Oscillating Contacts
Friction Induced Motion and Control of Friction

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Abstract

Friction as well as vibrations are integral parts of many technological applications. Their simultaneous appearance results in oscillating frictional contacts. While these are often a side effect of technical systems and result in troublesome effects like squeal, it may as well be a desired effect. Among others, the interaction of friction and vibration is used to transport objects or to control friction. In both cases the same methods apply to describe the frictional behaviour of the dynamic system. In case of dry friction, mostly the Coulomb friction law or more sophisticated so-called single state friction laws are used. Those laws do not consider the contact geometry. Therefore, its dynamical behaviour under alternate loadings is neglected and the contact properties are significantly simplified. Contrariwise, a numerical description of the entire contact area in a dynamic system using the finite element method or the boundary element method is very time consuming and thus not practical.

Consequently, in this work, the possibility of describing dynamic frictional oscillating contacts by the means of simple models as well as with the Method of Dimensionality Reduction (MDR) are explored. In addition systematic experimental investigations have been performed. Two classes of oscillating contacts will be examined. First the force-generating axis of the mobile nano robot RaMoNa is investigated. Due to asymmetric vibrations of contact spheres, a stick-slip motion is induced onto a force-generating runner. The contact interaction is described numerically. The results are compared to experimental data. For contact spheres of different dimensions the force-generation can be described by the chosen method, proving the method’s universal character. In addition a simplified model of the MDR gives a detailed insight into the principle of movement of the runner. This is used to invent and test a new, more efficient input signal to control the motion of the runner.

The second class of oscillating contacts addresses the control and reduction of sliding friction. Due to ultrasonic oscillations of the contact partners in different vibration directions the characteristic of the sliding friction is changed. Experiments on a pin-on-disc type test stand are systematically performed. The friction reducing influence of the vibrations for three vibration directions and a wide amplitude range is experimentally established. Furthermore, for all vibration directions, the overall tendencies observed in the experiments can be described using analytical models based only on Coulomb’s law.

For the vibration direction normal to the contact plane, the MDR is applied. Moreover for this macroscopic technical example the MDR exhibits a good comparability with the measured data, even for the simplest possible contact geometry.
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Nomenclature

Latin Letters

\( a \)  Contact radius \( \text{m} \)
\( A_0 \)  0-amplitude, maximum actuation amplitude of the runner for which no force generation can be observed \( \text{m} \)
\( A_v \)  Vibration amplitude of an oscillating contact \( \text{m} \)
\( A_{\text{max}} \)  Maximal actuation amplitude \( \text{m} \)
\( b \)  Radius of the flat part of a flattened sphere \( \text{m} \)
\( B_{1D} \)  1D amplitude spectrum \( \text{m} \)
\( c \)  Radius of the stick area in a contact \( \text{m} \)
\( C_M \)  Cattaneo-Mindlin coefficient \( \text{m} \)
\( C_{1D} \)  1D spectral density \( \text{m}^3 \)
\( C_{2D} \)  3D spectral density \( \text{m}^4 \)
\( d \)  Indentation depth \( \text{m} \)
\( E \)  Elastic modulus \( \text{Nm}^{-2} \)
\( E^* \)  Effective elastic modulus \( \text{Nm}^{-2} \)
\( f_0 \)  Natural frequency \( \text{s}^{-1} \)
\( F_n \)  Force normal to the contact plane \( \text{N} \)
\( f_n \)  Normal force of the springs used in the MDR \( \text{N} \)
\( F_p \)  Preload applied to the runner of RaMoNa \( \text{N} \)
\( F_x \)  Force in \( x \)-direction, usually in direction of motion \( \text{N} \)
\( f_x \)  Tangential force of the springs used in the MDR \( \text{N} \)
\( F_{\text{gen}} \)  Generated force by the stick-slip motion of RaMoNa’s \( x \)-axis \( \text{N} \)
\( F_g \)  Gravitational force \( \text{N} \)
\( F_{r,d} \)  Dynamic friction force \( \text{N} \)
\( F_{r,s} \)  Static friction force \( \text{N} \)
\( F_r \)  Friction force (no distinction between static and dynamic case) \( \text{N} \)
\( G \)  Shear modulus \( \text{Nm}^{-2} \)
\( g(.) \)  Equivalent one dimensional line \( \text{m} \)
\( G^* \)  Effective shear modulus \( \text{Nm}^{-2} \)
\( H \)  Hurst exponent \( 1 \)
\( h(.) \)  Profile of a three dimensional body \( \text{m} \)
\( h_{\text{rms}} \)  Root mean square roughness \( \text{m} \)
\( \Delta k_n \)  Normal stiffness of the springs used in the MDR \( \text{Nm}^{-1} \)
\( \Delta k_x \)  Tangential stiffness of the springs used in the MDR \( \text{Nm}^{-1} \)
\( K \)  Kinetic energy \( \text{Nm} \)
\( k_n \)  Normal contact stiffness \( \text{Nm}^{-1} \)
\( k_{fs} \)  Stiffness of the force sensor detecting \( F_{\text{gen}} \) \( \text{Nm}^{-1} \)
$k_{x,i}$ Tangential spring stiffness in the three spring model, $i = 1, 2$ Nm$^{-1}$

$k_{x,tot}$ Total tangential spring stiffness in the three spring model Nm$^{-1}$

$k_x$ Tangential contact stiffness Nm$^{-1}$

$\Delta l$ Amplitude of the changing length of the oscillating probe m

$L$ Characteristic length of a rough surface m

$l(t)$ Changing length of the oscillating probe m

$l_0$ Initial length of the oscillating probe m

$m_e$ Equivalent mass kg

$p_n$ Prefactor defining the shape of a rotational symmetric body m$^{-1}$

$q$ Wave number m$^{-1}$

$q_n$ Distributed load in normal direction resulting from the MDR Nm$^{-1}$

$q_x$ Distributed load in tangential direction resulting from the MDR Nm$^{-1}$

$T$ Period of an oscillation s

$s_i$ Slip distance between runner and springs in the three spring model, $i = 1, 2$ m

$U$ Potential energy Nm

$u_e$ Equilibrium displacement of springs in the three spring model m

$u_i$ Maximal tangential displacement of springs before slip occurs in the three spring model, $i = 1, 2$ m

$u_x$ Displacement of a contact’s surface in $x$-direction m

$u_y$ Displacement of a contact’s surface in $y$-direction m

$u_z$ Displacement of a contact’s surface in $z$-direction m

$u_s$ Static equilibrium position in the three spring model m

$u_{x,max}$ Presliding, maximal displacement of a contact before slip occurs m

$x_e$ Elastic displacement of contact in Dahl’s friction law m

$x_p$ Plastic displacement of contact in Dahl’s friction law m

$x_{e,ps}$ Presliding displacement in Dahl’s friction law m

$x_{e,ss}$ Elastic steady state displacement in Dahl’s friction law m

$\dot{v}$ Velocity of the ends of the oscillating probe ms$^{-1}$

$\langle v \rangle$ Mean sliding velocity of the oscillating probe ms$^{-1}$

$W$ Work Nm

$w(x, t)$ Deflection of a beam m

$\Delta x$ Distance between the springs of the MDR m

$x$-axis Positioning/force-generating axis of RaMoNa m

$x_s$ Sawtooth displacement of the ruby hemispheres m

$(\cdot)'$ Derivative with respect to a specific (location) variable

$(\cdot)^\prime$ Derivative with respect to time

$(\cdot)^{\langle \cdot \rangle}$ Mean value averaged over one oscillation period

$(\cdot)^\ddagger$ Dimensionless variable

$(\cdot)^\ast$ Vector
Greek Letters

- $\alpha$: Prefactor in Dahl’s friction law to introduce elastic displacement
- $\gamma$: Constant in Cerutti solution
- $\Gamma(.)$: Gamma function
- $\eta_1$: Elastic viscous damping in Dahl’s friction law (kg s$^{-1}$)
- $\eta_2$: Viscous damping in Dahl’s friction law (kg s$^{-1}$)
- $\kappa_n$: Transforming factor in the MDR for a rotational symmetric body
- $\lambda$: Fitting parameter for the contact stiffness for the OOP vibrations
- $\langle \mu \rangle$: Friction coefficient under the influence of oscillation
- $\mu$: Mean friction coefficient (no distinction between static and dynamic case)
- $\mu_0$: Friction coefficient between disc and probe without oscillation
- $\mu_d$: Dynamic friction coefficient
- $\mu_s$: Static friction coefficient
- $\nu$: Poisson’s ratio
- $\rho$: Density (kg m$^{-3}$)
- $\sigma$: Normal pressure (normal to the contact plane) (Nm$^{-2}$)
- $\tau$: Shear stress in the contact plane, if not differently indicated in direction of motion (Nm$^{-2}$)
- $\vartheta$: Angle at the centre of rotating disc defining the probes geometry (°)
- $\omega$: Natural circular frequency (rad s$^{-1}$)

Abbreviations

- BEM: Boundary Element Method
- IPI: In plane in direction of motion
- IPP: In plane perpendicular to the direction of motion
- OOP: Out of plane
- CEIM: Friction model named after the inventors C. Edeler and I. Meyer
- LuGre: Lund and Grenoble friction model
- MDR: Method of Dimensionality Reduction
- RaMoNa: Rapid mobile platform with nanometre precision
1. Introduction

Contact mechanics and friction are an integral part of technical applications but also of everyday life. In this work frictional contacts under alternating loadings will be explored. A simple everyday example is walking; classical technical applications are vibrating band conveyors used to transport goods or vibration compactors. External vibrations in interplay with friction are used either to transport objects or to actively control the frictional behaviour. An example for the latter one are vibrations used to loosen a frictional contact as exemplarily a screw, or a vibrating cell phone sliding of a table.

The basic processes in contacts loaded with an alternating force are similar, regardless if it results in transportation or in a control of friction. To illustrate this, let us consider the simple system of two elastic bodies in contact as schematically shown in figure 1.1. The oscillations can be introduced either by applying varying forces or by moving the body normally or tangentially to the contact plane (or it can be a superposition of either of the above oscillations). In the following we will distinguish between microscopic movement (movement performed on the time scale smaller than the oscillation period) and macroscopic movement (averaged over the period of an oscillation).

\[ F_n \]
\[ F_x \]

Figure 1.1.: Two elastic bodies in contact; the upper one is loaded with a normal force \( F_n \) and a tangential force \( F_x \).

Depending on the properties of the contacting surfaces as well as the exact time dependence of the external forces, several effects are possible and of interest for many technical applications. If the loading and the substrate are symmetric with respect to
the \( x \)-axis (for example, only the (oscillating) normal force is present) no macroscopic directed motion occurs due to oscillations. Note that this definition of symmetry has to be considered for a whole vibration amplitude. Only if a force is symmetric with respect to time and place no directed motion is possible. If the symmetry is broken, a macroscopic motion may be induced. The symmetry breaking can occur basically in two different ways:

1. By introducing an (oscillating) horizontal force, which clearly introduces a preferred direction. In this case the motion may occur only as long as the horizontal force is acting\(^1\). The average tangential component of the force is interpreted as friction force. This case is referred to in literature as influence of vibrations on the friction force or as active control of friction.

2. Asymmetry of the system may be introduced even if the average value of the external horizontal force is zero and the normal force is constant. In those cases a movement without average external force may occur. These induced motions are often referred to as friction drives.

In the following, examples of symmetric and asymmetric load cases and the resulting qualitative behaviour of the systems are discussed in more detail.

**Symmetric Systems**

a. If the bodies are pressed against each other with a constant normal force \( F_n \) and subjected to a tangential force \( F_x \) oscillating in a symmetric fashion (e.g. as a sine or cosine function of time), then no net movement takes place. During one period the body is similarly moving back and forth with a zero net movement. The only physical effect of interest in this case is the damping due to partial slip in the contact area, a problem going back to works by Mindlin \[1\].

b. The symmetry also remains unbroken if the normal force is oscillating according to an arbitrary law, and the tangential force vanishes.

**Asymmetric Systems**

c. The most obvious example of an asymmetric loading is applying a tangential force, creating a moving tangential frictional contact.

d. When on the other hand \( F_n \) is constant and \( F_x \) is of an asymmetric form, e.g. an asymmetric triangle wave with average vanishing tangential force over one oscillation period, a directed macroscopic movement may be induced. This is the basic working principle of a stick-slip drive.

e. If both, normal force \( F_n \) and tangential force \( F_x \) do oscillate, then the asymmetry can be introduced by a particular choice of the phase shift between two oscillations. Due to the phase shift, the horizontal force reaches at one point of the oscillation the critical value and a motion in horizontal direction starts. This case is realized, for example, in walking or in travelling wave motors.

\(^1\)This is valid for symmetrical material properties. Discussion of asymmetrical properties see below.
f. Finally, asymmetry can be introduced through asymmetry of material properties of the contacting bodies, for example if the coefficient of friction is different for movements in positive and negative $x$-direction. These kinds of drives are known as ratchets.

Critical Value of Asymmetry and Presliding

The presence of asymmetry in a system is a necessary but not sufficient condition for a directed macroscopic motion. In many cases, the asymmetry must achieve a certain critical value before a directed macroscopic motion starts. In the case of the asymmetry due to external tangential force, this threshold is called the static friction force. In the case of an asymmetric saw-like tangential loading, it is known as the $0$-amplitude or presliding.

Before this critical value is reached no macroscopic movement of the system occurs. But, due to an elastic deformation of the frictional partners, there is always some microscopic movement. In the context of frictional contacts, it is often called presliding and may be important for such applications as precise micro and nano positioning devices as well as if considering structural damping.

Thus, contacts subjected to superimposed static and oscillating forces, cover both the situations: problems which are traditionally handled as friction control and problems which are classified as drives. There is no clear distinction between friction control systems and friction drives. As a matter of fact any continuous transition between these two limiting cases is imaginable. It is therefore not only logical but also necessary to consider both problems from the same point of view and with the use of the same contact mechanical models.

In particular this work will focus on three cases:

1. The asymmetric excitation of a stick-slip drive with an asymmetric sawtooth tangential load $F_x$ and a constant normal force (chapter 3).

2. Active control of friction with a constant tangential force and a harmonic normal load (chapter 4.3).

3. Active control of friction with a constant normal force and a harmonic oscillating tangential force (chapter 4.4 and chapter 4.5).

Objective

In all three case studies, experimental work will be compared with analytical and numerical models of the devices. The ultimate gain would be a qualitative and quantitative accurate description of the measured data, resulting in a deep understanding of the systems. This could on the one hand lead to an amelioration of the existing devices in case of the nano robot RaMoNa. On the other hand, regarding the active control of friction, this work could result in practical implementations, e.g. suppressing cornering squeal of trains or in building the fastest sled on the hill for my nephew.
Another aim of this work is to apply a new numerical method to describe the movement of nano drives and to also investigate the friction reducing effect of an oscillating normal force in a dynamic contact. Due to the high computational cost, the numerical treatment of the full dynamically changing contact configuration, meaning the change in normal and shear stress, is most often discarded with respect to ‘simple’ friction laws. In the simplest case these laws consider only one material parameter, namely the friction coefficient, or in the more advanced cases multiple friction parameters including contact properties like tangential stiffness. However, no matter how sophisticated advanced frictional laws may be, they will never be able to model the complex contact configuration. The numerical method, namely the Method of Dimensionality Reduction (MDR), applied in this work reduces the three dimensional contact geometry to a one dimensional line, which even allows numerical computations of dynamically changing contacts. The performed numerical description of the friction drive and the active control of friction are described exemplarily and in detail with the aim in mind to allow the reader to apply the method to their own specific contact problem.
Consequently this work commences with an introduction of commonly used friction laws and continues with a summary of the physical description of the tangential contact for different contact configurations as well as the main ideas of the MDR in chapter 2.
Chapter 3 addresses the description of the force-generating friction drive RaMoNa. The chapter itself is organized as follows: First an overview of existing concepts of micro robots is given, before the nano positioning device RaMoNa is introduced. Then the physical model of the device is described, prior to discussing numerical and experimental data for different simulation parameters. The chapter concludes with the description of the drive’s movement by a simplified semi-analytical model allowing for approximations of the drive’s characteristic values, as well as inventing a method enhancing its performance.
Chapter 4 is devoted to the description of the active control of friction by ultrasonic oscillations. First an overview on the achieved insight of this topic over the last decades is given, before the experimental set-up used for the performed experiments is introduced. Then, successively for three different oscillation directions, the theoretical models and the underlining experimental data are described and compared. The chapter closes with a summary of the won insights.
The thesis concludes by recapitulating the major results of chapter 3 and chapter 4 and providing an outlook on work worth pursuing to complement the presented results.
2. Dry Friction and Tangential Contacts

Leonardo Da Vinci (1452-1519) conducted the first systematic scientific description of friction. But it was not until Amontons [2] (1699) and Coulomb [3] (1781) that further investigations on the nature of friction were recorded. Their work resulted in a law in mathematical form:

\[ F_{r,s} \leq \mu_s F_n \]  \hspace{1cm} (2.1)
\[ F_{r,d} = \mu_d F_n \text{sgn}(v_{rel}) \]  \hspace{1cm} (2.2)

According to this law the static and dynamic friction force, \( F_{r,s} \) and \( F_{r,d} \), respectively, are dependent only on the normal force \( F_n \) and a static or dynamic friction coefficient, \( \mu_s \) and \( \mu_d \) (Amonton did not differentiate between static and dynamic friction). The friction coefficient is only dependent on the material pairing. As such, according to this law, the friction force is not dependent on contact area or sliding velocity. Additionally the friction force points in the opposite direction of the implied motion. However, reducing Coulomb’s work to this simple law would be a grievous simplification, since Coulomb, as his studies show, was already well aware that friction is a complex phenomenon. It is dependent on uncountable factors, e.g. temperature, change in the configuration of contact area, and contact history, to name just a few.

In 1949 Bowden and Tabor [4] introduced a theory which explains the universality of Coulomb’s law by the interaction of rough surfaces. This theory states that if two surfaces come in contact, only asperities are in contact, and so called micro contacts are responsible for the friction interaction. The real contact area is therefore much smaller than the assumed one and is proportional to the normal force. The friction coefficient is then the ratio of the shear stress and the material hardness.

However, even if Coulomb’s law has a great universality and is generally applicable to gain an overall concept of the friction interaction in a dynamic contact regarding the system, one is obliged to consider more refined models to accurately describe the desired friction mechanisms.

There are two major approaches to describe friction with more accuracy. First, so called single state friction laws are used. They consider the friction force to be a result of the one considered state of the friction interaction. Depending on the system, different mechanisms such as Strubeck effects [5, 6], material viscosity or contact time [7] can be considered in a mathematical description. Second, the friction is seen as a direct consequence of the changing contact area. The overall friction force is thus the direct effect of the contact’s shear stress distribution.

The macroscopic systems considered in this thesis, employed to investigate the friction reduction by ultrasonic oscillation, can be qualitatively described using Coulomb’s law. The nano stick-slip drives however, cannot be accurately investigated using this law, since multiple effects, i.e. presliding, have to be considered in the friction description due
to the contact’s nanoscopic nature. The approach of choice is to describe the tangential contact, and as such its entire contact area configuration dynamically using the MDR. Since until now only single state friction laws have been used to describe these drives, this chapter will start with a description of some single state laws. After this, the theory of a tangential contact with slip will be considered. It will furthermore be described how the MDR can be used to not only describe circular contact areas, but also axial symmetrical shapes, as well as fractal rough surfaces.

Concluding, the discrepancies of the MDR and the single state friction laws will be summarized.

