OPTIMIZED SPHERICAL SOUND SOURCE FOR AURALIZATION WITH ARBITRARY SOURCE DIRECTIVITY

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ABSTRACT

The auralization of measured room impulse responses (RIRs) is traditionally bound to the directivity of the source as well as of the receiver. For the comparability of room acoustical measurements ISO 3382 requires the source and the receiver to be of an omni-directional directivity. Other source directivity patterns cannot be auralized using RIRs obtained this way.

In order to include the spatial information the room impulse response has either to be measured with a sound source of the desired directivity or - assuming the room to be a linear time-invariant system - it can be generated by superposing a set of measurements with a source of known directivity. The advantage of the latter method is that it generates a set of RIRs that can be used to derive the RIR for an arbitrary directivity up to a certain spherical harmonic order in post processing.

This article describes a superposition method and a specialized measurement source for the measurement of room impulse responses for an arbitrary source directivity and discusses their capabilities and the limitations. The measurement source was developed using an analytical model. The directivity patterns used for the post processing originate from high-resolution measurements of the actual device. The deviation compared to the analytical model is analyzed regarding the radiation pattern and the achievable synthesis accuracy.

1. INTRODUCTION

The results of room impulse response measurements are inextricably linked with the directivity of the employed sources and receivers. To ensure the comparability of measured standardized room acoustical parameters, ISO 3382 requires the sources and receivers to have an omni-directional directivity [1]. By excluding the influence of any directivity it neglects important information for realistic auralizations and room acoustical analysis besides the standardized parameters [2].

A sequential synthesis method employing an optimized measurement source was developed in previous research [2, 3, 4]. The method allows for the synthesis of room transfer functions of sources with an arbitrary directivity in hindsight of an extensive measurement. It complies with the requirements of the ISO 3382 and simultaneously gathers all source directivity related information about the room.

Conventional measurement sources such as dodecahedron loudspeakers are not well suited for the required measurements [4]. The optimized measurement source was developed to provide the required radiation features and to speed-up the measurement process. During the development of the optimized source its directivity was simulated using an analytical model of a vibrating cap on a sphere [5].

For room acoustical applications the synthesis method and source are described in [6]. This article focuses on the comparability of the real source with the analytical model used during the development.

2. SYNTHESIS OF ROOM TRANSFER FUNCTIONS

The *target room transfer function* for a source with a certain *target directivity* can be synthesized by superposing single room transfer functions obtained in several physical orientations with a measurement source of known directivity [2, 3]. Using a large set of orientations greatly enhances the spatial resolution of the possible target directivity patterns. The weights for the superposition can be derived from the known target directivity and the measurement source directivity in the spherical harmonics domain. The required computational steps have to be executed separately for every frequency. To enhance the readability of the equations the frequency dependence is omitted in this article.

2.1. Spherical Harmonics

In all further considerations, ϑ and φ are the elevation and the azimuth angle of a spherical coordinate system with r being its radius. Two-dimensional square-integrable functions $f(\vartheta, \varphi)$ on the surface of a unit sphere in $\Re^3$ can be represented using *spherical*

harmonics. The complex functions

$$Y_n^m(\vartheta,\varphi) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}} \cdot P_n^m(\cos\vartheta) \cdot e^{\mathrm{j}m\varphi} \quad (1)$$

span the space of scalar functions on the unit sphere [5]. Herein, P_n^m is the *associated Lengendre function* of the first kind of the mth degree in the nth order [7].

The functions Y_n^m can be weighted with individual coefficients $\hat{f}_n^m$ and superposed to yield the shape of the directional function $f(\vartheta,\varphi)$ in an operation called *spherical harmonic expansion* [5]

$$f(\vartheta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \hat{f}_n^m \cdot Y_n^m(\vartheta,\varphi). \quad (2)$$

To obtain the coefficients for a function the *spherical harmonic transform*

$$\hat{f}_n^m = \oint_{S^2} f(\vartheta,\varphi) \cdot \overline{Y_n^m(\vartheta,\varphi)}\,\mathrm{d}\Omega \quad (3)$$

has to be performed [5].

