A Turbulent Jet in Counterflow

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Frau Prof. Dr. M. Yoda

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Berlin 2000
D 83
to the memory of Prof. H.E. Fiedler
Zusammenfassung


Die vorliegenden Beobachtungen legen die Aufteilung des Strömungsfeldes in ein Nah- und ein Fernfeld nahe. Im strahlähnlichem Nahfeld unterdrückt die Gegenströmung die Wirbelpaarung und verringert die typische Frequenz des Strahles. Auch wenn im Fernfeld starke unregelmässige und niederfrequente dreidimensionale Fluktuationen entstehen, kann das zeitgemittelte Strömungsfeld durch eine einfache Überlagerung eines Strahles und einer gleichförmigen Strömung annähernd modelliert werden. Eine weitere Mischungserhöhung kann durch passive Kontrolle mit nicht-runden Düsen erreicht werden.
Abstract

A turbulent jet in counterflow, i.e. a jet issuing into a uniform stream of opposite direction, produces a rather complex flowfield and was therefore up to now only scarcely investigated. Nevertheless, some interesting features - among which its enhanced dilution - motivate further studies of this flow configuration, also in view of possible practical applications. The recent developments in techniques such as particle image velocimetry (PIV) and laser-induced fluorescence (LIF), combined with advanced signal analysis methods (e.g. proper orthogonal decomposition - POD), allow a further advance in the understanding of this phenomenon. The present work was therefore planned with the aim of expanding the basis of data and knowledge available on the turbulent jet in counterflow, thereby providing a deeper insight in some of its aspects.

Velocity and passive scalar concentration measurements of a turbulent jet in counterflow were carried out in a water channel by means of LDA, LIF, and PIV. By investigating the flowfield under different boundary and initial conditions, it was possible to determine their effect on the flow and to define which conditions should be required in order to obtain a "standard" flow for a consistent comparison of results. A characterization of the flow was given by determining the extent of the mixing region, and its scaling with the jet-to-counterflow velocity ratio. The dilution enhancement produced by the counterflow is indicated by a faster centerline decay, a faster jet growth, and a faster decay of the primary instability structures. The last one is due to the enhanced formation of secondary instability structures and to the appearance of higher order (azimuthal) instability modes.

It was observed that the flowfield can be divided into two distinct regions: a near, inner field and an outer, far field. In the near field, which is similar to that of a free jet, the counterflow can suppress vortex pairing and reduce the typical frequency of the jet. The far field is characterized by strong and slow three-dimensional fluctuations showing complex dynamics. Nevertheless, the time-averaged flowfield can be approximately predicted by a very simple model, through the superposition of a jet and a uniform stream. It was also shown that a further dilution enhancement can be obtained through passive flow control by using non-circular (and especially square) nozzles.
Acknowledgement

At first I would like to recall the memory of my advisor, Prof. Dr.-Ing. H.E. Fiedler, who gave me the opportunity to work on this interesting project and guided me throughout the research. His advice and many stimulating suggestions and ideas were a fundamental contribution to the outcome of this work. The freedom he gave in the organization of the work and his open relationship with all co-workers created a jovial, encouraging, and fruitful atmosphere in the working group. Caring and responsible, while courageously fighting against the overwhelming illness, he stood at our side till his last days. For this, and for having had the chance of profiting from his teaching and his example, I feel greatly indebted to him.

I would also like to express my gratitude to Prof. Dr.-Ing. H.-H. Fernholz for taking over the supervision of this project after Prof. Fiedler's death, thereby allowing me to complete this work. I wish to thank Prof. Dr. M. Yoda for designing and building the experimental setup, for laying – with her own work – the foundation for the present investigation, for her advise and suggestions, and for accepting to be a member of the PhD committee. Thanks are due to Prof. Dr. rer. nat. A. Dillmann as well for chairing the committee.

All colleagues in the Werk III lab offered precious help and their experience, which allowed many problems to be solved. Among them deserve special mention Bastian Blümel for providing the software for image processing and PIV, Lutz Taubert for developing the hardware for PIV and simultaneous LIF, and Guiren Wang, who built the excitation mechanism. Special thanks go also to René Spieweg and Rainer Nagel for their friendly help and support.

In the Hermann-Föttinger-Institut für Strömungsmechanik (HFI), the help of Dr. rer. nat. B. Sammler for the software on POD was highly appreciated. I am also grateful to Dr.-Ing. B. Lehmann of DLR (Deutsches Zentrum für Luft- und Raumfahrt) for his tips and many interesting discussions on laser, optics, and LDA. Since this is an experimental project, it could not have been carried out without the support of the machine shop of the institute, whose helpful and capable workers performed an excellent job on many occasions. In the HFI, finally, I wish to thank Dipl.-Ing. A. Leutz, Mrs. L. Lindemann and the rest of the staff for their help and administration work.
My thanks go also to the Versuchsanstalt für Wasserbau und Schiffbau (VWS) of the Technische Universität Berlin, which accommodated the experimental setup, and in particular to Dr.-Ing. H.-D. Stinzing, who lent us the LDA equipment.

For the part of the work which was done at the University of Hong Kong (HKU), I would like to thank Prof. N.W.M. Ko and Dr. K.M. Lam for their advise and supervision, Dr. C.H. Chan for the interesting discussions, and the staff of the Hydraulics Lab for their help in setting up the experiment.

The present work resulted from my activity as a guest scholar at the Hermann-Föttinger-Institut für Strömungsmechanik of the Technische Universität Berlin (TUB), which was sponsored by DAAD (Deutscher Akademischer Austauschdienst - German Academic Exchange Program), the Rotary Foundation, and the European Commission, TMR Program, under contract Nr. ERBFMBI-CT97-1915. DAAD also funded the collaboration with the University of Hong Kong, which allowed some further discussion and comparisons of results. The financial support granted by these institutions is highly appreciated.

Last but not least, let me thank my family for their irreplaceable support, and especially my wife Yanzi for her love and constant encouragement.
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<th>Description</th>
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<tr>
<td>$A$</td>
<td>Excitation amplitude $= \sqrt{\frac{u^2}{U_j}}$</td>
</tr>
<tr>
<td>$A_\alpha$</td>
<td>${\tilde{a}_i}$ matrix (in POD analysis)</td>
</tr>
<tr>
<td>$B$</td>
<td>Counterflow width $= 2 \times$ channel hydraulic diameter</td>
</tr>
<tr>
<td>$C$</td>
<td>Correlation matrix (in POD analysis)</td>
</tr>
<tr>
<td>$C$</td>
<td>Time averaged concentration $= \overline{C}(x, r)$ or $C(x, y)$</td>
</tr>
<tr>
<td>$C_j$</td>
<td>Initial concentration (at jet exit) $= C_{max}(0) = C(0, 0)$</td>
</tr>
<tr>
<td>$C_{max}$</td>
<td>Centerline concentration $= C(x, 0)$</td>
</tr>
<tr>
<td>$C_{xp}$</td>
<td>Centerline concentration at $x = x_p$ $(= C_{max}(x_p) = C(x_p, 0))$</td>
</tr>
<tr>
<td>$D$</td>
<td>Nozzle exit diameter</td>
</tr>
<tr>
<td>$D$</td>
<td>Scalar diffusivity</td>
</tr>
<tr>
<td>$L$</td>
<td>Number of modes for POD reconstruction</td>
</tr>
<tr>
<td>$M$</td>
<td>mesh size (for grids, screens or honeycombs)</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of vectors $\tilde{a}_i$ (in POD analysis)</td>
</tr>
<tr>
<td>$N$</td>
<td>Dimension of vectors $\tilde{a}_i$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Total volume flux $= \int_0^y 2\pi y U , dy$</td>
</tr>
<tr>
<td>$Re_0$</td>
<td>Counterflow-based Reynolds number $=</td>
</tr>
<tr>
<td>$Re_M$</td>
<td>Mesh-based Reynolds number $=</td>
</tr>
<tr>
<td>$Re_j$</td>
<td>Jet-based Reynolds number $= U_j D/\nu$</td>
</tr>
<tr>
<td>$Sc$</td>
<td>Schmidt number $= \nu/D$</td>
</tr>
<tr>
<td>$St$</td>
<td>Jet-based Strouhal number $= fD/U_j$</td>
</tr>
<tr>
<td>$Tu$</td>
<td>Turbulence level $= \sqrt{\frac{u'^2}{U}}$</td>
</tr>
<tr>
<td>$U$</td>
<td>Time averaged axial velocity $= U(x, r)$ or $U(x, y)$</td>
</tr>
<tr>
<td>$U_j$</td>
<td>Jet exit velocity $= U_{max}(0) = U(0, 0)$</td>
</tr>
<tr>
<td>$U_{max}$</td>
<td>Jet centerline velocity $= U(x, 0)$</td>
</tr>
<tr>
<td>$U_0$</td>
<td>Counterflow velocity (negative in our reference system)</td>
</tr>
<tr>
<td>$Z$</td>
<td>Jet-to-counterflow momentum flux ratio $= \left(\frac{U_j D}{</td>
</tr>
<tr>
<td>$b$</td>
<td>Half-velocity width $= \text{width at } (U - U_0)/(U_{max} - U_0) = 0.5$</td>
</tr>
<tr>
<td>$b_c$</td>
<td>Half-concentration width $= \text{width at } C/C_{max} = 0.5$</td>
</tr>
<tr>
<td>$c$</td>
<td>Instantaneous concentration $= C + c'$</td>
</tr>
<tr>
<td>$c'$</td>
<td>Concentration fluctuation</td>
</tr>
<tr>
<td>$d_w$</td>
<td>Wire or web thickness (for screens or grids, respectively)</td>
</tr>
<tr>
<td>$f$</td>
<td>Excitation or characteristic frequency</td>
</tr>
<tr>
<td>$k$</td>
<td>Constant of the hyperbolic decay</td>
</tr>
<tr>
<td>$k'$</td>
<td>Constant of linearity in relationship $x_p$ vs. $\alpha$</td>
</tr>
</tbody>
</table>
\( r \)  
Radial coordinate (see Fig. 1.1)

\( u \)  
Instantaneous velocity = \( U + u' \)

\( u' \)  
Velocity fluctuation

\( x \)  
Axial coordinate (see Figs. 1.1 and 2.1)

\( x_c \)  
Potential core length = \( x \) at which \( U_{\text{max}}/U_j = 0.9 \)

\( x_h \)  
Half concentration length = \( x \) at which \( C_{\text{max}}/C_j = 2/(1 + \alpha) \)

\( x_p \)  
Average penetration length

\( x_Q \)  
\( x \)-coordinate of maximum extension of \( Q = 0 \) dividing line

\( x_U \)  
\( x \)-coordinate of maximum extension of \( U = 0 \) dividing line

\( y \)  
Cartesian coordinate (see Fig. 2.1)

\( y_Q \)  
\( y \)-coordinate of maximum extension of \( Q = 0 \) dividing line

\( y_U \)  
\( y \)-coordinate of maximum extension of \( U = 0 \) dividing line

\( z \)  
Cartesian coordinate (see Fig. 2.1)

\( \Delta t \)  
Time interval between the two PIV exposures

\( \Delta x \)  
Spacing between grid and reference location

\( \alpha \)  
Jet-to-counterflow velocity ratio = \( U_j/|U_0| \)

\( \tilde{\alpha} \)  
Given vector field for POD analysis

\( \tilde{\beta} \)  
Eigenvector

\( \tilde{\gamma} \)  
Eigenfunction

\( \delta \)  
characteristic length scale for large-scale structures

\( \lambda \)  
Eigenvalue

\( \lambda_B \)  
Batchelor length scale = \( \lambda_K/Sc^{1/2} \)

\( \lambda_K \)  
Kolmogorov length scale \( \approx 10\delta/Re^{3/4} \)

\( \nu \)  
Kinematic viscosity

\( \sigma \)  
Solidity = \( d_w/M(2 - d_w/M) \)

\( \tau \)  
Convective time scale

\( \Theta \)  
Angle between jet and counterflow direction

\( \omega_z \)  
\( z \)-component of vorticity = \( \partial v'/\partial x - \partial u'/\partial y \)
**List of abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCD</td>
<td>Charged-couple device</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier transform</td>
</tr>
<tr>
<td>HFI</td>
<td>Hermann-Föttinger-Institut für Strömungsmechanik</td>
</tr>
<tr>
<td>HKU</td>
<td>The University of Hong Kong</td>
</tr>
<tr>
<td>LIF</td>
<td>Laser-induced fluorescence</td>
</tr>
<tr>
<td>LDA</td>
<td>Laser Doppler anemometry</td>
</tr>
<tr>
<td>pdf</td>
<td>Probability density function</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle image velocimetry</td>
</tr>
<tr>
<td>POD</td>
<td>Proper orthogonal decomposition</td>
</tr>
<tr>
<td>rms</td>
<td>Root mean square</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>TUB</td>
<td>Technische Universität Berlin</td>
</tr>
</tbody>
</table>
1. Introduction

The turbulent round jet is a basic free shear flow, and, as such, has been extensively studied. In many practical applications, however, the jet does not issue into quiescent fluid, interacting instead with an external stream.

Depending on the relative direction of the jet and of the external flow, different phenomena and characteristics appear, which allow the following distinction among jets in co-, cross-, and counterflow. While the interaction with a stream at zero or small angle (coflow) mainly affects jet growth and entrainment, the structures are strongly modified and large streamwise vortices appear when the angle increases (crossflow). These vortices are responsible for the typical kidney shape of the cross section in the far field. When the angle between the jet and the external stream approaches 180° (counterflow), the flow pattern of the jet changes drastically, due to the reversal of the main flow direction after the interaction with the counterflow.

The experimental and theoretical difficulties related to flow reversal and to the pronounced instability explain why, although jets in coflow or crossflow have been widely investigated over the past years (Andreopoulos & Rodi (1984), Nickels & Perry (1996)), relatively few studies are available on jets in a counterflowing uniform stream. Nevertheless, the same features that are responsible for the increased complexity of the flow also contribute to enhance its dilution efficiency, making this flow configuration interesting for combustion or mixing processes in general.

The practical application of the jet in counterflow, however, has been hindered by the inadequate knowledge of its dynamics and of its control possibilities. Further studies are therefore necessary to complete the available information, with the aim of deepening the understanding of the strong complexity and instability of the flow, and of the possibility
to control it. This is the objective of the present work, which – based on the previous projects carried out in this institute (König & Fiedler (1991), Yoda & Fiedler (1996)) – presents the results of an experimental investigation of this phenomenon, regarding both the velocity and the concentration field. Data are collected both under standard conditions and after the modification of initial or boundary conditions, in order to evaluate their effect on the flow and their applicability for flow control. The data set is used also as a basis for further analysis, in order to describe and understand the dynamics of the flow.

1.1 Description of the flowfield

Based on the sketch of the flow geometry shown in Fig. 1.1, the main variables and parameters used in the study of the jet in counterflow are defined as follows:

- \( U_j \) Jet exit velocity
- \( U_0 \) Counterflow velocity (negative in our reference system)
- \( \alpha \) Jet-to-counterflow velocity ratio = \( U_j/|U_0| \)
- \( D \) Nozzle exit diameter
- \( B \) Counterflow width = 2 × channel hydraulic diameter
- \( x_p \) Average penetration length
- \( Re_j \) Jet-based Reynolds number = \( U_jD/\nu \)
- \( St \) Jet-based Strouhal number = \( fD/U_j \)

Since the setup is axisymmetric, a cylindrical coordinate system is used. A Cartesian system will be used in order to define the plane in which the measurements were taken (for the orientation of the Cartesian system with respect to the channel the reader should refer to Fig. 2.1). The \( x \)-axis is used as a reference to define the forward and downstream direction, which correspond therefore to the direction of the jet flow at the nozzle.

The sketch in Fig. 1.1 represents the time-averaged flowfield as shown in Fig. 3.2.a, but for a description of the phenomenon also the instantaneous flowfield has to be considered. Depending on the value of the velocity ratio \( \alpha \), two typical flow patterns can be observed (Hopkins & Robertson (1967), König & Fiedler (1991), Yoda & Fiedler (1996)).
1.1. Description of the flowfield

![Diagram of flowfield]

**Fig. 1.1:** Sketch of the average flowfield and definitions; $-$ $U = 0$ dividing streamline, $--$ stagnation stream surface.

For conciseness and for consistency with the existing literature, these will simply be called the *stable case* and the *unstable case*, even if these definitions are not based on the classical concept of stability for turbulent flows. Each of these flow patterns actually presents several features, which are briefly described here based on some series of instantaneous images from LIF flow visualization (Fig. 3.17 and 3.18). The stable and unstable case may coexist for some $\alpha$-values (further considerations on this problem can be found in Sec. 3.2.1), but the differences appear more clearly if they are observed at different $\alpha$-values.

For very low $\alpha$-values (*stable case* – Fig. 3.17) the jet forms one single vortex ring at the nozzle exit, which is axisymmetric and sheds regularly. The penetration is generally small and the interface between jet and counterflow fluid is sharp, so that the mixing occurs only into this vortex, as entrainment, and in the wake of the vortex itself.

Above a certain $\alpha$-value, (*unstable case* – Fig. 3.18) the flow shows a larger average penetration and strong axial and radial fluctuations. The flowfield can be divided into two main regions (Yoda & Fiedler (1996)). In the near field the behavior is similar to that of a jet issuing into a stagnant ambient. In the far field, as the velocity decays, the jet tip interacts with the counterflowing stream. Due to this interaction, the jet is deflected to one side and, with further dilution, convected backwards. The orientation and amplitude of the jet deflection are not fixed, instead the jet tip appears to oscillate with a low frequency in a disordered and unstable pattern (König & Fiedler (1991), Yoda...
1. Introduction

& Fiedler (1996)). This feature is causing the greatest difficulties in the investigation of the flow and in the interpretation of its results, as well as in practical applications. Owing to these fluctuations a large difference appears between the strongly asymmetric and unsteady instantaneous flowfield and the averaged one, which instead shows a good symmetry (see Fig. 3.1 and Fig. 3.2.a).

The determination of the boundaries of the mixing region, which is of fundamental importance for several practical applications, is possible through the definition of the average penetration length \( x_p \) and of the dividing streamlines shown in Fig. 1.1.

1.2 Literature review

The first studies of the turbulent jet in counterflow date back to the fifties and early sixties (Arendt, Babcock & Schuster (1956), Vulis & Leonteva (1955), Sui & Ivanov (1959), Sui (1961), Ilizarova & Ginevskii (1962)). Reviews of these early investigations, completed with more experiments and some empirical or analytical models, were presented by Sekundov (1969) and by Beltaos & Rajaratnam (1973). More recent investigations were performed by Lam (1991), Lam, Tang & Ko (1991), Chan & Lam (1994), Lam & Chan (1995), Lam & Chan (1997), Chan & Lam (1998), and Chan (1998), as well as by König & Fiedler (1991) and Yoda & Fiedler (1996).

A schematic review of all the available literature about jets in counterflow is given in Tab. 1.1. Since one of the most interesting applications of this flow configuration is an aerodynamic flameholder for combustion processes, a series of investigations was dedicated to the study of a jet in counterflow with combustion. These papers are more concerned with flame stabilization than with the fluid mechanics of the problem and are listed at the end of Tab. 1.1. Since the physical phenomenon is different, investigations of supersonic jets in counterflow (Romeo & Sterrett (1963), Baron & Alzner (1963), Schiff (1976), Moraes (1982)), the interaction of jets with a counterflowing shock wave (Hermening (1999)), jets whose fluid is different from that of the counterflow (Entov & Yarin (1980)), and jets impinging on a counterflowing jet of the same size (Rolon et al. (1991), Sardi, Taylor & Whitelaw (1998)) are not included.
In a few studies (Vulis & Leonteva (1955), Sui & Ivanov (1959), Sui (1961), Ilizarova & Ginevskii (1962)) the counterflow consisted of a jet of larger dimension; in all other works it consisted of a pipe or channel flow, either at the inlet or fully developed. In both cases it is important to determine the influence of the flow boundaries – or of the channel walls – on the jet flow. Sui & Ivanov (1959) suggested that the condition \( B/x_p \leq 2.5 \) should be satisfied for the counterflow to be considered indefinitely wide. Sekundov (1969) used the same parameter and found a threshold value of 2 from his analytical model. Morgan, Brinkworth & Evans (1976) conducted an analysis of this problem using different nozzle and counterflow sizes, and proposed the jet-to-counterflow momentum flux ratio \( Z \) as a parameter. Their condition

\[
Z = \left( \frac{U_j D}{|U_0| B} \right)^2 < 0.25
\]

(1.1)

can be compared to the ones mentioned above if the linear relationship

\[
\frac{x_p}{D} = k'\alpha = k' \frac{U_j}{|U_0|}
\]

(1.2)

is assumed to be valid (as will be discussed below).