2.1. Single State Friction Laws

As does the Coulomb law, single state friction laws consider the friction interface as a single spot interaction. In their mathematical description they allow a dependence of multiple factors, e.g. contact stiffness, viscous friction, Stribeck effect, and temperature. These models are used to represent all different kinds of frictional contacts. An early survey of approaches was delivered by Armstrong-Hélouvry in 1994 [8]. In this thesis the friction interaction of stick-slip drives is described, and thus only laws able to characterize these devices will be considered. An important characteristic of the models of choice is the existence of presliding. It defines a microscopic displacement \( u_{x, \text{max}} \) of the frictional contact before total slip occurs.

The law most frequently applied to microscopic devices is the Dahl model and its extension to the Stribeck effect, the LuGre model see ([9] and [10]). The further extension is the so called Elasto-Plastic model [11]. To better understand the differences between these models, they are brought into a generalized form. As such they possess a displacement \( x \), which consist of an elastic part \( x_e \) and a plastic part \( x_p \) such that the overall displacement

\[
x = x_e + x_p. \tag{2.3}
\]

The corresponding friction law is defined as

\[
F_r = k_x x_e + \eta_1 \dot{x}_e + \eta_2 \ddot{x}. \tag{2.4}
\]

The constants \( k_x \), \( \eta_1 \), and \( \eta_2 \) represent the tangential contact stiffness, elastic viscous damping and viscous damping, respectively. The plastic displacement is defined implicitly, as well as the possible Stribeck effect, by

\[
\dot{x}_e = \dot{x} \left( 1 - \alpha(x_e, \dot{x}) \frac{x_e}{x_{e,ss}} \right). \tag{2.5}
\]

Here \( x_{e,ss} \) is the steady state displacement, denoting the elastic displacement in the single state sliding regime, where \( F_{ss}(\dot{x}) = k_x x_{e,ss}(\dot{x}) \). Moreover, \( \alpha(x_e, \dot{x}) \) is the defining characteristic of the model, if considering presliding. To investigate the crucial presliding characteristic of these models, the following displacements are defined.

\[
\begin{align*}
\dot{x} = \dot{x}_e & \Rightarrow \dot{x}_p = 0 & \text{elastic displacement} & \tag{2.6} \\
\dot{x} = \dot{x}_e + \dot{x}_p & \text{mixed elastic and plastic displacement} & \tag{2.7} \\
\dot{x} = \dot{x}_p & \Rightarrow \dot{x}_e = 0 & \text{plastic displacement} & \tag{2.8}
\end{align*}
\]
2.1. Single State Friction Laws

Considering the elastic state and thus the displacement $\dot{x}_e = \dot{x}$, with equation (2.5) it follows that
\[
\alpha(x_e, \dot{x}) \frac{x_{e,ps}}{x_{e,ss}} = 0.
\] (2.9)

Pure elastic presliding can only be achieved if $x_{e,ps} \neq 0$ for every $\dot{x}$.
Consequently $\alpha(x_e, \dot{x}) = 0$ for $x_e = x_{e,ps}$. The following single state models, which can all be seen as special cases of the above model, have been developed to describe frictional contacts to control applications with displacements in a microscopic regime.

**Dahl Model**  This model was developed in 1968 [12] and takes no viscous effects into account, such that $\eta_1 = 0$ and $\eta_2 = 0$. Additionally the implicit plastic displacement is defined as
\[
\dot{x}_e = \dot{x} \left(1 - \frac{x_e}{x_{e,r}}\right),
\] (2.10)
where $x_{e,r} = \frac{F_c}{x_e} \text{sgn}(\dot{x})$. $F_c$ is the Coulomb friction force. Thus, this model has no presliding, since $\alpha \neq 0$ for all $x_e$ and the friction force equals the Coulomb friction force in case of sliding. A development of this model to take more frictional properties into account is the LuGre Model.

**LuGre Model**  This model was introduced by scientists in Lund and Grénoble ([9] and [10]). Compared to Dahl’s friction model it extended the model by viscous effects as introduced in equation (2.5) and additionally took the Stibbeck effect into account, namely a friction force $F_{ss}(\dot{x})$ decreasing with respect to the sliding velocity. The implicit definition of displacements due to plastic deformation for this model is:
\[
\dot{x}_e = \dot{x} \left(1 - \frac{x_e}{x_{e,ss}}\right).
\] (2.11)
However, this model still takes no elastic deformation and an inherent displacement $x_e$ into account, since $\alpha(x_e, \dot{x}) = 1$ for all sliding velocities.

**Elasto-Plastic Model**  This model was developed due to the LuGre model not including elastic deformation. The idea of elastically deformed asperity contacts was firstly introduced for this specific Dahl model and resulted in the additional elastic displacement $x_e$ allowing this model to describe the effect of presliding. The corresponding model [11] introduced in equation (2.3) allows the coefficient $\alpha(x_e, \dot{x})$ to include elastic displacements. Figure 2.1 shows this coefficient as a function of the elastic displacement $x_e$.

The coefficient $\alpha$ is chosen such that until an elastic displacement $x_{e,ps}$ is reached, the displacement is only elastic. This state can be identified with the contact being still in a stick state, the overall velocity of the contact is small. If $x_e > x_{e,ps}$, the maximal displacement will be in a regime where the displacement is elasto-plastic until a steady state displacement $x_{e,ss}$ is reached. Then the plastic regime begins. If $x_{e,ps} = x_{e,ss}$, no elasto-plastic displacement would be possible.
To better understand the nature of these models and the further development into the CEIM model (see below), a recapitulation of all empiric parameters is useful and depicted in Table 2.1. In total seven empiric parameters are used, of which some can be measured, as $\mu$ or $k_x$, while others have to be chosen arbitrarily, e.g. the presliding displacement $x_{e,ps}$. The presliding distance was initially introduced due to the lack of the model to cover the effect of presliding. However, the choice of the value remains arbitrary. It is often even used as fitting parameter and not as system immanent property. Consequently, if the parameters are chosen well, and this does not mean chosen to be realistic, a good fit for every system to be modelled can be achieved. The CEIM model represents one of these models.

### Table 2.1: Empiric parameters of the Elasto-Plastic model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Physical meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_x$</td>
<td>tangential contact stiffness</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>contact specific viscosity</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>lubrication viscosity</td>
</tr>
<tr>
<td>$x_{e,ss}$</td>
<td>steady state displacement, computable from Strubeck curve $F_{ss}$</td>
</tr>
<tr>
<td>$x_{e,ps}$</td>
<td>elastic presliding</td>
</tr>
<tr>
<td>$\mu$</td>
<td>friction coefficient, mostly implicitly used in $F_{ss}$</td>
</tr>
</tbody>
</table>

CEIM Model

This model was developed by Christoph Edeler in his PhD thesis [13]. It can be understood as an extension of the Elasto-Plastic model with the focus to describe the $x$-axis of the stick-slip drive RaMoNa (see chapter 3.2). This is done by reducing the model to the relevant properties. Since the friction is dry, $\eta_2 = 0$, the Strubeck effect is neglected due to its small influence and the steady state friction is assumed to be the Coulomb friction. Finally the presliding distance is assumed to be related to the steady state distance such that $x_{e,ss} = 2x_{e,ps}$. No motivation why the latter is applicable is given. The initial motivation of the model is claimed to be related to empirical findings indicating that the presliding and the contact stiffness are highly dependent on the preload, as well as on the material properties, Elastic modulus $E$ and the Poisson’s ratio $\nu$. Taking these influences into account, $x_{e,ps}$ is a function of the preload, $E$ and $\nu$ as well as six fitting parameters. The contact stiffness is also a function of the preload, $E$ and $\nu$.
Interestingly the author refers to the maximal displacement of a tangential contact $u_{x,\text{max}}$ and claims that this is highly related to the value of $x_{e,ps}$, which he relates to the so called 0-amplitude of the drive. That $u_{x,\text{max}}$ can be used to derive an expression for the 0-amplitude will be one of the main results of chapter 3.

Consequently, single state friction models can only be an approximation of better or worse quality. Two reasons indicate this fact. First, friction between two bodies is highly dependent on the contact properties. The contact itself has multiple areas, which are changing rapidly especially in dynamic systems. A single state model cannot take these effects into account. Secondly, to be able to cover these gaps, multiple friction mechanisms are implemented in the model. This is an advantage as well as a disadvantage, since more mechanism increase the accuracy, but also the amount of empirical or fitting parameters which are difficult to determine. Consequently, single state models are an approximation and can at the utmost only be as accurate as the user is careful to choose the appropriate empirical values.

2.2. Classical Tangential Contact Theory

A more sophisticated model to describe friction is the consideration of the tangential contact configuration for different contact geometries. This is done by derivation of stress, strain and displacement configuration of the contact area described as a continuum. At each point in the contact area where sliding occurs, Coulomb's law is assumed to be valid:

\[ \tau(x, y) = \mu \sigma(x, y). \] (2.12)

Consequently, if the right pressure distribution $\sigma(x, y)$ is found, the overall friction force becomes $\mu F_n$ in case of pure slip. It appears that nothing is won in consequence, and this is certainly true for high sliding velocities. For the transition between stick to slip state however, the model describes the contact area as consisting of a stick and a slip area. This results in a microscopic displacement of the contact surface $u_x$ before slip occurs. Thus, the model describes not only a realistic contact area, but also implicitly defines the presliding as maximal displacement $u_{x,\text{max}}$ before total slip occurs. If this theory is coupled with a dynamic model of the system of interest, effects of dynamic stick-slip motion can be described in better detail. In literature, a simplified model of the contact geometry as a line contact with (viscous) elastic properties was previously introduced for example for ultrasonic motors [14]. However, even if these theories are drawing a more realistic picture of the real contact mechanics, they mostly disregard the overall contact geometry.

One way to include the contact configuration and its stress distributions is to create an equivalent line contact to combine the classic tangential theory for different contact geometries in the MDR. Doing so, static and dynamic systems can be accurately and efficiently modelled and implemented. Consequently a brief summary of the classical contact theory will be given here, prior to describing the MDR.
2.2.1. Classical Contact Theory for a Sphere

The classical tangential contact theory for an elastic sphere pressed into an elastic half space with partial slip was independently found by Cattaneo [15] in 1938 and Mindlin [16] in 1949. An overview of their findings can be found in [17]. The basic idea is to find a tangential and normal stress distribution for a given tangential displacement of an elastic sphere in contact with an elastic half space, initially loaded by a normal force $F_n$ and a tangential force $F_x$ (see figure 2.2).

Assuming elastically similar materials it is

$$\frac{1 - 2\nu_1}{G_1} = \frac{1 - 2\nu_2}{G_2},$$  \hspace{1cm} (2.13)

where $G$ is the shear modulus, $\nu$ the Poisson’s ratio. The normal and tangential contact problems can be assumed to be decoupled. Decoupled means that two elastic bodies, which are not necessarily spheres, under a tangential load, with a tangential traction $\tau_1(x, y) = -\tau_2(x, y)$ also have the same normal displacement on the surface at this point: $u_{z,1}(x, y) = -u_{z,2}(x, y)$. This follows directly from Boussinesq and Cerutti (see [18] and [19]). It implies nothing more than the overall contact area being defined by the normal contact problem only and thus the tangential contact properties are decoupled from the normal problem. Furthermore it can be shown that the coupling of normal and tangential contact problem can be neglected if deflections perpendicular to the sliding direction are small and the friction coefficient is considerably smaller than unity [20, 21]. Thus in this thesis an independence of both contact problems will be assumed, such that the entire solution might be interpreted as a superposition of both solutions, the one for the normal and the one for the tangential contact problem.

For a sphere approximated as a parabolic contact, the normal contact problem is solved according to Hertz [22]. For a given parabolic profile at any point $(x, y)$ in the contact area, with its radius $r = \sqrt{x^2 + y^2}$ the normal displacement, pressure distribution, and...
normal force, are:

\[ u_z(r) = d - \frac{r^2}{2R} \quad (2.14) \]

\[ \sigma(r) = \sigma_0 \left( 1 - \frac{r^2}{a^2} \right)^{1/2} \quad \text{with} \]

\[ \sigma_0 = \frac{2}{\pi} E^* \frac{a}{R} \quad \text{and} \]

\[ F_n = \frac{4}{3} E^* \frac{a^3}{R} \quad (2.17) \]

Here, \( E^* = \left( \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \right)^{-1} \) is the effective elastic modulus, \( R \) the sphere’s radius, \( d \) is the indentation depth and \( a \) is the contact radius with

\[ a^2 = Rd. \quad (2.18) \]

The tangential contact problem is solved using again the potential theory by Boussinesq [19] and Cerutti [18]. For simplicity only one of the contacting bodies is assumed to be elastic. For a shear stress distribution

\[ \tau = \tau^{(1)} + \tau^{(2)}, \]

\[ \tau = \tau_1 \left( 1 - \left( \frac{r}{a} \right)^2 \right)^{1/2} - \tau_2 \left( 1 - \left( \frac{r}{c} \right)^2 \right)^{1/2}. \]

the corresponding displacement in direction of the tangential force \( F_x \) is

\[ u_x(r) = \frac{\tau_1 \pi}{32Ga} \left[ 4(2-\nu)a^2 - (4-3\nu)x^2 - (4-\nu)y^2 \right] - \frac{\tau_2 \pi}{32Gc} \left[ 4(2-\nu)c^2 - (4-3\nu)x^2 - (4-\nu)y^2 \right]. \quad (2.19) \]

Note that \( c \) denotes the radius of the contact area in which the contact sticks, it is:

\[ r \leq c \quad \text{stick} \quad u_x(r) = \text{const.}, \quad (2.20) \]

\[ c < r \leq a \quad \text{slip} \quad \tau(r) = \mu \sigma(r). \quad (2.21) \]

Figure 2.3.: Stress distribution and corresponding contact area of a contact with contact radius \( a \) and a radius of the stick region \( c \).
Figure 2.3 shows the pressure distribution $\sigma$ within the contact area along with the shear stress distribution $\tau$ and the corresponding stick and slip area. Since $\tau^{(2)}$ is only defined in the stick region it follows
\[ \tau_1 = \mu \sigma_0. \]  
(2.22)

Moreover, since the displacement in the stick region is constant for each point $(x, y)$, equation (2.21) yields:
\[ \tau_2 = \tau_1 \frac{c}{a}. \]  
(2.23)

The tangential displacement of the contacts surface in the stick area thus becomes
\[ u_x = \mu \frac{E^*}{G^*} \left( d - \frac{c^2}{R} \right). \]  
(2.24)

Here, $G^* = \frac{(2 - \nu_1)}{4G_1} + \frac{(2 - \nu_2)}{4G_2}$ is the effective shear modulus. This result thus represents the solution for two elastic bodies in contact. Important to note is the maximal displacement before total slip occurs when $c = 0$:
\[ u_{x,max} = \mu \frac{E^*}{G^*} d. \]  
(2.25)

The constant $C_M = \frac{E^*}{G^*}$ is the so-called Cattaneo-Mindlin coefficient. In addition the overall tangential force is computed by integrating the shear stress over the contact area:
\[ F_x = \frac{2}{3} \pi \left( \tau_1 a^2 - \tau_2 c^2 \right) = \frac{2\pi a^2}{3} \mu \sigma_0 \left( 1 - \frac{c^3}{a^3} \right). \]  
(2.26)

With equation (2.16) and (2.17) the tangential force yields
\[ F_x = \mu F_n \left( 1 - \frac{c^3}{a^3} \right), \]  
(2.27)

which is equivalent to Coulomb’s law in case of total slip $c = 0$.

Note that for known forces, $F_n$ and $F_x$, with the result of the Hertzian contact problem, $u_x$ and $c$ can be directly determined.

### 2.2.2. Classical Theory for Rotational Symmetric Bodies

While the Cattaneo-Mindlin theory showed for a parabolic contact that the tangential contact problem could be solved for elastically equivalent materials as soon as the solution for the normal contact problem was found, a generalization of the theory was not achieved until the end of the last century by Jäger \[23\] and Ciavarella \[24, 25, 26\]. Jäger developed his theory for the contact of two bodies of the same material for a superposition of flat punches, which can represent any axial symmetric body. Ciavarella on the other hand developed his theory based on the Cerutti and Boussinesq approach for loaded half planes. He showed the solution for arbitrary bodies with Poisson’s ratio...
\( \nu_1 = \nu_2 = 0 \). He also found the solution for the tangential contact of two axial symmetric bodies of elastically similar materials according to equation (2.13). This is equivalent to the requirement that Dundurs second constant vanishes [27]. The solutions for tangential stress, tangential force and tangential displacement interestingly are all dependent on the difference of the normal contact defining quantities: pressure, normal force and indentation depth, for the contact, subtracted by the parameters for a fictional contact with contact radius \( c \). The governing equations thus read:

\[ \tau(r) = -\mu [\sigma(a, r) - \sigma(c, r)] \] (2.28)
\[ F_x = \mu [F_n(a) - F_n(c)] \] (2.29)
\[ u_x = \mu E^* \frac{G^*}{d(a) - d(c)}. \] (2.30)

Here, \( \sigma, F_n \) and \( d \) are the normal pressure, normal force and indentation depth respectively, defining the normal contact problem for a circular contact area of radius \( a \) and a stick region \( c \).

For the sake of clarity it should be once more remarked that Coulomb’s law as in equation (2.21) is supposed to be valid in the slip region. This means that the tangential traction has the same direction as the tangentially applied force \( F_x \). This is a simplification, since in reality, there are displacements in the slip area \( u_x \), as well as \( u_y \). However, the latter have been shown to be neglectable (see again [20, 21]).

The maximal displacement before sliding is again reached for \( c = 0 \) and is

\[ u_{x,\text{max}} = \mu E^* \frac{G^*}{d}. \] (2.31)

This correlation can be directly derived for any isotropic contact area [28, 29]. The Boussinesq and Cerutti solution for two materials in contact with a contact area \( S \) are given in [25] to be

\[ u_z(x, y) = \frac{1}{\pi E^*} \int_0^1 \int_S \frac{\sigma(\xi, \eta)}{s} \frac{d\xi}{\eta} d\eta \] with \((x, y) \in S \) (2.32)

for the vertical displacement and

\[ u_x(x, y) = \frac{1}{\pi E^*} \int_0^1 \int_S \left( \frac{1}{s} + \frac{\gamma (\xi - x)^2}{s^3} \right) \tau(\xi, \eta) d\xi d\eta \] with \((x, y) \in S \) (2.33)

for the horizontal displacement in direction of the applied force. Here,

\[ s^2 = (x - \xi)^2 + (y - \eta)^2 \] and
\[ \gamma = \left( \frac{\nu_1}{G_1} + \frac{\nu_2}{G_2} \right) \frac{E^*}{2}. \] (2.34)

For rotational symmetric profiles as well as for isotropic profiles, for example self-affine fractal rough surfaces, the integrals can be written in polar coordinates, which gives:

\[ u_z(r) = \frac{1}{\pi E^*} \int_0^1 \int_S \frac{\sigma(s)}{s} r^2 \frac{d\theta}{ds} ds, \] (2.36)
\[ u_x(r) = \frac{1}{\pi E^*} \int_0^1 \int_S \left( \frac{1}{s} + \gamma \cos^2 \theta \right) \tau(s) r^2 d\theta ds \] (2.37)
Since \( \sigma \) and \( \tau \) do not depend on \( \theta \) for isotropic surfaces, integrating both integrals within the contact region with respect to \( \theta \) yields

\[
\begin{align*}
 u_z(r) &= \frac{2}{E^*} \int_s \frac{\sigma(s)}{s} ds \\
 u_x(r) &= \frac{2}{E^*} \int_s \left(1 + \frac{\gamma}{2}\right) \frac{\tau(s)}{s} ds,
\end{align*}
\]

where the latter integral assumes a mean value for the cosine term. This assumption is correct if the maximal indentation depth can be assumed to lie in the centre of the contact and the further height distributions are of isotropic character. In case of total slip, when \( u_z = d \) and \( u_x = u_{x,\text{max}} \), Coulomb’s law applies and \( \tau(r) = \mu \sigma(r) \). In addition it can be shown that \( 1 + \frac{\gamma}{2} = \frac{E^*}{G^*} \) (see appendix A). Finally using equation (2.38) in (2.39), correlation (2.31) is derived. Thus it is also valid for rotational symmetric and every other isotropic surfaces in contact.