The coefficients can be stored consecutively in a *coefficient vector* $\hat{\mathbf{f}}$, the spherical harmonics in the *function matrix* $\mathbf{Y}$ and the sampled values of the spatial function in a *value vector* $\mathbf{f}$. The expansion and transform are simplified to matrix multiplications

$$\mathbf{f} = \mathbf{Y} \cdot \hat{\mathbf{f}} \quad (4)$$

$$\hat{\mathbf{f}} = \mathbf{Y}^+ \cdot \mathbf{f}, \quad (5)$$

with $\mathbf{Y}^+$ being a generalized inverse of $\mathbf{Y}$ resulting in a not generally unique and exact result for the transform.

A spherical harmonic transformed function can be rotated by an angle α about the z-axis by multiplication with the *Euler rotation term* $e^{-\mathrm{j}m\alpha}$. Rotations about the y-axis require a full *Wigner-D rotation* [8].

Spherical harmonics offer a unified description of a sound source directivity regardless of the distribution of the measurement points. Combined with its efficient rotation, this makes spherical harmonics the calculation method of choice for the synthesis method presented here.

2.2. Synthesis Method

The single room transfer functions obtained with the measurement source in all physical orientations O in a specific acoustical environment are stored in the *frequency response vector*

$$\mathbf{h} = [h_1, h_2, \ldots, h_O]\,. \quad (6)$$

The goal is to synthesize the room transfer function for the target directivity. Therefore, the single room transfer functions are superposed applying a *weighting vector* $\mathbf{g}_T$ resulting in the room transfer function

$$h_{\mathrm{T}} = \mathbf{h} \cdot \mathbf{g}_T, \quad (7)$$

for the desired target directivity. The directivity of the measurement source can be described in a *directivity matrix*

$$\hat{\mathbf{D}} = \left[\hat{\mathbf{d}}_1\,\hat{\mathbf{d}}_2 \cdots \hat{\mathbf{d}}_O\right], \quad (8)$$

containing the respective SH-coefficients of the directivity of the measurement source in all physical orientations O as column vectors. A synthesized directivity coefficient vector

$$\hat{\mathbf{d}} = \hat{\mathbf{D}} \cdot \mathbf{g} \quad (9)$$

can be generated by weighting and superposing the single directivity columns of the directivity matrix. For a given *target directivity* $\hat{\mathbf{d}}_T$ the weighting vector

$$\mathbf{g}_T = \hat{\mathbf{D}}^+ \cdot \hat{\mathbf{d}}_T \quad (10)$$

can be found by multiplication with the generalized inverse of the directivity matrix $\hat{\mathbf{D}}$. These weights can be applied in Eq. (7) to obtain the room transfer function of the desired target directivity.

3. SOURCE DEVELOPMENT

The weighting vector $\mathbf{g}_T$ in Eq. (10) is calculated from the spherical harmonics transformed directivity pattern of the measurement source. The directivity pattern has to contain sufficiently large coefficients in every spherical harmonic order to generate valid weights.

The spherical harmonic coefficients of the directivity of any electro-acoustical source are determined by the size of the entire source and the aperture angle of the transducer in its enclosure [5]. This suggests the design of a new source for the synthesis method.

Eq. (10) projects spherical harmonic coefficients into weights for spatial orientations. This is an analogy to the discrete spherical harmonic expansion. Several spatial sampling strategies have been introduced to efficiently perform this operation [9]. The physical orientations of the source should be selected according to one of these strategies.

3.1. Simulation Model

A suitable measurement source can virtually be of any shape. Here, the shape is chosen to be spherical, allowing for rapid prototyping applying an analytical simulation model. In this model a transducer on a sphere is simplified as a radially vibrating cap [5]. It has to be noted that this model does not take into account partial modes or the physical interaction of transducers in a common volume. Based on the model it is possible to successively calculate the *squared aperture magnitude* and the radiated sound pressure.

3.2. Aperture Magnitude

The *aperture function*

$$a(\vartheta,\varphi) = a(\vartheta) = 1 - \varepsilon\left(\vartheta - \alpha/2\right) \quad (11)$$

of a single membrane on the north pole of a sphere spanning the aperture angle α can be formulated as a continuous function on the sphere. $\varepsilon(x)$ is the *Heaviside function* (or *unit step function*). The spherical harmonic coefficients of this function can be expressed as [10]

$$\hat{a}_n^m = \begin{cases} \sqrt{\pi(2n+1)}\int_{\cos(\alpha/2)}^{1} P_n(x)\,\mathrm{d}x & \text{if } m = 0 \\ 0 & \text{otherwise} \end{cases}. \quad (12)$$

$P_n(x)$ is the *Legendre polynomial* of the order n. The aperture can be rotated to any orientation applying the *Wigner-D* rotation [8].

$$E_{\mathrm{a}}(n) = \sum_{m=-n}^{n} |\hat{a}_n^m|^2 \quad (13)$$

describes the frequency independent squared magnitude of the aperture per spherical harmonic order n created by a specific aperture [11].