**Extent of the mixing region**

All investigations agree on the determination of the axial extent of the mixing region, *i.e.* of the average penetration length \( x_p \). The linearity between average penetration length and velocity ratio (Eq. 1.2) was first shown by Arendt, Babcock & Schuster (1956) using dimensional analysis and was verified experimentally by several investigators. The value of the linearity constant \( k' \) varies, according to the several authors, in the range 2.4–2.9 (see Tab. 1.1); as suggested by Rajaratnam (1976), however, 2.7 can be taken as a reference value. The linearity is usually valid for \( \alpha \)-values larger than 3–4, and holds up to \( Z = 0.25 \). Above this value the slope decreases, and for \( Z > 2.5 \) a new power coefficient 1/3 was found by Morgan, Brinkworth & Evans (1976). The same authors also investigated the effect of the Reynolds number on this relationship and found no influence, as long as it was larger than about 3,000 for the jet and 10,000 for the counterflow.
<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Investigation</th>
<th>Comments</th>
<th>$\alpha$</th>
<th>$Re_j$</th>
<th>$Z_{max}$</th>
<th>$k'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vulis &amp; Leonteva</td>
<td>1955</td>
<td>experimental &amp; analytical</td>
<td>air, 3-hole-probe</td>
<td>2.1-2.6</td>
<td>84,000</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Arendt et al.</td>
<td>1956</td>
<td>experimental</td>
<td>air, Pitot, dimens. analysis</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.4</td>
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<tr>
<td>Sui, Ivanov</td>
<td>1959</td>
<td>experimental &amp; analytical</td>
<td>air, velocity meas.</td>
<td>2-20</td>
<td>1,000-13,300</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Ilizarova &amp; Ginevskii</td>
<td>1962</td>
<td>experimental &amp; analytical</td>
<td>air, velocity &amp; pressure meas.</td>
<td>2.8-11.5</td>
<td>80,000-150,000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hopkins &amp; Robertson</td>
<td>1967</td>
<td>experimental &amp; analytical</td>
<td>2-D jet, inviscid model without mixing</td>
<td>1-3</td>
<td>46,000</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Margason</td>
<td>1968</td>
<td>experimental</td>
<td>air, visualizations, several angles</td>
<td>1.23-12</td>
<td>-</td>
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<td>Sekundov</td>
<td>1969</td>
<td>experimental &amp; analytical</td>
<td>model: 3 zones &amp; linear profiles</td>
<td>1-</td>
<td>-</td>
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<tr>
<td>Beltaos &amp; Rajaratnam</td>
<td>1973</td>
<td>experimental &amp; analytical</td>
<td>review, empirical models</td>
<td>10.8</td>
<td>24,800</td>
<td>-</td>
<td>2.6</td>
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<td>Morgan et al.</td>
<td>1976</td>
<td>experimental</td>
<td>water visualizations, containment effect</td>
<td>1-160</td>
<td>&gt;3,000</td>
<td>0.08-6</td>
<td>2.5</td>
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<td>Oron &amp; Abuaf</td>
<td>1977</td>
<td>analytical</td>
<td>b.l. approx.</td>
<td>6-20</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>Peck</td>
<td>1981</td>
<td>numerical</td>
<td>$k-\epsilon$ model</td>
<td>1-20</td>
<td>-</td>
<td>-</td>
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<td>Majumdar &amp; Bhaduri</td>
<td>1981</td>
<td>experimental &amp; numerical</td>
<td>air, pitot, effective viscosity</td>
<td>7.85-25.4</td>
<td>-</td>
<td>0.76</td>
<td>-</td>
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<tr>
<td>Elgobashy et al.</td>
<td>1981</td>
<td>numerical &amp; experimental</td>
<td>air/CO, cold/heated jet, LDA, temp. &amp; conc. meas., $k-\epsilon$ model</td>
<td>9-20.4</td>
<td>11,000-12,500</td>
<td>0.21</td>
<td>-</td>
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<tr>
<td>Name</td>
<td>Year(s)</td>
<td>Methodology</td>
<td>Flame Stabilization</td>
<td>Conditions</td>
<td>Flame Speed (m/s)</td>
<td>Stability Limit (m/s)</td>
<td>Notes</td>
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<td>Solfrank, Moeller</td>
<td>1985, 1987</td>
<td>experimental</td>
<td>air, heated jet, Pitot &amp; thermocouples</td>
<td>1.3-2.3, 1700-3000</td>
<td>0.01</td>
<td>-</td>
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<td>König &amp; Fiedler</td>
<td>1991</td>
<td>experimental</td>
<td>air, smoke visualizations</td>
<td>1-8, 5000-20000</td>
<td>0.01</td>
<td>2.7</td>
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<td>Lam, Chan</td>
<td>1991, 1998</td>
<td>experimental &amp; (stability)</td>
<td>water, LDA, LIF, model</td>
<td>3-20, 3000-20000</td>
<td>0.25-0.44</td>
<td>2.6-2.9</td>
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<td>Yoda &amp; Fiedler</td>
<td>1996</td>
<td>experimental</td>
<td>water, calibrated LIF</td>
<td>1.3-10, 1700-4900</td>
<td>0.063</td>
<td>2.8</td>
<td></td>
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<tr>
<td>Present investigation</td>
<td>1999</td>
<td>experimental</td>
<td>water, LDA, calibrated LIF, PIV</td>
<td>1.3-50, 1700-13000</td>
<td>0.063</td>
<td>2.5-2.7</td>
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**flame stabilization studies (jet with combustion)**

<table>
<thead>
<tr>
<th>Name</th>
<th>Year(s)</th>
<th>Methodology</th>
<th>Flame Stabilization</th>
<th>Conditions</th>
<th>Flame Speed (m/s)</th>
<th>Stability Limit (m/s)</th>
<th>Notes</th>
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<tr>
<td>Schaffer &amp; Cambell</td>
<td>1955, 1956</td>
<td>experimental</td>
<td>premixed propane + air or oxygen, schlieren</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Bellamy et al.</td>
<td>1968</td>
<td>analytical &amp; experimental</td>
<td>flame stability calculation</td>
<td>-</td>
<td>-</td>
<td>0.08</td>
<td>-</td>
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<td>Yamaguchi et al.</td>
<td>1971</td>
<td>experimental</td>
<td>air + propane, temp. &amp; chem. meas.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Peck &amp; Samuelsen</td>
<td>1977, 1982</td>
<td>analytical &amp; experimental</td>
<td>premixed methane/air, temp. &amp; chem. meas.</td>
<td>17, 11300</td>
<td>0.002</td>
<td>-</td>
<td>-</td>
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<td>McDannel et al.</td>
<td>1982</td>
<td>experimental</td>
<td>propane/air, temp. &amp; chem. meas.</td>
<td>9-18, 6000-12000</td>
<td>0.002</td>
<td>-</td>
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</table>

**Tab. 1.1:** Literature review
An estimation of the radial extent of the mixing region appears more problematic, since it is difficult to correctly define and to experimentally determine the jet boundaries. The dividing line between forward and backward flow \( U = 0 \) line, —— in Fig. 1.1, is easily defined, but cannot be taken as representative of the external border of the mixing region, since any fluid with a negative velocity greater than the counterflow value has also mixed with jet fluid. The definition of the boundary as the line along which the velocity has reached the undisturbed value of the counterflow (or, as is usual for mixing layers, 95\% of it) is more meaningful but too difficult to determine experimentally due to the strong fluctuations of the outer "edge" of the flow and to its high sensitivity to small changes in the experimental conditions (Yoda & Fiedler (1996)). Beltaos & Rajaratnam (1973) proposed instead to define the stagnation stream surface as the line along which the total (= forward + backward) momentum flux \( Q \) is zero \( (Q = 0 \) line, —— in Fig. 1.1). This corresponds to the locus of the points \( y = y^* \) satisfying the condition

\[
Q = \int_0^{y^*} 2\pi yU \, dy = 0 ,
\]

which can be calculated from the velocity profiles. A precise calculation based on this definition requires therefore velocity measurements over the whole axial section of the flow. Also the stagnation stream surface does not correspond to the exact edge of the mixing region, yet it seems the most reasonable and reliable approximation.

An estimate of the size and shape of the mixing region can be obtained by the values of \( x_Q \) and \( y_Q \), which correspond to the \( x- \) and \( y- \)coordinate of the point of maximum width of the \( Q = 0 \) line. Beltaos & Rajaratnam (1973) found the constant values 0.75 \( x_p \) and 0.3 \( x_p \) for \( x_Q \) and \( y_Q \), respectively, with their empirical model. These values were verified with measurements in one single case, which was used by Beltaos & Rajaratnam (1973) to empirically adjust the model. No further experimental data are available to verify these values, since the rest of the literature results on the radial extent are based upon flow visualizations, rather than velocity data. König & Fiedler (1991) determined the point of maximum radial extent visually from long-time exposures of smoke visualizations and obtained for the \( x- \)coordinate a value which is increasing with \( \alpha \) and reaches an almost constant value of about 0.7 \( x_p \) for high \( \alpha- \)values. The \( y- \)coordinate decreases instead with \( \alpha \) until about 0.4 \( x_p \), as in Lam & Chan (1995), who determined
the boundary from LIF visualizations as the line on which the brightness of the image falls to \( e^{-1} \) times its centerline value. Yoda & Fiedler (1996), who used instead the line corresponding to 10\% of the centerline concentration from calibrated LIF, obtained for its maximum half-width a roughly constant value of about 0.42 \( x_p \) for 3.4 \( \leq \alpha \leq 10 \).

In order to point out once more the strong instability of this flow, it is interesting to consider the coordinates of the instantaneous maximum penetration point \((x'_p, y'_p)\), as was done by Lam & Chan (1997). These showed a large scatter (about 12\% standard deviation for \( x'_p \) and about 15\% for \( y'_p \)), and the average value of \( x'_p \) did not always correspond with the penetration length from the averaged images, \( x_p \). Similarly, the value of the instantaneous maximum half-width and its \( x \)-coordinate showed about 11\% and 13\% standard deviation, respectively. Also for the maximum half-widths there was no exact correspondence between the average of the values from instantaneous images and the value from the averaged image, which was reported by Lam & Chan (1995). This feature can be crucial, for example, in environmental applications dealing with pollutant dispersion, since the instantaneous penetration and maximum half-width can exceed the mean values of up to 30\% and 100\%, respectively (Lam & Chan (1997)).

**Velocity (and concentration) profiles**

The centerline \( U \)-velocity (or concentration) decay, as well as the axial penetration, were provided by many of the investigators. The centerline velocity or concentration decay is roughly proportional to the inverse of the downstream distance \( x \) (hyperbolic), similarly to the jet in quiescent fluid, with some changes of curvature in the region 0.8-1 \( x_p \). In a few cases further and stronger non-uniformities appear (Lam (1991), Peck (1981)), but this seems to be the result of inaccurate measurements. Beltaos & Rajaratnam (1973) therefore used a hyperbolic decay as the assumption for their model and chose a value of 5.83 for its constant. Compared with the value 6.3 suggested by Rajaratnam (1976) for jets in quiescent fluid, this shows that the decay is faster with counterflow. Similarly, for the centerline concentration decay, the constant was determined to be equal to 4 by Yoda & Fiedler (1996), against the value of 5.4 for jets without counterflow (Dahm & Dimotakis (1990)).
Radial profiles can be used for verifying self-similarity and for determining the jet width and growth rate, which are useful for comparison with jets without counterflow. Some radial profiles of the $U$-velocity were measured by Sui & Ivanov (1959) and by Beltaos & Rajaratnam (1973), who stated that similarity exists and suggested an approximation for the radial profiles based on the potential flow solution of a point source in a uniform stream. A more precise observation of the profiles, as reported by Chan (1998), showed that the self-similarity is limited to the inner jet region, while at the radial edges of the flow the profiles do not collapse to a single curve. This was confirmed by the concentration profiles measured by Yoda & Fiedler (1996), who limited the self-similar behavior both radially to the inner region, as well as axially to the $x/x_p < 0.7$ zone.

For the normalization of the radial profiles the half-velocity width $b$ is calculated, similarly to the case of jets without counterflow, as the $y$-coordinate at which

$$\frac{U - U_0}{U_{max} - U_0} = 0.5$$

(only Chan (1998) and Lam & Chan (1997) used the fraction $e^{-1}$ instead of 0.5). The growth rate of a jet is usually calculated as the slope of the line representing the half-velocity width $b$ after the jet has reached self-similarity. As seen above, in the case of a jet in counterflow self-similarity is never achieved totally, therefore the growth rate cannot be unequivocally determined, but still $b$ can be used to evaluate the effect of the counterflow on jet growth. The results of Sui (1961) and of Beltaos & Rajaratnam (1973), who also proposed an equation to describe it, show that the jet growth is faster than for a jet in quiescent fluid, and that the growth rate increases with $x$. A faster growth than in the case without counterflow is confirmed also by the measurements of Chan (1998). These extend also to higher $x$-values (with the definition of Eq. 1.4 it is possible to calculate $b$ also for $x > x_p$, even if its interpretation becomes problematic), showing that for $x/x_p > 0.7$ the growth rate starts decreasing with $x$.

Proposed models

Several models have been proposed to represent the velocity field generated by the interaction of a jet and a counterflowing stream, yielding
in many cases good qualitative or approximate quantitative agreement with the experimental values. It seems however that, up to now, a precise quantitative prediction of the flow was not possible.

Vulis & Leonteva (1955), corresponding to their experimental setup, modeled the flow by the superposition of two turbulent jets using Tollmien’s theory as presented by Abramovich (1963) but obtained only a qualitative agreement with their experimental results. Ginevskii (1962) used an empirical analytic model containing some simplifying assumptions, such as zero boundary layer at the nozzle and constant pressure, which are dubious. The results produced by this model are in agreement only with the data of Ilizarova & Ginevskii (1962), but not with those of other authors.

Hopkins & Robertson (1967) performed a kinematic analysis through a modified Helmholtz free-streamline theory. The hypothesis made in this work (of ignoring viscous or turbulent mixing across the dividing streamline) allows some qualitative prediction in the low-α cases to which this investigation was restricted. Also in these cases, however, the results considerably deviate from the experimental ones.

The empirical analytic model presented by Sekundov (1969) is based on the integral equations of conservation of flow rate and momentum and divides the flowfield into different zones, assuming a linear velocity profile within each of them. As dividing lines along y, the edge of the potential core, of the forward flow, and of the unperturbed flow are taken. Along x a jet-like propagation is assumed to be valid until \( U_{max} = |U_0| \), while downstream of this point the static pressure changes sharply as the width of the forward flow region is reduced as an effect of the flow deflection and retardation. This model predicts the penetration length in good agreement with experimental results, also in cases where the confinement effect of the channel walls is not negligible, but not all of the results were compared with experiments.

Beltaos & Rajaratnam (1973) proposed different empirical models for the region upstream of \( x_p \) (jet flow, hyperbolic centerline velocity decay with constant = 5.83) and downstream of it (potential flow solution of a point source in uniform stream). These models can predict experimental data approximately, but a more detailed comparison shows their limits in many aspects.

The analytical model suggested by Oron & Abuaf (1977) makes use
of the boundary layer approximation and assumes a constant static pressure, calculating the dynamic viscosity as in Schlichting (1968). The results were compared by the authors with the experiments of Sui (1961) and found in good agreement, but their validity is limited to the region in which the centerline velocity $U_{max}$ is larger than $0.22 \, |U_0|$, and therefore not in the proximity of and downstream of $x_p$.

The most comprehensive model available at present was developed by Chan (Chan & Lam (1994), Chan & Lam (1998), Chan (1998), Chan & Lam (1999)) to predict the whole velocity and concentration field. In order to obtain the centerline velocity decay, within the potential core the flow is ”compressed” as a consequence of the interaction of the jet with the counterflow. Downstream of it a Lagrangian formulation is used to model the advection produced by the counterflow on the particles of jet fluid. The jet width and the radial velocity and concentration profiles are modeled through the conservation of momentum and volume flux, and some empirical assumptions and adjustments are made after the comparison with the experimental results. The agreement with experimental data is in general good, but the prediction of the average penetration length does not show the linear relationship obtained experimentally.

Most of the models described above are very complex and yield equations which cannot always be solved explicitly. A very simple model based on the superposition of a jet flow and of a uniform counterflowing stream was instead proposed by Yoda & Fiedler (1996). For $\alpha \gg 1$ this model reproduces correctly the linearity of the penetration length relationship, including its proportionality constant $k'$ of about 2.9.

Peck (1981) and Elgobashi et al. (1981) used the $k - \epsilon$ model for a numerical simulation. The former author predicted penetration length and radial profiles with qualitative agreement, but the centerline velocity decay showed some irregularities. The latter author also measured the temperature and the concentration fields, and his calculation yielded fair agreement with the experiments for the mean velocities, but some discrepancies for mean temperature and concentration, as well as for the values of fluctuations.

Majumdar & Bhaduri (1981) performed a numerical simulation by solving the differential equations for the stream function and vorticity and by modeling the effect of turbulence through an empirically adjusted effective viscosity. Only the centerline velocity decay was
predicted accurately, while the radial profiles do not agree with the experimental results.

**Dynamics and stability**

The main reason for the difficulties in modeling the flow lies in the complexity of the flow dynamics and in the limited understanding of it. The available information on the dynamics of the phenomenon is therefore reviewed next.

Hopkins & Robertson (1967) first pointed out the presence of the stable and unstable case (see Sec. 1.1), giving the value 1.74 of the velocity ratio $\alpha$ as the limit between the two (for a 2-D jet). The phenomenon was investigated for axisymmetric jets by König & Fiedler (1991) – who set the limit at about 1.4 – and by Yoda & Fiedler (1996), who suggested a value between 1.3 and 1.4. The latter authors also pointed out that for $\alpha$-values just above that limit the unstable case starts appearing but still coexists with the stable one. The fraction of the total time in which the unstable case occurs increases with $\alpha$, until for $\alpha > 3.4$ the stable case disappears totally.

Yoda & Fiedler (1996) found that the regular axisymmetric vortex shedding in the stable case for low velocity ratios occurs at about 3–5 $Hz$, corresponding to a jet-based Strouhal number $St = fD/U_j$ of about 0.2–0.3. The same authors also showed that, for large velocity ratios $\alpha$ (unstable case), the characteristic frequency of the jet fluctuations in the far field lies below 1 $Hz$. König & Fiedler (1991) reported a weak dependence of this frequency on $\alpha$, but gave neither frequency values nor a further description of this dependence. For large velocity ratios the near field is similar to that of a free jet (Yoda & Fiedler (1996)), therefore its typical frequency can be expected to approximately correspond with that of the preferred mode. The only data available on this frequency partially confirm this assumption (as discussed more thoroughly in Sec. 3.2.2) and were obtained by Strykowski & Niccum (1991) and Strykowski & Wilcoxon (1993). Their experiments actually refer to the counter-current mixing layer generated by annular suction around a nozzle: this does not correspond totally with a jet in counterflow, but can somehow approximate its near field.

König & Fiedler (1991) reported also the different response of the stable and unstable flow case to excitation. The stable case (for $\alpha <
1.4) can be considerably influenced by axial excitation, which produces distinct vortex rings (0\textsuperscript{th} mode) at excitation amplitudes \( A < 15\% \) and causes the jet to break off the nozzle at larger amplitudes (\( A > 20\% \)). For the stable case also radial excitation, either alone or combined with the axial one, can generate a typical pattern (the helix corresponding to the 1\textsuperscript{st} mode). For the unstable case both kinds of excitation appear to have no influence on the far field of the flow. In the near field, instead, axial excitation produces regular but short-lived vortex rings.

Lam, Tang & Ko (1991) performed a spatial stability investigation of the jet in counterflow by modifying the inviscid linear stability theory suggested by Michalke & Hermann (1982) for jets in a co-flowing stream. Even if limited to weak opposing streams (\emph{i.e.} high velocity ratios, \( \alpha > 10 \)), these results show that the counterflow enhances the instability of the flow, increasing the amplification rate. At the same time the range of unstable frequencies is reduced and shifted to lower values.

**Applications**

Both the faster axial decay due to the ”compression” of the jet column and the stirring generated by the oscillations of the jet tip contribute to increase the dilution produced by a jet in counterflow. This dilution enhancement is the most evident characteristic of a jet in counterflow and is exploited in many of the proposed practical applications.

Possible fields of application of this flow can be found in environmental, chemical, or process engineering, \emph{e.g.} for wastewater or pollutants disposal (Lam (1991)), flow rate measurements in pipelines (Hutton & Spencer (1960)), mixing reactors (Morgan, Brinkworth & Evans (1976), or in-duct burners. In combustion processes, it has been proposed as an igniter in catalytic gas turbine combustors (Anderson et al. (1981)) and, in propulsion, as an aerodynamic flameholder in afterburners of jet engines (Filippi (1958)). The application for flame stabilization motivated several studies of a single jet in counterflow (Schaffer & Cambell (1955), Bellamy, Barron & O’Loughlin (1968), Yamaguchi, Maki & Imamura (1971), Peck & Samuelsen (1982), McDannel, Peterson & Samuelsen (1982) – also listed in Tab. 1.1) and of combinations of two jets at various angles (Duclos, Schaffer & Cambell (1957)).
Other possible applications include V/STOL airplanes and thrust reversal (Peck (1981), Margason (1968)), thrust vectoring of supersonic jets (Washington et al. (1996), Strykowski, Krothapalli & Forliti (1996)), drag reduction (Wuest & von Trotha (1964)), and ejection of coolant gas at the nose of a bluff body (Stalder & Inouye (1956), Warren (1960)).

1.3 Experimental approach

The flowfield generated by the interaction of a jet with a counterflow is characterized by regions which always have the same sign of velocity, whether positive for forward flow or negative for reverse flow, and regions where the sign of the velocity varies over time. The measuring technique used must therefore be able to resolve directional ambiguity is therefore needed. For the preliminary investigation of the flow, for the calibration of the setup, and for measurements in which a high spatial and temporal resolution is needed, frequency-shifted LDA was used. Instantaneous 2-D velocity fields and averaged flowfields were measured by means of cross-correlation based PIV.

Flow visualization is useful for the observation and for a qualitative description of the phenomenon. With appropriate calibration, LIF can be used for obtaining quantitative information on the concentration field of a passive scalar (jet fluid dye) as well (Koochesfahani & Dimotakis (1986)). For this purpose, calibrated LIF visualizations were performed in both axial and radial planes of the jet.

The recent developments in PIV and LIF (Raffel, Willert & Kompenhans (1998), Karasso & Mungal (1996)) have made each of them an important technique in the experimental investigation of turbulent flow phenomena. Given their compatibility, the next step is the combination of LIF and PIV to yield simultaneous information on both vector (velocity) and passive scalar (concentration) field. A method for simultaneous LIF and PIV imaging with the use of two synchronized cameras and a scanner system has been developed recently by Taubert (1997) and was applied to the study of the jet in counterflow with the twofold purpose of obtaining more information about this flow as well as of testing the technique in a different and rather challenging case.
The application of PIV and LIF, which can produce complete 2-D data of the instantaneous velocity and concentration fields, allows some considerations on the large-scale vortical structures. Further information on the dynamics of the flow can be extracted from these data through advanced signal analysis methods such as POD (Proper-Orthogonal-Decomposition).

POD is a method for low-dimensional representation of dynamical systems in a space with an optimized orthonormal basis. The procedure, also known as the Karhunen-Loève expansion, found application in several fields, such as pattern recognition, statistics, and data compression. It was first introduced for the study of turbulent flows by Lumley (1967) and, in particular with the snapshot form of POD suggested by Sirovich (1987), is useful in helping to recognize and characterize coherent structures objectively (Glauser & George (1986), Sirovich, Kirby & Winter (1990), Rajaee, Karlsson & Sirovich (1994), Berkooz, Holmes & Lumley (1993)). For its application spatio-temporal data over the whole flow field are needed. In cases where a clear periodicity could be imposed through forcing, these were obtained by phase averaging (Hilberg (1992), Rajaee, Karlsson & Sirovich (1994)). Otherwise, a series of simultaneous 2-D flow fields is needed, and this is exactly what PIV is able to provide.

Dilution, stirring, and mixing

For a correct interpretation of the results obtained from the present concentration measurements, it is important to clearly distinguish between the terms “stirring” and “mixing”. The definition given by Aref & Balachandar (1989), as reported by Broadwell & Mungal (1991), reads:

“For convenience we shall follow a terminology suggested by Eckart in which stirring signifies the process whereby fluids are distributed more uniformly within a given domain, e.g., stirring is a process of stretching of intermaterial area. Mixing, on the other hand, is the process of diffusion of substances across intermaterial surfaces. Stirring can promote mixing by creating more intermaterial surface area. Mixing depends on material properties, such as diffusivities, whereas stirring is a purely kinematical aspect dependent on flow parameters. Indeed, it is possible to stir fluids that do not mix at all.”