### 2.3. The Method of Dimensionality Reduction (MDR)

If the friction force of dynamic contacts is understood as the sum of the shear stress in the contact area, it is evident that the shear stress has to be known for each considered moment of the contact interaction. This itself raises multiple complications. First, for the considered profiles in contact, after the Cattaneo-Mindlin theory, at least the normal pressure distribution to compute the shear stress is required. However, analytical solutions of these problems are rare, such that numerical methods such as the Boundary Element Method (BEM) have to be used. The drawback is that they are slow even for static problems and thus not applicable for dynamic simulations. Another approach is the Method of Dimensionality Reduction (MDR). The idea of the method is to map the three dimensional (rough) profile onto a one dimensional line which is pressed into an elastic foundation, such that the overall governing quantities are exactly mapped.

The advantage of this method is obvious, the dimension reduction will provide a much faster numerical approximation then e.g. the BEM. The method was first introduced in the PhD thesis of Thomas Geike [30] and in [31]. The following papers are all published by the work group around Prof. V.L. Popov and established the further usage of the method [32, 33, 34, 35, 36, 37, 38]. Furthermore, in his PhD thesis Markus Hess derived the exact correlations for axial symmetric bodies in various normal contact configurations [39]. An introduction to the method and to its applications has been given in the book [40]. Recently another group of scientist has published a different approach for the method in [41].

In the subsequent chapters, the MDR will be used to model dynamic interaction in the contact of different frictional partners. Thus, the MDR will be briefly introduced in the following subsections.

#### 2.3.1. Hertzian Contact and Tangential Contact for a Sphere

The idea of the MDR is very simple. For a three dimensional profile \( h(x, y) \) an equivalent line \( g(x) \) is found. This line is pressed into an elastic foundation with distinctive springs
and normal and tangential stiffness

\[ \Delta k_n = E^* \Delta x \quad \text{and} \quad \Delta k_x = G^* \Delta x, \]  

respectively. Here \( \Delta x \) is the distance between the equidistantly positioned springs in the elastic foundation. The aim is to ensure the validity of the governing equations of the three dimensional contact. For a Hertzian contact with a superposed tangential contact force \( F_x \), the systems are depicted in figures 2.4a and 2.4b.

Each spring has a normal and tangential force

\[ f_n(x) = \Delta k_n u_z(x) \quad \text{and} \quad f_x(x) = \begin{cases} \Delta k_x u_z(x) & \text{for } |x| < c \\ \mu f_n(x) & \text{for } c \leq |x| < a \\ 0 & \text{elsewise.} \end{cases} \]  

Here, analogous to the three dimensional case, \( c \) defines the stick region of the contact. In the context of the MDR \( u_z(x) \) and \( u_x(x) \) denote the horizontal and vertical displacements of the springs. In addition, Coulomb’s law is assumed to be valid in the slip area of the contact. The distributed loads in the contact are defined as:

\[ q_n(x) = \frac{f_n}{\Delta x} = E^* u_z(x) \quad \text{and} \quad q_x(x) = \begin{cases} G^* u_z(x) & \text{for } |x| < c \\ \mu E^* u_z(x) & \text{for } c \leq |x| < a \\ 0 & \text{elsewise.} \end{cases} \]  

The aim is now to find \( R_1 \), such that for the contact length \( a \) of the elastic foundation, equations (2.18) and (2.17) are valid for the normal contact, as well as equations (2.24) and (2.27) for the tangential contact problem.


**Normal Contact**

The profile of the parabolic line is given by:

\[ g(x) = \frac{x^2}{2R_1}. \]  

(2.46)

If the line is pressed into the elastic foundation with an indentation depth \( d \) the normal deflection of the springs is

\[ u_z(x) = d - \frac{x^2}{2R_1}. \]  

(2.47)

The equivalent of equation (2.18) is

\[ u_z(a) = 0 \]  

\[ \Rightarrow a^2 = 2R_1 d. \]  

(2.49)

And indeed, if one chooses \( R_1 = R_2 \), the normal force

\[ F_n = \int_{-a}^{a} q_x(x)dx = \frac{4}{3} E^* a^3 \frac{R}{R} \]  

is exactly the same as in equation (2.17).

**Tangential Contact**

It remains to show that \( u_x \) in the stick area and \( F_x \) overall correspond to the classical problem if \( R_1 = R_2 \). For the spring forces, if Coulomb’s law applies, it is

\[ f_x(c) = \mu f_n(c) \]  

\[ \Leftrightarrow u_x = \mu \frac{E^*}{G^*} \left( d - \frac{c^3}{2R_1} \right) = \mu \frac{E^*}{G^*} \left( d - \frac{c^3}{R} \right), \]  

(2.52)

as in equation (2.24). In addition the tangential force \( F_x \) yields

\[ F_x = 2 \int_{-a}^{a} q_x(x)dx = 2 \int_{0}^{c} G^* u_x(x)dx + 2 \int_{c}^{a} \mu \left( d - \frac{x^2}{2R_1} \right) E^* dx \]  

\[ = \frac{2\mu E^* a^3}{3R_1} \left( 1 - \frac{c^3}{a^3} \right). \]  

(2.54)

With \( R_1 = R_2 \) this is the exact result of equation (2.27).

**2.3.2. Normal and Tangential Contact for Rotational Symmetric Bodies**

As shown in the previous section, the aim of the MDR is to find a one dimensional line which, if pressed into an elastic foundation, reproduces the main contact quantities of the original three dimensional contact. For the rotational symmetric contacts this mapping can be found. A proof of the exactness can be found in [40] in chapter 17 for the normal contact and chapter 18 for the tangential contact. All proofs for the normal contact problems were first introduced in [39].

The main ideas are illustrated here.
2.3. The Method of Dimensionality Reduction (MDR)

Normal Contact

A rotational symmetric profile is given by
\[ h(r) = p_n r^n, \]  
where \( n \in \mathbb{R}^+ \) and \( p_n \in \mathbb{R}^+ \). Then the equivalent one dimensional profile is given by
\[ g(x) = \kappa_n p_n |x|^n, \quad \text{with} \]
\[ \kappa_n = \frac{\sqrt{\pi}}{2} n \Gamma\left(\frac{n}{2}\right) \frac{n \Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}. \]

Here, \( \Gamma \) denotes the Gamma-Function
\[ \Gamma(n) = \int_0^\infty \xi^{n-1} e^{-\xi} d\xi. \]

Furthermore, it was shown that the profile \( g(x) \) can be expressed as:
\[ g(x) = x \int_0^x \frac{h'(r)}{\sqrt{x^2 - r^2}} dr. \]

Certainly, any three dimensional rotational symmetric or one dimensional axial symmetric profile can be achieved by a superposition of other symmetric profiles.

Exactly analogous to the case of the sphere, \( k_n, f_n, q_n, \) and \( u_z \) are defined as in equations (2.40), (2.42), (2.44), and (2.47) and the normal force \( F_n \) is again computed as
\[ F_n = \int_{-a}^a q_n(x)dx = \int_{-a}^a (d - g(x)) E^* dx, \]
which yields for the general profile as in equation (2.56):
\[ F_n = 2 E^* \frac{n}{n+1} \kappa_n p_n a^{n+1}. \]

The fundamental stress distribution of the three dimensional contact can be extracted from the one dimensional contact configuration. It derives from the line load \( q_n(x) \) and is given by
\[ \sigma(r) = \int_0^\infty \frac{q_n'(x)}{\sqrt{x^2 - r^2}} dr. \]

Tangential Contact

For the three dimensional axial symmetric contact it was claimed in chapter 2.2.2 that the tangential contact is known, as soon as a description for the corresponding normal contact has been found. As it turns out, the exact same claim is true for the MDR. It can consequently be assumed that for an axial symmetric profile \( h(r) \) and its equivalent
one dimensional profile \( g(x) \) with the definitions as in equations (2.41), (2.43), (2.45), and (2.24) for \( k_x, f_x, q_x, \) and \( u_x \) the tangential force yields

\[
F_x = 2 \int_0^a q_x(x)dx = 2 \int_c^e G^* u_x(x)dx + 2 \int_c^e \mu E^* u_z(x)dx .
\]

(2.63)

The unknown radius of the stick area \( c \) and the tangential displacement \( u_x(x) \) can be won from Coulomb’s law

\[
f_x(c) = \mu f_n(c) .
\]

(2.64)

For a general profile as given in equation (2.56) this yields

\[
u_x = \mu E^* \kappa_n p_n (a^n - p^n)
\]

(2.66)
in complete agreement with equation (2.30). The tangential force then becomes

\[
F_x = \mu 2E^* \kappa_n p_n (a^n - c^n) + \mu 2E^* \kappa_n p_n \int_c^a (a^n - x^n) dx
\]

(2.67)

\[
= \mu 2E^* \frac{n}{n+1} \kappa_n p_n (a^{n+1} - c^{n+1}) ,
\]

(2.68)
in direct agreement with equation (2.29).

Concluding, it has been shown in [40] that the shear traction can be expressed as:

\[
\tau(r) = \frac{1}{\pi r} \int_r^a \frac{xq_x}{\sqrt{x^2 - r^2}} dx .
\]

(2.69)

2.3.3. Introduction to Randomly Rough Fractal Surfaces

Bowden and Tabor [4] already pointed towards the irrevocable fact that almost all surfaces are rough, and hence only a few actual asperities are in contact with each other. They found the illustrative formulation that ‘putting two solids together is rather like turning Switzerland upside down and standing it on Austria’. Still the description and numerical implementation of rough surfaces was and still is an important research area [35, 42, 43, 44, 45, 46, 47]. An extensive description of the work in progress can be found in [48]. In the following a brief introduction on the class of fractal rough surfaces, and how they are mapped onto a fractal one dimensional line, which may be used in the MDR is given. Furthermore, a method will be introduced, allowing to map the fractal line onto an axial symmetric line equivalent to the ones in chapter 2.3.2. A detailed description can once again be found in [40].

Rough surfaces, which are statistically isotropic can be characterized by a full Fourier spectrum. This can be related to the power spectrum \( C_{2D} \) for a surface and \( C_{1D} \) for a line,

\[
C_{2D}(\vec{q}) = \frac{1}{(2\pi)^2} \int \langle h(\vec{x})h(\vec{0}) \rangle e^{-i\vec{q}\cdot\vec{x}} dA
\]

(2.70)
and

\[ C_{1D}(q) = \frac{1}{(2\pi)^2} \int_S \langle g(x)g(0) \rangle e^{-iqx} \, dx, \quad (2.71) \]

respectively. Here, \( \langle \cdot \rangle \) denotes the average over a statistical ensemble and \( h(\vec{x}) \) is the height profile, with a zero position such that \( \langle h \rangle = 0 \). In addition, \( \vec{q} \) denotes the wave vector. The special class of fractal rough surfaces, which will be considered in this thesis, have the characteristic that their statistic properties remain unchanged under scaling. This can be formally expressed as

\[ h_m(x, y) = \Phi^H h(x/\Phi, y/\Phi). \quad (2.72) \]

Here \( \Phi \) is an arbitrary factor, and \( H \) is the Hurst exponent. In case of a Hurst exponent of \( H \in [0, 2] \) the power spectra can be expressed as:

\[ C_{2D} = C_0(Lq)^{-2H-2}. \quad (2.73) \]

Here \( L \) is the characteristic length of the contact region. For the generation of a one dimensional profile which has the same properties as the two dimensional, the correlation between \( C_{1D} \) and \( C_{2D} \) was derived as

\[ C_{1D} = \Lambda(H) q C_{2D}, \quad (2.74) \]

with \( \Lambda(H) \) being a transforming factor. In case of any rough surfaces, this rule implies that the general transformation rule for the creation of (any) equivalent rough line might suffice a (not yet known) integral transformation similar to the one in equation (2.59). Finally, the surface topography can be numerically created using the identities:

\[ g(x) = \sum_q B_{1D}(q)e^{i(q(x)+\phi(q))}, \text{ with } \quad (2.75) \]
\[ B_{1D} = \sqrt{\frac{2\pi}{L}} C_{1D}(q), \quad (2.76) \]

where \( \phi(q) \) is a randomized phase in \([0, 2\pi]\). Notably random rough surfaces can thus be computed in its one dimensional formulation. However, since they are random a computation must be executed over a representative ensemble. This can be avoided if the contra intuitive mapping of a random line onto an axial symmetric line is applied:

\[ g(x) = Q_{1D}|x|^H, \text{ with } \quad (2.77) \]
\[ Q_{1D} = \left( \frac{2(H+1)}{1.9412L} \right)^{H+1} \left( \frac{(H+1)L}{2H} \right)^{H+1/2} h_{\text{rms}}. \quad (2.78) \]

Here, \( h_{\text{rms}} \) denotes the rms roughness of the considered profile.

In this specific work most contact configurations have a sphere as one contact partner. Fortunately, it has been shown in [49] how a superposition of a rough surface and a sphere can be treated in case of a normal contact problem. In general, the new one dimensional line is then a superposition of equation (2.59) and (2.77). The intuitive
result of the contact behaving like a rough surface for small normal forces and behaving like a sphere contact in case of large normal forces is shown. It has not yet been shown that the mapping onto an equivalent one dimensional profile as well as a superposition of a rough line and an axial symmetric profile is applicable for the tangential contact and its characteristic values. Since the Cattaneo-Mindlin theory suggests that the tangential contact is solved for a rotational symmetric body as soon as the normal contact configuration is derived, this approach will be also followed for this contact axial symmetric one dimensional contact configuration.

2.4. Friction Laws and Dynamic Tangential Contacts

The previous sections described how different approaches are exploited to describe same phenomenon. To gain a better overview on the major difference between the description of friction by single state laws and tangential contact theory, they will be explicitly stated here.

**Contact Scaling** While a tangential contact theory always takes the geometry and thus the scale of the contact partners into account, single state friction laws assume an independence of friction of the contact area. They consider it only through a change of the empiric parameter.

**Physical Properties** Single state friction laws can be almost infinitely expanded regarding imaginable physical properties. This is on the one hand an advantage, since in frictional contacts numerous physical phenomena are at play. On the other hand, if a single state friction law is applied, one has to gain a profound understanding of the friction mechanisms in order to reasonably chose these parameters. The tangential contact theory is based on two major assumptions: Coulomb’s law and a profound theory of elasticity. Consequently, it can only deliver as good results as the contact can be characterized by dry friction of one of the considered shapes of contacting bodies. However, in the framework of the MDR, it has been shown that at least for the normal contact problem viscous effects and adhesion can be considered [50, 40].

**Presliding** The description of a microscopic displacement before total slip occurs can be included in a single state friction law. It may be accurate if the presliding distance \( x_{ps} \) is chosen to be close to \( u_{x_{max}} \). However, presliding is always integrated in the tangential contact theory. Furthermore, for axial symmetric and isotropic contacts the presliding distance is directly dependent only on the friction coefficient, the elastic properties of the materials in contact and the indentation depth.
3. Stick-Slip Drives

This chapter focuses on the effect of alternate loading with asymmetric excitation on friction by investigating so-called stick-slip drives. In particular, a force-generating drive of microscopic dimension is examined. First, an introduction on their working principle and application range is given. Then the nano positioning or force-generating device RaMoNa (Rapid mobile platform with nanometer precision) will be introduced. This special drive will then be thoroughly investigated by modelling the force-generating $x$-axis by means of a macroscopic overall model in conjunction with a microscopic model based on the classical tangential contact theory combined with the MDR.

3.1. Introduction to Stick-Slip Drives

The stick-slip principle is a well-known mechanism which describes a friction couple that has two distinctive phases. During the stick phase, both bodies do not overcome the static friction force, while in the slip phase both bodies slip with respect to each. Uncountable physical applications of this phenomenon on macro, micro, and nano scale are known. This effect can be beneficial, as for stick-slip drives, or completely undesired. An example for the latter case is the possible induction of brake squeal [51]. In figure 3.1 the overall working principle of a stick-slip drive is illustrated. The rod is excited as exemplarily shown by a sawtooth function $x_s(t)$. In its slow phase, the rod and the mass $m$ stick together, the static friction force is not overcome. During the fast portion of the sawtooth excitation, due to its inertia, the mass presumably remains in its current position, while the rod is performing a fast backward movement. Differently addressed, in the quasi static slow sawtooth part, the inertia force can be neglected and the static friction force is not overcome, while in the fast part of the sawtooth motion of the rod, the inertia force overcomes the static friction force and slipping occurs. This explains why in literature the stick-slip drives are often referred to as inertia drives (this nomenclature is even more suitable in cases where the stick phase becomes a slower slip phase as in [52, 53]).

![Figure 3.1: Simple model to illustrate the stick-slip principle](image-url)
The stick-slip principle is probably one of the most important mechanisms to induce directed motion. Widely known also as inertia slip, it is used for example in transportation of macroscopic objects. The application range of linear motors is wide. They are by today uncountable, since they are used in read and write units of storage devices, driving devices in machine tools, as well as auto mobile devices such as electrical power windows, to name just a few. A good overview of linear drives can be found in [54] and the references within. However, in the late 1980th, with the development of micro robotic devices, the need of precise positioning devices arose. The stick-slip principle was a relatively easy approach, since it has two major characteristics: a theoretically infinite forward motion and a relatively easy mechanical concept. The first concepts were introduced by Pohl [55], Anders et al. [56], as well as Besocke [57]. The device by Pohl has a similar concept to the one in figure 3.1, where the rod is a so called support which is excited over piezo elements. The initial device could precisely position a rather macroscopic object up to 1 kg in weight.

The device GEL by Anders et al. follows a different concept, by allowing a transportation in the entire horizontal plane by superposing two translational movements (see figure 3.2a). This is achieved by placing a specimen on a in quadrants separated piezo disc in a frame and contracting inversely two neighbour quadrants in opposite directed sawtooth motions. Depending on which neighbours are connected to a sawtooth signal, perpendicular translational steps of the specimen are induced. They can overall lead to a movement to a specific position of the specimen on the disc as long as all quadrants are in contact with it. A further development of the technique even allowed a three dimensional movement of the specimen.

Even though the device invented by Besocke as shown in figure 3.2b is a scanning tunnelling microscope (STM), the overall schematic features lead the way for multiple other positioning devices. The specimen to be scanned is placed onto three piezo rods with metal spheres on top, providing a stable table for the specimen. The rods themselves are hollow piezo ceramics with an electrode inside and four on the outside. By applying the same sawtooth voltage to the opposite outer electrodes, the rods bend and can move the sample in the horizontal plane as well as perform a certain deflection around the vertical axis.