The squared aperture magnitude features a distinct maximum and several minima due to the *Legendre polynomial* $P_n(x)$. Reducing the aperture angle shifts the extremes towards higher orders n while simultaneously decreasing the absolute squared magnitude.

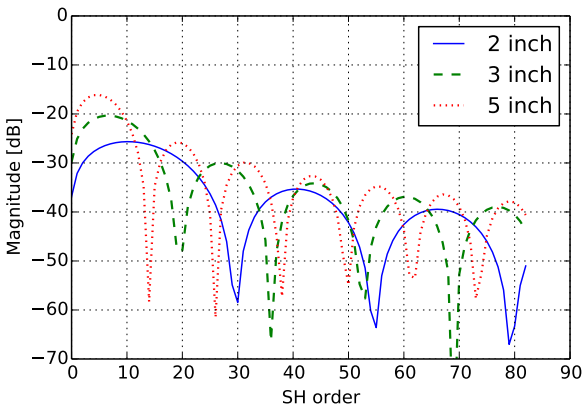


Figure 1: *Squared magnitude of the aperture functions of three transducer types on a sphere with radius* $r = 0.2\,\mathrm{m}$.

The radiation of a spherical harmonic order depends on the frequency and the size of the source (see 3.3). Thus, the squared magnitude depicted in Figure 1 describes the theoretical squared magnitude of each aperture, which is also a subject to an order-dependent low pass during the radiation. The squared magnitude in orders above 10 for the 5 inch transducer has to be neglected due to its frequency range, motivating the use of the 3 inch transducer.

3.3. Spherical Radiation

With the speed of sound c and the membrane displacement ξ, the aperture coefficient vector $\hat{\mathbf{a}}$ can be converted into a surface velocity coefficient vector [10]

$$\hat{\mathbf{v}}_{\mathrm{sphere}} = \mathrm{j}\,kc \cdot \xi \cdot \hat{\mathbf{a}}. \tag{14}$$

The *wave number* k introduces a frequency dependence. The surface velocity $\hat{\mathbf{v}}_{\mathrm{sphere}}$ can be transformed into the radiated sound pressure

$$\hat{\mathbf{p}}(r_{\mathrm{obs}}) = \rho_0 c^2 k \cdot \frac{h_n(k\,r_{\mathrm{obs}})}{h_n'(k\,r_{\mathrm{sphere}})} \cdot \xi \cdot \hat{\mathbf{a}} \tag{15}$$

at an arbitrary observation distance r_{obs} by multiplication with the acoustic impedance and the radial wave propagation [5]. The functions h_n and h_n' are the spherical *Hankel function* of the second kind in the nth order and its derivative, respectively. Due to their properties, a sound source of a certain size can only radiate a limited range of orders at a particular frequency.

3.4. Source

The radius of the optimized source is $r = 0.2\,\mathrm{m}$. Due to the minima of the associated Legendre function in Eq. (11) a combination of three different transducer sizes (2, 3 and 5 inch) is used to generate a sufficiently large squared aperture magnitude in a wide range of spherical harmonic orders.

The 2 inch and 3 inch transducers are placed in a way that approximates a *Gaussian* sampling strategy of the order 11. The 5 inch transducers are placed accordingly in an order of 3. The physical locations are chosen to cover all required elevations for each transducer type. All azimuthal sampling points are reached by rotating the measurement source to 24 positions around the z-axis. A measurement for a source directivity of a spherical harmonic order of 11 generates 672 room impulses.

An additional tilt of the source to a second elevation and a rotation of the source to 48 positions in both elevations generates an approximation of a Gaussian sampling strategy of the order 23. This measurement generates 2688 room impulse responses.

The real behavior of the transducers such as membrane modes and deviations in the effective membrane area is expected to change the directivity of the real source compared to the simulated directivity.

