork_285	159.5M	63.1M	152.9	14400.0	124.5M	124.5M	0.0	12394.5	160.2M	69.2M	131.4	14400.0
dem_100_newyork_286	175.9M	76.1M	131.1	14401.1	159.6M	144.1M	10.7	14401.6	176.8M	90.2M	96.0	14400.0
dem_100_newyork_287	161.1M	75.9M	112.3	14400.0	158.7M	144.1M	10.1	14403.5	182.4M	89.7M	103.3	14400.0
dem_100_newyork_288	214.8M	87.8M	144.6	14453.5	197.0M	175.3M	12.4	14400.0	236.5M	107.7M	119.6	14400.0
dem_100_newyork_289	136.4M	63.4M	115.1	14452.0	122.8M	111.3M	10.3	14400.0	134.6M	72.6M	85.4	14400.0
dem_100_newyork_290	192.6M	77.9M	147.1	14400.0	177.5M	163.8M	8.4	14400.0	207.7M	95.2M	118.1	14400.0
dem_100_newyork_291	118.4M	52.9M	123.9	14400.0	103.3M	103.3M	0.0	6029.5	110.4M	65.3M	69.0	14400.0
dem_100_newyork_292	167.3M	61.4M	172.6	14400.0	151.2M	125.8M	20.2	14400.0	167.3M	69.6M	140.5	14400.0
dem_100_newyork_293	174.4M	83.2M	109.6	14400.0	156.6M	138.8M	12.8	14401.5	191.1M	90.6M	110.9	14400.0
dem_100_newyork_294	154.6M	68.0M	127.4	14400.0	134.9M	126.5M	6.6	14401.4	170.9M	77.2M	121.5	14400.0
dem_100_newyork_295	175.7M	78.3M	124.5	14466.6	159.9M	159.9M	0.0	11150.7	188.5M	96.6M	95.1	14400.0
dem_100_newyork_296	148.1M	57.9M	155.5	14400.0	130.7M	119.8M	9.1	14401.8	152.1M	67.6M	125.1	14400.0
dem_100_newyork_297	141.3M	69.8M	102.5	14400.0	133.6M	122.6M	9.0	14400.0	157.5M	75.1M	109.6	14400.5
dem_100_newyork_298	141.5M	52.1M	171.4	14400.0	125.7M	113.6M	10.7	14401.9	144.5M	63.8M	126.6	14400.0
dem_100_newyork_299	168.4M	72.1M	133.6	14400.0	142.3M	128.7M	10.5	14401.9	145.5M	83.1M	75.2	14400.0
dem_100_newyork_300	161.9M	74.3M	118.0	14400.0	140.0M	140.0M	0.0	10019.7	188.8M	80.1M	135.6	14400.0
dem_100_newyork_301	170.3M	75.2M	126.4	14400.0	158.6M	158.6M	0.0	13214.9	175.8M	89.5M	96.5	14400.0
dem_100_newyork_302	215.4M	89.2M	141.6	14400.2	187.9M	178.7M	5.2	14400.0	210.1M	111.2M	88.9	14400.0
dem_100_newyork_303	170.5M	81.0M	110.5	14400.4	162.8M	151.9M	7.2	14400.0	165.6M	100.7M	64.5	14400.0
dem_100_newyork_304	180.3M	67.6M	166.7	14400.0	166.9M	147.6M	13.1	14402.1	196.9M	91.0M	116.3	14400.0
dem_100_newyork_305	146.2M	66.7M	119.2	14400.5	138.9M	126.0M	10.2	14400.0	174.5M	73.6M	137.1	14400.0
dem_100_newyork_306	171.2M	58.4M	192.9	14400.2	157.8M	138.1M	14.3	14400.0	174.2M	78.9M	120.6	14400.0
dem_100_newyork_307	169.4M	79.0M	114.5	14400.0	155.4M	155.4M	0.0	12368.1	197.9M	87.4M	126.5	14400.2
dem_100_newyork_308	165.4M	79.5M	108.0	14400.6	162.0M	144.9M	11.8	14401.8	176.9M	93.6M	88.9	14400.0
dem_100_newyork_309	163.9M	66.4M	146.8	14400.0	154.3M	154.3M	0.0	8563.5	184.6M	84.9M	117.5	14400.0
dem_100_newyork_310	108.7M	66.9M	62.6	14400.0	103.9M	103.9M	0.0	10133.1	122.3M	75.6M	61.7	14400.0
dem_100_newyork_311	135.4M	71.7M	88.7	14400.7	131.5M	131.5M	0.0	7671.1	179.3M	85.9M	108.6	14400.0
dem_100_newyork_312	198.2M	79.6M	148.8	14451.7	154.8M	135.0M	14.6	14400.0	171.7M	98.0M	75.1	14400.0
dem_100_newyork_313	201.7M	81.0M	148.8	14400.1	174.7M	155.2M	12.6	14403.6	237.0M	94.2M	151.6	14400.5
dem_100_newyork_314	193.4M	79.4M	143.4	14465.3	178.8M	163.0M	9.7	14400.0	187.0M	106.5M	75.5	14400.0
dem_100_newyork_315	122.6M	47.9M	155.9	14400.0	102.3M	102.3M	0.0	7921.3	115.6M	55.6M	107.8	14400.0
dem_100_newyork_316	122.4M	53.0M	131.1	14400.0	109.4M	109.4M	0.0	9331.1	131.2M	60.9M	115.4	14400.0
dem_100_newyork_317	154.7M	69.5M	122.7	14400.0	146.4M	146.2M	0.2	6314.3	159.5M	89.1M	78.9	14400.0
dem_100_newyork_318	175.0M	72.9M	140.0	14400.0	162.5M	140.5M	15.7	14400.0	180.5M	91.3M	97.8	14400.0
dem_100_newyork_319	169.6M	84.8M	99.9	14400.0	158.4M	142.4M	11.2	14400.0	173.8M	94.2M	84.5	14400.0
dem_100_newyork_320	127.3M	59.9M	112.4	14400.0	111.8M	111.8M	0.0	7782.5	135.3M	62.2M	117.7	14400.0
dem_100_newyork_321	162.8M	68.9M	136.1	14400.0	150.8M	129.7M	16.3	14400.0	171.2M	84.0M	103.8	14400.0
dem_100_newyork_322	158.5M	69.7M	127.4	14400.3	140.4M	121.1M	15.9	14401.7	159.0M	83.4M	90.5	14400.0
dem_100_newyork_323	125.6M	57.0M	120.2	14400.0	111.6M	111.5M	0.0	5046.8	121.1M	63.9M	89.5	14400.0
dem_100_newyork_324	94.7M	57.1M	66.0	14400.0	91.5M	91.5M	0.0	12166.2	104.5M	61.9M	68.9	14400.0
dem_100_newyork_325	185.5M	91.6M	102.5	14400.0	179.4M	157.7M	13.7	14404.8	197.3M	113.8M	73.4	14400.0
dem_100_newyork_326	189.2M	82.4M	129.7	14400.0	182.9M	168.6M	8.5	14400.0	196.3M	105.6M	85.9	14400.0
dem_100_newyork_327	95.9M	48.6M	97.2	14400.0	88.8M	88.8M	0.0	10198.2	103.6M	50.3M	105.9	14400.0
dem_100_newyork_328	131.9M	61.5M	114.4	14400.0	122.1M	122.1M	0.0	9242.5	134.4M	73.9M	81.9	14400.0
dem_100_newyork_329	142.6M	56.5M	152.6	14400.0	119.2M	119.2M	0.0	9084.7	131.0M	67.3M	94.7	14400.8
dem_100_newyork_330	100.6M	45.5M	121.2	14400.4	94.8M	86.5M	9.6	14400.0	106.4M	54.4M	95.5	14400.0
dem_100_newyork_331	197.9M	77.9M	154.0	14465.6	177.9M	160.3M	11.0	14400.0	203.9M	100.1M	103.6	14400.0
dem_100_newyork_332	148.3M	62.0M	139.1	14400.0	118.9M	107.7M	10.4	14401.9	145.2M	67.4M	115.3	14400.0
dem_100_newyork_333	153.9M	76.0M	102.6	14400.0	148.1M	148.1M	0.0	11240.5	159.4M	87.2M	82.8	14400.0
dem_100_newyork_334	155.2M	68.9M	125.4	14400.0	146.0M	131.2M	11.3	14400.0	167.3M	85.9M	94.7	14400.0
dem_100_newyork_335	174.3M	63.8M	173.1	14400.0	150.0M	126.0M	19.1	14400.0	161.0M	80.9M	99.1	14400.0
dem_100_newyork_336	170.2M	82.1M	107.3	14400.0	164.6M	140.1M	17.5	14400.0	200.9M	95.1M	111.2	14400.0
dem_100_newyork_337	150.4M	71.3M	111.0	14400.0	140.9M	122.6M	14.9	14400.0	166.1M	76.3M	117.7	14400.2
dem_100_newyork_338	161.3M	68.9M	134.0	14445.1	142.2M	124.1M	14.6	14400.0	148.3M	81.9M	81.0	14400.0
dem_100_newyork_339	158.7M	72.1M	120.0	14400.3	145.4M	131.1M	10.9	14400.0	169.1M	90.1M	87.7	14400.0
dem_100_newyork_340	176.0M	75.2M	134.0	14400.0	161.8M	139.2M	16.2	14400.0	197.0M	84.9M	132.0	14400.0
dem_100_newyork_341	135.2M	51.0M	164.9	14400.0	107.4M	107.4M	0.0	9114.4	128.2M	61.3M	109.2	14400.0
dem_100_newyork_342	123.5M	55.8M	121.5	14400.2	110.9M	100.3M	10.6	14402.7	132.5M	56.2M	135.6	14400.0
dem_100_newyork_343	176.8M	81.1M	117.9	14449.7	163.1M	137.3M	18.8	14402.7	175.0M	89.9M	94.6	14400.0
dem_100_newyork_344	111.1M	58.8M	89.0	14400.0	108.6M	108.6M	0.0	5924.0	111.2M	71.2M	56.1	14400.0
dem_100_newyork_345	157.2M	63.2M	148.9	14400.0	147.2M	123.7M	18.9	14400.0	165.4M	75.4M	119.2	14400.0
dem_100_newyork_346	111.9M	66.3M	68.8	14400.0	106.1M	94.2M	12.7	14400.0	113.3M	69.1M	63.9	14400.0
dem_100_newyork_347	199.5M	89.2M	123.6	14400.5	184.8M	184.8M	0.0	12965.4	202.7M	107.9M	87.8	14400.0
dem_100_newyork_348	134.4M	52.1M	158.1	14400.0	112.4M	101.3M	10.9	14402.8	120.8M	55.7M	117.0	14400.0
dem_100_newyork_349	117.4M	53.0M	121.3	14400.0	100.6M	90.0M	11.8	14403.0	121.2M	59.2M	104.8	14400.0
dem_100_newyork_350	172.8M	80.7M	114.0	14400.0	164.0M	164.0M	0.0	9962.2	178.4M	100.1M	78.2	14400.0
dem_100_newyork_351	163.3M	62.5M	161.2	14400.3	150.7M	133.7M	12.7	14401.8	190.4M	75.8M	151.1	14400.0
dem_100_newyork_352	183.5M	92.3M	98.8	14400.0	172.4M	172.3M	0.1	13280.1	181.7M	107.4M	69.1	14400.0
dem_100_newyork_353	163.5M	65.2M	151.0	14400.0	143.7M	129.7M	10.8	14401.5	159.5M	78.0M	104.4	14400.0
dem_100_newyork_354	142.1M	71.0M	100.3	14400.0	138.7M	138.7M	0.0	5982.7	173.4M	87.1M	99.2	14400.0

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Table A.35: Comparison of discrete models on *New York* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_355	171.6M	73.1M	134.9	14400.2	147.3M	131.4M	12.1	14408.0	156.3M	83.0M	88.4	14400.0
dem_100_newyork_356	154.5M	73.7M	109.7	14400.0	154.8M	127.7M	21.2	14402.8	175.8M	87.8M	100.2	14400.0
dem_100_newyork_357	204.6M	86.1M	137.5	14400.0	168.7M	151.9M	11.1	14400.0	222.6M	102.2M	117.8	14400.0
dem_100_newyork_358	170.7M	82.4M	107.1	14400.1	160.6M	156.3M	2.7	14401.1	171.8M	98.5M	74.4	14400.0
dem_100_newyork_359	204.3M	83.2M	145.7	14400.0	166.0M	139.5M	19.1	14402.6	196.2M	90.6M	116.5	14400.0
dem_100_newyork_360	164.1M	77.0M	113.1	14400.0	162.0M	140.6M	15.3	14402.8	171.4M	91.9M	86.5	14400.0
dem_100_newyork_361	130.2M	62.6M	108.0	14400.0	120.5M	120.5M	0.0	5447.6	135.3M	75.9M	78.2	14400.0
dem_100_newyork_362	164.3M	62.9M	161.3	14400.0	143.2M	122.0M	17.4	14400.0	155.3M	79.0M	96.5	14400.0
dem_100_newyork_363	143.8M	76.0M	89.3	14400.0	136.0M	136.0M	0.0	12269.1	156.4M	87.9M	77.9	14400.0
dem_100_newyork_364	141.5M	69.3M	104.2	14400.4	124.0M	124.0M	0.0	13662.0	150.5M	81.1M	85.7	14400.0
dem_100_newyork_365	83.1M	53.4M	55.6	14400.0	81.8M	74.4M	9.9	14400.0	96.4M	55.1M	75.0	14400.2
dem_100_newyork_366	109.5M	53.9M	103.0	14400.0	103.9M	103.6M	0.3	2368.9	128.3M	62.6M	105.1	14400.0
dem_100_newyork_367	105.9M	51.4M	106.1	14400.0	94.1M	94.1M	0.0	3663.2	124.0M	56.0M	121.5	14400.0
dem_100_newyork_368	133.5M	56.1M	138.0	14400.0	126.9M	115.4M	9.9	14400.0	166.2M	63.2M	162.8	14400.0
dem_100_newyork_369	193.1M	85.9M	124.8	14400.0	173.9M	160.0M	8.7	14400.0	208.8M	104.4M	100.1	14400.0
dem_100_newyork_370	178.4M	89.2M	100.0	14400.7	171.1M	171.1M	0.0	9048.0	181.9M	105.3M	72.7	14400.0
dem_100_newyork_371	151.0M	72.8M	107.4	14447.7	142.8M	128.3M	11.3	14402.0	174.6M	86.2M	102.6	14400.0
dem_100_newyork_372	110.1M	42.8M	157.1	14400.0	95.4M	95.4M	0.0	10604.5	111.4M	49.5M	125.3	14400.0
dem_100_newyork_373	192.1M	84.6M	127.1	14400.0	176.2M	155.9M	13.0	14400.0	194.4M	96.5M	101.5	14400.0
dem_100_newyork_374	194.3M	81.1M	139.5	14400.0	186.3M	166.6M	11.8	14402.8	203.0M	104.2M	94.9	14400.0
dem_100_newyork_375	188.0M	76.4M	146.0	14466.1	162.7M	144.6M	12.6	14402.8	189.7M	92.3M	105.4	14400.0
dem_100_newyork_376	145.7M	49.4M	195.3	14400.0	115.1M	115.1M	0.0	2528.3	164.3M	59.8M	174.9	14400.0
dem_100_newyork_377	130.8M	60.0M	118.0	14400.3	119.4M	114.0M	4.7	14400.8	142.1M	68.6M	107.1	14400.0
dem_100_newyork_378	158.7M	73.1M	117.3	14400.0	153.3M	136.4M	12.4	14402.7	179.6M	79.2M	126.8	14400.0
dem_100_newyork_379	132.2M	60.3M	119.2	14400.0	118.3M	118.3M	0.0	5758.1	143.1M	68.7M	108.2	14400.0
dem_100_newyork_380	113.9M	63.9M	78.3	14400.0	105.4M	105.4M	0.0	2739.2	116.6M	72.8M	60.1	14400.2
dem_100_newyork_381	196.3M	70.7M	177.6	14453.5	172.2M	149.6M	15.1	14400.0	191.9M	80.3M	138.9	14400.0
dem_100_newyork_382	141.6M	66.5M	113.1	14400.0	123.0M	116.6M	5.4	14401.3	153.8M	75.8M	103.0	14400.0
dem_100_newyork_383	150.3M	67.8M	121.7	14400.0	136.7M	136.7M	0.0	12552.9	178.5M	73.2M	143.9	14400.0
dem_100_newyork_384	143.3M	61.9M	131.6	14400.0	131.2M	131.2M	0.0	10118.3	146.7M	76.4M	92.0	14400.1
dem_100_newyork_385	211.5M	88.8M	138.3	14400.0	187.1M	175.3M	6.7	14400.0	202.2M	105.3M	92.1	14400.0
dem_100_newyork_386	182.4M	76.7M	137.7	14444.2	154.4M	154.4M	0.0	8322.6	169.9M	90.9M	87.0	14400.0
dem_100_newyork_387	136.0M	61.0M	122.9	14400.0	123.6M	123.6M	0.0	7962.0	142.8M	71.2M	100.6	14400.0
dem_100_newyork_388	169.8M	72.6M	133.8	14400.0	141.7M	130.1M	8.9	14400.0	165.9M	86.3M	92.2	14400.1
dem_100_newyork_389	133.9M	54.8M	144.4	14400.0	123.6M	123.6M	0.0	6700.1	133.8M	60.8M	120.2	14400.0
dem_100_newyork_390	162.3M	76.5M	112.2	14400.0	155.0M	155.0M	0.0	6174.5	172.4M	100.3M	71.9	14400.0
dem_100_newyork_391	167.0M	78.0M	114.0	14400.4	150.9M	135.1M	11.7	14404.9	206.4M	88.5M	133.1	14400.0
dem_100_newyork_392	187.8M	71.1M	164.3	14400.3	149.2M	140.7M	6.0	14400.0	169.4M	87.1M	94.4	14400.0
dem_100_newyork_393	141.4M	61.7M	129.2	14400.0	129.1M	118.3M	9.1	14400.0	151.1M	67.5M	124.0	14400.0
dem_100_newyork_394	222.6M	85.9M	159.0	14400.2	197.6M	177.4M	11.4	14402.1	217.3M	111.5M	94.9	14400.0
dem_100_newyork_395	150.4M	80.2M	87.6	14400.0	146.7M	140.0M	4.8	14401.4	177.4M	86.2M	105.9	14411.6
dem_100_newyork_396	158.6M	71.7M	121.1	14400.0	141.8M	141.8M	0.0	13970.8	169.4M	85.3M	98.5	14400.0
dem_100_newyork_397	197.6M	83.2M	137.6	14400.0	176.4M	160.3M	10.0	14403.1	228.2M	100.1M	128.0	14400.4
dem_100_newyork_398	160.6M	75.6M	112.6	14400.0	156.5M	141.3M	10.7	14400.0	169.6M	89.1M	90.4	14400.0
dem_100_newyork_399	132.4M	62.0M	113.6	14400.0	113.6M	113.6M	0.0	6859.6	117.8M	69.0M	70.7	14400.0
dem_100_newyork_400	136.7M	52.0M	162.6	14400.0	106.4M	106.4M	0.0	12636.4	138.4M	59.8M	131.5	14400.0
dem_100_newyork_401	140.0M	67.0M	109.0	14400.1	137.9M	126.9M	8.7	14400.0	183.7M	76.8M	139.1	14400.1
dem_100_newyork_402	156.6M	57.8M	170.8	14400.0	142.3M	132.6M	7.3	14400.0	158.3M	68.9M	129.9	14400.0
dem_100_newyork_403	140.8M	54.9M	156.4	14400.2	116.0M	116.0M	0.0	4199.3	149.9M	62.5M	139.8	14400.0
dem_100_newyork_404	181.7M	74.9M	142.4	14400.0	165.5M	143.0M	15.7	14400.0	215.1M	85.1M	152.6	14400.0
dem_100_newyork_405	191.1M	80.0M	138.8	14465.9	172.0M	153.8M	11.9	14402.0	204.4M	103.9M	96.6	14400.0
dem_100_newyork_406	162.1M	69.6M	132.9	14400.0	151.0M	151.0M	0.0	13757.9	172.3M	80.5M	114.1	14400.0
dem_100_newyork_407	188.4M	83.5M	125.6	14459.3	167.2M	150.3M	11.2	14400.0	195.3M	98.4M	98.4	14400.0
dem_100_newyork_408	153.6M	73.0M	110.4	14400.0	144.9M	132.5M	9.4	14400.0	166.5M	82.6M	101.6	14400.0
dem_100_newyork_409	143.3M	78.0M	83.7	14400.0	130.9M	130.9M	0.0	4454.0	141.1M	92.5M	52.6	14400.0
dem_100_newyork_410	180.0M	87.4M	106.0	14400.0	170.0M	164.8M	3.2	14400.3	183.3M	111.1M	65.0	14400.0
dem_100_newyork_411	150.9M	69.4M	117.4	14400.0	144.1M	128.3M	12.3	14402.7	174.2M	78.5M	122.0	14400.0
dem_100_newyork_412	168.7M	74.3M	127.1	14400.0	151.6M	138.8M	9.3	14400.0	185.9M	95.6M	94.4	14400.0
dem_100_newyork_413	180.2M	87.9M	104.8	14400.1	177.0M	170.1M	4.1	14400.2	182.4M	108.8M	67.7	14400.0
dem_100_newyork_414	75.8M	37.9M	99.9	14400.0	68.9M	68.9M	0.0	4185.9	72.5M	42.6M	70.2	14400.0
dem_100_newyork_415	128.9M	53.3M	141.9	14400.0	119.0M	108.7M	9.4	14401.2	133.6M	62.8M	112.7	14400.0
dem_100_newyork_416	185.6M	75.4M	146.3	14400.0	171.6M	155.5M	10.4	14401.2	205.3M	93.5M	119.6	14400.0
dem_100_newyork_417	132.9M	67.4M	97.3	14400.0	127.1M	127.1M	0.0	6819.2	146.6M	73.3M	100.2	14400.0
dem_100_newyork_418	179.4M	76.4M	134.9	14400.0	160.7M	142.5M	12.7	14400.0	182.0M	89.0M	104.4	14400.0
dem_100_newyork_419	157.5M	61.3M	157.2	14400.0	148.2M	129.5M	14.4	14400.0	152.0M	81.0M	87.6	14400.0
dem_100_newyork_420	153.8M	68.0M	126.1	14400.4	132.5M	132.5M	0.0	12339.4	158.9M	79.0M	101.1	14400.0
dem_100_newyork_421	157.7M	62.8M	151.1	14400.2	136.3M	123.0M	10.9	14402.3	151.3M	70.7M	113.9	14400.0
dem_100_newyork_422	179.9M	76.3M	135.6	14400.0	167.7M	142.4M	17.7	14400.0	188.1M	85.2M	120.7	14400.0
dem_100_newyork_423	160.1M	82.4M	94.4	14400.0	149.2M	149.2M	0.0	13134.5	194.9M	93.3M	108.8	14400.0
dem_100_newyork_424	164.0M	72.7M	125.7	14400.6	155.5M	144.5M	7.6	14401.6	206.2M	88.2M	133.7	14400.0
dem_100_newyork_425	166.4M	85.4M	94.7	14400.0	164.9M	164.8M	0.1	14395.5	167.1M	108.3M	54.3	14400.0
dem_100_newyork_426	160.0M	69.6M	130.0	14400.0	145.0M	125.5M	15.6	14403.6	168.7M	87.6M	92.6	14400.0
dem_100_newyork_427	135.6M	58.9M	130.0	14400.0	125.0M	125.0M	0.0	4985.7	144.5M	68.1M	112.1	14400.0
dem_100_newyork_428	164.5M	82.3M	99.8	14400.0	154.8M	137.3M	12.7	14400.0	173.9M	94.9M	83.3	14400.3
dem_100_newyork_429	166.0M	67.9M	144.6	14400.4	125.7M	125.6M	0.0	6888.6	132.4M	73.1M	81.1	14400.0
dem_100_newyork_430	173.2M	79.8M	117.2	14449.2	164.2M	155.6M	5.5	14400.0	168.0M	97.2M	72.8	14400.0
dem_100_newyork_431	144.7M	63.0M	129.6	14400.0	130.8M	130.8M	0.0	4643.4	139.6M	75.6M	84.7	14400.0
dem_100_newyork_432	164.7M	69.7M	136.3	14400.0	154.5M	131.6M	17.4	14400.0	166.6M	79.0M	111.0	14400.0

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Table A.35: Comparison of discrete models on *New York* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_433	161.4M	74.0M	118.1	14400.0	149.6M	143.5M	4.2	14400.0	167.1M	85.4M	95.6	14400.0
dem_100_newyork_434	171.1M	83.4M	105.1	14400.0	165.3M	140.4M	17.7	14403.5	188.5M	93.9M	100.7	14400.0
dem_100_newyork_435	184.7M	72.1M	156.0	14452.1	160.5M	142.0M	13.0	14403.0	170.0M	89.8M	89.3	14400.0
dem_100_newyork_436	131.6M	58.5M	125.0	14442.0	109.6M	109.6M	0.0	4321.4	140.3M	68.5M	104.7	14400.0
dem_100_newyork_437	157.2M	78.5M	100.3	14400.2	155.3M	140.0M	10.9	14403.0	162.5M	93.9M	73.1	14400.0
dem_100_newyork_438	145.8M	60.4M	141.3	14400.0	118.5M	110.0M	7.8	14400.0	144.8M	71.1M	103.7	14400.0
dem_100_newyork_439	164.8M	72.2M	128.2	14400.0	151.2M	137.5M	10.0	14401.6	158.2M	77.7M	103.5	14400.0
dem_100_newyork_440	144.3M	68.8M	109.6	14400.0	132.1M	132.1M	0.0	8821.4	161.5M	78.8M	105.0	14400.1
dem_100_newyork_441	111.8M	51.9M	115.5	14400.0	103.5M	103.5M	0.0	13915.0	106.0M	59.0M	79.8	14400.0
dem_100_newyork_442	158.7M	76.3M	108.1	14460.9	146.5M	135.3M	8.2	14400.0	164.5M	90.7M	81.3	14400.0
dem_100_newyork_443	130.5M	58.3M	123.7	14400.0	114.0M	104.9M	8.6	14400.0	139.2M	68.1M	104.4	14400.0
dem_100_newyork_444	159.8M	59.7M	167.6	14400.0	144.8M	128.4M	12.8	14402.6	154.1M	71.0M	117.1	14400.0
dem_100_newyork_445	179.6M	74.7M	140.3	14400.0	163.9M	146.0M	12.3	14402.8	185.1M	90.6M	104.3	14400.0
dem_100_newyork_446	169.0M	75.2M	124.7	14450.3	147.8M	127.3M	16.1	14402.1	164.5M	85.5M	92.4	14400.0
dem_100_newyork_447	128.5M	59.0M	117.8	14448.8	124.6M	124.4M	0.1	11655.4	164.5M	65.1M	152.7	14400.0
dem_100_newyork_448	138.6M	56.9M	143.6	14400.0	123.7M	111.3M	11.2	14400.0	138.1M	66.0M	109.1	14400.0
dem_100_newyork_449	167.2M	67.4M	147.9	14400.3	163.7M	141.5M	15.7	14403.2	192.1M	85.6M	124.5	14400.0
dem_100_newyork_450	170.0M	85.7M	98.5	14450.5	162.6M	143.2M	13.6	14400.0	198.5M	97.3M	104.0	14400.0
dem_100_newyork_451	148.6M	77.8M	91.0	14400.0	147.5M	147.5M	0.0	13140.3	165.0M	96.1M	71.8	14400.0
dem_100_newyork_452	167.6M	57.8M	190.0	14445.3	135.6M	125.9M	7.7	14401.0	179.4M	68.9M	160.3	14400.2
dem_100_newyork_453	176.3M	71.5M	146.4	14400.0	160.0M	141.0M	13.5	14400.0	178.4M	84.2M	111.9	14400.0
dem_100_newyork_454	175.8M	76.1M	131.2	14400.0	156.9M	141.4M	11.0	14400.0	182.2M	90.4M	101.6	14400.0
dem_100_newyork_455	186.1M	88.1M	111.2	14400.1	174.4M	174.4M	0.0	5945.5	192.0M	117.8M	62.9	14400.0
dem_100_newyork_456	184.8M	80.7M	129.0	14400.2	171.1M	154.9M	10.5	14400.0	196.3M	98.7M	98.8	14400.0
dem_100_newyork_457	161.7M	72.1M	124.2	14400.0	152.3M	142.3M	7.1	14400.0	181.7M	83.4M	117.8	14400.5
dem_100_newyork_458	132.3M	55.7M	137.8	14400.0	113.4M	102.4M	10.8	14400.0	113.4M	59.9M	89.3	14400.0
dem_100_newyork_459	141.3M	65.5M	115.8	14400.0	117.1M	117.1M	0.0	9672.4	121.3M	75.0M	61.8	14400.0
dem_100_newyork_460	202.2M	81.7M	147.6	14400.0	174.0M	161.2M	7.9	14402.5	192.5M	92.3M	108.5	14400.0
dem_100_newyork_461	118.5M	68.2M	73.7	14400.0	114.0M	114.0M	0.0	9442.9	120.4M	81.7M	47.2	14400.0
dem_100_newyork_462	180.2M	77.4M	132.9	14400.3	172.1M	142.8M	20.5	14403.6	184.1M	95.8M	92.2	14400.0
dem_100_newyork_463	125.0M	51.3M	143.6	14400.0	107.8M	107.8M	0.0	3171.0	135.9M	55.7M	144.1	14400.0
dem_100_newyork_464	194.3M	90.0M	115.9	14400.0	180.1M	162.9M	10.6	14400.0	197.7M	109.9M	80.0	14400.0
dem_100_newyork_465	194.1M	74.5M	160.6	14400.5	181.6M	158.7M	14.4	14400.0	195.5M	95.6M	104.5	14400.0
dem_100_newyork_466	162.3M	72.8M	123.1	14400.0	145.4M	127.2M	14.4	14402.6	181.0M	85.6M	111.4	14400.0
dem_100_newyork_467	170.4M	72.1M	136.2	14400.5	158.8M	150.2M	5.8	14400.0	193.1M	88.1M	119.1	14400.0
dem_100_newyork_468	157.0M	61.5M	155.2	14447.1	125.0M	125.0M	0.0	4251.2	162.4M	74.1M	119.3	14400.0
dem_100_newyork_469	192.2M	88.6M	116.8	14400.0	182.1M	162.4M	12.2	14400.0	208.6M	111.8M	86.6	14400.2
dem_100_newyork_470	141.6M	68.8M	105.9	14400.0	138.9M	125.8M	10.4	14401.9	155.2M	73.5M	111.2	14400.0
dem_100_newyork_471	182.6M	76.1M	140.0	14400.1	160.4M	139.5M	15.0	14401.5	186.3M	89.8M	107.4	14400.0
dem_100_newyork_472	201.7M	69.9M	188.6	14400.0	168.9M	148.4M	13.8	14400.0	209.7M	88.3M	137.5	14400.0
dem_100_newyork_473	141.3M	71.6M	97.3	14400.2	129.2M	115.1M	12.3	14400.0	159.0M	83.1M	91.3	14400.0
dem_100_newyork_474	172.1M	72.2M	138.4	14464.9	163.4M	147.8M	10.5	14400.0	248.5M	81.6M	204.5	14400.0
dem_100_newyork_475	205.9M	85.5M	140.8	14400.0	184.9M	163.4M	13.2	14403.8	207.3M	112.6M	84.2	14400.0
dem_100_newyork_476	133.2M	62.0M	115.0	14400.0	124.1M	124.1M	0.0	9928.1	138.5M	70.3M	96.9	14400.0
dem_100_newyork_477	131.8M	65.1M	102.4	14400.0	116.9M	116.9M	0.0	10271.6	144.8M	76.3M	89.8	14400.0
dem_100_newyork_478	145.2M	70.4M	106.3	14475.7	139.8M	133.6M	4.6	14400.0	157.5M	85.0M	85.4	14400.0
dem_100_newyork_479	191.7M	85.9M	123.1	14400.4	168.3M	156.3M	7.7	14405.0	173.2M	97.8M	77.1	14400.0
dem_100_newyork_480	147.9M	74.7M	98.1	14400.0	125.1M	125.1M	0.0	2181.4	136.1M	84.0M	62.0	14400.0
dem_100_newyork_481	163.7M	75.1M	117.9	14466.3	152.0M	152.0M	0.0	12310.8	167.6M	86.1M	94.7	14400.0
dem_100_newyork_482	169.8M	70.8M	139.8	14400.0	160.9M	149.1M	7.9	14400.0	176.0M	91.4M	92.5	14400.0
dem_100_newyork_483	174.8M	70.4M	148.4	14400.0	163.5M	152.2M	7.4	14400.0	191.2M	89.6M	113.3	14400.0
dem_100_newyork_484	154.5M	70.8M	118.1	14400.0	149.4M	128.6M	16.2	14400.0	159.7M	84.8M	88.2	14400.0
dem_100_newyork_485	155.1M	67.3M	130.5	14400.0	133.8M	122.1M	9.6	14403.6	156.0M	79.2M	96.9	14400.0
dem_100_newyork_486	198.4M	90.7M	118.8	14400.0	194.7M	194.5M	0.1	14377.3	243.8M	113.8M	114.3	14400.0
dem_100_newyork_487	168.4M	64.6M	160.9	14400.0	145.3M	130.8M	11.1	14401.5	165.6M	78.8M	110.1	14400.0
dem_100_newyork_488	160.8M	65.8M	144.3	14400.0	144.5M	123.1M	17.4	14403.0	152.0M	81.2M	87.1	14400.0
dem_100_newyork_489	221.5M	96.9M	128.5	14448.8	214.0M	183.0M	17.0	14403.9	235.9M	131.0M	80.0	14400.0
dem_100_newyork_490	171.3M	64.7M	164.7	14400.0	144.0M	126.8M	13.5	14402.6	154.1M	80.2M	92.3	14400.2
dem_100_newyork_491	164.9M	78.0M	111.5	14400.0	159.3M	159.1M	0.1	9773.2	185.6M	91.9M	102.0	14400.0
dem_100_newyork_492	110.5M	63.1M	75.1	14400.0	101.7M	98.0M	3.8	14400.2	102.7M	64.5M	59.2	14400.0
dem_100_newyork_493	175.2M	66.2M	164.7	14400.2	161.4M	143.5M	12.5	14400.0	172.7M	74.0M	133.4	14400.0
dem_100_newyork_494	117.7M	53.3M	120.8	14400.0	111.6M	102.4M	9.0	14400.0	144.7M	63.1M	129.2	14400.0
dem_100_newyork_495	176.5M	82.2M	114.6	14400.0	172.1M	151.8M	13.4	14400.0	175.7M	108.9M	61.3	14400.0
dem_100_newyork_496	124.8M	56.8M	119.8	14448.2	107.8M	107.8M	0.0	5385.9	138.4M	64.6M	114.2	14400.0
dem_100_newyork_497	159.7M	83.1M	92.2	14400.0	154.2M	154.2M	0.0	13858.5	164.7M	99.8M	64.9	14400.0
dem_100_newyork_498	142.9M	63.1M	126.5	14400.0	124.8M	124.8M	0.0	12295.8	151.7M	65.6M	131.3	14400.0
dem_100_newyork_499	180.9M	85.8M	110.9	14400.1	181.0M	159.9M	13.2	14400.0	189.7M	106.7M	77.9	14400.0
dem_100_newyork_500	138.3M	69.4M	99.2	14400.0	135.9M	122.2M	11.2	14400.0	151.3M	83.4M	81.4	14400.0

Table A.36: Detailed results of the split-pipe models on *New York* with $\mathcal{B} = 100$ as summarized in Figure 3.17a and Table 3.4e. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_1	145.1M	144.9M	0.1	4.1	145.1M	144.9M	0.1	3.3
dem_100_newyork_2	141.7M	141.5M	0.1	2.9	141.7M	141.5M	0.1	4.8
dem_100_newyork_3	171.6M	171.5M	0.1	4.9	171.6M	171.5M	0.1	4.1
dem_100_newyork_4	158.3M	158.2M	0.1	3.0	158.3M	158.2M	0.1	2.5
dem_100_newyork_5	149.1M	149.0M	0.1	1.3	149.1M	149.0M	0.1	1.6
dem_100_newyork_6	159.0M	158.9M	0.1	4.0	159.0M	158.9M	0.1	2.8
dem_100_newyork_7	130.1M	130.1M	0.0	1.4	130.1M	130.0M	0.1	1.2
dem_100_newyork_8	168.8M	168.6M	0.1	3.0	168.8M	168.6M	0.1	3.0
dem_100_newyork_9	104.7M	104.6M	0.1	1.8	104.7M	104.6M	0.1	1.9
dem_100_newyork_10	176.8M	176.6M	0.1	3.2	176.7M	176.6M	0.1	2.6
dem_100_newyork_11	149.3M	149.2M	0.1	3.5	149.3M	149.2M	0.1	3.2
dem_100_newyork_12	184.7M	184.5M	0.1	6.6	184.7M	184.5M	0.1	5.5
dem_100_newyork_13	176.7M	176.5M	0.1	5.8	176.7M	176.5M	0.1	5.3
dem_100_newyork_14	120.0M	119.8M	0.1	7.1	120.0M	119.8M	0.1	4.0
dem_100_newyork_15	139.9M	139.8M	0.0	2.5	139.9M	139.8M	0.1	1.9
dem_100_newyork_16	145.3M	145.1M	0.1	2.2	145.3M	145.1M	0.1	1.9
dem_100_newyork_17	139.8M	139.7M	0.1	3.5	139.8M	139.7M	0.1	3.5
dem_100_newyork_18	134.1M	134.0M	0.1	2.2	134.1M	134.0M	0.1	2.5
dem_100_newyork_19	175.5M	175.4M	0.1	2.2	175.5M	175.4M	0.1	1.9
dem_100_newyork_20	169.8M	169.7M	0.1	3.0	169.8M	169.7M	0.1	3.1
dem_100_newyork_21	137.5M	137.4M	0.0	1.9	137.5M	137.4M	0.0	1.5
dem_100_newyork_22	138.5M	138.3M	0.1	1.3	138.5M	138.3M	0.1	1.3
dem_100_newyork_23	181.8M	181.6M	0.1	2.3	181.8M	181.6M	0.1	2.4
dem_100_newyork_24	76.5M	76.4M	0.0	0.9	76.5M	76.4M	0.0	0.8
dem_100_newyork_25	169.8M	169.6M	0.1	3.6	169.7M	169.5M	0.1	2.7
dem_100_newyork_26	160.0M	159.9M	0.1	5.0	160.0M	159.9M	0.1	4.7
dem_100_newyork_27	125.1M	125.0M	0.1	1.1	125.1M	125.0M	0.1	1.1
dem_100_newyork_28	150.6M	150.5M	0.1	3.2	150.6M	150.5M	0.1	3.3
dem_100_newyork_29	175.9M	175.7M	0.1	2.7	175.9M	175.7M	0.1	2.6
dem_100_newyork_30	136.2M	136.0M	0.1	2.0	136.2M	136.0M	0.1	1.5
dem_100_newyork_31	153.6M	153.5M	0.1	5.6	153.6M	153.5M	0.1	5.8
dem_100_newyork_32	132.1M	132.0M	0.1	2.0	132.1M	132.0M	0.1	2.8
dem_100_newyork_33	178.2M	178.0M	0.1	4.3	178.2M	178.0M	0.1	4.2
dem_100_newyork_34	131.1M	131.0M	0.1	3.2	131.1M	131.0M	0.1	3.4
dem_100_newyork_35	166.6M	166.4M	0.1	3.0	166.5M	166.4M	0.1	4.2
dem_100_newyork_36	146.4M	146.3M	0.1	3.5	146.4M	146.3M	0.1	4.2
dem_100_newyork_37	141.6M	141.5M	0.1	7.0	141.6M	141.5M	0.1	7.6
dem_100_newyork_38	162.4M	162.2M	0.1	3.8	162.4M	162.2M	0.1	4.8
dem_100_newyork_39	142.0M	141.8M	0.1	4.0	142.0M	141.8M	0.1	4.0
dem_100_newyork_40	119.3M	119.2M	0.1	4.5	119.3M	119.2M	0.1	3.9
dem_100_newyork_41	155.6M	155.5M	0.1	2.7	155.6M	155.5M	0.1	2.4
dem_100_newyork_42	157.9M	157.8M	0.1	2.5	157.9M	157.8M	0.1	2.9
dem_100_newyork_43	128.1M	128.0M	0.1	2.3	128.1M	128.1M	0.0	2.4
dem_100_newyork_44	127.5M	127.4M	0.1	1.2	127.5M	127.4M	0.1	1.1
dem_100_newyork_45	129.1M	129.0M	0.1	2.6	129.1M	129.0M	0.1	2.6
dem_100_newyork_46	109.4M	109.3M	0.0	1.1	109.4M	109.3M	0.1	0.9
dem_100_newyork_47	187.2M	187.0M	0.1	3.7	187.2M	187.0M	0.1	3.9
dem_100_newyork_48	112.5M	112.4M	0.1	1.0	112.5M	112.4M	0.1	0.8
dem_100_newyork_49	156.8M	156.7M	0.1	2.1	156.7M	156.6M	0.1	1.3
dem_100_newyork_50	139.3M	139.2M	0.1	6.1	139.3M	139.2M	0.1	6.3
dem_100_newyork_51	159.8M	159.6M	0.1	4.3	159.8M	159.6M	0.1	4.8
dem_100_newyork_52	120.8M	120.7M	0.1	6.4	120.8M	120.7M	0.1	5.6
dem_100_newyork_53	186.5M	186.3M	0.1	3.0	186.5M	186.3M	0.1	3.9
dem_100_newyork_54	146.3M	146.1M	0.1	4.5	146.3M	146.2M	0.1	5.8
dem_100_newyork_55	152.2M	152.1M	0.0	1.1	152.2M	152.1M	0.1	1.3
dem_100_newyork_56	133.4M	133.3M	0.1	1.4	133.4M	133.3M	0.1	1.3
dem_100_newyork_57	194.8M	194.7M	0.1	3.7	194.8M	194.7M	0.1	3.2
dem_100_newyork_58	149.8M	149.8M	0.0	2.8	149.8M	149.7M	0.1	2.1
dem_100_newyork_59	125.1M	125.0M	0.1	3.1	125.1M	125.0M	0.1	3.0
dem_100_newyork_60	205.8M	205.6M	0.1	3.5	205.8M	205.6M	0.1	3.6
dem_100_newyork_61	154.0M	153.8M	0.1	2.3	154.0M	153.8M	0.1	2.4
dem_100_newyork_62	146.7M	146.6M	0.1	1.6	146.7M	146.6M	0.1	1.9
dem_100_newyork_63	154.0M	153.9M	0.0	1.5	154.0M	154.0M	0.0	1.3
dem_100_newyork_64	101.4M	101.3M	0.1	2.3	101.4M	101.3M	0.1	2.7
dem_100_newyork_65	196.4M	196.2M	0.1	3.1	196.3M	196.1M	0.1	3.0
dem_100_newyork_66	146.2M	146.0M	0.1	1.6	146.2M	146.0M	0.1	1.5
dem_100_newyork_67	127.7M	127.6M	0.1	1.8	127.7M	127.6M	0.1	1.9
dem_100_newyork_68	159.3M	159.2M	0.0	4.2	159.3M	159.2M	0.0	2.7
dem_100_newyork_69	164.6M	164.4M	0.1	2.2	164.6M	164.4M	0.1	1.8
dem_100_newyork_70	108.2M	108.0M	0.1	1.6	108.2M	108.1M	0.1	1.7
dem_100_newyork_71	149.7M	149.5M	0.1	2.1	149.7M	149.6M	0.1	2.0
dem_100_newyork_72	134.6M	134.4M	0.1	1.2	134.6M	134.4M	0.1	1.1
dem_100_newyork_73	187.8M	187.6M	0.1	2.5	187.8M	187.7M	0.1	3.2
dem_100_newyork_74	179.5M	179.3M	0.1	2.2	179.5M	179.3M	0.1	2.5

continued on next page

Table A.36: Comparison of split-pipe models on *New York* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_75	117.4M	117.3M	0.1	1.4	117.4M	117.3M	0.1	1.4
dem_100_newyork_76	144.6M	141.8M	1.9	4.8	142.0M	141.8M	0.1	2.4
dem_100_newyork_77	128.7M	128.6M	0.1	2.5	129.1M	128.6M	0.4	3.4
dem_100_newyork_78	161.3M	161.1M	0.1	4.6	161.1M	161.1M	0.0	4.2
dem_100_newyork_79	143.9M	143.8M	0.1	7.2	143.9M	143.8M	0.1	5.3
dem_100_newyork_80	122.3M	122.2M	0.1	5.6	122.3M	122.2M	0.1	4.5
dem_100_newyork_81	110.5M	110.4M	0.1	2.0	110.5M	110.4M	0.1	1.8
dem_100_newyork_82	165.8M	165.7M	0.1	2.1	165.8M	165.7M	0.1	1.9
dem_100_newyork_83	108.5M	108.5M	0.0	1.2	108.5M	108.4M	0.1	0.8
dem_100_newyork_84	112.0M	111.9M	0.1	3.1	112.0M	111.9M	0.1	3.4
dem_100_newyork_85	148.1M	148.0M	0.1	4.8	148.1M	148.0M	0.1	4.3
dem_100_newyork_86	175.8M	175.8M	0.0	2.0	175.8M	175.7M	0.0	2.1
dem_100_newyork_87	186.0M	185.9M	0.0	5.8	186.0M	185.8M	0.1	2.2
dem_100_newyork_88	144.1M	144.0M	0.1	1.0	144.1M	144.0M	0.1	1.0
dem_100_newyork_89	124.7M	124.6M	0.1	0.7	124.7M	124.6M	0.1	0.8
dem_100_newyork_90	139.1M	139.1M	0.0	1.2	139.1M	139.1M	0.1	1.1
dem_100_newyork_91	161.5M	161.3M	0.1	5.0	161.5M	161.3M	0.1	4.2
dem_100_newyork_92	169.8M	169.7M	0.1	6.2	169.8M	169.7M	0.1	6.1
dem_100_newyork_93	177.9M	177.7M	0.1	2.7	177.9M	177.7M	0.1	2.3
dem_100_newyork_94	141.0M	140.9M	0.1	2.2	141.0M	140.9M	0.1	2.7
dem_100_newyork_95	163.0M	162.8M	0.1	2.9	163.0M	162.9M	0.1	3.5
dem_100_newyork_96	156.9M	156.7M	0.1	4.4	156.9M	156.7M	0.1	4.2
dem_100_newyork_97	157.8M	157.7M	0.1	5.2	157.8M	157.7M	0.1	4.0
dem_100_newyork_98	87.9M	87.9M	0.1	2.6	87.9M	87.9M	0.1	1.3
dem_100_newyork_99	179.8M	179.7M	0.1	3.0	179.8M	179.7M	0.1	3.1
dem_100_newyork_100	148.7M	148.5M	0.1	4.5	148.7M	148.5M	0.1	3.7
dem_100_newyork_101	167.4M	167.3M	0.1	4.0	167.4M	167.3M	0.1	4.0
dem_100_newyork_102	138.7M	137.7M	0.7	3.0	137.8M	137.8M	0.0	3.8
dem_100_newyork_103	129.8M	129.7M	0.1	3.0	129.8M	129.7M	0.1	2.5
dem_100_newyork_104	152.9M	152.9M	0.0	2.7	152.9M	152.8M	0.1	1.8
dem_100_newyork_105	145.4M	145.2M	0.1	3.0	145.4M	145.3M	0.1	2.1
dem_100_newyork_106	109.9M	109.8M	0.1	3.7	109.9M	109.8M	0.1	1.5
dem_100_newyork_107	145.2M	145.0M	0.1	3.4	145.2M	145.0M	0.1	2.9
dem_100_newyork_108	111.9M	111.8M	0.1	2.5	111.9M	111.8M	0.1	2.0
dem_100_newyork_109	179.8M	179.6M	0.1	5.0	179.8M	179.7M	0.1	4.6
dem_100_newyork_110	126.6M	126.5M	0.1	1.9	126.6M	126.5M	0.1	1.8
dem_100_newyork_111	172.4M	172.3M	0.1	2.7	172.4M	172.3M	0.1	2.2
dem_100_newyork_112	157.3M	157.2M	0.1	3.8	157.3M	157.2M	0.1	3.8
dem_100_newyork_113	155.0M	154.9M	0.1	7.2	155.0M	154.9M	0.1	3.2
dem_100_newyork_114	144.8M	144.7M	0.1	3.0	144.8M	144.8M	0.0	2.4
dem_100_newyork_115	120.4M	120.4M	0.0	2.9	120.4M	120.3M	0.0	1.6
dem_100_newyork_116	166.4M	166.3M	0.1	4.2	166.4M	166.2M	0.1	3.4
dem_100_newyork_117	148.3M	148.2M	0.1	3.2	148.3M	148.2M	0.0	2.4
dem_100_newyork_118	132.5M	132.4M	0.1	5.5	132.5M	132.4M	0.1	6.0
dem_100_newyork_119	123.5M	123.4M	0.1	2.9	123.5M	123.4M	0.1	2.5
dem_100_newyork_120	168.2M	168.0M	0.1	5.2	168.2M	168.2M	0.0	2.6
dem_100_newyork_121	147.3M	147.1M	0.1	4.8	147.3M	147.1M	0.1	4.7
dem_100_newyork_122	141.4M	141.2M	0.1	4.1	141.4M	141.2M	0.1	3.8
dem_100_newyork_123	162.1M	162.0M	0.1	5.5	162.1M	162.0M	0.1	3.6
dem_100_newyork_124	149.4M	149.3M	0.1	4.1	149.4M	149.3M	0.1	4.3
dem_100_newyork_125	135.8M	135.7M	0.1	1.9	135.8M	135.7M	0.1	1.1
dem_100_newyork_126	186.1M	186.1M	0.0	3.3	186.2M	186.0M	0.1	2.8
dem_100_newyork_127	191.8M	191.7M	0.1	3.1	191.8M	191.6M	0.1	2.1
dem_100_newyork_128	155.4M	155.2M	0.1	4.4	155.4M	155.2M	0.1	3.9
dem_100_newyork_129	129.3M	129.1M	0.1	2.9	129.3M	129.1M	0.1	2.6
dem_100_newyork_130	183.2M	183.1M	0.1	5.1	183.2M	183.1M	0.1	5.6
dem_100_newyork_131	119.4M	119.3M	0.1	5.5	119.4M	119.2M	0.1	5.6
dem_100_newyork_132	178.6M	178.4M	0.1	4.1	178.6M	178.4M	0.1	4.0
dem_100_newyork_133	129.2M	129.0M	0.1	2.5	129.2M	129.0M	0.1	1.9
dem_100_newyork_134	174.3M	174.1M	0.1	5.2	174.3M	174.1M	0.1	4.5
dem_100_newyork_135	148.0M	147.8M	0.1	2.4	148.0M	147.8M	0.1	2.3
dem_100_newyork_136	166.8M	166.6M	0.1	4.3	166.8M	166.6M	0.1	3.5
dem_100_newyork_137	133.0M	132.9M	0.1	3.1	133.0M	132.9M	0.1	3.8
dem_100_newyork_138	158.9M	158.7M	0.1	3.2	158.9M	158.7M	0.1	2.1
dem_100_newyork_139	143.7M	143.6M	0.1	4.2	143.7M	143.7M	0.0	5.1
dem_100_newyork_140	143.2M	143.0M	0.1	3.1	143.2M	143.1M	0.1	3.5
dem_100_newyork_141	134.4M	134.3M	0.1	1.9	134.4M	134.3M	0.1	1.7
dem_100_newyork_142	119.5M	119.3M	0.1	3.3	119.5M	119.3M	0.1	2.8
dem_100_newyork_143	172.0M	171.9M	0.1	2.2	172.0M	171.9M	0.1	1.4
dem_100_newyork_144	173.9M	173.8M	0.1	3.4	173.9M	173.8M	0.1	3.8
dem_100_newyork_145	145.3M	145.2M	0.1	2.0	145.3M	145.2M	0.1	2.8
dem_100_newyork_146	121.1M	121.0M	0.1	3.9	121.1M	121.0M	0.1	2.5
dem_100_newyork_147	176.6M	176.5M	0.1	3.6	176.6M	176.5M	0.1	3.6
dem_100_newyork_148	144.4M	144.4M	0.0	3.0	144.4M	144.3M	0.1	1.9
dem_100_newyork_149	154.4M	154.2M	0.1	11.1	154.4M	154.2M	0.1	10.8
dem_100_newyork_150	154.5M	154.4M	0.1	2.1	154.5M	154.3M	0.1	1.4
dem_100_newyork_151	154.2M	154.0M	0.1	3.1	154.2M	154.0M	0.1	3.3
dem_100_newyork_152	105.5M	105.4M	0.1	2.0	105.5M	105.4M	0.1	1.3

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Table A.36: Comparison of split-pipe models on *New York* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_153	115.9M	115.9M	0.0	2.6	115.9M	115.8M	0.1	2.0
dem_100_newyork_154	129.7M	129.6M	0.1	1.6	129.7M	129.6M	0.1	1.2
dem_100_newyork_155	174.0M	173.8M	0.1	3.5	174.0M	173.8M	0.1	3.0
dem_100_newyork_156	172.9M	172.8M	0.1	2.5	172.9M	172.9M	0.0	3.2
dem_100_newyork_157	167.2M	167.1M	0.1	4.1	167.2M	167.1M	0.1	4.6
dem_100_newyork_158	147.0M	146.9M	0.1	4.2	147.0M	146.9M	0.1	4.2
dem_100_newyork_159	161.5M	161.3M	0.1	3.7	161.5M	161.3M	0.1	3.4
dem_100_newyork_160	121.5M	121.4M	0.1	3.3	121.5M	121.4M	0.1	2.8
dem_100_newyork_161	130.4M	130.2M	0.1	6.9	130.4M	130.2M	0.1	5.8
dem_100_newyork_162	170.8M	170.6M	0.1	4.5	170.8M	170.6M	0.1	3.8
dem_100_newyork_163	137.4M	137.3M	0.1	3.8	137.4M	137.3M	0.1	2.6
dem_100_newyork_164	185.3M	185.1M	0.1	4.1	185.3M	185.1M	0.1	4.3
dem_100_newyork_165	152.2M	152.0M	0.1	5.7	152.2M	152.0M	0.1	4.2
dem_100_newyork_166	165.1M	164.9M	0.1	2.9	165.1M	164.9M	0.1	3.0
dem_100_newyork_167	179.4M	179.2M	0.1	6.3	179.4M	179.3M	0.1	7.0
dem_100_newyork_168	150.2M	150.1M	0.1	5.0	150.2M	150.1M	0.1	5.2
dem_100_newyork_169	130.9M	130.7M	0.1	5.0	130.9M	130.7M	0.1	5.6
dem_100_newyork_170	155.6M	155.5M	0.1	3.5	155.6M	155.5M	0.1	3.4
dem_100_newyork_171	142.6M	142.5M	0.1	2.3	142.6M	142.5M	0.1	1.8
dem_100_newyork_172	131.8M	131.7M	0.1	3.0	131.8M	131.7M	0.1	4.4
dem_100_newyork_173	182.0M	181.8M	0.1	5.1	182.0M	181.8M	0.1	7.1
dem_100_newyork_174	132.8M	132.7M	0.1	1.3	132.8M	132.7M	0.1	1.0
dem_100_newyork_175	164.5M	164.3M	0.1	3.7	164.5M	164.3M	0.1	3.2
dem_100_newyork_176	174.8M	174.7M	0.1	2.5	174.9M	174.7M	0.1	1.9
dem_100_newyork_177	153.7M	153.5M	0.1	4.5	153.7M	153.5M	0.1	2.8
dem_100_newyork_178	180.1M	179.9M	0.1	2.2	180.1M	179.9M	0.1	2.4
dem_100_newyork_179	140.5M	140.4M	0.1	2.1	140.5M	140.4M	0.1	1.2
dem_100_newyork_180	140.3M	140.3M	0.0	2.0	140.3M	140.3M	0.0	1.3
dem_100_newyork_181	163.2M	163.1M	0.1	4.4	163.2M	163.1M	0.1	4.1
dem_100_newyork_182	119.7M	119.6M	0.1	1.8	119.7M	119.6M	0.1	1.2
dem_100_newyork_183	186.6M	186.4M	0.1	4.4	186.6M	186.4M	0.1	3.2
dem_100_newyork_184	136.5M	134.1M	1.8	2.9	134.2M	134.0M	0.1	1.6
dem_100_newyork_185	143.7M	143.5M	0.1	7.3	143.7M	143.5M	0.1	6.7
dem_100_newyork_186	155.8M	155.6M	0.1	3.0	155.7M	155.6M	0.1	1.4
dem_100_newyork_187	157.7M	157.6M	0.1	2.8	157.7M	157.6M	0.1	2.2
dem_100_newyork_188	172.0M	171.9M	0.1	6.3	172.0M	171.8M	0.1	4.3
dem_100_newyork_189	149.3M	149.3M	0.0	3.4	149.3M	149.2M	0.1	2.9
dem_100_newyork_190	172.4M	172.2M	0.1	4.9	172.4M	172.2M	0.1	4.5
dem_100_newyork_191	119.8M	119.7M	0.1	2.6	119.7M	119.7M	0.1	1.3
dem_100_newyork_192	151.6M	151.4M	0.1	6.7	151.6M	151.4M	0.1	6.0
dem_100_newyork_193	163.3M	163.2M	0.1	3.0	163.3M	163.1M	0.1	2.3
dem_100_newyork_194	142.8M	142.8M	0.0	2.0	142.8M	142.8M	0.0	1.4
dem_100_newyork_195	174.3M	174.1M	0.1	2.5	174.4M	174.2M	0.1	2.3
dem_100_newyork_196	112.9M	112.8M	0.1	2.5	112.9M	112.8M	0.1	1.9
dem_100_newyork_197	172.6M	172.4M	0.1	4.8	172.6M	172.4M	0.1	3.5
dem_100_newyork_198	126.0M	125.9M	0.1	2.1	126.0M	125.9M	0.1	1.7
dem_100_newyork_199	128.5M	128.4M	0.0	2.2	128.5M	128.3M	0.1	1.9
dem_100_newyork_200	165.8M	165.7M	0.1	3.9	165.8M	165.7M	0.1	4.5
dem_100_newyork_201	131.6M	131.5M	0.1	4.8	131.6M	131.5M	0.1	3.8
dem_100_newyork_202	156.1M	156.0M	0.1	3.1	156.1M	156.0M	0.1	2.3
dem_100_newyork_203	124.4M	124.3M	0.1	4.6	124.4M	124.3M	0.1	4.8
dem_100_newyork_204	162.6M	162.4M	0.1	3.6	162.6M	162.4M	0.1	3.5
dem_100_newyork_205	168.8M	168.7M	0.1	5.0	168.8M	168.7M	0.1	5.1
dem_100_newyork_206	131.7M	131.6M	0.1	3.7	131.7M	131.6M	0.1	3.5
dem_100_newyork_207	145.3M	145.2M	0.1	6.1	145.3M	145.2M	0.1	4.5
dem_100_newyork_208	102.2M	102.1M	0.1	1.6	102.2M	102.1M	0.1	1.1
dem_100_newyork_209	177.8M	177.6M	0.1	5.2	177.7M	177.5M	0.1	4.1
dem_100_newyork_210	162.2M	162.1M	0.1	1.6	162.2M	162.1M	0.1	1.6
dem_100_newyork_211	188.6M	188.4M	0.1	3.7	188.6M	188.4M	0.1	3.3
dem_100_newyork_212	148.3M	148.1M	0.1	3.3	148.3M	148.1M	0.1	3.5
dem_100_newyork_213	148.2M	148.1M	0.1	3.2	148.2M	148.1M	0.1	3.0
dem_100_newyork_214	106.8M	106.8M	0.0	2.2	106.8M	106.7M	0.1	2.1
dem_100_newyork_215	152.9M	152.8M	0.0	2.7	152.9M	152.7M	0.1	1.5
dem_100_newyork_216	154.0M	153.9M	0.1	2.2	154.0M	153.9M	0.1	2.7
dem_100_newyork_217	173.0M	172.8M	0.1	4.2	173.0M	172.8M	0.1	4.0
dem_100_newyork_218	146.0M	145.9M	0.1	3.5	146.0M	145.9M	0.1	3.5
dem_100_newyork_219	153.2M	153.0M	0.1	3.4	153.2M	153.0M	0.1	3.6
dem_100_newyork_220	142.8M	142.7M	0.1	2.0	142.8M	142.7M	0.1	1.8
dem_100_newyork_221	154.4M	154.3M	0.1	3.0	154.4M	154.3M	0.1	2.5
dem_100_newyork_222	161.7M	161.5M	0.1	2.7	161.7M	161.5M	0.1	1.8
dem_100_newyork_223	95.0M	94.9M	0.1	1.2	95.0M	95.0M	0.1	1.4
dem_100_newyork_224	153.4M	153.3M	0.1	3.5	153.4M	153.3M	0.1	3.3
dem_100_newyork_225	101.4M	101.3M	0.1	0.8	101.4M	101.3M	0.1	0.8
dem_100_newyork_226	122.2M	122.1M	0.1	1.9	122.2M	122.1M	0.1	1.4
dem_100_newyork_227	135.8M	135.7M	0.1	3.7	135.8M	135.7M	0.1	3.3
dem_100_newyork_228	138.0M	137.9M	0.1	2.6	138.0M	137.9M	0.1	2.1
dem_100_newyork_229	129.9M	129.9M	0.0	1.2	129.9M	129.8M	0.1	1.1
dem_100_newyork_230	166.2M	166.1M	0.1	4.3	166.2M	166.1M	0.1	3.6

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Table A.36: Comparison of split-pipe models on *New York* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_231	119.8M	119.7M	0.1	2.5	119.8M	119.7M	0.1	1.9
dem_100_newyork_232	147.2M	147.0M	0.1	2.9	147.2M	147.0M	0.1	3.3
dem_100_newyork_233	125.4M	125.3M	0.1	2.2	125.4M	125.3M	0.0	3.4
dem_100_newyork_234	155.7M	155.6M	0.1	4.3	155.7M	155.5M	0.1	3.0
dem_100_newyork_235	138.7M	138.5M	0.1	2.6	138.7M	138.5M	0.1	2.4
dem_100_newyork_236	161.7M	161.5M	0.1	3.8	161.7M	161.5M	0.1	4.3
dem_100_newyork_237	151.7M	151.6M	0.1	3.1	151.7M	151.6M	0.1	2.5
dem_100_newyork_238	162.1M	161.9M	0.1	1.3	162.1M	161.9M	0.1	1.1
dem_100_newyork_239	185.4M	185.3M	0.1	4.3	185.4M	185.3M	0.1	4.4
dem_100_newyork_240	147.6M	147.5M	0.1	2.8	147.6M	147.5M	0.1	3.2
dem_100_newyork_241	134.8M	134.6M	0.1	4.8	134.8M	134.6M	0.1	4.5
dem_100_newyork_242	134.7M	134.6M	0.0	2.1	134.7M	134.6M	0.0	1.5
dem_100_newyork_243	155.9M	155.8M	0.1	4.1	155.9M	155.8M	0.1	2.3
dem_100_newyork_244	145.4M	145.2M	0.1	2.2	145.4M	145.2M	0.1	1.8
dem_100_newyork_245	145.0M	144.8M	0.1	2.5	145.0M	144.8M	0.1	2.8
dem_100_newyork_246	152.0M	151.1M	0.6	1.8	151.2M	151.2M	0.0	1.9
dem_100_newyork_247	170.4M	170.3M	0.1	2.4	170.4M	170.3M	0.1	2.4
dem_100_newyork_248	123.3M	123.2M	0.1	3.6	123.3M	123.2M	0.1	2.0
dem_100_newyork_249	172.9M	172.7M	0.1	3.1	172.9M	172.7M	0.1	4.5
dem_100_newyork_250	146.4M	146.3M	0.1	2.2	146.4M	146.3M	0.1	1.8
dem_100_newyork_251	122.1M	121.9M	0.1	3.0	122.1M	121.9M	0.1	2.4
dem_100_newyork_252	127.1M	127.0M	0.1	2.9	127.1M	127.0M	0.1	3.2
dem_100_newyork_253	157.5M	157.4M	0.1	4.7	157.5M	157.3M	0.1	4.8
dem_100_newyork_254	158.2M	158.1M	0.1	1.9	158.2M	158.1M	0.1	1.8
dem_100_newyork_255	172.4M	172.3M	0.1	3.5	172.4M	172.3M	0.1	2.8
dem_100_newyork_256	171.9M	171.8M	0.1	6.5	171.9M	171.8M	0.1	4.3
dem_100_newyork_257	110.4M	110.3M	0.1	4.6	110.4M	110.3M	0.1	4.2
dem_100_newyork_258	124.5M	124.3M	0.1	2.2	124.5M	124.3M	0.1	2.0
dem_100_newyork_259	99.3M	99.3M	0.1	1.7	99.3M	99.3M	0.1	1.5
dem_100_newyork_260	196.1M	195.9M	0.1	2.0	196.1M	195.9M	0.1	1.8
dem_100_newyork_261	158.1M	158.0M	0.1	6.8	158.1M	158.0M	0.1	5.6
dem_100_newyork_262	120.7M	120.7M	0.0	1.9	120.7M	120.6M	0.1	1.2
dem_100_newyork_263	164.5M	164.4M	0.1	4.2	164.5M	164.4M	0.1	5.0
dem_100_newyork_264	130.8M	130.7M	0.1	1.8	130.8M	130.7M	0.1	1.7
dem_100_newyork_265	149.3M	149.2M	0.1	4.1	149.3M	149.2M	0.1	4.7
dem_100_newyork_266	132.1M	132.0M	0.1	2.7	132.1M	132.0M	0.1	2.2
dem_100_newyork_267	129.9M	129.8M	0.1	2.2	129.9M	129.8M	0.1	1.9
dem_100_newyork_268	143.5M	143.5M	0.1	1.6	143.5M	143.4M	0.1	1.6
dem_100_newyork_269	156.1M	156.0M	0.1	2.2	156.1M	156.0M	0.1	2.1
dem_100_newyork_270	163.9M	163.7M	0.1	3.9	163.9M	163.7M	0.1	3.5
dem_100_newyork_271	150.4M	150.2M	0.1	3.4	150.4M	150.2M	0.1	2.3
dem_100_newyork_272	147.0M	146.9M	0.1	2.2	147.0M	146.8M	0.1	1.9
dem_100_newyork_273	141.7M	141.6M	0.1	1.5	141.7M	141.6M	0.1	1.3
dem_100_newyork_274	153.3M	153.1M	0.1	3.5	153.3M	153.1M	0.1	2.9
dem_100_newyork_275	162.2M	162.1M	0.1	3.1	162.2M	162.1M	0.1	2.7
dem_100_newyork_276	186.1M	185.9M	0.1	3.8	186.1M	186.0M	0.0	6.6
dem_100_newyork_277	157.3M	157.1M	0.1	2.9	157.3M	157.1M	0.1	2.7
dem_100_newyork_278	177.3M	177.2M	0.0	9.3	177.4M	177.3M	0.1	13.5
dem_100_newyork_279	140.8M	140.7M	0.1	3.9	140.8M	140.7M	0.1	3.7
dem_100_newyork_280	149.5M	149.4M	0.1	3.1	149.5M	149.4M	0.1	2.3
dem_100_newyork_281	150.5M	150.4M	0.1	2.2	150.5M	150.4M	0.1	1.7
dem_100_newyork_282	161.8M	161.7M	0.1	2.3	161.8M	161.7M	0.1	3.3
dem_100_newyork_283	133.6M	133.6M	0.0	3.8	133.6M	133.5M	0.1	2.8
dem_100_newyork_284	148.4M	148.3M	0.1	2.4	148.4M	148.3M	0.1	2.4
dem_100_newyork_285	123.7M	123.6M	0.1	1.6	123.7M	123.6M	0.1	1.6
dem_100_newyork_286	158.5M	158.3M	0.1	2.2	158.5M	158.3M	0.1	2.5
dem_100_newyork_287	157.2M	157.0M	0.1	1.4	157.2M	157.0M	0.1	1.5
dem_100_newyork_288	195.5M	195.3M	0.1	3.9	195.5M	195.3M	0.1	3.8
dem_100_newyork_289	121.8M	121.6M	0.1	2.3	121.8M	121.7M	0.1	2.1
dem_100_newyork_290	175.9M	175.7M	0.1	3.2	175.9M	175.7M	0.1	3.3
dem_100_newyork_291	102.5M	102.4M	0.1	0.8	102.5M	102.4M	0.1	0.8
dem_100_newyork_292	150.1M	149.9M	0.1	4.9	150.1M	149.9M	0.1	4.4
dem_100_newyork_293	155.7M	155.6M	0.1	1.7	155.7M	155.6M	0.1	1.5
dem_100_newyork_294	133.9M	133.7M	0.1	5.7	133.9M	133.7M	0.1	5.0
dem_100_newyork_295	159.1M	158.9M	0.1	3.2	159.1M	158.9M	0.1	3.7
dem_100_newyork_296	129.1M	129.0M	0.1	3.5	129.1M	129.0M	0.1	3.3
dem_100_newyork_297	131.6M	131.5M	0.1	3.3	131.6M	131.5M	0.1	3.4
dem_100_newyork_298	125.2M	125.1M	0.1	3.2	125.2M	125.1M	0.1	3.2
dem_100_newyork_299	139.9M	139.8M	0.1	3.1	139.9M	139.8M	0.1	2.4
dem_100_newyork_300	138.9M	138.7M	0.1	2.1	138.8M	138.7M	0.1	2.5
dem_100_newyork_301	157.6M	157.5M	0.0	3.1	157.6M	157.4M	0.1	3.5
dem_100_newyork_302	187.3M	187.1M	0.1	2.1	187.3M	187.2M	0.1	1.6
dem_100_newyork_303	161.2M	161.0M	0.1	2.6	161.2M	161.0M	0.1	1.8
dem_100_newyork_304	164.3M	164.1M	0.1	2.3	164.3M	164.1M	0.1	2.4
dem_100_newyork_305	137.5M	137.4M	0.1	2.0	137.5M	137.4M	0.1	2.1
dem_100_newyork_306	155.8M	155.6M	0.1	3.9	155.8M	155.6M	0.1	2.9
dem_100_newyork_307	154.2M	154.1M	0.1	3.7	154.2M	154.1M	0.1	2.8
dem_100_newyork_308	160.7M	160.6M	0.1	3.5	160.7M	160.6M	0.1	4.0

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Table A.36: Comparison of split-pipe models on *New York* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_309	152.4M	152.2M	0.1	1.8	152.4M	152.2M	0.1	1.9
dem_100_newyork_310	102.8M	102.8M	0.0	2.4	102.8M	102.7M	0.1	1.9
dem_100_newyork_311	131.1M	130.4M	0.5	2.1	130.5M	130.4M	0.1	1.4
dem_100_newyork_312	154.0M	153.8M	0.1	3.6	154.0M	153.9M	0.0	5.8
dem_100_newyork_313	172.7M	172.5M	0.1	3.4	172.7M	172.5M	0.1	3.7
dem_100_newyork_314	177.5M	177.4M	0.1	1.1	177.5M	177.4M	0.1	1.2
dem_100_newyork_315	99.8M	99.8M	0.1	1.6	99.8M	99.8M	0.1	1.6
dem_100_newyork_316	107.0M	106.9M	0.1	2.8	107.0M	106.9M	0.1	2.8
dem_100_newyork_317	145.6M	145.5M	0.1	1.5	145.6M	145.5M	0.1	1.5
dem_100_newyork_318	161.3M	161.2M	0.1	5.6	161.3M	161.2M	0.1	4.3
dem_100_newyork_319	156.3M	156.2M	0.1	1.9	156.3M	156.1M	0.1	1.9
dem_100_newyork_320	109.8M	109.7M	0.1	2.1	109.8M	109.7M	0.1	1.6
dem_100_newyork_321	148.8M	148.6M	0.1	3.2	148.8M	148.6M	0.1	4.2
dem_100_newyork_322	139.1M	138.9M	0.1	2.5	139.1M	139.0M	0.1	2.3
dem_100_newyork_323	110.3M	110.2M	0.1	3.0	110.3M	110.2M	0.1	3.5
dem_100_newyork_324	91.1M	91.0M	0.1	1.5	91.1M	91.0M	0.1	1.1
dem_100_newyork_325	177.1M	177.0M	0.1	4.2	177.1M	177.0M	0.1	3.3
dem_100_newyork_326	180.6M	180.5M	0.1	2.1	180.6M	180.5M	0.1	2.3
dem_100_newyork_327	87.9M	87.9M	0.1	1.2	87.9M	87.9M	0.1	1.4
dem_100_newyork_328	120.5M	120.4M	0.1	1.2	120.5M	120.4M	0.1	1.3
dem_100_newyork_329	117.4M	117.3M	0.1	1.3	117.4M	117.3M	0.1	1.4
dem_100_newyork_330	93.9M	93.8M	0.1	2.7	93.9M	93.8M	0.1	3.0
dem_100_newyork_331	175.9M	175.7M	0.1	2.9	175.9M	175.8M	0.1	4.5
dem_100_newyork_332	117.5M	117.3M	0.1	2.8	117.5M	117.3M	0.1	2.5
dem_100_newyork_333	146.9M	146.7M	0.1	1.9	146.9M	146.7M	0.1	1.6
dem_100_newyork_334	143.6M	143.5M	0.1	2.8	143.6M	143.5M	0.1	3.1
dem_100_newyork_335	147.6M	147.4M	0.1	6.6	147.6M	147.5M	0.0	9.7
dem_100_newyork_336	162.9M	162.7M	0.1	5.0	162.9M	162.7M	0.1	4.5
dem_100_newyork_337	138.6M	138.5M	0.1	3.5	138.6M	138.5M	0.1	3.8
dem_100_newyork_338	141.2M	141.1M	0.1	2.6	141.2M	141.1M	0.1	2.3
dem_100_newyork_339	144.3M	144.1M	0.1	2.6	144.3M	144.1M	0.1	2.3
dem_100_newyork_340	159.9M	159.7M	0.1	3.0	159.9M	159.7M	0.1	2.7
dem_100_newyork_341	106.0M	105.9M	0.1	1.5	106.0M	105.9M	0.1	1.5
dem_100_newyork_342	109.1M	108.9M	0.1	2.3	109.1M	109.0M	0.1	2.9
dem_100_newyork_343	161.2M	161.0M	0.1	2.8	161.2M	161.0M	0.1	3.8
dem_100_newyork_344	107.8M	107.7M	0.1	0.9	107.8M	107.7M	0.0	1.0
dem_100_newyork_345	145.7M	145.6M	0.1	5.4	145.7M	145.6M	0.1	5.0
dem_100_newyork_346	105.5M	105.4M	0.1	1.3	105.5M	105.4M	0.1	1.3
dem_100_newyork_347	183.7M	183.6M	0.1	3.3	183.7M	183.6M	0.1	2.8
dem_100_newyork_348	111.0M	110.9M	0.1	3.8	111.0M	110.9M	0.1	3.8
dem_100_newyork_349	99.6M	99.5M	0.1	2.9	99.6M	99.5M	0.1	3.5
dem_100_newyork_350	163.3M	163.2M	0.1	1.9	163.3M	163.1M	0.1	2.0
dem_100_newyork_351	149.4M	149.2M	0.1	3.6	149.4M	149.2M	0.1	4.2
dem_100_newyork_352	170.0M	169.9M	0.1	2.8	170.0M	169.8M	0.1	2.2
dem_100_newyork_353	142.9M	142.8M	0.1	3.5	142.9M	142.8M	0.1	4.5
dem_100_newyork_354	137.9M	137.8M	0.1	1.2	139.0M	137.8M	0.9	1.4
dem_100_newyork_355	145.4M	145.3M	0.1	4.6	145.4M	145.2M	0.1	3.4
dem_100_newyork_356	151.7M	151.5M	0.1	5.8	151.7M	151.5M	0.1	5.9
dem_100_newyork_357	167.3M	167.3M	0.0	4.7	167.3M	167.2M	0.1	3.5
dem_100_newyork_358	159.5M	159.3M	0.1	2.5	159.5M	159.4M	0.1	2.2
dem_100_newyork_359	165.2M	165.0M	0.1	3.3	165.2M	165.0M	0.1	3.7
dem_100_newyork_360	160.0M	159.9M	0.1	6.2	160.0M	159.8M	0.1	5.0
dem_100_newyork_361	118.8M	118.7M	0.1	1.2	118.8M	118.7M	0.1	1.3
dem_100_newyork_362	141.1M	140.9M	0.1	5.3	141.1M	140.9M	0.1	4.7
dem_100_newyork_363	134.3M	134.1M	0.1	2.2	134.3M	134.1M	0.1	2.0
dem_100_newyork_364	123.2M	123.1M	0.1	0.8	123.2M	123.1M	0.1	0.8
dem_100_newyork_365	79.8M	79.7M	0.1	1.2	79.8M	79.8M	0.1	1.2
dem_100_newyork_366	103.2M	103.1M	0.1	0.7	103.2M	103.1M	0.1	0.8
dem_100_newyork_367	92.6M	92.5M	0.1	0.8	92.6M	92.5M	0.1	0.9
dem_100_newyork_368	126.5M	126.4M	0.1	1.2	126.5M	126.3M	0.1	1.1
dem_100_newyork_369	172.9M	172.7M	0.1	3.6	172.9M	172.7M	0.1	3.9
dem_100_newyork_370	170.5M	170.4M	0.1	2.0	170.5M	170.4M	0.1	1.8
dem_100_newyork_371	142.1M	142.0M	0.1	3.5	142.1M	142.0M	0.1	3.6
dem_100_newyork_372	94.9M	94.8M	0.1	1.5	94.9M	94.8M	0.1	1.6
dem_100_newyork_373	174.3M	174.1M	0.1	3.2	174.3M	174.1M	0.1	3.2
dem_100_newyork_374	184.7M	184.4M	0.2	3.7	184.5M	184.3M	0.1	2.9
dem_100_newyork_375	161.2M	161.1M	0.1	2.3	161.2M	161.1M	0.1	2.5
dem_100_newyork_376	113.5M	113.5M	0.1	1.7	113.5M	113.4M	0.1	1.7
dem_100_newyork_377	118.0M	117.9M	0.1	1.5	118.0M	117.9M	0.1	1.6
dem_100_newyork_378	151.6M	151.4M	0.1	2.4	151.6M	151.4M	0.1	3.3
dem_100_newyork_379	116.8M	116.7M	0.1	1.2	116.8M	116.7M	0.1	1.1
dem_100_newyork_380	105.2M	105.1M	0.1	1.2	105.2M	105.1M	0.1	1.1
dem_100_newyork_381	167.6M	167.5M	0.1	4.0	167.6M	167.5M	0.1	3.7
dem_100_newyork_382	121.3M	121.2M	0.1	1.3	121.3M	121.2M	0.1	1.1
dem_100_newyork_383	135.5M	135.4M	0.1	1.1	135.5M	135.5M	0.0	1.4
dem_100_newyork_384	130.8M	130.7M	0.1	1.0	130.8M	130.7M	0.1	1.1
dem_100_newyork_385	186.6M	186.4M	0.1	3.9	186.6M	186.4M	0.1	3.9
dem_100_newyork_386	152.9M	152.8M	0.1	2.4	152.9M	152.8M	0.1	2.4

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Table A.36: Comparison of split-pipe models on *New York* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_387	122.8M	122.8M	0.1	1.0	122.8M	122.8M	0.1	1.1
dem_100_newyork_388	141.2M	141.0M	0.1	6.0	141.2M	141.0M	0.1	6.2
dem_100_newyork_389	122.8M	122.7M	0.1	2.2	122.8M	122.7M	0.1	1.8
dem_100_newyork_390	153.2M	153.2M	0.0	2.0	153.2M	153.2M	0.0	1.5
dem_100_newyork_391	149.4M	149.3M	0.1	3.2	149.4M	149.2M	0.1	3.4
dem_100_newyork_392	147.9M	147.8M	0.1	2.7	147.9M	147.8M	0.1	2.1
dem_100_newyork_393	128.1M	128.0M	0.1	2.6	128.1M	128.0M	0.1	2.7
dem_100_newyork_394	196.2M	196.0M	0.1	5.0	196.2M	196.0M	0.1	4.0
dem_100_newyork_395	145.2M	145.0M	0.1	5.4	145.2M	145.0M	0.1	4.2
dem_100_newyork_396	140.5M	140.3M	0.1	2.9	140.5M	140.4M	0.1	2.5
dem_100_newyork_397	174.7M	174.5M	0.1	2.7	174.7M	174.5M	0.1	2.8
dem_100_newyork_398	154.2M	154.0M	0.1	3.6	154.2M	154.0M	0.1	3.7
dem_100_newyork_399	112.9M	112.8M	0.1	1.2	112.9M	112.8M	0.1	1.2
dem_100_newyork_400	105.6M	105.5M	0.1	1.5	105.6M	105.5M	0.1	1.3
dem_100_newyork_401	137.3M	137.2M	0.1	1.6	137.3M	137.2M	0.1	1.6
dem_100_newyork_402	139.6M	139.5M	0.1	1.6	139.6M	139.5M	0.1	1.6
dem_100_newyork_403	114.3M	114.2M	0.1	1.1	114.3M	114.2M	0.1	1.0
dem_100_newyork_404	164.7M	164.6M	0.1	5.3	164.7M	164.6M	0.0	8.7
dem_100_newyork_405	170.4M	170.4M	0.0	9.2	170.4M	170.3M	0.1	3.8
dem_100_newyork_406	149.9M	149.8M	0.1	3.5	150.0M	149.8M	0.1	3.8
dem_100_newyork_407	165.2M	165.0M	0.1	2.9	165.2M	165.0M	0.1	2.6
dem_100_newyork_408	143.2M	143.1M	0.1	3.9	143.2M	143.1M	0.1	4.1
dem_100_newyork_409	129.7M	129.6M	0.0	1.2	129.7M	129.6M	0.0	1.3
dem_100_newyork_410	169.0M	168.9M	0.1	1.4	169.0M	168.9M	0.1	1.1
dem_100_newyork_411	141.9M	141.8M	0.1	3.2	141.9M	141.8M	0.1	2.8
dem_100_newyork_412	149.7M	149.7M	0.0	2.7	149.7M	149.6M	0.1	1.4
dem_100_newyork_413	174.5M	174.3M	0.1	1.8	174.5M	174.3M	0.1	1.4
dem_100_newyork_414	67.8M	67.8M	0.1	1.6	67.8M	67.8M	0.1	0.9
dem_100_newyork_415	114.4M	114.3M	0.1	2.0	114.4M	114.3M	0.1	1.4
dem_100_newyork_416	168.4M	168.2M	0.1	3.1	168.4M	168.2M	0.1	3.0
dem_100_newyork_417	126.1M	126.0M	0.1	1.4	126.1M	126.0M	0.1	0.9
dem_100_newyork_418	158.6M	158.4M	0.1	5.5	158.6M	158.4M	0.1	5.2
dem_100_newyork_419	146.6M	146.5M	0.1	2.8	146.6M	146.5M	0.1	2.9
dem_100_newyork_420	131.0M	130.9M	0.1	2.1	131.0M	130.9M	0.1	1.7
dem_100_newyork_421	134.8M	134.7M	0.1	4.1	135.3M	134.7M	0.4	5.4
dem_100_newyork_422	164.6M	164.4M	0.1	3.8	164.6M	164.4M	0.1	4.4
dem_100_newyork_423	146.9M	146.8M	0.1	1.5	146.9M	146.8M	0.1	1.6
dem_100_newyork_424	153.4M	153.2M	0.1	4.5	153.4M	153.2M	0.1	4.6
dem_100_newyork_425	164.2M	164.2M	0.0	1.9	164.2M	164.1M	0.1	1.2
dem_100_newyork_426	144.5M	144.4M	0.1	3.0	144.5M	144.4M	0.1	3.0
dem_100_newyork_427	124.5M	124.4M	0.1	1.3	124.5M	124.4M	0.1	0.9
dem_100_newyork_428	154.4M	154.3M	0.1	2.8	154.4M	154.3M	0.1	2.4
dem_100_newyork_429	123.4M	123.3M	0.1	1.4	123.4M	123.3M	0.1	1.0
dem_100_newyork_430	163.2M	163.1M	0.1	3.5	163.2M	163.1M	0.1	4.6
dem_100_newyork_431	129.9M	129.8M	0.1	1.3	129.9M	129.8M	0.1	1.1
dem_100_newyork_432	152.1M	152.0M	0.1	3.2	152.1M	152.0M	0.1	2.8
dem_100_newyork_433	148.6M	148.4M	0.1	2.5	148.6M	148.4M	0.1	1.4
dem_100_newyork_434	162.9M	162.8M	0.1	4.2	162.9M	162.8M	0.1	4.3
dem_100_newyork_435	158.8M	158.7M	0.1	5.0	158.8M	158.7M	0.1	5.5
dem_100_newyork_436	109.2M	109.1M	0.1	1.5	109.2M	109.1M	0.1	1.6
dem_100_newyork_437	153.2M	153.1M	0.1	3.0	153.2M	153.1M	0.1	2.7
dem_100_newyork_438	117.2M	117.0M	0.1	1.3	117.2M	117.0M	0.1	1.2
dem_100_newyork_439	148.8M	148.7M	0.1	3.5	148.8M	148.7M	0.1	3.6
dem_100_newyork_440	130.8M	130.6M	0.1	1.4	130.8M	130.6M	0.1	1.4
dem_100_newyork_441	102.5M	102.4M	0.1	1.2	102.5M	102.4M	0.1	1.2
dem_100_newyork_442	145.6M	145.4M	0.1	5.0	145.5M	145.4M	0.1	3.6
dem_100_newyork_443	113.0M	112.9M	0.1	3.0	113.0M	112.9M	0.1	2.7
dem_100_newyork_444	144.0M	143.9M	0.1	4.4	144.0M	143.9M	0.1	6.0
dem_100_newyork_445	162.4M	162.2M	0.1	4.0	162.4M	162.2M	0.1	3.9
dem_100_newyork_446	146.1M	146.0M	0.1	1.6	146.1M	146.0M	0.1	2.0
dem_100_newyork_447	123.1M	123.0M	0.1	1.5	123.1M	123.0M	0.1	1.6
dem_100_newyork_448	122.8M	122.7M	0.1	4.4	122.8M	122.7M	0.1	4.8
dem_100_newyork_449	162.3M	162.1M	0.1	3.4	162.3M	162.1M	0.1	3.4
dem_100_newyork_450	161.1M	161.0M	0.1	1.9	161.1M	161.0M	0.1	1.9
dem_100_newyork_451	147.1M	147.0M	0.1	2.3	147.1M	147.0M	0.1	2.3
dem_100_newyork_452	133.8M	133.7M	0.1	1.7	133.8M	133.7M	0.1	2.2
dem_100_newyork_453	157.4M	157.3M	0.1	2.9	157.4M	157.2M	0.1	2.3
dem_100_newyork_454	156.0M	155.9M	0.1	4.0	156.0M	155.9M	0.1	4.0
dem_100_newyork_455	172.4M	172.4M	0.0	1.3	172.4M	172.4M	0.0	1.7
dem_100_newyork_456	169.8M	169.6M	0.1	3.1	169.8M	169.6M	0.1	3.0
dem_100_newyork_457	151.7M	151.6M	0.1	3.0	151.7M	151.6M	0.0	3.7
dem_100_newyork_458	112.8M	112.7M	0.1	1.9	112.8M	112.7M	0.0	2.8
dem_100_newyork_459	116.0M	115.9M	0.1	3.0	116.0M	115.9M	0.1	3.1
dem_100_newyork_460	173.1M	172.9M	0.1	3.2	173.1M	172.9M	0.1	2.7
dem_100_newyork_461	112.6M	112.6M	0.1	1.1	112.6M	112.5M	0.1	1.1
dem_100_newyork_462	167.5M	167.4M	0.1	2.9	167.5M	167.4M	0.1	3.2
dem_100_newyork_463	106.8M	106.7M	0.1	1.1	106.8M	106.7M	0.1	1.2
dem_100_newyork_464	179.0M	178.9M	0.1	2.5	179.0M	178.8M	0.1	2.5

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Table A.36: Comparison of split-pipe models on *New York* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork_465	180.1M	180.0M	0.1	6.2	180.1M	180.0M	0.1	6.9
dem_100_newyork_466	144.9M	144.7M	0.1	2.5	144.9M	144.7M	0.1	2.4
dem_100_newyork_467	157.6M	157.4M	0.1	2.7	157.6M	157.4M	0.1	2.2
dem_100_newyork_468	123.7M	123.6M	0.1	3.1	123.7M	123.5M	0.1	2.5
dem_100_newyork_469	180.5M	180.3M	0.1	3.7	180.5M	180.4M	0.1	3.7
dem_100_newyork_470	137.6M	137.5M	0.1	3.6	137.6M	137.5M	0.1	3.4
dem_100_newyork_471	159.5M	159.3M	0.1	4.3	159.5M	159.3M	0.1	4.8
dem_100_newyork_472	167.9M	167.7M	0.1	9.3	167.9M	167.7M	0.1	6.2
dem_100_newyork_473	126.9M	126.8M	0.1	1.5	126.9M	126.8M	0.1	1.8
dem_100_newyork_474	162.3M	162.1M	0.1	3.0	162.3M	162.1M	0.1	2.9
dem_100_newyork_475	182.1M	181.9M	0.1	4.4	182.1M	181.9M	0.1	4.3
dem_100_newyork_476	123.2M	123.1M	0.1	2.7	123.2M	123.0M	0.1	2.3
dem_100_newyork_477	115.2M	115.1M	0.1	1.3	115.2M	115.1M	0.1	1.3
dem_100_newyork_478	138.6M	138.5M	0.1	2.3	138.6M	138.5M	0.1	1.7
dem_100_newyork_479	166.9M	166.8M	0.1	1.9	166.9M	166.8M	0.1	2.1
dem_100_newyork_480	124.6M	124.5M	0.1	1.2	124.6M	124.5M	0.1	1.0
dem_100_newyork_481	150.4M	150.2M	0.1	2.9	150.4M	150.2M	0.1	3.5
dem_100_newyork_482	159.6M	159.5M	0.1	2.6	159.6M	159.5M	0.1	2.6
dem_100_newyork_483	161.4M	161.3M	0.1	2.2	161.4M	161.3M	0.1	2.2
dem_100_newyork_484	147.6M	147.5M	0.1	3.5	147.6M	147.5M	0.1	3.2
dem_100_newyork_485	132.7M	132.4M	0.2	2.0	132.4M	132.3M	0.1	1.5
dem_100_newyork_486	193.3M	193.1M	0.1	2.0	193.3M	193.2M	0.1	2.2
dem_100_newyork_487	143.8M	143.6M	0.1	2.8	143.7M	143.6M	0.1	2.7
dem_100_newyork_488	142.1M	142.0M	0.1	2.0	142.1M	142.0M	0.1	2.4
dem_100_newyork_489	211.1M	210.9M	0.1	2.5	211.1M	210.9M	0.1	2.7
dem_100_newyork_490	143.0M	142.8M	0.1	8.2	143.0M	142.8M	0.1	9.2
dem_100_newyork_491	158.3M	158.3M	0.0	2.2	158.3M	158.3M	0.0	2.8
dem_100_newyork_492	100.8M	100.7M	0.1	0.9	100.8M	100.8M	0.1	0.9
dem_100_newyork_493	160.8M	160.6M	0.1	3.7	160.8M	160.7M	0.1	4.2
dem_100_newyork_494	110.0M	110.0M	0.1	2.1	110.0M	109.9M	0.1	2.0
dem_100_newyork_495	170.8M	170.7M	0.1	2.1	170.8M	170.7M	0.1	1.4
dem_100_newyork_496	107.5M	107.4M	0.1	1.0	107.5M	107.4M	0.1	0.9
dem_100_newyork_497	153.9M	153.8M	0.0	1.5	153.9M	153.8M	0.0	1.8
dem_100_newyork_498	124.4M	124.4M	0.0	1.5	124.4M	124.3M	0.1	1.2
dem_100_newyork_499	177.5M	177.4M	0.1	2.3	177.5M	177.4M	0.1	2.1
dem_100_newyork_500	133.8M	133.7M	0.1	2.5	133.8M	133.7M	0.1	2.5

Table A.37: Detailed results of the discrete models on *New York* with $\mathcal{B} = 200$ as summarized in Figure 3.11b and Table 3.3e. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_1	403.0M	168.7M	138.8	14400.0	385.6M	385.6M	0.0	5898.1	428.4M	254.5M	68.3	14400.0
dem_200_newyork_2	422.2M	242.7M	74.0	14400.0	407.3M	407.3M	0.0	906.2	436.8M	307.4M	42.1	14400.0
dem_200_newyork_3	394.7M	201.2M	96.1	14400.0	376.0M	376.0M	0.0	5224.7	431.8M	252.9M	70.7	14400.0
dem_200_newyork_4	439.3M	159.6M	175.2	14400.0	424.3M	424.3M	0.0	607.8	465.8M	282.9M	64.6	14400.0
dem_200_newyork_5	465.7M	204.1M	128.2	14400.0	418.0M	418.0M	0.0	643.8	450.4M	281.3M	60.1	14400.0
dem_200_newyork_6	453.0M	161.9M	179.7	14400.0	418.7M	418.7M	0.0	2095.0	472.6M	273.5M	72.8	14400.0
dem_200_newyork_7	447.7M	224.8M	99.2	14432.0	386.2M	386.2M	0.0	736.9	408.5M	296.5M	37.8	14400.0
dem_200_newyork_8	376.6M	171.9M	119.1	14400.3	369.9M	369.9M	0.0	3695.0	393.3M	235.2M	67.3	14400.3
dem_200_newyork_9	462.5M	196.2M	135.7	14400.0	461.2M	461.2M	0.0	879.8	490.5M	350.3M	40.0	14400.0
dem_200_newyork_10	365.0M	170.0M	114.7	14400.0	348.6M	348.6M	0.0	2325.8	368.1M	223.5M	64.7	14400.0
dem_200_newyork_11	454.0M	199.0M	128.2	14400.0	441.9M	441.9M	0.0	1515.4	454.5M	370.6M	22.7	14400.0
dem_200_newyork_12	403.3M	219.4M	83.8	14400.0	396.2M	396.2M	0.0	2091.7	1e+20	267.1M	–	14400.0
dem_200_newyork_13	458.3M	277.7M	65.0	14400.8	458.3M	458.3M	0.0	588.4	464.2M	304.9M	52.2	14400.0
dem_200_newyork_14	403.8M	222.8M	81.3	14400.0	396.7M	396.7M	0.0	3460.9	431.0M	282.6M	52.5	14400.0
dem_200_newyork_15	466.1M	238.5M	95.4	14423.6	452.9M	452.9M	0.0	1618.1	467.8M	352.8M	32.6	14400.0
dem_200_newyork_16	386.6M	189.0M	104.6	14400.2	375.7M	373.9M	0.5	3502.8	404.9M	236.5M	71.2	14400.0
dem_200_newyork_17	393.3M	168.6M	133.3	14400.0	382.8M	382.8M	0.0	966.6	1e+20	228.3M	–	14400.0
dem_200_newyork_18	496.0M	259.1M	91.5	14400.3	487.6M	487.6M	0.0	1611.9	1e+20	355.0M	–	14400.0
dem_200_newyork_19	359.4M	166.5M	115.9	14400.0	342.4M	342.4M	0.0	6172.6	393.6M	216.5M	81.8	14400.0
dem_200_newyork_20	448.1M	178.8M	150.6	14400.0	442.9M	442.9M	0.0	998.3	1e+20	303.3M	–	14400.0
dem_200_newyork_21	428.9M	217.4M	97.3	14431.2	414.5M	414.0M	0.1	3557.3	458.4M	333.0M	37.7	14400.0
dem_200_newyork_22	328.6M	153.9M	113.5	14400.0	325.8M	325.8M	0.0	5732.9	359.0M	193.1M	85.9	14400.0
dem_200_newyork_23	1e+20	324.6M	–	14400.0	458.1M	458.1M	0.0	1148.7	464.4M	320.8M	44.8	14400.0
dem_200_newyork_24	490.1M	190.2M	157.7	14400.0	448.8M	448.8M	0.0	2303.6	472.2M	273.3M	72.8	14400.0
dem_200_newyork_25	1e+20	264.0M	–	14400.0	1e+20	1e+20	0.0	194.3	1e+20	1e+20	0.0	1071.7
dem_200_newyork_26	474.2M	221.0M	114.5	14400.0	472.1M	472.1M	0.0	3123.5	1e+20	346.5M	–	14400.0
dem_200_newyork_27	454.6M	178.4M	154.9	14400.0	454.4M	454.4M	0.0	661.0	479.7M	318.3M	50.7	14400.0

continued on next page

Table A.37: Comparison of discrete models on *New York* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_28	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	665.8	1e+20	1e+20	0.0	131.2
dem_200_newyork_29	489.9M	212.9M	130.2	14400.0	480.3M	480.1M	0.0	742.2	504.9M	375.3M	34.6	14400.0
dem_200_newyork_30	393.6M	166.7M	136.1	14400.0	380.0M	380.0M	0.0	3262.2	391.6M	261.6M	49.7	14400.0
dem_200_newyork_31	463.6M	281.9M	64.4	14400.0	426.6M	426.6M	0.0	2702.2	442.3M	348.4M	27.0	14400.0
dem_200_newyork_32	1e+20	430.4M	–	14400.0	1e+20	1e+20	0.0	108.8	1e+20	1e+20	0.0	86.3
dem_200_newyork_33	418.5M	174.4M	140.0	14400.1	399.1M	399.1M	0.0	5220.1	434.2M	287.1M	51.3	14400.0
dem_200_newyork_34	452.9M	197.4M	129.4	14400.0	441.1M	441.1M	0.0	921.1	454.2M	328.2M	38.4	14400.0
dem_200_newyork_35	501.1M	238.4M	110.2	14400.0	494.3M	494.3M	0.0	210.2	494.3M	386.7M	27.9	14400.0
dem_200_newyork_36	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	140.9	1e+20	1e+20	0.0	48.3
dem_200_newyork_37	465.6M	240.3M	93.7	14438.8	427.8M	427.8M	0.0	310.0	431.5M	329.5M	30.9	14400.0
dem_200_newyork_38	443.1M	165.9M	167.0	14400.0	421.6M	421.6M	0.0	2323.3	436.9M	284.3M	53.7	14400.0
dem_200_newyork_39	522.2M	216.0M	141.8	14400.0	521.6M	521.6M	0.0	379.2	521.6M	521.6M	0.0	4802.6
dem_200_newyork_40	477.1M	182.5M	161.4	14400.0	462.0M	462.0M	0.0	416.6	468.4M	358.7M	30.6	14400.0
dem_200_newyork_41	1e+20	271.0M	–	14400.0	1e+20	1e+20	0.0	93.5	1e+20	1e+20	0.0	3665.9
dem_200_newyork_42	431.3M	221.1M	95.1	14400.0	427.6M	427.6M	0.0	3957.3	1e+20	297.8M	–	14400.0
dem_200_newyork_43	469.8M	206.0M	128.0	14400.0	469.1M	469.1M	0.0	840.5	475.9M	348.8M	36.4	14400.0
dem_200_newyork_44	358.4M	139.0M	157.9	14449.2	339.7M	339.7M	0.0	6951.0	434.8M	204.6M	112.6	14400.0
dem_200_newyork_45	1e+20	248.0M	–	14400.0	413.7M	412.5M	0.3	8149.6	1e+20	349.3M	–	14400.0
dem_200_newyork_46	436.9M	152.9M	185.6	14400.0	425.2M	425.2M	0.0	2724.0	563.9M	262.3M	115.0	14400.0
dem_200_newyork_47	1e+20	321.8M	–	14400.0	502.2M	502.2M	0.0	619.5	502.2M	502.2M	0.0	3018.8
dem_200_newyork_48	453.2M	156.5M	189.7	14400.0	450.4M	450.4M	0.0	1480.8	464.2M	310.5M	49.5	14400.0
dem_200_newyork_49	422.3M	154.6M	173.3	14400.1	402.9M	402.9M	0.0	6170.4	444.2M	223.4M	98.8	14400.0
dem_200_newyork_50	376.2M	131.0M	187.1	14400.0	368.7M	368.7M	0.0	1487.2	469.7M	207.6M	126.3	14400.0
dem_200_newyork_51	501.1M	209.8M	138.8	14400.0	468.7M	468.7M	0.0	596.6	474.5M	395.8M	19.9	14400.0
dem_200_newyork_52	393.9M	184.2M	113.9	14400.0	384.3M	384.3M	0.0	2774.5	423.5M	264.7M	60.0	14400.0
dem_200_newyork_53	456.4M	290.2M	57.3	14400.0	455.1M	455.1M	0.0	418.7	455.1M	455.1M	0.0	14308.1
dem_200_newyork_54	478.0M	171.7M	178.3	14400.0	438.0M	438.0M	0.0	1200.2	449.8M	327.0M	37.5	14400.0
dem_200_newyork_55	458.1M	458.1M	0.0	596.7	458.1M	458.1M	0.0	247.5	458.1M	458.1M	0.0	3987.0
dem_200_newyork_56	470.3M	316.5M	48.6	14400.0	467.8M	467.8M	0.0	545.1	1e+20	344.7M	–	14400.0
dem_200_newyork_57	404.1M	151.2M	167.3	14400.0	387.1M	387.1M	0.0	3185.6	1e+20	209.5M	–	14400.1
dem_200_newyork_58	497.2M	387.3M	28.4	14400.0	497.2M	497.2M	0.0	587.2	497.2M	417.0M	19.2	14400.0
dem_200_newyork_59	358.9M	132.0M	172.0	14400.0	337.5M	337.5M	0.0	2189.7	1e+20	215.1M	–	14400.0
dem_200_newyork_60	476.7M	244.6M	94.9	14400.0	476.7M	476.7M	0.0	567.1	476.7M	412.3M	15.6	14400.0
dem_200_newyork_61	479.9M	233.5M	105.5	14400.0	479.9M	479.9M	0.0	1448.6	1e+20	390.7M	–	14400.0
dem_200_newyork_62	389.8M	166.5M	134.2	14400.0	380.7M	380.7M	0.0	1951.5	1e+20	241.4M	–	14400.0
dem_200_newyork_63	430.2M	246.1M	74.8	14400.0	416.2M	416.2M	0.0	978.7	434.4M	326.2M	33.2	14400.0
dem_200_newyork_64	1e+20	1e+20	0.0	1.5	1e+20	1e+20	0.0	78.2	1e+20	1e+20	0.0	44.7
dem_200_newyork_65	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	108.7	1e+20	1e+20	0.0	2631.7
dem_200_newyork_66	515.6M	300.3M	71.7	14400.0	509.9M	509.9M	0.0	424.5	509.9M	509.9M	0.0	5098.2
dem_200_newyork_67	435.6M	167.8M	159.6	14400.0	429.7M	429.7M	0.0	1697.7	435.5M	289.4M	50.5	14400.0
dem_200_newyork_68	484.1M	207.0M	133.8	14433.5	480.5M	480.5M	0.0	1832.8	487.0M	373.2M	30.5	14400.0
dem_200_newyork_69	1e+20	1e+20	0.0	6.3	1e+20	1e+20	0.0	203.6	1e+20	1e+20	0.0	1884.0
dem_200_newyork_70	405.2M	172.7M	134.7	14400.0	384.4M	384.4M	0.0	1730.4	386.4M	263.2M	46.8	14400.0
dem_200_newyork_71	443.0M	184.4M	140.3	14400.0	443.0M	443.0M	0.0	345.5	469.1M	359.2M	30.6	14400.0
dem_200_newyork_72	434.8M	193.4M	124.8	14400.0	428.5M	428.5M	0.0	1720.2	471.2M	277.1M	70.0	14400.0
dem_200_newyork_73	1e+20	464.6M	–	14400.0	1e+20	1e+20	0.0	74.1	1e+20	1e+20	0.0	454.6
dem_200_newyork_74	377.8M	170.1M	122.1	14400.0	366.9M	366.9M	0.0	6952.7	426.9M	227.9M	87.3	14400.0
dem_200_newyork_75	427.1M	297.2M	43.7	14400.0	419.3M	419.3M	0.0	1862.2	419.3M	419.3M	0.0	9570.6
dem_200_newyork_76	1e+20	262.3M	–	14400.0	1e+20	1e+20	0.0	163.2	1e+20	483.5M	–	14400.0
dem_200_newyork_77	432.0M	191.2M	125.9	14400.0	419.6M	419.6M	0.0	685.7	1e+20	320.8M	–	14400.0
dem_200_newyork_78	476.6M	169.9M	180.5	14400.0	457.7M	457.7M	0.0	468.3	515.4M	309.8M	66.3	14400.0
dem_200_newyork_79	1e+20	1e+20	0.0	1.5	1e+20	1e+20	0.0	66.3	1e+20	1e+20	0.0	57.0
dem_200_newyork_80	514.2M	281.7M	82.5	14400.0	501.6M	501.6M	0.0	300.8	501.6M	501.6M	0.0	10847.6
dem_200_newyork_81	455.3M	171.3M	165.8	14400.2	452.1M	451.8M	0.1	3772.5	481.1M	331.2M	45.3	14400.0
dem_200_newyork_82	1e+20	228.1M	–	14400.0	462.3M	462.3M	0.0	573.6	464.3M	371.1M	25.1	14400.0
dem_200_newyork_83	403.0M	144.9M	178.1	14400.6	388.9M	388.9M	0.0	2708.6	458.6M	231.6M	98.1	14400.0
dem_200_newyork_84	522.7M	243.7M	114.5	14431.9	516.2M	516.2M	0.0	1519.1	516.7M	438.6M	17.8	14400.0
dem_200_newyork_85	382.7M	155.7M	145.9	14400.3	364.9M	364.9M	0.0	2389.4	425.1M	222.4M	91.1	14400.0
dem_200_newyork_86	534.8M	534.8M	0.0	13677.8	534.8M	534.8M	0.0	292.8	534.8M	534.8M	0.0	474.7
dem_200_newyork_87	436.6M	232.0M	88.2	14400.0	425.5M	425.5M	0.0	1167.2	1e+20	334.0M	–	14400.0
dem_200_newyork_88	446.7M	246.6M	81.2	14400.0	436.3M	436.3M	0.0	2402.1	438.7M	379.1M	15.7	14400.0
dem_200_newyork_89	411.4M	151.4M	171.7	14400.0	410.0M	410.0M	0.0	3385.7	434.1M	286.5M	51.5	14400.0
dem_200_newyork_90	491.3M	211.3M	132.5	14400.0	485.9M	485.9M	0.0	716.9	1e+20	390.1M	–	14400.0
dem_200_newyork_91	1e+20	456.7M	–	14400.0	1e+20	1e+20	0.0	594.6	1e+20	1e+20	0.0	145.5
dem_200_newyork_92	479.3M	221.0M	116.8	14400.0	471.9M	471.9M	0.0	864.8	482.6M	377.7M	27.8	14400.0
dem_200_newyork_93	425.1M	160.4M	165.0	14400.0	408.3M	408.3M	0.0	3332.2	433.6M	243.2M	78.3	14400.0
dem_200_newyork_94	480.4M	280.6M	71.2	14400.0	472.6M	472.6M	0.0	1001.8	472.6M	384.4M	22.9	14400.0
dem_200_newyork_95	376.7M	208.8M	80.4	14400.0	372.9M	372.9M	0.0	499.2	392.0M	294.9M	32.9	14400.0
dem_200_newyork_96	496.8M	255.9M	94.1	14400.0	494.0M	494.0M	0.0	265.6	524.3M	409.7M	28.0	14400.0
dem_200_newyork_97	465.8M	205.3M	126.8	14400.0	446.3M	446.3M	0.0	1164.7	447.7M	372.8M	20.1	14400.0
dem_200_newyork_98	469.8M	217.7M	115.8	14400.0	468.6M	468.6M	0.0	2116.0	509.6M	348.2M	46.4	14400.0
dem_200_newyork_99	508.7M	218.9M	132.4	14400.0	478.9M	478.9M	0.0	160.2	478.9M	399.7M	19.8	14400.0
dem_200_newyork_100	1e+20	236.1M	–	14400.0	1e+20	1e+20	0.0	235.8	1e+20	1e+20	0.0	7735.6
dem_200_newyork_101	516.0M	231.0M	123.4	14400.0	508.0M	508.0M	0.0	1282.7	522.5M	388.0M	34.7	14400.0
dem_200_newyork_102	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	77.5	1e+20	1e+20	0.0	92.0
dem_200_newyork_103	453.8M	240.1M	89.0	14400.0	442.9M	442.9M	0.0	802.8	451.7M	346.5M	30.4	14400.0
dem_200_newyork_104	454.9M	255.7M	77.9	14432.6	452.4M	452.4M	0.0	1460.2	467.6M	361.3M	29.4	14400.0
dem_200_newyork_105	463.2M	245.0M	89.1	14400.0	459.4M	459.4M	0.0	1026.3	475.8M	338.3M	40.6	14400.0

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Table A.37: Comparison of discrete models on *New York* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_106	496.5M	223.8M	121.9	14400.0	496.5M	496.5M	0.0	353.0	496.5M	452.5M	9.7	14400.0
dem_200_newyork_107	518.8M	208.9M	148.3	14400.0	518.8M	518.8M	0.0	147.8	518.8M	472.6M	9.8	14400.0
dem_200_newyork_108	461.9M	185.4M	149.2	14400.0	458.5M	458.5M	0.0	679.3	1e+20	320.6M	–	14400.0
dem_200_newyork_109	1e+20	278.8M	–	14400.0	1e+20	1e+20	0.0	259.1	1e+20	1e+20	0.0	8165.0
dem_200_newyork_110	400.9M	184.1M	117.8	14400.0	395.2M	395.2M	0.0	1796.8	418.3M	293.0M	42.8	14400.0
dem_200_newyork_111	437.8M	208.4M	110.1	14400.0	437.2M	437.2M	0.0	749.7	441.8M	382.0M	15.7	14400.0
dem_200_newyork_112	1e+20	223.0M	–	14400.0	504.2M	504.2M	0.0	274.4	504.2M	504.2M	0.0	3162.8
dem_200_newyork_113	1e+20	284.4M	–	14400.0	1e+20	1e+20	0.0	377.3	1e+20	475.8M	–	14400.0
dem_200_newyork_114	490.1M	206.6M	137.2	14400.0	490.1M	490.1M	0.0	352.1	490.1M	490.1M	0.0	10267.9
dem_200_newyork_115	420.7M	192.5M	118.6	14400.0	397.1M	397.1M	0.0	1968.2	462.6M	238.5M	93.9	14400.0
dem_200_newyork_116	465.4M	368.8M	26.2	14400.0	465.0M	465.0M	0.0	1143.1	465.0M	465.0M	0.0	1458.1
dem_200_newyork_117	1e+20	234.8M	–	14400.0	494.8M	494.4M	0.1	347.2	494.4M	494.4M	0.0	9020.0
dem_200_newyork_118	432.2M	168.3M	156.8	14400.4	429.9M	429.9M	0.0	2921.4	1e+20	271.0M	–	14400.0
dem_200_newyork_119	1e+20	312.2M	–	14400.0	1e+20	1e+20	0.0	281.9	1e+20	1e+20	0.0	97.8
dem_200_newyork_120	431.0M	213.5M	101.9	14400.0	421.6M	421.6M	0.0	6553.1	498.7M	291.4M	71.1	14400.0
dem_200_newyork_121	477.2M	216.9M	120.1	14400.0	475.8M	475.8M	0.0	577.5	490.2M	380.7M	28.8	14400.0
dem_200_newyork_122	444.6M	184.9M	140.5	14400.0	426.7M	426.7M	0.0	6920.2	434.4M	304.9M	42.5	14400.0
dem_200_newyork_123	476.5M	233.1M	104.5	14400.0	471.9M	471.8M	0.0	894.0	496.7M	387.4M	28.2	14400.0
dem_200_newyork_124	409.4M	171.7M	138.4	14400.0	390.9M	390.9M	0.0	1801.8	412.0M	251.9M	63.5	14400.0
dem_200_newyork_125	482.4M	246.5M	95.7	14400.0	473.5M	473.5M	0.0	504.8	1e+20	360.3M	–	14400.0
dem_200_newyork_126	473.0M	193.5M	144.5	14400.0	462.4M	462.4M	0.0	1506.1	1e+20	322.5M	–	14400.0
dem_200_newyork_127	447.5M	169.5M	164.0	14400.0	435.9M	435.9M	0.0	653.3	1e+20	317.0M	–	14400.0
dem_200_newyork_128	1e+20	248.4M	–	14400.0	380.8M	380.8M	0.0	7162.7	426.2M	300.1M	42.0	14400.0
dem_200_newyork_129	492.1M	214.8M	129.1	14400.0	489.5M	489.5M	0.0	237.7	1e+20	400.0M	–	14400.0
dem_200_newyork_130	387.2M	179.9M	115.3	14400.0	380.1M	380.1M	0.0	1366.4	384.0M	282.4M	36.0	14400.0
dem_200_newyork_131	429.5M	180.4M	138.0	14400.0	426.5M	426.5M	0.0	1129.3	431.5M	342.4M	26.0	14400.0
dem_200_newyork_132	449.5M	226.6M	98.3	14400.0	432.6M	432.6M	0.0	2318.2	467.1M	323.8M	44.3	14400.0
dem_200_newyork_133	461.7M	179.1M	157.8	14400.0	460.8M	460.8M	0.0	578.8	472.5M	357.3M	32.3	14400.0
dem_200_newyork_134	1e+20	268.4M	–	14400.0	1e+20	1e+20	0.0	127.7	1e+20	1e+20	0.0	102.2
dem_200_newyork_135	424.6M	196.0M	116.6	14400.0	424.6M	424.6M	0.0	1606.4	433.7M	351.1M	23.5	14400.0
dem_200_newyork_136	441.8M	177.3M	149.1	14435.4	435.4M	435.4M	0.0	1291.8	473.6M	282.1M	67.8	14400.0
dem_200_newyork_137	1e+20	1e+20	0.0	13409.8	1e+20	1e+20	0.0	67.2	1e+20	1e+20	0.0	72.5
dem_200_newyork_138	1e+20	187.5M	–	14400.0	461.9M	461.9M	0.0	1162.6	465.9M	379.6M	22.8	14400.0
dem_200_newyork_139	473.3M	224.4M	110.9	14400.0	469.5M	469.5M	0.0	1198.5	503.1M	357.4M	40.8	14400.0
dem_200_newyork_140	472.5M	193.6M	144.0	14400.0	463.2M	463.2M	0.0	869.4	480.3M	347.5M	38.2	14400.0
dem_200_newyork_141	480.3M	287.8M	66.9	14400.0	446.4M	446.4M	0.0	1322.1	478.4M	333.8M	43.4	14400.0
dem_200_newyork_142	391.5M	178.0M	120.0	14400.0	382.6M	382.6M	0.0	1316.5	400.2M	264.9M	51.1	14400.0
dem_200_newyork_143	498.4M	257.0M	94.0	14400.2	472.9M	472.9M	0.0	720.2	478.6M	395.8M	20.9	14400.0
dem_200_newyork_144	452.9M	188.6M	140.2	14400.0	444.4M	444.4M	0.0	1048.8	444.4M	444.4M	0.0	6842.0
dem_200_newyork_145	460.0M	150.9M	204.8	14400.0	441.3M	441.3M	0.0	1907.6	1e+20	247.7M	–	14400.0
dem_200_newyork_146	443.2M	155.4M	185.2	14400.0	412.8M	412.8M	0.0	5335.9	485.7M	218.6M	122.2	14400.0
dem_200_newyork_147	487.5M	195.7M	149.1	14400.0	473.3M	473.3M	0.0	472.9	473.3M	473.3M	0.0	8823.3
dem_200_newyork_148	434.5M	218.1M	99.3	14400.0	432.5M	432.5M	0.0	3343.8	482.4M	339.5M	42.1	14400.0
dem_200_newyork_149	1e+20	373.9M	–	14400.0	1e+20	1e+20	0.0	76.8	1e+20	1e+20	0.0	143.0
dem_200_newyork_150	513.2M	233.9M	119.4	14431.7	490.2M	490.2M	0.0	1233.9	490.2M	490.2M	0.0	10559.7
dem_200_newyork_151	1e+20	383.6M	–	14400.0	1e+20	1e+20	0.0	544.0	1e+20	517.5M	–	14400.0
dem_200_newyork_152	501.4M	216.3M	131.8	14400.0	501.4M	501.3M	0.0	1967.6	501.4M	501.4M	0.0	8891.0
dem_200_newyork_153	465.4M	191.1M	143.6	14400.0	440.1M	439.8M	0.1	1214.6	459.1M	293.5M	56.4	14400.0
dem_200_newyork_154	458.5M	209.3M	119.1	14400.0	445.4M	445.4M	0.0	1745.1	464.3M	299.8M	54.9	14400.0
dem_200_newyork_155	406.6M	136.7M	197.4	14400.0	372.0M	372.0M	0.0	2091.9	396.9M	212.4M	86.9	14400.0
dem_200_newyork_156	382.1M	150.7M	153.6	14400.0	357.5M	357.5M	0.0	914.2	379.6M	231.8M	63.7	14400.0
dem_200_newyork_157	446.4M	178.0M	150.8	14438.6	424.3M	424.3M	0.0	1809.3	436.3M	250.6M	74.1	14400.0
dem_200_newyork_158	508.7M	508.7M	0.0	6.5	508.7M	508.5M	0.1	1946.9	508.7M	508.4M	0.1	1201.3
dem_200_newyork_159	1e+20	183.9M	–	14400.0	1e+20	1e+20	0.0	205.9	1e+20	1e+20	0.0	6006.1
dem_200_newyork_160	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	237.2	1e+20	1e+20	0.0	7946.1
dem_200_newyork_161	1e+20	204.4M	–	14400.0	441.5M	441.3M	0.0	580.2	486.6M	344.1M	41.4	14400.0
dem_200_newyork_162	478.0M	211.1M	126.4	14400.0	467.4M	467.4M	0.0	667.7	490.9M	386.7M	26.9	14400.0
dem_200_newyork_163	364.0M	149.9M	142.9	14400.1	355.6M	355.6M	0.0	10523.9	375.3M	219.1M	71.3	14400.0
dem_200_newyork_164	370.4M	127.9M	189.5	14400.0	335.4M	335.4M	0.0	7766.8	1e+20	167.5M	–	14400.0
dem_200_newyork_165	490.2M	243.9M	101.0	14400.0	477.8M	477.8M	0.0	195.0	477.8M	477.8M	0.0	4856.9
dem_200_newyork_166	474.8M	253.4M	87.4	14400.0	469.4M	469.4M	0.0	762.7	496.2M	384.6M	29.0	14400.0
dem_200_newyork_167	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	197.9	1e+20	1e+20	0.0	84.8
dem_200_newyork_168	364.2M	140.5M	159.2	14400.0	353.5M	353.5M	0.0	5276.7	369.9M	215.8M	71.4	14400.0
dem_200_newyork_169	1e+20	1e+20	0.0	264.1	1e+20	1e+20	0.0	309.9	1e+20	1e+20	0.0	13964.6
dem_200_newyork_170	467.5M	183.6M	154.7	14400.0	455.2M	455.2M	0.0	346.3	506.7M	317.8M	59.4	14400.0
dem_200_newyork_171	506.8M	269.5M	88.0	14400.4	485.2M	484.9M	0.1	844.4	515.3M	408.5M	26.1	14400.0
dem_200_newyork_172	420.5M	159.7M	163.2	14400.0	405.4M	405.4M	0.0	3607.4	422.8M	256.0M	65.2	14400.0
dem_200_newyork_173	394.1M	263.8M	49.4	14400.3	389.4M	389.3M	0.0	1345.0	404.1M	305.0M	32.5	14400.0
dem_200_newyork_174	476.3M	205.5M	131.8	14400.0	473.2M	473.2M	0.0	1283.9	484.7M	383.6M	26.3	14400.0
dem_200_newyork_175	494.4M	228.9M	115.9	14400.2	493.2M	493.2M	0.0	517.3	1e+20	358.5M	–	14400.0
dem_200_newyork_176	1e+20	1e+20	0.0	2.7	1e+20	1e+20	0.0	280.5	1e+20	1e+20	0.0	156.2
dem_200_newyork_177	525.9M	224.3M	134.5	14400.0	518.3M	518.3M	0.0	654.0	518.3M	518.3M	0.0	6476.1
dem_200_newyork_178	538.7M	238.2M	126.2	14400.0	517.5M	517.5M	0.0	1549.0	517.5M	517.5M	0.0	10398.1
dem_200_newyork_179	464.5M	209.1M	122.2	14400.0	454.7M	454.7M	0.0	679.3	1e+20	297.0M	–	14400.0
dem_200_newyork_180	425.7M	215.8M	97.3	14400.0	425.2M	425.2M	0.0	1005.4	464.6M	268.4M	73.1	14400.0
dem_200_newyork_181	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	211.2	1e+20	1e+20	0.0	206.4
dem_200_newyork_182	383.0M	195.0M	96.4	14400.0	378.2M	378.2M	0.0	841.9	418.1M	279.6M	49.5	14400.0
dem_200_newyork_183	537.3M	537.3M	0.0	11.8	538.7M	537.3M	0.3	2043.7	537.3M	537.3M	0.0	235.1

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Table A.37: Comparison of discrete models on *New York* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_184	450.5M	204.3M	120.5	14400.0	437.0M	436.8M	0.0	549.8	478.3M	323.4M	47.9	14400.0
dem_200_newyork_185	1e+20	384.2M	–	14400.0	1e+20	1e+20	0.0	928.2	1e+20	1e+20	0.0	2099.3
dem_200_newyork_186	467.2M	195.7M	138.8	14400.0	451.1M	451.1M	0.0	865.3	1e+20	317.9M	–	14400.0
dem_200_newyork_187	427.5M	197.4M	116.5	14400.0	405.7M	405.7M	0.0	664.5	425.7M	268.1M	58.8	14400.0
dem_200_newyork_188	1e+20	228.0M	–	14400.0	401.3M	401.3M	0.0	2574.6	433.1M	309.5M	39.9	14400.0
dem_200_newyork_189	479.7M	441.4M	8.7	14400.0	479.7M	479.7M	0.0	1184.3	479.7M	479.7M	0.0	2221.1
dem_200_newyork_190	506.4M	255.3M	98.3	14400.0	504.9M	504.9M	0.0	216.0	1e+20	379.8M	–	14400.0
dem_200_newyork_191	1e+20	195.2M	–	14400.0	376.5M	376.5M	0.0	5422.4	411.8M	275.1M	49.7	14400.0
dem_200_newyork_192	438.6M	168.1M	160.8	14400.0	398.2M	398.2M	0.0	2884.1	463.2M	249.4M	85.7	14400.0
dem_200_newyork_193	434.2M	179.3M	142.2	14400.0	430.8M	430.8M	0.0	748.1	436.1M	299.4M	45.7	14400.0
dem_200_newyork_194	392.1M	158.3M	147.7	14400.3	386.0M	386.0M	0.0	3346.9	438.4M	250.6M	75.0	14400.0
dem_200_newyork_195	1e+20	224.7M	–	14400.0	1e+20	1e+20	0.0	131.9	1e+20	1e+20	0.0	806.5
dem_200_newyork_196	1e+20	204.8M	–	14400.0	385.0M	385.0M	0.0	1666.9	415.4M	243.5M	70.6	14400.0
dem_200_newyork_197	374.1M	116.1M	222.3	14400.0	350.0M	350.0M	0.0	6283.8	1e+20	168.3M	–	14400.1
dem_200_newyork_198	1e+20	232.1M	–	14400.0	1e+20	1e+20	0.0	171.6	1e+20	1e+20	0.0	7998.9
dem_200_newyork_199	449.1M	170.6M	163.3	14400.0	434.0M	434.0M	0.0	971.4	465.9M	271.8M	71.4	14400.0
dem_200_newyork_200	512.7M	206.1M	148.8	14400.0	493.9M	493.9M	0.0	752.4	494.1M	447.7M	10.4	14400.0
dem_200_newyork_201	518.3M	270.6M	91.5	14400.0	507.0M	507.0M	0.0	735.3	507.0M	507.0M	0.0	11525.4
dem_200_newyork_202	434.3M	182.1M	138.4	14400.0	425.5M	425.1M	0.1	2445.8	456.3M	282.8M	61.4	14400.0
dem_200_newyork_203	387.8M	231.0M	67.8	14400.0	380.2M	380.2M	0.0	1449.2	403.5M	252.8M	59.6	14400.0
dem_200_newyork_204	478.5M	249.5M	91.8	14400.0	477.2M	477.2M	0.0	901.4	488.4M	365.2M	33.7	14400.0
dem_200_newyork_205	420.5M	185.5M	126.6	14400.5	374.8M	374.8M	0.0	4432.8	379.1M	244.8M	54.8	14400.0
dem_200_newyork_206	451.6M	164.7M	174.2	14400.8	441.9M	441.9M	0.0	2199.8	1e+20	271.7M	–	14400.0
dem_200_newyork_207	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	537.9	1e+20	1e+20	0.0	115.0
dem_200_newyork_208	1e+20	297.5M	–	14400.0	1e+20	1e+20	0.0	609.9	1e+20	1e+20	0.0	8319.1
dem_200_newyork_209	442.7M	225.5M	96.3	14400.0	418.2M	418.2M	0.0	2562.7	480.5M	338.1M	42.1	14400.0
dem_200_newyork_210	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	137.9	1e+20	1e+20	0.0	144.4
dem_200_newyork_211	503.9M	414.6M	21.5	14400.0	502.9M	502.9M	0.0	386.4	502.9M	502.9M	0.0	7825.3
dem_200_newyork_212	426.3M	272.5M	56.4	14400.0	424.3M	424.3M	0.0	1471.1	439.2M	353.8M	24.1	14400.0
dem_200_newyork_213	432.8M	165.8M	161.1	14400.0	399.9M	399.9M	0.0	4191.5	425.8M	257.4M	65.4	14400.0
dem_200_newyork_214	436.9M	250.8M	74.2	14400.0	378.7M	378.7M	0.0	2240.4	430.0M	277.1M	55.2	14400.0
dem_200_newyork_215	442.4M	192.1M	130.3	14400.0	439.1M	439.1M	0.0	2593.7	447.4M	363.8M	23.0	14400.0
dem_200_newyork_216	471.0M	295.7M	59.3	14400.0	467.8M	467.8M	0.0	551.9	467.8M	379.1M	23.4	14400.0
dem_200_newyork_217	421.3M	176.8M	138.3	14400.0	416.2M	416.2M	0.0	957.5	452.3M	311.1M	45.4	14400.0
dem_200_newyork_218	397.2M	294.0M	35.1	14400.0	384.2M	382.3M	0.5	7963.0	414.3M	306.8M	35.0	14400.0
dem_200_newyork_219	443.3M	182.0M	143.5	14400.0	439.2M	439.2M	0.0	1149.7	504.0M	332.9M	51.4	14400.0
dem_200_newyork_220	419.2M	184.8M	126.8	14400.0	417.1M	417.1M	0.0	3213.0	432.2M	334.3M	29.3	14400.0
dem_200_newyork_221	460.1M	208.8M	120.4	14400.0	452.7M	452.3M	0.1	1863.8	502.4M	323.0M	55.5	14400.0
dem_200_newyork_222	473.0M	226.8M	108.6	14400.0	450.9M	450.9M	0.0	1544.5	1e+20	335.5M	–	14400.0
dem_200_newyork_223	521.2M	225.4M	131.2	14400.0	482.6M	482.6M	0.0	955.4	1e+20	381.6M	–	14400.0
dem_200_newyork_224	393.2M	151.1M	160.2	14400.6	362.9M	362.9M	0.0	8069.7	398.3M	219.3M	81.6	14400.0
dem_200_newyork_225	393.5M	190.8M	106.2	14400.0	382.1M	381.8M	0.1	1333.8	445.3M	261.7M	70.2	14400.0
dem_200_newyork_226	506.5M	211.4M	139.6	14400.0	500.2M	500.2M	0.0	291.5	500.2M	500.2M	0.0	2665.3
dem_200_newyork_227	451.9M	172.3M	162.3	14400.0	445.9M	445.9M	0.0	2419.1	486.0M	301.5M	61.2	14400.0
dem_200_newyork_228	393.1M	143.0M	174.9	14400.0	376.0M	376.0M	0.0	5275.9	436.1M	222.8M	95.7	14400.0
dem_200_newyork_229	458.3M	176.9M	159.1	14400.0	453.8M	453.8M	0.0	1313.2	460.0M	361.7M	27.2	14400.0
dem_200_newyork_230	421.4M	199.0M	111.7	14400.0	416.3M	416.3M	0.0	1032.4	455.4M	331.3M	37.5	14400.0
dem_200_newyork_231	453.5M	151.3M	199.7	14400.0	440.9M	440.9M	0.0	1887.3	1e+20	258.6M	–	14400.0
dem_200_newyork_232	485.2M	201.2M	141.1	14400.0	462.1M	462.1M	0.0	1822.2	502.8M	329.7M	52.5	14400.0
dem_200_newyork_233	409.6M	329.9M	24.2	14400.0	404.6M	404.6M	0.0	3026.6	404.6M	404.4M	0.0	12653.4
dem_200_newyork_234	414.9M	157.4M	163.6	14400.5	399.7M	399.7M	0.0	2189.6	1e+20	226.4M	–	14400.0
dem_200_newyork_235	1e+20	146.0M	–	14400.0	395.1M	395.1M	0.0	748.9	1e+20	235.4M	–	14400.0
dem_200_newyork_236	440.6M	277.3M	58.9	14400.0	435.0M	434.7M	0.1	2011.1	467.7M	370.4M	26.3	14400.0
dem_200_newyork_237	1e+20	220.0M	–	14400.0	377.6M	377.6M	0.0	3187.6	405.1M	277.6M	45.9	14400.0
dem_200_newyork_238	399.1M	141.0M	183.1	14400.5	395.1M	395.1M	0.0	2830.7	431.5M	259.3M	66.4	14400.0
dem_200_newyork_239	487.6M	252.0M	93.5	14400.0	483.5M	483.5M	0.0	1765.3	501.6M	387.8M	29.3	14400.0
dem_200_newyork_240	492.3M	313.9M	56.9	14400.0	474.4M	474.2M	0.0	2188.9	474.2M	474.2M	0.0	1298.3
dem_200_newyork_241	500.3M	228.2M	119.2	14400.0	499.2M	499.2M	0.0	473.6	499.2M	499.2M	0.0	13196.9
dem_200_newyork_242	466.7M	174.3M	167.7	14400.0	454.7M	454.7M	0.0	1510.1	493.3M	297.9M	65.6	14400.0
dem_200_newyork_243	487.6M	216.2M	125.5	14400.0	477.8M	477.8M	0.0	698.3	488.1M	362.7M	34.6	14400.0
dem_200_newyork_244	488.9M	488.9M	0.0	3134.5	488.9M	488.9M	0.0	568.5	488.9M	488.9M	0.0	1232.1
dem_200_newyork_245	370.7M	180.4M	105.4	14400.0	364.8M	364.8M	0.0	3959.1	428.5M	238.9M	79.3	14400.6
dem_200_newyork_246	397.1M	183.4M	116.5	14400.0	380.1M	380.1M	0.0	2277.2	388.6M	257.7M	50.8	14400.0
dem_200_newyork_247	1e+20	255.3M	–	14400.0	426.0M	426.0M	0.0	998.2	430.7M	363.5M	18.5	14400.0
dem_200_newyork_248	1e+20	1e+20	0.0	5.9	1e+20	1e+20	0.0	112.2	1e+20	1e+20	0.0	3585.2
dem_200_newyork_249	459.7M	198.7M	131.3	14400.0	459.7M	459.7M	0.0	602.1	476.6M	366.5M	30.0	14400.0
dem_200_newyork_250	1e+20	200.8M	–	14400.0	542.8M	542.8M	0.0	410.4	542.8M	542.8M	0.0	11176.4
dem_200_newyork_251	359.3M	167.5M	114.5	14400.1	354.0M	353.9M	0.0	4038.6	399.1M	221.3M	80.3	14400.0
dem_200_newyork_252	450.9M	220.8M	104.2	14400.0	445.7M	445.7M	0.0	1669.2	1e+20	333.0M	–	14400.0
dem_200_newyork_253	383.7M	139.0M	176.0	14400.1	370.8M	370.8M	0.0	818.1	400.9M	204.0M	96.5	14400.0
dem_200_newyork_254	419.7M	194.7M	115.6	14400.0	415.7M	415.7M	0.0	932.4	465.8M	349.6M	33.3	14400.0
dem_200_newyork_255	1e+20	1e+20	0.0	3.3	1e+20	1e+20	0.0	114.2	1e+20	1e+20	0.0	115.7
dem_200_newyork_256	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	116.3	1e+20	1e+20	0.0	73.6
dem_200_newyork_257	1e+20	222.4M	–	14400.0	1e+20	1e+20	0.0	252.8	1e+20	1e+20	0.0	12687.4
dem_200_newyork_258	1e+20	353.0M	–	14400.0	1e+20	1e+20	0.0	203.8	1e+20	1e+20	0.0	155.1
dem_200_newyork_259	512.2M	512.2M	0.0	4588.6	512.2M	512.2M	0.0	189.2	512.2M	512.2M	0.0	218.1
dem_200_newyork_260	483.7M	248.9M	94.3	14400.0	482.8M	482.8M	0.0	326.4	484.5M	413.5M	17.2	14400.0
dem_200_newyork_261	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	187.2	1e+20	1e+20	0.0	311.7

continued on next page

Table A.37: Comparison of discrete models on *New York* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_262	442.5M	182.9M	141.9	14400.0	416.3M	416.3M	0.0	4919.1	449.4M	268.9M	67.1	14400.4
dem_200_newyork_263	427.7M	172.2M	148.3	14400.0	423.2M	423.2M	0.0	1138.5	436.4M	304.2M	43.4	14400.0
dem_200_newyork_264	471.9M	174.8M	169.9	14400.0	434.3M	434.3M	0.0	610.1	443.8M	283.0M	56.8	14400.0
dem_200_newyork_265	417.4M	166.5M	150.7	14444.8	409.5M	409.5M	0.0	6450.8	1e+20	235.5M	–	14400.0
dem_200_newyork_266	460.5M	201.5M	128.5	14400.0	456.1M	455.8M	0.1	1644.9	479.7M	340.3M	41.0	14400.0
dem_200_newyork_267	405.1M	180.1M	124.9	14400.0	377.7M	377.7M	0.0	2998.2	1e+20	229.7M	–	14400.0
dem_200_newyork_268	407.3M	193.8M	110.2	14400.0	396.4M	396.4M	0.0	3318.2	413.7M	277.6M	49.0	14400.0
dem_200_newyork_269	444.9M	214.1M	107.8	14400.0	426.9M	426.9M	0.0	1360.3	436.7M	324.8M	34.4	14400.0
dem_200_newyork_270	447.4M	224.3M	99.4	14400.0	440.9M	440.9M	0.0	662.2	454.9M	336.0M	35.4	14400.0
dem_200_newyork_271	411.0M	237.0M	73.4	14400.0	409.6M	409.6M	0.0	3208.6	424.2M	292.0M	45.3	14400.0
dem_200_newyork_272	438.5M	204.3M	114.7	14400.0	422.8M	422.8M	0.0	3837.5	471.7M	291.5M	61.8	14400.0
dem_200_newyork_273	1e+20	220.5M	–	14400.0	489.8M	489.8M	0.0	130.2	489.8M	489.8M	0.0	4336.6
dem_200_newyork_274	401.9M	129.4M	210.5	14454.9	396.2M	396.2M	0.0	3573.2	1e+20	219.7M	–	14400.1
dem_200_newyork_275	440.0M	166.0M	165.0	14400.0	431.3M	431.3M	0.0	1146.5	1e+20	315.3M	–	14400.0
dem_200_newyork_276	303.1M	169.2M	79.1	14400.3	295.9M	295.9M	0.0	1028.6	319.0M	208.0M	53.4	14400.0
dem_200_newyork_277	529.7M	276.0M	91.9	14400.0	529.7M	529.7M	0.0	234.1	529.7M	529.7M	0.0	148.1
dem_200_newyork_278	313.1M	133.3M	134.8	14400.0	305.3M	305.3M	0.0	4727.8	311.3M	181.3M	71.7	14400.0
dem_200_newyork_279	460.3M	273.4M	68.4	14400.0	441.8M	439.7M	0.5	7787.3	1e+20	357.6M	–	14400.0
dem_200_newyork_280	1e+20	263.0M	–	14400.0	1e+20	1e+20	0.0	84.8	1e+20	1e+20	0.0	92.2
dem_200_newyork_281	429.6M	309.7M	38.7	14400.0	429.6M	428.8M	0.2	2990.8	454.7M	380.5M	19.5	14400.0
dem_200_newyork_282	450.0M	191.4M	135.1	14400.0	438.3M	438.3M	0.0	1068.2	468.6M	289.4M	61.9	14400.0
dem_200_newyork_283	1e+20	224.8M	–	14400.0	434.4M	434.4M	0.0	836.8	434.4M	434.4M	0.0	13912.2
dem_200_newyork_284	1e+20	216.1M	–	14400.0	439.3M	439.3M	0.0	1959.7	468.4M	337.2M	38.9	14400.0
dem_200_newyork_285	490.5M	193.2M	153.8	14400.0	482.7M	482.5M	0.0	670.3	482.7M	482.7M	0.0	8930.5
dem_200_newyork_286	451.6M	321.3M	40.6	14400.0	426.6M	426.6M	0.0	5114.5	1e+20	360.5M	–	14400.0
dem_200_newyork_287	422.9M	222.6M	90.0	14400.0	388.5M	388.5M	0.0	2972.7	438.4M	276.2M	58.7	14400.0
dem_200_newyork_288	1e+20	1e+20	0.0	3.3	1e+20	1e+20	0.0	406.3	1e+20	1e+20	0.0	265.2
dem_200_newyork_289	441.7M	215.0M	105.4	14400.0	397.4M	397.4M	0.0	2734.7	458.8M	284.7M	61.1	14400.0
dem_200_newyork_290	417.3M	197.2M	111.6	14400.4	402.4M	402.4M	0.0	1765.2	451.1M	284.4M	58.6	14400.0
dem_200_newyork_291	372.2M	162.2M	129.4	14400.0	336.4M	336.4M	0.0	3939.2	385.3M	204.7M	88.2	14400.0
dem_200_newyork_292	482.1M	302.3M	59.5	14400.0	478.7M	478.7M	0.0	502.6	1e+20	372.2M	–	14400.0
dem_200_newyork_293	503.8M	202.5M	148.9	14427.9	503.4M	503.4M	0.0	219.2	505.8M	410.7M	23.2	14400.0
dem_200_newyork_294	459.0M	187.9M	144.2	14400.0	447.4M	447.4M	0.0	505.0	475.7M	305.1M	55.9	14400.0
dem_200_newyork_295	448.6M	227.2M	97.4	14400.0	432.2M	432.0M	0.0	1068.7	437.1M	321.2M	36.1	14400.0
dem_200_newyork_296	453.2M	209.4M	116.4	14400.0	453.2M	453.2M	0.0	762.1	456.6M	362.2M	26.1	14400.0
dem_200_newyork_297	460.1M	177.1M	159.7	14400.0	438.5M	438.5M	0.0	1744.8	474.6M	258.0M	83.9	14400.0
dem_200_newyork_298	329.6M	137.7M	139.3	14400.0	324.3M	324.3M	0.0	7157.6	356.0M	184.9M	92.6	14400.0
dem_200_newyork_299	510.5M	275.4M	85.4	14400.0	477.8M	477.8M	0.0	1240.0	484.5M	398.2M	21.7	14400.0
dem_200_newyork_300	480.6M	197.2M	143.7	14400.0	461.3M	461.3M	0.0	1807.1	480.4M	325.1M	47.8	14400.0
dem_200_newyork_301	1e+20	200.5M	–	14400.0	518.0M	518.0M	0.0	486.8	518.7M	431.4M	20.2	14400.0
dem_200_newyork_302	385.1M	211.6M	82.0	14400.0	359.8M	359.8M	0.0	3157.6	396.6M	276.8M	43.3	14400.0
dem_200_newyork_303	428.4M	265.0M	61.7	14400.0	423.7M	422.9M	0.2	1368.8	1e+20	356.5M	–	14400.0
dem_200_newyork_304	457.5M	183.8M	149.0	14400.0	454.7M	454.7M	0.0	1638.8	487.9M	336.3M	45.1	14400.0
dem_200_newyork_305	473.2M	240.9M	96.4	14400.0	464.0M	464.0M	0.0	2640.8	1e+20	320.3M	–	14400.0
dem_200_newyork_306	441.1M	224.6M	96.4	14400.0	437.6M	437.6M	0.0	1307.5	490.3M	284.5M	72.3	14400.0
dem_200_newyork_307	403.5M	219.7M	83.6	14400.0	386.8M	386.8M	0.0	1832.5	399.6M	304.5M	31.2	14400.0
dem_200_newyork_308	347.9M	151.8M	129.2	14400.0	339.4M	339.1M	0.1	5764.4	1e+20	191.0M	–	14400.0
dem_200_newyork_309	427.0M	183.8M	132.4	14400.0	419.4M	419.4M	0.0	1339.0	1e+20	307.5M	–	14400.0
dem_200_newyork_310	486.2M	197.6M	146.1	14400.0	476.2M	476.2M	0.0	1519.7	483.0M	355.3M	35.9	14400.0
dem_200_newyork_311	1e+20	185.2M	–	14400.0	1e+20	1e+20	0.0	223.6	1e+20	1e+20	0.0	12870.5
dem_200_newyork_312	441.5M	178.2M	147.7	14400.0	423.2M	423.2M	0.0	1854.7	445.5M	280.1M	59.1	14400.0
dem_200_newyork_313	457.6M	251.6M	81.9	14400.0	450.0M	449.8M	0.0	1355.7	450.7M	369.0M	22.2	14400.0
dem_200_newyork_314	444.2M	343.9M	29.2	14400.0	432.0M	432.0M	0.0	1661.9	457.5M	365.3M	25.2	14400.0
dem_200_newyork_315	1e+20	238.9M	–	14400.0	1e+20	1e+20	0.0	147.2	1e+20	1e+20	0.0	3304.5
dem_200_newyork_316	1e+20	446.3M	–	14400.0	1e+20	1e+20	0.0	115.7	1e+20	1e+20	0.0	185.5
dem_200_newyork_317	1e+20	453.5M	–	14400.0	1e+20	1e+20	0.0	171.7	1e+20	1e+20	0.0	142.6
dem_200_newyork_318	388.2M	212.8M	82.4	14400.0	381.7M	381.7M	0.0	3487.6	413.2M	281.2M	46.9	14400.0
dem_200_newyork_319	449.0M	228.9M	96.2	14400.0	406.0M	406.0M	0.0	3279.8	514.8M	304.2M	69.2	14400.1
dem_200_newyork_320	1e+20	1e+20	0.0	10864.5	1e+20	1e+20	0.0	71.0	1e+20	1e+20	0.0	8562.5
dem_200_newyork_321	453.3M	176.8M	156.4	14400.0	437.0M	437.0M	0.0	1466.0	448.4M	286.5M	56.5	14400.0
dem_200_newyork_322	1e+20	230.1M	–	14400.0	528.3M	528.3M	0.0	302.3	528.3M	528.3M	0.0	355.8
dem_200_newyork_323	435.1M	225.1M	93.2	14425.5	432.2M	432.2M	0.0	602.6	1e+20	320.2M	–	14400.0
dem_200_newyork_324	1e+20	321.5M	–	14400.0	1e+20	1e+20	0.0	264.0	1e+20	1e+20	0.0	201.6
dem_200_newyork_325	1e+20	207.1M	–	14400.0	1e+20	1e+20	0.0	209.2	1e+20	466.3M	–	14400.0
dem_200_newyork_326	434.2M	214.3M	102.6	14400.0	426.0M	426.0M	0.0	947.5	426.0M	386.4M	10.2	14400.0
dem_200_newyork_327	430.1M	159.2M	170.2	14400.0	426.9M	426.5M	0.1	4076.9	478.7M	284.3M	68.4	14400.0
dem_200_newyork_328	501.0M	207.6M	141.3	14400.0	494.2M	494.2M	0.0	576.8	494.2M	415.8M	18.8	14400.0
dem_200_newyork_329	1e+20	1e+20	0.0	65.3	1e+20	1e+20	0.0	266.3	1e+20	1e+20	0.0	587.6
dem_200_newyork_330	431.4M	206.2M	109.3	14400.0	411.0M	411.0M	0.0	8194.1	447.5M	297.7M	50.3	14400.0
dem_200_newyork_331	554.7M	361.2M	53.6	14400.0	532.9M	532.6M	0.1	859.9	532.9M	532.9M	0.0	9172.2
dem_200_newyork_332	478.8M	217.1M	120.5	14400.0	478.1M	478.1M	0.0	203.1	478.1M	478.1M	0.0	354.9
dem_200_newyork_333	509.7M	408.0M	24.9	14400.0	509.7M	509.7M	0.0	342.6	509.7M	460.0M	10.8	14400.0
dem_200_newyork_334	410.3M	163.1M	151.5	14400.0	394.7M	394.7M	0.0	2784.6	408.7M	284.3M	43.8	14400.0
dem_200_newyork_335	436.9M	183.8M	137.7	14400.0	436.2M	436.2M	0.0	1474.5	448.0M	309.6M	44.7	14400.0
dem_200_newyork_336	384.5M	215.2M	78.7	14400.0	382.4M	382.4M	0.0	2697.8	431.1M	270.4M	59.4	14400.0
dem_200_newyork_337	437.8M	183.5M	138.6	14400.0	435.6M	435.6M	0.0	2278.6	445.2M	337.9M	31.8	14400.0
dem_200_newyork_338	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	135.8	1e+20	1e+20	0.0	53.4
dem_200_newyork_339	460.0M	199.3M	130.9	14400.0	437.2M	437.2M	0.0	2995.4	1e+20	295.4M	–	14400.0

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Table A.37: Comparison of discrete models on *New York* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_340	1e+20	1e+20	0.0	2.7	1e+20	1e+20	0.0	90.2	1e+20	1e+20	0.0	129.8
dem_200_newyork_341	491.1M	214.4M	129.0	14400.0	448.2M	448.2M	0.0	1194.7	1e+20	275.5M	–	14400.0
dem_200_newyork_342	1e+20	197.1M	–	14400.0	524.8M	524.8M	0.0	279.0	1e+20	415.9M	–	14400.0
dem_200_newyork_343	461.7M	207.8M	122.2	14400.0	449.2M	449.2M	0.0	1319.5	462.0M	308.8M	49.6	14400.0
dem_200_newyork_344	477.8M	185.4M	157.6	14400.0	462.8M	462.8M	0.0	850.6	462.8M	462.8M	0.0	9132.9
dem_200_newyork_345	502.6M	254.0M	97.9	14400.0	502.6M	502.6M	0.0	526.1	502.6M	502.6M	0.0	9962.5
dem_200_newyork_346	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	184.9	1e+20	1e+20	0.0	423.7
dem_200_newyork_347	378.3M	145.6M	159.8	14400.0	361.3M	361.3M	0.0	2754.6	1e+20	210.1M	–	14400.0
dem_200_newyork_348	492.6M	417.9M	17.9	14400.0	483.9M	483.9M	0.0	366.4	483.9M	483.9M	0.0	637.8
dem_200_newyork_349	488.3M	223.1M	118.8	14400.0	483.2M	483.2M	0.0	964.1	484.2M	375.6M	28.9	14400.0
dem_200_newyork_350	374.4M	181.8M	105.9	14400.0	371.2M	371.1M	0.0	2003.5	403.6M	228.4M	76.7	14400.0
dem_200_newyork_351	387.8M	137.5M	182.0	14400.0	377.4M	377.4M	0.0	1585.5	1e+20	214.5M	–	14400.0
dem_200_newyork_352	1e+20	231.0M	–	14400.0	524.3M	524.3M	0.0	362.1	524.3M	524.3M	0.0	5907.5
dem_200_newyork_353	503.5M	203.7M	147.2	14400.0	493.0M	493.0M	0.0	219.8	493.0M	493.0M	0.0	5000.1
dem_200_newyork_354	1e+20	223.6M	–	14400.3	429.8M	429.8M	0.0	3508.4	482.8M	333.5M	44.7	14400.0
dem_200_newyork_355	1e+20	1e+20	0.0	6870.5	1e+20	1e+20	0.0	135.6	1e+20	1e+20	0.0	103.2
dem_200_newyork_356	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	78.3	1e+20	1e+20	0.0	53.1
dem_200_newyork_357	396.6M	187.5M	111.5	14434.6	394.2M	394.2M	0.0	5950.9	401.8M	267.7M	50.1	14400.0
dem_200_newyork_358	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	103.5	1e+20	1e+20	0.0	314.7
dem_200_newyork_359	474.3M	201.9M	134.9	14400.0	439.2M	439.2M	0.0	1240.7	441.1M	341.8M	29.1	14400.0
dem_200_newyork_360	458.8M	255.4M	79.7	14400.0	452.0M	452.0M	0.0	646.2	458.5M	358.1M	28.0	14400.0
dem_200_newyork_361	418.1M	152.7M	173.7	14400.0	404.7M	404.5M	0.0	1407.1	1e+20	237.0M	–	14400.0
dem_200_newyork_362	1e+20	234.3M	–	14400.0	470.2M	470.2M	0.0	504.8	470.2M	470.2M	0.0	334.5
dem_200_newyork_363	473.0M	236.2M	100.2	14400.0	459.8M	459.7M	0.0	1989.0	459.7M	459.7M	0.0	3104.7
dem_200_newyork_364	531.9M	480.6M	10.7	14400.0	531.9M	531.9M	0.0	150.1	531.9M	531.9M	0.0	754.3
dem_200_newyork_365	422.4M	169.5M	149.3	14400.0	398.1M	398.1M	0.0	4492.0	409.6M	262.2M	56.2	14400.0
dem_200_newyork_366	455.4M	187.2M	143.2	14441.4	436.6M	436.6M	0.0	651.8	456.0M	305.7M	49.2	14400.0
dem_200_newyork_367	472.7M	225.4M	109.7	14400.0	467.6M	467.6M	0.0	874.5	1e+20	318.0M	–	14400.2
dem_200_newyork_368	486.3M	343.7M	41.5	14400.0	470.9M	470.7M	0.0	2113.9	1e+20	388.1M	–	14400.0
dem_200_newyork_369	443.8M	281.6M	57.6	14400.0	439.9M	439.9M	0.0	1211.6	439.9M	379.2M	16.0	14400.0
dem_200_newyork_370	1e+20	370.5M	–	14400.0	1e+20	1e+20	0.0	81.0	1e+20	1e+20	0.0	706.3
dem_200_newyork_371	421.5M	184.3M	128.7	14400.2	408.9M	408.8M	0.0	2194.4	1e+20	259.4M	–	14400.0
dem_200_newyork_372	1e+20	253.2M	–	14400.0	396.1M	396.1M	0.0	2723.9	397.7M	319.8M	24.4	14400.0
dem_200_newyork_373	426.5M	149.5M	185.3	14400.0	398.4M	398.4M	0.0	4934.4	428.0M	241.4M	77.3	14400.0
dem_200_newyork_374	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	416.7	1e+20	1e+20	0.0	69.8
dem_200_newyork_375	470.3M	214.4M	119.3	14400.0	453.2M	453.2M	0.0	932.6	465.6M	334.2M	39.3	14400.0
dem_200_newyork_376	425.5M	179.2M	137.4	14400.0	422.0M	422.0M	0.0	1996.0	466.2M	294.0M	58.6	14400.0
dem_200_newyork_377	350.1M	117.1M	199.1	14400.0	328.4M	328.4M	0.0	2803.1	1e+20	162.6M	–	14400.6
dem_200_newyork_378	419.5M	149.6M	180.4	14400.0	418.2M	418.2M	0.0	3368.7	465.8M	241.8M	92.7	14400.0
dem_200_newyork_379	406.0M	170.7M	137.8	14400.7	395.6M	395.6M	0.0	1919.2	409.9M	259.2M	58.2	14400.0
dem_200_newyork_380	436.3M	179.1M	143.6	14400.0	436.3M	436.3M	0.0	1271.7	446.5M	304.1M	46.8	14400.0
dem_200_newyork_381	337.7M	145.2M	132.5	14400.0	322.2M	322.2M	0.0	7451.0	398.6M	197.9M	101.4	14400.0
dem_200_newyork_382	486.1M	314.3M	54.7	14400.0	486.1M	486.1M	0.0	288.4	486.1M	486.1M	0.0	2264.7
dem_200_newyork_383	453.4M	194.1M	133.6	14400.0	446.8M	446.8M	0.0	1328.5	479.8M	329.7M	45.5	14400.0
dem_200_newyork_384	1e+20	218.6M	–	14400.0	1e+20	1e+20	0.0	183.4	1e+20	1e+20	0.0	12352.1
dem_200_newyork_385	423.3M	191.0M	121.6	14443.9	403.1M	403.1M	0.0	807.0	457.1M	279.9M	63.3	14400.0
dem_200_newyork_386	402.8M	182.9M	120.2	14400.0	391.6M	391.6M	0.0	4475.1	417.9M	238.2M	75.4	14400.0
dem_200_newyork_387	449.3M	203.7M	120.6	14400.0	438.8M	438.8M	0.0	1515.1	478.0M	338.6M	41.1	14400.0
dem_200_newyork_388	344.1M	135.4M	154.2	14400.0	313.2M	313.2M	0.0	3907.6	372.6M	189.1M	97.0	14400.1
dem_200_newyork_389	431.1M	221.2M	94.9	14400.0	426.6M	425.9M	0.2	1237.8	473.2M	311.3M	52.0	14400.0
dem_200_newyork_390	1e+20	399.5M	–	14400.0	1e+20	1e+20	0.0	242.6	1e+20	1e+20	0.0	164.7
dem_200_newyork_391	396.7M	179.2M	121.4	14400.0	368.0M	368.0M	0.0	8618.1	419.2M	243.9M	71.8	14400.0
dem_200_newyork_392	487.0M	211.3M	130.4	14400.0	476.1M	476.1M	0.0	804.1	477.5M	369.2M	29.3	14400.0
dem_200_newyork_393	1e+20	223.9M	–	14400.0	497.5M	497.5M	0.0	254.2	497.5M	497.5M	0.0	7853.2
dem_200_newyork_394	411.4M	123.5M	233.2	14400.3	394.8M	394.8M	0.0	5393.6	1e+20	200.7M	–	14400.0
dem_200_newyork_395	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	92.8	1e+20	1e+20	0.0	1632.0
dem_200_newyork_396	476.1M	199.5M	138.6	14400.0	440.6M	440.6M	0.0	6038.6	486.4M	288.4M	68.6	14400.0
dem_200_newyork_397	523.4M	523.4M	0.0	177.3	523.4M	523.4M	0.0	333.6	523.4M	523.4M	0.0	303.3
dem_200_newyork_398	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	92.1	1e+20	1e+20	0.0	61.2
dem_200_newyork_399	344.7M	123.1M	180.0	14400.0	329.7M	329.6M	0.0	4695.5	346.9M	174.4M	98.9	14400.0
dem_200_newyork_400	418.6M	205.4M	103.8	14400.0	417.5M	417.5M	0.0	1238.9	465.0M	280.4M	65.8	14400.0
dem_200_newyork_401	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	85.7	1e+20	1e+20	0.0	3244.7
dem_200_newyork_402	1e+20	235.0M	–	14400.0	423.9M	423.9M	0.0	1563.7	440.6M	288.2M	52.9	14400.0
dem_200_newyork_403	416.5M	183.4M	127.1	14400.0	400.8M	400.8M	0.0	1179.0	430.4M	286.9M	50.0	14400.0
dem_200_newyork_404	1e+20	1e+20	0.0	4764.0	1e+20	1e+20	0.0	110.8	1e+20	1e+20	0.0	147.5
dem_200_newyork_405	1e+20	208.1M	–	14400.0	407.3M	407.3M	0.0	3916.7	407.8M	298.1M	36.8	14400.0
dem_200_newyork_406	1e+20	213.6M	–	14400.0	1e+20	1e+20	0.0	251.7	1e+20	1e+20	0.0	9987.1
dem_200_newyork_407	388.8M	190.6M	103.9	14400.0	386.2M	386.2M	0.0	2624.9	440.2M	270.4M	62.8	14400.0
dem_200_newyork_408	485.7M	318.1M	52.7	14442.4	485.7M	485.7M	0.0	563.3	485.7M	485.7M	0.0	512.0
dem_200_newyork_409	1e+20	444.2M	–	14400.0	1e+20	1e+20	0.0	76.2	1e+20	1e+20	0.0	220.4
dem_200_newyork_410	442.4M	194.4M	127.5	14425.1	439.6M	439.6M	0.0	1419.7	455.9M	347.4M	31.2	14400.0
dem_200_newyork_411	501.3M	236.9M	111.6	14400.0	496.2M	496.2M	0.0	586.0	496.2M	414.5M	19.7	14400.0
dem_200_newyork_412	463.5M	188.9M	145.3	14400.0	452.8M	452.8M	0.0	950.6	469.1M	341.9M	37.2	14400.0
dem_200_newyork_413	445.0M	243.0M	83.1	14400.0	443.9M	443.9M	0.0	867.9	460.1M	409.4M	12.4	14400.0
dem_200_newyork_414	476.6M	226.6M	110.3	14400.0	474.0M	474.0M	0.0	853.8	488.7M	314.0M	55.7	14400.0
dem_200_newyork_415	509.0M	264.2M	92.7	14400.0	499.0M	499.0M	0.0	503.4	499.0M	407.5M	22.5	14400.0
dem_200_newyork_416	1e+20	193.8M	–	14400.0	1e+20	1e+20	0.0	152.0	1e+20	1e+20	0.0	1158.7
dem_200_newyork_417	1e+20	316.7M	–	14400.0	1e+20	1e+20	0.0	88.0	1e+20	1e+20	0.0	71.5

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Table A.37: Comparison of discrete models on *New York* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_418	448.2M	232.6M	92.7	14400.0	444.0M	444.0M	0.0	1677.8	457.5M	371.9M	23.0	14400.0
dem_200_newyork_419	1e+20	209.7M	–	14400.0	1e+20	1e+20	0.0	426.8	1e+20	421.7M	–	14400.0
dem_200_newyork_420	439.1M	201.9M	117.4	14400.0	420.8M	420.8M	0.0	2511.3	498.7M	297.5M	67.6	14400.0
dem_200_newyork_421	398.5M	149.8M	166.1	14400.0	372.0M	371.9M	0.0	4375.5	383.5M	243.2M	57.7	14400.0
dem_200_newyork_422	346.6M	131.2M	164.1	14400.0	333.1M	333.1M	0.0	9198.5	392.1M	189.8M	106.5	14400.0
dem_200_newyork_423	490.9M	255.0M	92.5	14400.0	481.1M	481.1M	0.0	2395.2	511.4M	388.2M	31.7	14400.0
dem_200_newyork_424	1e+20	266.3M	–	14400.0	1e+20	1e+20	0.0	340.7	1e+20	1e+20	0.0	191.3
dem_200_newyork_425	424.6M	328.0M	29.4	14400.3	425.9M	407.6M	4.5	14400.0	423.7M	360.3M	17.6	14400.0
dem_200_newyork_426	439.5M	226.3M	94.1	14400.0	413.7M	413.7M	0.0	700.2	430.5M	275.0M	56.6	14400.0
dem_200_newyork_427	452.1M	174.2M	159.5	14443.1	417.3M	417.3M	0.0	3713.3	1e+20	245.4M	–	14400.0
dem_200_newyork_428	434.1M	288.8M	50.3	14419.6	432.4M	432.4M	0.0	253.2	438.5M	362.2M	21.1	14400.0
dem_200_newyork_429	478.5M	202.8M	135.9	14400.0	471.5M	471.5M	0.0	1690.0	495.5M	326.4M	51.8	14400.0
dem_200_newyork_430	415.3M	227.0M	83.0	14400.1	396.9M	396.9M	0.0	4584.6	427.1M	301.2M	41.8	14400.0
dem_200_newyork_431	486.1M	319.9M	52.0	14400.0	486.1M	486.1M	0.0	234.9	491.3M	360.1M	36.4	14400.0
dem_200_newyork_432	466.0M	206.6M	125.6	14400.2	455.6M	455.6M	0.0	1511.2	455.6M	455.6M	0.0	9995.1
dem_200_newyork_433	1e+20	236.2M	–	14400.0	1e+20	1e+20	0.0	86.0	1e+20	1e+20	0.0	3368.0
dem_200_newyork_434	426.0M	277.4M	53.6	14400.0	424.7M	424.7M	0.0	2101.9	463.8M	344.7M	34.6	14400.0
dem_200_newyork_435	425.2M	165.4M	157.1	14400.0	418.7M	418.7M	0.0	1880.7	464.6M	255.7M	81.7	14400.0
dem_200_newyork_436	1e+20	257.8M	–	14400.0	1e+20	1e+20	0.0	294.5	1e+20	448.8M	–	14400.0
dem_200_newyork_437	1e+20	266.8M	–	14400.0	425.0M	425.0M	0.0	1863.6	482.2M	343.3M	40.5	14400.0
dem_200_newyork_438	452.2M	177.1M	155.4	14400.0	443.9M	443.9M	0.0	2928.8	517.4M	325.2M	59.1	14400.0
dem_200_newyork_439	427.3M	211.9M	101.7	14400.0	362.9M	362.9M	0.0	4233.5	424.6M	254.3M	67.0	14400.0
dem_200_newyork_440	478.0M	260.8M	83.3	14400.0	460.8M	460.2M	0.1	1937.3	481.9M	380.1M	26.8	14400.0
dem_200_newyork_441	1e+20	240.2M	–	14400.0	489.6M	489.6M	0.0	243.4	490.1M	428.0M	14.5	14400.0
dem_200_newyork_442	479.1M	197.4M	142.7	14400.8	447.5M	447.5M	0.0	990.8	477.1M	280.1M	70.3	14400.0
dem_200_newyork_443	442.2M	164.5M	168.8	14400.0	419.4M	419.4M	0.0	1020.3	1e+20	241.3M	–	14400.0
dem_200_newyork_444	485.8M	252.6M	92.3	14418.3	458.0M	458.0M	0.0	748.3	501.0M	393.4M	27.4	14400.0
dem_200_newyork_445	406.9M	184.9M	120.1	14400.0	379.1M	379.1M	0.0	7548.7	442.7M	235.9M	87.6	14400.0
dem_200_newyork_446	415.3M	177.1M	134.4	14400.0	411.6M	411.6M	0.0	3652.6	1e+20	236.9M	–	14400.0
dem_200_newyork_447	433.2M	197.0M	119.9	14439.3	426.3M	426.3M	0.0	1086.2	448.5M	292.5M	53.3	14400.0
dem_200_newyork_448	1e+20	243.3M	–	14400.0	503.1M	502.8M	0.1	144.9	502.8M	502.8M	0.0	3943.2
dem_200_newyork_449	441.4M	162.9M	170.9	14400.1	425.3M	425.3M	0.0	1310.1	441.3M	250.1M	76.5	14400.0
dem_200_newyork_450	387.5M	209.8M	84.7	14400.0	383.2M	383.2M	0.0	4927.7	1e+20	264.4M	–	14400.0
dem_200_newyork_451	549.9M	549.9M	0.0	1137.7	549.9M	549.9M	0.0	1197.2	549.9M	549.9M	0.0	482.9
dem_200_newyork_452	503.8M	253.6M	98.7	14400.0	500.2M	500.2M	0.0	438.3	505.1M	412.4M	22.5	14400.0
dem_200_newyork_453	407.8M	192.1M	112.3	14400.0	404.7M	404.7M	0.0	1142.6	414.8M	288.4M	43.8	14400.0
dem_200_newyork_454	405.2M	191.8M	111.2	14400.0	395.7M	395.7M	0.0	1530.6	1e+20	274.4M	–	14400.0
dem_200_newyork_455	1e+20	330.0M	–	14400.0	1e+20	1e+20	0.0	201.7	1e+20	1e+20	0.0	6876.2
dem_200_newyork_456	497.8M	226.0M	120.3	14400.0	496.1M	496.1M	0.0	299.7	496.1M	496.1M	0.0	294.4
dem_200_newyork_457	466.2M	200.0M	133.1	14400.0	462.0M	462.0M	0.0	901.7	479.1M	360.6M	32.9	14400.0
dem_200_newyork_458	451.6M	217.7M	107.4	14400.0	438.1M	437.9M	0.1	993.2	479.0M	343.3M	39.5	14400.0
dem_200_newyork_459	1e+20	246.5M	–	14400.0	1e+20	1e+20	0.0	173.0	1e+20	1e+20	0.0	280.2
dem_200_newyork_460	424.1M	156.1M	171.7	14400.4	402.9M	402.9M	0.0	2067.6	1e+20	226.5M	–	14400.0
dem_200_newyork_461	480.7M	211.3M	127.5	14400.0	458.5M	458.5M	0.0	892.1	497.9M	367.1M	35.6	14400.0
dem_200_newyork_462	1e+20	419.2M	–	14400.0	497.3M	497.3M	0.0	535.4	497.6M	456.9M	8.9	14400.0
dem_200_newyork_463	417.1M	245.9M	69.6	14400.2	408.5M	408.5M	0.0	3645.2	469.6M	311.3M	50.8	14400.0
dem_200_newyork_464	374.6M	242.4M	54.6	14454.3	362.7M	362.7M	0.0	2113.1	376.3M	260.2M	44.7	14400.0
dem_200_newyork_465	460.7M	214.9M	114.4	14400.0	457.1M	457.1M	0.0	1951.1	460.9M	370.5M	24.4	14400.0
dem_200_newyork_466	439.8M	163.7M	168.7	14400.0	393.2M	393.2M	0.0	3108.6	1e+20	253.7M	–	14400.0
dem_200_newyork_467	406.0M	150.6M	169.6	14400.0	387.0M	387.0M	0.0	3482.1	402.9M	257.4M	56.5	14400.0
dem_200_newyork_468	1e+20	280.1M	–	14400.0	1e+20	1e+20	0.0	261.3	1e+20	1e+20	0.0	9778.3
dem_200_newyork_469	494.3M	284.4M	73.8	14400.0	487.7M	487.7M	0.0	681.6	487.7M	487.7M	0.0	4737.4
dem_200_newyork_470	1e+20	241.5M	–	14400.0	457.2M	457.2M	0.0	456.0	470.1M	375.4M	25.2	14400.0
dem_200_newyork_471	515.7M	214.2M	140.8	14400.2	514.0M	514.0M	0.0	167.9	514.9M	424.8M	21.2	14400.0
dem_200_newyork_472	489.8M	211.1M	132.0	14400.0	477.0M	477.0M	0.0	1566.5	494.0M	321.8M	53.5	14400.0
dem_200_newyork_473	426.2M	306.6M	39.0	14400.0	425.5M	425.2M	0.1	9598.1	428.9M	366.9M	16.9	14400.0
dem_200_newyork_474	1e+20	287.7M	–	14400.0	1e+20	1e+20	0.0	253.9	1e+20	1e+20	0.0	1710.0
dem_200_newyork_475	399.3M	157.3M	153.8	14400.0	391.7M	391.7M	0.0	2175.8	431.5M	221.8M	94.6	14400.0
dem_200_newyork_476	1e+20	313.0M	–	14400.0	1e+20	1e+20	0.0	146.3	1e+20	1e+20	0.0	8352.8
dem_200_newyork_477	519.1M	519.1M	0.0	17.4	519.1M	519.1M	0.0	1099.0	519.1M	519.1M	0.0	3861.7
dem_200_newyork_478	389.5M	165.1M	135.9	14400.0	382.8M	382.8M	0.0	621.9	416.8M	239.0M	74.4	14400.0
dem_200_newyork_479	485.2M	349.7M	38.8	14400.0	485.2M	485.2M	0.0	207.6	485.2M	485.2M	0.0	12208.1
dem_200_newyork_480	1e+20	216.6M	–	14400.0	486.9M	486.9M	0.0	613.7	1e+20	352.6M	–	14400.0
dem_200_newyork_481	483.5M	214.6M	125.3	14400.0	473.8M	473.8M	0.0	396.7	473.8M	473.8M	0.0	9874.4
dem_200_newyork_482	417.8M	132.9M	214.5	14446.9	409.4M	409.4M	0.0	1062.8	427.4M	230.8M	85.2	14400.0
dem_200_newyork_483	423.5M	190.0M	122.9	14400.0	381.9M	381.9M	0.0	5682.9	398.3M	239.1M	66.6	14400.0
dem_200_newyork_484	415.7M	188.3M	120.8	14400.0	396.9M	396.9M	0.0	2202.0	414.8M	280.3M	48.0	14400.0
dem_200_newyork_485	455.5M	239.2M	90.5	14400.0	455.2M	455.2M	0.0	1366.1	500.2M	339.5M	47.4	14400.0
dem_200_newyork_486	1e+20	1e+20	0.0	1065.8	1e+20	1e+20	0.0	187.6	1e+20	1e+20	0.0	56.6
dem_200_newyork_487	1e+20	168.5M	–	14400.0	415.3M	415.3M	0.0	1588.8	1e+20	260.6M	–	14400.0
dem_200_newyork_488	507.4M	210.8M	140.7	14400.0	475.9M	475.9M	0.0	2574.0	1e+20	327.4M	–	14400.0
dem_200_newyork_489	325.5M	117.6M	176.8	14400.0	313.0M	313.0M	0.0	6291.5	1e+20	151.8M	–	14400.0
dem_200_newyork_490	444.1M	153.1M	190.1	14400.0	417.5M	417.5M	0.0	1986.0	437.8M	284.5M	53.9	14400.0
dem_200_newyork_491	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	118.5	1e+20	1e+20	0.0	86.3
dem_200_newyork_492	496.9M	177.4M	180.1	14400.0	492.7M	492.7M	0.0	743.6	512.5M	397.7M	28.9	14400.0
dem_200_newyork_493	427.6M	155.2M	175.6	14400.0	427.6M	427.6M	0.0	1910.6	1e+20	265.3M	–	14400.0
dem_200_newyork_494	1e+20	175.6M	–	14400.0	409.5M	409.5M	0.0	1290.4	1e+20	276.4M	–	14400.0
dem_200_newyork_495	424.5M	162.8M	160.8	14400.4	397.4M	397.4M	0.0	1204.2	458.7M	242.6M	89.0	14400.2

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Table A.37: Comparison of discrete models on *New York* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_496	414.0M	188.7M	119.4	14434.9	399.4M	399.4M	0.0	979.8	1e+20	258.7M	–	14400.0
dem_200_newyork_497	448.1M	306.7M	46.1	14400.0	434.5M	434.5M	0.0	655.4	441.3M	370.2M	19.2	14400.0
dem_200_newyork_498	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	104.3	1e+20	1e+20	0.0	36.9
dem_200_newyork_499	418.6M	209.9M	99.4	14400.0	417.9M	417.9M	0.0	1386.9	1e+20	316.2M	–	14400.0
dem_200_newyork_500	377.8M	146.2M	158.3	14400.1	354.1M	354.1M	0.0	8306.0	393.1M	212.2M	85.2	14400.0

