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On Project Scheduling with Irregular Starting Time Costs

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Abstract

Maniezzo and Mingozzi (Operations Research Letters 25 (1999), pp. 175–182) study a project scheduling problem with irregular starting time costs. Starting from the assumption that its computational complexity status is open, they develop a branch-and-bound procedure and they identify special cases that are solvable in polynomial time. In this note, we present a collection of previously established results which show that the general problem is solvable in polynomial time. This collection may serve as a useful guide to the literature, since this polynomial-time solvability has been rediscovered in different contexts or using different methods. In addition, we briefly review some related results for specializations and generalizations of the problem.

Keywords: Scheduling; Project scheduling; Integer programming; Network flow; Minimum cut; Minimum weight closure

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1 Introduction

Maniezzo and Mingozzi [21] consider the problem of finding a minimum-cost schedule for a set $V = \{1,\dots,n\}$ of precedence-constrained jobs which have starting time dependent costs. A schedule must respect the given precedence constraints, and each job $j \in V$ incurs a cost of $w_j(t)$ if it is started at time t. Here, $t \in I := \{0,1,\dots,T\}$, and T denotes the planning horizon. Since no restrictions are imposed on the cost functions $w_j(t)$, the objective function $\sum_{j\in V}\sum_{t\in I} w_j(t)$ generalizes many popular (regular and irregular) objective functions. One example is the maximization of the net present value (e.g. in [32]), where a cash flow of w_j is associated with every job, α is an interest rate, and $w_j(t) = -w_j \exp(-\alpha t)$. Another important special case is given by linear earliness-tardiness costs, or more general, by (piecewise linear) convex cost functions $w_j(t)$. The problem with arbitrary cost functions $w_j(t)$ owes its significance to a good part to its appearance as a subproblem in the computation of bounds on the objective function value for different resource-constrained project scheduling problems, e.g. in [7, 9, 17, 20, 24, 31, 33]. In [21], Maniezzo and Mingozzi suggest that the computational complexity status of this problem is open. On this account, they show that the following two special cases can be solved in polynomial time: cost functions $w_j(t)$ which are monotonous in t, and precedence constraints in the form of an out-tree. The case with monotonous cost functions is obviously solvable through longest path calculations. Maniezzo and Mingozzi propose a dynamic programming algorithm of running time

$O(nT)$ for the case of out-tree precedence constraints. In addition, following earlier work by Christofides, Alvarez-Valdés and Tamarit [7] on the same problem, they develop a lower bound as well as a branch-and-bound procedure for the general case. Maniezzo and Mingozzi's lower bound is obtained by extracting an out-tree from the given precedence constraints, and by penalizing the violation of the neglected constraints in a Lagrangian fashion.

In this note, we give a historical synopsis of previously established results which show that the general problem considered in [7] and [21] is solvable in polynomial time, and we point out interrelations between them. We found proofs implying this result (for varying levels of generality) in [3, 4, 5, 8, 14, 17, 22, 24, 31, 33]. Most of them have co-existed in the literature, apparently without anyone making the connection. We hope that this note will help to establish this connection. It is organized according to the different techniques that have been used. First, the integrality of the linear programming relaxations of two popular integer programming formulations implies that a more general problem can be solved in polynomial time, namely with temporal constraints in the form of arbitrary (i.e., positive and negative) time lags. These results will be summarized in Sections 2.1 and 2.2. In Section 2.3, we then report on different reductions to minimum cut problems which result in algorithms with running time $O(nmT^2 \log(n^2T/m))$, for the generalized problem. Here, m is the number of temporal constraints.