As discussed further in Sec. 3.1.4, the limitation in spatial resolu-
tion does not allow the present “passive scalar” concentration measurements to distinguish between stirring and mixing as “chemical product” concentration measurements would do (Karasso & Mungal (1996)). Only the average concentration of the passive scalar at the resolution scale can be determined correctly, and its decrease relative to the undiluted jet fluid will be referred to as “dilution”.

1.4 Motivation and objectives

As seen above, due to the complexity of the flow and to the corresponding scarcity of available literature, many problems are still open in the study of the jet in counterflow. Yet some interesting features of this flow, among which the enhanced dilution, suggest that it should be taken into consideration for practical application. The present study is therefore motivated by the need of improving the knowledge about this phenomenon not only out of theoretical interest for such a basic flow configuration but also with respect to its possible applications for practical problems.

The measurement and data processing methods presented in the previous section allow new investigations of the jet in counterflow, which are likely to provide a better understanding of it and to help to solve some of its open problems. The use of different experimental techniques should allow to determine limitations and capabilities of each method, so that information can be extracted with the technique which is most suitable for that purpose. At the same time, an improvement in reliability can be reached by comparing the results obtained from different methods.

The questions which remain unclear are concerned with a correct description of the flow features and with their interpretation. In particular, motivated by the corresponding inadequacies of the present knowledge, the work has the following goals.

- The available literature data were obtained under different initial and/or boundary conditions, therefore a systematic study of the effect of these conditions on the flow is needed.

- The description of the average flowfield is not satisfactory as far as the (radial) extent of the mixing region is concerned. Since its
determination is easier from visualization experiments but more meaningful from velocity measurements, it should be attempted to combine the information from PIV and from LIF in order to solve this problem.

- Since one of the typical features of this flow is dilution enhancement, its quantification would be of primary importance, especially in view of practical applications. Both experimental difficulties (high resolution requirements) and theoretical ones (choice of the representative parameters and interpretation of the results) hinder the pursuit of accurate results. However, at least some indirect or approximate evaluation should be attempted.

- Although a few of the proposed models can reproduce some features of the averaged flowfield, this differs significantly from its instantaneous realizations. A deeper investigation of the dynamics of the flow, focused both on the jet-like structures of the near field and on the three-dimensional fluctuations in the far field, is therefore necessary.

- For possible applications, it would be interesting to further enhance the dilution. The possibility of flow control by passive and active methods (e.g. non-circular nozzle exit cross section, excitation) should therefore be investigated.

Even if the present work cannot provide a satisfactory solution of all problems listed above, it should allow a deeper insight into some aspects of the phenomenon and its results should form a more solid basis for further work.
2. Experimental facilities and methods

2.1 Experimental setup

2.1.1 Water tunnel (counterflow facility)

A recirculating water tunnel was built for the investigation of the jet in counterflow (Yoda & Fiedler (1996)). The facility, sketched schematically in Fig. 2.1, is made mostly of PVC, while the 30×30 cm wide and 120 cm long test section had glass walls on three sides to allow easy optical access. The counterflow was generated by a pump, and its velocity in the test section \( |U_0| \) could be adjusted up to a maximum value of 0.13 m/s \( (U_0 = -0.13 \text{ m/s}) \) by electronic control of the motor speed\(^1\).

Since the main flow in the test section was in vertical direction, buoyancy effects were minimized. This facility also had the advantage of being very compact. It presented some problems, however, due to the presence of a 90 degree bend with cross section increase just upstream of the test section. This caused a non-uniform distribution of the counterflow in the test section (Fig. 2.2). The velocity profiles showed values on the outer side of the bend lying about 5% higher than those on the inner side, and the background turbulence level had values of about 4 to 5%.

Several arrangements of the settling devices were tested in order to improve this situation, and the best results were obtained with the

\(^1\) A detailed list of equipment and components used for the experiments can be found in App. A
configuration shown in Fig. 2.1. After the first honeycomb with 5 \( \text{mm} \) mesh size and 70 \( \text{mm} \) thickness (1) two sets of turning vanes with 25 and 4 \( \text{mm} \) spacing were placed on the upstream (2) and the downstream (3) bends on the channel bottom, respectively. At the entrance of the test section, a further honeycomb (4) with the same measures as the first one preceded two screens with mesh size of 2 and 1.5 \( \text{mm} \), respectively. The first screen (5) had a solidity \( \sigma = 0.44 \) and was placed at a distance of 20 \( \text{mm} \) downstream of the honeycomb. The second screen (6), with a solidity \( \sigma = 0.31 \), was 70 \( \text{mm} \) downstream of the first one. The velocity and turbulence profiles (Fig. 2.2) show that the non-uniformity was reduced to about 0.3 \% and the freestream turbulence level was decreased to 1.6\% (all values refer to the standard case with counterflow velocity \( |U_0| = 0.13 \text{ m/s} \)).

A further disturbance of the flow was introduced by the vibrations of the motor of the pump. This happened at frequencies corresponding to that of the selected motor speed, which ranged in the present exper-
2.1. Experimental setup

![Graphs showing velocity and turbulence profiles at x = 10 cm](image)

**Fig. 2.2:** Counterflow velocity and turbulence profiles at $x = 10$ cm before (◦) and after (●) the modification.

Instruments from 7 to 35 $Hz$. The effect of these disturbances was evaluated by analyzing the spectra of the velocity fluctuations (see Sec. 3.2.2 and 5.2) and was determined to be of minor importance with the exception of a few cases, in which the vibration frequency was very close to that of the preferred mode of the jet.

2.1.2 The jet flow

The jet flow was generated by a submersible pump that could be placed either in a separate tank (when the jet fluid was dyed) or in the main channel (for a uniform concentration of seeding particles in both streams). The mass flow could be adjusted by a valve with the help of a rotameter, which was calibrated by LDA measurements.

A brass tube with a length of 60 cm and an internal diameter of 40 mm served both as a settling chamber and as a nozzle holder (Fig. 2.3). It was mounted on the axis of the test section as shown in Fig. 2.1 and could be tilted around the z-axis. Great attention was therefore paid to avoid misalignment between jet and counterflow, and the angle was checked before each experiment using a water level mounted on the jet holder. All nozzles had a 40 mm long contraction with a fifth order polynomial inner contour and an outer form corresponding to the stagnation streamline obtained from the potential flow calculation of a point source in a uniform flow. Honeycombs with 2 mm mesh diameter and 20 mm thickness were placed in every nozzle, 10 mm upstream of
Fig. 2.3: Sketch of nozzle holder and excitation mechanism.

Fig. 2.4: (a) Jet average (○) and rms (●) velocity profiles at $x/D = 0.5$ with $D = 10$ mm and $\alpha = 4$. (b) Corresponding turbulence intensity.

The contraction (20 mm for non-axisymmetric nozzles). Three round nozzles with exit diameter $D = 10$, $5$, and $2$ mm, respectively, were used to obtain jet-to-counterflow velocity ratios ranging from 1.3 to 50. A straight pipe could be fitted at the exit lip of the 10 mm nozzle in order to investigate the effect of the initial boundary layer thickness. Experiments were carried out also using nozzles with elliptic and square exit cross section. In both cases the area contraction and the exit cross section were equivalent to those of the 10 mm round nozzle, and the inner cross section shape was adjusted progressively with the same polynomial law used for the inner contours.

As shown in Fig. 2.4 for a sample case, the jet had a top hat exit profile and a freestream turbulence level below 1%.
At the top end of the settling chamber a flange connected either an end plate with only a ventilating valve or the excitation mechanism (Fig. 2.3), consisting of a rubber membrane connected to a loudspeaker, which is operated by a function generator. A sealed fluid reservoir (pressure reservoir) maintained the same pressure on both sides of the membrane under any working condition and therefore allowed the loudspeaker to operate always under optimum conditions. A carrier frequency bridge connected to a capacitive measurement system (consisting of two capacitor plates moving with the membrane and a fixed one between them) was used to monitor the membrane displacement, but it was considered more reliable to perform the calibration of the excitation directly with LDA measurements at the nozzle exit.

The calibration showed that the excitation can be operated in the frequency range from 20 to 200 Hz, with amplitudes up to about 10%. For lower frequencies higher amplitudes could be obtained, but also harmonics of the original sinusoidal signal appeared and, below 10 Hz, the amplifier started to approach its limit and the sinusoid showed some asymmetry in its shape. For low frequencies, additional difficulties in the analysis of the excitation response arose from other sources of fluctuations in the same frequency range. The power spectra analysis revealed in fact a series of peaks around 12 and 15 Hz, independent of the jet velocity and increasing in intensity with the counterflow velocity, which must therefore be assumed to have originated from some vibration of the nozzle holder.

2.2 Experimental methods

2.2.1 LDA

In addition to the experiments carried out at the Technische Universität Berlin (TUB), some LDA measurements were performed in the flume of the Hydraulics Lab of the Department of Civil Engineering at HKU. The same jet setup and nozzles were used in the two facilities, in order to investigate possible differences produced by the different characteristics of the counterflow. In the following, therefore, also a brief description of the HKU flume is given together with the specifications of the LDA measuring systems used at TUB and at HKU.
2. Experimental facilities and methods

Measurements at TUB

Pointwise velocity measurements were obtained with a one-component Dantec LDA system, based on the 55X optics and a counter processor (Fig. 2.5). A 60 $mW$ He-Ne laser from Spectra Physics was used, and the optics consisted of beam separator, Bragg cell for frequency shifting, beam displacer, beam expander, and front lens with 600 $mm$ focal length. The system had to be operated in forward scattering in order to obtain a Doppler signal of good quality. Since with this configuration a common traversing system for laser, optics and photomultiplier would have been too heavy or unstable and could not be set up, the photomultiplier had to be moved and adjusted manually each time. The Doppler signal, shifted by a frequency shifter, was fed to the counter processor, whose sample-and-hold analog velocity output was acquired on a PC through a 12 bit A/D board.

With the given setup and its maximum allowed beam spacing of 39 $mm$, the probe volume was about 3.6 $mm$ long and 0.23 $mm$ in diameter, with a fringe spacing of 5.05 $\mu m$. The water filter connected usually to the channel water supply was disconnected for LDA experiments, so that the impurities guaranteed a good burst rate in the Doppler signal and no further seeding was required.

The frequency shift was varied in the range -100 to 100 $Hz$ and chosen so as to maximize the resolution of the counter processor in the frequency range of interest. Typically, data rates of 800 to 1200 $Hz$ with validation rates of 40 to 70% could be obtained, depending on the flow region. The acquisition was carried out at a rate of 200 $Hz$ and with 16384 samples, corresponding to a sampling time of about 82 $s$. In the proximity of the mean penetration length, however, this sampling time was not sufficient to obtain good statistics, and the measurement was repeated up to four times, bringing the total measuring time to about 330 $s$. For the measurement of power spectra, instead, a sampling rate of 200 to 400 $Hz$ was used, according to the frequencies of interest, and the number of samples was extended to 32768.

Measurements at HKU

The experiments at HKU were carried out in a 10 $m$ long and 30 $cm$ wide laboratory flume with recirculating flow (for more detailed infor-
2.2. Experimental methods

![Diagram of LDA system at TUB](image)

**Fig. 2.5:** Sketch of LDA system at TUB.

...mation see also Chan (1998)). For the present measurements, it was filled up to a level of 30 cm and the jet nozzle with its brass tube holder was fixed at mid-depth and around mid-length of the flume. The counterflow could be adjusted through an intercept valve and a weir, while the jet flow, fed from a constant head tank, was adjusted with a rotameter. Both the counterflow and the jet velocity were monitored by LDA. The measurements were performed with a counterflow velocity of either 0.10 or 0.13 m/s, corresponding to turbulence intensities between 5% and 6%.

The LDA system available at HKU was based on a Dantec FiberFlow system and a correlation Doppler signal processor (FVA). The light of a Spectra Physics 5 W water cooled Argon-ion laser (operated at 2 W) was transmitted to the optical probe through a transmitter, fiber manipulators, and optical fibers. A front lens with focal length of 310 mm produced a measuring volume of 2.5 mm in length and 0.15 mm in diameter. The system was capable of 2-D measurements, but only one component was used, with the blue beam of 488 nm wavelength. The operation in backscattering mode, together with the...
use of optical fibers, allowed to use a compact, fully automated 3-D traversing system.

The FVA processor contains all the electronics and a software interface (FLOWARE) to control photomultiplier sensitivity, frequency shifting, traversing, and data acquisition, which saved the data directly in a PC. By using a polycrystalline powder with a nominal diameter of 10 μm as seeding, a data rate always above 200 Hz could be obtained. A fixed number of 16384 data samples was acquired, which however did not correspond to a fixed sampling time, since every validated burst was recorded at its arrival time. For the spectral analysis of the velocity data it was therefore necessary to take into account the fact that the sampling frequency was not constant, which was done by the software.

2.2.2 LIF

Laser-induced-fluorescence (LIF) visualizations were also carried out in an axial or a radial plane of the jet with the equipment sketched in Fig. 2.6. The light sheet was produced by a Spectra Physics 5 W Argon-ion laser and a lens system. Two groups of spherical lenses
with focal length of 10 / 100 mm and 40 / 80 mm, respectively, were used to focus the laser beam to a thickness between 1 and 1.5 mm in the measuring section, while two cylinder lenses of 40 and 10 mm focal length spread the beam to a wide sheet. Since the direct view of radial planes was obstructed by the nozzle and its holder on one end of the test section and by the turning vanes and settling devices on the other, the images had to be taken from the side. In order to reduce the resulting strong distortion and the effect of the refraction index on the air/glass interface, a box with a slanted observation window was constructed. Filled with water, this box worked as a prism and allowed to obtain images of good quality and negligible distortion.

Disodium fluorescein (\(C_{20}H_{10}Na_2O_5\)) with a concentration of \(0.5 - 5 \times 10^{-6} M\) (molar concentration) was used to dye the jet fluid. At this concentration, and with the available light sheet intensities, the relationship between concentration and intensity of the emitted light (\(\approx\) gray scale value of digitized image) is linear and attenuation is negligible (Koochesfahani & Dimotakis (1986), Yoda & Fiedler (1996)). It is therefore possible, by calibration, to use the visualization images for a quantitative determination of the passive scalar concentration field. This was done by keeping record of the quantity of dye which was progressively fed into the channel and by recording, at the beginning and at the end of each run, an image of the counterflow alone (i.e. without jet flow).

An orange filter (see Taubert (1997) for its transmittance curve) placed in front of the camera eliminated the wavelengths corresponding to the incident laser light, so that only the emitted fluorescence was recorded. A reference frame positioned on the plane of the laser sheet before each experimental run allowed the exact scaling and positioning of the images. It could have also served to quantify the image distortion in order to eventually reduce it via a coordinate transformation during post-processing, but this appeared to be unnecessary.

Images were acquired by a CCD camera and a Betacam VCR. They were digitized by a frame grabber and fed to a SGI workstation for further processing. An exposure time of 1 ms was chosen, as the best compromise between image sharpness and brightness. The images were digitized with 256 gray scale levels and had a size of 768×576 pixels, the larger dimension being parallel to the x-axis. This corresponded to a spatial resolution of 0.05 mm for the visualizations of the near
field, otherwise of 0.12 to 0.3 mm, depending on the velocity ratio $\alpha$, for the images of the whole flowfield. The temporal resolution was limited to the video frequency of 25 Hz and series of 400 images were generally acquired, obtaining a record length of 16 s. In order to reduce the influence of low frequency fluctuations, the statistical analysis of the average flowfield was instead conducted with a reduced sampling frequency of about 2–4 Hz, thereby obtaining a sampling time of 96–192 s.

2.2.3 PIV

A sketch of the measurement system is shown in Fig. 2.7. The basic optical and electronic equipment as well as the methodology is the same as described in Taubert (1997). The light sheet is generated by the same laser used for LIF, and its beam is focused to about 2 mm and
scanned through the axial plane of the jet. The camera is synchronized with the scanners to obtain two exposures corresponding to the two half-frames of every video image. The time interval between the two exposures, $\Delta t$, could be adjusted to a minimum of 2.4 ms, which is the value used in the present experiments. The shutter of the camera was left open, so that the exposure time corresponded to the time in which the laser beam scanned the measuring window, which was equal to 0.8 ms. The tracer particles were VESTOSINT 7182 polyamide spheres from Degussa-Hüls with a nominal diameter of 20 $\mu$m and a specific gravity of about 1.02.

The method for image scaling, the time resolution, and the sampling rates and times are the same as for LIF, while spatial resolution is lower and depends on the PIV data processing (see Sec. 2.3.2)

### Simultaneous PIV and LIF

The basic setup is the same as used for PIV (Fig. 2.7), with the addition of a camera for LIF, synchronized with the first one, and of a fluorescent dye in the jet fluid as in Sec. 2.2.2. Here rhodamine 6G ($C_{28}H_{31}N_2O_3Cl$) was used, since it allowed a better color separation than fluorescein. Its concentration ($2.5 \times 10^{-7} M$) was, as in the LIF experiments, low enough to be in the range where linear dependence exists between fluorescence intensity and concentration. The reference frame used for image scaling served in this case also to check and to correct the correspondence between the areas captured by the two cameras.

The color filter placed in front of the LIF camera removed virtually all the PIV particles from the LIF-image. While Taubert (1997) also used a filter for the PIV camera, here this was not used, otherwise the particle density in the image would have been reduced too drastically. The fluorescence in the background, however, seemed neither to affect the quality of the PIV results nor to increase the number of outliers. No further image enhancement was required here after digitization, in contrast to Taubert (1997). Both differences in the procedure can be explained by the fact that in the present case the size of the measuring area was much larger than in Taubert (1997) so that the particle images appeared less bright, but at the same time also the undesired side effect of light scattering by the PIV particles was reduced.
2.3 Data processing

2.3.1 LIF image processing

The background was removed from each series of images by subtracting the corresponding light sheet intensity image, obtained as the average between the images of the counterflow alone (without jet) recorded before and after each experimental run. The concentration of the passive scalar was then obtained by normalizing the data by the same light sheet intensity image. At the same time, in order to recover the original range of gray scale values, the data were multiplied by the average gray scale value of the light sheet intensity image. For a statistical analysis one average image and one rms image were determined from each image series, by calculating the average and rms gray scale values pixelwise. For a few points along the axis and along some radial profiles also the histogram of the gray scale value distribution during the sampling time (corresponding to the concentration pdf) was determined.

A series of 400 images was used for each experiment in order to calculate the flow statistics, corresponding to a sampling time of at least 96 s (see Sec. 2.2.2). With the near field convective time scales estimated according to Yoda & Fiedler (1996) as \( \tau = 2D/(U_j + U_0) \), the sampling time was about 200–115,000 \( \tau \) in the different cases, the lower values corresponding to the lower velocity ratios. For the far field, it is suggested to estimate the convective time scale as the time needed for an element of jet fluid to reach \( x = x_p \). This time can be estimated using Eq. 3.5 (with \( k' = 5.83 \) and \( U_{max} = 0 \)) as \( \tau \simeq 2.9 U_j D/U_0^2 \) or \( \tau \simeq 6.7 \sqrt{Z} \), where \( Z \) is the jet-to-counterflow momentum flux ratio \((U_j D/|U_0|B)^2\). A sampling time of 96 s corresponds therefore to 60-360 \( \tau \), the minimum value being for the maximum allowed momentum flux ratio \((\sqrt{Z} = 0.25)\).

Usually at least 1,000 convective time scales are necessary to obtain well-converged statistics up to the first order fluctuations in a turbulent flow, but a value of a few hundred is enough if only the averages are needed. It appears therefore that the average values are accurate over the whole flowfield only as long as the momentum flux ratio is well below its maximum value. Since the sampling time of 96 s instead of the standard 160–192 s was used only in a few cases at low \( \sqrt{Z} \),
however, we can assume that the average values were acceptable in all cases presented here. In the near field, for sufficiently large values of $\alpha$, also good estimates of the the fluctuations could be obtained.

Tests were performed using a progressively increasing number of images and observing the corresponding development of the parameters that characterize the extent of the mixing region. The number of images required to obtain an error (defined as the difference from the value obtained with all 400 images) of less than 5% was between 80 and 200 for $x_p$, between 70 and 90 for $y_U$, and between 90 and 200 for $y_Q$. Only for the axial coordinates $x_U$ and $x_Q$ larger numbers of images were required, between 150 and 350 at the different velocity ratios.

The average penetration length $x_p$ and the average radial extent of the mixing region were determined from the average image of axial flow visualizations. Here a percentage of the maximum gray scale value ($\sim$ initial jet fluid concentration) had to be chosen as the threshold for the dividing line, but its determination is not straightforward (the reader is therefore referred to Sec. 3.1.1 for a more thorough discussion). The average penetration length was computed by applying this percentage to the concentration decay profile along the jet axis, and the jet axis was determined, for each $x$-position, as the $y$-value at which the radial concentration profile had its maximum. This procedure can therefore be applied also when the jet axis does not correspond to the $x$-axis. This is usually the case when a small angle between jet and counterflow is present, but may occur also with no angle, due to the high sensitivity of the flow to sideways disturbances – especially at low velocity ratios. The two dividing lines corresponding to the forward flow region and to the stagnation streamline (see again Sec. 3.1.1 for details) were determined by applying the above percentage to every radial profile. As a measure of the extent of the mixing region, for each of these two dividing lines the coordinates of the point of maximum radial extent, $(x_U, y_U)$ and $(x_Q, y_Q)$, were calculated.

In the case of radial flow visualizations, the jet axis was determined as the center of gravity of the area formed by the points whose gray scale values lay above a certain threshold. The radii (i.e. the distances from the jet axis to the external edge of this area) and the distribution of the gray scale values within this area allowed the calculation of parameters such as the shape parameter and the cross section non-
uniformity. These parameters are defined in Sec. 3.1.4, where they are used to study the effect of the counterflow on the cross section of the jet.

2.3.2 PIV data processing

The odd and even lines of each acquired image were separated, yielding two single exposures corresponding to the two half-frames. Displacements were then calculated through cross-correlation with a program based on PIVWARE (Westerweel (1993)) and further extended in this institute to improve iteratively the accuracy through a window shifting procedure (Blümel & Taubert (1996)). An interrogation window size of 32×32 pixels with 50% overlapping in each direction was chosen. The CCD size of 768×576 pixels and the image size of about 90×68 mm yielded a spatial resolution of about 2 mm for the smaller velocity ratios. The larger image area needed for the higher velocity ratios resulted in a poorer resolution of up to 5 mm.

The $\omega_z$ vorticity component was calculated in each point of the velocity field through the circulation along the 9 surrounding points, as described in Taubert (1997). The statistics of the flowfield over the time series were calculated in each point of the 16×16 pixel grid. For the average values a vector representation could be used, while rms values, Reynolds stresses, and vorticity were represented either by gray scale values or by contour lines. This representation of the whole 2-D field allows only a qualitative analysis, while quantitative considerations can be made on axial or radial profiles. However, if quantitative results are needed, the poor spatial resolution has to be taken into account and no good accuracy can be obtained for the velocity fluctuations.