Two other devices by the name of Abalone and NanoCrab (figure 3.2c and 3.2d) were developed by Zesch et al. [58]. Abalone is a drive with two translational and one rotational degree of freedom in the horizontal plane. It consists of two frames, one outer heavier frame in which a smaller frame is bedded over three piezo stack actuators which itself are connected via elastic joints to each frame. The inner platform frame rests on the ground. The heavier outer frame is used as inertia mass such that, if a sawtooth voltage is applied to the piezo stacks, the whole robot starts to move. NanoCrab is a rotational motor. Simplified, it is a sphere which is mounted between five ruby hemispheres, which themselves are tangentially moved by shear piezo elements. The rotor can thus move in all possible directions.

Mariotto created a very similar device [64], where the movable rod was placed between three shear piezo elements in a triangular set-up.

Scientists around Martel at the MIT developed the so-called NanoWalker [59, 60]. This device is schematically depicted in figure 3.2e. It consists of a cube with an embedded
3.1. Introduction to Stick-Slip Drives

Figure 3.2.: Concepts of nano positioning devices
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Chapter 3. Stick-Slip Drives

electronic which is mounted on three hollow piezo rods. Due to the voltage application at the five electrodes of the piezo rods, different bending modes can be achieved, which lead to successive steps of each rod, resulting in an overall motion in the horizontal plane. From another point of view this can be regarded as the STM by Besocke, but upside down.

Multiple set-ups of micro robots especially for nano positioning were developed at the École Polytechnique Fédérale de Lausanne (EPFL) in the Laboratoires des systèmes robotique [61, 62, 63]. Among others the authors developed a micro and nano handling device as part of the MiCRoN project. The locomotion device of this robot consists of two steel frames where the inner flexible one stands on three sapphire half spheres. On the outer frames side’s, four piezo elements are fixated. Depending on the applied voltage configuration, the platform can slide in the two translational directions of the horizontal plane as well as rotate. Both modes are shown in figure 3.2f.

All those drives are performing their duty with high enough precision. However, they were and often still are black boxes regarding the understanding of their physical properties and thus a profound understanding of their dynamics is still lacking. This is why this chapter does not focus on a new concept for a robot, but in the physical description of the still to be introduced, exciting device RaMoNa. While a proof of concept is a good point to start, it does not provide a deeper understanding of the frictional dynamics of the systems. If an amelioration of the concept is however desired a profound description of the working principle may result in possible performance improvements [65]. Consequently it remains to introduce RaMoNa, before the new method to describe the drives macroscopic and microscopic mechanics will be presented.

3.2. Stick-Slip Drive: RaMoNa

The investigated nano positioning device RaMoNa is a development of the Carl von Ossietzky University in Oldenburg at the Department of Microrobotics and Control Engineering. All experiments presented in this chapter were conducted at the institute by Ha Xuan Nguyen.

3.2.1. Experimental Set-Up

To analyse the effect of vibrations on a friction contact, more precisely the movement generating properties of an asymmetric excitation, the nano positioning device RaMoNa will be exemplarily investigated. Since the whole drive is complex to describe due to its five degrees of freedom, this work focuses on the description of the positioning axis. Nevertheless, the concept of the entire drive deserves an introduction. It was invented as a positioning device at the University of Oldenburg by the Department of Microrobotics and was first presented in [66]. Since then it has been refined in multiple ways. In his PhD thesis, C. Edeler [13] described the device and investigated the positioning axis, which he transformed into a force-generating device. The description of RaMoNa will thus be a brief summary of his work.

The device has a geometry of a cube with 2 cm edge length as depicted in figure 3.3. The silver runner is responsible for picking and placing the nano particles. This axis
will be referenced from here on as $x$-axis.

![Image of RaMoNa device](image1)

Figure 3.3.: The nano positioning device RaMoNa

The locomotion platform for a translational and rotational movement in the $y$, $z$-plane is shown in figure 3.4. There are three symmetrically distributed steel spheres mounted on the actuators via magnets. The steel spheres are actuated such that they rotate independently and thus RaMoNa is enabled to move in the entire horizontal plane.

![Image of locomotion platform](image2)

Figure 3.4.: Locomotion platform of RaMoNa

The rotation of the steel spheres is induced by three sophisticated laser structured piezo actuators, one of which is schematically shown in figure 3.5a. Each actuator consists of three ruby hemispheres mounted on electrodes. The electrodes itself are actuated such that the ruby hemispheres rotate and stick or slip with respect to a the steel sphere (see figure 3.5a and 3.5b).

The influence of the sawtooth actuation of the ruby hemispheres on the motion of the drive is the subject of this work. However the locomotion platform is a complicated system consisting of nine ruby hemispheres in interplay with three steel spheres, which itself are in contact with the surface. Consequently, as a first step of the drives description, this work focuses on the positioning axis, the $x$-axis of the drive.
To achieve clarity on the properties of the drive, Edeler further simplified the structure of the axis and developed a force-generating stick-slip drive with one translational degree of freedom. The schematics are depicted in figure 3.6. This picture shows the side view of the new device where the vertical axis of figure 3.3 has been rotated and represents now the horizontal axis. The steel runner is mounted between four piezo-actuators, two on top and two on the bottom. The upper actuators are so called B-type actuators and consist of one laser structured piezo element each on which a ruby hemisphere is glued. The lower actuators are of so called A-type and consist of two piezo actuators each with two ruby hemispheres per piezo. The two actuators on top and on the bottom, respectively, are mounted with an 90° angle with respect to each other. The geometry of this arrangement can be seen in figure 3.6. The lower piezo actuators are pressed via a spring onto the runner, with guidance of a linear bearing. The applied preload $F_p$ is measured with a force sensor (not depicted) of type sensor nano 17 ATI.

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(a) Laser structured piezo actuator for mounting the steel sphere
(b) Rotation principle of actuator and steel sphere

Figure 3.5.: Schematic view of the locomotion platform’s actuator

(a) Side view of the $x$-axis of RaMoNa
(b) Rear view of the $x$-axis of RaMoNa and runner’s geometry

Figure 3.6.: Set-up of the investigative force-generating $x$-axis
If a synchronous sawtooth voltage is applied to the actuators, the ruby hemispheres are rotated accordingly and induce a directed motion onto runner, which itself is pressed onto the force sensor. This motion is measured twofold. Firstly, a laser interferometer (SIOS SP-S120) measures the displacement of the runner and secondly, the Honeywell Model 31 miniature load cell measures the generated force (see again figure 3.6). To ensure a smooth contact between runner and force sensor, another ruby hemisphere at the contact point is embedded in a silicon wafer.

### 3.2.2. Experimental Results and Main Properties

The sawtooth voltage used for the experiments was \( \pm 150 \text{ V} \) with a slew rate, or differently addressed the maximal back step velocity of the sawtooth function is \( 300 \text{ V} \mu \text{s}^{-1} \). To determine the overall physical length of the amplitude in nm, the experiments are performed for different preloads without force generation. The runner’s step width is then detected by recording only the first stick-slip step of the runner and finding an average value of multiple measurements. In figure 3.7 the sawtooth function used along with the corresponding runner’s displacement is shown for a preload \( F_p = 1 \text{ N} \).

![Figure 3.7.: Sawtooth actuation and runner’s displacement](image)

The stick-slip principle is clearly distinguishable. During the slow part of the sawtooth motion, the runner sticks to the ruby hemispheres and is dragged along. In the fast portion of the sawtooth motion of the actuators, due to its inertia force, the runner and hemispheres no longer stick for the whole back step amplitude. Both slide against each other and an overall forward motion is achieved. Notably, vibrations are induced after each back step of the actuator. These vibrations are caused by the elastic properties of the runner–actuator contact as well as elastic effects of the overall macroscopic set-up of the test stand.

For the force-generating device, the deflection of the runner and the generated force as a function of time is depicted in figure 3.8. At the beginning of the experiment the runner and the force sensor are not in contact with each other. Thus, no force is generated and the displacement rises linearly with time. After 1.25 s at a displacement of
1.2 μm the generated force increases drastically. It is thus assumed that at this point the runner comes into contact with the force sensor. With each step the runner takes, the force rises until at 1.4 s a static level of displacement and generated force is reached. This maximal generated force corresponds with a new equilibrium condition, where the generated force and the tangential force induced by the actuators on the runner are in average equal over one sawtooth period. This maximal generateable force will be denoted as $F_{\text{gen}}$. At the end of one measurement the runner is moved backwards, the generated force reduces to zero.

To analyse the device and compare the physical modelling introduced in chapter 3.3 with the experimental data, the experiments have been performed for different radii of the ruby hemispheres: 0.25 mm, 0.5 mm, and 1 mm. The detailed results will be shown in chapter 3.4 but to explain the overall properties of the drive, values for the medium radius 0.5 mm and the preloads 0.1 N, 0.4 N, 0.7 N, and 1 N are shown in figure 3.9.

The generated force $F_{\text{gen}}$ is depicted as a function of the sawtooth function’s amplitude. To ensure the reproducibility, all experiments have been repeated ten times. The graphs are plotted for their average values, the error bars indicate the deviation of the measured data. All error bars throughout this thesis represent the maximal and minimal measured values. It is observable that the reproducibility is better, the lower the preload. A reason for this may be the influence of the preload on the dynamic behaviour of the test stand. The larger the preload, the higher the possibly implied vibration amplitudes. These vibrations disturb the set-up of the test stand, and thus good repeatability is no longer given. The reproducibility of the experiments better the larger the radii of the ruby hemispheres and the lower the generated forces (see appendix B). If the spheres are larger the runner is mounted more stable into the device, which underlines the hypothesis above.
3.2 Stick-Slip Drive: RaMoNa

Figure 3.9.: Generated force $F_{\text{gen}}$ as a function of the actuation amplitude

For all four preloads $F_p$, it exists a minimum amplitude up until which no force is generated. This amplitude is larger, the greater the preloads become and is called 0-amplitude. After this amplitude, the generated force rises and for lower preloads reaches a saturated level in the observed amplitude range. These are the two main characteristics, describing the graphs for all considered preloads and radii.

**0-Amplitude** For each considered preload $F_p$ exists a minimal value of the amplitude until no force is generated. This amplitude is called 0-amplitude and denoted as $A_0$. As will be shown within this chapter, this amplitude is closely related to the displacement of a contact before total slip occurs, $u_{x,\text{max}}$ of equation (2.31).

**Maximal Generated Force** This phenomenon is not clearly visible for all considered preloads, due to the limited amplitude range of the experimental set-up, but it is clearly visible for $F_{\text{gen}}=0.1$ N. After the force generation starts, the force reaches a steady level. For this specific case the maximal generated force is approximately $F_p = 40$ mN and is achieved for all amplitudes larger than 62 nm. The effect can be explained as follows: The amount of energy transported into the system by the sawtooth actuation is clearly dependent on its amplitude. Thus the new equilibrium position and the constant force level of figure 3.8 depends on the amplitude as well. The higher the amplitude, the higher the force. However, there exists a maximal generatable force. Independent of the amplitude, the overall energy, which can be induced into the system by the actuators, cannot exceed the overall friction force of all contacts. Hence the maximal possible generatable force of the $x$-axis is limited to a value of approximately $2\mu F_n$. Here $F_n$
denotes the absolute value of the normal force on all actuators of one side of the runner. Since the contacts are in a 45° angle to the preload $F_p$, the normal force of the contacts is not equal to $F_p$. Regarding figure 3.10 and the fact that two hemispheres are in contact on the side where $F_{n,1}$ applies and four hemispheres on the opposite side it is

$$F_p = F_n \sin(45°) = 2F_{n,1} \frac{1}{\sqrt{2}} = 4F_{n,2} \frac{1}{\sqrt{2}}. \quad (3.1)$$

Consequently $F_n = F_p \sqrt{2}$. Note that the overall friction force applied to the system is $2 \mu F_n$, since $F_p$ is applied from both sides of the runner. Besides the gravitational force $F_g$ has been neglected. This force is small compared to $F_p$, and has a negligible influence on the runner’s dynamics, as we will see in chapter 3.4. Interestingly, the maximal generated force at the force sensor for one specific actuation amplitude is independent of how the force sensor is physically modelled. Thus, as will be shown in the next section, a simple model of the force sensor, as e.g. a linear spring, is sufficient.

**Other Influences on the Force-Generating Device** In his thesis, C. Edeler [13] devoted chapter 3 on how the force is generated and which system specific entities, such as the runner’s mass, frequency of the sawtooth signal or slew rate are influencing the force generation. His results are that all influences can be kept minimal, if the slew rate is large, the runner’s mass is small and the frequencies are kept low. Moreover according to Edeler’s results, roughness has a minimal effect on the runner’s displacement and the force generation.

**Properties of the Drive** For the modelling of the drive, the material parameters of table 3.1 are used.
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<table>
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<tr>
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<td>0.2</td>
</tr>
<tr>
<td>static friction coefficient $\mu_s$</td>
<td>0.3</td>
</tr>
<tr>
<td>radius of the ruby hemisphere $R$</td>
<td>0.25 mm, 0.5 mm, and 1 mm</td>
</tr>
<tr>
<td>stiffness of the force sensor $k_{fs}$</td>
<td>$25 \cdot 10^3$ N m$^{-1}$</td>
</tr>
</tbody>
</table>

Table 3.1.: Properties of the force-generating device

The material parameters of Elastic modulus and Poisson’s ratio are standard literature values. The friction coefficient $\mu$ was experimentally found as part of C. Edeler’s dissertation in [67]. For an evaluation of the microscopic physical model, the device was equipped with actuators mounted with ruby hemispheres of different radii. The stiffness of the force sensor $k_{fs}$ was provided by its manufacturer Honeywell.

### 3.3. Physical Modelling

The physical modelling of the $x$-axis of RaMoNa will be conducted with a hybrid model, consisting of a macroscopic part describing the overall dynamics of the drive and a second part describing the induced friction on the runner. Two possibilities seem appropriate: first, a specific friction law (e.g. as described in chapter 2.1), which could cover especially the main characteristics of the drive, such as the zero amplitude, is possible. Secondly, the whole contact area could be numerically simulated, using for example the BEM. This attempt is already slow when computing a single static contact. Thus the dynamic simulation of six contacts changing their properties dynamically would take decades to be computed even with the fastest computers. Instead the MDR will be applied, which allows, due to the high reduction of the dimension, a dynamic simulation of the contact areas. This attempt is already slow when computing a single static contact. Thus the dynamic simulation of six contacts changing their properties dynamically would take decades to be computed even with the fastest computers. Instead the MDR will be applied, which allows, due to the high reduction of the dimension, a dynamic simulation of the contact areas. This macroscopic and microscopic model was first introduced in [40, 68, 69] for the parabolic contact. For the flattened sphere the theoretical foundations of the microscopic model as well as for the rough sphere can be found in [40]. The idea and results for the rough sphere model applied to the dynamic modelling of the runner’s dynamics have been published in [65] by me and co-authors.

#### 3.3.1. Macroscopic Model

To describe the overall movement of the runner, simple Newtonian mechanics are suitable. The corresponding system is depicted in figure 3.11. In this model the movement of the spheres is approximated by a translational displacement of the spheres by a sawtooth law $x_s(t)$. Realistically, the spheres rotate around the centre of the mass. For small displacements with respect to the contact area, a translational movement is a very good approximation as will be revealed in chapter 3.4. The displacement of the
runner is specified by the equation of motion:

\[ m\ddot{x} = -k_{fs}x - F_r, \tag{3.2} \]

where \( F_r \) is the overall friction force. The displacement \( x \) is equivalent to the distance tracked by the laser vibrometer, the force \( k_{fs}x \) is the generated force \( F_{gen} \) recorded by the force sensor. The modelling of the force sensor as a linear spring is reasonable, regarding the material properties in table 3.1: the force sensor’s stiffness is given as a linear spring with \( k_{fs} = 25 \cdot 10^3 \text{ N m}^{-1} \), which is small compared to the stiffness of the silicon ruby contact stiffness. The latter can be computed as \( k_n = 2aE^* \). The maximal contact radius is given by \( a_{max} = \frac{3\cdot2\mu F_n R}{4E^*} \approx 1 \cdot 10^{-5} \text{ m} \). Here the maximal normal force is assumed to be \( 2\mu F_n \), according to equation (3.1), with the values for ruby of table 3.1 and an Elastic modulus, as well as a Poisson’s ratio of silicon of \( E = 188 \text{ GPa} \) and \( \nu = 0.22 \), respectively (70). The contact stiffness computes to \( k_n \approx 4 \cdot 10^6 \text{ N m}^{-1} \). With the force sensor’s stiffness being two orders smaller than the stiffness of the contact, the latter can thus be neglected in the series connection of the two ‘springs’.

The induced force itself is dependent on the normal forces and the computed friction forces in the contacts. However, as we have seen in chapter 3.2, the actuation is done by six ruby hemispheres. Thus, in figure 3.11 each depicted sphere represents two real ruby hemispheres. Moreover, if upper and lower force have a total normal force of \( 2F_{n,1} \), then \( F_{n,1} = 2F_{n,2} \).

The movement of the runner can be computed using a numerical integration. The method used in this work is the Velocity Verlet algorithm (71). The friction force \( F_r \) is a result of the contacts of the hemispheres and can be modelled and described in multiple ways.

### 3.3.2. Microscopic Contact Model

To solve the equation of motion (3.2) the friction force needs to be computed in each time step. The MDR is implemented to do so. Recalling equations (2.42) and (2.43) the
3.3. Physical Modelling

forces of each spring in the contact can be denoted as $f_{n,i}$ and $f_{x,i}$. The overall friction force is the summation of all tangential spring forces:

$$F_r = \sum_{i=1}^{N} f_{x,i}. \quad (3.3)$$

Here, $N$ is the number of all springs in contact. In each time step, each spring has to be considered if it is stick or slip state. The latter applies, if

$$f_{x,i} = \Delta k_x u_{x,i} > \mu f_{n,i} = \Delta k_n u_{z,i}. \quad (3.4)$$

In this case the spring starts to slide and rests again in a new stick position, for which again $f_{x,i} = \mu f_{n,i}$ holds. Furthermore the new displacement of the spring yields

$$u_{x,i} = \mu f_{n,i} = \mu E^* u_{z,i}. \quad (3.5)$$

It is therefore evident that the vertical and horizontal displacements have to be known to compute the friction force. The tangential displacement is originally relative to the runner $x$ with respect to the displacement of the actuator $x_s$, $u_x = x - x_s$. It can thus be written:

$$u_{x,i} = \begin{cases} 
  x - x_s & \text{stick} \\
  \mu E^* u_{z,i} & \text{slip}.
\end{cases} \quad (3.6)$$

In addition, to determine $f_{n,i}$ one has to know the vertical displacement of each spring. If small deformations are assumed, which is reasonable in small contacts and for small time steps, the change in vertical displacement is numerically neglectable. Thus to implement the tangential contact problem only the vertical deflection of the original contact $u_z(x) = d - g(x)$ and thus the equivalent one dimensional line has to be identified. Consequently the algorithm presented in figure 3.12 is extraordinarily powerful, since in changing only $u_{z,i}$, a whole new contact geometry is exploitable. The relevant geometries for the considered system will be briefly discussed here.