2 Solution Techniques

We first introduce some additional notation. A temporal constraint between two jobs i and j is an inequality of the form $S_j \geq S_i + d_{ij}$. Here, S_j and S_i denote the starting times of jobs j and i, respectively, and the integer number d_{ij}, $-\infty \leq d_{ij} < \infty$, imposes a time lag between them. Note that ordinary precedence constraints arise as the special case $S_j \geq S_i + p_i$, where $p_i \geq 0$ denotes the processing time of job i. We assume throughout that the given temporal constraints are consistent, i. e., the digraph $G = (V, A)$ with $A = \{(i, j) | d_{ij} > -\infty\}$ and arc lengths d_{ij} does not contain a directed cycle of positive length. Given the temporal constraints and the time horizon T, it is easy to compute earliest possible and latest possible starting times for each job $j \in V$. For convenience of notation, however, we simply assume throughout the text that variables with time indices outside these boundaries are fixed at values which ensure that no job is started at an infeasible time.

2.1 Integer programming formulation I

The following integer program represents one formulation of the project scheduling problem with irregular starting time costs. We use binary variables z_{jt}, $j \in V$, $t \in I = \{0, 1, \ldots, T\}$, with the intended meaning that $z_{jt} = 1$ if job j is started by time t and $z_{jt} = 0$, otherwise. To the best of the authors' knowledge, this type of variables for modeling scheduling problems was originally introduced by Pritsker and Watters [28]. Using these variables, the problem reads as follows.

$$\begin{aligned}
\min \quad & \sum_j \sum_t \overline{w}_j(t)\, z_{jt} && && (1)\\
\text{s. t.} \quad & z_{jT} = 1, && j \in V, && (2)\\
& z_{jt} - z_{j,t+1} \leq 0, && j \in V,\ t \in I, && (3)\\
& z_{j,t+d_{ij}} - z_{it} \leq 0, && (i,j) \in A,\ t \in I, && (4)\\
& z_{jt} \geq 0, && j \in V,\ t \in I, && (5)\\
& z_{jt} \text{ integer}, && j \in V,\ t \in I. && (6)
\end{aligned}$$

Here, $\overline{w}_j(t) := w_j(t) - w_j(t+1)$ for all $j \in V$ and $t \in I$, where $w_j(T+1) := 0$. Gröflin, Liebling and Prodon observed in the context of their work on pipeline scheduling with out-tree precedence constraints [14] that the constraint matrix of (3) – (4) is the arc-node incidence matrix of a digraph. In particular, it is totally unimodular. This implies that the linear programming relaxation of the above integer program is integral (as was also observed in [3, 8, 17, 22, 31, 33] in various contexts). Hence, the scheduling problem is solvable in polynomial time. Moreover, the dual linear program to (1) – (5) can be solved as a minimum-cost flow problem [14, 17, 31, 33]. In fact, Gröflin, Liebling and Prodon [14] presented a network flow type algorithm that solves the pipeline scheduling problem with out-tree precedence constraints in $O(nT)$ time. Their pipeline scheduling problem can be interpreted as follows: It is a scheduling problem with irregular starting time costs, zero time lags ($d_{ij} = 0$) which form an out-tree and (2) is relaxed to $z_{jt} \leq 1$ for all $j \in V$ and $t \in I$ (i.e., jobs may not be scheduled at all). With minor modifications, however, their algorithm also applies to the problem with constraints (2), arbitrary d_{ij}, and out–tree precedence constraints. A different algorithm was proposed by Roundy, Maxwell, Herer, Tayur, and Getzler [31] for the case where the precedence constraints consist of independent chains. This special case arises from a Lagrangian relaxation of the job-shop scheduling problem.

2.2 Integer programming formulation II

Pritsker, Watters and Wolfe [29] were likely the first to use variables x_{jt} ($j \in V, t \in I$), where $x_{jt} = 1$ if job j is started at time t and $x_{jt} = 0$, otherwise. The problem now reads as follows.