2.3.3 POD analysis

A simplified description of POD, based on the work of Sirovich & Kirby (1987), Hilberg (1992) and Huang (1993), is given in the following as a help in the interpretation of the present results. More precise and detailed information on POD in general can be found, e.g., in Sirovich (1987).
2.3. Data processing

Given a series of \( M \) vectors \( \{ \vec{\alpha}_i \} \) of dimension \( N \), the basic POD idea is to find a coordinate system \( \{ \vec{\beta}_k \} \) in which the ensemble \( \{ \vec{\alpha}_i \} \) is optimally represented, \( i.e. \) on which it has, on average, the largest projection. The POD procedure consists then in finding an orthonormal basis on which the mean quadratic scalar product

\[
\lambda_k = \sum_{i=1}^{N} \left( \vec{\alpha}_i \cdot \vec{\beta}_k \right)^2
\]

(2.1)
is maximized (with decreasing maxima \( \lambda_{k+1} \leq \lambda_k \)), which corresponds to solving the eigenvalue problem

\[
C \vec{\beta}_k = \lambda_k \vec{\beta}_k, \quad k = 1, 2, ..., M
\]

(2.2)
where the correlation matrix \( C \) is defined as

\[
C = A^T_{\alpha} A_{\alpha}
\]

(2.3)

\( A^T_{\alpha} \) is the transposed of \( A_{\alpha} \), which is the matrix representing the given vector field \( \{ \vec{\alpha}_i \} \)

\[
A_{\alpha} = \{ \vec{\alpha}_i \} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1M} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N1} & \alpha_{N2} & \cdots & \alpha_{NM} \end{pmatrix}
\]

(2.4)
The result of a POD analysis is then a set of eigenvectors \( \{ \vec{\beta}_k \} \) (the orthonormal basis) and eigenvalues \( \{ \lambda_k \} \), where \( k \) is the mode number. The eigenvectors, together with the given vectors \( \{ \vec{\alpha}_i \} \), allow the eigenfunctions to be calculated as

\[
\vec{\gamma}_k = A_{\alpha} \vec{\beta}_k
\]

(2.5)
and, through linear combination of the eigenfunctions with the eigenvectors as coefficients, it is possible to reconstruct the original vector field

\[
A_{\alpha} = \sum_{k=1}^{L} \vec{\gamma}_k \vec{\beta}_k
\]

(2.6)
to a given degree of approximation (\( i.e. \) taking into account a given number \( L \leq M \) of modes). In the case of flow data, each eigenfunction
(here also called eigenflow) represents a typical dynamical structure, while the corresponding eigenvalue represents its relative importance, or the portion of turbulent kinetic energy associated with it when the velocity fluctuations are the components of the vector \( \{ \tilde{\alpha}_i \} \).

The above description is for classical POD, but it is valid also for snapshot POD if one exchanges \( A \) vs. its transposed \( A^T \), i.e. \( M \) vs. \( N \) and space vs. time. In both cases the vectors \( \{ \tilde{\alpha}_i \} \) are time series of \( N \) points each measured simultaneously at \( M \) positions in space, and the number of independent solutions to the eigenvalue problem corresponds to the smaller of the two values \( M \) and \( N \). In contrast to classical POD, snapshot POD is therefore used when many spatial data are available for a limited number of time steps, i.e. when \( M > N \). This is the case of PIV, where a large number of vectors can be obtained in each image but time resolution is generally low and the total number of images that can be processed is limited by computer memory and time requirements. While the eigenvalues obtained from classical POD are typical time series of the given signal, whose distribution in space is given by the eigenvectors, in snapshot POD each eigenvalue corresponds to a ”snapshot” of a typical flow structure, and the eigenvectors represent its variation in time.

In the present case snapshot POD was calculated on series of both LIF images and PIV vector fields. Each \( \tilde{\alpha}_i \) vector consisted thereby of \( M \) points, where \( M \) is equal to the total number of pixels \( (768 \times 576 = 442,368) \) for LIF and to the number of grid points \( (35 \times 47 = 1,645) \) for PIV (see Sec. 2.3.2). The number of points for LIF caused too long computing times, therefore the images were downscaled to 30% of their original size in each direction, yielding \( 230 \times 173 = 39,790 \) grid points. The gray scale value representations for PIV suffered instead from the low spatial resolution, so that the PIV data were further refined by reducing the grid size to \( 8 \times 8 \) pixels, thereby obtaining 6580 points. The size \( N \) of each vector corresponded to the length of the time series, in both cases equal to 400.

As a result, 400 modes (eigenflows) were obtained, together with a series of 400 eigenvectors of dimension 400, each of them representing the variation of the corresponding eigenflow with time. When POD was applied to LIF data, the components of each \( \tilde{\alpha}_i \) vector were simply the gray scale values of each pixel of the ”caricatures” (the instantaneous images from which the averaged image had been subtracted),
which correspond, after calibration, to the concentration fluctuations \( c' \). The same results are obtained when using the instantaneous values \( c \) instead of \( c' \), with the only difference that the first mode corresponds to the mean flow, therefore all the mode numbers are shifted by 1 and the first eigenvalue has to be subtracted from the progressive and the total sum in order to obtain the same eigenvalue distribution.

POD was applied to PIV data according to the method of Huang (1993) both in the energy and the vorticity domain, i.e. constructing the correlation matrix either with the velocity fluctuations as

\[
C = A_{u'}^T A_{u'} + A_{v'}^T A_{v'}
\]  

(2.7)

or with the \( \omega'_z \) (= \( \partial u' / \partial x - \partial u' / \partial y \)) vorticity component as

\[
C = A_{\omega'_z}^T A_{\omega'_z}
\]  

(2.8)

The resulting eigenflows were represented as a vector field using both the \( u' \) and the \( v' \) component, i.e. calculating Eq. 2.5 once with \( A_{u'} \) and once with \( A_{v'} \). For the gray scale value representation of the approximate turbulent kinetic energy or of the \( \omega'_z \) vorticity field, instead, Eq. 2.5 was used with \( A_{u'} + A_{v'} \) or with \( A_{\omega'_z} \). For such gray scale value representations, as for the eigenflows from LIF (also called eigen-pictures), the gray color represents the neutral background, while the black and the white zones correspond to the flow structures, oscillating in time with the eigenvectors as coefficients. If the flow is reconstructed with one single mode, therefore, two gray scale value images a half period of oscillation apart will be the black/white-inverted image of each other.
3. The round turbulent jet in counterflow - base case

3.1 Statistical description of the flowfield

In this section the time-averaged flowfield and some statistical data of its fluctuations are described, based on data obtained along a single axial section of the flow. The validity of this simplification is discussed in Sec. 3.2.3, where visualizations on radial planes show that the time-averaged flow is axially symmetric, even if this is not the case for instantaneous flowfields (see Fig. 3.1).

3.1.1 Correspondence between velocity and concentration field

Since results originating from both velocity and passive scalar concentration measurements are presented, some preliminary remarks on the correspondence between velocity and concentration field are needed. For this purpose, simultaneous PIV and LIF experiments were carried out. For observations on average and rms values there is no need for the PIV and LIF experiments to be performed simultaneously. Nevertheless, it is preferable that also some series of simultaneous experiments be available as a reference in order to ensure that no other factors can affect the comparison of the results from the two methods.

A first visual check of the agreement between the two methods was made using instantaneous images and comparing the flow structures which can be identified with each of them. The superposition of a sample (inverted) instantaneous LIF-image and the corresponding vorticity
3.1. Statistical description of the flowfield

![Image](image.png)

**Fig. 3.1:** Instantaneous PIV/LIF image: superposition of instantaneous LIF image and corresponding PIV iso-vorticity lines; $\alpha=3.4$ and $D=10$ mm.

![Image](image.png)

**Fig. 3.2:** (a) Average of the LIF gray scale values together with two dividing streamlines calculated from PIV ($\quad U=0, \quad - - Q=0$). (b) Rms of the LIF gray scale values with superimposed contours of the rms of $U$-velocity fluctuations from PIV; $\alpha=2.2$ and $D=10$ mm.

Contours calculated from PIV (Fig. 3.1) shows a good correspondence between the vortical large-scale structures which can be identified from flow visualization and those defined by the iso-vorticity lines.

A qualitative comparison between statistics from simultaneous LIF and PIV data is shown in Fig. 3.2 for a sample case. In both figures the LIF results are represented by (inverted) gray levels, the number of which was reduced to 10 for simplicity. The distribution of rms values (Fig. 3.2.b) confirms the good qualitative agreement between the two methods, showing two regions of large fluctuations, one in the shear layer and the other where the velocity inversion takes place. From
3. The round turbulent jet in counterflow - base case

Fig. 3.3: Correspondence between velocity and concentration decay.

Fig. 3.2.a it can be seen that the dividing lines calculated from PIV have a similar shape to those of the LIF percentage contours and lie within the 20% and the 30% concentration boundary.

It appears therefore that a general agreement exists, but a precise relationship between velocity and concentration field is needed in order to allow also quantitative comparisons. The derivation of such a relationship from a complete model would involve problems similar to those encountered when trying to model the velocity field (see Sec. 1.2), therefore it is not followed up any further. Instead, the present objective is limited to the determination of some approximate correspondence which allows to obtain a consistent estimate of the extent of the mixing region also from passive scalar concentration measurements.

Correspondence of axial profiles

Based on the conservation of mean concentration flux it was shown for the free jet (Abramovich (1963)) that the centerline concentration decay is proportional to $1/x$ as is the centerline velocity decay, even if with a smaller constant (as verified also for the jet in counterflow by Beltaos & Rajaratnam (1973) and Yoda & Fiedler (1996)). Here it is assumed that the shapes of the centerline decay curves for velocity and for concentration are approximately similar. This means that, if both curves are normalized in the range [0,1] (see Fig. 3.3), the following relationship is obtained:

$$\frac{C_{xp}}{C_j} \sim \frac{|U_0|}{U_j - U_0} = \frac{1}{1 + \alpha}. \quad (3.1)$$

The fraction $1/(1 + \alpha)$ of the initial concentration (or gray scale value) is therefore used for calculating the average penetration length from
3.1. Statistical description of the flowfield

Fig. 3.4: Normalized profiles of centerline velocity decay and centerline concentration decay vs. $x/D$ (a) and $x/x_p$ (b).

the average images of calibrated LIF experiments. Due to the smaller constant in the centerline decay, however, an underestimation of $x_p$ is expected when using this method.

An approximate verification of the correspondence between the normalized centerline decay curves can be obtained from Fig. 3.4. By representing the data vs. $x/D$ (Fig. 3.4.a), it is evident that this method will tend to underestimate the penetration length (as appears also from Fig. 3.5). The agreement between PIV and LIF data after the normalization of $x$ with $x_p$ (Fig. 3.4.b) is however satisfactory, even if not complete. The large discrepancy for $\alpha$-values larger than 3.4 is due to the inadequacy of PIV for measuring large velocity gradients, which characterize the near field of the jet at these values of $\alpha$.

The proposed relationship can be verified also by calculating the penetration length from PIV and then by determining the corresponding percentage of the normalized centerline concentration decay. The points obtained in this way for different values of $\alpha$ are shown in Fig. 3.5: it can be seen that they fall usually below the $1/(1 + \alpha)$ curve, still the curve represents their behavior much better than a fixed percentage (which was used e.g. by Lam & Chan (1995)).

Correspondence of radial profiles

For the radial profiles there is no exact correspondence either between concentration and velocity field, the concentration decay being slower
Fig. 3.5: Fraction of initial concentration value at $x = x_p$.

(Abramovich (1963)). However, the present measurements show that this seems to compensate the difference in the axial profiles. In fact, if the fraction $1/(1 + \alpha)$ is used as the threshold value for calculating the line corresponding to the $U = 0$ dividing streamline from the concentration field $C/C_j$, the estimate of the radial extent from LIF agrees with that from PIV (see Sec. 3.1.3). This method therefore avoids the problem of the arbitrary choice of the threshold value (Lam & Chan (1995), Yoda & Fiedler (1996)).

### 3.1.2 Axial development

**Average penetration length**

Almost all the literature sources agree on the linearity of the relationship between the jet-to-counterflow velocity ratio $\alpha = U_j / |U_0|$ and the average penetration length $x_p$. The present results, shown in Fig. 3.6, confirm the linearity and suggest a value of about 2.5–2.7 for the proportionality constant. It also appears that the agreement among the different measurement techniques is generally satisfactory. Slightly lower values appear for the data from LIF, as seen in the previous section, and larger discrepancies appear for data obtained with the smaller nozzles at low velocity ratios. This can be explained by the difficulty of adjusting the rotameter for extremely small flow rates, especially for PIV and LIF experiments (where $U_j$ cannot be directly monitored as for LDA).
3.1. Statistical description of the flowfield

![Graphs showing penetration length vs. angle for low and high values of x, with LDA, PIV, and LIF data.](image)

**Fig. 3.6:** Average penetration length from LDA, PIV and LIF for low (a) and high (b) values of x (dotted symbols indicate very small flow rate – possible errors due to rotamer setting).

In Fig. 3.6 only data were shown which satisfy the condition $Z = (U_j D)^2 / (|U_0| B)^2 < 0.5$ prescribed by Morgan, Brinkworth & Evans (1976) to exclude the confinement effect of the channel walls. As shown in detail in Sec. 4.1, if this condition is not fulfilled, a departure from linearity with reduced $x_p$-values occurs. The condition implies that the departure from linearity occurs at increasing $\alpha$-values for decreasing $D/B$ ratios, as was verified by Morgan, Brinkworth & Evans (1976) and by the present experiments. The model proposed by Chan & Lam (1998), which predicts a strong departure from the usual linear relationship at a fixed $\alpha$ of about 20, appears therefore on this point in contrast with the experimental evidence.

For low values of $\alpha$, the departure from linearity is explained by the different dynamics of the phenomenon, with the appearance of a stable case with smaller penetration, described first by König & Fiedler (1991) and Yoda & Fiedler (1996) and discussed here more extensively in Sec. 3.2.1.

**Axial development of centerline velocity and concentration**

The axial development of the jet can be described by the profile of the average $U$-velocity and/or concentration along the centerline (centerline velocity and/or concentration decay) as shown in Fig. 3.4. The normalization of Fig. 3.4.b shows that, even if there are some irregu-
3. The round turbulent jet in counterflow - base case

Fig. 3.7: Axial profiles of average velocities measured with LDA for different $\alpha$ values for low (a) and high (b) values of $x$. BR indicates the models proposed by Beltaos & Rajaratnam (1973), ”model” indicates the present one.

Fig. 3.8: Inverse centerline decay from LDA (a) and LIF (b) and slopes for free jet (symbols as in Fig. 3.7).

imilarities between LIF and PIV data, the behavior is similar, so that LIF might be used to roughly estimate the velocity profiles in the case of high $\alpha$-values, where PIV fails to yield correct data in the near field.

A normalization procedure which lets all profiles collapse on the same curve (at least downstream of the potential core) was suggested by Beltaos & Rajaratnam (1973). This normalization is used in Fig. 3.7 to show the centerline velocity decay obtained from the present LDA measurements, together with the hyperbolic decay typical of free jets and some curves from the models discussed in Sec. 3.2.4. It can be noticed that the counterflow produces a faster decay than in the case of quiescent ambient fluid, and a quantification of this phenomenon is possible by representing the profiles in the inverse form (Fig. 3.8). In
Fig. 3.9: Axial profiles of velocity fluctuations (rms) measured with LDA for different $\alpha$ values, scaled with $U_{max} - U_0$ (a) and with $|U_0|$ (b) (symbols as in Fig. 3.7).

In this figure, for comparison, also the slopes corresponding to the free jet are displayed (as reported by Rajaratnam (1976) and by Dahm & Dimotakis (1990) for velocity and concentration decay, respectively). This shows that the decay slope is always larger than in the case without counterflow and further increases when moving downstream and when increasing the counterflow.

No observations are available in literature about the axial profiles of velocity fluctuations (rms). By considering the results of the present measurements (Fig. 3.9), it is interesting to notice that the profiles scale with $U_{max} - U_0$ for $x \leq x_p$ and with $|U_0|$ for $x \geq x_p$. This suggests the existence of two distinct regions with different dynamics, as discussed in Sec. 3.2.

Length of the potential core

According to measurements presented by Rajaratnam (1976), the length of the potential core $x_c$ is about 5 $D$ for a free jet. Chan & Lam (1998) estimated this length as 6.2 $D$ and proposed in their model, for the case with counterflow, $x_c = 6.2(\alpha - 1)/(\alpha + 1)$. Even if no experimental data were given to verify this relationship, it appears reasonable to assume that the effect produced by the counterflow on the near field of the jet will be a "compression" of the potential core, i.e. a reduction of its length. This corresponds in fact to the observed faster jet growth (Sec. 3.1.3) and faster decay (Sec. 3.1.4 and 3.2.2) of the primary instability structures (vortex rings or spanwise structures) due
Fig. 3.10: Length of the potential core from LDA and LIF measurements and comparison with model of Chan & Lam (1998).

to secondary instability (streamwise structures).

The core length can be estimated from the axial decay profiles by calculating the axial distance at which the velocity or concentration is reduced to a certain percentage (here 90%) of its initial value. The values obtained in the present experiments are shown in Fig. 3.10 together with the model of Chan & Lam (1998). It appears that the data measured in the HKU flume approximately follow that curve, while the ones measured at TUB are slightly lower, showing a better agreement with the data of Rajaratnam (1976). The differences between the measurements at HKU and those at TUB can be explained by the different freestream turbulence level (see Sec. 4.3).

3.1.3 Radial development

Boundaries of the mixing region

In view of possible practical applications of the jet in counterflow, it is important to determine the boundaries of the mixing region but – as discussed in Sec. 1.2 – this problem could not be solved up to now. The radial extent of the mixing region is also useful as a reference parameter when comparing a large number of different flow cases, as done here when investigating the effect of initial and boundary conditions (Chap. 4). Such a reference parameter should be both physically meaningful and easy to obtain, given the large amount of data.
3.1. Statistical description of the flowfield

The stagnation stream surface (defined as the line where the total momentum flux $Q$ is zero by Beltaos & Rajaratnam (1973)) seems to satisfy at best the first requirement, but requires data over the complete velocity field. LDA can provide these data only with extremely long acquisition times (if the whole field is well sampled) or with poor resolution (if just a few radial profiles are measured). PIV shows instead problems in regions with high gradients (see Fig. 3.4.b), which can be solved only by repeating the experiments with closeups of the near field, requiring again long acquisition and processing times.

The determination of the dividing lines is easier if it is based on calibrated LIF data, by making use of the relationship between velocity and concentration presented in Sec. 3.1.1. According to it, the points in which the concentration $C$ falls to the fraction $1/(1 + \alpha)$ of its initial value $C_j$ can be assumed to be equivalent to the dividing streamline $U = 0$. By integrating the concentration along $y$ as was done with the velocity when calculating the total volume flux (with the concentrations at radial positions outside the above defined dividing streamline taken as negative), a line equivalent to the $Q = 0$ line could be obtained. The separating lines calculated with these methods are shown in Fig. 3.11 for a few sample cases: as seen in Sec. 3.1.1, the penetration length is smaller for LIF than for PIV, but the shape of the curves and the radial extent appear to be similar for both methods.

The maximum half-width of these separating lines, which is usually taken as a parameter to characterize the radial extent of the mixing

Fig. 3.11: Boundaries of the mixing region from simultaneous PIV and LIF. —— and –– are $U=0$ and $Q=0$ line from PIV; empty and filled symbols are corresponding lines from LIF.
region, is shown in Fig. 3.12.a. Again there is agreement between the PIV and the LIF values calculated with the suggested method. Here we present also the values calculated from LIF with a fixed 10% limit, which show however a more irregular trend. In all cases the width decreases with $\alpha$ until $\alpha = 7.5$, then has an almost constant value of about 0.14 $x_p$ for the $U = 0$ boundary and about 0.22 $x_p$ for the $Q = 0$ one. The $x$-coordinate of the point of maximum half-width shows a close correspondence between the values of $x_U$ and those of $x_Q$, but no good agreement between the PIV and the LIF data. The values increase with $\alpha$ and – as for the $y$-coordinate – remain almost constant for $\alpha > 7.5$, lying in the range around 0.7 $x_p$.

Jet growth - similarity of radial profiles

The growth rate of a jet is usually calculated as the slope of the line representing the half-velocity width $b$ after the jet has reached self-similarity. A jet in counterflow never achieves true self-similarity; nevertheless, $b$ can still be used to estimate the growth rate and to evaluate the effect of the counterflow upon the jet growth. In the present case, both the half-velocity width $b$ and the half-concentration width $b_c$ were calculated and some of the curves obtained are presented in Fig. 3.13.a. Due to the counterflow the curves do not grow linearly, but reduce their slope when approaching $x = x_p$, so that a determination of the growth rate becomes problematic. In order to be far from the mixing layer region at low $x/D$ as well as from the inversion region at
**3.1. Statistical description of the flowfield**

*Fig. 3.13:* Simultaneous LIF and PIV: (a) Half-velocity and half-concentration width; (b) Self-similarity of velocity and concentration profiles ($\alpha=3.4$ and $D=10$ mm).

$x \simeq x_p$, only the interval between 0.4 and 0.8 $x_p$ was considered and the average slope in this range was determined. It appears that the widths obtained by LIF are generally larger than the ones obtained by PIV. The slope of the curves lies between 0.2 and 0.4 for LIF and between 0.1 and 0.2 for PIV. This is in both cases above the value of the growth rate for a jet in quiescent ambient ($\simeq 0.1$) and can be taken as a measure of the increased dilution rate caused by the counterflow.

An example of radial velocity and concentration profiles from PIV and from LIF, normalized with $U_{max}$ and $b$ (or $C_{max}$ and $b_c$, respectively) in order to verify self similarity, is shown in Fig. 3.13.b. The present results confirm the observation by Yoda & Fiedler (1996) that a region of jet-like self-similarity exists near the axis, while in the external region the profiles are not self-similar.

### 3.1.4 Dilution characteristics

No exact data or criteria are available to characterize and quantify mixing within the mixing region of a jet in counterflow, although this would be very useful for practical applications. Since mixing is the combination of large-scale stirring and molecular diffusion, however, statements about mixing can be made only if we are able to resolve the smallest diffusion length scales (Batchelor scales, from Batchelor (1959)).
The spatial resolution of the applied passive scalar measurement method lies in the order of magnitude of 0.1 \( mm \) (and 1 \( mm \) in the direction of the light sheet thickness, see LIF data in Sec. 2.2.2). The Kolmogorov length scales can be estimated through the relationship \( \lambda_K \approx 10 \delta / R e_2^{3/4} \) (Buch & Dahm (1996)), where \( \delta \) is the characteristic length scale \( \delta \). In the present case \( \delta \) (i.e. the jet width, as in Yoda (1992)) corresponds to about 2\( \delta \), therefore \( \lambda_K \) is varying in the range 0.1–1 \( mm \), depending on the axial position and the velocity ratio \( \alpha \). The Batchelor length scale \( (\lambda_B = \lambda_K / S c^{1/2}, \) with \( S c \approx 2000 \) for fluorescein in water as in the present case – see Buch & Dahm (1996)) is about 50 times smaller and cannot therefore be resolved.