**Vertical Displacement of a Sphere**

As described in chapter 2.3, the vertical displacement for a sphere in contact with an elastic half space yields

$$u_z(x) = d - \frac{x^2}{2R_1}, \quad (3.7)$$

which can be directly implemented in the algorithm of figure 3.12.
**In:** problem parameter
**Out:** runner’s displacement

- generate sawtooth function
  \[ x_s, u_{z,i}, \text{and } f_{n,i} = \Delta k_z u_{z,i} \]

- generate displacements \[ u_{x,i} = x - x_s \]

- compute each spring force
  \[ f_{x,i} = \Delta k_x u_{x,i} \]

- if \( f_{x,i} > \mu f_{n,i} \)
  - no
  - yes
    - set \( f_{x,i} = \mu f_{n,i} \) and \( u_{x,i} = \frac{\mu E^*}{G} u_{z,i} \)

- perform one time step of Verlet algorithm

- stop criterion is reached
  - no
  - yes

- return

---

**Vertical Displacement of a Sphere with Flattened Tip**

Due to wear effects, spheres in contacts flatten out and have a round flat tip with radius \( b \), as represented in figure 3.13a. The profile of the flattened tip is given in three dimensions by:

\[
h(r) = \begin{cases} 
0 & 0 \leq r < b \\
\frac{r^2 - b^2}{2R} & b \leq r \leq a 
\end{cases}.
\] (3.8)

With equation (2.59) the one dimensional equivalent line (see figure 3.13b) yields

\[
g(x) = \begin{cases} 
0 & 0 \leq |x| < b \\
\frac{|x|}{R} \sqrt{x^2 - b^2} & b \leq |x| \leq a 
\end{cases}.
\] (3.9)
3.3. Physical Modelling

which delivers the vertical displacements of the springs as

\[ u_z(x) = g(a) - g(x). \]  

(3.10)

For a known normal force \( F_n \), the contact radius can be derived by solving the requirement

\[ F_n(a) = E^* \int_{-a}^{a} u_z(x) dx = \frac{2E^*}{3R} (2a^2 + b^2) \sqrt{a^2 - b^2} \]

(3.11)

for \( a \).

**Vertical Displacement of a Sphere in Contact with a Rough Surface**

It might be interesting to investigate if a surface roughness of the runner has an influence on the force generation. Consequently, the displacement \( u_z \) of a spherical indenter on a rough surface has to be derived. The normal contact configuration of this problem has been investigated in [49]. Using the insights of chapter 2.3, the first result is that a rough surface and a sphere physically behave like a rough sphere pressed onto a flat surface. In addition a rough surface can be modelled as one dimensional indenter of analytical form using equation (2.46), (2.77) and (2.78):

\[ g(x) = \frac{x^2}{2R_1} + \left( \frac{2(H + 1)}{1.9412L} \right)^{H+1} \frac{(H+1) L}{2H} h_{rms} |x|^H. \]

(3.12)

This implies nothing more than a spherical contact geometry is superposed by a rough line, which itself is converted in an indenter shape used in the MDR. This profile is depicted in figure 3.13b. The displacement is then again \( u_z = g(a) - g(x) \), where \( a \) can be won analogously to equation (3.11).
Regarding the numerical validation of the experiments, the Hurst exponent $H$, the rms-roughness $h_{rms}$, and $L$ as shown in chapter 2.3 are required. Two approaches in acquiring these data will be presented. On the one hand, measurements of the runner’s surface will be performed, on the other hand, $H$ will be fitted, to simulate and evaluate the effects of an idealized rough surface.
3.4. Comparison of Numerical Simulation and Experiment

Every described numerical model is only as good as its comparisons to the experimental data. However, a physical model is bound to make certain approximations, which compromises the models quality. Therefore, in this section it will be discussed how the elementary model of the actuators approximated as translational moved spheres, can predict the characteristics of the micro drive, in particular the overall generated force of the drive as well as the 0-amplitude. After doing so, it will be analysed, how certain adjustments of the model, for example rotating spheres, flattened spheres, or rough runner surfaces, can enhance the numerical results further.

If not otherwise stated, the material parameters and properties of the device are as indicated in table 3.1. It should be noted that the only value, which might be considered a fitting parameter, is the friction coefficient \( \mu \). However, this parameter has also been experimentally examined for the material pairing in the work scope of C. Edeler’s thesis in [67]. It was found that the static friction coefficient is \( \mu_s = 0.3 \) and the dynamic one \( \mu_d = 0.2 \).

The experiments have been performed to demonstrate the universality of the numerical approach. A universality is given if the numerical results are valid for different contact sizes. Since a single state friction law is usually not taking the contact geometry into account, it would predict that the experimental results need to be independent of the contact geometry as well. Consequently, the experiments have been performed for different radii of the ruby hemispheres. Various preloads have been sampled as well. A listing of all performed experiments can be retrieved from table 3.2, the step width of the preload is 0.1 N. To attain a clear presentation, in this chapter only selected data sets will be compared to numerical simulations. To underline the good quality of the results all further experimental results along with some numerical data are depicted in appendix B.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Preload ( F_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R = 1 \text{ mm} )</td>
<td>( F_p = 0.1 \text{ N} - 1.2 \text{ N} )</td>
</tr>
<tr>
<td>( R = 0.5 \text{ mm} )</td>
<td>( F_p = 0.1 \text{ N} - 1 \text{ N} )</td>
</tr>
<tr>
<td>( R = 0.25 \text{ mm} )</td>
<td>( F_p = 0.1 \text{ N} - 0.5 \text{ N} )</td>
</tr>
</tbody>
</table>

Table 3.2.: Performed experiments for the force-generating device

The measurements for each preload have been repeated ten times. The graphs are plotted for the average values, but the measurement deviations are indicated in the graphs by the according error bars.

3.4.1. Translational Movement of the Spheres – Generated Force

The easiest approach to macroscopically model the contact interaction is by assuming a synchronous translational movement of the six spheres pressed with the preload \( F_p \) onto the runner. Former results of old experimental work along with numerical simula-
tions have been previously published in [68, 69]. New and extended experimental and numerical results have been presented in a compressed form in [72].

**Radius R=1 mm**  
The experimental and numerical results for the preloads $F_p = 0.1$ N, $F_p = 0.5$ N, $F_p = 1$ N are depicted in figure 3.15. As already discussed in chapter 3.2, both, numerical and experimental data indicate a 0-amplitude, for which no force is generated. Numerically and experimentally detected 0-amplitudes fit very well, only for the smallest preload a significant difference can be seen. A generated force level for the numerical data is only reached for $F_p = 0.1$ N. For $F_p = 0.5$ N, of the experimental data, a saturated force level is reached for approximately 200 N at the maximal measured amplitude of 202 nm. The remaining curves show no saturated force levels for the investigated amplitude range. This indicates that the simulated data overestimates the generated force. As a concluding remark: the error-bars indicate a good reproducibility of the experiments. A reason for the remaining relative errors might be the vibrations induced during the movement of the runner after a back step is performed. These vibrations may have a chaotic influence on the runner’s movement, which might be more significant for smaller amplitudes and larger preloads.

![Figure 3.15.: Generated force $F_{gen}$ as a function of the actuation amplitude, experimental and numerical data for $R=1$ mm and different preloads $F_p$.](image)

**Radius R=0.5 mm**  
In figure 3.16, the generated force is depicted for the actuator sphere radius of $R = 0.5$ mm. The maximal measured amplitude is 160 nm. Recurrently, the overall tendencies of 0-amplitude and saturated force level are visible. Again, the generated force level is overestimated for the simulations. Only in the case of this radius, the 0-amplitude is strictly smaller for the simulated values compared to the experimental data. Since the overall tendencies of experimental and numerical graphs fit well, it appears plausible that an undetected raise of preload in the experiments could be a reason for the deviations between numerical simulations and experiments.
3.4. Comparison of Numerical Simulation and Experiment

![Comparison of Numerical Simulation and Experiment](image)

**Figure 3.16.** Generated force $F_{gen}$ as a function of the actuation amplitude, experimental and numerical data for $R = 0.5$ mm and different preloads $F_p$

Radius $R=0.25$ mm For the experiments of this radius, only preloads up to $F_p = 0.5$ N have been investigated. The reproducibility of the experiments also becomes worse the larger the preloads are. However, the overall tendencies remain the same as for the other actuator sphere radii. The 0-amplitude is simulated well, while the generated force is again over-approximated.

![Comparison of Numerical Simulation and Experiment](image)

**Figure 3.17.** Generated force $F_{gen}$ as a function of the actuation amplitude, experimental and numerical data for $R = 0.25$ mm and different preloads $F_p$
Conclusion and Further Investigations  The simulations show a very good agreement with the experiments. While the generated force is overestimated for large preloads for all investigated radii, the 0-amplitude is well matched for all experiments. Although the intermittent radius of \( R = 0.5 \) mm does underestimate the 0-amplitude, the fact that the smaller and the larger radius does not, underlines the fact that the error lies within the experimental accuracy. It is exceptional that for three different contact sizes the model predicts the experiments well.

Following three different routes, the discrepancies between experiments and simulations are tried to be explained. First, the system immanent size 0-amplitude will be discussed in detail. Second, the influence of the dynamic macroscopic model will be studied, changing a few selected parameters of the model. Finally, the influence of the microscopic model is challenged by performing simulations for different contact geometries.

3.4.2. Translational Movement of the Spheres – 0-Amplitude

As described earlier, the 0-amplitude is specific amplitude of the sawtooth actuation for which no force is generated. If at all, only a very small displacement of the runner is measurable with the laser vibrometer. This effect can be assumed to be closely related to the presliding distance \( u_{x,\text{max}} \). Until the 0-amplitude is smaller than a mean presliding distance of lower and upper contacts, no slip occurs and no force is generated. After this amplitude is exceeded the contacts slides with an overall friction force \( F_r = \mu F_n \).

Figure 3.18 depicts the numerically computed 0-amplitude as well as the measured one as a function of the preload \( F_p \) for all investigated radii. In addition the presliding distances \( u_{x,\text{max},1} \) and \( u_{x,\text{max},2} \) for the contact forces \( F_{n,1} \) and \( F_{n,2} \) after equation (2.31) are plotted.

For all radii, the numerically simulated 0-amplitude lies in between the analytical presliding distances of the actuator contacts. This suggests that the 0-amplitude of the runner is a specific mean value of both presliding distances. This hypothesis is supported by the analysis of the simplified MDR model in chapter 3.5.1. For the radius \( R = 1 \) mm in figure 3.18, numerical and experimental 0-amplitude fit nicely. For the intermittent radius \( R = 0.5 \) mm, as before, the experimental 0-amplitude is larger than the computed one. Interestingly, both curves appear to be almost parallel, supporting a potential shift in the experimental set-up. Anew for \( R = 0.25 \) mm the numerical and experimental data agree, except for the larger preloads, which did not allow a good reproducibility of the experiments. However, the important result of figure 3.18 is a direct relation between 0-amplitude and presliding distance \( u_{x,\text{max}} \). Not only does the result define the origin of the 0-amplitude to be the presliding distance, but also underlines the quality of the MDR. For all radii, the 0-amplitude is simulated well which would not be possible if a single state friction law was implemented.
Figure 3.18.: 0-amplitude and $u_{x,\text{max}}$ for all investigated radii
3.4.3. Translational Movement of the Spheres – Influence of Macroscopic Model Parameters

Ensuring the quality of the introduced microscopic and macroscopic model, parameter changes of both models are beneficial. Additionally the following investigations might present multiple possibilities, how the numerical results can predict the real characteristics of the drive more accurately.

Changing the Sawtooth Actuation

The motion of the actuators is possibly one of the most important factors regarding the influence on the movement of the runner. Different inertia drives use different actuation modes, e.g. stick-slip and slip-slip [52, 53]. In addition a divergence from the ideal sawtooth like shape of the signal might have a significant influence on the runner’s movement. Until now it has been assumed that the voltage applied on the piezo actuators is loss free converted into an ideal sawtooth translational or rotational movement of the ruby hemispheres. Naturally this conversion has some losses, since leastwise the turning of the actuators will, due to inertia effects, not be able to follow the ideal tip of a sawtooth signal. Hence, a more realistic form of the sawtooth actuation was implemented, as depicted in figure 3.19a. The results for the simulations with and without flattened sawtooth actuation are presented in figure 3.19b.

The numerical simulations have been performed for $\varepsilon = T_2$. The real flattening will be much smaller than the actual time of the fast portion of the sawtooth function. However, even for this large flattening, no influence on the force generation can be detected. The assumption is made that the turning point of the piezo actuation is not changed, only the signal is flattened. Thus, the actual amplitude performed by the spheres is assumed to remain unchanged. The amplitude itself is measured by averaging the first step of the runner for multiple preloads without a force sensor connected upstream. Thus, the
3.4. Comparison of Numerical Simulation and Experiment

overall induced force remains the same. Regarding the force generation, even a smaller amplitude would only have a minimal influence and the results would be exactly the same graphs, parallel-shifted along the positive direction of the amplitude axis. Differently spoken, the amplitude reached in the stick phase determines the generated force, regardless of the shape of the actuation signal. The extreme case would be a symmetric sawtooth signal, resulting in no net movement. A change of the characteristic of the generated force due to a different signal shape with the same amplitude, during which actuator and runner stick, is not possible.

**Influence of the Gravitational Force of the Runner**

Until now an influence of a gravitational force on the runner has been neglected. Regarding figure 3.10 and considering this gravitational influence, the maximal normal force on the lower side actuators of the runner will be larger than assumed previously, while the upper ones will be smaller. Besides a possible effect of gravitation and the asymmetry of the actuators could be a rotational movement of the runner. The influence of this effect has been shown to be inferior in [68]. The change on the normal forces of the actuators is an addition or a subtraction of \( \frac{mg}{2\sqrt{2}} \) on the absolute value of the upper and lower normal forces, respectively.

![Figure 3.20: Generated force \( F_{\text{gen}} \) as a function of the actuation amplitude, numerical data for \( R = 1 \text{ mm} \) for a runner with and without influence of gravitation](image)

The influence of gravitation on the resulting generated force for the numerical simulations is shown in figure 3.20 for the radius \( R = 1 \text{ mm} \) and in figure 3.21 for a radius of \( R = 0.25 \text{ mm} \). While the overall values are not significantly different from the previous simulations, the results indicate that the preloads have an influence on the 0-amplitude. For the considered configuration, the maximal and the minimal normal force of the contacts increase and decrease, respectively. The changes of the 0-amplitude, in particular in figure 3.21, thus indicate that both normal forces influence the 0-amplitude of the system. An analytic approximation of the 0-amplitude will be given in chapter 3.5.1.
and supports this assumption.

![Graph showing generated force $F_{gen}$ as a function of the actuation amplitude, numerical data for $R = 0.25$ mm for a runner with and without influence of gravitation.](image)

**Figure 3.21.:** Generated force $F_{gen}$ as a function of the actuation amplitude, numerical data for $R = 0.25$ mm for a runner with and without influence of gravitation

### Dynamic and Static Coefficient of Friction

In single state friction laws the Strubeck effect \cite{5, 6} is often considered to influence the friction force. Although the Strubeck effect is well established for lubricated friction, already Coulomb was aware of the dependence of friction on the sliding velocity. Later the steady dependence of dry friction on the sliding velocity was established by Rabinowicz et. al. \cite{73}. Furthermore, in his work for the pairing steel ruby, C. Edeler \cite{13} reported a difference between the static and sliding coefficient of friction. In another thesis, Altpeter \cite{74} reported an even lower friction coefficient between steel and ruby. Consequently, the friction coefficient in the microscopic simulation model was adjusted such that the stick condition $f_x \leq \mu_s f_n$ depends on the static coefficient of friction $\mu_s = 0.3$, whereas the new deflection of the spring is computed using the dynamic coefficient of friction $\mu_d = 0.2$ as $u_x = \mu_d f_n/\Delta k_x$. The comparison of this simulation with and without neglecting the velocity dependence for the sphere radius of $R = 1$ mm is depicted in figure 3.22. The comparison of simulated data with only static as well as dynamic and static friction coefficient are shown in figure 3.23. To complement the results, the figures for the two remaining radii are given in appendix B.

Regarding the comparison of the simulation with and without velocity influence on friction, two impacts can be observed. First, the saturation of the generated force $F_{gen}$ is reached at a smaller value if a velocity dependence is considered. Assuming a maximal generatable force directly related to the overall friction force, by assuming a smaller dynamic coefficient of friction, the overall generatable force will be smaller as well.
3.4. Comparison of Numerical Simulation and Experiment

Figure 3.22.: Generated force $F_{gen}$ as a function of the actuation amplitude, numerical data for $R = 1$ mm for a runner with and without changing friction coefficient.

Figure 3.23.: Generated force $F_{gen}$ as a function of the actuation amplitude, numerical data for $R = 1$ mm for a runner with changing friction coefficient and experimental data.

On the other hand the 0-amplitude is decreased. This is a contra intuitive effect, since assuming the 0-amplitude to be related to the presliding distance $u_{x,max}$ it should remain unchanged. However, due to the numerical coverage of dynamic effects of the vibrating contacts, the interplay of dynamic changing slip distances can be modeled.
and the overall 0-amplitude is decreased. This can also be interpreted as a dominance of the slip areas regarding the overall contact dynamics of the slider.

### 3.4.4. Translational Movement of Flattened Spheres

Even if the contact spheres are wear resistant ruby hemispheres, it should be analysed how possible wear effects influence the characteristics of the drive. Typically abrasive wear results in a flattened sphere as illustrated in figure 3.13. Thus, the according microscopic model was implemented. The numerical results for the flattened spheres of radius $R = 1\,\text{mm}$ and $R = 0.5\,\text{mm}$ are shown in comparison with the results for regular spheres in figure 3.24 and 3.25, respectively. The simulations have been conducted for a ratio of radius of the flat part to the sphere’s radius of $b/R = 0.006$.

![Figure 3.24. Generated force $F_{gen}$ as a function of the actuation amplitude, numerical data for $R = 1\,\text{mm}$ for a sphere and a flattened sphere](image)

For $R = 1\,\text{mm}$ a smaller 0-amplitude for the flat sphere than the ideal sphere is observable, whereas the overall generated force level is larger for the investigated amplitudes. After equation (2.25), $u_{x,max}$ is smaller for a flat sphere than a sphere, if the indentation depth $d$ is smaller as well. This can be explained by presuming the indentation of a flattened sphere as an intermediate case between a sphere pressed into an elastic surface with a normal force $F_n$ and a contact radius $a$ and a flat punch of radius $a$ also loaded with the same normal force. The normal force as a function of the indentation of a flat circular punch is $F_n = 2aE^*d$, the one of a sphere is given in equation (2.17). For the same applied normal force it follows that the indentation depth of a flat punch $d_P$ is smaller than the one of a sphere $d_S$. Thus, for the indentation depth of a flattened sphere $d_{FS}$, it is $d_P < d_{FS} < d_S$, and therefore $u_{x,max,FS} < u_{x,max,S}$. We will see in chapter 3.5.3 that the generated force depends approximately on the ratio $A/u_{x,max}$. Hence for a smaller presliding distance, a larger force is generated, which is well established in figure 3.24.
3.4. Comparison of Numerical Simulation and Experiment

Figure 3.25.: Generated force $F_{gen}$ as a function of the actuation amplitude, numerical data for $R = 0.5$ mm for a sphere and a flattened sphere

For the smaller radius $R = 0.5$ mm the effect of a larger generated force and a smaller $u_{x,max}$ is negligible for the examined ratio $b/R$. If the contact radii are compared with the actual size of the computed $b$ for $R = 1$ mm, the flattened portion of the contact takes up 91% of the contact area, whereas the flattened portion for $R = 0.5$ mm takes up 66% of the contact area. A wear effect of ruby hemispheres with a 66% flattening of the contact area is even without further investigations of the exact wear mechanisms highly unrealistic. Consequently possible wear effects resulting only in a new indenter form of a flattened sphere, have no influence of the dynamics of this force-generating device.