$$\min \quad \sum_j \sum_t w_j(t)\, x_{jt} \tag{7}$$

$$\text{s.t.} \quad \sum_t x_{jt} = 1, \qquad j \in V, \tag{8}$$

$$\sum_{s=t}^{T} x_{is} + \sum_{s=0}^{t+d_{ij}-1} x_{js} \leq 1, \quad (i,j) \in A,\ t \in I, \tag{9}$$

$$x_{jt} \geq 0, \qquad j \in V,\ t \in I, \tag{10}$$

$$x_{jt} \text{ integer}, \qquad j \in V,\ t \in I. \tag{11}$$

Chaudhuri, Walker and Mitchell [5] showed that the linear programming relaxation of this integer programming formulation is integral as well. For this, they made use of the following graph-theoretic interpretation of the problem: Identify with every job-time pair (j,t) a node v_{jt} in an undirected graph. There are two different types of edges. First, all pairs of nodes which belong to the same job are connected. Second, for each temporal constraint $S_j \geq S_i + d_{ij}$ and each time t, there are edges between v_{it} and all nodes v_{js} with $s < t + d_{ij}$. In the resulting graph, any stable set (a set of pairwise non-adjacent nodes) of cardinality n corresponds to a feasible solution of the original scheduling problem: Job j is started at time t if node v_{jt} belongs to the stable set. Consequently, if we assign the cost coefficients $w_j(t)$ as weights to the nodes v_{jt}, a minimum-weight stable set of cardinality n yields an optimum schedule. If we assume that $d_{ik} \geq d_{ij} + d_{jk}$, this graph can easily be transitively oriented. It therefore is a comparability graph and its corresponding fractional stable set polytope is integral (see, e. g., [15, Chapter 9]). Since the inequalities (8) – (10) define a face of the fractional stable set polytope, it follows that LP relaxation (7) – (10) is integral as well.

The integrality of LP relaxation (7) – (10) can alternatively be proved from the integrality of LP relaxation (1) – (5) by a linear transformation between the z- and the x-variables which preserves integrality. This was pointed out in [3, 8, 22, 33].

Maniezzo and Mingozzi also consider an integer programming formulation in x-variables. Instead of using (9), they model temporal constraints in the way originally suggested by Pritsker, Watters and Wolfe [29]:

$$\sum_t t\,(x_{jt} - x_{it}) \geq d_{ij}, \quad (i,j) \in A. \tag{12}$$

Note that the LP relaxation (7), (8), (10), and (12) is weaker than (7) – (10); in particular, it is not integral in general. We refer to [33] for a simple counter-example with ordinary precedence constraints.

2.3 Reduction to a minimum cut problem

A direct transformation of the project scheduling problem with irregular starting time costs to a minimum cut problem was given by Chang and Edmonds [4], and also in [24]. Although Chang and

Edmonds restricted themselves to the case of precedence constraints and unit processing times (that is, $d_{ij} = 1$ for all $(i,j) \in A$), the transformation works for the general case. Their approach relies in fact on a transformation of the scheduling problem to the so-called minimum weight closure problem, which is well-known to be equivalent to the minimum cut problem [2, 4, 27, 30]. Incidentally, Gröflin, Liebling and Prodon also observed that the pipeline scheduling problem they studied in [14] is an instance of the minimum weight closure problem.

The minimum weight closure problem in a node-weighted digraph is the problem to find a subset C of nodes of minimum weight such that any arc (u,v) with $u \in C$ implies $v \in C$. With binary variables z_u, we obtain the following integer programming formulation.

$$\begin{array}{lll} \min & \sum_u w_u z_u & \\ \text{s.t.} & z_u - z_v \le 0 & \text{for all arcs } (u,v), \\ & z_u \in \{0,1\} & \text{for all nodes } u. \end{array}$$

In this way, the connection to the integer programming formulation discussed in Section 2.1 becomes apparent. It was also noticed in this context that the constraint matrix is totally unimodular (e.g. [30]).