This means that it is not possible to conclude whether jet and counterflow fluid are just stirred at a scale below the resolution scale or actually mixed. Therefore, only the average concentration of the passive scalar (dilution of jet fluid) at the resolution scale can be determined correctly. The estimates of concentration fluctuations are biased and tend to be underestimated, thereby overestimating dilution (Koochesfahani & Dimotakis (1986)).

As defined in Sec. 1.2, the term “dilution” will be used instead of “mixing”, which cannot be resolved. The following analysis is therefore restricted to an attempt to characterize large scale stirring and dilution within the mixing region and in the near field of the jet.

Characteristics of the near field

Radial LIF experiments were carried out in one section of the jet at \( x/D = 1.6 \), with a fixed jet velocity \( U_j = 0.286 \) m/s. The counterflow velocity \( U_0 \) was progressively increased from zero to the standard value of 0.13 m/s, obtaining velocity ratios in the range 2.2–\( \infty \). The radial LIF images can be used to study the decay of the primary structures in the core region (see also Sec. 3.2.2), and this can be taken as an indirect measure of the dilution enhancement due to the effect of the counterflow.

In the near field the concentration gradient between the forward and the backward flow region is large, therefore it is simple to define the boundary of the jet cross section through a threshold value for concentration. In order to determine the deviation from the initial, approx-
3.1. Statistical description of the flowfield

Fig. 3.14: Quantification of dilution in the near field from radial LIF images at \( x/D = 1.6 \): (a) shape parameter, (b) cross section non-uniformity.

imately circular shape of the cross section, the shape parameter was used. This is defined as the normalized perimeter-to-area ratio of the cross section, calculated for each instantaneous image and then averaged over time. In order to quantify the mass exchange between core and surrounding fluid, and therefore the entrainment, the cross section non-uniformity was defined instead. This parameter was determined by calculating, for each instantaneous image, the rms gray scale value over the cross section (i.e. the rms of the gray scale values of all the pixels of the cross section), by normalizing it with the average gray scale value over the cross section, and by averaging this over time.

The values of these two parameters are shown in Fig. 3.14 for different velocity ratios (results for different nozzle shapes are also shown in this figure, but they will be commented on in Sec. 5.1). The values of both parameters grow with increasing counterflow, reaching at \( \alpha = 2.2 \) a value about twice as large as without counterflow and indicating a strong increase in dilution. Since the shape parameter defines the deviation from the circular shape of the cross section due to the secondary structures, which are larger than the resolution scale of the measurements, it should not be affected by errors. The data on the cross section non-uniformity, which are based on concentration fluctuations, are - however - biased.

In these experiments, the jet-based Reynolds number \( Re_j \) remained constant, but the Batchelor length scale and the relative spatial resolution (i.e. the resolution relative to the Batchelor scale) changed with
\( \alpha \) due to the variation in jet width with the counterflow. In particular, with increasing \( \alpha \) (i.e. with decreasing \( |U_0| \)) the jet width and the Batchelor length scale decrease, therefore the relative resolution becomes poorer. The bias in the fluctuations, which causes a reduction of the rms values and of the value of the cross section non-uniformity, is therefore stronger for low \( |U_0| \) values. Keeping the effect of bias into account, the slope of the curves in Fig. 3.14.b might therefore be reduced, and a comparison of the values at different \( \alpha \) might be misleading.

**Characteristics of the mixing region**

In order to investigate the whole mixing region, LIF visualizations in the axial plane were considered. In each instantaneous image it is possible to define the boundary of the mixing region through a fixed percentage of the initial concentration. Unlike the case of radial images, however, here the determination of this percentage is quite arbitrary, and was set to 10%. In order to characterize the distribution of the passive scalar within the mixing region, both the average concentration and the rms of the concentration fluctuations in this area, averaged over time, were calculated.

Fig. 3.15.a shows that the average concentration in the considered area, calculated as described above, decreases with increasing velocity ratios. The trend is very similar to the percentage \( 1/(1 + \alpha) \) used in Sec. 3.1.1 for determining \( x_p \) and can be explained by considering that, with increasing \( \alpha \)-values, the mass flow rate of the counterflow fluid interacting with the jet increases more than the mass flow rate of the jet fluid itself. In order to verify this relationship one needs to determine the part of the counterflow which is interacting with the jet. An approximate estimate was attempted by assuming that this part should be a cylinder of radius \( y_Q \), and the data obtained showed the same trend as in Fig. 3.15.a but much lower values. A similar estimate using the radius \( y_U \) produced instead approximately the same values as \( 1/(1 + \alpha) \).

For an evaluation of dilution within the whole mixing region, the rms value of the concentration (as defined above) was used as a measure of the "non-uniformity". The values of this parameter obtained from the present measurements are shown in Fig. 3.15.b: with decreasing
3.1. Statistical description of the flowfield

![Graphs showing concentration and rms of concentration vs. \( \alpha \)](image)

**Fig. 3.15:** Global characteristics of the mixing region, from axial LIF images: (a) average concentration, (b) rms of concentration.

\( \alpha \) (i.e. with increasing counterflow) the values decrease, showing that the uniformity within the mixing region increases.

The influence of the inadequate spatial resolution is limited to the fluctuation data shown in Fig. 3.15.b and can be evaluated as in the case of the near field. In this case, the relative resolution is changing both along the \( x \) direction (within every image) and with \( \alpha \) (when comparing the results), so that an estimate of the bias becomes more difficult. Using, as an approximation, the width of the forward flow region \( 2y_U \) as a global estimate of the jet width \( \delta \), it results that the relative resolution decreases with \( \alpha \) for \( \alpha \leq 2.2 \), and then increases as \( \alpha^{-1/4} \). According to this estimate, the data of Fig. 3.15.b should remain valid, at least for \( \alpha \leq 2.2 \). Since a precise estimate of the bias is not possible, the results remain questionable.

A characterization of mixing is usually made by considering the probability density function (pdf) of the passive scalar and its evolution along a typical profile across the mixing layer (Koochesfahani & Dimotakis (1986), Dahm & Dimotakis (1990), Karasso & Mungal (1996)). Here a similar approach is applied to a profile along the jet axis and the evolution of the pdf curves for different velocity ratios is shown in Fig. 3.16. A conclusion on whether the pdf for larger \( \alpha \) is of ”marching” type, which would indicate that the dilution of the jet fluid occurs continuously along the axis, is not possible. In fact it was shown that pdf curves from spatially under-resolved concentration data can be (even qualitatively) misleading and resolution deterioration can cause
Fig. 3.16: Pdf of the relative concentration for $D=10$ mm and $\alpha=1.3$ (a), 1.6 (b), 2.2 (c), and 3.4 (d); $D=2$ mm and $\alpha=7.5$ (e) and 10 (f).

For low $\alpha$ values distinct peaks at $C \simeq C_j$ and $C \simeq 0$ coexist, indicating that the tip of the jet has a sharp edge with an abrupt change in concentration (corresponding to the weak dilution along the axis in the stable case) and is strongly fluctuating in its penetration (corresponding to the shift between stable and unstable flow condition). This observation is conservative with respect to the spatial resolution issue, i.e. if affected by bias it will not loose its validity.
3.2 Dynamics of the flow

When observing results from flow visualizations (e.g. Fig. 3.1), it appears immediately that the instantaneous images totally differ from the averaged ones (e.g. Fig. 3.2.a). Therefore, it is important to investigate the dynamics of this phenomenon, concentrating on its instantaneous realizations and fluctuations. A few general observations on the typical phenomena and on the differences between the near and the far field of the jet are presented in Sec. 3.2.1. The following two sections will focus on the two fields separately and analyze in more detail the primary and secondary structures in the near field, as well as the three-dimensional fluctuations in the far field. Sec. 3.2.4, finally, will discuss some possibilities of modeling the flow.

3.2.1 Description of some observed phenomena

Stable and unstable case

The existence of a stable and an unstable flow case on either side of the boundary $\alpha \simeq 1.3 - 1.4$ was already observed by König & Fiedler (1991) and further described by Yoda & Fiedler (1996) (see Sec. 1.2). A description of these two typical flow patterns was given in Sec. 1.1, based on axial flow visualizations by LIF at different velocity ratios (Figs. 3.17 and 3.18), and can be schematically summarized as follows:

*stable case*: small penetration, axisymmetry, and regular vortex shedding;

*unstable case*: large penetration, asymmetry, free jet-like near field, and irregular, slow, but strong fluctuations in far field.

The most complex dynamics is found in the proximity of the $\alpha \simeq 1.3 - 1.4$ boundary, where the stable and the unstable cases coexist. The shift between these two cases is therefore investigated by observing LIF visualizations in the axial plane for $\alpha = 1.3$, and the appearance of a third, different flow condition is described.

The typical observed evolution of the flow is illustrated by the sketch of Fig. 3.19 (see also Fig. 3.20). It appears that when the flow is
Fig. 3.17: Series of instantaneous LIF images: stable case for $\alpha=1.3$, time interval $= 0.08$ s (left to right, top to bottom).

Fig. 3.18: Series of instantaneous LIF images: unstable case for $\alpha=3.4$, time interval $= 0.12$ s (left to right, top to bottom).
3.2. Dynamics of the flow

![Diagram showing transitions among different flow cases](image)

**Fig. 3.19:** Sketch of the typical transitions among the different flow cases for $\alpha=1.3$; $R =$ ring vortices.

characterized by a single, symmetric vortex ring (*stable case*), it might show a few regular vortex shedding, then start to slowly extend downstream, while approximately retaining symmetry. On reaching larger penetration lengths, it becomes more sensitive to perturbations and shows asymmetry and flapping (*unstable case*). This can result into a break-up of the jet column and into the consequent reduction of penetration length. At this point, the cycle can restart with the stable case or – especially in cases where the jet flapping had created a large recirculation zone which acts now as an obstacle for the approaching counterflow – also a third typical condition may appear (*asymmetric case*). In it, the jet is bent to one side (possibly due to counterflow perturbation) and can remain in that position for a certain period of time, showing a flow (Fig. 3.21) which is similar to that obtained by imposing a small angle between jet and counterflow (see Sec. 4.5). This underlines the extreme sensitivity of the flow to small perturbations in the direction or uniformity of the counterflow.

Since the flow is three-dimensional and our visualizations are limited to one axial plane, it can be argued that the stable case (Fig. 3.17) might
just be the asymmetric case (Fig. 3.21) when the bending occurs on the plane perpendicular to the one we are observing. This doubt could be definitively removed only by performing three-dimensional visualization experiments. Nevertheless, we are convinced that axisymmetry does exist, both in the stable case and in the typical symmetric downstream growth towards the unstable case.

An example of the typical cycle described above and in the sketch of Fig. 3.19 is shown in the time series of instantaneous LIF images of Fig. 3.20. In this figure, the time \( t = 0s \) corresponds to the stable case, the range \( t = 0.4 - 1.6s \) to the symmetric growth, and \( t = 2 - 2.4s \) to the unstable case. At \( t = 2.8s \) the penetration is small again, and from this condition a new cycle, the stable case, or the asymmetric case might follow. This cycle is just an example of a possible event, which is typical but occurs only irregularly and shows only a weak periodicity. Spectra of LDA measurements along the jet axis (not shown here), displayed in fact only some small peaks, at frequencies around 0.1 and 0.9 Hz.

**Fig. 3.20:** Series of instantaneous LIF images: transition from the stable to the unstable case and vice versa for \( \alpha=1.3 \).
3.2. Dynamics of the flow

![Image of LIF images]

**Fig. 3.21:** Series of instantaneous LIF images: asymmetric case for $\alpha=1.3$, time interval = 0.12 s (left to right).

An attempt to explain this behavior could be made by applying the analogy with an elastic column subject to axial compression as suggested by Bejan (1981),(1982),(1995) for modeling the meandering path of free jets. The exact application of this method to the present case, however, is complicated by the time dependence of the axial force (*i.e.* the counterflow) and made questionable by the fact that, with the large amplitude jet flapping, the buckling of the jet column cannot be considered small any more. Still, this model can be used as an approximation for qualitatively explaining some of the observed phenomena.

For example, it seems reasonable to assume that when the jet column is thick, as during the symmetric growth, it will remain elastically more stable (*i.e.* symmetric). When the jet column is long and slender, as in the unstable case, it will be more likely to bend, and eventually break up. While for the elastic column equilibrium is indifferent, here it is unstable (as in the case of Bejan (1981)). This is shown by the fact that, once symmetry is broken, it can only be recovered through a new beginning of the cycle, *i.e.* going through the stable case again (note that in Fig. 3.19 there are no arrows from the unstable or asymmetric case directly to the symmetrical growing ones).

**Further observations**

With the help of simultaneous LIF and PIV data, some of the large-scale vortical structures can be identified and followed in successive
time steps (see Fig. 3.1). This is generally not possible in the near field, where the spatial resolution of PIV is inadequate (only a diffused region of positive and negative vorticity can be recognized on either side of the axis, but the single vortex rings cannot be resolved), and the fixed temporal resolution of both methods (25 Hz) is not sufficient to keep up with the typical vortex frequency. In the far field, vorticity can help to identify the flow structures and to follow their evolution in a series of images. It can be observed that the vorticity structures are usually deflected back by the counterflow on the same side of the axis on which they were generated. As the jet tip oscillates radially, however, it sometimes happens that couples of counter-rotating vortices are swept back together on one side of the axis.

A few observations allow the identification of different phenomena in the near field and in the far field of the jet. In the rms images of both the velocity and the vorticity field (Fig. 3.2.b) areas of strong fluctuations can be recognized in correspondence with the initial shear layer and with the wandering of the jet tip. When the axial profiles of the velocity fluctuations are considered, two different scalings appear in the near and the far field (Fig. 3.9). Even if the dynamics of this flow phenomenon is still far from being understood, the existence of these two distinct regions as well as the different scalings shown by the axial profiles suggest the division of the flowfield into two parts. For $x < x_p$ and in the proximity of the axis the flow is similar to a free jet, while in the far field and at the external radial boundary the flow behavior is determined by the interaction with the counterflow. Here, slow fluctuations of the main stream of jet fluid, characterized by asymmetry and strong stirring, can be observed.

### 3.2.2 Dynamics of the near field

**Primary instability**

The near field of the jet is characterized by the shear layer which develops at the nozzle edge and persists until the end of the potential core. The typical structures which can be found in this region are therefore the vortex rings (R in Fig. 3.19), generated by the primary instability of the axisymmetric shear layer. Since the development and decay of these structures play an important role in the dilution phe-
nomena (Sec. 3.1.4), it is interesting to investigate how their dynamics is affected by the counterflow. Similar work was done, as mentioned in Sec. 1.2, by Strykowski & Niccum (1991), Strykowski & Wilcoxon (1993) and Favre-Marinet & Boguslawski (1999) with the aim of studying the stability of countercurrent mixing layers and of investigating the possibility of jet control by counterflow.

The flow configuration considered by Strykowski & Niccum (1991), Strykowski & Wilcoxon (1993), and Favre-Marinet & Boguslawski (1999) was a circular jet with annular suction at the nozzle exit plane and with only a sharp and thin edge separating the jet nozzle from the suction gap. The counterflow generated by the suction was steady and uniform, and its effect on the jet was further increased by placing annular collars of different geometry to extend the outer edge of the suction gap in the downstream direction.

The phenomenon studied by Strykowski and coauthors and by Favre-Marinet & Boguslawski (1999) can be assumed to be similar to a jet in counterflow, as far as the near field is considered. Still, some basic differences exist: first, in the present case the nozzle has a blunt outer shape; second, the counterflow in the proximity of the nozzle exit is not uniform and steady as it is at the beginning of the test section. In fact, the near field lies in the wake of the large mixing region which is present in the far field, in the proximity of the jet tip, and strongly oscillates with it.

Even if the correspondence between the two configurations is only approximate, also in the present case a behavior similar to the one reported in Strykowski & Wilcoxon (1993) was found. In order to compare cases at different velocity ratios while keeping the jet-based Strouhal number $St = fD/U_j$ constant, LDA experiments were carried out with constant jet exit velocity $U_j$ and varying the counterflow velocity $U_0$. Power spectra from LDA measurements obtained in the same conditions (same velocity ratios and positions) as in Strykowski & Wilcoxon (1993) are shown in Fig. 3.22.

Without counterflow, a dominant frequency of about 24 Hz is present, whose peak progressively decays while the one at 12.5 Hz increases (Fig. 3.22.a). Note that the latter value is approximately half the former one, indicating that vortex pairing takes place. At a velocity ratio of 3.4 (Fig. 3.22.b) the dominant peak appears at a lower frequency (17 Hz) and pairing is inhibited (the frequency of the peak remains
3. The round turbulent jet in counterflow - base case

**Fig. 3.22:** Power spectra from LDA measurements on the jet axis at different $x/D$ positions with $D=10$ mm and $U_j=0.442$ m/s: (a) without and (b) with counterflow ($\alpha=3.4$).

constant). Since with increasing counterflow the core length decreases (see Sec. 3.1.2), the peak cannot be recognized any more for $x/D > 3$, whereas in the case without counterflow the primary structures leave their trace on the spectra also for $x/D > 5$.

An additional, sharp peak at 34.5 Hz appears in the case with counterflow, corresponding to the frequency of the motor of the counterflow pump. This disturbance, as mentioned in Sec. 2.1.1, could not be eliminated, but did not affect the results. This was verified by the fact that a repetition of the experiment in the flume of the University of Hong Kong, which is exempt from such motor vibrations, reproduced otherwise the results obtained at TUB.

As can be observed in flow visualizations, the suppression of vortex pairing is not always so strong as in the case shown in Fig. 3.22. At higher $\alpha$, where the effect of the counterflow is weaker, the suppression of vortex pairing is reduced or, above some critical $\alpha$, totally disappears. Still, the critical $\alpha$-value is not fixed but varies between 4 and 7 in experiments carried out at different (jet-based) Reynolds numbers.

In order to discuss the variation of the typical Strouhal numbers with the velocity ratio, the dominant frequency was obtained from the power spectra measured on the axis at a fixed distance from the nozzle ($x/D = 1$) under different flow conditions. The results are shown in Fig. 3.23, together with the corresponding ones from Strykowski &
3.2. Dynamics of the flow

Fig. 3.23: Strouhal number of the dominant frequency in power spectra from LDA measurements on the jet axis at $x/D=1$ (dotted symbols indicate possible errors due to pump frequency).

Wilcoxon (1993). It can be noticed that, even if there is a large scatter in the data, for each data series the $St$-values generally decrease from a value between 0.55 and 0.7 progressively towards 0.3 when the counterflow velocity $|U_0|$ is increased.

Secondary instability

The decay of the primary structures with increasing $x$ due to their instability gives rise to secondary or streamwise structures (see Corcos & Lin (1984), Taubert (1997)). This can be observed best by radial flow visualizations, as was done in Sec. 3.1.4. There it was found that the counterflow produces a faster decay of the vortex rings with the corresponding enhanced generation of streamwise structures.

With the help of POD, further information on the decay of the primary structures was obtained. Since the sampling rate was limited by the video frequency of 25 $Hz$, however, we could not follow exactly the dynamics of the near field structures, whose typical frequency was of the order of magnitude of 10 $Hz$ or higher. Still, POD can help recognize some typical patterns and their relative importance. Fig. 3.24 shows a few POD modes obtained from radial LIF-visualizations recorded 1.6 $D$ downstream of the nozzle exit, with a constant jet velocity $U_j = 0.286$ $m/s$ and different velocity ratios.
3. The round turbulent jet in counterflow - base case

\[
\begin{array}{ccc}
\alpha & \infty & 11.5 & 3.6 \\
N & 1 & & \\
& 3 & & \\
& 5 & & \\
& 30 & & \\
\end{array}
\]

**Fig. 3.24:** POD eigenpictures (\(N^{th}\) mode) from radial LIF-visualizations at \(x/D = 1.6\) with different \(\alpha\) values.

Without counterflow, only some small oscillations or deformations of the circular cross section can be seen, which appear like radial instability modes. When the counterflow is increased both the amplitude of the oscillations (i.e. the thickness of the black and white ring) and their complexity are increased too, as shown by the appearance of higher order (azimuthal) instability modes at \(\alpha = 3.6\). The increasing degree of complexity can be quantified by considering the corresponding eigenvalues (see Fig. 3.25). It appears that the first 10 modes can
Fig. 3.25: Progressive sum of eigenvalues as percentage of total sum from POD of radial LIF-visualizations at $x/D = 1.6$ with different $\alpha$ values (and nozzle shapes).

represent about 75\% of the total without counterflow, while this percentage is reduced to about 60, 50, and 44\% for $\alpha = 11.5$, 5.4, and 3.6, respectively.

3.2.3 Dynamics of the far field

Three-dimensionality of the fluctuations

It is to be expected that the radial fluctuations of the jet observed in the axial flow visualizations are three-dimensional. König & Fiedler (1991) through smoke visualizations, as well as Yoda & Fiedler (1996) through LIF, observed the motion of the jet on radial planes, reporting in fact oscillations of large amplitude and low frequency around and/or through the axis.

Here an attempt was made to describe the jet fluctuations through LIF experiments on a radial plane located at half the penetration length ($x/x_p = 0.5$). The jet cross section was defined by setting a threshold gray scale value which kept into account both forward and backward flowing jet fluid, and the jet axis was calculated as the center of gravity of this section.

The time traces of the jet axis position were analyzed in several ways without being able to determine any typical fluctuation pattern. By
Fig. 3.26: Position of the center of gravity of a radial LIF-visualization with $\alpha = 3.4$, cross section at $x/x_p = 0.5$. (a) Example of time traces (b) Positions assumed over a long time series.

visual inspection of the time traces, it is possible to recognize – at least for some periods of time (● in Fig. 3.26.a) – a slow movement around the axis, sometimes interrupted by small loops. This pattern is however unstable and can, as in this case (○ in Fig. 3.26.a), suddenly shift to a chaotic movement. The periodicity of the fluctuations was investigated through FFT-analysis of the time series of the coordinates of the jet axis position, but neither fixed frequency values nor relationships between frequency and $\alpha$ could be found. Still, a periodicity was present, with peaks in the low frequency range between 0.1 and 1 Hz. Over long time series (Fig. 3.26.b), the positions assumed by the jet axis show an axisymmetric distribution over the cross section. By calculating the histogram of the fluctuations of the $y$ and $z$-coordinates, a shape similar to a Gaussian one was found. The maximum values of these fluctuations approximately correspond to those of $y_U$ (maximum radial extent of the $U = 0$ dividing streamline).