3.5. Periphery

A turntable is used for the azimuthal rotation of the measurement source. To allow for an additional tilt the sphere is suspended on an axis piercing its eastern and western sides. The tilt is controlled by a step motor inside the sphere. The frame construction as seen in Figure 2 might have an impact on the directivity of the source.

Figure 2: *Optimized measurement source.*

4. DIRECTIVITY OF THE MEASUREMENT SOURCE

The calculation of the weighting vector $\mathbf{g}_T$ in Eq. (10) uses the directivity matrix $\hat{\mathbf{D}}$ in Eq. (8). The directivity of the transducers in a fixed orientation is measured in an anechoic chamber. The sound pressure at the measurement points is transformed into the spherical harmonic domain according to Eq. (5). The coefficient vectors for other source orientations are generated by multiplication with the Euler rotation term, as defined in 2.1.

It is crucial to choose a suitable sampling strategy for the measurement of the directivity. *Quadrature* samplings allow for a fast and exact spherical harmonic transformation of the sound pressure $\mathbf{p}$ at the sampling points into the directivity vector

$$\hat{\mathbf{d}} = \mathbf{Y}^H \cdot \mathrm{diag}(\mathbf{w}) \cdot \mathbf{p} \tag{16}$$

using the *quadrature weights* $\mathbf{w}$, as long as the spherical functions are order limited [12].

The measurement has to be done for a sufficiently high sampling order to prevent aliasing due to a violation of the required

order limitation. Different sampling strategies require different total numbers of spatial sampling points for a certain spherical harmonic order. The Gaussian quadrature sampling is easily usable and has a relatively high efficiency [9]. It is also quite robust against aliasing and useful for applications where rectangular samplings (a set of points at both constant azimuths and elevations) are beneficial [9].

All measurements are done using a Gaussian sampling strategy of the order $n = 82$. The elevation angle $\vartheta = 0°$ indicates an upward orientation in the spherical coordinate system, $\vartheta = 180°$ a downwards orientation, respectively.

The optimized source is placed on top of a turntable, allowing for a full $360°$ rotation in the azimuth angle φ. A swivel arm with a microphone is used to measure the sound pressure along one arc of sampling points down to $\vartheta = 90°$. This way, the upper hemisphere of the source radiation pattern is measured. Using the internal step motor to tilt the source by $180°$ allows for the subsequent measurement of the lower hemisphere. The directivity of each loudspeaker is measured separately, using an interleaved sweep measurement signal.

5. DEVIATION ANALYSIS

The directivity of the real source deviates from the simulated directivity simulated with the spherical cap model. For the synthesis method it is of interest to identify the deviation of the real directivity and to analyze its impact on the synthesis performance. For the directivity measurement it is necessary to gain knowledge about the radiated orders of the source to prevent aliasing effects in the directivity matrix $\hat{\mathbf{D}}$.

5.1. Directivity

Figure 3(a) exemplary shows the simulated and the respective measured directivity of a 5 inch transducer on the optimized measurement source at a frequency of 400 Hz. The transducer is orientated upwards at an angle of about 45 degrees. At this frequency the simulation and the measurement match quite well. An impact of the frame construction on the directivity cannot be identified.

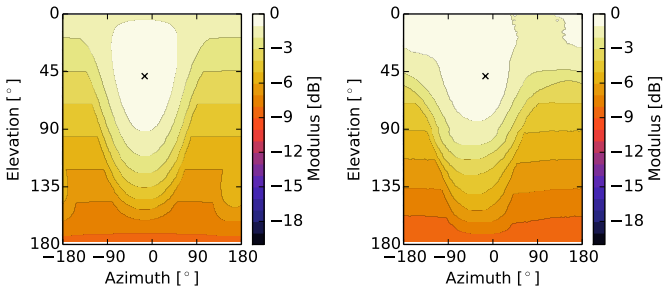
Figure 3(b) shows the same comparison at a frequency of 6400 Hz. Here, the measurement deviates clearly. Especially in angles below 90 degrees the radiation is changed. This effect can be explained with the measurement procedure. As described in section 4, the spherical body of the optimized source is tilted by $180°$ for the measurement of the lower hemisphere. In this position, the radiation of the regarded transducer is obstructed by the frame construction. Thus, the directivity can only be considered as measured correctly for the upper hemisphere and the measurement method introduces an additional error to the computation of the superposition weights in Eq. (10).