Table A.38: Detailed results of the split-pipe models on *New York* with $\mathcal{B} = 200$ as summarized in Figure 3.17b and Table 3.4e. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_1	382.9M	382.6M	0.1	1.7	382.9M	382.6M	0.1	3.0
dem_200_newyork_2	405.6M	405.6M	0.0	1.5	405.6M	405.6M	0.0	2.7
dem_200_newyork_3	374.2M	373.8M	0.1	2.2	374.0M	373.6M	0.1	3.2
dem_200_newyork_4	422.6M	422.5M	0.0	1.1	422.6M	422.5M	0.0	2.3
dem_200_newyork_5	416.8M	416.4M	0.1	1.0	416.8M	416.4M	0.1	1.8
dem_200_newyork_6	415.2M	414.9M	0.1	1.7	415.2M	414.9M	0.1	2.3
dem_200_newyork_7	384.8M	384.6M	0.1	0.9	384.8M	384.4M	0.1	2.4
dem_200_newyork_8	368.2M	367.8M	0.1	2.0	368.2M	367.8M	0.1	3.0
dem_200_newyork_9	457.4M	457.1M	0.1	1.0	457.4M	457.1M	0.1	2.0
dem_200_newyork_10	346.8M	346.8M	0.0	1.1	346.8M	346.8M	0.0	2.2
dem_200_newyork_11	439.4M	439.0M	0.1	1.1	439.3M	438.9M	0.1	2.4
dem_200_newyork_12	395.4M	395.1M	0.1	1.1	395.4M	395.1M	0.1	2.2
dem_200_newyork_13	456.9M	456.6M	0.1	0.9	456.9M	456.5M	0.1	1.6
dem_200_newyork_14	388.3M	388.0M	0.1	2.9	388.3M	388.0M	0.1	2.6
dem_200_newyork_15	448.2M	448.0M	0.0	1.1	448.3M	448.1M	0.1	2.6
dem_200_newyork_16	364.4M	364.2M	0.0	1.2	364.4M	364.1M	0.1	2.1
dem_200_newyork_17	381.0M	380.6M	0.1	1.1	381.0M	380.8M	0.0	2.1
dem_200_newyork_18	485.0M	484.6M	0.1	1.2	485.0M	484.6M	0.1	2.2
dem_200_newyork_19	340.3M	339.9M	0.1	1.9	340.3M	339.9M	0.1	3.1
dem_200_newyork_20	441.1M	441.0M	0.0	1.1	441.1M	440.8M	0.1	2.1
dem_200_newyork_21	403.7M	403.4M	0.1	1.3	403.7M	403.3M	0.1	2.2
dem_200_newyork_22	323.9M	323.6M	0.1	2.9	323.9M	323.6M	0.1	3.2
dem_200_newyork_23	453.8M	453.4M	0.1	1.4	453.8M	453.7M	0.0	3.4
dem_200_newyork_24	445.0M	444.6M	0.1	1.1	445.0M	444.6M	0.1	2.3
dem_200_newyork_25	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	9.2
dem_200_newyork_26	470.1M	469.7M	0.1	1.5	470.1M	469.9M	0.0	2.8
dem_200_newyork_27	451.6M	451.2M	0.1	1.0	451.6M	451.3M	0.1	2.5
dem_200_newyork_28	1e+20	1e+20	0.0	5.1	1e+20	1e+20	0.0	7.4
dem_200_newyork_29	477.0M	476.6M	0.1	1.1	477.1M	476.7M	0.1	2.5
dem_200_newyork_30	378.3M	378.0M	0.1	2.5	378.3M	378.0M	0.1	2.8
dem_200_newyork_31	425.1M	424.7M	0.1	0.9	425.1M	424.7M	0.1	1.7
dem_200_newyork_32	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	4.2
dem_200_newyork_33	396.9M	396.5M	0.1	1.0	396.9M	396.5M	0.1	2.3
dem_200_newyork_34	433.1M	432.7M	0.1	1.8	433.1M	432.8M	0.0	3.0
dem_200_newyork_35	491.3M	491.3M	0.0	0.8	491.3M	491.2M	0.0	2.2
dem_200_newyork_36	1e+20	1e+20	0.0	8.4	1e+20	1e+20	0.0	4.9
dem_200_newyork_37	427.2M	426.9M	0.1	0.9	427.2M	426.9M	0.1	1.9
dem_200_newyork_38	419.0M	418.6M	0.1	1.4	419.1M	418.7M	0.1	3.1
dem_200_newyork_39	517.8M	517.4M	0.1	0.8	517.8M	517.4M	0.1	2.2
dem_200_newyork_40	458.8M	458.4M	0.1	0.9	458.8M	458.6M	0.0	3.0
dem_200_newyork_41	1e+20	1e+20	0.0	3.3	1e+20	1e+20	0.0	4.6
dem_200_newyork_42	416.5M	416.1M	0.1	1.2	416.5M	416.1M	0.1	1.9
dem_200_newyork_43	466.2M	465.7M	0.1	1.0	466.2M	465.7M	0.1	2.2
dem_200_newyork_44	338.1M	337.9M	0.1	2.1	338.1M	337.8M	0.1	2.0
dem_200_newyork_45	401.8M	401.7M	0.0	1.1	401.8M	401.7M	0.0	2.2
dem_200_newyork_46	423.9M	423.5M	0.1	1.4	423.9M	423.5M	0.1	3.1
dem_200_newyork_47	487.9M	487.5M	0.1	0.7	487.9M	487.5M	0.1	2.2
dem_200_newyork_48	448.1M	447.7M	0.1	1.0	448.9M	448.1M	0.2	2.8
dem_200_newyork_49	400.5M	400.2M	0.1	1.3	400.5M	400.2M	0.1	2.1
dem_200_newyork_50	367.0M	366.7M	0.1	2.0	367.1M	366.7M	0.1	2.9
dem_200_newyork_51	467.6M	467.1M	0.1	1.1	467.6M	467.1M	0.1	1.6
dem_200_newyork_52	383.4M	383.1M	0.1	1.2	383.4M	383.1M	0.1	3.1
dem_200_newyork_53	447.8M	447.7M	0.0	0.8	447.7M	447.3M	0.1	2.0
dem_200_newyork_54	437.0M	436.6M	0.1	1.4	437.0M	436.6M	0.1	2.0
dem_200_newyork_55	449.9M	449.8M	0.0	0.8	449.9M	449.8M	0.0	1.6
dem_200_newyork_56	463.3M	463.0M	0.1	0.9	463.3M	463.0M	0.1	2.6
dem_200_newyork_57	386.0M	385.9M	0.0	1.4	386.0M	385.7M	0.1	2.6
dem_200_newyork_58	492.0M	491.9M	0.0	0.8	492.0M	491.9M	0.0	2.2

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Table A.38: Comparison of split-pipe models on *New York* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_59	336.6M	336.2M	0.1	1.5	336.6M	336.2M	0.1	3.0
dem_200_newyork_60	473.8M	473.4M	0.1	0.9	473.8M	473.3M	0.1	1.7
dem_200_newyork_61	471.0M	470.7M	0.1	1.0	471.0M	470.7M	0.1	1.7
dem_200_newyork_62	378.4M	378.0M	0.1	2.1	378.2M	377.8M	0.1	3.1
dem_200_newyork_63	413.3M	413.1M	0.0	1.8	413.2M	413.1M	0.0	2.7
dem_200_newyork_64	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	8.5
dem_200_newyork_65	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	3.4
dem_200_newyork_66	509.5M	509.0M	0.1	1.1	509.5M	509.0M	0.1	2.0
dem_200_newyork_67	428.0M	427.7M	0.1	1.1	428.0M	427.7M	0.1	2.6
dem_200_newyork_68	467.7M	467.7M	0.0	1.3	467.7M	467.6M	0.0	1.6
dem_200_newyork_69	1e+20	1e+20	0.0	5.2	1e+20	1e+20	0.0	6.2
dem_200_newyork_70	382.8M	382.6M	0.0	1.8	382.7M	382.3M	0.1	2.2
dem_200_newyork_71	441.7M	441.3M	0.1	1.3	441.7M	441.4M	0.1	2.4
dem_200_newyork_72	425.3M	425.0M	0.1	1.2	425.3M	425.0M	0.1	1.8
dem_200_newyork_73	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	8.8
dem_200_newyork_74	364.3M	364.0M	0.1	1.6	364.3M	364.0M	0.1	2.4
dem_200_newyork_75	415.9M	415.6M	0.1	1.1	415.9M	415.8M	0.0	2.0
dem_200_newyork_76	1e+20	1e+20	0.0	5.2	1e+20	1e+20	0.0	5.2
dem_200_newyork_77	417.1M	416.7M	0.1	1.3	417.1M	416.8M	0.1	2.3
dem_200_newyork_78	454.2M	453.8M	0.1	0.8	454.2M	453.8M	0.1	2.0
dem_200_newyork_79	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	3.7
dem_200_newyork_80	498.9M	498.5M	0.1	1.1	498.9M	498.5M	0.1	2.3
dem_200_newyork_81	448.0M	447.6M	0.1	1.0	448.0M	447.7M	0.1	2.6
dem_200_newyork_82	460.1M	459.7M	0.1	1.0	460.1M	459.7M	0.1	2.3
dem_200_newyork_83	387.5M	387.2M	0.1	1.4	387.7M	387.3M	0.1	2.5
dem_200_newyork_84	500.5M	500.0M	0.1	1.1	500.5M	500.3M	0.0	1.5
dem_200_newyork_85	364.2M	363.8M	0.1	2.0	364.3M	364.0M	0.1	3.6
dem_200_newyork_86	522.1M	521.6M	0.1	1.0	522.1M	521.6M	0.1	2.0
dem_200_newyork_87	418.0M	417.6M	0.1	1.3	418.3M	417.8M	0.1	3.2
dem_200_newyork_88	433.5M	433.1M	0.1	1.2	433.5M	433.1M	0.1	2.2
dem_200_newyork_89	408.3M	408.2M	0.0	1.9	408.3M	408.0M	0.1	2.5
dem_200_newyork_90	474.4M	474.1M	0.1	0.8	474.4M	474.1M	0.1	1.7
dem_200_newyork_91	1e+20	1e+20	0.0	7.8	1e+20	1e+20	0.0	4.2
dem_200_newyork_92	458.0M	457.6M	0.1	1.1	457.8M	457.3M	0.1	1.6
dem_200_newyork_93	405.5M	405.1M	0.1	1.6	405.5M	405.1M	0.1	3.6
dem_200_newyork_94	460.9M	460.6M	0.1	1.1	460.9M	460.6M	0.1	1.9
dem_200_newyork_95	372.4M	372.3M	0.0	1.1	372.4M	372.0M	0.1	2.2
dem_200_newyork_96	490.9M	490.5M	0.1	0.9	490.9M	490.5M	0.1	1.7
dem_200_newyork_97	443.3M	442.9M	0.1	0.8	443.3M	443.2M	0.0	2.6
dem_200_newyork_98	465.8M	465.4M	0.1	1.1	465.8M	465.4M	0.1	2.1
dem_200_newyork_99	476.4M	476.1M	0.1	0.8	476.4M	476.3M	0.0	2.0
dem_200_newyork_100	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	6.8
dem_200_newyork_101	502.5M	502.0M	0.1	1.2	502.8M	502.3M	0.1	2.1
dem_200_newyork_102	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	4.3
dem_200_newyork_103	440.0M	439.6M	0.1	1.2	440.0M	439.7M	0.1	2.0
dem_200_newyork_104	444.6M	444.2M	0.1	0.9	444.6M	444.2M	0.1	2.1
dem_200_newyork_105	448.1M	447.8M	0.1	1.0	448.0M	447.7M	0.1	2.9
dem_200_newyork_106	492.5M	492.1M	0.1	1.2	492.5M	492.1M	0.1	2.2
dem_200_newyork_107	515.6M	515.4M	0.0	0.8	515.6M	515.1M	0.1	2.4
dem_200_newyork_108	456.7M	456.3M	0.1	1.1	456.7M	456.6M	0.0	1.8
dem_200_newyork_109	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	5.5
dem_200_newyork_110	394.1M	393.9M	0.1	1.2	394.1M	393.8M	0.1	2.4
dem_200_newyork_111	434.7M	434.3M	0.1	1.0	434.7M	434.2M	0.1	2.6
dem_200_newyork_112	500.7M	500.3M	0.1	0.8	500.7M	500.3M	0.1	2.1
dem_200_newyork_113	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	6.4
dem_200_newyork_114	488.3M	488.1M	0.0	0.9	488.1M	488.1M	0.0	2.0
dem_200_newyork_115	394.8M	394.4M	0.1	1.5	394.8M	394.6M	0.0	2.6
dem_200_newyork_116	455.1M	454.7M	0.1	0.9	455.1M	454.7M	0.1	2.4
dem_200_newyork_117	490.0M	489.5M	0.1	0.8	490.0M	489.5M	0.1	2.2
dem_200_newyork_118	426.0M	425.7M	0.1	1.3	426.0M	425.7M	0.1	2.1
dem_200_newyork_119	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	4.1
dem_200_newyork_120	417.9M	417.5M	0.1	1.4	418.0M	417.6M	0.1	2.3
dem_200_newyork_121	473.8M	473.6M	0.0	0.8	473.8M	473.6M	0.0	2.2
dem_200_newyork_122	424.8M	424.5M	0.1	1.7	424.8M	424.4M	0.1	2.8
dem_200_newyork_123	466.8M	466.3M	0.1	1.0	466.8M	466.4M	0.1	2.0
dem_200_newyork_124	388.5M	388.4M	0.0	1.9	388.5M	388.2M	0.1	2.4
dem_200_newyork_125	471.2M	471.0M	0.0	1.3	471.2M	470.9M	0.1	2.2
dem_200_newyork_126	460.7M	460.4M	0.1	1.8	460.7M	460.3M	0.1	2.2
dem_200_newyork_127	432.3M	431.9M	0.1	1.0	432.3M	431.8M	0.1	1.9
dem_200_newyork_128	374.7M	374.4M	0.1	1.1	374.7M	374.4M	0.1	2.3
dem_200_newyork_129	486.3M	485.9M	0.1	0.7	486.3M	486.1M	0.0	2.5
dem_200_newyork_130	378.6M	378.2M	0.1	1.8	378.6M	378.2M	0.1	3.5
dem_200_newyork_131	425.1M	424.9M	0.0	1.2	425.1M	424.7M	0.1	2.4
dem_200_newyork_132	430.5M	430.1M	0.1	1.1	430.5M	430.1M	0.1	2.2
dem_200_newyork_133	458.2M	457.8M	0.1	1.2	458.2M	457.8M	0.1	2.4
dem_200_newyork_134	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	4.1
dem_200_newyork_135	421.7M	421.3M	0.1	0.7	421.9M	421.6M	0.1	2.3
dem_200_newyork_136	433.4M	433.1M	0.1	1.5	433.4M	433.2M	0.1	2.5

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Table A.38: Comparison of split-pipe models on *New York* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_137	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	7.0
dem_200_newyork_138	458.9M	458.5M	0.1	0.9	458.9M	458.5M	0.1	1.8
dem_200_newyork_139	467.3M	467.0M	0.1	0.7	467.3M	466.9M	0.1	2.5
dem_200_newyork_140	460.6M	460.2M	0.1	0.9	460.6M	460.2M	0.1	2.8
dem_200_newyork_141	443.3M	442.9M	0.1	0.9	443.3M	443.0M	0.1	1.6
dem_200_newyork_142	380.0M	379.7M	0.1	1.2	380.0M	379.7M	0.1	2.4
dem_200_newyork_143	468.9M	468.7M	0.1	0.8	468.9M	468.7M	0.0	2.4
dem_200_newyork_144	441.8M	441.4M	0.1	1.1	441.8M	441.4M	0.1	2.4
dem_200_newyork_145	439.5M	439.1M	0.1	1.5	439.5M	439.1M	0.1	2.5
dem_200_newyork_146	411.0M	410.6M	0.1	1.6	411.0M	410.7M	0.1	2.2
dem_200_newyork_147	471.4M	471.0M	0.1	1.0	471.4M	471.0M	0.1	1.9
dem_200_newyork_148	429.5M	429.1M	0.1	0.9	429.4M	429.0M	0.1	2.5
dem_200_newyork_149	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	3.2
dem_200_newyork_150	488.5M	488.1M	0.1	1.0	488.5M	488.1M	0.1	2.1
dem_200_newyork_151	1e+20	1e+20	0.0	5.1	1e+20	1e+20	0.0	5.4
dem_200_newyork_152	491.4M	491.4M	0.0	1.4	491.4M	491.1M	0.1	1.9
dem_200_newyork_153	436.7M	436.4M	0.1	1.3	436.7M	436.3M	0.1	2.6
dem_200_newyork_154	442.2M	442.0M	0.1	1.2	442.2M	441.9M	0.1	2.8
dem_200_newyork_155	369.3M	369.0M	0.1	2.1	369.3M	369.0M	0.1	2.3
dem_200_newyork_156	356.4M	356.1M	0.1	1.4	356.4M	356.1M	0.1	2.4
dem_200_newyork_157	423.6M	423.5M	0.0	1.5	423.6M	423.2M	0.1	2.8
dem_200_newyork_158	503.3M	503.0M	0.1	1.1	503.3M	503.3M	0.0	2.8
dem_200_newyork_159	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	5.9
dem_200_newyork_160	1e+20	1e+20	0.0	3.4	1e+20	1e+20	0.0	8.5
dem_200_newyork_161	438.5M	438.2M	0.1	1.1	438.5M	438.2M	0.1	2.5
dem_200_newyork_162	465.8M	465.4M	0.1	1.1	465.8M	465.3M	0.1	2.2
dem_200_newyork_163	353.9M	353.6M	0.1	2.0	353.9M	353.6M	0.1	2.7
dem_200_newyork_164	334.1M	333.8M	0.1	1.9	334.1M	333.8M	0.1	2.6
dem_200_newyork_165	473.6M	473.3M	0.1	0.7	473.6M	473.2M	0.1	1.9
dem_200_newyork_166	466.7M	466.4M	0.1	0.9	466.7M	466.4M	0.1	2.1
dem_200_newyork_167	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	3.2
dem_200_newyork_168	352.2M	352.0M	0.0	2.1	352.2M	351.8M	0.1	2.7
dem_200_newyork_169	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	7.6
dem_200_newyork_170	453.3M	452.8M	0.1	1.4	453.3M	452.9M	0.1	2.8
dem_200_newyork_171	481.9M	481.5M	0.1	0.8	481.9M	481.5M	0.1	2.0
dem_200_newyork_172	402.0M	401.7M	0.1	1.3	402.0M	401.7M	0.1	2.2
dem_200_newyork_173	387.7M	387.3M	0.1	1.6	387.6M	387.2M	0.1	2.1
dem_200_newyork_174	459.1M	458.8M	0.1	0.7	459.1M	458.7M	0.1	1.9
dem_200_newyork_175	491.2M	490.7M	0.1	1.0	491.2M	490.9M	0.1	8.0
dem_200_newyork_176	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	3.1
dem_200_newyork_177	514.8M	514.3M	0.1	1.0	514.8M	514.3M	0.1	1.4
dem_200_newyork_178	511.6M	511.2M	0.1	1.0	511.6M	511.2M	0.1	2.4
dem_200_newyork_179	453.1M	452.7M	0.1	1.3	453.1M	452.8M	0.1	3.3
dem_200_newyork_180	421.8M	421.4M	0.1	1.6	421.8M	421.5M	0.1	2.4
dem_200_newyork_181	1e+20	1e+20	0.0	8.3	1e+20	1e+20	0.0	3.1
dem_200_newyork_182	375.2M	375.0M	0.1	0.8	375.2M	375.0M	0.1	1.8
dem_200_newyork_183	535.2M	534.8M	0.1	1.1	535.2M	534.8M	0.1	8.4
dem_200_newyork_184	425.4M	425.1M	0.1	1.2	425.4M	425.1M	0.1	2.4
dem_200_newyork_185	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	3.9
dem_200_newyork_186	448.1M	447.8M	0.1	1.2	448.1M	447.7M	0.1	2.1
dem_200_newyork_187	401.7M	401.3M	0.1	1.3	401.7M	401.3M	0.1	3.0
dem_200_newyork_188	397.0M	396.7M	0.1	1.1	397.0M	396.7M	0.1	1.9
dem_200_newyork_189	471.2M	470.7M	0.1	0.9	471.2M	470.7M	0.1	2.1
dem_200_newyork_190	503.6M	503.3M	0.1	0.8	503.6M	503.6M	0.0	2.5
dem_200_newyork_191	375.4M	375.1M	0.1	1.1	375.4M	375.1M	0.1	2.5
dem_200_newyork_192	396.8M	396.5M	0.1	1.7	396.9M	396.5M	0.1	2.6
dem_200_newyork_193	429.1M	429.1M	0.0	1.0	429.1M	428.8M	0.1	2.4
dem_200_newyork_194	385.0M	384.7M	0.1	2.1	385.0M	384.7M	0.1	2.7
dem_200_newyork_195	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	3.5
dem_200_newyork_196	383.7M	383.3M	0.1	1.4	383.7M	383.3M	0.1	2.3
dem_200_newyork_197	347.7M	347.3M	0.1	2.0	347.7M	347.3M	0.1	2.9
dem_200_newyork_198	1e+20	1e+20	0.0	6.1	1e+20	1e+20	0.0	8.9
dem_200_newyork_199	430.9M	430.5M	0.1	1.2	430.9M	430.5M	0.1	2.5
dem_200_newyork_200	491.5M	491.0M	0.1	0.9	491.8M	491.3M	0.1	2.0
dem_200_newyork_201	493.8M	493.5M	0.1	0.9	493.8M	493.3M	0.1	1.8
dem_200_newyork_202	422.9M	422.5M	0.1	1.1	422.9M	422.5M	0.1	2.1
dem_200_newyork_203	370.5M	370.4M	0.0	1.6	370.5M	370.1M	0.1	2.1
dem_200_newyork_204	475.7M	475.2M	0.1	1.0	475.5M	475.2M	0.1	2.1
dem_200_newyork_205	366.3M	365.9M	0.1	2.2	366.2M	366.1M	0.0	3.4
dem_200_newyork_206	440.2M	439.7M	0.1	1.9	440.0M	439.7M	0.1	2.3
dem_200_newyork_207	1e+20	1e+20	0.0	4.9	1e+20	1e+20	0.0	5.8
dem_200_newyork_208	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	8.6
dem_200_newyork_209	408.4M	408.0M	0.1	1.1	408.4M	408.0M	0.1	2.2
dem_200_newyork_210	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	6.7
dem_200_newyork_211	499.2M	499.0M	0.0	1.0	499.2M	498.9M	0.1	3.2
dem_200_newyork_212	419.3M	418.9M	0.1	1.0	419.3M	419.1M	0.0	2.5
dem_200_newyork_213	396.8M	396.4M	0.1	2.6	396.8M	396.4M	0.1	3.5
dem_200_newyork_214	373.2M	372.9M	0.1	1.4	373.2M	372.9M	0.1	2.7

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Table A.38: Comparison of split-pipe models on *New York* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_215	437.3M	437.0M	0.1	1.2	437.3M	437.0M	0.1	2.2
dem_200_newyork_216	465.5M	465.1M	0.1	1.3	465.5M	465.0M	0.1	2.1
dem_200_newyork_217	412.8M	412.5M	0.1	1.2	412.8M	412.4M	0.1	2.9
dem_200_newyork_218	378.9M	378.6M	0.1	1.4	378.9M	378.6M	0.1	2.5
dem_200_newyork_219	437.8M	437.4M	0.1	1.0	437.8M	437.5M	0.1	2.4
dem_200_newyork_220	405.6M	405.2M	0.1	1.4	405.3M	405.0M	0.1	2.1
dem_200_newyork_221	450.4M	450.0M	0.1	1.0	450.2M	449.8M	0.1	2.2
dem_200_newyork_222	448.2M	447.9M	0.1	1.0	448.2M	447.9M	0.1	2.1
dem_200_newyork_223	480.5M	480.1M	0.1	0.9	480.5M	480.0M	0.1	2.2
dem_200_newyork_224	361.4M	360.8M	0.2	4.2	360.9M	360.7M	0.1	4.5
dem_200_newyork_225	380.3M	380.0M	0.1	1.6	380.3M	379.9M	0.1	2.3
dem_200_newyork_226	497.1M	496.6M	0.1	0.8	497.1M	496.6M	0.1	2.0
dem_200_newyork_227	444.1M	443.7M	0.1	2.7	443.9M	443.5M	0.1	4.2
dem_200_newyork_228	373.9M	373.6M	0.1	2.2	373.9M	373.6M	0.1	2.6
dem_200_newyork_229	451.4M	451.1M	0.1	1.2	451.4M	450.9M	0.1	1.9
dem_200_newyork_230	414.8M	414.5M	0.1	1.5	414.8M	414.5M	0.1	2.7
dem_200_newyork_231	439.1M	438.7M	0.1	1.5	439.3M	438.9M	0.1	3.2
dem_200_newyork_232	459.9M	459.6M	0.1	1.7	459.9M	459.6M	0.1	1.8
dem_200_newyork_233	400.5M	400.1M	0.1	1.1	400.5M	400.1M	0.1	2.4
dem_200_newyork_234	397.3M	397.0M	0.1	1.8	397.5M	397.1M	0.1	3.7
dem_200_newyork_235	394.6M	394.3M	0.1	1.0	394.6M	394.3M	0.1	2.3
dem_200_newyork_236	423.0M	422.8M	0.1	1.0	423.0M	422.6M	0.1	2.0
dem_200_newyork_237	372.7M	372.3M	0.1	1.9	372.7M	372.3M	0.1	2.7
dem_200_newyork_238	394.0M	393.6M	0.1	1.4	393.9M	393.5M	0.1	2.2
dem_200_newyork_239	468.3M	467.9M	0.1	0.9	468.3M	467.8M	0.1	1.5
dem_200_newyork_240	471.3M	470.9M	0.1	0.8	471.3M	470.9M	0.1	2.1
dem_200_newyork_241	491.1M	490.7M	0.1	0.7	491.1M	490.7M	0.1	1.6
dem_200_newyork_242	452.0M	451.8M	0.1	1.2	452.2M	451.9M	0.1	3.1
dem_200_newyork_243	475.6M	475.3M	0.1	1.2	475.6M	475.3M	0.1	1.9
dem_200_newyork_244	485.9M	485.8M	0.0	0.8	486.1M	485.8M	0.1	1.9
dem_200_newyork_245	364.0M	363.7M	0.1	1.1	364.2M	363.8M	0.1	2.4
dem_200_newyork_246	379.1M	378.7M	0.1	1.1	379.1M	378.7M	0.1	2.8
dem_200_newyork_247	420.3M	419.9M	0.1	1.1	420.3M	419.9M	0.1	2.2
dem_200_newyork_248	1e+20	1e+20	0.0	5.3	1e+20	1e+20	0.0	6.0
dem_200_newyork_249	458.6M	458.4M	0.0	1.4	458.6M	458.4M	0.0	2.0
dem_200_newyork_250	537.4M	537.4M	0.0	0.9	537.4M	537.4M	0.0	5.4
dem_200_newyork_251	352.0M	351.6M	0.1	1.7	352.0M	351.6M	0.1	2.9
dem_200_newyork_252	443.5M	443.0M	0.1	1.1	443.5M	443.0M	0.1	1.9
dem_200_newyork_253	368.3M	368.1M	0.1	0.9	368.3M	368.1M	0.1	2.6
dem_200_newyork_254	412.4M	411.1M	0.3	1.5	411.2M	410.9M	0.1	2.5
dem_200_newyork_255	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	3.2
dem_200_newyork_256	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	5.4
dem_200_newyork_257	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	5.8
dem_200_newyork_258	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	7.4
dem_200_newyork_259	509.8M	509.5M	0.1	0.7	509.8M	509.5M	0.1	1.3
dem_200_newyork_260	478.2M	477.8M	0.1	0.8	478.2M	477.8M	0.1	2.2
dem_200_newyork_261	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	7.1
dem_200_newyork_262	414.3M	413.9M	0.1	1.7	414.3M	413.9M	0.1	2.4
dem_200_newyork_263	421.3M	420.9M	0.1	1.2	421.3M	420.9M	0.1	2.1
dem_200_newyork_264	430.6M	430.5M	0.0	1.2	430.6M	430.5M	0.0	2.0
dem_200_newyork_265	406.5M	406.1M	0.1	2.0	406.3M	405.9M	0.1	2.8
dem_200_newyork_266	453.9M	453.5M	0.1	1.2	453.9M	453.5M	0.1	2.2
dem_200_newyork_267	375.2M	375.1M	0.0	1.3	375.2M	374.9M	0.1	2.5
dem_200_newyork_268	394.9M	394.6M	0.1	1.5	394.9M	394.5M	0.1	2.6
dem_200_newyork_269	425.5M	425.4M	0.0	1.4	425.5M	425.1M	0.1	2.3
dem_200_newyork_270	436.5M	436.0M	0.1	0.9	436.3M	435.9M	0.1	2.0
dem_200_newyork_271	397.9M	397.6M	0.1	1.2	397.9M	397.7M	0.0	1.9
dem_200_newyork_272	420.1M	419.7M	0.1	1.9	419.9M	419.5M	0.1	3.0
dem_200_newyork_273	487.4M	487.3M	0.0	0.9	487.4M	487.3M	0.0	2.8
dem_200_newyork_274	394.6M	394.3M	0.1	1.3	394.6M	394.3M	0.1	2.4
dem_200_newyork_275	427.9M	427.5M	0.1	1.2	427.9M	427.5M	0.1	2.5
dem_200_newyork_276	295.0M	294.7M	0.1	1.6	295.0M	294.7M	0.1	1.8
dem_200_newyork_277	520.6M	520.2M	0.1	1.2	520.4M	520.2M	0.0	7.9
dem_200_newyork_278	304.7M	304.4M	0.1	1.7	304.7M	304.5M	0.1	2.9
dem_200_newyork_279	434.0M	433.6M	0.1	1.2	433.8M	433.6M	0.1	1.7
dem_200_newyork_280	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	4.2
dem_200_newyork_281	427.2M	426.9M	0.1	1.0	427.3M	426.9M	0.1	1.4
dem_200_newyork_282	435.2M	435.0M	0.1	1.2	435.2M	434.8M	0.1	2.2
dem_200_newyork_283	426.4M	426.2M	0.1	0.9	426.4M	426.2M	0.1	2.1
dem_200_newyork_284	429.5M	429.2M	0.1	1.1	429.5M	429.2M	0.1	2.3
dem_200_newyork_285	478.6M	478.3M	0.1	0.9	478.6M	478.3M	0.1	1.7
dem_200_newyork_286	425.0M	424.7M	0.1	1.3	425.0M	424.7M	0.1	2.4
dem_200_newyork_287	385.1M	384.7M	0.1	1.7	384.8M	384.5M	0.1	2.2
dem_200_newyork_288	1e+20	1e+20	0.0	4.0	1e+20	1e+20	0.0	7.8
dem_200_newyork_289	395.3M	394.9M	0.1	1.7	395.3M	394.9M	0.1	2.0
dem_200_newyork_290	401.1M	400.7M	0.1	1.2	401.1M	400.7M	0.1	2.3
dem_200_newyork_291	334.7M	334.4M	0.1	2.4	334.7M	334.4M	0.1	2.5
dem_200_newyork_292	472.1M	471.8M	0.1	0.8	472.1M	471.8M	0.1	1.9

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Table A.38: Comparison of split-pipe models on *New York* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_293	501.3M	500.8M	0.1	1.0	501.3M	501.0M	0.0	2.0
dem_200_newyork_294	445.4M	445.0M	0.1	1.1	445.4M	445.1M	0.1	2.7
dem_200_newyork_295	425.5M	425.1M	0.1	1.1	425.7M	425.3M	0.1	1.8
dem_200_newyork_296	451.1M	450.6M	0.1	1.0	450.9M	450.5M	0.1	1.5
dem_200_newyork_297	435.2M	434.8M	0.1	1.6	435.3M	434.9M	0.1	2.3
dem_200_newyork_298	322.5M	322.1M	0.1	1.6	322.5M	322.2M	0.1	2.6
dem_200_newyork_299	477.1M	476.8M	0.1	1.2	477.1M	476.8M	0.1	2.3
dem_200_newyork_300	459.5M	459.1M	0.1	1.4	459.7M	459.3M	0.1	2.9
dem_200_newyork_301	516.8M	516.6M	0.0	1.3	516.7M	516.3M	0.1	1.6
dem_200_newyork_302	356.1M	355.9M	0.0	1.6	356.1M	355.9M	0.0	2.5
dem_200_newyork_303	416.3M	415.9M	0.1	1.2	416.3M	415.9M	0.1	1.8
dem_200_newyork_304	452.0M	451.6M	0.1	1.2	452.0M	451.6M	0.1	2.7
dem_200_newyork_305	461.9M	461.5M	0.1	1.4	461.8M	461.5M	0.1	1.9
dem_200_newyork_306	435.4M	435.0M	0.1	1.1	435.4M	435.1M	0.1	2.3
dem_200_newyork_307	384.7M	384.4M	0.1	1.3	384.7M	384.4M	0.1	2.7
dem_200_newyork_308	337.7M	337.3M	0.1	2.3	337.7M	337.3M	0.1	3.6
dem_200_newyork_309	417.3M	416.8M	0.1	1.5	417.3M	416.9M	0.1	3.4
dem_200_newyork_310	473.2M	472.8M	0.1	0.8	473.2M	472.8M	0.1	2.0
dem_200_newyork_311	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	5.4
dem_200_newyork_312	419.7M	419.3M	0.1	1.4	419.7M	419.3M	0.1	1.9
dem_200_newyork_313	448.3M	448.1M	0.0	1.2	448.3M	448.0M	0.1	2.3
dem_200_newyork_314	429.8M	429.4M	0.1	1.0	429.8M	429.4M	0.1	1.9
dem_200_newyork_315	1e+20	1e+20	0.0	5.3	1e+20	1e+20	0.0	6.3
dem_200_newyork_316	1e+20	1e+20	0.0	6.8	1e+20	1e+20	0.0	3.4
dem_200_newyork_317	1e+20	1e+20	0.0	8.9	1e+20	1e+20	0.0	7.0
dem_200_newyork_318	378.3M	378.2M	0.0	2.1	378.3M	378.2M	0.0	3.3
dem_200_newyork_319	400.2M	399.8M	0.1	1.2	400.0M	399.6M	0.1	2.3
dem_200_newyork_320	1e+20	1e+20	0.0	4.1	1e+20	1e+20	0.0	4.1
dem_200_newyork_321	435.5M	435.3M	0.0	1.4	435.5M	435.1M	0.1	1.8
dem_200_newyork_322	521.4M	521.1M	0.1	1.0	521.4M	521.0M	0.1	3.6
dem_200_newyork_323	430.2M	429.8M	0.1	1.8	430.2M	429.8M	0.1	1.8
dem_200_newyork_324	1e+20	1e+20	0.0	6.2	1e+20	1e+20	0.0	6.4
dem_200_newyork_325	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	6.5
dem_200_newyork_326	423.8M	423.4M	0.1	1.0	423.8M	423.4M	0.1	2.1
dem_200_newyork_327	425.6M	425.3M	0.1	3.0	425.5M	425.1M	0.1	2.5
dem_200_newyork_328	492.3M	491.8M	0.1	1.0	492.3M	491.9M	0.1	1.8
dem_200_newyork_329	1e+20	1e+20	0.0	10.5	1e+20	1e+20	0.0	4.0
dem_200_newyork_330	410.3M	409.9M	0.1	1.6	410.3M	409.9M	0.1	1.9
dem_200_newyork_331	519.0M	519.0M	0.0	0.8	519.0M	518.8M	0.0	2.2
dem_200_newyork_332	476.3M	476.2M	0.0	0.9	476.3M	476.2M	0.0	2.4
dem_200_newyork_333	507.9M	507.5M	0.1	0.8	508.4M	507.8M	0.1	2.7
dem_200_newyork_334	392.5M	392.1M	0.1	1.2	392.5M	392.1M	0.1	1.9
dem_200_newyork_335	434.7M	434.3M	0.1	1.1	434.7M	434.3M	0.1	2.0
dem_200_newyork_336	380.8M	380.4M	0.1	1.1	380.8M	380.4M	0.1	2.5
dem_200_newyork_337	423.9M	423.6M	0.1	1.1	423.9M	423.7M	0.1	2.4
dem_200_newyork_338	1e+20	1e+20	0.0	5.9	1e+20	1e+20	0.0	6.6
dem_200_newyork_339	434.9M	434.5M	0.1	1.6	435.0M	434.6M	0.1	2.6
dem_200_newyork_340	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	7.1
dem_200_newyork_341	446.9M	446.5M	0.1	1.3	447.0M	446.5M	0.1	2.2
dem_200_newyork_342	518.3M	518.0M	0.0	1.1	518.3M	518.0M	0.0	7.1
dem_200_newyork_343	446.4M	446.1M	0.1	1.6	446.2M	445.9M	0.1	2.3
dem_200_newyork_344	458.4M	458.1M	0.1	0.8	458.4M	458.1M	0.1	2.5
dem_200_newyork_345	496.8M	496.7M	0.0	0.7	496.8M	496.6M	0.0	2.0
dem_200_newyork_346	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	4.2
dem_200_newyork_347	359.2M	358.9M	0.1	1.8	359.2M	358.9M	0.1	2.4
dem_200_newyork_348	477.2M	477.1M	0.0	1.2	477.2M	477.2M	0.0	1.8
dem_200_newyork_349	479.9M	479.5M	0.1	1.1	479.9M	479.5M	0.1	2.4
dem_200_newyork_350	369.9M	369.5M	0.1	1.9	369.9M	369.5M	0.1	2.8
dem_200_newyork_351	375.5M	375.3M	0.0	2.2	375.5M	375.1M	0.1	2.3
dem_200_newyork_352	523.3M	523.3M	0.0	1.0	523.3M	523.3M	0.0	2.9
dem_200_newyork_353	491.2M	491.1M	0.0	1.0	491.2M	491.1M	0.0	2.1
dem_200_newyork_354	424.1M	423.8M	0.1	1.2	424.1M	423.8M	0.1	1.7
dem_200_newyork_355	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	3.3
dem_200_newyork_356	1e+20	1e+20	0.0	5.3	1e+20	1e+20	0.0	4.1
dem_200_newyork_357	392.1M	391.7M	0.1	1.7	391.9M	391.7M	0.1	2.7
dem_200_newyork_358	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	3.4
dem_200_newyork_359	436.0M	435.6M	0.1	1.5	435.9M	435.5M	0.1	2.6
dem_200_newyork_360	448.5M	448.2M	0.1	0.9	448.5M	448.2M	0.1	2.7
dem_200_newyork_361	401.1M	400.9M	0.1	1.2	401.1M	400.8M	0.1	1.6
dem_200_newyork_362	466.8M	466.6M	0.0	0.7	466.8M	466.6M	0.0	2.1
dem_200_newyork_363	446.1M	445.7M	0.1	0.9	446.1M	445.7M	0.1	1.6
dem_200_newyork_364	523.4M	523.1M	0.1	0.8	523.4M	522.9M	0.1	1.8
dem_200_newyork_365	396.1M	395.8M	0.1	1.9	396.3M	396.0M	0.1	3.1
dem_200_newyork_366	434.7M	434.3M	0.1	1.2	435.0M	434.6M	0.1	3.1
dem_200_newyork_367	465.5M	465.0M	0.1	1.8	465.5M	465.3M	0.0	2.5
dem_200_newyork_368	462.9M	462.6M	0.1	1.7	462.9M	462.6M	0.1	1.9
dem_200_newyork_369	437.0M	437.0M	0.0	0.9	437.0M	436.9M	0.0	1.7
dem_200_newyork_370	1e+20	1e+20	0.0	5.9	1e+20	1e+20	0.0	8.2

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Table A.38: Comparison of split-pipe models on *New York* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_371	407.8M	407.4M	0.1	1.7	407.8M	407.5M	0.1	3.2
dem_200_newyork_372	387.5M	387.2M	0.1	1.1	387.5M	387.2M	0.1	1.9
dem_200_newyork_373	396.0M	395.6M	0.1	2.5	395.7M	395.4M	0.1	2.3
dem_200_newyork_374	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	4.0
dem_200_newyork_375	448.3M	448.0M	0.1	1.7	448.3M	448.0M	0.1	2.8
dem_200_newyork_376	419.2M	418.9M	0.1	1.0	419.2M	418.9M	0.1	2.6
dem_200_newyork_377	326.8M	326.5M	0.1	1.9	326.8M	326.5M	0.1	2.1
dem_200_newyork_378	416.3M	415.9M	0.1	1.3	416.3M	415.9M	0.1	3.5
dem_200_newyork_379	394.2M	393.8M	0.1	1.3	394.2M	393.9M	0.1	2.2
dem_200_newyork_380	432.8M	432.5M	0.1	1.3	432.8M	432.4M	0.1	2.1
dem_200_newyork_381	320.1M	319.8M	0.1	1.4	320.1M	319.9M	0.1	2.3
dem_200_newyork_382	482.7M	482.3M	0.1	0.9	482.7M	482.3M	0.1	2.1
dem_200_newyork_383	442.2M	441.8M	0.1	1.2	442.2M	441.9M	0.1	1.5
dem_200_newyork_384	1e+20	1e+20	0.0	4.0	1e+20	1e+20	0.0	4.4
dem_200_newyork_385	401.5M	401.1M	0.1	1.2	401.5M	401.2M	0.1	2.7
dem_200_newyork_386	392.7M	390.1M	0.7	3.2	390.2M	389.8M	0.1	2.3
dem_200_newyork_387	436.4M	436.0M	0.1	1.3	436.4M	436.0M	0.1	2.3
dem_200_newyork_388	311.4M	311.1M	0.1	2.3	311.4M	311.2M	0.1	3.0
dem_200_newyork_389	424.3M	424.0M	0.1	1.2	424.4M	424.0M	0.1	2.8
dem_200_newyork_390	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	4.4
dem_200_newyork_391	367.0M	366.8M	0.1	1.6	367.0M	366.8M	0.1	2.6
dem_200_newyork_392	474.1M	473.8M	0.1	0.9	474.0M	473.6M	0.1	2.1
dem_200_newyork_393	495.3M	495.0M	0.1	0.8	495.3M	495.2M	0.0	2.3
dem_200_newyork_394	392.0M	391.7M	0.1	1.8	392.0M	391.7M	0.1	3.1
dem_200_newyork_395	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	3.5
dem_200_newyork_396	437.1M	436.7M	0.1	1.2	437.1M	436.8M	0.1	2.9
dem_200_newyork_397	514.7M	514.7M	0.0	0.6	514.7M	514.7M	0.0	1.6
dem_200_newyork_398	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	5.5
dem_200_newyork_399	327.0M	326.6M	0.1	1.9	327.0M	326.6M	0.1	3.4
dem_200_newyork_400	416.0M	415.7M	0.1	1.3	416.0M	415.7M	0.1	2.4
dem_200_newyork_401	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	7.7
dem_200_newyork_402	423.0M	422.6M	0.1	1.6	423.0M	422.6M	0.1	3.0
dem_200_newyork_403	392.0M	391.8M	0.1	1.4	392.0M	391.8M	0.1	2.0
dem_200_newyork_404	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	3.9
dem_200_newyork_405	405.1M	404.9M	0.0	1.2	405.1M	404.9M	0.0	2.7
dem_200_newyork_406	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	4.9
dem_200_newyork_407	382.4M	382.2M	0.1	1.2	382.4M	382.1M	0.1	2.1
dem_200_newyork_408	484.1M	483.7M	0.1	0.8	484.1M	483.7M	0.1	1.9
dem_200_newyork_409	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	5.5
dem_200_newyork_410	436.7M	436.2M	0.1	1.4	436.8M	436.4M	0.1	3.0
dem_200_newyork_411	494.3M	493.8M	0.1	0.9	494.3M	493.8M	0.1	2.3
dem_200_newyork_412	450.9M	450.5M	0.1	1.1	450.9M	450.5M	0.1	2.1
dem_200_newyork_413	442.8M	442.5M	0.1	0.9	442.8M	442.7M	0.0	2.4
dem_200_newyork_414	471.7M	471.3M	0.1	1.0	471.7M	471.2M	0.1	2.6
dem_200_newyork_415	494.0M	493.6M	0.1	1.4	493.9M	493.6M	0.1	1.8
dem_200_newyork_416	1e+20	1e+20	0.0	3.3	1e+20	1e+20	0.0	4.2
dem_200_newyork_417	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	4.6
dem_200_newyork_418	442.1M	442.0M	0.0	0.9	442.1M	441.7M	0.1	1.6
dem_200_newyork_419	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	12.0
dem_200_newyork_420	412.8M	412.5M	0.1	2.1	412.8M	412.4M	0.1	2.8
dem_200_newyork_421	369.0M	368.7M	0.1	2.0	369.0M	368.7M	0.1	2.7
dem_200_newyork_422	331.7M	331.4M	0.1	1.6	331.7M	331.4M	0.1	2.9
dem_200_newyork_423	477.3M	477.0M	0.1	1.1	477.3M	476.9M	0.1	2.0
dem_200_newyork_424	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	4.2
dem_200_newyork_425	416.3M	416.0M	0.1	1.7	416.3M	416.0M	0.1	2.9
dem_200_newyork_426	412.7M	412.3M	0.1	1.1	412.7M	412.3M	0.1	2.6
dem_200_newyork_427	415.5M	415.1M	0.1	1.5	415.5M	415.1M	0.1	2.1
dem_200_newyork_428	431.0M	430.8M	0.1	1.5	431.0M	430.7M	0.1	2.5
dem_200_newyork_429	468.6M	468.1M	0.1	1.2	468.6M	468.1M	0.1	2.2
dem_200_newyork_430	386.5M	386.1M	0.1	1.2	386.5M	386.1M	0.1	2.0
dem_200_newyork_431	481.8M	481.6M	0.0	0.7	481.8M	481.5M	0.1	2.5
dem_200_newyork_432	454.1M	453.7M	0.1	1.0	454.1M	453.7M	0.1	1.4
dem_200_newyork_433	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	3.2
dem_200_newyork_434	422.3M	422.1M	0.0	1.1	422.3M	422.1M	0.1	1.7
dem_200_newyork_435	417.6M	417.4M	0.1	1.4	417.6M	417.4M	0.1	2.5
dem_200_newyork_436	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	7.6
dem_200_newyork_437	417.0M	416.8M	0.1	1.4	417.0M	416.8M	0.1	2.1
dem_200_newyork_438	442.5M	442.1M	0.1	1.5	442.3M	441.9M	0.1	2.4
dem_200_newyork_439	361.4M	361.1M	0.1	2.3	361.2M	360.9M	0.1	2.2
dem_200_newyork_440	457.6M	457.3M	0.1	1.0	457.8M	457.3M	0.1	2.0
dem_200_newyork_441	488.8M	488.4M	0.1	1.5	488.8M	488.4M	0.1	2.6
dem_200_newyork_442	445.1M	444.6M	0.1	1.0	445.1M	444.6M	0.1	2.4
dem_200_newyork_443	417.8M	417.4M	0.1	3.2	417.4M	417.0M	0.1	3.8
dem_200_newyork_444	454.5M	454.4M	0.0	0.9	454.5M	454.4M	0.0	2.2
dem_200_newyork_445	378.0M	377.6M	0.1	1.3	378.0M	377.6M	0.1	2.4
dem_200_newyork_446	409.5M	409.2M	0.1	3.1	409.5M	409.2M	0.1	3.2
dem_200_newyork_447	425.6M	425.2M	0.1	1.0	425.5M	425.1M	0.1	2.5
dem_200_newyork_448	500.1M	500.0M	0.0	0.8	500.1M	499.8M	0.1	2.5

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Table A.38: Comparison of split-pipe models on *New York* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork_449	423.3M	423.2M	0.0	2.4	423.3M	422.9M	0.1	2.2
dem_200_newyork_450	380.7M	380.4M	0.1	1.7	380.7M	380.4M	0.1	2.7
dem_200_newyork_451	543.7M	543.5M	0.0	1.0	543.7M	543.3M	0.1	2.8
dem_200_newyork_452	493.3M	492.8M	0.1	0.8	493.3M	492.9M	0.1	2.2
dem_200_newyork_453	401.7M	401.3M	0.1	1.1	401.7M	401.3M	0.1	1.9
dem_200_newyork_454	393.7M	393.4M	0.1	1.1	393.7M	393.4M	0.1	2.2
dem_200_newyork_455	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	4.0
dem_200_newyork_456	492.9M	492.5M	0.1	0.8	492.9M	492.6M	0.0	1.7
dem_200_newyork_457	458.5M	458.2M	0.1	1.2	458.5M	458.2M	0.1	2.0
dem_200_newyork_458	434.5M	434.1M	0.1	1.2	434.5M	434.1M	0.1	2.6
dem_200_newyork_459	1e+20	1e+20	0.0	4.9	1e+20	1e+20	0.0	5.0
dem_200_newyork_460	401.3M	401.1M	0.0	1.6	401.2M	400.8M	0.1	2.1
dem_200_newyork_461	457.4M	457.2M	0.0	1.4	457.3M	457.0M	0.1	2.2
dem_200_newyork_462	487.7M	487.3M	0.1	0.7	487.7M	487.3M	0.1	1.8
dem_200_newyork_463	398.7M	398.3M	0.1	1.4	398.5M	398.2M	0.1	2.5
dem_200_newyork_464	355.1M	354.7M	0.1	2.0	355.0M	354.7M	0.1	2.5
dem_200_newyork_465	444.8M	444.5M	0.1	1.2	444.8M	444.4M	0.1	1.9
dem_200_newyork_466	392.2M	391.9M	0.1	1.7	392.2M	391.8M	0.1	2.0
dem_200_newyork_467	383.7M	383.4M	0.1	1.3	383.6M	383.3M	0.1	1.8
dem_200_newyork_468	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	10.1
dem_200_newyork_469	481.8M	481.4M	0.1	0.9	481.8M	481.4M	0.1	1.8
dem_200_newyork_470	455.6M	455.3M	0.1	0.8	455.6M	455.3M	0.1	2.6
dem_200_newyork_471	509.6M	509.2M	0.1	0.7	509.6M	509.2M	0.1	1.8
dem_200_newyork_472	474.2M	473.8M	0.1	1.7	474.2M	473.8M	0.1	2.1
dem_200_newyork_473	424.0M	423.7M	0.1	1.3	424.0M	423.9M	0.0	2.5
dem_200_newyork_474	1e+20	1e+20	0.0	5.2	1e+20	1e+20	0.0	3.6
dem_200_newyork_475	389.0M	388.8M	0.0	1.3	389.0M	388.6M	0.1	2.5
dem_200_newyork_476	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	9.9
dem_200_newyork_477	517.0M	516.6M	0.1	0.7	517.0M	516.6M	0.1	4.0
dem_200_newyork_478	381.5M	381.2M	0.1	1.2	381.5M	381.2M	0.1	2.2
dem_200_newyork_479	481.7M	481.2M	0.1	0.9	481.7M	481.2M	0.1	1.9
dem_200_newyork_480	483.9M	483.5M	0.1	0.8	483.9M	483.6M	0.0	1.8
dem_200_newyork_481	472.2M	471.7M	0.1	0.9	472.2M	471.7M	0.1	1.5
dem_200_newyork_482	408.1M	407.7M	0.1	1.2	408.1M	407.7M	0.1	2.7
dem_200_newyork_483	373.3M	373.0M	0.1	2.3	373.3M	373.0M	0.1	2.8
dem_200_newyork_484	394.9M	394.6M	0.1	1.5	395.0M	394.6M	0.1	2.8
dem_200_newyork_485	452.0M	451.8M	0.0	0.9	452.0M	451.8M	0.1	2.0
dem_200_newyork_486	1e+20	1e+20	0.0	5.3	1e+20	1e+20	0.0	4.3
dem_200_newyork_487	413.1M	412.7M	0.1	1.1	413.1M	412.8M	0.1	2.5
dem_200_newyork_488	471.3M	471.0M	0.1	1.3	471.3M	471.0M	0.1	2.3
dem_200_newyork_489	312.1M	311.8M	0.1	3.0	312.1M	311.8M	0.1	3.8
dem_200_newyork_490	415.0M	414.6M	0.1	1.1	415.0M	414.7M	0.1	2.1
dem_200_newyork_491	1e+20	1e+20	0.0	9.1	1e+20	1e+20	0.0	4.9
dem_200_newyork_492	490.1M	489.6M	0.1	0.9	490.1M	489.7M	0.1	2.3
dem_200_newyork_493	426.1M	425.7M	0.1	1.5	426.1M	425.7M	0.1	3.2
dem_200_newyork_494	405.6M	405.4M	0.0	1.1	405.6M	405.3M	0.1	2.4
dem_200_newyork_495	396.1M	395.8M	0.1	1.4	396.1M	395.8M	0.1	2.4
dem_200_newyork_496	398.0M	397.6M	0.1	1.7	397.9M	397.7M	0.1	3.0
dem_200_newyork_497	433.6M	433.2M	0.1	1.6	434.0M	433.5M	0.1	2.5
dem_200_newyork_498	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	5.8
dem_200_newyork_499	416.3M	415.9M	0.1	1.1	416.3M	415.9M	0.1	1.9
dem_200_newyork_500	351.3M	350.9M	0.1	1.6	351.3M	350.9M	0.1	2.8

Table A.39: Detailed results of the discrete models on *New York* with $\mathcal{B} = 500$ as summarized in Figure 3.11c and Table 3.3e. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_1	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_2	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_3	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_4	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_5	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_6	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_7	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_8	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_9	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_10	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_11	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0

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Table A.39: Comparison of discrete models on *New York* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_12	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_13	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_14	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_15	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_16	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_17	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_18	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_19	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_20	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_21	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_22	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_23	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_24	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_25	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_26	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_27	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_28	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_29	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_30	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_31	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_32	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_33	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_34	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_35	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_36	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_37	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_38	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_39	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_40	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_41	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_42	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_43	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_44	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_45	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_46	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_47	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_48	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_49	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_50	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_51	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_52	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_53	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_54	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_55	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_56	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_57	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_58	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_59	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_60	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_61	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_62	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_63	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_64	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_65	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_66	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_67	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_68	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_69	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_70	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_71	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_72	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_73	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_74	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_75	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_76	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_77	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_78	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_79	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_80	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_81	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_82	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_83	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_84	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_85	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_86	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_87	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_88	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_89	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0

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Table A.39: Comparison of discrete models on *New York* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_90	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_91	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_92	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_93	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_94	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_95	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_96	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_97	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_98	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_99	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_100	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_101	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_102	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_103	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_104	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_105	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_106	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_107	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_108	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_109	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_110	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_111	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_112	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_113	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_114	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_115	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_116	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_117	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_118	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_119	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_120	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_121	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_122	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_123	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_124	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_125	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_126	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_127	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_128	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_129	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_130	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_131	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_132	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_133	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_134	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_135	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_136	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_137	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_138	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_139	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_140	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_141	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_142	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_143	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_144	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_145	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_146	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_147	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_148	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_149	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_150	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_151	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_152	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_153	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_154	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_155	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_156	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_157	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_158	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_159	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_160	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_161	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_162	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_163	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_164	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_165	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_166	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_167	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0

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Table A.39: Comparison of discrete models on *New York* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_168	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_169	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_170	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_171	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_172	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_173	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_174	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_175	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_176	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_177	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_178	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_179	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_180	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_181	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_182	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_183	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_184	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_185	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_186	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_187	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_188	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_189	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_190	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_191	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_192	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_193	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_194	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_195	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_196	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_197	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_198	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_199	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_200	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_201	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_202	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_203	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_204	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_205	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_206	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_207	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_208	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_209	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_210	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_211	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_212	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_213	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_214	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_215	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_216	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_217	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_218	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_219	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_220	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_221	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_222	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_223	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_224	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_225	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_226	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_227	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_228	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_229	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_230	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_231	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_232	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_233	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_234	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_235	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_236	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_237	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_238	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_239	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_240	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_241	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_242	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_243	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_244	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_245	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0

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Table A.39: Comparison of discrete models on *New York* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_246	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_247	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_248	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_249	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_250	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_251	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_252	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_253	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_254	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_255	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_256	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_257	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_258	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_259	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_260	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_261	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_262	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_263	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_264	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_265	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_266	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_267	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_268	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_269	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_270	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_271	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_272	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_273	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_274	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_275	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_276	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_277	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_278	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_279	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_280	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_281	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_282	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_283	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_284	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_285	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_286	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_287	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_288	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_289	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_290	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_291	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_292	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_293	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_294	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_295	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_296	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_297	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_298	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_299	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_300	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_301	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_302	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_303	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_304	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_305	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_306	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_307	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_308	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_309	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_310	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_311	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_312	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_313	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_314	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_315	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_316	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_317	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_318	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_319	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_320	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_321	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_322	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_323	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0

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Table A.39: Comparison of discrete models on *New York* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_324	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_325	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_326	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_327	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_328	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_329	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_330	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_331	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_332	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_333	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_334	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_335	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_336	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_337	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_338	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_339	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_340	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_341	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_342	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_343	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_344	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_345	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_346	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_347	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_348	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_349	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_350	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_351	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_352	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_353	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_354	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_355	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_356	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_357	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_358	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_359	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_360	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_361	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_362	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_363	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_364	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_365	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_366	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_367	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_368	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_369	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_370	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_371	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_372	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_373	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_374	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_375	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_376	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_377	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_378	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_379	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_380	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_381	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_382	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_383	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_384	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_385	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_386	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_387	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_388	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_389	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_390	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_391	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_392	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_393	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_394	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_395	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_396	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_397	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_398	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_399	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_400	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_401	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0

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Table A.39: Comparison of discrete models on *New York* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_402	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_403	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_404	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_405	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_406	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_407	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_408	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_409	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_410	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_411	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_412	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_413	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_414	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_415	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_416	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_417	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_418	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_419	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_420	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_421	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_422	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_423	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_424	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_425	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_426	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_427	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_428	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_429	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_430	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_431	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_432	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_433	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_434	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_435	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_436	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_437	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_438	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_439	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_440	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_441	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_442	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_443	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_444	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_445	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_446	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_447	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_448	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_449	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_450	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_451	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_452	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_453	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_454	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_455	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_456	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_457	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_458	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_459	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_460	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_461	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_462	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_463	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_464	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_465	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_466	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_467	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_468	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_469	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_470	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_471	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_472	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_473	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_474	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_475	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_476	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_477	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_478	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_479	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0

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Table A.39: Comparison of discrete models on *New York* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_480	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_481	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_482	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_483	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_484	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_485	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_486	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_487	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_488	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_489	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_490	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_491	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_492	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_493	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_494	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_495	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_496	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_497	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_498	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0
dem_500_newyork_499	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.0
dem_500_newyork_500	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	0.0

Table A.40: Detailed results of the split-pipe models on *New York* with $\mathcal{B} = 500$ as summarized in Figure 3.17c and Table 3.4e. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_1	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_2	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_3	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_4	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_5	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_6	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_7	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_8	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_9	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_10	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_11	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_12	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_13	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_14	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_15	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_16	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_17	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_18	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_19	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_20	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_21	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_22	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_23	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_24	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_25	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_26	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_27	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_28	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_29	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_30	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_31	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_32	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_33	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_34	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_35	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_36	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_37	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_38	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_39	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_40	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_41	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_42	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.40: Comparison of split-pipe models on *New York* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_43	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_44	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_45	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_46	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_47	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_48	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_49	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_50	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_51	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_52	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_53	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_54	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_55	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_56	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_57	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_58	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_59	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_60	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_61	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_62	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_63	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_64	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_65	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_66	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_67	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_68	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_69	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_70	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_71	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_72	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_73	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_74	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_75	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_76	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_77	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_78	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_79	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_80	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_81	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_82	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_83	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_84	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_85	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_86	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_87	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_88	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_89	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_90	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_91	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_92	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_93	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_94	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_95	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_96	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_97	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_98	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_99	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_100	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_101	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_102	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_103	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_104	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_105	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_106	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_107	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_108	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_109	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_110	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_111	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_112	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_113	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_114	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_115	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_116	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_117	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_118	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_119	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_120	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.40: Comparison of split-pipe models on *New York* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_121	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_122	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_123	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_124	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_125	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_126	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_127	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_128	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_129	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_130	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_131	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_132	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_133	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_134	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_135	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_136	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_137	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_138	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_139	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_140	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_141	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_142	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_143	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_144	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_145	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_146	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_147	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_148	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_149	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_150	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_151	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_152	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_153	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_154	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_155	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_156	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_157	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_158	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_159	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_160	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_161	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_162	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_163	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_164	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_165	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_166	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_167	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_168	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_169	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_170	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_171	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_172	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_173	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_174	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_175	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_176	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_177	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_178	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_179	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_180	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_181	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_182	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_183	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_184	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_185	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_186	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_187	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_188	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_189	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_190	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_191	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_192	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_193	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_194	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_195	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_196	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_197	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_198	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.40: Comparison of split-pipe models on *New York* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_199	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_200	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_201	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_202	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_203	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_204	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_205	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_206	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_207	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_208	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_209	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_210	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_211	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_212	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_213	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_214	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_215	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_216	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_217	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_218	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_219	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_220	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_221	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_222	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_223	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_224	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_225	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_226	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_227	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_228	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_229	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_230	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_231	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_232	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_233	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_234	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_235	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_236	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_237	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_238	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_239	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_240	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_241	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_242	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_243	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_244	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_245	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_246	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_247	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_248	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_249	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_250	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_251	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_252	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_253	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_254	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_255	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_256	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_257	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_258	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_259	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_260	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_261	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_262	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_263	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_264	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_265	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_266	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_267	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_268	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_269	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_270	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_271	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_272	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_273	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_274	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_275	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_276	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.40: Comparison of split-pipe models on *New York* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_277	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_278	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_279	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_280	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_281	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_282	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_283	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_284	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_285	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_286	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_287	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_288	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_289	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_290	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_291	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_292	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_293	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_294	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_295	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_296	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_297	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_298	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_299	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_300	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_301	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_302	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_303	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_304	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_305	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_306	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_307	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_308	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_309	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_310	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_311	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_312	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_313	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_314	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_315	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_316	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_317	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_318	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_319	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_320	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_321	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_322	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_323	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_324	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_325	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_326	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_327	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_328	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_329	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_330	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_331	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_332	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_333	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_334	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_335	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_336	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_337	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_338	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_339	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_340	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_341	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_342	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_343	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_344	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_345	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_346	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_347	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_348	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_349	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_350	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_351	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_352	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_353	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_354	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

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Table A.40: Comparison of split-pipe models on *New York* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_355	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_356	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_357	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_358	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_359	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_360	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_361	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_362	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_363	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_364	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_365	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_366	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_367	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_368	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_369	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_370	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_371	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_372	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_373	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_374	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_375	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_376	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_377	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_378	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_379	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_380	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_381	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_382	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_383	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_384	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_385	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_386	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_387	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_388	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_389	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_390	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_391	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_392	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_393	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_394	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_395	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_396	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_397	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_398	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_399	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_400	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_401	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_402	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_403	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_404	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_405	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_406	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_407	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_408	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_409	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_410	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_411	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_412	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_413	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_414	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_415	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_416	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_417	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_418	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_419	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_420	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_421	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_422	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_423	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_424	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_425	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_426	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_427	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_428	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_429	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_430	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_431	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_432	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

continued on next page

Table A.40: Comparison of split-pipe models on *New York* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork_433	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_434	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_435	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_436	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_437	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_438	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_439	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_440	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_441	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_442	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_443	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_444	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_445	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_446	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_447	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_448	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_449	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_450	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_451	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_452	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_453	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_454	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_455	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_456	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_457	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_458	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_459	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_460	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_461	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_462	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_463	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_464	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_465	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_466	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_467	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_468	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_469	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_470	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_471	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_472	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_473	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_474	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_475	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_476	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_477	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_478	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_479	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_480	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_481	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_482	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_483	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_484	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_485	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_486	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_487	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_488	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_489	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_490	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_491	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_492	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_493	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_494	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_495	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_496	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_497	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_498	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_499	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0
dem_500_newyork_500	1e+20	1e+20	0.0	0.0	1e+20	1e+20	0.0	0.0

Table A.41: Detailed results of the discrete models on *New York+* with four additional arcs and $\mathcal{B} = 100$ as summarized in Figure 3.12a and Table 3.3f. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_1	59.2M	17.1M	245.2	14400.0	52.3M	16.0M	226.9	14400.0	36.4M	13.6M	167.5	14400.1
dem_100_newyork+_2	37.1M	25.5M	45.2	14400.0	39.4M	24.4M	61.4	14400.0	37.1M	19.6M	89.5	14400.0
dem_100_newyork+_3	56.8M	19.3M	194.7	14400.0	47.4M	23.3M	102.9	14400.0	39.2M	15.3M	156.3	14400.0
dem_100_newyork+_4	36.4M	20.9M	74.3	14400.0	32.8M	20.1M	62.7	14400.0	32.8M	13.4M	145.0	14400.0
dem_100_newyork+_5	52.0M	23.6M	120.0	14400.0	38.1M	29.1M	30.9	14400.0	38.3M	16.7M	128.9	14400.0
dem_100_newyork+_6	38.1M	22.7M	67.4	14400.0	38.1M	23.6M	61.1	14400.0	38.1M	15.2M	149.7	14400.0
dem_100_newyork+_7	50.8M	13.2M	284.7	14400.0	41.6M	11.9M	248.7	14400.0	29.6M	7455.1k	297.2	14400.0
dem_100_newyork+_8	38.5M	14.7M	162.7	14400.0	25.0M	14.2M	76.2	14400.0	26.2M	7412.8k	253.1	14400.0
dem_100_newyork+_9	20.0M	20.0M	0.0	1702.0	20.0M	13.0M	54.5	14400.1	20.0M	7130.6k	181.1	14400.0
dem_100_newyork+_10	47.3M	32.6M	45.4	14400.0	46.7M	33.1M	40.9	14400.0	46.8M	21.4M	119.0	14400.0
dem_100_newyork+_11	40.0M	18.6M	114.6	14400.0	36.0M	28.7M	25.5	14400.0	39.2M	13.0M	200.9	14400.0
dem_100_newyork+_12	48.3M	25.6M	88.6	14400.0	48.2M	36.7M	31.5	14400.0	48.2M	21.7M	122.6	14400.0
dem_100_newyork+_13	35.1M	25.9M	35.7	14400.0	32.8M	28.2M	16.1	14400.0	32.8M	12.5M	162.1	14400.0
dem_100_newyork+_14	27.5M	27.5M	0.0	7426.7	27.5M	24.7M	11.7	14400.0	28.9M	6810.1k	324.8	14400.0
dem_100_newyork+_15	25.3M	25.3M	0.0	5954.4	47.6M	14.1M	237.9	14400.0	25.3M	10.3M	145.1	14400.0
dem_100_newyork+_16	39.8M	16.0M	149.1	14400.0	28.9M	28.9M	0.0	7335.7	29.2M	11.9M	145.8	14400.0
dem_100_newyork+_17	39.2M	27.8M	40.7	14400.0	38.3M	26.4M	44.9	14400.0	38.3M	23.2M	65.2	14400.0
dem_100_newyork+_18	30.8M	10.4M	195.7	14400.0	30.7M	26.0M	18.2	14400.2	43.4M	5693.6k	662.7	14400.0
dem_100_newyork+_19	42.3M	23.4M	81.2	14400.3	111.3M	22.8M	388.8	14400.0	44.2M	20.1M	119.7	14400.0
dem_100_newyork+_20	40.2M	25.3M	59.2	14400.1	40.2M	28.5M	41.1	14404.4	40.2M	18.3M	120.1	14400.0
dem_100_newyork+_21	39.8M	20.3M	95.6	14400.0	70.0M	17.4M	302.5	14400.0	32.1M	15.5M	107.8	14400.0
dem_100_newyork+_22	31.0M	24.7M	25.6	14400.0	35.0M	16.0M	118.4	14400.0	31.1M	13.6M	129.1	14400.0
dem_100_newyork+_23	43.4M	26.8M	62.1	14400.0	43.4M	30.7M	41.4	14400.0	42.9M	17.8M	141.1	14400.0
dem_100_newyork+_24	20.0M	20.0M	0.0	292.9	20.0M	17.2M	16.8	14400.0	20.0M	20.0M	0.0	6601.1
dem_100_newyork+_25	58.3M	27.9M	109.1	14400.0	47.2M	39.8M	18.7	14400.0	47.2M	22.8M	107.0	14400.0
dem_100_newyork+_26	39.1M	32.2M	21.4	14400.0	39.1M	28.5M	37.5	14400.0	39.1M	20.7M	89.3	14400.0
dem_100_newyork+_27	36.8M	19.2M	91.9	14400.0	38.8M	17.4M	123.0	14400.0	30.0M	16.7M	79.1	14400.0
dem_100_newyork+_28	46.3M	22.0M	110.7	14400.0	46.4M	23.5M	97.7	14400.0	38.4M	21.5M	78.3	14400.0
dem_100_newyork+_29	69.2M	31.3M	121.2	14400.1	47.2M	35.9M	31.3	14400.0	47.2M	27.3M	72.9	14400.0
dem_100_newyork+_30	31.9M	31.9M	0.0	10121.8	31.9M	24.7M	29.1	14400.0	34.9M	13.3M	161.6	14400.0
dem_100_newyork+_31	38.7M	22.7M	70.6	14400.0	37.1M	37.1M	0.0	10406.3	37.7M	13.1M	188.0	14400.0
dem_100_newyork+_32	37.9M	15.4M	145.3	14400.0	24.6M	12.6M	96.2	14400.0	30.8M	7817.2k	293.4	14400.0
dem_100_newyork+_33	126.0M	18.8M	571.1	14400.0	33.8M	29.8M	13.7	14401.6	39.9M	15.2M	162.4	14400.0
dem_100_newyork+_34	32.9M	21.7M	51.7	14400.0	32.9M	25.0M	31.4	14400.0	32.9M	14.5M	127.1	14400.0
dem_100_newyork+_35	60.5M	27.0M	124.3	14400.0	45.6M	32.5M	40.2	14400.0	45.6M	25.3M	79.9	14400.0
dem_100_newyork+_36	49.7M	22.5M	120.6	14400.0	37.6M	30.9M	21.7	14400.0	37.7M	18.2M	108.0	14400.0
dem_100_newyork+_37	37.0M	18.1M	104.3	14400.0	34.9M	25.9M	34.6	14400.0	34.9M	14.0M	149.3	14400.0
dem_100_newyork+_38	39.1M	25.6M	52.9	14400.0	39.2M	25.3M	54.7	14400.0	40.2M	21.6M	86.3	14400.0
dem_100_newyork+_39	104.8M	21.3M	392.4	14400.0	34.9M	30.8M	13.3	14400.3	37.1M	15.1M	144.9	14400.0
dem_100_newyork+_40	30.2M	20.2M	49.9	14400.0	35.1M	13.7M	156.0	14400.0	28.0M	13.4M	108.9	14400.0
dem_100_newyork+_41	37.4M	20.5M	82.3	14400.0	35.1M	29.9M	17.4	14400.0	43.8M	14.9M	193.4	14400.0
dem_100_newyork+_42	42.4M	24.9M	70.4	14400.0	40.2M	25.9M	55.1	14400.0	40.2M	20.1M	100.1	14400.0
dem_100_newyork+_43	33.8M	21.6M	56.6	14400.0	33.8M	22.3M	51.7	14400.0	34.7M	15.8M	119.1	14400.0
dem_100_newyork+_44	33.8M	21.2M	59.7	14400.0	33.8M	27.9M	21.2	14403.9	57.5M	11.7M	392.4	14400.0
dem_100_newyork+_45	30.7M	30.7M	0.0	14245.4	30.7M	20.4M	50.3	14400.0	30.7M	15.9M	93.6	14400.0
dem_100_newyork+_46	19.8M	13.1M	51.5	14400.0	19.4M	4667.5k	316.0	14400.0	16.4M	6170.8k	165.6	14400.0
dem_100_newyork+_47	61.3M	22.8M	169.2	14400.0	42.9M	39.5M	8.7	14401.0	42.9M	14.9M	188.5	14400.0
dem_100_newyork+_48	90.0M	24.0M	274.4	14400.0	38.1M	22.9M	66.5	14400.0	40.3M	14.9M	170.5	14400.0
dem_100_newyork+_49	38.0M	22.9M	66.3	14400.0	39.1M	26.9M	45.3	14400.0	38.0M	18.3M	107.6	14400.0
dem_100_newyork+_50	37.9M	22.0M	71.9	14400.0	35.0M	25.3M	38.5	14400.0	36.0M	14.4M	150.0	14400.0
dem_100_newyork+_51	67.1M	19.3M	247.4	14400.0	34.9M	25.2M	38.4	14400.0	34.9M	17.6M	98.3	14400.0
dem_100_newyork+_52	32.9M	22.8M	44.2	14400.0	53.9M	19.2M	180.6	14400.0	33.1M	14.0M	137.0	14400.0
dem_100_newyork+_53	48.9M	27.7M	76.7	14400.0	48.3M	42.5M	13.6	14400.0	49.5M	22.5M	119.7	14400.0
dem_100_newyork+_54	32.9M	22.1M	49.0	14400.0	32.8M	22.0M	49.0	14400.0	32.9M	12.5M	164.2	14400.0
dem_100_newyork+_55	41.7M	15.9M	163.1	14400.0	45.3M	14.6M	210.9	14400.0	28.0M	9918.0k	182.5	14400.0
dem_100_newyork+_56	33.9M	20.1M	68.3	14400.0	33.9M	22.4M	51.3	14400.0	33.9M	14.9M	127.9	14400.0
dem_100_newyork+_57	52.9M	25.0M	111.3	14400.0	45.6M	33.6M	35.5	14403.8	43.8M	16.6M	164.1	14400.0
dem_100_newyork+_58	51.5M	21.7M	137.1	14400.0	38.1M	30.7M	24.0	14402.4	51.9M	11.6M	347.0	14400.0
dem_100_newyork+_59	23.7M	16.5M	43.3	14400.0	23.7M	13.2M	79.2	14400.0	23.7M	10.8M	118.1	14400.0
dem_100_newyork+_60	98.7M	21.9M	351.6	14400.0	44.6M	44.6M	0.0	10248.7	44.6M	16.0M	178.8	14400.0
dem_100_newyork+_61	49.7M	21.9M	126.5	14400.0	39.2M	34.2M	14.5	14400.1	39.9M	13.3M	200.3	14400.0
dem_100_newyork+_62	45.5M	22.5M	102.0	14400.0	37.0M	33.6M	10.1	14400.0	38.1M	16.7M	128.9	14400.0
dem_100_newyork+_63	47.2M	27.0M	74.9	14400.0	44.2M	33.0M	33.9	14400.0	45.6M	25.9M	76.1	14400.0
dem_100_newyork+_64	25.9M	25.9M	0.0	4246.6	25.9M	16.2M	59.6	14400.0	26.5M	5852.4k	352.1	14400.0
dem_100_newyork+_65	50.2M	24.6M	103.7	14400.0	43.5M	43.5M	0.0	9247.2	43.5M	16.3M	166.2	14400.0
dem_100_newyork+_66	69.2M	20.1M	244.9	14400.0	44.5M	32.0M	39.3	14400.0	45.6M	20.0M	128.1	14400.0
dem_100_newyork+_67	46.6M	20.9M	122.4	14400.0	39.9M	20.1M	98.7	14400.0	34.0M	15.4M	121.0	14400.0
dem_100_newyork+_68	38.7M	18.6M	108.1	14400.0	38.2M	38.2M	0.0	12097.4	39.1M	14.9M	162.7	14400.0
dem_100_newyork+_69	39.4M	25.4M	55.0	14400.0	38.1M	25.8M	47.7	14400.0	44.5M	14.7M	202.0	14400.0
dem_100_newyork+_70	30.5M	20.2M	50.8	14400.0	30.6M	12.6M	143.0	14400.0	28.0M	11.8M	137.1	14400.0
dem_100_newyork+_71	35.1M	26.5M	32.6	14400.0	33.9M	29.4M	15.2	14400.0	34.9M	16.9M	105.8	14400.0
dem_100_newyork+_72	151.1M	22.0M	587.0	14410.7	38.8M	33.5M	15.8	14400.0	39.2M	14.6M	167.7	14400.0
dem_100_newyork+_73	55.7M	19.9M	180.1	14400.0	38.1M	38.1M	0.0	8575.2	49.7M	10.0M	394.9	14400.0
dem_100_newyork+_74	20.3M	20.3M	0.0	1188.6	20.3M	18.2M	11.8	14400.0	20.3M	20.3M	0.0	8594.0

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Table A.41: Comparison of discrete models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_75	25.9M	18.9M	37.1	14400.0	26.7M	13.9M	91.5	14400.0	25.6M	11.5M	123.2	14400.0
dem_100_newyork+_76	48.3M	19.7M	144.6	14400.0	35.9M	25.5M	40.6	14400.0	36.0M	11.8M	205.8	14400.0
dem_100_newyork+_77	41.5M	20.2M	105.7	14400.0	1e+20	21.8M	–	14400.0	36.0M	16.2M	122.7	14400.0
dem_100_newyork+_78	115.9M	26.1M	343.8	14400.0	41.9M	39.9M	5.0	14400.1	43.4M	20.4M	112.9	14400.0
dem_100_newyork+_79	97.7M	21.1M	362.0	14400.0	39.1M	23.3M	68.0	14400.0	37.0M	15.6M	137.5	14400.0
dem_100_newyork+_80	36.6M	18.4M	98.6	14423.4	29.4M	13.4M	119.5	14405.2	29.9M	4857.3k	515.2	14400.0
dem_100_newyork+_81	28.9M	20.6M	40.1	14400.0	50.9M	13.6M	273.0	14400.0	36.5M	11.3M	222.1	14400.0
dem_100_newyork+_82	41.4M	26.1M	58.5	14400.0	41.2M	37.6M	9.6	14400.0	41.4M	20.6M	100.6	14400.0
dem_100_newyork+_83	31.6M	20.0M	58.0	14400.0	28.9M	17.7M	63.1	14400.0	28.9M	10.1M	185.8	14400.0
dem_100_newyork+_84	25.7M	25.7M	0.0	3976.7	25.7M	14.8M	73.3	14400.0	37.9M	6935.4k	445.9	14400.0
dem_100_newyork+_85	105.0M	25.9M	305.1	14424.4	38.1M	31.7M	20.2	14400.0	52.2M	19.4M	169.0	14400.0
dem_100_newyork+_86	66.4M	22.0M	202.0	14400.0	41.3M	32.7M	26.3	14400.0	41.3M	18.3M	126.4	14400.0
dem_100_newyork+_87	50.4M	27.1M	86.0	14400.0	50.4M	50.4M	0.0	10324.2	50.4M	26.8M	88.3	14400.0
dem_100_newyork+_88	37.0M	22.5M	64.2	14400.0	37.0M	32.8M	12.6	14401.2	37.0M	18.8M	96.6	14400.0
dem_100_newyork+_89	36.1M	28.4M	27.3	14400.0	36.1M	20.6M	75.7	14400.0	37.2M	16.9M	120.9	14400.0
dem_100_newyork+_90	27.4M	23.4M	17.0	14400.0	26.4M	17.0M	54.7	14400.0	36.3M	11.4M	217.1	14400.0
dem_100_newyork+_91	38.1M	24.2M	57.7	14400.0	38.7M	27.6M	40.1	14404.1	38.1M	18.3M	108.0	14400.0
dem_100_newyork+_92	60.9M	24.8M	146.1	14400.0	39.8M	33.5M	18.8	14400.0	51.9M	12.1M	329.7	14400.0
dem_100_newyork+_93	42.9M	27.4M	56.6	14400.0	44.4M	28.2M	57.2	14400.0	44.1M	21.5M	104.9	14400.0
dem_100_newyork+_94	61.4M	20.8M	195.7	14400.0	42.5M	42.5M	0.0	13856.1	59.3M	16.9M	250.3	14400.0
dem_100_newyork+_95	43.1M	24.8M	73.6	14400.0	37.0M	29.8M	24.2	14400.0	39.9M	13.0M	208.0	14400.0
dem_100_newyork+_96	43.7M	26.4M	65.2	14400.0	46.1M	26.9M	71.3	14400.0	42.4M	16.8M	152.1	14400.0
dem_100_newyork+_97	42.3M	26.5M	59.3	14400.0	41.2M	31.4M	31.1	14400.0	41.2M	22.7M	81.4	14400.0
dem_100_newyork+_98	39.2M	23.1M	69.4	14400.0	38.1M	24.4M	55.8	14400.0	38.1M	17.6M	116.0	14400.0
dem_100_newyork+_99	64.6M	26.4M	144.9	14400.0	46.2M	46.2M	0.0	13891.9	46.2M	25.2M	83.1	14400.0
dem_100_newyork+_100	55.4M	21.3M	159.6	14400.0	35.9M	33.7M	6.8	14400.0	35.9M	12.1M	196.4	14400.0
dem_100_newyork+_101	44.7M	26.3M	69.6	14400.0	44.0M	33.1M	32.7	14403.3	44.7M	19.3M	131.9	14400.0
dem_100_newyork+_102	27.4M	19.0M	44.7	14400.0	30.4M	15.0M	102.4	14400.0	26.5M	12.7M	108.4	14400.0
dem_100_newyork+_103	56.9M	16.7M	241.5	14400.0	26.9M	15.6M	72.6	14400.0	28.9M	11.1M	161.0	14400.0
dem_100_newyork+_104	38.1M	22.2M	71.7	14419.5	33.8M	24.0M	40.8	14400.0	34.9M	6389.4k	446.5	14400.0
dem_100_newyork+_105	33.0M	24.1M	37.1	14400.0	31.9M	31.9M	0.0	11979.5	38.2M	13.6M	180.9	14400.0
dem_100_newyork+_106	32.0M	22.1M	44.9	14400.0	47.3M	18.4M	157.4	14400.0	32.4M	12.7M	155.8	14400.0
dem_100_newyork+_107	48.1M	24.9M	93.0	14400.0	35.9M	31.5M	14.0	14400.0	35.9M	19.9M	80.6	14400.0
dem_100_newyork+_108	34.0M	22.2M	53.2	14400.0	27.7M	20.2M	37.2	14400.0	31.3M	14.1M	122.5	14400.0
dem_100_newyork+_109	55.0M	24.3M	125.8	14400.0	41.3M	35.3M	17.0	14400.0	45.9M	17.1M	167.7	14400.0
dem_100_newyork+_110	49.7M	23.6M	110.8	14400.0	40.2M	28.9M	39.2	14400.0	40.2M	16.3M	147.2	14400.0
dem_100_newyork+_111	125.2M	20.5M	509.7	14400.0	40.8M	30.8M	32.4	14400.0	46.9M	14.6M	220.5	14400.0
dem_100_newyork+_112	37.5M	20.5M	83.1	14400.0	46.7M	22.8M	104.4	14400.0	34.9M	15.6M	123.8	14400.0
dem_100_newyork+_113	32.8M	32.8M	0.0	8411.2	32.8M	32.8M	0.0	9214.4	37.8M	10.5M	260.4	14400.0
dem_100_newyork+_114	69.3M	21.4M	223.7	14423.8	42.4M	19.8M	114.3	14400.0	35.6M	17.2M	107.4	14400.0
dem_100_newyork+_115	64.6M	24.4M	164.5	14400.0	49.8M	49.8M	0.0	7408.0	64.8M	22.9M	183.0	14400.0
dem_100_newyork+_116	44.5M	25.2M	76.7	14400.0	106.7M	33.0M	222.9	14400.0	55.0M	18.7M	194.2	14400.0
dem_100_newyork+_117	31.3M	21.4M	46.5	14400.0	36.2M	14.3M	154.0	14400.0	30.2M	12.5M	142.1	14400.0
dem_100_newyork+_118	36.1M	20.9M	72.6	14400.0	36.1M	31.9M	13.2	14400.2	49.7M	17.1M	190.7	14400.0
dem_100_newyork+_119	102.5M	12.6M	714.5	14400.3	27.6M	23.1M	19.7	14400.0	31.4M	6904.0k	354.9	14400.0
dem_100_newyork+_120	31.9M	19.5M	63.3	14400.0	31.9M	29.1M	9.6	14400.0	33.3M	12.6M	163.5	14400.0
dem_100_newyork+_121	95.4M	22.2M	330.8	14422.9	38.1M	32.9M	15.8	14400.0	38.1M	19.3M	97.7	14400.0
dem_100_newyork+_122	48.5M	16.3M	198.1	14400.1	28.7M	17.8M	60.8	14400.0	32.4M	7300.6k	344.0	14400.0
dem_100_newyork+_123	41.2M	25.0M	64.7	14400.0	41.2M	31.1M	32.4	14400.0	41.3M	18.7M	121.0	14400.0
dem_100_newyork+_124	35.9M	25.2M	42.7	14400.0	44.3M	23.4M	89.5	14400.0	35.9M	12.4M	190.5	14400.0
dem_100_newyork+_125	30.2M	17.6M	71.2	14400.0	24.5M	15.2M	61.3	14400.0	25.1M	14.6M	71.5	14400.0
dem_100_newyork+_126	67.8M	25.7M	163.5	14400.0	46.2M	30.7M	50.3	14400.0	47.3M	19.7M	140.7	14400.0
dem_100_newyork+_127	48.3M	24.4M	97.8	14400.0	48.3M	32.4M	49.1	14400.0	49.5M	25.8M	92.0	14400.0
dem_100_newyork+_128	39.1M	29.1M	34.6	14420.0	38.0M	24.6M	54.4	14400.0	38.1M	16.5M	130.4	14400.0
dem_100_newyork+_129	38.7M	17.9M	115.8	14400.0	25.9M	14.2M	82.4	14400.0	27.8M	10.1M	175.8	14400.0
dem_100_newyork+_130	43.4M	30.3M	43.3	14400.0	43.4M	34.7M	25.0	14404.0	43.4M	23.8M	82.3	14400.0
dem_100_newyork+_131	34.8M	21.3M	63.9	14400.0	41.7M	16.9M	146.5	14400.0	30.5M	11.2M	171.6	14400.0
dem_100_newyork+_132	73.0M	20.4M	257.3	14400.0	41.3M	36.1M	14.2	14400.0	41.3M	20.4M	102.7	14400.0
dem_100_newyork+_133	36.5M	20.2M	80.9	14400.0	30.7M	19.9M	54.5	14400.0	30.7M	7163.8k	328.9	14400.0
dem_100_newyork+_134	36.6M	28.5M	28.5	14400.0	35.9M	24.6M	45.9	14400.0	38.1M	14.6M	160.8	14400.0
dem_100_newyork+_135	38.4M	22.0M	75.0	14400.6	37.7M	23.8M	58.1	14400.0	38.1M	10.4M	267.8	14400.0
dem_100_newyork+_136	83.2M	23.6M	252.5	14400.9	37.0M	23.8M	55.4	14400.0	37.0M	14.9M	149.2	14400.0
dem_100_newyork+_137	34.9M	23.2M	50.7	14400.0	34.9M	27.2M	28.5	14400.0	34.9M	10.9M	221.4	14400.0
dem_100_newyork+_138	60.3M	24.6M	144.9	14400.0	42.5M	38.0M	11.8	14400.0	42.5M	17.9M	136.8	14400.0
dem_100_newyork+_139	34.9M	24.0M	45.6	14400.0	33.9M	25.5M	33.1	14400.0	34.1M	12.1M	182.0	14400.0
dem_100_newyork+_140	38.1M	18.7M	103.8	14400.0	36.3M	26.9M	35.1	14400.0	37.4M	20.4M	83.7	14400.0
dem_100_newyork+_141	31.3M	20.8M	50.3	14400.0	28.7M	26.2M	9.8	14400.1	31.3M	13.1M	139.0	14400.0
dem_100_newyork+_142	54.3M	22.9M	136.7	14400.0	49.8M	19.3M	158.4	14400.0	33.9M	16.9M	100.2	14400.0
dem_100_newyork+_143	47.8M	24.6M	94.5	14400.0	39.1M	29.9M	31.0	14403.0	39.1M	19.2M	104.2	14400.0
dem_100_newyork+_144	61.3M	23.1M	165.6	14400.0	44.0M	32.3M	36.3	14400.0	44.1M	24.5M	79.7	14400.0
dem_100_newyork+_145	64.7M	21.7M	198.6	14400.0	37.2M	26.1M	42.4	14400.0	38.1M	20.7M	84.3	14400.0
dem_100_newyork+_146	31.2M	22.4M	39.0	14400.0	37.3M	16.2M	129.6	14400.0	30.8M	14.1M	119.5	14400.0
dem_100_newyork+_147	41.4M	23.9M	73.5	14400.0	39.8M	31.4M	26.8	14400.0	40.9M	20.1M	103.3	14400.0
dem_100_newyork+_148	38.1M	28.7M	33.0	14400.0	38.1M	27.1M	40.8	14400.0	38.1M	17.3M	120.2	14400.0
dem_100_newyork+_149	56.2M	19.7M	185.7	14400.0	41.3M	22.7M	81.9	14400.0	32.9M	10.9M	202.2	14400.0
dem_100_newyork+_150	48.1M	27.7M	73.7	14400.0	46.8M	32.2M	45.3	14400.0	46.8M	24.7M	89.6	14400.0
dem_100_newyork+_151	35.2M	25.5M	38.0	14400.0	33.9M	20.0M	69.7	14400.0	50.3M	10.2M	394.3	14400.0
dem_100_newyork+_152	25.5M	17.5M	45.8	14400.0	23.1M	11.4M	102.7	14400.0	23.3M	8356.1k	178.5	14400.0

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Table A.41: Comparison of discrete models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_153	33.2M	16.3M	103.8	14400.0	32.5M	13.8M	135.0	14406.1	29.7M	10.5M	182.4	14400.0
dem_100_newyork+_154	33.3M	19.3M	72.0	14400.0	27.5M	16.0M	71.3	14400.0	29.1M	14.0M	108.7	14400.0
dem_100_newyork+_155	114.8M	24.5M	368.6	14400.0	41.8M	27.4M	52.7	14400.0	41.9M	22.9M	82.7	14400.0
dem_100_newyork+_156	73.0M	28.5M	155.6	14420.1	47.2M	34.5M	36.7	14400.0	50.8M	26.6M	91.2	14400.0
dem_100_newyork+_157	76.7M	23.2M	231.0	14400.0	39.2M	27.5M	42.4	14400.0	39.2M	19.3M	102.8	14400.0
dem_100_newyork+_158	37.1M	25.0M	48.6	14400.0	36.7M	31.1M	17.8	14400.0	37.0M	13.6M	171.3	14400.0
dem_100_newyork+_159	61.5M	22.6M	171.6	14400.0	54.7M	20.2M	170.0	14400.0	39.9M	15.5M	157.4	14400.0
dem_100_newyork+_160	54.7M	24.0M	127.3	14400.0	39.1M	25.4M	54.4	14400.0	39.1M	20.7M	89.3	14400.0
dem_100_newyork+_161	40.0M	19.8M	101.5	14400.0	38.7M	35.2M	10.1	14400.0	39.5M	16.8M	134.6	14400.0
dem_100_newyork+_162	38.4M	23.4M	64.2	14400.0	37.6M	29.6M	27.1	14404.4	38.1M	12.3M	210.4	14400.0
dem_100_newyork+_163	58.1M	20.6M	182.5	14400.0	35.0M	25.7M	36.0	14400.0	38.3M	14.3M	167.4	14400.0
dem_100_newyork+_164	63.4M	25.7M	146.4	14400.0	46.1M	46.1M	0.0	9845.1	46.1M	19.5M	136.1	14400.0
dem_100_newyork+_165	51.9M	25.7M	102.0	14400.0	40.2M	28.9M	39.4	14403.5	40.3M	19.2M	110.0	14400.0
dem_100_newyork+_166	44.5M	30.6M	45.5	14400.0	45.0M	31.7M	42.1	14405.0	44.5M	20.1M	121.2	14400.0
dem_100_newyork+_167	46.2M	25.4M	81.9	14400.0	45.0M	33.5M	34.5	14400.0	46.2M	22.7M	103.1	14400.0
dem_100_newyork+_168	89.9M	18.5M	384.6	14400.0	31.7M	17.1M	85.3	14400.0	31.7M	7677.7k	313.0	14400.0
dem_100_newyork+_169	48.4M	23.8M	103.4	14400.0	34.2M	24.1M	41.5	14400.0	50.1M	15.7M	218.7	14400.0
dem_100_newyork+_170	28.7M	28.7M	0.0	8044.2	28.7M	24.7M	16.2	14400.0	28.7M	5744.1k	399.7	14400.0
dem_100_newyork+_171	31.8M	19.5M	63.3	14400.0	31.7M	23.6M	34.1	14404.2	31.7M	10.2M	210.3	14400.0
dem_100_newyork+_172	35.1M	22.1M	58.7	14400.0	39.8M	20.8M	91.9	14400.0	35.2M	10.2M	245.0	14400.0
dem_100_newyork+_173	54.2M	25.1M	115.6	14400.0	42.9M	39.6M	8.3	14400.0	42.9M	22.3M	92.3	14400.0
dem_100_newyork+_174	28.9M	19.4M	49.2	14400.0	58.7M	16.5M	255.8	14400.0	29.8M	8890.2k	235.7	14400.0
dem_100_newyork+_175	76.4M	23.8M	220.2	14400.0	43.5M	28.8M	51.1	14400.0	43.4M	21.5M	101.7	14400.0
dem_100_newyork+_176	32.1M	22.9M	40.2	14400.0	32.0M	20.0M	60.3	14400.0	33.8M	13.6M	148.9	14400.0
dem_100_newyork+_177	38.1M	23.7M	60.6	14400.0	38.1M	28.6M	33.1	14400.0	38.2M	16.5M	131.0	14400.0
dem_100_newyork+_178	47.6M	20.9M	127.2	14400.0	35.9M	28.2M	27.6	14400.0	35.9M	17.2M	109.3	14400.0
dem_100_newyork+_179	86.3M	21.2M	307.1	14400.1	35.9M	28.2M	27.3	14400.0	35.9M	19.6M	83.3	14400.0
dem_100_newyork+_180	31.2M	19.0M	63.8	14400.0	31.0M	18.2M	70.5	14400.0	31.0M	11.5M	169.9	14400.0
dem_100_newyork+_181	49.7M	25.7M	93.6	14400.0	38.2M	31.9M	19.5	14400.0	39.2M	20.4M	92.3	14400.0
dem_100_newyork+_182	28.7M	28.7M	0.0	8392.8	28.7M	24.2M	18.7	14400.0	30.1M	8960.3k	235.9	14400.0
dem_100_newyork+_183	45.5M	25.4M	79.4	14400.0	45.5M	45.5M	0.0	13523.2	46.1M	20.0M	130.8	14400.0
dem_100_newyork+_184	40.2M	25.2M	59.7	14400.0	40.2M	27.0M	48.9	14400.0	41.4M	16.7M	147.8	14400.0
dem_100_newyork+_185	50.4M	21.5M	134.1	14400.0	41.3M	22.6M	82.5	14400.0	40.2M	16.5M	144.1	14400.0
dem_100_newyork+_186	40.2M	28.2M	42.2	14400.0	40.2M	29.9M	34.3	14400.0	40.2M	19.2M	109.1	14400.0
dem_100_newyork+_187	27.2M	27.2M	0.0	10387.3	27.2M	18.4M	47.8	14400.0	27.5M	14.4M	91.3	14400.0
dem_100_newyork+_188	39.1M	24.2M	61.9	14400.0	39.1M	25.6M	53.0	14400.0	39.1M	19.6M	99.5	14400.0
dem_100_newyork+_189	34.9M	23.2M	50.5	14400.0	34.9M	24.3M	43.4	14400.0	34.9M	14.6M	139.9	14400.0
dem_100_newyork+_190	25.6M	17.0M	51.1	14400.0	23.8M	18.6M	28.5	14400.0	24.6M	9382.5k	162.5	14400.0
dem_100_newyork+_191	37.4M	26.2M	42.7	14400.0	39.6M	23.8M	66.4	14400.0	37.1M	20.2M	83.6	14400.0
dem_100_newyork+_192	49.2M	18.6M	165.3	14400.0	35.9M	35.9M	0.0	6962.3	47.6M	11.9M	300.6	14400.0
dem_100_newyork+_193	41.4M	23.7M	74.8	14400.0	41.3M	30.1M	37.1	14400.0	41.4M	17.9M	130.6	14400.0
dem_100_newyork+_194	66.4M	21.6M	207.4	14400.0	34.2M	23.3M	46.6	14400.0	33.1M	16.3M	102.6	14400.0
dem_100_newyork+_195	65.9M	28.3M	132.8	14400.0	45.1M	35.6M	26.8	14400.0	48.3M	19.2M	151.2	14400.0
dem_100_newyork+_196	36.3M	20.6M	76.6	14400.0	37.5M	13.2M	183.4	14400.0	30.1M	9154.8k	228.2	14400.0
dem_100_newyork+_197	40.3M	20.2M	99.6	14400.0	49.5M	28.1M	76.3	14400.0	41.3M	18.7M	120.6	14400.0
dem_100_newyork+_198	37.0M	26.7M	38.8	14400.0	36.1M	24.9M	45.1	14400.0	37.0M	17.5M	111.3	14400.0
dem_100_newyork+_199	51.9M	22.1M	135.0	14400.0	40.3M	36.0M	11.7	14400.0	42.1M	19.9M	111.2	14400.0
dem_100_newyork+_200	46.5M	22.8M	104.0	14400.0	57.1M	24.7M	131.4	14400.0	47.4M	19.0M	149.7	14400.0
dem_100_newyork+_201	29.8M	29.8M	0.0	12259.0	29.8M	25.9M	15.3	14401.7	30.7M	12.2M	151.2	14400.0
dem_100_newyork+_202	27.5M	8988.4k	205.9	14400.0	27.4M	17.2M	59.2	14404.9	28.5M	11.1M	157.5	14400.0
dem_100_newyork+_203	68.6M	23.3M	193.9	14400.0	35.0M	28.8M	21.8	14400.0	37.0M	18.9M	95.7	14400.0
dem_100_newyork+_204	49.6M	21.0M	136.3	14400.0	41.3M	31.1M	32.9	14400.0	42.7M	20.3M	110.9	14400.0
dem_100_newyork+_205	65.8M	26.1M	152.0	14400.0	41.2M	26.2M	57.1	14400.0	41.2M	17.2M	140.2	14400.0
dem_100_newyork+_206	45.1M	23.7M	90.5	14400.0	63.9M	25.3M	152.3	14400.0	37.0M	18.8M	97.0	14400.0
dem_100_newyork+_207	34.1M	19.6M	74.2	14400.0	32.8M	21.3M	54.0	14400.0	45.5M	8990.7k	405.8	14400.0
dem_100_newyork+_208	86.3M	21.8M	296.5	14400.0	37.1M	37.1M	0.0	11794.1	37.2M	16.0M	132.5	14400.0
dem_100_newyork+_209	60.0M	15.4M	288.3	14400.0	28.7M	20.8M	37.8	14400.0	28.7M	11.1M	157.5	14400.0
dem_100_newyork+_210	43.4M	24.8M	74.7	14400.0	42.4M	30.5M	39.4	14400.0	43.4M	18.3M	136.9	14400.0
dem_100_newyork+_211	130.7M	20.2M	547.7	14400.0	44.5M	31.2M	42.7	14402.4	45.7M	18.1M	152.7	14400.0
dem_100_newyork+_212	56.3M	23.7M	137.7	14400.0	37.7M	25.5M	47.6	14400.0	37.4M	15.6M	140.3	14400.0
dem_100_newyork+_213	39.8M	24.7M	61.0	14400.0	45.0M	25.2M	78.3	14400.0	39.3M	16.9M	132.8	14400.0
dem_100_newyork+_214	27.5M	22.1M	24.1	14400.0	36.1M	15.6M	130.8	14400.0	27.5M	14.7M	86.9	14400.0
dem_100_newyork+_215	49.7M	22.7M	119.1	14400.0	39.2M	30.4M	29.0	14400.0	49.7M	14.3M	248.5	14400.0
dem_100_newyork+_216	35.9M	26.8M	34.3	14400.0	35.9M	35.9M	0.0	9138.7	35.9M	15.9M	125.3	14400.0
dem_100_newyork+_217	39.2M	20.3M	92.8	14400.0	38.6M	36.4M	6.2	14400.8	39.2M	14.8M	164.2	14400.0
dem_100_newyork+_218	60.0M	23.5M	155.8	14400.0	40.2M	34.8M	15.7	14400.0	41.0M	19.0M	115.9	14400.0
dem_100_newyork+_219	47.6M	16.4M	189.4	14400.0	35.1M	30.5M	14.8	14400.0	35.9M	9933.4k	261.7	14400.0
dem_100_newyork+_220	102.5M	21.8M	369.4	14422.3	37.0M	26.2M	41.3	14400.0	48.7M	12.2M	299.4	14400.0
dem_100_newyork+_221	32.9M	22.1M	48.7	14400.0	32.8M	30.1M	8.9	14400.1	32.8M	12.7M	158.7	14400.0
dem_100_newyork+_222	65.9M	23.8M	176.8	14400.0	40.9M	40.9M	0.0	13791.8	41.3M	16.6M	148.2	14400.0
dem_100_newyork+_223	21.4M	21.4M	0.0	3673.4	24.9M	11.9M	108.1	14400.0	25.2M	10.7M	136.6	14400.0
dem_100_newyork+_224	50.3M	26.9M	87.1	14400.0	44.1M	39.0M	12.9	14400.0	44.1M	18.3M	140.6	14400.0
dem_100_newyork+_225	20.5M	16.0M	28.1	1774.6	16.0M	8436.5k	89.3	14400.0	16.0M	7393.0k	116.1	14400.0
dem_100_newyork+_226	38.2M	20.9M	83.0	14400.0	36.0M	22.4M	60.8	14400.0	34.4M	12.8M	168.1	14400.0
dem_100_newyork+_227	31.5M	16.4M	92.1	14400.0	29.7M	15.6M	90.2	14400.0	29.7M	9467.0k	213.5	14400.0
dem_100_newyork+_228	28.8M	26.0M	10.7	14400.0	56.2M	14.2M	296.3	14400.0	31.3M	12.0M	160.6	14400.0
dem_100_newyork+_229	33.0M	21.3M	54.8	14400.0	32.1M	15.5M	106.9	14400.0	30.1M	11.6M	158.9	14400.0
dem_100_newyork+_230	32.9M	32.9M	0.0	13418.3	32.9M	22.5M	46.0	14400.0	33.9M	12.2M	178.7	14400.0

continued on next page

Table A.41: Comparison of discrete models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_231	35.4M	20.1M	75.6	14400.0	30.8M	16.5M	87.4	14400.0	28.7M	15.8M	80.9	14400.0
dem_100_newyork+_232	52.7M	24.4M	116.5	14400.0	43.5M	30.1M	44.5	14400.0	44.1M	21.4M	105.8	14400.0
dem_100_newyork+_233	38.1M	24.9M	53.1	14400.0	38.0M	25.7M	47.8	14400.0	38.4M	13.7M	180.4	14400.0
dem_100_newyork+_234	31.9M	20.3M	57.7	14400.0	27.6M	23.9M	15.9	14400.0	27.6M	11.6M	137.5	14400.0
dem_100_newyork+_235	32.9M	19.0M	73.4	14400.0	60.3M	23.2M	159.8	14400.0	32.9M	12.9M	156.0	14400.0
dem_100_newyork+_236	40.5M	22.0M	84.0	14400.0	38.7M	23.2M	66.4	14400.0	39.3M	18.2M	116.6	14400.0
dem_100_newyork+_237	41.3M	28.5M	44.8	14400.0	49.6M	25.2M	96.8	14400.0	41.3M	21.0M	97.1	14400.0
dem_100_newyork+_238	100.4M	20.2M	397.8	14400.4	39.1M	26.4M	48.6	14400.0	39.1M	17.3M	126.1	14400.0
dem_100_newyork+_239	47.4M	19.4M	144.1	14400.0	34.0M	21.5M	57.9	14400.0	35.0M	12.2M	187.1	14400.0
dem_100_newyork+_240	41.4M	27.0M	53.5	14419.1	41.3M	24.3M	69.9	14400.0	41.3M	19.9M	108.2	14400.0
dem_100_newyork+_241	51.9M	24.9M	108.1	14400.0	40.8M	36.8M	10.6	14400.1	40.8M	16.8M	142.6	14400.0
dem_100_newyork+_242	33.8M	24.5M	38.1	14400.0	1e+20	17.2M	–	0.0	33.9M	14.3M	136.6	14400.0
dem_100_newyork+_243	67.6M	20.8M	224.6	14400.0	40.9M	26.7M	53.3	14400.0	43.7M	16.4M	165.8	14400.0
dem_100_newyork+_244	66.7M	19.2M	247.1	14400.6	28.8M	23.7M	21.5	14400.0	28.8M	10.1M	185.6	14400.0
dem_100_newyork+_245	33.8M	21.4M	58.1	14400.0	33.8M	21.2M	59.9	14400.0	34.9M	13.2M	164.4	14400.0
dem_100_newyork+_246	42.5M	22.2M	91.4	14400.0	44.2M	23.4M	88.9	14400.0	38.1M	16.9M	125.9	14400.0
dem_100_newyork+_247	42.5M	26.8M	58.6	14400.0	41.4M	30.1M	37.4	14400.0	41.5M	18.4M	126.4	14400.0
dem_100_newyork+_248	31.9M	22.6M	41.3	14400.0	31.9M	31.9M	0.0	12689.3	37.2M	17.2M	116.3	14400.0
dem_100_newyork+_249	41.2M	24.2M	70.6	14400.0	41.2M	30.0M	37.3	14400.0	41.2M	17.9M	129.8	14400.0
dem_100_newyork+_250	46.2M	23.2M	99.2	14400.0	37.4M	23.7M	58.0	14400.0	37.4M	19.6M	91.2	14400.0
dem_100_newyork+_251	39.0M	23.2M	68.4	14400.0	35.9M	22.5M	59.8	14400.0	35.9M	17.1M	110.2	14400.0
dem_100_newyork+_252	28.7M	16.7M	72.0	14400.0	54.7M	18.6M	193.3	14400.0	35.8M	12.6M	183.8	14400.0
dem_100_newyork+_253	66.2M	24.8M	166.6	14401.1	38.1M	26.9M	41.3	14400.0	40.4M	17.6M	129.3	14400.0
dem_100_newyork+_254	36.2M	13.2M	174.9	14400.0	31.7M	24.8M	27.8	14400.0	34.1M	11.5M	197.2	14400.0
dem_100_newyork+_255	67.3M	25.8M	160.9	14400.0	47.2M	43.9M	7.5	14400.0	52.6M	24.7M	112.9	14400.0
dem_100_newyork+_256	41.7M	22.4M	86.2	14400.0	34.9M	29.3M	19.3	14400.0	34.9M	16.9M	107.2	14400.0
dem_100_newyork+_257	28.0M	28.0M	0.0	7831.4	49.9M	18.2M	173.4	14400.0	28.7M	13.5M	111.9	14400.0
dem_100_newyork+_258	41.3M	25.9M	59.3	14400.0	65.5M	24.1M	172.1	14400.0	40.3M	23.3M	72.5	14400.0
dem_100_newyork+_259	23.6M	23.6M	0.0	4561.3	27.4M	13.9M	96.5	14400.0	23.6M	13.8M	70.3	14400.0
dem_100_newyork+_260	54.2M	27.6M	96.1	14400.0	43.5M	32.3M	34.7	14400.0	43.5M	17.8M	144.2	14400.0
dem_100_newyork+_261	41.2M	22.8M	80.4	14400.0	38.7M	38.7M	0.0	5827.8	40.9M	17.4M	135.3	14400.0
dem_100_newyork+_262	30.8M	21.2M	45.0	14400.0	29.7M	19.3M	53.5	14400.0	29.7M	4622.1k	542.0	14400.0
dem_100_newyork+_263	33.5M	16.5M	102.7	14400.0	30.7M	18.6M	65.3	14400.0	43.4M	7027.5k	518.2	14400.0
dem_100_newyork+_264	27.6M	27.6M	0.0	9879.3	27.7M	13.2M	110.3	14400.0	27.6M	10.9M	154.0	14400.0
dem_100_newyork+_265	44.6M	23.5M	89.7	14400.0	36.0M	31.9M	12.9	14400.0	36.1M	13.2M	174.2	14400.0
dem_100_newyork+_266	33.2M	22.6M	47.1	14400.0	33.5M	18.4M	81.8	14400.0	31.7M	16.1M	96.5	14400.0
dem_100_newyork+_267	45.3M	17.3M	161.9	14400.0	26.9M	12.9M	107.9	14400.0	29.0M	10.8M	169.3	14400.0
dem_100_newyork+_268	50.1M	19.3M	159.8	14400.0	35.7M	20.2M	77.0	14400.0	39.0M	15.9M	145.1	14400.0
dem_100_newyork+_269	32.8M	22.2M	47.5	14400.0	30.7M	21.7M	41.3	14400.0	30.7M	12.8M	139.3	14400.0
dem_100_newyork+_270	37.0M	22.0M	68.3	14400.0	40.2M	22.9M	75.5	14400.0	37.0M	16.3M	126.5	14400.0
dem_100_newyork+_271	34.9M	19.0M	83.9	14400.0	34.9M	24.6M	42.0	14406.3	35.6M	17.0M	109.5	14400.0
dem_100_newyork+_272	41.1M	20.4M	101.1	14400.0	37.2M	21.7M	71.5	14400.0	38.4M	13.9M	176.3	14400.0
dem_100_newyork+_273	46.1M	21.5M	114.3	14400.0	34.0M	23.8M	42.8	14400.0	38.9M	16.3M	138.7	14400.0
dem_100_newyork+_274	34.9M	25.5M	36.8	14400.0	33.8M	24.7M	36.8	14400.0	33.8M	15.5M	118.7	14400.0
dem_100_newyork+_275	76.4M	18.9M	304.1	14400.0	38.7M	38.7M	0.0	11273.6	39.3M	18.3M	114.5	14400.0
dem_100_newyork+_276	49.7M	21.5M	131.3	14400.5	38.1M	32.6M	16.8	14401.8	38.2M	15.8M	141.2	14400.0
dem_100_newyork+_277	42.5M	27.3M	55.5	14400.0	41.4M	37.3M	10.9	14400.0	41.4M	21.5M	92.4	14400.0
dem_100_newyork+_278	43.4M	29.2M	48.6	14400.0	52.8M	25.7M	105.3	14400.0	44.5M	21.2M	109.8	14400.0
dem_100_newyork+_279	43.4M	26.8M	62.1	14400.0	43.4M	35.7M	21.5	14400.0	43.4M	23.3M	86.1	14400.0
dem_100_newyork+_280	32.9M	32.9M	0.0	11613.0	32.9M	26.1M	26.0	14400.0	32.9M	14.2M	132.6	14400.0
dem_100_newyork+_281	51.8M	15.6M	233.0	14400.0	31.2M	16.4M	90.5	14400.0	25.7M	8918.7k	188.0	14400.0
dem_100_newyork+_282	34.9M	21.4M	62.8	14400.0	44.4M	21.8M	103.6	14400.0	34.9M	17.6M	98.5	14400.0
dem_100_newyork+_283	27.5M	17.4M	58.3	14400.0	23.7M	15.7M	50.3	14400.0	23.7M	8082.8k	192.6	14400.0
dem_100_newyork+_284	32.3M	11.4M	183.6	14400.0	29.8M	29.8M	0.0	9517.8	29.8M	6359.7k	369.1	14400.0
dem_100_newyork+_285	45.3M	14.8M	205.4	14400.0	42.0M	9646.8k	335.5	14400.0	27.6M	6386.8k	332.8	14400.0
dem_100_newyork+_286	39.1M	25.0M	56.4	14400.0	39.1M	29.3M	33.7	14400.0	39.1M	19.3M	103.2	14400.0
dem_100_newyork+_287	29.7M	16.2M	83.3	14400.0	28.7M	17.6M	63.0	14400.0	29.7M	6697.0k	343.1	14400.0
dem_100_newyork+_288	37.0M	21.6M	71.7	14400.0	36.1M	28.9M	25.0	14400.0	37.3M	12.9M	188.3	14400.0
dem_100_newyork+_289	45.1M	29.1M	54.9	14400.0	45.6M	30.6M	48.8	14400.0	46.3M	25.5M	81.5	14400.0
dem_100_newyork+_290	62.0M	20.0M	210.6	14400.0	39.2M	39.2M	0.0	3013.7	40.9M	18.9M	117.0	14400.0
dem_100_newyork+_291	36.3M	15.9M	128.3	14400.0	20.0M	17.1M	17.3	14400.1	20.0M	14.8M	35.5	14400.0
dem_100_newyork+_292	57.3M	21.0M	172.3	14400.0	38.9M	38.9M	0.0	11985.0	51.9M	15.1M	243.0	14400.0
dem_100_newyork+_293	33.8M	24.6M	37.3	14400.0	33.8M	26.0M	30.0	14400.0	33.8M	11.3M	199.5	14400.0
dem_100_newyork+_294	36.1M	20.4M	77.3	14400.0	36.0M	20.3M	77.8	14400.0	37.5M	15.1M	149.1	14400.0
dem_100_newyork+_295	33.2M	22.4M	48.0	14400.0	31.7M	27.7M	14.4	14402.2	31.7M	10.7M	195.9	14400.0
dem_100_newyork+_296	33.9M	33.9M	0.0	12031.4	54.9M	16.7M	228.7	14400.0	39.1M	8628.8k	353.7	14400.0
dem_100_newyork+_297	28.7M	28.7M	0.0	11797.0	28.7M	20.0M	43.3	14400.0	29.8M	9734.3k	206.5	14400.0
dem_100_newyork+_298	30.7M	17.6M	74.8	14400.0	26.8M	15.6M	71.6	14400.0	29.2M	6722.1k	334.6	14400.0
dem_100_newyork+_299	38.1M	26.7M	42.7	14400.0	38.1M	28.9M	31.8	14400.0	38.1M	14.0M	172.6	14400.0
dem_100_newyork+_300	33.2M	18.2M	82.5	14400.0	29.7M	18.7M	58.5	14400.0	35.2M	10.7M	228.7	14400.0
dem_100_newyork+_301	35.9M	21.3M	69.1	14400.0	35.9M	32.2M	11.5	14400.0	36.0M	15.6M	130.7	14400.0
dem_100_newyork+_302	54.3M	26.5M	105.1	14400.0	72.3M	30.1M	140.4	14400.0	55.0M	18.6M	195.0	14400.0
dem_100_newyork+_303	71.7M	22.4M	220.1	14417.5	37.1M	26.9M	38.0	14400.0	39.2M	17.6M	122.9	14400.0
dem_100_newyork+_304	53.4M	24.5M	117.6	14400.0	40.0M	21.9M	83.0	14400.0	38.1M	14.0M	171.7	14400.0
dem_100_newyork+_305	30.9M	17.2M	79.3	14400.0	29.8M	15.5M	92.4	14400.0	29.8M	11.0M	171.5	14400.0
dem_100_newyork+_306	34.9M	34.9M	0.0	12262.7	34.9M	34.9M	0.0	8094.2	35.1M	17.4M	101.5	14400.0
dem_100_newyork+_307	38.2M	21.8M	75.2	14400.0	41.4M	24.6M	68.2	14400.0	39.2M	16.4M	138.2	14400.0
dem_100_newyork+_308	47.4M	22.6M	110.1	14400.0	49.9M	20.8M	139.6	14400.0	34.9M	13.5M	159.3	14400.0

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Table A.41: Comparison of discrete models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_309	50.8M	23.8M	113.1	14400.0	37.1M	29.0M	28.1	14400.0	39.2M	17.1M	128.9	14400.0
dem_100_newyork+_310	36.6M	23.0M	59.0	14400.0	32.9M	22.3M	47.9	14400.0	33.0M	17.4M	90.0	14400.0
dem_100_newyork+_311	44.5M	20.4M	118.7	14400.0	41.9M	28.2M	48.4	14400.0	42.4M	15.6M	171.4	14400.0
dem_100_newyork+_312	152.5M	13.7M	1011.1	14400.0	36.0M	28.3M	27.0	14400.0	36.0M	12.2M	195.5	14400.0
dem_100_newyork+_313	42.3M	26.4M	60.3	14400.2	42.3M	31.4M	34.6	14400.0	63.6M	16.0M	297.0	14400.0
dem_100_newyork+_314	60.5M	24.7M	145.0	14400.0	41.3M	34.2M	20.8	14400.0	46.2M	19.6M	135.3	14400.0
dem_100_newyork+_315	32.4M	24.6M	31.5	14415.9	31.0M	22.4M	38.2	14400.0	31.4M	11.2M	180.6	14400.0
dem_100_newyork+_316	103.5M	17.1M	505.1	14400.2	31.9M	26.6M	20.0	14400.0	31.9M	9637.6k	231.5	14400.0
dem_100_newyork+_317	57.0M	26.9M	112.2	14400.0	39.3M	26.2M	50.1	14400.0	39.3M	21.0M	87.1	14400.0
dem_100_newyork+_318	59.3M	21.7M	172.5	14400.0	41.9M	30.8M	35.8	14402.4	59.3M	15.7M	276.7	14400.0
dem_100_newyork+_319	40.4M	28.1M	43.5	14400.0	40.3M	30.0M	34.4	14403.4	40.3M	22.9M	76.0	14400.0
dem_100_newyork+_320	52.5M	20.4M	158.0	14400.0	53.1M	24.3M	118.9	14400.0	45.0M	20.0M	125.0	14400.0
dem_100_newyork+_321	92.6M	22.6M	309.4	14438.0	38.0M	25.2M	50.7	14400.0	39.2M	19.8M	97.6	14400.0
dem_100_newyork+_322	33.1M	21.5M	53.4	14400.0	38.6M	20.4M	88.8	14400.0	33.9M	12.2M	177.5	14400.0
dem_100_newyork+_323	27.9M	27.9M	0.0	13769.1	28.6M	17.4M	64.0	14400.0	32.9M	14.4M	127.5	14400.0
dem_100_newyork+_324	20.0M	20.0M	0.0	946.3	22.0M	11.6M	89.7	14400.0	20.0M	8182.5k	145.0	14400.0
dem_100_newyork+_325	60.6M	22.3M	172.5	14400.0	40.3M	36.5M	10.4	14403.2	40.2M	19.2M	110.0	14400.0
dem_100_newyork+_326	40.4M	20.1M	101.0	14400.0	40.2M	30.0M	33.7	14400.0	40.2M	13.2M	204.2	14400.0
dem_100_newyork+_327	34.3M	20.1M	70.7	14400.0	32.8M	32.8M	0.1	14307.3	32.8M	10.8M	203.7	14400.0
dem_100_newyork+_328	23.5M	23.5M	0.0	1666.6	23.5M	13.4M	75.8	14400.0	24.4M	13.0M	87.0	14400.0
dem_100_newyork+_329	24.1M	24.1M	0.0	1831.4	51.0M	14.5M	250.8	14400.0	24.3M	10.5M	130.5	14400.0
dem_100_newyork+_330	31.0M	20.8M	49.3	14411.7	28.7M	16.7M	71.8	14400.0	30.2M	14.4M	109.7	14400.0
dem_100_newyork+_331	38.1M	24.1M	58.3	14400.0	38.1M	26.4M	44.3	14400.0	38.1M	16.7M	128.5	14400.0
dem_100_newyork+_332	33.1M	22.9M	44.6	14400.0	35.6M	20.2M	76.6	14400.0	35.7M	15.0M	138.4	14400.0
dem_100_newyork+_333	32.3M	21.2M	52.3	14400.0	31.2M	21.7M	43.7	14400.0	34.6M	14.3M	141.0	14400.0
dem_100_newyork+_334	34.1M	23.0M	48.3	14400.0	30.8M	26.7M	15.4	14400.0	30.8M	8719.1k	253.4	14400.0
dem_100_newyork+_335	31.8M	20.0M	58.7	14400.0	31.8M	26.9M	18.0	14400.0	31.8M	12.1M	162.8	14400.0
dem_100_newyork+_336	44.8M	22.9M	96.0	14400.0	38.4M	21.3M	79.7	14400.0	39.1M	16.1M	142.2	14400.0
dem_100_newyork+_337	38.6M	21.1M	83.0	14400.0	36.7M	26.4M	39.2	14400.0	42.7M	17.7M	141.2	14400.0
dem_100_newyork+_338	35.9M	22.1M	62.3	14400.0	35.9M	27.8M	29.1	14400.0	47.6M	11.8M	303.4	14400.0
dem_100_newyork+_339	34.2M	19.7M	73.3	14400.0	32.9M	24.3M	35.7	14400.0	32.9M	14.5M	126.7	14400.0
dem_100_newyork+_340	59.3M	25.1M	136.2	14400.0	39.2M	28.5M	37.8	14404.5	39.2M	16.3M	139.9	14400.0
dem_100_newyork+_341	31.0M	16.0M	93.6	14400.0	31.4M	12.3M	155.0	14400.0	27.4M	9512.4k	187.8	14400.0
dem_100_newyork+_342	64.7M	23.6M	173.9	14400.1	33.2M	26.1M	27.2	14404.6	32.8M	16.9M	93.8	14400.0
dem_100_newyork+_343	37.7M	20.7M	82.0	14400.0	42.6M	23.6M	80.7	14400.0	49.7M	13.5M	269.3	14400.0
dem_100_newyork+_344	23.3M	23.3M	0.0	2696.0	23.3M	19.4M	19.9	14400.1	23.3M	12.5M	85.6	14400.0
dem_100_newyork+_345	41.1M	21.0M	96.0	14400.0	30.8M	23.9M	29.1	14400.0	32.3M	10.5M	208.7	14400.0
dem_100_newyork+_346	30.3M	22.9M	32.4	14400.0	30.1M	17.9M	68.5	14400.0	30.3M	12.7M	137.7	14400.0
dem_100_newyork+_347	68.0M	26.5M	157.0	14400.0	47.7M	47.7M	0.0	9369.9	48.3M	24.2M	99.3	14400.0
dem_100_newyork+_348	35.3M	19.1M	85.2	14400.0	34.9M	25.1M	39.0	14405.0	47.1M	12.4M	281.7	14400.0
dem_100_newyork+_349	30.9M	30.9M	0.0	8182.9	1e+20	15.7M	–	14400.0	30.9M	13.3M	133.1	14400.0
dem_100_newyork+_350	45.9M	19.8M	132.2	14400.0	33.8M	22.3M	51.7	14400.0	34.2M	11.6M	194.4	14400.0
dem_100_newyork+_351	33.2M	19.4M	70.8	14400.0	29.7M	20.5M	45.1	14400.0	29.7M	12.0M	146.6	14400.0
dem_100_newyork+_352	43.5M	27.1M	60.1	14400.0	43.5M	29.2M	48.7	14400.0	43.5M	21.9M	99.0	14400.0
dem_100_newyork+_353	35.2M	20.2M	74.5	14400.0	33.9M	26.5M	28.1	14400.0	34.0M	10.9M	212.2	14400.0
dem_100_newyork+_354	33.0M	18.1M	82.5	14400.0	33.0M	23.8M	38.6	14400.0	33.0M	10.8M	205.0	14400.0
dem_100_newyork+_355	65.3M	25.7M	153.7	14400.0	48.4M	29.4M	64.3	14400.0	44.6M	21.5M	106.9	14400.0
dem_100_newyork+_356	51.3M	21.1M	142.5	14400.0	38.1M	24.8M	53.4	14400.0	38.1M	18.2M	109.6	14400.0
dem_100_newyork+_357	91.0M	21.4M	325.5	14400.0	44.0M	34.2M	28.7	14400.0	44.1M	19.4M	127.0	14400.0
dem_100_newyork+_358	40.2M	26.1M	54.0	14400.0	40.2M	28.2M	42.5	14400.0	40.2M	17.5M	130.2	14400.0
dem_100_newyork+_359	39.1M	25.0M	56.6	14400.0	39.4M	26.2M	50.7	14400.0	42.7M	17.8M	139.6	14400.0
dem_100_newyork+_360	37.6M	23.7M	59.0	14400.0	36.7M	22.1M	66.1	14400.0	35.9M	12.5M	187.4	14400.0
dem_100_newyork+_361	27.9M	27.9M	0.0	4380.8	52.1M	14.9M	250.0	14400.0	29.1M	11.5M	153.7	14400.0
dem_100_newyork+_362	46.7M	18.6M	150.3	14400.0	32.9M	27.9M	17.8	14400.1	34.9M	9863.4k	254.1	14400.0
dem_100_newyork+_363	38.2M	22.1M	72.8	14400.0	71.2M	21.4M	233.0	14400.0	34.9M	16.0M	118.3	14400.0
dem_100_newyork+_364	29.9M	29.9M	0.0	12289.2	29.9M	25.5M	17.0	14400.0	29.9M	12.1M	147.6	14400.0
dem_100_newyork+_365	27.6M	27.6M	0.0	5552.3	27.6M	20.0M	38.3	14400.0	27.6M	16.5M	67.4	14400.0
dem_100_newyork+_366	140.4M	19.1M	633.9	14400.0	61.5M	24.0M	156.7	14400.0	39.8M	15.0M	165.4	14400.0
dem_100_newyork+_367	16.0M	16.0M	0.0	2405.4	24.0M	6489.3k	269.7	14400.0	16.0M	10.6M	50.4	14400.0
dem_100_newyork+_368	32.9M	24.0M	37.3	14400.0	32.9M	20.6M	60.0	14400.0	34.5M	12.5M	175.1	14400.0
dem_100_newyork+_369	52.9M	26.5M	99.5	14400.0	46.7M	31.4M	48.8	14400.0	43.4M	21.0M	106.3	14400.0
dem_100_newyork+_370	38.2M	22.9M	67.2	14400.0	37.6M	22.5M	66.8	14400.0	37.6M	15.9M	136.1	14400.0
dem_100_newyork+_371	41.1M	23.8M	72.8	14400.0	36.1M	24.2M	49.3	14400.0	37.2M	23.0M	61.9	14400.0
dem_100_newyork+_372	21.9M	21.9M	0.0	2235.7	21.9M	14.1M	55.2	14400.0	22.2M	8839.1k	150.7	14400.0
dem_100_newyork+_373	53.3M	26.3M	102.5	14400.0	41.3M	41.3M	0.0	9049.0	41.3M	22.3M	85.2	14400.0
dem_100_newyork+_374	33.1M	20.8M	59.5	14400.0	32.0M	18.6M	72.6	14400.0	33.2M	14.6M	127.3	14400.0
dem_100_newyork+_375	33.8M	19.0M	77.6	14400.0	29.0M	17.7M	63.3	14400.0	32.9M	10.7M	206.6	14400.0
dem_100_newyork+_376	46.0M	19.2M	138.8	14400.0	41.2M	14.8M	178.8	14406.9	34.3M	13.1M	162.6	14400.0
dem_100_newyork+_377	33.6M	17.6M	91.2	14400.0	35.5M	17.0M	109.1	14400.0	30.7M	15.4M	100.0	14400.0
dem_100_newyork+_378	41.6M	23.8M	74.7	14400.0	37.1M	37.1M	0.0	11420.5	37.1M	17.6M	111.4	14400.0
dem_100_newyork+_379	67.1M	21.4M	213.4	14400.0	34.0M	24.0M	42.0	14400.0	40.8M	14.6M	178.5	14400.0
dem_100_newyork+_380	39.5M	19.8M	99.3	14400.0	23.8M	15.5M	54.0	14400.0	23.8M	15.4M	54.4	14400.0
dem_100_newyork+_381	40.3M	20.4M	96.9	14400.0	39.2M	32.5M	20.6	14400.0	39.2M	19.2M	104.2	14400.0
dem_100_newyork+_382	57.2M	21.5M	166.5	14400.0	42.4M	19.1M	121.9	14400.0	40.2M	12.0M	235.3	14400.0
dem_100_newyork+_383	30.8M	30.8M	0.0	10606.4	71.7M	16.4M	337.8	14400.0	31.7M	6173.4k	413.7	14400.0
dem_100_newyork+_384	31.8M	26.5M	19.9	14400.0	30.0M	25.8M	16.5	14400.1	31.6M	8855.9k	256.4	14400.0
dem_100_newyork+_385	57.7M	23.3M	147.7	14400.0	48.2M	43.1M	11.9	14400.1	48.2M	21.3M	126.4	14400.0
dem_100_newyork+_386	130.1M	23.6M	451.9	14400.0	40.3M	28.5M	41.5	14400.0	40.3M	18.4M	118.9	14400.0

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Table A.41: Comparison of discrete models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_387	27.9M	16.1M	73.7	14400.0	26.7M	10.3M	158.1	14400.0	25.0M	9179.4k	172.1	14400.0
dem_100_newyork+_388	31.8M	31.8M	0.0	12145.5	31.8M	20.2M	57.5	14400.0	35.5M	15.9M	122.6	14400.0
dem_100_newyork+_389	31.9M	31.9M	0.0	9093.9	31.9M	27.1M	17.9	14400.1	33.2M	13.1M	153.4	14400.0
dem_100_newyork+_390	42.1M	19.1M	120.1	14400.0	28.9M	18.6M	55.3	14400.0	28.9M	11.5M	151.2	14400.0
dem_100_newyork+_391	38.2M	24.8M	54.3	14400.0	38.1M	28.9M	31.8	14400.0	38.1M	19.9M	91.4	14400.0
dem_100_newyork+_392	51.0M	21.2M	141.0	14400.0	32.9M	20.1M	64.1	14400.0	40.2M	14.4M	178.2	14400.0
dem_100_newyork+_393	22.2M	22.1M	0.3	3548.3	22.1M	15.3M	44.6	14400.0	22.2M	7013.0k	215.9	14400.0
dem_100_newyork+_394	54.7M	26.7M	104.8	14400.0	44.1M	38.8M	13.6	14402.3	44.1M	23.2M	90.2	14400.0
dem_100_newyork+_395	35.1M	24.8M	41.4	14400.0	35.1M	23.4M	49.7	14400.0	38.4M	19.0M	102.2	14400.0
dem_100_newyork+_396	93.7M	13.8M	578.8	14400.0	41.4M	12.7M	225.2	14400.0	25.6M	8864.0k	188.9	14400.0
dem_100_newyork+_397	53.9M	27.2M	98.5	14400.0	141.4M	26.1M	442.0	14400.0	44.5M	23.6M	88.7	14400.0
dem_100_newyork+_398	38.5M	20.9M	83.9	14400.0	34.9M	28.8M	21.2	14403.1	34.9M	16.4M	112.1	14400.0
dem_100_newyork+_399	24.7M	17.9M	38.1	14400.0	35.0M	13.5M	158.1	14400.0	25.6M	6034.3k	324.4	14400.0
dem_100_newyork+_400	37.7M	27.9M	35.5	14400.0	35.9M	23.2M	54.6	14400.0	35.9M	14.7M	145.0	14400.0
dem_100_newyork+_401	64.1M	27.3M	135.3	14400.0	41.2M	29.4M	40.2	14400.0	41.2M	25.9M	59.4	14400.0
dem_100_newyork+_402	34.0M	20.7M	64.3	14400.0	32.9M	17.4M	89.6	14400.0	32.9M	11.6M	184.8	14400.0
dem_100_newyork+_403	23.8M	19.3M	23.4	14410.4	25.3M	12.0M	112.0	14400.0	24.1M	11.2M	115.6	14400.0
dem_100_newyork+_404	57.3M	26.4M	116.7	14400.0	45.5M	41.9M	8.6	14401.0	46.7M	21.8M	114.6	14400.0
dem_100_newyork+_405	38.1M	22.2M	71.5	14400.0	38.0M	27.1M	40.4	14400.0	38.0M	18.9M	101.4	14400.0
dem_100_newyork+_406	45.7M	19.7M	132.4	14420.6	33.9M	21.7M	56.5	14400.0	35.0M	10.7M	226.3	14400.0
dem_100_newyork+_407	41.0M	24.4M	68.1	14400.0	35.1M	25.5M	37.8	14400.0	35.2M	21.8M	61.4	14400.0
dem_100_newyork+_408	32.8M	19.6M	67.5	14400.0	32.8M	28.8M	13.9	14400.2	32.8M	10.8M	202.4	14400.0
dem_100_newyork+_409	35.1M	26.0M	34.8	14400.0	35.1M	26.4M	32.8	14404.1	35.1M	19.6M	78.8	14400.0
dem_100_newyork+_410	50.8M	22.2M	128.6	14400.0	40.2M	29.3M	37.2	14400.0	40.2M	18.9M	113.0	14400.0
dem_100_newyork+_411	37.7M	25.6M	47.4	14400.0	34.9M	22.9M	52.4	14400.0	37.3M	16.6M	124.7	14400.0
dem_100_newyork+_412	31.9M	25.2M	26.8	14400.0	31.9M	21.3M	49.9	14400.0	31.9M	12.5M	155.4	14400.0
dem_100_newyork+_413	44.7M	18.7M	138.6	14420.2	32.8M	28.8M	13.7	14400.0	34.0M	13.0M	161.4	14400.0
dem_100_newyork+_414	14.0M	14.0M	0.0	1679.7	14.0M	11.6M	20.4	14400.0	14.0M	14.0M	0.0	9046.8
dem_100_newyork+_415	111.2M	28.3M	293.0	14400.0	45.7M	33.6M	35.8	14400.0	46.1M	25.2M	82.7	14400.0
dem_100_newyork+_416	49.5M	26.3M	88.2	14400.0	48.2M	48.2M	0.0	5972.6	50.5M	25.4M	98.7	14400.0
dem_100_newyork+_417	33.1M	18.9M	74.5	14400.0	60.4M	17.6M	242.6	14400.0	33.9M	17.2M	96.9	14400.0
dem_100_newyork+_418	103.6M	20.1M	415.1	14400.0	36.8M	20.6M	78.2	14400.0	34.9M	14.9M	133.4	14400.0
dem_100_newyork+_419	28.3M	15.4M	84.1	14400.0	27.4M	27.4M	0.0	10643.1	27.4M	13.2M	106.7	14400.0
dem_100_newyork+_420	35.0M	23.0M	52.5	14400.0	35.0M	23.2M	50.6	14400.0	35.0M	12.7M	176.1	14400.0
dem_100_newyork+_421	35.0M	23.6M	48.1	14400.0	34.9M	34.9M	0.0	8811.2	34.9M	11.5M	202.8	14400.0
dem_100_newyork+_422	54.7M	23.1M	137.0	14400.0	42.5M	32.2M	31.9	14400.0	43.2M	16.0M	170.1	14400.0
dem_100_newyork+_423	32.8M	21.1M	55.3	14400.0	39.9M	20.0M	99.5	14408.4	32.8M	14.3M	129.5	14400.0
dem_100_newyork+_424	68.7M	19.7M	248.7	14400.0	34.0M	22.4M	51.8	14400.0	34.0M	9723.2k	250.0	14400.0
dem_100_newyork+_425	38.6M	29.7M	30.0	14400.0	35.2M	26.5M	33.1	14400.0	38.6M	14.3M	169.9	14400.0
dem_100_newyork+_426	36.1M	28.7M	25.8	14410.7	33.9M	19.8M	71.1	14400.0	35.0M	14.1M	148.6	14400.0
dem_100_newyork+_427	22.1M	17.4M	27.2	14400.0	20.0M	13.5M	48.9	14400.0	20.0M	7420.6k	170.1	14400.0
dem_100_newyork+_428	37.2M	26.8M	38.5	14400.0	37.1M	24.8M	49.7	14400.0	39.4M	19.9M	97.9	14400.0
dem_100_newyork+_429	35.2M	28.1M	25.1	14400.0	34.9M	28.4M	23.2	14405.9	34.9M	14.3M	144.9	14400.0
dem_100_newyork+_430	58.6M	14.3M	310.6	14400.0	74.7M	13.0M	475.9	14400.0	38.7M	9037.0k	328.4	14400.0
dem_100_newyork+_431	64.8M	23.1M	181.0	14400.0	36.0M	25.4M	41.8	14400.0	36.3M	15.5M	134.4	14400.0
dem_100_newyork+_432	46.1M	27.3M	68.9	14400.0	41.4M	26.8M	54.4	14400.0	41.9M	20.5M	104.5	14400.0
dem_100_newyork+_433	31.2M	21.1M	48.0	14400.0	32.8M	17.3M	89.7	14400.0	30.7M	9875.9k	210.8	14400.0
dem_100_newyork+_434	40.2M	24.0M	67.6	14400.0	40.2M	29.2M	37.7	14400.0	42.4M	17.9M	137.3	14400.0
dem_100_newyork+_435	67.7M	18.0M	275.2	14400.0	35.9M	35.9M	0.0	9526.0	35.9M	15.0M	139.2	14400.0
dem_100_newyork+_436	39.2M	29.4M	33.1	14412.5	38.1M	32.2M	18.2	14402.8	38.1M	21.8M	75.1	14400.0
dem_100_newyork+_437	33.9M	25.7M	31.8	14400.0	33.9M	21.1M	61.0	14400.0	33.9M	12.9M	163.3	14400.0
dem_100_newyork+_438	32.3M	24.3M	33.1	14413.4	30.8M	23.2M	33.1	14400.0	30.8M	20.7M	48.7	14400.0
dem_100_newyork+_439	109.9M	24.1M	356.4	14400.0	48.3M	43.8M	10.3	14400.0	48.5M	23.6M	105.5	14400.0
dem_100_newyork+_440	39.1M	24.9M	57.5	14400.0	39.1M	27.2M	44.0	14400.0	39.1M	17.7M	120.9	14400.0
dem_100_newyork+_441	25.9M	25.9M	0.0	12511.3	45.6M	13.6M	234.6	14400.0	29.1M	11.6M	150.7	14400.0
dem_100_newyork+_442	80.3M	23.5M	241.2	14400.0	40.3M	27.6M	46.1	14400.0	41.3M	17.2M	139.8	14400.0
dem_100_newyork+_443	24.1M	24.1M	0.0	10167.5	24.1M	14.4M	68.0	14400.0	24.3M	10.9M	122.0	14400.0
dem_100_newyork+_444	36.1M	21.7M	66.2	14400.0	35.7M	32.1M	11.4	14400.0	35.9M	15.3M	134.2	14400.0
dem_100_newyork+_445	63.6M	25.5M	148.9	14400.0	45.5M	39.1M	16.4	14400.0	47.9M	20.9M	129.1	14400.0
dem_100_newyork+_446	45.8M	26.8M	71.0	14400.0	39.9M	28.0M	42.4	14400.0	40.5M	21.4M	89.6	14400.0
dem_100_newyork+_447	40.4M	26.2M	54.2	14400.0	38.0M	30.1M	26.4	14400.0	38.1M	17.1M	122.3	14400.0
dem_100_newyork+_448	52.8M	10.4M	408.7	14400.0	28.9M	22.7M	27.1	14400.0	32.9M	14.1M	132.7	14400.0
dem_100_newyork+_449	67.4M	20.4M	230.4	14400.0	36.0M	36.0M	0.0	9128.6	36.0M	16.2M	121.8	14400.0
dem_100_newyork+_450	27.6M	27.6M	0.0	13685.6	34.9M	16.7M	109.2	14400.0	27.6M	13.9M	98.7	14400.0
dem_100_newyork+_451	41.2M	23.3M	77.1	14400.0	56.0M	17.7M	217.1	14400.0	32.9M	12.2M	169.8	14400.0
dem_100_newyork+_452	30.8M	19.6M	57.3	14400.0	31.7M	18.5M	71.0	14400.0	30.8M	8368.0k	268.2	14400.0
dem_100_newyork+_453	47.9M	26.2M	83.1	14400.0	46.2M	42.0M	10.0	14400.1	58.3M	21.6M	170.3	14400.0
dem_100_newyork+_454	36.1M	21.6M	66.8	14400.0	34.9M	21.8M	59.8	14400.0	34.9M	10.8M	222.3	14400.0
dem_100_newyork+_455	58.9M	22.2M	164.9	14400.0	71.5M	18.3M	290.0	14400.0	34.2M	16.6M	106.0	14400.0
dem_100_newyork+_456	63.0M	19.9M	215.8	14400.0	37.1M	37.1M	0.0	10549.4	49.7M	19.0M	162.2	14400.0
dem_100_newyork+_457	38.0M	21.9M	73.9	14400.0	37.1M	31.5M	18.1	14400.0	39.1M	18.7M	109.0	14400.0
dem_100_newyork+_458	38.4M	24.1M	59.4	14400.0	38.2M	23.4M	63.1	14400.0	38.2M	20.0M	91.0	14400.0
dem_100_newyork+_459	51.9M	22.7M	128.2	14400.0	39.9M	33.1M	20.4	14400.0	40.4M	19.7M	105.4	14400.0
dem_100_newyork+_460	48.3M	24.3M	98.6	14400.0	44.5M	31.3M	42.2	14400.0	46.2M	22.4M	106.3	14400.0
dem_100_newyork+_461	36.0M	29.6M	21.8	14400.0	36.0M	24.8M	45.5	14400.0	36.1M	20.8M	73.4	14400.0
dem_100_newyork+_462	54.0M	25.3M	113.6	14400.0	42.3M	42.3M	0.0	5704.9	42.3M	17.3M	144.9	14400.0
dem_100_newyork+_463	20.7M	20.7M	0.0	2493.2	20.7M	11.5M	80.5	14400.0	21.3M	11.2M	89.5	14400.0
dem_100_newyork+_464	47.7M	26.9M	77.7	14400.0	48.7M	34.8M	39.8	14403.0	48.2M	24.4M	97.5	14400.0

continued on next page

Table A.41: Comparison of discrete models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_465	148.3M	24.2M	513.5	14400.0	40.3M	40.3M	0.0	4568.1	40.3M	18.7M	115.9	14400.0
dem_100_newyork+_466	35.0M	21.8M	60.7	14400.0	34.9M	19.8M	76.4	14400.0	35.0M	13.7M	155.5	14400.0
dem_100_newyork+_467	37.8M	22.8M	65.9	14400.0	37.1M	29.8M	24.5	14400.0	38.1M	19.0M	100.9	14400.0
dem_100_newyork+_468	39.1M	20.6M	89.3	14400.0	33.3M	18.9M	75.9	14400.0	32.8M	14.0M	133.9	14400.0
dem_100_newyork+_469	46.4M	25.1M	84.6	14400.0	41.0M	36.0M	14.0	14400.3	41.4M	20.2M	104.8	14400.0
dem_100_newyork+_470	32.8M	21.4M	53.1	14400.0	31.8M	27.2M	16.9	14400.0	31.8M	9315.3k	241.3	14400.0
dem_100_newyork+_471	43.9M	26.1M	68.2	14400.0	43.4M	26.1M	66.6	14400.0	44.1M	21.8M	102.4	14400.0
dem_100_newyork+_472	33.3M	26.5M	25.7	14400.0	33.0M	21.3M	55.0	14400.0	40.6M	13.9M	191.7	14400.0
dem_100_newyork+_473	49.0M	23.9M	104.7	14400.0	44.6M	38.3M	16.3	14400.2	44.6M	20.1M	122.5	14400.0
dem_100_newyork+_474	83.0M	20.8M	299.0	14400.3	35.1M	32.6M	7.8	14400.0	55.7M	15.6M	256.1	14400.0
dem_100_newyork+_475	41.3M	23.5M	75.4	14400.0	56.6M	25.8M	119.0	14405.0	41.3M	17.8M	132.0	14400.0
dem_100_newyork+_476	32.1M	19.3M	66.5	14400.0	35.2M	18.0M	95.2	14400.0	31.9M	8198.0k	288.8	14400.0
dem_100_newyork+_477	26.7M	26.7M	0.0	6200.3	33.7M	14.0M	140.6	14400.0	26.7M	12.0M	122.6	14400.0
dem_100_newyork+_478	38.5M	17.1M	124.7	14400.0	35.9M	27.1M	32.4	14400.0	49.9M	15.4M	223.3	14400.0
dem_100_newyork+_479	42.5M	28.1M	51.6	14400.0	41.3M	28.1M	47.1	14400.0	41.4M	21.0M	96.8	14400.0
dem_100_newyork+_480	33.0M	25.1M	31.4	14400.0	34.6M	16.9M	105.0	14400.0	33.2M	8403.2k	295.0	14400.0
dem_100_newyork+_481	46.2M	26.8M	72.4	14400.0	45.0M	33.9M	32.6	14400.0	45.6M	22.0M	107.5	14400.0
dem_100_newyork+_482	41.5M	26.8M	54.9	14400.0	47.9M	22.6M	111.5	14400.0	41.2M	14.5M	183.9	14400.0
dem_100_newyork+_483	43.6M	21.9M	98.9	14400.0	42.4M	33.2M	27.7	14402.4	43.4M	17.8M	143.3	14400.0
dem_100_newyork+_484	114.7M	21.6M	429.9	14400.0	35.0M	22.4M	56.0	14400.0	35.0M	11.2M	212.2	14400.0
dem_100_newyork+_485	33.8M	22.1M	53.1	14400.0	50.5M	18.1M	179.0	14400.0	33.9M	11.1M	206.1	14400.0
dem_100_newyork+_486	47.9M	24.9M	92.5	14400.0	46.1M	32.2M	43.2	14400.0	46.8M	23.1M	102.4	14400.0
dem_100_newyork+_487	33.1M	33.1M	0.0	10177.0	33.1M	29.9M	10.4	14400.2	33.9M	13.8M	146.5	14400.0
dem_100_newyork+_488	221.2M	21.9M	910.1	14419.1	38.1M	31.6M	20.5	14400.0	60.0M	15.7M	280.8	14400.0
dem_100_newyork+_489	25.4M	25.4M	0.0	6932.1	25.4M	16.8M	51.1	14400.0	25.6M	14.5M	76.9	14400.0
dem_100_newyork+_490	30.2M	19.4M	56.2	14400.0	27.7M	27.7M	0.0	11440.0	41.2M	12.1M	240.0	14400.0
dem_100_newyork+_491	83.0M	20.3M	307.9	14400.0	34.0M	21.5M	57.8	14400.0	40.2M	12.7M	216.1	14400.0
dem_100_newyork+_492	12.1M	12.1M	0.0	258.0	12.1M	6014.9k	100.7	14400.0	15.7M	4359.7k	260.2	14400.0
dem_100_newyork+_493	49.7M	23.2M	113.9	14400.2	39.1M	29.1M	34.2	14400.0	39.1M	18.9M	107.0	14400.0
dem_100_newyork+_494	26.8M	26.8M	0.0	4002.8	26.8M	18.5M	45.0	14400.0	27.8M	5831.0k	377.5	14400.0
dem_100_newyork+_495	37.0M	20.6M	79.6	14400.0	37.0M	32.5M	14.0	14400.0	39.3M	12.3M	220.3	14400.0
dem_100_newyork+_496	41.6M	20.5M	102.7	14400.0	32.9M	21.4M	53.8	14400.0	32.9M	16.5M	98.8	14400.0
dem_100_newyork+_497	36.1M	19.4M	86.5	14400.1	35.6M	19.1M	86.8	14400.0	34.9M	14.4M	142.2	14400.0
dem_100_newyork+_498	36.0M	22.8M	57.6	14401.2	35.0M	25.1M	39.5	14400.0	35.0M	12.1M	188.1	14400.0
dem_100_newyork+_499	33.2M	18.4M	81.0	14400.0	28.7M	20.2M	42.1	14400.0	29.7M	9145.1k	224.5	14400.0
dem_100_newyork+_500	32.9M	18.6M	77.3	14400.0	31.7M	27.8M	14.2	14400.0	32.9M	12.4M	165.2	14400.0

Table A.42: Detailed results of the split-pipe models on *New York+* with four additional arcs and $\mathcal{B} = 100$ as summarized in Figure 3.18a and Table 3.4f. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_1	29.5M	29.4M	0.1	12239.9	29.5M	29.4M	0.1	841.4
dem_100_newyork+_2	36.4M	36.4M	0.1	7644.5	36.4M	36.4M	0.1	1846.0
dem_100_newyork+_3	37.6M	37.6M	0.1	5157.4	37.6M	37.6M	0.1	29.1
dem_100_newyork+_4	31.6M	31.6M	0.1	6131.1	31.6M	31.6M	0.1	762.2
dem_100_newyork+_5	36.6M	36.6M	0.0	12.0	36.6M	36.5M	0.1	30.8
dem_100_newyork+_6	36.9M	36.9M	0.1	22.3	36.9M	36.9M	0.1	111.5
dem_100_newyork+_7	26.0M	25.9M	0.1	1495.6	26.0M	25.9M	0.1	151.9
dem_100_newyork+_8	23.8M	23.8M	0.1	721.2	23.8M	23.8M	0.1	494.5
dem_100_newyork+_9	19.9M	19.9M	0.1	68.4	19.9M	19.9M	0.1	122.9
dem_100_newyork+_10	45.2M	45.2M	0.0	14.0	45.2M	45.1M	0.1	275.1
dem_100_newyork+_11	34.8M	34.8M	0.1	5156.6	34.8M	34.8M	0.1	89.7
dem_100_newyork+_12	47.2M	46.7M	1.1	14400.0	47.2M	47.1M	0.1	3660.9
dem_100_newyork+_13	32.4M	32.3M	0.1	478.7	32.4M	32.3M	0.1	305.2
dem_100_newyork+_14	26.6M	26.2M	1.6	14400.0	26.6M	26.6M	0.1	624.1
dem_100_newyork+_15	23.9M	23.9M	0.1	34.8	23.9M	23.9M	0.0	72.9
dem_100_newyork+_16	27.7M	27.7M	0.0	5.6	27.7M	27.6M	0.1	253.1
dem_100_newyork+_17	37.7M	37.6M	0.2	14400.0	37.7M	37.6M	0.1	323.5
dem_100_newyork+_18	30.3M	27.9M	8.4	14400.0	30.3M	30.3M	0.1	20.4
dem_100_newyork+_19	40.8M	40.7M	0.1	5184.5	40.8M	40.7M	0.1	2205.4
dem_100_newyork+_20	39.8M	39.8M	0.0	11.1	39.8M	39.8M	0.1	31.6
dem_100_newyork+_21	31.0M	31.0M	0.1	160.6	31.0M	31.0M	0.1	331.7
dem_100_newyork+_22	29.3M	29.3M	0.0	32.7	29.3M	29.3M	0.0	41.0
dem_100_newyork+_23	41.9M	40.5M	3.4	14400.0	41.6M	39.7M	4.9	14400.0
dem_100_newyork+_24	20.0M	20.0M	0.1	8.0	20.0M	20.0M	0.0	15.3
dem_100_newyork+_25	46.1M	46.1M	0.0	15.9	46.1M	46.1M	0.0	27.3
dem_100_newyork+_26	38.8M	38.7M	0.1	24.4	38.8M	38.7M	0.1	41.2
dem_100_newyork+_27	29.7M	29.7M	0.1	4990.1	29.7M	29.7M	0.1	71.3

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Table A.42: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_28	36.4M	36.2M	0.5	14400.0	36.4M	36.4M	0.1	59.3
dem_100_newyork+_29	46.1M	46.1M	0.0	17.4	46.1M	46.1M	0.1	33.5
dem_100_newyork+_30	30.0M	30.0M	0.1	5394.0	30.0M	30.0M	0.1	58.6
dem_100_newyork+_31	37.0M	36.2M	2.2	14400.0	36.6M	36.6M	0.1	363.8
dem_100_newyork+_32	23.6M	23.6M	0.1	16.5	23.6M	23.6M	0.0	46.4
dem_100_newyork+_33	33.2M	33.1M	0.1	20.0	33.2M	32.9M	0.7	14400.0
dem_100_newyork+_34	31.7M	31.3M	1.1	14400.0	31.7M	31.6M	0.1	2978.6
dem_100_newyork+_35	44.3M	44.1M	0.6	14400.0	44.3M	44.3M	0.1	82.9
dem_100_newyork+_36	37.1M	37.1M	0.1	60.7	37.1M	36.6M	1.4	14400.0
dem_100_newyork+_37	34.7M	34.7M	0.0	10.2	34.7M	34.7M	0.1	789.0
dem_100_newyork+_38	37.9M	37.9M	0.1	609.9	37.9M	37.9M	0.1	47.1
dem_100_newyork+_39	34.3M	34.2M	0.1	32.6	34.3M	34.2M	0.1	62.4
dem_100_newyork+_40	26.5M	26.5M	0.0	51.1	26.5M	26.5M	0.1	79.3
dem_100_newyork+_41	33.9M	33.9M	0.1	3877.4	33.9M	33.9M	0.0	23.1
dem_100_newyork+_42	39.6M	39.5M	0.1	8563.3	39.6M	39.5M	0.1	179.8
dem_100_newyork+_43	33.5M	32.2M	3.9	14400.0	33.5M	33.5M	0.1	76.2
dem_100_newyork+_44	32.5M	32.5M	0.0	8.3	32.5M	32.5M	0.1	94.9
dem_100_newyork+_45	30.6M	30.5M	0.1	201.6	30.6M	30.5M	0.1	191.4
dem_100_newyork+_46	16.0M	16.0M	0.0	2.9	16.0M	16.0M	0.1	31.4
dem_100_newyork+_47	42.6M	42.6M	0.0	10.5	42.6M	42.5M	0.1	29.8
dem_100_newyork+_48	37.2M	37.2M	0.1	651.4	37.2M	37.2M	0.1	12739.1
dem_100_newyork+_49	37.7M	37.7M	0.1	679.0	37.7M	37.7M	0.1	149.9
dem_100_newyork+_50	34.3M	34.3M	0.1	2473.7	34.3M	34.3M	0.1	88.3
dem_100_newyork+_51	34.2M	34.1M	0.1	32.1	34.2M	34.1M	0.1	122.0
dem_100_newyork+_52	31.3M	31.3M	0.1	274.7	31.3M	31.3M	0.1	127.1
dem_100_newyork+_53	47.4M	47.4M	0.0	17.4	47.4M	47.4M	0.1	68.2
dem_100_newyork+_54	31.8M	31.8M	0.1	149.2	31.8M	31.8M	0.1	471.4
dem_100_newyork+_55	26.7M	26.7M	0.1	7475.2	26.7M	26.7M	0.1	53.7
dem_100_newyork+_56	32.9M	32.9M	0.1	4862.5	32.9M	32.9M	0.1	374.6
dem_100_newyork+_57	41.9M	41.9M	0.0	45.9	41.9M	41.9M	0.1	60.3
dem_100_newyork+_58	37.9M	37.9M	0.1	892.5	37.9M	37.4M	1.3	14400.0
dem_100_newyork+_59	23.0M	23.0M	0.1	722.0	23.0M	23.0M	0.1	13.2
dem_100_newyork+_60	44.1M	44.1M	0.0	8.5	44.1M	44.0M	0.1	61.0
dem_100_newyork+_61	38.8M	38.8M	0.0	14.1	38.8M	38.8M	0.1	26.7
dem_100_newyork+_62	36.4M	36.4M	0.1	2484.2	36.4M	36.4M	0.1	199.8
dem_100_newyork+_63	43.0M	43.0M	0.0	12.2	43.0M	43.0M	0.1	176.1
dem_100_newyork+_64	24.8M	24.8M	0.1	1586.1	24.8M	24.8M	0.1	19.1
dem_100_newyork+_65	43.0M	43.0M	0.0	12.0	43.0M	43.0M	0.1	15.3
dem_100_newyork+_66	43.7M	43.6M	0.1	5425.6	43.7M	43.6M	0.1	494.1
dem_100_newyork+_67	31.5M	31.5M	0.1	3526.3	31.6M	12.4M	154.3	14400.0
dem_100_newyork+_68	38.0M	38.0M	0.0	19.3	38.0M	6325.6k	501.4	14400.0
dem_100_newyork+_69	36.6M	36.5M	0.1	21.6	36.6M	36.6M	0.0	68.8
dem_100_newyork+_70	23.8M	23.3M	2.3	14400.0	23.8M	23.8M	0.1	113.1
dem_100_newyork+_71	33.9M	33.8M	0.1	91.0	33.9M	33.3M	1.8	14400.0
dem_100_newyork+_72	38.1M	38.1M	0.0	11.7	38.1M	38.1M	0.0	25.1
dem_100_newyork+_73	37.7M	37.6M	0.1	1108.4	37.7M	37.7M	0.0	20.2
dem_100_newyork+_74	19.9M	19.3M	3.2	14400.0	19.8M	19.8M	0.1	1384.1
dem_100_newyork+_75	24.3M	24.3M	0.1	211.6	24.3M	24.3M	0.1	14.3
dem_100_newyork+_76	34.5M	34.5M	0.1	34.8	34.5M	34.5M	0.1	35.0
dem_100_newyork+_77	34.8M	34.8M	0.1	1252.7	34.8M	34.8M	0.1	35.7
dem_100_newyork+_78	41.7M	41.7M	0.1	16.5	41.7M	41.7M	0.1	2756.3
dem_100_newyork+_79	35.5M	35.3M	0.4	14400.0	35.5M	35.4M	0.1	222.1
dem_100_newyork+_80	22.5M	22.5M	0.1	12526.0	22.5M	22.5M	0.1	21.3
dem_100_newyork+_81	25.7M	25.7M	0.1	171.3	25.7M	25.7M	0.0	121.5
dem_100_newyork+_82	40.0M	40.0M	0.1	25.8	40.0M	40.0M	0.1	1138.4
dem_100_newyork+_83	26.9M	26.4M	1.8	14400.0	26.9M	26.9M	0.1	73.1
dem_100_newyork+_84	25.4M	25.4M	0.1	2975.8	25.4M	25.4M	0.1	43.5
dem_100_newyork+_85	37.4M	37.3M	0.1	30.6	37.4M	37.3M	0.1	177.9
dem_100_newyork+_86	40.7M	40.0M	1.9	14400.0	40.7M	40.7M	0.2	14400.0
dem_100_newyork+_87	49.9M	49.8M	0.1	29.3	49.9M	49.9M	0.1	35.1
dem_100_newyork+_88	36.8M	35.8M	2.8	14400.0	36.8M	36.8M	0.1	23.5
dem_100_newyork+_89	35.2M	35.2M	0.1	657.0	35.2M	35.2M	0.1	124.7
dem_100_newyork+_90	25.4M	25.4M	0.1	1043.9	25.4M	25.4M	0.1	67.7
dem_100_newyork+_91	37.2M	37.1M	0.1	551.7	37.2M	37.1M	0.1	1553.0
dem_100_newyork+_92	39.6M	39.6M	0.1	396.4	39.6M	39.6M	0.1	120.1
dem_100_newyork+_93	41.7M	41.6M	0.3	14400.0	41.7M	41.7M	0.1	248.9
dem_100_newyork+_94	42.2M	42.2M	0.0	10.2	42.2M	42.2M	0.0	18.4
dem_100_newyork+_95	36.3M	36.3M	0.1	10.7	36.3M	36.3M	0.1	1127.7
dem_100_newyork+_96	39.3M	39.3M	0.1	19.8	39.3M	39.3M	0.0	167.4
dem_100_newyork+_97	41.0M	41.0M	0.0	14.3	41.0M	40.9M	0.1	3193.8
dem_100_newyork+_98	37.2M	37.1M	0.1	816.1	37.2M	37.2M	0.0	26.0
dem_100_newyork+_99	45.7M	45.7M	0.0	13.7	45.7M	45.7M	0.1	45.2
dem_100_newyork+_100	35.5M	35.5M	0.0	8.0	35.5M	35.5M	0.1	187.8
dem_100_newyork+_101	43.3M	43.1M	0.6	14400.0	43.3M	43.3M	0.1	32.3
dem_100_newyork+_102	23.7M	23.7M	0.0	34.4	23.7M	23.6M	0.1	242.9
dem_100_newyork+_103	26.1M	26.1M	0.1	102.3	26.1M	26.1M	0.1	116.7
dem_100_newyork+_104	33.3M	33.2M	0.1	51.4	33.3M	33.2M	0.1	250.7
dem_100_newyork+_105	31.4M	31.4M	0.1	3499.0	31.4M	31.4M	0.0	98.7

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Table A.42: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_106	30.8M	30.8M	0.1	22.7	30.8M	30.8M	0.0	68.8
dem_100_newyork+_107	35.8M	35.8M	0.0	8.0	35.8M	6806.6k	426.5	14400.0
dem_100_newyork+_108	27.2M	27.1M	0.4	14400.0	27.2M	27.1M	0.1	22.7
dem_100_newyork+_109	40.6M	40.6M	0.1	14400.0	40.6M	40.6M	0.1	37.7
dem_100_newyork+_110	39.2M	39.2M	0.1	503.1	39.2M	39.2M	0.1	30.1
dem_100_newyork+_111	40.1M	39.7M	1.2	14400.0	40.1M	40.1M	0.1	3452.9
dem_100_newyork+_112	34.4M	34.3M	0.1	26.7	34.4M	34.3M	0.1	828.9
dem_100_newyork+_113	31.8M	31.8M	0.1	265.8	31.8M	31.8M	0.1	472.4
dem_100_newyork+_114	34.1M	34.0M	0.1	66.3	34.1M	34.0M	0.1	410.6
dem_100_newyork+_115	48.8M	48.8M	0.0	10.8	48.8M	48.7M	0.1	31.3
dem_100_newyork+_116	43.7M	43.7M	0.1	50.4	43.7M	43.2M	1.2	14400.0
dem_100_newyork+_117	27.5M	27.4M	0.1	87.5	27.5M	27.4M	0.1	23.6
dem_100_newyork+_118	36.0M	36.0M	0.0	6.0	36.0M	35.9M	0.1	192.4
dem_100_newyork+_119	27.2M	27.2M	0.1	10.7	27.2M	27.2M	0.1	222.6
dem_100_newyork+_120	31.3M	31.2M	0.1	14.5	31.3M	31.3M	0.0	20.2
dem_100_newyork+_121	37.2M	37.2M	0.0	8.3	37.2M	37.1M	0.1	43.6
dem_100_newyork+_122	28.4M	28.4M	0.1	1440.5	28.4M	28.4M	0.1	174.3
dem_100_newyork+_123	39.7M	39.6M	0.1	12457.4	39.7M	39.6M	0.1	260.6
dem_100_newyork+_124	35.8M	35.1M	2.1	14400.0	35.5M	35.5M	0.1	79.8
dem_100_newyork+_125	23.7M	23.7M	0.0	46.8	23.7M	23.7M	0.1	92.1
dem_100_newyork+_126	45.0M	45.0M	0.0	12.7	45.0M	44.9M	0.1	36.1
dem_100_newyork+_127	46.9M	46.9M	0.1	19.1	46.9M	46.6M	0.8	14400.0
dem_100_newyork+_128	37.3M	37.2M	0.1	17.9	37.3M	37.3M	0.1	46.4
dem_100_newyork+_129	25.2M	25.2M	0.1	91.9	25.2M	25.2M	0.1	25.8
dem_100_newyork+_130	42.5M	42.5M	0.0	16.9	42.5M	42.5M	0.1	28.2
dem_100_newyork+_131	28.3M	28.3M	0.1	1027.2	28.3M	28.3M	0.1	59.2
dem_100_newyork+_132	41.0M	41.0M	0.0	11.4	41.0M	39.9M	2.7	14400.0
dem_100_newyork+_133	29.2M	29.2M	0.1	682.4	29.2M	29.2M	0.1	29.2
dem_100_newyork+_134	34.4M	34.3M	0.1	7825.2	34.4M	34.3M	0.1	719.6
dem_100_newyork+_135	35.5M	35.5M	0.1	38.5	35.5M	35.5M	0.1	186.9
dem_100_newyork+_136	35.7M	35.7M	0.1	1724.8	35.7M	35.7M	0.1	50.1
dem_100_newyork+_137	34.9M	34.9M	0.1	65.8	34.9M	34.9M	0.0	16.3
dem_100_newyork+_138	42.3M	42.3M	0.0	7.3	42.3M	42.3M	0.0	16.4
dem_100_newyork+_139	33.4M	33.4M	0.0	7.0	33.4M	33.4M	0.1	2956.9
dem_100_newyork+_140	35.5M	35.4M	0.1	12.1	35.5M	35.4M	0.1	598.2
dem_100_newyork+_141	27.6M	27.6M	0.1	2069.1	27.6M	27.6M	0.1	530.1
dem_100_newyork+_142	33.2M	32.6M	1.8	14400.0	33.2M	33.2M	0.1	209.1
dem_100_newyork+_143	38.4M	38.3M	0.1	191.5	38.4M	18.2M	110.3	14400.0
dem_100_newyork+_144	43.9M	43.9M	0.0	5.2	43.9M	43.8M	0.1	4180.5
dem_100_newyork+_145	36.6M	36.6M	0.1	23.2	36.6M	36.6M	0.1	34.9
dem_100_newyork+_146	28.8M	28.8M	0.1	1577.2	28.8M	28.8M	0.1	29.2
dem_100_newyork+_147	40.0M	38.3M	4.4	14400.0	39.7M	39.6M	0.1	46.8
dem_100_newyork+_148	36.6M	36.6M	0.0	833.4	36.6M	36.5M	0.1	13.3
dem_100_newyork+_149	32.4M	32.4M	0.1	432.3	32.4M	32.4M	0.1	303.6
dem_100_newyork+_150	45.3M	45.3M	0.0	21.4	45.3M	45.3M	0.0	39.4
dem_100_newyork+_151	33.1M	33.1M	0.1	1647.8	33.1M	33.1M	0.1	244.2
dem_100_newyork+_152	21.7M	21.7M	0.1	2020.0	21.7M	21.7M	0.1	134.9
dem_100_newyork+_153	25.8M	24.5M	5.4	14400.0	25.8M	25.8M	0.1	20.8
dem_100_newyork+_154	26.5M	26.5M	0.1	67.7	26.5M	26.5M	0.1	3298.8
dem_100_newyork+_155	41.7M	41.7M	0.0	9.5	41.7M	41.6M	0.1	21.3
dem_100_newyork+_156	46.0M	45.9M	0.2	14400.0	46.0M	46.0M	0.1	31.7
dem_100_newyork+_157	37.9M	37.9M	0.1	6107.3	37.9M	37.9M	0.0	42.6
dem_100_newyork+_158	36.2M	36.2M	0.0	7.5	36.2M	36.2M	0.1	159.0
dem_100_newyork+_159	39.2M	39.2M	0.1	300.4	39.2M	39.2M	0.0	638.9
dem_100_newyork+_160	38.4M	38.4M	0.1	18.8	38.4M	38.4M	0.1	761.0
dem_100_newyork+_161	38.7M	38.2M	1.5	14400.0	38.7M	38.7M	0.0	29.1
dem_100_newyork+_162	37.2M	37.2M	0.0	12.1	37.2M	37.1M	0.1	47.7
dem_100_newyork+_163	34.0M	34.0M	0.1	392.2	34.0M	34.0M	0.0	27.0
dem_100_newyork+_164	45.5M	45.5M	0.0	18.1	45.5M	45.5M	0.1	28.9
dem_100_newyork+_165	38.8M	38.7M	0.1	1339.2	38.8M	38.7M	0.1	1420.1
dem_100_newyork+_166	43.8M	42.9M	2.1	14400.0	43.8M	43.7M	0.1	418.9
dem_100_newyork+_167	45.1M	43.9M	2.8	14400.0	44.6M	44.6M	0.1	494.7
dem_100_newyork+_168	31.4M	31.4M	0.1	361.0	31.4M	31.4M	0.1	19.1
dem_100_newyork+_169	32.7M	32.7M	0.1	3230.4	32.7M	32.7M	0.1	222.3
dem_100_newyork+_170	28.5M	28.5M	0.1	8.2	28.5M	28.5M	0.0	16.8
dem_100_newyork+_171	30.7M	30.7M	0.1	10.2	30.7M	30.7M	0.0	20.7
dem_100_newyork+_172	33.1M	33.0M	0.1	2862.8	33.1M	33.0M	0.1	2373.5
dem_100_newyork+_173	42.7M	42.7M	0.0	8.4	42.7M	42.7M	0.1	4829.5
dem_100_newyork+_174	28.1M	28.1M	0.0	34.4	28.1M	28.1M	0.1	85.2
dem_100_newyork+_175	42.2M	41.6M	1.3	14400.0	42.2M	42.1M	0.1	372.8
dem_100_newyork+_176	30.7M	30.6M	0.1	5743.5	30.7M	30.7M	0.0	37.3
dem_100_newyork+_177	37.7M	37.7M	0.1	11.8	37.7M	37.7M	0.1	39.9
dem_100_newyork+_178	35.9M	34.3M	4.6	14400.0	35.9M	35.9M	0.1	1795.5
dem_100_newyork+_179	35.6M	35.6M	0.1	7.7	35.6M	35.6M	0.1	2338.3
dem_100_newyork+_180	29.0M	28.9M	0.1	2693.8	29.0M	28.9M	0.1	103.6
dem_100_newyork+_181	37.8M	37.8M	0.1	1025.5	37.8M	37.8M	0.0	39.0
dem_100_newyork+_182	28.5M	28.5M	0.1	8.7	28.5M	28.5M	0.1	37.0
dem_100_newyork+_183	44.8M	44.8M	0.1	220.3	44.8M	44.8M	0.0	51.0