The uniformity of the flow over a radial section can be described at best through a statistical investigation of the whole radial LIF images. This was done both at $x/x_p = 0.5$ and at the fixed location $x/D = 2$ for checking the near field. Even if instantaneous images appear to be strongly asymmetric (Fig. 3.27.a,b) the average and rms images show axial symmetry, both in the near and in the far field (Fig. 3.27.c-f).
3.2. Dynamics of the flow

Fig. 3.27: Radial LIF-visualizations with $\alpha=3.4$ of sections at $x/D=2$ (a,c,e) and $x/x_p=0.5$, which corresponds, in this case, to $x/D=5.1$ (b,d,f). (a,b) Sample instantaneous image. (c,d) Average image. (e,f) rms image.

Dynamics of the fluctuations - POD analysis

An attempt of characterizing the fluctuations of the jet in the far field was made by considering the power spectra of velocity signals measured with LDA at different $x$-locations along the axis (Fig. 3.28.b).

These showed, in the far field, some peaks in the region below 1 Hz, in agreement with the typical frequencies obtained from the fluctuations
3. The round turbulent jet in counterflow - base case

**Fig. 3.28:** Power spectra for $\alpha=3.4$ from (a) eigenvectors from energy POD, (b) LDA measurements on the jet axis.

of the center of gravity in radial visualizations. A dependence of the dominant frequencies on the velocity ratio or on the axial coordinate could not be found, and no further information could be obtained through this method. Therefore, in order to gain a better insight into the dynamics of the jet fluctuations and to investigate their typical patterns, POD has been applied to series of PIV and LIF data.

Snapshot POD was first performed on PIV data in the energy domain and the eigenflows, *i.e.* the flow patterns corresponding to the single POD modes, were calculated (Fig. 3.29). If these are imagined to be superimposed to the average vector field and to oscillate with time with the corresponding eigenvectors as coefficients (*i.e.* if a single mode reconstruction is performed), a radial jet flapping results from the $1^{st}$ mode, while a periodic variation of the jet penetration can be observed from the $2^{nd}$ mode (Fig. 3.30). These eigenflows are not perfectly symmetric and combinations of these two appear with increasing asymmetry in the following modes, showing that these low frequency fluctuations are the most important pattern of the flow.

The characteristic frequencies of the POD modes (typical jet oscillations) can be deduced from the power spectra of the corresponding eigenvectors, which for the first three modes show peaks at about 0.13 and 0.60 $Hz$ (Fig. 3.28.a). For higher modes other typical fre-
3.2. Dynamics of the flow

![PIV vector fields with α = 3.4: (a) eigenflow of 1\textsuperscript{st} POD-mode; (b) eigenflow of 2\textsuperscript{nd} POD-mode (from energy POD - all coordinates normalized with jet diameter $D$).](image)

Fig. 3.29: PIV vector fields with $\alpha = 3.4$: (a) eigenflow of 1\textsuperscript{st} POD-mode; (b) eigenflow of 2\textsuperscript{nd} POD-mode (from energy POD - all coordinates normalized with jet diameter $D$).

... frequencies appear, always in the range 0.1 to 1 Hz. These values are confirmed also by POD in vorticity domain and by POD on simultaneously recorded LIF data, and are in agreement with the results from LDA spectra and from the fluctuations of the jet axis. As in those results, several different frequencies are present, showing the complexity of the flow. In comparison with the spectra from LDA, however, the advantage of POD is the possibility of associating each frequency to a typical pattern (POD-mode). Also, the interpretation is facilitated by the possibility of quantifying the relative importance of the POD-modes and therefore of the associated frequencies.

Some considerations on the portion of energy associated with each mode can be made by examining the development of the progressive sum of the eigenvalues as a percentage of the total sum (Fig. 3.31). The first ten modes alone contain on average about 60% of the total turbulent kinetic energy and the first twenty about 70%. This percentage increases for very low jet-to-counterflow velocity ratios, where the presence of the stable case (see Sec. 3.2.1) reduces the ”disordered” oscillations. The values remain however much lower than in cases where well-defined coherent structures with strong periodicity are present (e.g. 96 to 97% for the forced shear layers of Rajaee, Karlsson & Sirovich (1994) and Hilberg (1992)). This confirms the high complexity of this flowfield, since a higher number of modes are necessary to satisfactorily represent the whole phenomenon.

It can be seen that the eigenvalues from energy POD are much (in
3. The round turbulent jet in counterflow - base case

Fig. 3.30: Reconstruction of PIV vector fields with a single mode, time series during half period of fluctuation: (a) 1\textsuperscript{st} POD-mode; (b) 2\textsuperscript{nd} POD-mode (from energy POD - all coordinates normalized with jet diameter $D$).
average 25% higher than those from vorticity POD (definitions in Sec. 2.3.3). The fact that energy POD seems to yield a better representation of the whole phenomenon than vorticity POD was also observed by Huang (1993), who motivated it with the inherent inaccuracy of the process needed for calculating vorticity. Also, as in Huang (1993), the first eigenflow obtained from energy POD corresponds with that from vorticity POD but the similarity declines with increasing modes (Fig. 3.32.d,e). The gray scale value representation of the approximate turbulent kinetic energy shows that, as for the global rms images (Fig. 3.32.a,c), also for the single eigenflows a certain similarity exists between LIF and PIV (Fig. 3.32.d,f). For higher modes, which represent structures with smaller scales, the better spatial resolution of LIF becomes evident and allows to clearly recognize also the single ring vortices in the initial mixing layer, where both vorticity and energy POD show instead only alternate structure patterns.

### 3.2.4 A simple model

Several models have been reviewed in Sec. 1.2, the most complete of which was the one by Chan & Lam (1998). These authors obtained an equation for the centerline velocity decay based on the hypotheses of
3. The round turbulent jet in counterflow - base case

Fig. 3.32: Simultaneous PIV and LIF with $\alpha = 3.4$: (a) approximate turbulent kinetic energy $\overline{u'^2} + \overline{v'^2}$ from PIV and corresponding representation of eigenflows from energy POD (d) and vorticity POD (e); (b) $\omega_z$ vorticity; (c) rms-gray scale value from LIF and corresponding POD eigenpictures (f).
3.2. Dynamics of the flow

potential core compression in the near field and of backward convection by the counterflow in the rest of the flowfield (through a Lagrangian formulation). Using the conservation of momentum and volume flux, they extended the model to the whole velocity and concentration field (Chan (1998)).

This model shows a good qualitative, and in some cases also quantitative, agreement with experimental results, but presents also some major problems. In particular, as seen in Sec. 3.1.2, it fails to predict the linearity of the relationship between the jet-to-counterflow velocity ratio $\alpha$ and the average penetration length $x_p$. Instead, it shows a behavior similar to the one caused by the confinement effect of the channel walls even if such an effect is not accounted for by the model.

A new model is presented here, with the aim of showing that – in contrast with the extremely complex dynamics of the flow – the averaged centerline velocity decay can be approximately predicted also with a very simple hypothesis. The basic idea of modeling a jet in counterflow through the superposition of a jet and a uniform stream, used by Yoda & Fiedler (1996) to predict the average penetration length $x_p$, is therefore extended as follows.

The axial decay of the centerline velocity $U_{max}$ in a free jet is, when neglecting the virtual origin:

$$\frac{U_{max}}{U_j} = \frac{k}{x^*/D},$$  \hspace{1cm} (3.2)

where $*$ denotes that we are considering only the effect of the jet. As in Beltaos & Rajaratnam (1973), taking the counterflow velocity $U_0$ as a reference (which is equivalent to having a jet issuing at velocity $U_j - U_0$ from a nozzle in motion with velocity $|U_0|$), we substitute $U_{max}$ with $U_{max} - U_0$ and $U_j$ with $U_j - U_0$. Restricting our analysis to large velocity ratios ($\alpha \gg 1$), $U_j - U_0 \simeq U_j$, and therefore:

$$\frac{U_{max} - U_0}{U_j} = \frac{k}{x^*/D}. $$ \hspace{1cm} (3.3)

As in Yoda & Fiedler (1996), we assume that the counterflow will convect the jet fluid backwards, so that the position of a generic element of jet fluid, taking into account both jet and counterflow, is:

$$\frac{x}{D} = \frac{x^*}{D} - \frac{t^*|U_0|}{D},$$ \hspace{1cm} (3.4)
where \( t^* \) is the time needed by the jet fluid element to reach this position. This time can be calculated by integrating Eq. 3.3, as:

\[
t^* = \int_0^{x^*} \frac{x'}{kU_jD} \, dx' = \frac{kU_jD}{2(U_{\text{max}} - U_0)^2}.
\] (3.5)

By substituting this into Eq. 3.4 and making use of Eq. 3.3 to express \( x^*/D \), we obtain:

\[
x/D = \frac{kU_j}{U_{\text{max}} - U_0} - \frac{kU_j|U_0|}{2(U_{\text{max}} - U_0)^2}.
\] (3.6)

If we consider the stagnation point, where \( x = x_p \) and \( U_{\text{max}} = 0 \), Eq. 3.6 yields the relationship obtained by Yoda & Fiedler (1996):

\[
x_p/D = \frac{kU_j}{2|U_0|} = \frac{k\alpha}{2}.
\] (3.7)

If we solve Eq. 3.6 for \( U_{\text{max}} - U_0 \) and rearrange, the following relationship is obtained, which is valid for \( \alpha \gg 1 \) and \( D \ll x \leq x_p \):

\[
\left( \frac{U_{\text{max}} - U_0}{U_j} \right) \frac{x_p}{D} = \left( \frac{k}{2} \right) \frac{1 + \sqrt{1 - x/x_p}}{x/x_p}.
\] (3.8)

By choosing \( k = 5.83 \) – as suggested by Beltaos & Rajaratnam (1973) – Yoda & Fiedler (1996) obtained from Eq. 3.7 the relationship \( x_p/D = 2.9\alpha \), which exactly reflects the linearity between velocity ratio and jet penetration and also yields a reasonable estimate of the proportionality constant. By substituting \( k = 5.83 \) in Eq. 3.8 the model compares reasonably well with the experimental results downstream of the potential core, where they collapse on a single curve (Fig. 3.7). The improvement over the model suggested by Beltaos & Rajaratnam (1973) (Eq. 3.3) is negligible for low \( x \)-values (Fig. 3.7.a), but becomes evident as \( x \) increases (Fig. 3.7.b). The faster jet decay in case of the counterflow is indicated not only by the choice of a lower constant for the hyperbolic decay (5.83 instead of 6.3 as for free jets), but also by the function of Eq. 3.8. For increasing \( x \)-values, in fact, it progressively departs from the hyperbolic shape, further increasing the decay rate.
4. Influence of initial and boundary conditions

In order to understand some of the fundamental phenomena in the jet in counterflow, the response of this flow to changes in both initial and boundary conditions was investigated. The results are compared with the base case described in the previous chapter and grouped according to the type of modification of the standard flow conditions. In this chapter those boundary conditions are considered, which contribute to the definition of the “standard” conditions (base case) and of the limits within which these conditions are satisfied. As a second step, in Chap. 5 those modifications of the base case are considered, which could be used for controlling the flow and for further increasing dilution.

4.1 Flow confinement

As discussed in Sec. 1.2, external flow confinement due to the channel walls may cause a reduction of the penetration length and a departure from the standard linear dependence of $x_p$ from $\alpha$. Morgan, Brinkworth & Evans (1976) suggested to take the jet-to-counterflow momentum flux ratio $Z = (U_j D)^2/(U_0 B)^2$ to characterize confinement effects. Based on their experimental results, they found that $x_p/B$ varied linearly with $\sqrt{Z}$ for $\sqrt{Z} < 0.5$ ($Z < 0.25$, Eq. 1.1), and that $x_p/B$ was proportional to $\sqrt{Z}^{-1/3}$ for $\sqrt{Z} > 1.6$ ($Z > 2.5$).

In Fig. 4.1 our LDA data are compared with previous results, including those of Morgan, Brinkworth & Evans (1976). Since the present setup did not allow to reach very large values of $Z$, a Plexiglas cylindrical
4. Influence of initial and boundary conditions

![Graph](image)

\[ \sqrt{Z} = \frac{(U_j D)}{|U_0| B} \]

**Fig. 4.1:** Representation of the penetration length from present LDA measurements and from literature data with the normalization suggested by Morgan, Brinkworth & Evans (1976).

An insert of 125 mm internal diameter was used to further contract the test section. The corresponding data (indicated by "CT" in Fig. 4.1) allowed therefore to verify the relationship of power 1/3. From Fig. 4.1 it can be seen that a small departure from linearity can already be found at values of \( \sqrt{Z} \) below 0.5. For the definition of the base case the more restrictive condition \( \sqrt{Z} < 0.25 \) \((Z < 0.063)\) was therefore imposed. This corresponded to allowing a maximum \( \alpha \)-value of 7.5, 15, and 37.5 for the experiments with \( D = 10, 5, \) and 2 mm, respectively.

### 4.2 Reynolds number

Morgan, Brinkworth & Evans (1976) reported that no variation in the penetration length with Reynolds number was found, as long as the jet-based Reynolds number \( Re_j = U_j D / \nu \) and the counterflow-based Reynolds number \( Re_0 = |U_0| B / \nu \) exceeded the approximate values of \( 3 \times 10^3 \) and \( 10^4 \), respectively.

In the present case the counterflow-based Reynolds number \( Re_0 \) always exceeded \( 10^4 \), being equal to 39,000 for all the standard experiments. During the LDA experiments at HKU a series of axial profiles were also measured at the slightly lower \( Re_0 \) of 30,000. As expected, no appreciable difference was evident between the mean centerline velocities and velocity fluctuations at these two \( Re_0 \), as shown in Fig. 4.2.
Fig. 4.2: LDA axial profiles measured at HKU with $Re_0 = 39,000$ (lines) and 30,000 (symbols): average velocities (a) and velocity fluctuations (b).

The condition suggested by Morgan, Brinkworth & Evans (1976) for the jet-based Reynolds number ($Re_j > 3,000$) would correspond to a minimum $\alpha$-value of 2.3, 4.6, and 11.5 for the present experiments with $D = 10$, 5, and 2 mm, respectively. The lowest investigated $\alpha$-values of 1.3, 1.6, and 2.2 lie below this limit, but Reynolds number comparisons could not be made, since no nozzles with $D > 10$ mm were available. For larger $\alpha$-values the comparison of velocity as well as of turbulence profiles were instead made through LDA measurements with the different nozzles. Axial profiles of average velocity and velocity fluctuations were measured with LDA for jet-based Reynolds numbers ranging from about 2,000 to 10,000 (Fig. 4.3). Only minor discrepancies are evident over this range of $Re_j$, even for $Re_j$ values below 3,000, the minimum value suggested by Morgan, Brinkworth & Evans (1976). The present measurements confirm therefore the observations of Morgan, Brinkworth & Evans (1976), even if they suggest that the limit at $Re_j = 3,000$ might be too restrictive.

4.3  Background turbulence intensity

In order to investigate the effect of the counterflow freestream turbulence on the jet, experiments were carried out with artificially enhanced test section turbulence levels. Grids with square mesh and different blockage ratios – made of perforated metal plates – were inserted for
Fig. 4.3: LDA axial profiles measured with $D = 10$, 5, and 2 mm
(lines, empty, and filled symbols, respectively) and therefore different $Re_j$ values: (a) average velocities; (b) velocity fluctuations – symbols as in (a).

this purpose at the entrance of the test section, at a distance of 50
mm downstream of the last screen (n. 6 in Fig. 2.1), i.e. at $x = 58$
cm. Three different grids were tested, resulting in the flow conditions
shown in Fig. 4.4 (see also Tab. 4.1 for grids and reference flow data).

All experiments were carried out at the maximum speed of the pump
motor. The counterflow velocity in the test section varied therefore
with the blockage of each grid (see Fig. 4.4.a and Tab. 4.1). This required
the corresponding corrections to be made when analyzing the results. It also needs to be considered that the effects of grids on the flow,
a combination of turbulence generation and suppression mechanisms
(Corrsin (1963), Tan-Atichat, Nagib & Loehrke (1982)), produces a
turbulence level $Tu = \sqrt{u'^2/|U_0|}$ which is varying in streamwise direction. The data at $x = 10$ cm were therefore taken as a reference, since
this location is sufficiently far from the grid (as shown by the parameter $\Delta x/M$ in Tab. 4.1) and is downstream of the average penetration length for most of the studied cases. Since the solidity $\sigma$ lies around
the critical value of 0.5 for the 11 mm grid, and above it for the 6 mm
one, in those cases also some lateral inhomogeneity of the turbulence
intensity was found, with variations in $Tu$ of the order of 0.5 %.

The effect of freestream turbulence on a jet in coflow has been in-
vestigated by Fink (1975). By increasing the turbulence level of the
external stream to about 8%, he found a dilution increase indicated
4.3. Background turbulence intensity

![Graphs](image)

**Fig. 4.4:** LDA profiles measured on the axis of the test section with different grids positioned at \( x = 58 \text{ cm} \): average velocities (a) and turbulence intensity (b).

<table>
<thead>
<tr>
<th>( M ) [mm]</th>
<th>( d_w ) [mm]</th>
<th>( \sigma )</th>
<th>( Re_M )</th>
<th>( \Delta x/M )</th>
<th>( [U_0] ) [m/s]</th>
<th>( Tu ) [%]</th>
</tr>
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<td>80</td>
<td>0.110</td>
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<td>48</td>
<td>0.125</td>
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<td>43.6</td>
<td>0.118</td>
<td>3.4</td>
</tr>
</tbody>
</table>

**Tab. 4.1:** Grids geometry and flow data at \( x = 10 \text{ mm} \).

by a faster centerline velocity decay and by a faster growth of the jet width \( b \). Since the presence of a coflow normally causes a slower axial decay and a slower lateral growth, this means that the freestream turbulence is reducing the influence of the coflow on the jet.

If we assume that a higher freestream turbulence level is reducing the influence of the external stream on the jet, in the present case this would have the opposite effect to that on a jet in coflow. Here, in fact, it would mean that the increase in jet growth (and in dilution), typical of the jet in counterflow, is reduced. This appears to be confirmed by the half-concentration width, which is smaller than in the base case. For most of the parameters, however, the present measurements do not show large differences with respect to the base case (Fig. 4.5 and Fig. 4.6). This indicates that the flow is not strongly affected by the turbulence level generated by the grids if \( Tu \) is small as in the present case. A higher turbulence level should therefore be generated in order to investigate further the influence of \( Tu \) on the flowfield.
4. Influence of initial and boundary conditions

Fig. 4.5: Influence of freestream turbulence: average penetration length (a) and ratio between half-dilution length and average penetration length (b) from LIF experiments.

Fig. 4.6: Influence of freestream turbulence: maximum radial extent of mixing region (a) and corresponding x-coordinate (b) from LIF experiments. Data from $U = 0$ dividing line (empty symbols and solid line) and from $Q = 0$ dividing line (filled symbols and dashed line).

The hypothesis of a decrease in dilution – which goes hand in hand with a slower axial decay – is supported at higher velocity ratios also by the slight increase of the values of $x_p$, as well as $x_U$ and $x_Q$ (Fig. 4.5.a and Fig. 4.6.b). An increase in $x_p$ at all velocity ratios can be measured if $T_u$ is larger, as appears from the data obtained through LDA measurements in the HKU flume, where $T_u \approx 5 - 6\%$ (see Fig. 3.6).

A new parameter is introduced in order to characterize the shape of the centerline concentration decay, called **half-dilution length**. This is the distance $x_h$ needed to obtain half of the concentration decay which
4.4. Initial momentum thickness

corresponds to the average penetration length. From the present calibrated LIF experiments $x_p$ is determined as the x-coordinate at which \( C/C_j = 1/(1 + \alpha) \). Correspondingly, $x_h$ is defined as the x-coordinate at which the centerline average concentration from calibrated LIF reaches a fraction of its initial value equal to \( 2/(1 + \alpha) \). The ratio $x_h/x_p$ is therefore used as a parameter which can quantify the dilution occurring at low $x/D$-values with respect to the global one. The values in the grid experiments are generally lower than those of the base case, showing that freestream turbulence changes the shape of the centerline decay. Even if overall dilution is reduced, the part of it which occurs in the near field of the jet increases with respect to that occurring in the far field (Fig. 4.5.b).

By comparing the effects of the three different grids, it appears that the data obtained with the 10 mm grid are usually very close to those of the base case. The 6 mm grid produces approximately the same turbulence level as the 10 mm one, but is supercritical and has therefore a stronger effect. Larger variations from the values of the base case are therefore found with the 6 mm grid, as well as with the 11 mm one, which produces the largest enhancement of turbulence intensity.

A few observations on the dynamics of the phenomenon can be made by considering series of LIF visualizations, which reveal that higher turbulence levels cause a more unsteady and asymmetric flow pattern. At $\alpha = 1.3$, in fact, the occasional symmetric growth (see Fig. 3.19) can not be observed, while the asymmetric case appears more often than in the base case.

4.4 Initial momentum thickness

The shape of the jet velocity profile at the nozzle exit plane, characterized by the initial momentum thickness of the shear layer, can influence the development and the structures of free jets (Hussain & Zedan (1978), Taubert (1996)). The initial momentum thickness of the shear layer depends mainly on the boundary layer thickness in the nozzle at the exit plane. If the initial momentum thickness of the shear layer is defined as that determined at a location immediately downstream of the exit plane, also the boundary conditions around the nozzle (e.g. the outer shape of the nozzle) will influence this thickness.
4. Influence of initial and boundary conditions

![Graphs](image)

**Fig. 4.7:** Influence of boundary layer thickness and outer nozzle shape: average penetration length (a) and ratio between half-dilution length and average penetration length (b) from LIF experiments.

The boundary layer thickness at the jet exit section can therefore be changed by changing the shape of the inner and outer nozzle contours (e.g. by tripping the boundary layer as in Hussain & Zedan (1978) or adding a straight pipe downstream of the contraction as in Taubert (1996)), and changing the Reynolds number. Also in the present case (see Sec. 2.1.2), a straight pipe was inserted at the exit of the 10 mm nozzle, in order to study the effect of the boundary layer thickness at the jet exit plane on the flow. A brass pipe of 10 cm entrance length was used, obtaining a length-to-diameter ratio $L/D = 10$. At the exit plane the fully developed pipe flow with parabolic profile was not reached (Klein (1981)), but the boundary layer was nevertheless significantly thicker than at the nozzle exit.

Since the pipe wall was only 0.5 mm thick, the exit had a much sharper lip than the standard nozzle. As discussed above, the outer nozzle shape can also affect the initial momentum thickness, even if its influence can be expected to be much smaller than that of the boundary layer thickness. In order to study the influence of these two factors separately, experiments were also performed using a pipe collar with the same outer form as the original nozzle.