3.4.5. Translational Movement of the Rough Spheres

With regard to equation (2.77) and (2.78) the MDR can also be applied for a rough sphere. Two roughness parameters have to be known to gain the equivalent one dimensional profile $g(x)$. The root mean square of the height profile of the three dimensional surface $h_{rms}$ as well as the so-called Hurst exponent $H$ have to be established. The latter gives, according to equation (2.72), an insight on the self-affinity of the surface of the runner. In [75] it is shown, how contrary to former notions, surfaces can be interpreted and treated as fractally rough, up until a Hurst exponent of $H = 2$. However, surfaces with a Hurst exponent close to two have a large dominant wave length, and thus exhibit the characteristics of flat (curved) surface.

$H$ and $h_{rms}$ itself can be measured via atomic force microscopy, or in our case by a white light interferometer based on the principle of a Michelson interferometer. The used MicroMap 512 microscope measures the height profile of the surface in a 383$\mu$m by 299$\mu$m spot. Numerical analysis of this data provides $h_{rms}$ and $H$. Two measurements were taken on each of the four sides of the runner in contact with the ruby hemispheres. One measurement is exemplarily shown in figure 3.26. The average rms-roughness was
computed to be $h_{\text{rms}} = 0.92 \mu m$. The Hurst exponent $H = 1.7$ was derived by computing the power spectrum of the surfaces and using equation (2.73). Consequently, the influence of the roughness in the numerical simulations should be small.

For all three investigated radii, the difference of the computations with and without roughness are depicted in figures 3.27a, 3.27b, and 3.27c. As expected, the influence of roughness with the Hurst exponent $H = 1.7$ is marginal. Only for the smallest sphere radius $R = 0.25 \text{ mm}$ a small discrepancy can be observed.

For the measured roughness, no influence on the numerical model is detectable. It is however interesting to inquire if other artificial roughness parameters would show an influence. In an earlier work by me and our workgroup [65], it was suggested that the roughness can be used in a more empirical way to explain the discrepancies between numerical and experimental data for contact spheres with a radius $R = 0.5 \text{ mm}$. While the approach follows a rather free method to fit $H$ and $h_{\text{rms}}$, here only the Hurst exponent is assumed to be a free fitting parameter, while the rms-roughness is assumed to be the measured one $h_{\text{rms}} = 0.92 \mu m$. To fit $H$ the experimental 0-amplitude is fitted to the approximated 0-amplitudes of equation (3.26). The $H$ with the smallest residual was computed to be $H = 0.9$. The comparison between experiments and numerical computation for $R = 1 \text{ mm}$, $R = 0.5 \text{ mm}$ and $R = 0.25 \text{ mm}$ is depicted in figures 3.28a, 3.28b and 3.28c.

While for the larger radius the influence of the roughness is not overly improving the numerical findings, it does for radius $R = 0.5 \text{ mm}$. However, the results for the smallest radius are corrupted. Since all experiments use the same runner, a fitting of the roughness should improve the results for all performed computation. This is not the case and underlines the assumption that a structural experimental error occurred for the measurements of radius $R = 0.5 \text{ mm}$. 

Figure 3.26.: Rough surface of the runner, measured with a white light interferometer.
Figure 3.27.: Generated force, comparison of simulations for a sphere and a rough sphere, $H = 1.7$, $h_{rms} = 0.92 \mu m$
Figure 3.28.: Generated force, comparison of simulations and experiments for a rough sphere, $H = 0.9$, $h_{rms} = 0.92\mu m$. 

(a) $R = 1$ mm

(b) $R = 0.5$ mm

(c) $R = 0.25$ mm
3.4.6. Rotational Movement of the Spheres

The spheres have been assumed to perform a translational movement in the direction of motion of the runner. This is a valid simplification for the investigated amplitude range, considered preloads and contact geometry as is established in figures 3.29 and 3.30. The numerical values for the generated force are plotted for the simulation with translational movement of the spheres and rotational movement of the spheres. Rolling contacts differ from tangential contacts in position and form of the contact area. Explicitly the slip region becomes an ellipse rather than a sphere. The generated displacement for a specific tangential force $F_x$ might thus be diverging, if rolling is considered (see Chapter 2 or 76).

However, these effects are negligible if the contact area is large compared to the rolling distance. For both shown sphere radii, $R = 1 \text{ mm}$ and $R = 0.25 \text{ mm}$ no change in the generated force can be detected. This can be explained by comparing minimal contact radius and maximal applied amplitude. For the smallest contact radius investigated, namely for a preload of $F_p = 0.1 \text{ N}$ and a sphere radius $R = 0.25 \text{ mm}$, the contact radius is $a = 3.5 \mu m$, which is two orders larger than the maximal implemented amplitude of the sawtooth amplitude for this radius, $A_{max} = 96 \text{ nm}$. For the largest considered amplitude of $A_{max} = 202 \text{ nm}$, the smallest contact area for the sphere radius $R = 1 \text{ mm}$ is $a = 5.9 \mu m$ and still of one order larger than the maximal amplitude. Consequently no effect for the rational movement of the spheres can be detected.

![Figure 3.29.](image)

Figure 3.29.: Generated force $F_{gen}$ as a function of the actuation amplitude, numerical data for $R = 1 \text{ mm}$ for hemispheres with rotational and translational movement
Figure 3.30.: Generated force $F_{\text{gen}}$ as a function of the actuation amplitude, numerical data for $R = 0.25 \text{ mm}$ for hemispheres with rotational and translational movement.
3.5. Understanding the Runner’s Dynamics – Simple Analytical Considerations

The MDR offers the possibility to describe complicated (dynamic) contacts in an effective manner. However, the method is still too complex to gain an insight on the dynamic behaviour of the runner. This is useful to gain a better understanding of the robots movements and therefore to develop possible enhancements. Consequently inquiring the simplest possible MDR model of only one spring per actuator contact will reduce the insights on the contact mechanics, but allow for a better understanding of the runner’s overall dynamics.

As depicted in figure 3.31a the runner’s contact interactions will be simplified as an interaction between springs and runner, where the springs slide if the deflection of the tangential spring multiplied by the spring stiffness becomes larger than the Coulomb force of this contact. As in chapter 3.2, it is assumed that on the upper side of the contact two equivalent springs are applied and four on the lower side. In figure 3.31a the upper spring thus symbolizes two springs and the lower one four springs. On both sides, the springs have the same applied normal forces $F_{n,1}$ and $F_{n,2}$, respectively, and the same tangential spring stiffness $k_{x,1}$ and $k_{x,2}$. Each spring then starts to slide if $\mu F_{n,i} \leq k_{x,i} u_{x,max,i}$, with $i = 1, 2$. According to the different sections of the sawtooth actuation the movement is divided in three phases.

**Phase 1** describes the stick phase during the slow motion of the sawtooth actuation (see figure 3.31b). The actuators are shifted by an amplitude $A$ from the grey position in figure 3.31a to the black position. Due to the force sensor’s reacting force $F$ the actuator is deflected into an equilibrium position where

$$ F = 2 u_e k_{x,1} + 4 u_e k_{x,2}. \quad (3.13) $$

**Phase 2** describes the fast backwards movement of the actuators and is illustrated in figure 3.32. During this phase it is assumed that the runner rests due to inertia effects of its mass. When the actuators are moved again by a distance $A$, the springs are deflected in the opposite direction, until the maximal possible deflections $u_1 := u_{x,max,1}$ and $u_2 := u_{x,max,2}$ are reached. The remaining distance of the overall amplitude, $s_{2,1}$ and $s_{2,2}$, actuator and runner slide against each other. The overall amplitude is then the summation of the two spring deflections and the sliding distances:

$$ A = u_e + s_{2,1} + u_1 $$
$$ = u_e + s_{2,2} + u_2. \quad (3.15) $$
Phase 3 is an idealization of the movement of the runner as shown in figure. After the back step a vibration of the runner around a position of rest begins. This vibration is damped and vibrates around the new slow moving displacement of the actuator. The back step of the runner has two sources. First, after the back step of the actuator, the system has the same characteristics as a simple mass spring system with
initially deflected springs. Therefore, as indicated in figure 3.33a, the runner vibrates around a static, system specific equilibrium position $u_s$, which results in an overall back step of $2u_s$.

The back step $u_s$ can be determined by computing the position of rest of the equation of motion for the vibration:

$$m \ddot{x} = -2k_{x,1} (u_1 + \dot{x}) - 4k_{x,2} (u_2 + \dot{x}) - F.$$ \hspace{1cm} (3.16)

Here $\dot{x}$ represents the coordinate in the same direction as $x$, but initialized as zero at the beginning of phase 3. The static displacement for $\ddot{x} = 0$ yields:

$$u_s = \frac{-2k_{x,1} u_1 + 4k_{x,2} u_2 + F}{2k_{x,1} + 4k_{x,2}} = \frac{2\mu F_{n,1} + 4\mu F_{n,2} + F}{2k_{x,1} + 4k_{x,2}} = \frac{\mu F_n + F}{2k_{x,1} + 4k_{x,2}}.$$ \hspace{1cm} (3.17)

Here, $F_n = 2F_{n,1} + 4F_{n,1}$ is the overall applied normal force on the runner.

Second, due to the energy brought into the system by $F$, runner and actuator slide against each other and the runner performs an additional back step $s_3$. This can be approximated by the energy theorem:

$$(2u_s + s_3)F = \mu F_n s_3 \Rightarrow s_3 = \frac{2u_s F}{\mu F_n - F}.$$ \hspace{1cm} (3.18)

The overall distance the runner moved during one period is thus

$$x = A - u_e - 2u_s - s_3.$$ \hspace{1cm} (3.19)
3.5.1. Analytic Approximation of Generated Force and 0-Amplitude

The generated force $F_{gen}$ is attained, when the net displacement of the runner during one period becomes zero:

$$A - u_e = 2u_s + s_3$$  \hspace{1cm} (3.20)

$$\Leftrightarrow A - u_e = 2u_s + \frac{2u_sF_{gen}}{\mu F_n - F_{gen}}.$$  \hspace{1cm} (3.21)

Using equation (3.13) and defining a resulting stiffness of all contacts as

$$k_{x,\text{tot}} = 2k_{x,1} + 4k_{x,2} = \frac{2\mu F_{n,1}}{u_1} + \frac{4\mu F_{n,2}}{u_2},$$  \hspace{1cm} (3.22)

yields

$$A - \frac{F_{gen}}{k_{x,\text{tot}}} = 2\frac{\mu F_n + F_{gen}}{k_{x,\text{tot}}} \frac{\mu F_n}{\mu F_n - F_{gen}}.$$  \hspace{1cm} (3.23)

$$\Rightarrow F_{gen} = \frac{1}{2} (Ak_{x,\text{tot}} + 3\mu F_n) + \sqrt{\frac{1}{4} (Ak_{x,\text{tot}} + 3\mu F_n)^2 - Ak_{x,\text{tot}}\mu F_n + 2(\mu F_n)^2}.$$  \hspace{1cm} (3.24)

To gain a better understanding of the generatable force, this equation can be further simplified, by assuming that all deflections of the springs are equally $\hat{u}$. Then $k_{x,\text{tot}} = \frac{2\mu F_n}{\hat{u}}$ and $F_{gen}$ simplifies to

$$F_{gen} = \mu F_n \left[ \frac{A}{\hat{u}} + \frac{3}{2} - \sqrt{\left( \frac{A}{\hat{u}} + \frac{3}{2} \right)^2 + 2 \left( 1 - \frac{A}{\hat{u}} \right)} \right].$$  \hspace{1cm} (3.25)

The generated force depends on the overall applied friction force on the runner, $2\mu F_n$. In fact, for large amplitudes $A$ the generated force converges against $2\mu F_n$. Additionally, if the generated force is zero, using equation (3.23) and $4F_{n,2} = 2F_{n,1} = F_n$ an approximate expression for the zero amplitude can be derived:

$$A_0 \approx 2\frac{u_1 u_2}{u_1 + u_2}.$$  \hspace{1cm} (3.26)

This result is illustrated in figure 3.34. For all investigated radii, the experimental, the numerical and the analytical 0-amplitude are depicted. They all lie in a corridor spanned by $u_1$ of the contacts with the larger normal force and $u_2$ of the contacts with the smaller normal force.
3.5. Analytical Considerations

Figure 3.34.: 0-amplitude and $u_{x,\text{max}}$ for all investigated radii
Interestingly, the analytic approximation of $A_0$ is in all cases not distinguishable from the numerical one, underlining that a model of the runner using three springs is sensible. However as in chapter 3.4 also the numerical 0-amplitude fits well for $R = 1 \text{ mm}$ and $R = 0.25 \text{ mm}$ with the experimental ones, while the values for $R = 0.5 \text{ mm}$ suggest a divergence in the experimental set-up. Consequently, these considerations underline the assumption that the 0-amplitude computes as an average value of both presliding distances, $u_1$ and $u_2$.

### 3.5.2. Damping of the Vibration by Coulomb Friction

Friction dissipates energy; hence if vibrations are present they will be damped, as can be seen in figure 3.31c. Following the argumentation in chapter 12, it will be demonstrated how the MDR can be used to derive the same expression for the dissipated energy of an oscillating contact, as Mindlin did in [1]. Thence, this theory will be applied to the damped vibration of the runner after the actuator back step is performed.

#### Dissipated Energy of an Oscillating Contact

The first aim is to derive the formula of the dissipated energy $W$ of a parabolic indenter, which is pressed with $F_n$ into an elastic half space and then oscillated by an amplitude $2A_v$ during one oscillation period. In the MDR all spring elements are independent of each other. Consequently, initially the energy dissipation of one spring at an arbitrary position $x$ in the contact, as illustrated in figure 3.35, is considered.

![Figure 3.35.: The sliding motion of a cyclically moved contact spring](image)

In this figure $u_{x,s}(x)$ denotes the deflection of the spring, similar to the three spring model of the runner actuator system, before the spring slides against the surface, such that

$$\Delta k_x u_{x,s}(x) = \mu f_z = \mu \Delta k_n u_{z,s}(x) \quad (3.27)$$

holds. Using the Cattaneo-Mindlin coefficient $C_M = E^*/G^*$, it is

$$u_{x,s}(x) = \mu C_M u_{z,s}(x). \quad (3.28)$$
3.5. Analytical Considerations

If \( A_v > u_{x,\text{max}} \) the spring in figure 3.35 slides a total distance of \( 2 \cdot 2(A_v - u_{x,\text{max}}) \) during one cycle. The work done by the friction force between spring and runner then yields

\[
\Delta W(x) = 4(A_v - u_{x,s}(x))\mu\Delta k_n u_{z,s}(x) = 4(A_v - \mu C_M u_{z,s}(x))\mu\Delta k_n u_{z,s}(x). \quad (3.29)
\]

To gain an expression for the work of the overall contact, all springs at each position \( x \) have to be considered. An integration over the entire contact area using equations (2.40), (2.47), and (3.28) yields

\[
W = 2 \int_a^c 4\mu E^* \left( A_v - \mu C_M \left( d - \frac{x^2}{R} \right) \right) \left( d - \frac{x^2}{R} \right) dx. \quad (3.30)
\]

The limits of this integral are given by \( a = \sqrt{Rd} \) and, since \( \mu k_z u_{z,s}(c) = A_v k_x \) as well as using equation (2.47),

\[
c = \sqrt{Rd - \frac{A_v}{\mu C_M}}. \quad (3.31)
\]

Computing the integral and using equation (2.25) thus derives an expression for the overall work

\[
W = \frac{8E^*}{C_M^{3/2}} \sqrt{\frac{R}{\mu}} \left( u_{x,\text{max}} (A_v - u_{x,\text{max}}) \left( u_{x,\text{max}}^{1/2} - (u_{x,\text{max}} - A_v)^{1/2} \right) \right) + \frac{1}{3} \left( 2u_{x,\text{max}} - A_v \right) \left( u_{x,\text{max}}^{3/2} - (u_{x,\text{max}} - A_v)^{3/2} \right) + \frac{1}{5} \left( u_{x,\text{max}}^{5/2} - (u_{x,\text{max}} - A_v)^{5/2} \right). \quad (3.32)
\]

If \( A_v \) is small this expression can be approximated by a Taylor series using only the first dominant element

\[
W \approx \frac{2E^*}{3C_M^{2}\mu} \sqrt{R} \frac{1}{\sqrt{d}} \frac{A_v^2}{u_{x,\text{max}}} + O \left( \left( \frac{A_v}{u_{x,\text{max}}} \right)^4 \right). \quad (3.33)
\]

\[ \Rightarrow W \approx \frac{2E^*}{3C_M^{2}\mu} \sqrt{R} \frac{1}{\sqrt{d}} A_v^2. \quad (3.34) \]

This result is exactly the same as the one derived by Mindlin in [11].

**Damping of the Vibration Amplitude of the Runner**

Considering the system again as a simple spring mass system with six contact springs, the new aim is to derive an expression for the declining amplitude with respect to time. The maximal potential energy of all six springs is

\[
U = \frac{1}{2} k_{x,\text{tot}} A_v^2 = (k_{x,1} + 2k_{x,2}) A_v^2. \quad (3.35)
\]
During one cycle the change of the potential energy of all six springs must be equal to all dissipated energy per period $T$ such that

$$\frac{W}{T} = \frac{dU}{dt} = \frac{\Delta U}{T} = -\frac{2\beta}{T}\left(\frac{1}{\sqrt{d_1}} + \frac{2}{\sqrt{d_2}}\right)A_v^2.$$  \hfill (3.36)

Using equation (3.35) and the initial condition $A_v(0) = A_{v,0}$ a differential equation for the amplitude $A_v(t)$ can be derived and solved:

$$\frac{dA_v}{dt} = -\frac{2\beta}{Tk_{x,tot}}\left(\frac{1}{\sqrt{d_1}} + \frac{2}{\sqrt{d_2}}\right)A_v^2$$

$$\Rightarrow A_v(t) = \frac{A_{v,0}}{1 + A_{v,0}\frac{2\beta}{Tk_{x,tot}}\left(\frac{1}{\sqrt{d_1}} + \frac{2}{\sqrt{d_2}}\right)t}.$$  \hfill (3.37)

In figure 3.36 the result of equation (3.38) is shown for a back step vibration of the runner with contact spheres of radius $R = 0.5$ mm, preload $F_p = 0.25$ N and actuation amplitude of 160 nm. The initial amplitude was read from the graph to be $A_{v,0} = 18$ nm. The blue line shows a nicely fitting envelope of the overall vibration of the runner, until the runner is again further dragged by the slowly moving actuators. It is interesting to note that the vibrations the runner performs is similar to beat frequency. One vibration results of the elastic contacts and is accordingly damped by the presented theory. The origin of the second vibration is not known, possibly they are caused by all three elastic contacts influencing each other.

![Figure 3.36: Back step of the runner with envelope $A_v$](image)

### 3.5.3. Signal Shaping

The damped vibrations (see again figure 3.36) at the beginning of each stick phase reduce the possible performance of the positioning device. It decreases the possible speed of the positioning axis and its accuracy. Hence, preventing these vibrations altogether
3.5. Analytical Considerations

or at least minimizing them would be a worthwhile achievement. An approach which has been followed is called signal shaping and was introduced by Bergander in [78]. The actuator signal is transformed such that the actuators are moved in a position where the vibrations are not induced into the system. This approach is easily explained and implemented using the MDR, especially if the simplification of the contacts is done as in chapter 3.5. This section’s ideas have been first presented in [69] and follow the argumentation for the enhanced method of [72].