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Table A.42: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_184	39.3M	38.9M	0.9	14400.0	39.3M	39.3M	0.1	10411.1
dem_100_newyork+_185	39.1M	39.1M	0.1	3586.7	39.1M	39.1M	0.0	45.5
dem_100_newyork+_186	39.3M	39.3M	0.1	9519.4	39.3M	39.3M	0.1	360.8
dem_100_newyork+_187	25.6M	25.6M	0.1	1648.9	25.6M	25.6M	0.0	19.9
dem_100_newyork+_188	37.9M	37.9M	0.0	34.9	37.9M	37.9M	0.1	22.3
dem_100_newyork+_189	34.6M	34.6M	0.1	13.8	34.6M	34.6M	0.1	23.8
dem_100_newyork+_190	23.4M	23.3M	0.4	14400.0	23.4M	23.3M	0.1	34.0
dem_100_newyork+_191	35.6M	35.6M	0.0	15.4	35.6M	35.6M	0.0	25.9
dem_100_newyork+_192	35.7M	35.7M	0.0	9.0	35.7M	35.7M	0.1	137.9
dem_100_newyork+_193	39.8M	39.8M	0.1	1084.5	39.8M	39.8M	0.1	4505.2
dem_100_newyork+_194	32.2M	32.2M	0.0	16.1	32.2M	32.2M	0.0	36.8
dem_100_newyork+_195	44.1M	44.0M	0.1	16.2	44.1M	44.0M	0.1	20.6
dem_100_newyork+_196	28.7M	28.7M	0.1	6113.2	28.7M	28.7M	0.1	59.3
dem_100_newyork+_197	37.8M	37.8M	0.1	144.9	37.8M	37.8M	0.1	2557.6
dem_100_newyork+_198	35.4M	35.3M	0.1	21.8	35.4M	35.3M	0.1	185.6
dem_100_newyork+_199	40.2M	40.1M	0.3	14400.0	40.2M	40.2M	0.1	3194.4
dem_100_newyork+_200	40.5M	40.5M	0.1	26.9	40.5M	40.5M	0.1	38.3
dem_100_newyork+_201	29.2M	29.2M	0.1	4648.8	29.2M	29.2M	0.1	200.1
dem_100_newyork+_202	26.0M	26.0M	0.0	17.2	26.0M	26.0M	0.0	61.0
dem_100_newyork+_203	34.6M	34.6M	0.1	32.7	34.6M	34.6M	0.1	41.7
dem_100_newyork+_204	40.1M	40.0M	0.1	12738.1	40.1M	40.0M	0.1	62.9
dem_100_newyork+_205	40.5M	40.5M	0.1	394.3	40.5M	40.5M	0.1	324.2
dem_100_newyork+_206	36.2M	36.2M	0.1	5114.1	36.2M	36.1M	0.2	14400.0
dem_100_newyork+_207	32.6M	32.6M	0.1	172.7	32.6M	32.6M	0.1	670.0
dem_100_newyork+_208	35.7M	35.7M	0.1	4151.9	35.7M	35.7M	0.1	95.5
dem_100_newyork+_209	27.7M	27.7M	0.1	3214.6	27.7M	27.7M	0.1	20.6
dem_100_newyork+_210	41.8M	41.7M	0.3	14400.0	41.8M	41.8M	0.1	73.0
dem_100_newyork+_211	42.3M	42.2M	0.1	11.1	42.3M	25.0M	68.8	14400.0
dem_100_newyork+_212	35.6M	35.6M	0.1	978.3	35.6M	35.6M	0.1	19.1
dem_100_newyork+_213	39.0M	38.9M	0.2	14400.0	39.0M	38.9M	0.1	61.7
dem_100_newyork+_214	25.9M	25.9M	0.1	421.1	25.9M	25.9M	0.1	47.3
dem_100_newyork+_215	38.5M	38.5M	0.0	8.6	38.5M	38.4M	0.1	17.9
dem_100_newyork+_216	35.6M	35.6M	0.0	6.4	35.6M	30.5M	17.0	14400.0
dem_100_newyork+_217	38.3M	37.8M	1.3	14400.0	38.3M	38.2M	0.1	32.8
dem_100_newyork+_218	40.0M	40.0M	0.0	9.9	40.0M	40.0M	0.1	1341.6
dem_100_newyork+_219	34.8M	34.7M	0.1	8.0	34.8M	34.7M	0.1	13.9
dem_100_newyork+_220	35.6M	35.6M	0.1	14400.0	35.6M	35.6M	0.1	24.8
dem_100_newyork+_221	31.9M	31.9M	0.1	324.6	31.9M	31.9M	0.1	119.5
dem_100_newyork+_222	40.2M	40.2M	0.0	6.7	40.2M	40.2M	0.1	17.8
dem_100_newyork+_223	20.7M	20.7M	0.1	4427.1	20.7M	20.7M	0.1	422.8
dem_100_newyork+_224	43.9M	43.9M	0.0	8.9	43.9M	43.8M	0.1	205.4
dem_100_newyork+_225	14.8M	14.8M	0.0	1.4	14.8M	14.8M	0.1	625.2
dem_100_newyork+_226	32.5M	32.5M	0.1	5161.7	32.5M	32.5M	0.1	2120.5
dem_100_newyork+_227	28.9M	28.9M	0.0	20.2	28.9M	28.8M	0.4	14400.0
dem_100_newyork+_228	28.0M	28.0M	0.1	5183.7	28.0M	28.0M	0.1	768.5
dem_100_newyork+_229	28.1M	28.1M	0.0	165.4	28.1M	28.0M	0.1	156.6
dem_100_newyork+_230	32.3M	32.3M	0.1	24.9	32.3M	32.3M	0.1	112.1
dem_100_newyork+_231	27.2M	27.2M	0.1	12509.7	27.2M	27.2M	0.1	105.7
dem_100_newyork+_232	43.4M	43.4M	0.1	9492.6	43.4M	43.4M	0.1	21.9
dem_100_newyork+_233	36.7M	36.7M	0.1	27.5	36.7M	36.7M	0.1	47.7
dem_100_newyork+_234	27.2M	27.2M	0.1	898.3	27.2M	27.2M	0.1	84.8
dem_100_newyork+_235	31.6M	31.5M	0.0	15.2	31.6M	31.5M	0.1	32.3
dem_100_newyork+_236	38.6M	38.5M	0.1	12.1	38.6M	38.5M	0.1	942.9
dem_100_newyork+_237	40.4M	40.3M	0.1	622.5	40.4M	40.3M	0.1	376.9
dem_100_newyork+_238	37.1M	37.1M	0.0	10.1	37.1M	36.9M	0.5	14400.0
dem_100_newyork+_239	32.8M	32.8M	0.1	18.3	32.8M	32.8M	0.1	37.8
dem_100_newyork+_240	39.4M	39.4M	0.1	9908.3	39.4M	39.4M	0.1	53.6
dem_100_newyork+_241	40.6M	40.4M	0.5	14400.0	40.5M	5816.0k	596.3	14400.0
dem_100_newyork+_242	33.1M	33.1M	0.1	63.2	33.1M	33.1M	0.1	46.9
dem_100_newyork+_243	39.8M	39.7M	0.1	851.8	39.8M	39.7M	0.1	66.4
dem_100_newyork+_244	27.7M	27.7M	0.0	20.5	27.7M	27.7M	0.1	17.5
dem_100_newyork+_245	33.8M	33.7M	0.1	20.5	33.8M	33.7M	0.1	719.7
dem_100_newyork+_246	37.5M	37.5M	0.1	141.8	37.5M	37.5M	0.1	1003.7
dem_100_newyork+_247	40.2M	40.2M	0.1	48.4	40.2M	40.2M	0.1	190.1
dem_100_newyork+_248	31.2M	31.2M	0.0	5.6	31.2M	31.2M	0.1	7170.1
dem_100_newyork+_249	40.7M	40.6M	0.1	15.4	40.7M	40.6M	0.1	111.5
dem_100_newyork+_250	36.6M	36.5M	0.0	20.5	36.6M	36.5M	0.1	39.3
dem_100_newyork+_251	35.6M	35.6M	0.1	964.9	35.6M	35.6M	0.1	27.1
dem_100_newyork+_252	28.0M	28.0M	0.1	14400.0	28.0M	28.0M	0.1	216.6
dem_100_newyork+_253	37.3M	37.3M	0.1	884.4	37.3M	37.3M	0.1	163.0
dem_100_newyork+_254	31.2M	31.0M	0.5	14400.0	31.2M	31.1M	0.1	337.7
dem_100_newyork+_255	46.4M	46.4M	0.0	16.2	46.4M	46.3M	0.1	280.4
dem_100_newyork+_256	34.4M	34.4M	0.0	7.7	34.4M	34.4M	0.1	61.6
dem_100_newyork+_257	27.5M	27.5M	0.1	60.6	27.5M	27.5M	0.1	11.2
dem_100_newyork+_258	39.3M	39.2M	0.3	14400.0	39.3M	39.2M	0.1	14400.0
dem_100_newyork+_259	22.8M	22.7M	0.1	24.4	22.8M	22.7M	0.1	1092.4
dem_100_newyork+_260	43.1M	43.1M	0.0	9.9	43.1M	42.3M	1.8	14400.0
dem_100_newyork+_261	38.4M	38.3M	0.1	7369.9	38.4M	38.3M	0.1	11087.2

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Table A.42: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_262	29.5M	29.3M	0.8	14400.0	29.5M	29.5M	0.1	316.0
dem_100_newyork+_263	30.6M	30.6M	0.0	9.2	30.6M	30.6M	0.1	30.3
dem_100_newyork+_264	27.0M	27.0M	0.1	17.6	27.0M	27.0M	0.1	60.6
dem_100_newyork+_265	33.7M	33.7M	0.1	2629.4	33.7M	33.7M	0.1	543.7
dem_100_newyork+_266	30.1M	30.1M	0.1	1738.2	30.1M	29.6M	1.7	14400.0
dem_100_newyork+_267	25.1M	25.1M	0.1	217.6	25.1M	25.1M	0.1	20.3
dem_100_newyork+_268	33.1M	33.1M	0.1	847.6	33.1M	33.1M	0.1	441.5
dem_100_newyork+_269	30.5M	30.4M	0.1	483.8	30.5M	30.4M	0.1	8533.4
dem_100_newyork+_270	35.9M	35.9M	0.1	1278.7	35.9M	35.9M	0.0	25.0
dem_100_newyork+_271	34.5M	34.5M	0.1	5921.0	34.5M	34.5M	0.0	1257.0
dem_100_newyork+_272	31.9M	31.8M	0.1	22.6	31.9M	31.8M	0.1	33.9
dem_100_newyork+_273	33.0M	32.2M	2.6	14400.0	33.0M	33.0M	0.1	25.8
dem_100_newyork+_274	33.5M	33.5M	0.1	586.9	33.5M	33.5M	0.1	110.0
dem_100_newyork+_275	38.3M	38.3M	0.1	12.4	38.3M	38.3M	0.1	3712.0
dem_100_newyork+_276	37.9M	37.9M	0.0	14.2	37.9M	37.9M	0.0	16.6
dem_100_newyork+_277	40.1M	40.1M	0.0	11.2	40.1M	40.0M	0.1	19.6
dem_100_newyork+_278	42.5M	42.4M	0.1	14400.0	42.5M	42.4M	0.1	278.8
dem_100_newyork+_279	43.5M	42.0M	3.5	14400.0	43.0M	43.0M	0.1	677.0
dem_100_newyork+_280	32.6M	32.6M	0.0	144.4	32.6M	32.6M	0.0	18.9
dem_100_newyork+_281	24.6M	24.6M	0.0	10.9	24.6M	24.6M	0.0	37.0
dem_100_newyork+_282	34.1M	34.0M	0.1	189.4	34.1M	33.7M	1.1	14400.0
dem_100_newyork+_283	23.0M	23.0M	0.1	41.9	23.0M	23.0M	0.1	47.0
dem_100_newyork+_284	29.6M	29.5M	0.1	199.9	29.6M	29.5M	0.1	38.5
dem_100_newyork+_285	26.3M	26.2M	0.1	14400.0	26.3M	26.3M	0.0	397.4
dem_100_newyork+_286	38.5M	38.5M	0.0	10.4	38.5M	38.5M	0.0	34.0
dem_100_newyork+_287	27.6M	27.6M	0.1	349.4	27.6M	27.6M	0.0	35.5
dem_100_newyork+_288	35.5M	35.5M	0.1	1033.8	35.5M	35.5M	0.1	832.9
dem_100_newyork+_289	44.4M	44.3M	0.1	34.6	44.4M	44.3M	0.1	5174.6
dem_100_newyork+_290	39.2M	39.2M	0.1	6.8	39.2M	39.2M	0.1	25.0
dem_100_newyork+_291	19.5M	19.5M	0.1	293.2	19.5M	19.5M	0.1	32.0
dem_100_newyork+_292	38.7M	38.7M	0.1	8.4	38.7M	38.7M	0.1	15.3
dem_100_newyork+_293	33.6M	33.6M	0.1	37.6	33.6M	33.6M	0.1	73.9
dem_100_newyork+_294	34.4M	34.0M	1.3	14400.0	34.4M	34.4M	0.1	21.1
dem_100_newyork+_295	30.5M	30.5M	0.1	1102.2	30.5M	30.5M	0.1	469.8
dem_100_newyork+_296	32.6M	32.6M	0.1	598.1	32.6M	32.6M	0.1	65.2
dem_100_newyork+_297	28.6M	28.6M	0.1	18.7	28.6M	28.6M	0.1	67.9
dem_100_newyork+_298	26.5M	26.5M	0.1	12539.2	26.5M	26.5M	0.1	2997.8
dem_100_newyork+_299	36.9M	36.9M	0.1	60.1	36.9M	36.9M	0.0	68.5
dem_100_newyork+_300	28.8M	28.8M	0.1	1029.8	28.8M	28.8M	0.1	40.9
dem_100_newyork+_301	35.2M	35.2M	0.0	274.4	35.2M	35.2M	0.1	52.8
dem_100_newyork+_302	44.0M	43.9M	0.1	28.1	44.0M	43.9M	0.1	6853.2
dem_100_newyork+_303	36.8M	36.7M	0.1	637.0	36.8M	36.7M	0.1	673.6
dem_100_newyork+_304	36.9M	36.6M	0.8	14400.0	36.9M	36.8M	0.1	1965.0
dem_100_newyork+_305	29.2M	29.2M	0.1	866.2	29.2M	29.2M	0.1	41.7
dem_100_newyork+_306	34.2M	34.2M	0.0	4.8	34.2M	34.2M	0.0	15.5
dem_100_newyork+_307	37.7M	37.4M	0.8	14400.0	37.7M	37.7M	0.1	117.1
dem_100_newyork+_308	34.6M	34.6M	0.1	530.4	34.6M	34.6M	0.1	44.1
dem_100_newyork+_309	36.4M	36.4M	0.0	14.3	36.4M	36.4M	0.0	82.1
dem_100_newyork+_310	32.1M	32.1M	0.1	19.6	32.1M	32.1M	0.1	35.7
dem_100_newyork+_311	40.0M	40.0M	0.1	592.7	40.0M	40.0M	0.1	688.0
dem_100_newyork+_312	35.2M	35.2M	0.1	11.0	35.2M	35.2M	0.0	42.5
dem_100_newyork+_313	41.9M	41.9M	0.1	5701.8	41.9M	41.9M	0.1	577.2
dem_100_newyork+_314	40.3M	40.3M	0.0	8.4	40.3M	40.3M	0.1	478.3
dem_100_newyork+_315	30.4M	30.4M	0.0	6.1	30.4M	30.3M	0.1	285.8
dem_100_newyork+_316	30.2M	30.1M	0.1	29.8	30.2M	30.1M	0.1	124.8
dem_100_newyork+_317	38.6M	38.5M	0.1	441.7	38.6M	38.5M	0.1	224.4
dem_100_newyork+_318	41.5M	41.5M	0.0	11.7	41.5M	41.4M	0.1	186.1
dem_100_newyork+_319	39.2M	39.1M	0.1	676.8	39.2M	39.1M	0.1	97.3
dem_100_newyork+_320	36.8M	35.8M	2.8	14400.0	36.8M	36.7M	0.1	36.9
dem_100_newyork+_321	37.4M	37.3M	0.1	2431.7	37.4M	37.3M	0.1	3469.1
dem_100_newyork+_322	32.2M	32.2M	0.1	11.5	32.2M	32.2M	0.1	63.7
dem_100_newyork+_323	26.0M	26.0M	0.1	1548.1	26.0M	26.0M	0.1	616.7
dem_100_newyork+_324	19.2M	19.2M	0.1	30.6	19.2M	19.2M	0.1	103.5
dem_100_newyork+_325	39.9M	39.9M	0.0	7.8	39.9M	39.9M	0.1	5073.4
dem_100_newyork+_326	38.7M	38.1M	1.6	14400.0	38.7M	38.7M	0.0	40.3
dem_100_newyork+_327	32.2M	32.2M	0.1	17.3	32.2M	31.8M	1.2	14400.0
dem_100_newyork+_328	21.2M	21.2M	0.1	698.9	21.2M	21.2M	0.0	38.5
dem_100_newyork+_329	22.5M	22.5M	0.1	30.3	22.5M	22.5M	0.1	47.2
dem_100_newyork+_330	27.8M	27.8M	0.1	24.7	27.8M	27.8M	0.1	18.7
dem_100_newyork+_331	37.2M	36.6M	1.7	14400.0	37.1M	37.1M	0.0	105.0
dem_100_newyork+_332	31.3M	31.3M	0.1	36.6	31.3M	31.3M	0.1	71.4
dem_100_newyork+_333	30.5M	30.5M	0.0	41.7	30.5M	30.5M	0.0	51.1
dem_100_newyork+_334	30.7M	30.7M	0.1	25.3	30.7M	30.7M	0.1	64.9
dem_100_newyork+_335	31.5M	31.5M	0.1	751.5	31.5M	31.5M	0.1	61.0
dem_100_newyork+_336	34.4M	34.4M	0.1	14.7	34.4M	34.4M	0.1	1751.0
dem_100_newyork+_337	36.0M	35.9M	0.1	5139.4	36.0M	35.9M	0.1	12451.9
dem_100_newyork+_338	35.8M	35.8M	0.1	26.7	35.8M	35.8M	0.1	108.8
dem_100_newyork+_339	32.0M	32.0M	0.1	2435.3	32.0M	32.0M	0.1	92.0

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Table A.42: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_340	38.4M	38.4M	0.0	19.0	38.4M	38.3M	0.1	19.4
dem_100_newyork+_341	24.1M	24.1M	0.0	9747.1	24.1M	23.1M	4.2	14400.0
dem_100_newyork+_342	31.7M	31.7M	0.1	6.8	31.7M	31.7M	0.0	26.5
dem_100_newyork+_343	37.0M	37.0M	0.0	11.8	37.0M	37.0M	0.1	15.3
dem_100_newyork+_344	21.0M	20.9M	0.1	177.0	21.0M	20.9M	0.1	84.5
dem_100_newyork+_345	30.3M	30.3M	0.1	16.5	30.3M	30.3M	0.1	16.7
dem_100_newyork+_346	28.9M	28.9M	0.1	43.4	28.9M	28.9M	0.1	190.0
dem_100_newyork+_347	47.1M	47.1M	0.0	191.1	47.1M	47.1M	0.1	76.1
dem_100_newyork+_348	33.1M	33.1M	0.1	30.4	33.1M	33.1M	0.1	43.6
dem_100_newyork+_349	30.0M	29.9M	0.1	78.1	30.0M	29.9M	0.1	113.8
dem_100_newyork+_350	32.9M	32.9M	0.1	14.8	32.9M	32.9M	0.1	604.6
dem_100_newyork+_351	28.8M	28.7M	0.1	13001.2	28.8M	28.7M	0.1	5063.6
dem_100_newyork+_352	43.0M	43.0M	0.1	6765.3	43.0M	43.0M	0.0	77.9
dem_100_newyork+_353	33.1M	33.1M	0.1	145.8	33.1M	33.1M	0.1	115.1
dem_100_newyork+_354	32.4M	32.4M	0.1	4346.3	32.4M	32.4M	0.1	419.4
dem_100_newyork+_355	42.9M	42.9M	0.1	3151.4	42.9M	42.9M	0.1	31.7
dem_100_newyork+_356	36.7M	36.6M	0.1	3277.5	36.7M	36.7M	0.0	48.5
dem_100_newyork+_357	43.9M	43.9M	0.0	8.7	43.9M	43.9M	0.1	106.6
dem_100_newyork+_358	40.0M	40.0M	0.1	4723.2	40.0M	40.0M	0.1	54.6
dem_100_newyork+_359	37.8M	37.8M	0.1	244.7	37.8M	37.8M	0.1	277.9
dem_100_newyork+_360	35.1M	35.1M	0.1	6139.0	35.1M	35.1M	0.1	20.6
dem_100_newyork+_361	27.3M	27.2M	0.1	273.8	27.3M	27.2M	0.1	76.7
dem_100_newyork+_362	31.3M	31.3M	0.1	14064.8	31.3M	31.3M	0.1	51.0
dem_100_newyork+_363	34.2M	34.2M	0.1	244.7	34.2M	34.2M	0.1	36.8
dem_100_newyork+_364	28.6M	28.6M	0.1	220.5	28.6M	28.6M	0.1	175.2
dem_100_newyork+_365	25.9M	25.4M	2.2	14400.0	32.8M	6934.3k	372.7	14400.0
dem_100_newyork+_366	39.3M	38.0M	3.3	14400.0	39.3M	39.3M	0.1	36.5
dem_100_newyork+_367	14.9M	14.9M	0.1	24.5	14.9M	14.9M	0.1	51.9
dem_100_newyork+_368	32.1M	32.1M	0.1	18.6	32.1M	32.1M	0.1	434.4
dem_100_newyork+_369	42.1M	42.1M	0.1	622.3	42.1M	15.4M	173.3	14400.0
dem_100_newyork+_370	37.0M	36.1M	2.6	14400.0	37.0M	37.0M	0.1	53.2
dem_100_newyork+_371	35.5M	35.5M	0.1	812.3	35.5M	35.5M	0.1	45.2
dem_100_newyork+_372	20.8M	20.7M	0.1	345.1	20.8M	20.8M	0.0	7.9
dem_100_newyork+_373	41.1M	41.1M	0.0	9.7	41.1M	41.1M	0.1	19.6
dem_100_newyork+_374	31.8M	31.8M	0.1	422.8	31.8M	31.8M	0.1	30.0
dem_100_newyork+_375	27.5M	27.5M	0.1	9621.9	27.5M	27.5M	0.1	302.8
dem_100_newyork+_376	28.9M	28.8M	0.1	20.3	28.9M	28.8M	0.1	1387.7
dem_100_newyork+_377	26.9M	26.6M	1.0	14400.0	26.9M	26.9M	0.1	3220.1
dem_100_newyork+_378	36.0M	35.6M	1.0	14400.0	36.0M	35.9M	0.1	7562.4
dem_100_newyork+_379	31.8M	31.8M	0.1	8360.8	31.8M	31.8M	0.1	1358.0
dem_100_newyork+_380	23.7M	22.4M	5.7	14400.0	23.7M	23.7M	0.0	146.4
dem_100_newyork+_381	37.7M	37.7M	0.1	229.0	37.7M	37.7M	0.1	154.3
dem_100_newyork+_382	31.6M	31.6M	0.1	64.8	31.6M	31.6M	0.1	336.9
dem_100_newyork+_383	30.7M	30.7M	0.1	106.4	30.7M	30.6M	0.1	2810.3
dem_100_newyork+_384	28.3M	28.3M	0.1	111.1	28.3M	28.3M	0.1	147.1
dem_100_newyork+_385	47.1M	47.1M	0.0	14.5	47.1M	47.1M	0.1	203.5
dem_100_newyork+_386	39.7M	39.7M	0.1	9850.0	39.7M	39.7M	0.1	43.0
dem_100_newyork+_387	20.9M	20.9M	0.1	3171.8	20.9M	20.9M	0.1	1690.4
dem_100_newyork+_388	31.8M	31.8M	0.1	10.5	31.8M	31.8M	0.1	41.9
dem_100_newyork+_389	31.7M	31.7M	0.1	10.0	31.7M	31.7M	0.1	143.2
dem_100_newyork+_390	28.3M	28.2M	0.1	15.8	28.3M	28.3M	0.0	19.8
dem_100_newyork+_391	37.0M	37.0M	0.1	3120.8	37.0M	37.0M	0.1	53.1
dem_100_newyork+_392	32.7M	32.7M	0.1	34.1	32.7M	32.7M	0.1	91.4
dem_100_newyork+_393	20.7M	20.7M	0.1	160.9	20.7M	20.7M	0.1	16.3
dem_100_newyork+_394	43.9M	43.9M	0.0	9.1	43.9M	43.9M	0.0	40.0
dem_100_newyork+_395	33.7M	33.7M	0.0	18.9	33.7M	33.7M	0.0	43.9
dem_100_newyork+_396	25.0M	25.0M	0.1	584.4	25.0M	25.0M	0.1	32.6
dem_100_newyork+_397	42.6M	42.5M	0.2	14400.0	42.6M	42.6M	0.0	34.3
dem_100_newyork+_398	34.1M	34.1M	0.1	1987.4	34.1M	34.1M	0.1	165.6
dem_100_newyork+_399	24.0M	23.8M	1.1	14400.0	24.0M	23.8M	0.9	14400.0
dem_100_newyork+_400	35.3M	35.3M	0.0	8.8	46.4M	11.7M	297.9	14400.0
dem_100_newyork+_401	41.0M	40.9M	0.1	13.9	41.0M	19.9M	105.4	14400.0
dem_100_newyork+_402	32.3M	32.3M	0.1	960.7	32.3M	32.1M	0.7	14400.0
dem_100_newyork+_403	22.4M	22.4M	0.1	52.1	22.4M	22.4M	0.1	97.5
dem_100_newyork+_404	45.2M	45.0M	0.3	14400.0	45.2M	45.1M	0.1	260.4
dem_100_newyork+_405	36.9M	36.8M	0.1	31.3	36.9M	36.9M	0.0	47.6
dem_100_newyork+_406	32.8M	32.8M	0.1	9260.6	32.8M	32.8M	0.1	191.7
dem_100_newyork+_407	34.1M	34.1M	0.1	2085.1	34.1M	34.1M	0.1	29.6
dem_100_newyork+_408	32.2M	32.1M	0.1	827.4	32.2M	32.1M	0.1	118.3
dem_100_newyork+_409	33.9M	33.9M	0.1	84.1	33.9M	33.9M	0.1	44.2
dem_100_newyork+_410	39.5M	39.4M	0.1	48.7	39.5M	39.5M	0.1	190.6
dem_100_newyork+_411	34.2M	34.1M	0.1	239.4	34.2M	34.1M	0.1	41.1
dem_100_newyork+_412	30.9M	30.9M	0.1	14.9	30.9M	30.9M	0.1	85.7
dem_100_newyork+_413	31.9M	31.9M	0.1	13.3	31.9M	31.9M	0.1	21.4
dem_100_newyork+_414	13.7M	13.7M	0.1	31.3	13.7M	13.7M	0.1	253.1
dem_100_newyork+_415	45.2M	45.2M	0.0	15.6	45.2M	45.2M	0.1	1105.3
dem_100_newyork+_416	47.6M	47.6M	0.0	13.0	47.6M	47.6M	0.1	35.2
dem_100_newyork+_417	31.6M	31.5M	0.1	534.3	31.6M	31.5M	0.1	28.2

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Table A.42: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_418	34.7M	34.2M	1.6	14400.0	34.7M	34.7M	0.0	173.5
dem_100_newyork+_419	27.3M	27.2M	0.1	9.8	27.3M	27.2M	0.1	13.3
dem_100_newyork+_420	33.9M	33.9M	0.1	33.6	33.9M	33.9M	0.1	425.0
dem_100_newyork+_421	34.1M	34.0M	0.1	11.4	34.1M	34.0M	0.1	20.6
dem_100_newyork+_422	42.3M	42.3M	0.0	9.2	42.3M	41.8M	1.2	14400.0
dem_100_newyork+_423	32.0M	31.7M	0.7	14400.0	32.0M	31.9M	0.1	2143.6
dem_100_newyork+_424	32.4M	31.8M	1.9	14400.0	32.4M	32.4M	0.0	30.9
dem_100_newyork+_425	34.0M	34.0M	0.0	17.2	34.0M	34.0M	0.0	17.0
dem_100_newyork+_426	32.8M	32.8M	0.1	712.1	32.8M	32.8M	0.0	19.0
dem_100_newyork+_427	19.9M	19.8M	0.1	411.4	19.9M	19.8M	0.1	11.7
dem_100_newyork+_428	36.1M	36.1M	0.0	14.5	36.1M	36.1M	0.0	19.7
dem_100_newyork+_429	33.9M	33.9M	0.1	12.3	33.9M	33.9M	0.1	59.5
dem_100_newyork+_430	25.8M	25.7M	0.1	1151.6	25.8M	25.7M	0.1	852.5
dem_100_newyork+_431	35.0M	35.0M	0.1	1041.3	35.0M	35.0M	0.1	42.9
dem_100_newyork+_432	40.9M	40.5M	1.0	14400.0	40.8M	40.7M	0.1	53.7
dem_100_newyork+_433	29.9M	29.9M	0.1	15.4	29.9M	29.9M	0.1	308.4
dem_100_newyork+_434	39.4M	39.3M	0.1	22.4	39.3M	38.0M	3.6	14400.0
dem_100_newyork+_435	35.2M	35.2M	0.0	6.9	35.2M	16.5M	112.8	14400.0
dem_100_newyork+_436	37.1M	37.1M	0.1	578.3	37.1M	37.1M	0.1	25.9
dem_100_newyork+_437	32.5M	32.5M	0.1	1082.7	32.5M	32.5M	0.0	32.8
dem_100_newyork+_438	30.2M	30.2M	0.0	49.9	30.2M	30.2M	0.1	172.3
dem_100_newyork+_439	46.8M	46.8M	0.0	11.8	46.8M	46.8M	0.1	6095.4
dem_100_newyork+_440	38.2M	38.1M	0.1	22.9	38.2M	38.2M	0.1	34.9
dem_100_newyork+_441	25.4M	25.4M	0.1	22.0	25.4M	25.4M	0.0	75.1
dem_100_newyork+_442	39.2M	39.2M	0.1	15.3	39.2M	39.2M	0.1	1605.3
dem_100_newyork+_443	22.7M	22.6M	0.7	14400.0	22.7M	22.7M	0.0	16.4
dem_100_newyork+_444	35.3M	35.3M	0.0	6.8	35.3M	35.3M	0.1	20.8
dem_100_newyork+_445	44.5M	44.4M	0.1	4861.3	44.5M	10.9M	309.4	14400.0
dem_100_newyork+_446	39.5M	39.5M	0.0	11.4	39.5M	39.5M	0.0	37.0
dem_100_newyork+_447	37.3M	37.3M	0.1	10035.1	37.3M	37.3M	0.1	18.8
dem_100_newyork+_448	28.2M	28.1M	0.1	66.3	28.2M	28.1M	0.1	53.9
dem_100_newyork+_449	35.5M	35.4M	0.1	636.4	35.5M	35.4M	0.1	45.1
dem_100_newyork+_450	27.5M	27.5M	0.2	14400.0	27.5M	27.5M	0.1	24.5
dem_100_newyork+_451	31.9M	31.9M	0.0	4435.8	31.9M	31.9M	0.1	23.8
dem_100_newyork+_452	30.1M	30.1M	0.1	52.4	30.1M	30.1M	0.1	33.5
dem_100_newyork+_453	45.1M	45.1M	0.0	14.8	45.1M	44.8M	0.7	14400.0
dem_100_newyork+_454	34.2M	34.2M	0.1	1108.9	34.2M	34.2M	0.0	24.8
dem_100_newyork+_455	33.4M	33.4M	0.1	27.9	33.4M	33.4M	0.1	134.5
dem_100_newyork+_456	36.5M	36.5M	0.0	7.4	36.5M	36.5M	0.1	76.1
dem_100_newyork+_457	36.8M	36.8M	0.0	10.2	36.8M	36.7M	0.1	72.1
dem_100_newyork+_458	37.5M	37.4M	0.1	1380.5	37.5M	36.9M	1.7	14400.0
dem_100_newyork+_459	39.5M	39.5M	0.1	9.6	39.5M	39.5M	0.1	22.6
dem_100_newyork+_460	43.4M	43.4M	0.0	16.1	43.4M	43.4M	0.1	88.6
dem_100_newyork+_461	35.0M	35.0M	0.0	22.6	35.0M	35.0M	0.0	32.2
dem_100_newyork+_462	41.8M	41.8M	0.0	14.5	41.8M	40.7M	2.5	14400.0
dem_100_newyork+_463	18.7M	18.7M	0.1	371.8	18.7M	18.7M	0.1	25.1
dem_100_newyork+_464	47.0M	47.0M	0.0	14.8	47.0M	46.9M	0.1	97.5
dem_100_newyork+_465	40.2M	40.2M	0.0	6.7	40.2M	40.2M	0.1	14.0
dem_100_newyork+_466	34.1M	34.0M	0.1	3644.3	34.1M	33.7M	1.2	14400.0
dem_100_newyork+_467	36.8M	36.8M	0.1	9129.5	36.8M	36.8M	0.2	14400.0
dem_100_newyork+_468	30.9M	30.8M	0.1	2453.2	30.9M	30.8M	0.1	35.6
dem_100_newyork+_469	39.5M	39.4M	0.1	79.9	39.5M	39.4M	0.1	70.7
dem_100_newyork+_470	31.5M	31.5M	0.1	2512.6	31.5M	31.5M	0.0	20.3
dem_100_newyork+_471	42.7M	42.1M	1.5	14400.0	42.7M	42.7M	0.1	558.8
dem_100_newyork+_472	31.6M	31.0M	1.9	14400.0	31.6M	31.5M	0.1	103.7
dem_100_newyork+_473	44.2M	44.2M	0.0	9.8	44.2M	44.1M	0.1	298.9
dem_100_newyork+_474	34.7M	34.7M	0.0	7.1	34.7M	34.6M	0.1	33.9
dem_100_newyork+_475	40.7M	40.6M	0.1	22.5	40.7M	40.7M	0.0	59.2
dem_100_newyork+_476	29.3M	29.3M	0.1	1421.1	29.3M	29.3M	0.1	249.4
dem_100_newyork+_477	25.7M	25.7M	0.1	3494.1	25.7M	25.6M	0.3	14400.0
dem_100_newyork+_478	35.0M	33.9M	3.4	14400.0	35.0M	35.0M	0.1	338.2
dem_100_newyork+_479	40.3M	40.3M	0.1	19.9	40.3M	40.3M	0.1	37.0
dem_100_newyork+_480	29.3M	29.3M	0.1	847.7	29.3M	29.3M	0.1	404.4
dem_100_newyork+_481	44.6M	44.4M	0.3	14400.0	44.6M	44.5M	0.1	4983.0
dem_100_newyork+_482	39.1M	38.8M	0.7	14400.0	39.1M	39.1M	0.1	871.9
dem_100_newyork+_483	41.9M	41.8M	0.3	14400.0	41.9M	41.9M	0.1	1562.7
dem_100_newyork+_484	34.7M	34.7M	0.0	22.4	42.4M	29.2M	45.2	14400.0
dem_100_newyork+_485	33.1M	33.1M	0.1	986.0	33.1M	33.1M	0.1	488.0
dem_100_newyork+_486	45.2M	43.9M	3.0	14400.0	45.2M	45.2M	0.1	31.1
dem_100_newyork+_487	31.3M	31.3M	0.0	8.2	31.3M	31.3M	0.0	67.1
dem_100_newyork+_488	37.1M	37.0M	0.1	4925.5	37.1M	37.0M	0.1	41.4
dem_100_newyork+_489	24.6M	24.6M	0.1	1578.7	24.6M	24.6M	0.1	63.4
dem_100_newyork+_490	27.0M	27.0M	0.0	8.7	27.0M	27.0M	0.0	29.4
dem_100_newyork+_491	33.4M	33.3M	0.1	3511.3	33.4M	33.3M	0.1	215.9
dem_100_newyork+_492	10.1M	10.1M	0.0	1.0	10.1M	10.1M	0.0	1.1
dem_100_newyork+_493	38.0M	37.9M	0.1	434.7	38.0M	37.9M	0.1	175.6
dem_100_newyork+_494	25.9M	25.6M	1.3	14400.0	25.9M	25.9M	0.1	528.6
dem_100_newyork+_495	35.7M	35.5M	0.6	14400.0	35.7M	35.7M	0.0	30.2

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Table A.42: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 100$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_100_newyork+_496	32.1M	32.0M	0.3	14400.0	32.1M	32.1M	0.1	19.4
dem_100_newyork+_497	30.7M	30.7M	0.1	12.5	30.7M	30.7M	0.0	15.7
dem_100_newyork+_498	34.0M	34.0M	0.1	30.2	34.0M	34.0M	0.1	88.5
dem_100_newyork+_499	28.0M	27.9M	0.1	10.1	28.0M	27.9M	0.1	19.4
dem_100_newyork+_500	31.2M	31.2M	0.1	5137.6	31.2M	31.2M	0.1	21.0

Table A.43: Detailed results of the discrete models on *New York+* with four additional arcs and $\mathcal{B} = 200$ as summarized in Figure 3.12b and Table 3.3f. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_1	1e+20	43.1M	–	14400.0	108.7M	70.6M	53.9	14400.0	116.2M	44.0M	164.0	14400.0
dem_200_newyork+_2	154.1M	48.9M	214.9	14400.0	138.1M	112.5M	22.8	14400.0	1e+20	59.7M	–	14400.0
dem_200_newyork+_3	130.9M	48.4M	170.4	14400.0	96.7M	82.2M	17.6	14400.0	96.7M	49.8M	94.3	14400.0
dem_200_newyork+_4	1e+20	48.5M	–	14400.0	152.4M	90.4M	68.6	14400.0	1e+20	55.4M	–	14400.0
dem_200_newyork+_5	151.0M	53.6M	181.7	14400.5	210.9M	82.9M	154.4	14400.0	156.5M	57.7M	171.3	14400.0
dem_200_newyork+_6	1e+20	48.6M	–	14400.0	124.0M	85.6M	44.8	14400.0	1e+20	50.5M	–	14400.0
dem_200_newyork+_7	146.1M	47.7M	206.4	14400.0	98.7M	98.7M	0.0	10400.0	104.8M	49.0M	113.9	14400.0
dem_200_newyork+_8	130.5M	52.3M	149.4	14400.0	149.6M	81.3M	83.9	14400.0	130.5M	59.2M	120.4	14400.0
dem_200_newyork+_9	178.8M	43.5M	311.2	14400.0	134.4M	90.3M	48.9	14400.0	154.2M	47.0M	228.2	14400.0
dem_200_newyork+_10	218.7M	41.2M	430.2	14400.0	103.7M	72.2M	43.5	14400.0	105.4M	44.4M	137.4	14400.0
dem_200_newyork+_11	1e+20	51.0M	–	14401.2	130.3M	97.3M	33.9	14400.0	158.4M	55.4M	185.9	14400.0
dem_200_newyork+_12	169.5M	49.6M	241.9	14400.0	1e+20	86.5M	–	14400.0	1e+20	55.0M	–	14400.0
dem_200_newyork+_13	165.0M	53.1M	210.7	14400.0	152.6M	113.1M	34.8	14400.0	1e+20	58.7M	–	14400.3
dem_200_newyork+_14	166.4M	48.8M	240.7	14400.6	114.5M	69.6M	64.7	14400.0	126.8M	50.6M	150.6	14400.0
dem_200_newyork+_15	1e+20	50.7M	–	14400.0	151.2M	105.9M	42.8	14400.0	1e+20	59.2M	–	14400.0
dem_200_newyork+_16	127.4M	46.8M	172.6	14400.0	1e+20	57.4M	–	14400.0	105.4M	48.8M	116.0	14400.0
dem_200_newyork+_17	229.6M	40.4M	468.2	14400.0	119.4M	78.1M	52.9	14400.0	1e+20	40.9M	–	14400.0
dem_200_newyork+_18	1e+20	47.8M	–	14400.0	156.8M	99.6M	57.4	14400.0	1e+20	55.0M	–	14400.0
dem_200_newyork+_19	156.8M	49.3M	218.0	14400.0	107.7M	71.8M	49.9	14400.0	131.9M	49.1M	168.6	14400.0
dem_200_newyork+_20	180.4M	47.3M	281.5	14400.0	144.3M	103.1M	40.0	14400.0	1e+20	55.4M	–	14400.0
dem_200_newyork+_21	1e+20	47.4M	–	14400.0	148.2M	97.2M	52.5	14400.0	258.1M	60.4M	327.4	14400.0
dem_200_newyork+_22	147.4M	39.0M	277.4	14400.0	90.2M	71.1M	26.8	14400.0	102.6M	44.7M	129.3	14400.0
dem_200_newyork+_23	156.0M	55.0M	183.7	14400.0	135.1M	91.0M	48.5	14400.0	1e+20	55.7M	–	14400.0
dem_200_newyork+_24	180.3M	49.8M	261.8	14400.0	1e+20	94.0M	–	14400.0	1e+20	51.0M	–	14400.0
dem_200_newyork+_25	180.0M	47.0M	283.4	14400.0	163.5M	98.2M	66.6	14400.0	244.6M	54.7M	347.4	14400.3
dem_200_newyork+_26	1e+20	41.2M	–	14400.0	124.7M	104.4M	19.5	14400.0	186.4M	42.9M	334.5	14400.1
dem_200_newyork+_27	1e+20	48.3M	–	14400.0	156.7M	112.5M	39.3	14400.0	1e+20	64.1M	–	14400.3
dem_200_newyork+_28	177.2M	54.2M	226.7	14400.2	154.4M	102.1M	51.3	14400.0	219.7M	60.1M	265.4	14400.0
dem_200_newyork+_29	171.0M	51.7M	231.1	14400.0	135.2M	102.8M	31.4	14400.0	210.3M	56.0M	275.7	14400.0
dem_200_newyork+_30	182.8M	44.9M	307.0	14400.0	127.6M	80.8M	57.9	14400.0	1e+20	48.8M	–	14400.0
dem_200_newyork+_31	217.3M	48.1M	352.0	14400.0	123.5M	91.8M	34.5	14400.0	1e+20	51.7M	–	14400.0
dem_200_newyork+_32	198.6M	47.2M	320.7	14400.0	141.3M	106.6M	32.6	14400.0	1e+20	52.0M	–	14400.3
dem_200_newyork+_33	127.3M	45.4M	180.2	14400.0	116.4M	95.6M	21.7	14400.0	152.5M	47.7M	219.6	14400.0
dem_200_newyork+_34	186.9M	51.1M	265.9	14400.0	122.8M	86.5M	42.0	14400.0	1e+20	53.8M	–	14400.0
dem_200_newyork+_35	1e+20	48.3M	–	14400.0	153.6M	93.3M	64.6	14400.0	298.1M	56.9M	423.9	14400.0
dem_200_newyork+_36	223.4M	54.2M	311.9	14400.0	177.1M	125.1M	41.6	14400.0	1e+20	63.7M	–	14400.0
dem_200_newyork+_37	1e+20	43.5M	–	14400.0	125.4M	95.6M	31.2	14400.0	157.9M	48.0M	229.2	14400.0
dem_200_newyork+_38	158.6M	50.5M	214.4	14400.0	139.7M	109.2M	27.9	14400.0	1e+20	54.6M	–	14400.0
dem_200_newyork+_39	163.2M	44.0M	271.0	14400.0	146.5M	101.6M	44.2	14400.0	1e+20	54.1M	–	14400.0
dem_200_newyork+_40	1e+20	40.9M	–	14400.0	130.8M	83.3M	57.0	14400.0	130.4M	47.2M	176.1	14400.0
dem_200_newyork+_41	169.4M	51.9M	226.4	14400.0	148.4M	104.1M	42.6	14400.0	460.0M	56.0M	721.4	14400.0
dem_200_newyork+_42	126.4M	50.7M	149.5	14400.0	1e+20	73.4M	–	14400.0	139.1M	56.3M	147.2	14400.0
dem_200_newyork+_43	1e+20	45.4M	–	14400.1	255.6M	100.6M	154.2	14400.0	1e+20	52.5M	–	14400.0
dem_200_newyork+_44	1e+20	57.5M	–	14400.0	167.3M	105.6M	58.5	14403.3	1e+20	73.0M	–	14400.0
dem_200_newyork+_45	144.4M	37.8M	282.3	14400.0	92.1M	70.4M	30.9	14400.0	120.8M	41.3M	192.8	14400.0
dem_200_newyork+_46	167.8M	45.2M	271.1	14400.0	154.0M	101.1M	52.3	14400.0	1e+20	45.8M	–	14400.0
dem_200_newyork+_47	1e+20	45.9M	–	14400.0	139.1M	99.6M	39.7	14400.0	143.5M	56.3M	155.0	14400.0
dem_200_newyork+_48	168.2M	45.3M	271.2	14400.3	117.0M	84.7M	38.1	14400.0	1e+20	47.7M	–	14400.0
dem_200_newyork+_49	179.3M	45.3M	296.0	14400.0	1e+20	73.2M	–	14400.0	151.8M	46.9M	223.8	14400.0
dem_200_newyork+_50	137.7M	40.8M	237.7	14400.0	126.8M	79.1M	60.4	14402.8	126.0M	47.0M	168.0	14400.1
dem_200_newyork+_51	142.7M	56.2M	153.8	14400.0	142.0M	96.8M	46.7	14400.0	217.4M	64.9M	234.9	14400.0
dem_200_newyork+_52	228.3M	48.7M	368.5	14400.2	121.2M	90.8M	33.6	14400.0	1e+20	56.8M	–	14400.0
dem_200_newyork+_53	159.9M	48.2M	231.7	14400.0	118.3M	77.4M	52.8	14400.0	1e+20	49.4M	–	14400.0
dem_200_newyork+_54	171.0M	44.5M	283.9	14400.0	138.9M	93.5M	48.6	14400.0	1e+20	49.0M	–	14400.0
dem_200_newyork+_55	161.3M	55.3M	191.7	14400.1	139.6M	93.6M	49.2	14400.0	213.7M	64.8M	229.7	14400.0
dem_200_newyork+_56	216.7M	48.5M	347.0	14400.3	136.2M	106.4M	28.1	14400.0	1e+20	54.3M	–	14400.0
dem_200_newyork+_57	1e+20	41.7M	–	14400.0	117.6M	83.4M	41.0	14403.2	117.6M	44.0M	167.3	14400.0
dem_200_newyork+_58	145.0M	52.7M	175.1	14400.0	1e+20	95.8M	–	14400.0	144.4M	58.7M	146.2	14400.0

continued on next page

Table A.43: Comparison of discrete models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_59	1e+20	43.8M	–	0.0	124.2M	86.0M	44.4	14400.0	1e+20	49.1M	–	14400.0
dem_200_newyork+_60	227.2M	49.0M	363.8	14400.2	133.8M	109.2M	22.5	14400.0	1e+20	54.7M	–	14400.0
dem_200_newyork+_61	1e+20	50.6M	–	14400.0	132.8M	97.1M	36.7	14400.0	1e+20	52.4M	–	14400.0
dem_200_newyork+_62	1e+20	46.2M	–	14400.0	116.1M	84.4M	37.5	14400.0	130.0M	55.1M	136.0	14400.0
dem_200_newyork+_63	1e+20	51.5M	–	14400.0	209.2M	86.6M	141.5	14400.0	280.2M	59.4M	371.5	14400.0
dem_200_newyork+_64	171.3M	59.5M	188.0	14400.5	159.0M	97.6M	62.9	14400.0	1e+20	62.3M	–	14400.0
dem_200_newyork+_65	1e+20	50.6M	–	14400.0	160.2M	107.0M	49.7	14400.0	1e+20	57.7M	–	14400.0
dem_200_newyork+_66	167.9M	48.8M	244.3	14400.0	134.3M	108.0M	24.4	14401.6	243.7M	56.9M	328.5	14400.0
dem_200_newyork+_67	195.8M	48.0M	307.6	14400.0	118.0M	90.2M	30.9	14400.0	139.4M	58.8M	137.3	14400.0
dem_200_newyork+_68	1e+20	52.7M	–	14400.5	140.2M	99.0M	41.7	14400.0	341.3M	56.9M	500.2	14400.1
dem_200_newyork+_69	215.7M	52.7M	309.4	14400.7	160.1M	127.9M	25.2	14400.0	205.4M	65.4M	214.3	14400.0
dem_200_newyork+_70	148.6M	46.0M	222.8	14400.0	119.2M	88.6M	34.5	14400.0	130.5M	54.7M	138.8	14400.0
dem_200_newyork+_71	200.6M	45.3M	343.0	14400.0	126.3M	88.4M	42.9	14400.0	1e+20	51.0M	–	14400.0
dem_200_newyork+_72	159.5M	51.7M	208.5	14400.2	131.2M	86.6M	51.6	14400.0	1e+20	51.6M	–	14400.0
dem_200_newyork+_73	169.0M	51.3M	229.4	14401.0	131.5M	131.5M	0.0	12601.0	1e+20	51.8M	–	14400.0
dem_200_newyork+_74	126.2M	46.1M	173.7	14400.1	102.9M	77.0M	33.6	14402.0	240.3M	43.9M	447.8	14400.2
dem_200_newyork+_75	128.2M	53.4M	139.9	14400.6	103.7M	85.2M	21.8	14400.0	1e+20	56.3M	–	14400.0
dem_200_newyork+_76	1e+20	52.7M	–	14400.0	135.4M	115.4M	17.4	14400.0	309.8M	57.9M	435.5	14400.0
dem_200_newyork+_77	151.7M	44.5M	240.6	14400.0	104.6M	88.2M	18.7	14400.0	110.8M	48.5M	128.3	14400.0
dem_200_newyork+_78	208.2M	50.9M	308.8	14400.0	143.1M	108.0M	32.6	14400.0	1e+20	55.4M	–	14400.0
dem_200_newyork+_79	1e+20	53.8M	–	14400.0	154.0M	121.8M	26.4	14400.0	176.9M	66.7M	165.3	14400.0
dem_200_newyork+_80	1e+20	49.6M	–	14400.0	1e+20	99.0M	–	14400.0	1e+20	51.6M	–	14400.0
dem_200_newyork+_81	1e+20	47.8M	–	14400.0	139.1M	112.0M	24.2	14400.0	1e+20	48.5M	–	14400.0
dem_200_newyork+_82	1e+20	52.6M	–	14400.9	144.7M	108.2M	33.8	14400.0	1e+20	62.9M	–	14400.0
dem_200_newyork+_83	1e+20	44.7M	–	14400.0	126.0M	88.5M	42.4	14400.0	129.1M	52.6M	145.7	14400.4
dem_200_newyork+_84	94.6M	37.8M	150.6	14400.0	81.0M	59.8M	35.5	14403.0	205.3M	39.8M	415.9	14400.0
dem_200_newyork+_85	161.4M	50.4M	220.1	14400.3	132.2M	83.7M	58.0	14400.0	166.2M	52.8M	214.7	14400.0
dem_200_newyork+_86	163.4M	49.7M	228.6	14400.0	142.1M	120.7M	17.7	14400.0	164.0M	56.0M	193.1	14400.0
dem_200_newyork+_87	153.1M	41.9M	265.2	14400.0	109.4M	76.1M	43.7	14400.0	117.7M	42.6M	176.2	14400.0
dem_200_newyork+_88	171.1M	51.3M	233.6	14400.0	126.0M	86.2M	46.2	14400.0	1e+20	51.0M	–	14400.0
dem_200_newyork+_89	1e+20	39.8M	–	14400.5	108.2M	108.2M	0.0	11593.7	1e+20	48.9M	–	14400.0
dem_200_newyork+_90	181.0M	47.4M	281.5	14400.0	110.7M	85.9M	28.8	14400.0	242.1M	53.6M	351.9	14400.0
dem_200_newyork+_91	242.2M	53.8M	350.6	14400.0	154.1M	111.0M	38.8	14400.0	1e+20	58.6M	–	14400.0
dem_200_newyork+_92	166.8M	47.2M	253.4	14400.0	1e+20	78.2M	–	14400.0	1e+20	48.7M	–	14400.0
dem_200_newyork+_93	133.6M	47.0M	184.4	14400.0	128.7M	88.0M	46.2	14400.0	140.5M	51.1M	174.9	14400.0
dem_200_newyork+_94	132.0M	51.9M	154.3	14401.7	135.9M	92.3M	47.3	14400.0	1e+20	58.4M	–	14400.0
dem_200_newyork+_95	1e+20	41.1M	–	14400.0	98.5M	60.9M	61.6	14400.0	110.4M	42.5M	159.5	14400.0
dem_200_newyork+_96	208.7M	47.3M	341.5	14400.2	134.2M	105.1M	27.7	14400.0	1e+20	51.3M	–	14400.1
dem_200_newyork+_97	151.5M	49.4M	206.8	14400.0	175.5M	87.7M	100.0	14404.8	1e+20	52.1M	–	14400.0
dem_200_newyork+_98	134.2M	45.8M	193.0	14400.0	145.5M	87.1M	67.1	14400.0	136.3M	46.6M	192.8	14400.0
dem_200_newyork+_99	150.3M	47.0M	219.5	14400.0	139.6M	120.0M	16.4	14400.0	1e+20	54.9M	–	14400.0
dem_200_newyork+_100	158.7M	57.5M	176.2	14400.0	159.7M	121.7M	31.1	14400.0	1e+20	69.5M	–	14400.0
dem_200_newyork+_101	1e+20	51.0M	–	14400.0	284.0M	93.1M	205.0	14400.0	376.5M	58.0M	549.3	14400.0
dem_200_newyork+_102	288.1M	51.5M	459.6	14400.0	153.7M	106.1M	44.8	14400.0	1e+20	61.4M	–	14400.0
dem_200_newyork+_103	1e+20	47.7M	–	14400.0	142.6M	107.0M	33.3	14400.0	1e+20	56.1M	–	14400.0
dem_200_newyork+_104	139.9M	51.4M	172.5	14400.0	138.3M	107.4M	28.8	14400.0	1e+20	52.9M	–	14400.0
dem_200_newyork+_105	164.1M	50.7M	223.7	14400.3	130.2M	87.6M	48.7	14400.0	1e+20	50.5M	–	14400.2
dem_200_newyork+_106	167.0M	49.7M	236.3	14400.0	141.8M	94.8M	49.6	14400.0	1e+20	60.8M	–	14400.0
dem_200_newyork+_107	1e+20	46.3M	–	14400.0	154.7M	116.7M	32.5	14400.0	1e+20	53.6M	–	14400.0
dem_200_newyork+_108	1e+20	46.5M	–	14401.7	138.8M	100.9M	37.5	14400.0	1e+20	52.3M	–	14400.0
dem_200_newyork+_109	203.2M	47.4M	328.5	14400.0	148.9M	110.8M	34.4	14400.0	1e+20	60.1M	–	14400.0
dem_200_newyork+_110	276.3M	48.6M	468.3	14400.0	177.2M	88.8M	99.4	14400.0	216.1M	52.6M	311.0	14400.0
dem_200_newyork+_111	124.1M	43.3M	186.3	14400.0	106.0M	106.0M	0.0	12643.9	1e+20	45.5M	–	14400.0
dem_200_newyork+_112	175.0M	49.6M	252.9	14400.0	170.9M	101.4M	68.5	14400.0	1e+20	61.1M	–	14400.1
dem_200_newyork+_113	1e+20	48.3M	–	14400.0	162.1M	105.9M	53.1	14400.0	1e+20	63.3M	–	14400.0
dem_200_newyork+_114	177.6M	47.9M	270.6	14400.0	145.8M	98.9M	47.5	14400.0	1e+20	51.4M	–	14400.0
dem_200_newyork+_115	160.0M	50.4M	217.7	14400.0	130.0M	88.4M	47.0	14400.0	1e+20	55.9M	–	14400.0
dem_200_newyork+_116	1e+20	49.2M	–	14400.0	108.0M	90.8M	18.9	14400.0	1e+20	49.1M	–	14400.0
dem_200_newyork+_117	440.5M	44.6M	886.7	14400.0	226.9M	114.6M	97.9	14400.0	1e+20	51.3M	–	14400.0
dem_200_newyork+_118	167.6M	46.6M	259.8	14400.0	141.8M	93.7M	51.4	14400.0	1e+20	54.4M	–	14400.0
dem_200_newyork+_119	1e+20	55.1M	–	14400.0	156.9M	100.7M	55.8	14400.0	235.2M	65.1M	261.2	14400.0
dem_200_newyork+_120	159.9M	50.3M	217.8	14400.0	1e+20	81.4M	–	14400.0	1e+20	53.6M	–	14400.0
dem_200_newyork+_121	196.1M	52.5M	273.7	14400.0	142.6M	104.0M	37.1	14403.0	1e+20	55.5M	–	14400.0
dem_200_newyork+_122	1e+20	45.6M	–	14400.0	131.8M	97.4M	35.4	14400.0	155.1M	52.9M	193.3	14400.0
dem_200_newyork+_123	252.1M	48.3M	421.3	14400.0	125.9M	97.0M	29.8	14402.2	125.9M	51.2M	146.0	14400.0
dem_200_newyork+_124	127.1M	45.3M	180.2	14400.0	121.6M	81.3M	49.5	14400.0	1e+20	48.7M	–	14400.0
dem_200_newyork+_125	159.7M	46.2M	245.7	14400.0	131.6M	88.2M	49.2	14400.0	1e+20	50.8M	–	14400.0
dem_200_newyork+_126	1e+20	49.9M	–	14400.0	1e+20	89.8M	–	14400.0	173.9M	56.2M	209.2	14400.0
dem_200_newyork+_127	160.9M	49.7M	223.7	14400.0	149.6M	94.0M	59.2	14400.0	1e+20	59.5M	–	14400.0
dem_200_newyork+_128	132.1M	50.7M	160.5	14400.0	100.0M	77.9M	28.3	14400.0	119.8M	47.5M	152.3	14400.1
dem_200_newyork+_129	231.8M	48.6M	377.3	14400.0	144.8M	96.4M	50.2	14400.0	164.3M	58.3M	181.8	14400.0
dem_200_newyork+_130	206.9M	46.5M	345.2	14400.0	103.5M	86.9M	19.2	14403.4	1e+20	46.4M	–	14400.0
dem_200_newyork+_131	223.1M	52.7M	323.0	14400.0	127.0M	101.9M	24.7	14400.0	131.3M	57.8M	127.3	14400.0
dem_200_newyork+_132	138.8M	49.8M	178.8	14400.0	147.0M	90.2M	62.9	14400.0	155.3M	59.7M	160.4	14400.0
dem_200_newyork+_133	1e+20	44.4M	–	14400.0	145.9M	94.9M	53.7	14400.0	1e+20	53.2M	–	14400.0
dem_200_newyork+_134	150.6M	50.1M	200.5	14400.0	127.8M	94.8M	34.8	14400.0	1e+20	56.8M	–	14400.0
dem_200_newyork+_135	179.6M	51.7M	247.7	14400.0	172.3M	93.1M	85.0	14400.0	132.2M	57.4M	130.3	14400.0
dem_200_newyork+_136	138.6M	51.0M	171.9	14400.1	122.7M	85.5M	43.4	14400.0	171.2M	49.6M	244.8	14400.0

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Table A.43: Comparison of discrete models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_137	205.1M	53.9M	280.8	14400.0	131.4M	102.9M	27.7	14400.0	1e+20	57.6M	–	14400.0
dem_200_newyork+_138	182.3M	49.7M	266.4	14400.0	1e+20	94.9M	–	14400.0	1e+20	59.4M	–	14400.0
dem_200_newyork+_139	193.0M	49.4M	291.1	14400.0	144.5M	95.7M	51.1	14400.0	1e+20	51.4M	–	14400.0
dem_200_newyork+_140	170.1M	49.7M	242.2	14400.2	139.6M	89.7M	55.7	14400.0	1e+20	55.0M	–	14400.0
dem_200_newyork+_141	161.8M	52.2M	209.7	14400.3	143.9M	90.3M	59.3	14400.0	1e+20	57.2M	–	14400.0
dem_200_newyork+_142	146.1M	49.2M	196.9	14400.0	131.6M	95.3M	38.1	14400.0	183.4M	55.3M	231.9	14400.0
dem_200_newyork+_143	148.5M	54.5M	172.4	14400.0	134.2M	91.0M	47.5	14402.9	1e+20	56.1M	–	14400.0
dem_200_newyork+_144	166.9M	50.0M	233.6	14400.8	167.1M	93.5M	78.7	14400.0	128.1M	57.2M	124.1	14400.0
dem_200_newyork+_145	157.1M	46.6M	237.0	14400.0	141.5M	93.6M	51.2	14400.0	1e+20	51.3M	–	14400.0
dem_200_newyork+_146	166.0M	46.8M	254.6	14400.0	134.9M	90.3M	49.4	14402.2	167.2M	52.4M	218.9	14400.0
dem_200_newyork+_147	181.5M	48.0M	277.8	14400.1	142.5M	95.3M	49.6	14400.0	1e+20	58.7M	–	14400.0
dem_200_newyork+_148	132.0M	54.1M	144.1	14400.3	127.6M	91.6M	39.4	14400.0	1e+20	57.7M	–	14400.0
dem_200_newyork+_149	179.1M	49.2M	264.3	14400.0	161.0M	105.4M	52.8	14400.0	1e+20	59.4M	–	14400.0
dem_200_newyork+_150	127.6M	42.8M	198.1	14400.0	121.1M	85.2M	42.1	14400.0	124.6M	48.1M	159.1	14400.0
dem_200_newyork+_151	213.8M	48.0M	345.0	14400.0	148.6M	100.5M	47.9	14400.0	1e+20	53.6M	–	14400.3
dem_200_newyork+_152	174.1M	51.2M	239.9	14400.0	171.2M	108.9M	57.2	14400.0	1e+20	55.9M	–	14400.0
dem_200_newyork+_153	154.5M	50.4M	206.4	14400.0	134.7M	96.0M	40.3	14400.0	159.0M	49.5M	221.1	14400.0
dem_200_newyork+_154	195.7M	52.6M	272.3	14400.0	145.9M	112.7M	29.5	14400.0	171.4M	59.0M	190.6	14400.0
dem_200_newyork+_155	190.9M	44.4M	329.7	14400.2	116.9M	83.2M	40.6	14400.0	133.8M	51.1M	161.6	14400.0
dem_200_newyork+_156	125.5M	39.5M	218.0	14400.0	106.7M	65.8M	62.1	14400.0	131.8M	45.3M	190.8	14400.0
dem_200_newyork+_157	1e+20	47.8M	–	14400.0	131.9M	104.4M	26.3	14400.0	1e+20	58.0M	–	14400.0
dem_200_newyork+_158	149.3M	51.9M	187.6	14401.1	123.1M	90.8M	35.5	14400.0	173.0M	54.6M	217.2	14400.0
dem_200_newyork+_159	180.2M	49.3M	265.8	14400.0	160.1M	105.9M	51.1	14400.0	1e+20	61.0M	–	14400.0
dem_200_newyork+_160	1e+20	51.5M	–	14400.2	143.2M	102.2M	40.1	14400.0	1e+20	64.3M	–	14400.0
dem_200_newyork+_161	1e+20	50.8M	–	14400.0	130.3M	89.4M	45.7	14403.8	1e+20	55.7M	–	14400.0
dem_200_newyork+_162	144.2M	49.7M	190.0	14400.0	134.0M	95.5M	40.3	14400.0	154.8M	56.5M	174.1	14400.1
dem_200_newyork+_163	157.3M	44.8M	251.1	14400.3	99.4M	81.6M	21.9	14403.3	102.5M	44.3M	131.7	14400.0
dem_200_newyork+_164	128.3M	47.5M	169.9	14400.0	169.4M	80.8M	109.5	14400.0	1e+20	47.7M	–	14400.0
dem_200_newyork+_165	217.2M	53.1M	309.3	14400.0	126.1M	104.7M	20.5	14400.0	1e+20	58.8M	–	14400.0
dem_200_newyork+_166	171.4M	46.0M	272.4	14400.0	141.5M	99.4M	42.4	14400.0	1e+20	45.9M	–	14400.0
dem_200_newyork+_167	144.2M	56.5M	155.4	14400.0	125.0M	94.0M	32.9	14400.0	1e+20	57.9M	–	14400.0
dem_200_newyork+_168	158.8M	45.9M	245.8	14400.0	101.8M	79.1M	28.7	14400.0	107.1M	50.0M	113.9	14400.0
dem_200_newyork+_169	222.6M	54.4M	309.5	14400.0	207.3M	129.4M	60.2	14400.0	1e+20	60.4M	–	14400.0
dem_200_newyork+_170	144.2M	47.2M	205.6	14400.0	151.7M	90.0M	68.5	14400.0	1e+20	53.2M	–	14400.0
dem_200_newyork+_171	142.0M	49.1M	189.3	14400.0	136.4M	98.2M	38.9	14403.0	1e+20	48.6M	–	14400.0
dem_200_newyork+_172	169.6M	50.6M	234.9	14400.0	142.5M	102.8M	38.6	14400.0	1e+20	60.4M	–	14400.0
dem_200_newyork+_173	1e+20	37.4M	–	0.0	98.3M	73.9M	32.9	14400.0	104.0M	50.3M	107.0	14400.0
dem_200_newyork+_174	131.1M	47.2M	177.6	14400.0	128.1M	92.7M	38.1	14403.2	1e+20	50.5M	–	14400.0
dem_200_newyork+_175	144.6M	48.0M	201.0	14400.0	149.9M	109.4M	37.1	14400.0	1e+20	55.7M	–	14400.0
dem_200_newyork+_176	191.3M	50.2M	281.1	14400.0	142.6M	89.9M	58.7	14400.0	146.7M	57.8M	154.0	14400.0
dem_200_newyork+_177	1e+20	45.4M	–	14400.0	150.1M	103.5M	45.0	14400.0	1e+20	55.2M	–	14400.0
dem_200_newyork+_178	177.4M	54.9M	222.9	14400.0	152.7M	114.6M	33.3	14400.0	1e+20	59.8M	–	14400.1
dem_200_newyork+_179	1e+20	48.6M	–	14400.1	137.5M	113.4M	21.2	14400.0	1e+20	51.1M	–	14400.0
dem_200_newyork+_180	134.3M	44.6M	201.3	14400.0	107.2M	80.3M	33.5	14400.0	1e+20	46.7M	–	14400.0
dem_200_newyork+_181	302.8M	58.3M	419.0	14401.0	160.3M	125.1M	28.1	14401.8	238.4M	73.8M	223.1	14400.2
dem_200_newyork+_182	142.0M	51.7M	174.6	14400.0	140.5M	93.2M	50.7	14400.0	135.1M	55.5M	143.6	14400.0
dem_200_newyork+_183	163.7M	50.0M	227.1	14400.0	150.4M	110.4M	36.2	14400.0	172.1M	52.7M	226.4	14400.0
dem_200_newyork+_184	171.7M	51.0M	237.0	14400.0	114.9M	83.0M	38.5	14400.0	1e+20	53.8M	–	14400.0
dem_200_newyork+_185	178.7M	54.7M	227.0	14400.3	143.0M	116.7M	22.6	14400.0	1e+20	60.6M	–	14400.0
dem_200_newyork+_186	133.4M	47.9M	178.6	14400.0	131.6M	98.7M	33.3	14400.0	443.1M	50.6M	775.7	14400.0
dem_200_newyork+_187	108.3M	51.3M	111.3	14400.0	110.2M	83.3M	32.3	14400.0	1e+20	51.7M	–	14400.0
dem_200_newyork+_188	192.7M	48.4M	297.8	14400.0	101.9M	81.5M	25.0	14402.3	125.8M	48.0M	162.3	14400.0
dem_200_newyork+_189	159.3M	53.9M	195.6	14400.0	157.3M	91.6M	71.7	14400.7	127.3M	63.4M	100.8	14400.0
dem_200_newyork+_190	167.0M	51.1M	226.5	14400.3	166.9M	120.7M	38.3	14400.0	1e+20	57.2M	–	14400.0
dem_200_newyork+_191	149.5M	51.8M	188.7	14400.0	114.8M	76.8M	49.5	14400.0	1e+20	57.2M	–	14400.2
dem_200_newyork+_192	162.5M	52.4M	210.2	14400.0	117.7M	89.0M	32.2	14400.0	1e+20	53.8M	–	14400.0
dem_200_newyork+_193	149.5M	46.0M	224.9	14400.0	1e+20	102.9M	–	14400.0	1e+20	51.2M	–	14400.0
dem_200_newyork+_194	144.1M	47.7M	201.9	14400.0	119.9M	85.0M	41.1	14400.0	1e+20	50.8M	–	14400.0
dem_200_newyork+_195	1e+20	47.2M	–	14400.0	176.3M	124.8M	41.3	14400.0	1e+20	59.0M	–	14400.0
dem_200_newyork+_196	155.7M	43.1M	261.6	14400.0	115.1M	76.3M	50.8	14400.0	198.8M	46.4M	328.5	14400.0
dem_200_newyork+_197	167.0M	46.4M	260.0	14400.0	119.6M	88.0M	36.0	14400.0	1e+20	46.0M	–	14400.0
dem_200_newyork+_198	182.2M	52.2M	249.2	14401.2	153.0M	112.1M	36.5	14400.0	243.6M	60.3M	304.1	14400.0
dem_200_newyork+_199	171.5M	57.6M	197.7	14400.8	175.7M	123.8M	41.9	14400.0	1e+20	62.3M	–	14400.0
dem_200_newyork+_200	1e+20	48.0M	–	14400.0	150.7M	105.6M	42.7	14400.0	1e+20	52.9M	–	14400.0
dem_200_newyork+_201	1e+20	46.0M	–	14400.0	133.9M	93.9M	42.6	14400.0	1e+20	57.2M	–	14400.0
dem_200_newyork+_202	172.2M	51.0M	237.7	14400.0	162.7M	88.5M	83.9	14400.0	150.9M	53.5M	181.9	14400.0
dem_200_newyork+_203	140.0M	42.6M	228.6	14400.0	96.4M	73.8M	30.5	14400.0	102.9M	47.4M	117.2	14400.0
dem_200_newyork+_204	180.5M	47.7M	278.5	14400.0	146.3M	103.9M	40.8	14400.0	210.0M	57.1M	267.9	14400.0
dem_200_newyork+_205	125.2M	49.3M	153.9	14400.0	105.6M	75.8M	39.2	14400.0	119.3M	47.3M	152.0	14400.0
dem_200_newyork+_206	1e+20	42.9M	–	14400.0	135.1M	103.2M	31.0	14400.0	1e+20	44.6M	–	14400.0
dem_200_newyork+_207	190.3M	50.0M	280.2	14401.5	144.4M	114.5M	26.1	14400.0	1e+20	56.4M	–	14400.0
dem_200_newyork+_208	137.6M	44.8M	207.0	14400.0	136.1M	90.7M	50.0	14400.0	1e+20	46.0M	–	14400.0
dem_200_newyork+_209	125.7M	50.8M	147.3	14400.0	104.5M	83.4M	25.2	14401.7	154.3M	52.6M	193.5	14400.1
dem_200_newyork+_210	1e+20	56.2M	–	14400.0	159.6M	111.5M	43.1	14400.0	1e+20	60.0M	–	14400.0
dem_200_newyork+_211	179.1M	53.5M	234.8	14400.0	131.5M	98.7M	33.3	14400.0	238.7M	58.6M	307.2	14400.4
dem_200_newyork+_212	144.8M	51.2M	183.0	14400.0	136.0M	83.8M	62.3	14400.0	1e+20	53.2M	–	14400.0
dem_200_newyork+_213	147.2M	46.7M	215.3	14400.0	207.9M	77.1M	169.8	14400.0	1e+20	48.2M	–	14400.0
dem_200_newyork+_214	116.7M	45.8M	154.9	14400.0	101.6M	73.4M	38.4	14400.0	1e+20	41.8M	–	14400.0

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Table A.43: Comparison of discrete models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_215	1e+20	46.9M	–	14400.0	1e+20	80.0M	–	14400.0	132.0M	55.1M	139.5	14400.0
dem_200_newyork+_216	207.5M	48.7M	326.3	14400.0	145.4M	105.6M	37.6	14400.0	290.4M	57.8M	402.8	14400.0
dem_200_newyork+_217	134.5M	49.3M	172.6	14400.0	114.7M	84.5M	35.8	14400.0	1e+20	52.6M	–	14400.0
dem_200_newyork+_218	198.8M	48.2M	312.7	14400.0	96.1M	70.8M	35.9	14400.0	128.0M	43.5M	194.2	14400.0
dem_200_newyork+_219	126.9M	49.5M	156.2	14400.0	120.2M	88.4M	36.1	14400.0	180.1M	57.4M	213.7	14400.0
dem_200_newyork+_220	143.7M	48.0M	199.4	14400.0	104.9M	104.9M	0.0	13410.2	1e+20	49.5M	–	14400.0
dem_200_newyork+_221	227.6M	45.0M	405.5	14400.0	105.9M	79.1M	33.9	14400.0	1e+20	46.1M	–	14400.0
dem_200_newyork+_222	1e+20	53.6M	–	14400.0	136.2M	112.5M	21.1	14401.9	1e+20	56.5M	–	14400.0
dem_200_newyork+_223	170.8M	52.0M	228.4	14400.0	1e+20	90.0M	–	14400.0	1e+20	57.9M	–	14400.1
dem_200_newyork+_224	131.5M	47.0M	179.9	14400.0	131.5M	87.7M	50.0	14400.0	1e+20	48.7M	–	14400.0
dem_200_newyork+_225	127.6M	52.7M	142.3	14400.0	119.6M	87.7M	36.3	14400.0	133.0M	55.8M	138.3	14400.0
dem_200_newyork+_226	1e+20	47.0M	–	14400.0	146.8M	107.5M	36.5	14402.7	1e+20	57.0M	–	14400.0
dem_200_newyork+_227	239.0M	44.6M	435.6	14400.0	136.8M	83.7M	63.4	14400.0	1e+20	47.1M	–	14400.0
dem_200_newyork+_228	154.5M	47.1M	228.3	14400.0	140.6M	82.9M	69.6	14400.0	1e+20	49.1M	–	14400.0
dem_200_newyork+_229	153.2M	48.9M	213.0	14400.0	131.9M	103.4M	27.6	14400.0	1e+20	52.0M	–	14400.0
dem_200_newyork+_230	213.1M	43.2M	393.2	14400.0	108.1M	76.5M	41.5	14400.0	1e+20	46.8M	–	14400.0
dem_200_newyork+_231	174.5M	43.0M	305.9	14442.5	283.1M	110.9M	155.3	14400.0	1e+20	45.3M	–	14400.0
dem_200_newyork+_232	1e+20	49.0M	–	14400.0	139.7M	96.7M	44.5	14400.0	1e+20	52.3M	–	14400.0
dem_200_newyork+_233	102.6M	47.7M	115.1	14400.0	94.1M	78.0M	20.7	14401.6	151.8M	49.1M	208.9	14400.0
dem_200_newyork+_234	1e+20	48.5M	–	14400.0	132.0M	95.9M	37.7	14400.0	1e+20	51.5M	–	14400.0
dem_200_newyork+_235	137.0M	42.9M	219.3	14400.0	110.0M	90.0M	22.2	14400.0	236.9M	49.2M	381.3	14400.0
dem_200_newyork+_236	121.9M	53.0M	130.1	14400.0	121.9M	107.6M	13.3	14401.2	162.5M	56.8M	186.3	14400.4
dem_200_newyork+_237	225.7M	47.3M	377.6	14400.0	101.9M	78.3M	30.1	14400.0	103.6M	48.9M	112.0	14400.0
dem_200_newyork+_238	129.6M	44.1M	193.8	14400.0	129.6M	86.3M	50.2	14400.0	1e+20	48.7M	–	14400.0
dem_200_newyork+_239	161.1M	50.2M	220.6	14400.0	132.1M	97.2M	35.9	14400.0	179.6M	52.0M	245.1	14400.1
dem_200_newyork+_240	141.6M	53.6M	164.2	14400.0	130.1M	89.3M	45.7	14402.8	133.1M	55.7M	139.1	14400.2
dem_200_newyork+_241	182.1M	53.4M	241.1	14400.0	151.9M	126.0M	20.6	14400.0	1e+20	64.3M	–	14400.0
dem_200_newyork+_242	199.1M	46.0M	332.6	14400.0	136.2M	95.8M	42.2	14400.0	1e+20	47.4M	–	14400.0
dem_200_newyork+_243	159.7M	44.6M	258.1	14400.5	138.9M	87.8M	58.2	14400.0	149.3M	52.2M	185.8	14400.0
dem_200_newyork+_244	1e+20	50.5M	–	14400.0	136.2M	106.2M	28.3	14402.0	174.2M	55.5M	213.9	14400.0
dem_200_newyork+_245	138.5M	47.0M	194.8	14400.0	110.6M	85.5M	29.4	14400.0	243.9M	49.6M	391.9	14400.0
dem_200_newyork+_246	125.7M	47.5M	164.7	14400.0	119.9M	78.3M	53.2	14400.0	385.7M	52.8M	631.1	14400.0
dem_200_newyork+_247	110.1M	50.2M	119.4	14400.1	103.0M	103.0M	0.0	14064.1	172.7M	51.2M	237.5	14400.0
dem_200_newyork+_248	171.0M	52.3M	227.2	14400.6	177.6M	111.1M	59.8	14400.0	166.8M	61.0M	173.3	14400.0
dem_200_newyork+_249	182.8M	46.5M	293.1	14400.4	148.2M	91.8M	61.4	14400.0	166.2M	55.6M	198.7	14400.3
dem_200_newyork+_250	143.4M	52.9M	171.2	14400.0	143.4M	106.8M	34.3	14400.0	1e+20	60.4M	–	14400.0
dem_200_newyork+_251	119.4M	46.3M	157.8	14400.0	112.0M	75.6M	48.1	14400.0	124.5M	45.4M	174.1	14400.0
dem_200_newyork+_252	158.9M	52.3M	203.9	14400.0	166.5M	84.7M	96.5	14400.0	128.2M	56.7M	126.3	14400.0
dem_200_newyork+_253	125.5M	43.6M	188.0	14400.0	111.0M	74.3M	49.5	14400.0	117.7M	44.5M	164.4	14400.1
dem_200_newyork+_254	183.4M	46.6M	293.2	14400.0	135.4M	93.1M	45.5	14400.0	1e+20	51.4M	–	14400.0
dem_200_newyork+_255	226.8M	45.7M	396.5	14400.0	136.9M	90.5M	51.3	14400.0	168.7M	55.5M	204.1	14400.0
dem_200_newyork+_256	116.6M	46.0M	153.2	14400.4	93.2M	71.9M	29.7	14400.0	93.2M	46.1M	102.4	14400.0
dem_200_newyork+_257	1e+20	51.3M	–	14400.0	182.8M	132.6M	37.9	14400.0	1e+20	58.0M	–	14400.0
dem_200_newyork+_258	224.8M	53.0M	324.2	14400.0	161.4M	116.6M	38.4	14400.0	158.9M	68.1M	133.5	14400.0
dem_200_newyork+_259	130.6M	46.8M	179.4	14400.0	130.3M	130.3M	0.0	13634.7	156.2M	54.7M	185.8	14400.0
dem_200_newyork+_260	130.4M	51.6M	152.8	14400.0	117.6M	89.7M	31.0	14400.0	254.8M	57.1M	346.0	14400.0
dem_200_newyork+_261	1e+20	47.5M	–	14400.2	209.9M	118.3M	77.5	14400.0	219.9M	64.0M	243.5	14400.0
dem_200_newyork+_262	178.2M	48.9M	264.4	14400.0	129.1M	97.5M	32.4	14400.0	1e+20	51.5M	–	14400.0
dem_200_newyork+_263	189.8M	47.9M	296.5	14400.0	123.3M	88.2M	39.7	14400.0	1e+20	50.9M	–	14400.0
dem_200_newyork+_264	150.9M	49.9M	202.4	14400.0	134.4M	103.1M	30.4	14400.0	1e+20	51.4M	–	14400.0
dem_200_newyork+_265	1e+20	48.2M	–	14400.0	178.4M	120.2M	48.4	14400.0	1e+20	55.5M	–	14400.0
dem_200_newyork+_266	274.6M	42.4M	548.0	14400.0	126.4M	91.5M	38.1	14400.0	1e+20	43.4M	–	14400.0
dem_200_newyork+_267	180.8M	50.9M	255.0	14400.0	120.6M	86.1M	40.1	14400.0	1e+20	51.9M	–	14400.0
dem_200_newyork+_268	102.5M	39.8M	157.6	14400.0	96.9M	96.9M	0.0	13226.2	1e+20	41.3M	–	14400.0
dem_200_newyork+_269	142.3M	44.1M	222.5	14400.0	115.5M	82.3M	40.4	14400.0	119.5M	45.3M	163.8	14400.2
dem_200_newyork+_270	174.4M	48.1M	262.3	14400.0	128.5M	94.1M	36.5	14400.0	1e+20	54.2M	–	14400.0
dem_200_newyork+_271	141.3M	46.3M	205.3	14400.0	111.8M	82.1M	36.2	14402.3	181.2M	53.2M	240.5	14400.0
dem_200_newyork+_272	156.7M	48.8M	221.3	14400.0	138.7M	93.5M	48.3	14400.0	1e+20	55.0M	–	14400.0
dem_200_newyork+_273	210.7M	47.2M	346.0	14400.0	158.0M	102.5M	54.1	14400.0	175.5M	64.6M	171.7	14400.0
dem_200_newyork+_274	1e+20	49.7M	–	14400.0	156.0M	112.1M	39.2	14400.0	1e+20	63.9M	–	14400.0
dem_200_newyork+_275	269.7M	54.3M	396.6	14400.0	136.2M	90.4M	50.7	14400.0	1e+20	56.6M	–	14400.0
dem_200_newyork+_276	205.1M	42.4M	384.1	14400.0	109.3M	62.9M	73.8	14400.0	1e+20	42.2M	–	14400.0
dem_200_newyork+_277	169.3M	54.9M	208.2	14400.0	155.9M	111.9M	39.2	14400.0	196.5M	63.5M	209.5	14400.0
dem_200_newyork+_278	122.1M	41.2M	196.1	14400.0	92.0M	72.4M	27.1	14400.0	118.2M	50.2M	135.7	14400.0
dem_200_newyork+_279	1e+20	49.4M	–	14400.0	122.9M	89.2M	37.8	14400.0	121.7M	51.7M	135.6	14400.0
dem_200_newyork+_280	1e+20	49.9M	–	14400.0	200.8M	119.6M	67.9	14400.0	1e+20	58.0M	–	14400.0
dem_200_newyork+_281	1e+20	50.7M	–	14400.0	121.0M	110.2M	9.8	14400.0	184.2M	58.9M	212.4	14400.0
dem_200_newyork+_282	1e+20	47.9M	–	14400.3	142.4M	92.6M	53.7	14400.0	213.4M	58.3M	265.9	14400.0
dem_200_newyork+_283	1e+20	36.7M	–	0.0	111.8M	87.3M	28.1	14400.0	1e+20	50.3M	–	14400.0
dem_200_newyork+_284	150.0M	49.3M	204.2	14400.0	148.4M	88.0M	68.7	14400.0	1e+20	50.1M	–	14400.0
dem_200_newyork+_285	214.5M	51.4M	317.5	14400.0	156.7M	115.9M	35.2	14400.0	1e+20	57.0M	–	14400.0
dem_200_newyork+_286	180.6M	51.8M	248.5	14400.1	119.4M	84.5M	41.3	14400.0	133.0M	59.1M	124.8	14400.0
dem_200_newyork+_287	220.5M	49.3M	347.5	14400.0	124.2M	91.3M	36.0	14400.0	1e+20	61.1M	–	14400.0
dem_200_newyork+_288	149.7M	45.3M	230.9	14400.0	134.8M	87.9M	53.4	14400.0	192.4M	49.6M	287.8	14400.0
dem_200_newyork+_289	116.7M	46.8M	149.4	14400.0	106.8M	70.7M	51.0	14400.0	116.2M	48.0M	142.2	14400.0
dem_200_newyork+_290	192.1M	49.0M	291.9	14400.0	127.6M	86.0M	48.4	14400.0	125.2M	51.8M	141.8	14400.0
dem_200_newyork+_291	202.9M	44.1M	359.8	14400.0	128.3M	90.1M	42.3	14400.0	1e+20	48.5M	–	14400.0
dem_200_newyork+_292	155.0M	49.3M	214.3	14400.0	131.8M	95.4M	38.0	14400.0	1e+20	52.3M	–	14400.0

continued on next page

Table A.43: Comparison of discrete models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_293	145.1M	47.2M	207.3	14400.0	129.3M	112.1M	15.3	14400.0	1e+20	46.9M	–	14400.0
dem_200_newyork+_294	1e+20	46.4M	–	14400.0	145.4M	91.4M	59.1	14400.0	1e+20	52.7M	–	14400.0
dem_200_newyork+_295	134.7M	52.3M	157.6	14400.0	206.4M	87.1M	137.1	14400.0	1e+20	54.4M	–	14400.0
dem_200_newyork+_296	178.4M	55.6M	220.5	14400.0	160.1M	96.8M	65.4	14400.0	230.9M	64.5M	258.1	14400.0
dem_200_newyork+_297	1e+20	44.2M	–	14400.0	136.2M	102.6M	32.7	14400.0	1e+20	51.8M	–	14400.0
dem_200_newyork+_298	105.5M	45.0M	134.7	14400.0	104.8M	76.9M	36.3	14400.0	119.8M	44.0M	172.3	14400.0
dem_200_newyork+_299	1e+20	47.2M	–	14400.0	1e+20	89.3M	–	14400.0	167.0M	51.1M	226.9	14400.0
dem_200_newyork+_300	1e+20	47.3M	–	14400.0	142.3M	99.9M	42.5	14400.0	172.3M	50.1M	243.7	14400.0
dem_200_newyork+_301	1e+20	45.4M	–	14400.0	147.1M	103.1M	42.7	14400.0	1e+20	60.8M	–	14400.0
dem_200_newyork+_302	131.7M	50.2M	162.5	14400.0	101.0M	80.3M	25.8	14400.0	102.2M	48.9M	109.1	14400.0
dem_200_newyork+_303	117.0M	46.8M	150.0	14400.1	103.3M	81.8M	26.3	14400.0	108.3M	52.8M	105.2	14400.0
dem_200_newyork+_304	142.5M	54.5M	161.5	14400.4	133.3M	106.8M	24.9	14401.9	140.1M	54.7M	156.3	14400.0
dem_200_newyork+_305	159.8M	50.5M	216.7	14400.0	135.7M	89.8M	51.1	14400.0	1e+20	60.4M	–	14400.0
dem_200_newyork+_306	182.2M	44.4M	310.1	14400.7	125.6M	85.4M	47.1	14400.0	120.6M	45.9M	162.5	14400.2
dem_200_newyork+_307	145.1M	49.9M	190.9	14400.0	105.9M	88.2M	20.1	14400.0	106.9M	53.7M	99.0	14400.0
dem_200_newyork+_308	158.2M	49.2M	221.5	14400.1	113.5M	79.6M	42.6	14400.0	221.1M	56.8M	289.2	14400.0
dem_200_newyork+_309	1e+20	49.4M	–	14400.0	129.5M	87.3M	48.4	14400.0	1e+20	51.6M	–	14400.0
dem_200_newyork+_310	184.3M	47.8M	285.7	14400.0	141.2M	111.1M	27.1	14400.0	1e+20	55.4M	–	14400.0
dem_200_newyork+_311	265.8M	52.3M	407.8	14400.9	170.1M	112.0M	52.0	14400.0	197.0M	63.7M	209.1	14400.0
dem_200_newyork+_312	138.4M	53.2M	160.2	14400.0	142.7M	97.3M	46.7	14400.0	225.9M	62.7M	260.6	14400.0
dem_200_newyork+_313	141.4M	55.5M	154.8	14400.0	177.6M	94.5M	88.0	14400.0	1e+20	61.2M	–	14400.0
dem_200_newyork+_314	171.0M	52.0M	228.6	14400.0	132.9M	80.8M	64.5	14400.0	173.6M	58.1M	198.6	14400.2
dem_200_newyork+_315	216.3M	46.6M	364.6	14400.0	177.2M	127.3M	39.2	14400.0	1e+20	62.3M	–	14400.0
dem_200_newyork+_316	421.5M	47.8M	782.6	14400.0	139.9M	99.6M	40.5	14400.0	1e+20	55.2M	–	14400.0
dem_200_newyork+_317	212.3M	50.3M	321.6	14400.0	137.7M	96.3M	42.9	14400.0	167.9M	55.7M	201.3	14400.0
dem_200_newyork+_318	114.8M	50.5M	127.4	14400.0	123.3M	80.5M	53.2	14400.0	136.7M	53.5M	155.7	14400.0
dem_200_newyork+_319	155.3M	50.2M	209.7	14400.0	126.7M	80.0M	58.4	14400.0	1e+20	50.6M	–	14400.0
dem_200_newyork+_320	224.9M	51.3M	338.8	14400.0	163.9M	107.5M	52.4	14400.0	318.8M	63.7M	400.3	14400.0
dem_200_newyork+_321	221.6M	49.4M	348.3	14400.0	186.3M	90.7M	105.4	14400.0	1e+20	50.7M	–	14400.0
dem_200_newyork+_322	232.4M	50.1M	363.7	14400.0	140.6M	120.2M	17.0	14401.4	167.2M	61.0M	174.2	14400.0
dem_200_newyork+_323	187.8M	52.1M	260.3	14400.0	158.9M	80.7M	97.0	14400.0	129.2M	55.3M	133.8	14400.1
dem_200_newyork+_324	143.2M	48.2M	197.1	14401.6	126.8M	84.6M	49.9	14401.9	131.4M	52.2M	151.5	14400.0
dem_200_newyork+_325	1e+20	48.0M	–	14400.0	167.7M	107.6M	55.8	14400.0	1e+20	59.9M	–	14400.0
dem_200_newyork+_326	135.5M	42.5M	218.7	14400.1	111.7M	86.4M	29.2	14400.0	117.9M	48.5M	142.9	14400.0
dem_200_newyork+_327	130.0M	45.9M	183.6	14400.0	125.4M	106.8M	17.4	14400.0	1e+20	48.4M	–	14400.0
dem_200_newyork+_328	213.5M	48.2M	343.0	14400.0	169.0M	104.6M	61.5	14400.0	267.5M	58.0M	361.4	14400.4
dem_200_newyork+_329	1e+20	44.8M	–	14400.0	129.9M	129.9M	0.0	6026.7	1e+20	52.4M	–	14400.1
dem_200_newyork+_330	126.6M	46.6M	171.7	14400.0	132.5M	87.0M	52.3	14400.0	1e+20	50.9M	–	14400.0
dem_200_newyork+_331	173.1M	49.7M	248.1	14400.0	132.9M	92.9M	43.1	14400.0	1e+20	55.1M	–	14400.0
dem_200_newyork+_332	164.1M	56.0M	193.2	14400.0	131.4M	98.4M	33.6	14400.0	1e+20	57.1M	–	14400.0
dem_200_newyork+_333	1e+20	47.4M	–	14400.0	161.4M	87.6M	84.3	14400.0	130.0M	55.4M	134.5	14400.0
dem_200_newyork+_334	175.2M	45.0M	288.9	14400.2	130.5M	80.9M	61.3	14400.0	1e+20	46.8M	–	14400.0
dem_200_newyork+_335	134.8M	51.1M	163.9	14400.0	130.1M	91.9M	41.6	14400.0	197.7M	56.1M	252.6	14400.0
dem_200_newyork+_336	160.5M	47.9M	235.0	14400.1	1e+20	84.0M	–	14400.0	1e+20	48.1M	–	14400.0
dem_200_newyork+_337	128.0M	49.5M	158.4	14400.0	117.0M	86.5M	35.2	14400.0	128.8M	52.8M	143.9	14400.0
dem_200_newyork+_338	393.1M	50.6M	676.5	14400.3	173.0M	119.4M	44.9	14400.0	1e+20	58.7M	–	14400.4
dem_200_newyork+_339	143.9M	51.9M	177.4	14400.0	127.3M	83.1M	53.1	14400.0	153.2M	49.2M	211.3	14400.0
dem_200_newyork+_340	163.1M	48.2M	238.5	14400.0	152.9M	117.9M	29.7	14403.0	1e+20	50.0M	–	14400.0
dem_200_newyork+_341	204.1M	49.8M	310.0	14400.0	187.6M	102.2M	83.6	14400.0	1e+20	53.5M	–	14400.0
dem_200_newyork+_342	1e+20	46.9M	–	14400.0	149.6M	100.4M	49.1	14400.0	1e+20	56.9M	–	14400.0
dem_200_newyork+_343	214.3M	45.7M	369.1	14400.1	127.3M	99.4M	28.1	14400.0	1e+20	49.6M	–	14400.0
dem_200_newyork+_344	215.7M	47.6M	353.3	14400.3	132.1M	102.1M	29.3	14400.0	183.8M	57.8M	218.0	14400.0
dem_200_newyork+_345	209.6M	44.8M	367.8	14400.5	137.2M	96.0M	42.8	14400.0	1e+20	56.1M	–	14400.0
dem_200_newyork+_346	218.5M	50.9M	328.9	14400.0	108.0M	86.3M	25.2	14400.0	229.4M	54.7M	319.0	14400.1
dem_200_newyork+_347	275.0M	41.6M	560.7	14400.0	125.3M	78.8M	59.0	14400.0	134.4M	51.5M	160.9	14400.0
dem_200_newyork+_348	1e+20	49.6M	–	14400.0	146.5M	92.4M	58.4	14400.0	186.2M	57.9M	221.9	14400.0
dem_200_newyork+_349	173.4M	49.9M	247.5	14400.0	144.4M	104.2M	38.6	14402.2	1e+20	58.4M	–	14400.0
dem_200_newyork+_350	1e+20	44.9M	–	14400.0	119.0M	88.3M	34.7	14400.0	133.3M	51.3M	159.9	14400.0
dem_200_newyork+_351	154.6M	49.1M	215.2	14400.0	132.3M	92.1M	43.7	14400.0	160.4M	58.4M	174.5	14400.0
dem_200_newyork+_352	228.3M	48.9M	366.9	14400.0	138.7M	101.8M	36.3	14400.0	143.7M	57.1M	151.5	14400.2
dem_200_newyork+_353	1e+20	51.8M	–	14400.0	149.5M	113.9M	31.3	14400.0	164.3M	65.9M	149.5	14400.0
dem_200_newyork+_354	203.4M	52.5M	287.7	14400.7	1e+20	83.0M	–	14400.0	123.9M	52.4M	136.4	14400.0
dem_200_newyork+_355	1e+20	50.7M	–	14400.0	183.1M	133.0M	37.7	14400.0	1e+20	63.2M	–	14400.0
dem_200_newyork+_356	1e+20	55.2M	–	14400.0	169.9M	105.8M	60.6	14400.0	211.1M	67.6M	212.3	14400.0
dem_200_newyork+_357	162.3M	42.1M	285.6	14400.0	116.3M	83.8M	38.8	14400.0	126.1M	45.4M	177.6	14400.0
dem_200_newyork+_358	204.7M	48.2M	324.8	14400.4	131.0M	130.9M	0.1	12081.8	216.7M	53.9M	302.4	14400.4
dem_200_newyork+_359	152.8M	47.7M	220.2	14400.0	110.2M	84.9M	29.7	14400.0	129.8M	46.0M	182.1	14400.0
dem_200_newyork+_360	1e+20	47.6M	–	14400.0	147.1M	96.7M	52.1	14400.0	1e+20	57.1M	–	14400.0
dem_200_newyork+_361	230.9M	46.9M	391.8	14400.2	132.9M	93.4M	42.3	14400.0	155.0M	50.9M	204.5	14400.0
dem_200_newyork+_362	150.4M	53.7M	179.9	14400.0	132.2M	88.6M	49.2	14400.0	152.5M	59.2M	157.4	14400.0
dem_200_newyork+_363	136.2M	54.0M	151.9	14400.0	140.8M	92.1M	52.9	14400.0	1e+20	56.1M	–	14400.0
dem_200_newyork+_364	173.2M	51.3M	237.3	14400.0	148.1M	99.9M	48.2	14400.0	179.6M	60.2M	198.5	14400.0
dem_200_newyork+_365	160.9M	53.5M	200.8	14400.0	230.5M	92.8M	148.5	14400.0	243.4M	61.9M	293.3	14400.0
dem_200_newyork+_366	1e+20	40.9M	–	14400.0	102.3M	78.4M	30.4	14401.9	130.8M	46.7M	180.2	14400.0
dem_200_newyork+_367	1e+20	48.2M	–	14400.0	138.4M	91.0M	52.1	14400.0	169.9M	59.7M	184.4	14400.2
dem_200_newyork+_368	139.5M	53.5M	161.0	14400.0	136.0M	91.1M	49.3	14400.0	139.6M	61.1M	128.4	14400.0
dem_200_newyork+_369	151.6M	48.2M	214.2	14400.0	118.0M	85.5M	38.1	14400.0	1e+20	47.1M	–	14400.0
dem_200_newyork+_370	1e+20	47.1M	–	14400.0	157.6M	110.9M	42.1	14400.0	1e+20	59.0M	–	14400.0

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Table A.43: Comparison of discrete models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_371	151.4M	42.4M	257.1	14401.0	111.5M	86.5M	28.9	14400.0	109.5M	44.7M	144.7	14400.0
dem_200_newyork+_372	136.4M	46.6M	192.6	14400.0	99.4M	78.6M	26.4	14400.0	99.4M	49.0M	103.1	14400.0
dem_200_newyork+_373	171.5M	47.8M	258.6	14400.0	171.4M	88.9M	92.8	14400.0	1e+20	51.9M	–	14400.0
dem_200_newyork+_374	1e+20	51.2M	–	14400.0	171.0M	106.7M	60.3	14400.0	1e+20	59.9M	–	14400.0
dem_200_newyork+_375	190.1M	50.4M	277.3	14400.0	140.2M	89.7M	56.3	14400.0	1e+20	49.8M	–	14400.0
dem_200_newyork+_376	242.4M	49.8M	386.8	14400.0	146.3M	92.9M	57.5	14400.0	160.0M	60.7M	163.5	14400.0
dem_200_newyork+_377	110.5M	37.7M	193.3	14400.3	97.2M	68.9M	41.1	14400.0	1e+20	38.4M	–	14400.0
dem_200_newyork+_378	1e+20	45.9M	–	14400.0	1e+20	93.4M	–	14400.0	211.1M	53.4M	295.3	14400.0
dem_200_newyork+_379	124.5M	51.3M	142.7	14400.0	165.2M	91.5M	80.7	14400.0	1e+20	55.5M	–	14400.0
dem_200_newyork+_380	171.3M	43.4M	294.6	14400.0	118.8M	89.0M	33.5	14400.0	1e+20	48.7M	–	14400.0
dem_200_newyork+_381	121.4M	48.5M	150.1	14400.0	115.2M	68.7M	67.8	14400.0	218.8M	50.3M	334.6	14400.0
dem_200_newyork+_382	231.2M	48.7M	374.6	14400.0	146.2M	116.1M	25.9	14401.9	177.5M	60.5M	193.4	14400.0
dem_200_newyork+_383	142.2M	49.0M	190.4	14400.0	129.2M	98.8M	30.7	14400.0	1e+20	51.1M	–	14400.0
dem_200_newyork+_384	1e+20	44.0M	–	14400.0	176.0M	97.2M	81.0	14403.5	1e+20	58.1M	–	14400.0
dem_200_newyork+_385	164.7M	51.2M	221.6	14400.0	135.8M	88.4M	53.7	14400.0	1e+20	53.3M	–	14400.0
dem_200_newyork+_386	125.5M	52.3M	139.8	14400.0	141.5M	84.4M	67.6	14400.0	1e+20	54.4M	–	14400.1
dem_200_newyork+_387	1e+20	50.9M	–	14400.0	127.8M	100.1M	27.7	14400.0	158.2M	56.9M	178.0	14400.0
dem_200_newyork+_388	130.4M	42.4M	207.9	14400.6	92.9M	71.0M	30.8	14400.0	102.1M	42.7M	139.3	14400.0
dem_200_newyork+_389	212.7M	49.8M	327.0	14400.0	134.5M	89.5M	50.2	14400.0	195.7M	51.5M	279.8	14400.0
dem_200_newyork+_390	1e+20	49.1M	–	14400.0	142.1M	116.6M	21.8	14400.0	1e+20	55.7M	–	14400.0
dem_200_newyork+_391	228.3M	48.4M	371.3	14400.0	129.1M	88.8M	45.4	14400.0	219.3M	51.5M	326.1	14400.2
dem_200_newyork+_392	1e+20	49.4M	–	14400.0	141.5M	110.1M	28.5	14402.4	1e+20	56.8M	–	14400.0
dem_200_newyork+_393	175.3M	50.4M	247.6	14400.0	148.1M	101.6M	45.7	14400.0	1e+20	57.6M	–	14400.0
dem_200_newyork+_394	175.1M	48.3M	262.1	14400.0	136.6M	86.8M	57.4	14400.0	1e+20	51.4M	–	14400.0
dem_200_newyork+_395	1e+20	46.3M	–	14402.2	162.4M	114.4M	42.0	14400.0	1e+20	55.1M	–	14400.0
dem_200_newyork+_396	130.8M	48.6M	169.3	14400.0	130.0M	90.8M	43.1	14400.0	173.9M	54.7M	217.8	14400.3
dem_200_newyork+_397	160.0M	53.2M	200.6	14400.0	130.7M	101.6M	28.6	14400.0	158.8M	54.3M	192.5	14400.0
dem_200_newyork+_398	1e+20	51.3M	–	14400.0	161.6M	110.1M	46.8	14400.0	1e+20	58.0M	–	14400.0
dem_200_newyork+_399	251.4M	50.8M	395.1	14400.5	129.3M	88.6M	45.9	14400.0	1e+20	51.0M	–	14400.0
dem_200_newyork+_400	1e+20	50.9M	–	14400.0	1e+20	81.9M	–	14400.0	146.4M	56.0M	161.6	14400.0
dem_200_newyork+_401	1e+20	53.0M	–	14400.0	163.0M	107.7M	51.3	14400.0	214.8M	60.6M	254.3	14400.5
dem_200_newyork+_402	181.5M	47.1M	285.1	14400.0	128.0M	96.8M	32.2	14400.0	1e+20	53.9M	–	14400.0
dem_200_newyork+_403	146.1M	48.4M	202.0	14400.0	120.5M	88.1M	36.8	14400.0	121.4M	56.7M	114.3	14400.0
dem_200_newyork+_404	251.2M	55.2M	355.1	14400.0	219.4M	110.0M	99.3	14400.0	251.8M	62.1M	305.4	14400.4
dem_200_newyork+_405	178.2M	46.1M	286.8	14400.6	107.3M	78.1M	37.4	14402.5	1e+20	49.9M	–	14400.0
dem_200_newyork+_406	1e+20	44.2M	–	14400.0	142.3M	95.7M	48.8	14400.0	228.3M	54.2M	320.9	14400.0
dem_200_newyork+_407	148.7M	48.4M	207.2	14400.0	131.9M	78.9M	67.1	14400.0	119.7M	50.2M	138.2	14400.0
dem_200_newyork+_408	176.8M	50.2M	252.0	14400.1	129.6M	83.6M	55.0	14400.0	1e+20	52.7M	–	14400.0
dem_200_newyork+_409	185.9M	49.8M	273.4	14400.0	154.2M	107.3M	43.7	14400.0	179.2M	63.6M	181.8	14400.0
dem_200_newyork+_410	142.4M	50.4M	182.7	14400.0	139.2M	94.8M	46.9	14400.0	1e+20	52.5M	–	14400.4
dem_200_newyork+_411	190.4M	44.0M	332.4	14400.0	90.8M	65.5M	38.6	14400.0	214.4M	46.1M	364.7	14400.0
dem_200_newyork+_412	138.2M	49.2M	180.7	14400.0	199.0M	82.9M	140.1	14400.0	1e+20	54.8M	–	14400.0
dem_200_newyork+_413	1e+20	46.4M	–	14400.0	97.8M	97.8M	0.0	8349.8	103.4M	48.9M	111.6	14400.0
dem_200_newyork+_414	161.2M	53.8M	199.4	14400.0	149.4M	101.1M	47.7	14400.0	1e+20	55.9M	–	14400.0
dem_200_newyork+_415	157.1M	48.3M	225.5	14400.0	146.4M	98.5M	48.6	14400.0	166.7M	53.7M	210.2	14400.0
dem_200_newyork+_416	1e+20	48.9M	–	14400.0	157.6M	113.0M	39.5	14400.0	1e+20	64.5M	–	14400.1
dem_200_newyork+_417	1e+20	48.9M	–	14400.0	151.6M	118.3M	28.1	14401.8	1e+20	61.6M	–	14400.0
dem_200_newyork+_418	130.4M	51.8M	151.7	14400.0	131.2M	81.5M	61.0	14400.0	151.7M	56.1M	170.5	14400.4
dem_200_newyork+_419	217.1M	51.5M	321.3	14400.0	256.5M	99.3M	158.5	14400.0	280.1M	68.1M	311.3	14400.0
dem_200_newyork+_420	171.0M	47.5M	260.1	14400.0	130.6M	82.2M	58.9	14400.0	1e+20	48.8M	–	14400.0
dem_200_newyork+_421	157.2M	48.3M	225.6	14400.0	108.4M	77.4M	40.1	14400.0	1e+20	49.6M	–	14400.0
dem_200_newyork+_422	125.7M	52.8M	138.3	14431.5	121.3M	89.7M	35.2	14403.0	122.0M	52.5M	132.4	14400.0
dem_200_newyork+_423	153.9M	55.9M	175.5	14401.0	139.7M	93.0M	50.2	14400.0	154.3M	64.9M	137.7	14400.0
dem_200_newyork+_424	197.0M	54.9M	258.6	14400.0	146.1M	146.1M	0.0	13964.3	1e+20	62.1M	–	14400.0
dem_200_newyork+_425	102.4M	48.7M	110.3	14400.0	97.0M	78.2M	24.0	14400.0	118.7M	50.4M	135.3	14400.0
dem_200_newyork+_426	126.7M	44.2M	186.9	14400.0	136.3M	89.3M	52.6	14400.0	1e+20	50.9M	–	14400.0
dem_200_newyork+_427	135.3M	45.9M	194.5	14400.0	123.8M	80.0M	54.7	14400.0	131.3M	48.1M	173.0	14400.0
dem_200_newyork+_428	1e+20	49.2M	–	14400.0	129.9M	99.4M	30.7	14400.0	1e+20	55.8M	–	14400.0
dem_200_newyork+_429	215.7M	56.5M	281.6	14400.2	162.9M	122.9M	32.5	14400.0	1e+20	69.9M	–	14400.0
dem_200_newyork+_430	141.4M	50.2M	181.4	14400.0	1e+20	74.6M	–	14401.8	120.9M	49.8M	142.8	14400.0
dem_200_newyork+_431	178.1M	55.3M	221.8	14400.0	142.7M	114.1M	25.0	14401.3	1e+20	59.8M	–	14400.2
dem_200_newyork+_432	150.0M	49.6M	202.1	14400.0	131.2M	106.2M	23.5	14400.0	190.1M	64.0M	197.0	14400.0
dem_200_newyork+_433	1e+20	46.4M	–	14400.0	174.6M	110.7M	57.8	14400.0	1e+20	54.9M	–	14400.0
dem_200_newyork+_434	178.2M	49.3M	261.8	14400.0	117.9M	88.7M	32.9	14400.0	131.0M	53.7M	144.0	14400.0
dem_200_newyork+_435	190.8M	46.4M	311.2	14400.0	133.7M	99.5M	34.4	14400.0	153.0M	51.8M	195.2	14400.0
dem_200_newyork+_436	1e+20	51.3M	–	14400.0	191.3M	117.9M	62.3	14400.0	259.5M	67.6M	284.0	14400.0
dem_200_newyork+_437	174.6M	49.9M	249.7	14400.1	119.7M	82.8M	44.6	14400.0	278.9M	49.9M	459.4	14400.0
dem_200_newyork+_438	170.8M	49.0M	248.9	14400.0	145.3M	101.7M	42.9	14400.0	1e+20	56.9M	–	14400.0
dem_200_newyork+_439	205.1M	47.7M	330.0	14437.3	85.5M	62.0M	37.9	14400.0	94.8M	45.0M	110.8	14400.0
dem_200_newyork+_440	124.6M	46.6M	167.4	14400.0	109.7M	93.7M	17.2	14400.0	235.4M	54.5M	331.8	14400.0
dem_200_newyork+_441	1e+20	51.2M	–	14400.0	138.8M	91.7M	51.4	14400.0	170.4M	60.6M	181.2	14400.0
dem_200_newyork+_442	1e+20	46.3M	–	14400.0	162.3M	94.9M	71.0	14400.0	156.6M	54.7M	186.2	14400.0
dem_200_newyork+_443	169.5M	45.6M	271.7	14400.0	119.1M	86.6M	37.5	14400.0	196.6M	46.3M	324.8	14400.0
dem_200_newyork+_444	1e+20	50.7M	–	14400.0	150.0M	108.2M	38.7	14400.0	1e+20	58.7M	–	14400.0
dem_200_newyork+_445	124.1M	51.0M	143.2	14400.1	136.5M	81.6M	67.3	14400.0	122.2M	50.2M	143.3	14400.0
dem_200_newyork+_446	130.2M	43.5M	199.4	14400.0	157.0M	80.7M	94.4	14400.0	306.6M	47.4M	546.8	14400.0
dem_200_newyork+_447	157.4M	53.2M	196.0	14400.0	140.0M	119.2M	17.5	14401.2	195.6M	57.6M	239.4	14400.0
dem_200_newyork+_448	1e+20	55.0M	–	14400.0	149.2M	105.2M	41.8	14400.0	215.7M	67.1M	221.6	14400.0

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Table A.43: Comparison of discrete models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_449	214.9M	43.6M	392.9	14400.0	130.5M	86.6M	50.7	14400.0	1e+20	45.5M	–	14400.0
dem_200_newyork+_450	141.5M	50.6M	179.7	14400.0	123.5M	92.2M	34.0	14403.0	255.1M	58.1M	339.4	14400.0
dem_200_newyork+_451	1e+20	56.6M	–	14400.0	1e+20	88.4M	–	14400.0	167.9M	61.4M	173.6	14400.0
dem_200_newyork+_452	1e+20	48.4M	–	14400.0	150.8M	117.4M	28.5	14400.0	1e+20	61.4M	–	14400.0
dem_200_newyork+_453	142.2M	49.8M	185.7	14400.0	115.6M	86.1M	34.3	14400.0	1e+20	54.4M	–	14400.0
dem_200_newyork+_454	1e+20	47.1M	–	0.0	108.2M	84.9M	27.5	14401.5	109.6M	53.5M	104.7	14400.0
dem_200_newyork+_455	192.8M	54.7M	252.5	14400.0	162.6M	112.5M	44.5	14400.0	1e+20	62.4M	–	14400.0
dem_200_newyork+_456	256.2M	52.3M	389.8	14400.0	141.0M	103.7M	36.0	14400.0	1e+20	60.4M	–	14400.2
dem_200_newyork+_457	135.8M	44.9M	202.4	14400.0	128.4M	91.3M	40.6	14400.0	300.4M	52.4M	473.5	14400.0
dem_200_newyork+_458	129.9M	52.9M	145.7	14400.0	1e+20	83.1M	–	14400.0	157.5M	58.3M	170.3	14400.0
dem_200_newyork+_459	166.7M	52.5M	217.5	14400.0	142.5M	111.1M	28.3	14400.0	1e+20	59.7M	–	14400.0
dem_200_newyork+_460	1e+20	43.8M	–	14400.0	134.7M	95.3M	41.4	14401.6	1e+20	50.7M	–	14400.0
dem_200_newyork+_461	1e+20	48.1M	–	14400.3	159.3M	90.2M	76.5	14400.0	130.1M	55.8M	133.0	14400.0
dem_200_newyork+_462	201.1M	46.3M	334.6	14400.0	111.1M	85.0M	30.7	14400.0	107.1M	57.0M	87.8	14400.0
dem_200_newyork+_463	131.5M	46.9M	180.6	14400.0	101.8M	82.2M	23.7	14400.0	103.8M	50.5M	105.7	14400.0
dem_200_newyork+_464	92.8M	35.9M	158.7	14400.3	86.8M	55.5M	56.3	14400.0	1e+20	32.9M	–	14400.0
dem_200_newyork+_465	1e+20	51.5M	–	14400.0	171.5M	84.0M	104.2	14400.0	126.5M	47.3M	167.6	14400.0
dem_200_newyork+_466	220.1M	42.3M	419.8	14400.0	105.8M	79.5M	33.1	14400.0	1e+20	43.4M	–	14400.0
dem_200_newyork+_467	153.9M	51.0M	201.7	14400.0	1e+20	80.0M	–	14400.0	1e+20	53.8M	–	14400.0
dem_200_newyork+_468	1e+20	46.5M	–	14400.0	117.9M	82.6M	42.8	14400.0	122.9M	50.3M	144.3	14400.0
dem_200_newyork+_469	1e+20	51.2M	–	14400.0	131.0M	90.5M	44.8	14400.0	1e+20	52.5M	–	14400.0
dem_200_newyork+_470	163.0M	52.6M	209.7	14400.0	328.3M	86.9M	277.9	14400.0	255.2M	59.3M	330.6	14400.0
dem_200_newyork+_471	1e+20	46.5M	–	14400.0	150.1M	105.7M	42.0	14400.0	1e+20	51.2M	–	14400.0
dem_200_newyork+_472	160.8M	45.8M	251.3	14400.6	139.9M	109.3M	28.0	14402.3	1e+20	50.3M	–	14400.0
dem_200_newyork+_473	145.9M	47.6M	206.7	14400.0	101.1M	68.5M	47.6	14400.0	103.9M	46.0M	125.7	14400.0
dem_200_newyork+_474	1e+20	47.6M	–	14400.0	153.1M	101.2M	51.3	14400.0	175.3M	56.5M	210.1	14400.1
dem_200_newyork+_475	1e+20	41.6M	–	14400.0	136.1M	82.5M	65.1	14400.0	129.4M	48.5M	166.6	14400.0
dem_200_newyork+_476	164.0M	45.4M	260.9	14400.0	145.5M	99.9M	45.7	14400.0	1e+20	50.9M	–	14400.0
dem_200_newyork+_477	1e+20	51.5M	–	14400.5	150.6M	103.3M	45.8	14400.0	1e+20	52.7M	–	14400.0
dem_200_newyork+_478	169.2M	41.3M	309.8	14400.0	107.4M	80.1M	34.0	14400.0	1e+20	45.0M	–	14400.0
dem_200_newyork+_479	171.5M	50.7M	237.9	14400.0	137.7M	111.1M	24.0	14400.0	1e+20	57.9M	–	14400.0
dem_200_newyork+_480	1e+20	47.0M	–	14400.0	168.6M	121.8M	38.3	14400.0	1e+20	55.0M	–	14400.0
dem_200_newyork+_481	153.2M	41.3M	271.2	14400.0	121.0M	83.0M	45.8	14400.0	326.1M	48.2M	576.5	14400.0
dem_200_newyork+_482	1e+20	50.3M	–	14400.0	146.9M	99.3M	47.9	14400.0	236.7M	60.0M	294.5	14400.0
dem_200_newyork+_483	101.8M	40.5M	151.6	14400.0	111.4M	62.3M	78.8	14400.0	162.4M	38.3M	324.1	14400.0
dem_200_newyork+_484	124.4M	48.9M	154.4	14400.0	112.6M	74.9M	50.4	14400.0	128.8M	50.0M	157.3	14400.0
dem_200_newyork+_485	158.8M	52.9M	200.3	14400.0	142.1M	102.2M	39.1	14400.0	1e+20	56.2M	–	14400.0
dem_200_newyork+_486	131.0M	52.2M	151.0	14400.0	128.5M	90.4M	42.1	14400.0	1e+20	56.5M	–	14400.0
dem_200_newyork+_487	133.9M	44.8M	198.7	14400.0	116.9M	93.2M	25.5	14403.3	1e+20	55.2M	–	14400.0
dem_200_newyork+_488	163.5M	54.1M	202.1	14400.7	142.5M	111.2M	28.2	14400.0	1e+20	62.9M	–	14400.1
dem_200_newyork+_489	159.3M	42.7M	273.1	14400.0	97.7M	70.2M	39.2	14400.0	1e+20	44.2M	–	14400.0
dem_200_newyork+_490	142.5M	44.6M	219.6	14400.0	122.9M	90.6M	35.7	14400.0	1e+20	52.0M	–	14400.0
dem_200_newyork+_491	218.2M	57.6M	278.5	14400.0	1e+20	112.5M	–	14400.0	416.7M	64.6M	544.8	14400.4
dem_200_newyork+_492	1e+20	49.6M	–	14400.0	160.4M	105.8M	51.6	14400.0	1e+20	64.1M	–	14400.0
dem_200_newyork+_493	166.9M	51.9M	221.8	14401.1	135.8M	104.7M	29.7	14400.0	156.2M	54.0M	189.5	14400.3
dem_200_newyork+_494	148.9M	49.6M	200.1	14400.0	117.7M	86.7M	35.7	14400.0	132.3M	54.5M	142.7	14400.0
dem_200_newyork+_495	219.0M	47.5M	361.5	14400.0	139.6M	95.6M	46.1	14400.0	287.4M	54.4M	428.2	14400.0
dem_200_newyork+_496	196.6M	42.8M	359.3	14400.0	106.9M	64.3M	66.3	14400.0	107.8M	39.7M	171.7	14400.0
dem_200_newyork+_497	1e+20	43.8M	–	14400.0	132.5M	97.6M	35.7	14400.0	1e+20	48.3M	–	14400.0
dem_200_newyork+_498	213.6M	54.8M	289.7	14400.0	146.9M	113.3M	29.6	14400.0	246.2M	62.3M	295.3	14400.0
dem_200_newyork+_499	127.2M	45.1M	182.3	14400.0	109.9M	86.8M	26.6	14400.0	218.6M	51.3M	326.3	14400.0
dem_200_newyork+_500	100.9M	40.4M	149.9	14400.0	91.8M	72.0M	27.6	14400.0	98.1M	37.7M	159.9	14400.0

Table A.44: Detailed results of the split-pipe models on *New York+* with four additional arcs and $\mathcal{B} = 200$ as summarized in Figure 3.18b and Table 3.4f. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_1	100.2M	100.1M	0.1	241.3	100.2M	100.1M	0.1	152.9
dem_200_newyork+_2	133.0M	133.0M	0.0	123.3	133.0M	133.0M	0.0	175.5
dem_200_newyork+_3	96.1M	94.8M	1.3	14400.0	96.1M	96.0M	0.1	216.1
dem_200_newyork+_4	131.5M	131.4M	0.0	564.2	131.5M	131.3M	0.1	580.6
dem_200_newyork+_5	133.6M	133.4M	0.1	251.6	133.6M	133.5M	0.0	94.1
dem_200_newyork+_6	119.9M	119.8M	0.1	181.0	119.9M	119.8M	0.1	2286.2
dem_200_newyork+_7	97.4M	96.5M	0.9	14400.0	97.3M	97.2M	0.1	230.9
dem_200_newyork+_8	116.4M	116.3M	0.1	223.3	116.4M	116.3M	0.1	229.4
dem_200_newyork+_9	131.4M	131.2M	0.1	152.1	131.4M	131.2M	0.1	60.0
dem_200_newyork+_10	97.2M	96.6M	0.6	14400.0	97.2M	97.1M	0.1	994.6
dem_200_newyork+_11	129.4M	129.3M	0.1	338.9	129.4M	129.3M	0.1	109.0

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Table A.44: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_12	133.8M	133.7M	0.1	339.8	133.8M	133.7M	0.1	122.5
dem_200_newyork+_13	146.9M	146.7M	0.1	157.0	146.9M	146.7M	0.1	71.3
dem_200_newyork+_14	108.1M	108.0M	0.1	720.4	108.1M	108.0M	0.1	288.4
dem_200_newyork+_15	134.9M	134.8M	0.1	192.9	134.9M	134.9M	0.0	113.3
dem_200_newyork+_16	101.5M	101.5M	0.0	13633.3	101.5M	101.4M	0.1	192.8
dem_200_newyork+_17	112.9M	112.8M	0.1	829.0	112.9M	112.8M	0.1	8567.4
dem_200_newyork+_18	149.1M	149.0M	0.1	107.6	149.1M	149.0M	0.1	121.4
dem_200_newyork+_19	101.7M	101.6M	0.1	1069.9	103.1M	63.5M	62.4	14400.0
dem_200_newyork+_20	140.6M	140.5M	0.1	136.0	140.6M	140.6M	0.0	156.3
dem_200_newyork+_21	140.4M	140.4M	0.0	4534.4	140.4M	140.3M	0.1	209.3
dem_200_newyork+_22	88.5M	88.4M	0.1	459.6	88.5M	86.3M	2.6	14400.0
dem_200_newyork+_23	132.2M	132.1M	0.1	576.3	132.2M	132.0M	0.1	828.5
dem_200_newyork+_24	136.9M	136.8M	0.1	75.1	136.9M	136.8M	0.1	135.4
dem_200_newyork+_25	143.8M	143.6M	0.1	170.1	143.8M	143.6M	0.1	88.1
dem_200_newyork+_26	123.1M	123.0M	0.1	113.5	123.1M	123.0M	0.1	106.1
dem_200_newyork+_27	155.5M	155.4M	0.1	150.9	155.5M	155.3M	0.1	78.0
dem_200_newyork+_28	142.3M	142.2M	0.1	202.5	142.3M	142.2M	0.1	94.9
dem_200_newyork+_29	133.3M	133.2M	0.1	189.9	133.3M	133.2M	0.1	61.1
dem_200_newyork+_30	111.7M	111.5M	0.1	249.0	111.7M	111.5M	0.1	182.2
dem_200_newyork+_31	121.9M	121.7M	0.1	230.0	121.9M	121.7M	0.1	249.5
dem_200_newyork+_32	138.7M	138.5M	0.1	66.0	138.7M	138.5M	0.1	56.8
dem_200_newyork+_33	114.6M	114.5M	0.1	216.6	114.6M	114.5M	0.1	70.5
dem_200_newyork+_34	112.0M	111.9M	0.1	5798.8	112.0M	80.1M	39.9	14400.0
dem_200_newyork+_35	126.8M	126.6M	0.1	149.6	126.8M	126.6M	0.1	268.5
dem_200_newyork+_36	155.7M	155.6M	0.1	68.1	155.7M	155.6M	0.1	98.6
dem_200_newyork+_37	124.2M	123.8M	0.3	14400.0	124.2M	124.0M	0.1	113.4
dem_200_newyork+_38	137.8M	137.7M	0.1	123.1	137.8M	137.7M	0.1	134.4
dem_200_newyork+_39	140.7M	140.6M	0.1	54.3	140.7M	140.6M	0.1	133.9
dem_200_newyork+_40	121.4M	121.3M	0.1	87.5	121.4M	121.3M	0.1	82.0
dem_200_newyork+_41	142.4M	142.3M	0.1	296.8	142.4M	142.3M	0.1	154.3
dem_200_newyork+_42	119.2M	119.2M	0.0	327.0	119.2M	119.1M	0.1	383.4
dem_200_newyork+_43	147.9M	147.8M	0.1	96.6	147.9M	147.8M	0.1	89.8
dem_200_newyork+_44	151.7M	151.5M	0.1	123.7	151.7M	151.6M	0.1	127.5
dem_200_newyork+_45	87.6M	87.4M	0.2	14400.0	87.6M	86.7M	1.0	14400.0
dem_200_newyork+_46	146.5M	146.3M	0.1	194.3	146.4M	146.3M	0.1	133.7
dem_200_newyork+_47	125.9M	125.7M	0.2	14400.0	125.8M	125.7M	0.1	1108.6
dem_200_newyork+_48	114.6M	114.5M	0.1	227.0	114.6M	114.5M	0.1	124.6
dem_200_newyork+_49	119.5M	119.4M	0.1	90.9	119.5M	68.6M	74.3	14400.0
dem_200_newyork+_50	105.3M	105.2M	0.1	195.2	105.3M	103.0M	2.2	14400.0
dem_200_newyork+_51	138.7M	138.6M	0.1	181.3	138.7M	138.6M	0.1	330.1
dem_200_newyork+_52	114.0M	113.9M	0.1	3208.2	114.0M	113.9M	0.1	335.0
dem_200_newyork+_53	105.5M	105.4M	0.1	1291.4	105.5M	105.4M	0.1	766.8
dem_200_newyork+_54	129.2M	129.1M	0.1	255.4	129.2M	129.1M	0.1	427.8
dem_200_newyork+_55	130.9M	130.7M	0.1	72.4	130.9M	130.8M	0.0	168.8
dem_200_newyork+_56	134.4M	134.3M	0.1	71.7	134.4M	134.2M	0.1	201.2
dem_200_newyork+_57	117.2M	115.0M	2.0	14400.0	115.5M	115.5M	0.0	131.7
dem_200_newyork+_58	122.5M	122.4M	0.1	12602.2	122.5M	122.4M	0.1	405.0
dem_200_newyork+_59	117.5M	117.4M	0.1	267.4	117.5M	117.4M	0.1	264.6
dem_200_newyork+_60	132.5M	132.3M	0.1	364.0	132.5M	132.3M	0.1	193.7
dem_200_newyork+_61	130.0M	129.7M	0.2	14400.0	130.0M	129.9M	0.1	286.9
dem_200_newyork+_62	112.2M	112.1M	0.1	2017.5	112.2M	112.1M	0.1	104.4
dem_200_newyork+_63	113.2M	84.5M	34.0	14400.0	113.2M	113.1M	0.1	2963.2
dem_200_newyork+_64	144.9M	144.8M	0.1	189.3	144.9M	144.8M	0.1	245.1
dem_200_newyork+_65	144.4M	144.3M	0.1	131.5	144.4M	144.3M	0.1	155.8
dem_200_newyork+_66	133.5M	133.3M	0.1	68.3	133.5M	133.5M	0.0	66.5
dem_200_newyork+_67	116.7M	116.6M	0.1	2902.4	116.7M	116.6M	0.1	114.5
dem_200_newyork+_68	136.7M	136.5M	0.1	120.8	136.7M	136.7M	0.0	87.6
dem_200_newyork+_69	157.9M	157.7M	0.1	102.4	157.9M	157.7M	0.1	102.4
dem_200_newyork+_70	115.4M	115.3M	0.1	934.7	115.4M	115.3M	0.1	815.9
dem_200_newyork+_71	116.7M	116.6M	0.1	107.2	116.7M	116.6M	0.1	67.0
dem_200_newyork+_72	126.6M	126.5M	0.1	454.9	126.6M	126.5M	0.1	302.3
dem_200_newyork+_73	129.3M	129.2M	0.1	68.8	129.3M	129.1M	0.1	51.5
dem_200_newyork+_74	100.3M	100.2M	0.1	1975.7	100.3M	100.2M	0.1	680.8
dem_200_newyork+_75	101.5M	101.4M	0.1	116.4	101.5M	101.4M	0.1	250.1
dem_200_newyork+_76	134.3M	134.2M	0.1	223.3	134.3M	134.3M	0.0	89.7
dem_200_newyork+_77	102.3M	102.2M	0.1	393.0	102.3M	102.3M	0.0	41.5
dem_200_newyork+_78	140.1M	140.1M	0.0	228.1	140.1M	139.9M	0.1	596.6
dem_200_newyork+_79	147.6M	147.5M	0.1	211.9	147.6M	147.5M	0.1	92.9
dem_200_newyork+_80	140.3M	140.2M	0.1	104.1	140.3M	140.2M	0.1	80.6
dem_200_newyork+_81	137.6M	137.5M	0.1	95.2	137.6M	137.5M	0.1	102.0
dem_200_newyork+_82	142.7M	142.5M	0.1	236.8	142.7M	142.5M	0.1	140.0
dem_200_newyork+_83	116.1M	116.0M	0.1	340.8	116.1M	116.0M	0.1	104.2
dem_200_newyork+_84	75.9M	75.8M	0.1	3525.4	75.9M	75.8M	0.1	1655.4
dem_200_newyork+_85	116.8M	116.7M	0.1	418.4	116.8M	116.7M	0.1	725.7
dem_200_newyork+_86	141.6M	141.5M	0.1	119.6	141.6M	141.5M	0.1	103.5
dem_200_newyork+_87	106.7M	106.6M	0.0	200.9	106.7M	106.6M	0.1	58.5
dem_200_newyork+_88	115.9M	115.8M	0.1	388.8	115.9M	115.8M	0.1	76.2
dem_200_newyork+_89	106.6M	106.5M	0.1	88.0	106.6M	106.6M	0.0	61.5

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Table A.44: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_90	107.9M	107.8M	0.1	167.6	107.9M	107.9M	0.0	49.4
dem_200_newyork+_91	149.9M	149.7M	0.1	92.5	149.8M	149.7M	0.1	138.7
dem_200_newyork+_92	124.0M	123.9M	0.1	126.8	124.0M	123.9M	0.1	405.8
dem_200_newyork+_93	124.1M	124.0M	0.1	239.4	124.1M	124.0M	0.1	750.6
dem_200_newyork+_94	125.4M	125.3M	0.1	13316.4	125.4M	125.3M	0.1	69.7
dem_200_newyork+_95	95.6M	95.1M	0.5	14400.0	95.6M	95.5M	0.1	463.7
dem_200_newyork+_96	131.3M	131.2M	0.1	62.1	131.3M	131.2M	0.1	68.4
dem_200_newyork+_97	121.0M	120.9M	0.1	118.2	121.0M	120.9M	0.1	6027.9
dem_200_newyork+_98	127.8M	127.7M	0.1	340.8	127.7M	127.6M	0.1	218.3
dem_200_newyork+_99	137.9M	137.8M	0.1	125.4	137.9M	137.7M	0.1	90.1
dem_200_newyork+_100	154.9M	154.7M	0.1	584.5	154.9M	154.8M	0.1	143.0
dem_200_newyork+_101	141.5M	141.4M	0.1	107.8	141.5M	141.4M	0.1	119.1
dem_200_newyork+_102	141.4M	141.3M	0.1	141.9	141.4M	141.3M	0.1	5554.2
dem_200_newyork+_103	136.9M	136.8M	0.1	252.7	136.9M	136.8M	0.1	127.0
dem_200_newyork+_104	136.2M	136.1M	0.1	293.1	136.2M	136.2M	0.0	107.6
dem_200_newyork+_105	127.1M	126.9M	0.1	239.0	127.1M	126.9M	0.1	132.6
dem_200_newyork+_106	135.2M	135.0M	0.1	544.8	135.2M	135.0M	0.1	87.1
dem_200_newyork+_107	152.8M	152.7M	0.1	71.9	152.8M	152.7M	0.1	76.1
dem_200_newyork+_108	135.4M	135.3M	0.1	59.5	135.4M	135.4M	0.0	345.2
dem_200_newyork+_109	147.6M	147.5M	0.1	110.2	147.6M	147.5M	0.1	186.3
dem_200_newyork+_110	112.2M	112.0M	0.1	610.8	112.2M	112.2M	0.0	270.4
dem_200_newyork+_111	105.1M	105.0M	0.1	164.2	105.1M	105.0M	0.1	188.8
dem_200_newyork+_112	145.7M	145.5M	0.1	120.9	145.6M	145.5M	0.1	126.6
dem_200_newyork+_113	151.8M	151.6M	0.1	138.0	151.7M	151.6M	0.1	345.5
dem_200_newyork+_114	144.5M	89.8M	60.9	14400.0	144.5M	144.4M	0.1	109.3
dem_200_newyork+_115	117.6M	117.5M	0.1	191.8	117.6M	117.5M	0.1	176.9
dem_200_newyork+_116	106.5M	106.4M	0.1	741.2	106.5M	106.4M	0.1	65.1
dem_200_newyork+_117	153.3M	153.2M	0.1	73.5	153.3M	153.2M	0.1	70.2
dem_200_newyork+_118	135.9M	135.8M	0.1	1481.8	135.9M	135.8M	0.1	169.4
dem_200_newyork+_119	147.3M	147.2M	0.1	93.6	147.3M	147.2M	0.1	67.3
dem_200_newyork+_120	131.2M	131.1M	0.1	973.7	131.2M	131.1M	0.1	193.5
dem_200_newyork+_121	140.0M	139.9M	0.1	403.4	140.0M	139.9M	0.1	150.5
dem_200_newyork+_122	130.4M	130.3M	0.1	80.5	130.4M	130.3M	0.1	66.0
dem_200_newyork+_123	124.8M	124.7M	0.1	179.1	124.8M	124.3M	0.4	14400.0
dem_200_newyork+_124	115.4M	115.3M	0.1	173.8	115.4M	115.2M	0.1	1138.2
dem_200_newyork+_125	129.2M	129.1M	0.1	80.1	129.2M	129.1M	0.1	76.4
dem_200_newyork+_126	136.6M	136.6M	0.0	195.4	136.6M	136.6M	0.0	226.8
dem_200_newyork+_127	130.5M	130.3M	0.1	941.4	130.5M	130.3M	0.1	587.6
dem_200_newyork+_128	96.9M	96.2M	0.7	14400.0	96.9M	72.9M	32.9	14400.0
dem_200_newyork+_129	132.7M	132.5M	0.1	385.8	132.6M	132.5M	0.1	396.9
dem_200_newyork+_130	102.5M	102.4M	0.1	2012.2	102.5M	102.4M	0.1	163.2
dem_200_newyork+_131	126.2M	126.1M	0.1	63.1	126.2M	126.1M	0.1	101.1
dem_200_newyork+_132	122.8M	122.8M	0.0	378.0	122.8M	122.8M	0.0	244.3
dem_200_newyork+_133	136.1M	135.9M	0.1	1036.9	136.0M	135.9M	0.1	438.2
dem_200_newyork+_134	120.0M	119.9M	0.1	120.1	120.0M	119.9M	0.1	8424.0
dem_200_newyork+_135	120.9M	120.8M	0.1	561.8	120.9M	120.8M	0.1	359.0
dem_200_newyork+_136	120.8M	120.6M	0.1	155.4	120.8M	120.6M	0.1	151.2
dem_200_newyork+_137	129.4M	129.3M	0.1	79.4	129.4M	129.3M	0.1	83.3
dem_200_newyork+_138	143.4M	143.3M	0.1	888.2	143.4M	143.3M	0.1	664.1
dem_200_newyork+_139	133.7M	133.6M	0.1	115.8	133.7M	133.5M	0.1	68.7
dem_200_newyork+_140	135.1M	135.0M	0.1	201.1	135.1M	135.0M	0.1	188.5
dem_200_newyork+_141	126.8M	126.7M	0.1	167.2	126.7M	126.6M	0.1	310.6
dem_200_newyork+_142	124.4M	124.1M	0.3	14400.0	124.4M	123.7M	0.6	14400.0
dem_200_newyork+_143	136.1M	81.7M	66.6	14400.0	132.1M	131.9M	0.1	124.7
dem_200_newyork+_144	122.8M	122.6M	0.1	2135.1	122.8M	122.6M	0.1	204.0
dem_200_newyork+_145	140.0M	139.9M	0.1	219.3	140.0M	139.9M	0.1	317.5
dem_200_newyork+_146	132.5M	132.5M	0.0	145.2	132.5M	132.4M	0.1	225.1
dem_200_newyork+_147	136.3M	136.2M	0.1	112.5	136.3M	136.3M	0.0	71.2
dem_200_newyork+_148	125.8M	125.7M	0.1	223.0	125.8M	124.3M	1.3	14400.0
dem_200_newyork+_149	144.0M	143.9M	0.1	1554.3	144.0M	143.9M	0.1	206.4
dem_200_newyork+_150	119.3M	119.2M	0.1	409.9	129.5M	75.5M	71.4	14400.0
dem_200_newyork+_151	141.9M	141.7M	0.1	51.6	141.9M	141.9M	0.0	86.6
dem_200_newyork+_152	148.9M	148.7M	0.1	201.2	148.9M	148.7M	0.1	92.4
dem_200_newyork+_153	130.5M	130.4M	0.1	137.5	130.5M	130.4M	0.1	300.1
dem_200_newyork+_154	144.5M	144.4M	0.1	571.6	144.5M	144.4M	0.1	707.3
dem_200_newyork+_155	112.2M	112.1M	0.1	492.3	112.2M	112.1M	0.1	237.8
dem_200_newyork+_156	98.6M	98.5M	0.1	1224.8	98.6M	97.1M	1.5	14400.0
dem_200_newyork+_157	131.1M	130.9M	0.1	3205.8	131.1M	131.0M	0.1	77.9
dem_200_newyork+_158	119.3M	119.1M	0.1	312.2	119.3M	119.3M	0.0	186.4
dem_200_newyork+_159	152.0M	134.9M	12.6	14400.0	150.2M	150.0M	0.1	217.8
dem_200_newyork+_160	140.3M	140.1M	0.1	77.6	140.3M	140.2M	0.1	60.2
dem_200_newyork+_161	127.4M	127.3M	0.1	175.6	127.4M	127.3M	0.1	180.1
dem_200_newyork+_162	132.7M	132.6M	0.1	105.1	132.7M	132.6M	0.1	89.6
dem_200_newyork+_163	96.4M	95.8M	0.7	14400.0	96.4M	96.3M	0.1	143.2
dem_200_newyork+_164	120.8M	120.7M	0.1	517.9	120.8M	120.7M	0.1	538.1
dem_200_newyork+_165	120.7M	120.5M	0.1	79.5	120.7M	120.6M	0.1	197.6
dem_200_newyork+_166	133.0M	132.9M	0.1	84.7	133.0M	132.8M	0.1	112.1
dem_200_newyork+_167	123.7M	123.6M	0.1	1362.8	123.7M	123.6M	0.1	1175.9

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Table A.44: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_168	99.4M	99.3M	0.1	5628.2	99.4M	99.3M	0.1	80.3
dem_200_newyork+_169	163.7M	163.6M	0.1	153.9	163.7M	163.6M	0.1	142.5
dem_200_newyork+_170	131.3M	131.1M	0.1	218.8	131.3M	131.2M	0.1	98.4
dem_200_newyork+_171	134.4M	134.3M	0.1	91.8	134.4M	134.2M	0.1	157.7
dem_200_newyork+_172	140.6M	140.5M	0.0	201.2	140.6M	140.4M	0.1	168.7
dem_200_newyork+_173	96.2M	96.1M	0.1	291.8	96.2M	96.2M	0.0	3391.7
dem_200_newyork+_174	126.3M	126.2M	0.1	100.1	126.3M	126.1M	0.1	157.3
dem_200_newyork+_175	141.8M	141.7M	0.1	95.9	141.8M	141.7M	0.1	103.0
dem_200_newyork+_176	122.8M	122.7M	0.1	109.8	122.8M	122.7M	0.1	147.4
dem_200_newyork+_177	144.0M	143.9M	0.1	269.0	144.0M	143.9M	0.1	242.0
dem_200_newyork+_178	151.4M	151.2M	0.1	2813.4	151.4M	151.2M	0.1	234.9
dem_200_newyork+_179	136.0M	135.9M	0.1	130.7	136.0M	135.9M	0.1	76.2
dem_200_newyork+_180	105.4M	105.3M	0.1	757.7	105.4M	105.3M	0.1	107.3
dem_200_newyork+_181	157.6M	157.4M	0.1	188.2	157.6M	157.4M	0.1	907.5
dem_200_newyork+_182	124.1M	124.0M	0.1	132.1	124.1M	124.1M	0.0	106.4
dem_200_newyork+_183	146.7M	146.5M	0.1	245.2	146.6M	146.5M	0.1	354.8
dem_200_newyork+_184	112.3M	112.2M	0.1	2319.0	112.3M	112.2M	0.1	2760.1
dem_200_newyork+_185	140.3M	140.1M	0.1	169.1	140.3M	140.1M	0.1	100.4
dem_200_newyork+_186	128.6M	128.5M	0.1	47.3	128.6M	128.5M	0.1	70.2
dem_200_newyork+_187	106.6M	106.5M	0.1	423.4	106.6M	106.5M	0.1	11900.6
dem_200_newyork+_188	100.8M	84.1M	19.8	14400.0	100.8M	83.4M	20.9	14400.0
dem_200_newyork+_189	123.7M	123.7M	0.0	136.8	123.7M	123.7M	0.0	99.7
dem_200_newyork+_190	153.6M	153.5M	0.1	146.4	153.6M	153.4M	0.1	115.3
dem_200_newyork+_191	107.4M	107.4M	0.0	202.7	107.4M	107.3M	0.1	1663.4
dem_200_newyork+_192	115.7M	115.6M	0.0	138.2	115.7M	115.6M	0.1	296.8
dem_200_newyork+_193	132.8M	132.7M	0.1	63.1	132.8M	132.7M	0.1	99.9
dem_200_newyork+_194	116.0M	115.9M	0.1	4847.3	116.0M	115.9M	0.1	449.3
dem_200_newyork+_195	166.9M	166.7M	0.1	96.0	166.9M	166.7M	0.1	94.1
dem_200_newyork+_196	105.4M	105.3M	0.1	311.3	105.4M	105.4M	0.0	143.9
dem_200_newyork+_197	118.2M	118.1M	0.1	481.9	118.2M	118.1M	0.1	142.5
dem_200_newyork+_198	149.9M	149.7M	0.1	482.2	149.9M	149.7M	0.1	156.3
dem_200_newyork+_199	170.9M	170.8M	0.1	165.8	170.9M	170.8M	0.1	100.8
dem_200_newyork+_200	146.8M	146.7M	0.1	107.1	146.8M	146.7M	0.1	135.2
dem_200_newyork+_201	129.3M	129.2M	0.1	186.1	129.3M	129.1M	0.1	580.8
dem_200_newyork+_202	135.8M	135.7M	0.1	295.1	135.8M	135.7M	0.1	520.0
dem_200_newyork+_203	94.4M	94.3M	0.1	422.1	94.4M	94.3M	0.1	177.6
dem_200_newyork+_204	139.5M	139.4M	0.1	99.6	139.5M	139.3M	0.1	154.0
dem_200_newyork+_205	100.6M	99.5M	1.1	14400.0	100.6M	100.5M	0.1	209.5
dem_200_newyork+_206	133.3M	133.2M	0.1	93.7	133.3M	133.2M	0.1	104.9
dem_200_newyork+_207	142.6M	142.4M	0.1	124.9	142.6M	142.4M	0.1	90.9
dem_200_newyork+_208	134.5M	134.4M	0.1	318.2	134.5M	134.3M	0.1	148.0
dem_200_newyork+_209	102.9M	102.8M	0.1	797.3	102.9M	100.3M	2.6	14400.0
dem_200_newyork+_210	153.0M	152.9M	0.1	113.2	153.0M	152.9M	0.1	162.8
dem_200_newyork+_211	130.4M	130.3M	0.1	118.0	130.4M	130.3M	0.1	243.2
dem_200_newyork+_212	116.7M	116.6M	0.1	124.5	116.7M	116.6M	0.1	456.0
dem_200_newyork+_213	120.3M	120.2M	0.1	331.4	120.3M	120.2M	0.1	185.8
dem_200_newyork+_214	98.7M	98.6M	0.1	4054.0	98.7M	98.6M	0.1	422.5
dem_200_newyork+_215	125.6M	125.5M	0.1	1786.8	125.6M	125.5M	0.1	4234.3
dem_200_newyork+_216	142.5M	142.2M	0.2	14400.0	142.3M	142.2M	0.1	670.2
dem_200_newyork+_217	108.7M	106.7M	1.8	14400.0	108.7M	108.6M	0.1	58.2
dem_200_newyork+_218	99.2M	79.0M	25.6	14400.0	94.9M	94.8M	0.1	6282.0
dem_200_newyork+_219	114.5M	114.4M	0.1	5964.6	114.5M	114.5M	0.1	78.9
dem_200_newyork+_220	103.9M	103.8M	0.1	5256.8	103.9M	103.8M	0.1	201.8
dem_200_newyork+_221	101.8M	87.4M	16.4	14400.0	101.8M	101.7M	0.1	116.5
dem_200_newyork+_222	134.6M	134.6M	0.0	362.0	134.6M	134.5M	0.1	323.3
dem_200_newyork+_223	146.6M	146.5M	0.1	263.6	146.6M	146.5M	0.1	108.1
dem_200_newyork+_224	128.1M	128.0M	0.1	119.3	128.1M	128.0M	0.1	65.0
dem_200_newyork+_225	111.1M	111.0M	0.1	2869.8	111.1M	111.0M	0.1	713.9
dem_200_newyork+_226	144.3M	144.1M	0.1	231.4	144.3M	144.1M	0.1	661.6
dem_200_newyork+_227	132.5M	132.4M	0.1	204.1	132.5M	132.4M	0.1	112.8
dem_200_newyork+_228	116.6M	116.6M	0.0	1088.7	116.6M	116.5M	0.1	105.7
dem_200_newyork+_229	130.6M	130.5M	0.1	79.0	130.6M	130.5M	0.1	122.2
dem_200_newyork+_230	105.2M	105.1M	0.1	98.1	105.2M	105.1M	0.1	355.2
dem_200_newyork+_231	139.4M	139.2M	0.1	95.0	139.4M	139.2M	0.1	77.0
dem_200_newyork+_232	136.2M	136.1M	0.1	178.3	136.2M	136.1M	0.1	115.2
dem_200_newyork+_233	92.2M	92.1M	0.1	191.0	92.2M	92.1M	0.1	188.7
dem_200_newyork+_234	127.5M	127.4M	0.1	137.1	127.5M	127.3M	0.1	5822.7
dem_200_newyork+_235	109.0M	108.9M	0.1	1520.2	109.0M	109.0M	0.0	65.1
dem_200_newyork+_236	120.2M	119.4M	0.7	14400.0	120.2M	120.1M	0.1	119.1
dem_200_newyork+_237	96.8M	96.7M	0.1	436.1	98.0M	72.8M	34.7	14400.0
dem_200_newyork+_238	120.0M	119.9M	0.1	66.2	120.0M	119.9M	0.1	88.6
dem_200_newyork+_239	129.9M	129.8M	0.1	76.4	129.9M	129.8M	0.1	134.5
dem_200_newyork+_240	124.9M	124.8M	0.1	378.7	124.9M	124.8M	0.1	151.9
dem_200_newyork+_241	151.4M	151.3M	0.1	99.6	151.4M	151.3M	0.1	113.8
dem_200_newyork+_242	130.6M	130.4M	0.1	63.5	130.6M	130.4M	0.1	62.6
dem_200_newyork+_243	121.9M	121.7M	0.1	4717.1	121.9M	121.7M	0.1	2870.7
dem_200_newyork+_244	134.6M	134.4M	0.1	86.1	134.5M	134.4M	0.1	101.5
dem_200_newyork+_245	107.7M	107.6M	0.1	327.1	107.7M	107.6M	0.1	318.8

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Table A.44: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_246	112.0M	111.8M	0.1	3887.2	112.0M	111.9M	0.1	250.4
dem_200_newyork+_247	101.6M	101.5M	0.1	196.5	101.5M	100.9M	0.6	14400.0
dem_200_newyork+_248	139.0M	138.9M	0.1	267.3	139.0M	138.9M	0.1	106.3
dem_200_newyork+_249	128.7M	128.6M	0.1	539.7	128.7M	128.6M	0.1	2895.6
dem_200_newyork+_250	141.9M	141.8M	0.1	477.3	141.9M	141.8M	0.1	178.0
dem_200_newyork+_251	104.9M	104.8M	0.1	1756.0	104.9M	104.8M	0.1	886.2
dem_200_newyork+_252	122.5M	122.4M	0.1	141.0	122.5M	122.4M	0.1	123.8
dem_200_newyork+_253	101.2M	101.2M	0.0	237.6	101.2M	101.1M	0.1	1319.9
dem_200_newyork+_254	133.3M	133.2M	0.1	361.5	133.3M	133.2M	0.1	117.4
dem_200_newyork+_255	134.8M	134.7M	0.1	179.4	134.8M	134.7M	0.1	127.4
dem_200_newyork+_256	91.3M	88.6M	3.0	14400.0	91.3M	91.2M	0.1	366.7
dem_200_newyork+_257	168.3M	168.1M	0.1	114.3	168.3M	168.1M	0.1	89.8
dem_200_newyork+_258	154.8M	154.7M	0.1	427.0	154.8M	154.7M	0.1	123.1
dem_200_newyork+_259	129.1M	129.0M	0.1	63.6	129.1M	129.0M	0.1	229.8
dem_200_newyork+_260	113.0M	112.9M	0.1	1053.3	113.0M	112.9M	0.1	169.9
dem_200_newyork+_261	158.0M	157.8M	0.1	108.0	158.0M	157.8M	0.1	55.1
dem_200_newyork+_262	128.2M	125.5M	2.2	14400.0	128.2M	128.1M	0.1	404.5
dem_200_newyork+_263	117.1M	117.0M	0.1	183.6	117.1M	117.0M	0.1	82.4
dem_200_newyork+_264	130.8M	130.7M	0.1	289.6	130.8M	130.7M	0.1	126.7
dem_200_newyork+_265	166.4M	166.3M	0.1	90.6	166.4M	166.2M	0.1	436.0
dem_200_newyork+_266	125.3M	125.2M	0.1	141.7	125.3M	125.2M	0.1	116.4
dem_200_newyork+_267	112.0M	111.8M	0.1	210.8	112.0M	111.9M	0.0	120.3
dem_200_newyork+_268	96.5M	96.4M	0.1	99.8	96.5M	94.2M	2.5	14400.0
dem_200_newyork+_269	110.1M	110.0M	0.1	60.9	110.1M	110.0M	0.1	72.3
dem_200_newyork+_270	126.3M	82.3M	53.4	14400.0	139.5M	82.4M	69.4	14400.0
dem_200_newyork+_271	106.1M	85.8M	23.6	14400.0	106.1M	105.9M	0.1	338.4
dem_200_newyork+_272	134.6M	134.5M	0.1	160.7	134.6M	134.4M	0.1	166.7
dem_200_newyork+_273	143.2M	143.1M	0.1	600.4	143.2M	143.1M	0.1	169.3
dem_200_newyork+_274	154.6M	154.4M	0.1	278.1	154.6M	154.4M	0.1	256.6
dem_200_newyork+_275	128.9M	128.8M	0.1	172.3	128.9M	128.8M	0.1	214.8
dem_200_newyork+_276	90.9M	88.2M	3.1	14400.0	90.9M	84.5M	7.6	14400.0
dem_200_newyork+_277	150.9M	150.8M	0.0	76.7	150.9M	150.7M	0.1	66.7
dem_200_newyork+_278	90.1M	90.0M	0.1	2369.7	90.1M	90.0M	0.1	2796.7
dem_200_newyork+_279	119.0M	118.9M	0.1	214.5	126.3M	63.2M	99.8	14400.0
dem_200_newyork+_280	163.2M	163.1M	0.1	137.8	163.2M	163.1M	0.1	109.7
dem_200_newyork+_281	119.5M	119.4M	0.1	92.2	119.5M	119.4M	0.1	104.8
dem_200_newyork+_282	134.1M	134.0M	0.1	140.5	134.1M	134.0M	0.1	116.4
dem_200_newyork+_283	108.7M	108.6M	0.1	674.3	108.7M	108.6M	0.1	128.4
dem_200_newyork+_284	118.8M	118.7M	0.1	290.5	118.8M	118.7M	0.1	233.0
dem_200_newyork+_285	153.6M	153.5M	0.1	105.3	153.6M	153.5M	0.1	74.7
dem_200_newyork+_286	113.9M	113.8M	0.1	271.0	113.9M	113.8M	0.1	131.2
dem_200_newyork+_287	122.2M	122.1M	0.1	904.9	122.2M	121.4M	0.6	14400.0
dem_200_newyork+_288	119.1M	119.0M	0.1	57.8	119.1M	119.1M	0.1	64.9
dem_200_newyork+_289	100.3M	100.2M	0.1	430.6	100.3M	100.2M	0.1	2560.3
dem_200_newyork+_290	115.7M	115.6M	0.1	689.7	115.7M	115.6M	0.1	229.5
dem_200_newyork+_291	126.1M	126.0M	0.1	529.1	126.1M	126.0M	0.1	143.4
dem_200_newyork+_292	126.9M	126.8M	0.1	124.9	126.9M	126.8M	0.1	127.2
dem_200_newyork+_293	127.9M	127.8M	0.1	92.9	127.9M	127.8M	0.1	321.1
dem_200_newyork+_294	123.6M	123.5M	0.1	101.1	123.6M	123.5M	0.1	276.0
dem_200_newyork+_295	127.5M	127.4M	0.1	705.4	127.5M	127.4M	0.1	693.8
dem_200_newyork+_296	140.3M	140.2M	0.1	75.1	140.3M	140.2M	0.1	90.9
dem_200_newyork+_297	135.2M	135.1M	0.0	148.9	135.2M	135.1M	0.1	119.9
dem_200_newyork+_298	97.6M	97.6M	0.0	234.5	97.6M	97.5M	0.1	135.4
dem_200_newyork+_299	137.3M	137.1M	0.1	182.9	137.3M	137.1M	0.1	82.0
dem_200_newyork+_300	138.1M	138.0M	0.1	183.8	138.1M	138.0M	0.1	100.4
dem_200_newyork+_301	145.9M	145.8M	0.1	351.6	145.9M	145.8M	0.1	114.8
dem_200_newyork+_302	99.0M	98.9M	0.1	649.8	99.0M	97.5M	1.5	14400.0
dem_200_newyork+_303	100.8M	79.3M	27.1	14400.0	100.8M	100.7M	0.1	147.3
dem_200_newyork+_304	132.6M	132.5M	0.1	181.5	132.6M	132.5M	0.1	122.4
dem_200_newyork+_305	134.2M	134.1M	0.1	388.5	134.2M	134.1M	0.1	183.9
dem_200_newyork+_306	114.8M	114.7M	0.1	125.3	114.8M	114.7M	0.1	59.9
dem_200_newyork+_307	105.1M	105.0M	0.1	152.4	105.1M	105.0M	0.1	140.3
dem_200_newyork+_308	103.9M	102.7M	1.2	14400.0	103.9M	103.8M	0.1	341.9
dem_200_newyork+_309	120.5M	120.4M	0.1	2493.1	120.5M	120.5M	0.0	504.3
dem_200_newyork+_310	140.5M	140.4M	0.1	52.1	140.5M	140.4M	0.1	59.3
dem_200_newyork+_311	158.1M	158.0M	0.1	377.7	158.1M	158.0M	0.1	347.4
dem_200_newyork+_312	132.9M	132.8M	0.1	1124.1	132.9M	132.8M	0.1	122.4
dem_200_newyork+_313	128.2M	128.1M	0.1	584.1	128.2M	128.1M	0.1	339.8
dem_200_newyork+_314	114.7M	114.6M	0.1	173.7	114.7M	114.6M	0.1	89.8
dem_200_newyork+_315	168.5M	168.4M	0.1	157.0	168.5M	168.4M	0.1	174.1
dem_200_newyork+_316	130.2M	130.1M	0.1	103.5	130.2M	130.1M	0.1	54.2
dem_200_newyork+_317	128.6M	128.6M	0.1	271.1	128.6M	128.5M	0.1	179.0
dem_200_newyork+_318	107.9M	107.8M	0.1	4472.5	107.9M	107.8M	0.1	3409.3
dem_200_newyork+_319	110.1M	110.0M	0.1	220.8	110.1M	110.0M	0.1	1077.2
dem_200_newyork+_320	154.5M	154.3M	0.1	138.4	154.5M	154.3M	0.1	97.5
dem_200_newyork+_321	135.4M	135.3M	0.1	145.9	135.4M	135.3M	0.1	113.3
dem_200_newyork+_322	139.0M	138.9M	0.1	251.7	139.0M	138.9M	0.1	98.8
dem_200_newyork+_323	112.6M	112.6M	0.1	2062.3	112.9M	74.8M	50.8	14400.0

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Table A.44: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_324	111.1M	111.1M	0.0	594.5	111.1M	111.0M	0.1	591.0
dem_200_newyork+_325	148.2M	148.0M	0.1	155.9	148.2M	148.1M	0.1	114.6
dem_200_newyork+_326	109.0M	108.9M	0.1	94.7	109.0M	108.9M	0.1	529.4
dem_200_newyork+_327	124.5M	124.4M	0.1	118.6	124.5M	124.4M	0.1	164.2
dem_200_newyork+_328	150.2M	150.1M	0.1	168.3	150.1M	150.0M	0.1	120.0
dem_200_newyork+_329	129.0M	128.9M	0.1	96.5	129.0M	129.0M	0.0	37.7
dem_200_newyork+_330	123.6M	123.6M	0.0	392.5	123.6M	123.5M	0.1	459.4
dem_200_newyork+_331	130.4M	130.3M	0.1	273.0	130.4M	130.3M	0.1	255.7
dem_200_newyork+_332	127.6M	126.5M	0.9	14400.0	127.6M	127.5M	0.1	101.4
dem_200_newyork+_333	114.8M	108.4M	6.0	14400.0	114.8M	114.7M	0.1	132.8
dem_200_newyork+_334	117.5M	117.4M	0.1	105.5	117.5M	117.4M	0.1	85.5
dem_200_newyork+_335	129.0M	128.9M	0.1	139.2	129.0M	129.0M	0.0	100.4
dem_200_newyork+_336	114.0M	113.9M	0.1	102.4	114.0M	114.0M	0.0	99.9
dem_200_newyork+_337	112.2M	112.2M	0.0	715.8	112.2M	112.1M	0.1	124.8
dem_200_newyork+_338	147.8M	147.7M	0.1	164.0	147.8M	147.7M	0.1	102.7
dem_200_newyork+_339	126.5M	126.4M	0.1	86.7	126.5M	126.4M	0.1	93.7
dem_200_newyork+_340	150.9M	150.9M	0.0	99.3	150.9M	150.7M	0.1	57.5
dem_200_newyork+_341	140.8M	140.7M	0.1	147.3	140.7M	140.6M	0.1	101.6
dem_200_newyork+_342	140.8M	140.7M	0.1	71.9	140.8M	140.6M	0.1	197.5
dem_200_newyork+_343	125.6M	125.5M	0.1	66.3	125.6M	125.5M	0.1	87.0
dem_200_newyork+_344	129.3M	129.1M	0.1	133.5	129.3M	129.1M	0.1	212.0
dem_200_newyork+_345	130.9M	130.8M	0.1	2399.5	130.9M	130.8M	0.1	99.3
dem_200_newyork+_346	105.3M	105.2M	0.1	152.6	105.3M	105.3M	0.0	72.0
dem_200_newyork+_347	112.0M	111.9M	0.1	1600.0	111.9M	111.8M	0.1	366.0
dem_200_newyork+_348	120.4M	120.3M	0.1	95.8	120.4M	120.3M	0.1	64.1
dem_200_newyork+_349	142.1M	141.9M	0.1	126.5	142.1M	141.9M	0.1	81.1
dem_200_newyork+_350	113.5M	113.4M	0.1	2881.4	113.4M	112.6M	0.7	14400.0
dem_200_newyork+_351	129.3M	129.1M	0.1	488.3	129.3M	129.1M	0.1	364.4
dem_200_newyork+_352	135.7M	135.6M	0.1	169.0	135.7M	135.6M	0.1	179.3
dem_200_newyork+_353	144.2M	144.0M	0.1	156.9	144.2M	144.0M	0.1	93.2
dem_200_newyork+_354	118.6M	118.5M	0.1	141.9	118.6M	118.5M	0.1	111.8
dem_200_newyork+_355	171.3M	171.2M	0.1	112.9	171.3M	171.1M	0.1	86.8
dem_200_newyork+_356	152.0M	151.8M	0.1	88.1	152.0M	151.8M	0.1	147.8
dem_200_newyork+_357	112.6M	112.5M	0.1	192.3	112.6M	112.5M	0.1	1783.7
dem_200_newyork+_358	130.3M	130.1M	0.1	85.3	130.3M	130.3M	0.0	51.2
dem_200_newyork+_359	108.9M	107.5M	1.3	14400.0	108.9M	108.8M	0.1	4212.7
dem_200_newyork+_360	124.1M	124.0M	0.1	106.4	124.1M	124.0M	0.1	153.4
dem_200_newyork+_361	130.6M	130.5M	0.1	229.5	130.5M	130.4M	0.1	577.9
dem_200_newyork+_362	126.8M	126.7M	0.1	159.3	126.8M	126.4M	0.3	14400.0
dem_200_newyork+_363	120.5M	120.4M	0.1	130.7	120.5M	120.4M	0.1	1453.4
dem_200_newyork+_364	132.6M	132.5M	0.1	472.1	132.6M	132.5M	0.1	71.0
dem_200_newyork+_365	133.6M	133.5M	0.0	446.6	133.6M	133.4M	0.1	166.2
dem_200_newyork+_366	101.5M	101.4M	0.1	319.3	101.5M	101.4M	0.1	68.0
dem_200_newyork+_367	124.8M	124.7M	0.1	208.1	124.8M	124.7M	0.1	215.1
dem_200_newyork+_368	127.1M	127.1M	0.0	317.3	127.1M	127.0M	0.1	232.4
dem_200_newyork+_369	112.2M	112.1M	0.1	316.2	112.2M	112.1M	0.1	122.0
dem_200_newyork+_370	147.7M	147.6M	0.1	125.1	147.7M	147.6M	0.1	89.7
dem_200_newyork+_371	104.0M	82.4M	26.2	14400.0	104.0M	103.9M	0.1	231.3
dem_200_newyork+_372	94.0M	93.9M	0.1	8536.8	94.0M	93.8M	0.2	14400.0
dem_200_newyork+_373	134.4M	134.3M	0.1	132.3	134.4M	134.3M	0.1	167.6
dem_200_newyork+_374	157.9M	157.7M	0.1	109.9	157.9M	157.7M	0.1	197.1
dem_200_newyork+_375	127.6M	127.5M	0.1	78.2	127.6M	127.5M	0.1	115.0
dem_200_newyork+_376	126.3M	126.1M	0.1	217.8	126.3M	126.1M	0.1	138.5
dem_200_newyork+_377	90.4M	85.7M	5.6	14400.0	90.4M	90.4M	0.1	2885.4
dem_200_newyork+_378	130.6M	130.5M	0.1	95.2	130.6M	130.5M	0.1	119.9
dem_200_newyork+_379	120.3M	120.2M	0.1	118.8	120.3M	120.2M	0.1	118.5
dem_200_newyork+_380	115.2M	115.1M	0.1	48.1	115.2M	115.1M	0.1	176.4
dem_200_newyork+_381	96.1M	96.0M	0.1	239.4	96.1M	68.6M	40.2	14400.0
dem_200_newyork+_382	145.1M	145.0M	0.0	109.6	145.0M	144.9M	0.1	115.7
dem_200_newyork+_383	125.0M	125.0M	0.0	227.0	125.0M	125.0M	0.0	79.5
dem_200_newyork+_384	136.0M	135.8M	0.1	121.4	136.0M	135.8M	0.1	139.0
dem_200_newyork+_385	110.2M	110.1M	0.1	6002.6	110.2M	110.1M	0.1	123.4
dem_200_newyork+_386	122.9M	122.8M	0.1	12673.2	122.9M	122.8M	0.1	1968.7
dem_200_newyork+_387	126.0M	125.9M	0.1	14400.0	126.0M	125.9M	0.1	84.1
dem_200_newyork+_388	91.8M	71.3M	28.7	14400.0	91.8M	91.7M	0.1	1045.5
dem_200_newyork+_389	119.7M	119.7M	0.0	198.2	119.7M	119.6M	0.1	110.1
dem_200_newyork+_390	141.2M	141.0M	0.1	55.6	141.2M	141.1M	0.1	66.3
dem_200_newyork+_391	115.8M	115.6M	0.2	14400.0	115.8M	114.6M	1.0	14400.0
dem_200_newyork+_392	135.6M	135.5M	0.1	144.0	135.6M	135.5M	0.1	119.6
dem_200_newyork+_393	142.0M	141.8M	0.1	1270.7	142.0M	141.8M	0.1	338.7
dem_200_newyork+_394	128.1M	128.1M	0.1	197.9	128.1M	128.0M	0.1	288.2
dem_200_newyork+_395	156.1M	155.9M	0.1	125.0	156.1M	156.0M	0.1	185.8
dem_200_newyork+_396	127.4M	127.2M	0.1	233.9	127.4M	127.2M	0.1	921.8
dem_200_newyork+_397	129.7M	129.5M	0.1	58.9	129.7M	129.6M	0.1	108.4
dem_200_newyork+_398	144.8M	144.6M	0.1	103.2	144.8M	144.6M	0.1	72.6
dem_200_newyork+_399	128.2M	128.1M	0.1	697.1	128.2M	128.1M	0.1	96.0
dem_200_newyork+_400	124.0M	123.8M	0.1	301.8	124.0M	123.8M	0.1	214.7
dem_200_newyork+_401	151.0M	150.9M	0.1	151.1	151.0M	150.9M	0.1	150.5

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Table A.44: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_402	127.1M	79.2M	60.6	14400.0	125.9M	79.2M	59.0	14400.0
dem_200_newyork+_403	114.0M	113.9M	0.1	6408.1	114.0M	113.9M	0.1	127.7
dem_200_newyork+_404	152.1M	152.0M	0.1	231.8	152.1M	152.0M	0.1	143.8
dem_200_newyork+_405	104.3M	104.0M	0.4	14400.0	104.3M	104.2M	0.1	51.5
dem_200_newyork+_406	135.9M	135.7M	0.1	157.4	135.9M	135.7M	0.1	161.2
dem_200_newyork+_407	115.4M	115.3M	0.1	416.6	115.4M	115.3M	0.1	187.0
dem_200_newyork+_408	120.4M	120.3M	0.1	504.3	120.3M	120.2M	0.1	2529.4
dem_200_newyork+_409	145.6M	145.5M	0.1	235.3	145.6M	145.5M	0.1	397.7
dem_200_newyork+_410	137.1M	137.0M	0.1	141.8	137.1M	136.9M	0.1	293.3
dem_200_newyork+_411	88.6M	86.1M	3.0	14400.0	88.6M	88.5M	0.1	989.0
dem_200_newyork+_412	134.5M	134.4M	0.1	167.2	134.5M	134.3M	0.1	89.2
dem_200_newyork+_413	96.4M	96.3M	0.1	168.7	96.4M	96.3M	0.1	53.9
dem_200_newyork+_414	147.5M	147.4M	0.1	160.2	147.5M	147.4M	0.1	249.9
dem_200_newyork+_415	142.7M	142.5M	0.1	90.9	142.6M	142.5M	0.1	121.9
dem_200_newyork+_416	151.9M	151.8M	0.1	267.2	151.9M	151.8M	0.1	385.7
dem_200_newyork+_417	151.3M	151.1M	0.1	156.5	151.3M	151.1M	0.1	105.3
dem_200_newyork+_418	127.1M	126.9M	0.1	328.2	127.1M	126.9M	0.1	83.4
dem_200_newyork+_419	140.1M	137.5M	1.9	14400.0	140.0M	139.8M	0.1	153.4
dem_200_newyork+_420	123.6M	123.5M	0.1	698.6	123.6M	123.4M	0.1	436.3
dem_200_newyork+_421	104.0M	103.9M	0.1	1179.3	103.9M	102.8M	1.1	14400.0
dem_200_newyork+_422	120.1M	119.9M	0.1	551.9	120.1M	119.9M	0.1	164.9
dem_200_newyork+_423	137.1M	136.9M	0.1	212.4	137.0M	136.9M	0.1	329.6
dem_200_newyork+_424	145.8M	145.7M	0.1	214.6	145.8M	145.7M	0.1	396.5
dem_200_newyork+_425	94.5M	94.4M	0.1	645.0	94.5M	94.4M	0.1	90.8
dem_200_newyork+_426	117.9M	117.8M	0.1	709.0	117.9M	117.8M	0.1	130.2
dem_200_newyork+_427	107.7M	107.6M	0.1	1207.6	107.7M	107.6M	0.1	130.0
dem_200_newyork+_428	128.1M	128.1M	0.0	204.8	128.1M	128.0M	0.1	193.5
dem_200_newyork+_429	151.6M	151.5M	0.1	88.9	151.6M	151.5M	0.1	85.4
dem_200_newyork+_430	111.3M	111.2M	0.1	121.2	111.3M	111.3M	0.0	190.4
dem_200_newyork+_431	141.3M	141.2M	0.1	140.2	141.3M	141.2M	0.1	126.1
dem_200_newyork+_432	127.3M	127.1M	0.1	545.9	127.3M	127.1M	0.1	109.5
dem_200_newyork+_433	152.5M	152.3M	0.1	156.0	152.5M	152.3M	0.1	97.7
dem_200_newyork+_434	111.2M	111.1M	0.1	66.9	111.2M	111.1M	0.1	127.1
dem_200_newyork+_435	131.5M	131.3M	0.1	93.7	131.5M	131.4M	0.1	92.9
dem_200_newyork+_436	160.6M	160.5M	0.1	192.4	160.6M	160.5M	0.1	231.9
dem_200_newyork+_437	110.8M	110.7M	0.1	126.8	110.8M	110.8M	0.0	121.8
dem_200_newyork+_438	143.4M	143.2M	0.1	424.5	143.4M	143.3M	0.1	143.9
dem_200_newyork+_439	84.3M	84.2M	0.1	9166.9	84.3M	55.1M	52.9	14400.0
dem_200_newyork+_440	108.6M	108.5M	0.1	85.4	108.6M	107.0M	1.5	14400.0
dem_200_newyork+_441	133.8M	133.7M	0.1	570.9	133.8M	133.7M	0.1	487.3
dem_200_newyork+_442	134.9M	134.7M	0.1	202.1	134.9M	134.7M	0.1	724.1
dem_200_newyork+_443	114.0M	113.9M	0.1	91.9	113.9M	113.8M	0.1	232.8
dem_200_newyork+_444	144.5M	144.3M	0.1	609.9	144.5M	144.3M	0.1	130.5
dem_200_newyork+_445	115.8M	115.8M	0.0	229.6	115.8M	115.8M	0.0	151.2
dem_200_newyork+_446	122.3M	122.2M	0.1	134.2	122.3M	122.2M	0.1	212.6
dem_200_newyork+_447	138.1M	138.0M	0.1	944.6	138.1M	138.1M	0.0	83.0
dem_200_newyork+_448	141.8M	141.6M	0.1	225.3	141.8M	141.6M	0.1	367.6
dem_200_newyork+_449	128.1M	127.9M	0.1	260.5	128.1M	128.0M	0.1	133.1
dem_200_newyork+_450	120.6M	120.5M	0.1	354.6	120.6M	120.6M	0.0	74.5
dem_200_newyork+_451	141.7M	141.6M	0.1	368.5	141.7M	141.6M	0.1	490.5
dem_200_newyork+_452	148.0M	147.9M	0.1	106.0	148.0M	147.9M	0.1	77.8
dem_200_newyork+_453	113.2M	113.2M	0.0	1556.1	113.2M	113.1M	0.1	216.7
dem_200_newyork+_454	105.3M	105.3M	0.0	76.0	105.3M	105.3M	0.0	89.6
dem_200_newyork+_455	150.7M	150.6M	0.1	145.2	150.7M	150.6M	0.1	165.1
dem_200_newyork+_456	140.3M	140.2M	0.1	212.9	140.3M	140.2M	0.1	78.4
dem_200_newyork+_457	127.7M	127.6M	0.1	146.4	127.7M	127.6M	0.1	179.5
dem_200_newyork+_458	126.5M	126.4M	0.1	156.5	126.5M	126.4M	0.1	201.4
dem_200_newyork+_459	140.6M	140.5M	0.1	106.4	140.6M	140.5M	0.1	75.7
dem_200_newyork+_460	131.3M	131.2M	0.1	136.7	131.3M	131.2M	0.1	94.6
dem_200_newyork+_461	123.9M	123.8M	0.1	185.6	123.9M	123.8M	0.1	1524.0
dem_200_newyork+_462	104.1M	85.5M	21.8	14400.0	104.1M	104.0M	0.1	13531.0
dem_200_newyork+_463	99.4M	99.3M	0.1	120.1	99.4M	99.3M	0.1	2585.3
dem_200_newyork+_464	84.7M	84.6M	0.1	5658.4	84.7M	84.6M	0.1	204.0
dem_200_newyork+_465	120.4M	120.3M	0.1	183.7	120.4M	120.3M	0.1	196.8
dem_200_newyork+_466	104.6M	104.5M	0.1	122.2	104.6M	104.5M	0.1	146.8
dem_200_newyork+_467	116.3M	116.1M	0.1	483.7	116.3M	116.0M	0.2	14400.0
dem_200_newyork+_468	113.0M	112.8M	0.1	339.7	112.9M	111.9M	0.9	14400.0
dem_200_newyork+_469	118.7M	118.6M	0.1	130.4	118.7M	118.6M	0.1	145.5
dem_200_newyork+_470	127.3M	127.2M	0.1	1426.9	127.3M	127.2M	0.1	134.2
dem_200_newyork+_471	141.8M	141.7M	0.1	50.0	141.8M	141.6M	0.1	62.8
dem_200_newyork+_472	138.0M	137.8M	0.1	165.4	138.0M	137.8M	0.1	168.6
dem_200_newyork+_473	90.7M	59.1M	53.5	14400.0	90.7M	59.1M	53.4	14400.0
dem_200_newyork+_474	150.8M	150.6M	0.1	274.9	150.8M	150.6M	0.1	129.9
dem_200_newyork+_475	113.0M	112.9M	0.1	428.6	113.0M	112.9M	0.1	169.1
dem_200_newyork+_476	140.7M	140.5M	0.1	38.2	140.7M	140.5M	0.1	47.2
dem_200_newyork+_477	133.2M	133.0M	0.1	230.2	133.2M	133.0M	0.1	142.0
dem_200_newyork+_478	104.4M	104.3M	0.1	7474.5	104.4M	104.3M	0.1	8403.0
dem_200_newyork+_479	132.9M	132.7M	0.1	142.4	132.9M	132.7M	0.1	78.4

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Table A.44: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 200$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_200_newyork+_480	163.6M	163.4M	0.1	132.3	163.5M	163.4M	0.1	147.2
dem_200_newyork+_481	113.5M	112.4M	1.0	14400.0	113.5M	113.4M	0.1	80.7
dem_200_newyork+_482	139.5M	139.3M	0.1	929.2	139.5M	139.3M	0.1	208.7
dem_200_newyork+_483	88.2M	51.3M	71.7	14400.0	88.2M	51.2M	72.3	14400.0
dem_200_newyork+_484	105.4M	105.3M	0.1	4580.1	105.4M	76.5M	37.8	14400.0
dem_200_newyork+_485	136.3M	136.3M	0.0	135.6	136.3M	136.2M	0.1	323.0
dem_200_newyork+_486	122.6M	122.5M	0.1	372.8	122.6M	122.5M	0.1	460.4
dem_200_newyork+_487	114.5M	114.4M	0.1	441.4	114.5M	114.4M	0.1	67.0
dem_200_newyork+_488	140.2M	140.1M	0.1	98.0	140.2M	140.1M	0.1	79.1
dem_200_newyork+_489	96.0M	95.7M	0.3	14400.0	96.0M	95.9M	0.1	73.9
dem_200_newyork+_490	119.9M	119.8M	0.1	117.0	119.9M	118.4M	1.3	14400.0
dem_200_newyork+_491	156.5M	156.4M	0.1	63.3	156.5M	156.4M	0.1	68.6
dem_200_newyork+_492	151.9M	151.7M	0.1	283.7	151.9M	151.7M	0.1	134.8
dem_200_newyork+_493	134.2M	134.1M	0.1	106.4	134.2M	134.1M	0.1	105.1
dem_200_newyork+_494	113.7M	113.6M	0.1	702.4	113.7M	113.6M	0.1	134.0
dem_200_newyork+_495	137.6M	137.4M	0.1	141.1	137.6M	137.4M	0.1	136.6
dem_200_newyork+_496	96.5M	93.2M	3.5	14400.0	96.5M	91.9M	5.0	14400.0
dem_200_newyork+_497	130.4M	130.4M	0.0	87.3	130.4M	130.3M	0.1	50.7
dem_200_newyork+_498	144.4M	144.3M	0.1	84.4	144.5M	144.4M	0.1	66.4
dem_200_newyork+_499	107.4M	107.3M	0.1	298.0	107.4M	107.3M	0.1	213.6
dem_200_newyork+_500	89.5M	89.5M	0.1	850.2	89.5M	89.5M	0.1	127.5