Experiments with a similar pipe were carried out by Yoda & Fiedler (1996) in the present setup without observing any large difference with respect to the standard case. Here a slight increase in the penetration length could be generally seen (Fig. 4.7.a), together with a decrease in the maximum radial extent (Fig. 4.8.a). No conclusions could be
4.4. Initial momentum thickness

Fig. 4.8: Influence of boundary layer thickness and outer nozzle shape: maximum radial extent of mixing region (a) and corresponding $x$-coordinate (b) from LIF experiments. Data from $U = 0$ dividing line (empty symbols and solid line) and from $Q = 0$ dividing line (filled symbols and dashed line).

made by observing the behavior of other parameters, such as $x_U$, $x_Q$, or $x_h/x_p$, which did not show any clear deviation from the standard case (Fig. 4.7.b and Fig. 4.8.b).

Both the variation of $x_p$ and the one of $y_U$ and $y_Q$ indicate a decrease in jet growth and therefore in dilution with the increase of the initial momentum thickness occurring in the cases with pipe. The reduced growth can be associated with the weaker velocity gradient in the jet exit section. The deviations from the base case appear larger when the pipe is mounted without collar. This can be explained by the fact that in this case the absence of the blunt body (and therefore of the $U = 0$ boundary condition in proximity of the exit edge) allows an increased entrainment by the jet at its exit. This further reduces the velocity gradient and increases the initial momentum thickness.

By observing a series of LIF visualizations further differences appear in the cases with increased initial momentum thickness. As shown in Fig. 4.9, separation occasionally occurs within the pipe (probably induced by strong sweeps of entrained counterflow fluid), producing an asymmetric and unsteady outflow at the nozzle. As a consequence, for $\alpha = 1.3$ the symmetrical case appears only very seldom, the vortex shedding is not regular, and the asymmetrical case cannot persist for long time intervals. Also at higher velocity ratios the ring vortices in the near field develop more irregularly and less symmetrically than in
Fig. 4.9: Series of instantaneous LIF images for the experiments with pipe at $\alpha = 1.3$, time interval $= 0.02$ s (left to right, top to bottom).

the base case, showing a very limited extent in radial direction. This is in part due to the reduced diffusion of vorticity, caused by the reduced velocity gradients.

4.5 Inclination angle

The sensitivity of the flow to small angles between jet and counterflow was already observed by Duclos, Schaffer & Cambell (1957) and Yoda & Fiedler (1996). Also during the present measurements of the base case it was noticed that, especially at low velocity ratios, a slight inclination angle could cause strong asymmetry in the flow.

In order to investigate systematically the effect of small angles between jet and counterflow, the jet holder was fixed at different positions along a beam extending in y-direction above the test section. By tilting the jet holder along the z-axis, angles $\theta$ of $2.4^\circ$, $4.7^\circ$, $7.1^\circ$, and $9.5^\circ$ were obtained, while keeping the center of the nozzle exit cross section on
4.5. Inclination angle

Fig. 4.10: Influence of angle: average penetration length (a) and ratio between half-dilution length and average penetration length (b) from LIF experiments.

Fig. 4.11: Influence of angle: maximum radial extent of mixing region (a) and corresponding $x$-coordinate (b) from LIF experiments. Data from $U = 0$ dividing line.

the axis of the test section. The orientation of the $x$-axis and of the video camera remained unchanged, so that the jet axis (determined as the point of maximum concentration in each radial profile) did not correspond any longer with the $x$-axis. The determination of the centerline concentration decay and the calculation of all the parameters, however, was based as before on the jet axis.

The effect of inclination angles appears especially at low velocity ratios, where the average penetration is reduced up to 40% of the value in the base case (Fig. 4.10.a). This corresponds to the data given by
Fig. 4.12: Average penetration length vs. inclination angle $\theta$ and linear curve fits.

Yoda & Fiedler (1996), who investigated the phenomenon for $\theta = 4.7^\circ$ and $9.5^\circ$, and $\alpha \leq 2.8$. As $\alpha$ increases, the reduction in $x_p$ from the base case is generally decreasing and, at $\alpha = 10$, small angles even produce larger $x_p$-values than for $\theta = 0^\circ$. This might be explained by the stabilization of the flow pattern caused by the angle. If the angle is large enough, as shown here by the case $\theta = 9.5^\circ$, the reduction in $x_p$ remains constant over the whole range of velocity ratios. By plotting the penetration data of Fig. 4.10.a vs. $\theta$, with $\alpha$ as a parameter, a better overview of the effect of inclination angles was obtained (Fig. 4.12). The slope of the linear fits lies between 0.12 and 0.28, on average around 0.2, and therefore indicates an average reduction in $x_p$ of $0.2D$ for each degree of the angle $\theta$. A linear fit is however not always appropriate, since at very high $\alpha$-values $x_p$ actually increases for small angles, and at low velocity ratios the reduction in $x_p$ with $\theta$ is faster than average for small $\theta$-values. Still, the value obtained for the average penetration reduction can be used for a rough estimate of the effect of $\theta$ on $x_p$.

While $x_p$ is decreasing, the maximum radial extent of the mixing region is increasing with $\theta$ up to 200% of the value in the base case for small $\alpha$-values (Fig. 4.11.a). The deviation from the base case is reduced as $\alpha$ increases, being about 10% at $\alpha = 10$. No considerations can be made about the axial position of the point of maximum radial extent, since it shows large variations without any clear trend (Fig. 4.11.b). It must be noted that, here, radial extent data were only determined from the dividing line $U = 0$. This is because the stagnation streamline $Q = 0$
is calculated by integrating the profiles under the assumption of axial symmetry of the mean flow, which is not valid when $\theta \neq 0^\circ$.

Even if the overall centerline concentration decays faster when $\theta \neq 0^\circ$, as indicated by the values of $x_p$, the parameter $x_h/x_p$ shows that the decay occurs largely at higher $x$-values. Especially for low velocity ratios, in fact, a much larger axial distance is required in order to obtain a half-decay of the concentration (Fig. 4.10.b).

An explanation of some of the observed differences from the base case can be obtained by examining series of LIF visualization images. Drastic changes in the dynamics appear already for the smallest angle $\theta = 2.4^\circ$: the unstable case with large penetration length disappears completely for low $\alpha$-values, causing the strong decrease observed in the values of $x_p$. At $\alpha = 1.3$ a combination of the stable and the asymmetrical case can be observed. Starting from a pattern in which one single, asymmetric vortex ring appears, two typical flow conditions may occur. The first one, with regular vortex shedding on the upwind side and a very large radial extent on the lee side, is very similar to the asymmetrical case occasionally observed at $\theta = 0^\circ$ (Fig. 3.21) and to the one shown in Fig. 4.13.b. The second one (Fig. 4.13.a) shows a periodical variation in jet penetration and a pattern which is similar to the transition between stable and unstable flow case (Fig. 3.20.a-d). Here the variation in jet penetration is smaller than in the base case, and the flow is always asymmetric. While on the upwind side one single vortex rolls up slowly and persists until the next cycle, on the lee side several smaller vortical structures develop in the shear layer which generates in the wake of the downstream recirculation. The appearance of larger vortices on the upwind side, where the counterflow is stronger, agrees with the general observation made in Sec. 3.2.2. There it was found that, by increasing the counterflow, the typical Strouhal number decreased and the structure size therefore increased (Fig. 3.23).

For $\theta \geq 4.7^\circ$ the flow patterns at low velocity ratios change further, becoming more regular and asymmetric. At $\alpha = 1.3$ only the asymmetric case can be observed (Fig. 4.13.b), which becomes however more regular than for $\theta = 0^\circ$ (Fig. 3.21) or $\theta = 2.4^\circ$. The vortex shedding on the upwind side occurs at a frequency of about 2.5 to 3.5 $Hz$, increasing within this range for increasing $\theta$. These values were determined by LIF visualizations and are about half the ones obtained by Yoda & Fiedler (1996) through hot wire measurements. Since those mea-
Fig. 4.13: Series of instantaneous LIF images with $\alpha=1.3$ and $D=10$ mm: (a) $\theta = 2.4^\circ$, time interval = 0.12 s; (b) $\theta = 4.7^\circ$, time interval = 0.08 s.

Measurements were taken on the jet axis and contain therefore also the fluctuations on the lee side, they might indicate that the frequency on the lee side is approximately double the one on the upwind side. This is confirmed by observing the LIF images, where 2–3 small vortices are shed on the upper (lee) side for 1 large vortex on the lower side.

At larger velocity ratios, where the unstable case is the standard flow pattern, the effect of the inclination angle appears to be twofold. On the one hand, with increasing angle the jet column is bent more easily by the counterflow towards the lee side. On the other hand, the amplitude of the sideways fluctuations of the jet tip is reduced, especially since the fluctuations towards the upwind side are inhibited. The balance between these two effects might explain the increase of the average penetration length for large $\alpha$ values at small angles, and its decrease at larger angles. The increased regularity of the flow and the attenuation of the sideways fluctuations also cause a reduction of the jet column break up phenomenon. This can explain the fact that the average concentration decay in the near field (respect to decay in the far field) is slower than in the base case (Fig. 4.10.b).
5. Flow control

Based on the insights in the dynamics of the flow obtained as described in Sec. 3.2, the possibility of controlling the flow in order to further enhance dilution was investigated. Attempts were made both of passive flow control, using non-axisymmetric nozzle shapes, and of active flow control, by axial excitation of the jet.

5.1 Passive flow control - nozzle shape

The use of non-axisymmetric nozzles as a method for passive flow control was reviewed by Fiedler & Fernholz (1990). The principle of this method is that, since self-induction of a vortex is inversely proportional to its radius of curvature, any non-circular geometry generates three-dimensional disturbances of the primary structures (vortex rings). As an effect, the vortex break-up and reconnection phenomena appear, accompanied by local mixing increase. Among the most effective configurations were found to be elliptic nozzles with small aspect-ratio and polygonal nozzles, where high instability modes are introduced at the corners.

It was therefore decided to carry out some experiments with a square nozzle and with an elliptic nozzle of 2:1 aspect-ratio, as described in Sec. 2.1.2. In both cases calibrated LIF experiments were carried out on two different axial planes: these were the ones containing the maximum and minimum diameter of the elliptic nozzle and those through the middle line and the diagonal of the square nozzle. In addition, LIF visualizations on radial planes allowed to observe the modifications induced by the different nozzles on the vortex rings in the near field.
Axial LIF experiments

In order to examine the typical mean flow parameters, the data obtained from axial LIF on the two different planes defined above were considered. Due to the large differences which sometimes appear between the data on these two planes, especially at low velocity ratios, this method is not always accurate. Only complete three-dimensional measurements of the flowfield could have produced exacts results, but were beyond the objectives of the present investigation.

Further considerations on the three-dimensionality of the phenomenon are however allowed also by the applied method, through the comparison of the results obtained on the two considered planes.

The average penetration length $x_p$ shows a clear reduction of penetration for both non-circular nozzles for $\alpha < 4$ (Fig. 5.1.a). For larger velocity ratios, $x_p$ is the same as in the base case or even larger. Similarly, the ratio $x_h/x_p$ is much smaller than for the circular nozzle at small velocity ratios, especially in the range $2 < \alpha < 4$ (Fig. 5.1.b), then increases for larger $\alpha$ values. This indicates that – in this range of velocity ratios – non-circular nozzles enhance dilution, as expected particularly in the near field, where even the jet in counterflow behaves much like a free jet.

The maximum radial extent of the mixing region is much larger than with the circular nozzle for very low values of $\alpha$, then becomes slightly smaller than the reference one for $2 < \alpha < 4$ (Fig. 5.2.a). Its axial positions $x_U$ or $x_Q$ show for low velocity ratios generally smaller values than for the round nozzle, confirming again the dilution enhancement in the near field (Fig. 5.2.b). As in Sec. 4.5, only radial extent data obtained from the $U = 0$ dividing line were considered. Also here, in fact, the assumption of axial symmetry of the mean flow, which needs to be made for calculating the stagnation streamline $Q = 0$, is not valid (at least in the near field).

The three-dimensional instabilities generated by the non-circular nozzles affect especially the near field of the jet, where these instabilities develop and modify the ring vortices. Further downstream the primary structures decay, and the flow tends to reach axial symmetry. The effect of non-circular nozzles on the flow is therefore stronger at small velocity ratios, where the whole mixing region lies within the near field of the jet. If $\alpha$ is too small, however, the instabilities do not
5.1. Passive flow control - nozzle shape

**Fig. 5.1:** Influence of nozzle shape: average penetration length (a) and ratio between half-dilution length and average penetration length (b) from LIF experiments.

**Fig. 5.2:** Influence of nozzle shape: maximum radial extent of mixing region (a) and corresponding x-coordinate (b) from LIF experiments. Data from $U = 0$ dividing line.

have a sufficient development length and their effect is not so strong. This explains the different behavior of the jet in the two ranges $\alpha < 2$ and $2 < \alpha < 4$. In the former, faster growth and earlier dilution with respect to the base case appear, as shown by the decrease in $x_p$ and $x_U$ together with the increases in $y_U$. In the latter $x_p$, $x_U$, and $y_U$ all decrease, indicating that dilution is further enhanced, since it needs not only a shorter length but also a smaller region to occur.

Due to the stronger effect of the three-dimensionality at small velocity ratios, it is there that also the largest differences between the square
and the elliptic nozzle occur. These differences appear especially in the
data on the maximum radial extent but do not show any clear trend,
so that a further interpretation is not possible.

Similarly, the differences between the results obtained for each non-
circular nozzle on the two different planes are largest at small velocity
ratios and appear at best when considering the radial growth of the jet.
This is done by observing the development of the half-concentration
width, as well as the values of the maximum radial extent and of its
x-coordinate (Fig. 5.2). In the section along the maximum diameter
of the elliptic nozzle the width is, at the jet exit, twice as large as
along the minimum diameter. The maximum radial extent is therefore
reached earlier and is, for low \( \alpha \) values, larger than that obtained from
the section along the minimum diameter. As reported also by Fiedler
& Fernholz (1990), the jet growth along this latter section is much
faster, so that the phenomenon of axis switching occurs and for large
velocity ratios the maximum radial extent becomes larger than on the
section along the maximum diameter. For the square nozzle, where the
two considered sections are only 45° apart, the interpretation is not
so clear. In general, however, the maximum radial extent is slightly
larger and is reached earlier on the section along the middle line than
on the section along the diagonal.

Radial LIF experiments

LIF experiments were carried out on a radial section at \( x/D=1.6 \) and
the images were processed using the methods described in Sec. 2.3.1
and Sec. 3.1.4. The positions assumed over time by the center of
gravity of the cross section of the jet did not show any typical pattern
also in the case of non-circular nozzles. For the elliptic nozzle the
distribution of these positions over long time series was however not
axisymmetric, but showed the existence of fluctuations with larger
amplitude in the plane of the smallest diameter.

The same parameters as in Sec. 3.1.4 were used to quantify the ef-
fect of passive flow control on dilution in the radial section of the jet
(Fig. 3.14). From these data it appears that the effect of the elliptic
nozzle is generally small, and only for large counterflow (small veloc-
ity ratio), an enhancement of 10\% maximum (5\% on average) can be
reached. The square nozzle produces instead an enhancement up to
### 5.1. Passive flow control - nozzle shape

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**Fig. 5.3:** POD eigenpictures (2\(^{nd}\) mode) from radial LIF-visualizations at \(x/D=1.6\) with different \(\alpha\) values and nozzle shapes.

20\% (and 10–15\% on average) at all investigated \(\alpha\) values, *i.e.* in the range 2.2–11.5.

The POD analysis used in the base case to investigate the vortex structures in the near field (Sec. 3.2.2) was applied also to radial LIF visualizations with non-axisymmetrical nozzles (see Fig. 5.3). The comparison of the eigenflows corresponding to a fixed POD mode (here the 2\(^{nd}\)) shows that the intensity of the secondary structures is not
strongly modified by the elliptic nozzle. With the square nozzle, instead, an increase in secondary structures (or in the instability of the primary ones) appears. This is indicated, qualitatively, by the strong increase in thickness of the black and white rings in Fig. 5.3, which occurs for the square nozzle already at \( \alpha = 5.4 \). For the other two nozzles, this appears only when the counterflow is further increased, at \( \alpha = 3.6 \). The same figure also shows, for the square nozzle, an increase in the complexity of the fluctuations and deformations of the primary structures. This is indicated by the alternating black and white zones around the circumference of the vortex ring, which show the presence of higher order (azimuthal) modes. This behavior can be explained by the fact that sharp corners imply stronger coupling of streamwise vorticity into azimuthal structures.

A quantitative evaluation of the degree of complexity of the structures can be obtained by examining the progressive sum of the eigenvalues for a fixed number of modes with respect to the total sum (Fig. 3.25). With the round nozzle, the percentage corresponding to the first 10 modes is equal to 75% without counterflow, 60% for \( \alpha = 11.5 \), and 50% for \( \alpha = 5.4 \). With the elliptic nozzle, these values increase slightly to 64% and 52% for \( \alpha = 11.5 \) and \( \alpha = 5.4 \), respectively. This indicates a small decrease in complexity which could be explained by the fact that with the elliptic nozzle a preferred fluctuation direction (in the plane of the smaller axis) was observed in the near field, which means more order in the fluctuations. The increased complexity of the flow with the square nozzle is also reflected by the percentage values, which decrease to 48% for \( \alpha = 11.5 \) and 43% for \( \alpha = 5.4 \).

### 5.2 Active flow control - axial excitation

As reported in Sec. 1.2, experiments on a jet in counterflow with axial and radial excitation have been carried out by König & Fiedler (1991). By axially exciting the jet at a frequency corresponding to a Strouhal number (defined either with the jet exit velocity \( U_j \) or with the velocity difference \( U_j - U_0 \)) of 0.37, some changes occurred both in the penetration length and in the radial extent of the mixing region. These variations, however, did not show any clear trend over the investigated range of velocity ratios, so that no conclusions could be reached. It
was therefore decided to carry out axial excitation experiments on a wider range of Strouhal numbers.

The axial excitation system described in Sec. 2.1.2 was used to investigate the range of Strouhal numbers, here defined as $St = fD/U_j$, between 0.1 and 5. Particularly small steps for the values of $St$ were used in the range between 0.2 and 0.6, where according to Fiedler & Fernholz (1990) and to the present measurements (see Fig. 3.23) the characteristic frequency of the jet should be found. Velocity ratios in the range 1.3–3.4 were investigated. The excitation amplitudes, defined as $A = \sqrt{u^2}/U_j$, were determined by LDA measurements at the jet exit and ranged usually between 5% and 10%. Higher $A$-values of up to 20-25% were reached only in a few cases at smaller $St$.

The interpretation of the results was complicated by the presence of fluctuations at the nozzle exit due to some structural resonance in the nozzle assembly, described in Sec. 2.1.2. Since these fluctuations were stronger during the excitation experiments than in those of the base case (where a simple end plate was fixed at the end of the nozzle holder) it must be assumed that they were amplified by the membrane used for the excitation. This caused, as can be noticed in the following, marked differences between the results of the base case and those where the excitation system was installed even if not operated.

Axial excitation produced in some cases large variations in the parameters used for the investigation, such as penetration length and radial extent of the mixing region. However, although the measurements systematically spanned a wide range of Strouhal numbers, no clear trends could be found in these variations and we did not observe a clear resonance in this flow over the range of $St$ investigated. This, together with the strong influence of the natural fluctuations, prevented a clear interpretation of the effect of axial excitation on the flow. No general conclusion could therefore be drawn from these experiments, and only some observations on a few sample cases will be presented.

The results obtained with excitation at $St = 0.36$ and 0.60 are chosen as examples, since these corresponded in many cases to clear modifications of the flow. It can be noticed that these values of $St$ approximately correspond to the limits of the range in which the dominant frequency in the near field of the jet is varying (see Fig. 3.23). Some results are presented in Figs. 5.4 and 5.5, where both the data obtained
with a plate fixed at the end of the nozzle holder ("base case"), and those obtained with the mounted but not operated excitation membrane ("$St = 0$") are shown as a reference. From these sample cases it appears that through axial excitation it is possible to obtain some reduction in the penetration length, at least at small velocity ratios. Typically, also a large increase in the radial extent of the mixing region is observed.

The data obtained for $\alpha=2.2$ often show a deviation from the trend of the other cases. By observing a series of LIF instantaneous images, it
appears that the dynamics of the phenomenon is strongly modified by the excitation (Fig. 5.6). Without excitation (Fig. 5.6.a) the vortex rings in the near field show a slow growth with some irregular pairing. Excitation at \( St = 0.6 \) enhances near field vortex ring formation: the vortex rings reach their maximum size immediately downstream of the jet exit and do not show pairing but simply decay as they travel downstream (Fig. 5.6.b). This rapid growth at low \( x \)-values can also be observed in the statistics of a series of images, as shown in Fig. 5.7 by the average and rms fluctuations of the gray scale values. Figs. 5.7.b and 5.7.d also show that excitation reduces the fluctuations in the far field, at least relatively to those in the near field.

Similar modifications of the vortex structures in the near field appear also at other excitation frequencies and for other velocity ratios, but produce a different global effect on the boundaries of the mixing region. This shows that the relationship between the dynamics of the near field and the flow in the far field is highly complex and remains under many aspects unclear.

For further attempts of active flow control, other forms of excitation should also be considered. The results obtained from POD on radial sections of the flow (Sec. 3.2.2) show that the counterflow tends to enhance higher order instability modes. Therefore, we can expect that orbital/helical excitation could have a stronger effect on this flow than axial excitation does.
Fig. 5.6: Series of instantaneous LIF images with \( \alpha = 2.2 \) and \( D = 10 \ mm \): (a) no excitation; (b) axial excitation at \( St = 0.6 \) and \( A \approx 10\% \) (time interval = 0.02 s).

Fig. 5.7: Comparison of statistics for \( \alpha = 2.2 \) and \( D = 10 \ mm \) without excitation (a,b) and with axial excitation at \( St = 0.6 \) and \( A \approx 10\% \) (c,d): average of the LIF gray scale values (a,c) and rms fluctuations (b,d).
6. Discussion and conclusions

In this chapter the main results of this thesis are reviewed and discussed, as well as the limitations of and errors associated with the measurement methods. The conclusions drawn from these results are presented, along with some suggestions for future work.

6.1 Experimental errors

Here, the potential sources of experimental error are discussed and, where possible, quantified.

Experimental setup

While existing problems of the facility regarding the uniformity of the counterflow were eliminated by improving the flow settling chamber design upstream of the test section (Sec. 2.1.1), the vibrations induced by the pump motor could not be eliminated and hence affected the measured power spectra. By repeating some measurements in the HKU flume, which was exempt from these vibrations, it was possible to check the validity of the results. Still we cannot exclude some influence in cases where the vibration frequency matched the typical frequencies of the flow and, in particular, that of the preferred mode of the jet. The cases in which this influence might have been possible are indicated in Fig. 3.23. In the excitation experiments, further spurious frequencies appeared, which were probably due to a structural resonance in the nozzle assembly. These frequencies were therefore imposed upon the flow as additional excitations on top of the actual excitation.