The digraph constructed by Chang and Edmonds [4] is in fact the one induced by the arc-node incidence matrix defined by constraints (3) and (4) of the z-formulation in Section 2.1. In other words, every job-time pair (j,t) corresponds to a node v_{jt}, and there are two different types of arcs. On the one hand, there is an arc $(v_{jt}, v_{j,t+1})$ for every job j and every point t in time. On the other hand, every temporal constraint $(i,j) \in A$ gives rise to arcs $(v_{j,t+d_{ij}}, v_{it})$, for all t. Finally, every vertex v_{jt} is assigned the weight $\overline{w}_j(t)$. The scheduling problem is equivalent to finding, in this digraph, a minimum-weight closure that contains the set $B := \{(j,T) : j \in V\}$. The latter constraint is easily enforced without changing the nature of the minimum weight closure problem as defined above; see [4] for a discussion. Therefore, the scheduling problem can be reduced to a minimum cut problem. If $\mathrm{M}(\alpha,\beta)$ is the running time for computing a minimum cut in a digraph with $\mathrm{O}(\alpha)$ nodes and $\mathrm{O}(\beta)$ arcs, this transformation results in an algorithm which solves the project scheduling problem with irregular starting time costs and arbitrary time lags in time $\mathrm{M}(nT, mT)$. (Recall that $m = |A|$ is the number of given temporal constraints, and $n = |V|$ is the number of jobs.) Using a push-relabel maximum flow algorithm [13], this yields an actual running time of $\mathrm{O}(nmT^2 \log(n^2T/m))$. If all weights $w_j(t)$ are integer and W is the largest absolute value among them, Goldberg and Rao's algorithm [12] leads to a running time of $\mathrm{O}(\min\{n^{2/3}mT^{5/3}, m^{3/2}T^{3/2}\} \log(n^2T/m) \log W)$.

The transformation in [24] was derived in the context of Lagrangian relaxation for resource-constrained project scheduling. It leads to a different, sparser minimum-cut digraph than the one obtained via the above described reduction to the minimum weight closure problem. It results in the same asymptotic time complexity, though.

Chang and Edmonds [4] additionally showed that every instance of the minimum cut problem can be reduced to an instance of the project scheduling problem with ordinary precedence constraints, unit processing times, and irregular starting time costs. (The reduction yields a scheduling problem with time horizon $T = 2$.) Hence, all three problems (project scheduling with irregular starting time costs discussed herein, minimum weight closure and minimum cut) are in fact equivalent.

3 Related Topics

We emphasize that the polynomiality results discussed in Section 2 of this note refer to instances of the scheduling problem which require an encoding length of $\Omega(nT)$. This is clearly the case for problems with general cost functions $w_j(t)$. However, this does not imply polynomial-time algorithms for problems which allow a more succinct encoding. To give an example, consider piecewise linear, convex cost functions $w_j(t)$, an important special case of which are linear earliness-tardiness costs. Instances from the latter class are used by Maniezzo and Mingozzi [21], among others, to evaluate the behavior of their branch-and-bound algorithm. There are algorithms to solve the scheduling problem with piecewise linear, convex cost functions in time polynomial in n and the number of breakpoints. Indeed, because the project scheduling problem with linear cost functions $w_j \cdot t$ can be solved as a linear program in starting time variables S_j, it follows from linear programming theory that the problem with piecewise linear, convex cost functions can be solved as a linear program as well, see, e.g., [25, Chapter 1] for details. This was, for instance, pointed out by Faaland and Schmitt [10] who also gave a combinatorial algorithm. Similar algorithms have recently been proposed in [6] and in [16]. On the other hand, the problem with piecewise linear, convex cost functions may also be seen as a special case of a convex cost integer dual network flow problem. This was observed by Wennink [34] in the context of job-shop scheduling problems and was exploited in full generality by Karzanov and McCormick [18] and Ahuja, Hochbaum and Orlin [1], among others. In the convex cost integer dual network flow problem, the time lags d_{ij} are considered as variables with associated convex cost functions as well. According to [1], this generalized problem can be solved in time $O(nm \log(n^2/m) \log(nT))$ by an adaption of the cost scaling algorithm for minimum cost flows.

It is gratifying to conclude this brief summary by mentioning that in 1961 Fulkerson [11] and Kelley [19] proposed network flow type methods to solve certain problems on project networks. In the time-cost tradeoff problem, (linear) cost functions are associated with the variables d_{ij} only. Fulkerson and Kelley computed the project cost curve through parametric solution of a minimum cost flow problem obtained as a dual of the linear programming formulation of the problem. Phillips and Dessouky [26] subsequently showed that the problem may be solved as a sequence of minimum cut problems.

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