Table A.45: Detailed results of the discrete models on *New York+* with four additional arcs and $\mathcal{B} = 500$ as summarized in Figure 3.12c and Table 3.3f. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_1	1e+20	1e+20	0.0	6.1	1e+20	1e+20	0.0	232.0	1e+20	1e+20	0.0	36.4
dem_500_newyork+_2	1e+20	1e+20	0.0	8.9	1e+20	1e+20	0.0	506.1	1e+20	1e+20	0.0	125.9
dem_500_newyork+_3	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	237.0	1e+20	1e+20	0.0	29.3
dem_500_newyork+_4	1e+20	1e+20	0.0	8.3	1e+20	1e+20	0.0	428.4	1e+20	1e+20	0.0	430.2
dem_500_newyork+_5	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	263.7	1e+20	1e+20	0.0	6.2
dem_500_newyork+_6	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	257.1	1e+20	1e+20	0.0	74.2
dem_500_newyork+_7	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	276.7	1e+20	1e+20	0.0	67.8
dem_500_newyork+_8	1e+20	356.3M	–	14400.0	1e+20	1e+20	0.0	932.2	1e+20	409.0M	–	14400.0
dem_500_newyork+_9	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	360.7	1e+20	1e+20	0.0	82.0
dem_500_newyork+_10	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	152.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_11	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	475.0	1e+20	1e+20	0.0	67.8
dem_500_newyork+_12	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	155.4	1e+20	1e+20	0.0	0.5
dem_500_newyork+_13	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	160.8	1e+20	1e+20	0.0	0.4
dem_500_newyork+_14	1e+20	1e+20	0.0	12.6	1e+20	1e+20	0.0	139.9	1e+20	1e+20	0.0	0.4
dem_500_newyork+_15	1e+20	1e+20	0.0	10.6	1e+20	1e+20	0.0	443.5	1e+20	1e+20	0.0	133.6
dem_500_newyork+_16	1e+20	1e+20	0.0	26.5	1e+20	1e+20	0.0	382.7	1e+20	1e+20	0.0	6275.4
dem_500_newyork+_17	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	157.6	1e+20	1e+20	0.0	0.4
dem_500_newyork+_18	1e+20	339.6M	–	14400.0	1e+20	1e+20	0.0	1370.2	1e+20	435.3M	–	14400.0
dem_500_newyork+_19	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	356.7	1e+20	1e+20	0.0	85.3
dem_500_newyork+_20	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	293.7	1e+20	1e+20	0.0	59.4
dem_500_newyork+_21	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	173.1	1e+20	1e+20	0.0	0.4
dem_500_newyork+_22	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	464.1	1e+20	1e+20	0.0	44.0
dem_500_newyork+_23	1e+20	1e+20	0.0	8.1	1e+20	1e+20	0.0	507.4	1e+20	1e+20	0.0	218.1
dem_500_newyork+_24	1e+20	1e+20	0.0	3.4	1e+20	1e+20	0.0	213.9	1e+20	1e+20	0.0	25.3
dem_500_newyork+_25	1e+20	1e+20	0.0	31.5	1e+20	1e+20	0.0	504.3	1e+20	1e+20	0.0	1876.9
dem_500_newyork+_26	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	245.5	1e+20	1e+20	0.0	42.3
dem_500_newyork+_27	1e+20	1e+20	0.0	1894.2	1e+20	1e+20	0.0	759.5	1e+20	574.9M	–	14400.0
dem_500_newyork+_28	1e+20	1e+20	0.0	10.4	1e+20	1e+20	0.0	574.0	1e+20	1e+20	0.0	57.6
dem_500_newyork+_29	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	31.3	1e+20	1e+20	0.0	0.3
dem_500_newyork+_30	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	149.1	1e+20	1e+20	0.0	0.4
dem_500_newyork+_31	1e+20	1e+20	0.0	8.8	1e+20	1e+20	0.0	410.5	1e+20	1e+20	0.0	229.2
dem_500_newyork+_32	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	208.5	1e+20	1e+20	0.0	0.6
dem_500_newyork+_33	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	203.4	1e+20	1e+20	0.0	15.6
dem_500_newyork+_34	1e+20	1e+20	0.0	21.0	1e+20	1e+20	0.0	411.8	1e+20	1e+20	0.0	517.1
dem_500_newyork+_35	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	7.9	1e+20	1e+20	0.0	0.3
dem_500_newyork+_36	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	350.4	1e+20	1e+20	0.0	43.9
dem_500_newyork+_37	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	483.5	1e+20	1e+20	0.0	158.1
dem_500_newyork+_38	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	223.6	1e+20	1e+20	0.0	29.0
dem_500_newyork+_39	1e+20	1e+20	0.0	1.9	1e+20	1e+20	0.0	233.4	1e+20	1e+20	0.0	42.5
dem_500_newyork+_40	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	505.4	1e+20	1e+20	0.0	229.9
dem_500_newyork+_41	1e+20	1e+20	0.0	11.7	1e+20	1e+20	0.0	449.6	1e+20	1e+20	0.0	182.4
dem_500_newyork+_42	1e+20	267.6M	–	14400.0	1e+20	1e+20	0.0	9756.5	1e+20	397.4M	–	14400.0

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Table A.45: Comparison of discrete models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_43	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	195.6	1e+20	1e+20	0.0	30.8
dem_500_newyork+_44	1e+20	1e+20	0.0	11.3	1e+20	1e+20	0.0	509.7	1e+20	1e+20	0.0	128.0
dem_500_newyork+_45	1e+20	1e+20	0.0	1.7	1e+20	1e+20	0.0	153.5	1e+20	1e+20	0.0	0.5
dem_500_newyork+_46	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	167.3	1e+20	1e+20	0.0	0.4
dem_500_newyork+_47	1e+20	473.2M	–	14400.0	1e+20	1e+20	0.0	1618.4	1e+20	419.6M	–	14400.0
dem_500_newyork+_48	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	15.7	1e+20	1e+20	0.0	0.4
dem_500_newyork+_49	1e+20	340.2M	–	14400.0	1e+20	1e+20	0.0	5045.8	1e+20	394.6M	–	14400.0
dem_500_newyork+_50	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	221.4	1e+20	1e+20	0.0	0.4
dem_500_newyork+_51	1e+20	387.4M	–	14400.0	1e+20	1e+20	0.0	646.9	1e+20	474.6M	–	14400.0
dem_500_newyork+_52	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	156.5	1e+20	1e+20	0.0	0.5
dem_500_newyork+_53	1e+20	324.8M	–	14400.0	1e+20	1e+20	0.0	1410.2	1e+20	432.1M	–	14400.0
dem_500_newyork+_54	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	9.4	1e+20	1e+20	0.0	0.3
dem_500_newyork+_55	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	50.7	1e+20	1e+20	0.0	0.3
dem_500_newyork+_56	1e+20	299.1M	–	14400.0	685.3M	685.3M	0.0	5798.4	1e+20	379.7M	–	14400.0
dem_500_newyork+_57	1e+20	1e+20	0.0	112.3	1e+20	1e+20	0.0	477.3	1e+20	1e+20	0.0	360.2
dem_500_newyork+_58	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	500.0	1e+20	1e+20	0.0	0.4
dem_500_newyork+_59	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	332.0	1e+20	1e+20	0.0	68.7
dem_500_newyork+_60	1e+20	1e+20	0.0	9.5	1e+20	1e+20	0.0	586.9	1e+20	1e+20	0.0	138.5
dem_500_newyork+_61	1e+20	310.8M	–	14400.0	1e+20	1e+20	0.0	13690.0	1e+20	332.2M	–	14400.0
dem_500_newyork+_62	1e+20	252.7M	–	14400.0	557.6M	500.3M	11.5	14401.4	1e+20	356.5M	–	14400.0
dem_500_newyork+_63	1e+20	1e+20	0.0	6.6	1e+20	1e+20	0.0	263.4	1e+20	1e+20	0.0	43.2
dem_500_newyork+_64	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	555.1	1e+20	1e+20	0.0	181.2
dem_500_newyork+_65	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	328.7	1e+20	1e+20	0.0	138.2
dem_500_newyork+_66	1e+20	1e+20	0.0	19.5	1e+20	1e+20	0.0	652.2	1e+20	1e+20	0.0	527.0
dem_500_newyork+_67	1e+20	1e+20	0.0	2.7	1e+20	1e+20	0.0	196.4	1e+20	1e+20	0.0	45.3
dem_500_newyork+_68	1e+20	390.3M	–	14400.0	1e+20	1e+20	0.0	568.3	1e+20	1e+20	0.0	5872.1
dem_500_newyork+_69	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	210.9	1e+20	1e+20	0.0	0.6
dem_500_newyork+_70	1e+20	1e+20	0.0	5949.7	1e+20	1e+20	0.0	681.6	1e+20	474.2M	–	14400.0
dem_500_newyork+_71	1e+20	1e+20	0.0	11.9	1e+20	1e+20	0.0	630.9	1e+20	1e+20	0.0	134.1
dem_500_newyork+_72	1e+20	1e+20	0.0	10.4	1e+20	1e+20	0.0	447.6	1e+20	1e+20	0.0	156.5
dem_500_newyork+_73	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	49.1	1e+20	1e+20	0.0	0.3
dem_500_newyork+_74	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	25.6	1e+20	1e+20	0.0	0.3
dem_500_newyork+_75	1e+20	1e+20	0.0	2.1	1e+20	1e+20	0.0	250.1	1e+20	1e+20	0.0	76.3
dem_500_newyork+_76	1e+20	1e+20	0.0	10.4	1e+20	1e+20	0.0	443.5	1e+20	1e+20	0.0	115.9
dem_500_newyork+_77	1e+20	490.5M	–	14400.0	1e+20	1e+20	0.0	3751.3	1e+20	380.8M	–	14400.0
dem_500_newyork+_78	1e+20	1e+20	0.0	3169.0	1e+20	1e+20	0.0	452.6	1e+20	1e+20	0.0	8329.5
dem_500_newyork+_79	1e+20	1e+20	0.0	5294.1	1e+20	1e+20	0.0	872.2	1e+20	543.7M	–	14400.0
dem_500_newyork+_80	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	14.2	1e+20	1e+20	0.0	0.4
dem_500_newyork+_81	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	270.6	1e+20	1e+20	0.0	0.9
dem_500_newyork+_82	1e+20	1e+20	0.0	15.6	1e+20	1e+20	0.0	663.4	1e+20	1e+20	0.0	164.3
dem_500_newyork+_83	1e+20	1e+20	0.0	13.6	1e+20	1e+20	0.0	596.2	1e+20	1e+20	0.0	214.2
dem_500_newyork+_84	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	343.6	1e+20	1e+20	0.0	0.5
dem_500_newyork+_85	1e+20	1e+20	0.0	10.0	1e+20	1e+20	0.0	804.0	1e+20	1e+20	0.0	107.1
dem_500_newyork+_86	1e+20	1e+20	0.0	41.1	1e+20	1e+20	0.0	676.2	1e+20	1e+20	0.0	585.7
dem_500_newyork+_87	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	244.7	1e+20	1e+20	0.0	0.4
dem_500_newyork+_88	1e+20	1e+20	0.0	44.7	1e+20	1e+20	0.0	805.7	1e+20	1e+20	0.0	1731.3
dem_500_newyork+_89	1e+20	1e+20	0.0	5.3	1e+20	1e+20	0.0	190.3	1e+20	1e+20	0.0	0.4
dem_500_newyork+_90	1e+20	1e+20	0.0	3.4	1e+20	1e+20	0.0	124.0	1e+20	1e+20	0.0	0.3
dem_500_newyork+_91	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	463.5	1e+20	1e+20	0.0	76.7
dem_500_newyork+_92	1e+20	1e+20	0.0	4.1	1e+20	1e+20	0.0	205.6	1e+20	1e+20	0.0	34.8
dem_500_newyork+_93	1e+20	1e+20	0.0	770.8	1e+20	1e+20	0.0	768.6	1e+20	431.5M	–	14400.0
dem_500_newyork+_94	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	167.8	1e+20	1e+20	0.0	0.4
dem_500_newyork+_95	1e+20	355.2M	–	14400.0	1e+20	1e+20	0.0	5359.8	1e+20	440.2M	–	14400.0
dem_500_newyork+_96	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	204.5	1e+20	1e+20	0.0	0.4
dem_500_newyork+_97	1e+20	1e+20	0.0	4.0	1e+20	1e+20	0.0	137.2	1e+20	1e+20	0.0	0.4
dem_500_newyork+_98	1e+20	1e+20	0.0	12.5	1e+20	1e+20	0.0	387.2	1e+20	1e+20	0.0	460.7
dem_500_newyork+_99	1e+20	1e+20	0.0	22.7	1e+20	1e+20	0.0	700.9	1e+20	1e+20	0.0	3016.9
dem_500_newyork+_100	1e+20	497.2M	–	14400.0	1e+20	1e+20	0.0	1419.4	1e+20	566.6M	–	14400.0
dem_500_newyork+_101	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	147.7	1e+20	1e+20	0.0	0.4
dem_500_newyork+_102	1e+20	1e+20	0.0	2.7	1e+20	1e+20	0.0	207.8	1e+20	1e+20	0.0	0.4
dem_500_newyork+_103	1e+20	1e+20	0.0	1296.0	1e+20	1e+20	0.0	582.6	1e+20	1e+20	0.0	10163.4
dem_500_newyork+_104	1e+20	1e+20	0.0	1.8	1e+20	1e+20	0.0	142.1	1e+20	1e+20	0.0	0.4
dem_500_newyork+_105	1e+20	1e+20	0.0	121.3	1e+20	1e+20	0.0	553.3	1e+20	1e+20	0.0	9921.0
dem_500_newyork+_106	1e+20	327.9M	–	14400.0	1e+20	1e+20	0.0	8421.3	1e+20	345.5M	–	14400.0
dem_500_newyork+_107	1e+20	1e+20	0.0	4.1	1e+20	1e+20	0.0	245.8	1e+20	1e+20	0.0	35.1
dem_500_newyork+_108	1e+20	1e+20	0.0	110.1	1e+20	1e+20	0.0	2610.9	1e+20	559.3M	–	14400.0
dem_500_newyork+_109	1e+20	1e+20	0.0	5.2	1e+20	1e+20	0.0	280.3	1e+20	1e+20	0.0	41.5
dem_500_newyork+_110	1e+20	1e+20	0.0	19.4	1e+20	1e+20	0.0	618.9	1e+20	1e+20	0.0	459.0
dem_500_newyork+_111	1e+20	1e+20	0.0	5.2	1e+20	1e+20	0.0	352.4	1e+20	1e+20	0.0	47.4
dem_500_newyork+_112	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	211.2	1e+20	1e+20	0.0	0.4
dem_500_newyork+_113	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	313.9	1e+20	1e+20	0.0	77.0
dem_500_newyork+_114	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	235.5	1e+20	1e+20	0.0	0.7
dem_500_newyork+_115	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	15.7	1e+20	1e+20	0.0	0.4
dem_500_newyork+_116	1e+20	355.1M	–	14400.0	1e+20	1e+20	0.0	894.6	1e+20	408.0M	–	14400.0
dem_500_newyork+_117	1e+20	251.4M	–	14400.0	1e+20	1e+20	0.0	4062.7	1e+20	271.4M	–	14400.0
dem_500_newyork+_118	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	448.7	1e+20	1e+20	0.0	135.0
dem_500_newyork+_119	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	221.7	1e+20	1e+20	0.0	1.5
dem_500_newyork+_120	1e+20	1e+20	0.0	1.7	1e+20	1e+20	0.0	378.2	1e+20	1e+20	0.0	0.4

continued on next page

Table A.45: Comparison of discrete models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_121	1e+20	1e+20	0.0	17.7	1e+20	1e+20	0.0	492.8	1e+20	1e+20	0.0	857.1
dem_500_newyork+_122	1e+20	1e+20	0.0	14.0	1e+20	1e+20	0.0	569.4	1e+20	1e+20	0.0	335.7
dem_500_newyork+_123	1e+20	1e+20	0.0	3.3	1e+20	1e+20	0.0	231.6	1e+20	1e+20	0.0	87.1
dem_500_newyork+_124	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	152.4	1e+20	1e+20	0.0	23.6
dem_500_newyork+_125	1e+20	1e+20	0.0	8.9	1e+20	1e+20	0.0	492.3	1e+20	1e+20	0.0	276.4
dem_500_newyork+_126	1e+20	1e+20	0.0	2.7	1e+20	1e+20	0.0	396.9	1e+20	1e+20	0.0	29.9
dem_500_newyork+_127	1e+20	323.5M	–	14400.0	1e+20	1e+20	0.0	3485.1	1e+20	350.9M	–	14400.0
dem_500_newyork+_128	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	273.4	1e+20	1e+20	0.0	31.2
dem_500_newyork+_129	1e+20	1e+20	0.0	12.7	1e+20	1e+20	0.0	395.6	1e+20	1e+20	0.0	163.7
dem_500_newyork+_130	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	462.3	1e+20	1e+20	0.0	75.3
dem_500_newyork+_131	1e+20	1e+20	0.0	4.9	1e+20	1e+20	0.0	182.5	1e+20	1e+20	0.0	15.4
dem_500_newyork+_132	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	145.1	1e+20	1e+20	0.0	0.5
dem_500_newyork+_133	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	311.8	1e+20	1e+20	0.0	39.7
dem_500_newyork+_134	1e+20	1e+20	0.0	9.4	1e+20	1e+20	0.0	359.8	1e+20	1e+20	0.0	73.5
dem_500_newyork+_135	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	213.7	1e+20	1e+20	0.0	24.3
dem_500_newyork+_136	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	238.8	1e+20	1e+20	0.0	0.4
dem_500_newyork+_137	1e+20	1e+20	0.0	2419.8	1e+20	1e+20	0.0	1008.1	1e+20	417.0M	–	14400.0
dem_500_newyork+_138	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	334.2	1e+20	1e+20	0.0	0.4
dem_500_newyork+_139	1e+20	1e+20	0.0	1.9	1e+20	1e+20	0.0	155.4	1e+20	1e+20	0.0	0.4
dem_500_newyork+_140	1e+20	467.5M	–	14400.0	1e+20	1e+20	0.0	1652.0	1e+20	484.2M	–	14400.0
dem_500_newyork+_141	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	0.2
dem_500_newyork+_142	1e+20	1e+20	0.0	20.2	1e+20	1e+20	0.0	584.4	1e+20	1e+20	0.0	801.4
dem_500_newyork+_143	1e+20	1e+20	0.0	18.0	1e+20	1e+20	0.0	474.1	1e+20	1e+20	0.0	464.1
dem_500_newyork+_144	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	25.0	1e+20	1e+20	0.0	0.5
dem_500_newyork+_145	1e+20	1e+20	0.0	6.2	1e+20	1e+20	0.0	444.7	1e+20	1e+20	0.0	191.2
dem_500_newyork+_146	1e+20	1e+20	0.0	18.0	1e+20	1e+20	0.0	610.5	1e+20	1e+20	0.0	757.0
dem_500_newyork+_147	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	174.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_148	1e+20	332.9M	–	14400.0	1e+20	1e+20	0.0	495.1	1e+20	457.3M	–	14400.0
dem_500_newyork+_149	1e+20	1e+20	0.0	42.4	1e+20	1e+20	0.0	361.3	1e+20	1e+20	0.0	3056.2
dem_500_newyork+_150	1e+20	313.0M	–	14400.0	1e+20	1e+20	0.0	1555.8	1e+20	334.6M	–	14400.0
dem_500_newyork+_151	1e+20	481.5M	–	14400.0	1e+20	1e+20	0.0	3485.5	1e+20	463.0M	–	14400.0
dem_500_newyork+_152	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	284.4	1e+20	1e+20	0.0	35.1
dem_500_newyork+_153	1e+20	301.7M	–	14400.0	1e+20	1e+20	0.0	1104.6	1e+20	391.6M	–	14400.0
dem_500_newyork+_154	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	248.5	1e+20	1e+20	0.0	63.4
dem_500_newyork+_155	1e+20	1e+20	0.0	6.1	1e+20	1e+20	0.0	210.4	1e+20	1e+20	0.0	47.8
dem_500_newyork+_156	1e+20	1e+20	0.0	1.8	1e+20	1e+20	0.0	379.0	1e+20	1e+20	0.0	64.2
dem_500_newyork+_157	1e+20	1e+20	0.0	2453.0	1e+20	1e+20	0.0	2159.2	1e+20	512.5M	–	14400.0
dem_500_newyork+_158	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	134.6	1e+20	1e+20	0.0	0.4
dem_500_newyork+_159	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	185.1	1e+20	1e+20	0.0	1.0
dem_500_newyork+_160	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	390.6	1e+20	1e+20	0.0	84.5
dem_500_newyork+_161	1e+20	1e+20	0.0	8.1	1e+20	1e+20	0.0	607.4	1e+20	1e+20	0.0	132.4
dem_500_newyork+_162	1e+20	338.0M	–	14400.0	1e+20	1e+20	0.0	422.3	1e+20	442.0M	–	14400.0
dem_500_newyork+_163	1e+20	1e+20	0.0	9.2	1e+20	1e+20	0.0	425.7	1e+20	1e+20	0.0	147.8
dem_500_newyork+_164	1e+20	1e+20	0.0	2.1	1e+20	1e+20	0.0	396.3	1e+20	1e+20	0.0	48.0
dem_500_newyork+_165	1e+20	1e+20	0.0	4.1	1e+20	1e+20	0.0	195.0	1e+20	1e+20	0.0	25.6
dem_500_newyork+_166	1e+20	1e+20	0.0	16.2	1e+20	1e+20	0.0	482.2	1e+20	1e+20	0.0	726.2
dem_500_newyork+_167	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	296.4	1e+20	1e+20	0.0	113.9
dem_500_newyork+_168	1e+20	1e+20	0.0	6.6	1e+20	1e+20	0.0	627.6	1e+20	1e+20	0.0	203.7
dem_500_newyork+_169	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	518.0	1e+20	1e+20	0.0	159.0
dem_500_newyork+_170	1e+20	1e+20	0.0	6.2	1e+20	1e+20	0.0	379.1	1e+20	1e+20	0.0	94.9
dem_500_newyork+_171	1e+20	1e+20	0.0	395.9	1e+20	1e+20	0.0	566.9	1e+20	1e+20	0.0	12896.5
dem_500_newyork+_172	1e+20	1e+20	0.0	1461.3	1e+20	1e+20	0.0	643.9	1e+20	1e+20	0.0	2794.6
dem_500_newyork+_173	1e+20	301.9M	–	14400.0	1e+20	1e+20	0.0	1745.5	1e+20	356.9M	–	14400.0
dem_500_newyork+_174	1e+20	386.0M	–	14400.0	1e+20	1e+20	0.0	1024.0	1e+20	418.9M	–	14400.0
dem_500_newyork+_175	1e+20	1e+20	0.0	18.6	1e+20	1e+20	0.0	560.0	1e+20	1e+20	0.0	245.7
dem_500_newyork+_176	1e+20	1e+20	0.0	8.7	1e+20	1e+20	0.0	384.0	1e+20	1e+20	0.0	75.9
dem_500_newyork+_177	1e+20	1e+20	0.0	6.6	1e+20	1e+20	0.0	679.7	1e+20	1e+20	0.0	186.9
dem_500_newyork+_178	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	293.5	1e+20	1e+20	0.0	54.5
dem_500_newyork+_179	1e+20	1e+20	0.0	7.4	1e+20	1e+20	0.0	543.0	1e+20	1e+20	0.0	884.2
dem_500_newyork+_180	1e+20	1e+20	0.0	9.3	1e+20	1e+20	0.0	413.1	1e+20	1e+20	0.0	239.0
dem_500_newyork+_181	1e+20	1e+20	0.0	4871.7	1e+20	1e+20	0.0	435.2	1e+20	532.6M	–	14400.0
dem_500_newyork+_182	1e+20	1e+20	0.0	11.3	1e+20	1e+20	0.0	192.1	1e+20	1e+20	0.0	0.4
dem_500_newyork+_183	1e+20	1e+20	0.0	6.4	1e+20	1e+20	0.0	705.4	1e+20	1e+20	0.0	170.9
dem_500_newyork+_184	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	203.7	1e+20	1e+20	0.0	41.0
dem_500_newyork+_185	1e+20	1e+20	0.0	198.1	1e+20	1e+20	0.0	518.2	1e+20	1e+20	0.0	255.2
dem_500_newyork+_186	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	11.6	1e+20	1e+20	0.0	0.3
dem_500_newyork+_187	1e+20	423.7M	–	14400.0	1e+20	1e+20	0.0	7555.8	1e+20	530.0M	–	14400.0
dem_500_newyork+_188	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	441.3	1e+20	1e+20	0.0	187.9
dem_500_newyork+_189	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	198.4	1e+20	1e+20	0.0	72.8
dem_500_newyork+_190	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	461.2	1e+20	1e+20	0.0	148.7
dem_500_newyork+_191	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	160.1	1e+20	1e+20	0.0	0.4
dem_500_newyork+_192	1e+20	1e+20	0.0	4.9	1e+20	1e+20	0.0	196.0	1e+20	1e+20	0.0	0.6
dem_500_newyork+_193	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	240.2	1e+20	1e+20	0.0	59.1
dem_500_newyork+_194	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	252.2	1e+20	1e+20	0.0	46.8
dem_500_newyork+_195	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	162.5	1e+20	1e+20	0.0	0.4
dem_500_newyork+_196	1e+20	1e+20	0.0	12.5	1e+20	1e+20	0.0	648.9	1e+20	1e+20	0.0	203.9
dem_500_newyork+_197	1e+20	1e+20	0.0	16.3	1e+20	1e+20	0.0	345.9	1e+20	1e+20	0.0	679.3
dem_500_newyork+_198	1e+20	328.7M	–	14400.0	1e+20	1e+20	0.0	1852.8	1e+20	421.5M	–	14400.0

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Table A.45: Comparison of discrete models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_199	1e+20	1e+20	0.0	4.1	1e+20	1e+20	0.0	164.8	1e+20	1e+20	0.0	29.4
dem_500_newyork+_200	1e+20	1e+20	0.0	4.0	1e+20	1e+20	0.0	389.6	1e+20	1e+20	0.0	66.8
dem_500_newyork+_201	1e+20	1e+20	0.0	822.9	1e+20	1e+20	0.0	403.1	1e+20	1e+20	0.0	5453.5
dem_500_newyork+_202	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	231.7	1e+20	1e+20	0.0	23.9
dem_500_newyork+_203	1e+20	1e+20	0.0	5408.4	1e+20	1e+20	0.0	1438.5	1e+20	530.6M	–	14400.0
dem_500_newyork+_204	1e+20	1e+20	0.0	4.0	1e+20	1e+20	0.0	149.4	1e+20	1e+20	0.0	13.3
dem_500_newyork+_205	1e+20	1e+20	0.0	2.1	1e+20	1e+20	0.0	182.6	1e+20	1e+20	0.0	0.4
dem_500_newyork+_206	1e+20	1e+20	0.0	7.5	1e+20	1e+20	0.0	421.7	1e+20	1e+20	0.0	301.5
dem_500_newyork+_207	1e+20	1e+20	0.0	3.4	1e+20	1e+20	0.0	189.8	1e+20	1e+20	0.0	49.7
dem_500_newyork+_208	1e+20	1e+20	0.0	12.7	1e+20	1e+20	0.0	612.1	1e+20	1e+20	0.0	88.8
dem_500_newyork+_209	1e+20	1e+20	0.0	17.4	1e+20	1e+20	0.0	512.5	1e+20	1e+20	0.0	3889.8
dem_500_newyork+_210	1e+20	1e+20	0.0	6.9	1e+20	1e+20	0.0	507.8	1e+20	1e+20	0.0	176.7
dem_500_newyork+_211	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	174.8	1e+20	1e+20	0.0	0.5
dem_500_newyork+_212	1e+20	316.0M	–	14400.0	1e+20	1e+20	0.0	1894.5	1e+20	373.8M	–	14400.0
dem_500_newyork+_213	1e+20	1e+20	0.0	1186.6	1e+20	1e+20	0.0	474.8	1e+20	1e+20	0.0	4431.6
dem_500_newyork+_214	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	182.7	1e+20	1e+20	0.0	35.0
dem_500_newyork+_215	1e+20	1e+20	0.0	30.3	1e+20	1e+20	0.0	486.2	1e+20	1e+20	0.0	1553.2
dem_500_newyork+_216	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	253.4	1e+20	1e+20	0.0	1.1
dem_500_newyork+_217	1e+20	1e+20	0.0	1.5	1e+20	1e+20	0.0	340.4	1e+20	1e+20	0.0	32.3
dem_500_newyork+_218	1e+20	418.5M	–	14400.0	1e+20	1e+20	0.0	1656.9	1e+20	487.1M	–	14400.0
dem_500_newyork+_219	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	234.4	1e+20	1e+20	0.0	1.1
dem_500_newyork+_220	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	250.8	1e+20	1e+20	0.0	33.3
dem_500_newyork+_221	1e+20	1e+20	0.0	893.6	1e+20	1e+20	0.0	441.6	1e+20	1e+20	0.0	3412.5
dem_500_newyork+_222	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	372.3	1e+20	1e+20	0.0	93.1
dem_500_newyork+_223	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	523.0	1e+20	1e+20	0.0	245.4
dem_500_newyork+_224	1e+20	1e+20	0.0	15.7	1e+20	1e+20	0.0	610.3	1e+20	1e+20	0.0	326.5
dem_500_newyork+_225	1e+20	1e+20	0.0	52.4	1e+20	1e+20	0.0	543.7	1e+20	1e+20	0.0	406.0
dem_500_newyork+_226	1e+20	1e+20	0.0	57.9	1e+20	1e+20	0.0	463.9	1e+20	1e+20	0.0	532.3
dem_500_newyork+_227	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	255.9	1e+20	1e+20	0.0	0.6
dem_500_newyork+_228	1e+20	1e+20	0.0	6.8	1e+20	1e+20	0.0	670.6	1e+20	1e+20	0.0	136.0
dem_500_newyork+_229	1e+20	1e+20	0.0	1651.1	1e+20	1e+20	0.0	550.8	1e+20	1e+20	0.0	2362.0
dem_500_newyork+_230	1e+20	1e+20	0.0	3.4	1e+20	1e+20	0.0	303.8	1e+20	1e+20	0.0	38.9
dem_500_newyork+_231	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	471.5	1e+20	1e+20	0.0	163.3
dem_500_newyork+_232	1e+20	1e+20	0.0	7.2	1e+20	1e+20	0.0	637.1	1e+20	1e+20	0.0	128.3
dem_500_newyork+_233	1e+20	1e+20	0.0	18.4	1e+20	1e+20	0.0	561.5	1e+20	1e+20	0.0	374.7
dem_500_newyork+_234	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	275.4	1e+20	1e+20	0.0	0.6
dem_500_newyork+_235	1e+20	1e+20	0.0	2.7	1e+20	1e+20	0.0	330.7	1e+20	1e+20	0.0	34.8
dem_500_newyork+_236	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	472.2	1e+20	1e+20	0.0	94.3
dem_500_newyork+_237	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	409.0	1e+20	1e+20	0.0	50.6
dem_500_newyork+_238	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	144.8	1e+20	1e+20	0.0	0.9
dem_500_newyork+_239	1e+20	1e+20	0.0	1941.5	1e+20	1e+20	0.0	770.3	1e+20	540.3M	–	14400.0
dem_500_newyork+_240	1e+20	1e+20	0.0	11.8	1e+20	1e+20	0.0	505.6	1e+20	1e+20	0.0	81.5
dem_500_newyork+_241	1e+20	1e+20	0.0	13.3	1e+20	1e+20	0.0	661.0	1e+20	1e+20	0.0	144.4
dem_500_newyork+_242	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	177.1	1e+20	1e+20	0.0	37.5
dem_500_newyork+_243	1e+20	1e+20	0.0	6.6	1e+20	1e+20	0.0	160.8	1e+20	1e+20	0.0	48.3
dem_500_newyork+_244	1e+20	1e+20	0.0	12895.0	1e+20	1e+20	0.0	887.3	1e+20	1e+20	0.0	11138.2
dem_500_newyork+_245	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	371.0	1e+20	1e+20	0.0	46.4
dem_500_newyork+_246	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	199.2	1e+20	1e+20	0.0	69.4
dem_500_newyork+_247	1e+20	338.1M	–	14400.0	1e+20	443.5M	–	14400.0	1e+20	329.3M	–	14400.0
dem_500_newyork+_248	1e+20	1e+20	0.0	11.1	1e+20	1e+20	0.0	432.4	1e+20	1e+20	0.0	144.8
dem_500_newyork+_249	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	14.0	1e+20	1e+20	0.0	0.3
dem_500_newyork+_250	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	220.5	1e+20	1e+20	0.0	76.1
dem_500_newyork+_251	1e+20	1e+20	0.0	15.8	1e+20	1e+20	0.0	473.8	1e+20	1e+20	0.0	336.0
dem_500_newyork+_252	1e+20	500.7M	–	14400.0	1e+20	1e+20	0.0	4072.4	1e+20	399.3M	–	14400.0
dem_500_newyork+_253	1e+20	487.4M	–	14400.0	1e+20	1e+20	0.0	846.7	1e+20	471.8M	–	14400.0
dem_500_newyork+_254	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	393.7	1e+20	1e+20	0.0	75.1
dem_500_newyork+_255	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	180.2	1e+20	1e+20	0.0	6.1
dem_500_newyork+_256	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	287.7	1e+20	1e+20	0.0	0.4
dem_500_newyork+_257	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	217.5	1e+20	1e+20	0.0	0.4
dem_500_newyork+_258	1e+20	1e+20	0.0	8.2	1e+20	1e+20	0.0	601.2	1e+20	1e+20	0.0	186.9
dem_500_newyork+_259	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	240.8	1e+20	1e+20	0.0	44.2
dem_500_newyork+_260	1e+20	339.6M	–	14400.0	1e+20	1e+20	0.0	885.2	1e+20	406.8M	–	14400.0
dem_500_newyork+_261	1e+20	1e+20	0.0	5.9	1e+20	1e+20	0.0	385.5	1e+20	1e+20	0.0	0.5
dem_500_newyork+_262	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	8.2	1e+20	1e+20	0.0	0.2
dem_500_newyork+_263	1e+20	1e+20	0.0	8.0	1e+20	1e+20	0.0	521.7	1e+20	1e+20	0.0	188.6
dem_500_newyork+_264	1e+20	1e+20	0.0	1.3	1e+20	1e+20	0.0	148.3	1e+20	1e+20	0.0	0.6
dem_500_newyork+_265	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	163.5	1e+20	1e+20	0.0	36.2
dem_500_newyork+_266	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	504.5	1e+20	1e+20	0.0	50.9
dem_500_newyork+_267	1e+20	1e+20	0.0	2.7	1e+20	1e+20	0.0	206.1	1e+20	1e+20	0.0	52.8
dem_500_newyork+_268	1e+20	525.1M	–	14400.0	1e+20	1e+20	0.0	547.3	1e+20	461.7M	–	14400.0
dem_500_newyork+_269	1e+20	1e+20	0.0	4.0	1e+20	1e+20	0.0	151.4	1e+20	1e+20	0.0	0.4
dem_500_newyork+_270	1e+20	1e+20	0.0	11.2	1e+20	1e+20	0.0	763.5	1e+20	1e+20	0.0	216.7
dem_500_newyork+_271	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	350.9	1e+20	1e+20	0.0	0.7
dem_500_newyork+_272	1e+20	1e+20	0.0	2.1	1e+20	1e+20	0.0	200.9	1e+20	1e+20	0.0	32.9
dem_500_newyork+_273	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	187.3	1e+20	1e+20	0.0	6.9
dem_500_newyork+_274	1e+20	376.6M	–	14400.0	1e+20	1e+20	0.0	955.5	1e+20	391.1M	–	14400.0
dem_500_newyork+_275	1e+20	1e+20	0.0	5.2	1e+20	1e+20	0.0	276.7	1e+20	1e+20	0.0	38.2
dem_500_newyork+_276	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	147.3	1e+20	1e+20	0.0	0.5

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Table A.45: Comparison of discrete models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_277	1e+20	1e+20	0.0	239.0	1e+20	1e+20	0.0	800.5	1e+20	681.7M	–	14400.0
dem_500_newyork+_278	1e+20	1e+20	0.0	8.1	1e+20	1e+20	0.0	590.7	1e+20	1e+20	0.0	245.9
dem_500_newyork+_279	1e+20	1e+20	0.0	246.2	1e+20	1e+20	0.0	551.6	1e+20	1e+20	0.0	8519.0
dem_500_newyork+_280	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	149.0	1e+20	1e+20	0.0	0.4
dem_500_newyork+_281	1e+20	1e+20	0.0	9.9	1e+20	1e+20	0.0	705.2	1e+20	1e+20	0.0	158.8
dem_500_newyork+_282	1e+20	1e+20	0.0	5911.5	1e+20	1e+20	0.0	890.2	1e+20	1e+20	0.0	1083.5
dem_500_newyork+_283	1e+20	1e+20	0.0	2.1	1e+20	1e+20	0.0	133.6	1e+20	1e+20	0.0	0.4
dem_500_newyork+_284	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	175.2	1e+20	1e+20	0.0	36.1
dem_500_newyork+_285	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	162.0	1e+20	1e+20	0.0	0.4
dem_500_newyork+_286	1e+20	1e+20	0.0	4.6	1e+20	1e+20	0.0	312.7	1e+20	1e+20	0.0	52.4
dem_500_newyork+_287	1e+20	1e+20	0.0	14315.9	1e+20	1e+20	0.0	916.0	1e+20	552.9M	–	14400.0
dem_500_newyork+_288	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	258.1	1e+20	1e+20	0.0	21.5
dem_500_newyork+_289	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	5.3	1e+20	1e+20	0.0	0.2
dem_500_newyork+_290	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	186.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_291	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	461.9	1e+20	1e+20	0.0	112.2
dem_500_newyork+_292	1e+20	284.6M	–	14400.0	1e+20	1e+20	0.0	1616.0	1e+20	389.6M	–	14400.0
dem_500_newyork+_293	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	238.2	1e+20	1e+20	0.0	38.6
dem_500_newyork+_294	1e+20	1e+20	0.0	4.9	1e+20	1e+20	0.0	185.1	1e+20	1e+20	0.0	25.0
dem_500_newyork+_295	1e+20	1e+20	0.0	18.8	1e+20	1e+20	0.0	697.0	1e+20	1e+20	0.0	189.5
dem_500_newyork+_296	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	32.0	1e+20	1e+20	0.0	0.3
dem_500_newyork+_297	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	161.3	1e+20	1e+20	0.0	3.5
dem_500_newyork+_298	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	331.3	1e+20	1e+20	0.0	78.0
dem_500_newyork+_299	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	202.7	1e+20	1e+20	0.0	52.7
dem_500_newyork+_300	1e+20	263.4M	–	14400.0	1e+20	596.5M	–	14400.0	1e+20	412.4M	–	14400.0
dem_500_newyork+_301	1e+20	1e+20	0.0	13355.3	1e+20	1e+20	0.0	717.4	1e+20	442.6M	–	14400.0
dem_500_newyork+_302	1e+20	390.9M	–	14400.0	1e+20	1e+20	0.0	738.8	1e+20	631.3M	–	14400.0
dem_500_newyork+_303	1e+20	1e+20	0.0	3085.7	1e+20	1e+20	0.0	1445.8	1e+20	501.5M	–	14400.0
dem_500_newyork+_304	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	403.9	1e+20	1e+20	0.0	35.0
dem_500_newyork+_305	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	332.7	1e+20	1e+20	0.0	1.0
dem_500_newyork+_306	1e+20	1e+20	0.0	77.3	1e+20	1e+20	0.0	1244.8	1e+20	584.0M	–	14400.0
dem_500_newyork+_307	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	148.2	1e+20	1e+20	0.0	0.4
dem_500_newyork+_308	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	141.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_309	1e+20	1e+20	0.0	36.6	1e+20	1e+20	0.0	530.0	1e+20	1e+20	0.0	582.1
dem_500_newyork+_310	1e+20	459.9M	–	14400.0	1e+20	1e+20	0.0	1115.3	1e+20	495.4M	–	14400.0
dem_500_newyork+_311	1e+20	337.4M	–	14400.0	1e+20	1e+20	0.0	522.8	1e+20	1e+20	0.0	13494.5
dem_500_newyork+_312	1e+20	1e+20	0.0	9.4	1e+20	1e+20	0.0	707.5	1e+20	1e+20	0.0	138.8
dem_500_newyork+_313	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	299.9	1e+20	1e+20	0.0	67.7
dem_500_newyork+_314	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	147.3	1e+20	1e+20	0.0	0.5
dem_500_newyork+_315	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	156.6	1e+20	1e+20	0.0	0.4
dem_500_newyork+_316	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	173.2	1e+20	1e+20	0.0	0.5
dem_500_newyork+_317	1e+20	1e+20	0.0	18.6	1e+20	1e+20	0.0	691.6	1e+20	1e+20	0.0	1519.4
dem_500_newyork+_318	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	20.2	1e+20	1e+20	0.0	0.3
dem_500_newyork+_319	1e+20	1e+20	0.0	4.1	1e+20	1e+20	0.0	222.2	1e+20	1e+20	0.0	39.8
dem_500_newyork+_320	1e+20	1e+20	0.0	1.5	1e+20	1e+20	0.0	163.1	1e+20	1e+20	0.0	0.4
dem_500_newyork+_321	1e+20	1e+20	0.0	6.8	1e+20	1e+20	0.0	450.4	1e+20	1e+20	0.0	101.8
dem_500_newyork+_322	1e+20	380.8M	–	14400.0	1e+20	1e+20	0.0	1229.2	1e+20	462.0M	–	14400.0
dem_500_newyork+_323	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	51.1	1e+20	1e+20	0.0	0.4
dem_500_newyork+_324	1e+20	326.8M	–	14400.0	1e+20	1e+20	0.0	1897.8	1e+20	444.3M	–	14400.0
dem_500_newyork+_325	1e+20	1e+20	0.0	8.8	1e+20	1e+20	0.0	689.7	1e+20	1e+20	0.0	271.4
dem_500_newyork+_326	1e+20	1e+20	0.0	16.0	1e+20	1e+20	0.0	519.8	1e+20	1e+20	0.0	184.0
dem_500_newyork+_327	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	307.5	1e+20	1e+20	0.0	39.3
dem_500_newyork+_328	1e+20	1e+20	0.0	12.3	1e+20	1e+20	0.0	504.2	1e+20	1e+20	0.0	872.1
dem_500_newyork+_329	1e+20	1e+20	0.0	2.1	1e+20	1e+20	0.0	286.9	1e+20	1e+20	0.0	0.6
dem_500_newyork+_330	1e+20	1e+20	0.0	189.4	1e+20	1e+20	0.0	571.3	1e+20	1e+20	0.0	3305.2
dem_500_newyork+_331	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	527.8	1e+20	1e+20	0.0	282.5
dem_500_newyork+_332	1e+20	1e+20	0.0	1.6	1e+20	1e+20	0.0	333.5	1e+20	1e+20	0.0	51.0
dem_500_newyork+_333	1e+20	1e+20	0.0	263.0	1e+20	1e+20	0.0	859.1	1e+20	1e+20	0.0	5767.0
dem_500_newyork+_334	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	585.0	1e+20	1e+20	0.0	132.6
dem_500_newyork+_335	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	263.9	1e+20	1e+20	0.0	17.7
dem_500_newyork+_336	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	400.5	1e+20	1e+20	0.0	51.4
dem_500_newyork+_337	1e+20	323.7M	–	14400.0	1e+20	1e+20	0.0	798.7	1e+20	405.0M	–	14400.0
dem_500_newyork+_338	1e+20	1e+20	0.0	12.4	1e+20	1e+20	0.0	940.2	1e+20	1e+20	0.0	443.4
dem_500_newyork+_339	1e+20	314.1M	–	14400.0	1e+20	1e+20	0.0	917.0	1e+20	420.2M	–	14400.0
dem_500_newyork+_340	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	171.3	1e+20	1e+20	0.0	0.4
dem_500_newyork+_341	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	301.6	1e+20	1e+20	0.0	95.3
dem_500_newyork+_342	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	27.8	1e+20	1e+20	0.0	0.3
dem_500_newyork+_343	1e+20	1e+20	0.0	11.5	1e+20	1e+20	0.0	563.4	1e+20	1e+20	0.0	125.3
dem_500_newyork+_344	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	177.0	1e+20	1e+20	0.0	0.4
dem_500_newyork+_345	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	0.2
dem_500_newyork+_346	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	565.4	1e+20	1e+20	0.0	66.1
dem_500_newyork+_347	1e+20	1e+20	0.0	11105.1	1e+20	1e+20	0.0	792.9	1e+20	1e+20	0.0	4152.2
dem_500_newyork+_348	1e+20	441.5M	–	14400.0	1e+20	1e+20	0.0	1320.1	1e+20	413.1M	–	14400.0
dem_500_newyork+_349	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	667.3	1e+20	1e+20	0.0	188.3
dem_500_newyork+_350	1e+20	1e+20	0.0	7.8	1e+20	1e+20	0.0	564.3	1e+20	1e+20	0.0	102.0
dem_500_newyork+_351	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	206.1	1e+20	1e+20	0.0	7.4
dem_500_newyork+_352	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	172.8	1e+20	1e+20	0.0	25.7
dem_500_newyork+_353	1e+20	1e+20	0.0	6.3	1e+20	1e+20	0.0	365.4	1e+20	1e+20	0.0	106.0
dem_500_newyork+_354	1e+20	325.2M	–	14400.0	1e+20	1e+20	0.0	947.7	1e+20	485.0M	–	14400.0

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Table A.45: Comparison of discrete models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_355	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	324.1	1e+20	1e+20	0.0	0.4
dem_500_newyork+_356	1e+20	1e+20	0.0	7271.7	1e+20	1e+20	0.0	657.4	1e+20	513.9M	–	14400.0
dem_500_newyork+_357	1e+20	1e+20	0.0	3.4	1e+20	1e+20	0.0	279.5	1e+20	1e+20	0.0	75.0
dem_500_newyork+_358	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	319.2	1e+20	1e+20	0.0	83.1
dem_500_newyork+_359	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	447.5	1e+20	1e+20	0.0	80.4
dem_500_newyork+_360	1e+20	340.8M	–	14400.0	1e+20	1e+20	0.0	668.3	1e+20	443.6M	–	14400.0
dem_500_newyork+_361	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	23.5	1e+20	1e+20	0.0	0.4
dem_500_newyork+_362	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	230.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_363	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	150.9	1e+20	1e+20	0.0	0.4
dem_500_newyork+_364	1e+20	1e+20	0.0	792.5	1e+20	1e+20	0.0	1265.1	1e+20	1e+20	0.0	9040.6
dem_500_newyork+_365	1e+20	315.8M	–	14400.0	1e+20	1e+20	0.0	873.7	1e+20	473.9M	–	14400.0
dem_500_newyork+_366	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	213.5	1e+20	1e+20	0.0	82.1
dem_500_newyork+_367	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	0.2
dem_500_newyork+_368	1e+20	1e+20	0.0	3.5	1e+20	1e+20	0.0	199.7	1e+20	1e+20	0.0	19.7
dem_500_newyork+_369	1e+20	349.1M	–	14400.0	1e+20	1e+20	0.0	753.4	1e+20	412.1M	–	14400.0
dem_500_newyork+_370	1e+20	1e+20	0.0	22.0	1e+20	1e+20	0.0	534.8	1e+20	1e+20	0.0	479.0
dem_500_newyork+_371	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	12.9	1e+20	1e+20	0.0	0.3
dem_500_newyork+_372	1e+20	1e+20	0.0	4742.6	1e+20	1e+20	0.0	643.9	1e+20	390.2M	–	14400.0
dem_500_newyork+_373	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	53.1	1e+20	1e+20	0.0	0.4
dem_500_newyork+_374	1e+20	402.1M	–	14400.0	1e+20	1e+20	0.0	914.4	1e+20	484.9M	–	14400.0
dem_500_newyork+_375	1e+20	1e+20	0.0	5.1	1e+20	1e+20	0.0	225.9	1e+20	1e+20	0.0	36.2
dem_500_newyork+_376	1e+20	1e+20	0.0	3.2	1e+20	1e+20	0.0	109.7	1e+20	1e+20	0.0	0.4
dem_500_newyork+_377	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	39.8	1e+20	1e+20	0.0	0.4
dem_500_newyork+_378	1e+20	1e+20	0.0	3.9	1e+20	1e+20	0.0	225.5	1e+20	1e+20	0.0	23.8
dem_500_newyork+_379	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	257.4	1e+20	1e+20	0.0	72.9
dem_500_newyork+_380	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	254.9	1e+20	1e+20	0.0	41.4
dem_500_newyork+_381	1e+20	316.5M	–	14400.0	1e+20	1e+20	0.0	1075.1	1e+20	373.7M	–	14400.0
dem_500_newyork+_382	1e+20	1e+20	0.0	2.6	1e+20	1e+20	0.0	254.0	1e+20	1e+20	0.0	0.7
dem_500_newyork+_383	1e+20	1e+20	0.0	3.3	1e+20	1e+20	0.0	226.6	1e+20	1e+20	0.0	0.5
dem_500_newyork+_384	1e+20	341.6M	–	14400.0	1e+20	1e+20	0.0	2008.2	1e+20	431.7M	–	14400.0
dem_500_newyork+_385	1e+20	1e+20	0.0	6.1	1e+20	1e+20	0.0	448.4	1e+20	1e+20	0.0	31.9
dem_500_newyork+_386	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	174.4	1e+20	1e+20	0.0	0.5
dem_500_newyork+_387	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	272.1	1e+20	1e+20	0.0	45.3
dem_500_newyork+_388	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	495.0	1e+20	1e+20	0.0	203.0
dem_500_newyork+_389	1e+20	1e+20	0.0	5.3	1e+20	1e+20	0.0	294.7	1e+20	1e+20	0.0	40.0
dem_500_newyork+_390	1e+20	1e+20	0.0	8.4	1e+20	1e+20	0.0	344.7	1e+20	1e+20	0.0	59.1
dem_500_newyork+_391	1e+20	1e+20	0.0	4.0	1e+20	1e+20	0.0	367.1	1e+20	1e+20	0.0	56.2
dem_500_newyork+_392	1e+20	294.9M	–	14400.0	670.0M	670.0M	0.0	8304.0	1e+20	329.9M	–	14400.0
dem_500_newyork+_393	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	331.6	1e+20	1e+20	0.0	66.9
dem_500_newyork+_394	1e+20	1e+20	0.0	285.6	1e+20	1e+20	0.0	793.1	1e+20	1e+20	0.0	7738.5
dem_500_newyork+_395	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	149.7	1e+20	1e+20	0.0	0.5
dem_500_newyork+_396	1e+20	1e+20	0.0	13.9	1e+20	1e+20	0.0	663.0	1e+20	1e+20	0.0	628.8
dem_500_newyork+_397	1e+20	1e+20	0.0	1.4	1e+20	1e+20	0.0	220.7	1e+20	1e+20	0.0	0.4
dem_500_newyork+_398	1e+20	1e+20	0.0	150.8	1e+20	1e+20	0.0	747.2	1e+20	1e+20	0.0	1939.1
dem_500_newyork+_399	1e+20	1e+20	0.0	2.7	1e+20	1e+20	0.0	183.8	1e+20	1e+20	0.0	0.5
dem_500_newyork+_400	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	28.6	1e+20	1e+20	0.0	0.3
dem_500_newyork+_401	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	495.2	1e+20	1e+20	0.0	568.9
dem_500_newyork+_402	1e+20	265.6M	–	14400.0	1e+20	1e+20	0.0	1579.7	1e+20	379.7M	–	14400.0
dem_500_newyork+_403	1e+20	1e+20	0.0	7.5	1e+20	1e+20	0.0	480.5	1e+20	1e+20	0.0	114.8
dem_500_newyork+_404	1e+20	349.9M	–	14400.0	1e+20	1e+20	0.0	1274.4	1e+20	398.7M	–	14400.0
dem_500_newyork+_405	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	229.9	1e+20	1e+20	0.0	4.3
dem_500_newyork+_406	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	318.1	1e+20	1e+20	0.0	2.1
dem_500_newyork+_407	1e+20	313.7M	–	14400.0	1e+20	1e+20	0.0	4589.9	1e+20	323.9M	–	14400.0
dem_500_newyork+_408	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	287.9	1e+20	1e+20	0.0	2.4
dem_500_newyork+_409	1e+20	272.1M	–	14400.0	1e+20	1e+20	0.0	941.8	1e+20	306.7M	–	14400.0
dem_500_newyork+_410	1e+20	1e+20	0.0	7.2	1e+20	1e+20	0.0	615.1	1e+20	1e+20	0.0	140.9
dem_500_newyork+_411	1e+20	1e+20	0.0	52.7	1e+20	1e+20	0.0	599.5	1e+20	1e+20	0.0	232.3
dem_500_newyork+_412	1e+20	1e+20	0.0	24.8	1e+20	1e+20	0.0	655.3	1e+20	1e+20	0.0	1476.5
dem_500_newyork+_413	1e+20	470.9M	–	14400.0	1e+20	1e+20	0.0	694.0	1e+20	1e+20	0.0	2919.1
dem_500_newyork+_414	1e+20	1e+20	0.0	2.7	1e+20	1e+20	0.0	124.6	1e+20	1e+20	0.0	0.4
dem_500_newyork+_415	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	219.7	1e+20	1e+20	0.0	0.5
dem_500_newyork+_416	1e+20	1e+20	0.0	3.6	1e+20	1e+20	0.0	230.4	1e+20	1e+20	0.0	26.1
dem_500_newyork+_417	1e+20	1e+20	0.0	4.0	1e+20	1e+20	0.0	301.8	1e+20	1e+20	0.0	33.1
dem_500_newyork+_418	1e+20	1e+20	0.0	4.6	1e+20	1e+20	0.0	480.1	1e+20	1e+20	0.0	143.3
dem_500_newyork+_419	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	316.0	1e+20	1e+20	0.0	107.0
dem_500_newyork+_420	1e+20	1e+20	0.0	17.4	1e+20	1e+20	0.0	744.3	1e+20	1e+20	0.0	741.0
dem_500_newyork+_421	1e+20	1e+20	0.0	7.3	1e+20	1e+20	0.0	138.3	1e+20	1e+20	0.0	0.4
dem_500_newyork+_422	1e+20	1e+20	0.0	2.4	1e+20	1e+20	0.0	342.7	1e+20	1e+20	0.0	85.4
dem_500_newyork+_423	1e+20	1e+20	0.0	6.7	1e+20	1e+20	0.0	638.5	1e+20	1e+20	0.0	333.1
dem_500_newyork+_424	1e+20	1e+20	0.0	21.9	1e+20	1e+20	0.0	626.9	1e+20	1e+20	0.0	461.6
dem_500_newyork+_425	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	9.9	1e+20	1e+20	0.0	0.3
dem_500_newyork+_426	1e+20	1e+20	0.0	4.2	1e+20	1e+20	0.0	212.4	1e+20	1e+20	0.0	0.5
dem_500_newyork+_427	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	254.6	1e+20	1e+20	0.0	69.1
dem_500_newyork+_428	1e+20	1e+20	0.0	3.3	1e+20	1e+20	0.0	194.0	1e+20	1e+20	0.0	33.0
dem_500_newyork+_429	1e+20	1e+20	0.0	8.5	1e+20	1e+20	0.0	552.5	1e+20	1e+20	0.0	104.3
dem_500_newyork+_430	1e+20	1e+20	0.0	2.2	1e+20	1e+20	0.0	198.3	1e+20	1e+20	0.0	49.5
dem_500_newyork+_431	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	0.3
dem_500_newyork+_432	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	11.7	1e+20	1e+20	0.0	0.3

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Table A.45: Comparison of discrete models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model A				Model B				Model C			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_433	1e+20	1e+20	0.0	28.2	1e+20	1e+20	0.0	650.6	1e+20	1e+20	0.0	393.9
dem_500_newyork+_434	1e+20	402.7M	–	14400.0	1e+20	1e+20	0.0	665.0	1e+20	421.2M	–	14400.0
dem_500_newyork+_435	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	606.6	1e+20	1e+20	0.0	123.2
dem_500_newyork+_436	1e+20	1e+20	0.0	3.4	1e+20	1e+20	0.0	194.0	1e+20	1e+20	0.0	0.8
dem_500_newyork+_437	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	630.4	1e+20	1e+20	0.0	84.3
dem_500_newyork+_438	1e+20	1e+20	0.0	18.9	1e+20	1e+20	0.0	418.0	1e+20	1e+20	0.0	372.0
dem_500_newyork+_439	1e+20	1e+20	0.0	9.9	1e+20	1e+20	0.0	545.7	1e+20	1e+20	0.0	206.3
dem_500_newyork+_440	1e+20	1e+20	0.0	109.1	1e+20	1e+20	0.0	526.4	1e+20	1e+20	0.0	547.0
dem_500_newyork+_441	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	156.0	1e+20	1e+20	0.0	0.4
dem_500_newyork+_442	1e+20	1e+20	0.0	15.9	1e+20	1e+20	0.0	576.2	1e+20	1e+20	0.0	247.3
dem_500_newyork+_443	1e+20	1e+20	0.0	222.7	1e+20	1e+20	0.0	745.7	1e+20	385.8M	–	14400.0
dem_500_newyork+_444	1e+20	1e+20	0.0	3.3	1e+20	1e+20	0.0	198.6	1e+20	1e+20	0.0	5.8
dem_500_newyork+_445	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	464.1	1e+20	1e+20	0.0	294.6
dem_500_newyork+_446	1e+20	1e+20	0.0	5.9	1e+20	1e+20	0.0	456.1	1e+20	1e+20	0.0	185.8
dem_500_newyork+_447	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	28.2	1e+20	1e+20	0.0	0.4
dem_500_newyork+_448	1e+20	1e+20	0.0	3.0	1e+20	1e+20	0.0	269.4	1e+20	1e+20	0.0	24.0
dem_500_newyork+_449	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	197.1	1e+20	1e+20	0.0	1.8
dem_500_newyork+_450	1e+20	1e+20	0.0	7.1	1e+20	1e+20	0.0	577.3	1e+20	1e+20	0.0	171.0
dem_500_newyork+_451	1e+20	1e+20	0.0	6.7	1e+20	1e+20	0.0	435.8	1e+20	1e+20	0.0	122.6
dem_500_newyork+_452	1e+20	1e+20	0.0	5.2	1e+20	1e+20	0.0	221.4	1e+20	1e+20	0.0	24.5
dem_500_newyork+_453	1e+20	1e+20	0.0	18.2	1e+20	1e+20	0.0	480.8	1e+20	1e+20	0.0	291.1
dem_500_newyork+_454	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	8.3	1e+20	1e+20	0.0	0.2
dem_500_newyork+_455	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	237.2	1e+20	1e+20	0.0	31.6
dem_500_newyork+_456	1e+20	290.2M	–	14400.0	1e+20	1e+20	0.0	2946.9	1e+20	389.0M	–	14400.0
dem_500_newyork+_457	1e+20	307.7M	–	14400.0	1e+20	1e+20	0.0	2415.9	1e+20	353.8M	–	14400.0
dem_500_newyork+_458	1e+20	1e+20	0.0	31.1	1e+20	1e+20	0.0	512.1	1e+20	1e+20	0.0	439.7
dem_500_newyork+_459	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	21.4	1e+20	1e+20	0.0	0.3
dem_500_newyork+_460	1e+20	1e+20	0.0	3.7	1e+20	1e+20	0.0	308.0	1e+20	1e+20	0.0	59.8
dem_500_newyork+_461	1e+20	1e+20	0.0	9.2	1e+20	1e+20	0.0	509.0	1e+20	1e+20	0.0	118.2
dem_500_newyork+_462	1e+20	1e+20	0.0	10.9	1e+20	1e+20	0.0	446.7	1e+20	1e+20	0.0	108.9
dem_500_newyork+_463	1e+20	1e+20	0.0	7.0	1e+20	1e+20	0.0	214.4	1e+20	1e+20	0.0	186.2
dem_500_newyork+_464	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	271.5	1e+20	1e+20	0.0	1.3
dem_500_newyork+_465	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	337.5	1e+20	1e+20	0.0	36.3
dem_500_newyork+_466	1e+20	1e+20	0.0	7.5	1e+20	1e+20	0.0	227.6	1e+20	1e+20	0.0	37.2
dem_500_newyork+_467	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	289.7	1e+20	1e+20	0.0	0.4
dem_500_newyork+_468	1e+20	1e+20	0.0	6.2	1e+20	1e+20	0.0	668.7	1e+20	1e+20	0.0	83.7
dem_500_newyork+_469	1e+20	1e+20	0.0	7.4	1e+20	1e+20	0.0	249.6	1e+20	1e+20	0.0	21.5
dem_500_newyork+_470	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	198.3	1e+20	1e+20	0.0	27.2
dem_500_newyork+_471	1e+20	1e+20	0.0	2.1	1e+20	1e+20	0.0	196.7	1e+20	1e+20	0.0	117.3
dem_500_newyork+_472	1e+20	1e+20	0.0	5.7	1e+20	1e+20	0.0	270.0	1e+20	1e+20	0.0	26.8
dem_500_newyork+_473	1e+20	1e+20	0.0	36.2	1e+20	1e+20	0.0	1610.2	1e+20	1e+20	0.0	1226.6
dem_500_newyork+_474	1e+20	1e+20	0.0	2.8	1e+20	1e+20	0.0	164.8	1e+20	1e+20	0.0	0.4
dem_500_newyork+_475	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	696.7	1e+20	1e+20	0.0	90.1
dem_500_newyork+_476	1e+20	1e+20	0.0	41.8	1e+20	1e+20	0.0	775.0	1e+20	1e+20	0.0	2060.0
dem_500_newyork+_477	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	297.1	1e+20	1e+20	0.0	29.7
dem_500_newyork+_478	1e+20	1e+20	0.0	1670.6	1e+20	1e+20	0.0	972.8	1e+20	1e+20	0.0	11462.9
dem_500_newyork+_479	1e+20	1e+20	0.0	0.2	1e+20	1e+20	0.0	19.2	1e+20	1e+20	0.0	0.3
dem_500_newyork+_480	1e+20	1e+20	0.0	2.9	1e+20	1e+20	0.0	211.5	1e+20	1e+20	0.0	1.0
dem_500_newyork+_481	1e+20	445.4M	–	14400.0	1e+20	1e+20	0.0	1780.4	1e+20	440.3M	–	14400.0
dem_500_newyork+_482	1e+20	1e+20	0.0	188.7	1e+20	1e+20	0.0	602.9	1e+20	1e+20	0.0	4930.9
dem_500_newyork+_483	1e+20	1e+20	0.0	556.6	1e+20	1e+20	0.0	517.0	1e+20	515.6M	–	14400.0
dem_500_newyork+_484	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	198.9	1e+20	1e+20	0.0	42.2
dem_500_newyork+_485	1e+20	453.6M	–	14400.0	1e+20	1e+20	0.0	692.4	1e+20	1e+20	0.0	11705.9
dem_500_newyork+_486	1e+20	1e+20	0.0	6485.9	1e+20	1e+20	0.0	760.2	1e+20	1e+20	0.0	3082.0
dem_500_newyork+_487	1e+20	327.8M	–	14400.0	1e+20	1e+20	0.0	2144.1	1e+20	315.5M	–	14400.0
dem_500_newyork+_488	1e+20	1e+20	0.0	2.0	1e+20	1e+20	0.0	220.3	1e+20	1e+20	0.0	1.1
dem_500_newyork+_489	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	607.8	1e+20	1e+20	0.0	60.6
dem_500_newyork+_490	1e+20	1e+20	0.0	5014.9	1e+20	1e+20	0.0	1524.8	1e+20	444.4M	–	14400.0
dem_500_newyork+_491	1e+20	1e+20	0.0	270.8	1e+20	1e+20	0.0	578.6	1e+20	1e+20	0.0	2524.2
dem_500_newyork+_492	1e+20	1e+20	0.0	2.5	1e+20	1e+20	0.0	298.8	1e+20	1e+20	0.0	0.4
dem_500_newyork+_493	1e+20	1e+20	0.0	0.1	1e+20	1e+20	0.0	10.1	1e+20	1e+20	0.0	0.3
dem_500_newyork+_494	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	150.0	1e+20	1e+20	0.0	0.4
dem_500_newyork+_495	1e+20	273.2M	–	14400.0	1e+20	1e+20	0.0	1111.3	1e+20	379.2M	–	14400.0
dem_500_newyork+_496	1e+20	1e+20	0.0	8.3	1e+20	1e+20	0.0	498.6	1e+20	1e+20	0.0	223.9
dem_500_newyork+_497	1e+20	1e+20	0.0	147.6	1e+20	1e+20	0.0	772.8	1e+20	1e+20	0.0	3772.3
dem_500_newyork+_498	1e+20	1e+20	0.0	3.1	1e+20	1e+20	0.0	353.2	1e+20	1e+20	0.0	73.2
dem_500_newyork+_499	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	303.5	1e+20	1e+20	0.0	193.8
dem_500_newyork+_500	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	182.5	1e+20	1e+20	0.0	0.5

Table A.46: Detailed results of the split-pipe models on *New York+* with four additional arcs and $\mathcal{B} = 500$ as summarized in Figure 3.18c and Table 3.4f. The table lists the primal bound, dual bound, optimality gap and solving time.

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_1	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_2	1e+20	1e+20	0.0	13.2	1e+20	1e+20	0.0	9.8
dem_500_newyork+_3	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_4	1e+20	1e+20	0.0	10.8	1e+20	1e+20	0.0	10.2
dem_500_newyork+_5	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_6	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_7	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_8	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	12.4
dem_500_newyork+_9	1e+20	1e+20	0.0	11.9	1e+20	1e+20	0.0	14.8
dem_500_newyork+_10	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_11	1e+20	1e+20	0.0	11.1	1e+20	1e+20	0.0	0.7
dem_500_newyork+_12	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.8
dem_500_newyork+_13	1e+20	1e+20	0.0	0.3	1e+20	1e+20	0.0	0.7
dem_500_newyork+_14	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_15	1e+20	1e+20	0.0	10.5	1e+20	1e+20	0.0	11.1
dem_500_newyork+_16	1e+20	1e+20	0.0	10.9	1e+20	1e+20	0.0	10.4
dem_500_newyork+_17	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_18	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	14.6
dem_500_newyork+_19	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_20	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.9
dem_500_newyork+_21	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_22	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_23	1e+20	1e+20	0.0	13.1	1e+20	1e+20	0.0	14.5
dem_500_newyork+_24	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.5
dem_500_newyork+_25	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	11.3
dem_500_newyork+_26	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_27	1e+20	1e+20	0.0	8.7	1e+20	1e+20	0.0	9.0
dem_500_newyork+_28	1e+20	1e+20	0.0	7.9	1e+20	1e+20	0.0	9.1
dem_500_newyork+_29	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_30	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_31	1e+20	1e+20	0.0	9.7	1e+20	1e+20	0.0	13.2
dem_500_newyork+_32	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_33	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_34	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	10.8
dem_500_newyork+_35	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_36	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_37	1e+20	1e+20	0.0	11.8	1e+20	1e+20	0.0	8.6
dem_500_newyork+_38	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_39	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_40	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_41	1e+20	1e+20	0.0	9.8	1e+20	1e+20	0.0	8.4
dem_500_newyork+_42	1e+20	1e+20	0.0	17.0	1e+20	1e+20	0.0	22.7
dem_500_newyork+_43	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_44	1e+20	1e+20	0.0	6.6	1e+20	1e+20	0.0	13.7
dem_500_newyork+_45	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.4
dem_500_newyork+_46	1e+20	1e+20	0.0	1.0	1e+20	1e+20	0.0	0.7
dem_500_newyork+_47	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	9.9
dem_500_newyork+_48	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.5
dem_500_newyork+_49	1e+20	1e+20	0.0	13.5	1e+20	1e+20	0.0	18.4
dem_500_newyork+_50	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_51	1e+20	1e+20	0.0	9.8	1e+20	1e+20	0.0	8.0
dem_500_newyork+_52	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_53	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	14.0
dem_500_newyork+_54	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.5
dem_500_newyork+_55	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.5
dem_500_newyork+_56	674.2M	673.6M	0.1	12.1	674.2M	673.6M	0.1	14.6
dem_500_newyork+_57	1e+20	1e+20	0.0	6.9	1e+20	1e+20	0.0	11.6
dem_500_newyork+_58	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_59	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_60	1e+20	1e+20	0.0	7.4	1e+20	1e+20	0.0	9.5
dem_500_newyork+_61	1e+20	1e+20	0.0	11.1	1e+20	1e+20	0.0	22.5
dem_500_newyork+_62	554.4M	553.9M	0.1	78.5	554.4M	553.9M	0.1	85.1
dem_500_newyork+_63	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_64	1e+20	1e+20	0.0	15.3	1e+20	1e+20	0.0	8.8
dem_500_newyork+_65	1e+20	1e+20	0.0	10.8	1e+20	1e+20	0.0	12.0
dem_500_newyork+_66	1e+20	1e+20	0.0	7.8	1e+20	1e+20	0.0	11.0
dem_500_newyork+_67	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.7
dem_500_newyork+_68	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	9.5
dem_500_newyork+_69	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_70	1e+20	1e+20	0.0	7.5	1e+20	1e+20	0.0	8.7
dem_500_newyork+_71	1e+20	1e+20	0.0	9.9	1e+20	1e+20	0.0	12.1
dem_500_newyork+_72	1e+20	1e+20	0.0	12.0	1e+20	1e+20	0.0	10.9
dem_500_newyork+_73	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.5
dem_500_newyork+_74	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6

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Table A.46: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_75	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_76	1e+20	1e+20	0.0	8.9	1e+20	1e+20	0.0	13.7
dem_500_newyork+_77	1e+20	1e+20	0.0	6.4	1e+20	1e+20	0.0	11.8
dem_500_newyork+_78	1e+20	1e+20	0.0	10.4	1e+20	1e+20	0.0	10.3
dem_500_newyork+_79	1e+20	1e+20	0.0	7.1	1e+20	1e+20	0.0	10.3
dem_500_newyork+_80	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.5
dem_500_newyork+_81	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_82	1e+20	1e+20	0.0	8.2	1e+20	1e+20	0.0	8.5
dem_500_newyork+_83	1e+20	1e+20	0.0	12.3	1e+20	1e+20	0.0	10.1
dem_500_newyork+_84	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_85	1e+20	1e+20	0.0	6.1	1e+20	1e+20	0.0	12.6
dem_500_newyork+_86	1e+20	1e+20	0.0	11.2	1e+20	1e+20	0.0	11.2
dem_500_newyork+_87	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_88	1e+20	1e+20	0.0	8.4	1e+20	1e+20	0.0	12.8
dem_500_newyork+_89	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_90	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_91	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_92	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_93	1e+20	1e+20	0.0	6.8	1e+20	1e+20	0.0	10.6
dem_500_newyork+_94	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.6
dem_500_newyork+_95	1e+20	1e+20	0.0	6.5	1e+20	1e+20	0.0	15.0
dem_500_newyork+_96	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_97	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_98	1e+20	1e+20	0.0	10.7	1e+20	1e+20	0.0	9.7
dem_500_newyork+_99	1e+20	1e+20	0.0	9.4	1e+20	1e+20	0.0	11.7
dem_500_newyork+_100	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	8.0
dem_500_newyork+_101	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_102	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_103	1e+20	1e+20	0.0	9.8	1e+20	1e+20	0.0	8.3
dem_500_newyork+_104	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.6
dem_500_newyork+_105	1e+20	1e+20	0.0	11.6	1e+20	1e+20	0.0	10.9
dem_500_newyork+_106	1e+20	1e+20	0.0	10.0	1e+20	1e+20	0.0	8.9
dem_500_newyork+_107	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_108	1e+20	1e+20	0.0	7.5	1e+20	1e+20	0.0	12.6
dem_500_newyork+_109	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_110	1e+20	1e+20	0.0	11.9	1e+20	1e+20	0.0	8.3
dem_500_newyork+_111	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_112	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_113	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_114	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.6
dem_500_newyork+_115	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_116	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	11.0
dem_500_newyork+_117	1e+20	1e+20	0.0	15.2	1e+20	1e+20	0.0	9.2
dem_500_newyork+_118	1e+20	1e+20	0.0	6.3	1e+20	1e+20	0.0	9.9
dem_500_newyork+_119	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.6
dem_500_newyork+_120	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_121	1e+20	1e+20	0.0	7.3	1e+20	1e+20	0.0	10.0
dem_500_newyork+_122	1e+20	1e+20	0.0	7.5	1e+20	1e+20	0.0	12.6
dem_500_newyork+_123	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.6
dem_500_newyork+_124	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_125	1e+20	1e+20	0.0	9.5	1e+20	1e+20	0.0	12.3
dem_500_newyork+_126	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_127	1e+20	1e+20	0.0	13.4	1e+20	1e+20	0.0	13.1
dem_500_newyork+_128	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.8
dem_500_newyork+_129	1e+20	1e+20	0.0	8.3	1e+20	1e+20	0.0	11.4
dem_500_newyork+_130	1e+20	1e+20	0.0	7.4	1e+20	1e+20	0.0	14.9
dem_500_newyork+_131	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_132	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_133	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_134	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_135	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	1.1
dem_500_newyork+_136	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_137	1e+20	1e+20	0.0	7.1	1e+20	1e+20	0.0	10.2
dem_500_newyork+_138	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_139	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_140	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	11.4
dem_500_newyork+_141	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.5
dem_500_newyork+_142	1e+20	1e+20	0.0	11.6	1e+20	1e+20	0.0	14.8
dem_500_newyork+_143	1e+20	1e+20	0.0	8.7	1e+20	1e+20	0.0	11.1
dem_500_newyork+_144	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_145	1e+20	1e+20	0.0	6.5	1e+20	1e+20	0.0	10.4
dem_500_newyork+_146	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	15.5
dem_500_newyork+_147	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.8
dem_500_newyork+_148	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	13.4
dem_500_newyork+_149	1e+20	1e+20	0.0	10.3	1e+20	1e+20	0.0	14.8
dem_500_newyork+_150	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	11.1
dem_500_newyork+_151	1e+20	1e+20	0.0	7.4	1e+20	1e+20	0.0	9.2
dem_500_newyork+_152	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6

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Table A.46: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_153	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	12.6
dem_500_newyork+_154	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_155	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.8
dem_500_newyork+_156	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_157	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	7.6
dem_500_newyork+_158	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_159	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_160	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_161	1e+20	1e+20	0.0	8.7	1e+20	1e+20	0.0	13.8
dem_500_newyork+_162	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	13.0
dem_500_newyork+_163	1e+20	1e+20	0.0	8.0	1e+20	1e+20	0.0	5.6
dem_500_newyork+_164	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_165	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_166	1e+20	1e+20	0.0	13.8	1e+20	1e+20	0.0	14.8
dem_500_newyork+_167	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.7
dem_500_newyork+_168	1e+20	1e+20	0.0	9.3	1e+20	1e+20	0.0	11.5
dem_500_newyork+_169	1e+20	1e+20	0.0	13.4	1e+20	1e+20	0.0	9.3
dem_500_newyork+_170	1e+20	1e+20	0.0	11.7	1e+20	1e+20	0.0	12.8
dem_500_newyork+_171	1e+20	1e+20	0.0	6.8	1e+20	1e+20	0.0	11.2
dem_500_newyork+_172	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	10.1
dem_500_newyork+_173	1e+20	1e+20	0.0	6.9	1e+20	1e+20	0.0	12.7
dem_500_newyork+_174	1e+20	1e+20	0.0	6.8	1e+20	1e+20	0.0	10.4
dem_500_newyork+_175	1e+20	1e+20	0.0	13.3	1e+20	1e+20	0.0	7.3
dem_500_newyork+_176	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	11.5
dem_500_newyork+_177	1e+20	1e+20	0.0	11.0	1e+20	1e+20	0.0	11.1
dem_500_newyork+_178	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_179	1e+20	1e+20	0.0	9.7	1e+20	1e+20	0.0	10.4
dem_500_newyork+_180	1e+20	1e+20	0.0	4.5	1e+20	1e+20	0.0	11.7
dem_500_newyork+_181	1e+20	1e+20	0.0	10.1	1e+20	1e+20	0.0	8.9
dem_500_newyork+_182	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_183	1e+20	1e+20	0.0	7.9	1e+20	1e+20	0.0	8.8
dem_500_newyork+_184	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_185	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	13.4
dem_500_newyork+_186	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_187	1e+20	1e+20	0.0	3.8	1e+20	1e+20	0.0	11.7
dem_500_newyork+_188	1e+20	1e+20	0.0	10.2	1e+20	1e+20	0.0	14.9
dem_500_newyork+_189	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.8
dem_500_newyork+_190	1e+20	1e+20	0.0	11.0	1e+20	1e+20	0.0	13.1
dem_500_newyork+_191	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_192	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_193	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_194	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_195	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_196	1e+20	1e+20	0.0	6.3	1e+20	1e+20	0.0	8.8
dem_500_newyork+_197	1e+20	1e+20	0.0	9.5	1e+20	1e+20	0.0	10.0
dem_500_newyork+_198	1e+20	1e+20	0.0	7.6	1e+20	1e+20	0.0	13.5
dem_500_newyork+_199	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_200	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_201	1e+20	1e+20	0.0	7.3	1e+20	1e+20	0.0	10.2
dem_500_newyork+_202	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_203	1e+20	1e+20	0.0	6.5	1e+20	1e+20	0.0	13.5
dem_500_newyork+_204	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.7
dem_500_newyork+_205	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_206	1e+20	1e+20	0.0	13.5	1e+20	1e+20	0.0	10.7
dem_500_newyork+_207	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_208	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	10.9
dem_500_newyork+_209	1e+20	1e+20	0.0	6.7	1e+20	1e+20	0.0	10.4
dem_500_newyork+_210	1e+20	1e+20	0.0	9.5	1e+20	1e+20	0.0	10.9
dem_500_newyork+_211	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_212	1e+20	1e+20	0.0	11.0	1e+20	1e+20	0.0	6.0
dem_500_newyork+_213	1e+20	1e+20	0.0	7.1	1e+20	1e+20	0.0	8.1
dem_500_newyork+_214	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_215	1e+20	1e+20	0.0	10.9	1e+20	1e+20	0.0	10.8
dem_500_newyork+_216	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_217	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_218	1e+20	1e+20	0.0	4.6	1e+20	1e+20	0.0	13.7
dem_500_newyork+_219	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_220	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_221	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	10.7
dem_500_newyork+_222	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.6
dem_500_newyork+_223	1e+20	1e+20	0.0	8.7	1e+20	1e+20	0.0	9.8
dem_500_newyork+_224	1e+20	1e+20	0.0	11.3	1e+20	1e+20	0.0	12.8
dem_500_newyork+_225	1e+20	1e+20	0.0	8.1	1e+20	1e+20	0.0	11.6
dem_500_newyork+_226	1e+20	1e+20	0.0	10.3	1e+20	1e+20	0.0	12.2
dem_500_newyork+_227	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.7
dem_500_newyork+_228	1e+20	1e+20	0.0	12.5	1e+20	1e+20	0.0	7.8
dem_500_newyork+_229	1e+20	1e+20	0.0	9.9	1e+20	1e+20	0.0	9.3
dem_500_newyork+_230	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7

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Table A.46: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_231	1e+20	1e+20	0.0	6.2	1e+20	1e+20	0.0	10.6
dem_500_newyork+_232	1e+20	1e+20	0.0	9.2	1e+20	1e+20	0.0	10.9
dem_500_newyork+_233	1e+20	1e+20	0.0	9.3	1e+20	1e+20	0.0	12.4
dem_500_newyork+_234	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_235	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_236	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_237	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_238	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_239	1e+20	1e+20	0.0	5.3	1e+20	1e+20	0.0	13.0
dem_500_newyork+_240	1e+20	1e+20	0.0	12.8	1e+20	1e+20	0.0	12.8
dem_500_newyork+_241	1e+20	1e+20	0.0	7.1	1e+20	1e+20	0.0	12.5
dem_500_newyork+_242	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_243	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.7
dem_500_newyork+_244	1e+20	1e+20	0.0	6.2	1e+20	1e+20	0.0	9.3
dem_500_newyork+_245	1e+20	1e+20	0.0	1.0	1e+20	1e+20	0.0	0.7
dem_500_newyork+_246	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.9
dem_500_newyork+_247	1e+20	1e+20	0.0	7.2	1e+20	1e+20	0.0	20.8
dem_500_newyork+_248	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_249	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_250	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_251	1e+20	1e+20	0.0	12.6	1e+20	1e+20	0.0	7.7
dem_500_newyork+_252	1e+20	1e+20	0.0	4.9	1e+20	1e+20	0.0	9.9
dem_500_newyork+_253	1e+20	1e+20	0.0	5.9	1e+20	1e+20	0.0	14.7
dem_500_newyork+_254	1e+20	1e+20	0.0	12.8	1e+20	1e+20	0.0	11.0
dem_500_newyork+_255	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_256	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_257	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_258	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	8.0
dem_500_newyork+_259	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.8
dem_500_newyork+_260	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	11.3
dem_500_newyork+_261	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.6
dem_500_newyork+_262	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_263	1e+20	1e+20	0.0	11.3	1e+20	1e+20	0.0	9.9
dem_500_newyork+_264	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.5
dem_500_newyork+_265	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_266	1e+20	1e+20	0.0	9.9	1e+20	1e+20	0.0	9.2
dem_500_newyork+_267	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_268	1e+20	1e+20	0.0	11.3	1e+20	1e+20	0.0	11.8
dem_500_newyork+_269	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_270	1e+20	1e+20	0.0	13.6	1e+20	1e+20	0.0	9.2
dem_500_newyork+_271	1e+20	1e+20	0.0	2.3	1e+20	1e+20	0.0	0.6
dem_500_newyork+_272	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_273	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_274	1e+20	1e+20	0.0	4.1	1e+20	1e+20	0.0	9.7
dem_500_newyork+_275	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_276	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_277	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	13.0
dem_500_newyork+_278	1e+20	1e+20	0.0	9.7	1e+20	1e+20	0.0	10.3
dem_500_newyork+_279	1e+20	1e+20	0.0	13.7	1e+20	1e+20	0.0	9.6
dem_500_newyork+_280	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_281	1e+20	1e+20	0.0	11.5	1e+20	1e+20	0.0	9.7
dem_500_newyork+_282	1e+20	1e+20	0.0	4.3	1e+20	1e+20	0.0	10.0
dem_500_newyork+_283	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.6
dem_500_newyork+_284	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_285	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.8
dem_500_newyork+_286	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_287	1e+20	1e+20	0.0	5.7	1e+20	1e+20	0.0	11.3
dem_500_newyork+_288	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_289	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_290	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.7
dem_500_newyork+_291	1e+20	1e+20	0.0	9.2	1e+20	1e+20	0.0	10.4
dem_500_newyork+_292	1e+20	1e+20	0.0	8.3	1e+20	1e+20	0.0	13.8
dem_500_newyork+_293	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_294	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_295	1e+20	1e+20	0.0	13.0	1e+20	1e+20	0.0	8.1
dem_500_newyork+_296	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.5
dem_500_newyork+_297	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_298	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_299	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_300	633.1M	632.4M	0.1	42.1	633.1M	632.4M	0.1	50.1
dem_500_newyork+_301	1e+20	1e+20	0.0	6.9	1e+20	1e+20	0.0	3.8
dem_500_newyork+_302	1e+20	1e+20	0.0	6.3	1e+20	1e+20	0.0	6.6
dem_500_newyork+_303	1e+20	1e+20	0.0	10.0	1e+20	1e+20	0.0	12.3
dem_500_newyork+_304	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.9
dem_500_newyork+_305	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_306	1e+20	1e+20	0.0	5.2	1e+20	1e+20	0.0	16.7
dem_500_newyork+_307	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_308	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6

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Table A.46: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_309	1e+20	1e+20	0.0	12.3	1e+20	1e+20	0.0	13.2
dem_500_newyork+_310	1e+20	1e+20	0.0	11.7	1e+20	1e+20	0.0	10.0
dem_500_newyork+_311	1e+20	1e+20	0.0	10.6	1e+20	1e+20	0.0	13.1
dem_500_newyork+_312	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	9.3
dem_500_newyork+_313	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_314	1e+20	1e+20	0.0	1.0	1e+20	1e+20	0.0	0.7
dem_500_newyork+_315	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_316	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_317	1e+20	1e+20	0.0	7.3	1e+20	1e+20	0.0	11.8
dem_500_newyork+_318	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_319	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_320	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.6
dem_500_newyork+_321	1e+20	1e+20	0.0	12.6	1e+20	1e+20	0.0	13.2
dem_500_newyork+_322	1e+20	1e+20	0.0	6.4	1e+20	1e+20	0.0	11.0
dem_500_newyork+_323	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.6
dem_500_newyork+_324	1e+20	1e+20	0.0	5.1	1e+20	1e+20	0.0	17.1
dem_500_newyork+_325	1e+20	1e+20	0.0	13.4	1e+20	1e+20	0.0	12.5
dem_500_newyork+_326	1e+20	1e+20	0.0	8.1	1e+20	1e+20	0.0	12.8
dem_500_newyork+_327	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_328	1e+20	1e+20	0.0	5.7	1e+20	1e+20	0.0	10.0
dem_500_newyork+_329	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_330	1e+20	1e+20	0.0	8.7	1e+20	1e+20	0.0	11.0
dem_500_newyork+_331	1e+20	1e+20	0.0	11.7	1e+20	1e+20	0.0	11.5
dem_500_newyork+_332	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_333	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	10.4
dem_500_newyork+_334	1e+20	1e+20	0.0	0.4	1e+20	1e+20	0.0	0.9
dem_500_newyork+_335	1e+20	1e+20	0.0	9.5	1e+20	1e+20	0.0	12.1
dem_500_newyork+_336	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_337	1e+20	1e+20	0.0	8.2	1e+20	1e+20	0.0	5.8
dem_500_newyork+_338	1e+20	1e+20	0.0	6.5	1e+20	1e+20	0.0	10.9
dem_500_newyork+_339	1e+20	1e+20	0.0	9.2	1e+20	1e+20	0.0	11.5
dem_500_newyork+_340	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_341	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_342	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_343	1e+20	1e+20	0.0	11.0	1e+20	1e+20	0.0	8.4
dem_500_newyork+_344	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_345	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_346	1e+20	1e+20	0.0	8.5	1e+20	1e+20	0.0	12.3
dem_500_newyork+_347	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	13.5
dem_500_newyork+_348	1e+20	1e+20	0.0	9.2	1e+20	1e+20	0.0	16.3
dem_500_newyork+_349	1e+20	1e+20	0.0	13.9	1e+20	1e+20	0.0	7.8
dem_500_newyork+_350	1e+20	1e+20	0.0	11.8	1e+20	1e+20	0.0	13.2
dem_500_newyork+_351	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_352	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_353	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_354	1e+20	1e+20	0.0	8.5	1e+20	1e+20	0.0	11.4
dem_500_newyork+_355	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_356	1e+20	1e+20	0.0	7.2	1e+20	1e+20	0.0	12.1
dem_500_newyork+_357	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.7
dem_500_newyork+_358	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_359	1e+20	1e+20	0.0	11.2	1e+20	1e+20	0.0	8.5
dem_500_newyork+_360	1e+20	1e+20	0.0	5.7	1e+20	1e+20	0.0	8.7
dem_500_newyork+_361	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.6
dem_500_newyork+_362	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_363	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_364	1e+20	1e+20	0.0	6.8	1e+20	1e+20	0.0	17.3
dem_500_newyork+_365	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	12.3
dem_500_newyork+_366	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_367	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.5
dem_500_newyork+_368	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.7
dem_500_newyork+_369	1e+20	1e+20	0.0	5.2	1e+20	1e+20	0.0	8.7
dem_500_newyork+_370	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	11.4
dem_500_newyork+_371	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_372	1e+20	1e+20	0.0	8.6	1e+20	1e+20	0.0	12.5
dem_500_newyork+_373	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_374	1e+20	1e+20	0.0	6.0	1e+20	1e+20	0.0	10.0
dem_500_newyork+_375	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_376	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_377	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.5
dem_500_newyork+_378	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_379	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_380	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_381	1e+20	1e+20	0.0	6.4	1e+20	1e+20	0.0	10.4
dem_500_newyork+_382	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_383	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_384	1e+20	1e+20	0.0	5.1	1e+20	1e+20	0.0	11.9
dem_500_newyork+_385	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_386	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7

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Table A.46: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_387	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_388	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_389	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_390	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	8.8
dem_500_newyork+_391	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_392	647.4M	646.8M	0.1	5.7	647.4M	646.8M	0.1	11.7
dem_500_newyork+_393	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.9
dem_500_newyork+_394	1e+20	1e+20	0.0	6.5	1e+20	1e+20	0.0	11.9
dem_500_newyork+_395	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_396	1e+20	1e+20	0.0	10.1	1e+20	1e+20	0.0	11.5
dem_500_newyork+_397	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.7
dem_500_newyork+_398	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	12.0
dem_500_newyork+_399	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_400	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_401	1e+20	1e+20	0.0	8.1	1e+20	1e+20	0.0	13.3
dem_500_newyork+_402	1e+20	1e+20	0.0	9.4	1e+20	1e+20	0.0	15.4
dem_500_newyork+_403	1e+20	1e+20	0.0	8.6	1e+20	1e+20	0.0	10.0
dem_500_newyork+_404	1e+20	1e+20	0.0	7.7	1e+20	1e+20	0.0	13.2
dem_500_newyork+_405	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_406	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_407	1e+20	1e+20	0.0	4.7	1e+20	1e+20	0.0	13.1
dem_500_newyork+_408	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_409	1e+20	1e+20	0.0	8.2	1e+20	1e+20	0.0	6.7
dem_500_newyork+_410	1e+20	1e+20	0.0	9.4	1e+20	1e+20	0.0	9.4
dem_500_newyork+_411	1e+20	1e+20	0.0	10.5	1e+20	1e+20	0.0	12.7
dem_500_newyork+_412	1e+20	1e+20	0.0	6.5	1e+20	1e+20	0.0	10.6
dem_500_newyork+_413	1e+20	1e+20	0.0	5.8	1e+20	1e+20	0.0	10.3
dem_500_newyork+_414	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_415	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_416	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_417	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_418	1e+20	1e+20	0.0	11.2	1e+20	1e+20	0.0	7.9
dem_500_newyork+_419	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_420	1e+20	1e+20	0.0	8.9	1e+20	1e+20	0.0	13.3
dem_500_newyork+_421	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_422	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_423	1e+20	1e+20	0.0	12.8	1e+20	1e+20	0.0	11.5
dem_500_newyork+_424	1e+20	1e+20	0.0	8.6	1e+20	1e+20	0.0	9.8
dem_500_newyork+_425	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.5
dem_500_newyork+_426	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.7
dem_500_newyork+_427	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_428	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_429	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_430	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_431	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.5
dem_500_newyork+_432	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_433	1e+20	1e+20	0.0	5.4	1e+20	1e+20	0.0	12.4
dem_500_newyork+_434	1e+20	1e+20	0.0	8.4	1e+20	1e+20	0.0	10.7
dem_500_newyork+_435	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	11.6
dem_500_newyork+_436	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_437	1e+20	1e+20	0.0	10.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_438	1e+20	1e+20	0.0	4.8	1e+20	1e+20	0.0	10.6
dem_500_newyork+_439	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	10.6
dem_500_newyork+_440	1e+20	1e+20	0.0	5.1	1e+20	1e+20	0.0	8.7
dem_500_newyork+_441	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_442	1e+20	1e+20	0.0	10.3	1e+20	1e+20	0.0	9.8
dem_500_newyork+_443	1e+20	1e+20	0.0	8.3	1e+20	1e+20	0.0	7.8
dem_500_newyork+_444	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_445	1e+20	1e+20	0.0	9.9	1e+20	1e+20	0.0	9.0
dem_500_newyork+_446	1e+20	1e+20	0.0	13.1	1e+20	1e+20	0.0	8.5
dem_500_newyork+_447	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_448	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_449	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_450	1e+20	1e+20	0.0	9.0	1e+20	1e+20	0.0	10.9
dem_500_newyork+_451	1e+20	1e+20	0.0	5.5	1e+20	1e+20	0.0	12.5
dem_500_newyork+_452	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.7
dem_500_newyork+_453	1e+20	1e+20	0.0	8.9	1e+20	1e+20	0.0	12.6
dem_500_newyork+_454	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_455	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_456	1e+20	1e+20	0.0	12.2	1e+20	1e+20	0.0	37.8
dem_500_newyork+_457	1e+20	1e+20	0.0	6.9	1e+20	1e+20	0.0	15.7
dem_500_newyork+_458	1e+20	1e+20	0.0	10.9	1e+20	1e+20	0.0	14.9
dem_500_newyork+_459	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_460	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_461	1e+20	1e+20	0.0	8.2	1e+20	1e+20	0.0	8.4
dem_500_newyork+_462	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_463	1e+20	1e+20	0.0	9.5	1e+20	1e+20	0.0	11.3
dem_500_newyork+_464	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6

continued on next page

Table A.46: Comparison of split-pipe models on *New York+* with $\mathcal{B} = 500$ (continued).

	Model D				Model E			
instance	Primal	Dual	Gap	Time	Primal	Dual	Gap	Time
dem_500_newyork+_465	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.8
dem_500_newyork+_466	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_467	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_468	1e+20	1e+20	0.0	13.1	1e+20	1e+20	0.0	10.7
dem_500_newyork+_469	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_470	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_471	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_472	1e+20	1e+20	0.0	0.9	1e+20	1e+20	0.0	0.7
dem_500_newyork+_473	1e+20	1e+20	0.0	8.1	1e+20	1e+20	0.0	9.5
dem_500_newyork+_474	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.8
dem_500_newyork+_475	1e+20	1e+20	0.0	10.6	1e+20	1e+20	0.0	11.1
dem_500_newyork+_476	1e+20	1e+20	0.0	4.4	1e+20	1e+20	0.0	8.7
dem_500_newyork+_477	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_478	1e+20	1e+20	0.0	5.9	1e+20	1e+20	0.0	14.4
dem_500_newyork+_479	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_480	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_481	1e+20	1e+20	0.0	5.6	1e+20	1e+20	0.0	12.7
dem_500_newyork+_482	1e+20	1e+20	0.0	9.9	1e+20	1e+20	0.0	9.9
dem_500_newyork+_483	1e+20	1e+20	0.0	6.1	1e+20	1e+20	0.0	12.9
dem_500_newyork+_484	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6
dem_500_newyork+_485	1e+20	1e+20	0.0	5.0	1e+20	1e+20	0.0	7.9
dem_500_newyork+_486	1e+20	1e+20	0.0	7.2	1e+20	1e+20	0.0	14.9
dem_500_newyork+_487	1e+20	1e+20	0.0	6.6	1e+20	1e+20	0.0	10.2
dem_500_newyork+_488	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_489	1e+20	1e+20	0.0	0.5	1e+20	1e+20	0.0	0.6
dem_500_newyork+_490	1e+20	1e+20	0.0	7.9	1e+20	1e+20	0.0	14.9
dem_500_newyork+_491	1e+20	1e+20	0.0	7.1	1e+20	1e+20	0.0	10.3
dem_500_newyork+_492	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_493	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_494	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.7
dem_500_newyork+_495	1e+20	1e+20	0.0	8.5	1e+20	1e+20	0.0	17.0
dem_500_newyork+_496	1e+20	1e+20	0.0	6.5	1e+20	1e+20	0.0	10.7
dem_500_newyork+_497	1e+20	1e+20	0.0	5.3	1e+20	1e+20	0.0	7.5
dem_500_newyork+_498	1e+20	1e+20	0.0	0.6	1e+20	1e+20	0.0	0.6
dem_500_newyork+_499	1e+20	1e+20	0.0	0.8	1e+20	1e+20	0.0	0.6
dem_500_newyork+_500	1e+20	1e+20	0.0	0.7	1e+20	1e+20	0.0	0.6

A.3 Aggregated Results of Discrete Models

The tables are structured as follows: each column contains the results of 500 different instances with the only variable parameter being the total network demand $\mathcal{B}$. The computational results within a column are grouped into different categories: In the section *All*, we provide a summary of the results of all 500 instances. The section *All opt* reports data for all instances for which all models under comparison have found an optimal solution. In the section *Only opt by*, we compare the models with respect to the instances that they alone were able to solve to global optimality.

In all cases, we report computational time by means of the shifted geometric mean (sgm) and the arithmetic mean (amm). For instances that are solved to optimality by all models, we additionally present the sgm of the number of branch-and-bound nodes.

	Data set	$\mathcal{B} = 100$			$\mathcal{B} = 200$			$\mathcal{B} = 500$			$\mathcal{B}$ =1,000		
	Models	A	B	C	A	B	C	A	B	C	A	B	C
All	time (amm)	80.4	69.7	446.9	552.9	552.2	1,652.4	6,093.5	5,962.8	9,187.8	3,480.6	3,307.6	4,319.1
	time (sgm)	9.1	39.1	20.3	41.4	159.9	106.7	1,424.6	2,380.3	3,691.8	88.4	151.0	113.2
All opt	# opt		493			464			209			48	
	time (amm)	29.8	64.4	248.7	136.8	336.0	741.2	582.5	1,632.7	2,548.0	784.9	2,783.5	3,602.1
	time (sgm)	7.8	37.6	17.7	27.0	134.3	71.4	155.4	634.0	673.9	301.5	1,248.8	1,055.0
	nodes (sgm)	293	305	564	2,047	2,140	5,015	33,296	26,614	151,517	99,209	60,268	364,978
Only opt by	# opt	0	1	0	1	8	0	45	73	0	9	33	0
	time (amm)	14,400.0	363.7	14,400.0	13,034.9	3,784.9	14,400.0	11,433.0	8,876.4	14,400.0	12,937.6	8,447.2	14,400.0
	time (sgm)	14,400.0	363.7	14,400.0	11,638.7	1,880.6	14,400.0	9,591.8	6,634.3	14,400.0	12,117.4	7,048.4	14,400.0

Table A.47: Aggregated results of discrete models on *Belgium* with different demand scenarios. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

	Data set	$\mathcal{B} = 50$			$\mathcal{B} = 100$			$\mathcal{B} = 500$			$\mathcal{B}$ =1,000		
	Models	A	B	C	A	B	C	A	B	C	A	B	C
All	time (amm)	9.9	119.7	203.4	7,781.9	9,681.5	14,245.3	14,400.0	14,400.0	14,400.0	10,264.6	9,932.3	10,391.5
	time (sgm)	4.0	17.2	12.8	3,923.0	7,777.7	14,068.8	14,400.0	14,400.0	14,400.0	1,794.8	3,176.8	2,085.0
All opt	# opt		496			9			0			0	
	time (amm)	6.9	76.2	88.9	55.7	1,168.0	5,802.7	-	-	-	-	-	-
	time (sgm)	3.7	16.1	11.7	45.0	1,024.7	3,949.2	-	-	-	-	-	-
	nodes (sgm)	60	159	284	2,689	14,905	415,265	-	-	-	-	-	-
Only opt by	# opt	0	0	0	45	26	0	0	0	0	0	2	0
	time (amm)	-	-	-	9,814.8	12,517.3	14,400.0	-	-	-	14,400.0	8,849.7	14,400.0
	time (sgm)	-	-	-	7,990.3	12,100.7	14,400.0	-	-	-	14,400.0	8,815.3	14,400.0

Table A.48: Aggregated results of discrete models on *Gaslib-40* with different demand scenarios. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

	Data set	$\mathcal{B} = 50$			$\mathcal{B} = 100$			$\mathcal{B} = 150$			$\mathcal{B} = 500$		
	Models	A	B	C	A	B	C	A	B	C	A	B	C
All	time (amm)	289.6	14,400.0	238.8	8,050.5	14,400.0	8,018.4	14,313.7	14,400.0	14,311.2	14,400.0	14,400.0	14,363.7
	time (sgm)	3.4	14,400.0	9.5	677.8	14,400.0	907.2	13,846.9	14,400.0	13,858.7	14,400.0	14,400.0	14,336.1
All opt	# opt		0			0			0			0	
	time (amm)	-	-	-	-	-	-	-	-	-	-	-	-
	time (sgm)	-	-	-	-	-	-	-	-	-	-	-	-
	nodes (sgm)	-	-	-	-	-	-	-	-	-	-	-	-
Only opt by	# opt	0	0	4	6	0	7	0		0	0		0
	time (amm)	14,400.0	-	6,831.1	11,576.8		10,389.8	-	-	-	-	-	-
	time (sgm)	14,400.0	-	5,781.5	10,478.9		8,892.0	-	-	-	-	-	-

Table A.49: Aggregated results of discrete models on *Gaslib-135* with different demand scenarios. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

	Data set	+0 Arcs			+2 Arcs			+4 Arcs		
	Models	A	B	C	A	B	C	A	B	C
All	time (amm)	80.4	69.7	446.9	4,688.8	4,726.2	9,094.5	8,489.5	7,306.5	11,352.2
	time (sgm)	9.1	39.1	20.3	1,368.6	2,069.0	4,766.3	3,492.2	4,329.8	6,399.7
All opt	# opt		493			221			117	
	time (amm)	29.8	64.4	248.7	545.8	859.6	2,632.2	753.0	1,440.6	2,148.4
	time (sgm)	7.8	37.6	17.7	249.5	585.5	1,213.8	258.0	787.3	495.3
	nodes (sgm)	294	305	564	33,817	8,509	146,470	20,679	7,216	25,610
Only opt by	# opt	0	1	0	22	39	0	24	129	0
	time (amm)	14,400.0	363.7	14,400.0	11,918.7	9,747.6	14,400.0	13,283.2	8,441.5	14,400.0
	time (sgm)	14,400.0	363.7	14,400.0	10,717.3	8,217.9	14,400.0	12,488.9	7,165.3	14,400.0

	Data set	+6 Arcs			+8 Arcs			+10 Arcs		
	Models	A	B	C	A	B	C	A	B	C
All	time (amm)	7,018.2	9,003.2	9,463.2	6,719.1	9,817.5	8,613.4	5,123.6	9,969.2	7,151.8
	time (sgm)	2,327.2	5,812.4	4,357.7	2,489.2	7,343.8	3,805.4	1,443.4	6,824.2	2,262.1
All opt	# opt		183			192			207	
	time (amm)	809.8	3,632.4	2,276.1	1,381.0	4,934.4	2,254.0	969.5	4,870.5	1,899.4
	time (sgm)	278.2	1,879.5	663.9	489.2	3,291.6	814.1	326.4	2,712.0	510.2
	nodes (sgm)	18,061	16,784	33,299	30,406	27,272	37,800	19,120	20,619	15,523
Only opt by	# opt	48	36	1	62	25	6	67	3	8
	time (amm)	9,907.2	12,030.4	14,358.8	8,811.9	12,824.1	13,859.5	5,925.5	14,154.0	13,321.2
	time (sgm)	8,101.0	11,101.1	14,352.9	6,849.8	12,245.5	12,989.9	3,866.6	13,979.7	11,744.4

Table A.50: Aggregated results of discrete models on *Circuit rank*. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

	Data set	$\mathcal{B} = 100$			$\mathcal{B} = 200$			$\mathcal{B} = 500$		
	Models	A	B	C	A	B	C	A	B	C
All	time (amm)	14,400.0	12,781.4	14,400.0	13,227.5	1,768.0	11,639.1	0.0	0.3	0.0
	time (sgm)	14,400.0	12,183.0	14,400.0	8,759.6	1,000.0	7,465.9	0.0	0.3	0.0
All opt	# opt	0			8			0		
	time (amm)	-	-	-	2,917.1	734.3	1,470.1	-	-	-
	time (sgm)	-	-	-	563.3	537.6	881.1	-	-	-
	nodes (sgm)	-	-	-	126,220	8,918	109,148	-	-	-
Only opt by	# opt	0	142	0	0	355	0	0	0	0
	time (amm)	14,400.0	8,903.2	14,400.0	14,400.0	2,190.2	14,400.0	-	-	-
	time (sgm)	14,400.0	8,185.9	14,400.0	14,400.0	1,570.5	14,400.0	-	-	-

Table A.51: Aggregated results of discrete models on *New York* with different demand scenarios. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

	Data set	$\mathcal{B} = 100$			$\mathcal{B} = 200$			$\mathcal{B} = 500$		
	Models	A	B	C	A	B	C	A	B	C
All	time (amm)	13,679.5	14,051.0	14,362.1	14,400.0	14,338.6	14,400.0	2170.1	709.2	2838.1
	time (sgm)	13,015.3	13,937.2	14,349.4	14,400.0	14,323.7	14,400.0	46.9	362.3	162.0
All opt	# opt	0			0			0		
	time (amm)	-	-	-	-	-	-	-	-	-
	time (sgm)	-	-	-	-	-	-	-	-	-
	nodes (sgm)	-	-	-	-	-	-	-	-	-
Only opt by	# opt	42	36	0	0	12	0	0	2	0
	time (amm)	10,694.8	12,308.0	14,400.0	14,400.0	11,833.0	14,400.0	14,400.0	7,051.2	14,400.0
	time (sgm)	8,858.3	11,831.8	14,400.0	14,400.0	11,535.0	14,400.0	14,400.0	6,939.2	14,400.0

Table A.52: Aggregated results of discrete models on *New York+*. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

A.4 Aggregated Results of Split-Pipe Models

The tables are structured as follows: each column contains the results of 500 different instances with the only variable parameter being the total network demand $\mathcal{B}$. The computational results within a column are grouped into different categories: In the section *All*, we provide a summary of the results of all 500 instances. The section *All opt* reports data for all instances for which all models under comparison have found an optimal solution. In the section *Only opt by*, we compare the models with respect to the instances that they alone were able to solve to global optimality.

In all cases, we report computational time by means of the shifted geometric mean (sgm) and the arithmetic mean (amm). For instances that are solved to optimality by all models, we additionally present the sgm of the number of branch-and-bound nodes.

	Data set	$\mathcal{B} = 100$		$\mathcal{B} = 200$		$\mathcal{B} = 500$		$\mathcal{B} = 1{,}000$	
	Models	D	E	D	E	D	E	D	E
All	time (amm)	1.0	0.7	256.2	169.9	5,850.9	5,891.4	3,655.4	3,533.2
All	time (sgm)	1.0	0.6	7.0	4.8	676.1	640.9	80.3	79.1
All opt	# opt	500		492		269		60	
All opt	time (amm)	1.0	0.7	84.0	42.0	841.4	951.2	1903.4	1657.4
All opt	time (sgm)	1.0	0.6	5.5	3.8	80.0	78.3	205.0	196.8
All opt	nodes (sgm)	24	21	240	197	16,381	15,153	50,809	53,529
Only opt by	# opt	0	0	2	4	49	46	17	22
Only opt by	time (amm)	-	-	9,662.2	5,916.3	8,554.7	8,457.3	10,335.6	9,148.2
Only opt by	time (sgm)	-	-	3,106.3	415.1	4,037.7	3,234.6	6,748.7	6,067.2

Table A.53: Aggregated results of split-pipe models on *Belgium* with different demand scenarios. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

	Data set	$\mathcal{B} = 50$		$\mathcal{B} = 100$		$\mathcal{B} = 500$		$\mathcal{B} = 1{,}000$	
	Models	D	E	D	E	D	E	D	E
All	time (amm)	32.4	3.2	4,045.7	3,071.0	9,273.6	9,002.8	6,154.6	5,477.9
All	time (sgm)	3.5	1.4	215.7	164.3	4,832.4	4,452.5	756.7	687.8
All opt	# opt	499		354		214		186	
All opt	time (amm)	3.6	3.2	375.3	341.7	3,291.6	2,738.2	5,434.6	4,247.7
All opt	time (sgm)	3.3	1.4	40.4	37.1	1,300.6	1,050.9	2,842.7	2,226.8
All opt	nodes (sgm)	42	16	4,270	4,040	157,062	33,984	338,695	285,031
Only opt by	# opt	0	1	17	59	23	30	20	57
Only opt by	time (amm)	14,400.0	4.5	11,605.1	5,349.1	10,890.4	10,570.0	12,434.0	10,905.4
Only opt by	time (sgm)	14,400.0	4.5	5,266.3	1,317.9	7,837.6	8,512.7	11,240.0	10,117.4

Table A.54: Aggregated results of split-pipe models on *GasLib-40* with different demand scenarios. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

	Data set	$\mathcal{B} = 50$		$\mathcal{B} = 100$		$\mathcal{B} = 150$		$\mathcal{B} = 500$	
	Models	D	E	D	E	D	E	D	E
All	time (amm)	5,817.6	365.8	11,121.0	8,017.4	14,344.4	14,313.9	14,400.0	14,400.0
	time (sgm)	2,257.4	13.8	6,461.6	841.6	14,209.3	13,879.2	14,400.0	14,400.0
All opt	# opt	392		149		2		0	
	time (amm)	3,489.4	52.4	3,477.0	30.7	505.7	75.6	-	-
	time (sgm)	1,368.4	10.6	983.6	13.3	505.7	34.5	-	-
	nodes (sgm)	51,661	114	30,822	157	21,772	685	-	-
Only one opt by	# opt 1	97	1	75	0	1	0	0	
	time (amm)	14,254.4	187.5	14,242.4	580.5	14,400.0	5.3	-	-
	time (sgm)	13,736.8	12.2	14,066.7	25.3	14,400.0	5.3	-	-

Table A.55: Aggregated results of split-pipe models on *GasLib-135* with different demand scenarios. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

	Data set	+0 Arcs		+2 Arcs		+4 Arcs		+6 Arcs		+8 Arcs		+10 Arcs	
	Models	D	E	D	E	D	E	D	E	D	E	D	E
All	time (amm)	1.0	0.7	10,188.8	7,963.9	12,928.2	10,819.0	13,766.4	9,781.5	14,087.2	9,346.4	13,634.8	6,224.3
	time (sgm)	1.0	0.6	3,192.4	1,173.1	9,611.2	3,952.4	11,843.6	2,790.4	12,944.1	2,675.5	11,243.6	844.2
All opt	# opt	500		101		35		22		12		29	
	time (amm)	1.0	0.7	761.7	477.2	1,087.5	1,015.0	2,047.6	246.8	1,364.9	172.1	1,275.8	23.4
	time (sgm)	1.0	0.6	91.2	34.4	258.5	37.2	237.8	14.5	160.3	32.0	194.0	10.9
	nodes (sgm)	24	21	21,956	4,642	71,439	4,495	29,888	1,023	19,818	2,046	17,299	582
Only opt by	# opt	0	0	54	134	23	101	4	148	0	182	1	279
	time (amm)	-	-	10,526.8	4,762.4	12,222.9	3,738.5	14,103.6	1,256.1	14,400.0	1,454.7	14,392.9	1,289.6
	time (sgm)	-	-	3,777.5	407.2	8,689.6	387.3	13,637.5	155.7	14,400.0	199.6	14,392.4	172.6

Table A.56: Aggregated results of split-pipe models on *Circuit rank*. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

	Data set	*New York*						*New York+*					
		$\mathcal{B} = 100$		$\mathcal{B} = 200$		$\mathcal{B} = 500$		$\mathcal{B} = 100$		$\mathcal{B} = 200$		$\mathcal{B} = 500$	
	Models	D	E	D	E	D	E	D	E	D	E	D	E
All	time (amm)	3.1	2.9	1.9	2.9	0.0	0.0	3,332.9	1,723.1	2,001.7	1,557.6	4.7	6.0
	time (sgm)	3.0	2.8	1.8	2.8	0.0	0.0	321.5	186.9	390.6	275.2	3.9	4.9
All opt	# opt	500		421		0		386		424		4	
	time (amm)	3.1	2.9	1.3	2.4	-	-	1288.4	513.7	629.4	418.4	34.6	40.4
	time (sgm)	3.0	2.8	1.3	2.4	-	-	157.5	113.6	250.3	185.9	25.6	31.8
	nodes (sgm)	698	677	182	176	-	-	33,885	24,442	51,672	36,524	4,881	4,497
Only opt by	# opt	0	0	0	0	0	0	36	72	28	37	0	0
	time (amm)	-	-	-	-	-	-	1,276.7	811.8	1,527.9	1,073.8	-	-
	time (sgm)	-	-	-	-	-	-	131.2	185.3	578.0	328.2	-	-

Table A.57: Aggregated results of split-pipe models on *New York* and *New York+*. The runtime is denoted in seconds for the shifted geometric mean (sgm) and the arithmetic mean (amm).

Appendix B

Tight Convex Relaxations for the Expansion Planning Problem

B.1 Aggregated Results of the ROOTGAP Experiment

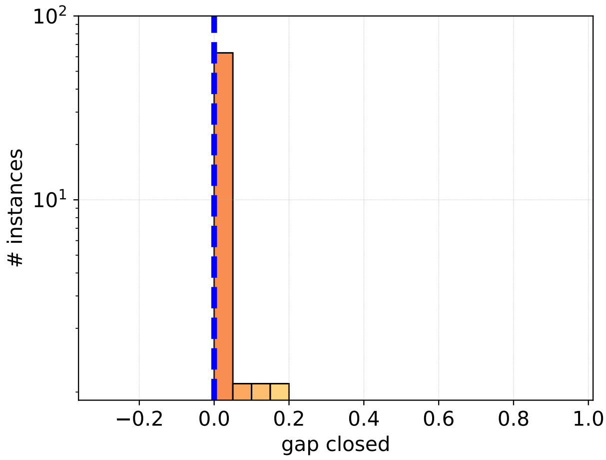
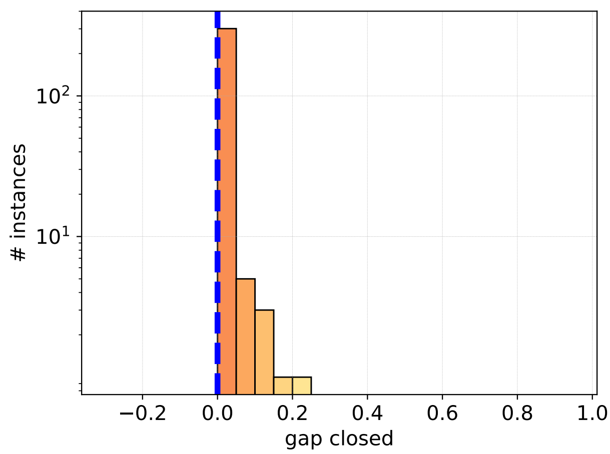
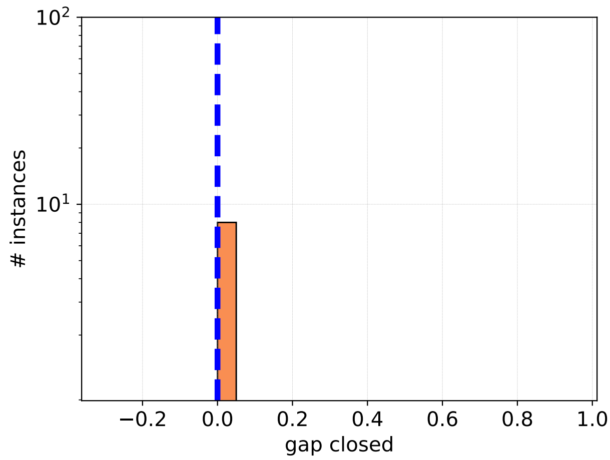


Figure B.1: Results for the *Gap closed* improvement of the ROOTGAP experiment on the *Belgium* test set, where each bar on the x-axis represents the relative improvement with the corresponding number of instances on the y-axis in logarithmic scale. Positive values on the x-axis represent improved dual bounds with *Enabled* and negative values indicate better dual bounds using *Disabled*. An instance having a *gap closed* value of 0.5 (on the x-axis) implies that the gap is closed by a factor of 2 using *Enabled* over *Disabled*.



(a) *Gap closed* improvement of the `ROOTGAP` experiment on *GasLib-40*



(b) *Gap closed* improvement of the `ROOTGAP` experiment on *Circuit rank*

Figure B.2: Results for the *Gap closed* improvement of the `ROOTGAP` experiment, where each bar on the x-axis represents the relative improvement with the corresponding number of instances on the y-axis in logarithmic scale. Positive values on the x-axis represent improved dual bounds with *Enabled* and negative values indicate better dual bounds using *Disabled*. An instance having a *gap closed* value of 0.5 (on the x-axis) implies that the gap is closed by a factor of 2 using *Enabled* over *Disabled*.

	# instances	gap closed (%)
	1,694	0.11
> 0% `change`	171	1.13
> 0% `better`	171	1.13
> 0% `worse`	0	–
> 2% `change`	19	4.28
> 2% `better`	19	4.28
> 2% `worse`	0	–
> 10% `change`	2	14.41
> 10% `better`	2	14.41
> 10% `worse`	0	–

Table B.1: Aggregated results of the `ROOTGAP` experiment on *Belgium*. The *gap closed* values are depicted on average for `ALL` instances, and for those instances where *Enabled* yields an improvement or deterioration of more than 0%, 2%, and 10% compared to *Disabled*, called `better` or `worse`. The category `change` corresponds to the union of `better` and `worse` instances.

	# instances	gap closed (%)
	844	0.93
> 0% `change`	518	1.51
> 0% `better`	518	1.51
> 0% `worse`	0	–
> 2% `change`	101	3.37
> 2% `better`	101	3.37
> 2% `worse`	0	–
> 10% `change`	5	15.53
> 10% `better`	5	15.53
> 10% `worse`	0	–

Table B.2: Aggregated results of the `ROOTGAP` experiment on *GasLib-40*. The *gap closed* values are depicted on average for `ALL` instances, and for those instances where *Enabled* yields an improvement or deterioration of more than 0%, 2%, and 10% compared to *Disabled*, called `better` or `worse`. The category `change` corresponds to the union of `better` and `worse` instances.

	# instances	gap closed (%)
	536	0.04
$> 0\%$ `change`	31	0.75
$> 0\%$ `better`	31	0.75
$> 0\%$ `worse`	0	–
$> 2\%$ `change`	5	2.88
$> 2\%$ `better`	5	2.88
$> 2\%$ `worse`	0	–
$> 10\%$ `change`	0	–
$> 10\%$ `better`	0	–
$> 10\%$ `worse`	0	–

Table B.3: Aggregated results of the `ROOTGAP` experiment on *Circuit rank*. The *gap closed* values are depicted on average for `ALL` instances, and for those instances where *Enabled* yields an improvement or deterioration of more than $0\%, 2\%$, and 10% compared to *Disabled*, called `better` or `worse`. The category `change` corresponds to the union of `better` and `worse` instances.

B.2 Aggregated Results of the `TREE` Experiment

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
`ALL`	500	500	500	0.99	-
`ALL OPT`	500	500	500	0.99	0.92
[1, `tlim`]	500	500	500	0.99	-
[10, `tlim`]	97	97	97	1.05	-
[100, `tlim`]	3	3	3	0.14	-
[1000, `tlim`]	0	-	-	-	-

Table B.4: Aggregated results of the `TREE` experiment on *Belgium* for a demand of $\mathcal{B} = 100$. The column “relative” reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

	# instances	*Enabled* # solved	*Disabled* # solved	time (relative)	B&B nodes (relative)
`ALL`	500	498	498	1.48	-
`ALL OPT`	497	497	497	1.49	1.52
[1, `tlim`]	499	498	498	1.49	-
[10, `tlim`]	328	327	327	1.69	-
[100, `tlim`]	71	70	70	3.02	-
[1000, `tlim`]	8	7	7	7.35	-

Table B.5: Aggregated results of the `TREE` experiment on *Belgium* for a demand of $\mathcal{B} = 200$. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

	# instances	*Enabled* # solved	*Disabled* # solved	time (relative)	B&B nodes (relative)
`ALL`	500	460	448	1.22	-
`ALL OPT`	422	422	442	1.18	1.33
[1, `tlim`]	481	455	443	1.24	-
[10, `tlim`]	462	436	424	1.24	-
[100, `tlim`]	333	307	295	1.31	-
[1000, `tlim`]	141	115	103	1.47	-

Table B.6: Aggregated results of the `TREE` experiment on *Belgium* for a demand of $\mathcal{B} = 500$. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

	# instances	*Enabled* # solved	*Disabled* # solved	time (relative)	B&B nodes (relative)
`ALL`	500	447	420	1.14	-
`ALL OPT`	413	413	413	1.15	1.18
[1, tlim]	155	148	121	1.42	-
[10, `tlim`]	152	145	118	1.43	-
[100, `tlim`]	135	128	101	1.46	-
[1000, `tlim`]	88	81	54	1.54	-

Table B.7: Aggregated results of the `TREE` experiment on *Belgium* for a demand of $\mathcal{B} = 1000$. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
ALL	500	497	497	1.16	-
ALL OPT	495	495	495	1.17	1.33
[1, tlim]	499	497	497	1.15	-
[10, tlim]	499	497	497	1.15	-
[100, tlim]	151	149	149	1.48	-
[1000, tlim]	8	6	6	1.18	-

Table B.8: Aggregated results of the TREE experiment on *GasLib-40* for a demand of $\mathcal{B} = 50$. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
ALL	500	448	429	1.17	-
ALL OPT	393	393	393	1.13	1.28
[1, tlim]	484	448	429	1.17	-
[10, tlim]	484	448	429	1.17	-
[100, tlim]	475	439	420	1.17	-
[1000, tlim]	203	167	148	1.31	-

Table B.9: Aggregated results of the TREE experiment on *GasLib-40* for a demand of $\mathcal{B} = 100$. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
ALL	500	45	45	1.0	-
ALL OPT	35	35	35	0.88	0.96
[1, tlim]	55	45	45	0.97	-
[10, tlim]	55	45	45	0.97	-
[100, tlim]	55	45	45	0.97	-
[1000, tlim]	54	44	44	0.97	-

Table B.10: Aggregated results of the TREE experiment on *GasLib-40* for a demand of $\mathcal{B} = 500$. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
ALL	500	191	191	1.01	-
ALL OPT	185	185	185	1.07	1.12
[1, tlim]	108	102	102	1.06	-
[10, tlim]	45	39	39	1.11	-
[100, tlim]	38	32	32	1.15	-
[1000, tlim]	28	22	22	0.98	-

Table B.11: Aggregated results of the TREE experiment on *GasLib-40* for a demand of $\mathcal{B} = 1{,}000$. The column “relative” reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
ALL	500	436	418	1.34	-
ALL OPT	369	369	369	1.24	1.27
[1, tlim]	485	436	418	1.35	-
[10, tlim]	485	436	418	1.35	-
[100, tlim]	333	284	266	1.49	-
[1000, tlim]	158	109	91	1.74	-

Table B.12: Aggregated results of the TREE experiment on *Circuit rank* with 2 additional arcs. The column “relative” reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
ALL	500	332	318	1.05	-
ALL OPT	244	244	244	1.05	1.02
[1, tlim]	406	332	318	1.06	-
[10, tlim]	406	332	318	1.06	-
[100, tlim]	406	332	318	1.06	-
[1000, tlim]	341	267	253	1.06	-

Table B.13: Aggregated results of the TREE experiment on *Circuit rank* with 4 additional arcs. The column “relative” reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
`ALL`	500	169	149	1.04	-
`ALL OPT`	103	103	103	1.03	1.05
[1, `tlim`]	215	169	149	1.10	-
[10, `tlim`]	215	169	149	1.10	-
[100, `tlim`]	215	169	149	1.10	-
[1000, `tlim`]	191	145	125	1.08	-

Table B.14: Aggregated results of the `TREE` experiment on *Circuit rank* with 6 additional arcs. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
`ALL`	500	61	60	1.01	-
`ALL OPT`	35	35	35	1.04	1.09
[1, `tlim`]	86	61	60	1.03	-
[10, `tlim`]	86	61	60	1.03	-
[100, `tlim`]	86	61	60	1.03	-
[1000, `tlim`]	85	60	59	1.03	-

Table B.15: Aggregated results of the `TREE` experiment on *Circuit rank* with 8 additional arcs. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

		Enabled	*Disabled*		
	# instances	# solved	# solved	time	B&B nodes
				relative	
`ALL`	500	43	28	1.0	-
`ALL OPT`	15	15	15	0.96	0.99
[1, `tlim`]	56	43	28	1.03	-
[10, `tlim`]	56	43	43	1.03	-
[100, `tlim`]	56	43	43	1.03	-
[1000, `tlim`]	56	43	43	1.03	-

Table B.16: Aggregated results of the `TREE` experiment on *Circuit rank* with 10 additional arcs. The column "relative" reports the change of solving time and branch-and-bound nodes relative to *Enabled*.

B.3 Detailed Results of the ROOTGAP and TREE Experiments

Table B.17: Detailed results for the ROOTGAP and TREE experiments on the *Belgium* test set, as summarized in Figure 4.19 and Tables 4.1 – 4.2. For the ROOTGAP experiment, the table lists the gap closed (GC) improvement. A hyphen indicates that the instance is either detected to be infeasible, no primal solution is known, or the dual bound is zero within a tolerance of $\epsilon = 10^{-6}$ for *Enabled* and *Disabled*. For the TREE experiment, the table shows the optimality gap in %, number of B&B-nodes and solving time in seconds for *Enabled* and *Disabled*.

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_belg_orig_1	0.0	0.0	408	4.3	0.0	299	3.6
dem_100_belg_orig_2	0.0	0.0	265	3.5	0.0	512	4.2
dem_100_belg_orig_3	0.0	0.0	175	4.2	0.0	289	4.1
dem_100_belg_orig_4	0.0	0.0	1489	7.1	0.0	1221	7.6
dem_100_belg_orig_5	0.0	0.0	294	5.2	0.0	453	5.5
dem_100_belg_orig_6	0.0	0.0	352	3.8	0.0	566	4.1
dem_100_belg_orig_7	0.0	0.0	770	6.0	0.0	299	4.9
dem_100_belg_orig_8	0.0	0.0	837	5.1	0.0	5460	14.6
dem_100_belg_orig_9	0.0	0.0	4052	16.8	0.0	4701	20.9
dem_100_belg_orig_10	0.0	0.0	308	4.4	0.0	191	3.4
dem_100_belg_orig_11	0.0	0.0	7437	25.4	0.0	2531	16.4
dem_100_belg_orig_12	0.0	0.0	461	5.8	0.0	3314	13.6
dem_100_belg_orig_13	0.0	0.0	326	4.7	0.0	275	4.1
dem_100_belg_orig_14	0.0	0.0	456	4.9	0.0	500	4.8
dem_100_belg_orig_15	0.0	0.0	4841	18.7	0.0	4614	18.0
dem_100_belg_orig_16	0.0	0.0	2022	8.5	0.0	3424	13.1
dem_100_belg_orig_17	0.0	0.0	1299	6.2	0.0	324	3.7
dem_100_belg_orig_18	0.0	0.0	532	4.4	0.0	324	4.1
dem_100_belg_orig_19	0.0	0.0	731	5.0	0.0	185	3.2
dem_100_belg_orig_20	0.0	0.0	463	4.1	0.0	359	3.5
dem_100_belg_orig_21	0.0	0.0	813	4.7	0.0	1697	7.2
dem_100_belg_orig_22	0.0	0.0	266	5.2	0.0	555	6.0
dem_100_belg_orig_23	0.01106	0.0	1383	6.9	0.0	275	3.8
dem_100_belg_orig_24	0.0	0.0	685	4.7	0.0	284	3.6
dem_100_belg_orig_25	0.0	0.0	445	4.8	0.0	684	5.2
dem_100_belg_orig_26	0.0	0.0	516	4.4	0.0	1918	9.5
dem_100_belg_orig_27	0.0	0.0	634	4.6	0.0	538	4.9
dem_100_belg_orig_28	0.0	0.0	454	4.1	0.0	4949	19.1
dem_100_belg_orig_29	0.0	0.0	292	4.8	0.0	330	4.8
dem_100_belg_orig_30	0.0	0.0	575	4.9	0.0	360	4.6
dem_100_belg_orig_31	0.00892	0.0	239	4.9	0.0	208	5.0
dem_100_belg_orig_32	0.00696	0.0	551	5.6	0.0	471	5.5
dem_100_belg_orig_33	0.0	0.0	309	4.5	0.0	283	4.8
dem_100_belg_orig_34	0.0	0.0	835	5.6	0.0	721	6.0
dem_100_belg_orig_35	0.0	0.0	521	4.8	0.0	321	4.2
dem_100_belg_orig_36	0.0	0.0	614	5.2	0.0	347	4.9
dem_100_belg_orig_37	0.0	0.0	436	4.5	0.0	270	4.2
dem_100_belg_orig_38	0.0	0.0	2359	12.2	0.0	2666	15.1
dem_100_belg_orig_39	0.0	0.0	1048	5.2	0.0	297	3.8
dem_100_belg_orig_40	0.0	0.0	2207	8.7	0.0	408	5.3
dem_100_belg_orig_41	0.0	0.0	174	5.5	0.0	430	5.7
dem_100_belg_orig_42	0.0	0.0	857	5.5	0.0	468	4.4
dem_100_belg_orig_43	0.0	0.0	360	4.1	0.0	412	4.1
dem_100_belg_orig_44	0.0	0.0	2573	12.4	0.0	5852	20.5
dem_100_belg_orig_45	0.0	0.0	402	4.2	0.0	706	4.7
dem_100_belg_orig_46	0.0	0.0	294	3.8	0.0	413	4.3
dem_100_belg_orig_47	0.0	0.0	308	4.7	0.0	395	5.0
dem_100_belg_orig_48	0.0	0.0	528	4.5	0.0	277	4.3
dem_100_belg_orig_49	0.0	0.0	373	4.0	0.0	388	4.3
dem_100_belg_orig_50	0.0	0.0	480	4.4	0.0	221	3.6
dem_100_belg_orig_51	0.0	0.0	161	4.4	0.0	158	4.5
dem_100_belg_orig_52	0.0	0.0	272	4.0	0.0	408	4.1
dem_100_belg_orig_53	0.0	0.0	549	4.7	0.0	329	4.2
dem_100_belg_orig_54	0.0	0.0	263	4.4	0.0	301	4.7
dem_100_belg_orig_55	0.0	0.0	626	5.3	0.0	488	5.0
dem_100_belg_orig_56	0.0	0.0	379	4.7	0.0	288	4.6
dem_100_belg_orig_57	0.0	0.0	471	5.0	0.0	233	4.9
dem_100_belg_orig_58	0.0	0.0	211	3.7	0.0	709	4.8
dem_100_belg_orig_59	0.0	0.0	271	4.2	0.0	180	3.9
dem_100_belg_orig_60	0.0	0.0	879	4.7	0.0	235	3.6

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_belg_orig_61	0.0	0.0	334	3.7	0.0	413	3.8
dem_100_belg_orig_62	0.0	0.0	392	3.7	0.0	233	3.2
dem_100_belg_orig_63	0.0	0.0	916	6.6	0.0	185	4.9
dem_100_belg_orig_64	0.0	0.0	187	5.9	0.0	226	5.7
dem_100_belg_orig_65	0.0	0.0	397	4.3	0.0	623	4.4
dem_100_belg_orig_66	0.0	0.0	745	5.3	0.0	661	4.7
dem_100_belg_orig_67	0.0	0.0	436	6.2	0.0	313	4.2
dem_100_belg_orig_68	0.0	0.0	1206	9.1	0.0	404	4.8
dem_100_belg_orig_69	0.0	0.0	1818	6.8	0.0	1979	8.9
dem_100_belg_orig_70	0.0	0.0	446	4.5	0.0	345	4.4
dem_100_belg_orig_71	0.0	0.0	4212	17.4	0.0	303	4.0
dem_100_belg_orig_72	0.0	0.0	264	4.6	0.0	470	4.6
dem_100_belg_orig_73	0.0	0.0	471	4.6	0.0	1046	6.1
dem_100_belg_orig_74	0.0	0.0	2595	11.2	0.0	8013	28.4
dem_100_belg_orig_75	0.0	0.0	1681	9.2	0.0	686	6.5
dem_100_belg_orig_76	0.0	0.0	603	5.7	0.0	1880	8.9
dem_100_belg_orig_77	0.0	0.0	1943	10.1	0.0	4890	16.5
dem_100_belg_orig_78	0.0	0.0	300	4.7	0.0	708	5.5
dem_100_belg_orig_79	0.0	0.0	423	3.8	0.0	455	4.2
dem_100_belg_orig_80	0.0	0.0	520	5.5	0.0	434	5.3
dem_100_belg_orig_81	0.0	0.0	940	6.1	0.0	6829	28.0
dem_100_belg_orig_82	0.0	0.0	307	5.4	0.0	273	5.3
dem_100_belg_orig_83	0.0	0.0	511	3.8	0.0	656	4.3
dem_100_belg_orig_84	0.0	0.0	250	4.6	0.0	297	4.6
dem_100_belg_orig_85	0.0	0.0	7900	21.9	0.0	8885	25.1
dem_100_belg_orig_86	0.0	0.0	731	5.8	0.0	442	4.4
dem_100_belg_orig_87	0.0	0.0	2276	9.9	0.0	292	4.3
dem_100_belg_orig_88	0.0	0.0	458	4.3	0.0	308	3.6
dem_100_belg_orig_89	0.0	0.0	629	5.4	0.0	297	4.4
dem_100_belg_orig_90	0.0	0.0	645	4.5	0.0	903	5.0
dem_100_belg_orig_91	0.0	0.0	214	4.2	0.0	695	5.7
dem_100_belg_orig_92	0.0	0.0	223	4.1	0.0	318	4.6
dem_100_belg_orig_93	0.0	0.0	351	4.2	0.0	320	4.5
dem_100_belg_orig_94	0.0	0.0	4831	13.7	0.0	657	5.0
dem_100_belg_orig_95	0.0	0.0	1001	5.5	0.0	262	4.0
dem_100_belg_orig_96	0.0	0.0	235	3.7	0.0	354	4.3
dem_100_belg_orig_97	0.0	0.0	258	4.1	0.0	263	4.4
dem_100_belg_orig_98	0.0	0.0	552	6.2	0.0	727	6.6
dem_100_belg_orig_99	0.0	0.0	994	5.7	0.0	1734	7.8
dem_100_belg_orig_100	0.0	0.0	946	5.4	0.0	26k	95.9
dem_100_belg_orig_101	0.0	0.0	4780	16.0	0.0	1919	9.2
dem_100_belg_orig_102	0.0	0.0	618	6.1	0.0	1846	8.3
dem_100_belg_orig_103	0.0	0.0	357	4.3	0.0	787	5.2
dem_100_belg_orig_104	0.0	0.0	1176	7.6	0.0	658	6.2
dem_100_belg_orig_105	0.0	0.0	577	4.6	0.0	541	4.0
dem_100_belg_orig_106	0.0	0.0	439	3.9	0.0	551	4.5
dem_100_belg_orig_107	0.0	0.0	367	4.9	0.0	303	4.8
dem_100_belg_orig_108	0.0	0.0	773	5.4	0.0	965	6.8
dem_100_belg_orig_109	0.0	0.0	720	4.5	0.0	276	4.0
dem_100_belg_orig_110	0.0	0.0	255	3.5	0.0	139	3.4
dem_100_belg_orig_111	0.0	0.0	203	4.2	0.0	312	4.5
dem_100_belg_orig_112	0.0	0.0	244	4.0	0.0	499	4.5
dem_100_belg_orig_113	0.0	0.0	477	3.9	0.0	631	4.7
dem_100_belg_orig_114	0.0	0.0	401	3.7	0.0	648	4.7
dem_100_belg_orig_115	0.15754	0.0	5639	20.2	0.0	275	4.0
dem_100_belg_orig_116	0.0	0.0	411	5.6	0.0	846	6.6
dem_100_belg_orig_117	0.0	0.0	199	4.5	0.0	367	4.5
dem_100_belg_orig_118	0.0	0.0	3008	14.3	0.0	3251	14.9
dem_100_belg_orig_119	0.0	0.0	311	4.7	0.0	8661	28.0
dem_100_belg_orig_120	0.0	0.0	2690	10.3	0.0	2623	11.1
dem_100_belg_orig_121	0.0	0.0	617	4.6	0.0	289	4.1
dem_100_belg_orig_122	0.0	0.0	522	4.8	0.0	193	4.0
dem_100_belg_orig_123	0.0	0.0	24k	105.5	0.0	21k	82.3
dem_100_belg_orig_124	0.0	0.0	475	4.9	0.0	260	4.5
dem_100_belg_orig_125	0.0	0.0	296	3.9	0.0	430	4.2
dem_100_belg_orig_126	0.0	0.0	4943	18.1	0.0	6187	20.7
dem_100_belg_orig_127	0.0	0.0	848	6.2	0.0	4510	17.2
dem_100_belg_orig_128	0.0	0.0	376	4.2	0.0	558	4.8
dem_100_belg_orig_129	0.0	0.0	6362	26.8	0.0	27k	89.0
dem_100_belg_orig_130	0.0	0.0	3451	16.6	0.0	11k	40.6
dem_100_belg_orig_131	0.0	0.0	10k	25.9	0.0	1783	8.5
dem_100_belg_orig_132	0.0	0.0	670	5.2	0.0	388	4.7
dem_100_belg_orig_133	0.0	0.0	598	4.9	0.0	764	5.5
dem_100_belg_orig_134	0.0	0.0	1505	9.3	0.0	472	5.8
dem_100_belg_orig_135	0.0	0.0	282	4.5	0.0	361	4.6
dem_100_belg_orig_136	0.0	0.0	1445	6.2	0.0	3613	14.7
dem_100_belg_orig_137	0.0	0.0	651	5.6	0.0	545	4.9

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_belg_orig_138	0.0	0.0	299	4.5	0.0	513	5.0
dem_100_belg_orig_139	0.0	0.0	287	4.3	0.0	279	4.0
dem_100_belg_orig_140	0.0	0.0	418	5.0	0.0	297	4.2
dem_100_belg_orig_141	0.0	0.0	3664	12.7	0.0	3610	13.5
dem_100_belg_orig_142	0.0	0.0	976	6.1	0.0	366	3.9
dem_100_belg_orig_143	0.0	0.0	1314	7.8	0.0	1161	8.2
dem_100_belg_orig_144	0.0	0.0	1225	7.6	0.0	1310	8.0
dem_100_belg_orig_145	0.0	0.0	4873	21.2	0.0	4795	20.1
dem_100_belg_orig_146	0.0	0.0	268	4.7	0.0	240	4.6
dem_100_belg_orig_147	0.0	0.0	650	5.2	0.0	1048	5.5
dem_100_belg_orig_148	0.0	0.0	254	4.2	0.0	413	4.3
dem_100_belg_orig_149	0.0	0.0	1209	6.8	0.0	998	6.4
dem_100_belg_orig_150	0.0	0.0	427	5.3	0.0	516	5.6
dem_100_belg_orig_151	0.0	0.0	289	3.9	0.0	461	5.6
dem_100_belg_orig_152	0.0	0.0	389	4.6	0.0	456	4.3
dem_100_belg_orig_153	0.0	0.0	3035	9.8	0.0	464	4.4
dem_100_belg_orig_154	0.0	0.0	254	4.2	0.0	635	5.2
dem_100_belg_orig_155	0.0	0.0	462	4.6	0.0	493	4.5
dem_100_belg_orig_156	0.0	0.0	416	4.9	0.0	434	4.8
dem_100_belg_orig_157	0.0	0.0	2882	13.2	0.0	321	5.1
dem_100_belg_orig_158	0.0	0.0	791	5.4	0.0	216	3.9
dem_100_belg_orig_159	0.0	0.0	223	4.6	0.0	508	5.3
dem_100_belg_orig_160	0.0	0.0	387	4.2	0.0	301	3.9
dem_100_belg_orig_161	0.0	0.0	286	5.7	0.0	361	5.5
dem_100_belg_orig_162	0.0	0.0	266	4.5	0.0	270	4.3
dem_100_belg_orig_163	0.0	0.0	295	3.9	0.0	280	4.0
dem_100_belg_orig_164	0.0	0.0	740	5.5	0.0	888	5.8
dem_100_belg_orig_165	0.0	0.0	321	3.9	0.0	339	4.2
dem_100_belg_orig_166	0.0	0.0	2031	10.9	0.0	3080	13.0
dem_100_belg_orig_167	0.0	0.0	783	6.6	0.0	410	4.5
dem_100_belg_orig_168	0.0	0.0	1198	6.1	0.0	383	6.0
dem_100_belg_orig_169	0.0	0.0	968	6.1	0.0	353	4.3
dem_100_belg_orig_170	0.0	0.0	382	5.4	0.0	321	5.5
dem_100_belg_orig_171	0.0	0.0	706	4.8	0.0	555	4.5
dem_100_belg_orig_172	0.05761	0.0	354	4.4	0.0	968	6.5
dem_100_belg_orig_173	0.0	0.0	457	5.7	0.0	347	5.3
dem_100_belg_orig_174	0.0	0.0	638	5.0	0.0	248	4.4
dem_100_belg_orig_175	0.0	0.0	619	6.7	0.0	390	4.4
dem_100_belg_orig_176	0.0	0.0	440	6.6	0.0	335	6.6
dem_100_belg_orig_177	0.0	0.0	517	4.0	0.0	4134	13.1
dem_100_belg_orig_178	0.0	0.0	650	6.6	0.0	1181	6.6
dem_100_belg_orig_179	0.0	0.0	454	4.6	0.0	400	4.1
dem_100_belg_orig_180	0.0	0.0	459	4.5	0.0	314	4.5
dem_100_belg_orig_181	0.0	0.0	2168	9.5	0.0	264	4.7
dem_100_belg_orig_182	0.0	0.0	784	5.2	0.0	925	5.3
dem_100_belg_orig_183	0.0	0.0	732	5.3	0.0	360	4.6
dem_100_belg_orig_184	0.0	0.0	181	3.9	0.0	1372	6.0
dem_100_belg_orig_185	0.0	0.0	379	3.6	0.0	329	3.8
dem_100_belg_orig_186	0.0	0.0	284	4.7	0.0	533	5.5
dem_100_belg_orig_187	0.0	0.0	429	4.7	0.0	255	4.4
dem_100_belg_orig_188	0.0	0.0	1078	6.1	0.0	2562	10.8
dem_100_belg_orig_189	0.0	0.0	441	3.9	0.0	267	3.6
dem_100_belg_orig_190	0.0	0.0	1401	6.9	0.0	2796	11.8
dem_100_belg_orig_191	0.0	0.0	741	5.3	0.0	351	4.3
dem_100_belg_orig_192	0.0	0.0	442	4.8	0.0	210	4.7
dem_100_belg_orig_193	0.0	0.0	401	4.4	0.0	542	4.4
dem_100_belg_orig_194	0.0	0.0	417	4.1	0.0	388	4.1
dem_100_belg_orig_195	0.0	0.0	671	5.8	0.0	1621	9.9
dem_100_belg_orig_196	0.0	0.0	324	4.8	0.0	418	4.9
dem_100_belg_orig_197	0.0	0.0	322	4.4	0.0	582	4.8
dem_100_belg_orig_198	0.0	0.0	2914	9.0	0.0	821	5.6
dem_100_belg_orig_199	0.0	0.0	902	5.4	0.0	265	3.6
dem_100_belg_orig_200	0.0	0.0	1611	7.6	0.0	1003	6.0
dem_100_belg_orig_201	0.0	0.0	2406	10.5	0.0	5161	19.6
dem_100_belg_orig_202	0.0	0.0	3053	12.8	0.0	586	7.1
dem_100_belg_orig_203	0.0	0.0	2066	11.9	0.0	4585	18.4
dem_100_belg_orig_204	0.0	0.0	1312	7.3	0.0	791	6.1
dem_100_belg_orig_205	0.0	0.0	842	4.8	0.0	397	3.9
dem_100_belg_orig_206	0.0	0.0	1193	7.3	0.0	1033	7.0
dem_100_belg_orig_207	0.0	0.0	3796	18.8	0.0	412	5.2
dem_100_belg_orig_208	0.0	0.0	2683	10.8	0.0	650	5.8
dem_100_belg_orig_209	0.0	0.0	731	4.5	0.0	844	5.3
dem_100_belg_orig_210	0.0	0.0	976	7.4	0.0	2199	10.8
dem_100_belg_orig_211	0.0	0.0	280	4.5	0.0	318	5.2
dem_100_belg_orig_212	0.0	0.0	443	4.6	0.0	1908	8.0
dem_100_belg_orig_213	0.0	0.0	169	4.2	0.0	542	5.3
dem_100_belg_orig_214	0.0	0.0	1341	6.2	0.0	291	3.2

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_belg_orig_215	0.0	0.0	249	3.7	0.0	286	3.8
dem_100_belg_orig_216	0.0	0.0	1561	7.4	0.0	1389	8.1
dem_100_belg_orig_217	0.0	0.0	304	4.1	0.0	247	4.4
dem_100_belg_orig_218	0.0	0.0	332	4.1	0.0	282	3.9
dem_100_belg_orig_219	0.0	0.0	4415	17.0	0.0	2772	14.3
dem_100_belg_orig_220	0.0	0.0	328	4.2	0.0	1036	7.2
dem_100_belg_orig_221	0.0	0.0	1617	8.0	0.0	4082	15.4
dem_100_belg_orig_222	0.0	0.0	465	5.0	0.0	461	4.4
dem_100_belg_orig_223	0.0	0.0	252	3.7	0.0	687	4.5
dem_100_belg_orig_224	0.0	0.0	298	4.3	0.0	391	4.4
dem_100_belg_orig_225	0.0	0.0	4297	16.5	0.0	3104	15.9
dem_100_belg_orig_226	0.0	0.0	1002	6.1	0.0	339	4.5
dem_100_belg_orig_227	0.0	0.0	1766	9.0	0.0	4080	16.7
dem_100_belg_orig_228	0.0	0.0	220	4.3	0.0	281	4.3
dem_100_belg_orig_229	0.0	0.0	248	4.2	0.0	379	4.1
dem_100_belg_orig_230	0.0	0.0	357	4.3	0.0	320	4.1
dem_100_belg_orig_231	0.0	0.0	284	4.1	0.0	283	3.8
dem_100_belg_orig_232	0.0	0.0	206	3.9	0.0	490	5.2
dem_100_belg_orig_233	0.0	0.0	342	3.9	0.0	592	4.5
dem_100_belg_orig_234	0.0	0.0	851	5.6	0.0	301	4.0
dem_100_belg_orig_235	0.0	0.0	925	6.5	0.0	249	4.8
dem_100_belg_orig_236	0.0	0.0	318	4.6	0.0	406	4.8
dem_100_belg_orig_237	0.0	0.0	672	5.2	0.0	3411	13.5
dem_100_belg_orig_238	0.0	0.0	642	4.6	0.0	386	4.3
dem_100_belg_orig_239	0.0	0.0	690	4.4	0.0	671	3.8
dem_100_belg_orig_240	0.0	0.0	477	5.3	0.0	421	4.4
dem_100_belg_orig_241	0.0	0.0	141k	273.2	0.0	919	7.5
dem_100_belg_orig_242	0.0	0.0	335	5.0	0.0	239	5.0
dem_100_belg_orig_243	0.0	0.0	826	5.3	0.0	402	4.8
dem_100_belg_orig_244	0.0	0.0	979	5.7	0.0	1572	7.1
dem_100_belg_orig_245	0.0	0.0	4355	15.8	0.0	938	5.2
dem_100_belg_orig_246	0.0	0.0	4676	14.7	0.0	2538	12.8
dem_100_belg_orig_247	0.0	0.0	841	6.9	0.0	674	7.6
dem_100_belg_orig_248	0.0	0.0	2476	9.2	0.0	611	5.0
dem_100_belg_orig_249	0.0	0.0	475	5.7	0.0	1942	10.1
dem_100_belg_orig_250	0.0	0.0	478	4.3	0.0	597	4.8
dem_100_belg_orig_251	0.0	0.0	1765	8.0	0.0	1457	6.5
dem_100_belg_orig_252	0.0	0.0	395	4.4	0.0	486	4.8
dem_100_belg_orig_253	0.0	0.0	472	4.7	0.0	385	4.4
dem_100_belg_orig_254	0.0	0.0	1344	5.8	0.0	3687	12.9
dem_100_belg_orig_255	0.0	0.0	253	4.0	0.0	573	5.0
dem_100_belg_orig_256	0.0	0.0	600	5.5	0.0	229	5.0
dem_100_belg_orig_257	0.0	0.0	317	5.1	0.0	335	5.4
dem_100_belg_orig_258	0.0	0.0	649	5.5	0.0	1313	7.6
dem_100_belg_orig_259	0.0	0.0	1522	7.8	0.0	738	5.2
dem_100_belg_orig_260	0.0	0.0	261	4.6	0.0	464	4.6
dem_100_belg_orig_261	0.0	0.0	460	4.3	0.0	495	4.2
dem_100_belg_orig_262	0.0	0.0	2841	12.4	0.0	341	5.1
dem_100_belg_orig_263	0.0	0.0	1988	7.4	0.0	797	5.7
dem_100_belg_orig_264	0.0	0.0	908	6.5	0.0	918	6.9
dem_100_belg_orig_265	0.0	0.0	802	7.5	0.0	830	6.0
dem_100_belg_orig_266	0.0	0.0	5107	16.5	0.0	607	6.5
dem_100_belg_orig_267	0.0	0.0	869	5.2	0.0	510	4.0
dem_100_belg_orig_268	0.0	0.0	245	4.5	0.0	290	4.5
dem_100_belg_orig_269	0.0	0.0	402	4.4	0.0	327	3.9
dem_100_belg_orig_270	0.0	0.0	1083	7.1	0.0	451	5.8
dem_100_belg_orig_271	0.0	0.0	1381	6.4	0.0	410	4.6
dem_100_belg_orig_272	0.0	0.0	409	4.5	0.0	495	4.9
dem_100_belg_orig_273	0.0	0.0	394	3.9	0.0	594	4.0
dem_100_belg_orig_274	0.0	0.0	314	5.8	0.0	244	5.3
dem_100_belg_orig_275	0.0	0.0	985	5.5	0.0	974	5.5
dem_100_belg_orig_276	0.0	0.0	598	4.2	0.0	297	4.0
dem_100_belg_orig_277	0.0	0.0	828	4.7	0.0	460	3.9
dem_100_belg_orig_278	0.0	0.0	1770	7.8	0.0	337	4.5
dem_100_belg_orig_279	0.0	0.0	436	4.1	0.0	219	4.6
dem_100_belg_orig_280	0.0	0.0	1072	7.3	0.0	4173	14.8
dem_100_belg_orig_281	0.0	0.0	822	6.4	0.0	1266	7.8
dem_100_belg_orig_282	0.0	0.0	505	4.2	0.0	287	3.7
dem_100_belg_orig_283	0.0	0.0	8413	35.8	0.0	814	6.2
dem_100_belg_orig_284	0.0	0.0	557	6.0	0.0	759	6.5
dem_100_belg_orig_285	0.0	0.0	504	5.8	0.0	7079	28.8
dem_100_belg_orig_286	0.0	0.0	2476	10.3	0.0	1160	6.1
dem_100_belg_orig_287	0.0	0.0	293	3.5	0.0	329	3.6
dem_100_belg_orig_288	0.0	0.0	1739	7.2	0.0	1216	6.6
dem_100_belg_orig_289	0.0	0.0	333	4.5	0.0	532	4.8
dem_100_belg_orig_290	0.0	0.0	542	4.8	0.0	327	4.6
dem_100_belg_orig_291	0.0	0.0	8969	26.5	0.0	3843	13.3

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_belg_orig_292	0.0	0.0	1017	5.0	0.0	441	4.0
dem_100_belg_orig_293	0.0	0.0	500	4.7	0.0	714	4.6
dem_100_belg_orig_294	0.0	0.0	297	5.2	0.0	332	5.6
dem_100_belg_orig_295	0.0	0.0	274	4.3	0.0	344	4.8
dem_100_belg_orig_296	0.0	0.0	3923	16.4	0.0	1017	6.4
dem_100_belg_orig_297	0.0	0.0	869	6.6	0.0	1128	7.3
dem_100_belg_orig_298	0.0	0.0	337	4.3	0.0	298	4.3
dem_100_belg_orig_299	0.0	0.0	1259	7.5	0.0	474	5.3
dem_100_belg_orig_300	0.0	0.0	5153	16.8	0.0	2022	10.7
dem_100_belg_orig_301	0.0	0.0	689	6.3	0.0	455	5.0
dem_100_belg_orig_302	0.0	0.0	503	5.5	0.0	998	6.6
dem_100_belg_orig_303	0.0	0.0	309	3.9	0.0	781	4.8
dem_100_belg_orig_304	0.0	0.0	459	4.7	0.0	433	4.1
dem_100_belg_orig_305	0.0	0.0	447	4.8	0.0	359	5.2
dem_100_belg_orig_306	0.0	0.0	396	3.6	0.0	270	3.3
dem_100_belg_orig_307	0.0	0.0	937	5.3	0.0	341	4.0
dem_100_belg_orig_308	0.0	0.0	581	5.0	0.0	323	4.5
dem_100_belg_orig_309	0.0	0.0	448	4.2	0.0	419	4.3
dem_100_belg_orig_310	0.0	0.0	320	4.5	0.0	412	4.6
dem_100_belg_orig_311	0.0	0.0	1717	8.1	0.0	22k	50.3
dem_100_belg_orig_312	0.0	0.0	813	4.2	0.0	235	3.3
dem_100_belg_orig_313	0.0	0.0	1056	7.3	0.0	1307	8.3
dem_100_belg_orig_314	0.0	0.0	2361	11.6	0.0	722	6.2
dem_100_belg_orig_315	0.0	0.0	536	4.8	0.0	389	4.3
dem_100_belg_orig_316	0.0	0.0	952	5.6	0.0	469	4.6
dem_100_belg_orig_317	0.0	0.0	275	5.0	0.0	450	4.9
dem_100_belg_orig_318	0.0	0.0	4691	14.8	0.0	5512	19.0
dem_100_belg_orig_319	0.0	0.0	888	5.3	0.0	278	4.3
dem_100_belg_orig_320	0.0	0.0	244	4.7	0.0	367	4.4
dem_100_belg_orig_321	0.0	0.0	309	4.5	0.0	510	5.0
dem_100_belg_orig_322	0.0	0.0	329	4.6	0.0	358	4.8
dem_100_belg_orig_323	0.0	0.0	546	4.5	0.0	435	4.5
dem_100_belg_orig_324	0.0	0.0	408	4.2	0.0	289	3.6
dem_100_belg_orig_325	0.0	0.0	587	4.8	0.0	681	4.6
dem_100_belg_orig_326	0.0	0.0	446	4.0	0.0	415	3.9
dem_100_belg_orig_327	0.0	0.0	531	4.5	0.0	346	3.9
dem_100_belg_orig_328	0.0	0.0	1284	7.2	0.0	1000	6.7
dem_100_belg_orig_329	0.0	0.0	324	5.0	0.0	393	5.2
dem_100_belg_orig_330	0.0	0.0	2435	13.4	0.0	381	5.0
dem_100_belg_orig_331	0.0	0.0	226	4.0	0.0	232	4.0
dem_100_belg_orig_332	0.0	0.0	397	4.0	0.0	433	4.3
dem_100_belg_orig_333	0.0	0.0	348	4.4	0.0	708	4.8
dem_100_belg_orig_334	0.0	0.0	620	5.6	0.0	290	4.7
dem_100_belg_orig_335	0.0	0.0	1881	6.8	0.0	471	4.3
dem_100_belg_orig_336	0.0	0.0	383	5.0	0.0	374	5.1
dem_100_belg_orig_337	0.01276	0.0	463	4.9	0.0	228	4.2
dem_100_belg_orig_338	0.0	0.0	391	3.5	0.0	493	3.7
dem_100_belg_orig_339	0.0	0.0	3991	17.6	0.0	3829	18.4
dem_100_belg_orig_340	0.0	0.0	449	5.8	0.0	1106	7.3
dem_100_belg_orig_341	0.0	0.0	525	4.5	0.0	283	3.9
dem_100_belg_orig_342	0.0	0.0	1163	5.0	0.0	268	3.7
dem_100_belg_orig_343	0.0	0.0	140	3.6	0.0	282	4.1
dem_100_belg_orig_344	0.0	0.0	620	4.4	0.0	315	3.7
dem_100_belg_orig_345	0.0	0.0	188	3.5	0.0	275	4.1
dem_100_belg_orig_346	0.0	0.0	466	4.0	0.0	775	4.0
dem_100_belg_orig_347	0.0	0.0	594	5.5	0.0	520	5.7
dem_100_belg_orig_348	0.0	0.0	401	4.1	0.0	660	4.5
dem_100_belg_orig_349	0.0	0.0	1617	8.3	0.0	2036	10.2
dem_100_belg_orig_350	0.0	0.0	561	4.3	0.0	305	4.1
dem_100_belg_orig_351	0.0	0.0	203	3.6	0.0	347	5.3
dem_100_belg_orig_352	0.0	0.0	182	3.7	0.0	391	4.3
dem_100_belg_orig_353	0.0	0.0	362	4.8	0.0	422	5.0
dem_100_belg_orig_354	0.0	0.0	232	3.9	0.0	691	5.4
dem_100_belg_orig_355	0.0	0.0	268	4.2	0.0	780	5.3
dem_100_belg_orig_356	0.0	0.0	1576	7.8	0.0	1243	7.7
dem_100_belg_orig_357	0.0	0.0	77k	155.2	0.0	2013	10.8
dem_100_belg_orig_358	0.0	0.0	7216	26.3	0.0	3524	17.0
dem_100_belg_orig_359	0.0	0.0	831	6.1	0.0	682	4.8
dem_100_belg_orig_360	0.0	0.0	1046	5.9	0.0	2338	10.8
dem_100_belg_orig_361	0.0	0.0	604	4.1	0.0	420	4.3
dem_100_belg_orig_362	0.0	0.0	6931	20.8	0.0	3711	15.4
dem_100_belg_orig_363	0.0	0.0	255	3.9	0.0	362	3.9
dem_100_belg_orig_364	0.0	0.0	614	4.8	0.0	2351	14.5
dem_100_belg_orig_365	0.0	0.0	423	4.0	0.0	930	5.3
dem_100_belg_orig_366	0.0	0.0	193	4.2	0.0	281	4.8
dem_100_belg_orig_367	0.0	0.0	450	5.2	0.0	324	4.5
dem_100_belg_orig_368	0.0	0.0	383	4.2	0.0	476	4.2

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_belg_orig_369	0.0	0.0	1819	9.1	0.0	361	4.8
dem_100_belg_orig_370	0.0	0.0	1829	8.3	0.0	2792	13.3
dem_100_belg_orig_371	0.0	0.0	523	5.1	0.0	335	4.5
dem_100_belg_orig_372	0.0	0.0	487	5.1	0.0	595	5.5
dem_100_belg_orig_373	0.0	0.0	4091	12.8	0.0	372	4.8
dem_100_belg_orig_374	0.0	0.0	218	3.5	0.0	160	3.5
dem_100_belg_orig_375	0.0	0.0	2399	9.0	0.0	5347	17.9
dem_100_belg_orig_376	0.0	0.0	1141	7.7	0.0	401	5.7
dem_100_belg_orig_377	0.0	0.0	550	4.8	0.0	622	4.1
dem_100_belg_orig_378	0.0	0.0	1087	6.7	0.0	1742	7.7
dem_100_belg_orig_379	0.0	0.0	502	4.6	0.0	362	4.3
dem_100_belg_orig_380	0.0	0.0	732	5.6	0.0	592	4.8
dem_100_belg_orig_381	0.0	0.0	293	4.7	0.0	570	5.6
dem_100_belg_orig_382	0.0	0.0	379	4.1	0.0	785	5.7
dem_100_belg_orig_383	0.0	0.0	424	5.2	0.0	157	4.1
dem_100_belg_orig_384	0.0	0.0	274	3.9	0.0	399	4.1
dem_100_belg_orig_385	0.0	0.0	884	5.7	0.0	731	5.4
dem_100_belg_orig_386	0.0	0.0	1463	6.8	0.0	310	4.8
dem_100_belg_orig_387	0.0	0.0	1074	5.5	0.0	1996	7.7
dem_100_belg_orig_388	0.0	0.0	379	4.3	0.0	484	4.5
dem_100_belg_orig_389	0.0	0.0	701	7.7	0.0	360	4.5
dem_100_belg_orig_390	0.0	0.0	543	4.8	0.0	2111	9.4
dem_100_belg_orig_391	0.0	0.0	4741	20.6	0.0	303	5.5
dem_100_belg_orig_392	0.0	0.0	754	4.8	0.0	1244	6.1
dem_100_belg_orig_393	0.0	0.0	615	4.7	0.0	376	4.1
dem_100_belg_orig_394	0.0	0.0	754	5.4	0.0	374	4.4
dem_100_belg_orig_395	0.0	0.0	758	6.0	0.0	4214	23.1
dem_100_belg_orig_396	0.0	0.0	424	4.6	0.0	1380	6.7
dem_100_belg_orig_397	0.0	0.0	1642	7.0	0.0	1071	5.8
dem_100_belg_orig_398	0.04608	0.0	427	4.6	0.0	333	4.3
dem_100_belg_orig_399	0.0	0.0	267	3.6	0.0	652	4.8
dem_100_belg_orig_400	0.0	0.0	339	5.7	0.0	555	6.0
dem_100_belg_orig_401	0.0	0.0	451	3.9	0.0	398	3.4
dem_100_belg_orig_402	0.0	0.0	3780	10.5	0.0	1030	5.4
dem_100_belg_orig_403	0.0	0.0	8321	25.6	0.0	1191	7.2
dem_100_belg_orig_404	0.0	0.0	588	4.7	0.0	552	4.5
dem_100_belg_orig_405	0.0	0.0	285	4.5	0.0	269	4.4
dem_100_belg_orig_406	0.0	0.0	1251	8.4	0.0	829	6.4
dem_100_belg_orig_407	0.0	0.0	219	4.2	0.0	357	4.0
dem_100_belg_orig_408	0.0	0.0	434	5.9	0.0	1531	8.1
dem_100_belg_orig_409	0.0	0.0	255	3.6	0.0	291	3.9
dem_100_belg_orig_410	0.0	0.0	178	5.1	0.0	628	6.5
dem_100_belg_orig_411	0.0	0.0	1080	6.3	0.0	3551	14.0
dem_100_belg_orig_412	0.0	0.0	345	3.9	0.0	352	4.3
dem_100_belg_orig_413	0.0	0.0	1888	8.8	0.0	555	4.7
dem_100_belg_orig_414	0.0	0.0	1725	7.8	0.0	1243	7.6
dem_100_belg_orig_415	0.0	0.0	266	4.4	0.0	318	4.6
dem_100_belg_orig_416	0.0	0.0	521	4.2	0.0	821	5.0
dem_100_belg_orig_417	0.0	0.0	1129	5.0	0.0	423	4.0
dem_100_belg_orig_418	0.0	0.0	316	3.5	0.0	327	4.5
dem_100_belg_orig_419	0.0	0.0	342	3.7	0.0	758	4.6
dem_100_belg_orig_420	0.0	0.0	371	4.9	0.0	398	5.2
dem_100_belg_orig_421	0.0	0.0	1422	8.0	0.0	482	4.2
dem_100_belg_orig_422	0.0	0.0	3821	14.6	0.0	6113	22.4
dem_100_belg_orig_423	0.0	0.0	600	6.2	0.0	1732	8.8
dem_100_belg_orig_424	0.0	0.0	7409	25.6	0.0	469	4.8
dem_100_belg_orig_425	0.0	0.0	398	5.5	0.0	281	4.9
dem_100_belg_orig_426	0.0	0.0	1711	8.0	0.0	777	5.0
dem_100_belg_orig_427	0.0	0.0	538	6.2	0.0	668	6.4
dem_100_belg_orig_428	0.0	0.0	771	6.0	0.0	426	5.8
dem_100_belg_orig_429	0.0	0.0	1037	5.7	0.0	2174	10.3
dem_100_belg_orig_430	0.0	0.0	221	4.1	0.0	474	4.3
dem_100_belg_orig_431	0.0	0.0	1003	5.4	0.0	303	3.9
dem_100_belg_orig_432	0.0	0.0	1085	5.9	0.0	1947	8.6
dem_100_belg_orig_433	0.0	0.0	1323	7.4	0.0	721	5.1
dem_100_belg_orig_434	0.0	0.0	1798	7.2	0.0	2785	14.3
dem_100_belg_orig_435	0.0	0.0	407	5.2	0.0	271	4.1
dem_100_belg_orig_436	0.0	0.0	430	5.6	0.0	2133	9.8
dem_100_belg_orig_437	0.0	0.0	4239	14.5	0.0	941	6.2
dem_100_belg_orig_438	0.0	0.0	661	5.1	0.0	1187	7.0
dem_100_belg_orig_439	0.0	0.0	331	3.6	0.0	1189	5.7
dem_100_belg_orig_440	0.00166	0.0	1019	8.9	0.0	293	5.1
dem_100_belg_orig_441	0.0	0.0	776	4.5	0.0	567	4.4
dem_100_belg_orig_442	0.0	0.0	441	4.9	0.0	313	4.6
dem_100_belg_orig_443	0.0	0.0	394	5.0	0.0	432	5.0
dem_100_belg_orig_444	0.0	0.0	248	5.9	0.0	761	6.6
dem_100_belg_orig_445	0.0	0.0	524	4.9	0.0	708	4.9

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_belg_orig_446	0.0	0.0	434	4.9	0.0	353	4.8
dem_100_belg_orig_447	0.0	0.0	2900	12.4	0.0	4740	16.8
dem_100_belg_orig_448	0.0	0.0	260	3.7	0.0	375	4.2
dem_100_belg_orig_449	0.0	0.0	376	4.6	0.0	605	4.8
dem_100_belg_orig_450	0.0	0.0	12k	30.7	0.0	3118	10.7
dem_100_belg_orig_451	0.0	0.0	163	3.7	0.0	255	4.0
dem_100_belg_orig_452	0.0	0.0	636	6.2	0.0	759	6.3
dem_100_belg_orig_453	0.0	0.0	419	5.7	0.0	259	4.2
dem_100_belg_orig_454	0.0	0.0	257	4.3	0.0	1331	8.4
dem_100_belg_orig_455	0.0	0.0	428	5.5	0.0	123	3.1
dem_100_belg_orig_456	0.0	0.0	757	6.2	0.0	243	3.6
dem_100_belg_orig_457	0.0	0.0	1638	10.2	0.0	321	6.1
dem_100_belg_orig_458	0.0	0.0	347	5.1	0.0	237	4.7
dem_100_belg_orig_459	0.0	0.0	315	3.9	0.0	604	4.9
dem_100_belg_orig_460	0.0	0.0	3626	19.2	0.0	3057	14.3
dem_100_belg_orig_461	0.0	0.0	581	4.4	0.0	425	3.8
dem_100_belg_orig_462	0.0	0.0	1483	6.8	0.0	906	5.9
dem_100_belg_orig_463	0.0	0.0	2975	11.7	0.0	659	5.8
dem_100_belg_orig_464	0.0	0.0	695	5.1	0.0	343	4.3
dem_100_belg_orig_465	0.0	0.0	258	4.4	0.0	391	4.6
dem_100_belg_orig_466	0.0	0.0	393	3.9	0.0	348	3.9
dem_100_belg_orig_467	0.0	0.0	869	5.3	0.0	815	4.5
dem_100_belg_orig_468	0.04229	0.0	371	5.7	0.0	215	3.9
dem_100_belg_orig_469	0.0	0.0	337	3.5	0.0	305	3.7
dem_100_belg_orig_470	0.0	0.0	632	5.9	0.0	791	6.2
dem_100_belg_orig_471	0.0	0.0	771	5.0	0.0	271	3.6
dem_100_belg_orig_472	0.0	0.0	361	4.5	0.0	275	4.9
dem_100_belg_orig_473	0.0	0.0	499	4.0	0.0	1445	6.3
dem_100_belg_orig_474	0.0	0.0	1072	6.7	0.0	227	4.0
dem_100_belg_orig_475	0.0	0.0	601	4.5	0.0	759	5.2
dem_100_belg_orig_476	0.0	0.0	1878	7.3	0.0	3191	9.3
dem_100_belg_orig_477	0.0	0.0	7634	21.1	0.0	34k	95.5
dem_100_belg_orig_478	0.0	0.0	388	4.0	0.0	336	4.0
dem_100_belg_orig_479	0.0	0.0	3325	11.7	0.0	3769	15.6
dem_100_belg_orig_480	0.0	0.0	317	4.3	0.0	231	4.1
dem_100_belg_orig_481	0.0	0.0	448	5.0	0.0	757	6.8
dem_100_belg_orig_482	0.0	0.0	741	5.7	0.0	7070	21.4
dem_100_belg_orig_483	0.0	0.0	410	5.4	0.0	520	5.6
dem_100_belg_orig_484	0.0	0.0	1740	8.0	0.0	4667	18.4
dem_100_belg_orig_485	0.0	0.0	371	4.4	0.0	244	4.6
dem_100_belg_orig_486	0.0	0.0	2137	9.6	0.0	341	4.8
dem_100_belg_orig_487	0.0	0.0	300	5.2	0.0	286	5.0
dem_100_belg_orig_488	0.0	0.0	1116	6.4	0.0	2990	14.5
dem_100_belg_orig_489	0.0	0.0	353	3.7	0.0	477	4.3
dem_100_belg_orig_490	0.0	0.0	411	4.2	0.0	222	4.2
dem_100_belg_orig_491	0.0	0.0	233	3.2	0.0	411	4.3
dem_100_belg_orig_492	0.0	0.0	222	3.7	0.0	293	3.6
dem_100_belg_orig_493	0.0	0.0	3598	13.4	0.0	581	5.2
dem_100_belg_orig_494	0.0	0.0	3352	14.5	0.0	5018	17.0
dem_100_belg_orig_495	0.0	0.0	438	4.0	0.0	1041	5.8
dem_100_belg_orig_496	0.0	0.0	353	4.0	0.0	333	4.5
dem_100_belg_orig_497	0.0	0.0	1658	8.8	0.0	2166	11.3
dem_100_belg_orig_498	0.0	0.0	575	4.6	0.0	222	4.1
dem_100_belg_orig_499	0.0	0.0	357	4.4	0.0	448	4.4
dem_100_belg_orig_500	0.0	0.0	374	4.7	0.0	458	5.1
dem_200_belg_orig_1	0.0	0.0	4822	18.0	0.0	42k	165.1
dem_200_belg_orig_2	0.0	0.0	39k	144.5	0.0	9566	35.6
dem_200_belg_orig_3	0.0	0.0	751	5.9	0.0	2776	12.1
dem_200_belg_orig_4	0.0	0.0	2305	9.8	0.0	7011	25.7
dem_200_belg_orig_5	0.0	0.0	12k	38.0	0.0	5441	20.8
dem_200_belg_orig_6	0.0	0.0	402	5.4	0.0	365	5.0
dem_200_belg_orig_7	0.0	0.0	32k	97.9	0.0	51k	198.5
dem_200_belg_orig_8	0.0	0.0	13k	43.0	0.0	565k	1546.8
dem_200_belg_orig_9	0.0	0.0	8031	32.1	0.0	29k	98.9
dem_200_belg_orig_10	0.0	0.0	1030	6.3	0.0	4115	18.2
dem_200_belg_orig_11	0.0	0.0	241	5.1	0.0	423	5.2
dem_200_belg_orig_12	0.0	0.0	5590	21.5	0.0	45k	197.9
dem_200_belg_orig_13	0.0	0.0	380	4.3	0.0	610	4.9
dem_200_belg_orig_14	0.0	0.0	393	5.7	0.0	314	5.4
dem_200_belg_orig_15	0.0	0.0	2698	12.6	0.0	5634	21.3
dem_200_belg_orig_16	0.0	8.3	844k	3600.0	7.1	792k	3600.0
dem_200_belg_orig_17	0.0	0.0	4671	17.3	0.0	4582	16.7
dem_200_belg_orig_18	0.0	0.0	144k	320.7	0.0	5201	20.5
dem_200_belg_orig_19	0.0	0.0	368	5.8	0.0	277	5.5
dem_200_belg_orig_20	0.0	0.0	521	6.1	0.0	174	4.0
dem_200_belg_orig_21	0.0	0.0	422	3.5	0.0	478	4.0
dem_200_belg_orig_22	0.0	0.0	3204	17.3	0.0	35k	162.9