It is of interest to analyze the similarity of the simulated and measured directivity over the whole frequency range. The similarity of two spherical functions can be quantified using the spherical correlation [13]

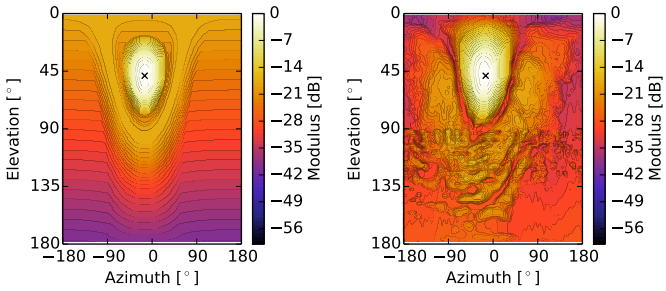
$$C(f,g) = \oint_{S^2} \overline{f(\vartheta,\varphi)}\, g(\vartheta,\varphi)\, \mathrm{d}\Omega. \tag{17}$$

The continuous integral can be expressed by a weighted summation of the quadrature sampling points as normalized correlation

$$\tilde{C} = \frac{\sum_{i=1}^{N} \overline{f(\vartheta_i,\varphi_i)} g(\vartheta_i,\varphi_i) w_i}{\sqrt{E_f E_g}}, \tag{18}$$



(a) Frequency of 400 Hz



(b) Frequency of 6400 Hz

Figure 3: *Simulated (left) and measured (right) directivity of a 5 inch transducer of the optimized measurement source. Center of membrane marked with a black cross.*

with the energy of the two spherical functions f and g sampled at N points, $E_x = \sum_{i=1}^{N} |x(\vartheta_i,\varphi_i)|^2 w_i$ and w_i being the weights for the Gaussian quadrature sampling.

The correlation between simulation and measurement for the same transducer as shown in Figure 3(a) and Figure 3(b) is depicted in Figure 4. The correlation is high for low frequencies, which confirms the general validity of the simplified analytic model. For frequencies above 4 kHz the correlation drops, confirming that a measured directivity should be used for the computations of the weights in Eq. (10).

Changes of the effective membrane area and membrane modes introduce effects which render the simplified consideration of a radially vibrating cap on a sphere invalid. Furthermore, the constructive frame of the measurement source cannot be considered acoustically transparent in these frequency ranges and changes the radiation, especially due to the explained measurement error induced by tilting the source.

5.2. Radiation

For the synthesis it is important that the measurement source is capable of radiating all required spherical harmonic orders for the computation of the synthesis weights as shown in Eq. (10). Aliasing effects during the directivity measurement would additionally distort the directivity matrix $\hat{\mathbf{D}}$. Therefore it is also important to know about the maximum spherical harmonic order radiated by the measurement source.

The radiation can be simulated with the spherical cap model. Eq. (15) yields the radiation of a transducer on a spherical sound source in spherical harmonics. To look at the radiation of more

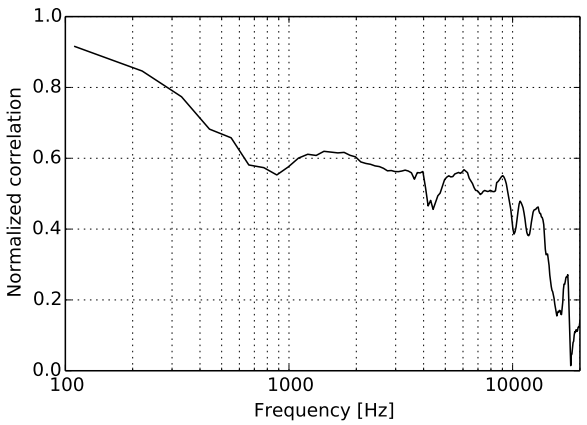


Figure 4: *Correlation between the simulation and measurement of a 5 inch transducer on the optimized measurement source.*

than one transducer, the magnitude of the coefficients of the radiation patterns can be summed up. Figure 5(a) shows the sum of the maximum magnitudes of each transducer in each order over the frequency. Since the absolute magnitude is of no interest, the data is normalized to the global maximum. The figure indicates the spherical harmonic orders that are radiated by the source. By rotating the source, the magnitude can be shifted to all coefficients within the respective order. The figure also indicates which orders have to be expected during the directivity measurement.