Considering the movement of the runner in three phases, two observations are significant. First, at the end of the slow part of the sawtooth motion, the runner is in an equilibrium position, which will not induce any vibrations into the system. In addition, after the back step of the actuator, before a new phase 1 starts, the springs are deflected such that a vibration of the runner is induced and overlays a new phase 1. However, moving the actuator quickly back into the equilibrium position of phase 1, before the vibration is able to start, will minimize the vibration. Additionally, the back step of the runner \(2u_s + s_3\) will be minimized. This principle is depicted in figure 3.37a.

The effect is of great importance for the special case of a positioning device, where \(F = 0\) N. Thus \(u_s = 0\) at the end of phase 1, the springs are not deflected. To move the spring in the undeflected equilibrium position the actuators have to be moved independently of each other quickly by

\[
\begin{align*}
    u_1 &= \frac{\mu F_{n,1}}{k_{x,1}} \\
    u_2 &= \frac{\mu F_{n,1}}{k_{x,2}}
\end{align*}
\]

The altered signal of the different actuator types are depicted in figure 3.37b and 3.37c.

To experimentally implement the new signal, the actuators were remodelled, such that they are still driven synchronously, but with different driving velocities to achieve the two distinct deflections of the upper and lower actuators. Additionally, \(u_1\) and \(u_2\) have to be explicitly validated. An experimental approach was used, although, the results could be won analytically using equation (2.31). To evaluate \(u_1\) and \(u_2\) the contact stiffnesses \(k_{x,1}\) and \(k_{x,2}\) have to be known, as the contacts are too small to measure the

Figure 3.37.: New signal for the performance improvement of the drive
contact stiffness experimentally. Assuming again a overall stiffness of the runner

\[ k_{x,\text{tot}} = 2k_{x,1} + 4k_{x,2}. \] (3.39)

After Mindlin [16] the tangential contact stiffness and normal contact stiffness are linearly dependent, \( k_x \propto k_n \). Additionally after Hertz [22] it is \( k_n \propto F_n^{1/3} \) and from \( F_{n,1} = 2F_{n,2} \) it follows that \( k_{x,1} = 2^{1/3}k_{x,2} \). Using this in equation (3.39) yields:

\[ k_{x,1} = \frac{k_{x,\text{tot}}}{2 \cdot 2^{2/3} + 4} \quad \text{and} \quad k_{x,2} = \frac{k_{x,\text{tot}}}{2 \cdot 2^{1/3} + 4}. \] (3.40)

(3.41)

Consequently, the total contact stiffness of the drive has to be measured. The runner is thus assumed to be a mass spring system as in equation (3.16), again with \( F = 0 \) N. It is then actuated by a simple impulse signal resulting in a vibration of the runner in its natural frequency \( f_0 \). Naturally, this frequency is related to the runner’s total stiffness by

\[ \omega^2 = (2\pi f_0)^2 = \frac{k_{x,\text{tot}}}{m}. \] (3.42)

The response of the system was measured and analysed by a Fast Fourier transform to gain the natural frequency, which leads to the displacements

\[ u_1 = 44 \text{ nm} \quad \text{and} \quad u_2 = 28 \text{ nm}. \] (3.43)

(3.44)

With these values a new actuation signal was implemented experimentally and numerically. The results for contact spheres of \( R = 0.5 \) mm are depicted in figures 3.38a and 3.38b, respectively. Here the displacements of the runner with respect to time for the initial actuator signals are compared with the ones of the optimized waveform.

![Figure 3.38.: Comparison of the runner’s displacement with and without shaped signal for a frequency of 100 Hz](image-url)
As anticipated, while the initial signal inhibits the usual stick-slip behaviour, the improved signal shows indeed the desired reduction of the vibration as well as an elimination of the overall back step. The experiments were performed for a moderate frequency of 100 Hz. As can be seen in the graphs, the vibrations are damped during one period of the stick-slip movement due to internal damping. Consequently, a reduction of the vibration is not crucial for small frequencies as it is for high frequencies, when the vibrations cannot be damped by the overall system. The effect of the optimized signal for a frequency of 1 kHz is illustrated in figure 3.39.

![Comparison of the runner's displacement with and without the shaped signal for a frequency of 1 kHz](image)

Figure 3.39.: Comparison of the runner’s displacement with and without the shaped signal for a frequency of 1 kHz

Again for one sawtooth period, a reduction of the residual vibration and of the overall back step can be observed for the optimized signal. It remains to implement these highly promising results into a functioning mobile platform and evaluate, if the concept of signal shaping allows for the desired performance improvements in practice.

### 3.6. Conclusions

In this chapter the numerical results for the friction drive RaMoNa were compared with high precision measurements, using the drive as a force-generating device. As a new approach, a numerical model describing the dynamically changing tangential contact, using the MDR was chosen. The crucial improvements in comparison with other models using so-called single state friction laws are twofold: First, the used model agrees qualitatively and mostly quantitatively with the measurements for different contact dimension. For single state friction laws, the measurable changes due to different contact sizes cannot be directly addressed, since they assume the friction properties to be independent of the geometry factors. Contrarily, the MDR takes the contact dimension into account and provides good results for all investigated contact sphere radii. Second,
all results were gained using no empirical or fitting parameters. All used material and geometrical parameters were measurable. 

Even though, numerical and experimental results match for the small and the large radius, $R = 0.25$ mm and $R = 1$ mm, stronger discrepancies are observable for the medium radius $R = 0.5$ mm. The fact that the larger and the smaller radius’ experiments fit nicely with the numerical simulations, and furthermore numerical and experimental results for the intermittent radius are parallel shifted, implies that an inaccurate measurement might be causing the discrepancies. While the $0$-amplitude is well predicted, the force level for large amplitudes is highly overestimated. This might be due a changing friction coefficient with rising sliding velocity or an additional dissipative effect not included in the physical model.

Furthermore, intensive parameter studies have been performed. Concluding it can be summarized that macroscopic as well as microscopic parameters do not influence the overall results of the model, when applied in a sensible range. Roughness, realistic actuation, and wear are just a few of the investigated effects not influencing the overall dynamics of the drive and underlining the robustness of the method in practical use. Moreover, the simple approximation of the drive by an actuation of three contact springs provided fundamental insights on the dynamic properties of the drive. It led to an improved actuation method damping the overall vibration of the runner after the backstep of the stick-slip movement. Besides Mindlin’s theory of damped oscillated contacts was derived and applied to the experimental data.

Concluding, using the MDR to describe dynamic tangential contacts and thus describing countless macroscopic configurations without the usage of multiple empirical or fitting parameters is very promising. Not only is a fast computation possible, but also simplified models can be introduced permitting detailed insights on the system dynamics.
4. Active Control of Friction and Friction Reduction

Friction might be reduced by oscillating one of the frictional partners. This chapter focuses on vibrations in a sinusoidal form in three different directions, two in the sliding plane and one normal to the sliding plane. To give an overview on the effect of friction reduction due to oscillations, first a summary of the effects reported in literature will be given. Then a pin-on-disc style test stand will be introduced, allowing measurements with oscillations in different directions. Each of the investigated directions will subsequently be discussed separately starting with a physical model of the system. The results of the analytical and numerical descriptions will then be compared to the performed measurements.

4.1. Introduction to Friction Reduction by Ultrasonic Oscillations

While the previous chapter covered the control of friction by (asymmetric) vibration patterns, the reduction of friction is achieved by non directed chaotic or harmonic oscillations. It is long known that high frequency oscillations reduce friction and have positive effects on technical processes such as wire drawing and cutting. In their 1966 studies on metal forming processes, Pohlman and Lehfeld [79] investigated the influence of ultrasonic vibrations in a friction couple in different directions. They found an approximate decrease in the friction force of factor three due to harmonic oscillations. Another early study was done by Mitskevitch [80] in 1968. He discussed the effects of harmonic oscillations on friction between a metal specimen and a vibrating plate, using Newtonian mechanics and Coulomb’s law. He concluded that using ultrasonic oscillations in the metal cutting process enhances the cutting stability and purifies the surface finish. In the case of metal drawing the drawing rate may be increased two- or threefold. Early work on the effect of vibration on metal forming processes has also been conducted by Eaves et al. [81]. Since then, much research and practical progress has been made and can be reviewed in [82, 83, 84].

While in this work the effect of oscillations on friction will be studied on the meso- and macroscopic scale the effect of friction reduction is also well known in nanotribology. Gao et al. [85] proposed a numerical model for a sheared, lubricated friction couple under small vibrations. A transition from a stick-slip friction regime to an ultra low kinetic friction regime is achieved. Other works include a theoretical study by Tshiprut et al. [86] in which a substrate is vibrated laterally causing friction reduction. The effect of friction reduction in an atomic force microscope has been investigated in [87] and [88]. While the work by Jeon and co-workers compares experimental and numerical results
for lubricated friction under vibration, the latter work by Socoliuc et al. investigates experimental findings on the influence of vibrations normal to the sliding plane. They conclude that friction is reduced or even vanishes with increasing vibration amplitudes. Capozza [89] performed numerical studies on the nanoscale concerning sliding friction of a specimen moved on a vibrating plate. In this study, it was shown how the vibration helps to free the interfaces from each other by inducing energy to free the surfaces out of the energy minimum.

For macroscopic and mesoscopic vibrations multiple studies of experimental or theoretical character have been done. On the one hand it is often differentiated between static and sliding friction, while on the other hand different vibration directions have to be distinguished. Figure 4.1 shows the three different vibration directions. If \( v \) is the sliding direction of the probe, three oscillation directions are defined: in plane perpendicular to the sliding direction (IPP); in plane in sliding direction (IPI); out of plane (OOP).

![Figure 4.1.: Possible vibration directions of an oscillating probe](image)

The effect of friction reduction due to in plane vibrations has often been assumed to lie in the change of relative velocity, due to an overlay of sliding velocity and vibration velocity and therefore a sing change of the friction force. Another approach assumes that a reduction of the presliding distance results in a reduction of energy needed to engage the system into a sliding motion, regardless if the vibration is IPI or IPP [90]. This assumption allows an immediate approximation concerning the relevant amplitudes needed for a specific system in the stick-slip regime to allow friction reduction by oscillations. Recalling that for a rotational symmetric or self affine rough tangential contact the presliding distance is

\[
u_{x,max} = \mu C_M d
\]

(4.1)

with given material properties, normal force, and friction coefficient, the magnitude of the amplitude can thus be easily estimated. For the OOP vibrations, the direct influence on the vibration is immanent, since the normal force proportional to the friction force is changing. In case of static friction the decisive normal force is not the mean value of the normal force, but its minimal value. In sliding friction the movable object achieves a stick-slip motion for normal forces smaller than the mean value, resulting in an overall friction reduction.
4.1. Introduction to Friction Reduction by Ultrasonic Oscillations

The effect of vibration on friction is used manifold: to reduce wear, as a dry friction lubricant and changing the general velocity dependence of the friction coefficient. Energy savings however are only possible for OOP vibrations. Systematic studies until today are numerous, an overview of the findings until today including the studies highly influential for this work is given below.

**Static Friction** The earliest work on reduction of static friction due to vibration dates back to 1959. The investigations by Fridman and Levesque [91] were performed for various frequencies (6-42 Hz) for OOP vibrations of a vibration plate on which a block was pulled. They found a decrease in friction force by up to 100% with increasing amplitude. This unrealistic decrease of friction force was explained by assuming the block to be floating due to the vibrations. Lenkiewicz [92] studied the effects of vibrations on static friction, but he investigated IPI vibrations. In an experimental work with a rotating drum and a block slider in a frequency range of 5-120 Hz and for various amplitudes he found a decrease of up to 80% of the friction coefficient. Together with Broniec [93] he determined the decrease of friction for OOP vibrations of up to 60%. One of the first theoretical models to describe the effect of vibrations on friction results from work by Hess and Soom [94, 95], who implemented a numerical and experimental study on the influence of harmonic and chaotic OOP oscillations on the static coefficient of friction. They used a model based on the assumption that the frictional force is proportional to the real contact area and found a decrease in friction of 10% for harmonic and 9% for stochastic oscillations. They concluded that no effect could be simulated if the simple Coulomb’s law was assumed. Recently Cheng [96] found a 23-50% decrease of friction through IPI vibrations of two plates in contact.

A systematic study on the static coefficient of friction for IPI vibrations was performed by Starcevic and Popov [97, 98] and in the PhD thesis by Starcevic [99]. On a linear tribometer they investigated the influence of ultrasonic oscillations for various amplitudes and different material pairings. The only materials tested which did not show an effect of friction reduction with rising vibration amplitude were rubber and aluminium. They even measured a negative friction coefficient, which was explained by the ratchet effect. In addition to the experimental work, a theoretical model was proposed [100] based on the Prandtl-Tomlinson model [101] supporting the experimental findings.

**Sliding Friction** Due to their multiple technical applications the studies concerning the influence of vibration on sliding friction have been more numerous. One of the ground breaking works has been written by Godfrey [102] in 1967. In an experimental set-up similar to the one used by Fridman and Levesque, he found a total decrease of sliding friction due to OOP vibrations. However, he explained the complete loss of vibration by jumping motions of the specimen. Regardless, a loss of contact is necessary, to achieve a complete loss of friction. Also in 1967 a work was published by Tolstoi [103] analysing the self-induced OOP vibrations of a sliding body. It is his hypothesis that the nature of a velocity dependent friction coefficient is highly influenced by self-induced vibrations. He states that if these vibrations are damped, the velocity dependence vanishes. He underlined his ideas by performing experiments in which he found a reduction of the static friction coefficient by 35-80%. The results he found were theoretically and
numerically studied by Tworzydlo and Becker [104]. Weishaupt [105] observed friction reduction due to OOP vibrations in his early experimental work. He implied that the results could be used to control friction and reduce wear. Another publication focusing on the positive effects of wear due to ultrasonic vibrations was conducted by Goto et al. [106]. Skare and Stahr [107] performed experiments to investigate static and dynamic friction influences by IPI vibrations on a linear tribometer. They also discovered a reduction of sliding friction, which gets more prominent with increasing vibration amplitude and frequency. Self-induced vibrations and its related effect of external excitation on friction have been theoretically studied by Thomson [108]. He uses Newtonian mechanics combined with Coulomb’s law to describe an IPI exited friction couple. He shows that high frequency oscillations may cancel out the negative slope of the friction coefficient with respect to sliding velocity. A couple of studies, which were highly influential for this work, have been performed by the work-group of Wallaschek [109, 110]. On a linear tribometer they tested the influence of ultrasonic IPI and IPP oscillations on sliding friction. They developed an analytical model based on Coulomb’s law which highly supports their experimental findings. More recently they investigated possible energy saving effects in soil-work [111]. As the researchers around Wallaschek, additionally Kumar and Hutchings [112] investigated IPI and IPP vibrations. Supporting their work with a theoretical model, they found a larger effect of friction reduction by IPI vibrations than by IPP vibrations. Tsai and Tseng [113] extended the model introduced by Wallaschek et al., such that any superposition of IPP and IPI vibration influences can be theoretically studied. They further improved the model by using Dahl’s friction model rather than Coulomb’s law. Leus and Gutowski [114, 115] followed the same approach. They performed experiments and described their model physically using Dahl’s model, but also by the Elasto-Plasic model. Both models have been compared to results using Coulomb’s law. Their results were drastically enhanced using Dahl’s model, but no significant improvements could be found by using the Elasto-Plasic model. Finally, very recently, Dong et al. published an experimental and theoretical work [116]. The influence of the superposition of OOP and IPI vibrations on friction have been measured in a macroscopic test stand. The applied friction model combines an asperity contact model with a tangential contact model based on the bending stiffness of a beam. Quite numerous parameters have been used for their theoretical model.

In [98] by Popov et al. the effect of ultrasonic vibration on sliding friction has been studied for vibrations in IPI direction. The pin-on-disc apparatus used to perform the experimental investigations is the same as the one used for the data measured for this work. To draw a complete picture of the influence of friction gained on the same test stand, parts of this publication are merged with new measurements and new theoretical investigations. In particular the measurements for large velocities as well as model 2 for the IPI oscillation direction have been published in this previous work.

4.2. Experimental Set-up

To measure the dynamic friction coefficient for the introduced vibration directions, IPI, IPP, and OOP, a pin-on-disc tribometer was used. The apparatus is schematically
depicted in figure 4.2, whereas a picture of the test stand is shown in figure 4.3. The device’s main features are a rotating disc, driven by a step motor and a guiding device with integrated force sensor pressuring a probe onto the rotating disc. In the figures a probe vibrating in OOP direction is mounted.

Disc and motor are one controllable unit. The step motor ensures a smooth rotational movement of the interchangeable discs. Thus, various materials could be tested. Furthermore a reduction gear box can be mounted, such that very slow velocities of 1 mm s\(^{-1}\) can be smoothly initiated.
The second component of the apparatus is the mounting device for the probes. It consists of a linear table on which a guiding device is placed. Fixed to one side of this device is a force sensor and the vibrating probe. On the other side, using a lever arm, a normal force $F_n$ can be applied to the probe.

![Figure 4.4: Oscillating probes, built of steel parts and piezo elements](image)

The probes themselves are assembled from steel and piezo elements as depicted in figure 4.4. To accomplish a harmonic excitation of the probe, the ultrasound generator measures the natural frequency $f_0$ of the probe, such that the applied voltage to the piezo elements is a sinusoidal function of frequency $f_0$ with adjustable amplitude. The ultrasonic movement of the probe can be measured with a Polytec laservibrometer of 8nm accuracy. The force sensor between guiding device and probe measures the orthogonal forces in sliding direction, $F_r$, the normal force $F_n$ and the (irrelevant) radial force $F_{rad}$.

![Figure 4.5: Rotating friction disc with probes oscillating in IPI, IPP, and OOP direction](image)

Figure 4.5 illustrates the experimental set-up for the different possible actuation vibrations. The probe used for IPI and IPP vibrations (see figures 4.4a, 4.5a and 4.5b) is identical and can be mounted in both perpendicular in plane directions. It has two contact points between which the piezo elements induce the vibration, changing the overall length of the probe $l(t)$. The parts in contact with the disc are fabricated of
4.3. Out Of Plane Oscillations (OOP)

This kind of asymmetric excitation by a harmonic, symmetric normal force and a constant tangential force results in a motion, as well as friction reduction. The overall concepts and a first theory are introduced by combining a macroscopic model with a simple macroscopic Coulomb’s law. A more sophisticated theory is derived, in analogy to chapter 3.3, by combining the macroscopic model with the MDR. Both theories are then compared to the performed experiments.