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_200_belg_orig_23	0.0	0.0	641	6.3	0.0	2673	11.4
dem_200_belg_orig_24	0.0	0.0	388	5.7	0.0	118	5.3
dem_200_belg_orig_25	0.0	0.0	5988	22.5	0.0	4171	16.5
dem_200_belg_orig_26	0.0	0.0	557	5.2	0.0	305	4.5
dem_200_belg_orig_27	0.0	0.0	195	3.7	0.0	457	4.0
dem_200_belg_orig_28	0.0	0.0	3630	14.1	0.0	2529	11.0
dem_200_belg_orig_29	0.0	0.0	1042	6.3	0.0	542	5.3
dem_200_belg_orig_30	0.0	0.0	7590	26.8	0.0	74k	183.2
dem_200_belg_orig_31	0.0	0.0	3538	13.9	0.0	3671	16.1
dem_200_belg_orig_32	0.0	0.0	620	5.0	0.0	941	5.2
dem_200_belg_orig_33	0.0	0.0	21k	93.5	0.0	21k	79.3
dem_200_belg_orig_34	0.0	0.0	1571	8.5	0.0	6457	25.0
dem_200_belg_orig_35	0.0	0.0	6138	22.9	0.0	15k	45.9
dem_200_belg_orig_36	0.0	0.0	327	5.4	0.0	409	5.7
dem_200_belg_orig_37	0.0	0.0	3623	18.8	0.0	1961	10.4
dem_200_belg_orig_38	0.0	0.0	440	6.0	0.0	452	5.9
dem_200_belg_orig_39	0.0	0.0	24k	65.4	0.0	15k	52.8
dem_200_belg_orig_40	0.0	0.0	5736	27.6	0.0	19k	79.0
dem_200_belg_orig_41	0.0	0.0	7082	38.2	0.0	14k	65.8
dem_200_belg_orig_42	0.0	0.0	8791	33.2	0.0	26k	137.9
dem_200_belg_orig_43	0.0	0.0	2691	12.9	0.0	1180	9.2
dem_200_belg_orig_44	0.0	0.0	4529	21.8	0.0	4575	21.7
dem_200_belg_orig_45	0.0	0.0	747	7.0	0.0	1382	9.2
dem_200_belg_orig_46	0.0	0.0	616	6.3	0.0	2086	10.2
dem_200_belg_orig_47	0.0	0.0	4421	17.9	0.0	2126	11.4
dem_200_belg_orig_48	0.0	0.0	3810	17.2	0.0	4669	17.0
dem_200_belg_orig_49	0.0	0.0	6093	23.5	0.0	23k	96.5
dem_200_belg_orig_50	0.0	0.0	339	5.0	0.0	3980	20.2
dem_200_belg_orig_51	0.0	0.0	31k	117.5	0.0	14k	60.7
dem_200_belg_orig_52	0.0	0.0	4287	14.1	0.0	9614	44.8
dem_200_belg_orig_53	0.0	0.0	4035	17.9	0.0	3243	16.7
dem_200_belg_orig_54	0.0	0.0	615	5.7	0.0	3463	15.6
dem_200_belg_orig_55	0.0	0.0	38k	170.5	0.0	29k	111.0
dem_200_belg_orig_56	0.0	0.0	263	5.6	0.0	322	5.8
dem_200_belg_orig_57	0.0	0.0	80k	192.7	0.0	406k	1476.8
dem_200_belg_orig_58	0.0	0.0	4756	17.2	0.0	21k	79.4
dem_200_belg_orig_59	0.0	0.0	940	5.1	0.0	2710	14.6
dem_200_belg_orig_60	0.0	0.0	296	5.3	0.0	958	6.5
dem_200_belg_orig_61	0.0	0.0	532	6.0	0.0	585	6.0
dem_200_belg_orig_62	0.0	0.0	1402	6.6	0.0	55k	142.0
dem_200_belg_orig_63	0.0	0.0	9101	45.4	0.0	10k	44.1
dem_200_belg_orig_64	0.0	0.0	620	6.3	0.0	2061	9.1
dem_200_belg_orig_65	0.0	0.0	1351	7.4	0.0	3421	13.3
dem_200_belg_orig_66	0.0	0.0	9884	50.6	0.0	21k	104.1
dem_200_belg_orig_67	0.0	0.0	910	6.6	0.0	2800	13.6
dem_200_belg_orig_68	0.0	0.0	9254	27.3	0.0	88k	327.4
dem_200_belg_orig_69	0.0	0.0	1532	8.3	0.0	1299	7.5
dem_200_belg_orig_70	0.0	0.0	405	4.6	0.0	181	4.0
dem_200_belg_orig_71	0.0	0.0	805	6.7	0.0	627	6.3
dem_200_belg_orig_72	0.0	0.0	10k	30.9	0.0	10k	36.2
dem_200_belg_orig_73	0.0	0.0	1131	8.4	0.0	402	5.7
dem_200_belg_orig_74	0.0	0.0	344	5.0	0.0	3111	14.6
dem_200_belg_orig_75	0.0	0.0	2579	11.4	0.0	24k	77.2
dem_200_belg_orig_76	0.0	0.0	67k	120.7	0.0	13k	55.8
dem_200_belg_orig_77	0.0	0.0	310	4.8	0.0	1099	6.5
dem_200_belg_orig_78	0.0	0.0	364	6.0	0.0	2968	12.9
dem_200_belg_orig_79	0.0	0.0	396	5.4	0.0	422	4.8
dem_200_belg_orig_80	0.0	0.0	345	4.2	0.0	583	5.1
dem_200_belg_orig_81	0.0	0.0	308	4.1	0.0	426	4.5
dem_200_belg_orig_82	0.0	0.0	11k	33.8	0.0	19k	49.8
dem_200_belg_orig_83	0.0	0.0	372	6.0	0.0	511	6.4
dem_200_belg_orig_84	0.0	0.0	792	5.9	0.0	467	5.0
dem_200_belg_orig_85	0.0	0.0	576	5.6	0.0	7626	27.6
dem_200_belg_orig_86	0.0	0.0	3341	10.3	0.0	3069	11.0
dem_200_belg_orig_87	0.0	0.0	4147	17.3	0.0	1167	8.0
dem_200_belg_orig_88	0.0	0.0	289	4.3	0.0	275	4.1
dem_200_belg_orig_89	0.0	0.0	11k	35.2	0.0	5030	22.7
dem_200_belg_orig_90	0.0	0.0	1491	7.8	0.0	2393	11.6
dem_200_belg_orig_91	0.0	0.0	781	6.3	0.0	23k	75.9
dem_200_belg_orig_92	0.0	0.0	6564	26.5	0.0	9443	33.7
dem_200_belg_orig_93	0.0	0.0	200	5.3	0.0	333	4.3
dem_200_belg_orig_94	0.0	0.0	6679	21.2	0.0	22k	61.4
dem_200_belg_orig_95	0.0	0.0	23k	66.4	0.0	78k	242.5
dem_200_belg_orig_96	0.0	0.0	56k	164.5	0.0	7557	24.8
dem_200_belg_orig_97	0.0	0.0	678	6.7	0.0	332	5.5
dem_200_belg_orig_98	0.0	0.0	17k	77.5	0.0	37k	120.8
dem_200_belg_orig_99	0.0	0.0	9864	31.3	0.0	41k	143.5

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_200_belg_orig_100	0.0	0.0	1275	8.4	0.0	516	6.2
dem_200_belg_orig_101	0.0	0.0	466	4.8	0.0	1312	7.3
dem_200_belg_orig_102	0.0	0.0	319	5.8	0.0	523	6.1
dem_200_belg_orig_103	0.0	0.0	6129	32.4	0.0	3480	18.7
dem_200_belg_orig_104	0.0	0.0	6913	32.4	0.0	17k	65.9
dem_200_belg_orig_105	0.0	0.0	382	5.8	0.0	637	6.6
dem_200_belg_orig_106	0.0	0.0	2372	11.3	0.0	1491	8.7
dem_200_belg_orig_107	0.0	0.0	2996	18.7	0.0	3861	17.6
dem_200_belg_orig_108	0.0	0.0	6794	21.1	0.0	8209	28.8
dem_200_belg_orig_109	0.0	0.0	1231	6.9	0.0	1708	9.5
dem_200_belg_orig_110	0.0	0.0	752	5.7	0.0	441	5.0
dem_200_belg_orig_111	0.0	0.0	778	5.2	0.0	4272	15.6
dem_200_belg_orig_112	0.0	0.0	248	4.5	0.0	978	6.0
dem_200_belg_orig_113	0.0	0.0	3747	17.8	0.0	66k	224.6
dem_200_belg_orig_114	0.0	0.0	1421	8.8	0.0	8069	30.4
dem_200_belg_orig_115	0.0	0.0	9395	27.9	0.0	58k	194.8
dem_200_belg_orig_116	0.0	0.0	215	4.2	0.0	399	4.7
dem_200_belg_orig_117	0.0	0.0	15k	61.4	0.0	8721	37.6
dem_200_belg_orig_118	0.0	0.0	691	6.4	0.0	370	6.0
dem_200_belg_orig_119	0.0	0.0	4691	16.1	0.0	58k	194.3
dem_200_belg_orig_120	0.0	0.0	6681	24.3	0.0	15k	50.7
dem_200_belg_orig_121	0.0	0.0	4958	16.1	0.0	2124	11.3
dem_200_belg_orig_122	0.0	0.0	404	6.1	0.0	791	6.7
dem_200_belg_orig_123	0.0	0.0	333	3.8	0.0	291	3.9
dem_200_belg_orig_124	0.0	0.0	610	5.5	0.0	3381	14.2
dem_200_belg_orig_125	0.0	0.0	1839	8.6	0.0	3725	16.2
dem_200_belg_orig_126	0.0	0.0	2166	13.3	0.0	2061	12.2
dem_200_belg_orig_127	0.0	0.0	1860	9.2	0.0	6233	25.9
dem_200_belg_orig_128	0.0	0.0	715	6.2	0.0	5097	19.5
dem_200_belg_orig_129	0.0	0.0	11k	48.9	0.0	950k	1959.0
dem_200_belg_orig_130	0.0	0.0	329	5.7	0.0	891	6.8
dem_200_belg_orig_131	0.0	0.0	444	5.5	0.0	393	5.3
dem_200_belg_orig_132	0.0	0.0	943	6.8	0.0	597	5.9
dem_200_belg_orig_133	0.0	0.0	35k	150.4	0.0	66k	204.6
dem_200_belg_orig_134	0.0	0.0	251	4.0	0.0	389	4.0
dem_200_belg_orig_135	0.0	0.0	1739	8.6	0.0	10k	34.8
dem_200_belg_orig_136	0.0	0.0	15k	50.5	0.0	103k	264.0
dem_200_belg_orig_137	0.0	0.0	3965	13.5	0.0	4732	14.7
dem_200_belg_orig_138	0.0	0.0	1108	7.2	0.0	192	5.3
dem_200_belg_orig_139	0.0	0.0	479	5.0	0.0	1103	6.3
dem_200_belg_orig_140	0.0	0.0	17k	58.3	0.0	29k	95.4
dem_200_belg_orig_141	0.0	0.0	994	5.6	0.0	380	4.5
dem_200_belg_orig_142	0.0	0.0	629	5.9	0.0	7971	33.0
dem_200_belg_orig_143	0.0	0.0	375	4.8	0.0	583	5.1
dem_200_belg_orig_144	0.0	0.0	5739	22.2	0.0	2840	14.6
dem_200_belg_orig_145	0.0	0.0	10k	31.7	0.0	14k	50.9
dem_200_belg_orig_146	0.0	0.0	531	5.5	0.0	911	8.1
dem_200_belg_orig_147	0.0	0.0	469	5.4	0.0	414	5.0
dem_200_belg_orig_148	0.0	0.0	13k	40.5	0.0	1021	7.9
dem_200_belg_orig_149	0.0	0.0	20k	64.9	0.0	34k	107.5
dem_200_belg_orig_150	0.0	0.0	1317	6.9	0.0	12k	44.6
dem_200_belg_orig_151	0.0	0.0	4727	17.0	0.0	43k	135.2
dem_200_belg_orig_152	0.0	0.0	926	6.4	0.0	1485	7.5
dem_200_belg_orig_153	0.0	0.0	483	6.0	0.0	3529	17.2
dem_200_belg_orig_154	0.0	0.0	2711	13.5	0.0	7159	33.3
dem_200_belg_orig_155	0.0	0.0	4954	19.5	0.0	6403	25.5
dem_200_belg_orig_156	0.0	0.0	4352	17.2	0.0	4103	16.9
dem_200_belg_orig_157	0.0	0.0	1265	8.2	0.0	2239	11.1
dem_200_belg_orig_158	0.0	0.0	18k	44.1	0.0	9201	33.1
dem_200_belg_orig_159	0.0	0.0	571	7.0	0.0	780	7.3
dem_200_belg_orig_160	0.0	0.0	2623	10.7	0.0	7358	33.3
dem_200_belg_orig_161	0.0	0.0	1481	10.3	0.0	4531	16.8
dem_200_belg_orig_162	0.0	0.0	733	4.8	0.0	3668	12.7
dem_200_belg_orig_163	0.0	0.0	2001	10.1	0.0	2969	14.0
dem_200_belg_orig_164	0.0	0.0	960	6.5	0.0	3780	16.9
dem_200_belg_orig_165	0.0	0.0	752	5.8	0.0	1241	6.7
dem_200_belg_orig_166	0.0	0.0	339	5.7	0.0	438	6.0
dem_200_belg_orig_167	0.0	0.0	8479	26.8	0.0	5707	22.8
dem_200_belg_orig_168	0.0	0.0	371	6.0	0.0	352	5.9
dem_200_belg_orig_169	0.0	33.7	709k	3600.0	0.0	10k	41.0
dem_200_belg_orig_170	0.0	0.0	298	5.4	0.0	3088	13.7
dem_200_belg_orig_171	0.0	0.0	546	5.8	0.0	305	5.0
dem_200_belg_orig_172	0.0	0.0	9212	34.5	0.0	10k	39.6
dem_200_belg_orig_173	0.0	0.0	1218	6.0	0.0	568	4.7
dem_200_belg_orig_174	0.0	0.0	1053	8.0	0.0	1280	8.6
dem_200_belg_orig_175	0.0	0.0	371	5.5	0.0	2079	11.7
dem_200_belg_orig_176	0.0	0.0	3872	17.0	0.0	552	5.2

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_200_belg_orig_177	0.0	0.0	10k	40.5	0.0	600k	2756.1
dem_200_belg_orig_178	0.0	0.0	292	5.6	0.0	791	6.9
dem_200_belg_orig_179	0.0	0.0	3445	17.6	0.0	1756	10.2
dem_200_belg_orig_180	0.0	0.0	444	6.3	0.0	4746	15.6
dem_200_belg_orig_181	0.0	0.0	422	4.5	0.0	337	4.4
dem_200_belg_orig_182	0.0	0.0	566	5.4	0.0	6991	25.8
dem_200_belg_orig_183	0.0	0.0	27k	111.7	0.0	46k	178.1
dem_200_belg_orig_184	0.0	0.0	7766	37.3	0.0	4676	18.1
dem_200_belg_orig_185	0.0	0.0	321	4.3	0.0	540	4.6
dem_200_belg_orig_186	0.0	0.0	8068	25.5	0.0	3965	16.2
dem_200_belg_orig_187	0.0	0.0	1549	7.5	0.0	5601	21.3
dem_200_belg_orig_188	0.0	0.0	255	5.4	0.0	471	5.8
dem_200_belg_orig_189	0.0	0.0	2654	14.4	0.0	42k	187.3
dem_200_belg_orig_190	0.0	0.0	7524	34.5	0.0	2814	13.5
dem_200_belg_orig_191	0.0	0.0	427	4.7	0.0	300	5.1
dem_200_belg_orig_192	0.0	0.0	279	4.1	0.0	268	4.1
dem_200_belg_orig_193	0.0	0.0	2821	16.3	0.0	17k	72.1
dem_200_belg_orig_194	0.0	0.0	599	7.1	0.0	520	6.9
dem_200_belg_orig_195	0.0	0.0	2844	14.1	0.0	277k	1265.6
dem_200_belg_orig_196	0.0	0.0	10k	30.8	0.0	12k	36.5
dem_200_belg_orig_197	0.0	0.0	7580	22.6	0.0	362	5.6
dem_200_belg_orig_198	0.0	0.0	71k	200.2	0.0	2665	13.1
dem_200_belg_orig_199	0.0	0.0	661	6.9	0.0	2406	13.3
dem_200_belg_orig_200	0.0	0.0	213	4.0	0.0	2391	11.9
dem_200_belg_orig_201	0.0	0.0	84k	324.7	0.0	129k	435.8
dem_200_belg_orig_202	0.0	0.0	1699	8.7	0.0	6959	29.7
dem_200_belg_orig_203	0.0	0.0	369	5.1	0.0	851	7.5
dem_200_belg_orig_204	0.0	0.0	9632	30.3	0.0	4736	19.5
dem_200_belg_orig_205	0.0	0.0	4177	16.9	0.0	4592	20.1
dem_200_belg_orig_206	0.0	0.0	3575	16.8	0.0	1206	7.8
dem_200_belg_orig_207	0.0	0.0	1998	10.2	0.0	3441	16.4
dem_200_belg_orig_208	0.0	0.0	7958	41.4	0.0	68k	301.9
dem_200_belg_orig_209	0.0	0.0	21k	87.4	0.0	15k	54.4
dem_200_belg_orig_210	0.0	0.0	415	5.0	0.0	408	5.2
dem_200_belg_orig_211	0.0	0.0	4671	21.8	0.0	73k	181.6
dem_200_belg_orig_212	0.0	0.0	4489	16.1	0.0	2693	12.2
dem_200_belg_orig_213	0.0	0.0	682	6.1	0.0	2530	12.8
dem_200_belg_orig_214	0.0	0.0	25k	67.3	0.0	12k	49.7
dem_200_belg_orig_215	0.0	0.0	525	5.3	0.0	451	4.9
dem_200_belg_orig_216	0.0	0.0	647	5.1	0.0	853	5.6
dem_200_belg_orig_217	0.0	0.0	258	4.7	0.0	580	5.4
dem_200_belg_orig_218	0.0	0.0	6901	25.1	0.0	43k	102.3
dem_200_belg_orig_219	0.0	0.0	872	6.4	0.0	2030	10.3
dem_200_belg_orig_220	0.0	0.0	2138	9.3	0.0	659	6.7
dem_200_belg_orig_221	0.0	0.0	1268	7.5	0.0	859	6.7
dem_200_belg_orig_222	0.0	0.0	499	4.7	0.0	2007	8.8
dem_200_belg_orig_223	0.0	0.0	348	4.7	0.0	292	4.7
dem_200_belg_orig_224	0.0	0.0	4781	19.0	0.0	6860	32.1
dem_200_belg_orig_225	0.0	0.0	2731	10.5	0.0	3641	13.1
dem_200_belg_orig_226	0.0	0.0	9884	39.4	1.2	1439k	3600.0
dem_200_belg_orig_227	0.0	0.0	476	4.8	0.0	360	4.7
dem_200_belg_orig_228	0.0	0.0	405	4.9	0.0	380	4.8
dem_200_belg_orig_229	0.0	0.0	1891	8.4	0.0	4772	18.7
dem_200_belg_orig_230	0.0	0.0	741	7.0	0.0	3432	14.6
dem_200_belg_orig_231	0.0	0.0	8288	25.6	0.0	1331	7.7
dem_200_belg_orig_232	0.0	0.0	2986	14.1	0.0	467	5.2
dem_200_belg_orig_233	0.0	0.0	1887	8.3	0.0	3716	17.9
dem_200_belg_orig_234	0.0	0.0	921	6.0	0.0	1727	7.1
dem_200_belg_orig_235	0.0	0.0	1176	7.1	0.0	330	5.5
dem_200_belg_orig_236	0.0	0.0	35k	107.0	0.0	21k	64.3
dem_200_belg_orig_237	0.0	0.0	1971	9.6	0.0	7553	30.3
dem_200_belg_orig_238	0.0	0.0	550	4.5	0.0	929	6.0
dem_200_belg_orig_239	0.0	0.0	1194	7.9	0.0	454	5.8
dem_200_belg_orig_240	0.0	0.0	1949	9.1	0.0	2313	13.6
dem_200_belg_orig_241	0.0	0.0	3126	9.7	0.0	4930	16.8
dem_200_belg_orig_242	0.0	0.0	2725	11.2	0.0	6653	26.1
dem_200_belg_orig_243	0.0	0.0	855	6.3	0.0	375	5.3
dem_200_belg_orig_244	0.0	0.0	4675	24.4	0.0	7301	27.7
dem_200_belg_orig_245	0.0	0.0	13k	59.7	0.0	21k	90.2
dem_200_belg_orig_246	0.0	0.0	552	5.1	0.0	234	4.5
dem_200_belg_orig_247	0.0	0.0	30k	79.2	0.0	42k	130.6
dem_200_belg_orig_248	0.0	0.0	30k	89.1	0.0	4653	17.8
dem_200_belg_orig_249	0.0	0.0	871	6.6	0.0	796	7.1
dem_200_belg_orig_250	0.0	0.0	703	5.7	0.0	419	4.9
dem_200_belg_orig_251	0.0	0.0	393	4.4	0.0	216	4.1
dem_200_belg_orig_252	0.0	0.0	2309	10.7	0.0	3285	17.5
dem_200_belg_orig_253	0.0	0.0	1754	9.3	0.0	3144	13.4

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_200_belg_orig_254	0.0	0.0	26k	84.3	0.0	2481	10.8
dem_200_belg_orig_255	0.0	0.0	2505	11.6	0.0	2160	10.2
dem_200_belg_orig_256	0.0	0.0	644	6.3	0.0	651	6.2
dem_200_belg_orig_257	0.0	0.0	1864	10.6	0.0	7927	36.3
dem_200_belg_orig_258	0.0	0.0	699	4.6	0.0	278	3.4
dem_200_belg_orig_259	0.0	0.0	5078	23.6	0.0	18k	63.9
dem_200_belg_orig_260	0.0	0.0	5161	19.6	0.0	1217	7.5
dem_200_belg_orig_261	0.0	0.0	1103	8.5	0.0	300	6.1
dem_200_belg_orig_262	0.0	0.0	2127	11.6	0.0	3372	16.1
dem_200_belg_orig_263	0.0	0.0	307	5.2	0.0	408	5.4
dem_200_belg_orig_264	0.0	0.0	379	5.2	0.0	439	5.2
dem_200_belg_orig_265	0.0	0.0	901	6.9	0.0	17k	65.1
dem_200_belg_orig_266	0.0	0.0	1068	9.0	0.0	274	5.3
dem_200_belg_orig_267	0.0	0.0	2760	13.5	0.0	4303	17.9
dem_200_belg_orig_268	0.0	0.0	13k	59.1	0.0	220k	928.0
dem_200_belg_orig_269	0.0	0.0	8718	24.2	0.0	25k	86.3
dem_200_belg_orig_270	0.0	0.0	13k	49.9	0.0	4090	21.0
dem_200_belg_orig_271	0.0	0.0	542	5.6	0.0	462	5.6
dem_200_belg_orig_272	0.0	0.0	1125	6.6	0.0	4611	18.6
dem_200_belg_orig_273	0.0	0.0	1168	6.7	0.0	3298	15.5
dem_200_belg_orig_274	0.0	0.0	4831	17.4	0.0	4357	18.4
dem_200_belg_orig_275	0.0	0.0	881	6.3	0.0	1783	10.5
dem_200_belg_orig_276	0.0	0.0	2677	12.6	0.0	937	7.1
dem_200_belg_orig_277	0.0	0.0	6884	19.9	0.0	3620	10.9
dem_200_belg_orig_278	0.0	0.0	237	4.1	0.0	407	4.2
dem_200_belg_orig_279	0.0	0.0	3556	15.3	0.0	7007	23.2
dem_200_belg_orig_280	0.0	0.0	161	4.3	0.0	340	4.7
dem_200_belg_orig_281	0.0	0.0	294	4.7	0.0	452	5.0
dem_200_belg_orig_282	0.0	0.0	1439	7.9	0.0	2098	8.4
dem_200_belg_orig_283	0.0	0.0	6186	33.9	0.0	17k	74.4
dem_200_belg_orig_284	0.0	0.0	36k	69.7	0.0	9891	43.3
dem_200_belg_orig_285	0.0	0.0	6586	22.4	0.0	8351	35.1
dem_200_belg_orig_286	0.0	0.0	9791	32.8	0.0	7421	29.2
dem_200_belg_orig_287	0.0	0.0	20k	105.8	0.0	170k	790.5
dem_200_belg_orig_288	0.0	0.0	1136	8.9	0.0	3256	21.0
dem_200_belg_orig_289	0.0	0.0	4924	22.5	0.0	41k	173.1
dem_200_belg_orig_290	0.0	0.0	7737	25.3	0.0	17k	54.5
dem_200_belg_orig_291	0.0	0.0	16k	46.8	0.0	22k	64.9
dem_200_belg_orig_292	0.0	0.0	4967	14.1	0.0	801	5.1
dem_200_belg_orig_293	0.0	0.0	1791	8.9	0.0	33k	93.7
dem_200_belg_orig_294	0.0	0.0	4830	18.4	0.0	2225	10.9
dem_200_belg_orig_295	0.0	0.0	2910	14.9	0.0	3842	14.6
dem_200_belg_orig_296	0.0	0.0	372	5.9	0.0	571	5.7
dem_200_belg_orig_297	0.0	0.0	9651	27.9	0.0	2977	13.5
dem_200_belg_orig_298	0.0	0.0	1873	9.7	0.0	1294	7.1
dem_200_belg_orig_299	0.0	0.0	7041	22.8	0.0	15k	50.7
dem_200_belg_orig_300	0.0	0.0	25k	79.3	0.0	20k	65.5
dem_200_belg_orig_301	0.0	0.0	1552	6.1	0.0	2411	8.4
dem_200_belg_orig_302	0.0	0.0	498	4.7	0.0	399	4.5
dem_200_belg_orig_303	0.0	0.0	3751	12.3	0.0	3457	13.6
dem_200_belg_orig_304	0.0	0.0	774	5.5	0.0	2178	10.7
dem_200_belg_orig_305	0.0	0.0	337	5.0	0.0	485	5.4
dem_200_belg_orig_306	0.0	0.0	1012	6.6	0.0	3038	13.3
dem_200_belg_orig_307	0.0	0.0	1281	6.5	0.0	306	3.3
dem_200_belg_orig_308	0.0	0.0	67k	226.9	0.0	49k	134.1
dem_200_belg_orig_309	0.0	0.0	19k	65.1	0.0	5883	27.0
dem_200_belg_orig_310	0.0	0.0	500	6.0	0.0	22k	58.4
dem_200_belg_orig_311	0.0	0.0	445	5.7	0.0	3175	14.2
dem_200_belg_orig_312	0.0	0.0	4479	19.8	0.0	6704	23.7
dem_200_belg_orig_313	0.0	0.0	4256	16.7	0.0	345	5.7
dem_200_belg_orig_314	0.0	0.0	12k	31.2	0.0	3074	9.9
dem_200_belg_orig_315	0.0	0.0	948	6.0	0.0	492	4.9
dem_200_belg_orig_316	0.0	0.0	487	5.3	0.0	632	5.6
dem_200_belg_orig_317	0.0	0.0	50k	140.7	0.0	5621	21.9
dem_200_belg_orig_318	0.0	0.0	450	4.7	0.0	371	4.2
dem_200_belg_orig_319	0.0	0.0	2351	12.9	0.0	15k	53.7
dem_200_belg_orig_320	0.0	0.0	2561	12.4	0.0	6542	22.5
dem_200_belg_orig_321	0.0	0.0	416	5.0	0.0	427	5.0
dem_200_belg_orig_322	0.0	0.0	11k	42.4	0.0	62k	232.1
dem_200_belg_orig_323	0.0	0.0	441	6.1	0.0	543	6.2
dem_200_belg_orig_324	0.0	0.0	8489	24.7	0.0	9825	32.7
dem_200_belg_orig_325	0.0	0.0	360	5.5	0.0	431	5.7
dem_200_belg_orig_326	0.0	0.0	815	6.6	0.0	712	6.1
dem_200_belg_orig_327	0.0	0.0	2971	12.1	0.0	5495	22.1
dem_200_belg_orig_328	0.0	0.0	4835	17.2	0.0	5651	22.4
dem_200_belg_orig_329	0.0	0.0	557	5.7	0.0	395	5.2
dem_200_belg_orig_330	0.0	0.0	4586	18.5	0.0	7103	25.9

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_200_belg_orig_331	0.0	0.0	1556	7.0	0.0	5974	23.7
dem_200_belg_orig_332	0.0	0.0	4188	15.0	0.0	1107	6.8
dem_200_belg_orig_333	0.0	0.0	313	5.0	0.0	316	5.3
dem_200_belg_orig_334	0.0	0.0	15k	45.5	0.0	18k	65.4
dem_200_belg_orig_335	0.0	0.0	349	5.7	0.0	483	6.2
dem_200_belg_orig_336	0.0	0.0	732	6.8	0.0	1581	8.4
dem_200_belg_orig_337	0.0	0.0	1499	7.6	0.0	759	5.6
dem_200_belg_orig_338	0.0	0.0	473	4.8	0.0	660	5.3
dem_200_belg_orig_339	0.0	0.0	3319	13.0	0.0	3545	16.9
dem_200_belg_orig_340	0.0	0.0	5730	21.8	0.0	7576	29.0
dem_200_belg_orig_341	0.0	0.0	44k	115.5	0.0	52k	147.9
dem_200_belg_orig_342	0.0	0.0	4873	28.3	0.0	9664	43.5
dem_200_belg_orig_343	0.0	0.0	538	5.0	0.0	602	5.5
dem_200_belg_orig_344	0.0	0.0	6481	23.3	0.0	33k	107.8
dem_200_belg_orig_345	0.0	0.0	1023	7.0	0.0	543	5.7
dem_200_belg_orig_346	0.0	0.0	722	6.4	0.0	413	5.7
dem_200_belg_orig_347	0.0	0.0	3542	12.4	0.0	2284	12.0
dem_200_belg_orig_348	0.0	0.0	236	3.7	0.0	188	3.6
dem_200_belg_orig_349	0.0	0.0	3608	20.5	0.0	2492	16.6
dem_200_belg_orig_350	0.0	0.0	9060	29.1	0.0	2736	11.1
dem_200_belg_orig_351	0.0	0.0	531	4.5	0.0	312	4.1
dem_200_belg_orig_352	0.0	0.0	5387	21.6	0.0	18k	49.4
dem_200_belg_orig_353	0.0	0.0	646	5.7	0.0	462	6.0
dem_200_belg_orig_354	0.0	0.0	471	5.8	0.0	495	5.7
dem_200_belg_orig_355	0.0	0.0	4671	21.7	0.0	3370	16.3
dem_200_belg_orig_356	0.0	0.0	741	4.7	0.0	524	4.7
dem_200_belg_orig_357	0.0	0.0	1903	7.7	0.0	5517	19.4
dem_200_belg_orig_358	0.0	0.0	3958	18.0	0.0	267	4.8
dem_200_belg_orig_359	0.0	0.0	2565	11.0	0.0	2107	9.1
dem_200_belg_orig_360	0.0	0.0	756	4.4	0.0	333	3.9
dem_200_belg_orig_361	0.0	0.0	1200	7.0	0.0	2496	12.7
dem_200_belg_orig_362	0.0	0.0	14k	39.5	0.0	715	6.8
dem_200_belg_orig_363	0.0	0.0	332	4.8	0.0	2133	11.7
dem_200_belg_orig_364	0.0	0.0	4151	16.6	0.0	4872	19.2
dem_200_belg_orig_365	0.0	0.0	815	6.5	0.0	1310	7.1
dem_200_belg_orig_366	0.0	0.0	17k	79.8	0.0	9766	52.9
dem_200_belg_orig_367	0.0	0.0	4364	21.6	0.0	6831	30.2
dem_200_belg_orig_368	0.0	0.0	9751	35.4	0.0	47k	155.7
dem_200_belg_orig_369	0.0	0.0	2199	9.4	0.0	4307	17.6
dem_200_belg_orig_370	0.0	0.0	5987	20.5	0.0	4826	18.1
dem_200_belg_orig_371	0.0	0.0	1205	7.4	0.0	1901	9.7
dem_200_belg_orig_372	0.0	0.0	514	5.2	0.0	459	4.9
dem_200_belg_orig_373	0.0	0.0	2004	9.4	0.0	3031	15.3
dem_200_belg_orig_374	0.0	0.0	1717	8.3	0.0	15k	46.6
dem_200_belg_orig_375	0.0	0.0	2051	8.9	0.0	7239	25.9
dem_200_belg_orig_376	0.0	0.0	320	4.4	0.0	252	4.3
dem_200_belg_orig_377	0.0	0.0	10k	51.4	0.0	17k	69.7
dem_200_belg_orig_378	0.0	0.0	12k	33.1	0.0	331	5.7
dem_200_belg_orig_379	0.0	0.0	7681	28.2	0.0	8638	28.3
dem_200_belg_orig_380	0.0	0.0	429	5.9	0.0	310	4.1
dem_200_belg_orig_381	0.0	0.0	2629	13.9	0.0	6803	32.8
dem_200_belg_orig_382	0.0	0.0	6231	24.9	0.0	3599	14.8
dem_200_belg_orig_383	0.0	0.0	7879	29.7	0.0	47k	148.0
dem_200_belg_orig_384	0.0	0.0	3986	16.4	0.0	4266	16.0
dem_200_belg_orig_385	0.0	0.0	4654	18.2	0.0	4241	19.4
dem_200_belg_orig_386	0.0	0.0	417k	1196.6	0.0	82k	279.9
dem_200_belg_orig_387	0.0	0.0	3267	17.0	0.0	4829	20.6
dem_200_belg_orig_388	0.0	0.0	3370	15.5	0.0	4140	19.3
dem_200_belg_orig_389	0.0	0.0	3451	13.8	0.0	3641	16.3
dem_200_belg_orig_390	0.0	0.0	1111	6.9	0.0	1708	7.7
dem_200_belg_orig_391	0.0	0.0	2991	13.8	0.0	3536	15.8
dem_200_belg_orig_392	0.0	0.0	459	5.1	0.0	399	4.9
dem_200_belg_orig_393	0.0	0.0	26k	105.5	0.0	16k	69.0
dem_200_belg_orig_394	0.0	0.0	1375	7.3	0.0	943	6.6
dem_200_belg_orig_395	0.0	0.0	969	6.5	0.0	432	6.2
dem_200_belg_orig_396	0.0	0.0	5848	27.3	0.0	5181	24.2
dem_200_belg_orig_397	0.0	0.0	6682	25.1	0.0	7375	22.5
dem_200_belg_orig_398	0.0	0.0	7372	29.7	0.0	4726	20.1
dem_200_belg_orig_399	0.0	0.0	3025	12.8	0.0	7774	29.2
dem_200_belg_orig_400	0.0	0.0	261	3.7	0.0	1136	5.3
dem_200_belg_orig_401	0.0	0.0	542	6.4	0.0	2082	10.4
dem_200_belg_orig_402	0.0	0.0	2474	10.1	0.0	1861	10.1
dem_200_belg_orig_403	0.0	0.0	38k	126.8	0.0	28k	87.2
dem_200_belg_orig_404	0.0	0.0	414	5.5	0.0	2011	9.9
dem_200_belg_orig_405	0.0	0.0	440	4.6	0.0	432	4.7
dem_200_belg_orig_406	0.0	0.0	3148	13.9	0.0	3841	14.2
dem_200_belg_orig_407	0.0	0.0	3212	12.7	0.0	3930	15.3

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_200_belg_orig_408	0.0	0.0	708	4.9	0.0	414	3.8
dem_200_belg_orig_409	0.0	0.0	5071	17.4	0.0	79k	239.4
dem_200_belg_orig_410	0.0	0.0	36k	138.3	0.0	23k	86.0
dem_200_belg_orig_411	0.0	0.0	417	4.5	0.0	479	5.0
dem_200_belg_orig_412	0.0	0.0	6295	20.4	0.0	4071	19.9
dem_200_belg_orig_413	0.0	0.0	419	4.0	0.0	1309	6.0
dem_200_belg_orig_414	0.0	0.0	1531	9.0	0.0	90k	345.6
dem_200_belg_orig_415	0.0	0.0	1254	6.8	0.0	3546	16.1
dem_200_belg_orig_416	0.0	0.0	262	5.6	0.0	337	5.7
dem_200_belg_orig_417	0.0	0.0	1085	7.6	0.0	26k	95.7
dem_200_belg_orig_418	0.0	0.0	8731	23.6	0.0	25k	96.0
dem_200_belg_orig_419	0.0	0.0	552	4.4	0.0	268	3.6
dem_200_belg_orig_420	0.0	0.0	1292	6.5	0.0	2428	13.9
dem_200_belg_orig_421	0.0	0.0	700	6.3	0.0	784	6.7
dem_200_belg_orig_422	0.0	0.0	610	5.0	0.0	1157	6.6
dem_200_belg_orig_423	0.0	0.0	6627	32.8	0.0	8041	32.1
dem_200_belg_orig_424	0.0	0.0	7050	40.8	0.0	66k	235.7
dem_200_belg_orig_425	0.0	0.0	599	6.3	0.0	1553	9.2
dem_200_belg_orig_426	0.0	0.0	5566	27.5	0.0	6313	30.2
dem_200_belg_orig_427	0.0	0.0	604	5.3	0.0	2861	13.2
dem_200_belg_orig_428	0.0	0.0	578	5.3	0.0	741	5.9
dem_200_belg_orig_429	0.0	0.0	715	5.2	0.0	738	5.4
dem_200_belg_orig_430	0.0	0.0	5326	18.6	0.0	50k	123.6
dem_200_belg_orig_431	0.0	0.0	21k	49.7	0.0	27k	102.1
dem_200_belg_orig_432	0.0	0.0	10k	30.2	0.0	14k	34.8
dem_200_belg_orig_433	0.0	0.0	1914	9.3	0.0	582	5.3
dem_200_belg_orig_434	0.0	0.0	358	4.2	0.0	236	3.3
dem_200_belg_orig_435	0.0	0.0	1371	8.6	0.0	1486	10.2
dem_200_belg_orig_436	0.0	0.0	28k	107.5	0.0	22k	66.8
dem_200_belg_orig_437	0.0	0.0	1992	9.6	0.0	134k	445.4
dem_200_belg_orig_438	0.0	0.0	1481	8.1	0.0	6816	33.9
dem_200_belg_orig_439	0.0	0.0	13k	44.4	0.0	77k	185.4
dem_200_belg_orig_440	0.0	0.0	10k	27.2	0.0	24k	80.2
dem_200_belg_orig_441	0.0	0.0	314	5.1	0.0	847	5.8
dem_200_belg_orig_442	0.0	0.0	3198	13.4	0.0	2435	13.5
dem_200_belg_orig_443	0.0	0.0	332	4.8	0.0	468	5.0
dem_200_belg_orig_444	0.0	0.0	5731	21.0	0.0	8137	42.4
dem_200_belg_orig_445	0.0	0.0	1821	9.2	0.0	2127	10.5
dem_200_belg_orig_446	0.0	0.0	999	6.3	0.0	380	5.0
dem_200_belg_orig_447	0.0	0.0	432	5.3	0.0	650	5.7
dem_200_belg_orig_448	0.0	0.0	8838	45.2	0.0	14k	58.1
dem_200_belg_orig_449	0.0	0.0	456	5.4	0.0	445	5.3
dem_200_belg_orig_450	0.0	0.0	13k	57.9	0.0	11k	49.6
dem_200_belg_orig_451	0.0	0.0	9602	33.2	0.0	25k	89.8
dem_200_belg_orig_452	0.0	0.0	747	7.0	0.0	12k	47.1
dem_200_belg_orig_453	0.0	0.0	10k	40.2	0.0	15k	57.2
dem_200_belg_orig_454	0.0	0.0	1687	8.5	0.0	56k	161.6
dem_200_belg_orig_455	0.0	0.0	816	6.3	0.0	3773	16.6
dem_200_belg_orig_456	0.0	0.0	3071	14.4	0.0	23k	84.1
dem_200_belg_orig_457	0.0	0.0	418	5.7	0.0	632	5.8
dem_200_belg_orig_458	0.0	0.0	99k	423.9	0.0	197k	642.1
dem_200_belg_orig_459	0.0	0.0	1844	9.7	0.0	5142	19.8
dem_200_belg_orig_460	0.0	0.0	764	6.3	0.0	5151	22.8
dem_200_belg_orig_461	0.0	0.0	6051	18.4	0.0	34k	98.8
dem_200_belg_orig_462	0.0	0.0	1497	7.7	0.0	110k	293.7
dem_200_belg_orig_463	0.0	0.0	317	5.3	0.0	489	5.7
dem_200_belg_orig_464	0.0	0.0	414	4.5	0.0	747	4.9
dem_200_belg_orig_465	0.0	0.0	381	5.2	0.0	555	5.5
dem_200_belg_orig_466	0.0	0.0	15k	47.4	0.0	18k	75.1
dem_200_belg_orig_467	0.0	0.0	325	4.5	0.0	353	4.6
dem_200_belg_orig_468	0.0	0.0	8369	38.0	0.0	27k	106.7
dem_200_belg_orig_469	0.0	0.0	8188	28.1	0.0	702	5.3
dem_200_belg_orig_470	0.0	0.0	15k	48.6	0.0	42k	126.2
dem_200_belg_orig_471	0.0	0.0	2052	8.3	0.0	3999	14.2
dem_200_belg_orig_472	0.0	0.0	237	4.6	0.0	453	5.2
dem_200_belg_orig_473	0.0	0.0	1402	8.2	0.0	1101	7.8
dem_200_belg_orig_474	0.0	0.0	21k	77.2	0.0	6269	28.5
dem_200_belg_orig_475	0.0	0.0	1640	9.0	0.0	7857	32.8
dem_200_belg_orig_476	0.0	0.0	540	5.2	0.0	3964	16.5
dem_200_belg_orig_477	0.0	0.0	12k	38.2	0.0	3437	15.1
dem_200_belg_orig_478	0.0	0.0	2071	10.4	0.0	13k	50.3
dem_200_belg_orig_479	0.0	0.0	1052	5.6	0.0	455	4.6
dem_200_belg_orig_480	0.0	0.0	25k	118.8	0.0	84k	366.8
dem_200_belg_orig_481	0.0	0.0	582	5.1	0.0	389	4.5
dem_200_belg_orig_482	0.0	0.0	3325	11.7	0.0	2531	13.8
dem_200_belg_orig_483	0.0	0.0	85k	409.4	0.0	15k	56.5
dem_200_belg_orig_484	0.0	0.0	648	6.4	0.0	81k	170.7

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_200_belg_orig_485	0.0	0.0	8080	35.0	0.0	1580	10.2
dem_200_belg_orig_486	0.0	0.0	484	4.8	0.0	498	4.8
dem_200_belg_orig_487	0.0	0.0	816	6.4	0.0	2719	12.2
dem_200_belg_orig_488	0.0	0.0	419	5.5	0.0	380	5.6
dem_200_belg_orig_489	0.0	0.0	461	4.8	0.0	611	4.8
dem_200_belg_orig_490	0.0	0.0	699	6.1	0.0	26k	59.2
dem_200_belg_orig_491	0.0	0.0	1093	6.7	0.0	17k	66.3
dem_200_belg_orig_492	0.0	0.0	2016	9.1	0.0	2395	10.9
dem_200_belg_orig_493	0.0	0.0	1341	7.7	0.0	460	6.0
dem_200_belg_orig_494	0.0	0.0	1545	7.9	0.0	6069	29.9
dem_200_belg_orig_495	0.0	0.0	2031	7.6	0.0	2291	11.3
dem_200_belg_orig_496	0.0	0.0	2768	10.5	0.0	2434	11.1
dem_200_belg_orig_497	0.0	0.0	443	4.6	0.0	367	4.7
dem_200_belg_orig_498	0.0	0.0	192	5.2	0.0	346	5.2
dem_200_belg_orig_499	0.0	0.0	6953	29.0	0.0	16k	62.1
dem_200_belg_orig_500	0.0	0.0	3631	17.1	0.0	3576	18.3
dem_500_belg_orig_1	0.0	0.0	14k	65.8	0.0	14k	54.5
dem_500_belg_orig_2	0.0	0.0	173k	677.4	0.0	21k	84.0
dem_500_belg_orig_3	0.0	0.0	25k	101.7	0.0	62k	191.0
dem_500_belg_orig_4	0.0	0.9	747k	3600.0	4.9	512k	3600.0
dem_500_belg_orig_5	0.0	0.0	16k	73.1	0.0	28k	89.6
dem_500_belg_orig_6	0.0	0.0	355k	1361.2	0.0	1196k	3103.8
dem_500_belg_orig_7	0.0	0.0	90k	364.0	0.0	44k	151.3
dem_500_belg_orig_8	0.0	0.0	12k	52.1	0.0	17k	68.7
dem_500_belg_orig_9	0.0	0.0	4581	22.4	0.0	4217	17.7
dem_500_belg_orig_10	0.0	0.0	259k	906.3	0.0	261k	983.6
dem_500_belg_orig_11	0.0	0.0	391	4.5	0.0	754	5.5
dem_500_belg_orig_12	0.0	0.0	1011	5.5	0.0	2216	8.9
dem_500_belg_orig_13	0.0	0.0	429k	1590.0	0.0	143k	461.4
dem_500_belg_orig_14	0.0	0.0	347k	1666.4	0.0	33k	126.0
dem_500_belg_orig_15	0.0	0.0	7670	34.9	0.0	8508	37.5
dem_500_belg_orig_16	0.0	0.7	1224k	3600.0	0.0	35k	135.7
dem_500_belg_orig_17	0.0	0.0	19k	55.5	0.0	85k	198.9
dem_500_belg_orig_18	0.0	7.2	722k	3600.0	0.0	122k	485.4
dem_500_belg_orig_19	0.0	0.0	77k	307.0	0.0	66k	217.6
dem_500_belg_orig_20	0.0	0.0	30k	151.2	0.0	73k	280.0
dem_500_belg_orig_21	0.0	0.0	16k	69.5	0.0	92k	303.2
dem_500_belg_orig_22	0.0	0.0	1625k	3600.0	0.0	12k	45.7
dem_500_belg_orig_23	0.0	0.0	90k	358.5	0.0	133k	523.5
dem_500_belg_orig_24	0.0	0.0	754k	2935.1	0.0	435k	1443.6
dem_500_belg_orig_25	0.0	0.0	18k	69.0	0.0	85k	319.0
dem_500_belg_orig_26	0.0	0.8	721k	3600.0	0.0	455k	1719.2
dem_500_belg_orig_27	0.0	0.0	23k	88.0	0.0	28k	96.8
dem_500_belg_orig_28	0.0	2.0	717k	3600.0	12.0	526k	3600.0
dem_500_belg_orig_29	0.0	0.0	26k	69.8	0.0	11k	48.4
dem_500_belg_orig_30	0.0	0.0	168k	562.1	1.6	1403k	3600.0
dem_500_belg_orig_31	0.0	0.0	10k	45.3	0.0	18k	62.3
dem_500_belg_orig_32	0.0	0.0	4961	25.5	0.0	30k	105.7
dem_500_belg_orig_33	0.00295	0.0	23k	144.7	2.7	687k	3600.0
dem_500_belg_orig_34	0.0	0.0	130k	471.6	0.0	74k	276.7
dem_500_belg_orig_35	0.0	0.0	6651	30.9	0.0	33k	114.3
dem_500_belg_orig_36	0.0	0.0	587k	2086.3	2.5	1039k	3600.0
dem_500_belg_orig_37	0.0	0.0	86k	259.0	1.5	887k	3600.0
dem_500_belg_orig_38	0.0	0.0	169k	502.1	0.6	1744k	3600.0
dem_500_belg_orig_39	0.0	0.3	1318k	3600.0	0.0	343k	831.8
dem_500_belg_orig_40	0.0	0.0	763k	2643.3	0.0	179k	543.0
dem_500_belg_orig_41	0.0	0.0	93k	335.0	0.0	539k	1513.1
dem_500_belg_orig_42	0.0	0.0	378k	1536.0	0.0	146k	506.3
dem_500_belg_orig_43	0.0	0.0	12k	56.9	0.0	13k	53.0
dem_500_belg_orig_44	0.0	0.0	683k	2347.3	0.0	347k	1209.5
dem_500_belg_orig_45	0.0	0.0	40k	255.7	0.0	17k	70.5
dem_500_belg_orig_46	0.0	0.0	878	6.8	0.0	381	4.0
dem_500_belg_orig_47	0.0	0.0	217k	802.4	0.0	58k	207.0
dem_500_belg_orig_48	0.0	0.0	78k	269.2	0.0	252k	739.6
dem_500_belg_orig_49	0.0	0.0	38k	199.3	0.0	93k	404.9
dem_500_belg_orig_50	0.0	0.0	205k	703.1	0.0	40k	137.4
dem_500_belg_orig_51	0.0	0.0	743k	2798.9	0.0	35k	130.5
dem_500_belg_orig_52	0.0	0.0	54k	150.7	0.0	169k	457.1
dem_500_belg_orig_53	0.0	0.0	151k	637.1	0.0	214k	709.1
dem_500_belg_orig_54	0.0	0.0	46k	200.3	0.0	294k	1097.4
dem_500_belg_orig_55	0.0	0.0	206k	643.6	0.0	41k	119.0
dem_500_belg_orig_56	0.0	0.0	8830	43.0	0.0	8338	39.3
dem_500_belg_orig_57	0.0	0.0	2514	12.2	0.0	5848	20.7
dem_500_belg_orig_58	0.0	0.4	1301k	3600.0	0.0	27k	86.3
dem_500_belg_orig_59	0.0	0.0	31k	116.3	0.0	28k	76.9
dem_500_belg_orig_60	0.0	0.0	30k	130.6	0.0	135k	507.8
dem_500_belg_orig_61	0.0	0.0	86k	231.8	1.3	1303k	3600.0

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_belg_orig_62	0.00035	0.0	88k	336.8	0.0	109k	416.8
dem_500_belg_orig_63	0.0065	0.0	173k	596.5	0.0	188k	544.2
dem_500_belg_orig_64	0.0	0.6	1625k	3600.0	0.0	295k	773.4
dem_500_belg_orig_65	0.0	0.0	3549	16.2	0.0	17k	65.7
dem_500_belg_orig_66	0.0	0.0	1241	8.2	0.0	9952	32.5
dem_500_belg_orig_67	0.0	0.0	3321	15.8	0.0	1954	9.9
dem_500_belg_orig_68	0.0	0.0	25k	88.2	0.0	55k	219.1
dem_500_belg_orig_69	0.0	0.0	378k	1033.2	0.0	855k	1806.1
dem_500_belg_orig_70	0.0	0.0	84k	288.1	0.0	38k	121.6
dem_500_belg_orig_71	0.0	0.0	848k	3071.4	0.0	206k	757.8
dem_500_belg_orig_72	0.0	0.0	10k	33.4	0.0	686	6.2
dem_500_belg_orig_73	0.0	0.0	55k	211.6	0.0	45k	192.3
dem_500_belg_orig_74	0.0	0.0	4314	18.8	0.0	10k	37.7
dem_500_belg_orig_75	0.0	0.0	79k	212.8	0.0	222k	617.9
dem_500_belg_orig_76	0.0	0.0	18k	73.5	0.0	93k	329.2
dem_500_belg_orig_77	0.0	0.0	42k	161.0	0.0	55k	243.9
dem_500_belg_orig_78	0.0	0.0	9140	43.5	0.0	40k	149.4
dem_500_belg_orig_79	0.0	0.0	467k	2540.1	0.0	98k	352.7
dem_500_belg_orig_80	0.0	0.0	27k	102.5	0.0	439k	1510.3
dem_500_belg_orig_81	0.0	0.0	112k	493.6	0.0	194k	764.7
dem_500_belg_orig_82	0.0	0.0	47k	180.2	0.0	29k	107.9
dem_500_belg_orig_83	0.0	0.0	160k	559.7	0.0	22k	69.9
dem_500_belg_orig_84	0.0	0.0	49k	178.7	0.0	56k	211.7
dem_500_belg_orig_85	0.0	0.0	4551	15.4	0.0	10k	41.3
dem_500_belg_orig_86	0.0	0.0	25k	92.9	0.0	48k	163.5
dem_500_belg_orig_87	0.0	0.0	377k	1187.4	0.0	118k	411.4
dem_500_belg_orig_88	0.0	0.0	7098	29.1	0.0	4891	21.0
dem_500_belg_orig_89	0.0	0.0	28k	88.2	0.0	11k	38.5
dem_500_belg_orig_90	0.0	28.4	645k	3600.0	0.0	77k	267.7
dem_500_belg_orig_91	0.0	0.0	15k	71.9	0.0	37k	142.9
dem_500_belg_orig_92	0.0	0.0	424k	1137.9	0.0	43k	153.1
dem_500_belg_orig_93	–	0.0	0	0.0	0.0	0	0.0
dem_500_belg_orig_94	0.0	0.0	157k	545.1	1.4	1453k	3600.0
dem_500_belg_orig_95	0.0	0.3	927k	3600.0	0.0	898k	3121.7
dem_500_belg_orig_96	0.0	0.0	520k	1997.4	1.7	1354k	3600.0
dem_500_belg_orig_97	0.0	0.0	4454	24.7	0.0	12k	51.7
dem_500_belg_orig_98	0.0	0.0	128k	486.1	0.0	106k	395.0
dem_500_belg_orig_99	0.0	0.0	183k	560.7	0.0	464k	1691.6
dem_500_belg_orig_100	0.0	18.1	511k	3600.0	0.0	55k	192.0
dem_500_belg_orig_101	0.0	0.0	834	5.9	0.0	8592	34.4
dem_500_belg_orig_102	0.0	0.0	49k	171.7	0.0	14k	53.3
dem_500_belg_orig_103	0.0	0.0	8362	36.2	0.0	18k	82.2
dem_500_belg_orig_104	0.0	0.0	8469	34.8	0.0	6566	32.9
dem_500_belg_orig_105	0.0	0.0	65k	251.6	0.0	35k	146.6
dem_500_belg_orig_106	0.0	0.0	12k	52.5	0.0	640k	2136.4
dem_500_belg_orig_107	0.0	0.0	371k	1156.3	0.0	897k	3150.4
dem_500_belg_orig_108	0.0	0.0	117k	436.4	0.0	419k	1314.7
dem_500_belg_orig_109	0.0	0.0	117k	519.1	0.0	203k	789.3
dem_500_belg_orig_110	0.0	0.0	119k	517.5	5.2	1065k	3600.0
dem_500_belg_orig_111	0.0	0.0	53k	201.2	0.0	13k	53.5
dem_500_belg_orig_112	0.0	0.0	33k	153.4	0.0	157k	572.3
dem_500_belg_orig_113	0.0	0.0	35k	150.8	0.0	41k	164.5
dem_500_belg_orig_114	0.0	0.0	15k	63.7	0.0	64k	219.8
dem_500_belg_orig_115	0.0	0.0	72k	281.3	0.0	108k	357.2
dem_500_belg_orig_116	0.0	0.0	15k	66.8	0.0	12k	54.4
dem_500_belg_orig_117	0.0	0.0	39k	134.8	0.0	7608	30.5
dem_500_belg_orig_118	0.0	0.0	370k	1243.5	0.0	72k	246.5
dem_500_belg_orig_119	0.0	0.0	7913	53.5	0.0	15k	55.8
dem_500_belg_orig_120	0.0	0.0	121k	518.3	0.0	157k	595.9
dem_500_belg_orig_121	0.0	0.0	14k	56.1	1.5	766k	3600.0
dem_500_belg_orig_122	0.0	0.0	3627	19.9	0.0	759k	1426.3
dem_500_belg_orig_123	0.0	0.0	5672	20.2	0.0	3100	14.3
dem_500_belg_orig_124	0.0	0.0	5761	28.3	0.0	46k	177.8
dem_500_belg_orig_125	0.0	0.2	821k	3600.0	0.0	99k	339.0
dem_500_belg_orig_126	0.0	0.0	715	5.9	0.0	694	6.0
dem_500_belg_orig_127	0.0	0.0	165k	640.1	0.0	70k	241.1
dem_500_belg_orig_128	0.0	0.0	12k	37.5	0.0	26k	84.7
dem_500_belg_orig_129	0.0	0.0	8231	33.5	1.7	1103k	3600.0
dem_500_belg_orig_130	0.0	0.0	277k	953.4	0.0	395k	1297.5
dem_500_belg_orig_131	0.0	0.0	5580	29.4	0.0	8171	34.0
dem_500_belg_orig_132	0.0	0.0	57k	214.5	128.6	1035k	3600.0
dem_500_belg_orig_133	0.0	0.0	14k	55.1	0.0	15k	56.2
dem_500_belg_orig_134	0.0	0.0	32k	118.0	0.0	14k	56.1
dem_500_belg_orig_135	0.0	0.0	43k	186.9	0.0	109k	366.3
dem_500_belg_orig_136	0.0	0.0	4964	23.6	0.0	5628	22.2
dem_500_belg_orig_137	0.0	0.0	14k	71.5	0.0	32k	126.6
dem_500_belg_orig_138	0.0	0.0	3922	19.7	0.0	2367	12.4

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_belg_orig_139	0.0	0.0	357k	1478.8	0.0	105k	418.6
dem_500_belg_orig_140	0.0	0.0	4903	25.3	0.0	15k	59.2
dem_500_belg_orig_141	0.0	0.0	65k	273.8	0.0	40k	141.6
dem_500_belg_orig_142	0.0	0.0	7358	27.8	0.0	60k	155.5
dem_500_belg_orig_143	0.0	0.0	129k	599.2	0.0	927k	3110.3
dem_500_belg_orig_144	0.0	0.0	47k	167.5	0.0	66k	208.2
dem_500_belg_orig_145	0.0	0.0	167k	673.6	0.6	1089k	3600.0
dem_500_belg_orig_146	0.0	0.0	198k	512.5	0.8	1580k	3600.0
dem_500_belg_orig_147	0.0	0.0	195k	897.9	0.0	229k	855.5
dem_500_belg_orig_148	0.0	0.0	7237	32.6	0.0	3072	13.7
dem_500_belg_orig_149	0.0	0.0	8211	29.6	0.0	10k	37.8
dem_500_belg_orig_150	0.0	0.0	3771	17.2	0.0	11k	39.0
dem_500_belg_orig_151	0.0	0.0	12k	54.7	0.0	239k	902.5
dem_500_belg_orig_152	0.0	0.0	10k	55.1	0.0	18k	82.7
dem_500_belg_orig_153	0.0	0.0	31k	89.3	0.0	14k	45.6
dem_500_belg_orig_154	0.0	0.0	546k	2013.1	0.0	88k	292.5
dem_500_belg_orig_155	0.0	0.0	32k	155.5	0.0	28k	114.7
dem_500_belg_orig_156	0.0	0.0	6839	36.0	0.0	13k	60.5
dem_500_belg_orig_157	0.0	0.0	97k	357.4	0.0	441k	1396.8
dem_500_belg_orig_158	0.0	0.0	6623	34.8	0.0	19k	85.0
dem_500_belg_orig_159	0.0	1.0	910k	3600.0	1.2	1106k	3600.0
dem_500_belg_orig_160	0.0	0.0	21k	84.5	0.0	36k	119.9
dem_500_belg_orig_161	0.0	0.0	28k	76.1	0.0	139k	346.1
dem_500_belg_orig_162	0.0	0.0	23k	82.9	0.0	12k	43.7
dem_500_belg_orig_163	0.0	4.9	1034k	3600.0	0.0	518k	1419.6
dem_500_belg_orig_164	0.0	0.0	56k	198.1	0.0	281k	884.2
dem_500_belg_orig_165	0.0	0.0	55k	233.4	6.2	799k	3600.0
dem_500_belg_orig_166	0.00325	0.0	210k	947.9	0.0	526k	2516.3
dem_500_belg_orig_167	0.0	0.0	573	5.3	0.0	518	5.4
dem_500_belg_orig_168	0.0	0.0	1538	10.7	0.0	2201	13.2
dem_500_belg_orig_169	0.0	0.0	33k	153.9	0.0	58k	212.9
dem_500_belg_orig_170	0.0	0.0	30k	103.1	0.0	4381	19.1
dem_500_belg_orig_171	0.0	0.0	35k	148.4	0.0	33k	141.9
dem_500_belg_orig_172	0.0	0.0	431k	970.2	0.0	14k	50.2
dem_500_belg_orig_173	0.0	0.0	225k	946.1	0.0	281k	902.3
dem_500_belg_orig_174	0.0	0.0	36k	148.8	0.0	517k	1631.6
dem_500_belg_orig_175	0.0	5.9	764k	3600.0	4.4	1444k	3600.0
dem_500_belg_orig_176	0.0	0.0	14k	67.1	0.0	3735	19.2
dem_500_belg_orig_177	0.0	0.0	266k	908.3	0.7	1237k	3600.0
dem_500_belg_orig_178	0.0	0.0	91k	330.2	0.0	25k	99.3
dem_500_belg_orig_179	0.0	0.0	519	5.6	0.0	649	5.6
dem_500_belg_orig_180	0.0	0.0	5497	24.0	0.0	1905	9.1
dem_500_belg_orig_181	0.0	0.0	115k	478.6	4.1	833k	3600.0
dem_500_belg_orig_182	0.0	0.0	188k	750.8	0.0	213k	908.8
dem_500_belg_orig_183	0.0	0.0	344k	1446.8	0.0	114k	501.8
dem_500_belg_orig_184	0.0	0.0	115k	477.3	0.0	491k	1841.9
dem_500_belg_orig_185	0.0	0.0	26k	120.7	0.0	14k	57.4
dem_500_belg_orig_186	0.0	0.0	6398	25.8	0.0	18k	51.8
dem_500_belg_orig_187	0.0	0.9	1059k	3600.0	1.0	1215k	3600.0
dem_500_belg_orig_188	0.0	0.0	6820	29.0	0.0	4002	15.9
dem_500_belg_orig_189	0.0	0.0	67k	286.8	0.0	528k	1507.8
dem_500_belg_orig_190	0.0	0.0	14k	52.0	0.0	7342	30.8
dem_500_belg_orig_191	0.0	0.0	60k	242.2	0.0	554k	1265.4
dem_500_belg_orig_192	0.0	0.0	43k	132.8	0.0	1192k	2288.5
dem_500_belg_orig_193	0.0	0.0	288k	903.3	0.0	149k	534.7
dem_500_belg_orig_194	0.0	0.0	1631	10.4	0.0	1816	9.9
dem_500_belg_orig_195	0.0	0.5	773k	3600.0	0.0	573k	1926.6
dem_500_belg_orig_196	0.0	0.0	3329	16.4	0.0	4689	22.8
dem_500_belg_orig_197	0.0	0.0	614k	2331.4	0.0	80k	293.9
dem_500_belg_orig_198	0.0	0.0	20k	86.8	0.0	6828	31.3
dem_500_belg_orig_199	0.0	0.0	327k	1236.7	0.0	231k	736.3
dem_500_belg_orig_200	0.0	0.0	56k	235.6	0.0	65k	234.2
dem_500_belg_orig_201	0.0	0.0	58k	203.2	0.0	1262k	3594.4
dem_500_belg_orig_202	0.0	0.0	1171	8.4	0.0	1138	6.8
dem_500_belg_orig_203	0.0	0.0	103k	271.2	0.0	920k	1988.8
dem_500_belg_orig_204	0.0	0.0	14k	51.1	0.0	17k	59.9
dem_500_belg_orig_205	0.0	0.6	1064k	3600.0	0.0	550k	1449.7
dem_500_belg_orig_206	0.0	0.0	14k	49.2	0.8	1635k	3600.0
dem_500_belg_orig_207	0.0	0.0	56k	210.9	0.0	190k	757.6
dem_500_belg_orig_208	–	0.0	0	0.0	0.0	0	0.0
dem_500_belg_orig_209	0.0	0.0	65k	183.8	0.0	1217k	3408.3
dem_500_belg_orig_210	0.0	5.9	600k	3600.0	0.0	41k	141.8
dem_500_belg_orig_211	0.0	0.0	789	6.4	0.0	204	5.0
dem_500_belg_orig_212	0.0	0.0	155k	524.9	0.0	245k	782.3
dem_500_belg_orig_213	0.0	0.0	6799	34.1	0.0	6142	29.9
dem_500_belg_orig_214	0.0	0.0	1032k	2458.7	0.1	1844k	3600.0
dem_500_belg_orig_215	0.0	0.0	26k	98.4	0.0	36k	121.6

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_belg_orig_216	0.0	0.0	2968	12.0	0.0	6316	22.1
dem_500_belg_orig_217	0.0	0.0	64k	241.2	0.0	75k	234.2
dem_500_belg_orig_218	0.0	0.0	9610	53.9	0.0	15k	71.4
dem_500_belg_orig_219	0.0	0.0	35k	133.9	0.0	499k	2133.2
dem_500_belg_orig_220	0.0	0.0	54k	198.5	0.0	222k	518.8
dem_500_belg_orig_221	0.0	0.0	22k	94.0	0.1	1629k	3600.0
dem_500_belg_orig_222	0.0	0.0	63k	143.2	0.0	104k	332.7
dem_500_belg_orig_223	0.0	0.0	6901	35.5	0.0	9421	37.9
dem_500_belg_orig_224	0.0	0.0	6541	34.1	0.0	97k	366.1
dem_500_belg_orig_225	0.0	0.0	31k	152.2	12.1	1163k	3600.0
dem_500_belg_orig_226	0.0	0.0	311k	933.1	0.0	301k	859.1
dem_500_belg_orig_227	0.0	0.0	118k	381.1	0.0	49k	136.0
dem_500_belg_orig_228	0.0	0.0	23k	99.0	0.0	44k	154.7
dem_500_belg_orig_229	0.0	0.0	63k	232.7	0.0	103k	324.4
dem_500_belg_orig_230	0.0	0.0	3127	13.6	0.0	3736	14.8
dem_500_belg_orig_231	0.0	0.0	32k	138.4	0.0	738k	2464.7
dem_500_belg_orig_232	0.0	0.0	97k	286.9	0.0	220k	588.3
dem_500_belg_orig_233	0.0	0.0	948k	1851.3	0.0	10k	44.4
dem_500_belg_orig_234	0.00352	0.0	86k	366.0	0.0	122k	545.8
dem_500_belg_orig_235	0.0	0.0	132k	372.2	0.2	1634k	3600.0
dem_500_belg_orig_236	0.0	0.0	6031	31.2	0.0	16k	63.7
dem_500_belg_orig_237	0.0	0.0	1196	9.5	0.0	1524	10.9
dem_500_belg_orig_238	0.0	0.0	18k	57.3	0.0	7748	24.5
dem_500_belg_orig_239	0.0	0.3	1211k	3600.0	0.7	1135k	3600.0
dem_500_belg_orig_240	0.0	0.0	792k	2801.0	1.1	1303k	3600.0
dem_500_belg_orig_241	0.0	0.8	751k	3600.0	0.0	56k	224.7
dem_500_belg_orig_242	0.0	0.0	5288	21.9	0.0	29k	114.0
dem_500_belg_orig_243	0.0	0.8	897k	3600.0	0.0	117k	494.9
dem_500_belg_orig_244	0.0	0.0	188k	697.1	0.0	351k	1374.1
dem_500_belg_orig_245	0.0	0.0	7856	23.1	0.0	898k	2741.6
dem_500_belg_orig_246	0.0	0.0	74k	263.2	0.0	11k	42.7
dem_500_belg_orig_247	0.0	0.0	19k	80.2	0.0	23k	92.8
dem_500_belg_orig_248	0.0	0.0	17k	62.8	0.0	21k	78.9
dem_500_belg_orig_249	0.0	0.0	12k	54.7	0.0	16k	71.1
dem_500_belg_orig_250	0.0	5.9	596k	3600.0	0.0	186k	637.3
dem_500_belg_orig_251	0.0	0.0	4699	19.6	0.0	8051	41.9
dem_500_belg_orig_252	0.0	0.0	1691	9.0	0.0	2328	14.2
dem_500_belg_orig_253	0.0	0.0	197k	772.6	1.5	862k	3600.0
dem_500_belg_orig_254	0.0	0.0	201k	799.0	0.0	196k	700.0
dem_500_belg_orig_255	0.0	0.0	8181	32.2	0.0	5571	22.8
dem_500_belg_orig_256	0.0	0.0	17k	75.2	0.0	15k	67.0
dem_500_belg_orig_257	0.0	0.0	586	5.8	0.0	911	5.9
dem_500_belg_orig_258	0.0	0.0	120k	352.3	0.0	510k	1448.4
dem_500_belg_orig_259	0.0	0.0	336k	998.3	0.0	46k	146.0
dem_500_belg_orig_260	0.0	0.0	696k	1656.7	3.4	860k	3600.0
dem_500_belg_orig_261	0.0	0.0	39k	197.2	0.0	343k	1231.3
dem_500_belg_orig_262	–	0.0	0	0.0	0.0	0	0.0
dem_500_belg_orig_263	0.0	2.4	1051k	3600.0	0.0	405k	1258.2
dem_500_belg_orig_264	0.0	0.0	17k	78.1	0.0	66k	259.3
dem_500_belg_orig_265	0.0	0.0	1521	5.8	0.0	21k	69.1
dem_500_belg_orig_266	0.0	0.0	3581	15.1	0.0	3551	14.7
dem_500_belg_orig_267	0.0	0.0	444k	1384.0	1.4	1017k	3600.0
dem_500_belg_orig_268	0.0	0.0	112k	306.7	–	2443k	3600.0
dem_500_belg_orig_269	0.0	0.0	66k	236.3	0.0	51k	162.6
dem_500_belg_orig_270	0.0	8.7	750k	3600.0	0.0	444k	1574.9
dem_500_belg_orig_271	0.0	0.0	18k	84.9	0.0	26k	100.5
dem_500_belg_orig_272	0.0	0.0	12k	37.4	0.0	287k	852.6
dem_500_belg_orig_273	0.0	0.0	66k	262.3	0.6	1242k	3600.0
dem_500_belg_orig_274	0.0	0.0	2141	9.7	0.0	108k	348.4
dem_500_belg_orig_275	0.0	0.0	14k	64.0	0.0	300k	644.3
dem_500_belg_orig_276	0.0	0.0	1007	6.8	0.0	220k	624.5
dem_500_belg_orig_277	0.0	0.0	39k	156.7	0.0	43k	145.4
dem_500_belg_orig_278	0.0	0.0	14k	63.7	0.0	5063	22.9
dem_500_belg_orig_279	0.0	0.0	27k	112.2	0.0	962k	2206.5
dem_500_belg_orig_280	0.0	0.0	349	5.1	0.0	330	5.3
dem_500_belg_orig_281	0.0	0.0	9571	37.9	0.0	12k	40.9
dem_500_belg_orig_282	0.0	0.0	52k	202.1	0.0	16k	56.2
dem_500_belg_orig_283	0.0	0.0	21k	90.8	0.0	16k	64.7
dem_500_belg_orig_284	0.0	0.7	977k	3600.0	16.1	683k	3600.0
dem_500_belg_orig_285	0.0	0.0	1351	8.2	0.0	2107	10.9
dem_500_belg_orig_286	0.0	0.0	4984	23.8	0.0	12k	45.3
dem_500_belg_orig_287	0.0	0.0	115k	363.3	0.0	134k	386.3
dem_500_belg_orig_288	0.0	0.0	332k	1520.8	0.0	380k	1648.5
dem_500_belg_orig_289	0.0	0.0	215k	875.0	0.0	119k	431.5
dem_500_belg_orig_290	0.0	0.0	709k	2775.5	0.0	98k	346.4
dem_500_belg_orig_291	0.0	0.0	53k	231.3	0.0	19k	76.4
dem_500_belg_orig_292	0.0	0.0	9369	30.7	0.0	2776	12.4

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_belg_orig_293	0.0	0.0	75k	253.7	0.0	874k	2571.0
dem_500_belg_orig_294	0.0	0.0	74k	308.9	0.0	161k	556.3
dem_500_belg_orig_295	0.0	0.0	13k	47.2	0.0	6370	25.3
dem_500_belg_orig_296	0.0	0.0	74k	235.7	0.0	103k	307.0
dem_500_belg_orig_297	0.0	0.0	62k	245.0	0.0	16k	70.4
dem_500_belg_orig_298	0.0	0.0	23k	86.6	0.0	243k	886.1
dem_500_belg_orig_299	0.00229	0.0	19k	105.1	0.0	52k	214.6
dem_500_belg_orig_300	0.0	0.0	231k	975.8	0.0	165k	621.7
dem_500_belg_orig_301	0.0	0.0	60k	220.0	0.0	79k	260.8
dem_500_belg_orig_302	0.0	0.0	235k	630.8	0.0	458k	1645.7
dem_500_belg_orig_303	0.0	0.0	37k	196.4	0.0	45k	190.7
dem_500_belg_orig_304	0.0	0.0	11k	82.0	0.0	11k	46.0
dem_500_belg_orig_305	0.0	0.0	49k	152.7	0.0	53k	146.2
dem_500_belg_orig_306	0.0	0.0	553k	2267.3	0.0	487k	1708.3
dem_500_belg_orig_307	0.0	0.3	1291k	3600.0	0.3	1279k	3600.0
dem_500_belg_orig_308	0.0	0.0	2504	15.5	0.0	14k	56.2
dem_500_belg_orig_309	0.0	0.0	1559	8.8	0.0	2975	12.8
dem_500_belg_orig_310	0.0	0.0	73k	245.3	0.0	56k	196.5
dem_500_belg_orig_311	0.0	0.0	67k	247.9	0.3	1230k	3600.0
dem_500_belg_orig_312	0.0	0.0	339k	1480.1	9.9	802k	3600.0
dem_500_belg_orig_313	0.0	0.0	7431	26.6	0.0	26k	97.4
dem_500_belg_orig_314	0.0	0.0	337k	1400.9	0.0	386k	1158.8
dem_500_belg_orig_315	0.0	0.0	60k	240.8	0.0	129k	473.7
dem_500_belg_orig_316	0.00421	5.3	760k	3600.0	0.0	175k	522.4
dem_500_belg_orig_317	0.0	0.0	33k	175.1	0.0	28k	123.2
dem_500_belg_orig_318	0.0	0.0	1299	8.5	0.0	1249	8.3
dem_500_belg_orig_319	0.0	0.0	21k	94.1	0.0	116k	401.8
dem_500_belg_orig_320	0.0	0.0	15k	76.9	0.0	24k	90.8
dem_500_belg_orig_321	0.0	0.0	3079	13.0	0.0	18k	59.0
dem_500_belg_orig_322	0.0	0.0	11k	44.1	0.0	7364	31.2
dem_500_belg_orig_323	0.0	0.0	27k	114.3	0.0	46k	177.5
dem_500_belg_orig_324	0.0	0.0	1101	6.8	0.0	991	6.8
dem_500_belg_orig_325	0.0	0.0	94k	277.4	0.0	50k	160.9
dem_500_belg_orig_326	0.0	0.0	8224	43.2	0.0	8399	39.5
dem_500_belg_orig_327	0.0062	0.0	9421	36.8	0.0	8631	35.2
dem_500_belg_orig_328	0.0	0.0	171k	658.5	0.0	263k	891.0
dem_500_belg_orig_329	0.0	0.0	14k	53.2	0.0	6834	24.4
dem_500_belg_orig_330	0.0	0.0	29k	119.4	0.0	29k	98.9
dem_500_belg_orig_331	0.0	0.0	552k	1172.4	0.0	97k	357.1
dem_500_belg_orig_332	0.0	0.0	6716	33.8	0.0	141k	401.5
dem_500_belg_orig_333	0.0	0.0	12k	59.3	0.0	21k	84.3
dem_500_belg_orig_334	0.0	0.0	6649	31.8	0.0	8508	33.9
dem_500_belg_orig_335	0.0	0.0	190k	500.7	0.0	282k	852.5
dem_500_belg_orig_336	0.0	0.0	2110	9.3	0.0	89k	273.0
dem_500_belg_orig_337	0.0	0.0	99k	315.8	0.0	411k	1085.0
dem_500_belg_orig_338	0.0	0.0	262k	929.8	0.0	317k	785.5
dem_500_belg_orig_339	0.0	0.0	65k	256.3	0.7	1225k	3600.0
dem_500_belg_orig_340	0.0	0.0	98k	281.0	0.0	187k	429.3
dem_500_belg_orig_341	0.0	0.0	15k	65.9	0.0	52k	198.3
dem_500_belg_orig_342	0.0	0.0	163k	475.7	0.0	23k	66.4
dem_500_belg_orig_343	0.0	0.3	962k	3600.0	0.2	904k	3600.0
dem_500_belg_orig_344	0.0	0.0	471	4.6	0.0	1363	7.5
dem_500_belg_orig_345	0.0	0.0	82k	382.1	0.0	449k	780.2
dem_500_belg_orig_346	0.0	0.0	520k	1975.4	0.0	186k	680.4
dem_500_belg_orig_347	0.0	0.0	44k	193.0	0.0	20k	81.3
dem_500_belg_orig_348	0.0	0.0	176k	652.8	0.0	266k	883.8
dem_500_belg_orig_349	0.00046	0.0	42k	193.2	0.0	85k	297.3
dem_500_belg_orig_350	0.0	0.0	59k	226.0	0.0	56k	165.7
dem_500_belg_orig_351	0.0	0.0	62k	201.6	0.0	58k	185.3
dem_500_belg_orig_352	–	0.0	0	0.0	0.0	0	0.0
dem_500_belg_orig_353	0.0	0.0	359k	1159.4	0.0	688k	1871.0
dem_500_belg_orig_354	0.0	0.0	2565	13.1	0.0	4306	18.8
dem_500_belg_orig_355	0.0	0.0	162k	663.8	0.0	138k	507.2
dem_500_belg_orig_356	0.0	0.0	25k	128.4	0.0	13k	53.5
dem_500_belg_orig_357	0.0	0.0	21k	89.7	0.0	61k	203.1
dem_500_belg_orig_358	0.0	0.0	49k	208.0	0.0	21k	105.8
dem_500_belg_orig_359	0.0	0.0	680k	2295.6	0.0	74k	290.8
dem_500_belg_orig_360	0.0	0.0	649	6.9	0.0	9841	34.8
dem_500_belg_orig_361	0.0	0.0	2344	14.0	0.0	2386	10.4
dem_500_belg_orig_362	0.0	0.9	1366k	3600.0	0.3	1397k	3600.0
dem_500_belg_orig_363	0.0	0.0	492k	1604.3	1.4	960k	3600.0
dem_500_belg_orig_364	0.0	0.0	36k	129.5	0.0	21k	82.7
dem_500_belg_orig_365	0.0	0.7	864k	3600.0	0.0	379k	1320.7
dem_500_belg_orig_366	0.0	0.0	58k	172.8	0.0	118k	340.0
dem_500_belg_orig_367	0.0	0.0	141k	436.0	0.0	36k	125.6
dem_500_belg_orig_368	0.0	0.0	270k	1117.8	0.0	35k	137.2
dem_500_belg_orig_369	0.0	0.0	61k	184.2	0.0	130k	395.9

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_belg_orig_370	0.0	10.9	682k	3600.0	0.0	405k	1356.8
dem_500_belg_orig_371	0.0	0.0	2991	14.8	0.0	6501	28.3
dem_500_belg_orig_372	0.0	0.0	1091	6.8	0.0	1771	7.5
dem_500_belg_orig_373	0.0	0.0	8974	41.0	0.0	55k	195.3
dem_500_belg_orig_374	0.0	0.0	24k	88.8	0.0	26k	75.9
dem_500_belg_orig_375	0.0	0.0	19k	78.0	0.0	21k	79.9
dem_500_belg_orig_376	0.0	0.0	325k	840.5	0.0	656k	1833.2
dem_500_belg_orig_377	0.0	0.0	292	4.8	0.0	1105	6.3
dem_500_belg_orig_378	0.0	0.0	166k	485.3	0.0	1175k	2582.1
dem_500_belg_orig_379	0.0	0.0	4081	18.6	0.0	6869	28.5
dem_500_belg_orig_380	0.0	0.0	209k	607.8	0.0	234k	710.4
dem_500_belg_orig_381	0.0	0.0	5518	31.9	0.0	6671	33.0
dem_500_belg_orig_382	0.0	0.0	10k	47.2	0.0	13k	59.4
dem_500_belg_orig_383	0.0	0.0	6643	33.2	0.0	2251	11.9
dem_500_belg_orig_384	0.0	0.0	2571	13.9	0.0	263k	721.1
dem_500_belg_orig_385	0.0	0.6	831k	3600.0	0.6	946k	3600.0
dem_500_belg_orig_386	0.0	0.0	124k	387.8	0.0	27k	83.8
dem_500_belg_orig_387	0.0	0.0	5915	26.7	0.0	3581	16.7
dem_500_belg_orig_388	0.0	0.0	82k	277.2	0.0	61k	212.9
dem_500_belg_orig_389	0.0	0.0	87k	301.4	0.0	259k	805.0
dem_500_belg_orig_390	0.0	4.8	1347k	3600.0	0.0	855k	2372.4
dem_500_belg_orig_391	0.0	0.0	4903	29.5	0.0	4510	20.2
dem_500_belg_orig_392	0.0	0.0	661k	1378.8	0.0	913k	2830.5
dem_500_belg_orig_393	0.0	0.0	18k	85.9	0.0	32k	138.0
dem_500_belg_orig_394	0.0	0.0	53k	247.0	0.0	45k	173.7
dem_500_belg_orig_395	0.0	0.0	94k	362.7	0.0	31k	111.7
dem_500_belg_orig_396	0.0	0.0	8252	37.2	0.0	5601	21.3
dem_500_belg_orig_397	0.0	0.0	301k	1141.0	1.8	944k	3600.0
dem_500_belg_orig_398	0.0	0.0	16k	73.9	0.0	32k	128.8
dem_500_belg_orig_399	0.0	0.0	223k	770.8	0.5	1271k	3600.0
dem_500_belg_orig_400	0.0	0.0	741k	2621.9	0.0	151k	486.4
dem_500_belg_orig_401	0.0	0.0	33k	142.5	0.0	11k	49.9
dem_500_belg_orig_402	0.0	0.0	13k	60.7	0.0	15k	64.4
dem_500_belg_orig_403	0.0	0.0	4172	16.1	0.0	11k	33.4
dem_500_belg_orig_404	0.0	0.0	15k	61.3	0.0	51k	161.5
dem_500_belg_orig_405	0.0	0.0	28k	128.4	0.0	60k	197.1
dem_500_belg_orig_406	0.0	0.0	40k	181.9	0.0	61k	227.1
dem_500_belg_orig_407	0.0	0.0	12k	55.7	0.0	32k	105.7
dem_500_belg_orig_408	0.0	0.0	42k	167.0	0.0	50k	166.1
dem_500_belg_orig_409	0.0	0.0	7918	25.7	0.0	3358	12.7
dem_500_belg_orig_410	0.0	0.0	1828	11.5	0.0	2018	13.9
dem_500_belg_orig_411	0.0	0.0	12k	97.9	0.0	9878	38.9
dem_500_belg_orig_412	0.0	0.0	55k	218.7	0.0	25k	98.1
dem_500_belg_orig_413	0.00262	0.0	75k	352.1	0.0	71k	277.5
dem_500_belg_orig_414	0.0	0.0	56k	197.0	0.0	11k	43.8
dem_500_belg_orig_415	0.0	0.0	348k	1745.9	0.0	456k	1990.8
dem_500_belg_orig_416	0.0	0.0	5824	28.1	0.0	16k	56.4
dem_500_belg_orig_417	0.0	0.0	2922	17.6	0.0	2131	14.3
dem_500_belg_orig_418	0.0	0.0	19k	55.3	0.0	9326	28.6
dem_500_belg_orig_419	0.0	0.0	2033	10.1	0.0	9101	36.9
dem_500_belg_orig_420	0.00367	0.0	195k	908.5	0.0	218k	919.1
dem_500_belg_orig_421	0.0	0.0	286k	1029.7	0.0	262k	830.5
dem_500_belg_orig_422	0.0	0.0	496k	1801.1	0.0	246k	905.9
dem_500_belg_orig_423	0.0	0.0	46k	196.3	0.0	82k	296.8
dem_500_belg_orig_424	0.0	0.0	3078	13.4	0.0	3471	14.0
dem_500_belg_orig_425	0.0	6.7	669k	3600.0	0.1	1187k	3600.0
dem_500_belg_orig_426	0.0	0.0	163k	628.7	0.0	168k	591.0
dem_500_belg_orig_427	0.0	0.0	325k	1274.4	0.0	72k	239.4
dem_500_belg_orig_428	0.0	0.0	99k	367.8	0.0	7253	31.3
dem_500_belg_orig_429	0.0	0.0	64k	265.1	0.0	283k	970.9
dem_500_belg_orig_430	0.0	0.0	918k	3018.5	0.0	380k	1130.9
dem_500_belg_orig_431	0.0	0.0	2851	13.0	0.0	76k	238.1
dem_500_belg_orig_432	0.0	0.0	312k	794.9	0.0	839k	2025.1
dem_500_belg_orig_433	0.0	7.2	671k	3600.0	7.7	917k	3600.0
dem_500_belg_orig_434	0.0	0.0	499	5.7	0.0	966	6.7
dem_500_belg_orig_435	0.0	0.0	147k	401.9	0.0	81k	278.0
dem_500_belg_orig_436	0.0	0.0	239k	870.2	0.0	163k	521.1
dem_500_belg_orig_437	0.0	0.0	11k	45.9	0.0	10k	43.9
dem_500_belg_orig_438	0.0	0.0	70k	239.8	2.8	964k	3600.0
dem_500_belg_orig_439	0.0	0.0	69k	168.0	0.0	8931	27.8
dem_500_belg_orig_440	0.0	0.0	12k	42.9	0.0	9327	27.5
dem_500_belg_orig_441	0.0	0.0	9441	33.7	0.0	14k	63.1
dem_500_belg_orig_442	0.0	0.0	9302	33.6	0.0	13k	39.5
dem_500_belg_orig_443	0.0	0.0	4631	19.0	0.0	7930	29.7
dem_500_belg_orig_444	0.0	0.0	80k	309.3	0.0	34k	133.0
dem_500_belg_orig_445	0.0	0.0	10k	38.9	0.0	7170	31.9
dem_500_belg_orig_446	0.0	0.0	11k	52.0	0.0	24k	99.9

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_belg_orig_447	0.0	0.0	1401	7.4	0.0	2251	11.2
dem_500_belg_orig_448	0.0	0.0	5921	24.0	0.0	200k	431.9
dem_500_belg_orig_449	0.0	0.0	7340	26.8	0.0	257k	776.9
dem_500_belg_orig_450	0.0	0.0	3834	21.6	0.0	8745	37.3
dem_500_belg_orig_451	0.0	0.0	2900	14.5	0.0	3510	16.0
dem_500_belg_orig_452	0.0	0.0	11k	52.0	0.0	11k	46.6
dem_500_belg_orig_453	0.0	0.0	19k	62.6	0.0	20k	72.7
dem_500_belg_orig_454	0.0	0.0	222k	949.7	0.0	115k	381.8
dem_500_belg_orig_455	0.0	0.0	5460	20.3	0.0	17k	60.9
dem_500_belg_orig_456	0.0	0.0	223k	858.1	0.0	216k	785.0
dem_500_belg_orig_457	0.0	0.0	200k	641.4	0.0	45k	153.3
dem_500_belg_orig_458	0.0	0.0	320k	1417.6	0.0	568k	2331.7
dem_500_belg_orig_459	0.0	0.0	1941	10.3	0.0	4348	18.2
dem_500_belg_orig_460	0.0	0.0	42k	168.2	0.0	49k	213.1
dem_500_belg_orig_461	0.00378	0.0	21k	117.3	0.0	466k	1762.0
dem_500_belg_orig_462	0.0	0.0	41k	151.2	0.0	22k	70.1
dem_500_belg_orig_463	0.0	0.0	108k	415.9	0.6	1425k	3600.0
dem_500_belg_orig_464	0.0	0.0	85k	297.7	0.0	77k	257.8
dem_500_belg_orig_465	0.0	0.0	79k	281.1	0.9	1326k	3600.0
dem_500_belg_orig_466	0.0	0.0	126k	369.8	0.0	66k	186.7
dem_500_belg_orig_467	0.0	0.0	9267	42.5	0.0	36k	130.1
dem_500_belg_orig_468	0.0	0.0	334	4.9	0.0	1809	8.1
dem_500_belg_orig_469	0.0	0.0	2081	9.6	0.0	3979	12.9
dem_500_belg_orig_470	0.0	0.0	153k	703.4	0.0	177k	620.1
dem_500_belg_orig_471	0.0	0.0	282k	976.6	0.0	426k	1598.5
dem_500_belg_orig_472	0.0	0.0	18k	70.5	0.0	111k	379.6
dem_500_belg_orig_473	0.0	0.0	293k	1208.0	0.0	669k	2635.4
dem_500_belg_orig_474	0.0	0.0	324k	1190.2	0.0	48k	177.4
dem_500_belg_orig_475	0.0	0.0	71k	242.8	0.0	62k	225.5
dem_500_belg_orig_476	0.0	0.0	1095k	3564.9	0.1	809k	3600.0
dem_500_belg_orig_477	0.0	0.0	38k	195.9	0.0	60k	218.0
dem_500_belg_orig_478	0.0	0.0	108k	463.8	0.0	79k	298.8
dem_500_belg_orig_479	0.0	0.0	24k	107.7	0.0	45k	162.0
dem_500_belg_orig_480	0.0	0.0	3604	15.3	0.0	1711	8.8
dem_500_belg_orig_481	0.0	0.0	1173	7.6	0.0	851	5.9
dem_500_belg_orig_482	–	0.0	0	0.0	0.0	0	0.0
dem_500_belg_orig_483	0.00151	0.0	176k	772.2	0.0	411k	1832.5
dem_500_belg_orig_484	0.0	0.0	26k	87.9	0.0	6605	27.1
dem_500_belg_orig_485	0.0	0.0	21k	82.0	0.0	17k	66.4
dem_500_belg_orig_486	0.0	0.0	1303k	3387.1	0.0	32k	117.5
dem_500_belg_orig_487	0.0	0.0	109k	424.2	0.0	70k	251.5
dem_500_belg_orig_488	0.0	0.0	767	6.0	0.0	1614	8.4
dem_500_belg_orig_489	0.0	0.0	7211	33.4	0.0	37k	116.6
dem_500_belg_orig_490	0.0	0.0	116k	447.5	0.0	191k	767.7
dem_500_belg_orig_491	0.0	0.0	50k	197.0	0.0	98k	334.2
dem_500_belg_orig_492	0.0	0.0	2521	11.7	0.0	2443	9.9
dem_500_belg_orig_493	0.0	1.7	1064k	3600.0	1.7	1092k	3600.0
dem_500_belg_orig_494	0.0	0.0	13k	60.5	0.0	17k	72.8
dem_500_belg_orig_495	0.0	2.0	1068k	3600.0	0.0	92k	303.1
dem_500_belg_orig_496	0.0	0.0	207k	789.7	0.0	284k	1000.0
dem_500_belg_orig_497	0.0	0.0	26k	98.2	0.0	32k	112.5
dem_500_belg_orig_498	0.0	0.0	61k	227.5	0.0	44k	175.3
dem_500_belg_orig_499	0.0	0.0	55k	283.0	0.0	13k	63.5
dem_500_belg_orig_500	0.0	3.6	678k	3600.0	0.0	795k	2630.2
dem_1000_belg_orig_1	0.01567	20.7	585k	3600.0	18.1	658k	3600.0
dem_1000_belg_orig_2	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_3	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_4	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_5	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_6	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_7	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_8	0.00093	0.0	302k	1222.5	0.1	979k	3600.0
dem_1000_belg_orig_9	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_10	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_11	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_12	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_13	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_14	0.0093	0.0	410k	2113.3	0.0	362k	1615.1
dem_1000_belg_orig_15	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_16	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_17	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_18	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_19	0.01388	0.0	264k	760.4	2.5	943k	3600.0
dem_1000_belg_orig_20	0.02456	0.0	427k	2206.9	0.0	235k	1228.3
dem_1000_belg_orig_21	0.00159	0.0	164k	836.0	0.0	511k	1862.8
dem_1000_belg_orig_22	0.01448	0.0	345k	1206.0	0.0	1093k	3057.9
dem_1000_belg_orig_23	0.0	0.8	1083k	3600.0	0.9	1234k	3600.0

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_belg_orig_24	0.02014	0.0	49k	308.9	0.0	390k	2195.2
dem_1000_belg_orig_25	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_26	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_27	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_28	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_29	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_30	0.00209	0.5	787k	3600.0	3.7	865k	3600.0
dem_1000_belg_orig_31	0.0	0.0	15k	45.8	0.0	3426	16.7
dem_1000_belg_orig_32	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_33	0.00223	0.2	726k	3600.0	0.4	892k	3600.0
dem_1000_belg_orig_34	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_35	0.00232	1.4	737k	3600.0	1.4	887k	3600.0
dem_1000_belg_orig_36	0.0	0.0	132k	344.1	0.0	53k	148.9
dem_1000_belg_orig_37	0.01014	0.0	595k	2686.2	0.5	804k	3600.0
dem_1000_belg_orig_38	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_39	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_40	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_41	–	–	1680k	3600.0	–	1424k	3600.0
dem_1000_belg_orig_42	0.0	0.0	319	5.1	0.0	860	6.0
dem_1000_belg_orig_43	0.0	1.3	972k	3600.0	0.0	198k	665.8
dem_1000_belg_orig_44	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_45	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_46	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_47	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_48	0.00631	0.0	147k	647.5	0.0	235k	824.2
dem_1000_belg_orig_49	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_50	0.00014	0.4	662k	3600.0	1.5	809k	3600.0
dem_1000_belg_orig_51	0.0	0.0	22k	109.7	0.0	466k	1007.5
dem_1000_belg_orig_52	0.00099	0.0	698k	3352.1	0.2	825k	3600.0
dem_1000_belg_orig_53	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_54	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_55	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_56	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_57	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_58	0.00137	0.0	29k	113.6	0.0	29k	102.2
dem_1000_belg_orig_59	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_60	0.00042	0.9	789k	3600.0	2.1	910k	3600.0
dem_1000_belg_orig_61	0.0	0.0	872k	2914.9	1.5	1452k	3600.0
dem_1000_belg_orig_62	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_63	0.13055	1.3	576k	3600.0	1.5	704k	3600.0
dem_1000_belg_orig_64	0.0	0.0	819k	1980.7	0.0	1535k	2798.2
dem_1000_belg_orig_65	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_66	0.00642	0.6	1121k	3600.0	4.2	747k	3600.0
dem_1000_belg_orig_67	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_68	0.00331	0.0	600k	2529.8	0.1	941k	3600.0
dem_1000_belg_orig_69	0.01693	0.0	625k	3168.2	0.2	934k	3600.0
dem_1000_belg_orig_70	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_71	0.00417	0.0	110k	419.5	0.0	211k	764.5
dem_1000_belg_orig_72	0.0	0.0	268k	1129.9	0.0	396k	1286.7
dem_1000_belg_orig_73	0.00998	0.0	820k	3594.5	0.6	925k	3600.0
dem_1000_belg_orig_74	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_75	0.00564	0.8	781k	3600.0	0.7	779k	3600.0
dem_1000_belg_orig_76	0.0119	0.4	688k	3600.0	2.3	736k	3600.0
dem_1000_belg_orig_77	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_78	0.0	0.3	826k	3600.0	0.0	135k	468.3
dem_1000_belg_orig_79	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_80	0.01128	0.0	859k	2889.9	0.6	834k	3600.0
dem_1000_belg_orig_81	0.00085	0.2	664k	3600.0	0.0	565k	1815.8
dem_1000_belg_orig_82	0.01702	0.0	206k	863.2	0.0	602k	1922.4
dem_1000_belg_orig_83	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_84	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_85	0.00355	0.0	670k	2513.3	0.0	236k	745.4
dem_1000_belg_orig_86	0.03573	0.0	381	13.4	0.0	1178	14.0
dem_1000_belg_orig_87	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_88	0.0	0.0	61k	240.2	0.0	94k	310.0
dem_1000_belg_orig_89	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_90	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_91	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_92	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_93	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_94	0.01452	0.0	284k	1261.5	0.9	748k	3600.0
dem_1000_belg_orig_95	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_96	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_97	0.00069	0.0	33k	149.2	0.0	70k	258.5
dem_1000_belg_orig_98	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_99	0.0	0.1	1122k	3600.0	0.2	1298k	3600.0
dem_1000_belg_orig_100	–	0.0	0	0.0	0.0	0	0.0

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_belg_orig_101	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_102	0.00081	0.0	870k	3246.5	0.0	737k	2606.8
dem_1000_belg_orig_103	0.00473	0.0	15k	57.5	0.0	32k	99.9
dem_1000_belg_orig_104	–	–	2158k	3600.0	–	2761k	3600.0
dem_1000_belg_orig_105	0.00324	4.4	635k	3600.0	4.2	761k	3600.0
dem_1000_belg_orig_106	0.01899	0.0	315k	1394.2	0.0	773k	2609.4
dem_1000_belg_orig_107	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_108	0.0	0.0	2121	11.6	0.0	2525	14.2
dem_1000_belg_orig_109	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_110	0.0	0.0	33k	123.2	0.0	11k	48.7
dem_1000_belg_orig_111	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_112	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_113	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_114	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_115	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_116	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_117	0.00619	0.0	6436	39.3	0.0	4437	25.2
dem_1000_belg_orig_118	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_119	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_120	0.04156	0.0	2982	17.9	0.0	19k	84.8
dem_1000_belg_orig_121	0.01143	0.0	809k	3588.1	2.0	792k	3600.0
dem_1000_belg_orig_122	0.02099	1.3	523k	3600.0	1.2	600k	3600.0
dem_1000_belg_orig_123	0.01194	0.0	219k	958.2	0.1	866k	3600.0
dem_1000_belg_orig_124	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_125	0.0178	0.0	35k	175.6	0.0	49k	283.8
dem_1000_belg_orig_126	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_127	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_128	0.0	0.0	26k	115.1	0.0	20k	71.8
dem_1000_belg_orig_129	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_130	0.0	0.0	916	7.1	0.0	25k	70.2
dem_1000_belg_orig_131	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_132	0.01108	0.0	584k	1947.7	0.0	319k	1078.6
dem_1000_belg_orig_133	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_134	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_135	0.0	0.0	123k	496.7	1.6	1096k	3600.0
dem_1000_belg_orig_136	0.00205	0.0	16k	71.8	0.0	18k	71.3
dem_1000_belg_orig_137	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_138	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_139	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_140	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_141	0.00449	0.0	111k	469.5	0.0	303k	969.3
dem_1000_belg_orig_142	0.0	0.0	11k	51.2	0.2	1635k	3600.0
dem_1000_belg_orig_143	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_144	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_145	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_146	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_147	0.0	2.7	822k	3600.0	0.0	350k	1203.4
dem_1000_belg_orig_148	0.01262	0.0	70k	448.5	0.0	485k	2070.1
dem_1000_belg_orig_149	0.00234	0.0	23k	120.4	0.0	11k	69.4
dem_1000_belg_orig_150	0.00774	0.0	390k	1711.4	0.0	768k	2854.5
dem_1000_belg_orig_151	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_152	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_153	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_154	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_155	0.00916	1.0	960k	3600.0	0.0	425k	1407.9
dem_1000_belg_orig_156	0.00824	0.0	1301k	3600.0	0.0	1224k	3600.0
dem_1000_belg_orig_157	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_158	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_159	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_160	0.00489	0.0	68k	233.1	0.0	201k	492.9
dem_1000_belg_orig_161	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_162	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_163	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_164	0.00236	1.6	678k	3600.0	1.4	851k	3600.0
dem_1000_belg_orig_165	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_166	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_167	0.01307	0.0	83k	439.8	0.0	112k	482.0
dem_1000_belg_orig_168	0.01035	0.0	188k	735.1	0.0	219k	731.3
dem_1000_belg_orig_169	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_170	0.00851	0.0	23k	68.3	0.0	7924	31.2
dem_1000_belg_orig_171	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_172	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_173	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_174	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_175	0.0	0.0	41k	113.1	0.0	51k	155.7
dem_1000_belg_orig_176	0.00325	0.0	455k	2138.7	0.0	494k	1985.2
dem_1000_belg_orig_177	–	0.0	0	0.0	0.0	0	0.0

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_belg_orig_178	0.0	0.0	2286	9.6	0.0	24k	64.8
dem_1000_belg_orig_179	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_180	0.0177	0.0	41k	241.3	0.0	96k	418.2
dem_1000_belg_orig_181	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_182	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_183	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_184	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_185	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_186	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_187	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_188	0.0096	0.0	497k	2342.9	0.0	575k	2310.3
dem_1000_belg_orig_189	0.0	0.0	148k	699.1	0.0	397k	1506.9
dem_1000_belg_orig_190	0.00471	0.0	174k	603.7	0.0	113k	458.2
dem_1000_belg_orig_191	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_192	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_193	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_194	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_195	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_196	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_197	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_198	0.0	0.0	51k	185.6	0.0	201k	704.2
dem_1000_belg_orig_199	0.01814	0.0	46k	308.7	0.0	345k	1928.4
dem_1000_belg_orig_200	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_201	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_202	0.0	0.0	11k	55.7	0.0	7009	27.6
dem_1000_belg_orig_203	0.0	0.0	1900	8.3	0.0	2355	10.8
dem_1000_belg_orig_204	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_205	0.00309	0.0	551k	1469.7	0.0	958k	2580.9
dem_1000_belg_orig_206	0.00352	0.0	123k	504.6	0.0	294k	950.5
dem_1000_belg_orig_207	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_208	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_209	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_210	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_211	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_212	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_213	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_214	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_215	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_216	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_217	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_218	0.0	0.0	27k	99.6	0.0	160k	370.2
dem_1000_belg_orig_219	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_220	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_221	0.00937	0.0	688k	2818.9	0.5	845k	3600.0
dem_1000_belg_orig_222	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_223	0.00478	0.0	725k	2822.1	0.6	945k	3600.0
dem_1000_belg_orig_224	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_225	0.00033	0.0	477k	1974.6	1.7	1032k	3600.0
dem_1000_belg_orig_226	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_227	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_228	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_229	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_230	0.00164	0.0	394k	1516.6	0.1	955k	3600.0
dem_1000_belg_orig_231	0.01176	0.0	57k	310.5	0.0	89k	418.0
dem_1000_belg_orig_232	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_233	0.02138	0.0	23k	174.7	0.0	18k	114.8
dem_1000_belg_orig_234	0.0183	0.0	73k	363.8	0.0	34k	166.1
dem_1000_belg_orig_235	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_236	0.00674	0.0	276k	1089.1	0.0	503k	1742.6
dem_1000_belg_orig_237	0.01554	0.0	103k	481.5	0.0	338k	1274.7
dem_1000_belg_orig_238	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_239	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_240	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_241	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_242	0.00656	0.4	680k	3600.0	0.9	821k	3600.0
dem_1000_belg_orig_243	0.0181	1.0	562k	3600.0	1.5	632k	3600.0
dem_1000_belg_orig_244	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_245	0.01852	0.0	18k	100.6	0.0	31k	161.3
dem_1000_belg_orig_246	0.02717	2.8	562k	3600.0	2.0	630k	3600.0
dem_1000_belg_orig_247	0.00518	0.0	80k	324.0	0.0	164k	618.5
dem_1000_belg_orig_248	0.00896	0.5	576k	3600.0	1.3	654k	3600.0
dem_1000_belg_orig_249	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_250	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_251	0.00196	2.0	684k	3600.0	3.3	789k	3600.0
dem_1000_belg_orig_252	0.03881	3.0	555k	3600.0	3.6	568k	3600.0
dem_1000_belg_orig_253	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_254	–	0.0	0	0.0	0.0	0	0.0

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_belg_orig_255	0.01054	0.0	498k	1995.9	0.0	1042k	3600.0
dem_1000_belg_orig_256	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_257	0.0	5.0	687k	3600.0	0.2	1216k	3600.0
dem_1000_belg_orig_258	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_259	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_260	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_261	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_262	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_263	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_264	0.00837	0.7	605k	3600.0	0.4	830k	3600.0
dem_1000_belg_orig_265	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_266	0.0	0.0	6466	26.7	0.0	8756	33.1
dem_1000_belg_orig_267	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_268	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_269	0.01593	0.0	311k	1907.6	0.1	761k	3600.0
dem_1000_belg_orig_270	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_271	0.00435	0.0	17k	87.6	0.0	89k	364.8
dem_1000_belg_orig_272	0.00436	0.0	50k	218.3	0.0	124k	441.4
dem_1000_belg_orig_273	0.00633	0.0	214k	1090.2	0.0	557k	2244.9
dem_1000_belg_orig_274	0.00023	0.0	428k	1945.3	0.2	901k	3600.0
dem_1000_belg_orig_275	0.00172	0.0	51k	292.0	0.0	551k	2422.1
dem_1000_belg_orig_276	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_277	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_278	0.02094	0.0	677k	3237.1	0.0	803k	3291.4
dem_1000_belg_orig_279	0.00376	0.0	157k	526.3	1.2	868k	3600.0
dem_1000_belg_orig_280	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_281	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_282	0.00875	0.0	28k	104.5	0.0	110k	356.4
dem_1000_belg_orig_283	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_284	0.01594	0.0	396k	1964.3	0.2	776k	3600.0
dem_1000_belg_orig_285	0.00581	0.0	336k	1615.2	0.0	823k	3290.7
dem_1000_belg_orig_286	0.00761	0.0	206k	894.1	0.0	553k	2061.7
dem_1000_belg_orig_287	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_288	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_289	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_290	0.009	0.1	795k	3600.0	0.8	838k	3600.0
dem_1000_belg_orig_291	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_292	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_293	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_294	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_295	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_296	0.0	0.0	4675	25.3	0.0	13k	38.0
dem_1000_belg_orig_297	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_298	0.00638	0.0	330k	1602.5	0.0	247k	927.0
dem_1000_belg_orig_299	0.0	0.0	79k	363.2	0.0	249k	1022.5
dem_1000_belg_orig_300	0.0	0.0	1681	9.6	0.0	1732	9.0
dem_1000_belg_orig_301	0.00271	0.0	50k	213.3	0.0	102k	379.4
dem_1000_belg_orig_302	0.0037	0.0	386k	1591.0	0.0	904k	3341.3
dem_1000_belg_orig_303	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_304	0.00484	0.0	403k	1637.1	0.0	940k	3099.7
dem_1000_belg_orig_305	0.00299	0.0	31k	123.9	0.0	370k	983.5
dem_1000_belg_orig_306	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_307	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_308	0.0	0.0	11k	46.3	0.0	10k	37.2
dem_1000_belg_orig_309	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_310	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_311	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_312	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_313	0.00204	0.0	697k	3366.1	0.7	833k	3600.0
dem_1000_belg_orig_314	0.0	0.0	475k	1834.7	0.2	1029k	3600.0
dem_1000_belg_orig_315	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_316	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_317	0.0	1.1	1030k	3600.0	0.0	141k	434.8
dem_1000_belg_orig_318	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_319	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_320	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_321	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_322	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_323	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_324	0.00385	0.0	83k	349.0	0.0	208k	719.9
dem_1000_belg_orig_325	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_326	0.00219	0.0	59k	258.1	0.0	144k	498.1
dem_1000_belg_orig_327	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_328	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_329	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_330	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_331	–	0.0	0	0.0	0.0	0	0.0

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_belg_orig_332	0.00492	0.0	268k	1084.7	0.0	465k	1599.8
dem_1000_belg_orig_333	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_334	0.0121	2.9	882k	3600.0	–	697k	3600.0
dem_1000_belg_orig_335	0.00354	0.0	685k	2463.1	0.0	428k	1573.1
dem_1000_belg_orig_336	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_337	0.01845	0.0	187k	764.2	0.0	820k	2603.4
dem_1000_belg_orig_338	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_339	0.0	0.8	1085k	3600.0	0.8	1233k	3600.0
dem_1000_belg_orig_340	0.00647	0.0	393k	1750.5	0.0	426k	1711.2
dem_1000_belg_orig_341	0.00408	0.0	47k	172.2	0.0	53k	183.4
dem_1000_belg_orig_342	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_343	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_344	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_345	0.02932	0.0	189k	892.1	0.2	835k	3600.0
dem_1000_belg_orig_346	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_347	0.01429	0.7	557k	3600.0	0.9	710k	3600.0
dem_1000_belg_orig_348	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_349	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_350	0.02046	0.0	81k	335.0	0.0	142k	487.7
dem_1000_belg_orig_351	0.00048	0.0	204k	875.9	0.0	166k	622.7
dem_1000_belg_orig_352	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_353	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_354	0.01277	0.8	589k	3600.0	2.0	802k	3600.0
dem_1000_belg_orig_355	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_356	0.00181	0.0	476k	1694.0	0.0	484k	1438.5
dem_1000_belg_orig_357	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_358	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_359	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_360	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_361	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_362	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_363	0.01353	0.8	699k	3600.0	1.4	809k	3600.0
dem_1000_belg_orig_364	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_365	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_366	0.00142	0.3	624k	3600.0	1.1	701k	3600.0
dem_1000_belg_orig_367	0.00116	0.1	856k	3600.0	2.1	816k	3600.0
dem_1000_belg_orig_368	0.00657	0.0	8641	53.6	0.0	13k	70.0
dem_1000_belg_orig_369	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_370	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_371	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_372	0.0	0.0	324k	1192.5	0.0	323k	1132.7
dem_1000_belg_orig_373	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_374	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_375	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_376	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_377	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_378	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_379	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_380	0.0	1.4	871k	3600.0	0.2	880k	3600.0
dem_1000_belg_orig_381	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_382	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_383	0.0	0.0	111k	356.1	0.0	286k	1095.5
dem_1000_belg_orig_384	0.0	0.0	177k	598.6	0.0	105k	334.5
dem_1000_belg_orig_385	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_386	0.00052	0.0	116k	581.5	0.0	106k	436.6
dem_1000_belg_orig_387	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_388	0.01284	0.1	631k	3600.0	1.0	711k	3600.0
dem_1000_belg_orig_389	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_390	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_391	0.03056	0.6	779k	3600.0	1.3	772k	3600.0
dem_1000_belg_orig_392	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_393	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_394	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_395	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_396	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_397	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_398	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_399	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_400	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_401	0.00372	0.0	80k	312.1	0.0	124k	449.9
dem_1000_belg_orig_402	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_403	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_404	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_405	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_406	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_407	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_408	–	0.0	0	0.0	0.0	0	0.0

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_belg_orig_409	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_410	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_411	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_412	0.0084	0.0	167k	623.0	0.0	113k	396.4
dem_1000_belg_orig_413	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_414	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_415	0.0	0.0	757k	2823.0	0.0	625k	2229.0
dem_1000_belg_orig_416	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_417	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_418	0.0	0.0	3050	13.3	0.0	3136	13.8
dem_1000_belg_orig_419	0.00336	0.0	217k	898.8	0.0	266k	824.7
dem_1000_belg_orig_420	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_421	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_422	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_423	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_424	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_425	0.0	0.0	53k	232.2	0.0	28k	114.8
dem_1000_belg_orig_426	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_427	0.0	0.0	937k	3244.7	0.6	1029k	3600.0
dem_1000_belg_orig_428	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_429	0.01209	1.2	1184k	3600.0	0.0	76k	260.9
dem_1000_belg_orig_430	0.00651	2.4	578k	3600.0	18.3	782k	3600.0
dem_1000_belg_orig_431	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_432	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_433	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_434	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_435	0.0	0.0	68k	240.5	0.0	29k	115.0
dem_1000_belg_orig_436	0.0	0.0	248k	1015.8	1.5	1192k	3600.0
dem_1000_belg_orig_437	0.00074	2.9	771k	3600.0	2.9	888k	3600.0
dem_1000_belg_orig_438	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_439	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_440	0.0	0.0	721	5.5	0.0	600	5.2
dem_1000_belg_orig_441	0.01325	0.0	21k	100.8	0.0	24k	118.5
dem_1000_belg_orig_442	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_443	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_444	0.0	0.0	33k	143.6	0.0	130k	546.3
dem_1000_belg_orig_445	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_446	0.00126	0.0	177k	735.8	0.0	401k	1402.2
dem_1000_belg_orig_447	0.00923	0.0	656k	3095.0	0.1	843k	3600.0
dem_1000_belg_orig_448	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_449	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_450	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_451	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_452	0.01061	0.0	39k	196.8	0.0	76k	298.1
dem_1000_belg_orig_453	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_454	0.00404	2.1	834k	3600.0	3.5	975k	3600.0
dem_1000_belg_orig_455	0.0	0.0	707k	3061.2	0.0	519k	1731.6
dem_1000_belg_orig_456	0.0	0.0	45k	185.2	1.4	1490k	3600.0
dem_1000_belg_orig_457	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_458	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_459	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_460	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_461	0.00153	0.0	275k	1178.0	0.0	787k	2469.8
dem_1000_belg_orig_462	0.01439	0.0	504k	2307.9	2.9	769k	3600.0
dem_1000_belg_orig_463	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_464	0.0	0.2	1277k	3600.0	0.4	1243k	3600.0
dem_1000_belg_orig_465	0.0164	0.0	239k	1287.6	0.0	231k	936.5
dem_1000_belg_orig_466	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_467	0.01576	0.0	415k	2139.6	0.0	882k	3500.1
dem_1000_belg_orig_468	0.01539	0.0	36k	202.8	0.0	74k	402.1
dem_1000_belg_orig_469	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_470	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_471	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_472	0.02652	0.2	1085k	3600.0	0.2	741k	3600.0
dem_1000_belg_orig_473	0.00676	2.1	785k	3600.0	0.9	939k	3600.0
dem_1000_belg_orig_474	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_475	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_476	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_477	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_478	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_479	0.02162	3.7	619k	3600.0	2.0	699k	3600.0
dem_1000_belg_orig_480	0.01693	0.0	273k	1205.6	0.5	787k	3600.0
dem_1000_belg_orig_481	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_482	0.01884	0.0	219k	1347.5	0.0	321k	1436.6
dem_1000_belg_orig_483	0.01755	0.0	417k	2233.6	9.4	870k	3600.0
dem_1000_belg_orig_484	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_485	–	0.0	0	0.0	0.0	0	0.0

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Table B.17: Results for ROOTGAP and TREE experiments on *Belgium* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_belg_orig_486	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_487	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_488	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_489	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_490	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_491	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_492	0.01728	0.7	620k	3600.0	1.4	725k	3600.0
dem_1000_belg_orig_493	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_494	0.00154	0.0	87k	348.4	0.0	156k	519.0
dem_1000_belg_orig_495	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_496	0.00288	0.0	782k	3412.2	0.8	901k	3600.0
dem_1000_belg_orig_497	0.00484	0.0	218k	914.8	0.0	864k	3158.3
dem_1000_belg_orig_498	0.00725	0.0	572k	2553.4	0.0	661k	2285.4
dem_1000_belg_orig_499	–	0.0	0	0.0	0.0	0	0.0
dem_1000_belg_orig_500	0.0101	0.7	663k	3600.0	1.6	710k	3600.0

Table B.18: Detailed results for the ROOTGAP and TREE experiments on the *GasLib-40* test set, as summarized in Figure 4.19 and Tables 4.1 – 4.2. For the ROOTGAP experiment, the table lists the gap closed (GC) improvement. A hyphen indicates that the instance is either detected to be infeasible, no primal solution is known, or the dual bound is zero within a tolerance of $\epsilon = 10^{-6}$ for *Enabled* and *Disabled*. For the TREE experiment, the table shows the optimality gap in %, number of B&B-nodes and solving time in seconds for *Enabled* and *Disabled*.

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_50_gaslib40_1	–	0.0	3886	34.9	0.0	5084	43.2
dem_50_gaslib40_2	–	0.0	9670	72.5	0.0	14k	98.8
dem_50_gaslib40_3	–	0.0	4419	33.9	0.0	4546	43.8
dem_50_gaslib40_4	–	0.0	2354	21.4	0.0	2421	22.5
dem_50_gaslib40_5	–	0.0	11k	87.9	0.0	6233	50.0
dem_50_gaslib40_6	–	0.0	7310	55.3	0.0	13k	91.8
dem_50_gaslib40_7	–	0.0	2508	23.0	0.0	10k	75.6
dem_50_gaslib40_8	–	0.0	8863	60.6	0.0	4818	38.2
dem_50_gaslib40_9	–	0.0	7198	56.2	0.0	8170	64.1
dem_50_gaslib40_10	–	0.0	4044	35.0	0.0	4168	35.7
dem_50_gaslib40_11	–	0.0	5316	45.1	0.0	6625	42.1
dem_50_gaslib40_12	–	0.0	3837	54.4	0.0	3876	34.2
dem_50_gaslib40_13	–	0.0	4541	36.3	0.0	3448	25.4
dem_50_gaslib40_14	–	0.0	2560	25.9	0.0	20k	124.0
dem_50_gaslib40_15	–	0.0	2684	25.4	0.0	4119	36.0
dem_50_gaslib40_16	–	0.0	5962	47.6	0.0	11k	80.1
dem_50_gaslib40_17	–	0.0	4923	57.5	0.0	6964	52.6
dem_50_gaslib40_18	–	0.0	2979	31.2	0.0	2079	21.0
dem_50_gaslib40_19	–	0.0	1621	18.1	0.0	5569	42.8
dem_50_gaslib40_20	–	0.0	3737	28.5	0.0	21k	153.7
dem_50_gaslib40_21	–	0.0	16k	98.7	0.0	2515	23.1
dem_50_gaslib40_22	–	0.0	2514	23.7	0.0	8664	62.2
dem_50_gaslib40_23	–	0.0	13k	136.4	0.0	6842	60.4
dem_50_gaslib40_24	–	0.0	11k	84.9	0.0	31k	179.9
dem_50_gaslib40_25	–	0.0	4365	39.2	0.0	2641	28.6
dem_50_gaslib40_26	–	0.0	7510	66.1	0.0	43k	226.1
dem_50_gaslib40_27	–	0.0	2718	25.2	0.0	3994	30.5
dem_50_gaslib40_28	–	0.0	16k	143.4	0.0	4780	34.9
dem_50_gaslib40_29	–	0.0	12k	117.6	0.0	5920	48.3
dem_50_gaslib40_30	–	0.0	1803	22.5	0.0	13k	108.1
dem_50_gaslib40_31	–	0.0	2973	24.0	0.0	3720	30.2
dem_50_gaslib40_32	–	0.0	4262	31.9	0.0	17k	113.7
dem_50_gaslib40_33	–	0.0	3772	38.2	0.0	25k	177.6
dem_50_gaslib40_34	–	0.0	2360	26.7	0.0	4045	37.4
dem_50_gaslib40_35	–	0.0	14k	96.4	0.0	10k	71.8
dem_50_gaslib40_36	–	0.0	15k	89.3	0.0	3246	24.4
dem_50_gaslib40_37	–	0.0	6528	52.9	0.0	6443	41.5

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_50_gaslib40_38	–	0.0	5465	49.5	0.0	68k	369.5
dem_50_gaslib40_39	–	0.0	6359	55.0	0.0	25k	208.8
dem_50_gaslib40_40	–	0.0	4344	41.7	0.0	1923	18.3
dem_50_gaslib40_41	–	0.0	32k	231.0	0.0	22k	178.8
dem_50_gaslib40_42	–	0.0	5350	42.1	0.0	8957	71.6
dem_50_gaslib40_43	–	0.0	6096	68.9	0.0	2266	26.1
dem_50_gaslib40_44	–	0.0	1711	19.4	0.0	1430	16.1
dem_50_gaslib40_45	–	0.0	1148	17.6	0.0	5857	42.1
dem_50_gaslib40_46	–	0.0	4904	36.4	0.0	3950	32.4
dem_50_gaslib40_47	–	0.0	20k	175.5	0.0	10k	80.0
dem_50_gaslib40_48	–	0.0	118k	594.3	0.0	2530	26.9
dem_50_gaslib40_49	–	0.0	2699	26.2	0.0	10k	95.1
dem_50_gaslib40_50	–	0.0	7556	56.3	0.0	3086	25.7
dem_50_gaslib40_51	–	0.0	17k	107.2	0.0	22k	180.1
dem_50_gaslib40_52	–	0.0	4130	39.3	0.0	5822	51.6
dem_50_gaslib40_53	–	0.0	2798	23.5	0.0	7506	53.1
dem_50_gaslib40_54	–	0.0	5220	44.0	0.0	2432	25.6
dem_50_gaslib40_55	–	0.0	1787	21.9	0.0	2457	24.5
dem_50_gaslib40_56	–	0.0	12k	88.7	0.0	10k	65.2
dem_50_gaslib40_57	–	0.0	74k	456.3	0.0	7619	59.2
dem_50_gaslib40_58	–	0.0	2026	22.5	0.0	4825	38.4
dem_50_gaslib40_59	–	0.0	2485	22.8	0.0	3826	27.3
dem_50_gaslib40_60	–	0.0	2049	22.3	0.0	3841	28.6
dem_50_gaslib40_61	–	0.0	13k	104.4	0.0	2945	25.4
dem_50_gaslib40_62	–	0.0	2151	22.9	0.0	3440	31.1
dem_50_gaslib40_63	–	0.0	13k	73.0	0.0	3655	31.3
dem_50_gaslib40_64	–	0.0	2665	24.9	0.0	8051	58.4
dem_50_gaslib40_65	–	0.0	1519	18.3	0.0	7151	75.7
dem_50_gaslib40_66	–	0.0	2383	23.0	0.0	8735	60.5
dem_50_gaslib40_67	–	0.0	4696	32.4	0.0	2102	20.4
dem_50_gaslib40_68	–	0.0	3155	31.1	0.0	18k	122.0
dem_50_gaslib40_69	–	0.0	2718	26.5	0.0	8437	57.7
dem_50_gaslib40_70	–	0.0	2335	24.0	0.0	7678	57.7
dem_50_gaslib40_71	–	0.0	7091	55.1	0.0	65k	402.4
dem_50_gaslib40_72	–	0.0	5923	73.0	0.0	2022	20.1
dem_50_gaslib40_73	–	0.0	4991	51.4	0.0	43k	241.2
dem_50_gaslib40_74	–	0.0	2847	24.7	0.0	6423	45.1
dem_50_gaslib40_75	–	0.0	4835	39.9	0.0	4759	33.4
dem_50_gaslib40_76	–	0.0	2611	26.9	0.0	170k	866.5
dem_50_gaslib40_77	–	0.0	14k	129.6	0.0	8712	72.7
dem_50_gaslib40_78	–	0.0	4237	43.5	0.0	4959	40.6
dem_50_gaslib40_79	–	0.0	19k	136.8	0.0	2946	26.4
dem_50_gaslib40_80	–	0.0	55k	371.8	0.0	27k	212.3
dem_50_gaslib40_81	–	0.0	10k	73.6	0.0	5845	45.9
dem_50_gaslib40_82	–	0.0	2213	22.6	0.0	1745	20.9
dem_50_gaslib40_83	–	0.0	5482	44.6	0.0	7088	48.4
dem_50_gaslib40_84	–	0.0	2158	21.4	0.0	4545	35.6
dem_50_gaslib40_85	–	0.0	3540	34.1	0.0	4950	41.9
dem_50_gaslib40_86	–	0.0	7596	53.4	0.0	32k	221.2
dem_50_gaslib40_87	–	0.0	4231	37.3	0.0	1909	20.6
dem_50_gaslib40_88	–	0.0	1436	18.4	0.0	1168	15.6
dem_50_gaslib40_89	–	0.0	6698	43.9	0.0	4571	35.4
dem_50_gaslib40_90	–	0.0	8643	64.1	0.0	4530	32.0
dem_50_gaslib40_91	–	0.0	3032	30.4	0.0	4429	30.6
dem_50_gaslib40_92	–	0.0	8686	61.3	0.0	6943	62.8
dem_50_gaslib40_93	–	0.0	64k	309.6	0.0	3419	33.9
dem_50_gaslib40_94	–	0.0	18k	113.8	0.0	4585	36.5
dem_50_gaslib40_95	–	0.0	15k	86.5	0.0	11k	57.8
dem_50_gaslib40_96	–	0.0	4239	37.9	0.0	17k	126.0
dem_50_gaslib40_97	–	0.0	4424	35.9	0.0	14k	81.4
dem_50_gaslib40_98	–	0.0	1926	18.4	0.0	14k	104.8
dem_50_gaslib40_99	–	0.0	24k	167.4	0.0	4646	41.6
dem_50_gaslib40_100	–	0.0	6376	88.6	0.0	6655	51.4
dem_50_gaslib40_101	–	0.0	5882	44.1	0.0	10k	67.3
dem_50_gaslib40_102	–	0.0	3985	39.2	0.0	8390	53.5
dem_50_gaslib40_103	–	0.0	3673	44.5	0.0	1181	16.3
dem_50_gaslib40_104	–	0.0	5997	41.5	0.0	2987	26.8
dem_50_gaslib40_105	–	0.0	3407	25.3	0.0	8066	64.5
dem_50_gaslib40_106	–	0.0	4013	29.3	0.0	5459	41.2
dem_50_gaslib40_107	–	0.0	7030	58.7	0.0	7390	50.3
dem_50_gaslib40_108	–	0.0	7115	49.4	0.0	5820	44.4
dem_50_gaslib40_109	–	0.0	5217	40.6	0.0	3039	21.6
dem_50_gaslib40_110	–	0.0	4442	39.3	0.0	2996	27.6
dem_50_gaslib40_111	–	0.0	5961	45.1	0.0	14k	123.0
dem_50_gaslib40_112	–	0.0	6064	52.7	0.0	12k	81.5
dem_50_gaslib40_113	–	0.0	2536	24.3	0.0	3154	27.9
dem_50_gaslib40_114	–	0.0	2504	25.3	0.0	3781	32.9

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_50_gaslib40_115	–	0.0	10k	97.6	0.0	4264	36.3
dem_50_gaslib40_116	–	0.0	3847	34.5	0.0	2780	22.5
dem_50_gaslib40_117	–	0.0	4640	37.6	0.0	8138	67.8
dem_50_gaslib40_118	–	0.0	2819	33.9	0.0	2225	22.7
dem_50_gaslib40_119	–	0.0	3064	28.1	0.0	4621	36.7
dem_50_gaslib40_120	–	0.0	3502	26.5	0.0	10k	75.0
dem_50_gaslib40_121	–	0.0	5050	38.7	0.0	1281	15.9
dem_50_gaslib40_122	–	0.0	3187	34.0	0.0	4692	35.4
dem_50_gaslib40_123	–	0.0	3202	35.4	0.0	3036	29.8
dem_50_gaslib40_124	–	0.0	5619	63.7	0.0	9618	75.9
dem_50_gaslib40_125	–	0.0	7588	50.7	0.0	7353	53.0
dem_50_gaslib40_126	–	0.0	5790	46.4	0.0	26k	152.4
dem_50_gaslib40_127	–	0.0	3705	45.2	0.0	3468	30.2
dem_50_gaslib40_128	–	0.0	1871	25.7	0.0	5660	45.5
dem_50_gaslib40_129	–	0.0	9230	68.9	0.0	2302	25.6
dem_50_gaslib40_130	–	0.0	6065	61.0	0.0	4790	38.2
dem_50_gaslib40_131	–	0.0	3326	28.1	0.0	3052	27.2
dem_50_gaslib40_132	–	0.0	3594	35.9	0.0	2794	23.9
dem_50_gaslib40_133	–	0.0	2818	26.5	0.0	6816	44.1
dem_50_gaslib40_134	–	0.0	3103	25.6	0.0	6398	46.5
dem_50_gaslib40_135	–	0.0	31k	128.0	0.0	10k	76.3
dem_50_gaslib40_136	–	0.0	2463	20.8	0.0	18k	122.0
dem_50_gaslib40_137	–	0.0	4150	35.0	0.0	5324	38.7
dem_50_gaslib40_138	–	0.0	37k	238.4	0.0	30k	229.2
dem_50_gaslib40_139	–	0.0	14k	106.0	0.0	26k	193.0
dem_50_gaslib40_140	–	0.0	5382	39.7	0.0	7098	52.2
dem_50_gaslib40_141	–	0.0	4999	42.3	0.0	9181	56.7
dem_50_gaslib40_142	–	0.0	4934	51.2	0.0	18k	107.0
dem_50_gaslib40_143	–	0.0	5968	48.7	0.0	6850	45.6
dem_50_gaslib40_144	–	0.0	8647	59.3	0.0	5189	40.3
dem_50_gaslib40_145	–	0.0	4912	43.8	0.0	7403	53.4
dem_50_gaslib40_146	–	0.0	6397	50.4	0.0	7319	52.2
dem_50_gaslib40_147	–	0.0	1801	20.1	0.0	7677	52.2
dem_50_gaslib40_148	–	0.0	7133	54.6	0.0	3806	31.5
dem_50_gaslib40_149	–	0.0	17k	112.0	0.0	2555	25.3
dem_50_gaslib40_150	–	0.0	2415	25.1	0.0	31k	228.9
dem_50_gaslib40_151	–	0.0	2623	27.4	0.0	10k	80.5
dem_50_gaslib40_152	–	0.0	13k	145.0	0.0	15k	115.9
dem_50_gaslib40_153	–	0.0	3059	27.4	0.0	2505	22.9
dem_50_gaslib40_154	–	0.0	5089	49.2	0.0	3448	26.2
dem_50_gaslib40_155	–	0.0	2074	18.9	0.0	36k	226.6
dem_50_gaslib40_156	–	0.0	10k	76.8	0.0	2855	25.0
dem_50_gaslib40_157	–	0.0	2838	36.1	0.0	5806	43.4
dem_50_gaslib40_158	–	0.0	2266	22.4	0.0	1390	19.6
dem_50_gaslib40_159	–	0.0	39k	261.3	19.9	827k	3600.0
dem_50_gaslib40_160	–	0.0	2969	26.0	0.0	7778	60.8
dem_50_gaslib40_161	–	0.0	2760	27.4	0.0	2334	26.0
dem_50_gaslib40_162	–	0.0	6137	57.8	0.0	6013	45.6
dem_50_gaslib40_163	–	0.0	4545	38.2	0.0	6567	48.9
dem_50_gaslib40_164	–	0.0	6412	46.4	0.0	14k	101.0
dem_50_gaslib40_165	–	0.0	6983	48.7	0.0	5787	48.4
dem_50_gaslib40_166	–	0.0	5334	44.8	0.0	3118	27.9
dem_50_gaslib40_167	–	0.0	5805	43.6	0.0	4953	38.9
dem_50_gaslib40_168	–	0.0	5822	38.1	0.0	19k	110.1
dem_50_gaslib40_169	0.01085	0.0	22k	153.2	0.0	120k	634.7
dem_50_gaslib40_170	–	0.0	10k	89.5	0.0	2401	21.6
dem_50_gaslib40_171	–	0.0	3531	25.8	0.0	11k	80.1
dem_50_gaslib40_172	–	0.0	3639	32.1	0.0	10k	59.3
dem_50_gaslib40_173	–	0.0	2676	27.2	0.0	14k	83.8
dem_50_gaslib40_174	–	0.0	5825	49.9	0.0	12k	90.6
dem_50_gaslib40_175	–	0.0	4646	33.1	0.0	4165	29.5
dem_50_gaslib40_176	–	0.0	4381	56.4	0.0	2716	21.9
dem_50_gaslib40_177	–	0.0	145k	591.2	0.0	5537	45.8
dem_50_gaslib40_178	–	0.0	1918	19.4	0.0	2180	22.6
dem_50_gaslib40_179	–	0.0	13k	109.6	0.0	5594	49.0
dem_50_gaslib40_180	–	0.0	10k	88.8	0.0	13k	97.6
dem_50_gaslib40_181	–	0.0	2279	22.0	0.0	2481	22.6
dem_50_gaslib40_182	–	0.0	6739	72.5	0.0	10k	65.3
dem_50_gaslib40_183	–	0.0	8103	78.5	0.0	4363	39.4
dem_50_gaslib40_184	–	0.0	5220	40.1	0.0	10k	79.6
dem_50_gaslib40_185	–	0.0	3665	27.9	0.0	3793	30.6
dem_50_gaslib40_186	–	0.0	3972	28.6	0.0	3167	26.9
dem_50_gaslib40_187	–	0.0	7386	58.7	0.0	10k	60.0
dem_50_gaslib40_188	–	2860.5	727k	3600.0	0.0	13k	99.2
dem_50_gaslib40_189	–	0.0	6769	62.7	0.0	6674	50.9
dem_50_gaslib40_190	–	0.0	8630	69.8	0.0	19k	122.5
dem_50_gaslib40_191	–	0.0	48k	303.2	0.0	17k	123.4

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_50_gaslib40_192	–	0.0	6631	52.4	0.0	16k	122.7
dem_50_gaslib40_193	–	0.0	3559	39.7	0.0	2348	24.1
dem_50_gaslib40_194	–	0.0	4534	34.0	0.0	3271	26.4
dem_50_gaslib40_195	–	0.0	1721	20.2	0.0	3210	25.4
dem_50_gaslib40_196	–	0.0	2031	23.0	0.0	3935	34.1
dem_50_gaslib40_197	–	0.0	3489	42.1	0.0	2698	22.1
dem_50_gaslib40_198	–	0.0	16k	121.0	0.0	9047	63.6
dem_50_gaslib40_199	–	0.0	1292	16.7	0.0	6711	53.5
dem_50_gaslib40_200	–	0.0	2436	19.8	0.0	5915	54.5
dem_50_gaslib40_201	–	0.0	3164	36.6	0.0	10k	67.0
dem_50_gaslib40_202	–	0.0	1846	19.2	0.0	9132	60.9
dem_50_gaslib40_203	–	0.0	2372	23.6	0.0	12k	65.5
dem_50_gaslib40_204	–	0.0	18k	124.2	0.0	12k	73.9
dem_50_gaslib40_205	–	0.0	3607	30.2	0.0	6412	46.6
dem_50_gaslib40_206	–	0.0	4672	36.7	0.0	11k	73.7
dem_50_gaslib40_207	–	0.0	3118	28.5	0.0	15k	103.4
dem_50_gaslib40_208	–	0.0	2417	22.3	0.0	8264	54.8
dem_50_gaslib40_209	–	0.0	8443	51.8	0.0	2466	21.8
dem_50_gaslib40_210	–	0.0	16k	137.7	0.0	10k	72.6
dem_50_gaslib40_211	–	0.0	6549	55.5	0.0	11k	71.9
dem_50_gaslib40_212	–	0.0	3060	46.5	0.0	2571	24.6
dem_50_gaslib40_213	–	0.0	3180	24.8	0.0	5623	35.1
dem_50_gaslib40_214	–	0.0	5335	45.0	0.0	2773	25.4
dem_50_gaslib40_215	–	0.0	1956	20.5	0.0	20k	133.4
dem_50_gaslib40_216	–	0.0	1644	21.0	0.0	1386	16.0
dem_50_gaslib40_217	–	0.0	1950	18.8	0.0	2474	20.9
dem_50_gaslib40_218	–	0.0	92k	628.8	0.0	18k	121.8
dem_50_gaslib40_219	–	0.0	40k	241.6	0.0	9251	61.7
dem_50_gaslib40_220	–	0.0	3624	27.0	0.0	2616	27.6
dem_50_gaslib40_221	–	0.0	8712	64.7	0.0	95k	419.9
dem_50_gaslib40_222	–	0.0	5178	42.2	0.0	8357	62.4
dem_50_gaslib40_223	–	0.0	5793	43.4	0.0	8148	67.4
dem_50_gaslib40_224	–	0.0	14k	141.7	0.0	74k	551.7
dem_50_gaslib40_225	–	0.0	2089	22.6	0.0	4487	35.2
dem_50_gaslib40_226	–	0.0	15k	93.0	0.0	5756	38.9
dem_50_gaslib40_227	0.13312	77.0	410k	3600.0	0.0	10k	80.7
dem_50_gaslib40_228	–	0.0	6473	54.9	0.0	3616	29.7
dem_50_gaslib40_229	–	0.0	5424	63.3	0.0	1282	19.5
dem_50_gaslib40_230	–	0.0	8570	65.3	0.0	9331	80.0
dem_50_gaslib40_231	–	0.0	10k	91.8	0.0	10k	65.5
dem_50_gaslib40_232	–	0.0	2202	27.3	0.0	14k	100.2
dem_50_gaslib40_233	–	0.0	2194	20.8	0.0	3373	27.0
dem_50_gaslib40_234	–	0.0	18k	118.1	0.0	2514	23.8
dem_50_gaslib40_235	–	0.0	27k	163.5	0.0	38k	261.1
dem_50_gaslib40_236	–	0.0	6163	40.3	0.0	12k	97.6
dem_50_gaslib40_237	–	0.0	2044	21.1	0.0	5483	39.3
dem_50_gaslib40_238	–	0.0	3745	32.0	0.0	3836	31.8
dem_50_gaslib40_239	–	0.0	58k	256.3	0.0	20k	126.4
dem_50_gaslib40_240	–	0.0	10k	77.0	0.0	9712	69.8
dem_50_gaslib40_241	–	0.0	5186	39.9	0.0	3676	33.0
dem_50_gaslib40_242	–	0.0	41k	258.0	0.0	14k	93.4
dem_50_gaslib40_243	–	0.0	2547	26.8	0.0	7901	56.0
dem_50_gaslib40_244	–	0.0	3863	26.9	0.0	13k	84.0
dem_50_gaslib40_245	–	0.0	11k	96.8	0.0	5832	46.1
dem_50_gaslib40_246	–	0.0	10k	92.4	0.0	8690	56.9
dem_50_gaslib40_247	–	0.0	5294	39.0	0.0	4008	33.6
dem_50_gaslib40_248	–	0.0	4320	47.8	0.0	39k	217.3
dem_50_gaslib40_249	–	0.0	4259	33.1	0.0	7123	47.2
dem_50_gaslib40_250	–	0.0	5450	47.4	0.0	3410	30.4
dem_50_gaslib40_251	–	0.0	6571	50.4	0.0	9937	57.1
dem_50_gaslib40_252	–	0.0	23k	156.5	0.0	25k	137.8
dem_50_gaslib40_253	–	0.0	13k	112.6	0.0	11k	83.5
dem_50_gaslib40_254	–	0.0	2182	22.3	0.0	5308	42.0
dem_50_gaslib40_255	–	0.0	6589	70.7	0.0	11k	88.0
dem_50_gaslib40_256	–	0.0	2221	28.4	0.0	7804	55.8
dem_50_gaslib40_257	–	0.0	2162	26.0	0.0	23k	147.0
dem_50_gaslib40_258	–	0.0	1720	21.8	0.0	8513	61.1
dem_50_gaslib40_259	–	0.0	7419	77.5	0.0	2847	29.1
dem_50_gaslib40_260	–	0.0	3873	36.9	0.0	4991	41.3
dem_50_gaslib40_261	0.0	0.0	2681	25.8	0.0	4025	43.8
dem_50_gaslib40_262	–	0.0	1333	19.0	0.0	3256	27.6
dem_50_gaslib40_263	–	0.0	1818	19.0	0.0	3643	28.7
dem_50_gaslib40_264	–	0.0	133k	816.4	0.0	508k	1633.2
dem_50_gaslib40_265	–	0.0	2842	33.5	0.0	4893	37.3
dem_50_gaslib40_266	–	0.0	11k	75.1	0.0	1717	19.7
dem_50_gaslib40_267	–	0.0	7098	70.7	0.0	12k	85.1
dem_50_gaslib40_268	–	0.0	5586	49.8	0.0	34k	237.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_50_gaslib40_269	0.0681	0.0	7860	69.7	0.0	17k	117.8
dem_50_gaslib40_270	–	0.0	1778	19.4	0.0	16k	102.6
dem_50_gaslib40_271	–	0.0	351k	1599.5	960.8	480k	3600.0
dem_50_gaslib40_272	–	0.0	6668	58.9	0.0	4801	42.1
dem_50_gaslib40_273	–	0.0	5197	40.7	0.0	2559	24.8
dem_50_gaslib40_274	–	0.0	6959	97.0	0.0	11k	71.0
dem_50_gaslib40_275	–	0.0	7130	53.4	0.0	16k	120.4
dem_50_gaslib40_276	–	0.0	2121	21.4	0.0	14k	103.0
dem_50_gaslib40_277	–	0.0	4961	41.3	0.0	6306	43.2
dem_50_gaslib40_278	–	0.0	3456	29.7	0.0	5658	38.8
dem_50_gaslib40_279	–	0.0	3094	27.4	0.0	4239	29.4
dem_50_gaslib40_280	–	0.0	2522	27.1	0.0	2549	23.3
dem_50_gaslib40_281	–	0.0	2018	20.9	0.0	10k	72.6
dem_50_gaslib40_282	–	0.0	2430	26.7	0.0	4281	30.6
dem_50_gaslib40_283	–	0.0	5115	44.8	0.0	9393	67.7
dem_50_gaslib40_284	–	0.0	18k	129.7	0.0	7822	51.1
dem_50_gaslib40_285	–	0.0	36k	246.3	0.0	51k	291.2
dem_50_gaslib40_286	–	0.0	10k	91.8	0.0	8523	60.8
dem_50_gaslib40_287	–	0.0	2881	23.8	0.0	18k	115.7
dem_50_gaslib40_288	–	0.0	3533	29.1	0.0	8409	47.8
dem_50_gaslib40_289	–	0.0	8188	94.3	0.0	2668	23.6
dem_50_gaslib40_290	–	0.0	5347	61.1	0.0	14k	115.2
dem_50_gaslib40_291	–	0.0	2703	34.5	0.0	10k	72.6
dem_50_gaslib40_292	–	0.0	5320	47.8	0.0	7650	56.4
dem_50_gaslib40_293	–	0.0	2781	26.2	0.0	7070	49.0
dem_50_gaslib40_294	–	0.0	4633	30.1	0.0	7325	57.3
dem_50_gaslib40_295	–	0.0	3689	33.8	0.0	136k	907.9
dem_50_gaslib40_296	–	0.0	11k	72.2	0.0	13k	86.2
dem_50_gaslib40_297	–	0.0	5908	57.5	0.0	1118	17.5
dem_50_gaslib40_298	–	0.0	3483	28.8	0.0	5759	46.4
dem_50_gaslib40_299	–	0.0	2784	28.0	0.0	7657	61.6
dem_50_gaslib40_300	–	0.0	2082	20.1	0.0	10k	70.3
dem_50_gaslib40_301	–	0.0	2227	21.1	0.0	12k	80.7
dem_50_gaslib40_302	–	0.0	14k	110.4	0.0	4768	36.8
dem_50_gaslib40_303	–	0.0	17k	117.5	0.0	1788	20.9
dem_50_gaslib40_304	–	0.0	6181	55.3	0.0	7213	51.8
dem_50_gaslib40_305	–	0.0	4797	34.8	0.0	1781	22.4
dem_50_gaslib40_306	–	0.0	7115	51.1	0.0	4955	47.8
dem_50_gaslib40_307	–	0.0	2882	26.2	0.0	23k	121.4
dem_50_gaslib40_308	–	0.0	3626	35.5	0.0	2248	23.4
dem_50_gaslib40_309	–	0.0	3639	33.9	0.0	16k	89.8
dem_50_gaslib40_310	–	0.0	10k	70.6	0.0	2771	25.8
dem_50_gaslib40_311	–	0.0	13k	109.0	0.0	3462	31.8
dem_50_gaslib40_312	–	0.0	13k	131.9	0.0	1991	19.2
dem_50_gaslib40_313	–	0.0	2631	19.8	0.0	11k	75.5
dem_50_gaslib40_314	–	0.0	900	15.9	0.0	3948	34.2
dem_50_gaslib40_315	–	0.0	10k	79.1	0.0	11k	79.9
dem_50_gaslib40_316	–	0.0	14k	143.9	0.0	25k	184.7
dem_50_gaslib40_317	–	0.0	2635	24.4	0.0	22k	166.3
dem_50_gaslib40_318	–	0.0	9254	68.6	0.0	8433	53.5
dem_50_gaslib40_319	–	0.0	20k	156.9	0.0	3908	33.0
dem_50_gaslib40_320	–	0.0	2329	26.7	0.0	3248	26.9
dem_50_gaslib40_321	–	0.0	3303	28.7	0.0	29k	141.2
dem_50_gaslib40_322	–	0.0	4292	40.4	0.0	18k	150.4
dem_50_gaslib40_323	–	0.0	8668	71.5	0.0	3183	24.3
dem_50_gaslib40_324	–	0.0	9599	79.6	0.0	16k	101.0
dem_50_gaslib40_325	–	0.0	5115	39.8	0.0	17k	101.2
dem_50_gaslib40_326	–	0.0	5149	49.7	0.0	2459	21.1
dem_50_gaslib40_327	–	0.0	4977	43.5	0.0	21k	136.2
dem_50_gaslib40_328	–	0.0	2683	23.1	0.0	2245	21.4
dem_50_gaslib40_329	–	0.0	7565	62.6	0.0	16k	119.0
dem_50_gaslib40_330	–	0.0	5727	40.5	0.0	4788	35.3
dem_50_gaslib40_331	–	0.0	2376	25.9	0.0	9648	59.1
dem_50_gaslib40_332	–	0.0	12k	60.0	0.0	3327	32.8
dem_50_gaslib40_333	–	0.0	4447	40.7	0.0	1546	18.7
dem_50_gaslib40_334	–	0.0	7649	65.5	0.0	9409	62.9
dem_50_gaslib40_335	–	0.0	1576	23.9	0.0	11k	97.5
dem_50_gaslib40_336	–	0.0	4989	43.5	0.0	49k	289.6
dem_50_gaslib40_337	–	0.0	6372	55.2	0.0	5816	47.4
dem_50_gaslib40_338	–	0.0	2152	25.1	0.0	3183	28.4
dem_50_gaslib40_339	–	0.0	6248	57.8	0.0	5418	40.1
dem_50_gaslib40_340	–	0.0	5595	32.9	0.0	18k	130.6
dem_50_gaslib40_341	–	0.0	4630	49.0	0.0	7910	51.3
dem_50_gaslib40_342	–	0.0	3523	46.6	0.0	23k	144.3
dem_50_gaslib40_343	–	0.0	3638	39.3	0.0	15k	124.0
dem_50_gaslib40_344	–	0.0	3246	27.6	0.0	5912	34.1
dem_50_gaslib40_345	–	0.0	3828	31.6	0.0	5679	49.8

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Table B.18: Results for `ROOTGAP` and `TREE` experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_50_gaslib40_346	–	0.0	1598	20.7	0.0	4437	37.7
dem_50_gaslib40_347	–	0.0	4529	50.8	0.0	1288	17.2
dem_50_gaslib40_348	–	0.0	18k	166.2	0.0	4251	31.9
dem_50_gaslib40_349	–	0.0	4214	38.9	0.0	1981	20.2
dem_50_gaslib40_350	–	0.0	15k	151.2	0.0	3624	31.5
dem_50_gaslib40_351	–	0.0	4685	51.9	0.0	9031	58.9
dem_50_gaslib40_352	–	0.0	4255	43.7	0.0	8430	59.2
dem_50_gaslib40_353	–	0.0	3175	33.2	0.0	2347	22.2
dem_50_gaslib40_354	–	0.0	3004	25.6	0.0	12k	82.7
dem_50_gaslib40_355	–	0.0	2948	35.7	0.0	12k	84.2
dem_50_gaslib40_356	–	0.0	2084	21.7	0.0	3760	28.1
dem_50_gaslib40_357	–	0.0	6642	48.1	0.0	6887	55.7
dem_50_gaslib40_358	–	0.0	1363	18.5	0.0	5919	46.2
dem_50_gaslib40_359	–	0.0	2300	23.7	0.0	8713	73.2
dem_50_gaslib40_360	–	0.0	3251	24.9	0.0	2894	25.2
dem_50_gaslib40_361	–	0.0	4690	44.0	0.0	7444	46.4
dem_50_gaslib40_362	–	0.0	6565	59.4	0.0	16k	116.5
dem_50_gaslib40_363	–	0.0	4571	48.2	0.0	3600	36.9
dem_50_gaslib40_364	–	0.0	6670	52.8	0.0	5400	34.7
dem_50_gaslib40_365	–	0.0	10k	87.7	0.0	9054	65.7
dem_50_gaslib40_366	–	0.0	3403	28.5	0.0	3753	35.2
dem_50_gaslib40_367	–	0.0	8021	51.9	0.0	16k	125.1
dem_50_gaslib40_368	–	0.0	20k	135.8	0.0	10k	80.0
dem_50_gaslib40_369	–	0.0	26k	177.0	0.0	5885	50.0
dem_50_gaslib40_370	–	0.0	2268	22.5	0.0	99k	766.0
dem_50_gaslib40_371	–	0.0	175k	1010.0	0.0	31k	190.6
dem_50_gaslib40_372	–	0.0	7973	65.9	0.0	980	14.0
dem_50_gaslib40_373	–	0.0	5468	48.5	0.0	11k	71.4
dem_50_gaslib40_374	–	0.0	2140	21.2	0.0	12k	80.7
dem_50_gaslib40_375	–	0.0	2831	27.1	0.0	9114	62.1
dem_50_gaslib40_376	–	0.0	2516	30.6	0.0	12k	81.1
dem_50_gaslib40_377	–	0.0	3425	27.7	0.0	3437	32.8
dem_50_gaslib40_378	–	0.0	7801	85.9	0.0	6889	47.5
dem_50_gaslib40_379	–	0.0	19k	140.0	0.0	5149	50.3
dem_50_gaslib40_380	–	0.0	3351	28.1	0.0	10k	83.6
dem_50_gaslib40_381	–	0.0	10k	77.2	0.0	50k	342.0
dem_50_gaslib40_382	–	0.0	4729	46.0	0.0	10k	68.6
dem_50_gaslib40_383	–	0.0	10k	83.2	0.0	281k	1809.2
dem_50_gaslib40_384	–	0.0	3865	35.1	0.0	2738	24.6
dem_50_gaslib40_385	–	0.0	3849	45.6	0.0	9704	73.5
dem_50_gaslib40_386	–	0.0	28k	160.9	0.0	29k	194.6
dem_50_gaslib40_387	–	0.0	3482	26.5	0.0	22k	142.2
dem_50_gaslib40_388	–	0.0	3868	32.7	0.0	2985	22.9
dem_50_gaslib40_389	–	0.0	12k	87.4	0.0	5091	36.6
dem_50_gaslib40_390	–	0.0	2283	24.0	0.0	6654	52.8
dem_50_gaslib40_391	–	0.0	2093	20.1	0.0	4241	32.1
dem_50_gaslib40_392	–	0.0	6667	55.6	0.0	11k	77.9
dem_50_gaslib40_393	–	0.0	11k	86.6	0.0	6424	46.3
dem_50_gaslib40_394	–	0.0	11k	78.3	0.0	27k	146.8
dem_50_gaslib40_395	–	0.0	29k	210.7	0.0	2643	27.4
dem_50_gaslib40_396	–	0.0	54k	331.1	0.0	5129	38.0
dem_50_gaslib40_397	–	0.0	12k	106.2	0.0	50k	310.0
dem_50_gaslib40_398	–	0.0	6400	47.4	0.0	3846	28.8
dem_50_gaslib40_399	0.0	0.0	2513	25.5	0.0	1573	14.4
dem_50_gaslib40_400	–	0.0	16k	131.4	0.0	13k	94.4
dem_50_gaslib40_401	–	0.0	3371	27.2	0.0	5487	42.1
dem_50_gaslib40_402	–	0.0	15k	96.6	0.0	26k	176.4
dem_50_gaslib40_403	–	0.0	10k	88.0	0.0	2327	22.1
dem_50_gaslib40_404	–	0.0	25k	202.6	0.0	15k	152.3
dem_50_gaslib40_405	–	0.0	6679	58.3	0.0	3820	26.3
dem_50_gaslib40_406	–	0.0	6439	53.5	0.0	3941	41.5
dem_50_gaslib40_407	–	0.0	5450	72.5	0.0	6198	46.4
dem_50_gaslib40_408	–	0.0	18k	156.1	0.0	20k	170.4
dem_50_gaslib40_409	–	0.0	5523	45.7	0.0	6231	49.5
dem_50_gaslib40_410	–	0.0	2461	25.4	0.0	17k	108.0
dem_50_gaslib40_411	–	0.0	4569	40.1	0.0	13k	101.5
dem_50_gaslib40_412	–	0.0	3596	48.9	0.0	2240	22.8
dem_50_gaslib40_413	–	0.0	3805	46.4	0.0	5073	41.0
dem_50_gaslib40_414	–	0.0	6721	59.8	0.0	2830	22.9
dem_50_gaslib40_415	–	0.0	5012	38.1	0.0	1952	19.6
dem_50_gaslib40_416	–	0.0	1781	24.1	0.0	5105	47.5
dem_50_gaslib40_417	–	0.0	2980	24.6	0.0	3336	27.6
dem_50_gaslib40_418	–	0.0	19k	153.5	0.0	3475	29.9
dem_50_gaslib40_419	–	0.0	3008	26.9	0.0	2004	19.3
dem_50_gaslib40_420	–	0.0	2495	26.4	0.0	2719	25.3
dem_50_gaslib40_421	–	0.0	2665	22.9	0.0	17k	125.3
dem_50_gaslib40_422	–	0.0	4131	35.4	0.0	6559	43.4

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_50_gaslib40_423	–	0.0	10k	61.0	0.0	6179	44.0
dem_50_gaslib40_424	–	0.0	2474	20.6	0.0	4386	37.2
dem_50_gaslib40_425	–	0.0	2967	27.4	0.0	6996	69.3
dem_50_gaslib40_426	–	0.0	3963	49.8	0.0	7401	63.7
dem_50_gaslib40_427	–	0.0	7349	58.2	0.0	2742	25.5
dem_50_gaslib40_428	–	0.0	4539	40.1	0.0	3685	29.0
dem_50_gaslib40_429	–	0.0	3328	27.9	0.0	17k	119.5
dem_50_gaslib40_430	–	0.0	6848	84.7	0.0	1882	19.1
dem_50_gaslib40_431	–	0.0	6702	56.9	0.0	2274	26.3
dem_50_gaslib40_432	–	0.0	2032	22.1	0.0	27k	193.2
dem_50_gaslib40_433	–	0.0	3757	33.5	0.0	16k	104.7
dem_50_gaslib40_434	–	0.0	2465	18.9	0.0	4115	32.0
dem_50_gaslib40_435	–	0.0	5264	48.8	0.0	6463	46.3
dem_50_gaslib40_436	–	0.0	4055	48.3	0.0	4056	31.7
dem_50_gaslib40_437	–	0.0	4201	45.0	0.0	6273	49.8
dem_50_gaslib40_438	–	0.0	1421	17.0	0.0	24k	174.0
dem_50_gaslib40_439	–	0.0	18k	165.4	0.0	12k	80.5
dem_50_gaslib40_440	–	0.0	4756	41.7	0.0	5232	37.2
dem_50_gaslib40_441	–	0.0	7586	67.4	0.0	43k	232.8
dem_50_gaslib40_442	–	0.0	4011	36.8	0.0	6252	48.5
dem_50_gaslib40_443	–	0.0	3106	29.4	0.0	9846	69.6
dem_50_gaslib40_444	–	0.0	9541	70.3	0.0	47k	321.4
dem_50_gaslib40_445	–	0.0	2238	22.1	0.0	6370	40.2
dem_50_gaslib40_446	–	0.0	6083	60.6	0.0	3519	31.0
dem_50_gaslib40_447	–	0.0	3725	31.2	0.0	4668	34.8
dem_50_gaslib40_448	–	0.0	14k	78.0	0.0	2183	21.6
dem_50_gaslib40_449	–	0.0	7224	54.1	0.0	21k	119.1
dem_50_gaslib40_450	–	0.0	2358	25.1	0.0	1607	19.9
dem_50_gaslib40_451	–	0.0	2641	24.9	0.0	12k	70.7
dem_50_gaslib40_452	–	0.0	9011	81.7	0.0	2962	25.7
dem_50_gaslib40_453	–	0.0	3951	52.5	0.0	2165	24.9
dem_50_gaslib40_454	–	0.0	4909	39.7	0.0	13k	97.2
dem_50_gaslib40_455	–	0.0	1861	27.0	0.0	2628	30.7
dem_50_gaslib40_456	–	0.0	4295	38.9	0.0	51k	329.2
dem_50_gaslib40_457	–	0.0	3581	30.9	0.0	5027	35.0
dem_50_gaslib40_458	–	0.0	17k	105.0	0.0	40k	258.6
dem_50_gaslib40_459	–	0.0	5918	47.5	0.0	3251	29.7
dem_50_gaslib40_460	–	0.0	6396	64.6	0.0	3767	34.8
dem_50_gaslib40_461	–	0.0	25k	161.6	0.0	2230	23.4
dem_50_gaslib40_462	–	11.2	699k	3600.0	83.9	601k	3600.0
dem_50_gaslib40_463	–	0.0	2780	22.1	0.0	4138	33.5
dem_50_gaslib40_464	–	0.0	4538	49.8	0.0	10k	89.4
dem_50_gaslib40_465	–	0.0	35k	313.4	0.0	35k	221.5
dem_50_gaslib40_466	–	0.0	5886	60.9	0.0	3553	32.8
dem_50_gaslib40_467	–	0.0	7106	62.6	0.0	20k	128.6
dem_50_gaslib40_468	–	0.0	2282	19.9	0.0	4206	33.4
dem_50_gaslib40_469	–	0.0	7213	56.4	0.0	4473	40.8
dem_50_gaslib40_470	–	0.0	5701	45.7	0.0	50k	320.9
dem_50_gaslib40_471	–	0.0	3289	31.4	0.0	2393	22.0
dem_50_gaslib40_472	–	0.0	9176	70.4	0.0	2752	22.3
dem_50_gaslib40_473	–	0.0	4848	56.5	0.0	2873	28.0
dem_50_gaslib40_474	–	0.0	4713	36.9	0.0	16k	93.0
dem_50_gaslib40_475	–	0.0	2690	29.2	0.0	28k	162.8
dem_50_gaslib40_476	–	0.0	4211	39.6	0.0	13k	108.3
dem_50_gaslib40_477	–	0.0	2062	20.1	0.0	3564	35.4
dem_50_gaslib40_478	–	0.0	5285	44.0	0.0	6793	46.7
dem_50_gaslib40_479	–	0.0	1757	20.2	0.0	17k	114.3
dem_50_gaslib40_480	–	0.0	4801	29.1	0.0	5021	41.1
dem_50_gaslib40_481	–	0.0	4637	41.6	0.0	1871	20.3
dem_50_gaslib40_482	–	0.0	6268	51.2	0.0	4483	43.3
dem_50_gaslib40_483	–	0.0	7979	66.4	0.0	5413	45.1
dem_50_gaslib40_484	–	0.0	6541	50.4	0.0	3906	29.1
dem_50_gaslib40_485	–	0.0	12k	91.6	0.0	3137	29.0
dem_50_gaslib40_486	–	0.0	3941	37.0	0.0	9798	75.7
dem_50_gaslib40_487	–	0.0	4804	47.0	0.0	19k	135.9
dem_50_gaslib40_488	–	0.0	9923	88.5	0.0	3259	31.5
dem_50_gaslib40_489	–	0.0	13k	140.2	0.0	2190	18.7
dem_50_gaslib40_490	–	0.0	9784	73.9	0.0	9467	89.2
dem_50_gaslib40_491	–	0.0	3175	26.5	0.0	29k	134.3
dem_50_gaslib40_492	–	0.0	24k	281.2	0.0	6113	50.2
dem_50_gaslib40_493	–	0.0	4737	50.1	0.0	2188	25.5
dem_50_gaslib40_494	–	0.0	2568	38.5	0.0	4688	37.0
dem_50_gaslib40_495	–	0.0	5023	53.5	0.0	14k	82.5
dem_50_gaslib40_496	–	0.0	3979	33.7	0.0	36k	223.6
dem_50_gaslib40_497	–	0.0	10k	80.1	0.0	109k	621.6
dem_50_gaslib40_498	–	0.0	2148	23.7	0.0	6555	51.8
dem_50_gaslib40_499	–	0.0	11k	91.2	0.0	413k	2170.2