Orders up to 80 at a frequency of 20 kHz are expected to be excited by the optimized measurement source. In certain orders the radiation is minimal, coinciding with the effects explained in 3.2.The order limitation at each frequency is determined by the size of the source, as explained in 3.3. A sampling strategy with an order of 82 should be sufficient to prevent aliasing during the directivity measurement.

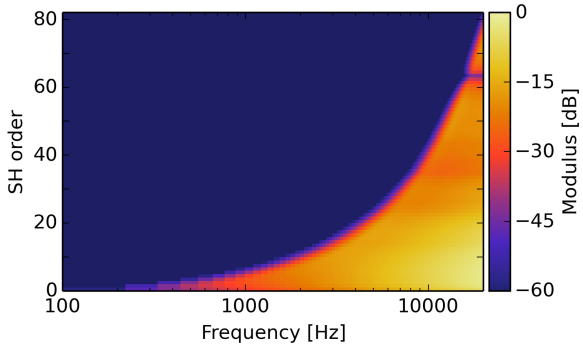
Figure 5(b) depicts the equivalent to Figure 5(a) for the measured directivity of the optimized source. The measurement has been done for one physical source orientation with a Gaussian sampling strategy of an order of 82. The data is normalized its global maximum.

A maximum order of 40 is radiated by the real source up to a frequency of 12 kHz. With the rotations to the measurement positions described in 3.4 this allows for the planned synthesis up to an order of 23.

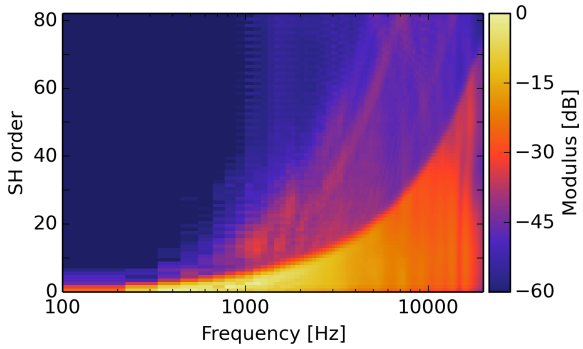
The slope of the maximum sum of the measurement is not as steep as in the simulation. A parallel slope of higher order and lower magnitude can be observed, indicating a virtual source of a larger radius. This effect is most probably caused by reflections at the frame construction which lead to a virtual enlargement of the source. Aliasing effects can be expected starting at 5 kHz.

6. CONCLUSIONS

In this article the design of a specialized measurement source for room acoustical measurements with arbitrarily given source directivity is addressed. The parameters of the source geometry are defined using an analytical model of a vibrating spherical cap in a perfect spherical housing. The source is designed to be mounted on a computerized turntable enhancing the spatial resolution with



(a) Simulation result, maximum values.



(b) Measurement result, maximum values.

Figure 5: *Radiation in spherical harmonic orders, limited to a dynamic range of 60 dB.*

a sequential measurement of different azimuthal orientations. Furthermore, the measurement source can be rotated around a horizontal axis, to increase the resolution in the elevation.

High-resolution directivity measurements of the actual prototype are used to confirm the results of the analytic calculation. The same measurements are used as a more realistic directivity $\hat{\mathbf{D}}$ for the synthesis of room impulse responses of arbitrary directivity patterns.

The device is expected to enhance the auralization of rooms for directive sound sources. Since a superposition approach is used, the desired directivity pattern can be synthesized in a post processing procedure. Even if room impulse responses with omnidirectional sources in accordance to ISO 3382 are desired, the presented measurement procedure can be used to enhance the result by synthesizing an even more omnidirectional sound source up to higher frequencies.

The prototype of the measurement source shows its capability of synthesizing directivity patterns up to a spherical harmonic order of about 23 for frequencies up to 12 kHz. This suggests great potential to improve auralizations of directive sound sources in measured acoustic environments. The measured directivity of the source is used for the computation of the synthesis weights. It has to be taken into account that for frequencies above 5 kHz this directivity contains errors due to the current frame construction and measurement procedure.

7. ACKNOWLEDGMENTS

The authors would like to thank the electrical workshop and the mechanical workshop of the ITA for their support during the construction of the measurement system as well as Ander Gaspar Perez Palacios for his commitment during the measurements and the post processing.

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