During the setup of the experiments, it was difficult to accurately set small volumetric flow rates on the rotameter with the calibrated valve.
6. Discussion and conclusions

This affected especially LIF and PIV experiments, where the actual velocity at the nozzle exit could not be monitored. It was estimated that at low velocity ratios with the smaller nozzles – i.e. at the minimum flow rates – the flow rate could deviate up to 20% from the set value. Cases affected by significant errors could be identified by checking parameters such as the penetration length (see Fig. 3.6). Errors in rotameter flow rate readings, due to heating of the water by the pumps during the experiments, were avoided by continuously monitoring the water temperature to avoid large temperature variations.

Even if particular care was taken to avoid misalignment between jet and counterflow (Sec. 2.1.2), the extreme sensitivity of the flow to the inclination angle could also have caused errors. The results were therefore always checked for symmetry, if possible during the experiments, otherwise during data processing. Data from any experimental run that showed asymmetry (other than during the investigation of the effect of the inclination angle – Sec. 4.5) were considered invalid and do not appear in this thesis.

LDA

The spatial resolution of the LDA data was limited by the size of the probe volume (Sec. 2.2.1). This limitation is clearly evident especially in the near field of the flow for experiments with the smallest (2 mm) nozzle. The error associated with probe volume angular orientation was negligible, and the error associated with positioning the probe (linear orientation error) was estimated to be less than 0.5 mm. The accuracy of the LDA counter was 1% and its digital output resolution was 0.2% of the selected range. The frequency shift was therefore adjusted according to the range of velocities to be measured in each point, in order to use the lowest possible output range and thereby maximize output resolution. Due to these manual adjustments and to the lack of an automatic traversing system (see Sec. 2.2.1), the use of the available LDA system was extremely unpractical. It was therefore limited to the preliminary investigations and to cases where a high sampling rate was needed. Since a sample-and-hold method was used in order to avoid bias due to the uneven particle arrival times, care was taken that the data rate was always much larger than the sampling rate in order to reduce inaccuracies.
6.1. Experimental errors

LIF

In both LIF and PIV, possible sources of errors were the alignment of the light sheet with the jet axis and the size scaling of the video images. Both adjustments were done with the help of the reference frame described in Sec. 2.2.2, which allowed to limit alignment errors to less than $1^\circ$ for inclination and $0.2 \text{ mm}$ for positioning. Scaling errors amounted to 0.2–0.5% for the different magnifications used to investigate different velocity ratios. Distortion effects due to the camera lens could have been adjusted by scaling as well, but this was unnecessary given the negligible distortion of the test grid images. A further source of errors common to both methods lies in CCD noise ($SNR < 0.2\%$), video tape noise ($SNR < 0.4\%$), and noise during the frame grabbing (digitalization) process. The tape errors are likely to have affected particularly the LIF images used for simultaneous PIV and LIF, since these had to be recorded on a S-VHS tape (whose quality is inferior to that of Betacam – $SNR < 0.7\%$) and later copied to a Betacam tape in order to allow automatic frame grabbing.

For the passive scalar concentration measurements, further sources of error lie in possible instantaneous variations in the laser power (10%) and laser beam profile during operation. Steady intensity variations, such as those due to light sheet spread and brightness variations over the image owing to flaws in the imaging lens and CCD, were eliminated by image post-processing. Because of the limited number of gray scale values and because the whole range was not always used, the uncertainty due to the discretization of the gray scale values was also a source of error, especially in the low concentration zones at the "edges" of the jet. As an example, when the gray scale values span the whole available range (i.e. 256 values) the discretization uncertainty is small ($< 0.4\%$) for the maximum concentration, but becomes a major source of error (about $\pm 4\%$) where the concentration is 5% of the maximum one.

Each image processing step may introduce errors that affect each parameter in a different way. Since a precise estimation of every error is very difficult, a global evaluation of the uncertainty was attempted by repeating some measurements several times. The number of repetitions was however not sufficient for a statistical analysis, and allowed only approximate estimates. The errors which occurred in the values
of the axial and radial extent of the mixing region were < \pm 5\%$. As observed in Sec. 2.3.1, the largest errors can be expected for the parameters $x_U$ and $x_Q$, which for low $\alpha$ showed differences up to $\pm 10\%$.

**PIV**

Although errors in the PIV data could have been introduced by irregular motion (e.g. wobble) of the scanner mirrors, the time interval between the two exposures $\Delta t$ monitored by photodiodes and an oscilloscope showed no temporal variations. The separation of each of the 25 $Hz$ frames into two interlaced fields required the subsequent reconstruction of full size images through interpolation. This was a possible source of errors in the determination of the position of the particles and/or of the correlation peaks, and may lead to inaccuracies in the determination of the displacement (error $< \pm 0.5$ pixels in the direction perpendicular to the image lines). The accuracy of the PIV calculation depends basically on the estimate of the cross-correlation peak. This was investigated in detail by Westerweel (1993), who found that the rms error generally lies below 2\% of the tracer particle size. With displacements smaller than 0.5 pixels (which can be obtained, e.g., through window shifting procedures as the one applied here) a further reduction of the uncertainty can be reached, the lower limit being around 0.01 pixels. After the detection of spurious vectors (outliers) through a local median test, these were removed, thereby smoothing the velocity field. This nonlinear smoothing procedure could produce some errors, which affected mainly the velocity fluctuations and not the average values.

The main deficiency of the PIV method was its lack of spatial resolution (compared with the spatial scales of the flow). Improvements could be obtained by zooming in and increasing the density of particle images (through increased seeding and/or laser power). With the available raw PIV images, the spatial resolution can be improved also during post-processing, by implementing other more advanced PIV processing techniques (see Raffel, Willert & Kompenhans (1998)). Possible improvements include applying – instead of or in addition to cross-correlation – methods such as neural networks or genetic algorithms, thereby combining the advantages of PIV and PTV (Particle-tracking-volumetry).
6.2 Summary and discussion of results

Definition of the base case

The jet in counterflow is strongly affected by boundary and initial conditions. When defining the minimum conditions required to obtain a "standard" flow (here called the "base case"), the following points should therefore be considered:

- **Flow confinement.** In order to avoid confinement due to the channel walls, a limit must be set on the jet-to-counterflow momentum flux ratio $Z$. For the linear dependence of $x_p$ on $\alpha$ to be guaranteed, the present results suggest that the condition $Z < 0.06$ should be fulfilled, instead of the less restrictive condition $Z < 0.25$ suggested by Morgan, Brinkworth & Evans (1976).

- **Reynolds number.** The absence of Reynolds number effects is guaranteed, according to Morgan, Brinkworth & Evans (1976), if the conditions on the counterflow-based and the jet-based Reynolds number $Re_0 > 10,000$ and $Re_j > 3,000$ are satisfied. The present measurements show, however, that the latter criterion can be relaxed to $Re_j > 2,000$.

- **Background turbulence intensity.** The influence of the counterflow freestream turbulence on the averaged flow appears to be small, at least when the turbulence level $Tu$ is below 3–4%. Above these $Tu$-values, however, a reduction of dilution and a consequent increase in jet penetration is to be expected. The present experiments, even if limited to a small range of $Tu$-values, suggest the condition $Tu < 3–4\%$ to avoid any effects associated with freestream turbulence.

- **Initial momentum thickness at the jet nozzle.** Although this effect could not be quantified as a function of the initial momentum thickness of the shear layer, it appears that an increase in momentum thickness at the jet nozzle exit reduces the jet dilution and increases the average penetration length $x_p$. Experiments show that this phenomenon depends not only on the boundary layer thickness in the nozzle but also, to a smaller extent, on the outer nozzle shape. The present investigation does not allow to
fix an exact limit value for the initial momentum thickness, but it suggests that a well-designed nozzle should be used in order to obtain as thin a shear layer as possible.

- **Inclination angle.** A precise alignment of jet and counterflow is crucial, since even a very small angle between the jet and counterflow directions results in a markedly different flow from the base case. The small velocity ratio cases are especially sensitive to inclination angle. Even a small misalignment angle results in an asymmetric flowfield with strong reduction in penetration and remarkable increase in the radial extent of the jet mixing region.

This discussion is based mainly on time-averaged parameters of the flowfield. The fluctuations of the flow are far more sensitive to variations in the initial and boundary conditions than the average flowfield. When investigating the flow dynamics, more attention and more restrictive conditions are therefore needed.

**Statistical description of the flow**

The data obtained in the base case of a round jet in uniform counterflow, defined by the above standard conditions, allowed the verification of literature results on the time-averaged flowfield. It was confirmed that, as long as the confinement due to the channel walls is negligible, the relationship between the axial jet penetration $x_p$ and $\alpha$ is linear. The collapse of the centerline velocity decay to a single curve — outside the potential core and for sufficiently large velocity ratios — was also verified, using the representation $(U - U_0)/U_j x_p/D$ vs. $x/x_p$ as suggested by Beltaos & Rajaratnam (1973). Radial profiles of both velocity and concentration confirmed the observation of Yoda & Fiedler (1996) that self-similarity exists only in the proximity of the jet axis.

By measuring axial profiles, new data on the reduction of the potential core length $x_c$ due to the counterflow were presented. These approximately follow the relationship $x_c \propto (\alpha - 1)/\alpha (\alpha + 1)$ used in the model of Chan & Lam (1998) but suggest a value of 5 instead of 6.2 for the proportionality constant. Considering the velocity fluctuations, it was found that, if $x$ is normalized by $x_p$, the axial profile of the centerline velocity fluctuations scales with $U - U_0$ for $x \leq x_p$ and with $|U_0|$ for $x \geq x_p$. 
No agreement was found in literature on the definition of the boundaries of the mixing region and on the estimation of its radial extent. Although locating the undisturbed "edge" of the flow remains problematic, it was suggested that the most meaningful and reliable boundary should be the *stagnation stream surface*, along which the total momentum flux $Q$ is zero. By using averaged field velocity measurements (PIV), it was shown that the maximum width $y_Q$ of the $Q = 0$ line decreases with $\alpha$, reaching asymptotically a value of about 0.22 $x_p$ for $\alpha > 7.5$. The downstream distance $x_Q$ at which the maximum width is reached increases with $\alpha$ and is equal to about 0.7 $x_p$ for $\alpha > 7.5$. The corresponding parameters $x_U$ and $y_U$ from the $U = 0$ dividing streamline showed the same behavior, indicating that—when $\alpha$ is large enough and the influence of the near field is small—the whole region scales with $x_p$ (and therefore linearly with $\alpha$).

In order to facilitate the determination of the parameters characterizing the radial extent, the approximate correspondence between the $U = 0$ dividing streamline and the $1/(1 + \alpha)$ iso-concentration line was proposed. This allowed to obtain dividing lines corresponding to $U = 0$ and to $Q = 0$ also from calibrated LIF experiments. Even if this method constantly underestimates the penetration length $x_p$, it allows a good estimation of the maximum width.

The dilution enhancement produced by the counterflow was observed through the faster axial decay and radial growth in comparison with a free jet, shown both by the velocity and by the concentration field. Evidence was also found that the counterflow enhances the decay of the vortex rings in the near field, but a single parameter for a precise and global quantification of the dilution enhancement could not be defined. Also, due to the insufficient spatial resolution of the applied methods, no observation could be made on mixing.

**Dynamics of the flow**

By analyzing time series of flow visualization images on an axial plane, it was possible to complete the observations of Yoda & Fiedler (1996) on the coexistence of a *stable case* and an *unstable case* at low velocity ratios. A third typical flow pattern was identified (asymmetric case) and it was attempted to describe the typical evolution paths of the flow (*i.e.* the typical transitions among these different cases). For higher
velocity ratios, where the flow is constantly in the unstable case, it was observed that two regions with different dynamics can be identified (near and far field).

Power spectra of velocity signals measured in the near field showed that the counterflow tends to suppress vortex pairing. By determining the dominant frequency at a fixed axial location, it was also observed that the typical Strouhal number is progressively decreasing from about 0.6 to about 0.3 when the counterflow is increased. Through POD analysis it was found that the counterflow increases the instability of the primary structures and promotes the formation of higher order (azimuthal) instability modes.

Radial flow visualizations showed that the flow in the far field is on average axisymmetric, but the jet axis strongly fluctuates with amplitudes that can be as large as the width of the average forward flow region \((U = 0\) dividing line). The typical frequency, determined through several independent methods, was in the range \(0.1\)–\(1\) \(Hz\) but did not show any preferred mode or any relationship with \(\alpha\). No typical pattern could be found for the fluctuations on radial planes of the jet, but POD on an axial plane indicated that the main modes (or typical patterns) correspond to a simple radial flapping and axial pulsation. The distribution of the energy associated with each mode showed that the dynamics of the far field fluctuations is very complex in comparison with that of other flowfields. This complexity is reduced, at low velocity ratios, by the presence of the stable case.

Even if the dynamics of the flow is extremely complex and still far from being understood, it was shown that the time averaged flow can be decently reproduced by models based on very simple assumptions. By extending the model of Yoda & Fiedler (1996), it was possible to predict the centerline velocity decay through the superposition of a jet and a uniform flow. In the range of validity of the model good agreement with experimental results was obtained. More complete models \(e.g.\) Chan & Lam (1998)) yield in some cases better results, but also show some inconsistencies and none of them can reproduce the whole flowfield without errors.
Flow control

Given the fact that the fluctuations in the far field of the jet show apparently no typical pattern and very low frequency, the possibility of flow control seems to be restricted to the near field. The present experiments show that the use of non-circular nozzles, and in particular of square ones, can strongly enhance the decay of the vortex rings in the near field and globally increase the jet dilution. Since the effect of the three-dimensionality introduced by the nozzle is limited to the near field, a dilution increase appears only for low velocity ratios ($\alpha < 4$), where most of the mixing occurs within the near field. Since the higher order instabilities generated by the non-circular nozzles need a certain development length, however, the strongest dilution increase occurs for $\alpha > 2$, where the mixing region extends sufficiently far downstream.

The results from flow control via axial excitation of the jet were inconclusive, in part due to the additional excitation introduced by errors in the experimental setup (see Sec. 6.1). Strong modifications of the vortex dynamics in the near field appear also with relatively small excitation amplitudes. These modifications produce sometimes large variations in the parameters characterizing the average flowfield. In particular, the jet penetration can be reduced, and the radial extent of the mixing region is in many cases strongly increased. The relationship between the modified vortex structure in the near field and its global effect on the flowfield remains however unclear. Similarly, even if the excitation frequency spanned a wide range of frequencies, no consistent relationship was found between the Strouhal number of the excitation and its effect on the flow.

6.3 Contributions

From a synthesis of the results discussed in the above section, it is possible to point out the main contributions of the present work to the research of the jet in counterflow phenomenon.

- The present investigation of the effect of boundary and initial conditions on the flow contributed to define which conditions should be required in order to obtain a "standard" flow for a consistent comparison of results.
• On the controversial point of the definition of a boundary for the mixing region, the present work suggests that the stagnation stream surface might be the most reliable estimate. Its maximum width $y_Q$ can be therefore taken as the characteristic (mean) radial length scale.

• A simple empirical, relationship was found, which allows an approximate but easier determination of $y_Q$ from measurements of the concentration field alone. The experimental data obtained allow the proposed relationship to be verified and show how the scaling of the extent of the mixing region varies with the velocity ratio $\alpha$.

• In the attempt of determining the characteristics of the mixing region, parameters were defined which can help to quantify the dilution enhancement produced by the counterflow.

• The present investigation provided new information on the dynamics of the flow. The transitions among the typical flow cases which appear at low velocity ratios were described. It was shown that, in the near field, the counterflow can suppress vortex pairing, reduce the typical frequency of the jet, and enhance the formation of secondary instability structures. Further evidence was given of the existence of two distinct flow regions: a near, inner field and an outer, far field.

• PIV was applied for the first time to study this flow field and produced a better description of the far field fluctuations. POD, which had never been used for this flow before, was performed on both PIV and LIF data, thereby allowing new observations on the dynamics of the flow.

• By extending an available model, a new model was developed to predict the (mean) centerline velocity decay by superposing a jet on a uniform flow. This showed that – in spite of the complexity of the instantaneous flow field – the mean flow can be approximately predicted through very simple relationships.

• Passive control of this flow through non-circular nozzle cross sections was investigated for the first time and allowed to further enhance dilution. Also axial excitation produced under some
conditions similar effects, which however could not be studied systematically.

- The secondary instability structures of a jet in counterflow had never been investigated in detail before. Here the application of POD allowed to study the effect of the counterflow on these streamwise vortices, both in the case of circular nozzle and in the other cases considered for passive flow control.

The present results also demonstrate the capabilities of modern field measurement techniques such as (simultaneous) PIV and LIF for the investigation of particularly complex flow fields, especially when advanced signal analysis methods (e.g. POD) are used to extract information from the measured data.

In general, this work contributes to expand the basis of data and knowledge available on the jet in counterflow. Since this flow has not been extensively explored, the present results may be useful to lay a more solid foundation for future work and to suggest which points would need or would be worth of a deeper investigation.

6.4 Concluding remarks and suggestions for future work

Some final conclusions that can be drawn from the present work and some hints and recommendations for future work on the jet in counterflow are summarized in the following.

- The flow is affected by counterflow width, jet and counterflow Reynolds number, counterflow turbulence level, initial momentum thickness, and inclination angle between jet and counterflow. In particular, even a slight misalignment between jet and counterflow can lead to large errors at low velocity ratios. In order to obtain a standard flowfield, these initial and boundary conditions should therefore be carefully checked. The present analysis could be further completed by quantifying the variation of the initial momentum thickness and by investigating a wider range of Reynolds numbers and turbulence levels.
• In view of possible practical applications, a more thorough exploration of the extent and characteristics of the mixing region of the jet in counterflow is required. At present, the most reliable definition of the boundary of the mixing region appears to be the stagnation stream surface. A semi-empirical relationship was suggested (Eq. 3.1), which is useful to compare and integrate the results obtained from the velocity and concentration fields. An attempt to improve this relationship might be worthwhile. Although some evidence of the dilution enhancement produced by the counterflow was found, a global parameter to quantify dilution does not yet exist. In addition, we were not able to resolve the smallest diffusion scales and therefore we could not distinguish actual mixing from stirring. In this direction, an approach through chemical reaction instead of passive scalar methods (as can be done with LIF by making use of the $pH$ sensitivity of fluorescent dyes – Karasso & Mungal (1996)) is recommended.

• Simple models show that the average flowfield is similar to the superposition of a jet and a uniform stream, but this does not represent the dynamics of the flow. For sufficiently large velocity ratios it is possible to distinguish a jet-like region in the near field from an outer (far field) region dominated by the interaction with the counterflow. In the near field it was therefore possible, by using some parameters and concepts “borrowed” from the free jet, to investigate the effect of the counterflow on the flow structures and to modify them through passive and active flow control. In addition to the cases investigated here, other non-circular nozzle cross sections might yield interesting results as far as dilution enhancement is concerned. Also active control appears to be worth investigating more extensively, especially by azimuthal excitation. The dynamics of the far field is still in large parts unclear. A few features of the far field fluctuations could be illustrated here, but further analysis is needed, as well as instantaneous three-dimensional measurements. These could help clear the doubts arising from the observation of the phenomenon on axial or radial planes alone. The investigation of the relationship between near and far field dynamics would also increase the possibilities of controlling dilution and greatly contribute to the understanding of this flow.
• Some observations on the typical flow patterns at low velocity ratios and on their dynamics were made by analyzing time series of flow visualizations. Additional information on the typical flow patterns and on the dynamics of large vortical structures might be obtained by considering three-dimensional visualizations, as well as simultaneous measurements of the velocity and concentration field. This could be done by further developing and applying the method of simultaneous PIV and LIF presented here.
Appendix

A. Equipment and components list

A.1 Experimental setup

A.1.1 Water channel

- OSIP FL 90 submersible pump
- KSB Etanorm G 50-125 pump
- ABB SAMI 04 MB4M2 motor frequency controller

A.1.2 Excitation

- Wavetek Model 166 function generator
- Soundcraft 30 W amplifier
- Philips PR 9307 carrier frequency bridge
- Soundcraft CN24L loudspeaker

A.2 LDA

A.2.1 LDA System at TUB

- Spectra Physics 60 mW He-Ne laser
- Dantec 55X optics
  - 55X41 mounting bench
  - 55X20 cover and retarder
- 55X24 beam splitter
- 55X29 Bragg cell section
- 55X28 beam displacer
- 55X30 backscatter section
- 55X32 beam translator
- 55X12 beam expander
- 55X58 600 mm front lens
- 55X08 PM (photomultiplier) section
- 55X34 PM optics

- Dantec 55L96 counter processor
- Dantec 55N10 frequency shifter
- Keithley DAS-1600 12 bit A/D board
- GOULD OS 4020 oscilloscope

A.2.2 LDA System at HKU

- Spectra Physics 5 W water cooled Argon-ion laser
- Dantec FiberFlow system
  - 60X41 transmitter
  - 60X24 fiber manipulators
  - 60X11 optical probe
  - 55X57 310 mm front lens
  - 5 m optical fibres
- Dantec 58N40 FVA enhanced Doppler signal processor
- Dantec 57H00 traversing system
- Dantec FLOWARE software

A.3 LIF and PIV

A.3.1 Laser, optics, and electronics

- Spectra Physics 5 W water cooled Ar-ion laser
A. Equipment and components list

- Spindler & Hoyer spherical lenses with focal length of 40 / 100 mm and 40 / 80 mm
- Spindler & Hoyer cylinder lenses of 10 and 40 mm focal length (for LIF only)
- Cambridge Technology 6390 galvanometer scanner (2)
- Cambridge Technology 603X control unit
- GOULD OS 4020 oscilloscope

A.3.2 Particles and chemicals

- Disodium fluorescein \((C_{20}H_{10}Na_2O_5)\) (for LIF)
- Rhodamine 6G \((C_{28}H_{31}N_2O_3Cl)\) (for LIF with simultaneous PIV)
- Degussa-Hüls VESTOSINT 7182 polyamide spheres (for PIV)
- BASF Cremophor A25 (for dilution of PIV particles)

A.3.3 Video system and image processing

- Sony DXC 930P CCD camera with:
  - Canon PH 10X8B F1.4, 8-80 mm objective
  - Nikon F1.8, 85 mm objective
- Panasonic NV-SX30EG CCD camera with F1.4, 3.9-66.3 mm objective and S-VHS tape (for LIF with simultaneous PIV)
- Courtyard CY 325 converter
- Sony PVW 2800P Betacam VCR
- Sony PVM 2042 QM monitor
- SGI D5-VF6U video framer (frame grabber)
- Videomedia V-LAN-S control network
- SGI Personal IRIS 4D35TG workstation
- SGI IRIS Indigo II workstation
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