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_50_gaslib40_500	–	0.0	30k	155.5	0.0	25k	185.1
dem_100_gaslib40_1	0.02013	0.0	79k	647.8	0.0	36k	316.5
dem_100_gaslib40_2	–	0.0	39k	350.4	0.0	59k	528.1
dem_100_gaslib40_3	0.0	0.0	71k	610.9	0.0	22k	181.6
dem_100_gaslib40_4	–	0.0	22k	231.0	0.0	57k	489.8
dem_100_gaslib40_5	–	0.0	10k	119.6	0.0	9083	94.3
dem_100_gaslib40_6	0.0	0.0	67k	611.9	0.0	86k	576.3
dem_100_gaslib40_7	0.0	0.0	18k	198.2	0.0	36k	382.1
dem_100_gaslib40_8	0.0	0.0	62k	716.1	0.0	30k	183.2
dem_100_gaslib40_9	0.0	–	452k	3600.0	0.0	53k	477.4
dem_100_gaslib40_10	–	0.0	47k	594.0	0.0	282k	2684.5
dem_100_gaslib40_11	0.0	0.0	27k	344.4	0.0	60k	586.7
dem_100_gaslib40_12	0.0	0.0	20k	193.3	0.0	44k	369.0
dem_100_gaslib40_13	0.0	0.0	24k	303.0	0.3	317k	3600.0
dem_100_gaslib40_14	–	0.0	22k	184.6	0.0	26k	212.7
dem_100_gaslib40_15	–	0.0	407k	2913.7	0.0	75k	686.4
dem_100_gaslib40_16	0.00456	0.0	18k	193.0	0.0	25k	241.6
dem_100_gaslib40_17	0.0	0.0	89k	1021.8	0.0	17k	204.8
dem_100_gaslib40_18	0.0	0.0	54k	352.6	0.0	71k	441.2
dem_100_gaslib40_19	0.0	0.0	7215	84.5	0.0	167k	1362.5
dem_100_gaslib40_20	–	0.0	305k	2449.1	0.0	62k	414.4
dem_100_gaslib40_21	–	0.0	77k	615.3	0.0	18k	190.2
dem_100_gaslib40_22	0.00243	0.0	14k	237.9	0.0	10k	107.1
dem_100_gaslib40_23	–	0.0	16k	175.4	–	343k	3600.0
dem_100_gaslib40_24	0.0	0.0	26k	325.6	0.0	51k	553.2
dem_100_gaslib40_25	0.0	0.0	173k	1569.5	0.0	203k	2306.9
dem_100_gaslib40_26	0.0	0.0	31k	282.4	0.0	5951	48.8
dem_100_gaslib40_27	–	41.1	412k	3600.0	206.9	313k	3600.0
dem_100_gaslib40_28	0.0	0.0	6036	79.6	0.0	90k	836.8
dem_100_gaslib40_29	0.0	0.0	31k	273.6	0.0	82k	606.6
dem_100_gaslib40_30	–	0.0	21k	212.5	86.4	425k	3600.0
dem_100_gaslib40_31	–	0.0	103k	1028.8	0.0	102k	760.2
dem_100_gaslib40_32	–	0.0	4181	49.4	0.0	31k	220.8
dem_100_gaslib40_33	0.02408	0.0	2081	26.1	0.0	38k	231.2
dem_100_gaslib40_34	0.13718	0.0	9708	95.5	0.0	12k	101.4
dem_100_gaslib40_35	0.0	0.0	24k	301.9	0.0	62k	562.4
dem_100_gaslib40_36	0.0	0.0	80k	856.2	0.0	326k	2963.9
dem_100_gaslib40_37	0.0	0.0	21k	196.7	0.0	402k	2770.8
dem_100_gaslib40_38	0.0	0.0	127k	1216.5	13.4	460k	3600.0
dem_100_gaslib40_39	0.0	0.0	32k	405.6	11.5	486k	3600.0
dem_100_gaslib40_40	0.0	0.0	39k	298.4	7.0	438k	3600.0
dem_100_gaslib40_41	0.0	0.0	15k	165.5	0.0	46k	315.4
dem_100_gaslib40_42	0.0	0.0	23k	251.4	0.0	71k	731.2
dem_100_gaslib40_43	–	0.0	201k	1361.1	0.0	113k	1024.2
dem_100_gaslib40_44	–	0.0	34k	285.8	117.5	616k	3600.0
dem_100_gaslib40_45	–	0.0	21k	192.8	0.0	220k	1596.0
dem_100_gaslib40_46	–	0.0	9270	97.8	0.0	24k	262.4
dem_100_gaslib40_47	0.00509	0.0	23k	233.3	0.0	21k	233.0
dem_100_gaslib40_48	0.0	0.0	177k	1108.5	0.0	28k	251.8
dem_100_gaslib40_49	0.0	0.0	23k	224.8	12.2	791k	3600.0
dem_100_gaslib40_50	0.0	0.0	36k	461.4	0.0	195k	1579.0
dem_100_gaslib40_51	–	0.0	189k	1630.7	0.0	153k	1239.7
dem_100_gaslib40_52	–	0.0	18k	192.7	0.0	46k	389.5
dem_100_gaslib40_53	0.0	0.0	55k	569.8	0.0	91k	810.9
dem_100_gaslib40_54	0.0	0.5	715k	3600.0	0.0	96k	885.6
dem_100_gaslib40_55	–	0.0	68k	642.8	82.9	991k	3600.0
dem_100_gaslib40_56	–	0.0	50k	533.7	0.0	88k	834.1
dem_100_gaslib40_57	–	0.0	337k	3095.7	25.3	506k	3600.0
dem_100_gaslib40_58	–	0.0	13k	128.7	0.0	120k	1161.5
dem_100_gaslib40_59	0.0	0.0	17k	212.0	0.0	60k	425.5
dem_100_gaslib40_60	0.02107	0.0	11k	99.2	0.0	4927	47.5
dem_100_gaslib40_61	0.0	0.0	20k	285.7	0.0	97k	982.1
dem_100_gaslib40_62	0.0	0.0	20k	162.9	0.0	92k	826.5
dem_100_gaslib40_63	0.0	0.0	44k	371.4	0.0	31k	248.7
dem_100_gaslib40_64	–	0.0	12k	163.0	0.0	185k	1364.1
dem_100_gaslib40_65	0.0	0.0	130k	929.4	0.0	103k	577.6
dem_100_gaslib40_66	0.0	183.6	301k	3600.0	0.0	43k	394.1
dem_100_gaslib40_67	0.0	0.0	211k	1922.9	42.2	291k	3600.0
dem_100_gaslib40_68	–	0.0	155k	1383.1	0.0	98k	893.7
dem_100_gaslib40_69	–	0.0	51k	661.7	0.0	60k	713.6
dem_100_gaslib40_70	0.0	–	426k	3600.0	168.0	429k	3600.0
dem_100_gaslib40_71	0.0	92.9	450k	3600.0	0.3	832k	3600.0
dem_100_gaslib40_72	–	0.0	24k	293.0	0.0	25k	228.4
dem_100_gaslib40_73	0.0	0.0	76k	818.7	0.0	91k	680.9
dem_100_gaslib40_74	0.0	0.0	59k	553.8	0.0	23k	195.4
dem_100_gaslib40_75	–	0.0	51k	446.4	0.0	73k	563.8
dem_100_gaslib40_76	0.0	0.0	28k	390.5	14.6	422k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_gaslib40_77	0.0	0.0	78k	712.8	0.0	34k	246.2
dem_100_gaslib40_78	0.0	0.0	130k	742.0	0.0	69k	691.8
dem_100_gaslib40_79	0.0	0.0	5969	80.3	0.0	53k	418.5
dem_100_gaslib40_80	0.0	0.0	143k	1400.6	0.0	385k	3282.1
dem_100_gaslib40_81	–	0.0	233k	2108.9	0.0	99k	1003.4
dem_100_gaslib40_82	0.0	0.0	312k	2373.6	66.8	397k	3600.0
dem_100_gaslib40_83	–	0.0	18k	180.2	–	616k	3600.0
dem_100_gaslib40_84	–	0.0	24k	168.6	0.0	15k	118.6
dem_100_gaslib40_85	–	1274.6	335k	3600.0	0.0	67k	576.2
dem_100_gaslib40_86	–	0.0	9891	79.9	0.0	50k	388.8
dem_100_gaslib40_87	–	0.0	33k	267.6	0.0	29k	291.1
dem_100_gaslib40_88	0.0	0.0	49k	528.4	0.0	96k	637.4
dem_100_gaslib40_89	0.0	0.0	26k	255.7	0.0	93k	725.8
dem_100_gaslib40_90	0.0	0.0	57k	490.0	0.0	49k	303.7
dem_100_gaslib40_91	0.0	0.0	22k	221.6	0.0	16k	191.4
dem_100_gaslib40_92	0.0	8.0	566k	3600.0	0.0	97k	794.5
dem_100_gaslib40_93	0.0	0.0	61k	638.9	0.0	39k	435.5
dem_100_gaslib40_94	0.0	0.0	23k	192.0	0.0	39k	376.3
dem_100_gaslib40_95	0.0	0.0	47k	477.7	29.7	603k	3600.0
dem_100_gaslib40_96	0.0	8.9	451k	3600.0	0.0	78k	590.9
dem_100_gaslib40_97	0.0	0.0	115k	804.9	0.0	52k	424.5
dem_100_gaslib40_98	0.0	93.6	304k	3600.0	16.1	309k	3600.0
dem_100_gaslib40_99	0.0	0.0	45k	456.2	0.0	12k	122.2
dem_100_gaslib40_100	0.03653	0.0	17k	148.4	0.0	49k	474.6
dem_100_gaslib40_101	0.008	0.0	9054	110.5	0.0	13k	102.0
dem_100_gaslib40_102	0.01109	0.0	34k	373.9	0.0	304k	2272.7
dem_100_gaslib40_103	0.0	0.0	15k	169.6	–	575k	3600.0
dem_100_gaslib40_104	0.0	0.0	15k	174.6	0.0	40k	415.0
dem_100_gaslib40_105	0.0	0.0	70k	600.4	0.0	86k	515.2
dem_100_gaslib40_106	0.0022	0.0	15k	183.6	0.0	232k	2188.3
dem_100_gaslib40_107	0.0	0.0	49k	420.7	0.0	39k	448.6
dem_100_gaslib40_108	0.0	0.0	17k	212.9	0.0	146k	1368.5
dem_100_gaslib40_109	0.00216	0.0	15k	137.4	0.0	32k	228.6
dem_100_gaslib40_110	0.0	0.0	94k	876.0	0.0	42k	336.0
dem_100_gaslib40_111	0.0	0.0	434k	3324.0	22.3	520k	3600.0
dem_100_gaslib40_112	0.0	0.0	48k	473.4	0.0	45k	444.3
dem_100_gaslib40_113	0.0	0.0	33k	312.7	0.0	10k	135.5
dem_100_gaslib40_114	0.0	0.0	23k	226.2	0.0	40k	317.0
dem_100_gaslib40_115	0.0	0.0	24k	239.2	0.0	32k	255.7
dem_100_gaslib40_116	0.0	0.0	166k	2172.3	8.2	299k	3600.0
dem_100_gaslib40_117	0.0	0.0	38k	308.2	0.0	15k	146.9
dem_100_gaslib40_118	–	0.0	30k	358.2	0.0	10k	109.5
dem_100_gaslib40_119	0.18252	0.0	29k	184.1	0.0	42k	235.7
dem_100_gaslib40_120	0.0	0.0	248k	2658.8	–	381k	3600.0
dem_100_gaslib40_121	0.0	0.0	63k	651.6	0.0	274k	1567.0
dem_100_gaslib40_122	0.0	0.0	39k	316.2	0.0	43k	322.1
dem_100_gaslib40_123	–	0.0	100k	776.1	0.0	7390	94.4
dem_100_gaslib40_124	–	0.0	45k	377.6	0.0	29k	271.0
dem_100_gaslib40_125	–	0.0	23k	234.2	0.0	15k	157.1
dem_100_gaslib40_126	0.00321	0.0	20k	177.5	0.0	96k	667.9
dem_100_gaslib40_127	0.0	0.0	91k	1000.6	0.0	60k	533.8
dem_100_gaslib40_128	–	0.0	110k	919.7	0.0	299k	2167.2
dem_100_gaslib40_129	–	0.0	53k	480.4	0.0	9664	82.3
dem_100_gaslib40_130	–	0.0	158k	1305.0	0.0	71k	551.7
dem_100_gaslib40_131	0.0	0.0	72k	769.8	0.0	122k	999.7
dem_100_gaslib40_132	0.0	0.0	12k	84.9	0.0	7894	76.4
dem_100_gaslib40_133	0.0	0.0	28k	312.4	0.0	150k	804.5
dem_100_gaslib40_134	–	0.0	137k	1169.4	0.0	149k	1192.2
dem_100_gaslib40_135	0.0	0.0	112k	858.6	0.0	86k	637.0
dem_100_gaslib40_136	0.0	0.0	94k	1041.8	–	761k	3600.0
dem_100_gaslib40_137	0.0	0.0	92k	881.2	0.0	322k	2488.0
dem_100_gaslib40_138	–	2.7	410k	3600.0	0.0	296k	2306.5
dem_100_gaslib40_139	0.0	0.0	12k	140.1	0.0	519k	3004.8
dem_100_gaslib40_140	–	0.0	20k	194.8	135.5	431k	3600.0
dem_100_gaslib40_141	0.0	0.0	101k	979.9	0.0	166k	1607.2
dem_100_gaslib40_142	0.0	0.0	11k	129.6	0.0	73k	714.5
dem_100_gaslib40_143	–	49.9	327k	3600.0	0.0	147k	1298.3
dem_100_gaslib40_144	0.0	0.0	17k	258.9	0.0	25k	226.1
dem_100_gaslib40_145	0.0	0.0	7711	93.5	0.0	60k	535.5
dem_100_gaslib40_146	–	0.0	16k	149.2	0.0	79k	1010.0
dem_100_gaslib40_147	–	0.0	21k	243.5	0.0	18k	178.5
dem_100_gaslib40_148	0.0	0.0	18k	213.5	0.0	27k	235.0
dem_100_gaslib40_149	0.0	0.0	14k	139.1	0.0	50k	294.9
dem_100_gaslib40_150	0.0	0.0	99k	1155.2	0.0	180k	1883.6
dem_100_gaslib40_151	0.0	0.0	12k	140.0	0.0	8281	73.3
dem_100_gaslib40_152	0.0	252.7	323k	3600.0	0.0	93k	790.8
dem_100_gaslib40_153	0.0	0.0	64k	825.2	8.8	710k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_gaslib40_154	–	91.8	334k	3600.0	0.0	129k	1247.9
dem_100_gaslib40_155	0.0	0.0	18k	230.1	0.0	291k	2721.2
dem_100_gaslib40_156	0.0	0.0	13k	179.5	0.0	30k	285.6
dem_100_gaslib40_157	0.0	0.0	50k	519.8	0.0	33k	294.6
dem_100_gaslib40_158	–	–	635k	3600.0	0.0	53k	345.0
dem_100_gaslib40_159	0.0	0.0	181k	1895.3	0.0	85k	745.9
dem_100_gaslib40_160	–	–	441k	3600.0	0.0	85k	1175.7
dem_100_gaslib40_161	0.0	0.0	128k	1366.6	–	333k	3600.0
dem_100_gaslib40_162	0.0	0.0	10k	77.6	0.0	10k	73.7
dem_100_gaslib40_163	0.06229	0.0	13k	103.0	0.0	6194	65.6
dem_100_gaslib40_164	0.0	0.0	48k	550.9	0.0	28k	266.2
dem_100_gaslib40_165	0.0	0.0	22k	225.6	0.0	91k	790.8
dem_100_gaslib40_166	0.0	0.0	23k	171.1	0.0	26k	162.4
dem_100_gaslib40_167	–	0.0	11k	140.0	0.0	681k	3490.7
dem_100_gaslib40_168	0.0	0.0	19k	214.0	–	700k	3600.0
dem_100_gaslib40_169	0.02519	0.0	1517	24.1	0.0	2202	28.6
dem_100_gaslib40_170	–	0.0	26k	266.5	0.0	114k	998.1
dem_100_gaslib40_171	0.0	0.0	211k	2088.2	109.6	388k	3600.0
dem_100_gaslib40_172	0.0	–	433k	3600.0	37.5	432k	3600.0
dem_100_gaslib40_173	0.0	0.0	4783	62.2	0.0	15k	178.8
dem_100_gaslib40_174	–	0.0	9411	93.0	0.0	16k	193.3
dem_100_gaslib40_175	0.0	0.0	92k	1204.0	116.8	313k	3600.0
dem_100_gaslib40_176	0.0	0.0	38k	382.0	–	443k	3600.0
dem_100_gaslib40_177	0.0	0.0	28k	257.5	0.0	30k	258.4
dem_100_gaslib40_178	0.0	0.0	96k	1103.0	24.4	245k	3600.0
dem_100_gaslib40_179	0.0	14.2	643k	3600.0	0.0	19k	215.1
dem_100_gaslib40_180	–	0.0	34k	265.1	0.0	18k	159.8
dem_100_gaslib40_181	0.0	0.0	147k	1537.9	9.1	313k	3600.0
dem_100_gaslib40_182	0.0	0.0	37k	485.2	0.0	53k	485.6
dem_100_gaslib40_183	0.0	58.6	319k	3600.0	0.3	418k	3600.0
dem_100_gaslib40_184	–	23.0	358k	3600.0	10.6	473k	3600.0
dem_100_gaslib40_185	–	0.0	14k	129.5	0.0	25k	214.9
dem_100_gaslib40_186	0.0	0.0	135k	850.5	0.0	150k	1305.4
dem_100_gaslib40_187	0.0	25.7	445k	3600.0	0.0	54k	498.7
dem_100_gaslib40_188	0.0	–	682k	3600.0	0.0	303k	2987.3
dem_100_gaslib40_189	0.0	0.0	21k	213.1	0.0	32k	309.1
dem_100_gaslib40_190	0.0	0.0	12k	106.0	0.0	10k	113.1
dem_100_gaslib40_191	0.0	0.0	10k	97.5	0.0	16k	136.4
dem_100_gaslib40_192	0.0	0.0	9361	106.3	0.0	34k	330.4
dem_100_gaslib40_193	–	0.0	10k	104.7	0.0	59k	480.8
dem_100_gaslib40_194	0.00317	0.0	32k	324.9	0.0	69k	514.9
dem_100_gaslib40_195	0.0	0.0	32k	409.1	66.7	292k	3600.0
dem_100_gaslib40_196	0.00185	0.0	4351	39.4	0.0	6717	61.0
dem_100_gaslib40_197	0.0	0.0	41k	325.4	0.0	23k	198.7
dem_100_gaslib40_198	0.02981	0.0	19k	161.3	0.0	10k	104.8
dem_100_gaslib40_199	0.0	0.0	35k	307.9	0.0	345k	3014.5
dem_100_gaslib40_200	0.0	0.0	278k	2582.8	0.0	88k	775.3
dem_100_gaslib40_201	0.0	0.0	221k	2015.8	0.0	44k	410.5
dem_100_gaslib40_202	–	0.0	29k	329.9	0.0	50k	489.5
dem_100_gaslib40_203	0.0	0.0	8721	76.5	0.0	8721	80.9
dem_100_gaslib40_204	–	0.0	18k	233.1	0.0	22k	238.1
dem_100_gaslib40_205	–	0.0	35k	315.6	0.0	43k	382.1
dem_100_gaslib40_206	0.0	0.0	66k	484.0	0.0	40k	299.7
dem_100_gaslib40_207	0.0	0.0	51k	631.1	0.0	66k	587.8
dem_100_gaslib40_208	0.0	0.0	25k	274.7	0.0	29k	324.8
dem_100_gaslib40_209	0.0	0.0	243k	2132.2	80.0	559k	3600.0
dem_100_gaslib40_210	–	0.0	60k	591.5	0.0	14k	146.8
dem_100_gaslib40_211	0.01896	0.0	16k	117.2	0.0	22k	144.9
dem_100_gaslib40_212	0.0	0.0	24k	208.4	0.0	8731	69.8
dem_100_gaslib40_213	0.0	0.0	25k	192.8	0.0	50k	391.0
dem_100_gaslib40_214	0.0	0.0	67k	696.6	0.0	10k	90.8
dem_100_gaslib40_215	0.0	0.0	19k	221.4	0.0	14k	132.1
dem_100_gaslib40_216	0.0	0.0	25k	247.7	0.0	74k	559.3
dem_100_gaslib40_217	0.0	0.0	273k	2896.1	–	357k	3600.0
dem_100_gaslib40_218	–	0.0	10k	80.4	0.0	17k	125.4
dem_100_gaslib40_219	0.0	0.0	21k	212.2	0.0	95k	678.5
dem_100_gaslib40_220	0.00187	0.0	64k	599.2	0.0	18k	213.3
dem_100_gaslib40_221	0.0	0.0	73k	756.8	0.0	85k	687.7
dem_100_gaslib40_222	–	0.0	17k	172.9	0.0	29k	282.1
dem_100_gaslib40_223	0.0	0.0	280k	2295.8	0.0	92k	790.0
dem_100_gaslib40_224	0.0	0.0	9800	91.1	0.0	41k	344.0
dem_100_gaslib40_225	–	0.0	7415	92.3	0.0	18k	170.0
dem_100_gaslib40_226	0.0	0.0	77k	548.8	0.0	227k	1728.4
dem_100_gaslib40_227	–	0.0	29k	259.5	0.0	133k	888.2
dem_100_gaslib40_228	0.12138	0.0	39k	226.1	0.0	9478	76.1
dem_100_gaslib40_229	–	–	418k	3600.0	0.0	106k	924.3
dem_100_gaslib40_230	0.0	16.7	320k	3600.0	6.4	285k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_gaslib40_231	–	0.0	263k	1967.9	0.0	102k	806.7
dem_100_gaslib40_232	0.0	0.0	25k	248.7	0.0	51k	369.7
dem_100_gaslib40_233	0.0	0.0	50k	589.3	0.0	87k	1012.3
dem_100_gaslib40_234	–	0.0	20k	165.5	0.0	28k	206.2
dem_100_gaslib40_235	0.0	0.0	324k	2507.1	0.0	54k	491.5
dem_100_gaslib40_236	–	1.2	314k	3600.0	0.0	71k	773.8
dem_100_gaslib40_237	0.0	0.0	28k	312.3	34.5	455k	3600.0
dem_100_gaslib40_238	0.0	0.0	55k	476.8	82.1	391k	3600.0
dem_100_gaslib40_239	0.0	0.0	16k	150.6	0.0	150k	1279.9
dem_100_gaslib40_240	0.0	0.0	14k	148.8	0.0	7025	71.5
dem_100_gaslib40_241	0.0	0.0	5898	60.4	0.0	82k	612.0
dem_100_gaslib40_242	0.0	0.0	41k	398.4	0.0	265k	3011.3
dem_100_gaslib40_243	–	0.0	10k	132.0	0.0	69k	836.0
dem_100_gaslib40_244	0.0	0.0	8844	116.4	0.0	17k	205.5
dem_100_gaslib40_245	0.0	0.0	140k	1883.7	21.7	350k	3600.0
dem_100_gaslib40_246	0.0	0.0	29k	335.2	0.0	43k	354.8
dem_100_gaslib40_247	0.0	0.0	207k	1867.3	0.4	323k	3600.0
dem_100_gaslib40_248	0.00046	0.0	61k	634.1	0.0	32k	366.5
dem_100_gaslib40_249	–	0.0	77k	1037.8	0.0	187k	1496.7
dem_100_gaslib40_250	0.0	39.3	359k	3600.0	29.6	373k	3600.0
dem_100_gaslib40_251	0.0	0.0	62k	555.7	0.0	15k	143.4
dem_100_gaslib40_252	0.0	0.0	104k	722.3	0.0	117k	940.7
dem_100_gaslib40_253	0.0	0.0	26k	177.5	0.0	52k	300.4
dem_100_gaslib40_254	0.0	0.0	29k	290.9	0.0	38k	388.4
dem_100_gaslib40_255	0.0	0.0	10k	103.4	0.0	62k	454.6
dem_100_gaslib40_256	0.0	0.0	11k	116.5	0.0	14k	114.0
dem_100_gaslib40_257	0.03415	0.0	18k	160.0	0.0	46k	407.7
dem_100_gaslib40_258	0.0	0.0	60k	757.0	20.4	300k	3600.0
dem_100_gaslib40_259	0.04048	0.0	11k	104.8	0.0	5777	38.9
dem_100_gaslib40_260	–	0.0	12k	140.2	0.0	45k	442.2
dem_100_gaslib40_261	0.0	0.0	80k	631.7	0.0	17k	135.3
dem_100_gaslib40_262	0.0	0.0	130k	1660.3	83.8	285k	3600.0
dem_100_gaslib40_263	0.0	0.0	71k	822.4	0.0	185k	1395.3
dem_100_gaslib40_264	0.00834	0.0	52k	450.9	0.0	17k	150.6
dem_100_gaslib40_265	0.0	–	435k	3600.0	0.0	107k	814.5
dem_100_gaslib40_266	0.0	0.0	21k	231.0	0.0	10k	112.6
dem_100_gaslib40_267	0.0	0.0	15k	182.3	0.0	87k	903.7
dem_100_gaslib40_268	–	0.0	19k	160.2	0.0	14k	124.7
dem_100_gaslib40_269	0.0	0.0	5395	88.8	0.0	27k	291.0
dem_100_gaslib40_270	0.00149	0.0	35k	317.4	0.0	19k	201.5
dem_100_gaslib40_271	0.0	0.0	7213	56.5	0.0	27k	173.6
dem_100_gaslib40_272	0.0	–	323k	3600.0	0.0	4301	45.6
dem_100_gaslib40_273	–	0.0	29k	284.1	0.0	32k	355.4
dem_100_gaslib40_274	0.0	13.2	630k	3600.0	60.1	305k	3600.0
dem_100_gaslib40_275	0.0	0.0	262k	3109.5	68.9	285k	3600.0
dem_100_gaslib40_276	0.0	0.0	77k	932.2	0.0	168k	1757.5
dem_100_gaslib40_277	–	0.0	22k	256.2	0.0	19k	181.8
dem_100_gaslib40_278	0.0	0.0	11k	151.3	0.0	185k	1632.0
dem_100_gaslib40_279	–	0.0	44k	482.3	0.0	114k	907.0
dem_100_gaslib40_280	0.0	0.0	61k	723.5	0.0	55k	483.1
dem_100_gaslib40_281	0.0	0.0	36k	413.7	0.0	200k	1607.7
dem_100_gaslib40_282	–	0.0	127k	910.6	0.0	55k	558.5
dem_100_gaslib40_283	0.0	0.0	93k	512.3	0.0	48k	329.4
dem_100_gaslib40_284	0.0	0.0	49k	478.2	0.0	76k	579.5
dem_100_gaslib40_285	0.0	0.0	5435	51.1	0.0	5578	50.0
dem_100_gaslib40_286	0.0	0.0	10k	122.2	0.0	11k	108.3
dem_100_gaslib40_287	0.0	0.0	52k	579.5	0.0	33k	268.4
dem_100_gaslib40_288	0.0	0.0	149k	1848.3	82.1	295k	3600.0
dem_100_gaslib40_289	0.0	13.0	690k	3600.0	0.0	295k	2120.9
dem_100_gaslib40_290	–	0.0	101k	785.5	0.0	18k	180.1
dem_100_gaslib40_291	0.0	0.0	166k	2231.7	0.0	245k	3210.6
dem_100_gaslib40_292	0.0	0.0	9572	142.9	0.0	7510	67.0
dem_100_gaslib40_293	0.02768	0.0	397k	3456.3	0.0	17k	125.3
dem_100_gaslib40_294	0.0	0.0	35k	292.9	0.0	197k	1671.0
dem_100_gaslib40_295	0.0	0.0	200k	1849.2	0.0	37k	412.8
dem_100_gaslib40_296	0.0	0.0	149k	1168.2	0.0	82k	578.6
dem_100_gaslib40_297	0.0	0.0	77k	818.5	0.0	33k	370.0
dem_100_gaslib40_298	0.0	0.0	83k	563.8	0.0	94k	878.8
dem_100_gaslib40_299	–	0.0	161k	1198.3	0.0	55k	474.3
dem_100_gaslib40_300	0.0	0.0	8736	93.0	0.0	11k	104.0
dem_100_gaslib40_301	0.0	0.0	31k	381.0	0.0	22k	247.8
dem_100_gaslib40_302	0.0	0.0	9013	91.3	0.0	15k	170.3
dem_100_gaslib40_303	0.0	31.1	243k	3600.0	32.0	319k	3600.0
dem_100_gaslib40_304	0.0	0.0	163k	1151.2	0.0	69k	593.6
dem_100_gaslib40_305	0.0	0.0	32k	319.2	0.0	14k	126.5
dem_100_gaslib40_306	0.0	0.0	41k	387.7	0.0	43k	442.5
dem_100_gaslib40_307	0.0	0.0	24k	264.4	0.0	31k	240.8

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_gaslib40_308	0.0	0.0	56k	424.1	0.0	45k	311.8
dem_100_gaslib40_309	0.00929	0.0	5733	47.6	0.0	38k	305.3
dem_100_gaslib40_310	–	0.0	26k	218.4	0.0	11k	78.3
dem_100_gaslib40_311	0.0	0.0	20k	198.0	0.0	40k	401.3
dem_100_gaslib40_312	0.0	0.0	32k	330.5	0.0	69k	466.1
dem_100_gaslib40_313	0.02359	0.0	42k	333.2	0.0	74k	659.2
dem_100_gaslib40_314	0.0	0.0	14k	134.3	0.0	28k	194.4
dem_100_gaslib40_315	–	22.2	222k	3600.0	0.0	155k	1443.4
dem_100_gaslib40_316	0.0	0.0	21k	220.3	0.0	36k	448.0
dem_100_gaslib40_317	0.0	0.0	179k	1768.5	0.0	285k	2385.7
dem_100_gaslib40_318	–	0.0	43k	437.6	0.0	66k	542.0
dem_100_gaslib40_319	–	0.0	8180	106.1	0.0	6722	50.4
dem_100_gaslib40_320	–	0.0	69k	798.1	0.0	19k	174.4
dem_100_gaslib40_321	–	–	305k	3600.0	0.0	341k	2796.4
dem_100_gaslib40_322	0.0	0.0	70k	813.6	0.0	95k	970.6
dem_100_gaslib40_323	–	0.0	16k	199.5	0.0	210k	1952.4
dem_100_gaslib40_324	0.0	0.0	61k	524.9	0.0	10k	90.2
dem_100_gaslib40_325	0.0	0.0	80k	807.1	0.0	277k	2029.5
dem_100_gaslib40_326	0.0	0.0	44k	426.3	0.0	8345	73.6
dem_100_gaslib40_327	0.20243	0.0	5237	83.7	0.0	51k	401.4
dem_100_gaslib40_328	0.0	0.0	28k	305.5	0.0	28k	254.2
dem_100_gaslib40_329	0.0	0.0	12k	146.6	0.0	90k	856.2
dem_100_gaslib40_330	0.00103	0.0	12k	161.9	0.0	42k	310.9
dem_100_gaslib40_331	0.0	0.0	83k	930.4	0.0	56k	505.0
dem_100_gaslib40_332	0.0	0.0	150k	1062.6	0.0	21k	188.2
dem_100_gaslib40_333	0.0	0.0	23k	255.5	0.0	9923	95.5
dem_100_gaslib40_334	–	0.0	57k	540.0	0.0	196k	1597.4
dem_100_gaslib40_335	–	0.0	26k	297.3	534.3	318k	3600.0
dem_100_gaslib40_336	0.0	0.0	41k	361.9	0.0	17k	143.5
dem_100_gaslib40_337	0.0	44.8	352k	3600.0	0.0	80k	781.8
dem_100_gaslib40_338	0.0	0.0	147k	1390.1	0.0	123k	1291.0
dem_100_gaslib40_339	0.0	0.0	8035	81.5	0.0	8321	88.4
dem_100_gaslib40_340	0.0	0.0	160k	1352.8	0.0	217k	1536.9
dem_100_gaslib40_341	–	0.0	79k	763.5	–	303k	3600.0
dem_100_gaslib40_342	0.00971	0.0	14k	125.8	0.0	12k	108.1
dem_100_gaslib40_343	0.0	0.0	103k	938.1	0.0	287k	2006.1
dem_100_gaslib40_344	0.01043	0.0	17k	153.4	0.0	59k	488.8
dem_100_gaslib40_345	0.01606	54.4	509k	3600.0	0.0	29k	209.1
dem_100_gaslib40_346	–	0.0	61k	600.8	0.0	24k	224.7
dem_100_gaslib40_347	0.0	0.0	11k	209.9	0.0	10k	81.1
dem_100_gaslib40_348	0.0	0.0	58k	715.4	0.0	97k	1043.7
dem_100_gaslib40_349	0.0	0.0	62k	913.8	0.0	292k	3220.4
dem_100_gaslib40_350	0.0	0.0	57k	547.6	0.0	378k	3499.6
dem_100_gaslib40_351	–	13.8	385k	3600.0	0.0	148k	1336.2
dem_100_gaslib40_352	–	0.0	10k	119.9	0.0	29k	320.8
dem_100_gaslib40_353	–	0.0	40k	336.1	0.0	61k	438.5
dem_100_gaslib40_354	–	0.0	6994	70.1	0.0	42k	391.7
dem_100_gaslib40_355	0.0	0.0	18k	223.5	0.0	173k	1401.0
dem_100_gaslib40_356	0.0	0.0	12k	102.0	0.0	18k	145.2
dem_100_gaslib40_357	–	37.7	424k	3600.0	0.0	71k	608.5
dem_100_gaslib40_358	0.0	0.0	55k	474.1	0.0	53k	428.5
dem_100_gaslib40_359	0.0	0.0	61k	448.2	0.0	155k	1331.5
dem_100_gaslib40_360	0.0	0.0	14k	161.4	0.0	20k	183.0
dem_100_gaslib40_361	0.0	0.0	42k	416.7	0.0	19k	226.1
dem_100_gaslib40_362	0.0	0.0	81k	858.6	0.0	94k	753.5
dem_100_gaslib40_363	–	0.0	17k	232.1	23.4	632k	3600.0
dem_100_gaslib40_364	0.0	0.0	224k	2276.6	0.0	22k	231.0
dem_100_gaslib40_365	–	0.0	61k	507.8	0.0	114k	770.0
dem_100_gaslib40_366	–	0.0	21k	199.9	0.0	127k	1049.4
dem_100_gaslib40_367	0.0	0.0	14k	116.8	0.0	17k	145.3
dem_100_gaslib40_368	0.0	0.0	91k	760.6	40.8	370k	3600.0
dem_100_gaslib40_369	–	0.0	150k	1745.8	0.0	41k	399.1
dem_100_gaslib40_370	0.0	0.0	7606	79.0	0.0	133k	1268.8
dem_100_gaslib40_371	–	27.8	318k	3600.0	0.0	221k	1695.7
dem_100_gaslib40_372	0.0	0.0	13k	142.7	0.0	18k	208.1
dem_100_gaslib40_373	0.0	0.0	73k	670.6	0.0	70k	717.9
dem_100_gaslib40_374	0.0	0.0	36k	380.6	–	347k	3600.0
dem_100_gaslib40_375	0.0	0.0	350k	2312.3	0.0	52k	381.1
dem_100_gaslib40_376	0.0	0.0	290k	2781.3	31.3	338k	3600.0
dem_100_gaslib40_377	0.0	0.0	51k	603.8	0.0	107k	960.1
dem_100_gaslib40_378	0.0	0.0	25k	291.7	0.0	44k	263.4
dem_100_gaslib40_379	0.0	0.0	10k	90.8	0.0	34k	212.5
dem_100_gaslib40_380	0.0	0.0	21k	165.3	0.0	50k	432.8
dem_100_gaslib40_381	0.0	0.0	65k	798.2	0.0	126k	1223.2
dem_100_gaslib40_382	0.0	0.0	224k	3240.1	0.0	53k	472.5
dem_100_gaslib40_383	–	0.0	300k	2632.8	0.0	39k	332.0
dem_100_gaslib40_384	–	0.0	90k	609.3	0.0	18k	174.6

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_gaslib40_385	0.0	0.0	51k	496.0	0.0	35k	263.7
dem_100_gaslib40_386	0.0	2.8	550k	3600.0	0.0	162k	1346.1
dem_100_gaslib40_387	0.0	0.0	83k	905.8	0.0	29k	331.1
dem_100_gaslib40_388	0.0	30.4	329k	3600.0	0.0	92k	1326.5
dem_100_gaslib40_389	–	0.0	47k	518.6	0.0	458k	3042.6
dem_100_gaslib40_390	0.0	0.0	172k	1526.4	0.0	18k	156.8
dem_100_gaslib40_391	–	0.0	49k	420.6	0.0	33k	321.3
dem_100_gaslib40_392	–	0.0	82k	1144.3	0.0	98k	863.1
dem_100_gaslib40_393	0.0	0.0	161k	1245.3	–	360k	3600.0
dem_100_gaslib40_394	0.0	0.0	61k	508.1	0.0	28k	220.3
dem_100_gaslib40_395	0.0	0.0	119k	921.7	0.0	23k	220.1
dem_100_gaslib40_396	0.0	0.0	46k	507.2	0.0	22k	222.8
dem_100_gaslib40_397	0.0	0.0	145k	853.8	0.0	39k	297.5
dem_100_gaslib40_398	0.0	0.0	153k	1410.5	0.0	19k	205.4
dem_100_gaslib40_399	–	0.0	33k	381.6	0.0	224k	2069.7
dem_100_gaslib40_400	0.0	0.0	262k	2869.4	0.0	128k	1422.5
dem_100_gaslib40_401	–	0.0	13k	157.7	0.0	15k	135.9
dem_100_gaslib40_402	0.0	0.0	9512	72.2	0.0	71k	504.7
dem_100_gaslib40_403	0.01011	0.0	10k	113.4	0.0	25k	211.6
dem_100_gaslib40_404	0.0	0.0	11k	126.8	0.0	18k	145.9
dem_100_gaslib40_405	–	0.0	35k	391.1	0.0	17k	126.2
dem_100_gaslib40_406	–	0.0	89k	830.5	0.0	11k	107.8
dem_100_gaslib40_407	0.0	0.0	16k	175.9	0.0	16k	161.8
dem_100_gaslib40_408	0.0	4.1	357k	3600.0	0.0	84k	757.1
dem_100_gaslib40_409	0.0	0.0	3590	50.4	0.0	9056	79.1
dem_100_gaslib40_410	0.0	0.0	8941	109.5	0.0	19k	219.4
dem_100_gaslib40_411	0.0	0.0	34k	433.3	0.0	109k	1231.6
dem_100_gaslib40_412	0.0	0.0	277k	3072.7	0.0	87k	730.8
dem_100_gaslib40_413	0.0	0.0	13k	123.6	0.0	14k	140.3
dem_100_gaslib40_414	0.0	0.0	41k	435.6	0.0	177k	1356.6
dem_100_gaslib40_415	–	0.0	6615	78.0	0.0	28k	232.7
dem_100_gaslib40_416	–	0.0	94k	1167.8	0.0	171k	2024.1
dem_100_gaslib40_417	–	0.0	146k	1706.3	0.0	119k	843.5
dem_100_gaslib40_418	0.0	0.0	56k	366.7	0.0	25k	190.9
dem_100_gaslib40_419	0.0	0.0	213k	2581.7	177.1	387k	3600.0
dem_100_gaslib40_420	0.0	0.0	341k	3420.0	0.0	53k	598.7
dem_100_gaslib40_421	0.0	0.0	29k	386.3	0.0	282k	3136.6
dem_100_gaslib40_422	0.00642	0.0	14k	129.6	0.0	203k	1588.5
dem_100_gaslib40_423	0.0	0.0	156k	1492.7	0.0	16k	173.8
dem_100_gaslib40_424	–	177.2	299k	3600.0	0.0	31k	290.9
dem_100_gaslib40_425	0.0	123.2	288k	3600.0	75.9	308k	3600.0
dem_100_gaslib40_426	–	0.0	216k	2129.7	18.7	558k	3600.0
dem_100_gaslib40_427	0.0	0.0	170k	1543.3	0.0	81k	842.3
dem_100_gaslib40_428	0.0	0.0	46k	419.4	0.0	100k	1138.7
dem_100_gaslib40_429	–	0.0	187k	1477.0	0.0	25k	251.4
dem_100_gaslib40_430	0.0	0.0	73k	829.0	0.0	397k	2815.7
dem_100_gaslib40_431	0.0	37.1	316k	3600.0	39.7	421k	3600.0
dem_100_gaslib40_432	0.0	0.0	53k	448.5	0.0	403k	2732.9
dem_100_gaslib40_433	–	36.1	649k	3600.0	11.4	684k	3600.0
dem_100_gaslib40_434	0.0	0.0	12k	126.8	0.0	13k	133.1
dem_100_gaslib40_435	–	0.0	202k	1760.1	0.0	4179	46.2
dem_100_gaslib40_436	–	0.0	7005	81.2	0.0	72k	700.7
dem_100_gaslib40_437	–	0.0	20k	247.1	0.0	16k	180.7
dem_100_gaslib40_438	0.0	0.0	13k	134.7	0.0	14k	122.3
dem_100_gaslib40_439	0.0	0.0	9945	92.3	–	516k	3600.0
dem_100_gaslib40_440	0.0	0.0	28k	300.5	274.5	360k	3600.0
dem_100_gaslib40_441	0.0	0.0	18k	184.3	0.0	39k	344.6
dem_100_gaslib40_442	0.0	169.8	322k	3600.0	56.7	431k	3600.0
dem_100_gaslib40_443	0.0	0.0	292k	2251.3	0.0	117k	1273.5
dem_100_gaslib40_444	0.0	0.0	116k	1252.8	0.0	70k	598.9
dem_100_gaslib40_445	–	0.0	32k	362.9	0.0	50k	375.2
dem_100_gaslib40_446	0.0	0.0	137k	890.0	0.0	37k	268.2
dem_100_gaslib40_447	0.0	0.6	282k	3600.0	76.1	302k	3600.0
dem_100_gaslib40_448	–	0.0	31k	285.2	0.0	43k	321.4
dem_100_gaslib40_449	–	0.0	21k	176.8	0.0	33k	328.1
dem_100_gaslib40_450	0.0	0.0	130k	1298.5	0.0	403k	3333.7
dem_100_gaslib40_451	–	0.0	22k	233.8	0.0	30k	252.5
dem_100_gaslib40_452	–	0.0	10k	121.1	0.0	39k	371.9
dem_100_gaslib40_453	–	0.0	49k	650.4	0.0	216k	1436.2
dem_100_gaslib40_454	–	37.3	937k	3600.0	0.0	20k	197.2
dem_100_gaslib40_455	0.0	0.0	57k	663.3	0.0	166k	1336.4
dem_100_gaslib40_456	0.01317	0.0	99k	697.0	0.0	43k	168.9
dem_100_gaslib40_457	0.0	0.0	15k	132.6	0.0	18k	128.4
dem_100_gaslib40_458	–	0.0	50k	547.1	0.0	89k	835.0
dem_100_gaslib40_459	0.0	0.0	16k	139.4	0.0	17k	143.8
dem_100_gaslib40_460	0.0	0.0	9153	104.7	0.0	9921	77.6
dem_100_gaslib40_461	–	0.0	69k	708.0	0.4	796k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_100_gaslib40_462	0.0	0.0	49k	517.7	0.0	98k	711.0
dem_100_gaslib40_463	0.0	0.0	9097	85.6	0.0	11k	126.3
dem_100_gaslib40_464	0.0	0.0	120k	1316.3	0.0	150k	961.9
dem_100_gaslib40_465	0.0	0.0	88k	800.8	0.0	22k	223.9
dem_100_gaslib40_466	0.0	0.0	66k	368.6	0.0	32k	283.8
dem_100_gaslib40_467	0.0	0.0	69k	691.4	0.0	25k	253.6
dem_100_gaslib40_468	0.0	0.0	9907	88.3	0.0	32k	165.5
dem_100_gaslib40_469	0.0	0.0	20k	199.0	0.0	46k	502.6
dem_100_gaslib40_470	0.0	0.0	24k	210.1	0.0	83k	635.4
dem_100_gaslib40_471	0.0	0.0	71k	518.8	0.0	129k	901.4
dem_100_gaslib40_472	0.0	0.0	40k	419.2	0.0	53k	580.0
dem_100_gaslib40_473	–	0.0	210k	2000.4	0.0	44k	384.5
dem_100_gaslib40_474	0.0	0.0	46k	509.0	0.0	80k	596.5
dem_100_gaslib40_475	0.0	–	469k	3600.0	0.0	35k	224.1
dem_100_gaslib40_476	–	0.0	15k	154.5	0.0	109k	860.0
dem_100_gaslib40_477	0.0	0.0	84k	1294.3	46.4	334k	3600.0
dem_100_gaslib40_478	0.0	0.0	34k	283.5	0.0	15k	129.8
dem_100_gaslib40_479	0.0	0.0	198k	1280.7	0.0	51k	456.6
dem_100_gaslib40_480	0.0	0.0	33k	326.8	0.0	109k	799.2
dem_100_gaslib40_481	–	0.0	22k	234.0	0.0	100k	791.3
dem_100_gaslib40_482	0.0	0.0	178k	2103.7	10.5	275k	3600.0
dem_100_gaslib40_483	–	0.0	210k	2132.1	0.0	13k	167.8
dem_100_gaslib40_484	0.01613	0.0	98k	735.2	0.0	27k	230.2
dem_100_gaslib40_485	–	0.0	28k	260.6	0.0	36k	279.4
dem_100_gaslib40_486	0.0	0.0	180k	2104.0	0.0	332k	2713.4
dem_100_gaslib40_487	0.0	0.0	54k	300.2	0.0	19k	152.9
dem_100_gaslib40_488	0.0	0.0	132k	1253.2	0.0	235k	1844.7
dem_100_gaslib40_489	0.0	0.0	14k	140.3	0.0	15k	144.2
dem_100_gaslib40_490	0.0	0.0	30k	295.2	0.0	50k	409.8
dem_100_gaslib40_491	0.0	153.1	1061k	3600.0	0.0	306k	2177.2
dem_100_gaslib40_492	0.0	0.0	289k	2648.6	0.0	85k	1107.9
dem_100_gaslib40_493	–	0.0	42k	352.9	0.0	97k	749.9
dem_100_gaslib40_494	0.0	0.0	41k	542.6	0.0	25k	203.1
dem_100_gaslib40_495	–	0.0	66k	621.8	0.0	60k	617.0
dem_100_gaslib40_496	–	0.0	299k	3118.6	19.7	521k	3600.0
dem_100_gaslib40_497	0.02063	0.0	3823	39.5	0.0	13k	127.5
dem_100_gaslib40_498	0.0	19.6	328k	3600.0	0.0	204k	1444.6
dem_100_gaslib40_499	0.0	21.4	441k	3600.0	0.0	119k	1228.9
dem_100_gaslib40_500	0.0	0.0	20k	207.6	0.0	21k	207.3
dem_500_gaslib40_1	0.00749	–	176k	3600.0	–	176k	3600.0
dem_500_gaslib40_2	0.0082	–	208k	3600.0	19.0	215k	3600.0
dem_500_gaslib40_3	0.00822	–	183k	3600.0	–	180k	3600.0
dem_500_gaslib40_4	0.0061	14.2	187k	3600.0	–	247k	3600.0
dem_500_gaslib40_5	–	–	177k	3600.0	–	181k	3600.0
dem_500_gaslib40_6	–	–	163k	3600.0	–	147k	3600.0
dem_500_gaslib40_7	0.00884	15.5	188k	3600.0	–	199k	3600.0
dem_500_gaslib40_8	–	–	183k	3600.0	–	173k	3600.0
dem_500_gaslib40_9	0.00922	–	168k	3600.0	–	189k	3600.0
dem_500_gaslib40_10	–	–	162k	3600.0	–	174k	3600.0
dem_500_gaslib40_11	0.01072	–	142k	3600.0	–	163k	3600.0
dem_500_gaslib40_12	0.01337	10.7	213k	3600.0	–	283k	3600.0
dem_500_gaslib40_13	0.01166	0.0	196k	2749.1	0.0	249k	3475.5
dem_500_gaslib40_14	–	–	156k	3600.0	–	155k	3600.0
dem_500_gaslib40_15	0.00945	17.7	223k	3600.0	20.4	248k	3600.0
dem_500_gaslib40_16	0.01586	0.0	123k	1330.1	0.0	131k	1209.8
dem_500_gaslib40_17	0.01257	–	197k	3600.0	18.1	217k	3600.0
dem_500_gaslib40_18	0.00868	–	200k	3600.0	17.9	249k	3600.0
dem_500_gaslib40_19	0.00603	0.0	232k	3093.4	–	277k	3600.0
dem_500_gaslib40_20	0.0104	22.7	235k	3600.0	15.8	293k	3600.0
dem_500_gaslib40_21	–	–	173k	3600.0	–	174k	3600.0
dem_500_gaslib40_22	–	–	146k	3600.0	–	154k	3600.0
dem_500_gaslib40_23	–	–	156k	3600.0	–	150k	3600.0
dem_500_gaslib40_24	–	–	163k	3600.0	–	182k	3600.0
dem_500_gaslib40_25	0.01207	0.0	142k	1516.1	0.0	194k	2038.1
dem_500_gaslib40_26	0.02619	0.0	284k	2318.3	0.0	318k	2474.9
dem_500_gaslib40_27	0.00655	16.2	164k	3600.0	–	155k	3600.0
dem_500_gaslib40_28	0.01148	–	217k	3600.0	15.1	194k	3600.0
dem_500_gaslib40_29	0.00822	9.7	210k	3600.0	–	229k	3600.0
dem_500_gaslib40_30	–	–	147k	3600.0	–	182k	3600.0
dem_500_gaslib40_31	0.00821	–	247k	3600.0	10.5	256k	3600.0
dem_500_gaslib40_32	0.01015	14.6	176k	3600.0	370.7	246k	3600.0
dem_500_gaslib40_33	0.00902	–	269k	3600.0	16.1	267k	3600.0
dem_500_gaslib40_34	0.00695	–	185k	3600.0	12.0	183k	3600.0
dem_500_gaslib40_35	0.01116	20.0	224k	3600.0	18.3	233k	3600.0
dem_500_gaslib40_36	0.01111	0.0	224k	2921.7	–	278k	3600.0
dem_500_gaslib40_37	0.00873	13.3	197k	3600.0	–	268k	3600.0
dem_500_gaslib40_38	0.01081	23.7	288k	3600.0	–	278k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_gaslib40_39	–	–	202k	3600.0	–	177k	3600.0
dem_500_gaslib40_40	0.01116	–	161k	3600.0	–	193k	3600.0
dem_500_gaslib40_41	–	–	215k	3600.0	–	241k	3600.0
dem_500_gaslib40_42	0.00715	–	188k	3600.0	–	196k	3600.0
dem_500_gaslib40_43	0.01112	0.1	252k	3600.0	0.1	290k	3600.0
dem_500_gaslib40_44	0.0092	–	203k	3600.0	11.8	219k	3600.0
dem_500_gaslib40_45	0.00759	11.9	176k	3600.0	–	194k	3600.0
dem_500_gaslib40_46	–	–	185k	3600.0	–	212k	3600.0
dem_500_gaslib40_47	–	–	164k	3600.0	–	147k	3600.0
dem_500_gaslib40_48	0.00836	17.8	157k	3600.0	–	176k	3600.0
dem_500_gaslib40_49	0.00868	–	198k	3600.0	–	215k	3600.0
dem_500_gaslib40_50	0.01826	0.0	149k	1800.7	0.0	131k	1437.1
dem_500_gaslib40_51	0.00945	12.2	160k	3600.0	–	178k	3600.0
dem_500_gaslib40_52	–	–	220k	3600.0	–	238k	3600.0
dem_500_gaslib40_53	0.00696	–	170k	3600.0	40.1	194k	3600.0
dem_500_gaslib40_54	0.01192	–	259k	3600.0	0.0	260k	3144.4
dem_500_gaslib40_55	0.00541	–	210k	3600.0	–	203k	3600.0
dem_500_gaslib40_56	–	–	143k	3600.0	–	157k	3600.0
dem_500_gaslib40_57	–	–	156k	3600.0	–	150k	3600.0
dem_500_gaslib40_58	0.01443	0.0	149k	2096.7	0.0	221k	2818.5
dem_500_gaslib40_59	0.00722	–	165k	3600.0	–	187k	3600.0
dem_500_gaslib40_60	0.01097	20.9	214k	3600.0	18.6	259k	3600.0
dem_500_gaslib40_61	–	–	176k	3600.0	–	157k	3600.0
dem_500_gaslib40_62	–	–	181k	3600.0	–	161k	3600.0
dem_500_gaslib40_63	–	–	156k	3600.0	–	170k	3600.0
dem_500_gaslib40_64	0.01864	0.0	258k	2996.2	0.0	165k	1879.2
dem_500_gaslib40_65	–	–	147k	3600.0	–	177k	3600.0
dem_500_gaslib40_66	–	–	167k	3600.0	–	172k	3600.0
dem_500_gaslib40_67	0.01163	0.0	177k	2651.1	0.0	183k	2407.8
dem_500_gaslib40_68	0.01013	24.7	214k	3600.0	–	191k	3600.0
dem_500_gaslib40_69	0.00304	250.9	212k	3600.0	332.7	274k	3600.0
dem_500_gaslib40_70	0.00887	14.7	208k	3600.0	8.9	259k	3600.0
dem_500_gaslib40_71	–	–	157k	3600.0	–	183k	3600.0
dem_500_gaslib40_72	–	–	159k	3600.0	–	167k	3600.0
dem_500_gaslib40_73	0.0087	25.0	222k	3600.0	11.3	262k	3600.0
dem_500_gaslib40_74	0.00692	–	170k	3600.0	–	174k	3600.0
dem_500_gaslib40_75	–	–	151k	3600.0	–	156k	3600.0
dem_500_gaslib40_76	–	–	156k	3600.0	–	171k	3600.0
dem_500_gaslib40_77	0.0102	–	211k	3600.0	10.2	227k	3600.0
dem_500_gaslib40_78	0.00818	–	181k	3600.0	13.5	177k	3600.0
dem_500_gaslib40_79	0.00977	21.8	208k	3600.0	14.0	240k	3600.0
dem_500_gaslib40_80	–	–	188k	3600.0	–	169k	3600.0
dem_500_gaslib40_81	0.01087	16.4	227k	3600.0	–	251k	3600.0
dem_500_gaslib40_82	–	–	180k	3600.0	–	204k	3600.0
dem_500_gaslib40_83	0.00776	–	188k	3600.0	–	205k	3600.0
dem_500_gaslib40_84	–	–	159k	3600.0	–	187k	3600.0
dem_500_gaslib40_85	0.01165	–	232k	3600.0	0.0	303k	3431.0
dem_500_gaslib40_86	0.00748	14.0	211k	3600.0	13.4	244k	3600.0
dem_500_gaslib40_87	0.00883	–	199k	3600.0	14.4	241k	3600.0
dem_500_gaslib40_88	0.02146	0.0	327k	3365.5	0.0	148k	1465.3
dem_500_gaslib40_89	–	–	145k	3600.0	–	151k	3600.0
dem_500_gaslib40_90	0.01239	0.0	178k	2453.8	0.0	49k	715.9
dem_500_gaslib40_91	–	–	167k	3600.0	–	220k	3600.0
dem_500_gaslib40_92	0.00771	17.2	216k	3600.0	19.6	251k	3600.0
dem_500_gaslib40_93	0.00959	268.3	259k	3600.0	30.2	261k	3600.0
dem_500_gaslib40_94	0.01256	23.8	256k	3600.0	–	247k	3600.0
dem_500_gaslib40_95	0.00741	17.9	194k	3600.0	17.0	213k	3600.0
dem_500_gaslib40_96	0.00879	38.6	227k	3600.0	25.3	286k	3600.0
dem_500_gaslib40_97	0.00483	47.8	149k	3600.0	57.2	181k	3600.0
dem_500_gaslib40_98	–	–	167k	3600.0	–	184k	3600.0
dem_500_gaslib40_99	0.01258	12.4	233k	3600.0	–	273k	3600.0
dem_500_gaslib40_100	–	–	174k	3600.0	–	224k	3600.0
dem_500_gaslib40_101	–	–	179k	3600.0	–	186k	3600.0
dem_500_gaslib40_102	–	–	148k	3600.0	–	188k	3600.0
dem_500_gaslib40_103	0.00753	–	198k	3600.0	19.3	219k	3600.0
dem_500_gaslib40_104	–	–	200k	3600.0	–	162k	3600.0
dem_500_gaslib40_105	0.01465	–	378k	3600.0	–	313k	3600.0
dem_500_gaslib40_106	0.00833	17.4	190k	3600.0	17.8	220k	3600.0
dem_500_gaslib40_107	–	–	186k	3600.0	–	197k	3600.0
dem_500_gaslib40_108	–	–	201k	3600.0	–	244k	3600.0
dem_500_gaslib40_109	–	–	179k	3600.0	–	199k	3600.0
dem_500_gaslib40_110	0.01478	–	275k	3600.0	0.0	242k	2729.8
dem_500_gaslib40_111	0.00432	–	191k	3600.0	–	210k	3600.0
dem_500_gaslib40_112	0.01122	–	178k	3600.0	14.0	193k	3600.0
dem_500_gaslib40_113	0.00867	–	205k	3600.0	17.2	232k	3600.0
dem_500_gaslib40_114	0.01281	4.3	293k	3600.0	4.0	403k	3600.0
dem_500_gaslib40_115	–	–	134k	3600.0	–	144k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_gaslib40_116	0.00858	–	183k	3600.0	–	229k	3600.0
dem_500_gaslib40_117	–	–	268k	3600.0	–	322k	3600.0
dem_500_gaslib40_118	0.01289	–	241k	3600.0	–	299k	3600.0
dem_500_gaslib40_119	–	–	179k	3600.0	–	185k	3600.0
dem_500_gaslib40_120	0.01186	0.0	236k	3304.3	0.0	161k	2197.4
dem_500_gaslib40_121	–	–	140k	3600.0	–	141k	3600.0
dem_500_gaslib40_122	0.01082	455.1	232k	3600.0	449.9	272k	3600.0
dem_500_gaslib40_123	0.01063	14.4	191k	3600.0	22.8	245k	3600.0
dem_500_gaslib40_124	–	–	163k	3600.0	–	170k	3600.0
dem_500_gaslib40_125	–	–	192k	3600.0	–	226k	3600.0
dem_500_gaslib40_126	–	–	243k	3600.0	–	252k	3600.0
dem_500_gaslib40_127	0.009	0.0	256k	3465.3	23.3	253k	3600.0
dem_500_gaslib40_128	0.00912	12.7	190k	3600.0	–	194k	3600.0
dem_500_gaslib40_129	0.009	0.0	234k	3240.7	0.0	188k	2170.8
dem_500_gaslib40_130	–	–	145k	3600.0	–	179k	3600.0
dem_500_gaslib40_131	0.01033	–	166k	3600.0	–	172k	3600.0
dem_500_gaslib40_132	–	–	201k	3600.0	–	175k	3600.0
dem_500_gaslib40_133	–	–	174k	3600.0	–	182k	3600.0
dem_500_gaslib40_134	–	–	166k	3600.0	–	170k	3600.0
dem_500_gaslib40_135	0.01386	–	148k	3600.0	–	188k	3600.0
dem_500_gaslib40_136	0.00992	18.9	206k	3600.0	–	234k	3600.0
dem_500_gaslib40_137	0.01219	–	221k	3600.0	9.7	198k	3600.0
dem_500_gaslib40_138	0.00961	3.2	257k	3600.0	0.0	290k	3470.4
dem_500_gaslib40_139	0.00814	18.8	200k	3600.0	19.3	204k	3600.0
dem_500_gaslib40_140	–	–	157k	3600.0	–	168k	3600.0
dem_500_gaslib40_141	0.00857	38.4	276k	3600.0	318.4	236k	3600.0
dem_500_gaslib40_142	0.00823	18.9	210k	3600.0	17.6	224k	3600.0
dem_500_gaslib40_143	0.00644	14.9	200k	3600.0	12.7	227k	3600.0
dem_500_gaslib40_144	–	–	259k	3600.0	–	271k	3600.0
dem_500_gaslib40_145	–	–	176k	3600.0	–	196k	3600.0
dem_500_gaslib40_146	0.00734	17.8	180k	3600.0	–	190k	3600.0
dem_500_gaslib40_147	0.0124	17.1	236k	3600.0	14.4	282k	3600.0
dem_500_gaslib40_148	0.00906	0.0	89k	1309.5	0.0	136k	1803.5
dem_500_gaslib40_149	–	–	175k	3600.0	–	180k	3600.0
dem_500_gaslib40_150	–	–	179k	3600.0	–	186k	3600.0
dem_500_gaslib40_151	0.01027	16.3	188k	3600.0	15.7	194k	3600.0
dem_500_gaslib40_152	–	–	195k	3600.0	–	219k	3600.0
dem_500_gaslib40_153	0.00935	0.0	166k	2253.1	0.0	163k	1976.3
dem_500_gaslib40_154	0.03569	0.0	194k	3574.1	0.0	200k	3307.4
dem_500_gaslib40_155	0.02173	0.3	504k	3600.0	0.0	374k	3139.1
dem_500_gaslib40_156	0.00751	–	145k	3600.0	–	161k	3600.0
dem_500_gaslib40_157	–	–	164k	3600.0	–	167k	3600.0
dem_500_gaslib40_158	0.01735	0.0	293k	3532.8	–	276k	3600.0
dem_500_gaslib40_159	0.0072	–	177k	3600.0	–	187k	3600.0
dem_500_gaslib40_160	0.00591	31.2	176k	3600.0	380.8	207k	3600.0
dem_500_gaslib40_161	0.01255	10.6	189k	3600.0	8.7	261k	3600.0
dem_500_gaslib40_162	0.00986	–	211k	3600.0	15.2	191k	3600.0
dem_500_gaslib40_163	–	–	168k	3600.0	–	207k	3600.0
dem_500_gaslib40_164	0.00869	6.5	266k	3600.0	19.4	197k	3600.0
dem_500_gaslib40_165	–	–	171k	3600.0	–	203k	3600.0
dem_500_gaslib40_166	0.00686	–	169k	3600.0	15.1	180k	3600.0
dem_500_gaslib40_167	0.00911	11.2	202k	3600.0	12.4	221k	3600.0
dem_500_gaslib40_168	–	–	151k	3600.0	–	156k	3600.0
dem_500_gaslib40_169	0.00468	23.0	200k	3600.0	12.3	202k	3600.0
dem_500_gaslib40_170	0.00858	–	179k	3600.0	7.7	199k	3600.0
dem_500_gaslib40_171	0.00851	–	203k	3600.0	–	194k	3600.0
dem_500_gaslib40_172	–	–	187k	3600.0	–	199k	3600.0
dem_500_gaslib40_173	0.00985	24.0	164k	3600.0	20.6	165k	3600.0
dem_500_gaslib40_174	0.00794	18.5	199k	3600.0	–	204k	3600.0
dem_500_gaslib40_175	–	–	144k	3600.0	–	166k	3600.0
dem_500_gaslib40_176	–	–	182k	3600.0	–	199k	3600.0
dem_500_gaslib40_177	–	–	152k	3600.0	–	184k	3600.0
dem_500_gaslib40_178	–	–	202k	3600.0	–	187k	3600.0
dem_500_gaslib40_179	0.01421	0.0	149k	2089.2	0.0	145k	1709.8
dem_500_gaslib40_180	0.00875	12.7	176k	3600.0	21.4	216k	3600.0
dem_500_gaslib40_181	–	–	263k	3600.0	–	267k	3600.0
dem_500_gaslib40_182	–	–	199k	3600.0	–	189k	3600.0
dem_500_gaslib40_183	0.01046	31.0	204k	3600.0	30.0	209k	3600.0
dem_500_gaslib40_184	0.00801	2.9	259k	3600.0	–	274k	3600.0
dem_500_gaslib40_185	–	–	204k	3600.0	–	191k	3600.0
dem_500_gaslib40_186	–	–	173k	3600.0	–	174k	3600.0
dem_500_gaslib40_187	0.00761	–	159k	3600.0	–	171k	3600.0
dem_500_gaslib40_188	0.01837	0.0	147k	2146.8	0.0	181k	2169.3
dem_500_gaslib40_189	0.01596	0.0	271k	2291.0	0.0	446k	3203.1
dem_500_gaslib40_190	0.00685	–	173k	3600.0	–	154k	3600.0
dem_500_gaslib40_191	–	–	192k	3600.0	–	184k	3600.0
dem_500_gaslib40_192	–	–	161k	3600.0	–	165k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_gaslib40_193	0.01013	24.4	263k	3600.0	–	278k	3600.0
dem_500_gaslib40_194	0.01499	–	182k	3600.0	–	209k	3600.0
dem_500_gaslib40_195	0.01415	0.0	116k	1568.2	0.0	160k	1773.4
dem_500_gaslib40_196	0.00828	8.1	182k	3600.0	–	179k	3600.0
dem_500_gaslib40_197	–	–	159k	3600.0	–	174k	3600.0
dem_500_gaslib40_198	–	–	170k	3600.0	–	184k	3600.0
dem_500_gaslib40_199	0.00779	18.1	205k	3600.0	–	246k	3600.0
dem_500_gaslib40_200	0.00909	16.3	160k	3600.0	–	171k	3600.0
dem_500_gaslib40_201	0.00964	19.9	191k	3600.0	–	230k	3600.0
dem_500_gaslib40_202	–	–	149k	3600.0	–	191k	3600.0
dem_500_gaslib40_203	–	–	220k	3600.0	–	258k	3600.0
dem_500_gaslib40_204	–	–	203k	3600.0	–	218k	3600.0
dem_500_gaslib40_205	0.01761	0.0	253k	2956.2	0.0	245k	2399.8
dem_500_gaslib40_206	–	–	259k	3600.0	–	278k	3600.0
dem_500_gaslib40_207	0.01021	–	211k	3600.0	20.2	213k	3600.0
dem_500_gaslib40_208	0.00818	20.2	209k	3600.0	16.3	245k	3600.0
dem_500_gaslib40_209	0.01304	–	257k	3600.0	21.5	244k	3600.0
dem_500_gaslib40_210	–	–	156k	3600.0	–	150k	3600.0
dem_500_gaslib40_211	0.00656	11.7	213k	3600.0	10.7	237k	3600.0
dem_500_gaslib40_212	0.00723	13.0	185k	3600.0	–	202k	3600.0
dem_500_gaslib40_213	0.01361	–	260k	3600.0	–	274k	3600.0
dem_500_gaslib40_214	0.00659	–	168k	3600.0	–	190k	3600.0
dem_500_gaslib40_215	0.01172	14.6	193k	3600.0	10.4	246k	3600.0
dem_500_gaslib40_216	–	–	164k	3600.0	–	162k	3600.0
dem_500_gaslib40_217	0.00806	15.0	180k	3600.0	13.6	202k	3600.0
dem_500_gaslib40_218	–	–	187k	3600.0	–	164k	3600.0
dem_500_gaslib40_219	0.00741	–	171k	3600.0	15.9	188k	3600.0
dem_500_gaslib40_220	–	–	242k	3600.0	–	257k	3600.0
dem_500_gaslib40_221	0.00888	21.3	217k	3600.0	428.5	256k	3600.0
dem_500_gaslib40_222	–	–	165k	3600.0	–	196k	3600.0
dem_500_gaslib40_223	0.01729	10.6	183k	3600.0	–	178k	3600.0
dem_500_gaslib40_224	–	–	265k	3600.0	–	234k	3600.0
dem_500_gaslib40_225	0.008	–	213k	3600.0	–	218k	3600.0
dem_500_gaslib40_226	0.0087	11.9	191k	3600.0	15.6	211k	3600.0
dem_500_gaslib40_227	–	–	178k	3600.0	–	173k	3600.0
dem_500_gaslib40_228	–	–	151k	3600.0	–	210k	3600.0
dem_500_gaslib40_229	–	–	195k	3600.0	–	160k	3600.0
dem_500_gaslib40_230	–	–	189k	3600.0	–	201k	3600.0
dem_500_gaslib40_231	–	–	204k	3600.0	–	264k	3600.0
dem_500_gaslib40_232	0.01401	0.3	231k	3600.0	17.4	227k	3600.0
dem_500_gaslib40_233	0.01851	–	169k	3600.0	–	150k	3600.0
dem_500_gaslib40_234	0.0097	17.7	211k	3600.0	18.6	225k	3600.0
dem_500_gaslib40_235	–	–	197k	3600.0	–	190k	3600.0
dem_500_gaslib40_236	0.0044	–	192k	3600.0	11.0	177k	3600.0
dem_500_gaslib40_237	–	–	169k	3600.0	–	183k	3600.0
dem_500_gaslib40_238	0.01603	–	274k	3600.0	–	286k	3600.0
dem_500_gaslib40_239	–	–	188k	3600.0	–	202k	3600.0
dem_500_gaslib40_240	–	–	157k	3600.0	–	157k	3600.0
dem_500_gaslib40_241	–	–	167k	3600.0	–	165k	3600.0
dem_500_gaslib40_242	–	–	152k	3600.0	–	168k	3600.0
dem_500_gaslib40_243	–	–	181k	3600.0	–	188k	3600.0
dem_500_gaslib40_244	–	–	165k	3600.0	–	206k	3600.0
dem_500_gaslib40_245	0.0108	18.8	178k	3600.0	–	252k	3600.0
dem_500_gaslib40_246	0.00953	–	212k	3600.0	21.0	222k	3600.0
dem_500_gaslib40_247	–	–	151k	3600.0	–	150k	3600.0
dem_500_gaslib40_248	–	–	156k	3600.0	–	151k	3600.0
dem_500_gaslib40_249	–	–	156k	3600.0	–	164k	3600.0
dem_500_gaslib40_250	0.01031	–	178k	3600.0	–	202k	3600.0
dem_500_gaslib40_251	0.00692	31.2	239k	3600.0	–	240k	3600.0
dem_500_gaslib40_252	0.00242	291.8	181k	3600.0	297.9	202k	3600.0
dem_500_gaslib40_253	0.01241	1.4	282k	3600.0	0.0	286k	3195.3
dem_500_gaslib40_254	–	–	164k	3600.0	–	168k	3600.0
dem_500_gaslib40_255	0.00767	–	196k	3600.0	–	215k	3600.0
dem_500_gaslib40_256	0.00952	18.1	212k	3600.0	–	274k	3600.0
dem_500_gaslib40_257	–	–	166k	3600.0	–	169k	3600.0
dem_500_gaslib40_258	0.01119	16.4	199k	3600.0	15.3	221k	3600.0
dem_500_gaslib40_259	–	–	171k	3600.0	–	198k	3600.0
dem_500_gaslib40_260	0.00773	17.2	194k	3600.0	–	266k	3600.0
dem_500_gaslib40_261	–	–	133k	3600.0	–	160k	3600.0
dem_500_gaslib40_262	0.00984	18.1	199k	3600.0	18.4	226k	3600.0
dem_500_gaslib40_263	–	–	161k	3600.0	–	182k	3600.0
dem_500_gaslib40_264	–	–	145k	3600.0	–	148k	3600.0
dem_500_gaslib40_265	–	–	225k	3600.0	–	270k	3600.0
dem_500_gaslib40_266	0.00404	–	262k	3600.0	–	258k	3600.0
dem_500_gaslib40_267	0.00615	17.3	183k	3600.0	13.1	205k	3600.0
dem_500_gaslib40_268	–	–	194k	3600.0	–	191k	3600.0
dem_500_gaslib40_269	0.00748	–	178k	3600.0	–	169k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_gaslib40_270	0.01637	0.0	96k	1181.3	0.0	64k	699.1
dem_500_gaslib40_271	0.01018	13.0	240k	3600.0	16.5	221k	3600.0
dem_500_gaslib40_272	–	–	257k	3600.0	–	201k	3600.0
dem_500_gaslib40_273	0.0106	0.0	185k	2673.9	0.0	173k	2240.3
dem_500_gaslib40_274	0.01228	3.2	260k	3600.0	0.3	303k	3600.0
dem_500_gaslib40_275	0.01061	0.4	255k	3600.0	–	288k	3600.0
dem_500_gaslib40_276	0.01784	17.7	249k	3600.0	–	283k	3600.0
dem_500_gaslib40_277	–	–	164k	3600.0	–	182k	3600.0
dem_500_gaslib40_278	0.01135	–	299k	3600.0	–	310k	3600.0
dem_500_gaslib40_279	0.00452	20.0	191k	3600.0	–	251k	3600.0
dem_500_gaslib40_280	–	–	164k	3600.0	–	141k	3600.0
dem_500_gaslib40_281	0.01652	0.0	122k	1299.7	0.0	101k	1208.3
dem_500_gaslib40_282	–	–	176k	3600.0	–	172k	3600.0
dem_500_gaslib40_283	0.00499	497.5	201k	3600.0	42.7	175k	3600.0
dem_500_gaslib40_284	0.00802	15.8	181k	3600.0	17.9	205k	3600.0
dem_500_gaslib40_285	0.01083	0.0	197k	3163.7	0.0	213k	3179.9
dem_500_gaslib40_286	0.008	38.0	205k	3600.0	423.1	237k	3600.0
dem_500_gaslib40_287	–	–	189k	3600.0	–	195k	3600.0
dem_500_gaslib40_288	–	–	153k	3600.0	–	183k	3600.0
dem_500_gaslib40_289	0.00732	13.4	189k	3600.0	–	205k	3600.0
dem_500_gaslib40_290	0.00782	0.0	133k	1857.1	0.0	136k	1672.0
dem_500_gaslib40_291	–	–	176k	3600.0	–	186k	3600.0
dem_500_gaslib40_292	–	–	169k	3600.0	–	169k	3600.0
dem_500_gaslib40_293	0.01195	–	172k	3600.0	9.6	183k	3600.0
dem_500_gaslib40_294	0.00861	–	210k	3600.0	21.8	236k	3600.0
dem_500_gaslib40_295	–	–	164k	3600.0	–	158k	3600.0
dem_500_gaslib40_296	–	–	222k	3600.0	–	242k	3600.0
dem_500_gaslib40_297	0.01494	13.5	168k	3600.0	–	192k	3600.0
dem_500_gaslib40_298	–	–	172k	3600.0	–	167k	3600.0
dem_500_gaslib40_299	0.00926	17.6	322k	3600.0	12.4	278k	3600.0
dem_500_gaslib40_300	0.01181	0.0	232k	3309.7	0.0	248k	3007.1
dem_500_gaslib40_301	0.00866	–	188k	3600.0	–	195k	3600.0
dem_500_gaslib40_302	–	–	145k	3600.0	–	153k	3600.0
dem_500_gaslib40_303	0.00964	14.3	182k	3600.0	8.1	189k	3600.0
dem_500_gaslib40_304	–	–	149k	3600.0	–	185k	3600.0
dem_500_gaslib40_305	0.00466	9.5	230k	3600.0	19.1	222k	3600.0
dem_500_gaslib40_306	–	–	231k	3600.0	–	262k	3600.0
dem_500_gaslib40_307	0.01317	11.1	157k	3600.0	–	170k	3600.0
dem_500_gaslib40_308	0.01645	–	270k	3600.0	2.4	296k	3600.0
dem_500_gaslib40_309	0.00967	12.3	180k	3600.0	–	178k	3600.0
dem_500_gaslib40_310	0.01141	0.0	122k	1492.5	0.0	222k	2542.1
dem_500_gaslib40_311	0.00929	–	175k	3600.0	9.0	188k	3600.0
dem_500_gaslib40_312	0.00766	243.6	190k	3600.0	25.8	168k	3600.0
dem_500_gaslib40_313	0.00684	–	172k	3600.0	24.7	181k	3600.0
dem_500_gaslib40_314	0.01097	31.1	270k	3600.0	31.4	341k	3600.0
dem_500_gaslib40_315	0.00826	–	176k	3600.0	11.1	208k	3600.0
dem_500_gaslib40_316	–	–	159k	3600.0	–	173k	3600.0
dem_500_gaslib40_317	0.01439	0.0	84k	1110.7	–	298k	3600.0
dem_500_gaslib40_318	–	–	163k	3600.0	–	155k	3600.0
dem_500_gaslib40_319	0.00685	–	190k	3600.0	17.7	250k	3600.0
dem_500_gaslib40_320	–	–	156k	3600.0	–	172k	3600.0
dem_500_gaslib40_321	0.00925	–	280k	3600.0	21.5	243k	3600.0
dem_500_gaslib40_322	0.01157	0.0	186k	2808.2	0.0	173k	2460.8
dem_500_gaslib40_323	–	–	167k	3600.0	–	162k	3600.0
dem_500_gaslib40_324	0.01161	0.0	113k	1434.7	0.0	178k	1848.0
dem_500_gaslib40_325	–	–	160k	3600.0	–	177k	3600.0
dem_500_gaslib40_326	0.00525	–	198k	3600.0	–	220k	3600.0
dem_500_gaslib40_327	–	–	204k	3600.0	–	232k	3600.0
dem_500_gaslib40_328	0.0089	19.6	182k	3600.0	17.7	222k	3600.0
dem_500_gaslib40_329	0.01674	6.2	290k	3600.0	6.5	249k	3600.0
dem_500_gaslib40_330	–	–	178k	3600.0	–	233k	3600.0
dem_500_gaslib40_331	0.01631	1.7	449k	3600.0	0.0	254k	2632.6
dem_500_gaslib40_332	0.00897	0.0	145k	2366.5	11.0	201k	3600.0
dem_500_gaslib40_333	–	–	189k	3600.0	–	197k	3600.0
dem_500_gaslib40_334	–	–	151k	3600.0	–	179k	3600.0
dem_500_gaslib40_335	0.00585	20.5	201k	3600.0	19.6	225k	3600.0
dem_500_gaslib40_336	–	–	253k	3600.0	–	284k	3600.0
dem_500_gaslib40_337	–	–	166k	3600.0	–	171k	3600.0
dem_500_gaslib40_338	0.00955	12.1	196k	3600.0	18.4	216k	3600.0
dem_500_gaslib40_339	0.00791	–	240k	3600.0	27.1	259k	3600.0
dem_500_gaslib40_340	0.0075	–	196k	3600.0	–	195k	3600.0
dem_500_gaslib40_341	–	–	147k	3600.0	–	152k	3600.0
dem_500_gaslib40_342	0.01163	23.7	281k	3600.0	3.8	230k	3600.0
dem_500_gaslib40_343	0.01583	–	159k	3600.0	9.0	178k	3600.0
dem_500_gaslib40_344	0.01807	191.8	161k	3600.0	28.1	139k	3600.0
dem_500_gaslib40_345	–	–	160k	3600.0	–	162k	3600.0
dem_500_gaslib40_346	–	–	182k	3600.0	–	174k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_gaslib40_347	–	–	174k	3600.0	–	193k	3600.0
dem_500_gaslib40_348	0.01035	19.4	252k	3600.0	13.1	214k	3600.0
dem_500_gaslib40_349	–	–	179k	3600.0	–	158k	3600.0
dem_500_gaslib40_350	0.00928	16.5	205k	3600.0	–	204k	3600.0
dem_500_gaslib40_351	0.00844	–	182k	3600.0	20.5	221k	3600.0
dem_500_gaslib40_352	0.006	440.8	186k	3600.0	18.3	178k	3600.0
dem_500_gaslib40_353	0.01486	0.0	132k	1785.9	0.0	72k	993.1
dem_500_gaslib40_354	–	–	172k	3600.0	–	212k	3600.0
dem_500_gaslib40_355	0.00764	–	163k	3600.0	–	180k	3600.0
dem_500_gaslib40_356	–	–	159k	3600.0	–	171k	3600.0
dem_500_gaslib40_357	0.00864	–	174k	3600.0	–	177k	3600.0
dem_500_gaslib40_358	–	–	157k	3600.0	–	145k	3600.0
dem_500_gaslib40_359	0.00644	–	162k	3600.0	–	162k	3600.0
dem_500_gaslib40_360	0.00799	–	174k	3600.0	–	177k	3600.0
dem_500_gaslib40_361	0.01697	0.0	63k	852.7	54.4	493k	3600.0
dem_500_gaslib40_362	0.01851	–	373k	3600.0	0.0	381k	3245.3
dem_500_gaslib40_363	0.02113	0.0	164k	1468.9	0.0	48k	462.6
dem_500_gaslib40_364	0.00667	–	179k	3600.0	–	209k	3600.0
dem_500_gaslib40_365	0.00873	28.3	240k	3600.0	28.5	316k	3600.0
dem_500_gaslib40_366	0.01853	0.0	120k	1537.2	0.0	100k	994.1
dem_500_gaslib40_367	0.01006	21.4	215k	3600.0	16.4	211k	3600.0
dem_500_gaslib40_368	0.01392	16.1	230k	3600.0	497.7	298k	3600.0
dem_500_gaslib40_369	0.00677	–	191k	3600.0	11.8	190k	3600.0
dem_500_gaslib40_370	0.00605	–	208k	3600.0	–	239k	3600.0
dem_500_gaslib40_371	–	–	179k	3600.0	–	242k	3600.0
dem_500_gaslib40_372	0.00863	–	175k	3600.0	21.4	207k	3600.0
dem_500_gaslib40_373	–	–	173k	3600.0	–	186k	3600.0
dem_500_gaslib40_374	0.01009	–	238k	3600.0	–	250k	3600.0
dem_500_gaslib40_375	0.01098	8.1	199k	3600.0	13.4	221k	3600.0
dem_500_gaslib40_376	–	–	166k	3600.0	–	196k	3600.0
dem_500_gaslib40_377	0.00682	24.2	196k	3600.0	21.1	217k	3600.0
dem_500_gaslib40_378	–	–	152k	3600.0	–	172k	3600.0
dem_500_gaslib40_379	0.01165	26.3	291k	3600.0	12.2	248k	3600.0
dem_500_gaslib40_380	0.01025	21.9	202k	3600.0	21.5	234k	3600.0
dem_500_gaslib40_381	–	–	165k	3600.0	–	177k	3600.0
dem_500_gaslib40_382	0.00868	48.3	218k	3600.0	15.4	203k	3600.0
dem_500_gaslib40_383	–	–	229k	3600.0	–	248k	3600.0
dem_500_gaslib40_384	–	–	195k	3600.0	–	151k	3600.0
dem_500_gaslib40_385	–	–	227k	3600.0	–	249k	3600.0
dem_500_gaslib40_386	0.00949	23.4	218k	3600.0	22.5	248k	3600.0
dem_500_gaslib40_387	0.01079	269.0	204k	3600.0	4.3	233k	3600.0
dem_500_gaslib40_388	–	–	151k	3600.0	–	160k	3600.0
dem_500_gaslib40_389	0.01849	136.3	168k	3600.0	32.3	176k	3600.0
dem_500_gaslib40_390	–	–	196k	3600.0	–	207k	3600.0
dem_500_gaslib40_391	0.00688	–	183k	3600.0	12.3	171k	3600.0
dem_500_gaslib40_392	0.0111	–	222k	3600.0	17.3	269k	3600.0
dem_500_gaslib40_393	–	–	165k	3600.0	–	189k	3600.0
dem_500_gaslib40_394	0.0111	–	154k	3600.0	–	193k	3600.0
dem_500_gaslib40_395	0.01628	23.7	165k	3600.0	181.1	169k	3600.0
dem_500_gaslib40_396	0.01203	209.8	191k	3600.0	35.3	181k	3600.0
dem_500_gaslib40_397	0.01142	15.2	213k	3600.0	–	249k	3600.0
dem_500_gaslib40_398	0.00721	–	175k	3600.0	12.0	181k	3600.0
dem_500_gaslib40_399	0.00685	–	208k	3600.0	–	226k	3600.0
dem_500_gaslib40_400	–	–	166k	3600.0	–	213k	3600.0
dem_500_gaslib40_401	–	–	161k	3600.0	–	147k	3600.0
dem_500_gaslib40_402	0.01262	–	174k	3600.0	14.6	169k	3600.0
dem_500_gaslib40_403	–	–	194k	3600.0	–	225k	3600.0
dem_500_gaslib40_404	–	–	169k	3600.0	–	167k	3600.0
dem_500_gaslib40_405	–	–	218k	3600.0	–	200k	3600.0
dem_500_gaslib40_406	0.00582	–	163k	3600.0	–	168k	3600.0
dem_500_gaslib40_407	–	–	187k	3600.0	–	232k	3600.0
dem_500_gaslib40_408	–	–	160k	3600.0	–	174k	3600.0
dem_500_gaslib40_409	–	–	143k	3600.0	–	165k	3600.0
dem_500_gaslib40_410	0.01051	13.2	164k	3600.0	11.0	211k	3600.0
dem_500_gaslib40_411	0.00695	–	185k	3600.0	–	187k	3600.0
dem_500_gaslib40_412	0.01176	13.1	165k	3600.0	–	189k	3600.0
dem_500_gaslib40_413	0.00864	14.0	214k	3600.0	15.6	224k	3600.0
dem_500_gaslib40_414	–	–	221k	3600.0	–	242k	3600.0
dem_500_gaslib40_415	0.00917	15.9	215k	3600.0	2.7	204k	3600.0
dem_500_gaslib40_416	0.0066	19.3	205k	3600.0	16.8	206k	3600.0
dem_500_gaslib40_417	–	–	147k	3600.0	–	172k	3600.0
dem_500_gaslib40_418	–	–	151k	3600.0	–	210k	3600.0
dem_500_gaslib40_419	–	–	244k	3600.0	–	210k	3600.0
dem_500_gaslib40_420	0.00994	246.3	171k	3600.0	248.4	191k	3600.0
dem_500_gaslib40_421	–	–	193k	3600.0	–	175k	3600.0
dem_500_gaslib40_422	0.00485	–	173k	3600.0	15.3	173k	3600.0
dem_500_gaslib40_423	0.00746	14.5	178k	3600.0	–	165k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_500_gaslib40_424	0.00859	16.5	192k	3600.0	18.9	231k	3600.0
dem_500_gaslib40_425	–	–	221k	3600.0	–	234k	3600.0
dem_500_gaslib40_426	0.01344	–	243k	3600.0	19.4	228k	3600.0
dem_500_gaslib40_427	0.01591	0.0	229k	2640.5	0.7	322k	3600.0
dem_500_gaslib40_428	0.01188	469.3	263k	3600.0	442.5	294k	3600.0
dem_500_gaslib40_429	0.0101	17.4	193k	3600.0	15.8	204k	3600.0
dem_500_gaslib40_430	0.00915	12.5	217k	3600.0	23.1	246k	3600.0
dem_500_gaslib40_431	–	–	160k	3600.0	–	164k	3600.0
dem_500_gaslib40_432	–	–	146k	3600.0	–	190k	3600.0
dem_500_gaslib40_433	0.00558	–	288k	3600.0	0.4	342k	3600.0
dem_500_gaslib40_434	0.00933	–	199k	3600.0	19.8	272k	3600.0
dem_500_gaslib40_435	–	–	187k	3600.0	–	217k	3600.0
dem_500_gaslib40_436	0.00563	–	188k	3600.0	–	193k	3600.0
dem_500_gaslib40_437	–	–	149k	3600.0	–	176k	3600.0
dem_500_gaslib40_438	–	–	234k	3600.0	–	282k	3600.0
dem_500_gaslib40_439	0.00804	–	151k	3600.0	–	167k	3600.0
dem_500_gaslib40_440	–	–	139k	3600.0	–	198k	3600.0
dem_500_gaslib40_441	0.01535	0.0	130k	1710.0	–	313k	3600.0
dem_500_gaslib40_442	0.01484	–	265k	3600.0	9.0	279k	3600.0
dem_500_gaslib40_443	–	–	189k	3600.0	–	198k	3600.0
dem_500_gaslib40_444	0.00888	–	227k	3600.0	–	263k	3600.0
dem_500_gaslib40_445	–	–	165k	3600.0	–	172k	3600.0
dem_500_gaslib40_446	–	–	162k	3600.0	–	168k	3600.0
dem_500_gaslib40_447	–	–	184k	3600.0	–	204k	3600.0
dem_500_gaslib40_448	0.01059	–	194k	3600.0	–	187k	3600.0
dem_500_gaslib40_449	0.00622	8.3	228k	3600.0	20.4	220k	3600.0
dem_500_gaslib40_450	0.02057	0.0	83k	946.2	0.0	96k	1032.3
dem_500_gaslib40_451	0.0111	9.6	161k	3600.0	–	184k	3600.0
dem_500_gaslib40_452	0.01919	27.2	153k	3600.0	24.4	162k	3600.0
dem_500_gaslib40_453	0.02323	0.0	23k	262.7	0.0	31k	313.1
dem_500_gaslib40_454	0.01111	–	177k	3600.0	11.1	175k	3600.0
dem_500_gaslib40_455	0.00872	3.5	215k	3600.0	15.1	220k	3600.0
dem_500_gaslib40_456	–	–	179k	3600.0	–	168k	3600.0
dem_500_gaslib40_457	0.00678	184.4	163k	3600.0	188.8	180k	3600.0
dem_500_gaslib40_458	0.01463	–	237k	3600.0	2.7	246k	3600.0
dem_500_gaslib40_459	–	–	156k	3600.0	–	170k	3600.0
dem_500_gaslib40_460	0.01719	0.0	110k	1350.6	0.0	171k	2012.5
dem_500_gaslib40_461	0.00643	4.8	205k	3600.0	3.4	242k	3600.0
dem_500_gaslib40_462	0.00903	–	240k	3600.0	18.8	249k	3600.0
dem_500_gaslib40_463	–	–	150k	3600.0	–	153k	3600.0
dem_500_gaslib40_464	–	–	169k	3600.0	–	182k	3600.0
dem_500_gaslib40_465	0.00839	13.9	179k	3600.0	–	195k	3600.0
dem_500_gaslib40_466	–	–	152k	3600.0	–	163k	3600.0
dem_500_gaslib40_467	0.00115	495.9	265k	3600.0	499.9	270k	3600.0
dem_500_gaslib40_468	–	–	197k	3600.0	–	210k	3600.0
dem_500_gaslib40_469	–	–	295k	3600.0	–	268k	3600.0
dem_500_gaslib40_470	0.00876	16.9	235k	3600.0	19.3	231k	3600.0
dem_500_gaslib40_471	–	–	161k	3600.0	–	177k	3600.0
dem_500_gaslib40_472	0.00684	–	176k	3600.0	–	166k	3600.0
dem_500_gaslib40_473	0.00795	14.0	182k	3600.0	–	205k	3600.0
dem_500_gaslib40_474	–	–	156k	3600.0	–	167k	3600.0
dem_500_gaslib40_475	0.00907	–	192k	3600.0	12.9	189k	3600.0
dem_500_gaslib40_476	0.0165	6.6	227k	3600.0	0.0	173k	2337.3
dem_500_gaslib40_477	–	–	168k	3600.0	–	177k	3600.0
dem_500_gaslib40_478	–	–	188k	3600.0	–	176k	3600.0
dem_500_gaslib40_479	–	–	189k	3600.0	–	170k	3600.0
dem_500_gaslib40_480	–	–	171k	3600.0	–	178k	3600.0
dem_500_gaslib40_481	0.01686	13.3	216k	3600.0	20.9	304k	3600.0
dem_500_gaslib40_482	0.01002	–	213k	3600.0	–	207k	3600.0
dem_500_gaslib40_483	–	–	243k	3600.0	–	274k	3600.0
dem_500_gaslib40_484	0.01002	20.9	231k	3600.0	–	232k	3600.0
dem_500_gaslib40_485	0.00746	0.0	241k	3422.4	6.4	241k	3600.0
dem_500_gaslib40_486	0.00701	–	182k	3600.0	18.5	201k	3600.0
dem_500_gaslib40_487	0.0106	18.9	230k	3600.0	13.2	238k	3600.0
dem_500_gaslib40_488	0.00618	–	160k	3600.0	–	194k	3600.0
dem_500_gaslib40_489	0.00754	–	239k	3600.0	–	201k	3600.0
dem_500_gaslib40_490	–	–	262k	3600.0	–	270k	3600.0
dem_500_gaslib40_491	–	–	191k	3600.0	–	160k	3600.0
dem_500_gaslib40_492	0.00825	18.8	207k	3600.0	–	201k	3600.0
dem_500_gaslib40_493	0.02239	–	280k	3600.0	0.0	333k	3421.4
dem_500_gaslib40_494	0.0101	–	267k	3600.0	16.9	280k	3600.0
dem_500_gaslib40_495	0.00905	–	181k	3600.0	–	175k	3600.0
dem_500_gaslib40_496	–	–	192k	3600.0	–	201k	3600.0
dem_500_gaslib40_497	0.00842	7.3	261k	3600.0	0.0	352k	3600.0
dem_500_gaslib40_498	–	–	161k	3600.0	–	153k	3600.0
dem_500_gaslib40_499	–	–	179k	3600.0	–	206k	3600.0
dem_500_gaslib40_500	0.0095	–	175k	3600.0	12.6	174k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_gaslib40_1	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_2	–	0.0	17	2.7	0.0	15	2.8
dem_1000_gaslib40_3	0.01131	8.8	141k	3600.0	13.7	148k	3600.0
dem_1000_gaslib40_4	–	–	144k	3600.0	–	156k	3600.0
dem_1000_gaslib40_5	–	0.0	63	4.4	0.0	33	4.1
dem_1000_gaslib40_6	–	–	147k	3600.0	–	130k	3600.0
dem_1000_gaslib40_7	–	0.0	1	1.8	0.0	1	1.5
dem_1000_gaslib40_8	0.04431	0.0	6945	91.5	0.0	11k	121.4
dem_1000_gaslib40_9	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_10	–	–	160k	3600.0	–	150k	3600.0
dem_1000_gaslib40_11	0.02121	2.1	162k	3600.0	1.9	207k	3600.0
dem_1000_gaslib40_12	0.01896	–	202k	3600.0	5.9	227k	3600.0
dem_1000_gaslib40_13	–	0.0	39	3.5	0.0	35	3.7
dem_1000_gaslib40_14	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_15	–	–	141k	3600.0	–	128k	3600.0
dem_1000_gaslib40_16	–	–	134k	3600.0	–	138k	3600.0
dem_1000_gaslib40_17	–	0.0	22k	354.8	0.0	49k	734.3
dem_1000_gaslib40_18	–	0.0	1	1.9	0.0	1	2.0
dem_1000_gaslib40_19	0.01564	–	187k	3600.0	3.4	236k	3600.0
dem_1000_gaslib40_20	–	–	141k	3600.0	–	149k	3600.0
dem_1000_gaslib40_21	–	–	145k	3600.0	–	176k	3600.0
dem_1000_gaslib40_22	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_23	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_24	–	0.0	203	6.3	0.0	249	6.9
dem_1000_gaslib40_25	–	0.0	33	3.4	0.0	39	3.5
dem_1000_gaslib40_26	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_27	–	0.0	53	3.6	0.0	39	3.7
dem_1000_gaslib40_28	0.01182	–	119k	3600.0	39.1	136k	3600.0
dem_1000_gaslib40_29	–	–	143k	3600.0	–	152k	3600.0
dem_1000_gaslib40_30	–	–	142k	3600.0	–	155k	3600.0
dem_1000_gaslib40_31	–	–	186k	3600.0	–	193k	3600.0
dem_1000_gaslib40_32	0.01933	16.1	139k	3600.0	17.3	154k	3600.0
dem_1000_gaslib40_33	–	–	221k	3600.0	–	244k	3600.0
dem_1000_gaslib40_34	0.0066	15.5	147k	3600.0	11.2	156k	3600.0
dem_1000_gaslib40_35	–	0.0	69	3.8	0.0	37	3.3
dem_1000_gaslib40_36	0.02329	2.5	176k	3600.0	2.5	198k	3600.0
dem_1000_gaslib40_37	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_38	0.02238	0.1	321k	3600.0	0.0	146k	1870.9
dem_1000_gaslib40_39	0.02216	0.6	208k	3600.0	0.2	249k	3600.0
dem_1000_gaslib40_40	0.01901	0.0	33k	455.3	0.0	51k	599.4
dem_1000_gaslib40_41	0.01482	0.0	100k	1486.2	0.0	126k	1732.7
dem_1000_gaslib40_42	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_43	0.04955	0.0	4781	64.9	0.0	4261	54.9
dem_1000_gaslib40_44	0.0198	1.5	169k	3600.0	2.8	193k	3600.0
dem_1000_gaslib40_45	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_46	–	–	200k	3600.0	–	169k	3600.0
dem_1000_gaslib40_47	–	0.0	73	5.9	0.0	75	5.9
dem_1000_gaslib40_48	–	–	154k	3600.0	–	153k	3600.0
dem_1000_gaslib40_49	–	–	148k	3600.0	–	161k	3600.0
dem_1000_gaslib40_50	–	–	139k	3600.0	–	155k	3600.0
dem_1000_gaslib40_51	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_52	0.02785	4.0	168k	3600.0	–	180k	3600.0
dem_1000_gaslib40_53	–	–	171k	3600.0	–	177k	3600.0
dem_1000_gaslib40_54	–	–	141k	3600.0	–	139k	3600.0
dem_1000_gaslib40_55	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_56	0.01002	7.4	207k	3600.0	–	191k	3600.0
dem_1000_gaslib40_57	–	0.0	25	3.1	0.0	77	4.5
dem_1000_gaslib40_58	–	–	143k	3600.0	–	159k	3600.0
dem_1000_gaslib40_59	–	0.0	73	4.0	0.0	79	4.1
dem_1000_gaslib40_60	0.00674	31.2	164k	3600.0	293.0	203k	3600.0
dem_1000_gaslib40_61	–	–	193k	3600.0	–	219k	3600.0
dem_1000_gaslib40_62	–	–	147k	3600.0	–	134k	3600.0
dem_1000_gaslib40_63	–	0.0	63	6.4	0.0	73	7.4
dem_1000_gaslib40_64	0.01974	6.1	203k	3600.0	–	218k	3600.0
dem_1000_gaslib40_65	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_66	–	0.0	33	4.1	0.0	33	4.3
dem_1000_gaslib40_67	0.01051	74.7	196k	3600.0	74.9	217k	3600.0
dem_1000_gaslib40_68	–	–	147k	3600.0	–	170k	3600.0
dem_1000_gaslib40_69	0.02119	–	183k	3600.0	–	204k	3600.0
dem_1000_gaslib40_70	–	–	137k	3600.0	–	144k	3600.0
dem_1000_gaslib40_71	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_72	0.01523	22.5	153k	3600.0	28.1	168k	3600.0
dem_1000_gaslib40_73	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_74	–	–	157k	3600.0	–	155k	3600.0
dem_1000_gaslib40_75	–	–	159k	3600.0	–	171k	3600.0
dem_1000_gaslib40_76	0.02406	0.5	186k	3600.0	1.9	203k	3600.0
dem_1000_gaslib40_77	0.01913	–	160k	3600.0	–	156k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_gaslib40_78	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_79	–	–	155k	3600.0	–	164k	3600.0
dem_1000_gaslib40_80	–	–	170k	3600.0	–	185k	3600.0
dem_1000_gaslib40_81	–	–	159k	3600.0	–	175k	3600.0
dem_1000_gaslib40_82	–	–	154k	3600.0	–	150k	3600.0
dem_1000_gaslib40_83	0.01622	109.4	154k	3600.0	20.4	154k	3600.0
dem_1000_gaslib40_84	0.00697	4.3	212k	3600.0	1.7	259k	3600.0
dem_1000_gaslib40_85	–	–	153k	3600.0	–	157k	3600.0
dem_1000_gaslib40_86	–	–	135k	3600.0	–	150k	3600.0
dem_1000_gaslib40_87	–	0.0	5191	54.7	0.0	141	6.8
dem_1000_gaslib40_88	–	–	145k	3600.0	–	165k	3600.0
dem_1000_gaslib40_89	0.01655	–	207k	3600.0	–	227k	3600.0
dem_1000_gaslib40_90	0.00635	126.7	167k	3600.0	120.1	180k	3600.0
dem_1000_gaslib40_91	–	–	137k	3600.0	–	140k	3600.0
dem_1000_gaslib40_92	–	0.0	81	6.4	0.0	199	8.5
dem_1000_gaslib40_93	0.02481	–	204k	3600.0	2.3	207k	3600.0
dem_1000_gaslib40_94	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_95	–	0.0	37	3.8	0.0	49	4.0
dem_1000_gaslib40_96	0.01406	0.0	4331	67.3	0.0	4541	56.3
dem_1000_gaslib40_97	–	–	155k	3600.0	–	152k	3600.0
dem_1000_gaslib40_98	–	–	147k	3600.0	–	170k	3600.0
dem_1000_gaslib40_99	–	–	160k	3600.0	–	159k	3600.0
dem_1000_gaslib40_100	–	0.0	67	3.8	0.0	155	4.9
dem_1000_gaslib40_101	–	–	140k	3600.0	–	125k	3600.0
dem_1000_gaslib40_102	–	0.0	71	4.2	0.0	67	3.6
dem_1000_gaslib40_103	0.02614	25.5	139k	3600.0	19.4	149k	3600.0
dem_1000_gaslib40_104	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_105	–	–	150k	3600.0	–	174k	3600.0
dem_1000_gaslib40_106	–	–	123k	3600.0	–	134k	3600.0
dem_1000_gaslib40_107	0.02121	–	150k	3600.0	9.4	149k	3600.0
dem_1000_gaslib40_108	0.02307	0.0	186k	2452.9	0.2	292k	3600.0
dem_1000_gaslib40_109	–	0.0	367	11.7	0.0	12k	209.4
dem_1000_gaslib40_110	0.02784	0.2	243k	3600.0	–	263k	3600.0
dem_1000_gaslib40_111	0.01794	–	172k	3600.0	–	176k	3600.0
dem_1000_gaslib40_112	–	–	153k	3600.0	–	143k	3600.0
dem_1000_gaslib40_113	0.01592	79.7	142k	3600.0	36.1	144k	3600.0
dem_1000_gaslib40_114	–	–	135k	3600.0	–	159k	3600.0
dem_1000_gaslib40_115	0.01716	0.0	216k	3349.5	2.3	261k	3600.0
dem_1000_gaslib40_116	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_117	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_118	–	–	147k	3600.0	–	142k	3600.0
dem_1000_gaslib40_119	–	0.0	81	7.8	0.0	117	7.6
dem_1000_gaslib40_120	–	–	165k	3600.0	–	175k	3600.0
dem_1000_gaslib40_121	–	–	122k	3600.0	–	150k	3600.0
dem_1000_gaslib40_122	–	–	153k	3600.0	–	165k	3600.0
dem_1000_gaslib40_123	–	–	142k	3600.0	–	144k	3600.0
dem_1000_gaslib40_124	0.01881	–	160k	3600.0	–	152k	3600.0
dem_1000_gaslib40_125	–	–	136k	3600.0	–	159k	3600.0
dem_1000_gaslib40_126	0.01089	72.4	303k	3600.0	0.0	63k	835.7
dem_1000_gaslib40_127	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_128	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_129	–	–	147k	3600.0	–	175k	3600.0
dem_1000_gaslib40_130	–	0.0	237	8.6	0.0	285	9.7
dem_1000_gaslib40_131	0.01253	0.0	124k	1697.1	0.2	293k	3600.0
dem_1000_gaslib40_132	–	–	145k	3600.0	–	184k	3600.0
dem_1000_gaslib40_133	–	–	141k	3600.0	–	150k	3600.0
dem_1000_gaslib40_134	0.02179	1.9	151k	3600.0	–	165k	3600.0
dem_1000_gaslib40_135	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_136	0.02441	3.9	164k	3600.0	–	180k	3600.0
dem_1000_gaslib40_137	0.02545	–	147k	3600.0	–	159k	3600.0
dem_1000_gaslib40_138	–	–	135k	3600.0	–	152k	3600.0
dem_1000_gaslib40_139	–	–	137k	3600.0	–	134k	3600.0
dem_1000_gaslib40_140	0.01086	0.0	105k	1620.2	0.0	160k	2182.0
dem_1000_gaslib40_141	–	–	216k	3600.0	–	221k	3600.0
dem_1000_gaslib40_142	–	–	142k	3600.0	–	157k	3600.0
dem_1000_gaslib40_143	–	–	141k	3600.0	–	167k	3600.0
dem_1000_gaslib40_144	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_145	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_146	0.01266	11.1	147k	3600.0	10.1	156k	3600.0
dem_1000_gaslib40_147	–	–	142k	3600.0	–	136k	3600.0
dem_1000_gaslib40_148	–	–	152k	3600.0	–	168k	3600.0
dem_1000_gaslib40_149	–	–	222k	3600.0	0.0	169k	1924.9
dem_1000_gaslib40_150	0.02488	1.6	189k	3600.0	–	215k	3600.0
dem_1000_gaslib40_151	0.01975	–	145k	3600.0	–	149k	3600.0
dem_1000_gaslib40_152	0.01086	4.7	222k	3600.0	3.8	245k	3600.0
dem_1000_gaslib40_153	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_154	0.00821	20.5	150k	3600.0	36.5	172k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_gaslib40_155	0.0255	–	146k	3600.0	9.0	161k	3600.0
dem_1000_gaslib40_156	–	0.0	71	7.4	0.0	57	6.8
dem_1000_gaslib40_157	0.0151	4.6	224k	3600.0	–	268k	3600.0
dem_1000_gaslib40_158	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_159	–	0.0	225	6.5	0.0	179	5.9
dem_1000_gaslib40_160	–	0.0	53	3.6	0.0	49	3.6
dem_1000_gaslib40_161	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_162	–	–	171k	3600.0	–	181k	3600.0
dem_1000_gaslib40_163	0.01206	0.0	66k	922.2	0.0	82k	943.4
dem_1000_gaslib40_164	–	–	139k	3600.0	–	143k	3600.0
dem_1000_gaslib40_165	–	–	154k	3600.0	–	160k	3600.0
dem_1000_gaslib40_166	0.02354	1.7	169k	3600.0	1.9	186k	3600.0
dem_1000_gaslib40_167	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_168	0.01772	4.4	215k	3600.0	4.5	220k	3600.0
dem_1000_gaslib40_169	0.01533	1.3	235k	3600.0	1.3	263k	3600.0
dem_1000_gaslib40_170	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_171	–	–	124k	3600.0	–	143k	3600.0
dem_1000_gaslib40_172	–	–	135k	3600.0	–	156k	3600.0
dem_1000_gaslib40_173	–	0.0	39	4.4	0.0	49	4.3
dem_1000_gaslib40_174	0.01474	157.7	168k	3600.0	27.2	158k	3600.0
dem_1000_gaslib40_175	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_176	–	–	142k	3600.0	–	164k	3600.0
dem_1000_gaslib40_177	–	–	136k	3600.0	–	151k	3600.0
dem_1000_gaslib40_178	0.00696	–	190k	3600.0	20.9	196k	3600.0
dem_1000_gaslib40_179	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_180	–	0.0	45	3.9	0.0	37	3.7
dem_1000_gaslib40_181	–	0.0	39	3.1	0.0	47	3.5
dem_1000_gaslib40_182	–	–	144k	3600.0	–	148k	3600.0
dem_1000_gaslib40_183	0.02105	–	145k	3600.0	1.3	181k	3600.0
dem_1000_gaslib40_184	0.05922	0.0	17k	252.7	0.0	42k	537.8
dem_1000_gaslib40_185	–	0.0	45	3.3	0.0	37	3.5
dem_1000_gaslib40_186	0.01225	1.5	198k	3600.0	2.6	210k	3600.0
dem_1000_gaslib40_187	–	0.0	45	4.1	0.0	41	4.3
dem_1000_gaslib40_188	–	0.0	67	3.4	0.0	41	3.1
dem_1000_gaslib40_189	–	0.0	47	3.5	0.0	45	3.1
dem_1000_gaslib40_190	–	–	147k	3600.0	–	166k	3600.0
dem_1000_gaslib40_191	0.01718	2.2	198k	3600.0	1.1	234k	3600.0
dem_1000_gaslib40_192	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_193	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_194	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_195	0.02471	2.0	153k	3600.0	2.6	177k	3600.0
dem_1000_gaslib40_196	–	–	201k	3600.0	–	256k	3600.0
dem_1000_gaslib40_197	–	0.0	177	8.1	0.0	187	7.9
dem_1000_gaslib40_198	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_199	–	–	138k	3600.0	–	159k	3600.0
dem_1000_gaslib40_200	0.01772	–	195k	3600.0	–	199k	3600.0
dem_1000_gaslib40_201	–	–	151k	3600.0	–	146k	3600.0
dem_1000_gaslib40_202	0.01851	1.5	166k	3600.0	–	174k	3600.0
dem_1000_gaslib40_203	–	0.0	15	2.8	0.0	13	2.8
dem_1000_gaslib40_204	0.01887	141.2	162k	3600.0	14.5	146k	3600.0
dem_1000_gaslib40_205	0.01188	–	285k	3600.0	6.4	277k	3600.0
dem_1000_gaslib40_206	0.02308	2.8	193k	3600.0	2.3	218k	3600.0
dem_1000_gaslib40_207	0.00619	–	242k	3600.0	2.6	224k	3600.0
dem_1000_gaslib40_208	0.02471	22.1	204k	3600.0	11.8	224k	3600.0
dem_1000_gaslib40_209	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_210	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_211	–	–	150k	3600.0	–	183k	3600.0
dem_1000_gaslib40_212	–	–	135k	3600.0	–	138k	3600.0
dem_1000_gaslib40_213	0.01643	14.0	145k	3600.0	17.1	149k	3600.0
dem_1000_gaslib40_214	0.02045	3.6	209k	3600.0	4.2	252k	3600.0
dem_1000_gaslib40_215	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_216	–	0.0	59	3.5	0.0	49	3.4
dem_1000_gaslib40_217	–	–	128k	3600.0	–	126k	3600.0
dem_1000_gaslib40_218	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_219	0.02286	85.7	149k	3600.0	16.9	146k	3600.0
dem_1000_gaslib40_220	–	–	140k	3600.0	–	141k	3600.0
dem_1000_gaslib40_221	0.0191	3.8	241k	3600.0	4.3	263k	3600.0
dem_1000_gaslib40_222	–	–	151k	3600.0	–	171k	3600.0
dem_1000_gaslib40_223	–	0.0	43	3.4	0.0	63	3.5
dem_1000_gaslib40_224	–	–	142k	3600.0	–	153k	3600.0
dem_1000_gaslib40_225	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_226	0.02504	10.2	161k	3600.0	96.2	192k	3600.0
dem_1000_gaslib40_227	–	–	129k	3600.0	–	149k	3600.0
dem_1000_gaslib40_228	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_229	–	–	144k	3600.0	–	140k	3600.0
dem_1000_gaslib40_230	0.02527	0.0	97k	1798.6	0.0	145k	2115.6
dem_1000_gaslib40_231	–	0.0	193	8.9	0.0	141	7.5

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_gaslib40_232	–	0.0	29	3.4	0.0	23	3.4
dem_1000_gaslib40_233	–	0.0	55	4.6	0.0	57	4.5
dem_1000_gaslib40_234	0.04531	0.0	21k	306.8	0.0	53k	619.5
dem_1000_gaslib40_235	–	0.0	163	7.2	0.0	379	10.3
dem_1000_gaslib40_236	0.02731	3.0	186k	3600.0	2.7	219k	3600.0
dem_1000_gaslib40_237	0.01603	2.6	228k	3600.0	2.8	224k	3600.0
dem_1000_gaslib40_238	0.02325	–	158k	3600.0	–	177k	3600.0
dem_1000_gaslib40_239	–	–	178k	3600.0	–	159k	3600.0
dem_1000_gaslib40_240	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_241	0.01881	101.3	160k	3600.0	–	128k	3600.0
dem_1000_gaslib40_242	0.00823	0.0	16k	148.0	0.0	17k	170.9
dem_1000_gaslib40_243	0.02443	–	162k	3600.0	2.9	182k	3600.0
dem_1000_gaslib40_244	0.02134	0.1	256k	3600.0	–	271k	3600.0
dem_1000_gaslib40_245	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_246	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_247	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_248	0.02133	–	173k	3600.0	3.1	207k	3600.0
dem_1000_gaslib40_249	–	0.0	31	5.5	0.0	37	5.5
dem_1000_gaslib40_250	–	–	183k	3600.0	–	156k	3600.0
dem_1000_gaslib40_251	0.02162	–	194k	3600.0	5.7	243k	3600.0
dem_1000_gaslib40_252	–	–	152k	3600.0	–	156k	3600.0
dem_1000_gaslib40_253	–	–	156k	3600.0	–	171k	3600.0
dem_1000_gaslib40_254	–	0.0	111	4.8	0.0	113	5.1
dem_1000_gaslib40_255	–	0.0	51	6.2	0.0	49	6.6
dem_1000_gaslib40_256	0.00947	5.4	173k	3600.0	–	174k	3600.0
dem_1000_gaslib40_257	–	0.0	51	4.2	0.0	159	5.2
dem_1000_gaslib40_258	0.01926	–	160k	3600.0	0.0	100k	1605.4
dem_1000_gaslib40_259	0.02701	172.7	163k	3600.0	35.5	158k	3600.0
dem_1000_gaslib40_260	–	0.0	1	2.0	0.0	1	2.0
dem_1000_gaslib40_261	0.01483	0.0	31k	390.1	0.0	76k	777.0
dem_1000_gaslib40_262	0.01968	1.2	176k	3600.0	2.2	168k	3600.0
dem_1000_gaslib40_263	0.0068	4.6	206k	3600.0	4.2	214k	3600.0
dem_1000_gaslib40_264	–	–	137k	3600.0	–	150k	3600.0
dem_1000_gaslib40_265	–	–	138k	3600.0	–	142k	3600.0
dem_1000_gaslib40_266	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_267	–	0.0	103	6.8	0.0	231	10.7
dem_1000_gaslib40_268	0.02817	0.0	144k	2405.3	0.0	277k	3499.2
dem_1000_gaslib40_269	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_270	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_271	0.02668	–	161k	3600.0	–	230k	3600.0
dem_1000_gaslib40_272	–	–	142k	3600.0	–	144k	3600.0
dem_1000_gaslib40_273	0.02115	–	168k	3600.0	–	192k	3600.0
dem_1000_gaslib40_274	–	–	145k	3600.0	–	145k	3600.0
dem_1000_gaslib40_275	0.03123	23.5	142k	3600.0	25.4	165k	3600.0
dem_1000_gaslib40_276	0.02227	1.1	187k	3600.0	1.9	207k	3600.0
dem_1000_gaslib40_277	0.02053	57.8	226k	3600.0	1.8	243k	3600.0
dem_1000_gaslib40_278	0.01009	15.7	176k	3600.0	13.6	213k	3600.0
dem_1000_gaslib40_279	–	–	213k	3600.0	–	221k	3600.0
dem_1000_gaslib40_280	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_281	0.02245	0.0	183k	2702.6	17.9	225k	3600.0
dem_1000_gaslib40_282	0.01848	4.2	200k	3600.0	3.7	227k	3600.0
dem_1000_gaslib40_283	–	–	127k	3600.0	–	146k	3600.0
dem_1000_gaslib40_284	–	–	132k	3600.0	–	179k	3600.0
dem_1000_gaslib40_285	–	0.0	91	7.7	0.0	129	8.3
dem_1000_gaslib40_286	–	–	144k	3600.0	–	140k	3600.0
dem_1000_gaslib40_287	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_288	–	–	138k	3600.0	–	135k	3600.0
dem_1000_gaslib40_289	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_290	0.01024	18.8	139k	3600.0	–	133k	3600.0
dem_1000_gaslib40_291	–	0.0	79	4.4	0.0	47	4.5
dem_1000_gaslib40_292	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_293	0.02419	–	154k	3600.0	2.2	169k	3600.0
dem_1000_gaslib40_294	0.02163	1.1	215k	3600.0	0.0	188k	2776.4
dem_1000_gaslib40_295	–	–	143k	3600.0	–	161k	3600.0
dem_1000_gaslib40_296	0.02327	0.9	200k	3600.0	14.5	236k	3600.0
dem_1000_gaslib40_297	0.01401	0.0	47k	598.2	0.0	202k	2537.6
dem_1000_gaslib40_298	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_299	–	–	131k	3600.0	–	153k	3600.0
dem_1000_gaslib40_300	0.02486	0.1	369k	3600.0	0.0	182k	2288.0
dem_1000_gaslib40_301	0.01418	–	251k	3600.0	5.1	223k	3600.0
dem_1000_gaslib40_302	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_303	0.0549	1.5	238k	3600.0	1.9	269k	3600.0
dem_1000_gaslib40_304	–	–	137k	3600.0	–	149k	3600.0
dem_1000_gaslib40_305	0.02187	–	131k	3600.0	–	145k	3600.0
dem_1000_gaslib40_306	–	0.0	97	6.5	0.0	315	9.4
dem_1000_gaslib40_307	–	–	226k	3600.0	–	253k	3600.0
dem_1000_gaslib40_308	–	–	138k	3600.0	–	151k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_gaslib40_309	0.01337	0.0	185k	2695.3	0.0	236k	2909.1
dem_1000_gaslib40_310	0.01082	–	222k	3600.0	–	235k	3600.0
dem_1000_gaslib40_311	–	–	139k	3600.0	–	148k	3600.0
dem_1000_gaslib40_312	–	–	155k	3600.0	–	168k	3600.0
dem_1000_gaslib40_313	–	–	146k	3600.0	–	143k	3600.0
dem_1000_gaslib40_314	–	–	148k	3600.0	–	157k	3600.0
dem_1000_gaslib40_315	0.01501	0.0	57k	1010.1	0.0	56k	846.2
dem_1000_gaslib40_316	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_317	–	–	144k	3600.0	–	124k	3600.0
dem_1000_gaslib40_318	–	0.0	49	3.4	0.0	47	3.2
dem_1000_gaslib40_319	–	0.0	63	3.4	0.0	59	3.4
dem_1000_gaslib40_320	0.03485	0.0	199k	2517.9	0.0	107k	1505.8
dem_1000_gaslib40_321	0.01917	2.1	161k	3600.0	2.7	165k	3600.0
dem_1000_gaslib40_322	–	0.0	75	3.8	0.0	81	3.8
dem_1000_gaslib40_323	–	–	149k	3600.0	–	158k	3600.0
dem_1000_gaslib40_324	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_325	–	–	133k	3600.0	–	153k	3600.0
dem_1000_gaslib40_326	–	–	160k	3600.0	–	163k	3600.0
dem_1000_gaslib40_327	–	0.0	61	3.0	0.0	61	3.2
dem_1000_gaslib40_328	0.01491	2.3	214k	3600.0	2.5	248k	3600.0
dem_1000_gaslib40_329	0.02427	15.9	140k	3600.0	81.6	164k	3600.0
dem_1000_gaslib40_330	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_331	0.02656	–	151k	3600.0	15.4	146k	3600.0
dem_1000_gaslib40_332	0.01594	5.4	237k	3600.0	–	265k	3600.0
dem_1000_gaslib40_333	0.02526	10.6	147k	3600.0	11.2	161k	3600.0
dem_1000_gaslib40_334	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_335	–	–	172k	3600.0	–	184k	3600.0
dem_1000_gaslib40_336	–	–	131k	3600.0	–	136k	3600.0
dem_1000_gaslib40_337	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_338	0.00713	6.7	171k	3600.0	–	173k	3600.0
dem_1000_gaslib40_339	0.0118	0.0	28k	329.5	0.0	28k	400.2
dem_1000_gaslib40_340	0.01807	3.3	170k	3600.0	2.4	177k	3600.0
dem_1000_gaslib40_341	–	–	143k	3600.0	–	145k	3600.0
dem_1000_gaslib40_342	–	–	159k	3600.0	–	164k	3600.0
dem_1000_gaslib40_343	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_344	–	0.0	175	9.0	0.0	105	8.0
dem_1000_gaslib40_345	–	–	167k	3600.0	–	165k	3600.0
dem_1000_gaslib40_346	–	–	134k	3600.0	–	142k	3600.0
dem_1000_gaslib40_347	0.01633	3.0	193k	3600.0	–	215k	3600.0
dem_1000_gaslib40_348	–	–	163k	3600.0	–	185k	3600.0
dem_1000_gaslib40_349	0.01816	–	155k	3600.0	17.7	171k	3600.0
dem_1000_gaslib40_350	0.02516	20.9	142k	3600.0	107.1	163k	3600.0
dem_1000_gaslib40_351	–	–	222k	3600.0	–	249k	3600.0
dem_1000_gaslib40_352	–	0.0	113	8.0	0.0	131	8.5
dem_1000_gaslib40_353	0.05132	0.0	67k	1199.3	0.0	92k	1148.4
dem_1000_gaslib40_354	–	–	136k	3600.0	–	162k	3600.0
dem_1000_gaslib40_355	–	–	195k	3600.0	–	207k	3600.0
dem_1000_gaslib40_356	–	–	144k	3600.0	–	139k	3600.0
dem_1000_gaslib40_357	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_358	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_359	–	–	146k	3600.0	–	165k	3600.0
dem_1000_gaslib40_360	0.01721	16.8	140k	3600.0	–	161k	3600.0
dem_1000_gaslib40_361	–	–	167k	3600.0	–	185k	3600.0
dem_1000_gaslib40_362	0.0208	0.0	134k	2181.3	0.0	120k	1664.2
dem_1000_gaslib40_363	–	–	190k	3600.0	–	209k	3600.0
dem_1000_gaslib40_364	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_365	0.00903	89.7	157k	3600.0	88.3	163k	3600.0
dem_1000_gaslib40_366	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_367	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_368	–	–	145k	3600.0	–	134k	3600.0
dem_1000_gaslib40_369	0.02105	0.0	113k	1749.9	0.0	145k	2116.6
dem_1000_gaslib40_370	–	–	137k	3600.0	–	154k	3600.0
dem_1000_gaslib40_371	–	–	145k	3600.0	–	155k	3600.0
dem_1000_gaslib40_372	–	–	133k	3600.0	–	137k	3600.0
dem_1000_gaslib40_373	–	–	134k	3600.0	–	159k	3600.0
dem_1000_gaslib40_374	–	–	128k	3600.0	–	138k	3600.0
dem_1000_gaslib40_375	–	–	145k	3600.0	–	149k	3600.0
dem_1000_gaslib40_376	0.01298	0.0	247k	3600.0	0.3	245k	3600.0
dem_1000_gaslib40_377	0.01625	–	187k	3600.0	–	185k	3600.0
dem_1000_gaslib40_378	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_379	–	–	149k	3600.0	–	140k	3600.0
dem_1000_gaslib40_380	–	–	161k	3600.0	–	165k	3600.0
dem_1000_gaslib40_381	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_382	–	–	127k	3600.0	–	141k	3600.0
dem_1000_gaslib40_383	0.01555	–	153k	3600.0	–	153k	3600.0
dem_1000_gaslib40_384	0.01087	3.3	229k	3600.0	3.1	259k	3600.0
dem_1000_gaslib40_385	–	0.0	0	0.0	0.0	0	0.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_gaslib40_386	0.03156	0.8	229k	3600.0	0.1	225k	3600.0
dem_1000_gaslib40_387	–	–	129k	3600.0	–	134k	3600.0
dem_1000_gaslib40_388	–	0.0	61	6.6	0.0	55	6.3
dem_1000_gaslib40_389	0.04599	0.0	214k	3033.0	0.0	203k	2446.0
dem_1000_gaslib40_390	–	–	141k	3600.0	–	135k	3600.0
dem_1000_gaslib40_391	–	–	157k	3600.0	–	157k	3600.0
dem_1000_gaslib40_392	0.01941	–	180k	3600.0	–	222k	3600.0
dem_1000_gaslib40_393	0.02372	14.0	173k	3600.0	11.2	193k	3600.0
dem_1000_gaslib40_394	–	0.0	71	5.3	0.0	155	6.5
dem_1000_gaslib40_395	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_396	0.01544	–	186k	3600.0	–	219k	3600.0
dem_1000_gaslib40_397	0.00621	89.9	168k	3600.0	114.3	181k	3600.0
dem_1000_gaslib40_398	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_399	–	–	179k	3600.0	–	185k	3600.0
dem_1000_gaslib40_400	0.02016	0.0	191k	3352.0	0.0	218k	3519.5
dem_1000_gaslib40_401	–	–	138k	3600.0	–	166k	3600.0
dem_1000_gaslib40_402	0.02099	0.2	272k	3600.0	0.3	279k	3600.0
dem_1000_gaslib40_403	–	–	163k	3600.0	–	166k	3600.0
dem_1000_gaslib40_404	0.01375	17.5	144k	3600.0	–	161k	3600.0
dem_1000_gaslib40_405	0.01564	–	147k	3600.0	–	161k	3600.0
dem_1000_gaslib40_406	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_407	–	–	133k	3600.0	–	144k	3600.0
dem_1000_gaslib40_408	–	0.0	65	4.1	0.0	89	4.7
dem_1000_gaslib40_409	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_410	0.02296	3.3	185k	3600.0	2.7	183k	3600.0
dem_1000_gaslib40_411	–	0.0	285	9.7	0.0	297	10.1
dem_1000_gaslib40_412	0.02303	5.5	230k	3600.0	1.8	253k	3600.0
dem_1000_gaslib40_413	–	0.0	141	7.6	0.0	601	14.8
dem_1000_gaslib40_414	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_415	–	–	157k	3600.0	–	195k	3600.0
dem_1000_gaslib40_416	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_417	0.02093	–	189k	3600.0	–	231k	3600.0
dem_1000_gaslib40_418	–	–	190k	3600.0	–	217k	3600.0
dem_1000_gaslib40_419	–	–	135k	3600.0	–	113k	3600.0
dem_1000_gaslib40_420	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_421	–	–	178k	3600.0	–	209k	3600.0
dem_1000_gaslib40_422	–	–	180k	3600.0	–	186k	3600.0
dem_1000_gaslib40_423	0.02421	15.8	178k	3600.0	16.1	195k	3600.0
dem_1000_gaslib40_424	–	–	148k	3600.0	–	153k	3600.0
dem_1000_gaslib40_425	–	–	151k	3600.0	–	151k	3600.0
dem_1000_gaslib40_426	0.0159	–	210k	3600.0	4.0	263k	3600.0
dem_1000_gaslib40_427	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_428	0.02388	15.5	146k	3600.0	–	135k	3600.0
dem_1000_gaslib40_429	0.00544	0.0	215k	2768.3	0.0	181k	2056.4
dem_1000_gaslib40_430	–	–	148k	3600.0	–	128k	3600.0
dem_1000_gaslib40_431	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_432	–	–	156k	3600.0	–	166k	3600.0
dem_1000_gaslib40_433	–	–	149k	3600.0	–	140k	3600.0
dem_1000_gaslib40_434	–	–	127k	3600.0	–	129k	3600.0
dem_1000_gaslib40_435	0.02502	6.3	205k	3600.0	1.6	237k	3600.0
dem_1000_gaslib40_436	–	–	140k	3600.0	–	161k	3600.0
dem_1000_gaslib40_437	–	0.0	33	2.7	0.0	35	2.7
dem_1000_gaslib40_438	–	–	159k	3600.0	–	179k	3600.0
dem_1000_gaslib40_439	0.01157	24.6	146k	3600.0	31.7	168k	3600.0
dem_1000_gaslib40_440	–	–	151k	3600.0	–	161k	3600.0
dem_1000_gaslib40_441	–	0.0	59	6.2	0.0	75	6.3
dem_1000_gaslib40_442	0.02194	0.0	54k	781.2	0.0	83k	1055.1
dem_1000_gaslib40_443	–	–	149k	3600.0	–	139k	3600.0
dem_1000_gaslib40_444	–	–	142k	3600.0	–	157k	3600.0
dem_1000_gaslib40_445	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_446	–	–	134k	3600.0	–	148k	3600.0
dem_1000_gaslib40_447	0.00576	159.4	205k	3600.0	165.1	223k	3600.0
dem_1000_gaslib40_448	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_449	–	–	171k	3600.0	–	198k	3600.0
dem_1000_gaslib40_450	–	0.0	51	3.2	0.0	543	8.7
dem_1000_gaslib40_451	0.01844	–	184k	3600.0	1.6	196k	3600.0
dem_1000_gaslib40_452	–	0.0	85	4.2	0.0	57	4.0
dem_1000_gaslib40_453	0.01342	19.9	140k	3600.0	22.1	137k	3600.0
dem_1000_gaslib40_454	–	–	141k	3600.0	–	161k	3600.0
dem_1000_gaslib40_455	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_456	–	–	165k	3600.0	–	188k	3600.0
dem_1000_gaslib40_457	–	–	134k	3600.0	–	146k	3600.0
dem_1000_gaslib40_458	0.0281	0.0	170k	2537.1	130.8	263k	3600.0
dem_1000_gaslib40_459	–	–	166k	3600.0	–	168k	3600.0
dem_1000_gaslib40_460	–	–	137k	3600.0	–	125k	3600.0
dem_1000_gaslib40_461	–	–	157k	3600.0	–	156k	3600.0
dem_1000_gaslib40_462	–	–	149k	3600.0	–	150k	3600.0

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Table B.18: Results for ROOTGAP and TREE experiments on *GasLib-40* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
dem_1000_gaslib40_463	–	–	152k	3600.0	–	137k	3600.0
dem_1000_gaslib40_464	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_465	0.01063	115.3	158k	3600.0	18.4	155k	3600.0
dem_1000_gaslib40_466	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_467	–	–	135k	3600.0	–	161k	3600.0
dem_1000_gaslib40_468	–	0.0	131	7.8	0.0	151	7.9
dem_1000_gaslib40_469	0.02749	0.0	91k	1640.8	–	203k	3600.0
dem_1000_gaslib40_470	0.01272	108.6	227k	3600.0	12.9	240k	3600.0
dem_1000_gaslib40_471	–	–	153k	3600.0	–	156k	3600.0
dem_1000_gaslib40_472	–	–	153k	3600.0	–	148k	3600.0
dem_1000_gaslib40_473	0.01361	21.1	199k	3600.0	11.8	228k	3600.0
dem_1000_gaslib40_474	–	0.0	17	2.9	0.0	19	2.9
dem_1000_gaslib40_475	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_476	–	0.0	27	2.5	0.0	45	3.1
dem_1000_gaslib40_477	–	–	136k	3600.0	–	147k	3600.0
dem_1000_gaslib40_478	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_479	0.02217	1.4	187k	3600.0	2.0	187k	3600.0
dem_1000_gaslib40_480	–	0.0	59	3.7	0.0	49	3.5
dem_1000_gaslib40_481	0.00693	0.0	174k	2009.7	0.0	101k	1167.1
dem_1000_gaslib40_482	–	0.0	33	6.1	0.0	61	6.4
dem_1000_gaslib40_483	–	–	151k	3600.0	–	165k	3600.0
dem_1000_gaslib40_484	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_485	0.02658	16.0	150k	3600.0	–	142k	3600.0
dem_1000_gaslib40_486	–	–	142k	3600.0	–	138k	3600.0
dem_1000_gaslib40_487	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_488	–	–	161k	3600.0	–	160k	3600.0
dem_1000_gaslib40_489	0.01	141.3	160k	3600.0	142.1	161k	3600.0
dem_1000_gaslib40_490	–	–	156k	3600.0	–	144k	3600.0
dem_1000_gaslib40_491	–	–	215k	3600.0	–	226k	3600.0
dem_1000_gaslib40_492	–	–	147k	3600.0	–	141k	3600.0
dem_1000_gaslib40_493	0.01204	1.4	294k	3600.0	1.6	279k	3600.0
dem_1000_gaslib40_494	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_495	–	–	138k	3600.0	–	140k	3600.0
dem_1000_gaslib40_496	–	0.0	0	0.0	0.0	0	0.0
dem_1000_gaslib40_497	0.01147	–	164k	3600.0	28.5	181k	3600.0
dem_1000_gaslib40_498	–	–	156k	3600.0	–	147k	3600.0
dem_1000_gaslib40_499	0.03217	–	192k	3600.0	2.5	226k	3600.0
dem_1000_gaslib40_500	–	0.0	0	0.0	0.0	0	0.0

Table B.19: Detailed results for the ROOTGAP and TREE experiments on the *Circuit rank* test set, as summarized in Figure 4.19 and Tables 4.1 – 4.2. For the ROOTGAP experiment, the table lists the gap closed (GC) improvement. A hyphen indicates that the instance is either detected to be infeasible, no primal solution is known, or the dual bound is zero within a tolerance of $\epsilon = 10^{-6}$ for *Enabled* and *Disabled*. For the TREE experiment, the table shows the optimality gap in %, number of B&B-nodes and solving time in seconds for *Enabled* and *Disabled*.

	ROOTGAP	TREE'					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+2arcs_1	0.0	0.0	4239	44.7	0.0	13k	105.8
belg+2arcs_2	0.0	2.5	1308k	3600.0	0.0	30k	241.6
belg+2arcs_3	0.0	0.0	54k	528.5	0.0	287k	2354.4
belg+2arcs_4	0.0	11.4	494k	3600.0	3.6	591k	3600.0
belg+2arcs_5	0.0	0.0	142k	1192.1	0.1	682k	3600.0
belg+2arcs_6	0.0	0.0	4583	43.1	0.3	1078k	3600.0
belg+2arcs_7	0.0	0.0	103k	1108.0	20.6	324k	3600.0
belg+2arcs_8	0.0	0.0	11k	75.7	0.0	1321	15.1
belg+2arcs_9	0.0	0.0	12k	131.6	0.6	732k	3600.0
belg+2arcs_10	0.0	0.0	26k	282.8	6.2	523k	3600.0
belg+2arcs_11	0.0	0.0	2991	28.8	0.0	7334	79.3
belg+2arcs_12	0.0	1.3	773k	3600.0	3.6	520k	3600.0
belg+2arcs_13	0.0	0.0	15k	150.8	0.0	10k	101.7
belg+2arcs_14	0.0	0.0	1564	18.3	0.0	4256	37.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+2arcs_15	0.0	0.0	1822	20.1	0.0	18k	75.5
belg+2arcs_16	0.0	0.0	120k	853.7	0.0	12k	105.4
belg+2arcs_17	0.0	0.0	1272	17.0	0.0	1520	17.8
belg+2arcs_18	0.0	7.4	735k	3600.0	31.2	289k	3600.0
belg+2arcs_19	0.0	0.0	7657	73.0	0.0	18k	166.0
belg+2arcs_20	0.0	0.0	8134	67.0	0.0	4727	38.8
belg+2arcs_21	0.0	0.0	2283	21.5	0.0	3842	32.5
belg+2arcs_22	0.0	0.0	153k	1453.0	0.0	222k	2013.5
belg+2arcs_23	0.0	0.0	8677	76.7	0.0	8033	60.4
belg+2arcs_24	0.0	0.0	34k	310.9	0.0	287k	1424.8
belg+2arcs_25	0.0	0.0	3171	26.1	0.0	5702	51.1
belg+2arcs_26	0.0	0.0	53k	535.4	0.0	189k	1709.4
belg+2arcs_27	0.0	0.0	3522	33.6	0.0	1787	21.9
belg+2arcs_28	0.0	5.3	564k	3600.0	0.0	53k	511.6
belg+2arcs_29	0.0	3.8	455k	3600.0	0.0	168k	1479.2
belg+2arcs_30	0.0	0.0	8118	73.7	8.5	763k	3600.0
belg+2arcs_31	0.0	0.0	1950	19.1	0.0	12k	96.5
belg+2arcs_32	0.0	0.0	51k	529.9	0.0	8498	82.4
belg+2arcs_33	0.0	4.3	553k	3600.0	0.0	26k	230.7
belg+2arcs_34	0.0	0.0	1825	18.6	0.0	4447	38.6
belg+2arcs_35	0.0	0.0	11k	125.4	2.2	499k	3600.0
belg+2arcs_36	0.0	0.0	51k	490.3	5.2	776k	3600.0
belg+2arcs_37	0.0	0.0	21k	217.1	0.3	741k	3600.0
belg+2arcs_38	0.0	0.0	4613	44.9	0.0	3171	32.8
belg+2arcs_39	0.0	0.0	13k	111.2	0.0	28k	209.4
belg+2arcs_40	0.0	0.0	320k	2314.6	0.0	230k	1616.7
belg+2arcs_41	0.0	0.0	19k	168.8	0.0	21k	202.7
belg+2arcs_42	0.0	0.0	9904	92.7	0.0	4861	56.9
belg+2arcs_43	0.0	0.0	18k	169.3	0.0	7281	69.6
belg+2arcs_44	0.0	0.0	69k	599.3	0.6	748k	3600.0
belg+2arcs_45	0.0	0.0	1707	18.5	0.0	3295	26.4
belg+2arcs_46	0.0	0.0	16k	171.0	3.7	768k	3600.0
belg+2arcs_47	0.0	0.0	22k	208.6	0.0	357k	3154.3
belg+2arcs_48	0.0	0.0	5601	49.9	0.0	6456	58.2
belg+2arcs_49	0.0	0.0	11k	93.8	0.0	13k	85.0
belg+2arcs_50	0.0	0.0	58k	479.9	0.0	83k	628.3
belg+2arcs_51	0.0	0.0	8141	77.3	0.0	34k	218.4
belg+2arcs_52	0.0	0.0	2491	25.4	0.0	429k	1475.8
belg+2arcs_53	0.0	0.0	8115	74.1	0.0	33k	203.9
belg+2arcs_54	0.0	0.0	44k	478.2	0.0	33k	353.2
belg+2arcs_55	0.0	0.0	23k	230.6	0.0	48k	547.6
belg+2arcs_56	0.0	0.0	24k	245.2	0.0	18k	175.1
belg+2arcs_57	0.0	0.0	56k	455.4	0.0	16k	144.7
belg+2arcs_58	0.0	0.0	6691	69.9	0.0	9945	87.4
belg+2arcs_59	0.0	0.0	8487	69.0	0.0	6581	50.4
belg+2arcs_60	0.0	0.0	10k	106.3	0.0	9710	101.7
belg+2arcs_61	0.0	0.0	5986	61.2	0.0	42k	240.0
belg+2arcs_62	0.0	0.0	21k	214.2	0.0	9724	110.1
belg+2arcs_63	0.0	4.8	338k	3600.0	0.0	221k	2206.7
belg+2arcs_64	0.0	0.0	1533	16.6	0.0	1889	21.4
belg+2arcs_65	0.0	2.6	444k	3600.0	0.0	69k	596.0
belg+2arcs_66	0.0	53.0	478k	3600.0	0.0	49k	276.8
belg+2arcs_67	0.0	2.5	875k	3600.0	0.0	7371	80.3
belg+2arcs_68	0.0	0.1	1175k	3600.0	0.0	28k	233.7
belg+2arcs_69	0.0	0.0	235k	2625.7	0.0	107k	1149.3
belg+2arcs_70	0.0	0.0	2211	24.4	0.0	4781	42.8
belg+2arcs_71	0.0	0.0	113k	786.4	0.0	4994	55.1
belg+2arcs_72	0.0	1.7	777k	3600.0	0.0	29k	233.3
belg+2arcs_73	0.0	0.0	8476	88.9	0.0	3515	35.5
belg+2arcs_74	0.0	0.0	33k	178.9	0.0	5488	55.8
belg+2arcs_75	0.0	11.0	787k	3600.0	0.0	18k	128.1
belg+2arcs_76	0.0	0.0	3191	25.4	1.3	908k	3600.0
belg+2arcs_77	0.0	0.0	98k	824.6	0.0	15k	131.2
belg+2arcs_78	0.0	0.0	12k	93.0	0.0	5827	53.7
belg+2arcs_79	0.0	0.0	28k	232.8	0.0	5795	48.1
belg+2arcs_80	0.0	0.0	55k	360.5	0.0	157k	807.5
belg+2arcs_81	0.0	0.0	1650	19.9	0.0	3076	28.8
belg+2arcs_82	0.0	0.0	1645	18.1	0.0	8973	72.7
belg+2arcs_83	0.0	0.0	6686	63.8	0.2	947k	3600.0
belg+2arcs_84	0.0	0.0	20k	199.7	20.0	542k	3600.0
belg+2arcs_85	0.0	0.0	3185	34.7	19.0	729k	3600.0
belg+2arcs_86	0.0	0.0	5352	39.6	0.0	2936	34.5
belg+2arcs_87	0.0	0.0	15k	134.6	0.2	988k	3600.0
belg+2arcs_88	0.0	0.0	2929	31.0	0.0	6311	56.8
belg+2arcs_89	0.0	0.0	46k	395.0	0.0	79k	661.9
belg+2arcs_90	0.0	0.1	756k	3600.0	0.0	5167	40.2
belg+2arcs_91	0.0	0.0	8174	69.9	0.0	10k	97.4

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+2arcs_92	0.0	0.0	161k	1515.7	0.0	53k	588.6
belg+2arcs_93	0.0	8.2	494k	3600.0	0.0	398k	3107.5
belg+2arcs_94	0.0	0.0	5051	52.5	0.0	9316	97.3
belg+2arcs_95	0.0	0.0	3331	29.4	0.0	2396	24.8
belg+2arcs_96	0.0	0.0	13k	110.0	0.0	19k	145.5
belg+2arcs_97	0.0	0.0	328k	918.7	0.0	1281	18.3
belg+2arcs_98	0.0	0.0	217k	1975.2	0.0	21k	270.1
belg+2arcs_99	0.0	0.0	13k	121.5	0.0	40k	351.3
belg+2arcs_100	0.0	0.0	13k	142.6	0.0	34k	337.1
belg+2arcs_101	0.0	0.0	11k	78.2	0.0	11k	103.9
belg+2arcs_102	0.0	0.0	10k	94.0	0.0	796k	2799.5
belg+2arcs_103	0.0	0.0	116k	1060.2	0.0	79k	749.9
belg+2arcs_104	0.0	0.0	36k	367.6	0.0	55k	482.8
belg+2arcs_105	0.0	0.0	2203	22.3	0.0	1521	16.1
belg+2arcs_106	0.0	0.0	59k	544.3	0.0	25k	280.8
belg+2arcs_107	0.0	0.0	73k	803.5	19.3	346k	3600.0
belg+2arcs_108	0.0	0.0	93k	1163.3	0.0	3956	42.4
belg+2arcs_109	0.0	0.0	13k	146.5	0.0	22k	221.5
belg+2arcs_110	0.0	0.0	59k	635.5	0.0	67k	664.8
belg+2arcs_111	0.0	0.0	4490	39.0	0.0	4491	40.9
belg+2arcs_112	0.0	0.0	1723	19.8	0.0	3777	31.7
belg+2arcs_113	0.0	0.0	2282	22.3	0.0	7798	56.7
belg+2arcs_114	0.0	0.0	313k	1136.8	0.0	7178	58.1
belg+2arcs_115	0.0	0.0	94k	889.0	1.6	430k	3600.0
belg+2arcs_116	0.0	0.0	17k	133.1	0.0	2224	24.2
belg+2arcs_117	0.0	0.0	282k	3117.3	0.0	82k	765.4
belg+2arcs_118	0.0	0.0	14k	140.1	0.0	48k	375.2
belg+2arcs_119	0.0	0.0	6260	61.6	0.0	8792	68.6
belg+2arcs_120	0.0	0.0	5392	40.3	0.0	4554	31.0
belg+2arcs_121	0.0	0.0	3853	39.5	0.0	9110	81.3
belg+2arcs_122	0.0	0.0	23k	222.6	0.0	12k	129.7
belg+2arcs_123	0.0	0.4	1240k	3600.0	0.0	1548	17.1
belg+2arcs_124	0.0	0.0	86k	923.2	0.0	309k	2410.7
belg+2arcs_125	0.0	0.0	3012	26.3	0.0	2731	28.9
belg+2arcs_126	0.0	0.0	6337	67.1	0.0	11k	94.2
belg+2arcs_127	0.0	0.0	13k	111.5	1.1	676k	3600.0
belg+2arcs_128	0.0	0.0	4976	65.7	0.0	28k	216.8
belg+2arcs_129	0.0	0.0	9423	87.2	0.0	9081	71.9
belg+2arcs_130	0.0	0.0	60k	638.4	0.0	50k	461.2
belg+2arcs_131	0.0	0.0	5752	49.9	0.0	3942	32.6
belg+2arcs_132	0.0	0.0	5473	48.0	0.0	6200	53.1
belg+2arcs_133	0.0	0.0	5539	50.6	0.0	44k	378.5
belg+2arcs_134	0.0	0.0	44k	479.1	0.0	16k	182.6
belg+2arcs_135	0.0	0.0	1882	24.4	0.0	4087	41.8
belg+2arcs_136	0.0	0.0	36k	335.9	0.0	55k	498.5
belg+2arcs_137	0.0	0.0	26k	218.8	0.0	11k	113.5
belg+2arcs_138	0.0	0.0	18k	178.9	0.0	34k	378.4
belg+2arcs_139	0.0	0.0	3642	35.3	0.3	1003k	3600.0
belg+2arcs_140	0.0	0.0	12k	73.0	0.0	3517	33.6
belg+2arcs_141	0.0	4.0	453k	3600.0	0.0	41k	397.8
belg+2arcs_142	0.0	0.0	60k	630.8	0.0	30k	292.6
belg+2arcs_143	0.0	0.0	11k	94.8	0.0	56k	424.2
belg+2arcs_144	0.0	0.0	30k	319.1	0.0	50k	479.7
belg+2arcs_145	0.0	0.0	2691	28.1	0.0	12k	87.3
belg+2arcs_146	0.0	0.0	9217	71.9	0.0	6431	50.5
belg+2arcs_147	0.0	19.3	388k	3600.0	0.0	7885	91.8
belg+2arcs_148	0.0	0.0	24k	177.4	1.5	536k	3600.0
belg+2arcs_149	0.0	0.0	66k	634.1	5.6	313k	3600.0
belg+2arcs_150	0.0	0.0	11k	95.5	0.0	268k	2252.0
belg+2arcs_151	0.0	0.0	8040	64.8	0.0	1565	17.7
belg+2arcs_152	0.0	0.0	5261	54.6	0.0	3036	32.6
belg+2arcs_153	0.0	0.0	4467	41.0	0.0	5810	50.6
belg+2arcs_154	0.0	0.0	3430	32.1	0.4	1020k	3600.0
belg+2arcs_155	0.0	0.0	9692	97.6	0.0	17k	201.2
belg+2arcs_156	0.0	0.0	3833	39.0	0.0	48k	401.2
belg+2arcs_157	0.0	0.0	2721	26.9	0.0	3252	37.2
belg+2arcs_158	0.0	0.0	66k	739.3	0.0	32k	314.6
belg+2arcs_159	0.0	0.0	2889	27.4	0.0	4249	32.5
belg+2arcs_160	0.0	0.0	4887	41.6	0.0	3941	35.7
belg+2arcs_161	0.0	0.0	2681	24.4	0.0	9744	61.0
belg+2arcs_162	0.0	0.0	2484	29.1	0.0	9439	79.1
belg+2arcs_163	0.0	7.2	368k	3600.0	0.0	45k	431.6
belg+2arcs_164	0.0	0.0	1194	14.7	0.0	1547	18.4
belg+2arcs_165	0.0	0.0	40k	356.0	0.0	47k	284.4
belg+2arcs_166	0.0	0.0	22k	224.0	0.0	55k	537.2
belg+2arcs_167	0.0	0.0	59k	590.0	7.7	488k	3600.0
belg+2arcs_168	0.0	0.0	7293	50.6	0.0	3037	25.3

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+2arcs_169	0.0	0.0	8676	93.1	0.0	247k	1525.4
belg+2arcs_170	0.0	0.0	1790	22.9	0.0	23k	216.8
belg+2arcs_171	0.0	3.6	563k	3600.0	1.7	502k	3600.0
belg+2arcs_172	0.0	0.0	5190	46.4	0.2	954k	3600.0
belg+2arcs_173	0.0	0.0	92k	858.7	0.0	113k	1072.6
belg+2arcs_174	0.0	0.0	2317	20.2	0.0	1418	16.5
belg+2arcs_175	0.0	0.0	52k	531.7	0.6	886k	3600.0
belg+2arcs_176	0.0	0.0	28k	236.8	0.0	11k	112.1
belg+2arcs_177	0.0	0.0	83k	553.5	0.0	108k	951.2
belg+2arcs_178	0.0	0.2	862k	3600.0	0.0	2459	28.9
belg+2arcs_179	0.0	0.1	1079k	3600.0	0.0	2711	33.2
belg+2arcs_180	0.0	0.0	1813	18.3	0.0	13k	79.2
belg+2arcs_181	0.0	0.0	160k	701.4	0.0	10k	115.7
belg+2arcs_182	0.0	17.0	409k	3600.0	0.0	11k	136.5
belg+2arcs_183	0.0	0.0	65k	644.6	0.0	160k	1477.1
belg+2arcs_184	0.0	0.0	985k	3600.0	1.1	854k	3600.0
belg+2arcs_185	0.0	0.0	3255	31.5	0.0	6179	47.8
belg+2arcs_186	0.0	–	425k	3600.0	0.0	119k	517.5
belg+2arcs_187	0.0	0.0	17k	173.1	0.0	27k	280.1
belg+2arcs_188	0.0	0.0	1071	14.6	0.0	28k	165.8
belg+2arcs_189	0.0	5.2	679k	3600.0	15.2	466k	3600.0
belg+2arcs_190	0.0	0.0	31k	288.8	0.0	40k	354.6
belg+2arcs_191	0.0	0.0	3630	33.5	0.0	6122	42.3
belg+2arcs_192	0.0	0.0	4988	50.6	0.0	11k	110.4
belg+2arcs_193	0.0	0.0	9617	92.0	0.0	10k	95.2
belg+2arcs_194	0.0	0.0	8628	65.3	0.0	1797	19.3
belg+2arcs_195	0.0	0.0	12k	112.9	0.0	44k	293.7
belg+2arcs_196	0.0	0.0	7865	66.3	0.5	1063k	3600.0
belg+2arcs_197	0.0	0.0	78k	825.1	–	313k	3600.0
belg+2arcs_198	0.0	0.0	89k	650.5	0.0	318k	2515.8
belg+2arcs_199	0.0	0.0	5660	44.8	0.0	5218	52.0
belg+2arcs_200	0.0	0.0	3261	35.2	0.0	6731	66.4
belg+2arcs_201	0.0	0.0	11k	105.0	0.0	53k	422.3
belg+2arcs_202	0.0	0.0	4381	33.7	5.1	1419k	3600.0
belg+2arcs_203	0.0	0.0	34k	317.9	0.6	512k	3600.0
belg+2arcs_204	0.0	0.0	2474	23.8	0.9	855k	3600.0
belg+2arcs_205	0.0	0.0	4609	47.8	0.0	3851	33.7
belg+2arcs_206	0.0	0.0	7293	68.2	0.0	6581	70.0
belg+2arcs_207	0.0	0.0	12k	109.6	0.0	2345	25.4
belg+2arcs_208	0.0	0.0	88k	899.1	0.0	17k	200.1
belg+2arcs_209	0.0	0.0	35k	368.9	0.0	89k	819.5
belg+2arcs_210	0.0	0.0	9065	84.4	0.0	75k	757.8
belg+2arcs_211	0.0	0.0	6770	76.8	0.5	853k	3600.0
belg+2arcs_212	0.0	0.0	9971	66.4	0.0	12k	85.4
belg+2arcs_213	0.0	0.0	46k	399.8	0.0	397k	2990.2
belg+2arcs_214	0.0	0.3	972k	3600.0	0.0	120k	1210.3
belg+2arcs_215	0.0	0.0	4790	42.2	0.0	20k	187.4
belg+2arcs_216	0.0	0.0	40k	349.8	0.0	44k	367.0
belg+2arcs_217	0.0	0.0	14k	149.1	0.0	7655	91.3
belg+2arcs_218	0.0	0.0	16k	167.2	0.0	23k	269.2
belg+2arcs_219	0.0	0.0	9102	84.4	0.0	10k	92.2
belg+2arcs_220	0.0	0.0	10k	95.9	0.0	25k	223.7
belg+2arcs_221	0.0	0.0	10k	80.5	0.0	2638	20.2
belg+2arcs_222	0.0	0.0	2606	30.8	0.0	2100	31.0
belg+2arcs_223	0.0	0.0	12k	88.3	0.0	2842	32.0
belg+2arcs_224	0.0	0.0	2796	35.5	0.0	12k	138.8
belg+2arcs_225	0.0	0.0	2671	28.4	0.0	18k	195.1
belg+2arcs_226	0.0	0.5	867k	3600.0	0.0	85k	846.6
belg+2arcs_227	0.0	0.0	2141	21.6	0.0	7136	76.4
belg+2arcs_228	0.0	0.0	9981	75.5	0.0	17k	125.8
belg+2arcs_229	0.0	0.0	17k	184.8	9.1	601k	3600.0
belg+2arcs_230	0.0	0.0	20k	251.2	0.0	33k	336.2
belg+2arcs_231	0.0	0.0	4882	37.9	0.0	1063	18.0
belg+2arcs_232	0.0	0.0	7305	73.8	0.0	3408	39.7
belg+2arcs_233	0.0	11.7	465k	3600.0	2.6	338k	3600.0
belg+2arcs_234	0.0	0.0	3062	33.0	0.0	6980	75.0
belg+2arcs_235	0.0	0.0	2331	25.8	0.0	3916	36.8
belg+2arcs_236	0.0	7.9	663k	3600.0	0.0	12k	104.4
belg+2arcs_237	0.0	0.0	6378	54.3	0.0	28k	193.2
belg+2arcs_238	0.0	0.0	8294	59.5	0.0	12k	82.7
belg+2arcs_239	0.0	0.0	4206	40.5	0.0	6647	71.1
belg+2arcs_240	0.0	0.0	33k	256.6	1.2	859k	3600.0
belg+2arcs_241	0.0	0.0	2170	23.9	0.0	13k	103.2
belg+2arcs_242	0.0	0.0	9893	94.0	0.0	10k	58.0
belg+2arcs_243	0.0	0.0	17k	186.1	0.0	249k	601.4
belg+2arcs_244	0.0	0.0	33k	306.8	0.0	269k	2436.1
belg+2arcs_245	0.0	0.0	26k	158.1	0.0	4884	51.2

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+2arcs_246	0.0	0.0	16k	189.2	0.2	1019k	3600.0
belg+2arcs_247	0.0	0.3	883k	3600.0	0.0	3974	40.5
belg+2arcs_248	0.0	2.8	581k	3600.0	0.0	216k	2138.3
belg+2arcs_249	0.0	0.0	1323	18.6	0.0	5553	44.0
belg+2arcs_250	0.0	0.0	2883	26.9	0.0	12k	124.9
belg+2arcs_251	0.0	0.0	8876	96.2	0.0	55k	592.8
belg+2arcs_252	0.0	0.0	15k	152.6	0.0	28k	265.6
belg+2arcs_253	0.0	2.7	834k	3600.0	1.7	639k	3600.0
belg+2arcs_254	0.0	0.0	48k	512.2	5.9	427k	3600.0
belg+2arcs_255	0.0	0.0	1496	17.3	0.0	2265	21.9
belg+2arcs_256	0.0	0.0	25k	241.4	0.0	38k	365.1
belg+2arcs_257	0.0	0.0	15k	140.4	0.0	20k	184.9
belg+2arcs_258	0.0	0.0	2325	24.5	0.0	16k	124.5
belg+2arcs_259	0.0	0.0	9033	82.1	1.4	656k	3600.0
belg+2arcs_260	0.0	13.2	332k	3600.0	0.0	44k	396.9
belg+2arcs_261	0.0	0.0	2773	26.1	0.0	1885	18.6
belg+2arcs_262	0.0	0.0	5211	47.8	0.0	2119	21.8
belg+2arcs_263	0.0	0.0	72k	677.8	0.0	74k	719.2
belg+2arcs_264	0.0	0.0	3277	35.5	0.0	5225	57.5
belg+2arcs_265	0.0	0.0	366k	3526.6	1.1	275k	3600.0
belg+2arcs_266	0.0	0.0	6023	68.1	0.0	6237	67.5
belg+2arcs_267	0.0	0.0	5225	56.4	0.0	12k	95.3
belg+2arcs_268	0.0	2.8	912k	3600.0	9.7	680k	3600.0
belg+2arcs_269	0.0	0.0	21k	216.9	0.0	36k	408.1
belg+2arcs_270	0.0	0.0	2440	26.0	0.0	45k	316.4
belg+2arcs_271	0.0	0.0	83k	886.8	0.0	128k	842.6
belg+2arcs_272	0.0	0.0	160k	1670.9	0.0	78k	871.1
belg+2arcs_273	0.0	0.0	32k	297.5	0.0	31k	312.0
belg+2arcs_274	0.0	0.0	3179	33.8	0.0	6159	61.1
belg+2arcs_275	0.0	0.0	2655	25.1	0.0	225k	1331.3
belg+2arcs_276	0.0	0.0	7031	60.8	0.0	12k	108.6
belg+2arcs_277	0.0	19.0	399k	3600.0	0.0	55k	497.2
belg+2arcs_278	0.0	0.0	22k	164.7	0.0	2071	18.2
belg+2arcs_279	0.0	0.0	9686	77.9	0.0	6738	44.5
belg+2arcs_280	0.0	0.0	12k	123.7	0.0	19k	168.3
belg+2arcs_281	0.0	0.0	6496	41.3	0.0	5835	47.0
belg+2arcs_282	0.0	0.0	33k	374.7	0.0	26k	298.9
belg+2arcs_283	0.0	4.4	531k	3600.0	0.0	20k	185.8
belg+2arcs_284	0.0	0.0	38k	415.1	0.0	10k	133.5
belg+2arcs_285	0.0	0.0	7080	66.6	0.0	2236	21.8
belg+2arcs_286	0.0	0.0	2278	24.1	1.0	610k	3600.0
belg+2arcs_287	0.0	0.0	6057	60.1	0.6	844k	3600.0
belg+2arcs_288	0.0	0.0	43k	496.7	0.0	52k	566.6
belg+2arcs_289	0.0	50.2	322k	3600.0	0.0	43k	428.8
belg+2arcs_290	0.0	0.0	18k	176.4	0.0	22k	222.7
belg+2arcs_291	0.0	0.0	224k	1725.4	0.0	3504	34.2
belg+2arcs_292	0.0	0.0	13k	107.0	0.0	2184	23.3
belg+2arcs_293	0.0	0.0	8099	83.4	0.0	32k	305.3
belg+2arcs_294	0.0	0.0	8159	72.0	0.0	7133	68.4
belg+2arcs_295	0.0	0.0	7804	71.9	0.0	17k	112.2
belg+2arcs_296	0.0	0.0	75k	722.7	0.0	26k	272.4
belg+2arcs_297	0.0	0.0	9467	101.9	0.0	41k	458.8
belg+2arcs_298	0.0	0.3	523k	3600.0	0.0	44k	425.6
belg+2arcs_299	0.0	0.0	20k	168.2	0.4	885k	3600.0
belg+2arcs_300	0.0	0.0	27k	279.9	0.0	38k	329.5
belg+2arcs_301	0.0	0.0	2562	22.2	0.0	6420	53.6
belg+2arcs_302	0.0	0.0	2961	34.4	0.0	6550	70.6
belg+2arcs_303	0.0	0.0	98k	762.7	0.0	364k	2952.5
belg+2arcs_304	0.0	0.0	21k	279.8	0.0	2991	31.2
belg+2arcs_305	0.0	0.0	31k	146.5	0.0	4566	41.0
belg+2arcs_306	0.0	0.0	17k	180.0	38.3	331k	3600.0
belg+2arcs_307	0.0	0.0	12k	127.1	0.0	7850	70.0
belg+2arcs_308	0.0	0.0	9017	61.4	0.0	10k	74.9
belg+2arcs_309	0.0	0.0	91k	881.2	0.0	285k	1717.5
belg+2arcs_310	0.0	0.0	12k	114.2	0.0	51k	575.2
belg+2arcs_311	0.0	0.0	2795	27.8	0.0	1962	20.7
belg+2arcs_312	0.0	0.0	4483	37.8	0.0	3327	29.7
belg+2arcs_313	0.0	0.0	8451	78.5	0.0	8080	97.0
belg+2arcs_314	0.0	1.2	1015k	3600.0	0.0	28k	222.0
belg+2arcs_315	0.0	0.0	2931	32.0	8.3	371k	3600.0
belg+2arcs_316	0.0	0.0	3880	32.6	0.0	6051	62.8
belg+2arcs_317	0.0	0.0	3727	39.5	0.0	383k	2701.4
belg+2arcs_318	0.0	0.0	5141	50.9	0.0	17k	130.1
belg+2arcs_319	0.0	0.0	8091	86.1	0.0	6191	59.7
belg+2arcs_320	0.0	0.0	11k	93.5	0.0	30k	234.7
belg+2arcs_321	0.0	0.0	2678	21.6	0.0	1056	14.3
belg+2arcs_322	0.0	0.0	108k	1104.6	0.0	128k	1194.4

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+2arcs_323	0.0	0.0	10k	88.3	0.0	71k	388.0
belg+2arcs_324	0.0	0.0	2657	21.7	0.0	70k	231.0
belg+2arcs_325	0.0	0.0	16k	141.2	0.0	39k	460.9
belg+2arcs_326	0.0	0.0	2865	36.2	0.0	1854	22.9
belg+2arcs_327	0.0	0.0	5599	53.8	0.0	5791	56.9
belg+2arcs_328	0.0	0.0	2643	30.2	0.0	2383	25.8
belg+2arcs_329	0.0	0.0	7372	66.9	0.0	4256	37.8
belg+2arcs_330	0.0	0.0	74k	430.5	0.0	7927	91.3
belg+2arcs_331	0.0	0.0	1596	17.3	0.0	562	12.6
belg+2arcs_332	0.0	9.3	698k	3600.0	8.7	408k	3600.0
belg+2arcs_333	0.0	0.0	22k	179.8	0.0	159k	703.1
belg+2arcs_334	0.0	0.0	9781	78.7	0.0	66k	624.4
belg+2arcs_335	0.0	0.0	4221	42.6	0.0	4353	46.7
belg+2arcs_336	0.0	0.0	11k	130.6	0.0	35k	330.2
belg+2arcs_337	0.0	0.0	2914	25.8	0.0	3004	27.0
belg+2arcs_338	0.0	0.0	1757	20.0	0.0	3431	34.7
belg+2arcs_339	0.0	0.0	5869	62.4	0.0	97k	796.0
belg+2arcs_340	0.0	0.0	4551	54.5	0.0	47k	462.4
belg+2arcs_341	0.0	0.0	8615	91.7	0.0	32k	294.2
belg+2arcs_342	0.0	0.0	69k	677.2	31.2	355k	3600.0
belg+2arcs_343	0.0	0.0	52k	268.9	0.0	8052	77.0
belg+2arcs_344	0.0	0.0	4999	54.8	0.0	6857	68.7
belg+2arcs_345	0.0	0.0	52k	501.8	0.0	47k	461.9
belg+2arcs_346	0.0	0.0	1841	18.9	0.0	4819	37.9
belg+2arcs_347	0.0	0.3	1054k	3600.0	0.0	457k	2124.8
belg+2arcs_348	0.0	0.0	5254	44.7	0.0	4426	36.7
belg+2arcs_349	0.0	0.0	48k	538.0	0.0	70k	840.0
belg+2arcs_350	0.0	12.2	422k	3600.0	0.0	101k	910.7
belg+2arcs_351	0.0	0.0	2081	21.4	0.0	9840	57.2
belg+2arcs_352	0.0	0.0	76k	591.7	2.5	584k	3600.0
belg+2arcs_353	0.0	0.0	4124	40.0	0.0	10k	99.9
belg+2arcs_354	0.0	0.0	1965	20.1	0.0	4594	41.8
belg+2arcs_355	0.0	0.0	1175	15.3	0.0	3361	33.2
belg+2arcs_356	0.0	0.0	2531	24.6	0.0	5795	52.3
belg+2arcs_357	0.0	0.0	2356	27.4	0.0	3170	30.3
belg+2arcs_358	0.0	0.0	46k	440.4	0.0	54k	612.5
belg+2arcs_359	0.0	0.0	2479	22.6	0.0	3251	23.1
belg+2arcs_360	0.0	0.0	6014	41.8	0.0	3382	28.5
belg+2arcs_361	0.0	0.0	24k	243.3	0.0	7947	69.0
belg+2arcs_362	0.0	0.0	4320	33.5	0.0	3897	33.5
belg+2arcs_363	0.0	0.0	55k	599.0	0.0	11k	106.5
belg+2arcs_364	0.0	0.0	2938	28.6	0.0	3348	23.8
belg+2arcs_365	0.0	0.0	101k	1089.5	0.0	270k	2701.1
belg+2arcs_366	0.0	0.0	39k	455.4	0.0	74k	698.0
belg+2arcs_367	0.0	5.8	611k	3600.0	0.0	11k	110.7
belg+2arcs_368	0.0	0.0	12k	97.2	0.0	48k	325.2
belg+2arcs_369	0.0	0.0	2084	22.6	0.0	1824	18.2
belg+2arcs_370	0.0	11.4	563k	3600.0	–	333k	3600.0
belg+2arcs_371	0.0	5.8	740k	3600.0	0.6	619k	3600.0
belg+2arcs_372	0.0	0.0	13k	141.1	2.3	352k	3600.0
belg+2arcs_373	0.0	0.0	4520	42.2	0.0	23k	240.4
belg+2arcs_374	4e-05	0.0	3597	34.2	0.0	2260	25.6
belg+2arcs_375	0.0	0.0	2345	22.2	0.9	1098k	3600.0
belg+2arcs_376	0.0	0.0	35k	337.2	11.2	507k	3600.0
belg+2arcs_377	0.0	0.0	185k	1785.8	0.0	228k	2362.4
belg+2arcs_378	0.0	0.0	1438	18.1	0.0	16k	117.5
belg+2arcs_379	0.0	0.0	16k	137.2	0.0	16k	147.7
belg+2arcs_380	0.0	10.3	396k	3600.0	0.0	13k	125.6
belg+2arcs_381	0.0	0.0	8039	72.2	0.0	22k	189.6
belg+2arcs_382	0.0	0.0	25k	275.3	6.8	919k	3600.0
belg+2arcs_383	0.0	0.0	77k	273.5	0.0	14k	156.8
belg+2arcs_384	0.0	0.0	2777	26.5	0.0	3191	26.6
belg+2arcs_385	0.0	0.0	3237	30.0	0.0	4173	37.6
belg+2arcs_386	0.0	2.4	484k	3600.0	0.0	55k	763.6
belg+2arcs_387	0.0	1.6	702k	3600.0	0.0	167k	1662.7
belg+2arcs_388	0.0	0.0	5912	57.8	0.0	13k	126.5
belg+2arcs_389	0.0	0.0	3071	31.8	18.2	855k	3600.0
belg+2arcs_390	0.0	0.0	3501	34.3	0.0	4230	36.2
belg+2arcs_391	0.0	0.0	39k	354.6	0.0	6431	58.7
belg+2arcs_392	0.0	3.2	801k	3600.0	7.5	954k	3600.0
belg+2arcs_393	0.0	0.0	42k	404.2	0.0	87k	589.2
belg+2arcs_394	0.0	0.0	2188	20.4	0.0	12k	63.7
belg+2arcs_395	0.0	0.0	7837	50.9	0.0	6668	47.0
belg+2arcs_396	0.0	0.0	3358	35.9	0.0	5233	55.3
belg+2arcs_397	0.0	0.0	88k	788.1	1.1	486k	3600.0
belg+2arcs_398	0.0	0.0	2237	22.4	0.0	18k	121.4
belg+2arcs_399	0.0	33.6	405k	3600.0	0.0	12k	111.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+2arcs_400	0.0	0.0	13k	96.8	0.0	7194	44.7
belg+2arcs_401	0.0	0.0	12k	147.2	6.3	530k	3600.0
belg+2arcs_402	0.0	0.0	17k	170.2	0.0	17k	172.2
belg+2arcs_403	0.0	0.0	132k	1130.1	0.0	55k	533.4
belg+2arcs_404	0.0	0.0	27k	258.1	0.0	14k	129.1
belg+2arcs_405	0.0	0.0	214k	2086.6	2.1	451k	3600.0
belg+2arcs_406	0.0	0.0	10k	114.2	0.0	4793	61.0
belg+2arcs_407	0.0	0.0	15k	130.8	0.0	39k	330.7
belg+2arcs_408	0.0	0.0	799	14.2	0.0	3513	33.9
belg+2arcs_409	0.0	0.0	680k	2400.0	0.0	18k	165.6
belg+2arcs_410	0.0	0.0	132k	1168.7	0.0	22k	224.1
belg+2arcs_411	0.0	0.0	4501	36.4	0.0	2494	27.1
belg+2arcs_412	0.0	0.0	7971	59.3	0.0	38k	336.6
belg+2arcs_413	0.0	15.0	406k	3600.0	0.0	422k	3066.4
belg+2arcs_414	0.0	0.0	18k	167.0	0.0	100k	904.6
belg+2arcs_415	0.0	0.0	5015	41.4	0.0	4671	34.2
belg+2arcs_416	0.0	0.0	59k	607.9	0.0	49k	480.3
belg+2arcs_417	0.0	0.0	14k	119.8	0.0	11k	135.3
belg+2arcs_418	0.0	0.0	13k	110.8	0.0	18k	141.3
belg+2arcs_419	0.0	0.0	19k	139.3	0.0	7111	67.3
belg+2arcs_420	0.0	0.0	6153	57.8	0.0	1524	18.5
belg+2arcs_421	0.0	34.2	982k	3600.0	0.0	52k	528.5
belg+2arcs_422	0.0	0.0	9554	68.7	0.0	2330	22.5
belg+2arcs_423	0.0	0.0	3881	36.0	0.0	11k	111.7
belg+2arcs_424	0.0	0.0	316k	2431.7	0.0	190k	1570.9
belg+2arcs_425	0.0	0.0	86k	474.8	0.0	20k	144.5
belg+2arcs_426	0.0	0.0	125k	1296.0	53.7	308k	3600.0
belg+2arcs_427	0.0	0.0	34k	286.1	0.0	7668	80.3
belg+2arcs_428	0.0	0.0	4310	32.8	0.0	8360	78.3
belg+2arcs_429	0.0	0.0	26k	194.4	0.0	26k	228.1
belg+2arcs_430	0.0	0.0	9201	55.5	0.0	4076	32.5
belg+2arcs_431	0.0	0.0	9438	68.1	0.5	998k	3600.0
belg+2arcs_432	0.0	0.0	1961	20.0	0.0	2369	22.6
belg+2arcs_433	0.0	0.0	2471	19.8	0.0	35k	244.9
belg+2arcs_434	0.0	9.3	668k	3600.0	0.0	20k	176.3
belg+2arcs_435	0.0	0.0	18k	171.4	0.0	30k	323.8
belg+2arcs_436	0.0	6.7	501k	3600.0	0.3	820k	3600.0
belg+2arcs_437	0.0	0.0	23k	210.6	2.9	615k	3600.0
belg+2arcs_438	0.0	0.0	1751	20.2	0.0	2701	27.6
belg+2arcs_439	0.0	0.0	12k	138.8	0.0	19k	201.3
belg+2arcs_440	0.0	0.0	18k	150.5	0.0	4838	55.1
belg+2arcs_441	0.0	0.0	1421	18.0	0.0	2453	23.8
belg+2arcs_442	0.0	0.0	2508	29.2	0.0	36k	290.6
belg+2arcs_443	0.0	0.0	16k	159.3	0.0	20k	147.0
belg+2arcs_444	7e-05	0.0	4864	44.6	0.0	8069	69.5
belg+2arcs_445	0.0	0.0	55k	504.9	0.6	412k	3600.0
belg+2arcs_446	0.0	0.0	3101	30.5	0.0	3672	47.0
belg+2arcs_447	0.0	0.0	1616	19.3	0.0	16k	85.5
belg+2arcs_448	0.0	0.0	94k	819.8	3.9	336k	3600.0
belg+2arcs_449	0.0	0.0	13k	112.9	–	1300k	3600.0
belg+2arcs_450	0.0	0.0	69k	716.6	0.0	167k	1799.3
belg+2arcs_451	0.0	0.0	6607	47.2	0.0	1858	16.4
belg+2arcs_452	0.0	0.0	19k	196.9	0.0	1414	18.7
belg+2arcs_453	0.0	0.0	185k	1488.1	0.0	54k	404.8
belg+2arcs_454	0.0	0.0	12k	126.4	0.0	13k	123.2
belg+2arcs_455	0.0	0.0	5696	50.8	0.0	3587	32.6
belg+2arcs_456	0.0	0.0	17k	160.8	0.8	822k	3600.0
belg+2arcs_457	0.0	0.0	103k	951.4	0.0	40k	515.8
belg+2arcs_458	0.0	3.8	847k	3600.0	11.8	333k	3600.0
belg+2arcs_459	0.0	0.0	58k	577.8	0.0	63k	577.1
belg+2arcs_460	0.0	0.0	2391	22.2	0.0	1879	16.7
belg+2arcs_461	0.0	0.0	15k	147.2	0.0	142k	1150.4
belg+2arcs_462	0.0	0.0	2434	25.3	0.0	8773	104.4
belg+2arcs_463	0.0	0.0	6963	64.5	0.0	12k	156.3
belg+2arcs_464	0.0	0.0	6109	56.7	0.0	2920	32.8
belg+2arcs_465	0.0	0.0	16k	162.9	0.0	197k	1845.6
belg+2arcs_466	0.0	0.0	8306	89.2	0.0	21k	207.7
belg+2arcs_467	0.0	0.0	464k	3260.9	5.5	282k	3600.0
belg+2arcs_468	0.0	35.4	560k	3600.0	0.0	40k	438.8
belg+2arcs_469	0.0	0.0	3929	32.7	0.0	1767	19.0
belg+2arcs_470	0.0	0.0	2576	23.0	0.0	5588	49.6
belg+2arcs_471	0.0	0.0	14k	105.1	0.0	10k	81.5
belg+2arcs_472	0.0	40.9	553k	3600.0	0.0	1494	16.6
belg+2arcs_473	0.0	0.0	66k	829.0	0.0	163k	1583.9
belg+2arcs_474	0.0	0.0	24k	101.8	0.0	58k	462.1
belg+2arcs_475	0.0	0.0	7810	81.3	0.9	616k	3600.0
belg+2arcs_476	0.0	0.0	7671	79.2	0.0	13k	134.6

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+2arcs_477	0.0	0.0	3645	34.5	0.0	3546	35.5
belg+2arcs_478	0.0	0.0	582k	2051.8	0.0	21k	212.0
belg+2arcs_479	0.0	0.0	247k	2144.3	13.8	333k	3600.0
belg+2arcs_480	0.0	0.0	17k	174.4	0.0	29k	351.5
belg+2arcs_481	0.0	0.0	11k	67.7	0.0	3005	30.3
belg+2arcs_482	0.0	0.0	8697	105.7	1.5	705k	3600.0
belg+2arcs_483	0.0	–	835k	3600.0	0.0	55k	530.7
belg+2arcs_484	0.0	0.0	19k	151.9	0.0	3064	31.9
belg+2arcs_485	0.0	36.3	850k	3600.0	0.0	7098	78.1
belg+2arcs_486	0.0	0.0	8387	86.8	0.0	15k	149.8
belg+2arcs_487	0.0	0.0	5930	77.1	0.0	23k	220.2
belg+2arcs_488	0.0	0.3	870k	3600.0	0.0	27k	179.1
belg+2arcs_489	0.0	0.0	17k	155.2	0.0	13k	119.4
belg+2arcs_490	0.0	0.0	6309	62.2	0.0	20k	167.6
belg+2arcs_491	0.0	0.0	11k	108.1	0.0	13k	125.2
belg+2arcs_492	0.0	0.0	90k	694.1	19.0	523k	3600.0
belg+2arcs_493	0.0	0.0	10k	83.9	0.2	691k	3600.0
belg+2arcs_494	0.0	0.0	6921	75.2	0.0	17k	116.1
belg+2arcs_495	0.0	22.7	469k	3600.0	0.0	71k	617.7
belg+2arcs_496	0.0	0.0	1772	20.2	0.0	3250	33.0
belg+2arcs_497	0.0	0.0	22k	265.9	0.0	6508	61.8
belg+2arcs_498	0.0	0.0	8366	97.9	0.0	21k	157.1
belg+2arcs_499	0.0	14.8	916k	3600.0	0.0	103k	783.5
belg+2arcs_500	0.0	0.0	79k	652.6	2.2	543k	3600.0
belg+4arcs_1	–	–	224k	3600.0	–	202k	3600.0
belg+4arcs_2	–	0.0	37k	492.1	0.0	22k	288.2
belg+4arcs_3	–	142.5	173k	3600.0	117.1	200k	3600.0
belg+4arcs_4	–	–	226k	3599.1	0.0	83k	1206.0
belg+4arcs_5	–	–	264k	3600.0	24.3	203k	3600.0
belg+4arcs_6	–	–	304k	3600.0	0.0	64k	1030.0
belg+4arcs_7	–	72.5	244k	3600.0	0.0	201k	3251.3
belg+4arcs_8	–	0.0	9900	112.7	0.0	49k	478.0
belg+4arcs_9	–	0.0	113k	1608.9	0.0	198k	2598.6
belg+4arcs_10	–	0.0	60k	781.6	0.0	58k	832.5
belg+4arcs_11	–	24.2	247k	3600.0	65.2	214k	3600.0
belg+4arcs_12	–	0.0	172k	2535.1	0.0	158k	2311.0
belg+4arcs_13	–	0.0	94k	1330.4	67.3	220k	3600.0
belg+4arcs_14	–	0.0	195k	2755.1	0.0	108k	1788.1
belg+4arcs_15	–	0.0	76k	703.1	0.0	44k	517.5
belg+4arcs_16	–	–	199k	3600.0	–	244k	3600.0
belg+4arcs_17	–	0.0	119k	1416.4	0.1	475k	3600.0
belg+4arcs_18	–	0.0	197k	2449.8	0.0	163k	2255.3
belg+4arcs_19	–	0.0	46k	598.8	0.0	83k	1232.6
belg+4arcs_20	–	–	292k	3600.0	0.0	223k	3202.6
belg+4arcs_21	–	0.0	27k	382.6	1241.2	297k	3600.0
belg+4arcs_22	–	0.0	143k	2028.8	0.0	97k	1407.7
belg+4arcs_23	–	0.0	61k	911.6	1243.6	473k	3600.0
belg+4arcs_24	–	0.0	180k	1905.5	0.0	46k	618.2
belg+4arcs_25	–	0.0	148k	1659.9	83.5	427k	3600.0
belg+4arcs_26	–	0.0	54k	762.7	0.0	254k	3010.6
belg+4arcs_27	–	0.0	35k	428.3	0.0	76k	811.1
belg+4arcs_28	–	0.0	181k	2611.6	0.0	206k	3042.5
belg+4arcs_29	–	0.0	93k	1052.6	0.0	45k	612.6
belg+4arcs_30	–	0.0	121k	1815.0	0.0	159k	1829.6
belg+4arcs_31	–	0.0	117k	1191.0	0.0	156k	1711.0
belg+4arcs_32	–	0.0	220k	2538.3	93.6	386k	3600.0
belg+4arcs_33	–	0.0	155k	2584.9	1114.1	641k	3600.0
belg+4arcs_34	0.0	0.0	8387	101.4	0.0	20k	248.8
belg+4arcs_35	–	0.0	124k	1845.7	0.0	91k	1479.0
belg+4arcs_36	–	110.0	230k	3600.0	0.0	35k	512.6
belg+4arcs_37	–	–	320k	3600.0	288.2	251k	3600.0
belg+4arcs_38	–	0.0	287k	1850.6	0.0	120k	1478.5
belg+4arcs_39	–	0.0	85k	1166.7	0.0	248k	2798.8
belg+4arcs_40	–	0.0	201k	3243.7	–	226k	3600.0
belg+4arcs_41	–	0.0	228k	2579.1	1.2	239k	3600.0
belg+4arcs_42	–	–	400k	3600.0	0.0	279k	3537.4
belg+4arcs_43	–	11.4	250k	3600.0	0.0	208k	3558.7
belg+4arcs_44	0.0	0.0	37k	448.0	0.0	21k	273.0
belg+4arcs_45	–	0.0	132k	1302.8	0.0	127k	1527.9
belg+4arcs_46	–	0.0	256k	3366.9	0.0	55k	974.9
belg+4arcs_47	–	0.0	125k	1553.0	0.0	121k	1611.6
belg+4arcs_48	–	26.3	677k	3600.0	0.0	233k	2892.0
belg+4arcs_49	–	0.0	116k	1295.4	0.0	59k	709.5
belg+4arcs_50	–	1.0	279k	3600.0	6.1	349k	3600.0
belg+4arcs_51	–	0.0	39k	465.8	0.0	21k	255.9
belg+4arcs_52	–	27.6	254k	3600.0	–	282k	3600.0
belg+4arcs_53	–	0.0	236k	3126.1	87.2	356k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+4arcs_54	–	0.0	109k	1671.9	22.1	231k	3600.0
belg+4arcs_55	–	0.0	53k	684.5	0.0	70k	913.4
belg+4arcs_56	–	0.0	71k	916.3	28.0	236k	3600.0
belg+4arcs_57	–	–	374k	3600.0	116.5	273k	3600.0
belg+4arcs_58	–	0.0	251k	3491.5	58.2	201k	3600.0
belg+4arcs_59	–	0.0	194k	2270.3	0.0	45k	525.0
belg+4arcs_60	–	6.6	462k	3600.0	0.0	100k	1273.1
belg+4arcs_61	–	3.7	265k	3600.0	0.0	211k	2964.0
belg+4arcs_62	–	74.1	203k	3600.0	83.2	209k	3600.0
belg+4arcs_63	–	0.0	29k	404.8	0.0	26k	353.1
belg+4arcs_64	–	0.0	105k	1347.6	0.0	73k	814.0
belg+4arcs_65	–	0.0	67k	856.8	0.0	85k	1154.9
belg+4arcs_66	–	0.0	79k	705.5	0.0	60k	781.0
belg+4arcs_67	–	0.0	131k	1429.3	0.0	157k	1336.7
belg+4arcs_68	–	0.0	97k	1660.5	86.8	360k	3600.0
belg+4arcs_69	–	0.0	240k	3576.3	0.0	196k	2227.1
belg+4arcs_70	–	0.0	163k	2353.2	0.0	73k	1053.7
belg+4arcs_71	–	0.0	219k	3087.2	0.0	98k	1476.5
belg+4arcs_72	–	579.6	323k	3600.0	139.4	240k	3600.0
belg+4arcs_73	0.0	0.0	11k	142.9	0.0	17k	196.9
belg+4arcs_74	–	0.0	228k	3237.2	–	240k	3600.0
belg+4arcs_75	–	79.5	236k	3600.0	123.2	238k	3600.0
belg+4arcs_76	0.00713	0.0	110k	1421.1	0.0	90k	1216.5
belg+4arcs_77	–	0.0	65k	916.4	0.0	50k	598.5
belg+4arcs_78	–	0.0	55k	619.0	310.5	272k	3600.0
belg+4arcs_79	–	157.5	237k	3600.0	561.5	281k	3600.0
belg+4arcs_80	–	0.0	137k	1748.4	0.0	173k	2404.8
belg+4arcs_81	–	21.4	233k	3600.0	89.1	405k	3600.0
belg+4arcs_82	–	0.0	24k	271.1	0.0	6502	69.7
belg+4arcs_83	–	0.0	314k	3600.0	–	221k	3600.0
belg+4arcs_84	0.0	562.2	187k	3600.0	0.0	63k	917.5
belg+4arcs_85	–	0.0	166k	1821.6	0.0	122k	1739.8
belg+4arcs_86	–	108.8	227k	3600.0	–	780k	3600.0
belg+4arcs_87	–	0.0	229k	3282.1	0.0	144k	1828.7
belg+4arcs_88	–	–	190k	3600.0	120.4	293k	3600.0
belg+4arcs_89	–	0.0	167k	2289.1	–	194k	3600.0
belg+4arcs_90	–	5.9	276k	3600.0	5.3	258k	3600.0
belg+4arcs_91	–	–	221k	3600.0	80.7	208k	3600.0
belg+4arcs_92	–	0.0	256k	2947.1	0.0	103k	1346.8
belg+4arcs_93	–	38.3	290k	3600.0	2.8	268k	3600.0
belg+4arcs_94	–	0.5	315k	3600.0	–	443k	3600.0
belg+4arcs_95	–	0.0	212k	2470.2	0.0	198k	2849.6
belg+4arcs_96	–	96.0	271k	3600.0	83.3	254k	3600.0
belg+4arcs_97	–	0.0	111k	1227.9	204.3	306k	3600.0
belg+4arcs_98	–	0.0	175k	2577.0	–	270k	3600.0
belg+4arcs_99	–	59.4	215k	3600.0	101.3	345k	3600.0
belg+4arcs_100	–	0.0	56k	732.8	0.0	95k	1478.1
belg+4arcs_101	–	0.0	105k	1395.7	61.9	392k	3600.0
belg+4arcs_102	–	0.0	204k	2883.6	0.0	94k	1414.9
belg+4arcs_103	–	0.0	66k	918.5	–	358k	3600.0
belg+4arcs_104	–	0.0	20k	233.1	0.0	41k	448.9
belg+4arcs_105	–	0.0	44k	622.9	0.0	267k	3122.5
belg+4arcs_106	–	0.0	52k	767.6	0.0	202k	2075.7
belg+4arcs_107	–	0.0	71k	1061.1	57.3	335k	3600.0
belg+4arcs_108	–	0.0	10k	125.9	0.0	12k	141.8
belg+4arcs_109	–	0.0	174k	2327.6	92.1	244k	3600.0
belg+4arcs_110	–	0.0	68k	1024.9	0.0	214k	3218.6
belg+4arcs_111	–	3.8	270k	3600.0	0.0	160k	2013.1
belg+4arcs_112	–	0.0	68k	930.8	0.0	126k	1662.3
belg+4arcs_113	–	Large	374k	3600.0	0.0	59k	619.6
belg+4arcs_114	–	1.9	347k	3600.0	0.0	108k	1158.7
belg+4arcs_115	–	54.5	221k	3600.0	74.6	224k	3600.0
belg+4arcs_116	–	0.0	272k	2923.3	0.0	63k	802.0
belg+4arcs_117	–	230.6	438k	3600.0	0.0	220k	3246.3
belg+4arcs_118	–	0.0	65k	712.9	0.0	45k	533.3
belg+4arcs_119	–	8.3	388k	3600.0	0.0	718k	3449.3
belg+4arcs_120	–	0.0	138k	1473.0	53.6	293k	3600.0
belg+4arcs_121	–	0.0	100k	1194.3	0.0	62k	602.7
belg+4arcs_122	–	0.0	138k	1600.6	0.0	111k	1296.3
belg+4arcs_123	–	235.4	331k	3600.0	0.0	152k	1533.7
belg+4arcs_124	–	0.0	49k	753.6	0.0	171k	2244.8
belg+4arcs_125	–	0.0	271k	3371.7	240.6	284k	3600.0
belg+4arcs_126	–	0.0	153k	1967.2	20.0	269k	3600.0
belg+4arcs_127	–	0.0	270k	2837.4	0.0	155k	2240.9
belg+4arcs_128	–	17.2	251k	3600.0	22.2	244k	3600.0
belg+4arcs_129	–	0.0	81k	1159.3	0.0	131k	1903.2
belg+4arcs_130	–	0.0	74k	908.8	100.0	271k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+4arcs_131	–	0.0	367k	2668.7	20.9	234k	3600.0
belg+4arcs_132	–	0.0	87k	821.4	0.0	67k	942.5
belg+4arcs_133	0.0	0.0	36k	478.2	0.0	111k	1539.4
belg+4arcs_134	–	65.3	291k	3600.0	54.8	210k	3600.0
belg+4arcs_135	–	0.0	33k	343.8	0.0	51k	586.2
belg+4arcs_136	–	–	226k	3600.0	–	203k	3600.0
belg+4arcs_137	–	60.0	480k	3600.0	–	502k	3600.0
belg+4arcs_138	–	0.0	234k	2992.9	73.6	255k	3600.0
belg+4arcs_139	–	28.0	254k	3600.0	0.0	172k	2023.1
belg+4arcs_140	–	249.2	277k	3600.0	250.4	260k	3600.0
belg+4arcs_141	–	–	219k	3600.0	0.0	206k	3064.8
belg+4arcs_142	–	0.0	208k	2795.2	89.1	234k	3600.0
belg+4arcs_143	–	0.0	89k	1055.3	0.0	66k	872.2
belg+4arcs_144	–	0.0	75k	1118.2	0.0	32k	428.5
belg+4arcs_145	–	68.4	320k	3600.0	305.1	296k	3600.0
belg+4arcs_146	–	0.0	143k	1823.5	148.5	245k	3600.0
belg+4arcs_147	–	0.0	128k	1914.8	0.0	75k	1255.4
belg+4arcs_148	–	12.2	222k	3600.0	77.3	208k	3600.0
belg+4arcs_149	–	0.0	48k	637.5	0.0	202k	3522.6
belg+4arcs_150	–	57.4	280k	3600.0	0.0	95k	1540.3
belg+4arcs_151	–	0.0	71k	981.2	348.6	256k	3600.0
belg+4arcs_152	–	0.0	83k	1002.2	0.0	84k	1083.7
belg+4arcs_153	–	0.0	101k	1338.8	0.0	138k	2100.7
belg+4arcs_154	–	0.0	164k	1737.3	0.0	88k	1190.2
belg+4arcs_155	–	–	240k	3600.0	–	503k	3600.0
belg+4arcs_156	–	0.0	169k	1926.7	0.0	104k	1144.5
belg+4arcs_157	–	0.0	96k	966.7	0.0	160k	1876.8
belg+4arcs_158	–	0.0	113k	1766.2	–	373k	3600.0
belg+4arcs_159	–	65.5	208k	3600.0	152.0	266k	3600.0
belg+4arcs_160	–	30.6	266k	3600.0	193.7	313k	3600.0
belg+4arcs_161	–	2.0	265k	3600.0	100.5	252k	3600.0
belg+4arcs_162	–	0.0	255k	3125.2	0.0	272k	2825.1
belg+4arcs_163	–	0.0	145k	1914.3	0.0	281k	3263.6
belg+4arcs_164	–	0.0	68k	662.2	0.0	261k	1853.0
belg+4arcs_165	–	0.0	54k	622.4	0.0	53k	592.2
belg+4arcs_166	–	0.0	33k	429.4	0.0	86k	1429.3
belg+4arcs_167	–	0.0	184k	3176.5	0.0	124k	1877.0
belg+4arcs_168	–	0.6	317k	3600.0	0.0	123k	1409.8
belg+4arcs_169	–	0.0	80k	1061.1	0.0	148k	1935.0
belg+4arcs_170	–	0.0	107k	1384.6	0.0	101k	1197.5
belg+4arcs_171	–	0.0	160k	2209.3	0.0	113k	1429.0
belg+4arcs_172	–	53.7	368k	3600.0	0.0	192k	2206.9
belg+4arcs_173	–	0.0	63k	873.0	0.0	70k	842.5
belg+4arcs_174	–	0.0	198k	1923.7	0.0	227k	3142.0
belg+4arcs_175	–	–	275k	3600.0	86.1	212k	3600.0
belg+4arcs_176	–	2.0	251k	3600.0	0.0	232k	3449.8
belg+4arcs_177	–	0.0	65k	959.3	0.0	56k	946.0
belg+4arcs_178	–	–	643k	3600.0	0.0	117k	1384.6
belg+4arcs_179	–	19.0	519k	3600.0	0.0	96k	1648.7
belg+4arcs_180	–	357.6	275k	3600.0	546.6	314k	3600.0
belg+4arcs_181	–	0.0	138k	1715.5	0.0	105k	1669.9
belg+4arcs_182	–	0.0	99k	1224.3	0.0	180k	2055.3
belg+4arcs_183	–	0.0	29k	390.4	0.0	32k	435.0
belg+4arcs_184	–	0.0	96k	1328.8	0.0	74k	1069.0
belg+4arcs_185	–	0.0	88k	1125.7	0.0	138k	1507.5
belg+4arcs_186	–	68.2	405k	3600.0	151.9	237k	3600.0
belg+4arcs_187	–	63.2	204k	3600.0	–	186k	3600.0
belg+4arcs_188	–	0.0	38k	495.1	0.0	82k	1245.5
belg+4arcs_189	–	0.0	176k	2373.8	0.0	152k	2230.9
belg+4arcs_190	–	0.0	174k	2473.3	174.0	273k	3600.0
belg+4arcs_191	–	0.0	73k	938.4	0.0	67k	849.1
belg+4arcs_192	–	0.0	255k	3379.1	28.8	277k	3600.0
belg+4arcs_193	–	0.0	24k	273.1	0.0	232k	2557.9
belg+4arcs_194	–	32.7	425k	3600.0	98.2	267k	3600.0
belg+4arcs_195	–	0.0	121k	1141.1	0.0	83k	1014.4
belg+4arcs_196	–	16.1	225k	3600.0	71.2	203k	3600.0
belg+4arcs_197	–	0.0	141k	2152.0	0.0	214k	3138.3
belg+4arcs_198	–	0.0	222k	1604.0	0.0	41k	552.7
belg+4arcs_199	–	0.0	84k	1275.8	23.7	262k	3600.0
belg+4arcs_200	–	95.3	263k	3600.0	9.5	216k	3600.0
belg+4arcs_201	–	0.0	155k	2108.4	75.1	208k	3600.0
belg+4arcs_202	–	0.0	85k	1094.0	–	743k	3600.0
belg+4arcs_203	–	23.4	211k	3600.0	0.0	178k	2781.8
belg+4arcs_204	–	0.0	243k	3582.6	0.0	211k	2972.7
belg+4arcs_205	–	16.2	290k	3600.0	0.0	231k	2615.2
belg+4arcs_206	–	0.0	145k	1895.9	0.0	126k	1726.0
belg+4arcs_207	–	0.0	320k	2939.5	–	265k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+4arcs_208	–	0.0	113k	1629.8	84.1	197k	3600.0
belg+4arcs_209	–	0.0	21k	311.5	189.7	197k	3600.0
belg+4arcs_210	–	0.0	104k	1371.1	–	812k	3600.0
belg+4arcs_211	–	99.5	211k	3600.0	0.0	142k	2085.1
belg+4arcs_212	–	0.0	46k	585.0	0.0	91k	1105.5
belg+4arcs_213	–	60.0	222k	3600.0	61.9	268k	3600.0
belg+4arcs_214	–	0.0	64k	939.8	36.7	277k	3600.0
belg+4arcs_215	–	0.0	76k	936.6	0.0	113k	1375.0
belg+4arcs_216	–	0.0	222k	2960.2	26.7	218k	3600.0
belg+4arcs_217	–	0.0	96k	1078.1	0.0	138k	1700.6
belg+4arcs_218	–	0.0	67k	1018.4	0.0	223k	2497.9
belg+4arcs_219	–	0.0	69k	1155.3	0.0	168k	2404.4
belg+4arcs_220	–	0.0	262k	3001.6	–	251k	3600.0
belg+4arcs_221	–	0.0	73k	781.4	0.0	60k	772.0
belg+4arcs_222	–	0.0	117k	1413.4	0.0	224k	2972.2
belg+4arcs_223	–	168.3	289k	3600.0	0.0	83k	1203.3
belg+4arcs_224	–	100.7	271k	3600.0	–	254k	3600.0
belg+4arcs_225	–	550.1	781k	3600.0	0.0	201k	2874.0
belg+4arcs_226	–	31.5	179k	3600.0	57.5	215k	3600.0
belg+4arcs_227	–	0.0	139k	1855.2	74.4	241k	3600.0
belg+4arcs_228	–	15.8	416k	3600.0	71.7	229k	3600.0
belg+4arcs_229	–	81.6	251k	3600.0	–	263k	3600.0
belg+4arcs_230	–	24.4	240k	3600.0	0.0	104k	1472.7
belg+4arcs_231	–	96.2	228k	3600.0	0.0	130k	1875.6
belg+4arcs_232	–	0.0	70k	917.8	0.0	13k	197.4
belg+4arcs_233	–	166.0	248k	3600.0	104.6	306k	3600.0
belg+4arcs_234	–	0.0	139k	1629.1	0.0	216k	3037.9
belg+4arcs_235	–	0.0	205k	2590.8	0.0	114k	1362.2
belg+4arcs_236	–	13.9	256k	3600.0	0.0	190k	2898.7
belg+4arcs_237	–	0.0	142k	1446.5	0.0	154k	1750.1
belg+4arcs_238	–	0.0	168k	2378.1	18.7	218k	3600.0
belg+4arcs_239	–	0.0	229k	2833.2	0.0	136k	1925.9
belg+4arcs_240	–	8.0	230k	3600.0	0.0	276k	3569.9
belg+4arcs_241	–	0.0	146k	2086.2	87.7	296k	3600.0
belg+4arcs_242	–	0.0	59k	759.2	0.0	177k	2290.6
belg+4arcs_243	–	0.0	113k	1673.1	0.0	172k	2511.6
belg+4arcs_244	–	0.0	39k	396.8	0.0	33k	384.2
belg+4arcs_245	–	0.0	47k	554.4	0.0	155k	2071.9
belg+4arcs_246	–	0.0	81k	1126.6	0.0	220k	2768.4
belg+4arcs_247	–	0.0	244k	3424.2	0.0	199k	2553.2
belg+4arcs_248	–	0.0	95k	1220.4	94.5	187k	3600.0
belg+4arcs_249	–	0.0	171k	2057.0	19.5	231k	3600.0
belg+4arcs_250	–	106.0	298k	3600.0	0.0	245k	3551.5
belg+4arcs_251	–	0.0	90k	1192.4	0.0	97k	1238.0
belg+4arcs_252	–	0.0	29k	323.8	0.0	110k	1522.0
belg+4arcs_253	–	17.4	220k	3600.0	–	396k	3600.0
belg+4arcs_254	–	73.1	252k	3600.0	22.2	218k	3600.0
belg+4arcs_255	0.04443	0.0	11k	146.2	0.0	71k	895.8
belg+4arcs_256	–	0.0	113k	1436.6	0.0	118k	1285.8
belg+4arcs_257	–	0.0	83k	1406.2	0.0	180k	2118.3
belg+4arcs_258	–	0.0	52k	618.4	0.0	203k	2532.2
belg+4arcs_259	–	0.0	66k	712.3	0.0	95k	922.1
belg+4arcs_260	–	0.0	169k	2631.4	110.9	252k	3600.0
belg+4arcs_261	–	0.0	46k	519.0	0.0	17k	208.7
belg+4arcs_262	–	0.0	155k	1826.4	122.5	289k	3600.0
belg+4arcs_263	–	75.2	237k	3600.0	0.0	83k	1263.7
belg+4arcs_264	–	0.0	97k	1338.5	0.0	151k	1966.8
belg+4arcs_265	–	79.1	217k	3600.0	13.6	224k	3600.0
belg+4arcs_266	–	0.0	95k	1389.4	11.9	292k	3600.0
belg+4arcs_267	–	–	204k	3600.0	–	190k	3600.0
belg+4arcs_268	–	0.0	154k	2251.5	18.2	237k	3600.0
belg+4arcs_269	–	1.3	400k	3600.0	0.0	195k	3224.6
belg+4arcs_270	–	0.0	45k	610.2	20.1	619k	3600.0
belg+4arcs_271	–	0.0	68k	951.9	0.0	195k	1841.5
belg+4arcs_272	–	0.0	44k	623.4	0.0	71k	921.4
belg+4arcs_273	–	79.9	251k	3600.0	75.6	281k	3600.0
belg+4arcs_274	–	0.0	39k	400.6	202.8	665k	3600.0
belg+4arcs_275	–	–	249k	3600.0	0.0	261k	2960.6
belg+4arcs_276	–	–	344k	3600.0	–	636k	3600.0
belg+4arcs_277	–	194.8	283k	3600.0	0.0	247k	3163.4
belg+4arcs_278	–	0.0	104k	1146.0	0.0	43k	494.4
belg+4arcs_279	–	114.4	289k	3600.0	0.0	110k	1162.5
belg+4arcs_280	–	0.0	109k	1603.6	6.5	222k	3600.0
belg+4arcs_281	–	0.0	147k	1892.6	235.0	304k	3600.0
belg+4arcs_282	–	0.0	226k	3257.9	0.0	73k	1000.7
belg+4arcs_283	–	0.0	101k	796.7	0.0	68k	997.8
belg+4arcs_284	–	0.0	112k	1289.5	156.5	350k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+4arcs_285	–	0.0	72k	888.5	0.0	72k	869.2
belg+4arcs_286	–	0.0	187k	2156.6	0.0	141k	2166.7
belg+4arcs_287	–	0.0	143k	1543.2	0.0	27k	336.8
belg+4arcs_288	–	0.0	22k	323.1	0.0	19k	250.0
belg+4arcs_289	–	30.0	230k	3600.0	227.3	216k	3600.0
belg+4arcs_290	–	4.0	270k	3600.0	0.0	130k	2048.3
belg+4arcs_291	–	–	204k	3600.0	103.0	202k	3600.0
belg+4arcs_292	–	1733.3	454k	3600.0	0.0	25k	254.7
belg+4arcs_293	0.0	0.0	34k	497.2	0.0	41k	661.5
belg+4arcs_294	–	0.0	126k	1132.7	0.0	96k	892.7
belg+4arcs_295	–	0.0	123k	1215.1	26.1	204k	3600.0
belg+4arcs_296	–	0.0	59k	809.6	0.0	90k	1076.0
belg+4arcs_297	–	0.0	31k	343.1	0.0	96k	1216.7
belg+4arcs_298	–	0.0	47k	699.1	0.0	46k	700.3
belg+4arcs_299	–	–	216k	3600.0	–	215k	3600.0
belg+4arcs_300	–	0.0	22k	322.5	0.0	40k	674.7
belg+4arcs_301	–	0.0	100k	1295.3	0.0	61k	789.7
belg+4arcs_302	–	0.1	350k	3600.0	114.6	217k	3600.0
belg+4arcs_303	–	0.0	100k	1295.5	0.0	32k	459.5
belg+4arcs_304	–	54.8	535k	3600.0	0.0	175k	2738.2
belg+4arcs_305	–	0.0	153k	2056.9	22.1	252k	3600.0
belg+4arcs_306	–	0.0	78k	1023.7	104.5	421k	3600.0
belg+4arcs_307	–	0.0	69k	975.6	1589.3	300k	3600.0
belg+4arcs_308	0.01333	0.0	178k	2396.2	111.5	543k	3600.0
belg+4arcs_309	–	20.6	213k	3600.0	7.8	229k	3600.0
belg+4arcs_310	–	0.0	47k	618.9	0.0	31k	358.7
belg+4arcs_311	–	0.0	185k	2065.9	0.0	117k	1547.5
belg+4arcs_312	–	0.0	89k	1121.3	9.4	275k	3600.0
belg+4arcs_313	–	41.1	336k	3600.0	–	209k	3600.0
belg+4arcs_314	–	–	227k	3600.0	–	247k	3600.0
belg+4arcs_315	–	112.2	271k	3600.0	109.3	206k	3600.0
belg+4arcs_316	–	0.0	76k	961.3	0.0	212k	2616.8
belg+4arcs_317	–	82.6	221k	3600.0	1.4	264k	3600.0
belg+4arcs_318	0.0	0.0	37k	463.9	0.0	58k	778.9
belg+4arcs_319	–	0.0	147k	1104.0	0.0	249k	1851.2
belg+4arcs_320	–	0.0	37k	435.7	0.0	86k	1016.5
belg+4arcs_321	–	0.0	113k	846.7	0.0	51k	630.9
belg+4arcs_322	–	0.0	92k	1446.5	0.0	136k	1921.6
belg+4arcs_323	–	0.0	154k	2256.3	268.1	260k	3600.0
belg+4arcs_324	–	0.0	231k	2591.1	29.9	222k	3600.0
belg+4arcs_325	–	0.0	183k	2581.7	42.2	245k	3600.0
belg+4arcs_326	–	0.0	177k	2087.8	83.4	266k	3600.0
belg+4arcs_327	–	257.2	281k	3600.0	0.0	16k	199.5
belg+4arcs_328	–	0.0	110k	1310.9	0.0	181k	1608.5
belg+4arcs_329	–	0.0	94k	964.6	0.0	77k	860.7
belg+4arcs_330	–	0.0	28k	376.1	0.0	74k	914.2
belg+4arcs_331	–	0.0	59k	754.9	0.0	138k	1734.3
belg+4arcs_332	–	51.9	200k	3600.0	–	260k	3600.0
belg+4arcs_333	–	0.0	48k	543.9	0.0	93k	1197.4
belg+4arcs_334	–	0.0	335k	3600.0	28.8	289k	3600.0
belg+4arcs_335	–	0.0	276k	3468.5	0.0	346k	2971.9
belg+4arcs_336	–	0.0	78k	1130.7	0.0	50k	647.7
belg+4arcs_337	–	163.6	286k	3600.0	0.0	246k	3359.5
belg+4arcs_338	–	0.0	90k	1258.1	0.0	229k	2564.0
belg+4arcs_339	–	13.6	210k	3600.0	77.9	252k	3600.0
belg+4arcs_340	–	71.9	204k	3600.0	70.3	235k	3600.0
belg+4arcs_341	–	0.0	104k	1299.8	0.0	51k	724.1
belg+4arcs_342	–	0.0	113k	1811.2	0.0	182k	2718.5
belg+4arcs_343	–	18.8	294k	3600.0	167.9	196k	3600.0
belg+4arcs_344	–	0.0	21k	309.2	0.0	75k	841.5
belg+4arcs_345	–	0.0	104k	1668.7	0.0	77k	1199.3
belg+4arcs_346	–	130.4	489k	3600.0	131.5	231k	3600.0
belg+4arcs_347	–	87.4	340k	3600.0	82.0	220k	3600.0
belg+4arcs_348	–	–	499k	3600.0	0.0	30k	385.4
belg+4arcs_349	–	–	266k	3600.0	0.0	163k	2417.2
belg+4arcs_350	–	0.0	122k	1399.5	0.0	270k	2805.9
belg+4arcs_351	–	0.0	262k	2543.0	0.0	150k	2491.1
belg+4arcs_352	–	54.4	263k	3600.0	–	207k	3600.0
belg+4arcs_353	–	0.0	228k	2861.6	71.0	247k	3600.0
belg+4arcs_354	–	0.0	271k	3104.8	0.0	212k	2464.7
belg+4arcs_355	–	0.0	184k	2012.3	0.0	35k	496.9
belg+4arcs_356	–	0.0	158k	2348.8	0.0	192k	2184.1
belg+4arcs_357	–	0.0	152k	2173.4	109.7	275k	3600.0
belg+4arcs_358	–	0.0	126k	2174.0	0.0	188k	3086.6
belg+4arcs_359	–	0.0	132k	1721.6	0.0	120k	1734.1
belg+4arcs_360	–	0.0	34k	460.2	0.0	67k	798.8
belg+4arcs_361	–	–	200k	3600.0	67.6	214k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+4arcs_362	–	0.1	337k	3600.0	0.0	126k	1783.5
belg+4arcs_363	–	0.0	208k	2410.8	0.0	301k	3181.5
belg+4arcs_364	–	0.0	138k	1750.4	0.0	71k	1006.2
belg+4arcs_365	–	78.4	418k	3600.0	86.5	446k	3600.0
belg+4arcs_366	–	0.0	140k	1938.7	0.0	123k	1459.5
belg+4arcs_367	–	0.0	47k	506.2	0.0	45k	562.6
belg+4arcs_368	–	154.4	228k	3600.0	299.8	307k	3600.0
belg+4arcs_369	–	0.0	84k	1089.2	0.0	82k	933.6
belg+4arcs_370	–	0.0	37k	421.3	0.0	56k	694.2
belg+4arcs_371	–	105.1	233k	3600.0	0.0	192k	2724.2
belg+4arcs_372	–	0.0	58k	798.0	0.0	88k	1175.9
belg+4arcs_373	–	0.0	91k	1309.4	0.0	63k	1032.4
belg+4arcs_374	–	0.0	172k	2095.0	0.0	105k	1262.3
belg+4arcs_375	–	0.0	144k	1801.4	0.0	57k	668.7
belg+4arcs_376	–	0.0	64k	924.3	0.0	48k	619.2
belg+4arcs_377	–	260.2	192k	3600.0	37.2	208k	3600.0
belg+4arcs_378	–	–	232k	3600.0	162.2	223k	3600.0
belg+4arcs_379	–	0.0	277k	3131.6	0.0	139k	2035.6
belg+4arcs_380	–	0.0	256k	2327.7	43.2	269k	3600.0
belg+4arcs_381	–	44.7	266k	3600.0	–	207k	3600.0
belg+4arcs_382	–	0.0	202k	2727.9	63.2	219k	3600.0
belg+4arcs_383	0.02241	0.0	74k	753.2	0.0	68k	927.7
belg+4arcs_384	–	0.0	175k	2525.1	0.0	86k	1183.7
belg+4arcs_385	–	0.0	121k	1274.2	0.0	175k	2129.4
belg+4arcs_386	–	65.9	201k	3600.0	–	301k	3600.0
belg+4arcs_387	–	60.8	227k	3600.0	0.0	185k	2788.4
belg+4arcs_388	0.0	0.0	49k	569.7	0.0	102k	1559.1
belg+4arcs_389	–	0.0	63k	659.7	0.0	83k	959.7
belg+4arcs_390	–	0.0	198k	2735.2	0.0	184k	2336.6
belg+4arcs_391	–	0.0	114k	1472.0	0.0	69k	735.2
belg+4arcs_392	–	–	742k	3600.0	0.0	55k	518.7
belg+4arcs_393	–	0.0	50k	512.2	0.0	34k	383.6
belg+4arcs_394	–	0.0	137k	1583.3	0.0	128k	1465.9
belg+4arcs_395	–	0.0	168k	1788.7	–	256k	3600.0
belg+4arcs_396	–	809.0	300k	3600.0	0.0	65k	948.4
belg+4arcs_397	–	0.0	125k	1715.8	0.0	108k	1072.2
belg+4arcs_398	–	27.7	249k	3600.0	0.0	208k	2390.5
belg+4arcs_399	–	0.0	155k	2120.1	0.0	280k	2719.5
belg+4arcs_400	–	0.0	151k	1724.4	208.3	264k	3600.0
belg+4arcs_401	–	129.8	280k	3600.0	0.0	155k	2303.5
belg+4arcs_402	–	0.0	60k	608.7	0.0	60k	722.0
belg+4arcs_403	–	0.0	17k	197.1	0.0	26k	350.5
belg+4arcs_404	–	0.0	112k	1544.3	0.0	119k	1943.0
belg+4arcs_405	–	0.0	38k	444.2	0.0	21k	291.0
belg+4arcs_406	–	277.3	230k	3600.0	0.0	112k	1699.2
belg+4arcs_407	–	0.0	179k	1902.9	61.7	199k	3600.0
belg+4arcs_408	–	0.0	22k	204.3	0.0	15k	190.2
belg+4arcs_409	–	0.0	18k	205.2	0.0	33k	439.2
belg+4arcs_410	–	190.5	229k	3600.0	0.0	104k	1363.8
belg+4arcs_411	–	4.6	271k	3600.0	97.4	253k	3600.0
belg+4arcs_412	–	0.0	186k	2204.3	0.0	168k	2151.8
belg+4arcs_413	–	105.2	279k	3600.0	0.0	226k	3593.6
belg+4arcs_414	–	0.0	52k	656.7	0.0	42k	460.4
belg+4arcs_415	–	60.3	333k	3600.0	0.0	70k	829.9
belg+4arcs_416	–	0.0	81k	1130.7	0.0	135k	2012.6
belg+4arcs_417	–	0.0	126k	1141.2	0.0	135k	1330.0
belg+4arcs_418	–	0.0	228k	3128.0	78.1	235k	3600.0
belg+4arcs_419	–	250.0	282k	3600.0	–	413k	3600.0
belg+4arcs_420	–	0.0	77k	962.9	0.0	51k	643.5
belg+4arcs_421	–	–	561k	3600.0	0.0	301k	3594.4
belg+4arcs_422	–	58.2	329k	3600.0	73.6	268k	3600.0
belg+4arcs_423	–	0.0	33k	466.2	0.0	51k	815.9
belg+4arcs_424	–	0.0	159k	2533.0	88.5	250k	3600.0
belg+4arcs_425	–	0.0	130k	1787.0	0.0	122k	1432.8
belg+4arcs_426	–	127.2	257k	3600.0	152.0	203k	3600.0
belg+4arcs_427	–	0.0	74k	895.3	0.0	107k	1273.4
belg+4arcs_428	–	0.0	178k	1854.5	249.5	265k	3600.0
belg+4arcs_429	–	0.0	74k	1134.9	0.0	173k	1793.0
belg+4arcs_430	–	86.7	271k	3600.0	105.5	212k	3600.0
belg+4arcs_431	–	–	232k	3600.0	–	189k	3600.0
belg+4arcs_432	–	0.0	108k	1269.2	0.0	88k	928.5
belg+4arcs_433	–	0.0	154k	1519.5	0.0	53k	660.3
belg+4arcs_434	–	0.0	197k	2399.8	0.0	101k	1559.2
belg+4arcs_435	–	98.2	275k	3600.0	126.5	234k	3600.0
belg+4arcs_436	–	0.0	27k	343.8	0.0	21k	288.5
belg+4arcs_437	–	146.1	255k	3600.0	166.9	235k	3600.0
belg+4arcs_438	–	0.0	270k	2874.5	0.0	160k	2022.1

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+4arcs_439	–	14.6	212k	3600.0	0.0	187k	2907.6
belg+4arcs_440	–	79.8	217k	3600.0	45.6	215k	3600.0
belg+4arcs_441	–	0.0	105k	1277.5	1.7	273k	3600.0
belg+4arcs_442	–	Large	476k	3600.0	0.0	141k	1907.5
belg+4arcs_443	–	50.0	276k	3600.0	0.0	68k	1003.6
belg+4arcs_444	–	–	422k	3600.0	0.0	151k	1694.5
belg+4arcs_445	–	0.0	155k	2202.7	0.0	210k	2581.4
belg+4arcs_446	–	0.0	69k	959.5	0.0	158k	1697.9
belg+4arcs_447	–	0.0	241k	3028.8	–	389k	3600.0
belg+4arcs_448	–	39.8	195k	3600.0	18.7	193k	3600.0
belg+4arcs_449	–	0.0	77k	1018.4	0.0	66k	1050.0
belg+4arcs_450	–	0.0	44k	576.2	0.0	40k	383.5
belg+4arcs_451	–	0.0	208k	2176.9	0.0	48k	458.8
belg+4arcs_452	–	0.0	71k	827.9	0.0	105k	1506.7
belg+4arcs_453	–	0.0	67k	786.9	0.0	88k	788.9
belg+4arcs_454	–	0.0	42k	582.4	0.0	87k	830.2
belg+4arcs_455	–	0.0	83k	840.3	0.0	94k	1049.6
belg+4arcs_456	–	86.5	231k	3600.0	0.0	324k	3600.0
belg+4arcs_457	–	0.0	64k	876.7	0.0	183k	1896.6
belg+4arcs_458	–	71.3	290k	3600.0	31.8	322k	3600.0
belg+4arcs_459	–	0.0	238k	3289.1	466.7	220k	3600.0
belg+4arcs_460	–	0.0	79k	776.3	0.0	103k	1165.7
belg+4arcs_461	–	102.8	202k	3600.0	135.1	247k	3600.0
belg+4arcs_462	–	0.0	115k	1265.6	0.0	91k	1004.2
belg+4arcs_463	–	–	590k	3600.0	0.0	127k	1386.8
belg+4arcs_464	–	5.9	316k	3600.0	0.0	141k	1570.0
belg+4arcs_465	–	62.6	325k	3600.0	0.0	217k	3481.6
belg+4arcs_466	–	0.0	78k	1150.5	0.0	108k	1440.5
belg+4arcs_467	–	0.0	55k	690.0	0.0	145k	2416.2
belg+4arcs_468	–	176.5	209k	3600.0	124.4	215k	3600.0
belg+4arcs_469	–	0.0	65k	719.5	117.7	269k	3600.0
belg+4arcs_470	–	0.0	171k	2267.7	14.4	251k	3600.0
belg+4arcs_471	–	0.0	108k	1539.2	0.0	172k	2216.8
belg+4arcs_472	–	0.0	80k	1011.0	0.0	103k	1336.0
belg+4arcs_473	–	79.4	240k	3600.0	0.0	118k	1650.3
belg+4arcs_474	–	4.7	384k	3600.0	0.0	50k	674.8
belg+4arcs_475	–	0.0	40k	458.6	0.0	40k	442.2
belg+4arcs_476	–	0.0	134k	1531.0	0.0	64k	836.5
belg+4arcs_477	–	1.1	269k	3600.0	0.0	197k	2098.0
belg+4arcs_478	–	104.3	245k	3600.0	0.0	138k	2273.4
belg+4arcs_479	–	0.0	302k	3513.2	0.0	43k	657.8
belg+4arcs_480	0.0	0.0	34k	402.0	0.0	21k	299.1
belg+4arcs_481	–	0.0	142k	1955.8	97.3	255k	3600.0
belg+4arcs_482	–	0.0	105k	1494.7	0.0	99k	1268.5
belg+4arcs_483	–	0.0	203k	3182.4	0.0	34k	440.8
belg+4arcs_484	–	0.0	112k	1128.6	0.0	75k	934.4
belg+4arcs_485	–	–	427k	3600.0	0.0	174k	2455.5
belg+4arcs_486	–	94.1	233k	3600.0	0.0	151k	2453.5
belg+4arcs_487	–	39.3	284k	3600.0	0.0	65k	658.1
belg+4arcs_488	–	41.3	337k	3600.0	0.0	59k	779.1
belg+4arcs_489	–	0.0	272k	3152.2	0.0	118k	1195.0
belg+4arcs_490	–	0.0	229k	3287.4	0.0	34k	388.1
belg+4arcs_491	–	0.0	181k	2527.5	344.8	364k	3600.0
belg+4arcs_492	–	0.0	37k	403.6	0.0	42k	535.4
belg+4arcs_493	–	99.6	272k	3600.0	0.0	292k	3560.8
belg+4arcs_494	–	0.0	105k	1364.1	0.0	157k	2137.7
belg+4arcs_495	–	0.0	172k	2923.4	97.8	251k	3600.0
belg+4arcs_496	–	–	508k	3600.0	127.1	276k	3600.0
belg+4arcs_497	–	0.0	121k	1746.9	0.0	112k	1484.0
belg+4arcs_498	–	91.1	187k	3600.0	13.8	230k	3600.0
belg+4arcs_499	–	0.0	106k	1361.4	0.0	67k	885.7
belg+4arcs_500	–	0.0	45k	649.3	1088.0	245k	3600.0
belg+6arcs_1	–	–	191k	3600.0	–	209k	3600.0
belg+6arcs_2	–	–	185k	3600.0	0.0	175k	2757.4
belg+6arcs_3	–	144.9	221k	3600.0	14.3	281k	3600.0
belg+6arcs_4	–	195.4	232k	3600.0	2030.4	207k	3600.0
belg+6arcs_5	–	–	203k	3600.0	–	222k	3600.0
belg+6arcs_6	–	117.7	222k	3600.0	423.2	204k	3600.0
belg+6arcs_7	–	141.0	208k	3600.0	25.2	200k	3600.0
belg+6arcs_8	–	0.0	45k	583.9	0.0	39k	488.2
belg+6arcs_9	–	–	229k	3600.0	174.8	212k	3600.0
belg+6arcs_10	–	14.4	308k	3600.0	352.3	302k	3600.0
belg+6arcs_11	–	411.4	198k	3600.0	370.2	199k	3600.0
belg+6arcs_12	0.01378	0.0	166k	2619.3	0.0	92k	1143.3
belg+6arcs_13	–	182.6	250k	3600.0	0.0	179k	2451.0
belg+6arcs_14	–	–	210k	3600.0	–	199k	3600.0
belg+6arcs_15	–	0.0	25k	291.9	0.0	75k	737.3

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+6arcs_16	–	–	195k	3600.0	–	208k	3600.0
belg+6arcs_17	–	252.2	207k	3600.0	–	322k	3600.0
belg+6arcs_18	–	0.0	209k	3597.0	0.0	74k	1018.3
belg+6arcs_19	–	0.0	146k	2038.0	431.9	226k	3600.0
belg+6arcs_20	–	284.5	194k	3600.0	–	174k	3600.0
belg+6arcs_21	–	0.0	54k	858.0	0.0	98k	1504.8
belg+6arcs_22	–	252.8	187k	3600.0	55.9	210k	3600.0
belg+6arcs_23	–	0.0	152k	2271.3	652.7	219k	3600.0
belg+6arcs_24	0.00079	0.0	78k	1334.0	246.8	319k	3600.0
belg+6arcs_25	–	–	242k	3600.0	1313.5	368k	3600.0
belg+6arcs_26	–	0.0	161k	2800.5	–	255k	3600.0
belg+6arcs_27	–	–	228k	3600.0	–	237k	3600.0
belg+6arcs_28	–	222.1	293k	3600.0	–	238k	3600.0
belg+6arcs_29	–	0.0	22k	279.5	0.0	69k	895.5
belg+6arcs_30	0.00641	0.0	203k	1810.0	169.9	327k	3600.0
belg+6arcs_31	–	455.5	215k	3600.0	–	212k	3600.0
belg+6arcs_32	–	–	208k	3600.0	179.9	195k	3600.0
belg+6arcs_33	–	1644.1	253k	3600.0	1074.9	202k	3600.0
belg+6arcs_34	–	–	289k	3600.0	0.0	111k	1770.2
belg+6arcs_35	–	771.8	234k	3600.0	342.5	233k	3600.0
belg+6arcs_36	–	0.0	144k	2073.9	154.9	215k	3600.0
belg+6arcs_37	–	836.6	221k	3600.0	834.3	210k	3600.0
belg+6arcs_38	–	–	222k	3600.0	45.9	254k	3600.0
belg+6arcs_39	–	–	234k	3600.0	0.0	96k	1503.0
belg+6arcs_40	–	360.7	186k	3600.0	397.0	174k	3600.0
belg+6arcs_41	–	–	374k	3600.0	–	256k	3600.0
belg+6arcs_42	–	–	192k	3600.0	229.9	204k	3600.0
belg+6arcs_43	–	771.7	218k	3600.0	272.5	231k	3600.0
belg+6arcs_44	–	0.0	125k	1748.1	0.0	56k	820.2
belg+6arcs_45	–	0.0	59k	820.2	0.0	100k	1385.0
belg+6arcs_46	–	0.0	85k	1265.8	0.0	184k	2660.4
belg+6arcs_47	–	198.5	203k	3600.0	0.0	206k	3025.4
belg+6arcs_48	–	61.3	225k	3600.0	254.2	222k	3600.0
belg+6arcs_49	–	0.0	132k	1409.1	0.0	71k	823.9
belg+6arcs_50	–	194.4	216k	3600.0	8.4	225k	3600.0
belg+6arcs_51	–	0.0	198k	2898.4	0.0	201k	3400.4
belg+6arcs_52	–	0.0	100k	1405.3	345.8	207k	3600.0
belg+6arcs_53	–	248.6	283k	3600.0	–	308k	3600.0
belg+6arcs_54	–	0.0	50k	703.3	–	172k	3600.0
belg+6arcs_55	0.00019	0.0	135k	2062.2	286.2	194k	3600.0
belg+6arcs_56	–	0.0	139k	2059.6	1097.6	195k	3600.0
belg+6arcs_57	–	365.9	249k	3600.0	392.3	291k	3600.0
belg+6arcs_58	–	1447.9	202k	3600.0	424.0	213k	3600.0
belg+6arcs_59	–	0.0	58k	730.9	0.0	80k	969.7
belg+6arcs_60	–	0.0	230k	2618.4	0.0	177k	3180.3
belg+6arcs_61	–	126.1	244k	3600.0	87.3	264k	3600.0
belg+6arcs_62	–	–	295k	3600.0	–	199k	3600.0
belg+6arcs_63	–	–	174k	3600.0	0.0	107k	1626.7
belg+6arcs_64	–	383.4	227k	3600.0	1552.7	247k	3600.0
belg+6arcs_65	–	–	219k	3600.0	–	241k	3600.0
belg+6arcs_66	0.00217	0.0	137k	1979.6	45.8	235k	3600.0
belg+6arcs_67	–	59.7	328k	3600.0	0.0	262k	3455.7
belg+6arcs_68	–	248.2	233k	3600.0	289.4	212k	3600.0
belg+6arcs_69	–	–	203k	3600.0	–	185k	3600.0
belg+6arcs_70	–	381.6	271k	3600.0	249.1	242k	3600.0
belg+6arcs_71	–	0.0	185k	3058.4	385.0	244k	3600.0
belg+6arcs_72	0.02922	0.0	59k	893.9	0.0	113k	1984.7
belg+6arcs_73	–	0.0	101k	1306.9	0.0	82k	1287.8
belg+6arcs_74	–	–	170k	3600.0	–	165k	3600.0
belg+6arcs_75	–	146.4	249k	3600.0	–	240k	3600.0
belg+6arcs_76	0.001	0.0	174k	2211.4	0.0	118k	1704.2
belg+6arcs_77	–	713.4	199k	3600.0	1332.9	242k	3600.0
belg+6arcs_78	–	0.0	134k	1934.3	0.0	144k	1975.1
belg+6arcs_79	–	874.1	224k	3600.0	0.0	263k	3445.3
belg+6arcs_80	–	1284.9	298k	3600.0	0.0	102k	1341.5
belg+6arcs_81	–	202.8	232k	3600.0	–	209k	3600.0
belg+6arcs_82	–	0.0	37k	496.0	1425.5	322k	3600.0
belg+6arcs_83	–	–	247k	3600.0	–	181k	3600.0
belg+6arcs_84	0.02801	0.0	110k	1877.2	0.0	134k	1941.0
belg+6arcs_85	–	184.4	208k	3600.0	–	398k	3600.0
belg+6arcs_86	–	–	246k	3600.0	–	229k	3600.0
belg+6arcs_87	–	–	198k	3600.0	129.5	291k	3600.0
belg+6arcs_88	–	–	294k	3600.0	–	189k	3600.0
belg+6arcs_89	–	–	168k	3600.0	–	274k	3600.0
belg+6arcs_90	–	–	202k	3600.0	–	172k	3600.0
belg+6arcs_91	–	139.6	242k	3600.0	–	202k	3600.0
belg+6arcs_92	–	140.2	182k	3600.0	–	263k	3600.0

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Table B.19: Results for `ROOTGAP` and `TREE` experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+6arcs_93	–	–	232k	3600.0	–	225k	3600.0
belg+6arcs_94	–	0.0	192k	2959.0	0.0	223k	2891.6
belg+6arcs_95	–	0.0	116k	1478.7	0.0	151k	1759.9
belg+6arcs_96	–	174.5	250k	3600.0	–	186k	3600.0
belg+6arcs_97	–	883.7	229k	3600.0	1130.6	232k	3600.0
belg+6arcs_98	–	342.4	161k	3600.0	–	196k	3600.0
belg+6arcs_99	–	–	196k	3600.0	–	221k	3600.0
belg+6arcs_100	–	0.0	153k	2022.6	349.2	210k	3600.0
belg+6arcs_101	–	0.0	194k	3126.9	0.0	154k	2202.0
belg+6arcs_102	–	0.0	217k	3376.6	14.4	250k	3600.0
belg+6arcs_103	–	290.5	174k	3600.0	659.3	185k	3600.0
belg+6arcs_104	–	–	188k	3600.0	255.6	203k	3600.0
belg+6arcs_105	–	0.0	196k	2850.6	0.0	178k	2390.3
belg+6arcs_106	–	284.7	188k	3600.0	239.4	204k	3600.0
belg+6arcs_107	–	–	201k	3600.0	0.0	102k	1503.0
belg+6arcs_108	–	Large	241k	3600.0	1648.6	250k	3600.0
belg+6arcs_109	–	–	242k	3600.0	330.3	198k	3600.0
belg+6arcs_110	–	0.0	90k	1506.4	0.0	54k	799.5
belg+6arcs_111	–	–	212k	3600.0	–	211k	3600.0
belg+6arcs_112	–	684.1	237k	3600.0	525.4	228k	3600.0
belg+6arcs_113	–	0.0	75k	790.7	0.0	79k	974.8
belg+6arcs_114	–	298.0	212k	3600.0	261.6	247k	3600.0
belg+6arcs_115	–	–	187k	3600.0	–	190k	3600.0
belg+6arcs_116	–	0.0	169k	2709.1	–	261k	3600.0
belg+6arcs_117	–	382.6	209k	3600.0	504.8	174k	3600.0
belg+6arcs_118	–	0.0	52k	750.7	0.0	75k	1272.7
belg+6arcs_119	–	–	197k	3600.0	–	244k	3600.0
belg+6arcs_120	–	1666.0	279k	3600.0	1118.2	277k	3600.0
belg+6arcs_121	–	0.0	48k	833.3	0.0	156k	1996.1
belg+6arcs_122	–	–	216k	3600.0	519.4	236k	3600.0
belg+6arcs_123	–	0.0	41k	551.1	0.0	101k	1037.5
belg+6arcs_124	–	549.6	248k	3600.0	–	232k	3600.0
belg+6arcs_125	–	0.0	48k	692.2	0.0	54k	602.6
belg+6arcs_126	–	580.2	238k	3600.0	37.8	265k	3600.0
belg+6arcs_127	–	404.2	219k	3600.0	58.6	255k	3600.0
belg+6arcs_128	–	0.0	253k	3491.9	0.0	221k	3171.1
belg+6arcs_129	–	–	225k	3600.0	0.0	151k	2015.1
belg+6arcs_130	–	0.0	72k	976.5	0.0	139k	2189.2
belg+6arcs_131	–	–	214k	3600.0	669.7	195k	3600.0
belg+6arcs_132	–	–	201k	3600.0	1384.1	190k	3600.0
belg+6arcs_133	–	0.0	124k	1834.6	83.3	256k	3600.0
belg+6arcs_134	–	323.1	191k	3600.0	166.4	206k	3600.0
belg+6arcs_135	–	0.0	135k	1663.4	2656.1	323k	3600.0
belg+6arcs_136	–	–	148k	3600.0	–	159k	3600.0
belg+6arcs_137	–	–	161k	3600.0	–	232k	3600.0
belg+6arcs_138	–	0.0	67k	894.7	0.0	216k	3561.3
belg+6arcs_139	–	2738.4	347k	3600.0	233.4	256k	3600.0
belg+6arcs_140	–	867.3	217k	3600.0	747.2	240k	3600.0
belg+6arcs_141	–	–	188k	3600.0	145.1	206k	3600.0
belg+6arcs_142	–	–	231k	3600.0	345.8	210k	3600.0
belg+6arcs_143	–	–	244k	3600.0	732.4	226k	3600.0
belg+6arcs_144	–	201.0	243k	3600.0	–	245k	3600.0
belg+6arcs_145	–	–	240k	3600.0	243.6	220k	3600.0
belg+6arcs_146	–	1986.3	206k	3600.0	–	209k	3600.0
belg+6arcs_147	–	–	346k	3600.0	–	339k	3600.0
belg+6arcs_148	–	–	182k	3600.0	–	236k	3600.0
belg+6arcs_149	–	–	203k	3600.0	1415.3	239k	3600.0
belg+6arcs_150	–	–	279k	3600.0	67.8	209k	3600.0
belg+6arcs_151	–	0.0	66k	900.2	0.0	78k	1020.3
belg+6arcs_152	–	0.0	227k	3130.8	–	206k	3600.0
belg+6arcs_153	–	0.0	117k	1579.8	0.0	98k	1434.8
belg+6arcs_154	–	0.0	140k	2009.3	85.6	216k	3600.0
belg+6arcs_155	–	–	217k	3600.0	–	194k	3600.0
belg+6arcs_156	–	–	224k	3600.0	365.6	203k	3600.0
belg+6arcs_157	–	0.0	148k	2236.0	254.6	199k	3600.0
belg+6arcs_158	–	0.0	236k	3576.3	0.0	246k	3243.7
belg+6arcs_159	–	1599.3	207k	3600.0	527.6	217k	3600.0
belg+6arcs_160	–	0.0	172k	2298.7	–	270k	3600.0
belg+6arcs_161	–	–	260k	3600.0	–	338k	3600.0
belg+6arcs_162	–	39.2	239k	3600.0	199.6	212k	3600.0
belg+6arcs_163	–	–	196k	3600.0	–	192k	3600.0
belg+6arcs_164	–	0.0	31k	370.4	0.0	32k	360.0
belg+6arcs_165	–	–	203k	3600.0	–	226k	3600.0
belg+6arcs_166	–	0.0	33k	442.6	0.0	65k	990.3
belg+6arcs_167	–	114.1	194k	3600.0	248.6	206k	3600.0
belg+6arcs_168	–	–	220k	3600.0	–	273k	3600.0
belg+6arcs_169	–	529.9	196k	3600.0	123.8	222k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+6arcs_170	–	0.0	108k	1657.5	0.0	53k	680.5
belg+6arcs_171	–	–	212k	3600.0	–	243k	3600.0
belg+6arcs_172	–	242.0	365k	3600.0	–	311k	3600.0
belg+6arcs_173	–	214.7	208k	3600.0	216.0	202k	3600.0
belg+6arcs_174	–	0.0	165k	2403.2	0.0	233k	3331.4
belg+6arcs_175	0.00049	279.7	219k	3600.0	0.0	130k	2194.7
belg+6arcs_176	–	456.6	234k	3600.0	0.0	181k	2712.3
belg+6arcs_177	–	0.0	195k	3499.8	0.0	120k	1767.3
belg+6arcs_178	–	68.7	249k	3600.0	–	232k	3600.0
belg+6arcs_179	–	–	427k	3600.0	–	219k	3600.0
belg+6arcs_180	–	248.3	263k	3600.0	2579.1	260k	3600.0
belg+6arcs_181	–	0.0	229k	3335.0	0.0	209k	3211.5
belg+6arcs_182	–	0.0	101k	1579.6	0.0	157k	2269.5
belg+6arcs_183	–	1036.8	237k	3600.0	405.9	199k	3600.0
belg+6arcs_184	–	–	208k	3600.0	–	219k	3600.0
belg+6arcs_185	–	741.2	226k	3600.0	–	235k	3600.0
belg+6arcs_186	–	693.0	233k	3600.0	9345.7	234k	3600.0
belg+6arcs_187	–	–	221k	3600.0	–	228k	3600.0
belg+6arcs_188	–	0.0	91k	1123.2	0.0	87k	1203.7
belg+6arcs_189	–	428.8	220k	3600.0	–	196k	3600.0
belg+6arcs_190	–	629.8	160k	3600.0	–	197k	3600.0
belg+6arcs_191	–	–	236k	3600.0	362.9	202k	3600.0
belg+6arcs_192	–	627.6	209k	3600.0	921.7	224k	3600.0
belg+6arcs_193	–	0.0	48k	686.4	0.0	131k	1622.2
belg+6arcs_194	–	–	238k	3600.0	284.7	193k	3600.0
belg+6arcs_195	–	643.9	222k	3600.0	1220.2	279k	3600.0
belg+6arcs_196	–	–	162k	3600.0	–	173k	3600.0
belg+6arcs_197	–	–	178k	3600.0	160.6	291k	3600.0
belg+6arcs_198	–	21.2	234k	3600.0	318.1	216k	3600.0
belg+6arcs_199	–	299.6	316k	3600.0	48.4	260k	3600.0
belg+6arcs_200	–	47.5	216k	3600.0	1.4	344k	3600.0
belg+6arcs_201	–	–	199k	3600.0	1709.7	197k	3600.0
belg+6arcs_202	–	579.4	232k	3600.0	–	218k	3600.0
belg+6arcs_203	–	48.3	224k	3600.0	67.6	244k	3600.0
belg+6arcs_204	–	0.0	30k	381.7	0.0	60k	827.9
belg+6arcs_205	–	1032.3	239k	3600.0	–	212k	3600.0
belg+6arcs_206	–	687.6	220k	3600.0	–	231k	3600.0
belg+6arcs_207	–	175.8	199k	3600.0	–	241k	3600.0
belg+6arcs_208	–	–	225k	3600.0	–	205k	3600.0
belg+6arcs_209	–	1888.6	204k	3600.0	Large	221k	3600.0
belg+6arcs_210	–	0.0	179k	2217.9	0.0	51k	732.1
belg+6arcs_211	–	1286.6	209k	3600.0	–	197k	3600.0
belg+6arcs_212	–	140.1	199k	3600.0	2651.0	242k	3600.0
belg+6arcs_213	–	–	239k	3600.0	–	630k	3600.0
belg+6arcs_214	–	51.6	191k	3600.0	1549.3	159k	3600.0
belg+6arcs_215	0.00113	0.0	122k	1788.9	0.0	108k	1221.4
belg+6arcs_216	–	–	312k	3600.0	1152.8	270k	3600.0
belg+6arcs_217	–	0.0	77k	1079.0	1666.7	242k	3600.0
belg+6arcs_218	–	–	178k	3600.0	220.7	229k	3600.0
belg+6arcs_219	–	388.4	218k	3600.0	0.0	139k	2299.4
belg+6arcs_220	–	–	165k	3600.0	–	212k	3600.0
belg+6arcs_221	–	0.0	38k	518.0	0.0	57k	680.9
belg+6arcs_222	–	–	203k	3600.0	–	201k	3600.0
belg+6arcs_223	–	–	208k	3600.0	878.2	209k	3600.0
belg+6arcs_224	–	265.6	204k	3600.0	159.5	237k	3600.0
belg+6arcs_225	–	0.0	29k	382.5	0.0	43k	466.1
belg+6arcs_226	–	–	228k	3600.0	–	265k	3600.0
belg+6arcs_227	–	0.0	118k	1485.4	0.0	83k	1057.2
belg+6arcs_228	–	–	212k	3600.0	–	198k	3600.0
belg+6arcs_229	–	0.0	98k	1262.5	265.7	228k	3600.0
belg+6arcs_230	–	0.0	40k	468.2	0.0	103k	1841.2
belg+6arcs_231	–	–	233k	3600.0	251.8	329k	3600.0
belg+6arcs_232	–	585.0	187k	3600.0	74.6	236k	3600.0
belg+6arcs_233	–	–	207k	3600.0	–	227k	3600.0
belg+6arcs_234	–	0.0	221k	3409.2	40.3	253k	3600.0
belg+6arcs_235	–	530.8	245k	3600.0	–	226k	3600.0
belg+6arcs_236	–	–	197k	3600.0	2277.5	454k	3600.0
belg+6arcs_237	–	0.0	51k	612.6	0.0	46k	578.3
belg+6arcs_238	–	–	193k	3600.0	–	242k	3600.0
belg+6arcs_239	–	230.8	221k	3600.0	–	242k	3600.0
belg+6arcs_240	–	194.2	196k	3600.0	–	204k	3600.0
belg+6arcs_241	–	218.2	277k	3600.0	197.7	207k	3600.0
belg+6arcs_242	–	–	243k	3600.0	–	221k	3600.0
belg+6arcs_243	–	0.0	41k	472.8	0.0	14k	182.9
belg+6arcs_244	–	411.9	195k	3600.0	54.4	179k	3600.0
belg+6arcs_245	–	0.0	90k	1420.4	0.0	79k	1150.0
belg+6arcs_246	–	362.1	228k	3600.0	299.4	239k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+6arcs_247	–	–	269k	3600.0	0.0	224k	2932.2
belg+6arcs_248	–	22.9	188k	3600.0	–	212k	3600.0
belg+6arcs_249	–	1449.4	219k	3600.0	0.0	188k	3091.6
belg+6arcs_250	–	–	242k	3600.0	–	301k	3600.0
belg+6arcs_251	–	0.0	118k	1899.7	64.5	268k	3600.0
belg+6arcs_252	–	517.0	199k	3600.0	–	192k	3600.0
belg+6arcs_253	–	0.0	183k	2998.9	0.0	166k	2424.9
belg+6arcs_254	–	349.4	222k	3600.0	183.1	205k	3600.0
belg+6arcs_255	–	0.0	301k	2646.2	0.0	54k	731.3
belg+6arcs_256	–	0.0	176k	2528.0	0.0	46k	617.2
belg+6arcs_257	–	4.2	266k	3600.0	81.1	207k	3600.0
belg+6arcs_258	–	0.0	255k	2688.9	0.0	155k	2204.5
belg+6arcs_259	–	0.0	241k	3322.7	–	233k	3600.0
belg+6arcs_260	–	0.0	144k	1924.5	0.0	33k	412.4
belg+6arcs_261	–	0.0	21k	273.6	0.0	264k	3435.4
belg+6arcs_262	–	0.0	31k	349.7	0.0	65k	792.7
belg+6arcs_263	–	40.4	264k	3600.0	118.6	218k	3600.0
belg+6arcs_264	–	238.6	202k	3600.0	388.2	261k	3600.0
belg+6arcs_265	–	–	225k	3600.0	176.7	244k	3600.0
belg+6arcs_266	–	0.0	35k	417.4	0.0	72k	944.2
belg+6arcs_267	–	35.3	238k	3600.0	–	226k	3600.0
belg+6arcs_268	–	–	313k	3600.0	216.7	364k	3600.0
belg+6arcs_269	–	212.5	190k	3600.0	130.4	224k	3600.0
belg+6arcs_270	–	–	184k	3600.0	–	222k	3600.0
belg+6arcs_271	–	0.0	222k	3407.5	0.0	230k	3592.3
belg+6arcs_272	–	–	217k	3600.0	214.3	215k	3600.0
belg+6arcs_273	–	184.9	218k	3600.0	194.5	227k	3600.0
belg+6arcs_274	–	0.0	119k	1759.1	0.0	178k	2440.9
belg+6arcs_275	–	0.0	70k	972.0	3121.8	272k	3600.0
belg+6arcs_276	–	0.0	147k	2100.4	0.0	175k	2013.2
belg+6arcs_277	0.01687	0.0	130k	1971.9	0.0	177k	2228.7
belg+6arcs_278	–	–	271k	3600.0	0.0	111k	1648.6
belg+6arcs_279	–	390.2	284k	3600.0	1279.9	251k	3600.0
belg+6arcs_280	–	235.9	302k	3600.0	0.0	190k	2733.7
belg+6arcs_281	–	361.9	222k	3600.0	445.1	220k	3600.0
belg+6arcs_282	–	0.0	129k	2204.7	–	257k	3600.0
belg+6arcs_283	0.00123	0.0	152k	2647.6	272.0	185k	3600.0
belg+6arcs_284	–	48.1	257k	3600.0	1209.5	327k	3600.0
belg+6arcs_285	–	–	196k	3600.0	–	264k	3600.0
belg+6arcs_286	–	566.3	233k	3600.0	581.2	267k	3600.0
belg+6arcs_287	–	0.0	233k	3558.7	0.0	81k	1021.1
belg+6arcs_288	–	0.0	63k	899.8	408.7	175k	3600.0
belg+6arcs_289	–	275.7	216k	3600.0	–	221k	3600.0
belg+6arcs_290	–	–	215k	3600.0	235.0	206k	3600.0
belg+6arcs_291	–	1150.5	266k	3600.0	244.5	286k	3600.0
belg+6arcs_292	–	0.0	88k	981.4	0.0	55k	645.1
belg+6arcs_293	–	0.0	182k	2472.1	238.0	230k	3600.0
belg+6arcs_294	–	69.0	240k	3600.0	0.0	77k	1122.6
belg+6arcs_295	–	–	811k	3600.0	–	220k	3600.0
belg+6arcs_296	–	62.5	263k	3600.0	0.0	193k	2940.2
belg+6arcs_297	–	0.0	91k	1535.0	1514.5	225k	3600.0
belg+6arcs_298	–	177.3	207k	3600.0	233.2	236k	3600.0
belg+6arcs_299	–	–	198k	3600.0	–	238k	3600.0
belg+6arcs_300	–	–	196k	3600.0	–	179k	3600.0
belg+6arcs_301	–	0.0	183k	2462.3	552.3	299k	3600.0
belg+6arcs_302	–	0.0	203k	2939.1	213.7	246k	3600.0
belg+6arcs_303	–	102.0	216k	3600.0	–	216k	3600.0
belg+6arcs_304	–	0.0	129k	2104.4	177.5	244k	3600.0
belg+6arcs_305	–	0.0	238k	3290.2	–	200k	3600.0
belg+6arcs_306	–	–	238k	3600.0	0.0	81k	1139.9
belg+6arcs_307	–	89.4	220k	3600.0	157.4	215k	3600.0
belg+6arcs_308	0.00328	0.0	82k	1052.0	0.0	53k	704.0
belg+6arcs_309	–	257.9	175k	3600.0	218.4	198k	3600.0
belg+6arcs_310	–	30.8	569k	3600.0	0.0	67k	844.1
belg+6arcs_311	–	0.0	203k	3395.2	1421.4	266k	3600.0
belg+6arcs_312	–	1050.1	237k	3600.0	–	280k	3600.0
belg+6arcs_313	–	279.3	297k	3600.0	224.2	223k	3600.0
belg+6arcs_314	–	–	215k	3600.0	–	162k	3600.0
belg+6arcs_315	–	236.9	268k	3600.0	0.0	233k	3481.7
belg+6arcs_316	–	392.1	212k	3600.0	403.5	254k	3600.0
belg+6arcs_317	–	0.0	185k	2781.9	0.0	210k	3399.8
belg+6arcs_318	–	124.5	315k	3600.0	0.0	147k	2191.9
belg+6arcs_319	–	0.0	234k	3305.2	0.0	193k	2824.6
belg+6arcs_320	–	0.0	59k	791.9	0.0	113k	1754.3
belg+6arcs_321	–	0.0	49k	635.6	0.0	99k	1281.5
belg+6arcs_322	–	809.0	216k	3600.0	–	215k	3600.0
belg+6arcs_323	–	–	227k	3600.0	1057.0	240k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+6arcs_324	–	0.0	96k	1527.0	0.0	74k	1006.3
belg+6arcs_325	–	–	210k	3600.0	21.4	353k	3600.0
belg+6arcs_326	–	4.7	276k	3600.0	–	376k	3600.0
belg+6arcs_327	–	0.0	38k	520.6	805.5	398k	3600.0
belg+6arcs_328	–	0.0	58k	850.2	0.0	20k	244.8
belg+6arcs_329	–	0.0	175k	2320.3	–	1015k	3600.0
belg+6arcs_330	–	0.0	59k	1010.6	0.0	105k	1428.6
belg+6arcs_331	–	0.0	200k	2987.9	962.7	234k	3600.0
belg+6arcs_332	–	127.7	200k	3600.0	163.5	189k	3600.0
belg+6arcs_333	–	660.7	229k	3600.0	655.9	229k	3600.0
belg+6arcs_334	–	–	255k	3600.0	–	232k	3600.0
belg+6arcs_335	–	–	224k	3600.0	729.2	219k	3600.0
belg+6arcs_336	–	0.0	113k	2000.3	–	212k	3600.0
belg+6arcs_337	–	0.0	156k	2494.7	0.0	155k	2010.7
belg+6arcs_338	–	126.6	219k	3600.0	289.4	202k	3600.0
belg+6arcs_339	–	162.4	277k	3600.0	–	191k	3600.0
belg+6arcs_340	–	–	176k	3600.0	–	246k	3600.0
belg+6arcs_341	–	–	206k	3600.0	–	212k	3600.0
belg+6arcs_342	–	–	167k	3600.0	–	184k	3600.0
belg+6arcs_343	–	–	240k	3600.0	–	223k	3600.0
belg+6arcs_344	–	0.0	202k	2856.8	101.9	235k	3600.0
belg+6arcs_345	–	0.0	63k	911.5	0.0	64k	1022.3
belg+6arcs_346	–	0.0	198k	2320.6	417.8	221k	3600.0
belg+6arcs_347	–	–	271k	3600.0	–	231k	3600.0
belg+6arcs_348	–	2260.9	205k	3600.0	–	237k	3600.0
belg+6arcs_349	–	198.4	234k	3600.0	0.0	176k	2823.0
belg+6arcs_350	–	2229.3	250k	3600.0	0.0	82k	1254.8
belg+6arcs_351	–	–	225k	3600.0	–	204k	3600.0
belg+6arcs_352	–	180.7	202k	3600.0	–	335k	3600.0
belg+6arcs_353	–	0.0	279k	3533.8	0.0	250k	3436.9
belg+6arcs_354	–	0.0	127k	1833.0	4282.8	271k	3600.0
belg+6arcs_355	–	0.0	39k	613.9	0.0	104k	1685.8
belg+6arcs_356	–	–	217k	3600.0	464.8	205k	3600.0
belg+6arcs_357	–	221.3	226k	3600.0	275.2	261k	3600.0
belg+6arcs_358	–	190.1	245k	3600.0	–	214k	3600.0
belg+6arcs_359	–	0.0	66k	800.2	0.0	41k	573.9
belg+6arcs_360	–	–	230k	3600.0	–	255k	3600.0
belg+6arcs_361	–	–	277k	3600.0	125.0	223k	3600.0
belg+6arcs_362	–	–	384k	3600.0	0.0	193k	2106.2
belg+6arcs_363	–	3237.6	302k	3600.0	47.1	261k	3600.0
belg+6arcs_364	–	386.9	203k	3600.0	346.3	207k	3600.0
belg+6arcs_365	–	0.0	150k	2022.6	95.3	234k	3600.0
belg+6arcs_366	4e-05	232.8	365k	3600.0	0.0	123k	1592.8
belg+6arcs_367	–	0.0	202k	3501.2	0.0	114k	1959.5
belg+6arcs_368	–	0.0	168k	2473.9	0.0	145k	1949.1
belg+6arcs_369	–	0.0	15k	193.5	0.0	43k	523.4
belg+6arcs_370	0.00182	0.0	57k	809.4	0.0	97k	1228.3
belg+6arcs_371	–	226.0	210k	3600.0	527.7	261k	3600.0
belg+6arcs_372	–	0.0	119k	1565.9	–	211k	3600.0
belg+6arcs_373	–	0.0	78k	1047.0	81.5	214k	3600.0
belg+6arcs_374	–	175.0	238k	3600.0	–	234k	3600.0
belg+6arcs_375	–	0.0	60k	810.5	0.0	95k	1246.8
belg+6arcs_376	–	–	180k	3600.0	–	262k	3600.0
belg+6arcs_377	–	–	198k	3600.0	41.0	225k	3600.0
belg+6arcs_378	–	0.0	340k	2953.4	567.7	582k	3600.0
belg+6arcs_379	–	–	185k	3600.0	–	219k	3600.0
belg+6arcs_380	–	242.0	281k	3600.0	0.0	203k	2746.8
belg+6arcs_381	–	–	220k	3600.0	–	201k	3600.0
belg+6arcs_382	–	–	556k	3600.0	0.9	307k	3600.0
belg+6arcs_383	0.02002	0.0	48k	720.5	0.0	81k	1021.3
belg+6arcs_384	–	701.9	279k	3600.0	–	213k	3600.0
belg+6arcs_385	–	–	296k	3600.0	–	243k	3600.0
belg+6arcs_386	–	–	217k	3600.0	–	203k	3600.0
belg+6arcs_387	–	567.4	204k	3600.0	1021.3	184k	3600.0
belg+6arcs_388	–	1002.1	197k	3600.0	209.9	217k	3600.0
belg+6arcs_389	–	0.0	219k	3142.0	1791.0	198k	3600.0
belg+6arcs_390	–	–	221k	3600.0	223.4	216k	3600.0
belg+6arcs_391	–	0.0	152k	2356.8	–	194k	3600.0
belg+6arcs_392	–	0.0	126k	1423.5	885.3	254k	3600.0
belg+6arcs_393	–	421.9	198k	3600.0	0.0	148k	1913.8
belg+6arcs_394	–	1114.0	244k	3600.0	–	240k	3600.0
belg+6arcs_395	–	–	216k	3600.0	–	184k	3600.0
belg+6arcs_396	0.00666	31.5	285k	3600.0	0.0	86k	1340.3
belg+6arcs_397	–	–	203k	3600.0	–	179k	3600.0
belg+6arcs_398	–	0.0	115k	1478.2	3595.6	264k	3600.0
belg+6arcs_399	–	0.0	16k	200.7	0.0	47k	530.9
belg+6arcs_400	–	1914.1	242k	3600.0	0.0	216k	3295.6

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+6arcs_401	–	0.0	59k	880.0	0.0	232k	3341.2
belg+6arcs_402	–	–	266k	3600.0	0.0	124k	1758.2
belg+6arcs_403	–	–	195k	3600.0	–	223k	3600.0
belg+6arcs_404	–	25.4	268k	3600.0	–	274k	3600.0
belg+6arcs_405	–	162.2	218k	3600.0	–	229k	3600.0
belg+6arcs_406	–	0.0	229k	3085.9	326.8	212k	3600.0
belg+6arcs_407	–	46.2	247k	3600.0	267.4	236k	3600.0
belg+6arcs_408	–	0.0	122k	1595.4	0.0	105k	1502.0
belg+6arcs_409	–	0.0	138k	1809.3	0.0	136k	2380.3
belg+6arcs_410	–	229.7	215k	3600.0	–	233k	3600.0
belg+6arcs_411	–	–	198k	3600.0	961.8	257k	3600.0
belg+6arcs_412	–	0.0	48k	719.4	0.0	43k	526.3
belg+6arcs_413	–	75.7	192k	3600.0	–	208k	3600.0
belg+6arcs_414	–	344.1	215k	3600.0	756.9	229k	3600.0
belg+6arcs_415	–	0.0	182k	1932.5	0.0	44k	515.7
belg+6arcs_416	–	277.3	248k	3600.0	0.0	126k	1901.2
belg+6arcs_417	–	359.7	235k	3600.0	613.5	205k	3600.0
belg+6arcs_418	–	–	230k	3600.0	–	232k	3600.0
belg+6arcs_419	–	84.6	233k	3600.0	553.7	222k	3600.0
belg+6arcs_420	–	–	205k	3600.0	–	219k	3600.0
belg+6arcs_421	–	838.0	225k	3600.0	351.4	203k	3600.0
belg+6arcs_422	–	921.2	236k	3600.0	988.7	243k	3600.0
belg+6arcs_423	–	–	195k	3600.0	395.1	194k	3600.0
belg+6arcs_424	–	0.0	169k	2838.0	196.3	257k	3600.0
belg+6arcs_425	–	4475.4	329k	3600.0	214.9	237k	3600.0
belg+6arcs_426	–	–	188k	3600.0	407.6	186k	3600.0
belg+6arcs_427	–	0.0	191k	2745.2	0.0	145k	1898.8
belg+6arcs_428	–	–	244k	3600.0	321.0	249k	3600.0
belg+6arcs_429	–	–	221k	3600.0	–	258k	3600.0
belg+6arcs_430	–	–	255k	3600.0	173.6	204k	3600.0
belg+6arcs_431	–	–	218k	3600.0	–	212k	3600.0
belg+6arcs_432	–	0.0	24k	316.1	0.0	49k	606.7
belg+6arcs_433	–	0.0	221k	3006.3	0.0	60k	732.0
belg+6arcs_434	–	–	254k	3600.0	272.2	216k	3600.0
belg+6arcs_435	–	56.1	283k	3600.0	–	508k	3600.0
belg+6arcs_436	–	–	207k	3600.0	0.0	179k	3263.5
belg+6arcs_437	–	11.9	226k	3600.0	–	200k	3600.0
belg+6arcs_438	–	–	649k	3600.0	0.0	189k	3024.5
belg+6arcs_439	–	–	179k	3600.0	–	204k	3600.0
belg+6arcs_440	–	–	174k	3600.0	–	234k	3600.0
belg+6arcs_441	–	–	208k	3600.0	–	208k	3600.0
belg+6arcs_442	–	0.0	19k	249.0	0.0	41k	492.4
belg+6arcs_443	–	–	218k	3600.0	684.9	205k	3600.0
belg+6arcs_444	–	0.0	228k	3181.9	0.0	254k	3051.8
belg+6arcs_445	–	159.1	206k	3600.0	–	217k	3600.0
belg+6arcs_446	–	555.0	230k	3600.0	0.0	159k	1869.2
belg+6arcs_447	–	0.0	274k	3376.7	–	251k	3600.0
belg+6arcs_448	–	–	198k	3600.0	–	174k	3600.0
belg+6arcs_449	–	0.0	111k	1492.2	0.0	174k	2472.2
belg+6arcs_450	–	0.0	121k	1990.8	0.0	75k	1121.7
belg+6arcs_451	–	0.0	53k	725.0	0.0	94k	1131.5
belg+6arcs_452	–	0.0	198k	2419.2	–	207k	3600.0
belg+6arcs_453	–	34.8	295k	3600.0	–	175k	3600.0
belg+6arcs_454	–	1623.8	201k	3600.0	9325.6	304k	3600.0
belg+6arcs_455	–	308.0	255k	3600.0	0.0	119k	1541.3
belg+6arcs_456	–	–	219k	3600.0	–	192k	3600.0
belg+6arcs_457	–	0.0	124k	2193.5	0.0	37k	532.5
belg+6arcs_458	–	0.0	39k	572.7	181.2	257k	3600.0
belg+6arcs_459	–	104.1	205k	3600.0	150.6	199k	3600.0
belg+6arcs_460	–	0.0	23k	268.6	0.0	17k	218.7
belg+6arcs_461	–	–	215k	3600.0	–	254k	3600.0
belg+6arcs_462	–	–	214k	3600.0	785.7	219k	3600.0
belg+6arcs_463	0.0001	0.0	94k	1575.4	–	323k	3600.0
belg+6arcs_464	–	506.4	203k	3600.0	77.8	259k	3600.0
belg+6arcs_465	–	0.0	129k	1438.1	0.0	54k	731.4
belg+6arcs_466	–	1390.8	193k	3600.0	0.0	94k	1563.8
belg+6arcs_467	–	–	260k	3600.0	0.0	151k	2779.8
belg+6arcs_468	–	–	181k	3600.0	–	177k	3600.0
belg+6arcs_469	–	0.0	202k	3044.7	1555.0	246k	3600.0
belg+6arcs_470	–	0.0	114k	1549.9	665.4	263k	3600.0
belg+6arcs_471	–	595.1	237k	3600.0	0.0	187k	2772.4
belg+6arcs_472	–	1076.8	217k	3600.0	4411.2	301k	3600.0
belg+6arcs_473	–	–	203k	3600.0	436.3	179k	3600.0
belg+6arcs_474	–	918.1	222k	3600.0	–	219k	3600.0
belg+6arcs_475	–	966.1	249k	3600.0	0.0	87k	1460.7
belg+6arcs_476	–	–	246k	3600.0	278.0	229k	3600.0
belg+6arcs_477	–	49.5	221k	3600.0	–	233k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+6arcs_478	–	233.0	237k	3600.0	241.3	234k	3600.0
belg+6arcs_479	–	1194.8	231k	3600.0	418.1	213k	3600.0
belg+6arcs_480	–	–	208k	3600.0	893.3	181k	3600.0
belg+6arcs_481	–	–	223k	3600.0	255.0	182k	3600.0
belg+6arcs_482	–	12.7	225k	3600.0	–	260k	3600.0
belg+6arcs_483	–	–	260k	3600.0	–	189k	3600.0
belg+6arcs_484	–	Large	292k	3600.0	1877.1	254k	3600.0
belg+6arcs_485	–	–	232k	3600.0	0.0	211k	3440.4
belg+6arcs_486	–	0.0	124k	1834.4	–	191k	3600.0
belg+6arcs_487	–	859.5	241k	3600.0	1126.3	213k	3600.0
belg+6arcs_488	–	0.0	190k	2590.7	1080.7	230k	3600.0
belg+6arcs_489	–	226.0	252k	3600.0	–	197k	3600.0
belg+6arcs_490	–	671.8	226k	3600.0	0.0	229k	3406.3
belg+6arcs_491	–	216.0	221k	3600.0	2528.8	317k	3600.0
belg+6arcs_492	–	733.6	212k	3600.0	258.0	233k	3600.0
belg+6arcs_493	–	–	205k	3600.0	–	245k	3600.0
belg+6arcs_494	–	0.0	87k	1246.6	–	207k	3600.0
belg+6arcs_495	–	–	207k	3600.0	–	217k	3600.0
belg+6arcs_496	–	–	228k	3600.0	–	212k	3600.0
belg+6arcs_497	–	0.0	83k	1271.8	357.3	398k	3600.0
belg+6arcs_498	–	190.9	255k	3600.0	0.0	197k	3303.7
belg+6arcs_499	–	–	209k	3600.0	0.0	127k	1556.1
belg+6arcs_500	–	0.0	178k	2866.3	378.0	221k	3600.0
belg+8arcs_1	–	–	209k	3600.0	549.3	237k	3600.0
belg+8arcs_2	–	–	195k	3600.0	1264.1	172k	3600.0
belg+8arcs_3	–	–	222k	3600.0	2297.0	182k	3600.0
belg+8arcs_4	–	413.3	194k	3600.0	–	187k	3600.0
belg+8arcs_5	–	–	196k	3600.0	–	190k	3600.0
belg+8arcs_6	–	–	200k	3600.0	–	180k	3600.0
belg+8arcs_7	–	–	200k	3600.0	–	207k	3600.0
belg+8arcs_8	–	0.0	88k	1070.1	0.0	86k	1398.3
belg+8arcs_9	–	–	203k	3600.0	–	222k	3600.0
belg+8arcs_10	–	–	202k	3600.0	–	179k	3600.0
belg+8arcs_11	–	–	215k	3600.0	–	164k	3600.0
belg+8arcs_12	–	–	234k	3600.0	–	209k	3600.0
belg+8arcs_13	–	71.8	221k	3600.0	–	183k	3600.0
belg+8arcs_14	–	–	175k	3600.0	–	170k	3600.0
belg+8arcs_15	–	0.0	77k	1119.2	0.0	129k	1592.0
belg+8arcs_16	–	–	160k	3600.0	–	165k	3600.0
belg+8arcs_17	–	–	180k	3600.0	–	170k	3600.0
belg+8arcs_18	–	–	239k	3600.0	275.9	203k	3600.0
belg+8arcs_19	–	0.0	206k	3245.7	Large	228k	3600.0
belg+8arcs_20	–	–	181k	3600.0	–	160k	3600.0
belg+8arcs_21	–	Large	202k	3600.0	0.0	114k	1794.4
belg+8arcs_22	–	–	212k	3600.0	–	167k	3600.0
belg+8arcs_23	–	–	231k	3600.0	Large	237k	3600.0
belg+8arcs_24	0.00071	–	183k	3600.0	0.0	128k	1794.0
belg+8arcs_25	–	1662.2	208k	3600.0	–	207k	3600.0
belg+8arcs_26	–	81.4	287k	3600.0	0.0	186k	2935.6
belg+8arcs_27	–	–	214k	3600.0	–	206k	3600.0
belg+8arcs_28	–	–	220k	3600.0	–	169k	3600.0
belg+8arcs_29	–	0.0	149k	2206.8	0.0	51k	760.5
belg+8arcs_30	–	409.7	230k	3600.0	0.0	227k	3178.8
belg+8arcs_31	–	–	194k	3600.0	–	216k	3600.0
belg+8arcs_32	–	–	182k	3600.0	–	199k	3600.0
belg+8arcs_33	–	–	347k	3600.0	2754.3	229k	3600.0
belg+8arcs_34	–	0.0	173k	2274.3	0.0	165k	2211.1
belg+8arcs_35	–	–	178k	3600.0	–	210k	3600.0
belg+8arcs_36	–	–	234k	3600.0	–	202k	3600.0
belg+8arcs_37	–	0.0	114k	1873.8	0.0	173k	2895.2
belg+8arcs_38	–	–	227k	3600.0	–	276k	3600.0
belg+8arcs_39	–	–	177k	3600.0	4125.6	260k	3600.0
belg+8arcs_40	–	–	214k	3600.0	–	195k	3600.0
belg+8arcs_41	–	–	197k	3600.0	–	223k	3600.0
belg+8arcs_42	–	–	212k	3600.0	–	219k	3600.0
belg+8arcs_43	–	–	195k	3600.0	–	209k	3600.0
belg+8arcs_44	–	1347.4	181k	3600.0	117.1	283k	3600.0
belg+8arcs_45	–	0.0	141k	2098.4	–	247k	3600.0
belg+8arcs_46	–	–	237k	3600.0	1359.9	211k	3600.0
belg+8arcs_47	–	–	174k	3600.0	–	173k	3600.0
belg+8arcs_48	–	–	244k	3600.0	–	214k	3600.0
belg+8arcs_49	–	–	229k	3600.0	–	222k	3600.0
belg+8arcs_50	–	–	195k	3600.0	–	191k	3600.0
belg+8arcs_51	–	–	200k	3600.0	–	265k	3600.0
belg+8arcs_52	–	–	199k	3600.0	–	200k	3600.0
belg+8arcs_53	–	1502.8	218k	3600.0	367.0	192k	3600.0
belg+8arcs_54	–	–	169k	3600.0	–	198k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+8arcs_55	–	–	246k	3600.0	–	203k	3600.0
belg+8arcs_56	–	–	170k	3600.0	–	208k	3600.0
belg+8arcs_57	–	–	203k	3600.0	–	240k	3600.0
belg+8arcs_58	–	–	165k	3600.0	–	185k	3600.0
belg+8arcs_59	–	–	264k	3600.0	0.0	94k	1349.5
belg+8arcs_60	–	–	216k	3600.0	717.9	209k	3600.0
belg+8arcs_61	–	–	226k	3600.0	–	205k	3600.0
belg+8arcs_62	–	–	191k	3600.0	–	262k	3600.0
belg+8arcs_63	–	–	191k	3600.0	–	175k	3600.0
belg+8arcs_64	–	–	188k	3600.0	–	200k	3600.0
belg+8arcs_65	–	–	184k	3600.0	–	193k	3600.0
belg+8arcs_66	0.00206	0.0	175k	2671.6	0.0	159k	2078.6
belg+8arcs_67	–	–	244k	3600.0	–	192k	3600.0
belg+8arcs_68	–	–	198k	3600.0	–	207k	3600.0
belg+8arcs_69	–	–	217k	3600.0	–	165k	3600.0
belg+8arcs_70	–	–	246k	3600.0	–	207k	3600.0
belg+8arcs_71	–	346.9	246k	3600.0	1163.6	203k	3600.0
belg+8arcs_72	–	0.0	181k	2079.0	0.0	142k	1881.9
belg+8arcs_73	–	0.0	117k	1968.7	2723.0	246k	3600.0
belg+8arcs_74	–	–	183k	3600.0	–	274k	3600.0
belg+8arcs_75	–	–	196k	3600.0	1838.4	233k	3600.0
belg+8arcs_76	–	69.9	237k	3600.0	1549.3	275k	3600.0
belg+8arcs_77	–	–	212k	3600.0	–	215k	3600.0
belg+8arcs_78	–	1460.0	270k	3600.0	–	206k	3600.0
belg+8arcs_79	–	–	196k	3600.0	1926.6	206k	3600.0
belg+8arcs_80	–	3969.9	244k	3600.0	0.0	215k	3542.7
belg+8arcs_81	–	–	194k	3600.0	–	177k	3600.0
belg+8arcs_82	–	0.0	109k	1564.2	–	243k	3600.0
belg+8arcs_83	–	–	189k	3600.0	–	184k	3600.0
belg+8arcs_84	–	–	202k	3600.0	–	321k	3600.0
belg+8arcs_85	–	–	185k	3600.0	–	179k	3600.0
belg+8arcs_86	–	–	197k	3600.0	–	226k	3600.0
belg+8arcs_87	–	–	186k	3600.0	–	172k	3600.0
belg+8arcs_88	–	–	177k	3600.0	–	182k	3600.0
belg+8arcs_89	–	–	228k	3600.0	–	181k	3600.0
belg+8arcs_90	–	–	188k	3600.0	–	203k	3600.0
belg+8arcs_91	–	–	211k	3600.0	–	204k	3600.0
belg+8arcs_92	–	951.8	248k	3600.0	–	183k	3600.0
belg+8arcs_93	–	–	248k	3600.0	–	195k	3600.0
belg+8arcs_94	–	–	183k	3600.0	–	198k	3600.0
belg+8arcs_95	–	1797.8	248k	3600.0	0.0	106k	1360.7
belg+8arcs_96	–	–	168k	3600.0	–	196k	3600.0
belg+8arcs_97	–	–	192k	3600.0	1237.0	216k	3600.0
belg+8arcs_98	–	–	211k	3600.0	–	202k	3600.0
belg+8arcs_99	–	–	219k	3600.0	–	184k	3600.0
belg+8arcs_100	–	0.0	222k	2974.8	573.1	205k	3600.0
belg+8arcs_101	–	–	178k	3600.0	–	217k	3600.0
belg+8arcs_102	–	0.0	140k	2064.6	85.3	195k	3600.0
belg+8arcs_103	–	–	158k	3600.0	–	193k	3600.0
belg+8arcs_104	–	–	174k	3600.0	–	180k	3600.0
belg+8arcs_105	–	–	197k	3600.0	2073.2	203k	3600.0
belg+8arcs_106	–	–	204k	3600.0	601.4	195k	3600.0
belg+8arcs_107	–	–	188k	3600.0	0.0	218k	3361.1
belg+8arcs_108	–	0.0	115k	1990.3	0.0	147k	2050.8
belg+8arcs_109	–	1835.1	207k	3600.0	4194.2	223k	3600.0
belg+8arcs_110	–	76.4	217k	3600.0	759.7	215k	3600.0
belg+8arcs_111	–	–	162k	3600.0	–	203k	3600.0
belg+8arcs_112	–	–	221k	3600.0	–	228k	3600.0
belg+8arcs_113	–	0.0	40k	621.4	0.0	132k	2009.2
belg+8arcs_114	–	–	180k	3600.0	–	182k	3600.0
belg+8arcs_115	–	–	187k	3600.0	–	215k	3600.0
belg+8arcs_116	–	–	225k	3600.0	–	207k	3600.0
belg+8arcs_117	–	–	210k	3600.0	–	176k	3600.0
belg+8arcs_118	–	0.0	176k	3026.2	4432.2	183k	3600.0
belg+8arcs_119	–	–	185k	3600.0	–	193k	3600.0
belg+8arcs_120	–	0.0	224k	3503.3	0.0	232k	3425.1
belg+8arcs_121	–	0.0	183k	2597.8	93.6	260k	3600.0
belg+8arcs_122	–	–	186k	3600.0	0.0	172k	2317.6
belg+8arcs_123	–	0.0	139k	1943.2	–	262k	3600.0
belg+8arcs_124	–	–	216k	3600.0	–	216k	3600.0
belg+8arcs_125	–	–	204k	3600.0	–	208k	3600.0
belg+8arcs_126	–	–	197k	3600.0	3258.7	219k	3600.0
belg+8arcs_127	–	–	207k	3600.0	–	212k	3600.0
belg+8arcs_128	–	–	200k	3600.0	–	329k	3600.0
belg+8arcs_129	–	0.0	107k	1502.1	–	262k	3600.0
belg+8arcs_130	–	–	236k	3600.0	–	177k	3600.0
belg+8arcs_131	–	–	217k	3600.0	–	185k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+8arcs_132	–	–	203k	3600.0	–	208k	3600.0
belg+8arcs_133	–	–	328k	3600.0	2939.4	196k	3600.0
belg+8arcs_134	–	–	207k	3600.0	–	208k	3600.0
belg+8arcs_135	–	409.7	196k	3600.0	–	248k	3600.0
belg+8arcs_136	–	–	153k	3600.0	–	176k	3600.0
belg+8arcs_137	–	–	196k	3600.0	–	172k	3600.0
belg+8arcs_138	–	–	199k	3600.0	–	163k	3600.0
belg+8arcs_139	–	3978.6	214k	3600.0	–	177k	3600.0
belg+8arcs_140	–	0.0	174k	2632.5	0.0	290k	3277.1
belg+8arcs_141	–	–	207k	3600.0	–	268k	3600.0
belg+8arcs_142	–	–	182k	3600.0	91.6	201k	3600.0
belg+8arcs_143	–	–	194k	3600.0	–	192k	3600.0
belg+8arcs_144	–	–	228k	3600.0	–	200k	3600.0
belg+8arcs_145	–	–	202k	3600.0	693.1	214k	3600.0
belg+8arcs_146	–	–	197k	3600.0	–	218k	3600.0
belg+8arcs_147	–	–	195k	3600.0	1252.3	231k	3600.0
belg+8arcs_148	–	–	188k	3600.0	–	164k	3600.0
belg+8arcs_149	–	–	173k	3600.0	–	169k	3600.0
belg+8arcs_150	–	689.3	227k	3600.0	–	169k	3600.0
belg+8arcs_151	–	0.0	69k	987.7	1207.5	248k	3600.0
belg+8arcs_152	–	–	215k	3600.0	709.2	215k	3600.0
belg+8arcs_153	–	62.8	205k	3600.0	–	194k	3600.0
belg+8arcs_154	–	–	181k	3600.0	–	197k	3600.0
belg+8arcs_155	–	–	188k	3600.0	197.7	201k	3600.0
belg+8arcs_156	–	–	195k	3600.0	–	311k	3600.0
belg+8arcs_157	–	5115.4	200k	3600.0	–	175k	3600.0
belg+8arcs_158	–	1990.0	230k	3600.0	3435.0	224k	3600.0
belg+8arcs_159	–	–	188k	3600.0	1594.6	198k	3600.0
belg+8arcs_160	–	0.0	53k	790.6	0.0	64k	852.9
belg+8arcs_161	–	–	187k	3600.0	–	196k	3600.0
belg+8arcs_162	–	–	186k	3600.0	–	200k	3600.0
belg+8arcs_163	–	–	213k	3600.0	294.5	209k	3600.0
belg+8arcs_164	–	0.0	43k	579.5	0.0	121k	1471.4
belg+8arcs_165	–	–	201k	3600.0	–	186k	3600.0
belg+8arcs_166	–	0.0	85k	1306.6	0.0	52k	717.6
belg+8arcs_167	–	–	224k	3600.0	–	194k	3600.0
belg+8arcs_168	–	–	205k	3600.0	–	231k	3600.0
belg+8arcs_169	–	–	181k	3600.0	–	198k	3600.0
belg+8arcs_170	–	–	176k	3600.0	84.5	234k	3600.0
belg+8arcs_171	–	–	204k	3600.0	–	276k	3600.0
belg+8arcs_172	–	0.0	52k	805.6	0.0	158k	2221.8
belg+8arcs_173	–	–	199k	3600.0	–	199k	3600.0
belg+8arcs_174	–	–	195k	3600.0	–	220k	3600.0
belg+8arcs_175	–	–	192k	3600.0	–	196k	3600.0
belg+8arcs_176	–	–	197k	3600.0	3444.7	236k	3600.0
belg+8arcs_177	–	771.6	185k	3600.0	–	207k	3600.0
belg+8arcs_178	–	–	180k	3600.0	–	185k	3600.0
belg+8arcs_179	–	–	211k	3600.0	–	222k	3600.0
belg+8arcs_180	–	–	228k	3600.0	–	245k	3600.0
belg+8arcs_181	–	0.0	100k	1660.3	717.1	222k	3600.0
belg+8arcs_182	–	861.3	204k	3600.0	1507.5	236k	3600.0
belg+8arcs_183	–	–	206k	3600.0	–	198k	3600.0
belg+8arcs_184	–	Large	233k	3600.0	0.0	114k	1545.2
belg+8arcs_185	–	–	229k	3600.0	–	221k	3600.0
belg+8arcs_186	–	–	190k	3600.0	–	217k	3600.0
belg+8arcs_187	–	–	201k	3600.0	–	178k	3600.0
belg+8arcs_188	–	0.0	231k	3352.9	0.0	162k	2501.2
belg+8arcs_189	–	–	193k	3600.0	–	250k	3600.0
belg+8arcs_190	–	–	145k	3600.0	3101.4	173k	3600.0
belg+8arcs_191	–	–	215k	3600.0	–	160k	3600.0
belg+8arcs_192	–	–	215k	3600.0	–	198k	3600.0
belg+8arcs_193	–	–	192k	3600.0	0.0	171k	2172.5
belg+8arcs_194	–	–	210k	3600.0	–	242k	3600.0
belg+8arcs_195	–	–	213k	3600.0	–	230k	3600.0
belg+8arcs_196	–	–	206k	3600.0	–	264k	3600.0
belg+8arcs_197	–	–	214k	3600.0	–	186k	3600.0
belg+8arcs_198	–	–	205k	3600.0	–	310k	3600.0
belg+8arcs_199	–	0.0	188k	3006.7	–	196k	3600.0
belg+8arcs_200	–	–	196k	3600.0	423.7	202k	3600.0
belg+8arcs_201	–	–	188k	3600.0	–	357k	3600.0
belg+8arcs_202	–	–	212k	3600.0	–	195k	3600.0
belg+8arcs_203	–	863.2	211k	3600.0	–	181k	3600.0
belg+8arcs_204	–	0.0	53k	702.1	0.0	101k	1187.6
belg+8arcs_205	–	–	197k	3600.0	9863.4	215k	3600.0
belg+8arcs_206	–	–	181k	3600.0	–	190k	3600.0
belg+8arcs_207	–	–	176k	3600.0	–	193k	3600.0
belg+8arcs_208	–	–	191k	3600.0	–	194k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+8arcs_209	–	3774.1	190k	3600.0	–	170k	3600.0
belg+8arcs_210	–	–	210k	3600.0	482.6	208k	3600.0
belg+8arcs_211	–	–	217k	3600.0	788.4	234k	3600.0
belg+8arcs_212	–	–	212k	3600.0	–	208k	3600.0
belg+8arcs_213	–	–	163k	3600.0	1044.9	242k	3600.0
belg+8arcs_214	–	–	184k	3600.0	–	186k	3600.0
belg+8arcs_215	–	–	210k	3600.0	795.4	212k	3600.0
belg+8arcs_216	–	193.0	188k	3600.0	1792.2	195k	3600.0
belg+8arcs_217	–	–	211k	3600.0	–	262k	3600.0
belg+8arcs_218	–	–	165k	3600.0	–	175k	3600.0
belg+8arcs_219	–	8.8	231k	3600.0	250.7	237k	3600.0
belg+8arcs_220	–	–	165k	3600.0	–	197k	3600.0
belg+8arcs_221	–	0.0	77k	1205.5	0.0	54k	618.6
belg+8arcs_222	–	–	187k	3600.0	–	183k	3600.0
belg+8arcs_223	–	972.6	180k	3600.0	983.8	191k	3600.0
belg+8arcs_224	–	1966.1	206k	3600.0	–	183k	3600.0
belg+8arcs_225	–	0.0	71k	1014.9	0.0	132k	1555.6
belg+8arcs_226	–	–	199k	3600.0	–	198k	3600.0
belg+8arcs_227	–	–	195k	3600.0	2827.1	212k	3600.0
belg+8arcs_228	–	–	190k	3600.0	–	178k	3600.0
belg+8arcs_229	–	–	199k	3600.0	–	183k	3600.0
belg+8arcs_230	–	–	169k	3600.0	–	183k	3600.0
belg+8arcs_231	–	–	167k	3600.0	301.6	198k	3600.0
belg+8arcs_232	–	–	194k	3600.0	–	208k	3600.0
belg+8arcs_233	–	–	213k	3600.0	1799.3	214k	3600.0
belg+8arcs_234	–	1540.5	202k	3600.0	1552.3	217k	3600.0
belg+8arcs_235	–	–	200k	3600.0	1141.9	233k	3600.0
belg+8arcs_236	–	–	208k	3600.0	–	208k	3600.0
belg+8arcs_237	–	0.0	39k	632.1	0.0	138k	1821.0
belg+8arcs_238	–	–	183k	3600.0	–	190k	3600.0
belg+8arcs_239	–	–	187k	3600.0	266.1	207k	3600.0
belg+8arcs_240	–	–	181k	3600.0	–	199k	3600.0
belg+8arcs_241	–	–	207k	3600.0	–	211k	3600.0
belg+8arcs_242	–	–	199k	3600.0	–	198k	3600.0
belg+8arcs_243	–	0.0	131k	2003.0	0.0	167k	2406.4
belg+8arcs_244	–	–	195k	3600.0	–	256k	3600.0
belg+8arcs_245	–	–	230k	3600.0	0.0	191k	3077.9
belg+8arcs_246	–	–	187k	3600.0	–	203k	3600.0
belg+8arcs_247	–	0.0	134k	1687.5	0.0	148k	2094.3
belg+8arcs_248	–	–	192k	3600.0	–	245k	3600.0
belg+8arcs_249	–	–	198k	3600.0	–	266k	3600.0
belg+8arcs_250	–	–	196k	3600.0	–	209k	3600.0
belg+8arcs_251	–	2490.6	257k	3600.0	–	208k	3600.0
belg+8arcs_252	–	–	171k	3600.0	–	233k	3600.0
belg+8arcs_253	–	–	206k	3600.0	–	186k	3600.0
belg+8arcs_254	–	763.6	201k	3600.0	–	325k	3600.0
belg+8arcs_255	–	0.0	162k	2906.4	–	219k	3600.0
belg+8arcs_256	–	–	190k	3600.0	–	221k	3600.0
belg+8arcs_257	–	–	253k	3600.0	–	214k	3600.0
belg+8arcs_258	–	–	201k	3600.0	–	191k	3600.0
belg+8arcs_259	–	–	199k	3600.0	–	251k	3600.0
belg+8arcs_260	–	2103.5	264k	3600.0	0.0	90k	1364.5
belg+8arcs_261	–	646.4	221k	3600.0	2866.1	219k	3600.0
belg+8arcs_262	–	0.0	155k	2250.0	0.0	136k	1618.7
belg+8arcs_263	–	–	205k	3600.0	–	391k	3600.0
belg+8arcs_264	–	–	195k	3600.0	–	196k	3600.0
belg+8arcs_265	–	–	202k	3600.0	–	191k	3600.0
belg+8arcs_266	–	0.0	217k	3311.2	0.0	91k	1305.3
belg+8arcs_267	–	–	205k	3600.0	–	264k	3600.0
belg+8arcs_268	–	–	219k	3600.0	–	243k	3600.0
belg+8arcs_269	–	461.5	203k	3600.0	–	207k	3600.0
belg+8arcs_270	–	–	220k	3600.0	–	194k	3600.0
belg+8arcs_271	–	–	185k	3600.0	–	212k	3600.0
belg+8arcs_272	–	–	209k	3600.0	–	172k	3600.0
belg+8arcs_273	–	256.6	197k	3600.0	1349.1	192k	3600.0
belg+8arcs_274	–	–	187k	3600.0	5027.9	246k	3600.0
belg+8arcs_275	–	0.0	169k	2741.2	Large	242k	3600.0
belg+8arcs_276	–	99.2	244k	3600.0	0.0	160k	2349.7
belg+8arcs_277	–	–	225k	3600.0	2255.7	229k	3600.0
belg+8arcs_278	–	0.0	171k	2458.6	0.0	169k	2067.6
belg+8arcs_279	–	3416.8	192k	3600.0	–	263k	3600.0
belg+8arcs_280	–	–	182k	3600.0	–	218k	3600.0
belg+8arcs_281	–	–	176k	3600.0	–	194k	3600.0
belg+8arcs_282	–	–	188k	3600.0	–	195k	3600.0
belg+8arcs_283	–	–	176k	3600.0	–	179k	3600.0
belg+8arcs_284	–	–	213k	3600.0	–	240k	3600.0
belg+8arcs_285	–	–	200k	3600.0	–	245k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+8arcs_286	–	195.6	223k	3600.0	–	219k	3600.0
belg+8arcs_287	–	–	197k	3600.0	291.5	216k	3600.0
belg+8arcs_288	–	–	206k	3600.0	–	216k	3600.0
belg+8arcs_289	–	495.6	200k	3600.0	–	172k	3600.0
belg+8arcs_290	–	2050.8	233k	3600.0	–	190k	3600.0
belg+8arcs_291	–	–	202k	3600.0	–	238k	3600.0
belg+8arcs_292	–	3586.2	216k	3600.0	0.0	183k	2670.0
belg+8arcs_293	–	1986.4	209k	3600.0	Large	516k	3600.0
belg+8arcs_294	–	–	200k	3600.0	0.0	190k	2404.9
belg+8arcs_295	–	–	217k	3600.0	–	196k	3600.0
belg+8arcs_296	–	2290.9	303k	3600.0	–	200k	3600.0
belg+8arcs_297	–	341.4	213k	3600.0	359.2	195k	3600.0
belg+8arcs_298	–	2763.0	201k	3600.0	–	197k	3600.0
belg+8arcs_299	–	–	209k	3600.0	–	211k	3600.0
belg+8arcs_300	–	–	179k	3600.0	–	143k	3600.0
belg+8arcs_301	–	–	221k	3600.0	–	200k	3600.0
belg+8arcs_302	–	272.1	218k	3600.0	–	182k	3600.0
belg+8arcs_303	–	–	171k	3600.0	–	210k	3600.0
belg+8arcs_304	–	–	181k	3600.0	–	201k	3600.0
belg+8arcs_305	–	–	182k	3600.0	–	200k	3600.0
belg+8arcs_306	–	0.0	216k	3485.2	0.0	199k	3079.3
belg+8arcs_307	–	–	218k	3600.0	–	188k	3600.0
belg+8arcs_308	–	–	224k	3600.0	–	188k	3600.0
belg+8arcs_309	–	–	169k	3600.0	–	195k	3600.0
belg+8arcs_310	–	0.0	117k	1506.2	0.0	224k	3276.7
belg+8arcs_311	–	–	204k	3600.0	2657.8	209k	3600.0
belg+8arcs_312	–	–	195k	3600.0	–	198k	3600.0
belg+8arcs_313	–	–	190k	3600.0	–	179k	3600.0
belg+8arcs_314	–	–	202k	3600.0	–	204k	3600.0
belg+8arcs_315	–	–	203k	3600.0	–	195k	3600.0
belg+8arcs_316	–	–	251k	3600.0	–	215k	3600.0
belg+8arcs_317	–	–	180k	3600.0	–	235k	3600.0
belg+8arcs_318	–	–	190k	3600.0	–	219k	3600.0
belg+8arcs_319	–	77.0	212k	3600.0	–	221k	3600.0
belg+8arcs_320	–	0.0	127k	1904.2	–	213k	3600.0
belg+8arcs_321	–	0.0	166k	2301.1	–	242k	3600.0
belg+8arcs_322	–	–	334k	3600.0	–	173k	3600.0
belg+8arcs_323	–	–	214k	3600.0	–	172k	3600.0
belg+8arcs_324	–	–	213k	3600.0	933.9	253k	3600.0
belg+8arcs_325	–	0.0	189k	3118.5	–	262k	3600.0
belg+8arcs_326	–	–	201k	3600.0	–	313k	3600.0
belg+8arcs_327	–	0.0	179k	2435.7	0.0	150k	2010.3
belg+8arcs_328	–	0.0	233k	2524.1	0.0	46k	682.0
belg+8arcs_329	–	–	176k	3600.0	–	220k	3600.0
belg+8arcs_330	–	–	216k	3600.0	–	220k	3600.0
belg+8arcs_331	–	1391.1	205k	3600.0	Large	212k	3600.0
belg+8arcs_332	–	–	186k	3600.0	–	184k	3600.0
belg+8arcs_333	–	–	206k	3600.0	–	254k	3600.0
belg+8arcs_334	–	868.8	185k	3600.0	–	205k	3600.0
belg+8arcs_335	–	–	185k	3600.0	–	186k	3600.0
belg+8arcs_336	–	–	195k	3600.0	–	221k	3600.0
belg+8arcs_337	–	1340.0	183k	3600.0	–	222k	3600.0
belg+8arcs_338	–	–	199k	3600.0	–	275k	3600.0
belg+8arcs_339	–	–	223k	3600.0	–	176k	3600.0
belg+8arcs_340	–	–	209k	3600.0	–	196k	3600.0
belg+8arcs_341	–	–	210k	3600.0	–	182k	3600.0
belg+8arcs_342	–	–	172k	3600.0	–	162k	3600.0
belg+8arcs_343	–	–	209k	3600.0	–	198k	3600.0
belg+8arcs_344	–	–	211k	3600.0	–	174k	3600.0
belg+8arcs_345	–	264.3	219k	3600.0	–	190k	3600.0
belg+8arcs_346	–	–	184k	3600.0	–	180k	3600.0
belg+8arcs_347	–	–	167k	3600.0	–	153k	3600.0
belg+8arcs_348	–	–	260k	3600.0	–	206k	3600.0
belg+8arcs_349	–	2344.6	213k	3600.0	Large	206k	3600.0
belg+8arcs_350	–	3080.6	285k	3600.0	Large	239k	3600.0
belg+8arcs_351	–	–	168k	3600.0	–	205k	3600.0
belg+8arcs_352	–	–	244k	3600.0	–	196k	3600.0
belg+8arcs_353	–	865.8	220k	3600.0	–	254k	3600.0
belg+8arcs_354	–	–	218k	3600.0	2453.8	214k	3600.0
belg+8arcs_355	–	2455.7	219k	3600.0	3060.6	216k	3600.0
belg+8arcs_356	–	–	185k	3600.0	–	183k	3600.0
belg+8arcs_357	–	–	184k	3600.0	–	166k	3600.0
belg+8arcs_358	–	256.9	222k	3600.0	–	158k	3600.0
belg+8arcs_359	–	Large	270k	3600.0	0.0	166k	2135.3
belg+8arcs_360	–	–	192k	3600.0	–	189k	3600.0
belg+8arcs_361	–	–	230k	3600.0	–	169k	3600.0
belg+8arcs_362	–	0.0	194k	3229.2	0.0	159k	2692.2

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+8arcs_363	–	–	179k	3600.0	–	220k	3600.0
belg+8arcs_364	–	2335.7	196k	3600.0	–	212k	3600.0
belg+8arcs_365	–	0.0	126k	1822.6	471.7	205k	3600.0
belg+8arcs_366	–	Large	241k	3600.0	–	279k	3600.0
belg+8arcs_367	–	–	208k	3600.0	0.0	178k	2948.9
belg+8arcs_368	–	Large	249k	3600.0	–	262k	3600.0
belg+8arcs_369	–	Large	284k	3600.0	–	243k	3600.0
belg+8arcs_370	0.00177	0.0	148k	2232.1	Large	210k	3600.0
belg+8arcs_371	–	0.0	176k	2953.4	–	312k	3600.0
belg+8arcs_372	–	–	218k	3600.0	–	181k	3600.0
belg+8arcs_373	–	90.0	196k	3600.0	–	214k	3600.0
belg+8arcs_374	–	–	237k	3600.0	1734.7	194k	3600.0
belg+8arcs_375	–	87.9	230k	3600.0	326.1	224k	3600.0
belg+8arcs_376	–	–	193k	3600.0	328.1	201k	3600.0
belg+8arcs_377	–	–	164k	3600.0	1509.4	208k	3600.0
belg+8arcs_378	–	–	191k	3600.0	–	168k	3600.0
belg+8arcs_379	–	–	165k	3600.0	–	177k	3600.0
belg+8arcs_380	–	–	191k	3600.0	–	193k	3600.0
belg+8arcs_381	–	–	219k	3600.0	–	205k	3600.0
belg+8arcs_382	–	285.3	197k	3600.0	–	201k	3600.0
belg+8arcs_383	0.00845	0.0	237k	3453.6	1166.5	223k	3600.0
belg+8arcs_384	–	–	213k	3600.0	–	209k	3600.0
belg+8arcs_385	–	–	215k	3600.0	–	191k	3600.0
belg+8arcs_386	–	–	213k	3600.0	–	205k	3600.0
belg+8arcs_387	–	4558.1	199k	3600.0	2684.9	187k	3600.0
belg+8arcs_388	–	313.5	207k	3600.0	7171.6	211k	3600.0
belg+8arcs_389	–	–	200k	3600.0	Large	244k	3600.0
belg+8arcs_390	–	–	199k	3600.0	–	170k	3600.0
belg+8arcs_391	–	–	186k	3600.0	–	187k	3600.0
belg+8arcs_392	–	–	191k	3600.0	Large	205k	3600.0
belg+8arcs_393	–	11.2	233k	3600.0	297.2	253k	3600.0
belg+8arcs_394	–	–	206k	3600.0	–	202k	3600.0
belg+8arcs_395	–	–	179k	3600.0	–	196k	3600.0
belg+8arcs_396	–	–	208k	3600.0	0.0	150k	2985.6
belg+8arcs_397	–	–	177k	3600.0	–	242k	3600.0
belg+8arcs_398	–	0.0	47k	555.0	0.0	109k	1522.8
belg+8arcs_399	–	0.0	97k	1269.9	0.0	53k	794.7
belg+8arcs_400	–	–	195k	3600.0	–	252k	3600.0
belg+8arcs_401	–	6971.3	228k	3600.0	126.2	242k	3600.0
belg+8arcs_402	–	4960.2	277k	3600.0	–	193k	3600.0
belg+8arcs_403	–	–	178k	3600.0	–	178k	3600.0
belg+8arcs_404	–	–	212k	3600.0	2236.6	251k	3600.0
belg+8arcs_405	–	–	207k	3600.0	–	188k	3600.0
belg+8arcs_406	–	–	202k	3600.0	–	197k	3600.0
belg+8arcs_407	–	–	213k	3600.0	–	306k	3600.0
belg+8arcs_408	–	–	222k	3600.0	0.0	166k	2551.7
belg+8arcs_409	–	–	211k	3600.0	–	178k	3600.0
belg+8arcs_410	–	–	265k	3600.0	–	171k	3600.0
belg+8arcs_411	–	–	209k	3600.0	–	198k	3600.0
belg+8arcs_412	–	–	310k	3600.0	0.0	208k	2848.0
belg+8arcs_413	–	–	196k	3600.0	–	189k	3600.0
belg+8arcs_414	–	–	181k	3600.0	–	164k	3600.0
belg+8arcs_415	–	Large	238k	3600.0	0.0	203k	2945.2
belg+8arcs_416	–	–	193k	3600.0	–	205k	3600.0
belg+8arcs_417	–	–	205k	3600.0	–	215k	3600.0
belg+8arcs_418	–	–	192k	3600.0	–	200k	3600.0
belg+8arcs_419	–	–	224k	3600.0	–	224k	3600.0
belg+8arcs_420	–	–	202k	3600.0	8555.2	212k	3600.0
belg+8arcs_421	–	1097.8	209k	3600.0	–	216k	3600.0
belg+8arcs_422	–	–	203k	3600.0	–	226k	3600.0
belg+8arcs_423	–	–	200k	3600.0	–	256k	3600.0
belg+8arcs_424	–	–	216k	3600.0	–	252k	3600.0
belg+8arcs_425	–	334.4	229k	3600.0	–	191k	3600.0
belg+8arcs_426	–	–	159k	3600.0	–	171k	3600.0
belg+8arcs_427	–	–	212k	3600.0	3829.0	294k	3600.0
belg+8arcs_428	–	–	216k	3600.0	–	208k	3600.0
belg+8arcs_429	–	–	237k	3600.0	–	197k	3600.0
belg+8arcs_430	–	–	175k	3600.0	–	218k	3600.0
belg+8arcs_431	–	–	186k	3600.0	–	174k	3600.0
belg+8arcs_432	–	0.0	48k	716.9	0.0	109k	1413.1
belg+8arcs_433	–	6017.3	219k	3600.0	2814.8	208k	3600.0
belg+8arcs_434	–	1086.8	195k	3600.0	–	203k	3600.0
belg+8arcs_435	–	–	194k	3600.0	–	243k	3600.0
belg+8arcs_436	–	–	170k	3600.0	–	188k	3600.0
belg+8arcs_437	–	1334.9	230k	3600.0	–	230k	3600.0
belg+8arcs_438	–	302.7	215k	3600.0	–	518k	3600.0
belg+8arcs_439	–	–	178k	3600.0	–	187k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+8arcs_440	–	–	166k	3600.0	–	175k	3600.0
belg+8arcs_441	–	–	182k	3600.0	–	187k	3600.0
belg+8arcs_442	–	0.0	206k	2955.9	0.0	111k	1388.4
belg+8arcs_443	–	–	207k	3600.0	–	184k	3600.0
belg+8arcs_444	–	2288.0	194k	3600.0	–	252k	3600.0
belg+8arcs_445	–	–	175k	3600.0	–	178k	3600.0
belg+8arcs_446	–	–	210k	3600.0	–	209k	3600.0
belg+8arcs_447	–	–	178k	3600.0	–	200k	3600.0
belg+8arcs_448	–	–	225k	3600.0	–	149k	3600.0
belg+8arcs_449	–	331.6	276k	3600.0	2103.9	197k	3600.0
belg+8arcs_450	–	–	204k	3600.0	–	225k	3600.0
belg+8arcs_451	–	–	185k	3600.0	Large	215k	3600.0
belg+8arcs_452	–	352.0	197k	3600.0	–	191k	3600.0
belg+8arcs_453	–	–	181k	3600.0	0.0	250k	3310.8
belg+8arcs_454	–	–	201k	3600.0	–	197k	3600.0
belg+8arcs_455	–	–	202k	3600.0	–	238k	3600.0
belg+8arcs_456	–	579.4	241k	3600.0	–	199k	3600.0
belg+8arcs_457	–	–	179k	3600.0	Large	255k	3600.0
belg+8arcs_458	0.00026	45.2	283k	3600.0	0.0	162k	2441.7
belg+8arcs_459	–	–	176k	3600.0	–	244k	3600.0
belg+8arcs_460	–	0.0	99k	1268.9	Large	242k	3600.0
belg+8arcs_461	–	–	225k	3600.0	–	226k	3600.0
belg+8arcs_462	–	–	192k	3600.0	–	220k	3600.0
belg+8arcs_463	–	0.0	199k	2815.0	–	198k	3600.0
belg+8arcs_464	–	–	229k	3600.0	–	345k	3600.0
belg+8arcs_465	–	0.0	112k	1587.9	0.0	78k	1110.8
belg+8arcs_466	–	–	188k	3600.0	–	225k	3600.0
belg+8arcs_467	–	91.8	186k	3600.0	–	175k	3600.0
belg+8arcs_468	–	–	172k	3600.0	–	146k	3600.0
belg+8arcs_469	–	5515.8	233k	3600.0	4348.3	224k	3600.0
belg+8arcs_470	–	2388.1	300k	3600.0	0.0	87k	1287.2
belg+8arcs_471	–	–	207k	3600.0	–	191k	3600.0
belg+8arcs_472	–	–	204k	3600.0	–	187k	3600.0
belg+8arcs_473	–	–	164k	3600.0	–	184k	3600.0
belg+8arcs_474	–	–	217k	3600.0	3336.4	265k	3600.0
belg+8arcs_475	–	–	186k	3600.0	–	171k	3600.0
belg+8arcs_476	–	–	198k	3600.0	4917.8	212k	3600.0
belg+8arcs_477	–	–	193k	3600.0	–	220k	3600.0
belg+8arcs_478	–	–	182k	3600.0	2999.1	224k	3600.0
belg+8arcs_479	–	–	210k	3600.0	–	258k	3600.0
belg+8arcs_480	–	4180.1	213k	3600.0	510.6	194k	3600.0
belg+8arcs_481	–	–	169k	3600.0	–	172k	3600.0
belg+8arcs_482	–	–	230k	3600.0	–	206k	3600.0
belg+8arcs_483	–	2545.4	188k	3600.0	–	197k	3600.0
belg+8arcs_484	–	3291.3	193k	3600.0	8329.7	222k	3600.0
belg+8arcs_485	–	0.0	137k	2334.3	–	240k	3600.0
belg+8arcs_486	–	–	199k	3600.0	–	188k	3600.0
belg+8arcs_487	–	896.5	201k	3600.0	–	195k	3600.0
belg+8arcs_488	–	Large	258k	3600.0	Large	279k	3600.0
belg+8arcs_489	–	–	257k	3600.0	–	202k	3600.0
belg+8arcs_490	–	–	185k	3600.0	–	199k	3600.0
belg+8arcs_491	–	3388.9	216k	3600.0	–	190k	3600.0
belg+8arcs_492	–	2612.5	212k	3600.0	–	223k	3600.0
belg+8arcs_493	–	–	224k	3600.0	–	202k	3600.0
belg+8arcs_494	–	0.0	171k	2434.6	–	177k	3600.0
belg+8arcs_495	–	–	202k	3600.0	–	182k	3600.0
belg+8arcs_496	–	–	213k	3600.0	–	236k	3600.0
belg+8arcs_497	–	565.6	206k	3600.0	–	263k	3600.0
belg+8arcs_498	–	–	177k	3600.0	2045.0	211k	3600.0
belg+8arcs_499	–	–	212k	3600.0	–	228k	3600.0
belg+8arcs_500	–	–	184k	3600.0	970.5	185k	3600.0
belg+10arcs_1	–	–	183k	3600.0	549.3	237k	3600.0
belg+10arcs_2	–	–	181k	3600.0	1264.1	172k	3600.0
belg+10arcs_3	–	–	181k	3600.0	2297.0	182k	3600.0
belg+10arcs_4	–	–	215k	3600.0	–	187k	3600.0
belg+10arcs_5	–	–	159k	3600.0	–	190k	3600.0
belg+10arcs_6	–	–	182k	3600.0	–	180k	3600.0
belg+10arcs_7	–	–	176k	3600.0	–	207k	3600.0
belg+10arcs_8	–	167.3	223k	3600.0	0.0	86k	1398.3
belg+10arcs_9	–	–	181k	3600.0	–	222k	3600.0
belg+10arcs_10	–	Large	212k	3600.0	–	179k	3600.0
belg+10arcs_11	–	–	204k	3600.0	–	164k	3600.0
belg+10arcs_12	–	0.0	222k	3086.5	–	209k	3600.0
belg+10arcs_13	–	–	195k	3600.0	–	183k	3600.0
belg+10arcs_14	–	–	188k	3600.0	–	170k	3600.0
belg+10arcs_15	–	0.0	168k	2371.1	0.0	129k	1592.0
belg+10arcs_16	–	–	190k	3600.0	–	165k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+10arcs_17	–	–	190k	3600.0	–	170k	3600.0
belg+10arcs_18	–	–	200k	3600.0	275.9	203k	3600.0
belg+10arcs_19	–	–	192k	3600.0	Large	228k	3600.0
belg+10arcs_20	–	–	186k	3600.0	–	160k	3600.0
belg+10arcs_21	–	–	188k	3600.0	0.0	114k	1794.4
belg+10arcs_22	–	–	204k	3600.0	–	167k	3600.0
belg+10arcs_23	–	Large	201k	3600.0	Large	237k	3600.0
belg+10arcs_24	–	–	228k	3600.0	0.0	128k	1794.0
belg+10arcs_25	–	–	203k	3600.0	–	207k	3600.0
belg+10arcs_26	–	–	171k	3600.0	0.0	186k	2935.6
belg+10arcs_27	–	–	207k	3600.0	–	206k	3600.0
belg+10arcs_28	–	–	187k	3600.0	–	169k	3600.0
belg+10arcs_29	–	0.0	182k	2733.8	0.0	51k	760.5
belg+10arcs_30	–	–	205k	3600.0	0.0	227k	3178.8
belg+10arcs_31	–	–	205k	3600.0	–	216k	3600.0
belg+10arcs_32	–	–	179k	3600.0	–	199k	3600.0
belg+10arcs_33	–	–	230k	3600.0	2754.3	229k	3600.0
belg+10arcs_34	–	58.0	255k	3600.0	0.0	165k	2211.1
belg+10arcs_35	–	–	204k	3600.0	–	210k	3600.0
belg+10arcs_36	–	–	193k	3600.0	–	202k	3600.0
belg+10arcs_37	–	–	212k	3600.0	0.0	173k	2895.2
belg+10arcs_38	–	–	209k	3600.0	–	276k	3600.0
belg+10arcs_39	–	0.0	187k	3492.0	4125.6	260k	3600.0
belg+10arcs_40	–	4592.6	272k	3600.0	–	195k	3600.0
belg+10arcs_41	–	–	185k	3600.0	–	223k	3600.0
belg+10arcs_42	–	–	193k	3600.0	–	219k	3600.0
belg+10arcs_43	–	–	220k	3600.0	–	209k	3600.0
belg+10arcs_44	–	–	195k	3600.0	117.1	283k	3600.0
belg+10arcs_45	–	–	189k	3600.0	–	247k	3600.0
belg+10arcs_46	–	–	169k	3600.0	1359.9	211k	3600.0
belg+10arcs_47	–	–	184k	3600.0	–	173k	3600.0
belg+10arcs_48	–	–	207k	3600.0	–	214k	3600.0
belg+10arcs_49	–	Large	220k	3600.0	–	222k	3600.0
belg+10arcs_50	–	–	185k	3600.0	–	191k	3600.0
belg+10arcs_51	–	–	168k	3600.0	–	265k	3600.0
belg+10arcs_52	–	–	179k	3600.0	–	200k	3600.0
belg+10arcs_53	–	–	172k	3600.0	367.0	192k	3600.0
belg+10arcs_54	–	–	164k	3600.0	–	198k	3600.0
belg+10arcs_55	–	–	246k	3600.0	–	203k	3600.0
belg+10arcs_56	–	–	183k	3600.0	–	208k	3600.0
belg+10arcs_57	–	–	200k	3600.0	–	240k	3600.0
belg+10arcs_58	–	0.0	153k	2250.4	–	185k	3600.0
belg+10arcs_59	–	–	229k	3600.0	0.0	94k	1349.5
belg+10arcs_60	–	0.0	223k	3330.6	717.9	209k	3600.0
belg+10arcs_61	–	0.0	206k	3265.3	–	205k	3600.0
belg+10arcs_62	–	–	245k	3600.0	–	262k	3600.0
belg+10arcs_63	–	–	151k	3600.0	–	175k	3600.0
belg+10arcs_64	–	–	190k	3600.0	–	200k	3600.0
belg+10arcs_65	–	–	204k	3600.0	–	193k	3600.0
belg+10arcs_66	–	–	202k	3600.0	0.0	159k	2078.6
belg+10arcs_67	–	–	193k	3600.0	–	192k	3600.0
belg+10arcs_68	–	–	213k	3600.0	–	207k	3600.0
belg+10arcs_69	–	–	209k	3600.0	–	165k	3600.0
belg+10arcs_70	–	–	218k	3600.0	–	207k	3600.0
belg+10arcs_71	–	0.0	154k	2492.6	1163.6	203k	3600.0
belg+10arcs_72	–	–	203k	3600.0	0.0	142k	1881.9
belg+10arcs_73	–	–	219k	3600.0	2723.0	246k	3600.0
belg+10arcs_74	–	–	186k	3600.0	–	274k	3600.0
belg+10arcs_75	–	–	222k	3600.0	1838.4	233k	3600.0
belg+10arcs_76	–	–	177k	3600.0	1549.3	275k	3600.0
belg+10arcs_77	–	Large	171k	3600.0	–	215k	3600.0
belg+10arcs_78	–	–	196k	3600.0	–	206k	3600.0
belg+10arcs_79	–	–	191k	3600.0	1926.6	206k	3600.0
belg+10arcs_80	–	–	181k	3600.0	0.0	215k	3542.7
belg+10arcs_81	–	–	184k	3600.0	–	177k	3600.0
belg+10arcs_82	–	–	185k	3600.0	–	243k	3600.0
belg+10arcs_83	–	–	185k	3600.0	–	184k	3600.0
belg+10arcs_84	–	6892.8	202k	3600.0	–	321k	3600.0
belg+10arcs_85	–	–	169k	3600.0	–	179k	3600.0
belg+10arcs_86	–	–	171k	3600.0	–	226k	3600.0
belg+10arcs_87	–	–	180k	3600.0	–	172k	3600.0
belg+10arcs_88	–	–	190k	3600.0	–	182k	3600.0
belg+10arcs_89	–	–	172k	3600.0	–	181k	3600.0
belg+10arcs_90	–	–	169k	3600.0	–	203k	3600.0
belg+10arcs_91	–	–	218k	3600.0	–	204k	3600.0
belg+10arcs_92	–	–	184k	3600.0	–	183k	3600.0
belg+10arcs_93	–	–	196k	3600.0	–	195k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+10arcs_94	–	–	177k	3600.0	–	198k	3600.0
belg+10arcs_95	–	–	189k	3600.0	0.0	106k	1360.7
belg+10arcs_96	–	Large	194k	3600.0	–	196k	3600.0
belg+10arcs_97	–	–	199k	3600.0	1237.0	216k	3600.0
belg+10arcs_98	–	–	190k	3600.0	–	202k	3600.0
belg+10arcs_99	–	–	207k	3600.0	–	184k	3600.0
belg+10arcs_100	–	–	174k	3600.0	573.1	205k	3600.0
belg+10arcs_101	–	–	205k	3600.0	–	217k	3600.0
belg+10arcs_102	–	Large	223k	3600.0	85.3	195k	3600.0
belg+10arcs_103	–	–	181k	3600.0	–	193k	3600.0
belg+10arcs_104	–	–	206k	3600.0	–	180k	3600.0
belg+10arcs_105	–	7238.7	203k	3600.0	2073.2	203k	3600.0
belg+10arcs_106	–	–	194k	3600.0	601.4	195k	3600.0
belg+10arcs_107	–	–	168k	3600.0	0.0	218k	3361.1
belg+10arcs_108	–	–	225k	3600.0	0.0	147k	2050.8
belg+10arcs_109	–	0.0	182k	3052.8	4194.2	223k	3600.0
belg+10arcs_110	–	Large	196k	3600.0	759.7	215k	3600.0
belg+10arcs_111	–	–	199k	3600.0	–	203k	3600.0
belg+10arcs_112	–	–	357k	3600.0	–	228k	3600.0
belg+10arcs_113	–	–	232k	3600.0	0.0	132k	2009.2
belg+10arcs_114	–	Large	205k	3600.0	–	182k	3600.0
belg+10arcs_115	–	–	234k	3600.0	–	215k	3600.0
belg+10arcs_116	–	–	190k	3600.0	–	207k	3600.0
belg+10arcs_117	–	–	162k	3600.0	–	176k	3600.0
belg+10arcs_118	–	9203.0	223k	3600.0	4432.2	183k	3600.0
belg+10arcs_119	–	–	189k	3600.0	–	193k	3600.0
belg+10arcs_120	–	0.0	343k	3519.2	0.0	232k	3425.1
belg+10arcs_121	–	–	231k	3600.0	93.6	260k	3600.0
belg+10arcs_122	–	–	215k	3600.0	0.0	172k	2317.6
belg+10arcs_123	–	–	199k	3600.0	–	262k	3600.0
belg+10arcs_124	–	Large	279k	3600.0	–	216k	3600.0
belg+10arcs_125	–	–	175k	3600.0	–	208k	3600.0
belg+10arcs_126	–	–	207k	3600.0	3258.7	219k	3600.0
belg+10arcs_127	–	2419.3	207k	3600.0	–	212k	3600.0
belg+10arcs_128	–	–	209k	3600.0	–	329k	3600.0
belg+10arcs_129	–	–	247k	3600.0	–	262k	3600.0
belg+10arcs_130	–	–	213k	3600.0	–	177k	3600.0
belg+10arcs_131	–	–	180k	3600.0	–	185k	3600.0
belg+10arcs_132	–	–	216k	3600.0	–	208k	3600.0
belg+10arcs_133	–	–	182k	3600.0	2939.4	196k	3600.0
belg+10arcs_134	–	–	186k	3600.0	–	208k	3600.0
belg+10arcs_135	–	252.4	219k	3600.0	–	248k	3600.0
belg+10arcs_136	–	–	189k	3600.0	–	176k	3600.0
belg+10arcs_137	–	–	192k	3600.0	–	172k	3600.0
belg+10arcs_138	–	–	168k	3600.0	–	163k	3600.0
belg+10arcs_139	–	–	187k	3600.0	–	177k	3600.0
belg+10arcs_140	–	–	164k	3600.0	0.0	290k	3277.1
belg+10arcs_141	–	–	177k	3600.0	–	268k	3600.0
belg+10arcs_142	–	–	186k	3600.0	91.6	201k	3600.0
belg+10arcs_143	–	–	192k	3600.0	–	192k	3600.0
belg+10arcs_144	–	–	230k	3600.0	–	200k	3600.0
belg+10arcs_145	–	0.0	129k	2100.8	693.1	214k	3600.0
belg+10arcs_146	–	–	199k	3600.0	–	218k	3600.0
belg+10arcs_147	–	–	246k	3600.0	1252.3	231k	3600.0
belg+10arcs_148	–	–	223k	3600.0	–	164k	3600.0
belg+10arcs_149	–	–	240k	3600.0	–	169k	3600.0
belg+10arcs_150	–	–	193k	3600.0	–	169k	3600.0
belg+10arcs_151	–	–	185k	3600.0	1207.5	248k	3600.0
belg+10arcs_152	–	–	174k	3600.0	709.2	215k	3600.0
belg+10arcs_153	–	–	201k	3600.0	–	194k	3600.0
belg+10arcs_154	–	0.0	182k	2915.9	–	197k	3600.0
belg+10arcs_155	–	–	186k	3600.0	197.7	201k	3600.0
belg+10arcs_156	–	–	194k	3600.0	–	311k	3600.0
belg+10arcs_157	–	0.0	173k	2251.5	–	175k	3600.0
belg+10arcs_158	–	Large	183k	3600.0	3435.0	224k	3600.0
belg+10arcs_159	–	–	216k	3600.0	1594.6	198k	3600.0
belg+10arcs_160	–	–	212k	3600.0	0.0	64k	852.9
belg+10arcs_161	–	–	216k	3600.0	–	196k	3600.0
belg+10arcs_162	–	–	188k	3600.0	–	200k	3600.0
belg+10arcs_163	–	–	190k	3600.0	294.5	209k	3600.0
belg+10arcs_164	–	0.0	136k	2519.9	0.0	121k	1471.4
belg+10arcs_165	–	–	204k	3600.0	–	186k	3600.0
belg+10arcs_166	–	–	233k	3600.0	0.0	52k	717.6
belg+10arcs_167	–	Large	192k	3600.0	–	194k	3600.0
belg+10arcs_168	–	–	207k	3600.0	–	231k	3600.0
belg+10arcs_169	–	–	163k	3600.0	–	198k	3600.0
belg+10arcs_170	–	–	220k	3600.0	84.5	234k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+10arcs_171	–	–	202k	3600.0	–	276k	3600.0
belg+10arcs_172	–	–	213k	3600.0	0.0	158k	2221.8
belg+10arcs_173	–	–	170k	3600.0	–	199k	3600.0
belg+10arcs_174	–	–	182k	3600.0	–	220k	3600.0
belg+10arcs_175	–	–	229k	3600.0	–	196k	3600.0
belg+10arcs_176	–	0.0	188k	2525.1	3444.7	236k	3600.0
belg+10arcs_177	–	–	186k	3600.0	–	207k	3600.0
belg+10arcs_178	–	–	207k	3600.0	–	185k	3600.0
belg+10arcs_179	–	–	182k	3600.0	–	222k	3600.0
belg+10arcs_180	–	–	183k	3600.0	–	245k	3600.0
belg+10arcs_181	–	–	213k	3600.0	717.1	222k	3600.0
belg+10arcs_182	–	–	230k	3600.0	1507.5	236k	3600.0
belg+10arcs_183	–	–	179k	3600.0	–	198k	3600.0
belg+10arcs_184	–	–	207k	3600.0	0.0	114k	1545.2
belg+10arcs_185	–	–	199k	3600.0	–	221k	3600.0
belg+10arcs_186	–	–	201k	3600.0	–	217k	3600.0
belg+10arcs_187	–	–	174k	3600.0	–	178k	3600.0
belg+10arcs_188	–	0.0	163k	2835.6	0.0	162k	2501.2
belg+10arcs_189	–	–	171k	3600.0	–	250k	3600.0
belg+10arcs_190	–	–	202k	3600.0	3101.4	173k	3600.0
belg+10arcs_191	–	–	210k	3600.0	–	160k	3600.0
belg+10arcs_192	–	–	209k	3600.0	–	198k	3600.0
belg+10arcs_193	–	–	176k	3600.0	0.0	171k	2172.5
belg+10arcs_194	–	–	187k	3600.0	–	242k	3600.0
belg+10arcs_195	–	–	228k	3600.0	–	230k	3600.0
belg+10arcs_196	–	–	169k	3600.0	–	264k	3600.0
belg+10arcs_197	–	–	167k	3600.0	–	186k	3600.0
belg+10arcs_198	–	8906.7	212k	3600.0	–	310k	3600.0
belg+10arcs_199	–	–	190k	3600.0	–	196k	3600.0
belg+10arcs_200	–	–	173k	3600.0	423.7	202k	3600.0
belg+10arcs_201	–	–	172k	3600.0	–	357k	3600.0
belg+10arcs_202	–	1859.4	202k	3600.0	–	195k	3600.0
belg+10arcs_203	–	–	183k	3600.0	–	181k	3600.0
belg+10arcs_204	–	557.7	230k	3600.0	0.0	101k	1187.6
belg+10arcs_205	–	–	194k	3600.0	9863.4	215k	3600.0
belg+10arcs_206	–	–	178k	3600.0	–	190k	3600.0
belg+10arcs_207	–	–	210k	3600.0	–	193k	3600.0
belg+10arcs_208	–	–	189k	3600.0	–	194k	3600.0
belg+10arcs_209	–	–	219k	3600.0	–	170k	3600.0
belg+10arcs_210	–	–	175k	3600.0	482.6	208k	3600.0
belg+10arcs_211	–	–	214k	3600.0	788.4	234k	3600.0
belg+10arcs_212	–	–	191k	3600.0	–	208k	3600.0
belg+10arcs_213	–	–	162k	3600.0	1044.9	242k	3600.0
belg+10arcs_214	–	–	211k	3600.0	–	186k	3600.0
belg+10arcs_215	–	–	179k	3600.0	795.4	212k	3600.0
belg+10arcs_216	–	–	200k	3600.0	1792.2	195k	3600.0
belg+10arcs_217	–	Large	180k	3600.0	–	262k	3600.0
belg+10arcs_218	–	–	175k	3600.0	–	175k	3600.0
belg+10arcs_219	–	–	200k	3600.0	250.7	237k	3600.0
belg+10arcs_220	–	–	182k	3600.0	–	197k	3600.0
belg+10arcs_221	–	0.0	180k	3254.3	0.0	54k	618.6
belg+10arcs_222	–	–	211k	3600.0	–	183k	3600.0
belg+10arcs_223	–	–	192k	3600.0	983.8	191k	3600.0
belg+10arcs_224	–	–	194k	3600.0	–	183k	3600.0
belg+10arcs_225	–	0.0	155k	2353.8	0.0	132k	1555.6
belg+10arcs_226	–	–	219k	3600.0	–	198k	3600.0
belg+10arcs_227	–	–	192k	3600.0	2827.1	212k	3600.0
belg+10arcs_228	–	–	188k	3600.0	–	178k	3600.0
belg+10arcs_229	–	–	174k	3600.0	–	183k	3600.0
belg+10arcs_230	–	–	206k	3600.0	–	183k	3600.0
belg+10arcs_231	–	–	206k	3600.0	301.6	198k	3600.0
belg+10arcs_232	–	–	180k	3600.0	–	208k	3600.0
belg+10arcs_233	–	–	174k	3600.0	1799.3	214k	3600.0
belg+10arcs_234	–	–	204k	3600.0	1552.3	217k	3600.0
belg+10arcs_235	–	–	368k	3600.0	1141.9	233k	3600.0
belg+10arcs_236	–	3170.8	210k	3600.0	–	208k	3600.0
belg+10arcs_237	–	0.0	199k	3453.3	0.0	138k	1821.0
belg+10arcs_238	–	–	181k	3600.0	–	190k	3600.0
belg+10arcs_239	–	–	179k	3600.0	266.1	207k	3600.0
belg+10arcs_240	–	–	181k	3600.0	–	199k	3600.0
belg+10arcs_241	–	–	207k	3600.0	–	211k	3600.0
belg+10arcs_242	–	5684.0	210k	3600.0	–	198k	3600.0
belg+10arcs_243	–	0.0	181k	2860.5	0.0	167k	2406.4
belg+10arcs_244	–	–	181k	3600.0	–	256k	3600.0
belg+10arcs_245	–	–	166k	3600.0	0.0	191k	3077.9
belg+10arcs_246	–	4351.3	209k	3600.0	–	203k	3600.0
belg+10arcs_247	–	Large	214k	3600.0	0.0	148k	2094.3

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+10arcs_248	–	–	177k	3600.0	–	245k	3600.0
belg+10arcs_249	–	–	181k	3600.0	–	266k	3600.0
belg+10arcs_250	–	–	186k	3600.0	–	209k	3600.0
belg+10arcs_251	–	–	305k	3600.0	–	208k	3600.0
belg+10arcs_252	–	–	205k	3600.0	–	233k	3600.0
belg+10arcs_253	–	–	186k	3600.0	–	186k	3600.0
belg+10arcs_254	–	7220.4	208k	3600.0	–	325k	3600.0
belg+10arcs_255	–	0.0	177k	3142.3	–	219k	3600.0
belg+10arcs_256	–	–	159k	3600.0	–	221k	3600.0
belg+10arcs_257	–	–	180k	3600.0	–	214k	3600.0
belg+10arcs_258	–	2726.9	201k	3600.0	–	191k	3600.0
belg+10arcs_259	–	–	202k	3600.0	–	251k	3600.0
belg+10arcs_260	–	3396.7	214k	3600.0	0.0	90k	1364.5
belg+10arcs_261	–	–	206k	3600.0	2866.1	219k	3600.0
belg+10arcs_262	–	–	223k	3600.0	0.0	136k	1618.7
belg+10arcs_263	–	–	174k	3600.0	–	391k	3600.0
belg+10arcs_264	–	–	208k	3600.0	–	196k	3600.0
belg+10arcs_265	–	–	183k	3600.0	–	191k	3600.0
belg+10arcs_266	–	0.0	234k	3518.8	0.0	91k	1305.3
belg+10arcs_267	–	–	185k	3600.0	–	264k	3600.0
belg+10arcs_268	–	–	177k	3600.0	–	243k	3600.0
belg+10arcs_269	–	–	198k	3600.0	–	207k	3600.0
belg+10arcs_270	–	–	183k	3600.0	–	194k	3600.0
belg+10arcs_271	–	–	204k	3600.0	–	212k	3600.0
belg+10arcs_272	–	–	211k	3600.0	–	172k	3600.0
belg+10arcs_273	–	–	178k	3600.0	1349.1	192k	3600.0
belg+10arcs_274	–	Large	190k	3600.0	5027.9	246k	3600.0
belg+10arcs_275	–	–	198k	3600.0	Large	242k	3600.0
belg+10arcs_276	–	–	232k	3600.0	0.0	160k	2349.7
belg+10arcs_277	–	Large	196k	3600.0	2255.7	229k	3600.0
belg+10arcs_278	–	–	261k	3600.0	0.0	169k	2067.6
belg+10arcs_279	–	0.0	138k	1938.8	–	263k	3600.0
belg+10arcs_280	–	–	189k	3600.0	–	218k	3600.0
belg+10arcs_281	–	2215.9	197k	3600.0	–	194k	3600.0
belg+10arcs_282	–	–	190k	3600.0	–	195k	3600.0
belg+10arcs_283	–	–	182k	3600.0	–	179k	3600.0
belg+10arcs_284	–	–	197k	3600.0	–	240k	3600.0
belg+10arcs_285	–	Large	214k	3600.0	–	245k	3600.0
belg+10arcs_286	–	–	190k	3600.0	–	219k	3600.0
belg+10arcs_287	–	–	181k	3600.0	291.5	216k	3600.0
belg+10arcs_288	–	–	242k	3600.0	–	216k	3600.0
belg+10arcs_289	–	–	167k	3600.0	–	172k	3600.0
belg+10arcs_290	–	0.0	172k	2527.7	–	190k	3600.0
belg+10arcs_291	–	–	213k	3600.0	–	238k	3600.0
belg+10arcs_292	–	–	179k	3600.0	0.0	183k	2670.0
belg+10arcs_293	–	0.0	148k	2796.9	Large	516k	3600.0
belg+10arcs_294	–	–	208k	3600.0	0.0	190k	2404.9
belg+10arcs_295	–	Large	198k	3600.0	–	196k	3600.0
belg+10arcs_296	–	–	206k	3600.0	–	200k	3600.0
belg+10arcs_297	–	–	194k	3600.0	359.2	195k	3600.0
belg+10arcs_298	–	–	171k	3600.0	–	197k	3600.0
belg+10arcs_299	–	–	166k	3600.0	–	211k	3600.0
belg+10arcs_300	–	–	184k	3600.0	–	143k	3600.0
belg+10arcs_301	–	0.0	167k	2556.5	–	200k	3600.0
belg+10arcs_302	–	–	180k	3600.0	–	182k	3600.0
belg+10arcs_303	–	–	206k	3600.0	–	210k	3600.0
belg+10arcs_304	–	–	194k	3600.0	–	201k	3600.0
belg+10arcs_305	–	–	186k	3600.0	–	200k	3600.0
belg+10arcs_306	–	–	200k	3600.0	0.0	199k	3079.3
belg+10arcs_307	–	–	188k	3600.0	–	188k	3600.0
belg+10arcs_308	–	–	191k	3600.0	–	188k	3600.0
belg+10arcs_309	–	–	155k	3600.0	–	195k	3600.0
belg+10arcs_310	–	0.0	206k	3448.9	0.0	224k	3276.7
belg+10arcs_311	–	–	209k	3600.0	2657.8	209k	3600.0
belg+10arcs_312	–	–	193k	3600.0	–	198k	3600.0
belg+10arcs_313	–	–	200k	3600.0	–	179k	3600.0
belg+10arcs_314	–	–	180k	3600.0	–	204k	3600.0
belg+10arcs_315	–	–	180k	3600.0	–	195k	3600.0
belg+10arcs_316	–	–	339k	3600.0	–	215k	3600.0
belg+10arcs_317	–	–	218k	3600.0	–	235k	3600.0
belg+10arcs_318	–	–	209k	3600.0	–	219k	3600.0
belg+10arcs_319	–	0.0	191k	3156.6	–	221k	3600.0
belg+10arcs_320	–	–	170k	3600.0	–	213k	3600.0
belg+10arcs_321	–	0.0	138k	2151.4	–	242k	3600.0
belg+10arcs_322	–	–	224k	3600.0	–	173k	3600.0
belg+10arcs_323	–	–	199k	3600.0	–	172k	3600.0
belg+10arcs_324	–	–	207k	3600.0	933.9	253k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+10arcs_325	–	–	190k	3600.0	–	262k	3600.0
belg+10arcs_326	–	–	177k	3600.0	–	313k	3600.0
belg+10arcs_327	–	–	200k	3600.0	0.0	150k	2010.3
belg+10arcs_328	–	–	218k	3600.0	0.0	46k	682.0
belg+10arcs_329	–	–	222k	3600.0	–	220k	3600.0
belg+10arcs_330	–	–	167k	3600.0	–	220k	3600.0
belg+10arcs_331	–	–	246k	3600.0	Large	212k	3600.0
belg+10arcs_332	–	–	169k	3600.0	–	184k	3600.0
belg+10arcs_333	–	0.0	208k	3169.0	–	254k	3600.0
belg+10arcs_334	–	–	175k	3600.0	–	205k	3600.0
belg+10arcs_335	–	–	191k	3600.0	–	186k	3600.0
belg+10arcs_336	–	–	196k	3600.0	–	221k	3600.0
belg+10arcs_337	–	–	179k	3600.0	–	222k	3600.0
belg+10arcs_338	–	5219.3	204k	3600.0	–	275k	3600.0
belg+10arcs_339	–	–	223k	3600.0	–	176k	3600.0
belg+10arcs_340	–	–	246k	3600.0	–	196k	3600.0
belg+10arcs_341	–	–	172k	3600.0	–	182k	3600.0
belg+10arcs_342	–	–	154k	3600.0	–	162k	3600.0
belg+10arcs_343	–	0.0	127k	1882.3	–	198k	3600.0
belg+10arcs_344	–	0.0	119k	2220.9	–	174k	3600.0
belg+10arcs_345	–	–	213k	3600.0	–	190k	3600.0
belg+10arcs_346	–	–	178k	3600.0	–	180k	3600.0
belg+10arcs_347	–	–	175k	3600.0	–	153k	3600.0
belg+10arcs_348	–	–	201k	3600.0	–	206k	3600.0
belg+10arcs_349	–	–	185k	3600.0	Large	206k	3600.0
belg+10arcs_350	–	–	171k	3600.0	Large	239k	3600.0
belg+10arcs_351	–	–	183k	3600.0	–	205k	3600.0
belg+10arcs_352	–	–	176k	3600.0	–	196k	3600.0
belg+10arcs_353	–	–	190k	3600.0	–	254k	3600.0
belg+10arcs_354	–	7151.5	193k	3600.0	2453.8	214k	3600.0
belg+10arcs_355	–	–	211k	3600.0	3060.6	216k	3600.0
belg+10arcs_356	–	–	207k	3600.0	–	183k	3600.0
belg+10arcs_357	–	–	189k	3600.0	–	166k	3600.0
belg+10arcs_358	–	–	183k	3600.0	–	158k	3600.0
belg+10arcs_359	–	3787.8	212k	3600.0	0.0	166k	2135.3
belg+10arcs_360	–	–	178k	3600.0	–	189k	3600.0
belg+10arcs_361	–	–	191k	3600.0	–	169k	3600.0
belg+10arcs_362	–	–	274k	3600.0	0.0	159k	2692.2
belg+10arcs_363	–	–	198k	3600.0	–	220k	3600.0
belg+10arcs_364	–	0.0	115k	1597.7	–	212k	3600.0
belg+10arcs_365	–	2178.0	193k	3600.0	471.7	205k	3600.0
belg+10arcs_366	–	–	175k	3600.0	–	279k	3600.0
belg+10arcs_367	–	–	179k	3600.0	0.0	178k	2948.9
belg+10arcs_368	–	7692.1	194k	3600.0	–	262k	3600.0
belg+10arcs_369	–	–	207k	3600.0	–	243k	3600.0
belg+10arcs_370	–	Large	193k	3600.0	Large	210k	3600.0
belg+10arcs_371	–	–	272k	3600.0	–	312k	3600.0
belg+10arcs_372	–	–	211k	3600.0	–	181k	3600.0
belg+10arcs_373	–	–	211k	3600.0	–	214k	3600.0
belg+10arcs_374	–	–	174k	3600.0	1734.7	194k	3600.0
belg+10arcs_375	–	–	201k	3600.0	326.1	224k	3600.0
belg+10arcs_376	–	–	174k	3600.0	328.1	201k	3600.0
belg+10arcs_377	–	–	167k	3600.0	1509.4	208k	3600.0
belg+10arcs_378	–	–	182k	3600.0	–	168k	3600.0
belg+10arcs_379	–	–	300k	3600.0	–	177k	3600.0
belg+10arcs_380	–	–	202k	3600.0	–	193k	3600.0
belg+10arcs_381	–	–	169k	3600.0	–	205k	3600.0
belg+10arcs_382	–	–	182k	3600.0	–	201k	3600.0
belg+10arcs_383	–	–	216k	3600.0	1166.5	223k	3600.0
belg+10arcs_384	–	4546.4	203k	3600.0	–	209k	3600.0
belg+10arcs_385	–	–	219k	3600.0	–	191k	3600.0
belg+10arcs_386	–	–	188k	3600.0	–	205k	3600.0
belg+10arcs_387	–	Large	205k	3600.0	2684.9	187k	3600.0
belg+10arcs_388	–	–	207k	3600.0	7171.6	211k	3600.0
belg+10arcs_389	–	–	195k	3600.0	Large	244k	3600.0
belg+10arcs_390	–	–	222k	3600.0	–	170k	3600.0
belg+10arcs_391	–	–	193k	3600.0	–	187k	3600.0
belg+10arcs_392	–	0.0	125k	2264.2	Large	205k	3600.0
belg+10arcs_393	–	–	195k	3600.0	297.2	253k	3600.0
belg+10arcs_394	–	–	197k	3600.0	–	202k	3600.0
belg+10arcs_395	–	–	208k	3600.0	–	196k	3600.0
belg+10arcs_396	–	–	398k	3600.0	0.0	150k	2985.6
belg+10arcs_397	–	–	175k	3600.0	–	242k	3600.0
belg+10arcs_398	–	–	226k	3600.0	0.0	109k	1522.8
belg+10arcs_399	–	0.0	192k	3461.1	0.0	53k	794.7
belg+10arcs_400	–	3714.1	196k	3600.0	–	252k	3600.0
belg+10arcs_401	–	–	307k	3600.0	126.2	242k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+10arcs_402	–	5897.8	192k	3600.0	–	193k	3600.0
belg+10arcs_403	–	–	174k	3600.0	–	178k	3600.0
belg+10arcs_404	–	–	208k	3600.0	2236.6	251k	3600.0
belg+10arcs_405	–	–	187k	3600.0	–	188k	3600.0
belg+10arcs_406	–	–	181k	3600.0	–	197k	3600.0
belg+10arcs_407	–	3477.9	196k	3600.0	–	306k	3600.0
belg+10arcs_408	–	–	206k	3600.0	0.0	166k	2551.7
belg+10arcs_409	–	Large	189k	3600.0	–	178k	3600.0
belg+10arcs_410	–	–	188k	3600.0	–	171k	3600.0
belg+10arcs_411	–	–	252k	3600.0	–	198k	3600.0
belg+10arcs_412	–	–	197k	3600.0	0.0	208k	2848.0
belg+10arcs_413	–	–	175k	3600.0	–	189k	3600.0
belg+10arcs_414	–	–	191k	3600.0	–	164k	3600.0
belg+10arcs_415	–	–	197k	3600.0	0.0	203k	2945.2
belg+10arcs_416	–	–	175k	3600.0	–	205k	3600.0
belg+10arcs_417	–	–	227k	3600.0	–	215k	3600.0
belg+10arcs_418	–	–	192k	3600.0	–	200k	3600.0
belg+10arcs_419	–	–	205k	3600.0	–	224k	3600.0
belg+10arcs_420	–	–	222k	3600.0	8555.2	212k	3600.0
belg+10arcs_421	–	Large	194k	3600.0	–	216k	3600.0
belg+10arcs_422	–	–	210k	3600.0	–	226k	3600.0
belg+10arcs_423	–	–	185k	3600.0	–	256k	3600.0
belg+10arcs_424	–	–	177k	3600.0	–	252k	3600.0
belg+10arcs_425	–	–	194k	3600.0	–	191k	3600.0
belg+10arcs_426	–	–	172k	3600.0	–	171k	3600.0
belg+10arcs_427	–	1961.8	192k	3600.0	3829.0	294k	3600.0
belg+10arcs_428	–	Large	194k	3600.0	–	208k	3600.0
belg+10arcs_429	–	–	171k	3600.0	–	197k	3600.0
belg+10arcs_430	–	–	184k	3600.0	–	218k	3600.0
belg+10arcs_431	–	–	173k	3600.0	–	174k	3600.0
belg+10arcs_432	–	0.0	207k	3390.5	0.0	109k	1413.1
belg+10arcs_433	–	–	207k	3600.0	2814.8	208k	3600.0
belg+10arcs_434	–	–	222k	3600.0	–	203k	3600.0
belg+10arcs_435	–	–	201k	3600.0	–	243k	3600.0
belg+10arcs_436	–	–	237k	3600.0	–	188k	3600.0
belg+10arcs_437	–	967.9	220k	3600.0	–	230k	3600.0
belg+10arcs_438	–	–	196k	3600.0	–	518k	3600.0
belg+10arcs_439	–	–	209k	3600.0	–	187k	3600.0
belg+10arcs_440	–	–	178k	3600.0	–	175k	3600.0
belg+10arcs_441	–	–	194k	3600.0	–	187k	3600.0
belg+10arcs_442	–	–	224k	3600.0	0.0	111k	1388.4
belg+10arcs_443	–	0.0	130k	2180.9	–	184k	3600.0
belg+10arcs_444	–	3425.3	238k	3600.0	–	252k	3600.0
belg+10arcs_445	–	–	176k	3600.0	–	178k	3600.0
belg+10arcs_446	–	–	193k	3600.0	–	209k	3600.0
belg+10arcs_447	–	–	224k	3600.0	–	200k	3600.0
belg+10arcs_448	–	–	150k	3600.0	–	149k	3600.0
belg+10arcs_449	–	–	259k	3600.0	2103.9	197k	3600.0
belg+10arcs_450	–	–	236k	3600.0	–	225k	3600.0
belg+10arcs_451	–	–	183k	3600.0	Large	215k	3600.0
belg+10arcs_452	–	–	196k	3600.0	–	191k	3600.0
belg+10arcs_453	–	–	163k	3600.0	0.0	250k	3310.8
belg+10arcs_454	–	–	189k	3600.0	–	197k	3600.0
belg+10arcs_455	–	–	175k	3600.0	–	238k	3600.0
belg+10arcs_456	–	–	191k	3600.0	–	199k	3600.0
belg+10arcs_457	–	0.0	63k	1132.9	Large	255k	3600.0
belg+10arcs_458	–	–	183k	3600.0	0.0	162k	2441.7
belg+10arcs_459	–	–	175k	3600.0	–	244k	3600.0
belg+10arcs_460	–	Large	234k	3600.0	Large	242k	3600.0
belg+10arcs_461	–	–	202k	3600.0	–	226k	3600.0
belg+10arcs_462	–	0.0	170k	2686.7	–	220k	3600.0
belg+10arcs_463	–	–	185k	3600.0	–	198k	3600.0
belg+10arcs_464	–	–	180k	3600.0	–	345k	3600.0
belg+10arcs_465	–	–	224k	3600.0	0.0	78k	1110.8
belg+10arcs_466	–	–	156k	3600.0	–	225k	3600.0
belg+10arcs_467	–	–	175k	3600.0	–	175k	3600.0
belg+10arcs_468	–	–	160k	3600.0	–	146k	3600.0
belg+10arcs_469	–	0.0	113k	1876.2	4348.3	224k	3600.0
belg+10arcs_470	–	–	171k	3600.0	0.0	87k	1287.2
belg+10arcs_471	–	–	234k	3600.0	–	191k	3600.0
belg+10arcs_472	–	0.0	245k	3332.8	–	187k	3600.0
belg+10arcs_473	–	–	197k	3600.0	–	184k	3600.0
belg+10arcs_474	–	–	210k	3600.0	3336.4	265k	3600.0
belg+10arcs_475	–	Large	181k	3600.0	–	171k	3600.0
belg+10arcs_476	–	–	180k	3600.0	4917.8	212k	3600.0
belg+10arcs_477	–	–	205k	3600.0	–	220k	3600.0
belg+10arcs_478	–	–	209k	3600.0	2999.1	224k	3600.0

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Table B.19: Results for ROOTGAP and TREE experiments on *Circuit rank* (continued).

	ROOTGAP	TREE					
		Enabled			*Disabled*		
instance	GC	Gap	Nodes	Time	Gap	Nodes	Time
belg+10arcs_479	–	Large	196k	3600.0	–	258k	3600.0
belg+10arcs_480	–	3952.2	236k	3600.0	510.6	194k	3600.0
belg+10arcs_481	–	–	201k	3600.0	–	172k	3600.0
belg+10arcs_482	–	–	215k	3600.0	–	206k	3600.0
belg+10arcs_483	–	–	172k	3600.0	–	197k	3600.0
belg+10arcs_484	–	0.0	115k	2041.7	8329.7	222k	3600.0
belg+10arcs_485	–	–	169k	3600.0	–	240k	3600.0
belg+10arcs_486	–	–	162k	3600.0	–	188k	3600.0
belg+10arcs_487	–	–	200k	3600.0	–	195k	3600.0
belg+10arcs_488	–	–	202k	3600.0	Large	279k	3600.0
belg+10arcs_489	–	9481.8	195k	3600.0	–	202k	3600.0
belg+10arcs_490	–	–	186k	3600.0	–	199k	3600.0
belg+10arcs_491	–	–	217k	3600.0	–	190k	3600.0
belg+10arcs_492	–	–	182k	3600.0	–	223k	3600.0
belg+10arcs_493	–	–	191k	3600.0	–	202k	3600.0
belg+10arcs_494	–	–	212k	3600.0	–	177k	3600.0
belg+10arcs_495	–	0.0	78k	1335.2	–	182k	3600.0
belg+10arcs_496	–	–	212k	3600.0	–	236k	3600.0
belg+10arcs_497	–	–	204k	3600.0	–	263k	3600.0
belg+10arcs_498	–	–	182k	3600.0	2045.0	211k	3600.0
belg+10arcs_499	–	–	164k	3600.0	–	228k	3600.0
belg+10arcs_500	–	–	175k	3600.0	970.5	185k	3600.0

Appendix C

An MINLP Model for the Transient Control Problem

C.1 Alternative Discretization of the Continuity Equation

The discretization of Equation (5.6a) in Section 5.1 is based on the application of "quadrature rules". In the following, we illustrate an alternative approach that yields the same algebraic formulations, however, it uses "finite differences" instead, i.e.,

$$\frac{d}{dx}f(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \frac{f(x+h)-f(x)}{h} + \mathcal{O}(h),\ h>0. \tag{C.1}$$

Then Equation (5.6a) can be reformulated

$$\begin{aligned} &\frac{A}{{\mathrm{c_s}}^2}\partial_t p(x,t) + \partial_x q(x,t) = 0 \\ \overset{(C.1)}{\longrightarrow}\quad &\frac{A}{{\mathrm{c_s}}^2}\frac{p(x,t_1)-p(x,t_0)}{t_1-t_0} + \frac{q(L,t)-q(0,t)}{L} = 0 \quad \forall x\in[0,L],\ \forall t\in[t_0,t_1]. \end{aligned} \tag{C.2}$$

Setting $t := t_1$, then (C.2) transforms to

$$\frac{A}{{\mathrm{c_s}}^2}\frac{p(x,t_1)-p(x,t_0)}{t_1-t_0} + \frac{q(L,t_1)-q(0,t_1)}{L} = 0 \quad \forall x\in[0,L]. \tag{C.3}$$

Defining the left hand side of (C.3) as

$$lhs(x) := \frac{A}{{\mathrm{c_s}}^2}\frac{p(x,t_1)-p(x,t_0)}{t_1-t_0} + \frac{q(L,t_1)-q(0,t_1)}{L} \quad \forall x\in[0,L],$$

and calculating the arithmetic mean of $lhs(x)$ with respect to its interval bounds 0 and L, i.e.,

$$\frac{lhs(L)+lhs(0)}{2},$$

finally yields the equivalent discretized continuity equation as given by (5.11)

$$\frac{A}{\mathrm{c_s}^2}\left(\frac{p_{r,t_1} - p_{r,t_0}}{t_1 - t_0} + \frac{p_{\ell,t_1} - p_{\ell,t_0}}{t_1 - t_0}\right) + 2\,\frac{q_{r,t_1} - q_{\ell,t_1}}{L} = 0,$$

when setting $\tau := (t_1 - t_0)$ for the time difference and using the notation for the pressure and flow variables introduced in paragraph *Discretization in time and space* in Section 5.1.

Appendix D

Aggregation of Transient Gas Networks

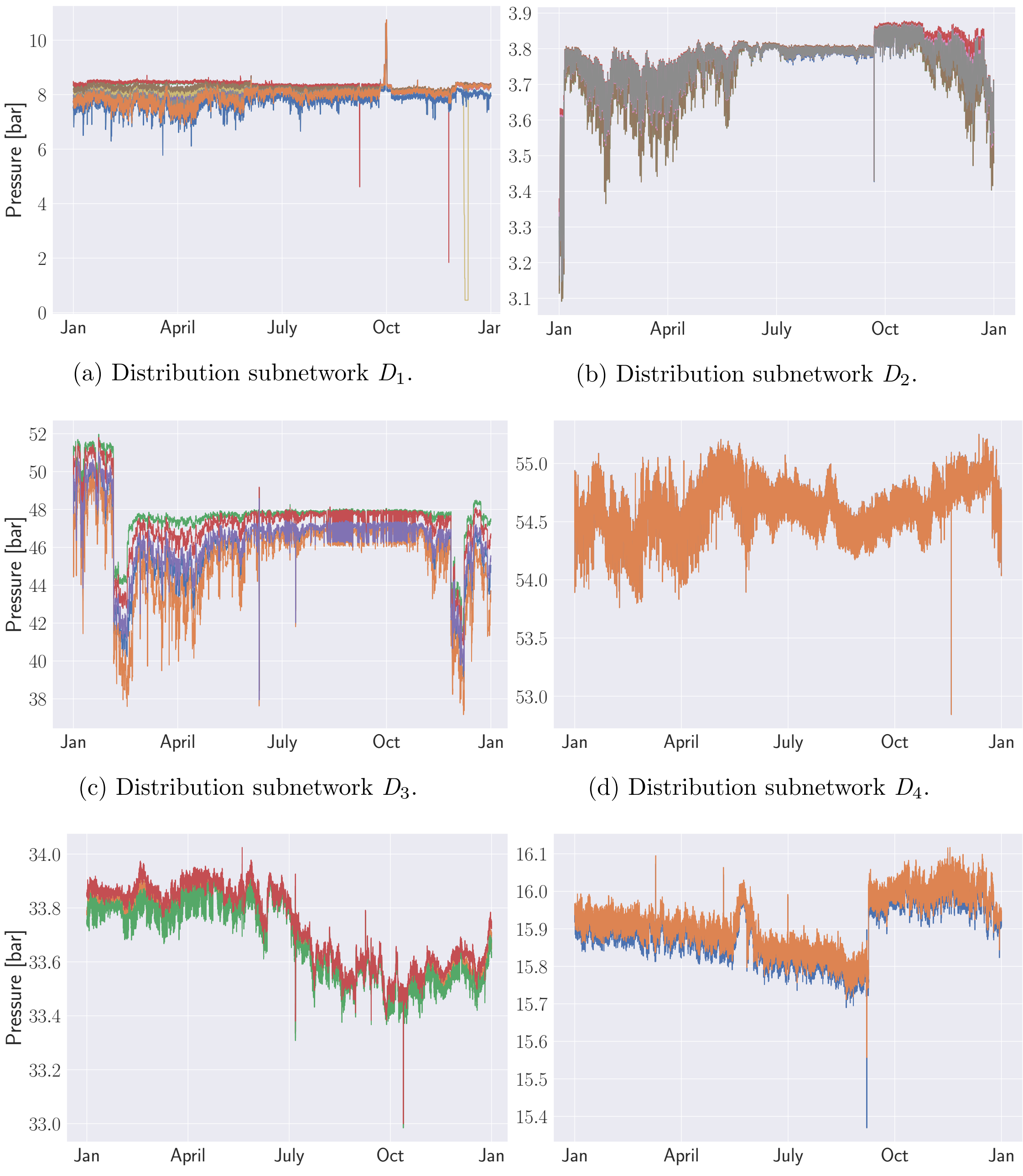
D.1 The Role of line-Pack in Distribution Subnetworks

D.1.1 Studying the usage of distribution subnetworks as gas storages

Table D.1 summarizes the numerical results for the difference of inflow and outflow (col. $\sum_{a \in \mathcal{A}_C^{in}} q_a - \sum_{v \in \mathcal{V}_C^-} b_v$) and the smoothed version (col. *Convolution*) of the six distribution subnetworks, as shown in Figure 6.1.

	$\sum_{a \in \mathcal{A}_C^{in}} q_a - \sum_{v \in \mathcal{V}_C^-} \lvert b_v \rvert$			Convolution		
Cases	AD	mean AD	mean MAD	AD	mean AD	mean MAD
D_1	7.990	0.164	5.9e−05	0.179	0.079	4.6e−05
D_2	1.482	1.057	5.4e−03	1.100	1.043	5.4e−03
D_3	106.719	1.048	1.9e−03	1.315	0.657	1.9e−03
D_4	0.181	1.8e−03	7.2e−06	0.002	8.2e−04	5.0e−06
D_5	0.370	5.7e−03	5.9e−06	0.006	3.2e−03	7.1e−06
D_6	0.530	1.4e−03	1.3e−06	0.003	1.1e−03	1.0e−06

Table D.1: Statistics on the inflow and outflow values of the distribution subnetworks $D_1, ..., D_6$. Here, the measures correspond to the absolute deviation (AD), the daily absolute deviation (mean AD), and the daily mean absolute deviation (mean MAD). Both latter measures are averaged over 365 days, where an entire day corresponds to 96 time steps with a time granularity of 15-minutes, see the explanations in Section 6.1.3. All measures are denoted as mass flow in [kg/s].



(a) Distribution subnetwork D_1.

(b) Distribution subnetwork D_2.

(c) Distribution subnetwork D_3.

(d) Distribution subnetwork D_4.

(e) Distribution subnetwork D_5.

(f) Distribution subnetwork D_6.

Figure D.1: Historical pressure values in distribution subnetworks $D_1, ..., D_6$, where the colors correspond to different subnetwork nodes. Pressure values are provided every 15 minutes over a duration of 365 days resulting in 35,040 time steps.

D.1.2 Studying the possibility to store gas in distribution subnetworks

In the experiment in Section 6.3.3, we have seen that line-pack in distribution subnetworks is rather inconsequential and not really used to buffer network operations. In this experiment, we study whether distribution subnetworks provide in any way the possibility to be used as storages. To this end, we investigate how fast the six distribution subnetworks $D_1, \ldots, D_6$ from Table 6.2 run empty compared to a selected main trunk of the transmission network. Under the assumption that line-pack does not play a significant role in distribution subnetworks, we expect that the process of running dry is rather fast in distribution subnetworks and that it might last considerably longer for the main trunk.

In order to determine the needed time until drainage, we run simulations using real-world data. Among others, this data encompasses the described topology for the distribution subnetworks. Moreover, we select for each subnetwork D_i with $i \in \{1, \ldots, 6\}$ a time interval $[t_i, T_i]$ and

i) initialize the pressure values at t_i of all nodes in the distribution subnetworks with historical values, i.e., p_{v,t_i} for all $v \in \mathcal{V}_{D_i}$, and

ii) specify demand values $(b_{v,t})_{v \in \mathcal{V}^-_{D_i}, t \in [t_i, T_i]}$ that follow a historical flow profile.

To simulate the process of running dry, we restrict the distribution subnetworks to zero inflow, i.e., $b_{v,t} = 0$ with $t \in [t_i, T_i]$ for all $v \in \mathcal{V}^{in}_{D_i}$ and for all $i \in \{1, ..., 6\}$.

As transmission line, we select a main trunk between two compressor stations taken from the same network as the distribution subnetworks. The pipeline has a length of 219 [km] and a diameter of 0.938 [m]. We simulate the transmission trunk in the same way as the distribution subnetworks by selecting a time interval and then assigning historical values to the initial pressure values at its end nodes. Similarly, the outflow pattern at its tail node follows a historical demand profile, which, in this case, is around five times higher than the exit flows of all considered distribution subnetworks together. Again, to simulate the process of running dry, no inflow into the head node of the trunk takes place. As simulation tool, we use the DAE solver mentioned in Section 6.1.4.

Here, we restrict the representation of the results to a visualization of the pressure profiles at both incident nodes of the transmission pipeline in Figure D.2 and to the subnetworks' nodes in Figure D.3. Please note that this experiment cannot easily be automated given that the simulator used, does not support event detection to determine the exact time when the pressure at the first node turns negative. Instead, this simulation has to be carried out carefully, because the simulator is not able to handle singularities, which arise when pressures turn zero. Especially when pressures turn negative, then the simulated results do not admit a physical interpretation.

The needed time until running dry can easily be recognized by means of the pressure profile that first hits its lower bound. Here, all nodes of the subnetworks and the main trunk have throughout a lower pressure bound of zero. Figure D.3 shows that the needed time until drainage varies roughly between 10 and 90 minutes for the different distribution subnetworks, as opposed to the transmission line, which requires around 10 hours, see Figure D.2. This means that all distribution subnetworks need comparatively short time to run empty compared to the major trunk of the transmission network. Hence, the results confirm our expectation that single distribution subnetworks do not significantly contribute as storages in the network. As a consequence, we deduce that distribution subnetworks cannot effectively be filled in order to enable self-supply for longer periods in the future. Instead, distribution subnetworks have to be supplied rather instantly to satisfy their consumption. Nevertheless, our aggregation approach even takes into account the impact of all distribution subnetworks together on the gas storage of the entire network by representing each distribution subnetwork by a single storage node.

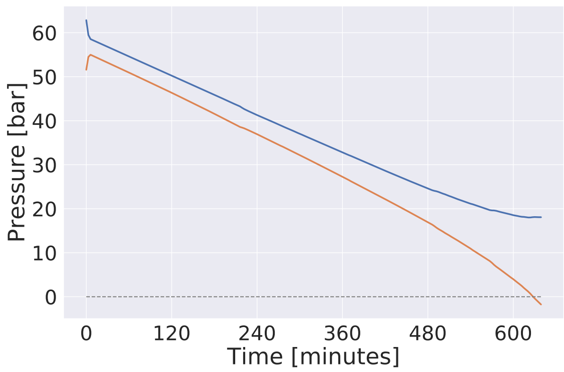
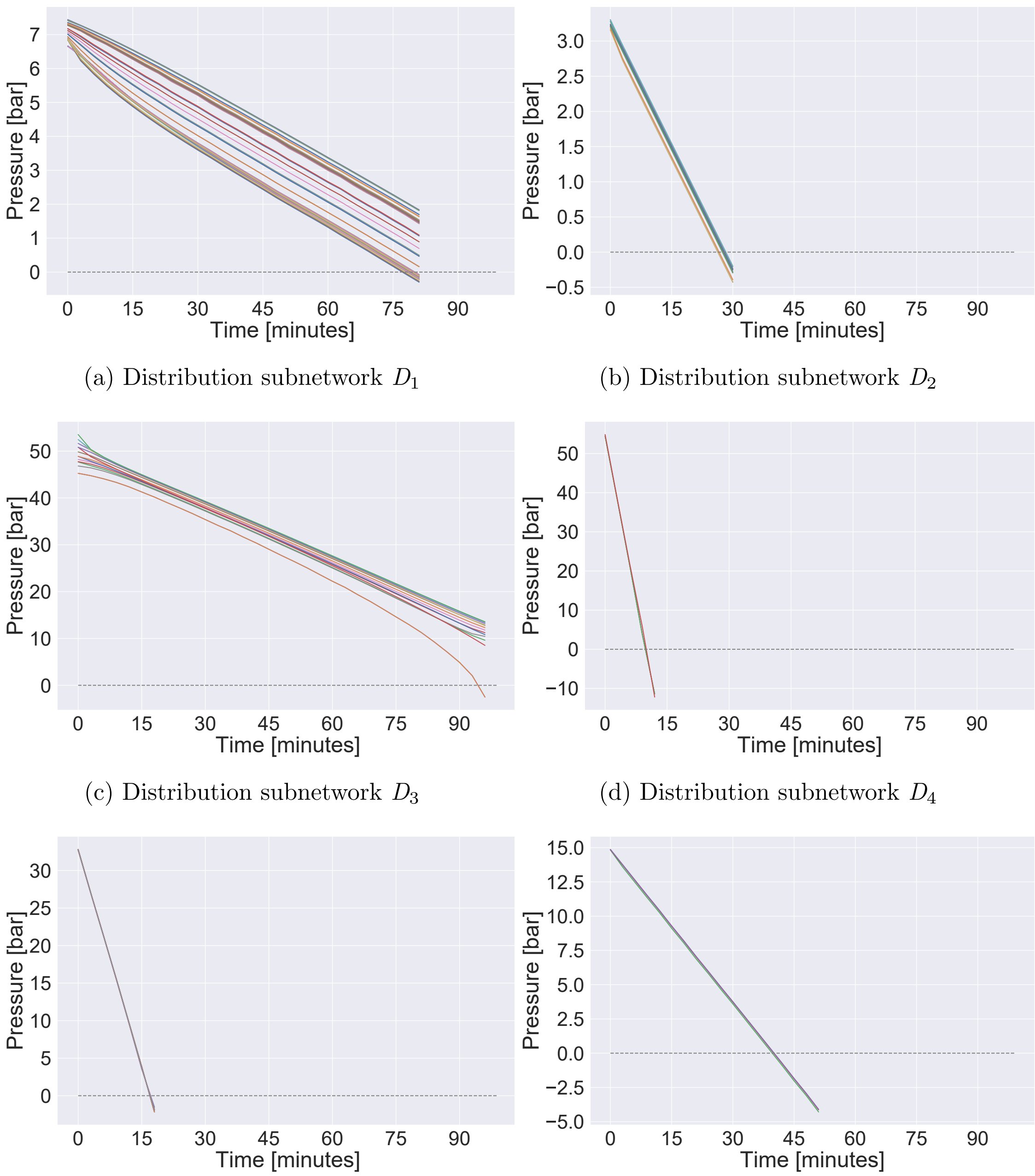


Figure D.2: Simulation results of draining a main transmission trunk, where the pressure profiles at the head node are displayed in blue and at the tail node in red. The selected transmission line represents a main trunk between two compressor stations and has a length of 219 [km] and diameter of 0.938 [m] and is taken from a real-world network.



(a) Distribution subnetwork D_1

(b) Distribution subnetwork D_2

(c) Distribution subnetwork D_3

(d) Distribution subnetwork D_4

(e) Distribution subnetwork D_5

(f) Distribution subnetwork D_6

Figure D.3: Simulation results of draining the distribution subnetworks $D_1, ..., D_6$. The colors illustrate the pressure profiles of the different subnetwork nodes.

D.2 Results on Transient Serial Pipe Merging

	Pipe a			Pipe b			Merged pipe c		
Pipes	α_a	β_a	γ_a	α_b	β_b	γ_b	α_c	β_c	γ_c
S_1	$2.86e{-}04$	$-6.50e{-}03$	$7.09e{-}04$	$2.95e{-}04$	$-6.14e{-}03$	$6.87e{-}04$	$1.46e{-}04$	$-1.26e{-}02$	$1.40e{-}03$
S_2	$6.50e{-}04$	$1.04e{-}02$	$2.16e{-}03$	$1.26e{-}02$	$1.70e{-}05$	$1.12e{-}04$	$6.18e{-}04$	$1.04e{-}02$	$2.27e{-}03$
S_3	$1.74e{-}04$	$1.11e{-}02$	$2.81e{-}04$	$1.62e{-}04$	$7.96e{-}03$	$3.00e{-}04$	$8.39e{-}05$	$1.91e{-}02$	$5.82e{-}04$
S_4	$2.00e{-}04$	$9.33e{-}04$	$5.30e{-}04$	$1.97e{-}04$	$1.96e{-}02$	$5.37e{-}04$	$9.92e{-}05$	$2.05e{-}02$	$1.06e{-}03$
S_5	$1.70e{-}04$	$4.58e{-}04$	$3.67e{-}04$	$1.70e{-}04$	$-5.68e{-}04$	$3.65e{-}04$	$8.50e{-}05$	$-1.10e{-}04$	$7.31e{-}04$
S_6	$1.74e{-}04$	$5.51e{-}03$	$5.73e{-}04$	$3.05e{-}02$	$2.04e{-}04$	$3.27e{-}06$	$1.73e{-}04$	$5.71e{-}03$	$5.76e{-}04$

Table D.2: Pipe parameters α, β and γ of the serial pipes a and b and the merged pipe c used in the evaluation of the serial merging approach for S_1, ..., S_6. Here, parameter α is scaled by a value of $1e{-}05$ and γ by a value of $1e{-}10$, since pressure values are denoted in [bar] and mass flow values in [kg/s] in the simulation model.

D.3 Proofs for Merging Active Elements

Here, we present the proofs of Propositions 6.8.1 – 6.8.6 from Section 6.8.

Recall from Chapter 5 that a valve $a = (\ell, r) \in \mathcal{A}_{va}$ is modeled by

$$\underline{q}_a \, z_{a,t} \le q_{a,t} \le \overline{q}_a \, z_{a,t} \qquad \forall t \in \mathcal{T}, \tag{D.1}$$

$$(\underline{p}_\ell - \overline{p}_r)(1 - z_{a,t}) \le p_{\ell,t} - p_{r,t} \le (\overline{p}_\ell - \underline{p}_r)(1 - z_{a,t}) \qquad \forall t \in \mathcal{T}, \tag{D.2}$$

$$z_{a,t} \in \{0, 1\} \qquad \forall t \in \mathcal{T}, \tag{D.3}$$

see also Equations (5.22) – (5.25), where the binary variable $z_{a,t}$ indicates whether the valve is open ($z_{a,t} = 1$) or closed ($z_{a,t} = 1$).

A control valve $a = (\ell, r) \in \mathcal{A}_{cv}$ is modeled by

$$z_{a,t} = z_{a,t}^{ac} + z_{a,t}^{bp} \qquad \forall t \in \mathcal{T}, \tag{D.4}$$

$$p_{r,t} - p_{\ell,t} \le (\overline{p}_r - \underline{p}_\ell)\,(1 - z_{a,t}^{ac}) \qquad \forall t \in \mathcal{T}, \tag{D.5}$$

$$(\underline{p}_\ell - \overline{p}_r)\,(1 - z_{a,t}^{bp}) \le p_{\ell,t} - p_{r,t} \le (\overline{p}_\ell - \underline{p}_r)\,(1 - z_{a,t}^{bp}) \qquad \forall t \in \mathcal{T}, \tag{D.6}$$

$$\underline{q}_a(1 - z_{a,t}^{ac}) \le q_{a,t} \qquad \forall t \in \mathcal{T}, \tag{D.7}$$

$$\underline{q}_a z_{a,t} \le q_{a,t} \le \overline{q}_a z_{a,t} \qquad \forall t \in \mathcal{T}, \tag{D.8}$$

$$z_{a,t}, z_{a,t}^{ac}, z_{a,t}^{bp} \in \{0, 1\} \qquad \forall t \in \mathcal{T}, \tag{D.9}$$

see also Equations (5.27) – (5.34), where the binary variable $z_{a,t}$ indicates whether the control valve is open ($z_{a,t} = 1$) or closed ($z_{a,t} = 1$), while the binary variables $z_{a,t}^{ac}, z_{a,t}^{bp}$ are used to model the open modes *active* and *in bypass*.

Note that we only apply the serial merge of two active elements $a = (\ell, m)$ and $b = (m, r)$, if the demand at the intermediate node m is zero. Then, as a consequence of the flow conservation at node m holds $q_{a,t} = q_{b,t}$, which allows us to define for the merged pipe $c = (\ell, r)$:

$$q_{c,t} := q_{a,t} = q_{b,t}, \qquad \underline{q}_c := \max\{\underline{q}_a, \underline{q}_b\}, \qquad \overline{q}_c := \min\{\overline{q}_a, \overline{q}_b\}. \tag{D.10}$$

Similarly, for the parallel merge of two active elements $a = (\ell, r)$ and $b = (\ell, r)$, we set for the merged pipe $c = (\ell, r)$

$$q_{c,t} := q_{a,t} + q_{b,t}, \qquad \underline{q}_c := \underline{q}_a + \underline{q}_b, \qquad \overline{q}_c := \overline{q}_a + \overline{q}_b. \tag{D.11}$$

The propositions state that parallel and serial elements can be replaced by new structurally equivalent elements. In our context, this means that for given operation modes z_a and z_b, there exists throughout an operation mode z_c of the new element that leads to the same physical behavior, and vice versa. Here, the operation modes z_a and z_b uniquely determine z_c, whereas the reverse direction does not necessarily hold, i.e., a particular operation mode of the merged element c might induce different possible operation modes of the elements a and b. Since the proofs are rather technical, we restrict them to the direction that given modes z_a and z_b imply z_c.

D.3.1 Proofs for merging valves

Proposition D.3.1 (Merging parallel valves)**.** *Given two parallel valves a and b between nodes ℓ and r. Then a and b can be equivalently replaced by a new valve $c = (\ell, r)$.*

Proof. We split the different possible operation modes $z_{a,t}, z_{b,t}$ into the following cases:

i) $(z_{a,t} = 1 \wedge z_{b,t} = 1) \vee (z_{a,t} = 1 \wedge z_{b,t} = 0) \vee (z_{a,t} = 0 \wedge z_{b,t} = 1) \iff z_{c,t} = 1,$

ii) $z_{a,t} = 0 \wedge z_{b,t} = 0 \iff z_{c,t} = 0,$

and show that the merged valve c satisfies (D.1) – (D.3).

(i) Let us w.l.o.g restrict to the case that both valves are open, i.e., $z_{a,t} = 1 \wedge z_{b,t} = 1$. The other cases $z_{a,t} = 1 \wedge z_{b,t} = 0$ and $z_{a,t} = 0 \wedge z_{b,t} = 1$ follow analogously. From $z_{a,t} = z_{b,t} = 1$ it follows that $\underline{q}_a \le q_{a,t} \le \overline{q}_a$ and $\underline{q}_b \le q_{b,t} \le \overline{q}_b$. Then (D.1) holds by setting $z_{c,t} := 1$ together with $q_c, \underline{q}_c, \overline{q}_c$ as given in (D.11). Furthermore $z_{a,t} = z_{b,t} = 1$ implies $p_{\ell,t} = p_{r,t}$ and thus (D.2) holds with $z_{c,t} = 1$.

(ii) From $z_{a,t} = 0 \wedge z_{b,t} = 0$ it follows that $q_{a,t} = q_{b,t} = 0$. Setting $z_{c,t} := 0$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.11), then (D.1) holds. Moreover $z_{a,t} = z_{b,t} = 0$ and $z_{c,t} = 0$ imply $\underline{p}_\ell - \overline{p}_r \le p_{\ell,t} - p_{r,t} \le \overline{p}_\ell - \underline{p}_r$, hence (D.2) is fulfilled. □

Proposition D.3.2 (Merging serial valves)**.** *Let $a = (\ell, m)$ and $b = (m, r)$ be two serial valves with $deg(m) = 2$ and $b_{m,t} = 0$ for all $t \in \mathcal{T}$ and let $\underline{p}_\ell = \underline{p}_m = \underline{p}_r$ and $\overline{p}_\ell = \overline{p}_m = \overline{p}_r$. Then a and b can be equivalently replaced by a new valve $c = (\ell, r)$.*

Proof. We split the different possible operation modes $z_{a,t}, z_{b,t}$ into the following cases:

i) $(z_{a,t} = 0 \wedge z_{b,t} = 0) \vee (z_{a,t} = 1 \wedge z_{b,t} = 0) \vee (z_{a,t} = 0 \wedge z_{b,t} = 0) \iff z_{c,t} = 0,$

ii) $z_{a,t} = 1 \wedge z_{b,t} = 1 \iff z_{c,t} = 1,$

and show that the merged valve c satisfies (D.1) – (D.3).

(i) Let us w.l.o.g restrict to the case that both valves are closed, i.e., $z_{a,t} = 0 \wedge z_{b,t} = 0$. The other cases $z_{a,t} = 0 \wedge z_{b,t} = 1$ and $z_{a,t} = 1 \wedge z_{b,t} = 0$ follow analogously. From $z_{a,t} = z_{b,t} = 0$ it follows that $q_{a,t} = q_{b,t} = 0$. Then, (D.1) holds by setting $z_{c,t} := 0$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10). Moreover, $z_{a,t} = z_{b,t} = 0$ implies $\underline{p}_\ell - \overline{p}_m \le p_{\ell,t} - p_{m,t} \le \overline{p}_\ell - \underline{p}_m$ and $\underline{p}_m - \overline{p}_r \le p_{m,t} - p_{r,t} \le \overline{p}_m - \underline{p}_r$. Provided that nodes ℓ, m, r have the same pressure bounds, then $\underline{p}_\ell - \overline{p}_r \le p_{\ell,t} - p_{r,t} \le \overline{p}_\ell - \underline{p}_r$ holds and hence (D.2) is satisfied with $z_{c,t} = 0$.

(ii) From $z_{a,t} = 1 \wedge z_{b,t} = 1$ it follows that $q_{a,t} = q_{b,t}$. By setting $z_{c,t} := 1$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10), it follows that (D.1) is fulfilled. Furthermore $z_{a,t} = 1$ and $z_{b,t} = 1$ lead to $p_{\ell,t} = p_{m,t} = p_{r,t}$. Likewise $z_{c,t} = 1$ implies $p_{\ell,t} = p_{r,t}$ and hence (D.2) holds with $z_{c,t} = 1$. □

D.3.2 Proofs for merging control valves

Compared to valves and pipes, the orientation of control valves is crucial, since control valves reduce pressure in the flow direction. Hence, we apply parallel and serial merges only to control valves that have the same orientation.

Proposition D.3.3 (Merging parallel control valves)**.** *Given two control valves a and b between nodes ℓ and r. Then a and b can be equivalently replaced by a new control valve $c = (\ell, r)$.*

Proof. We split the different possible operation modes $z_{a,t}, z_{b,t}$ into the following cases:

i) $z_{a,t} = 0 \ \wedge \ z_{b,t} = 0 \quad \iff \quad z_{c,t} = 0,$

ii) $z^{bp}_{a,t} = 1 \ \wedge \ z^{bp}_{b,t} = 1 \quad \iff \quad z^{bp}_{c,t} = 1,$

iii) $z^{ac}_{a,t} = 1 \ \wedge \ z^{ac}_{b,t} = 1 \quad \iff \quad z^{ac}_{c,t} = 1,$

iv) $(z^{ac}_{a,t} = 1 \ \wedge \ z_{b,t} = 0) \vee \ (z_{a,t} = 0 \ \wedge \ z^{ac}_{b,t} = 1) \quad \iff \quad z^{ac}_{c,t} = 1,$

v) $(z^{bp}_{a,t} = 1 \ \wedge \ z_{b,t} = 0) \vee \ (z_{a,t} = 0 \ \wedge \ z^{bp}_{b,t} = 1) \quad \iff \quad z^{bp}_{c,t} = 1,$

vi) $(z^{ac}_{a,t} = 1 \ \wedge \ z^{bp}_{b,t} = 1) \vee \ (z^{bp}_{a,t} = 1 \ \wedge \ z^{ac}_{b,t} = 1) \quad \iff \quad z^{bp}_{c,t} = 1,$

and show that the merged valve c satisfies (D.4) – (D.9).

(i) $z_{a,t} = 0 \ \wedge \ z_{b,t} = 0$:

From $z_{a,t} = z_{b,t} = 0$ it follows that $q_{a,t} = q_{b,t} = 0$. Then, by setting $z_{c,t} := 0$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.11), Inequalities (D.7) and (D.8) are satisfied. Moreover, (D.4) holds with $z^{bp}_{c,t} = z^{ac}_{c,t} = 0$. Finally, from $z^{ac}_{a,t} = z^{bp}_{a,t} = 0$ and $z^{ac}_{b,t} = z^{bp}_{b,t} = 0$ it follows that $\underline{p}_\ell - \overline{p}_r \le p_{\ell,t} - p_{r,t} \le \overline{p}_\ell - \underline{p}_r$ and thus (D.5) and (D.6) are satisfied.

(ii) $z^{bp}_{a,t} = 1 \ \wedge \ z^{bp}_{b,t} = 1$:

The modes $z^{bp}_{a,t} = z^{bp}_{b,t} = 1$ imply $p_{\ell,t} = p_{r,t}$, and hence by setting $z^{bp}_{c,t} := 1$, (D.5) and (D.6) hold. With $z^{bp}_{c,t} := 1$ and $z^{ac}_{c,t} := 0$, (D.4) is satisfied with $z_{c,t} = z^{bp}_{c,t} + z^{ac}_{c,t} = 1$. Moreover $z^{bp}_{a,t} = z^{bp}_{b,t} = 1$ imply $\underline{q}_a \le q_{a,t} \le \overline{q}_a$ and $\underline{q}_b \le q_{b,t} \le \overline{q}_b$, then, with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.11), Inequalities (D.7) and (D.8) are fulfilled with $z^{bp}_{c,t} = 1$.

(iii) $z^{ac}_{a,t} = 1 \ \wedge \ z^{ac}_{b,t} = 1$:

From $z^{ac}_{a,t} = z^{ac}_{b,t} = 1$ ($z_{a,t} = z_{b,t} = 1$) and $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.11), together with $z_{c,t} := 1$ it follows that (D.7) and (D.8) hold. Moreover, $z^{ac}_{a,t} = z^{ac}_{b,t} = 1$ imply $0 \le p_{\ell,t} - p_{r,t} \le \overline{p}_\ell - \underline{p}_r$ and hence (D.5) and (D.6) are fulfilled with $z^{ac}_{c,t} := 1$ and $z^{bp}_{c,t} := 0$. Finally, (D.4) is satisfied with $z_{c,t} = z^{ac}_{c,t} + z^{bp}_{c,t} = 1$.

(iv) Let us w.l.o.g restrict to $z^{ac}_{a,t} = 1 \ \wedge \ z_{b,t} = 0$. The other case $z_{a,t} = 0 \ \wedge \ z^{ac}_{b,t} = 1$ follows analogously. Then, it follows that $0 \le q_{a,t} \le \overline{q}_a$ and $q_{b,t} = 0$. For $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.11), Inequalities (D.7) and (D.8) hold with $z^{ac}_{c,t} := 1$. Moreover, $z^{ac}_{a,t} = 1$ leads to $0 \le p_{\ell,t} - p_{r,t} \le \overline{p}_\ell - \underline{p}_r$ and thus (D.5) and (D.6) are fulfilled with $z^{ac}_{c,t} := 1$ and $z^{bp}_{c,t} := 0$. Finally, (D.4) is satisfied with $z_{c,t} = z^{ac}_{c,t} + z^{bp}_{c,t} = 1$.

(v) Let us w.l.o.g restrict to $z^{bp}_{a,t} = 1 \ \wedge \ z_{b,t} = 0$. The other case $z_{a,t} = 0 \ \wedge \ z^{bp}_{b,t} = 1$ follows analogously. Then, it follows that $0 \le q_{a,t} \le \overline{q}_a$ and $q_{b,t} = 0$. With $q_{c,t}, \underline{q}_c, \overline{q}_c$ from (D.11) and by setting $z^{bp}_{c,t} := 1$, then (D.7) and (D.8) hold. Moreover, $z^{bp}_{a,t} = 1$ leads to $p_{\ell,t} = p_{r,t}$ and hence (D.5) and (D.6) are fulfilled with $z^{bp}_{c,t} = 1$ and $z^{ac}_{c,t} = 0$. Finally, (D.4) is satisfied with $z_{c,t} = z^{ac}_{c,t} + z^{bp}_{c,t} = 1$.

(vi) Let us w.l.o.g restrict to $z_{a,t}^{ac} = 1 \wedge z_{b,t}^{bp} = 1$. The other case $z_{a,t}^{bp} = 1 \wedge z_{b,t}^{ac} = 1$ follows analogously. Then, it follows that $0 \leq q_{a,t} \leq \overline{q}_a$ and $\underline{q}_b \leq q_{b,t} \leq \overline{q}_b$. By setting $z_{c,t}^{bp} := 1$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.11), then (D.7) and (D.8) hold. Moreover, $z_{a,t}^{bp} = 1$ yields $p_{\ell,t} = p_{r,t}$ and hence (D.5) and (D.6) are fulfilled with $z_{c,t}^{bp} = 1$ and $z_{c,t}^{ac} = 0$. Finally, (D.4) is satisfied with $z_{c,t} = z_{c,t}^{ac} + z_{c,t}^{bp} = 1$. □

Proposition D.3.4 (Merging serial control valves)**.** *Let $a = (\ell, m)$ and $b = (m, r)$ be two serial control valves with $deg(m) = 2$ and $b_{m,t} = 0$ for all $t \in \mathcal{T}$ and let $\underline{p}_\ell = \underline{p}_m = \underline{p}_r$ and $\overline{p}_\ell = \overline{p}_m = \overline{p}_r$. Then a and b can be equivalently replaced by a new control valve $c = (\ell, r)$.*

Proof. We split the different possible operation modes $z_{a,t}, z_{b,t}$ into the following cases:

i) $z_{a,t} = 0 \wedge z_{b,t} = 0 \iff z_{c,t} = 0,$

ii) $z_{a,t}^{bp} = 1 \wedge z_{b,t}^{bp} = 1 \iff z_{c,t}^{bp} = 1,$

iii) $z_{a,t}^{ac} = 1 \wedge z_{b,t}^{ac} = 1 \iff z_{c,t}^{ac} = 1,$

iv) $(z_{a,t}^{ac} = 1 \wedge z_{b,t} = 0) \vee (z_{a,t} = 0 \wedge z_{b,t}^{ac} = 1) \iff z_{c,t} = 0,$

v) $(z_{a,t}^{bp} = 1 \wedge z_{b,t} = 0) \vee (z_{a,t} = 0 \wedge z_{b,t}^{bp} = 1) \iff z_{c,t} = 0,$

vi) $(z_{a,t}^{ac} = 1 \wedge z_{b,t}^{bp} = 1) \vee (z_{a,t}^{bp} = 1 \wedge z_{b,t}^{ac} = 1) \iff z_{c,t}^{ac} = 1,$

and show that the merged valve c satisfies (D.4) – (D.9).

(i) $z_{a,t} = 0 \wedge z_{b,t} = 0$:

From $z_{a,t} = z_{b,t} = 0$ it follows that $q_{a,t} = q_{b,t} = 0$. By setting $z_{c,t} := 0$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as given in (D.10), Inequalities (D.7) and (D.8) are fulfilled. The closed modes $z_{a,t} = z_{b,t} = 0$ imply $(\underline{p}_\ell - \overline{p}_m) \leq p_{\ell,t} - p_{m,t} \leq (\overline{p}_\ell - \underline{p}_m)$ and $(\underline{p}_m - \overline{p}_r) \leq p_{m,t} - p_{r,t} \leq (\overline{p}_m - \underline{p}_r)$. Provided that all nodes have the same pressure bounds, then $(\underline{p}_\ell - \overline{p}_r) \leq p_{\ell,t} - p_{r,t} \leq (\overline{p}_\ell - \underline{p}_r)$ and also (D.5) and (D.6) hold with $z_{c,t}^{ac} = 0$ and $z_{c,t}^{bp} = 0$.

(ii) $z_{a,t}^{bp} = 1 \wedge z_{b,t}^{bp} = 1$:

By setting $z_{c,t}^{bp} := 1$ (i.e. $z_{c,t} = 1$), (D.7) and (D.8) hold with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10). Moreover $z_{a,t}^{bp} = z_{b,t}^{bp} = 1$ implies $p_{\ell,t} = p_{m,t} = p_{r,t}$. In the same way, it follows from $z_{c,t}^{bp} = 1$ that $p_{\ell,t} = p_{r,t}$ and thus (D.5) and (D.6) are satisfied.

(iii) $z_{a,t}^{ac} = 1 \wedge z_{b,t}^{ac} = 1$:

Setting $z_{c,t}^{ac} := 1$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10), then (D.7) and (D.8) hold. Furthermore, $z_{a,t}^{ac} = z_{b,t}^{ac} = 1$ implies $(\underline{p}_\ell - \overline{p}_m) \leq p_{\ell,t} - p_{m,t} \leq (\overline{p}_\ell - \underline{p}_m)$ and $(\underline{p}_m - \overline{p}_r) \leq p_{m,t} - p_{r,t} \leq (\overline{p}_m - \underline{p}_r)$. Provided that all nodes ℓ, m, r have the same pressure bounds, it follows that $(\underline{p}_\ell - \overline{p}_r) \leq p_{\ell,t} - p_{r,t} \leq (\overline{p}_\ell - \underline{p}_r)$ and hence (D.5) and (D.6) hold with $z_{c,t}^{ac} = 1$ (and $z_{c,t}^{bp} = 0$).

(iv) Let us w.l.o.g restrict to $z_{a,t}^{ac} = 1 \wedge z_{b,t} = 0$. The other case $z_{a,t} = 0 \wedge z_{b,t}^{ac} = 1$ follows analogously. The modes $z_{a,t}^{ac} = 1$ and $z_{b,t} = 0$ imply $0 \leq q_{a,t}$ and $q_{b,t} = 0$. In fact, considering the flow conservation $q_{a,t} - q_{b,t} = b_{m,t}$ at node m with $b_{m,t}, q_{b,t} = 0$

implies $q_{a,t} = 0$. Setting $z_{c,t} := 0$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10), then (D.7) and (D.8) hold. Moreover, $z^{ac}_{a,t} = 1$ and $z_{b,t} = 0$ imply $p_{\ell,t} \leq p_{m,t}$ and $\underline{p}_m - \overline{p}_r \leq p_{m,t} - p_{r,t} \leq \overline{p}_m - \underline{p}_r$. Then, under the requirement that the nodes ℓ, m, r have the same pressure bounds, it follows with $z_{c,t} = 0$ that (D.5) and (D.6).

(v) Let us w.l.o.g restrict to $z^{bp}_{a,t} = 1 \wedge z_{b,t} = 0$. The other case $z_{a,t} = 0 \wedge z^{bp}_{b,t} = 1$ follows analogously. The modes $z^{bp}_{a,t} = 1$ and $z_{b,t} = 0$ imply $\underline{q}_a \leq q_{a,t} \leq \overline{q}_a$ and $q_{b,t} = 0$. In fact, considering the flow conservation $q_{a,t} - q_{b,t} = b_{m,t}$ at node m with $b_{m,t} = 0$ implies $q_{a,t} = 0$. Setting $z_{c,t} := 0$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10), then (D.7) and (D.8) hold. Moreover, $z^{bp}_{a,t} = 1$ and $z_{b,t} = 0$ imply $p_{\ell,t} = p_{m,t}$ and $\underline{p}_m - \overline{p}_r \leq p_{m,t} - p_{r,t} \leq \overline{p}_m - \underline{p}_r$. Then, under the requirement that the nodes ℓ, m, r have the same pressure bounds, it follows with $z_{c,t} = 0$ that (D.5) and (D.6).

(vi) Let us w.l.o.g restrict to $z^{ac}_{a,t} = 1 \wedge z^{bp}_{b,t} = 1$. The other case $z^{bp}_{a,t} = 1 \wedge z^{ac}_{b,t} = 1$ follows analogously. The modes $z^{ac}_{a,t} = z^{bp}_{b,t} = 1$ imply $0 \leq q_{a,t} \leq \overline{q}_a$ and $\underline{q}_b \leq q_{b,t} \leq \overline{q}_b$. Setting $z^{ac}_{c,t} := 1$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10), then (D.7) and (D.8) hold. Moreover, $z^{ac}_{a,t} = z^{bp}_{b,t} = 1$ implies $p_{m,t} = p_{r,t}$ and $\underline{p}_\ell - \overline{p}_m \leq p_{\ell,t} - p_{m,t} \leq \overline{p}_\ell - \underline{p}_m$. Provided that nodes ℓ, m, r have the same pressure bounds, it follows that $\underline{p}_\ell - \overline{p}_r \leq p_{\ell,t} - p_{r,t} \leq \overline{p}_\ell - \underline{p}_r$ and hence (D.5) and (D.6) hold with $z^{ac}_{c,t} = 1$ (and $z^{bp}_{c,t} = 0$). □

D.3.3 Proofs for merging valves and control valves

Proposition D.3.5 (Merging parallel valves and control valves)**.** *Let a be a valve and let b be a control valve between nodes ℓ and r. Then a and b can be equivalently replaced by a new control valve $c = (\ell, r)$.*

Proof. In the following, let w.l.o.g be a the valve and b the control valve. We split the different possible operation modes $z_{a,t}, z_{b,t}$ into the following cases:

i) $z_{a,t} = 0 \wedge z_{b,t} = 0 \iff z_{c,t} = 0,$

ii) $z_{a,t} = 0 \wedge z^{bp}_{b,t} = 1 \iff z^{bp}_{c,t} = 1,$

iii) $z_{a,t} = 0 \wedge z^{ac}_{b,t} = 1 \iff z^{ac}_{c,t} = 1,$

iv) $z_{a,t} = 1 \wedge z_{b,t} = 0 \iff z^{bp}_{c,t} = 1,$

v) $z_{a,t} = 1 \wedge z^{bp}_{b,t} = 1 \iff z^{bp}_{c,t} = 1,$

vi) $z_{a,t} = 1 \wedge z^{ac}_{b,t} = 1 \iff z^{bp}_{c,t} = 1,$

and show that the merged control valve c fulfills (D.4) – (D.9).

(i) $z_{a,t} = 0 \wedge z_{b,t} = 0$:

The modes imply $q_{a,t} = q_{b,t} = 0$. Then, with $z_{c,t} := 0$ and $q_{c,t}, \underline{q}_c, \overline{q}_c$ as given in (D.11) it follows that (D.7) and (D.8). Furthermore, $z_{a,t} = z_{b,t} = 0$ implies $\underline{p}_\ell - \overline{p}_r \leq p_{\ell,t} - p_{r,t} \leq \overline{p}_\ell - \underline{p}_r$ and thus (D.5) and (D.6) are satisfied with $z_{c,t} = 0$.

(ii) $z_{a,t} = 0 \wedge z^{bp}_{b,t} = 1$:

The modes imply $q_{a,t} = 0$ and $\underline{q}_b \leq q_{b,t} \leq \overline{q}_b$. Then, with $z^{bp}_{c,t} := 1$ and $q_{c,t}, \underline{q}_c, \overline{q}_c$ as given in (D.11) it follows that (D.7) and (D.8). Furthermore, $z_{a,t} = 0$ implies $\underline{p}_\ell - \overline{p}_r \leq p_{\ell,t} - p_{r,t} \leq \overline{p}_\ell - \underline{p}_r$ and $z^{bp}_{b,t} = 1$ implies $p_{\ell,t} = p_{r,t}$. Hence, (D.5) and (D.6) are satisfied with $z^{bp}_{c,t} = 1$.

(iii) $z_{a,t} = 0 \ \wedge \ z^{ac}_{b,t} = 1$:

The modes imply $q_{a,t} = 0$ and $0 \leq q_{b,t} \leq \overline{q}_b$. Then, with $z^{ac}_{c,t} = 1$ and $q_{c,t}, \underline{q}_c, \overline{q}_c$ as given in (D.11) it follows that (D.7) and (D.8). Furthermore, $z_{a,t} = 0$ implies $\underline{p}_\ell - \overline{p}_r \leq p_{\ell,t} - p_{r,t} \leq \overline{p}_\ell - \underline{p}_r$ and $z^{ac}_{b,t} = 1$ implies $p_{\ell,t} \geq p_{r,t}$. Hence, (D.5) and (D.6) are satisfied with $z^{ac}_{c,t} = 1$.

(iv) $z_{a,t} = 1 \ \wedge \ z_{b,t} = 0$:

The modes imply $\underline{q}_a \leq q_{a,t} \leq \overline{q}_a$ and $0 = q_{b,t}$. Then, with $z^{bp}_{c,t} := 1$ and $q_{c,t}, \underline{q}_c, \overline{q}_c$ as given in (D.11) it follows that (D.7) and (D.8). Furthermore, $z_{a,t} = 1$ implies $p_{\ell,t} = p_{r,t}$ and $z_{b,t} = 0$ implies $\underline{p}_\ell - \overline{p}_r \leq p_{\ell,t} - p_{r,t} \leq \overline{p}_\ell - \underline{p}_r$. Hence, (D.5) and (D.6) are satisfied with $z^{bp}_{c,t} = 1$.

(v) $z_{a,t} = 1 \ \wedge \ z^{bp}_{b,t} = 1$:

The modes imply $\underline{q}_a \leq q_{a,t} \leq \overline{q}_a$ and $\underline{q}_b \leq q_{b,t} \leq \overline{q}_b$. Then, with $z^{bp}_{c,t} := 1$ and $q_{c,t}, \underline{q}_c, \overline{q}_c$ as given in (D.11) it follows that (D.7) and (D.8). Furthermore, $z_{a,t} = 1$ and $z^{bp}_{b,t} = 1$ imply $p_{\ell,t} = p_{r,t}$. Hence, (D.5) and (D.6) are satisfied with $z^{bp}_{c,t} = 1$.

(vi) $z_{a,t} = 1 \ \wedge \ z^{ac}_{b,t} = 1$:

The modes imply $\underline{q}_a \leq q_{a,t} \leq \overline{q}_a$ and $0 \leq q_{b,t} \leq \overline{q}_b$. Then, with $z^{bp}_{c,t} := 1$ and $q_{c,t}, \underline{q}_c, \overline{q}_c$ as given in (D.11) it follows that (D.7) and (D.8). Furthermore, $z_{a,t} = 1$ implies $p_{\ell,t} = p_{r,t}$ and $z^{ac}_{b,t} = 1$ implies $0 \leq p_{\ell,t} - p_{r,t} \leq \overline{p}_\ell - \underline{p}_r$. Hence, (D.5) and (D.6) are satisfied with $z^{bp}_{c,t} = 1$. □

Proposition D.3.6 (Merging serial valves and control valves)**.** *Let $a = (\ell, m)$ be a valve and $b = (m, r)$ a control valve with $deg(m) = 2$ and $b_{m,t} = 0$ for all $t \in \mathcal{T}$ and let $\underline{p}_\ell = \underline{p}_m = \underline{p}_r$ and $\overline{p}_\ell = \overline{p}_m = \overline{p}_r$. Then a and b can be equivalently replaced by a new control valve $c = (\ell, r)$.*

Proof. In the following, let w.l.o.g be a the valve and b the control valve. Nevertheless, the parallel merge is independent of the order of the valve and control valve. We split the different possible operation modes $z_{a,t}, z_{b,t}$ into the following cases:

$$\begin{aligned}
&\text{i)} && z_{a,t} = 0 \ \wedge \ z_{b,t} = 0 && \Longleftrightarrow && z_{c,t} = 0,\\
&\text{ii)} && z_{a,t} = 0 \ \wedge \ z^{bp}_{b,t} = 1 && \Longleftrightarrow && z_{c,t} = 0,\\
&\text{iii)} && z_{a,t} = 0 \ \wedge \ z^{ac}_{b,t} = 1 && \Longleftrightarrow && z_{c,t} = 0,\\
&\text{iv)} && z_{a,t} = 1 \ \wedge \ z_{b,t} = 0 && \Longleftrightarrow && z_{c,t} = 0,\\
&\text{v)} && z_{a,t} = 1 \ \wedge \ z^{bp}_{b,t} = 1 && \Longleftrightarrow && z^{bp}_{c,t} = 1,\\
&\text{vi)} && z_{a,t} = 1 \ \wedge \ z^{ac}_{b,t} = 1 && \Longleftrightarrow && z^{ac}_{c,t} = 1,
\end{aligned}$$

and show that the merged control valve c fulfills (D.4) – (D.9).

(i) $z_{a,t} = 0 \ \wedge \ z_{b,t} = 0$:

From $z_{a,t} = z_{b,t} = 0$ it follows that $q_{a,t} = q_{b,t} = 0$. By setting $z_{c,t} = 0$ and $q_{c,t}, \underline{q}_c, \overline{q}_c$ from (D.10), Inequalities (D.7) and (D.8) are fulfilled. Furthermore, $z_{a,t} = 0$ implies $\underline{p}_\ell - \overline{p}_m \le p_{\ell,t} - p_{m,t} \le \overline{p}_\ell - \underline{p}_m$ and $z_{b,t} = 0$ implies $\underline{p}_m - \overline{p}_r \le p_{m,t} - p_{r,t} \le \overline{p}_m - \underline{p}_r$. Provided that all nodes have the same pressure bounds, then $\underline{p}_\ell - \overline{p}_r \le p_{\ell,t} - p_{r,t} \le \overline{p}_\ell - \underline{p}_r$ and also (D.5) and (D.6) are satisfied with $z_{c,t} = 0$.

(ii) $z_{a,t} = 0 \ \wedge \ z^{bp}_{b,t} = 1$:

The modes $z_{a,t} = 0$ and $z^{bp}_{b,t} = 1$ imply $q_{a,t} = 0$ and $\underline{q}_b \le q_{b,t} \le \overline{q}_b$. In fact, from the flow conservation $q_{a,t} - q_{b,t} = b_{m,t}$ at node m with $b_{m,t} = 0$ it follows that $q_{a,t} = 0$. Setting $z_{c,t} := 0$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10), then (D.7) and (D.8) hold. Moreover, $z_{a,t} = 0$ and $z^{bp}_{b,t} = 1$ imply $\underline{p}_\ell - \overline{p}_m \le p_{\ell,t} - p_{m,t} \le \overline{p}_\ell - \underline{p}_m$ $p_{m,t} = p_{r,t}$. Then, under the requirement that the nodes ℓ, m, r have the same pressure bounds, it follows with $z_{c,t} = 0$ that (D.5) and (D.6).

(iii) $z_{a,t} = 0 \ \wedge \ z^{ac}_{b,t} = 1$:

The modes $z_{a,t} = 0$ and $z^{ac}_{b,t} = 1$ imply $q_{a,t} = 0$ and $q_{b,t} \le 0$. In fact, considering the flow conservation $q_{a,t} - q_{b,t} = b_{m,t}$ at node m with $b_{m,t} = 0$ implies $q_{b,t} = 0$. Setting $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10), then (D.7) and (D.8) hold. Moreover, $z_{a,t} = 0$ and $z^{ac}_{b,t} = 1$ imply $(\underline{p}_\ell - \overline{p}_m) \le p_{\ell,t} - p_{m,t} \le \overline{p}_\ell - \underline{p}_m$ and $p_{m,t} \le p_{r,t}$. Then, under the requirement that the nodes ℓ, m, r have the same pressure bounds, it follows that (D.5) and (D.6).

(iv) $z_{a,t} = 1 \ \wedge \ z_{b,t} = 0$:

The modes $z_{a,t} = 1$ and $z_{b,t} = 0$ imply $\underline{q}_a \le q_{a,t} \le \overline{q}_a$ and $q_{b,t} = 0$. In fact, considering the flow conservation $q_{a,t} - q_{b,t} = b_{m,t}$ at node m, with $b_{m,t} = 0$ implies $q_{a,t} = 0$. Setting $z^{bp}_{c,t} := 1$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10), then (D.7) and (D.8) hold. Moreover, $z_{a,t} = 1$ and $z_{b,t} = 0$ imply $p_{\ell,t} = p_{m,t}$ and $\underline{p}_m - \overline{p}_r \le p_{m,t} - p_{r,t} \le \overline{p}_m - \underline{p}_r$. Then, under the requirement that the nodes ℓ, m, r have the same pressure bounds, it follows with $z^{bp}_{c,t} = 1$ that (D.5) and (D.6).

(v) $z_{a,t} = 1 \ \wedge \ z^{bp}_{b,t} = 1$:

Setting $z^{bp}_{c,t} := 1$, then (D.7) and (D.8) hold with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10). Moreover $z^{bp}_{a,t} = z^{bp}_{b,t} = 1$ implies $p_{\ell,t} = p_{m,t} = p_{r,t}$. Setting $z^{bp}_{c,t} := 1$, then by the same token it follows that $p_{\ell,t} = p_{r,t}$ and thus (D.5) and (D.6) hold.

(vi) $z_{a,t} = 1 \ \wedge \ z^{ac}_{b,t} = 1$:

Setting $z^{ac}_{c,t} := 1$ together with $q_{c,t}, \underline{q}_c, \overline{q}_c$ as defined in (D.10), then (D.7) and (D.8) hold. Furthermore, $z_{a,t} = 1$ and $z^{ac}_{b,t} = 1$ imply $\underline{p}_\ell = \overline{p}_m$ and $\underline{p}_m - \overline{p}_r \le p_{m,t} - p_{r,t} \le \overline{p}_m - \underline{p}_r$. Provided that all nodes ℓ, m, r have the same pressure bounds, it follows that $\underline{p}_\ell - \overline{p}_r \le p_{\ell,t} - p_{r,t} \le \overline{p}_\ell - \underline{p}_r$ and hence (D.5) and (D.6) hold with $z^{ac}_{c,t} = 1$ (and $z^{bp}_{c,t} = 0$). □

Appendix E

Computational Study of *Transient Control Problem*

E.1 Computational Results

Table E.1: HEUR specific statistics on the number of `Affected` instances that require unfixings of the initial station modes $\texttt{stationSol}_{t=1,\ldots,|\mathcal{T}|}$ in some SubMINLPt-solve, but do not utilize other station mode fixings. These instances hold $\texttt{nrCurrentIterationCalls}_t \leq 1$ for all time iterations $t \in \mathcal{T}$ in Algorithm 5. For these instances, the table shows the number of required unfixings $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t$ of the initial station modes $\texttt{stationSol}_{t=1,\ldots,|\mathcal{T}|}$.

		# instances		
		Net 1	*Net 2*	*Net 3*
	# `Affected` instances	500	490	1383
# instances, where $\forall t \in \mathcal{T}$ holds $\texttt{nrCurrentIterationCalls}_t \leq 1$	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 0$	500	214	48
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 1$	0	55	72
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 2$	0	29	54
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 3$	0	11	91
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 4$	0	6	92
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 5$	0	1	85
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 6$	0	1	92
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 7$	0	6	72
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 8$	0	3	100
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 9$	0	1	79
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 10$	0	2	54
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 11$	0	2	59
	# instances with $\sum_{t\in\mathcal{T}} \texttt{nrCurrentIterationCalls}_t = 12$	0	0	70

Table E.2: Average number of $\texttt{MIP}_{t,\dots,|\mathcal{T}|}$-model solutions.

	networks		
	Net 1	*Net 2*	*Net 3*
# (avg) solutions of $\texttt{MIP}_{t=1,\dots,|\mathcal{T}|}$	2	3	3
# (avg) solutions of $\texttt{MIP}_{t=2,\dots,|\mathcal{T}|}$	-	2	2
# (avg) solutions of $\texttt{MIP}_{t=3,\dots,|\mathcal{T}|}$	-	1	2
# (avg) solutions of $\texttt{MIP}_{t=4,\dots,|\mathcal{T}|}$	-	2	3
# (avg) solutions of $\texttt{MIP}_{t=5,\dots,|\mathcal{T}|}$	-	2	3
# (avg) solutions of $\texttt{MIP}_{t=6,\dots,|\mathcal{T}|}$	-	2	3
# (avg) solutions of $\texttt{MIP}_{t=7,\dots,|\mathcal{T}|}$	-	2	3
# (avg) solutions of $\texttt{MIP}_{t=8,\dots,|\mathcal{T}|}$	-	2	3
# (avg) solutions of $\texttt{MIP}_{t=9,\dots,|\mathcal{T}|}$	-	2	3
# (avg) solutions of $\texttt{MIP}_{t=10,\dots,|\mathcal{T}|}$	-	2	2
# (avg) solutions of $\texttt{MIP}_{t=11,\dots,|\mathcal{T}|}$	-	3	1
# (avg) solutions of $\texttt{MIP}_{t=12,\dots,|\mathcal{T}|}$	-	1	1

Table E.3: Detailed results of the *Net 1* test set, as summarized in Tables 8.3 – 8.5 and Figure 8.3. For activating the heuristic HEUR (*Enabled*) and SCIP default (*Disabled*), the table lists the solving time (col. *Time*), and the objective function value (col. *Obj*), which represents the number of station mode changes. Further, it shows HEUR specific information about the highest achieved time step (col. *Round*), the number of backward moves (col. *Back*), and whether partitioning of any $\texttt{MIP}_{t,\cdots,|\mathcal{T}|}$-model occured (col. *Decompose-MIP*), indicated by True.

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
1	12.7	0	12	0	–	3600.0	–
2	13.0	0	12	0	–	3600.0	–
3	13.0	0	12	0	–	3600.0	–
4	13.2	0	12	0	–	3600.0	–
5	13.5	0	12	0	–	3600.0	–
6	12.1	0	12	0	–	3600.0	–
7	12.5	0	12	0	–	3600.0	–
8	12.3	0	12	0	–	3600.0	–
9	12.8	0	12	0	–	3600.0	–
10	12.8	0	12	0	–	3600.0	–
11	12.6	0	12	0	–	3600.0	–
12	13.0	0	12	0	–	3600.0	–
13	12.4	0	12	0	–	3600.0	–
14	12.5	0	12	0	–	3600.0	–
15	12.4	0	12	0	–	3600.0	–
16	12.9	0	12	0	–	3600.0	–
17	12.7	0	12	0	–	3600.0	–
18	12.7	0	12	0	–	3600.0	–
19	12.5	0	12	0	–	3600.0	–
20	13.2	0	12	0	–	3600.0	–
21	13.1	0	12	0	–	3600.0	–
22	12.5	0	12	0	–	3600.0	–
23	13.2	0	12	0	–	3600.0	–
24	14.2	0	12	0	–	3600.0	–
25	13.3	0	12	0	–	3600.0	–
26	13.2	0	12	0	–	3600.0	–
27	12.8	0	12	0	–	3600.0	–
28	13.2	0	12	0	–	3600.0	–
29	12.5	0	12	0	–	3600.0	–

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Table E.3: Detailed results of the *Net 1* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
30	12.6	0	12	0	–	3600.0	–
31	12.6	0	12	0	–	3600.0	–
32	12.1	0	12	0	–	3600.0	–
33	12.5	0	12	0	–	3600.0	–
34	12.6	0	12	0	–	3600.0	–
35	13.2	0	12	0	–	3600.0	–
36	12.7	0	12	0	–	3600.0	–
37	12.8	0	12	0	–	3600.0	–
38	12.8	0	12	0	–	3600.0	–
39	12.2	0	12	0	–	3600.0	–
40	12.5	0	12	0	–	3600.0	–
41	12.2	0	12	0	–	3600.0	–
42	13.1	0	12	0	–	3600.0	–
43	13.0	0	12	0	–	3600.0	–
44	12.2	0	12	0	–	3600.0	–
45	12.6	0	12	0	–	3600.0	–
46	13.1	0	12	0	–	3600.0	–
47	12.6	0	12	0	–	3600.0	–
48	12.7	0	12	0	–	3600.0	–
49	13.2	0	12	0	–	3600.0	–
50	12.8	0	12	0	–	3600.0	–
51	12.6	0	12	0	–	3600.0	–
52	12.6	0	12	0	–	3600.0	–
53	13.6	0	12	0	–	3600.0	–
54	12.5	0	12	0	–	3600.0	–
55	13.4	0	12	0	–	3600.0	–
56	12.3	0	12	0	–	3600.0	–
57	13.1	0	12	0	–	3600.0	–
58	13.2	0	12	0	–	3600.0	–
59	13.1	0	12	0	–	3600.0	–
60	13.6	0	12	0	–	3600.0	–
61	12.3	0	12	0	–	3600.0	–
62	12.6	0	12	0	–	3600.0	–
63	11.9	0	12	0	–	3600.0	–
64	13.6	0	12	0	–	3600.0	–
65	12.6	0	12	0	–	3600.0	–
66	12.8	0	12	0	–	3600.0	–
67	13.0	0	12	0	–	3600.0	–
68	12.8	0	12	0	–	3600.0	–
69	12.8	0	12	0	–	3600.0	–
70	12.2	0	12	0	–	3600.0	–
71	12.1	0	12	0	–	3600.0	–
72	12.2	0	12	0	–	3600.0	–
73	13.0	0	12	0	–	3600.0	–
74	13.2	0	12	0	–	3600.0	–
75	12.6	0	12	0	–	3600.0	–
76	12.8	0	12	0	–	3600.0	–
77	12.5	0	12	0	–	3600.0	–
78	12.2	0	12	0	–	3600.0	–
79	12.4	0	12	0	–	3600.0	–
80	12.6	0	12	0	–	3600.0	–
81	12.9	0	12	0	–	3600.0	–
82	12.5	0	12	0	–	3600.0	–
83	13.0	0	12	0	–	3600.0	–
84	13.1	0	12	0	–	3600.0	–
85	12.3	0	12	0	–	3600.0	–
86	13.2	0	12	0	–	3600.0	–
87	13.2	0	12	0	–	3600.0	–
88	12.8	0	12	0	–	3600.0	–
89	13.3	0	12	0	–	3600.0	–
90	13.0	0	12	0	–	3600.0	–
91	12.8	0	12	0	–	3600.0	–
92	12.6	0	12	0	–	3600.0	–
93	12.7	0	12	0	–	3600.0	–
94	13.0	0	12	0	–	3600.0	–
95	13.7	0	12	0	–	3600.0	–
96	12.9	0	12	0	–	3600.0	–
97	12.7	0	12	0	–	3600.0	–
98	12.6	0	12	0	–	3600.0	–
99	12.3	0	12	0	–	3600.0	–
100	13.0	0	12	0	–	3600.0	–
101	12.5	0	12	0	–	3600.0	–
102	12.4	0	12	0	–	3600.0	–
103	12.7	0	12	0	–	3600.0	–
104	13.3	0	12	0	–	3600.0	–
105	12.8	0	12	0	–	3600.0	–
106	13.0	0	12	0	–	3600.0	–
107	13.2	0	12	0	–	3600.0	–

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Table E.3: Detailed results of the *Net 1* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
108	12.7	0	12	0	–	3600.0	–
109	12.9	0	12	0	–	3600.0	–
110	12.7	0	12	0	–	3600.0	–
111	12.7	0	12	0	–	3600.0	–
112	12.5	0	12	0	–	3600.0	–
113	12.9	0	12	0	–	3600.0	–
114	12.8	0	12	0	–	3600.0	–
115	12.7	0	12	0	–	3600.0	–
116	12.1	0	12	0	–	3600.0	–
117	13.5	0	12	0	–	3600.0	–
118	12.6	0	12	0	–	3600.0	–
119	12.8	0	12	0	–	3600.0	–
120	12.9	0	12	0	–	3600.0	–
121	12.4	0	12	0	–	3600.0	–
122	13.2	0	12	0	–	3600.0	–
123	12.2	0	12	0	–	3600.0	–
124	12.5	0	12	0	–	3600.0	–
125	12.4	0	12	0	–	3600.0	–
126	12.4	0	12	0	–	3600.0	–
127	12.8	0	12	0	–	3600.0	–
128	12.7	0	12	0	–	3600.0	–
129	12.5	0	12	0	–	3600.0	–
130	12.3	0	12	0	–	3600.0	–
131	12.4	0	12	0	–	3600.0	–
132	12.4	0	12	0	–	3600.0	–
133	13.3	0	12	0	–	3600.0	–
134	13.0	0	12	0	–	3600.0	–
135	12.6	0	12	0	–	3600.0	–
136	13.0	0	12	0	–	3600.0	–
137	12.1	0	12	0	–	3600.0	–
138	12.4	0	12	0	–	3600.0	–
139	13.1	0	12	0	–	3600.0	–
140	12.7	0	12	0	–	3600.0	–
141	12.5	0	12	0	–	3600.0	–
142	12.4	0	12	0	–	3600.0	–
143	12.6	0	12	0	–	3600.0	–
144	12.4	0	12	0	–	3600.0	–
145	12.7	0	12	0	–	3600.0	–
146	13.3	0	12	0	–	3600.0	–
147	12.8	0	12	0	–	3600.0	–
148	13.1	0	12	0	–	3600.0	–
149	12.8	0	12	0	–	3600.0	–
150	12.4	0	12	0	–	3600.0	–
151	13.1	0	12	0	–	3600.0	–
152	13.9	0	12	0	–	3600.0	–
153	12.9	0	12	0	–	3600.0	–
154	12.2	0	12	0	–	3600.0	–
155	12.6	0	12	0	–	3600.0	–
156	12.6	0	12	0	–	3600.0	–
157	12.8	0	12	0	–	3600.0	–
158	12.3	0	12	0	–	3600.0	–
159	12.5	0	12	0	–	3600.0	–
160	12.6	0	12	0	–	3600.0	–
161	12.8	0	12	0	–	3600.0	–
162	12.5	0	12	0	–	3600.0	–
163	12.6	0	12	0	–	3600.0	–
164	12.4	0	12	0	–	3600.0	–
165	12.4	0	12	0	–	3600.0	–
166	12.5	0	12	0	–	3600.0	–
167	12.8	0	12	0	–	3600.0	–
168	12.4	0	12	0	–	3600.0	–
169	12.2	0	12	0	–	3600.0	–
170	12.6	0	12	0	–	3600.0	–
171	12.5	0	12	0	–	3600.0	–
172	12.6	0	12	0	–	3600.0	–
173	12.1	0	12	0	–	3600.0	–
174	13.0	0	12	0	–	3600.0	–
175	13.1	0	12	0	–	3600.0	–
176	13.2	0	12	0	–	3600.0	–
177	12.5	0	12	0	–	3600.0	–
178	13.1	0	12	0	–	3600.0	–
179	12.8	0	12	0	–	3600.0	–
180	13.8	0	12	0	–	3600.0	–
181	12.6	0	12	0	–	3600.0	–
182	12.5	0	12	0	–	3600.0	–
183	12.4	0	12	0	–	3600.0	–
184	12.3	0	12	0	–	3600.0	–
185	12.7	0	12	0	–	3600.0	–

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Table E.3: Detailed results of the *Net 1* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
186	12.7	0	12	0	–	3600.0	–
187	12.3	0	12	0	–	3600.0	–
188	12.8	0	12	0	–	3600.0	–
189	13.5	0	12	0	–	3600.0	–
190	13.2	0	12	0	–	3600.0	–
191	12.9	0	12	0	–	3600.0	–
192	12.4	0	12	0	–	3600.0	–
193	12.8	0	12	0	–	3600.0	–
194	13.3	0	12	0	–	3600.0	–
195	12.6	0	12	0	–	3600.0	–
196	12.7	0	12	0	–	3600.0	–
197	12.5	0	12	0	–	3600.0	–
198	12.5	0	12	0	–	3600.0	–
199	12.3	0	12	0	–	3600.0	–
200	13.1	0	12	0	–	3600.0	–
201	12.7	0	12	0	–	3600.0	–
202	12.7	0	12	0	–	3600.0	–
203	13.6	0	12	0	–	3600.0	–
204	13.0	0	12	0	–	3600.0	–
205	13.1	0	12	0	–	3600.0	–
206	12.4	0	12	0	–	3600.0	–
207	13.5	0	12	0	–	3600.0	–
208	12.5	0	12	0	–	3600.0	–
209	12.4	0	12	0	–	3600.0	–
210	12.8	0	12	0	–	3600.0	–
211	13.0	0	12	0	–	3600.0	–
212	12.9	0	12	0	–	3600.0	–
213	12.7	0	12	0	–	3600.0	–
214	12.3	0	12	0	–	3600.0	–
215	12.0	0	12	0	–	3600.0	–
216	12.9	0	12	0	–	3600.0	–
217	12.8	0	12	0	–	3600.0	–
218	12.4	0	12	0	–	3600.0	–
219	13.0	0	12	0	–	3600.0	–
220	13.3	0	12	0	–	3600.0	–
221	12.3	0	12	0	–	3600.0	–
222	13.3	0	12	0	–	3600.0	–
223	12.8	0	12	0	–	3600.0	–
224	12.6	0	12	0	–	3600.0	–
225	12.4	0	12	0	–	3600.0	–
226	12.5	0	12	0	–	3600.0	–
227	12.8	0	12	0	–	3600.0	–
228	12.9	0	12	0	–	3600.0	–
229	13.0	0	12	0	–	3600.0	–
230	12.2	0	12	0	–	3600.0	–
231	12.0	0	12	0	–	3600.0	–
232	12.8	0	12	0	–	3600.0	–
233	13.0	0	12	0	–	3600.0	–
234	11.5	0	12	0	–	3600.0	–
235	13.2	0	12	0	–	3600.0	–
236	12.5	0	12	0	–	3600.0	–
237	13.0	0	12	0	–	3600.0	–
238	12.4	0	12	0	–	3600.0	–
239	12.5	0	12	0	–	3600.0	–
240	12.3	0	12	0	–	3600.0	–
241	13.3	0	12	0	–	3600.0	–
242	13.6	0	12	0	–	3600.0	–
243	12.3	0	12	0	–	3600.0	–
244	12.8	0	12	0	–	3600.0	–
245	12.4	0	12	0	–	3600.0	–
246	12.7	0	12	0	–	3600.0	–
247	12.6	0	12	0	–	3600.0	–
248	12.4	0	12	0	–	3600.0	–
249	12.8	0	12	0	–	3600.0	–
250	12.5	0	12	0	–	3600.0	–
251	12.4	0	12	0	–	3600.0	–
252	13.2	0	12	0	–	3600.0	–
253	12.6	0	12	0	–	3600.0	–
254	12.7	0	12	0	–	3600.0	–
255	12.5	0	12	0	–	3600.0	–
256	12.3	0	12	0	–	3600.0	–
257	12.1	0	12	0	–	3600.0	–
258	13.8	0	12	0	–	3600.0	–
259	12.5	0	12	0	–	3600.0	–
260	12.6	0	12	0	–	3600.0	–
261	12.9	0	12	0	–	3600.0	–
262	12.8	0	12	0	–	3600.0	–
263	12.3	0	12	0	–	3600.0	–

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Table E.3: Detailed results of the *Net 1* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
264	12.8	0	12	0	–	3600.0	–
265	13.8	0	12	0	–	3600.0	–
266	13.1	0	12	0	–	3600.0	–
267	13.1	0	12	0	–	3600.0	–
268	13.4	0	12	0	–	3600.0	–
269	13.2	0	12	0	–	3600.0	–
270	12.4	0	12	0	–	3600.0	–
271	12.5	0	12	0	–	3600.0	–
272	13.4	0	12	0	–	3600.0	–
273	13.2	0	12	0	–	3600.0	–
274	12.8	0	12	0	–	3600.0	–
275	11.9	0	12	0	–	3600.0	–
276	12.5	0	12	0	–	3600.0	–
277	12.9	0	12	0	–	3600.0	–
278	12.8	0	12	0	–	3600.0	–
279	12.7	0	12	0	–	3600.0	–
280	12.8	0	12	0	–	3600.0	–
281	12.4	0	12	0	–	3600.0	–
282	13.2	0	12	0	–	3600.0	–
283	13.2	0	12	0	–	3600.0	–
284	12.7	0	12	0	–	3600.0	–
285	13.1	0	12	0	–	3600.0	–
286	11.8	0	12	0	–	3600.0	–
287	13.1	0	12	0	–	3600.0	–
288	13.3	0	12	0	–	3600.0	–
289	13.0	0	12	0	–	3600.0	–
290	13.3	0	12	0	–	3600.0	–
291	13.1	0	12	0	–	3600.0	–
292	12.2	0	12	0	–	3600.0	–
293	13.2	0	12	0	–	3600.0	–
294	13.1	0	12	0	–	3600.0	–
295	12.4	0	12	0	–	3600.0	–
296	13.1	0	12	0	–	3600.0	–
297	12.5	0	12	0	–	3600.0	–
298	12.9	0	12	0	–	3600.0	–
299	12.8	0	12	0	–	3600.0	–
300	12.7	0	12	0	–	3600.0	–
301	12.7	0	12	0	–	3600.0	–
302	12.4	0	12	0	–	3600.0	–
303	12.3	0	12	0	–	3600.0	–
304	12.3	0	12	0	–	3600.0	–
305	12.6	0	12	0	–	3600.0	–
306	13.0	0	12	0	–	3600.0	–
307	12.9	0	12	0	–	3600.0	–
308	12.6	0	12	0	–	3600.0	–
309	12.7	0	12	0	–	3600.0	–
310	12.9	0	12	0	–	3600.0	–
311	12.7	0	12	0	–	3600.0	–
312	12.9	0	12	0	–	3600.0	–
313	12.6	0	12	0	–	3600.0	–
314	12.7	0	12	0	–	3600.0	–
315	12.2	0	12	0	–	3600.0	–
316	12.8	0	12	0	–	3600.0	–
317	12.2	0	12	0	–	3600.0	–
318	12.2	0	12	0	–	3600.0	–
319	12.5	0	12	0	–	3600.0	–
320	12.7	0	12	0	–	3600.0	–
321	12.2	0	12	0	–	3600.0	–
322	12.7	0	12	0	–	3600.0	–
323	13.2	0	12	0	–	3600.0	–
324	12.4	0	12	0	–	3600.0	–
325	12.3	0	12	0	–	3600.0	–
326	12.8	0	12	0	–	3600.0	–
327	14.9	0	12	0	–	3600.0	–
328	12.7	0	12	0	–	3600.0	–
329	13.2	0	12	0	–	3600.0	–
330	12.3	0	12	0	–	3600.0	–
331	13.0	0	12	0	–	3600.0	–
332	12.7	0	12	0	–	3600.0	–
333	13.8	0	12	0	–	3600.0	–
334	12.8	0	12	0	–	3600.0	–
335	12.8	0	12	0	–	3600.0	–
336	13.2	0	12	0	–	3600.0	–
337	13.0	0	12	0	–	3600.0	–
338	12.3	0	12	0	–	3600.0	–
339	12.5	0	12	0	–	3600.0	–
340	13.3	0	12	0	–	3600.0	–
341	12.3	0	12	0	–	3600.0	–

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Table E.3: Detailed results of the *Net 1* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
342	13.5	0	12	0	–	3600.0	–
343	12.9	0	12	0	–	3600.0	–
344	14.1	0	12	0	–	3600.0	–
345	12.7	0	12	0	–	3600.0	–
346	12.9	0	12	0	–	3600.0	–
347	13.2	0	12	0	–	3600.0	–
348	12.7	0	12	0	–	3600.0	–
349	12.9	0	12	0	–	3600.0	–
350	12.7	0	12	0	–	3600.0	–
351	12.9	0	12	0	–	3600.0	–
352	12.8	0	12	0	–	3600.0	–
353	12.8	0	12	0	–	3600.0	–
354	12.6	0	12	0	–	3600.0	–
355	11.8	0	12	0	–	3600.0	–
356	12.9	0	12	0	–	3600.0	–
357	12.9	0	12	0	–	3600.0	–
358	13.1	0	12	0	–	3600.0	–
359	12.6	0	12	0	–	3600.0	–
360	12.1	0	12	0	–	3600.0	–
361	13.1	0	12	0	–	3600.0	–
362	12.6	0	12	0	–	3600.0	–
363	13.0	0	12	0	–	3600.0	–
364	12.8	0	12	0	–	3600.0	–
365	13.3	0	12	0	–	3600.0	–
366	12.4	0	12	0	–	3600.0	–
367	12.3	0	12	0	–	3600.0	–
368	12.2	0	12	0	–	3600.0	–
369	12.7	0	12	0	–	3600.0	–
370	13.1	0	12	0	–	3600.0	–
371	14.1	0	12	0	–	3600.0	–
372	12.9	0	12	0	–	3600.0	–
373	13.1	0	12	0	–	3600.0	–
374	12.6	0	12	0	–	3600.0	–
375	12.6	0	12	0	–	3600.0	–
376	13.1	0	12	0	–	3600.0	–
377	13.1	0	12	0	–	3600.0	–
378	12.5	0	12	0	–	3600.0	–
379	12.2	0	12	0	–	3600.0	–
380	12.6	0	12	0	–	3600.0	–
381	12.0	0	12	0	–	3600.0	–
382	13.4	0	12	0	–	3600.0	–
383	12.6	0	12	0	–	3600.0	–
384	12.4	0	12	0	–	3600.0	–
385	13.3	0	12	0	–	3600.0	–
386	12.7	0	12	0	–	3600.0	–
387	13.8	0	12	0	–	3600.0	–
388	12.2	0	12	0	–	3600.0	–
389	12.7	0	12	0	–	3600.0	–
390	12.7	0	12	0	–	3600.0	–
391	12.1	0	12	0	–	3600.0	–
392	13.0	0	12	0	–	3600.0	–
393	13.5	0	12	0	–	3600.0	–
394	12.3	0	12	0	–	3600.0	–
395	12.6	0	12	0	–	3600.0	–
396	12.9	0	12	0	–	3600.0	–
397	12.2	0	12	0	–	3600.0	–
398	13.1	0	12	0	–	3600.0	–
399	12.8	0	12	0	–	3600.0	–
400	12.6	0	12	0	–	3600.0	–
401	12.5	0	12	0	–	3600.0	–
402	13.7	0	12	0	–	3600.0	–
403	12.1	0	12	0	–	3600.0	–
404	13.4	0	12	0	–	3600.0	–
405	12.7	0	12	0	–	3600.0	–
406	12.3	0	12	0	–	3600.0	–
407	13.0	0	12	0	–	3600.0	–
408	12.9	0	12	0	–	3600.0	–
409	12.2	0	12	0	–	3600.0	–
410	12.5	0	12	0	–	3600.0	–
411	12.3	0	12	0	–	3600.0	–
412	13.0	0	12	0	–	3600.0	–
413	12.1	0	12	0	–	3600.0	–
414	13.2	0	12	0	–	3600.0	–
415	12.6	0	12	0	–	3600.0	–
416	12.7	0	12	0	–	3600.0	–
417	12.6	0	12	0	–	3600.0	–
418	13.4	0	12	0	–	3600.0	–
419	13.6	0	12	0	–	3600.0	–

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Table E.3: Detailed results of the *Net 1* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
420	13.1	0	12	0	–	3600.0	–
421	12.0	0	12	0	–	3600.0	–
422	12.5	0	12	0	–	3600.0	–
423	12.2	0	12	0	–	3600.0	–
424	13.5	0	12	0	–	3600.0	–
425	13.4	0	12	0	–	3600.0	–
426	12.2	0	12	0	–	3600.0	–
427	12.8	0	12	0	–	3600.0	–
428	12.9	0	12	0	–	3600.0	–
429	12.1	0	12	0	–	3600.0	–
430	13.1	0	12	0	–	3600.0	–
431	13.1	0	12	0	–	3600.0	–
432	12.4	0	12	0	–	3600.0	–
433	12.8	0	12	0	–	3600.0	–
434	12.3	0	12	0	–	3600.0	–
435	12.8	0	12	0	–	3600.0	–
436	12.6	0	12	0	–	3600.0	–
437	12.6	0	12	0	–	3600.0	–
438	13.3	0	12	0	–	3600.0	–
439	13.0	0	12	0	–	3600.0	–
440	12.3	0	12	0	–	3600.0	–
441	12.2	0	12	0	–	3600.0	–
442	12.3	0	12	0	–	3600.0	–
443	12.4	0	12	0	–	3600.0	–
444	13.0	0	12	0	–	3600.0	–
445	12.9	0	12	0	–	3600.0	–
446	13.1	0	12	0	–	3600.0	–
447	13.1	0	12	0	–	3600.0	–
448	12.8	0	12	0	–	3600.0	–
449	12.6	0	12	0	–	3600.0	–
450	12.5	0	12	0	–	3600.0	–
451	13.0	0	12	0	–	3600.0	–
452	12.0	0	12	0	–	3600.0	–
453	12.5	0	12	0	–	3600.0	–
454	12.4	0	12	0	–	3600.0	–
455	13.0	0	12	0	–	3600.0	–
456	12.8	0	12	0	–	3600.0	–
457	12.7	0	12	0	–	3600.0	–
458	13.2	0	12	0	–	3600.0	–
459	13.8	0	12	0	–	3600.0	–
460	13.0	0	12	0	–	3600.0	–
461	12.5	0	12	0	–	3600.0	–
462	13.1	0	12	0	–	3600.0	–
463	12.2	0	12	0	–	3600.0	–
464	12.3	0	12	0	–	3600.0	–
465	12.8	0	12	0	–	3600.0	–
466	13.4	0	12	0	–	3600.0	–
467	12.8	0	12	0	–	3600.0	–
468	13.6	0	12	0	–	3600.0	–
469	12.6	0	12	0	–	3600.0	–
470	12.2	0	12	0	–	3600.0	–
471	13.0	0	12	0	–	3600.0	–
472	13.0	0	12	0	–	3600.0	–
473	12.5	0	12	0	–	3600.0	–
474	13.1	0	12	0	–	3600.0	–
475	12.8	0	12	0	–	3600.0	–
476	12.4	0	12	0	–	3600.0	–
477	12.5	0	12	0	–	3600.0	–
478	12.9	0	12	0	–	3600.0	–
479	13.1	0	12	0	–	3600.0	–
480	12.5	0	12	0	–	3600.0	–
481	12.5	0	12	0	–	3600.0	–
482	12.4	0	12	0	–	3600.0	–
483	12.8	0	12	0	–	3600.0	–
484	12.8	0	12	0	–	3600.0	–
485	12.4	0	12	0	–	3600.0	–
486	12.8	0	12	0	–	3600.0	–
487	12.3	0	12	0	–	3600.0	–
488	13.7	0	12	0	–	3600.0	–
489	12.4	0	12	0	–	3600.0	–
490	13.3	0	12	0	–	3600.0	–
491	13.3	0	12	0	–	3600.0	–
492	12.6	0	12	0	–	3600.0	–
493	13.0	0	12	0	–	3600.0	–
494	12.6	0	12	0	–	3600.0	–
495	13.3	0	12	0	–	3600.0	–
496	13.0	0	12	0	–	3600.0	–
497	12.3	0	12	0	–	3600.0	–

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Table E.3: Detailed results of the *Net 1* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
498	12.8	0	12	0	–	3600.0	–
499	13.2	0	12	0	–	3600.0	–
500	12.8	0	12	0	–	3600.0	–

Table E.4: Detailed results of the *Net 2* test set, as summarized in Tables 8.3 – 8.5 and Figure 8.3. For activating the heuristic HEUR (*Enabled*) and SCIP default (*Disabled*), the table lists the solving time (col. *Time*), and the objective function value (col. *Obj*), which represents the number of station mode changes. Further, it shows HEUR specific information about the highest achieved time step (col. *Round*), the number of backward moves (col. *Back*), and whether partitioning of any $\texttt{MIP}_{t,\cdots,|\mathcal{T}|}$-model occured (col. *Decompose-MIP*), indicated by True.

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
1	53.9	0	12	0	–	3600.0	–
2	74.1	0	12	0	–	3600.0	–
3	250.0	5	12	0	–	3600.0	–
4	111.0	0	12	0	–	3600.0	–
5	225.2	0	12	0	–	3600.0	–
6	54.8	0	12	0	–	3600.0	–
7	52.2	0	12	0	–	3600.0	–
8	248.8	8	12	0	–	3600.0	–
9	160.7	6	12	1	–	3600.0	–
10	625.5	4	12	0	–	3600.0	–
11	485.3	9	12	1	–	3600.0	–
12	221.2	5	12	1	–	3600.0	–
13	570.2	8	12	0	–	3600.0	–
14	551.4	9	12	1	–	3600.0	–
15	161.8	4	12	1	–	3600.0	–
16	226.0	1	12	0	–	3600.0	–
17	56.2	0	12	0	–	3600.0	–
18	61.1	0	12	0	–	3600.0	–
19	82.8	0	12	0	–	3600.0	–
20	243.3	0	12	0	–	3600.0	–
21	270.9	4	12	1	–	3600.0	–
22	385.1	10	12	0	True	3600.0	–
23	417.5	8	12	1	–	3600.0	–
24	121.4	0	12	0	–	3600.0	–
25	51.0	0	12	0	–	3600.0	–
26	412.4	5	12	0	–	3600.0	–
27	249.1	4	12	0	–	3600.0	–
28	38.4	0	12	0	–	3600.0	–
29	48.5	0	12	0	–	3600.0	–
30	464.2	9	12	1	–	3600.0	–
31	62.6	0	12	0	–	3600.0	–
32	1264.4	22	12	1	–	3600.0	–
33	55.9	0	12	0	–	3600.0	–
34	749.3	1	12	0	–	3600.0	–
35	547.8	10	12	3	–	3600.0	–
36	247.2	5	12	0	–	3600.0	–
37	560.2	9	12	3	–	3600.0	–
38	47.2	0	12	0	–	3600.0	–
39	982.6	4	12	0	–	3600.0	–
40	247.2	5	12	1	–	3600.0	–
41	265.3	0	12	0	–	3600.0	–
42	90.0	0	12	0	–	3600.0	–
43	73.0	0	12	0	–	3600.0	–
44	154.8	4	12	1	–	3600.0	–
45	47.3	0	12	0	–	3600.0	–
46	611.4	14	12	0	–	3600.0	–
47	38.1	0	12	0	–	3600.0	–
48	80.1	0	12	0	–	3600.0	–
49	54.5	0	12	0	–	3600.0	–
50	46.1	0	12	0	–	3600.0	–
51	44.4	0	12	0	–	3600.0	–
52	537.1	7	12	1	True	3600.0	–

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Table E.4: Detailed results of the *Net 2* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
53	911.2	12	12	2	True	3600.0	–
54	773.3	1	12	0	–	3600.0	–
55	62.1	0	12	0	–	3600.0	–
56	55.6	0	12	0	–	3600.0	–
57	338.5	12	12	1	–	3600.0	–
58	977.9	12	12	1	–	3600.0	–
59	568.3	7	12	1	True	3600.0	–
60	433.6	5	12	1	–	3600.0	–
61	233.7	0	12	0	–	3600.0	–
62	40.5	0	12	0	–	3600.0	–
63	61.4	0	12	0	–	3600.0	–
64	153.7	0	12	0	–	3600.0	–
65	141.4	0	12	0	–	3600.0	–
66	42.5	0	12	0	–	3600.0	–
67	219.8	5	12	0	–	3600.0	–
68	878.8	6	12	1	–	3600.0	–
69	318.1	10	12	0	–	3600.0	–
70	768.2	7	12	0	–	3600.0	–
71	44.2	0	12	0	–	3600.0	–
72	870.5	10	12	1	–	3600.0	–
73	51.8	0	12	0	–	3600.0	–
74	436.6	6	12	0	–	3600.0	–
75	994.6	5	12	2	–	3600.0	–
76	527.9	7	12	0	True	3600.0	–
77	240.5	0	12	0	–	3600.0	–
78	515.9	4	12	1	–	3600.0	–
79	44.8	0	12	0	–	3600.0	–
80	155.6	0	12	0	–	3600.0	–
81	373.3	5	12	1	–	3600.0	–
82	168.2	0	12	0	–	3600.0	–
83	48.1	0	12	0	–	3600.0	–
84	339.4	9	12	1	–	3600.0	–
85	354.3	10	12	2	–	3600.0	–
86	46.3	0	12	0	–	3600.0	–
87	349.2	9	12	1	–	3600.0	–
88	234.0	8	12	0	–	3600.0	–
89	163.7	0	12	0	–	3600.0	–
90	52.8	0	12	0	–	3600.0	–
91	504.4	16	12	0	–	3600.0	–
92	52.0	0	12	0	–	3600.0	–
93	419.8	12	12	0	–	3600.0	–
94	40.5	0	12	0	–	3600.0	–
95	224.9	0	12	0	–	3600.0	–
96	65.2	0	12	0	–	3600.0	–
97	268.3	12	12	1	–	3600.0	–
98	137.1	0	12	0	–	3600.0	–
99	54.5	0	12	0	–	3600.0	–
100	242.8	0	12	0	–	3600.0	–
101	52.6	0	12	0	–	3600.0	–
102	996.3	20	12	0	–	3600.0	–
103	174.4	9	12	0	–	3600.0	–
104	251.3	0	12	0	–	3600.0	–
105	60.1	0	12	0	–	3600.0	–
106	408.1	9	12	1	–	3600.0	–
107	58.3	0	12	0	–	3600.0	–
108	53.1	0	12	0	–	3600.0	–
109	42.0	0	12	0	–	3600.0	–
110	340.3	8	12	1	–	3600.0	–
111	184.1	0	12	0	–	3600.0	–
112	278.4	9	12	1	–	3600.0	–
113	229.8	10	12	0	–	3600.0	–
114	260.8	6	12	0	–	3600.0	–
115	163.0	0	12	0	–	3600.0	–
116	67.9	0	12	0	–	3600.0	–
117	354.6	5	12	0	–	3600.0	–
118	34.6	0	12	0	–	3600.0	–
119	63.3	0	12	0	–	3600.0	–
120	272.8	3	12	0	–	3600.0	–
121	52.8	0	12	0	–	3600.0	–
122	46.6	0	12	0	–	3600.0	–
123	428.1	0	12	0	–	3600.0	–
124	140.3	0	12	0	–	3600.0	–
125	64.9	0	12	0	–	3600.0	–
126	242.2	5	12	1	–	3600.0	–
127	241.2	3	12	0	–	3600.0	–
128	43.8	0	12	0	–	3600.0	–
129	44.0	0	12	0	–	3600.0	–
130	41.5	0	12	0	–	3600.0	–

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Table E.4: Detailed results of the *Net 2* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
131	78.6	0	12	0	–	3600.0	–
132	46.1	0	12	0	–	3600.0	–
133	329.9	5	12	0	–	3600.0	–
134	51.3	0	12	0	–	3600.0	–
135	138.7	10	12	1	–	3600.0	–
136	241.0	9	12	1	–	3600.0	–
137	59.6	0	12	0	–	3600.0	–
138	547.2	8	12	1	–	3600.0	–
139	248.8	9	12	1	–	3600.0	–
140	657.0	8	12	1	–	3600.0	–
141	45.5	0	12	0	–	3600.0	–
142	69.8	0	12	0	–	3600.0	–
143	360.9	10	12	1	–	3600.0	–
144	323.0	4	12	0	–	3600.0	–
145	301.8	0	12	0	–	3600.0	–
146	56.3	0	12	0	–	3600.0	–
147	40.0	0	12	0	–	3600.0	–
148	106.3	0	12	0	–	3600.0	–
149	440.3	8	12	1	–	3600.0	–
150	45.3	0	12	0	–	3600.0	–
151	501.3	7	12	1	–	3600.0	–
152	36.8	0	12	0	–	3600.0	–
153	418.9	9	12	0	–	3600.0	–
154	402.9	5	12	1	–	3600.0	–
155	48.7	0	12	0	–	3600.0	–
156	341.6	14	12	0	–	3600.0	–
157	59.1	0	12	0	–	3600.0	–
158	99.1	0	12	0	–	3600.0	–
159	62.3	0	12	0	–	3600.0	–
160	141.9	0	12	0	–	3600.0	–
161	206.4	0	12	0	–	3600.0	–
162	85.9	0	12	0	–	3600.0	–
163	360.2	5	12	0	–	3600.0	–
164	278.9	6	12	2	–	3600.0	–
165	878.1	7	12	0	–	3600.0	–
166	52.9	0	12	0	–	3600.0	–
167	56.4	0	12	0	–	3600.0	–
168	246.1	9	12	0	–	3600.0	–
169	48.1	0	12	0	–	3600.0	–
170	65.7	0	12	0	–	3600.0	–
171	49.9	0	12	0	–	3600.0	–
172	241.0	10	12	0	–	3600.0	–
173	52.0	0	12	0	–	3600.0	–
174	54.8	0	12	0	–	3600.0	–
175	56.2	0	12	0	–	3600.0	–
176	85.3	0	12	0	–	3600.0	–
177	348.6	8	12	0	–	3600.0	–
178	102.9	0	12	0	–	3600.0	–
179	267.8	7	12	1	–	3600.0	–
180	73.0	0	12	0	–	3600.0	–
181	237.5	4	12	0	–	3600.0	–
182	94.3	0	12	0	–	3600.0	–
183	242.4	6	12	0	–	3600.0	–
184	288.9	4	12	0	–	3600.0	–
185	65.8	0	12	0	–	3600.0	–
186	59.8	0	12	0	–	3600.0	–
187	420.3	5	12	0	–	3600.0	–
188	75.9	0	12	0	–	3600.0	–
189	330.1	3	12	1	–	3600.0	–
190	384.4	12	12	0	–	3600.0	–
191	82.5	0	12	0	–	3600.0	–
192	127.8	0	12	0	–	3600.0	–
193	247.4	10	12	0	–	3600.0	–
194	281.3	9	12	0	–	3600.0	–
195	39.4	0	12	0	–	3600.0	–
196	422.2	6	12	0	–	3600.0	–
197	332.5	7	12	0	–	3600.0	–
198	508.6	7	12	1	–	3600.0	–
199	48.3	0	12	0	–	3600.0	–
200	807.1	12	12	0	–	3600.0	–
201	151.0	0	12	0	–	3600.0	–
202	144.0	0	12	0	–	3600.0	–
203	164.3	0	12	0	–	3600.0	–
204	1580.4	10	12	1	–	3600.0	–
205	83.3	0	12	0	–	3600.0	–
206	42.6	0	12	0	–	3600.0	–
207	79.6	0	12	0	–	3600.0	–
208	312.7	4	12	1	–	3600.0	–

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Table E.4: Detailed results of the *Net 2* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
209	46.2	0	12	0	–	3600.0	–
210	512.1	5	12	1	–	3600.0	–
211	500.1	5	12	1	–	3600.0	–
212	46.6	0	12	0	–	3600.0	–
213	52.6	0	12	0	–	3600.0	–
214	298.7	6	12	1	–	3600.0	–
215	33.4	0	12	0	–	3600.0	–
216	603.4	9	12	4	–	3600.0	–
217	721.0	9	12	2	–	3600.0	–
218	330.3	0	12	0	–	3600.0	–
219	266.3	0	12	0	–	3600.0	–
220	166.6	0	12	0	–	3600.0	–
221	179.7	0	12	0	–	3600.0	–
222	56.3	0	12	0	–	3600.0	–
223	362.4	8	12	1	–	3600.0	–
224	107.6	0	12	0	–	3600.0	–
225	409.6	6	12	2	–	3600.0	–
226	375.9	5	12	1	–	3600.0	–
227	464.1	10	12	1	–	3600.0	–
228	162.1	5	12	0	–	3600.0	–
229	48.0	0	12	0	–	3600.0	–
230	333.7	5	12	0	–	3600.0	–
231	264.7	0	12	0	–	3600.0	–
232	43.4	0	12	0	–	3600.0	–
233	246.2	5	12	1	–	3600.0	–
234	135.2	0	12	0	–	3600.0	–
235	622.5	6	12	0	–	3600.0	–
236	155.1	0	12	0	–	3600.0	–
237	353.4	4	12	1	–	3600.0	–
238	510.8	4	12	1	–	3600.0	–
239	538.9	9	12	2	–	3600.0	–
240	129.1	0	12	0	–	3600.0	–
241	38.4	0	12	0	–	3600.0	–
242	629.0	6	12	1	–	3600.0	–
243	240.6	2	12	1	–	3600.0	–
244	78.2	1	12	0	–	3600.0	–
245	142.8	4	12	1	–	3600.0	–
246	235.4	5	12	0	–	3600.0	–
247	36.9	0	12	0	–	3600.0	–
248	91.0	0	12	0	–	3600.0	–
249	48.1	0	12	0	–	3600.0	–
250	46.2	0	12	0	–	3600.0	–
251	47.0	0	12	0	–	3600.0	–
252	168.3	2	12	0	–	3600.0	–
253	1145.8	5	12	0	–	3600.0	–
254	514.8	10	12	1	–	3600.0	–
255	46.5	0	12	0	–	3600.0	–
256	273.5	5	12	1	–	3600.0	–
257	231.1	4	12	1	–	3600.0	–
258	35.1	0	12	0	–	3600.0	–
259	67.1	0	12	0	–	3600.0	–
260	1155.3	15	12	0	–	3600.0	–
261	170.7	0	12	0	–	3600.0	–
262	66.6	0	12	0	–	3600.0	–
263	35.3	0	12	0	–	3600.0	–
264	52.6	0	12	0	–	3600.0	–
265	192.5	0	12	0	–	3600.0	–
266	115.2	0	12	0	–	3600.0	–
267	496.3	10	12	1	–	3600.0	–
268	902.5	7	12	1	–	3600.0	–
269	64.1	0	12	0	–	3600.0	–
270	535.0	6	12	1	–	3600.0	–
271	241.7	3	12	0	–	3600.0	–
272	995.1	7	12	0	–	3600.0	–
273	849.4	5	12	0	–	3600.0	–
274	343.7	4	12	1	–	3600.0	–
275	146.1	0	12	0	–	3600.0	–
276	66.8	0	12	0	–	3600.0	–
277	85.3	0	12	0	–	3600.0	–
278	46.4	0	12	0	–	3600.0	–
279	360.1	4	12	0	–	3600.0	–
280	176.0	6	12	1	–	3600.0	–
281	234.3	0	12	0	–	3600.0	–
282	894.1	16	12	1	–	3600.0	–
283	373.8	10	12	1	–	3600.0	–
284	1138.0	12	12	2	–	3600.0	–
285	52.2	0	12	0	–	3600.0	–
286	676.8	10	12	1	–	3600.0	–

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Table E.4: Detailed results of the *Net 2* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
287	37.8	0	12	0	–	3600.0	–
288	42.8	0	12	0	–	3600.0	–
289	305.5	0	12	0	–	3600.0	–
290	180.5	0	12	0	–	3600.0	–
291	53.6	0	12	0	–	3600.0	–
292	41.4	0	12	0	–	3600.0	–
293	277.6	5	12	1	–	3600.0	–
294	450.1	9	12	1	–	3600.0	–
295	432.2	10	12	1	–	3600.0	–
296	427.2	6	12	2	–	3600.0	–
297	61.8	0	12	0	–	3600.0	–
298	419.1	12	12	1	–	3600.0	–
299	972.4	13	12	1	–	3600.0	–
300	144.3	4	12	1	–	3600.0	–
301	139.3	0	12	0	–	3600.0	–
302	251.0	4	12	1	–	3600.0	–
303	55.4	0	12	0	–	3600.0	–
304	604.8	12	12	2	–	3600.0	–
305	55.8	0	12	0	–	3600.0	–
306	150.5	0	12	0	–	3600.0	–
307	766.4	5	12	1	–	3600.0	–
308	795.0	5	12	0	–	3600.0	–
309	33.8	0	12	0	–	3600.0	–
310	462.7	8	12	0	True	3600.0	–
311	149.1	0	12	0	–	3600.0	–
312	248.5	4	12	0	–	3600.0	–
313	146.7	0	12	0	–	3600.0	–
314	186.2	6	12	1	–	3600.0	–
315	156.1	0	12	0	–	3600.0	–
316	234.5	0	12	0	–	3600.0	–
317	460.3	10	12	2	–	3600.0	–
318	294.9	4	12	1	–	3600.0	–
319	271.8	10	12	1	–	3600.0	–
320	299.4	6	12	1	–	3600.0	–
321	48.2	0	12	0	–	3600.0	–
322	244.5	0	12	0	–	3600.0	–
323	206.0	0	12	0	–	3600.0	–
324	62.0	0	12	0	–	3600.0	–
325	145.3	0	12	0	–	3600.0	–
326	253.6	0	12	0	–	3600.0	–
327	52.5	0	12	0	–	3600.0	–
328	187.1	0	12	0	–	3600.0	–
329	228.8	0	12	0	–	3600.0	–
330	387.2	8	12	1	–	3600.0	–
331	54.8	0	12	0	–	3600.0	–
332	241.3	10	12	1	–	3600.0	–
333	55.4	0	12	0	–	3600.0	–
334	647.2	6	12	1	–	3600.0	–
335	1317.5	25	12	1	–	3600.0	–
336	63.7	2	12	0	–	3600.0	–
337	653.3	11	12	2	–	3600.0	–
338	106.1	0	12	0	–	3600.0	–
339	250.1	1	12	0	–	3600.0	–
340	3600.0	–	2	1	–	3600.0	–
341	57.1	0	12	0	–	3600.0	–
342	257.1	2	12	0	–	3600.0	–
343	73.4	0	12	0	–	3600.0	–
344	512.8	7	12	0	–	3600.0	–
345	869.3	9	12	1	–	3600.0	–
346	144.8	3	12	0	–	3600.0	–
347	515.3	7	12	1	–	3600.0	–
348	50.5	0	12	0	–	3600.0	–
349	175.0	0	12	0	–	3600.0	–
350	57.0	0	12	0	–	3600.0	–
351	217.6	5	12	0	–	3600.0	–
352	301.7	5	12	0	–	3600.0	–
353	268.5	6	12	0	–	3600.0	–
354	435.3	7	12	0	–	3600.0	–
355	246.1	9	12	1	–	3600.0	–
356	249.4	7	12	0	–	3600.0	–
357	770.1	5	12	1	–	3600.0	–
358	3600.0	–	1	0	–	3600.0	–
359	168.9	6	12	1	–	3600.0	–
360	58.8	0	12	0	–	3600.0	–
361	974.2	7	12	1	–	3600.0	–
362	49.6	0	12	0	–	3600.0	–
363	51.2	0	12	0	–	3600.0	–
364	147.4	0	12	0	–	3600.0	–

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Table E.4: Detailed results of the *Net 2* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
365	84.8	0	12	0	–	3600.0	–
366	463.7	8	12	2	–	3600.0	–
367	334.7	2	12	0	–	3600.0	–
368	857.1	9	12	0	–	3600.0	–
369	142.7	4	12	1	–	3600.0	–
370	52.1	0	12	0	–	3600.0	–
371	41.7	0	12	0	–	3600.0	–
372	754.0	7	12	1	–	3600.0	–
373	3600.0	–	1	0	–	3600.0	–
374	812.1	9	12	1	–	3600.0	–
375	69.5	0	12	0	–	3600.0	–
376	3600.0	–	1	0	–	3600.0	–
377	606.0	19	12	1	–	3600.0	–
378	47.2	0	12	0	–	3600.0	–
379	56.2	0	12	0	–	3600.0	–
380	164.7	5	12	1	–	3600.0	–
381	661.4	13	12	2	–	3600.0	–
382	448.7	13	12	2	–	3600.0	–
383	190.7	0	12	0	–	3600.0	–
384	1229.8	12	12	0	–	3600.0	–
385	78.7	0	12	0	–	3600.0	–
386	66.3	0	12	0	–	3600.0	–
387	60.6	0	12	0	–	3600.0	–
388	829.2	8	12	1	–	3600.0	–
389	149.8	10	12	0	–	3600.0	–
390	341.2	0	12	0	–	3600.0	–
391	66.8	0	12	0	–	3600.0	–
392	968.9	8	12	1	True	3600.0	–
393	400.7	5	12	1	–	3600.0	–
394	370.1	4	12	1	–	3600.0	–
395	61.9	0	12	0	–	3600.0	–
396	57.3	0	12	0	–	3600.0	–
397	107.8	0	12	0	–	3600.0	–
398	172.0	6	12	1	–	3600.0	–
399	923.7	17	12	2	–	3600.0	–
400	355.0	0	12	0	–	3600.0	–
401	42.1	0	12	0	–	3600.0	–
402	49.1	0	12	0	–	3600.0	–
403	283.5	8	12	1	–	3600.0	–
404	289.1	0	12	0	–	3600.0	–
405	449.3	5	12	0	–	3600.0	–
406	127.2	0	12	0	–	3600.0	–
407	401.2	9	12	2	–	3600.0	–
408	527.2	8	12	0	True	3600.0	–
409	139.6	7	12	0	–	3600.0	–
410	322.3	2	12	0	–	3600.0	–
411	528.7	7	12	1	–	3600.0	–
412	151.2	5	12	1	–	3600.0	–
413	62.4	0	12	0	–	3600.0	–
414	1222.1	10	12	2	–	3600.0	–
415	75.3	0	12	0	–	3600.0	–
416	64.5	0	12	0	–	3600.0	–
417	51.4	0	12	0	–	3600.0	–
418	104.6	0	12	0	–	3600.0	–
419	3600.0	–	1	0	–	3600.0	–
420	39.9	0	12	0	–	3600.0	–
421	134.5	4	12	0	–	3600.0	–
422	145.4	0	12	0	–	3600.0	–
423	44.0	0	12	0	–	3600.0	–
424	1216.9	17	12	6	–	3600.0	–
425	30.8	0	12	0	–	3600.0	–
426	235.5	6	12	0	–	3600.0	–
427	51.6	0	12	0	–	3600.0	–
428	413.9	6	12	0	–	3600.0	–
429	154.3	0	12	0	–	3600.0	–
430	746.3	6	12	1	–	3600.0	–
431	75.1	0	12	0	–	3600.0	–
432	170.8	0	12	0	–	3600.0	–
433	72.5	0	12	0	–	3600.0	–
434	3600.0	–	1	0	–	3600.0	–
435	234.1	9	12	0	–	3600.0	–
436	1144.5	11	12	0	–	3600.0	–
437	42.1	0	12	0	–	3600.0	–
438	43.8	0	12	0	–	3600.0	–
439	446.6	10	12	1	–	3600.0	–
440	99.9	0	12	0	–	3600.0	–
441	1173.3	12	12	0	–	3600.0	–
442	240.7	6	12	1	–	3600.0	–

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Table E.4: Detailed results of the *Net 2* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
443	3600.0	–	1	0	–	3600.0	–
444	88.3	0	12	0	–	3600.0	–
445	173.7	4	12	1	–	3600.0	–
446	60.0	0	12	0	–	3600.0	–
447	725.4	5	12	0	–	3600.0	–
448	149.1	9	12	1	–	3600.0	–
449	438.8	5	12	2	–	3600.0	–
450	229.6	3	12	0	–	3600.0	–
451	53.3	0	12	0	–	3600.0	–
452	40.4	0	12	0	–	3600.0	–
453	3600.0	–	1	0	–	3600.0	–
454	327.6	4	12	0	–	3600.0	–
455	525.3	7	12	2	–	3600.0	–
456	800.8	7	12	1	–	3600.0	–
457	53.3	0	12	0	–	3600.0	–
458	44.5	0	12	0	–	3600.0	–
459	65.4	0	12	0	–	3600.0	–
460	52.5	0	12	0	–	3600.0	–
461	438.9	11	12	2	–	3600.0	–
462	278.0	9	12	1	–	3600.0	–
463	53.5	0	12	0	–	3600.0	–
464	574.2	5	12	1	–	3600.0	–
465	3600.0	–	2	1	–	3600.0	–
466	277.1	0	12	0	–	3600.0	–
467	149.2	0	12	0	–	3600.0	–
468	90.0	0	12	0	–	3600.0	–
469	44.7	0	12	0	–	3600.0	–
470	320.2	9	12	0	–	3600.0	–
471	438.9	4	12	0	–	3600.0	–
472	345.3	5	12	0	–	3600.0	–
473	57.8	0	12	0	–	3600.0	–
474	52.1	0	12	0	–	3600.0	–
475	3600.0	–	1	0	–	3600.0	–
476	955.1	9	12	2	–	3600.0	–
477	76.8	0	12	0	–	3600.0	–
478	51.8	0	12	0	–	3600.0	–
479	576.1	6	12	1	–	3600.0	–
480	53.0	0	12	0	–	3600.0	–
481	167.7	5	12	1	–	3600.0	–
482	519.0	9	12	2	–	3600.0	–
483	75.5	0	12	0	–	3600.0	–
484	75.8	0	12	0	–	3600.0	–
485	224.7	4	12	0	–	3600.0	–
486	58.8	0	12	0	–	3600.0	–
487	229.1	0	12	0	–	3600.0	–
488	438.2	8	12	1	–	3600.0	–
489	341.1	8	12	0	–	3600.0	–
490	57.4	0	12	0	–	3600.0	–
491	310.2	12	12	0	–	3600.0	–
492	133.0	3	12	0	–	3600.0	–
493	53.0	0	12	0	–	3600.0	–
494	932.3	7	12	1	–	3600.0	–
495	46.4	0	12	0	–	3600.0	–
496	114.1	0	12	0	–	3600.0	–
497	248.7	10	12	1	–	3600.0	–
498	569.0	8	12	0	–	3600.0	–
499	315.3	5	12	1	–	3600.0	–
500	167.2	0	12	0	–	3600.0	–

Table E.5: Detailed results of the *Net 3* test set, as summarized in Tables 8.3 – 8.5 and Figure 8.3. All 1440 instances are chronologically ordered, where the start time of consecutive instances is one hour ahead covering in total 60 consecutive days. For activating the heuristic HEUR (*Enabled*) and SCIP default (*Disabled*), the table lists the solving time (col. *Time*), and the objective function value (col. *Obj*), which represents the number of station mode changes. Further, it shows HEUR specific information about the highest achieved time step (col. *Round*), the number of backward moves (col. *Back*), and whether partitioning of any $\mathtt{MIP}_{t,\cdots,|\mathcal{T}|}$-model occured (col. *Decompose-MIP*), indicated by True.

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
1	1489.8	11	12	0	–	3600.0	–
2	3600.0	–	5	9	True	3600.0	–
3	650.2	18	12	0	–	3600.0	–
4	1790.8	8	12	1	–	3600.0	–
5	2513.1	19	12	3	–	3600.0	–
6	2066.6	11	12	0	–	3600.0	–
7	1718.7	24	12	1	True	3600.0	–
8	3600.0	–	7	31	–	3600.0	–
9	1400.2	12	12	0	–	3600.0	–
10	1064.7	10	12	0	–	3600.0	–
11	418.7	0	12	0	–	3600.0	–
12	572.5	15	12	0	–	3600.0	–
13	511.2	5	12	0	–	3600.0	–
14	94.7	0	12	0	–	3600.0	–
15	1573.3	3	12	0	–	3600.0	–
16	1827.1	8	12	0	–	3600.0	–
17	1907.2	12	12	1	–	3600.0	–
18	1322.4	7	12	0	–	3600.0	–
19	352.9	13	12	0	–	3600.0	–
20	332.1	1	12	0	–	3600.0	–
21	335.8	1	12	0	–	3600.0	–
22	1492.8	10	12	1	–	3600.0	–
23	1511.1	19	12	1	–	3600.0	–
24	1117.5	12	12	0	True	3600.0	–
25	1157.3	13	12	0	–	3600.0	–
26	1615.2	20	12	0	True	3600.0	–
27	3600.0	–	8	2	True	3600.0	–
28	1548.0	13	12	0	–	3600.0	–
29	2039.5	18	12	5	–	3600.0	–
30	1358.8	9	12	0	–	3600.0	–
31	1029.8	6	12	0	–	3600.0	–
32	1015.3	10	12	0	–	3600.0	–
33	2216.3	7	12	1	–	3600.0	–
34	1551.5	12	12	0	True	3600.0	–
35	693.4	12	12	1	–	3600.0	–
36	924.7	4	12	0	–	3600.0	–
37	1649.5	15	12	0	–	3600.0	–
38	1507.4	12	12	0	–	3600.0	–
39	1709.7	15	12	2	–	3600.0	–
40	2129.2	14	12	2	–	3600.0	–
41	1765.7	8	12	0	–	3600.0	–
42	2562.9	17	12	4	True	3600.0	–
43	778.2	2	12	0	–	3600.0	–
44	1609.2	6	12	0	–	3600.0	–
45	1263.4	12	12	0	–	3600.0	–
46	1229.3	11	12	1	True	3600.0	–
47	819.4	8	12	0	–	3600.0	–
48	1816.3	11	12	0	–	3600.0	–
49	2876.9	25	12	13	True	3600.0	–
50	809.4	13	12	0	–	3600.0	–
51	1386.6	11	12	1	–	3600.0	–
52	1217.3	12	12	1	–	3600.0	–
53	691.4	6	12	0	–	3600.0	–
54	1619.1	13	12	0	True	3600.0	–
55	929.9	8	12	0	–	3600.0	–
56	2292.8	20	12	1	–	3600.0	–
57	714.4	9	12	1	–	3600.0	–
58	1229.5	9	12	1	True	3600.0	–
59	1582.1	8	12	0	–	3600.0	–
60	787.5	11	12	0	–	3600.0	–
61	1311.8	11	12	1	–	3600.0	–
62	1193.1	5	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
63	173.6	0	12	0	–	3600.0	–
64	1149.3	10	12	0	True	3600.0	–
65	1506.2	5	12	0	–	3600.0	–
66	1532.5	12	12	0	–	3600.0	–
67	1581.7	10	12	0	–	3600.0	–
68	1541.0	14	12	0	–	3600.0	–
69	2021.6	11	12	2	–	3600.0	–
70	2737.1	11	12	2	–	3600.0	–
71	1360.3	20	12	0	–	3600.0	–
72	838.2	12	12	1	–	3600.0	–
73	1976.5	6	12	0	–	3600.0	–
74	1429.6	11	12	1	–	3600.0	–
75	1375.2	15	12	1	True	3600.0	–
76	1962.0	24	12	6	–	3600.0	–
77	583.2	0	12	0	–	3600.0	–
78	2063.2	11	12	0	–	3600.0	–
79	1123.5	11	12	0	–	3600.0	–
80	1103.0	15	12	1	–	3600.0	–
81	812.2	12	12	0	–	3600.0	–
82	1014.0	7	12	0	–	3600.0	–
83	2006.8	18	12	2	–	3600.0	–
84	2880.5	12	12	5	–	3600.0	–
85	1305.0	14	12	0	–	3600.0	–
86	1086.9	5	12	0	–	3600.0	–
87	1345.3	18	12	0	–	3600.0	–
88	2162.0	10	12	0	–	3600.0	–
89	1626.3	19	12	0	True	3600.0	–
90	828.8	3	12	0	–	3600.0	–
91	1656.0	8	12	0	–	3600.0	–
92	1530.8	6	12	0	–	3600.0	–
93	2206.8	11	12	2	–	3600.0	–
94	1343.4	17	12	0	–	3600.0	–
95	2082.8	11	12	0	–	3600.0	–
96	187.8	16	12	0	–	3600.0	–
97	2446.6	13	12	1	True	3600.0	–
98	1901.1	16	12	0	–	3600.0	–
99	1278.6	14	12	0	–	3600.0	–
100	735.3	2	12	0	–	3600.0	–
101	1246.2	11	12	0	–	3600.0	–
102	960.9	9	12	0	–	3600.0	–
103	1109.8	11	12	0	–	3600.0	–
104	1705.2	2	12	0	–	3600.0	–
105	2648.8	19	12	4	–	3600.0	–
106	1266.7	11	12	0	–	3600.0	–
107	3279.3	9	12	5	–	3600.0	–
108	501.1	10	12	0	–	3600.0	–
109	1986.5	17	12	3	–	3600.0	–
110	1968.2	13	12	6	–	3600.0	–
111	2096.2	2	12	0	True	3600.0	–
112	697.9	15	12	1	–	3600.0	–
113	1440.3	14	12	1	–	3600.0	–
114	1744.8	4	12	0	–	3600.0	–
115	3407.8	20	12	4	True	3600.0	–
116	2974.9	7	12	14	–	3600.0	–
117	1758.5	18	12	2	True	3600.0	–
118	1377.8	5	12	0	–	3600.0	–
119	725.6	11	12	1	True	3600.0	–
120	2077.4	10	12	7	–	3600.0	–
121	3151.9	17	12	5	True	3600.0	–
122	1285.5	14	12	0	–	3600.0	–
123	916.0	10	12	0	–	3600.0	–
124	1468.8	9	12	1	True	3600.0	–
125	1664.0	3	12	0	–	3600.0	–
126	1813.3	16	12	1	True	3600.0	–
127	1074.6	1	12	0	–	3600.0	–
128	1481.1	11	12	0	True	3600.0	–
129	2276.6	18	12	2	True	3600.0	–
130	1269.7	14	12	0	–	3600.0	–
131	2013.2	15	12	0	True	3600.0	–
132	2787.6	20	12	3	–	3600.0	–
133	137.8	0	12	0	–	3600.0	–
134	666.2	10	12	0	–	3600.0	–
135	3542.2	11	12	9	–	3600.0	–
136	1709.7	11	12	0	–	3600.0	–
137	208.3	0	12	0	–	3600.0	–
138	302.9	16	12	0	–	3600.0	–
139	928.4	10	12	0	–	3600.0	–
140	3570.1	26	12	9	True	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
141	1957.3	11	12	0	–	3600.0	–
142	1306.6	12	12	0	–	3600.0	–
143	1273.6	23	12	0	–	3600.0	–
144	967.7	17	12	0	–	3600.0	–
145	1667.4	13	12	1	–	3600.0	–
146	1622.2	32	12	0	True	3600.0	–
147	1380.5	11	12	0	–	3600.0	–
148	294.4	0	12	0	–	3600.0	–
149	3455.2	22	12	6	True	3600.0	–
150	518.0	0	12	0	–	3600.0	–
151	1105.4	9	12	1	–	3600.0	–
152	1897.9	19	12	0	–	3600.0	–
153	1600.1	17	12	0	–	3600.0	–
154	872.7	7	12	2	True	3600.0	–
155	633.2	3	12	0	–	3600.0	–
156	704.4	8	12	1	–	3600.0	–
157	2106.9	4	12	0	True	3600.0	–
158	1057.6	8	12	0	–	3600.0	–
159	1432.3	13	12	0	–	3600.0	–
160	1426.8	17	12	0	–	3600.0	–
161	1425.6	13	12	1	–	3600.0	–
162	1492.0	14	12	2	–	3600.0	–
163	3274.3	16	12	4	–	3600.0	–
164	211.1	20	12	0	–	3600.0	–
165	1539.0	18	12	0	–	3600.0	–
166	2110.2	12	12	0	True	3600.0	–
167	1383.0	15	12	0	–	3600.0	–
168	1574.4	12	12	1	–	3600.0	–
169	1644.8	8	12	0	–	3600.0	–
170	804.7	10	12	1	–	3600.0	–
171	1690.4	6	12	0	–	3600.0	–
172	2643.2	30	12	2	–	3600.0	–
173	871.9	12	12	0	–	3600.0	–
174	1197.5	13	12	1	True	3600.0	–
175	1853.4	13	12	4	True	3600.0	–
176	1898.5	17	12	3	–	3600.0	–
177	1059.7	16	12	0	–	3600.0	–
178	1741.0	26	12	0	–	3600.0	–
179	1715.7	9	12	0	–	3600.0	–
180	1171.4	10	12	1	True	3600.0	–
181	1811.2	15	12	3	–	3600.0	–
182	793.9	12	12	0	True	3600.0	–
183	1539.2	17	12	0	–	3600.0	–
184	1324.5	13	12	1	True	3600.0	–
185	1753.1	13	12	0	–	3600.0	–
186	2213.0	14	12	14	True	3600.0	–
187	90.8	0	12	0	–	3600.0	–
188	1771.2	9	12	0	–	3600.0	–
189	1768.5	13	12	1	True	3600.0	–
190	771.6	14	12	0	–	3600.0	–
191	988.8	8	12	0	–	3600.0	–
192	1067.7	15	12	1	True	3600.0	–
193	1732.5	3	12	0	–	3600.0	–
194	1589.4	15	12	0	–	3600.0	–
195	1071.3	11	12	1	–	3600.0	–
196	478.7	10	12	0	–	3600.0	–
197	1566.3	13	12	0	True	3600.0	–
198	1886.9	26	12	1	–	3600.0	–
199	208.5	0	12	0	–	3600.0	–
200	1462.5	6	12	1	–	3600.0	–
201	763.2	10	12	0	–	3600.0	–
202	2771.1	12	12	3	–	3600.0	–
203	2367.6	16	12	1	True	3600.0	–
204	559.9	1	12	0	–	3600.0	–
205	718.1	11	12	0	–	3600.0	–
206	1643.4	7	12	0	–	3600.0	–
207	833.7	19	12	0	–	3600.0	–
208	140.3	0	12	0	–	3600.0	–
209	1714.2	19	12	0	–	3600.0	–
210	2783.1	13	12	7	–	3600.0	–
211	176.9	9	12	0	–	3600.0	–
212	428.7	4	12	0	–	3600.0	–
213	1386.2	11	12	1	–	3600.0	–
214	1900.6	44	12	0	True	3600.0	–
215	1740.2	7	12	0	–	3600.0	–
216	1128.6	7	12	0	–	3600.0	–
217	1964.5	8	12	0	True	3600.0	–
218	1677.6	3	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
219	2092.9	11	12	1	–	3600.0	–
220	2057.2	12	12	3	–	3600.0	–
221	873.2	0	12	0	–	3600.0	–
222	2055.8	7	12	0	–	3600.0	–
223	1176.6	6	12	0	–	3600.0	–
224	957.0	11	12	1	True	3600.0	–
225	1390.3	17	12	0	True	3600.0	–
226	1004.0	8	12	0	–	3600.0	–
227	404.1	3	12	0	–	3600.0	–
228	998.9	7	12	0	–	3600.0	–
229	250.8	0	12	0	–	3600.0	–
230	2301.6	18	12	2	–	3600.0	–
231	2040.9	14	12	0	–	3600.0	–
232	1644.3	10	12	0	–	3600.0	–
233	831.6	4	12	0	–	3600.0	–
234	1629.2	16	12	1	–	3600.0	–
235	1566.7	7	12	0	–	3600.0	–
236	1248.6	8	12	0	–	3600.0	–
237	294.9	0	12	0	–	3600.0	–
238	830.2	13	12	0	–	3600.0	–
239	1680.9	6	12	2	–	3600.0	–
240	1304.3	12	12	0	–	3600.0	–
241	798.8	12	12	0	–	3600.0	–
242	494.3	0	12	0	–	3600.0	–
243	1994.5	28	12	1	–	3600.0	–
244	499.9	12	12	0	–	3600.0	–
245	1868.5	7	12	1	True	3600.0	–
246	1364.9	15	12	0	–	3600.0	–
247	1163.4	14	12	0	–	3600.0	–
248	1519.2	18	12	0	–	3600.0	–
249	1537.2	12	12	1	–	3600.0	–
250	1258.2	8	12	0	–	3600.0	–
251	2357.8	15	12	0	True	3600.0	–
252	1419.4	16	12	0	–	3600.0	–
253	1946.0	16	12	0	True	3600.0	–
254	340.0	3	12	0	–	3600.0	–
255	1461.2	13	12	0	–	3600.0	–
256	1549.8	21	12	0	True	3600.0	–
257	3397.8	12	12	4	–	3600.0	–
258	1268.7	20	12	1	–	3600.0	–
259	1384.2	11	12	0	–	3600.0	–
260	2207.2	12	12	3	True	3600.0	–
261	2121.6	6	12	0	–	3600.0	–
262	1322.2	8	12	0	–	3600.0	–
263	1704.6	18	12	0	–	3600.0	–
264	2739.9	16	12	2	–	3600.0	–
265	2310.4	8	12	5	–	3600.0	–
266	897.8	14	12	0	–	3600.0	–
267	1532.3	13	12	0	–	3600.0	–
268	1237.3	6	12	0	–	3600.0	–
269	2183.3	16	12	1	–	3600.0	–
270	387.1	0	12	0	–	3600.0	–
271	2830.5	7	12	4	True	3600.0	–
272	2495.9	12	12	5	–	3600.0	–
273	1643.2	12	12	1	–	3600.0	–
274	1295.5	14	12	0	–	3600.0	–
275	1870.6	10	12	0	–	3600.0	–
276	1110.1	12	12	0	True	3600.0	–
277	1215.7	15	12	0	–	3600.0	–
278	408.1	10	12	0	–	3600.0	–
279	1431.8	13	12	0	–	3600.0	–
280	1423.5	18	12	0	–	3600.0	–
281	1223.2	7	12	0	–	3600.0	–
282	1487.4	7	12	0	–	3600.0	–
283	205.6	0	12	0	–	3600.0	–
284	1640.6	20	12	1	True	3600.0	–
285	559.5	18	12	0	–	3600.0	–
286	1894.2	12	12	2	–	3600.0	–
287	924.8	10	12	0	–	3600.0	–
288	1094.5	19	12	0	–	3600.0	–
289	1630.8	12	12	0	–	3600.0	–
290	394.9	0	12	0	–	3600.0	–
291	1573.3	8	12	0	–	3600.0	–
292	1391.8	10	12	0	–	3600.0	–
293	1522.4	7	12	0	–	3600.0	–
294	3058.6	22	12	3	–	3600.0	–
295	1265.9	5	12	0	True	3600.0	–
296	1451.7	7	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
297	1672.1	11	12	0	True	3600.0	–
298	329.3	3	12	0	–	3600.0	–
299	1507.4	5	12	0	–	3600.0	–
300	1329.6	11	12	0	–	3600.0	–
301	1758.6	21	12	0	True	3600.0	–
302	1991.7	11	12	5	–	3600.0	–
303	966.2	8	12	0	–	3600.0	–
304	1399.7	14	12	0	–	3600.0	–
305	438.0	19	12	0	–	3600.0	–
306	1795.0	19	12	0	–	3600.0	–
307	1696.4	4	12	0	–	3600.0	–
308	3324.2	12	12	3	–	3600.0	–
309	1817.4	6	12	0	–	3600.0	–
310	833.6	18	12	0	–	3600.0	–
311	1587.5	6	12	0	–	3600.0	–
312	950.8	5	12	0	–	3600.0	–
313	1114.3	8	12	2	–	3600.0	–
314	884.5	11	12	0	True	3600.0	–
315	931.1	14	12	0	–	3600.0	–
316	754.5	12	12	0	True	3600.0	–
317	1543.8	8	12	1	–	3600.0	–
318	970.4	18	12	0	–	3600.0	–
319	1479.8	7	12	0	–	3600.0	–
320	319.0	20	12	0	–	3600.0	–
321	2882.9	14	12	2	–	3600.0	–
322	1294.3	15	12	0	–	3600.0	–
323	1571.2	14	12	2	–	3600.0	–
324	688.5	11	12	0	–	3600.0	–
325	1517.9	13	12	1	True	3600.0	–
326	1537.4	11	12	0	–	3600.0	–
327	1383.1	12	12	1	–	3600.0	–
328	333.3	16	12	0	–	3600.0	–
329	1780.4	10	12	3	True	3600.0	–
330	1415.8	11	12	0	–	3600.0	–
331	450.5	2	12	0	–	3600.0	–
332	579.2	18	12	1	–	3600.0	–
333	1690.8	20	12	0	True	3600.0	–
334	201.6	0	12	0	–	3600.0	–
335	1534.7	25	12	1	–	3600.0	–
336	754.0	20	12	0	–	3600.0	–
337	1832.2	2	12	0	–	3600.0	–
338	1251.6	19	12	1	–	3600.0	–
339	1190.9	15	12	0	–	3600.0	–
340	1470.9	12	12	0	–	3600.0	–
341	600.4	10	12	0	–	3600.0	–
342	1177.5	16	12	0	–	3600.0	–
343	708.9	4	12	0	–	3600.0	–
344	1962.2	12	12	4	True	3600.0	–
345	2692.9	15	12	2	–	3600.0	–
346	2004.8	7	12	0	–	3600.0	–
347	1147.3	8	12	1	–	3600.0	–
348	585.0	6	12	0	–	3600.0	–
349	1488.0	6	12	1	–	3600.0	–
350	1385.7	14	12	0	–	3600.0	–
351	2100.1	18	12	7	True	3600.0	–
352	1417.1	14	12	0	True	3600.0	–
353	1941.7	3	12	0	True	3600.0	–
354	496.9	11	12	0	–	3600.0	–
355	1453.9	11	12	1	–	3600.0	–
356	743.1	2	12	0	–	3600.0	–
357	1761.3	13	12	1	–	3600.0	–
358	2418.9	14	12	14	True	3600.0	–
359	1126.1	6	12	0	–	3600.0	–
360	1900.7	4	12	0	–	3600.0	–
361	2283.0	9	12	3	–	3600.0	–
362	1904.4	15	12	0	–	3600.0	–
363	910.1	15	12	0	–	3600.0	–
364	1592.6	14	12	0	True	3600.0	–
365	1395.7	11	12	0	–	3600.0	–
366	260.7	0	12	0	–	3600.0	–
367	2716.1	24	12	3	–	3600.0	–
368	1611.1	14	12	0	True	3600.0	–
369	329.5	8	12	0	–	3600.0	–
370	3600.0	–	9	26	True	3600.0	–
371	1387.3	8	12	2	–	3600.0	–
372	1398.3	17	12	0	True	3600.0	–
373	2568.9	8	12	9	–	3600.0	–
374	1633.5	18	12	1	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
375	1485.8	7	12	0	–	3600.0	–
376	1615.2	14	12	1	–	3600.0	–
377	1845.7	11	12	1	True	3600.0	–
378	914.9	11	12	1	–	3600.0	–
379	822.7	14	12	0	True	3600.0	–
380	1658.0	13	12	0	–	3600.0	–
381	933.2	3	12	0	–	3600.0	–
382	1803.2	12	12	2	–	3600.0	–
383	553.0	2	12	0	–	3600.0	–
384	1151.2	17	12	0	–	3600.0	–
385	2096.9	21	12	1	–	3600.0	–
386	1259.2	10	12	0	–	3600.0	–
387	1904.5	6	12	0	–	3600.0	–
388	2442.0	13	12	2	–	3600.0	–
389	1579.7	19	12	0	–	3600.0	–
390	1914.7	17	12	1	–	3600.0	–
391	409.8	0	12	0	–	3600.0	–
392	1516.6	20	12	0	–	3600.0	–
393	1981.4	12	12	1	–	3600.0	–
394	1531.3	19	12	0	–	3600.0	–
395	1792.7	7	12	0	True	3600.0	–
396	1231.4	6	12	1	True	3600.0	–
397	720.9	20	12	1	–	3600.0	–
398	2085.1	8	12	0	True	3600.0	–
399	1016.1	17	12	0	–	3600.0	–
400	1693.0	17	12	1	–	3600.0	–
401	1400.8	6	12	0	True	3600.0	–
402	1893.5	3	12	0	–	3600.0	–
403	705.8	10	12	0	–	3600.0	–
404	2285.2	22	12	11	True	3600.0	–
405	1097.8	9	12	0	–	3600.0	–
406	869.3	14	12	1	–	3600.0	–
407	1571.6	18	12	0	–	3600.0	–
408	2141.2	4	12	0	True	3600.0	–
409	1438.2	11	12	1	True	3600.0	–
410	995.1	16	12	0	–	3600.0	–
411	1516.3	19	12	0	–	3600.0	–
412	1271.1	11	12	0	–	3600.0	–
413	1375.9	16	12	1	–	3600.0	–
414	2018.5	6	12	0	–	3600.0	–
415	513.2	1	12	0	–	3600.0	–
416	618.1	18	12	0	–	3600.0	–
417	504.1	20	12	0	–	3600.0	–
418	783.7	8	12	0	–	3600.0	–
419	1747.2	3	12	0	–	3600.0	–
420	763.4	0	12	0	–	3600.0	–
421	984.0	14	12	0	–	3600.0	–
422	1368.1	9	12	1	–	3600.0	–
423	1414.5	15	12	1	–	3600.0	–
424	1271.1	19	12	1	–	3600.0	–
425	1300.7	15	12	0	–	3600.0	–
426	1910.3	12	12	1	True	3600.0	–
427	1242.3	1	12	0	–	3600.0	–
428	1400.3	24	12	0	–	3600.0	–
429	2608.3	12	12	2	–	3600.0	–
430	2417.9	22	12	3	–	3600.0	–
431	1073.9	11	12	1	–	3600.0	–
432	1951.2	23	12	0	True	3600.0	–
433	1605.7	15	12	1	–	3600.0	–
434	1875.8	12	12	2	True	3600.0	–
435	1564.6	15	12	1	–	3600.0	–
436	776.5	3	12	0	–	3600.0	–
437	1780.8	12	12	1	–	3600.0	–
438	1212.4	8	12	0	–	3600.0	–
439	1499.5	10	12	0	–	3600.0	–
440	325.0	0	12	0	–	3600.0	–
441	1567.4	8	12	0	–	3600.0	–
442	1909.4	12	12	0	–	3600.0	–
443	1827.3	10	12	3	–	3600.0	–
444	1871.2	10	12	0	–	3600.0	–
445	2082.4	8	12	0	–	3600.0	–
446	1639.4	12	12	0	–	3600.0	–
447	1304.5	11	12	0	–	3600.0	–
448	1435.4	17	12	0	–	3600.0	–
449	1115.0	6	12	0	–	3600.0	–
450	361.0	16	12	0	–	3600.0	–
451	1922.7	17	12	0	–	3600.0	–
452	1835.0	15	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
453	1900.9	15	12	0	True	3600.0	–
454	1821.7	2	12	0	–	3600.0	–
455	776.6	4	12	0	–	3600.0	–
456	2860.4	23	12	3	–	3600.0	–
457	1063.3	15	12	0	–	3600.0	–
458	1305.8	4	12	0	True	3600.0	–
459	1051.0	21	12	0	–	3600.0	–
460	1680.2	14	12	0	True	3600.0	–
461	1683.0	4	12	0	–	3600.0	–
462	1197.1	8	12	0	True	3600.0	–
463	1488.5	20	12	0	–	3600.0	–
464	3600.0	–	1	0	True	3600.0	–
465	324.9	4	12	0	–	3600.0	–
466	856.9	11	12	0	–	3600.0	–
467	315.9	16	12	0	–	3600.0	–
468	735.5	4	12	0	–	3600.0	–
469	304.9	14	12	0	–	3600.0	–
470	2147.8	12	12	2	–	3600.0	–
471	346.2	2	12	0	–	3600.0	–
472	332.4	19	12	0	–	3600.0	–
473	694.6	2	12	0	–	3600.0	–
474	2142.8	14	12	0	True	3600.0	–
475	1184.7	7	12	0	–	3600.0	–
476	1940.3	11	12	0	–	3600.0	–
477	1173.0	13	12	0	–	3600.0	–
478	1827.8	12	12	0	–	3600.0	–
479	850.2	11	12	0	–	3600.0	–
480	1800.8	6	12	6	True	3600.0	–
481	1933.8	29	12	2	–	3600.0	–
482	905.6	9	12	0	–	3600.0	–
483	606.5	15	12	0	–	3600.0	–
484	1671.7	5	12	0	–	3600.0	–
485	1308.2	12	12	0	–	3600.0	–
486	1257.6	11	12	0	–	3600.0	–
487	846.2	6	12	0	–	3600.0	–
488	717.9	6	12	0	–	3600.0	–
489	1825.7	4	12	0	–	3600.0	–
490	923.6	15	12	0	–	3600.0	–
491	1122.4	18	12	0	–	3600.0	–
492	2095.5	24	12	5	True	3600.0	–
493	326.1	1	12	1	–	3600.0	–
494	1217.0	10	12	0	True	3600.0	–
495	1832.9	9	12	0	True	3600.0	–
496	1390.0	14	12	0	True	3600.0	–
497	832.9	10	12	0	–	3600.0	–
498	215.7	0	12	0	–	3600.0	–
499	1064.2	11	12	1	–	3600.0	–
500	1044.8	10	12	0	–	3600.0	–
501	3600.0	–	3	4	–	3600.0	–
502	1785.1	18	12	1	True	3600.0	–
503	688.7	3	12	0	–	3600.0	–
504	306.8	10	12	0	–	3600.0	–
505	1775.3	17	12	0	–	3600.0	–
506	2077.6	8	12	12	–	3600.0	–
507	1067.4	10	12	0	–	3600.0	–
508	2825.2	12	12	3	–	3600.0	–
509	1116.9	10	12	0	–	3600.0	–
510	608.5	0	12	0	–	3600.0	–
511	1196.9	1	12	0	–	3600.0	–
512	2101.8	4	12	0	True	3600.0	–
513	335.7	2	12	0	–	3600.0	–
514	1623.4	7	12	0	–	3600.0	–
515	932.7	11	12	0	–	3600.0	–
516	1570.5	4	12	0	–	3600.0	–
517	937.0	10	12	0	True	3600.0	–
518	324.5	0	12	0	–	3600.0	–
519	1125.7	13	12	0	–	3600.0	–
520	593.8	8	12	0	–	3600.0	–
521	3600.0	–	4	8	–	3600.0	–
522	770.3	11	12	0	–	3600.0	–
523	1650.8	12	12	0	–	3600.0	–
524	1871.3	12	12	0	–	3600.0	–
525	1909.3	9	12	0	–	3600.0	–
526	1496.9	12	12	1	–	3600.0	–
527	1178.4	15	12	0	–	3600.0	–
528	1285.4	12	12	0	True	3600.0	–
529	1530.3	14	12	1	–	3600.0	–
530	2233.1	17	12	2	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
531	1177.0	16	12	3	True	3600.0	–
532	1393.4	15	12	2	–	3600.0	–
533	920.5	11	12	0	–	3600.0	–
534	460.0	6	12	0	–	3600.0	–
535	1503.9	5	12	0	–	3600.0	–
536	812.3	20	12	0	–	3600.0	–
537	1626.2	13	12	0	–	3600.0	–
538	2111.0	15	12	1	True	3600.0	–
539	1678.3	12	12	0	–	3600.0	–
540	981.6	12	12	2	–	3600.0	–
541	531.4	4	12	0	–	3600.0	–
542	1174.3	4	12	0	–	3600.0	–
543	1510.0	22	12	0	–	3600.0	–
544	2131.3	11	12	8	True	3600.0	–
545	1852.3	9	12	0	–	3600.0	–
546	3600.0	–	4	3	True	3600.0	–
547	942.1	0	12	0	–	3600.0	–
548	574.2	7	12	0	–	3600.0	–
549	1870.2	13	12	1	–	3600.0	–
550	1493.8	15	12	1	–	3600.0	–
551	2764.1	18	12	4	–	3600.0	–
552	2127.7	23	12	2	–	3600.0	–
553	1945.4	4	12	0	–	3600.0	–
554	2371.9	13	12	9	–	3600.0	–
555	588.0	12	12	0	–	3600.0	–
556	1101.6	12	12	0	–	3600.0	–
557	1059.0	18	12	0	–	3600.0	–
558	815.6	9	12	0	–	3600.0	–
559	1422.4	11	12	0	–	3600.0	–
560	1321.8	4	12	0	–	3600.0	–
561	1173.2	7	12	0	–	3600.0	–
562	1242.1	14	12	0	–	3600.0	–
563	1220.4	12	12	1	True	3600.0	–
564	1820.4	12	12	0	–	3600.0	–
565	267.5	0	12	0	–	3600.0	–
566	1933.7	6	12	0	–	3600.0	–
567	2672.9	25	12	20	–	3600.0	–
568	2934.6	14	12	4	–	3600.0	–
569	1310.5	10	12	0	–	3600.0	–
570	740.4	10	12	0	–	3600.0	–
571	1138.5	20	12	0	–	3600.0	–
572	1582.7	11	12	1	–	3600.0	–
573	2171.1	15	12	1	–	3600.0	–
574	965.8	11	12	0	–	3600.0	–
575	1528.9	20	12	1	–	3600.0	–
576	1024.1	9	12	0	–	3600.0	–
577	1719.1	12	12	0	–	3600.0	–
578	593.1	10	12	0	–	3600.0	–
579	1342.0	12	12	0	–	3600.0	–
580	1630.0	13	12	0	–	3600.0	–
581	266.2	0	12	0	–	3600.0	–
582	1887.0	12	12	1	True	3600.0	–
583	1622.3	11	12	0	–	3600.0	–
584	1384.7	9	12	0	–	3600.0	–
585	756.1	18	12	0	–	3600.0	–
586	729.6	11	12	1	–	3600.0	–
587	88.0	0	12	0	–	3600.0	–
588	3233.4	7	12	4	–	3600.0	–
589	2422.4	9	12	0	True	3600.0	–
590	2124.5	9	12	0	–	3600.0	–
591	1222.8	12	12	0	–	3600.0	–
592	2911.3	19	12	2	–	3600.0	–
593	3362.9	11	12	8	True	3600.0	–
594	1933.5	8	12	3	True	3600.0	–
595	1571.7	13	12	0	True	3600.0	–
596	1315.9	8	12	0	–	3600.0	–
597	919.0	19	12	0	–	3600.0	–
598	870.6	11	12	0	–	3600.0	–
599	1179.1	20	12	1	–	3600.0	–
600	1771.9	18	12	0	–	3600.0	–
601	3101.8	15	12	4	True	3600.0	–
602	901.9	14	12	0	–	3600.0	–
603	1661.7	8	12	0	–	3600.0	–
604	2382.5	14	12	1	–	3600.0	–
605	875.8	16	12	0	–	3600.0	–
606	1591.5	8	12	0	–	3600.0	–
607	1022.9	12	12	0	–	3600.0	–
608	752.7	18	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
609	1326.3	12	12	0	–	3600.0	–
610	2285.5	11	12	2	–	3600.0	–
611	786.1	15	12	0	–	3600.0	–
612	2351.3	12	12	1	True	3600.0	–
613	1410.8	16	12	0	True	3600.0	–
614	1163.6	5	12	0	–	3600.0	–
615	2153.4	11	12	0	–	3600.0	–
616	214.9	14	12	0	–	3600.0	–
617	2259.3	16	12	2	–	3600.0	–
618	1636.5	7	12	0	–	3600.0	–
619	1482.4	20	12	0	–	3600.0	–
620	66.0	0	12	0	–	3600.0	–
621	1716.1	3	12	0	–	3600.0	–
622	1633.5	15	12	0	True	3600.0	–
623	1224.7	13	12	0	–	3600.0	–
624	2892.2	12	12	2	–	3600.0	–
625	1690.2	6	12	0	–	3600.0	–
626	1934.0	11	12	1	–	3600.0	–
627	1905.1	10	12	0	–	3600.0	–
628	3504.9	11	12	6	–	3600.0	–
629	3600.0	–	2	1	True	3600.0	–
630	1769.4	8	12	0	–	3600.0	–
631	1999.6	26	12	0	True	3600.0	–
632	2540.2	33	12	2	–	3600.0	–
633	1788.3	11	12	3	–	3600.0	–
634	1690.4	12	12	0	–	3600.0	–
635	3600.0	–	5	6	–	3600.0	–
636	1896.5	12	12	0	–	3600.0	–
637	1485.9	4	12	0	True	3600.0	–
638	3600.0	–	1	0	–	3600.0	–
639	2044.9	11	12	1	–	3600.0	–
640	836.0	7	12	0	–	3600.0	–
641	1751.7	7	12	0	–	3600.0	–
642	1069.4	12	12	0	–	3600.0	–
643	1766.9	19	12	1	–	3600.0	–
644	434.9	7	12	0	–	3600.0	–
645	532.4	1	12	0	–	3600.0	–
646	779.6	4	12	0	–	3600.0	–
647	885.3	6	12	0	True	3600.0	–
648	2279.2	10	12	3	True	3600.0	–
649	2009.0	38	12	0	True	3600.0	–
650	3600.0	–	1	0	–	3600.0	–
651	1435.7	11	12	0	–	3600.0	–
652	1577.3	12	12	0	–	3600.0	–
653	1221.3	10	12	0	–	3600.0	–
654	1723.0	13	12	1	True	3600.0	–
655	2143.8	11	12	0	–	3600.0	–
656	1753.5	13	12	0	–	3600.0	–
657	1499.5	4	12	0	–	3600.0	–
658	1911.5	9	12	0	–	3600.0	–
659	3600.0	–	1	0	–	3600.0	–
660	1091.9	14	12	0	–	3600.0	–
661	1840.8	18	12	2	–	3600.0	–
662	3600.0	–	10	3	True	3600.0	–
663	2001.4	4	12	0	–	3600.0	–
664	1189.2	12	12	0	–	3600.0	–
665	2088.4	15	12	0	True	3600.0	–
666	200.3	0	12	0	–	3600.0	–
667	1682.2	14	12	2	–	3600.0	–
668	3600.0	–	5	8	–	3600.0	–
669	1580.7	17	12	0	–	3600.0	–
670	1408.2	7	12	0	–	3600.0	–
671	492.4	0	12	0	–	3600.0	–
672	435.0	10	12	0	–	3600.0	–
673	3600.0	–	6	42	True	3600.0	–
674	1669.3	18	12	1	–	3600.0	–
675	3162.1	12	12	6	–	3600.0	–
676	2235.7	9	12	0	–	3600.0	–
677	2037.7	20	12	2	–	3600.0	–
678	2059.6	11	12	10	True	3600.0	–
679	2128.3	7	12	2	True	3600.0	–
680	1334.6	20	12	0	–	3600.0	–
681	1571.3	12	12	2	True	3600.0	–
682	902.4	12	12	0	–	3600.0	–
683	1859.4	12	12	0	True	3600.0	–
684	1341.0	16	12	0	–	3600.0	–
685	1292.9	14	12	0	–	3600.0	–
686	585.7	18	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
687	2659.4	18	12	3	–	3600.0	–
688	2012.2	5	12	0	–	3600.0	–
689	1928.3	38	12	0	True	3600.0	–
690	997.7	13	12	1	–	3600.0	–
691	3600.0	–	2	1	–	3600.0	–
692	1653.9	12	12	0	–	3600.0	–
693	3449.0	12	12	3	–	3600.0	–
694	2138.1	11	12	0	–	3600.0	–
695	1415.3	16	12	0	–	3600.0	–
696	2599.6	17	12	2	True	3600.0	–
697	1411.9	17	12	0	–	3600.0	–
698	987.5	11	12	0	–	3600.0	–
699	1529.5	6	12	0	–	3600.0	–
700	86.9	0	12	0	–	3600.0	–
701	816.9	9	12	0	–	3600.0	–
702	936.4	5	12	0	–	3600.0	–
703	208.6	0	12	0	–	3600.0	–
704	2224.6	9	12	0	–	3600.0	–
705	1653.1	12	12	0	–	3600.0	–
706	1113.5	15	12	1	True	3600.0	–
707	949.1	8	12	0	–	3600.0	–
708	2324.8	12	12	2	True	3600.0	–
709	3435.0	14	12	4	True	3600.0	–
710	1286.6	11	12	1	–	3600.0	–
711	3600.0	–	8	1	True	3600.0	–
712	1348.5	10	12	0	–	3600.0	–
713	1967.0	6	12	0	True	3600.0	–
714	654.3	3	12	0	–	3600.0	–
715	2337.5	14	12	0	True	3600.0	–
716	1929.7	10	12	0	–	3600.0	–
717	830.4	17	12	1	–	3600.0	–
718	1065.7	14	12	1	–	3600.0	–
719	2453.0	15	12	1	True	3600.0	–
720	2019.0	14	12	0	–	3600.0	–
721	1074.6	11	12	0	–	3600.0	–
722	202.2	15	12	0	–	3600.0	–
723	1065.6	15	12	0	–	3600.0	–
724	2208.4	5	12	0	True	3600.0	–
725	785.7	18	12	0	–	3600.0	–
726	1318.2	10	12	0	–	3600.0	–
727	1990.5	29	12	2	–	3600.0	–
728	1306.4	15	12	0	True	3600.0	–
729	2069.3	12	12	0	True	3600.0	–
730	2116.2	10	12	0	True	3600.0	–
731	1314.9	11	12	2	–	3600.0	–
732	3600.0	–	1	0	True	3600.0	–
733	2128.1	6	12	0	–	3600.0	–
734	2189.2	16	12	7	True	3600.0	–
735	2188.1	13	12	0	True	3600.0	–
736	284.4	0	12	0	–	3600.0	–
737	1595.9	12	12	2	–	3600.0	–
738	3433.4	15	12	3	–	3600.0	–
739	1274.0	9	12	0	–	3600.0	–
740	1963.7	11	12	0	True	3600.0	–
741	955.3	10	12	2	–	3600.0	–
742	1080.1	13	12	0	–	3600.0	–
743	2343.1	17	12	8	–	3600.0	–
744	2117.2	14	12	3	–	3600.0	–
745	1725.8	27	12	1	True	3600.0	–
746	2252.9	11	12	5	–	3600.0	–
747	3425.7	10	12	7	True	3600.0	–
748	336.7	6	12	0	–	3600.0	–
749	568.5	16	12	0	–	3600.0	–
750	1106.6	12	12	1	–	3600.0	–
751	3063.4	12	12	2	True	3600.0	–
752	1468.0	15	12	0	–	3600.0	–
753	2265.4	20	12	0	True	3600.0	–
754	525.3	3	12	0	–	3600.0	–
755	3600.0	–	11	0	True	3600.0	–
756	1921.2	19	12	0	–	3600.0	–
757	1649.3	11	12	0	–	3600.0	–
758	530.5	7	12	0	–	3600.0	–
759	3600.0	–	2	1	True	3600.0	–
760	1723.2	12	12	0	–	3600.0	–
761	1192.1	13	12	0	–	3600.0	–
762	1920.5	17	12	0	True	3600.0	–
763	1174.0	14	12	0	–	3600.0	–
764	2602.8	12	12	11	True	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
765	2257.6	14	12	0	–	3600.0	–
766	2161.6	4	12	0	True	3600.0	–
767	410.3	0	12	0	–	3600.0	–
768	1511.2	13	12	0	–	3600.0	–
769	1274.4	13	12	0	–	3600.0	–
770	784.0	11	12	0	True	3600.0	–
771	1682.5	11	12	0	–	3600.0	–
772	1349.5	20	12	0	–	3600.0	–
773	1685.3	4	12	0	–	3600.0	–
774	2280.7	9	12	4	–	3600.0	–
775	1042.4	14	12	0	–	3600.0	–
776	1369.5	14	12	0	–	3600.0	–
777	3034.0	8	12	5	–	3600.0	–
778	1764.7	12	12	0	–	3600.0	–
779	1755.3	12	12	1	–	3600.0	–
780	3353.7	12	12	4	–	3600.0	–
781	1000.9	14	12	0	–	3600.0	–
782	698.9	14	12	0	–	3600.0	–
783	459.7	0	12	0	–	3600.0	–
784	492.0	11	12	0	–	3600.0	–
785	309.9	8	12	0	–	3600.0	–
786	1928.1	15	12	1	–	3600.0	–
787	3333.8	17	12	4	True	3600.0	–
788	1761.5	8	12	0	–	3600.0	–
789	898.4	5	12	1	–	3600.0	–
790	2172.5	8	12	0	–	3600.0	–
791	1504.7	1	12	0	–	3600.0	–
792	1713.8	18	12	0	–	3600.0	–
793	1640.1	3	12	0	–	3600.0	–
794	468.8	6	12	0	–	3600.0	–
795	1602.0	6	12	0	True	3600.0	–
796	626.0	4	12	0	–	3600.0	–
797	1774.9	8	12	0	–	3600.0	–
798	1281.3	20	12	0	–	3600.0	–
799	1150.3	7	12	0	–	3600.0	–
800	963.7	11	12	0	–	3600.0	–
801	1663.2	6	12	0	–	3600.0	–
802	549.5	3	12	0	–	3600.0	–
803	326.1	1	12	0	–	3600.0	–
804	1588.2	17	12	0	True	3600.0	–
805	1311.9	10	12	0	–	3600.0	–
806	305.4	17	12	0	–	3600.0	–
807	450.5	15	12	0	–	3600.0	–
808	1495.8	21	12	0	–	3600.0	–
809	1911.6	22	12	0	True	3600.0	–
810	1722.0	11	12	0	–	3600.0	–
811	1564.8	11	12	0	–	3600.0	–
812	940.0	14	12	0	–	3600.0	–
813	1891.9	19	12	0	–	3600.0	–
814	1389.1	11	12	0	–	3600.0	–
815	1531.2	13	12	0	–	3600.0	–
816	2033.3	8	12	2	–	3600.0	–
817	507.3	7	12	0	–	3600.0	–
818	2059.7	5	12	0	–	3600.0	–
819	1570.5	19	12	1	–	3600.0	–
820	1174.1	15	12	0	–	3600.0	–
821	326.1	4	12	0	–	3600.0	–
822	2395.8	13	12	2	True	3600.0	–
823	1651.5	10	12	0	–	3600.0	–
824	1943.4	14	12	1	–	3600.0	–
825	1653.0	21	12	1	–	3600.0	–
826	1349.0	16	12	1	–	3600.0	–
827	1353.2	9	12	0	–	3600.0	–
828	2030.4	18	12	2	–	3600.0	–
829	1143.6	19	12	2	True	3600.0	–
830	1845.3	2	12	0	–	3600.0	–
831	1141.0	9	12	0	–	3600.0	–
832	1756.8	8	12	0	–	3600.0	–
833	757.5	6	12	0	–	3600.0	–
834	2473.1	11	12	4	–	3600.0	–
835	1204.7	12	12	0	–	3600.0	–
836	1801.0	12	12	0	–	3600.0	–
837	1656.5	8	12	0	–	3600.0	–
838	1453.1	17	12	0	–	3600.0	–
839	1310.0	19	12	0	–	3600.0	–
840	464.3	4	12	0	–	3600.0	–
841	1871.2	2	12	0	–	3600.0	–
842	1470.7	8	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
843	1808.3	19	12	1	–	3600.0	–
844	3600.0	–	1	0	True	3600.0	–
845	760.2	8	12	1	–	3600.0	–
846	1657.1	20	12	0	–	3600.0	–
847	926.7	20	12	1	–	3600.0	–
848	675.7	2	12	0	–	3600.0	–
849	2012.8	12	12	0	True	3600.0	–
850	2909.9	17	12	5	–	3600.0	–
851	989.0	8	12	0	–	3600.0	–
852	840.5	4	12	0	–	3600.0	–
853	565.8	10	12	0	–	3600.0	–
854	2057.0	26	12	2	–	3600.0	–
855	1681.9	8	12	1	–	3600.0	–
856	1391.5	19	12	0	True	3600.0	–
857	1535.9	10	12	0	True	3600.0	–
858	944.9	9	12	0	–	3600.0	–
859	572.8	3	12	0	–	3600.0	–
860	1369.1	8	12	0	–	3600.0	–
861	1831.2	19	12	0	–	3600.0	–
862	2179.2	14	12	9	True	3600.0	–
863	670.9	16	12	0	–	3600.0	–
864	1779.8	8	12	0	–	3600.0	–
865	1516.3	13	12	0	–	3600.0	–
866	1008.9	13	12	0	–	3600.0	–
867	3526.3	23	12	3	–	3600.0	–
868	1046.1	11	12	1	–	3600.0	–
869	623.4	20	12	0	–	3600.0	–
870	1857.0	11	12	4	–	3600.0	–
871	467.8	18	12	0	–	3600.0	–
872	1380.2	14	12	0	True	3600.0	–
873	1611.9	11	12	1	–	3600.0	–
874	831.9	14	12	0	True	3600.0	–
875	1951.3	8	12	0	–	3600.0	–
876	1485.1	15	12	0	–	3600.0	–
877	2862.0	11	12	3	–	3600.0	–
878	1912.2	5	12	0	–	3600.0	–
879	1379.2	15	12	0	–	3600.0	–
880	1321.6	12	12	0	–	3600.0	–
881	713.7	12	12	0	–	3600.0	–
882	1964.1	12	12	0	True	3600.0	–
883	1642.2	11	12	0	–	3600.0	–
884	1456.5	12	12	0	–	3600.0	–
885	1953.7	19	12	6	True	3600.0	–
886	2005.0	16	12	0	–	3600.0	–
887	1232.0	7	12	0	–	3600.0	–
888	1701.5	11	12	1	–	3600.0	–
889	1142.9	9	12	0	–	3600.0	–
890	1992.5	16	12	2	True	3600.0	–
891	1149.7	18	12	0	–	3600.0	–
892	1290.5	16	12	1	–	3600.0	–
893	1656.5	9	12	0	–	3600.0	–
894	1363.5	11	12	1	–	3600.0	–
895	1081.2	5	12	3	–	3600.0	–
896	179.6	0	12	0	–	3600.0	–
897	2875.2	17	12	10	True	3600.0	–
898	3600.0	–	4	3	True	3600.0	–
899	2245.4	6	12	0	–	3600.0	–
900	1623.2	12	12	0	–	3600.0	–
901	1290.0	16	12	0	–	3600.0	–
902	1450.9	15	12	1	–	3600.0	–
903	1098.5	9	12	0	–	3600.0	–
904	3600.0	–	9	15	True	3600.0	–
905	1367.6	13	12	1	–	3600.0	–
906	1647.8	15	12	1	–	3600.0	–
907	1594.2	7	12	0	–	3600.0	–
908	1396.2	12	12	0	–	3600.0	–
909	1291.1	7	12	0	–	3600.0	–
910	701.1	16	12	0	–	3600.0	–
911	1669.3	13	12	0	–	3600.0	–
912	1331.6	12	12	1	True	3600.0	–
913	2062.5	12	12	0	True	3600.0	–
914	1135.3	19	12	0	True	3600.0	–
915	906.5	16	12	0	–	3600.0	–
916	1543.3	20	12	2	–	3600.0	–
917	1616.8	11	12	2	–	3600.0	–
918	3600.0	–	3	4	–	3600.0	–
919	195.0	0	12	0	–	3600.0	–
920	3600.0	–	5	8	True	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
921	1530.9	13	12	0	–	3600.0	–
922	1131.2	5	12	0	–	3600.0	–
923	481.3	11	12	0	–	3600.0	–
924	2799.5	13	12	3	True	3600.0	–
925	754.2	19	12	0	True	3600.0	–
926	944.5	5	12	0	–	3600.0	–
927	2199.9	11	12	2	True	3600.0	–
928	941.0	6	12	0	–	3600.0	–
929	1334.3	2	12	0	–	3600.0	–
930	827.1	14	12	0	–	3600.0	–
931	1330.3	11	12	0	–	3600.0	–
932	1711.8	8	12	2	–	3600.0	–
933	809.3	4	12	0	–	3600.0	–
934	1632.8	16	12	0	–	3600.0	–
935	1385.4	24	12	0	–	3600.0	–
936	2929.2	8	12	4	–	3600.0	–
937	1867.0	10	12	0	–	3600.0	–
938	1754.0	12	12	3	True	3600.0	–
939	1705.8	12	12	0	–	3600.0	–
940	1535.5	19	12	0	–	3600.0	–
941	3600.0	–	2	1	True	3600.0	–
942	485.9	1	12	0	–	3600.0	–
943	399.1	14	12	0	–	3600.0	–
944	941.7	14	12	0	–	3600.0	–
945	2136.0	7	12	0	–	3600.0	–
946	2211.0	22	12	2	–	3600.0	–
947	1229.3	8	12	0	–	3600.0	–
948	1162.7	13	12	0	True	3600.0	–
949	1800.0	17	12	1	–	3600.0	–
950	1642.0	13	12	2	–	3600.0	–
951	922.0	14	12	0	–	3600.0	–
952	2263.7	11	12	7	True	3600.0	–
953	1116.5	12	12	0	True	3600.0	–
954	2928.0	15	12	6	True	3600.0	–
955	2143.4	19	12	4	True	3600.0	–
956	1114.0	10	12	2	–	3600.0	–
957	1045.4	8	12	0	–	3600.0	–
958	519.6	4	12	0	–	3600.0	–
959	953.3	14	12	0	True	3600.0	–
960	3600.0	–	4	3	True	3600.0	–
961	1612.1	11	12	3	–	3600.0	–
962	895.9	10	12	0	–	3600.0	–
963	564.5	18	12	0	–	3600.0	–
964	1409.3	13	12	2	–	3600.0	–
965	1436.2	10	12	0	–	3600.0	–
966	1081.9	10	12	0	–	3600.0	–
967	3170.3	17	12	6	–	3600.0	–
968	1239.2	17	12	1	–	3600.0	–
969	2065.1	10	12	1	True	3600.0	–
970	504.1	9	12	0	–	3600.0	–
971	418.8	18	12	0	True	3600.0	–
972	775.3	8	12	0	–	3600.0	–
973	332.6	5	12	0	–	3600.0	–
974	928.8	11	12	0	True	3600.0	–
975	1906.2	10	12	0	True	3600.0	–
976	1328.8	17	12	2	–	3600.0	–
977	3600.0	–	1	0	True	3600.0	–
978	3600.0	–	1	0	True	3600.0	–
979	1561.8	4	12	0	–	3600.0	–
980	1045.5	12	12	1	–	3600.0	–
981	1523.2	16	12	0	–	3600.0	–
982	2410.1	7	12	2	–	3600.0	–
983	404.5	14	12	0	–	3600.0	–
984	951.1	18	12	0	–	3600.0	–
985	1644.2	12	12	1	True	3600.0	–
986	1622.3	14	12	3	–	3600.0	–
987	3030.7	13	12	2	–	3600.0	–
988	1228.0	10	12	0	–	3600.0	–
989	1253.7	19	12	1	True	3600.0	–
990	859.4	14	12	0	True	3600.0	–
991	3155.2	16	12	3	True	3600.0	–
992	1021.5	18	12	2	–	3600.0	–
993	2147.7	19	12	0	True	3600.0	–
994	1021.3	10	12	0	–	3600.0	–
995	935.7	12	12	0	–	3600.0	–
996	497.2	10	12	0	–	3600.0	–
997	987.7	14	12	0	–	3600.0	–
998	1188.9	4	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
999	1700.6	23	12	0	True	3600.0	–
1000	1509.4	18	12	0	–	3600.0	–
1001	843.2	17	12	0	–	3600.0	–
1002	585.9	11	12	0	–	3600.0	–
1003	1446.4	21	12	0	–	3600.0	–
1004	1229.9	22	12	1	–	3600.0	–
1005	1819.0	9	12	0	–	3600.0	–
1006	355.4	3	12	0	–	3600.0	–
1007	809.2	14	12	0	–	3600.0	–
1008	1392.7	14	12	0	–	3600.0	–
1009	46.9	0	12	0	–	3600.0	–
1010	1176.4	4	12	0	–	3600.0	–
1011	2082.8	17	12	1	–	3600.0	–
1012	1246.8	3	12	0	–	3600.0	–
1013	463.7	6	12	0	–	3600.0	–
1014	732.2	14	12	0	–	3600.0	–
1015	1157.5	10	12	0	True	3600.0	–
1016	1207.7	9	12	0	–	3600.0	–
1017	1110.2	15	12	0	–	3600.0	–
1018	2401.1	14	12	3	–	3600.0	–
1019	3128.8	18	12	5	True	3600.0	–
1020	1062.0	11	12	0	–	3600.0	–
1021	755.2	12	12	0	–	3600.0	–
1022	1738.8	13	12	1	–	3600.0	–
1023	1673.3	2	12	0	–	3600.0	–
1024	1881.2	13	12	0	–	3600.0	–
1025	1834.0	16	12	0	–	3600.0	–
1026	1438.7	11	12	1	–	3600.0	–
1027	1590.9	5	12	0	–	3600.0	–
1028	3600.0	–	2	2	–	3600.0	–
1029	3600.0	–	3	2	True	3600.0	–
1030	1837.5	10	12	0	–	3600.0	–
1031	1973.5	8	12	0	–	3600.0	–
1032	3600.0	–	2	2	–	3600.0	–
1033	1223.4	14	12	0	True	3600.0	–
1034	1827.2	4	12	0	–	3600.0	–
1035	1595.0	17	12	0	True	3600.0	–
1036	1799.6	4	12	0	–	3600.0	–
1037	1657.2	6	12	0	–	3600.0	–
1038	823.4	11	12	0	True	3600.0	–
1039	1592.4	13	12	0	True	3600.0	–
1040	1485.5	20	12	0	–	3600.0	–
1041	1425.0	11	12	0	–	3600.0	–
1042	921.4	12	12	0	–	3600.0	–
1043	191.1	12	12	0	–	3600.0	–
1044	1002.9	2	12	0	–	3600.0	–
1045	2112.2	11	12	0	–	3600.0	–
1046	3600.0	–	4	3	True	3600.0	–
1047	1278.3	4	12	0	–	3600.0	–
1048	2048.0	17	12	0	–	3600.0	–
1049	3600.0	–	7	9	True	3600.0	–
1050	2383.6	14	12	2	–	3600.0	–
1051	66.8	0	12	0	–	3600.0	–
1052	954.6	7	12	0	–	3600.0	–
1053	1891.9	19	12	6	True	3600.0	–
1054	1297.2	24	12	0	–	3600.0	–
1055	1016.0	2	12	0	–	3600.0	–
1056	2210.1	18	12	4	–	3600.0	–
1057	2374.0	12	12	4	–	3600.0	–
1058	1603.8	14	12	2	True	3600.0	–
1059	1123.3	14	12	1	–	3600.0	–
1060	824.8	7	12	0	–	3600.0	–
1061	1751.4	9	12	0	–	3600.0	–
1062	3600.0	–	2	1	True	3600.0	–
1063	1700.5	12	12	0	True	3600.0	–
1064	1561.1	13	12	0	–	3600.0	–
1065	1185.8	18	12	0	–	3600.0	–
1066	1591.4	10	12	0	–	3600.0	–
1067	2536.6	10	12	3	–	3600.0	–
1068	323.6	10	12	0	–	3600.0	–
1069	2019.7	16	12	1	–	3600.0	–
1070	665.7	8	12	0	–	3600.0	–
1071	2050.7	10	12	1	–	3600.0	–
1072	976.1	8	12	0	–	3600.0	–
1073	1402.3	15	12	1	–	3600.0	–
1074	1459.6	11	12	0	True	3600.0	–
1075	949.0	15	12	1	–	3600.0	–
1076	1358.7	14	12	1	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
1077	1772.0	14	12	0	–	3600.0	–
1078	1528.7	12	12	0	–	3600.0	–
1079	1353.5	20	12	0	–	3600.0	–
1080	760.4	11	12	0	–	3600.0	–
1081	143.9	0	12	0	–	3600.0	–
1082	1404.7	11	12	0	–	3600.0	–
1083	2508.3	15	12	0	–	3600.0	–
1084	1963.9	11	12	1	True	3600.0	–
1085	1691.7	15	12	0	–	3600.0	–
1086	1813.8	13	12	4	True	3600.0	–
1087	2123.1	8	12	0	True	3600.0	–
1088	1648.8	9	12	0	–	3600.0	–
1089	947.2	9	12	0	–	3600.0	–
1090	1630.6	14	12	0	–	3600.0	–
1091	3600.0	–	2	1	–	3600.0	–
1092	1051.8	10	12	0	–	3600.0	–
1093	1225.4	15	12	0	–	3600.0	–
1094	1639.0	15	12	0	–	3600.0	–
1095	1081.3	11	12	0	–	3600.0	–
1096	2004.1	11	12	2	–	3600.0	–
1097	2146.6	8	12	0	–	3600.0	–
1098	536.8	16	12	0	–	3600.0	–
1099	1313.2	16	12	0	–	3600.0	–
1100	1780.0	11	12	0	–	3600.0	–
1101	1821.5	11	12	0	–	3600.0	–
1102	691.1	13	12	0	–	3600.0	–
1103	1404.7	14	12	0	–	3600.0	–
1104	1929.8	4	12	0	–	3600.0	–
1105	1377.1	10	12	1	–	3600.0	–
1106	1909.5	5	12	0	–	3600.0	–
1107	586.5	8	12	0	–	3600.0	–
1108	2122.8	11	12	1	–	3600.0	–
1109	1631.1	8	12	0	–	3600.0	–
1110	1833.6	14	12	1	–	3600.0	–
1111	1301.5	22	12	0	–	3600.0	–
1112	1815.8	11	12	0	–	3600.0	–
1113	1529.1	12	12	0	–	3600.0	–
1114	1993.7	12	12	0	–	3600.0	–
1115	435.1	14	12	0	–	3600.0	–
1116	412.5	11	12	0	–	3600.0	–
1117	968.8	9	12	0	–	3600.0	–
1118	3594.4	12	12	7	–	3600.0	–
1119	173.2	0	12	0	–	3600.0	–
1120	1129.4	0	12	0	True	3600.0	–
1121	1071.9	8	12	0	–	3600.0	–
1122	1912.8	4	12	0	–	3600.0	–
1123	2133.2	27	12	2	True	3600.0	–
1124	1412.9	12	12	0	–	3600.0	–
1125	1192.7	19	12	0	–	3600.0	–
1126	940.2	8	12	0	–	3600.0	–
1127	2054.6	11	12	2	True	3600.0	–
1128	1166.0	13	12	0	True	3600.0	–
1129	573.6	11	12	0	–	3600.0	–
1130	936.8	11	12	0	–	3600.0	–
1131	2072.9	12	12	9	True	3600.0	–
1132	946.6	18	12	0	–	3600.0	–
1133	1719.8	13	12	0	–	3600.0	–
1134	921.8	7	12	0	–	3600.0	–
1135	3600.0	–	2	1	–	3600.0	–
1136	2335.7	8	12	7	–	3600.0	–
1137	1947.5	15	12	0	–	3600.0	–
1138	2138.3	6	12	0	True	3600.0	–
1139	1060.2	10	12	1	–	3600.0	–
1140	1398.0	14	12	0	–	3600.0	–
1141	2371.4	19	12	5	–	3600.0	–
1142	3197.0	12	12	5	True	3600.0	–
1143	2113.7	23	12	6	True	3600.0	–
1144	419.5	18	12	0	–	3600.0	–
1145	3600.0	–	3	4	True	3600.0	–
1146	2208.8	12	12	1	–	3600.0	–
1147	1940.5	19	12	0	–	3600.0	–
1148	580.2	16	12	1	–	3600.0	–
1149	1264.4	16	12	0	–	3600.0	–
1150	446.3	14	12	0	–	3600.0	–
1151	2331.2	6	12	0	True	3600.0	–
1152	1855.9	14	12	0	True	3600.0	–
1153	3600.0	–	2	1	True	3600.0	–
1154	1365.3	14	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
1155	1874.3	16	12	2	–	3600.0	–
1156	800.4	15	12	0	True	3600.0	–
1157	1835.3	18	12	2	–	3600.0	–
1158	2076.6	11	12	0	True	3600.0	–
1159	1122.3	12	12	1	–	3600.0	–
1160	1684.8	5	12	0	–	3600.0	–
1161	1349.2	9	12	0	–	3600.0	–
1162	1832.0	8	12	0	–	3600.0	–
1163	1237.5	16	12	0	–	3600.0	–
1164	1027.0	6	12	0	–	3600.0	–
1165	1656.5	11	12	0	–	3600.0	–
1166	1314.2	16	12	0	–	3600.0	–
1167	254.1	10	12	0	–	3600.0	–
1168	1888.4	11	12	1	True	3600.0	–
1169	1079.1	15	12	1	–	3600.0	–
1170	1323.1	19	12	0	–	3600.0	–
1171	1513.2	15	12	0	True	3600.0	–
1172	857.9	8	12	0	–	3600.0	–
1173	1398.8	11	12	1	True	3600.0	–
1174	869.4	8	12	0	–	3600.0	–
1175	1024.0	18	12	0	–	3600.0	–
1176	1311.3	10	12	0	–	3600.0	–
1177	869.4	13	12	0	–	3600.0	–
1178	1358.5	25	12	0	True	3600.0	–
1179	2175.2	14	12	0	–	3600.0	–
1180	469.7	13	12	0	–	3600.0	–
1181	573.0	0	12	0	–	3600.0	–
1182	745.1	18	12	1	–	3600.0	–
1183	1255.2	12	12	0	–	3600.0	–
1184	3600.0	–	2	1	True	3600.0	–
1185	3337.4	15	12	15	–	3600.0	–
1186	1854.3	11	12	2	–	3600.0	–
1187	156.3	0	12	0	–	3600.0	–
1188	302.1	0	12	0	–	3600.0	–
1189	1428.9	9	12	0	–	3600.0	–
1190	1991.5	18	12	3	–	3600.0	–
1191	1714.4	14	12	0	–	3600.0	–
1192	1018.6	8	12	0	–	3600.0	–
1193	1890.3	9	12	1	–	3600.0	–
1194	2765.9	16	12	11	True	3600.0	–
1195	3600.0	–	6	14	–	3600.0	–
1196	2306.9	12	12	4	True	3600.0	–
1197	512.1	2	12	0	–	3600.0	–
1198	1928.4	7	12	0	–	3600.0	–
1199	3600.0	–	2	1	–	3600.0	–
1200	1810.6	6	12	0	–	3600.0	–
1201	95.4	0	12	0	–	3600.0	–
1202	1235.2	13	12	0	–	3600.0	–
1203	1793.6	18	12	1	–	3600.0	–
1204	1291.5	11	12	0	–	3600.0	–
1205	2127.9	26	12	6	–	3600.0	–
1206	523.9	2	12	0	–	3600.0	–
1207	1829.4	20	12	0	True	3600.0	–
1208	857.4	2	12	0	–	3600.0	–
1209	2136.8	21	12	6	True	3600.0	–
1210	1426.1	20	12	0	–	3600.0	–
1211	2206.2	15	12	1	True	3600.0	–
1212	1300.3	19	12	1	–	3600.0	–
1213	1645.5	9	12	0	–	3600.0	–
1214	1423.6	7	12	0	–	3600.0	–
1215	934.8	14	12	0	–	3600.0	–
1216	1688.9	11	12	0	–	3600.0	–
1217	1863.3	14	12	0	–	3600.0	–
1218	1451.5	13	12	0	–	3600.0	–
1219	2851.4	18	12	2	True	3600.0	–
1220	2562.4	14	12	3	–	3600.0	–
1221	567.5	20	12	0	–	3600.0	–
1222	3600.0	–	2	2	–	3600.0	–
1223	1517.0	11	12	0	–	3600.0	–
1224	1046.7	11	12	0	–	3600.0	–
1225	2107.0	13	12	0	True	3600.0	–
1226	1646.2	20	12	0	–	3600.0	–
1227	2059.0	25	12	1	True	3600.0	–
1228	2666.2	11	12	4	–	3600.0	–
1229	1526.2	27	12	0	–	3600.0	–
1230	275.1	0	12	0	–	3600.0	–
1231	322.0	2	12	0	–	3600.0	–
1232	1876.0	19	12	0	True	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
1233	1261.7	19	12	2	–	3600.0	–
1234	3600.0	–	3	2	True	3600.0	–
1235	1801.9	11	12	2	True	3600.0	–
1236	1704.1	4	12	0	–	3600.0	–
1237	2603.4	13	12	2	True	3600.0	–
1238	972.0	9	12	1	–	3600.0	–
1239	1769.3	11	12	1	–	3600.0	–
1240	708.0	11	12	0	–	3600.0	–
1241	842.6	11	12	0	–	3600.0	–
1242	1480.2	9	12	0	–	3600.0	–
1243	1794.0	10	12	0	–	3600.0	–
1244	2253.8	8	12	0	True	3600.0	–
1245	1842.1	15	12	0	–	3600.0	–
1246	1176.5	15	12	0	–	3600.0	–
1247	1073.4	7	12	0	–	3600.0	–
1248	1118.3	14	12	0	–	3600.0	–
1249	1603.6	8	12	0	–	3600.0	–
1250	1810.2	12	12	1	–	3600.0	–
1251	3600.0	–	3	2	–	3600.0	–
1252	3600.0	–	2	1	–	3600.0	–
1253	991.1	8	12	0	–	3600.0	–
1254	962.0	9	12	0	–	3600.0	–
1255	1764.5	11	12	0	–	3600.0	–
1256	1503.3	9	12	0	–	3600.0	–
1257	1587.9	13	12	0	–	3600.0	–
1258	1763.4	12	12	0	–	3600.0	–
1259	1011.6	20	12	0	–	3600.0	–
1260	1536.2	11	12	1	True	3600.0	–
1261	318.6	0	12	0	–	3600.0	–
1262	1759.5	7	12	0	–	3600.0	–
1263	1491.2	21	12	0	True	3600.0	–
1264	3600.0	–	2	1	True	3600.0	–
1265	2022.9	20	12	0	True	3600.0	–
1266	2540.1	15	12	3	–	3600.0	–
1267	1087.9	17	12	0	–	3600.0	–
1268	2480.2	10	12	0	–	3600.0	–
1269	1653.5	20	12	1	–	3600.0	–
1270	1288.0	1	12	0	True	3600.0	–
1271	1685.0	9	12	0	–	3600.0	–
1272	862.8	15	12	0	–	3600.0	–
1273	1316.9	14	12	0	True	3600.0	–
1274	997.8	10	12	0	–	3600.0	–
1275	1329.4	19	12	0	–	3600.0	–
1276	3600.0	–	10	15	–	3600.0	–
1277	1665.0	16	12	0	–	3600.0	–
1278	1254.0	20	12	2	–	3600.0	–
1279	1940.1	12	12	1	–	3600.0	–
1280	1494.7	11	12	0	–	3600.0	–
1281	1792.2	12	12	0	–	3600.0	–
1282	110.1	0	12	0	–	3600.0	–
1283	1758.1	11	12	1	–	3600.0	–
1284	1574.3	3	12	0	–	3600.0	–
1285	1044.4	13	12	0	–	3600.0	–
1286	1333.3	14	12	0	–	3600.0	–
1287	1780.7	10	12	0	–	3600.0	–
1288	1913.5	7	12	0	–	3600.0	–
1289	1071.8	11	12	2	–	3600.0	–
1290	1581.5	9	12	0	–	3600.0	–
1291	602.5	10	12	0	–	3600.0	–
1292	315.0	18	12	0	–	3600.0	–
1293	3600.0	–	2	1	True	3600.0	–
1294	69.9	0	12	0	–	3600.0	–
1295	329.1	3	12	0	–	3600.0	–
1296	2284.3	9	12	5	–	3600.0	–
1297	1078.0	14	12	0	–	3600.0	–
1298	2472.6	19	12	3	–	3600.0	–
1299	1528.2	15	12	0	–	3600.0	–
1300	393.8	14	12	0	True	3600.0	–
1301	1931.6	7	12	0	–	3600.0	–
1302	1381.0	9	12	0	–	3600.0	–
1303	1210.9	20	12	0	–	3600.0	–
1304	1774.3	15	12	1	–	3600.0	–
1305	571.4	6	12	0	–	3600.0	–
1306	876.8	15	12	0	–	3600.0	–
1307	2522.9	11	12	4	True	3600.0	–
1308	1482.0	16	12	0	True	3600.0	–
1309	1416.7	9	12	0	–	3600.0	–
1310	658.8	9	12	0	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
1311	1216.0	11	12	0	–	3600.0	–
1312	839.3	8	12	0	–	3600.0	–
1313	2189.8	12	12	1	–	3600.0	–
1314	1612.5	11	12	3	–	3600.0	–
1315	996.3	18	12	0	–	3600.0	–
1316	1168.3	20	12	0	True	3600.0	–
1317	2460.8	15	12	4	–	3600.0	–
1318	1351.3	14	12	0	True	3600.0	–
1319	1885.6	12	12	0	True	3600.0	–
1320	587.1	19	12	0	–	3600.0	–
1321	1970.3	18	12	1	–	3600.0	–
1322	2347.5	18	12	1	True	3600.0	–
1323	2136.2	14	12	7	–	3600.0	–
1324	1280.5	14	12	0	True	3600.0	–
1325	1107.2	8	12	0	–	3600.0	–
1326	1667.8	18	12	0	–	3600.0	–
1327	1017.1	13	12	0	–	3600.0	–
1328	1439.9	10	12	0	True	3600.0	–
1329	2252.4	27	12	1	–	3600.0	–
1330	1858.3	13	12	0	–	3600.0	–
1331	2591.6	12	12	2	–	3600.0	–
1332	1880.9	6	12	0	–	3600.0	–
1333	1429.4	14	12	0	–	3600.0	–
1334	1659.8	22	12	1	True	3600.0	–
1335	176.2	0	12	0	–	3600.0	–
1336	1103.4	7	12	0	–	3600.0	–
1337	1170.4	13	12	0	–	3600.0	–
1338	2611.3	12	12	1	True	3600.0	–
1339	1895.6	5	12	0	–	3600.0	–
1340	395.1	17	12	1	–	3600.0	–
1341	206.5	0	12	0	–	3600.0	–
1342	2425.6	33	12	2	True	3600.0	–
1343	1595.8	8	12	0	–	3600.0	–
1344	1976.7	21	12	0	True	3600.0	–
1345	56.7	0	12	0	–	3600.0	–
1346	1267.0	16	12	1	True	3600.0	–
1347	1855.3	19	12	0	–	3600.0	–
1348	2500.4	4	12	0	–	3600.0	–
1349	1220.2	19	12	0	–	3600.0	–
1350	1175.4	13	12	0	True	3600.0	–
1351	2402.8	16	12	2	–	3600.0	–
1352	1575.0	15	12	1	–	3600.0	–
1353	1569.9	16	12	0	–	3600.0	–
1354	3600.0	–	1	0	True	3600.0	–
1355	1459.6	8	12	0	–	3600.0	–
1356	1180.5	19	12	0	–	3600.0	–
1357	1870.2	8	12	4	–	3600.0	–
1358	2108.4	9	12	1	–	3600.0	–
1359	1787.9	4	12	0	–	3600.0	–
1360	231.5	0	12	0	–	3600.0	–
1361	1797.6	15	12	0	–	3600.0	–
1362	1015.3	7	12	0	–	3600.0	–
1363	1025.9	12	12	0	–	3600.0	–
1364	1031.3	11	12	0	True	3600.0	–
1365	2210.1	14	12	0	True	3600.0	–
1366	961.1	15	12	1	–	3600.0	–
1367	3600.0	–	9	13	True	3600.0	–
1368	979.5	4	12	0	–	3600.0	–
1369	2042.5	8	12	0	–	3600.0	–
1370	1104.8	13	12	0	True	3600.0	–
1371	336.6	3	12	0	–	3600.0	–
1372	2101.7	14	12	4	–	3600.0	–
1373	1476.7	10	12	0	–	3600.0	–
1374	3600.0	–	2	1	–	3600.0	–
1375	1034.4	11	12	0	–	3600.0	–
1376	1499.5	17	12	1	–	3600.0	–
1377	3120.2	20	12	4	–	3600.0	–
1378	119.3	0	12	0	–	3600.0	–
1379	1837.9	7	12	0	–	3600.0	–
1380	1394.1	0	12	0	–	3600.0	–
1381	962.3	21	12	0	–	3600.0	–
1382	707.6	12	12	0	–	3600.0	–
1383	1516.0	8	12	0	–	3600.0	–
1384	620.9	8	12	0	–	3600.0	–
1385	618.6	3	12	0	–	3600.0	–
1386	695.2	3	12	0	–	3600.0	–
1387	1511.3	14	12	0	–	3600.0	–
1388	3600.0	–	2	2	–	3600.0	–

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Table E.5: Detailed results of the *Net 3* test set (continued).

	Enabled					*Disabled*	
Instance	Time	Obj	Round	Back	Decompose-MIP	Time	Obj
1389	1508.1	11	12	1	–	3600.0	–
1390	886.8	11	12	1	–	3600.0	–
1391	808.1	4	12	0	–	3600.0	–
1392	1667.0	3	12	0	–	3600.0	–
1393	1638.7	20	12	0	–	3600.0	–
1394	1003.2	7	12	0	–	3600.0	–
1395	1603.5	10	12	0	–	3600.0	–
1396	1487.3	6	12	0	–	3600.0	–
1397	1939.1	15	12	6	–	3600.0	–
1398	1258.3	12	12	0	–	3600.0	–
1399	944.2	8	12	1	True	3600.0	–
1400	1840.6	11	12	1	–	3600.0	–
1401	1929.9	4	12	0	True	3600.0	–
1402	207.9	0	12	0	–	3600.0	–
1403	2197.2	11	12	13	True	3600.0	–
1404	1633.9	19	12	0	–	3600.0	–
1405	2675.2	11	12	4	–	3600.0	–
1406	1958.3	19	12	5	–	3600.0	–
1407	1894.5	4	12	0	–	3600.0	–
1408	316.3	0	12	0	–	3600.0	–
1409	1820.5	13	12	1	–	3600.0	–
1410	1530.7	16	12	0	–	3600.0	–
1411	444.8	2	12	0	–	3600.0	–
1412	736.5	13	12	0	–	3600.0	–
1413	1100.4	15	12	0	–	3600.0	–
1414	1759.1	17	12	1	–	3600.0	–
1415	515.9	1	12	0	–	3600.0	–
1416	3600.0	–	3	2	True	3600.0	–
1417	347.1	10	12	0	–	3600.0	–
1418	1671.8	11	12	0	–	3600.0	–
1419	186.1	0	12	0	–	3600.0	–
1420	641.9	0	12	0	–	3600.0	–
1421	381.3	0	12	0	–	3600.0	–
1422	2997.4	11	12	5	True	3600.0	–
1423	606.3	14	12	0	True	3600.0	–
1424	1889.1	15	12	1	–	3600.0	–
1425	1300.6	8	12	0	–	3600.0	–
1426	1172.2	18	12	0	–	3600.0	–
1427	1996.8	17	12	2	True	3600.0	–
1428	1666.7	6	12	0	True	3600.0	–
1429	1838.6	30	12	0	True	3600.0	–
1430	3600.0	–	2	1	–	3600.0	–
1431	2181.9	13	12	0	True	3600.0	–
1432	1667.5	12	12	2	–	3600.0	–
1433	3600.0	–	6	10	True	3600.0	–
1434	1339.3	6	12	0	–	3600.0	–
1435	2607.8	17	12	3	–	3600.0	–
1436	1857.8	12	12	0	–	3600.0	–
1437	1140.8	16	12	0	–	3600.0	–
1438	1439.6	11	12	1	–	3600.0	–
1439	1652.7	13	12	1	–	3600.0	–
1440	1825.0	18	12	1	–	3600.0	–