4.3.1. Physical Modelling – Macroscopic Model

As in the case of the friction drives, to physically describe the pin-on-disc set-up a macroscopic model is paired with a microscopic one describing the friction interaction between the rotating disc and the oscillating probe. In case of the oscillation normal to the sliding plane, a macroscopic model can be derived using Newtonian mechanics paired with a model as depicted in figure 4.6. In contrast to the tribometer, not the disc is moving with constant velocity, but the probe is dragged with a force $F$ over an arrested surface. While the real probe is screwed to a vibrating probe, its model is assumed to be a freely moving mass $m$, excited by a normal force $F_n(t) = F_{n,0} + \Delta F_n \cos \omega t$. Here, $F_{n,0}$ is the average normal force measured by the force sensor, while $\Delta F_n \cos \omega t$ is the additional oscillation induced by the piezo elements induced vibration. The relation between the oscillating length of the probe and the amplitude of the oscillating part of the normal force can be approximated using the contact stiffness of the sphere disc interaction: $\Delta F_n \cos \omega t = 2E^*a\Delta l \cos \omega t$. Here, $a$ is the contact radius of the Hertz contact with a normal load $F_{n,0}$. The equation of motion of this macroscopic system is:

$$m\ddot{x} = F - F_r.$$ (4.2)

The friction force $F_r$ will be computed using a single state friction law or a numeric approach.
Figure 4.6.: Macroscopic model of the pin-on-disc tribometer with OOP oscillations

**Mass of the Probe**  
The part of the probe in contact with the disc is a sphere glued onto a screw, which itself is screwed onto the body holding the piezo elements. It might be assumed that the mass \( m \) is governed only by the real mass of the screw. However, the mass \( m \) of the probe is not independent with respect to the rest of the structure. An easy approximation of an equivalent mass can be computed with the method of Rayleigh-Ritz (see [119], chapter 7). For the first eigenfrequency the specimen is modelled as a beam, which has a clamped support on one side and a free end on the other side. The deflection curve of the 'beam' is denoted as \( w(x, t) \). Choosing a polynomial of third degree as ansatz function it is:

\[
 w(x, t) = \left( \hat{a}x^2 + \hat{b}x^3 \right) \varphi(t), \quad (4.3)
\]

which fulfils the first boundary condition \( w(0, t) = 0 \). The second boundary condition \( w''(l, t) = 0 \) is fulfilled if \( \hat{b} = -\frac{\hat{a}}{3} \). Then the ansatz function yields

\[
 w(x, t) = \left( x^2 - \frac{1}{3}x^3 \right) \hat{a}\varphi(t). \quad (4.4)
\]

Since the displacement of the rod has to be derived with respect to the surface, the ansatz function has to be presented as function of the displacement \( w(l, t) \). Thus, the remaining constant yields \( \hat{a} = \frac{3}{2} \frac{w(l, t)}{\varphi(t)} \) and the ansatz function reads:

\[
 w(x, t) = \left( \frac{3}{2} \left( \frac{x}{l} \right)^2 - \frac{1}{2} \left( \frac{x}{l} \right)^3 \right) w(l, t). \quad (4.5)
\]

The kinetic energy of the beam is defined as

\[
 K = \frac{1}{2} \rho \int_0^l A(x) \dot{w}(x, t)^2 \, dx. \quad (4.6)
\]

Here, \( \rho \) is the material density and \( A(x) \) is the cross section of the beam. Assuming a constant cross section over the length \( l \) of the beam, the kinetic energy derives to:

\[
 K = \frac{1}{2} \rho A \left( \frac{33}{144} \right) m_e w^2(l, t). \quad (4.7)
\]
Recalling that the Rayleigh-Ritz procedure finds an equivalent mass spring system for the first eigenform of the beam, the equivalent mass \( m_e \), with respect to the deflection at the end of the beam can be directly read from above equation. For a steel rod of \( l = 1.6 \) cm and a diameter of \( d = 6 \) mm, one centimetre length and \( \rho_{\text{steel}} = 7860 \) kg m\(^{-3}\), \( m_e = 1 \) g, which is roughly the same mass as the one of the screw and the steel ball itself, namely 3 g.

However, since the real specimen is a steel screw the cross section is not continuous. Assuming two distinct cross sections with the length \( l_1 = 10 \) mm and \( l_2 = 6 \) mm, with the diameters \( r_1 = 3 \) mm and \( r_2 = 5 \) mm respectively, the kinetic energy reads

\[
K = \frac{1}{2} \rho \pi r_1^2 \int_0^{l_1} \dot{w}(x,t)^2 \, dx + \frac{1}{2} \rho \pi r_2^2 \int_{l_1}^{l} \dot{w}(x,t)^2 \, dx
\]

\[
K = \frac{1}{2} \dot{w}^2(l,t) \rho \pi \left[ (r_1^2 - r_2^2) \left( \frac{9l_1^5}{20l^4} - \frac{l_1^6}{4l^5} + \frac{l_1^7}{28l^6} \right) + r_2^2 \frac{33}{144} \right].
\]

Here, \( l = l_1 + l_2 \) and the equivalent mass computes to \( m_e = 2.1 \) g. This mass will subsequently be used for the numerical modelling.

### 4.3.2. Physical Modelling – Coulomb’s Friction Law

The overall movement of the probe, and the effect of friction reduction for this asymmetric excitation, are already well explainable, if Coulomb’s friction law is applied. This has been done in [117] and will be recapitulated here. The friction force in equation (4.2) can be expressed as \( F_r = \mu_0(F_n,0 + \Delta F_n \cos \omega t) \). Here \( \mu_0 \) denotes the friction coefficient between probe and surface without oscillation. The friction reduction is a result of a stick-slip motion when the dragging force \( F \) is smaller than the average friction force \( \mu_0 F_{n,0} \). The pattern of motion is exhibited in Figure 4.7. The figure shows the oscillating friction force and the dragging force \( F \). The depicted constant force \( F \) is equal to the friction force at \( t = t_1 \). At this point the mass \( m \) starts to slide until inertia and friction force become again smaller than the friction force \( F_r \) at \( t = t_2 \). This results in a stick-slip motion. The green force interval, where \( \mu_0 (F_{n,0} - \Delta F_n) < F < \mu_0 F_{n,0} \) marks all dragging forces, resulting in a stick-slip motion. In the blue interval only slip occurs. Naturally, the limiting case is a dragging force \( F \) which is equal to the average friction force \( \mu_0 F_{n,0} \). This intuitive interpretation can be mathematically derived by solving the
equation of motion \( \ddot{x} = \ddot{f} - (1 + \cos \tilde{t}) \). For a more general consideration of the influencing parameter, i.e. gaining a characteristic for the mean friction coefficient \( \langle \mu \rangle = F(\langle v \rangle)/F_{n,0} \) with respect to the mean sliding velocity \( \langle v \rangle \), the equation of motion is brought into a dimensionless form:

\[
\ddot{x}' = \ddot{f} - (1 + \cos \tilde{t}) .
\] (4.8)

Here, \( \tilde{x} = \frac{m\omega^2}{\mu_0\Delta F_n} x, \tilde{f} = \frac{F}{\mu_0\Delta F_n} - \frac{F_{n,0}}{\Delta F_n} + 1 \) and \( \tilde{t} = \omega t \). Additionally \( \tilde{x}' \) denotes the derivation with respect to \( \tilde{t} \).

The aim is to find a semi-analytical expression for the mean sliding velocity averaged over one oscillation period:

\[
\langle v \rangle = \frac{\mu_0\Delta F_n}{m\omega} \langle \tilde{x}' \rangle = \frac{\mu_0\Delta F_n}{m\omega} \frac{\tilde{x}(\tilde{t}_2) - \tilde{x}(\tilde{t}_1)}{2\pi} .
\] (4.9)

Consequently the start and end time of the slip movement \( \tilde{t}_1 \) and \( \tilde{t}_2 \) have to be computed. In the dimensionless form, the mass starts to slide once the pulling force is larger than the friction force, or differently addressed, if the acceleration is still zero:

\[
\tilde{f} = 1 + \cos \tilde{t}_1 \quad \tilde{t}_1 = \arccos(\tilde{f} - 1) .
\] (4.10)

Dimensionless velocity and displacement are evaluated by integration of equation (4.8) with the initial conditions \( \tilde{x}(0) = 0 \) and \( \tilde{x}'(0) = 0 \):

\[
\tilde{x}'(\tilde{t}) = (\tilde{f} - 1)(\tilde{t} - \tilde{t}_1) + \sin \tilde{t}_1 - \sin \tilde{t} \quad \text{and} \quad \tilde{x}(\tilde{t}) = \frac{(\tilde{f} - 1)}{2}(\tilde{t} - \tilde{t}_1)^2 + (\tilde{t} - \tilde{t}_1)\sin \tilde{t}_1 + \cos \tilde{t} - \cos \tilde{t}_1 .
\] (4.12)

The terminating time of the slip motion \( \tilde{t}_2 \) is reached when the velocity of the probe reaches zero again (\( \tilde{x}'(\tilde{t}_2) = 0 \)), resulting in a transcendent equation

\[
\tilde{f} - 1 = \frac{\sin \tilde{t}_2 - \sin \tilde{t}_1}{\tilde{t}_2 - \tilde{t}_1} ,
\] (4.14)

which can only be solved graphically or numerically. The average dimensionless velocity is \( \langle \tilde{x}' \rangle = \tilde{x}(\tilde{t}_2)/2\pi \). Solving for \( \tilde{f} \) thus delivers the desired correlation \( \langle \mu \rangle (\langle v \rangle) \), if re-transformed in the dimensional form. Yet, a consideration of the limiting cases of this equation reveals much about the characteristic reduction in friction.

\( \tilde{f} = 0 \) is equivalent to the case when the dragging force \( F \) equals the static friction force \( F_r = \mu_0(F_{n,0} - \Delta F_n) \) for which \( \tilde{t}_1 = \tilde{t}_2 \) and a slip movement is not yet achieved. Consequently \( \langle \tilde{x}' \rangle = 0 \) (see again figure 4.7). In the scope of this work \( \Delta F_n \) is always assumed to be smaller than \( F_{n,0} \). This assures that no loss of contact takes place.
\( \tilde{f} = 1 \) represents the transition from stick-slip to a pure slip motion when \( F = \mu_0 F_{n,0} \). In this case \( t_2 = t_1 + 2\pi \), the duration of the motion is one vibration period, after which the movement stops and starts simultaneously. The mean velocity in this case is \( \langle \dot{x}' \rangle = 1 \).

The numerical solution of equation (4.8) is exemplarily presented in figure 4.8 for an average normal force \( F_{n,0} = 9.5 \) N, a friction coefficient \( \mu_0 = 0.4 \), a mass \( m_e = 2.1 \) g, and a natural frequency of 30 kHz.

The two characteristic cases for \( \tilde{f} = 0 \) and \( \tilde{f} = 1 \) are clearly distinguishable. For \( \tilde{f} = 0 \) the curves intersect with the \( y \)-axis with \( \langle \mu \rangle \) = 0 and

\[
\langle \mu \rangle = \frac{F}{\mu_0 F_{n,0}} = 1 - \frac{\Delta F_n}{F_{n,0}}.
\]

(4.15)

In addition, for \( \langle \mu \rangle = \mu_0 \Delta F_n/m_0 \omega \), the curves converge against the unreduced friction coefficient and it is \( \langle \mu \rangle/\mu_0 = 1 \).

Figure 4.8.: Normalized friction reduction – OOP model 1; solid lines are the numerical solution of equation (4.8), whereas the hardly distinguishable dashed lines are the results of the explicit approximate solution of equation (4.22).

As has been shown in [120], chapter 18, an approximation for \( \tilde{f}(\langle x' \rangle) \) and accordingly \( \langle \mu \rangle(\langle v \rangle) \) can be found. Considering first small dragging forces \( \tilde{f} \), the cosine of the differential equation (4.8) can be derived into a Taylor series around \( \tilde{t} = \pi \) as

\[
\cos \tilde{t} = -1 + \left( \tilde{t} - \pi \right)^2 / 2 + O(\tilde{t}^4).
\]

The equation of motion transforms into:

\[
\ddot{x}'' = \tilde{f} - \frac{\tilde{t}_{*}^2}{2},
\]

(4.16)

with \( \tilde{t}_{*} = \tilde{t} - \pi \). The beginning time of the slip phase at \( \ddot{x}''(\tilde{t}_{*1}) = 0 \) is \( \tilde{t}_{*1} = -\sqrt{2\tilde{f}} \) and
the velocity and displacement yield with the previous initial conditions

\[ \tilde{x}' = \tilde{f} (\tilde{t}^* - \tilde{t}_1^*) - \frac{\tilde{t}^3 - \tilde{t}_1^3}{6} \]  
\[ \tilde{x} = \tilde{f} \left( \frac{\tilde{t}^2}{2} - \tilde{t}_1^* \tilde{t}^* \right) - \frac{\tilde{t}^4 - 4\tilde{t}_1^{*3} \tilde{t}^*}{24} . \]  

The mass ceases to slide when \( \tilde{x}(\tilde{t}_2^*) = 0 \) at \( \tilde{t}_2^* = -2\tilde{t}_1^* \). The mean velocity is

\[ \langle \tilde{x}' \rangle = \frac{\tilde{x}(\tilde{t}_2^*) - \tilde{x}(\tilde{t}_1^*)}{2\pi} = \frac{9\tilde{f}}{4\pi} \]  
\[ \tilde{f} = \sqrt{\frac{4\pi}{9} \langle \tilde{x}' \rangle} . \]  

Since this last result is only an approximation for small forces \( \tilde{f} \), a more general solution was found using a correcting summand found by an evolutionary fitting algorithm:

\[ \tilde{f} = \sqrt{\frac{4\pi}{9} \langle \tilde{x}' \rangle} + \left( 1 - \sqrt{\frac{4\pi}{9} \langle \tilde{x}' \rangle} \right)^{6/5} . \]  

This explicit form \( \tilde{f} \) can be written in the original variables as

\[ \frac{\langle \mu \rangle}{\mu_0} = 1 - \frac{\Delta F_n}{F_{n,0}} + \frac{\Delta F_n}{F_{n,0}} \left[ \sqrt{\frac{4\pi}{9} \frac{m\omega}{\mu_0 \Delta F_n} \langle v \rangle} + \left( 1 - \sqrt{\frac{4\pi}{9} \langle \tilde{v} \rangle} \right) \left( \frac{m\omega}{\mu_0 \Delta F_n} \langle v \rangle \right)^{6/5} \right] . \]  

It is also plotted in figure 4.8 as dashed lines, hardly distinguishable from the numerical solution of the equation of motion (solid lines).

4.3.3. Physical Modelling – Method of Dimensionality Reduction

As for the stick-slip drives, the single state laws, the Coulomb’s law of the previous model is not taking any contact geometries or elasticity of the materials into account. The probe vibrating in OOP direction, as depicted in figure 4.6, is a sphere, or in case of wear, possibly a flattened sphere. Consequently, it is a rotational symmetric body and the friction force of the equation of motion (4.2) can be computed using the MDR, analogously as described for stick-slip drives in chapter 3.3. In this case, one has to implement a vibrating normal force \( F_n(t) = F_{n,0} + \Delta F_n \cos \omega t \) and consequently a changing contact area with indentation depth \( d \), vertical displacement \( u_z \) and horizontal displacement \( u_x \).

An algorithm used to compute the solution of the equation of motion using the MDR is shown in figure 4.9. First, an equivalent 1D profile \( g(x) \) needs to be found. This can be done for different contact geometries: sphere, flattened sphere, and rough sphere, as described in chapter 3.3. Using \( g(x) \) and a known normal force \( F_n \), the vertical displacement \( u_{z,i} \) can be found using equation (3.10). This contact line is discretized using equidistant springs with a distance \( \Delta x \) with respect to each other. The vertical and
4.3. Out Of Plane Oscillations (OOP)

**In:** problem parameter, \( N, m, F_{n,0}, \Delta F_n, F, R, E^*, \nu \)

**Out:** \( \langle v \rangle \)

1. set \( t = t + \Delta t \)
2. compute \( F_n(t) = F_{n,0} + \Delta F_n \cos \omega t \)
3. compute \( u_{z,i} \)
4. generate displacements \( u_{x,i} = u_{x,i} + dx \)
5. compute each spring force
   \[
   f_{n,i} = \Delta k_z u_{z,i} \\
   f_{x,i} = \Delta k_x u_{x,i}
   \]
6. if \( f_{x,i} > \mu f_{n,i} \)
   - yes
     - set \( f_{x,i} = \mu f_{n,i} \) and \( u_{x,i} = \mu \frac{E^*}{G} u_{z,i} \)
   - no
7. compute friction force \( F_r = \sum_i f_{x,i} \)
8. solve \( m \ddot{x} = F - F_r \)
9. compute \( x, v \) and \( dx \)
10. if \( t > \frac{2\pi}{\omega} \)
    - no
    - yes
    - compute \( \langle v \rangle \)
11. return

Figure 4.9.: Algorithm to compute the friction force \( F_r \) using the MDR to solve the equation of motion (4.2) for oscillations in the OOP direction
horizontal stiffness of the springs have to be set according to equations (2.40) and (2.41), respectively, such that the spring forces can be computed. The horizontal displacement of the springs \( u_{x,i} \) changes in each time step by the incremental displacement of the mass \( d_{x} \). In every time step, each spring has to be evaluated, if it is in stick or in slip state, namely if Coulomb’s law is locally valid and \( f_{x,i} \leq \mu_{0}f_{z,i} \). If the spring is not sticking a new displacement \( u_{x,i} \) is computed. To finally solve the equation of motion using the Verlet algorithm, \( F_{r} \) is computed as a summation of all horizontal spring forces.

The algorithm has been implemented in MATLAB for the parameter presented in table 4.1.

The results for the normalized friction coefficient with respect to sliding velocity for a sphere in contact with the rotating disc is presented for different ratios of \( \Delta F_{n} / F_{n,0} \) in figure 4.10. As for the model using Coulomb’s law to compute the friction force, the static friction is reduced by the oscillation according to equation (4.15). In contrast to this simple theory however, the maximal sliding velocity for which a reduction of friction is still observable, is much higher than for the simpler model (compare figure 4.8).

![Figure 4.10.: Normalized friction reduction – OOP model using the MDR for a sphere](image)

**4.3.4. Comparison of Theory and Experiments**

**Experimental Results**

The measurements for the vibration in OOP direction have been performed with a steel sphere on a steel disc. The set-up’s properties are summarized in table 4.1. To proof the reproducibility of the experimental data, all measurements have been performed four times with interchanging sequences of velocities or amplitudes. The measurements were performed for a test stand with and without reduction gear box, such that small and large sliding velocities can be examined. The modification of the test stand resulted in overall different values of the friction coefficient, even though the
same materials had been used. For instance, the mean friction coefficient for small velocities without excitation is $\mu_{0,\text{slow}} = 0.44$, while it is $\mu_{0,\text{fast}} = 0.33$ for the measurements without reduction gear box. A reason for this behaviour might be an overall smoother characteristic of the test stand due to structural damping induced by the additional reduction gear box. Additionally, the probe had to be rotated between experiments, resulting in a changed orientation of the tip with respect to sliding velocity. To sustain a comparability between the measurements, the friction coefficients are normalized with respect to $\mu_{0,\text{slow}}$ and $\mu_{0,\text{fast}}$. The plots for the absolute values for all single measurements as well as a plot indicating the error range of the measurements can be found in appendix C. The mean normalized values are shown in figure 4.11. Circles represent mean values for measurements with reduction gear box, while asterisks illustrate measurements without reduction gear box.

Figure 4.11a shows the measured amplitudes with respect to the sliding velocity. On the one hand the graph indicates a steady amplitude with respect to the sliding velocity. Only a very slight increase with increasing velocity is visible, which will presumably not influence the measurements of the friction force. On the other hand, it was possible to match the amplitudes measured with and without reduction gear box. In figure 4.11b, the mean values for the normalized friction coefficient are presented. The measurements with and without reduction gear box coincide well except for the smallest amplitudes. For those amplitudes of the experiments with reduction gear box a reproducibility was not established (see appendix C), which is assumingly the reason for this discrepancy. The dark blue curve shows the frictional behaviour without excitation. It exhibits a decreasing friction coefficient with increasing sliding velocity. The friction coefficient converges against a steady value. It is important to note that the steady value is smaller than the friction coefficient against which the curves with excitation converge. The reason for this lies in a faulty measurement, which was not excluded from the entire data sets. In figure C.3 the average friction coefficient without excitation is small, compared to the values measured with excitation. It also does not exhibit a decreasing slope. This is due to the fact that the cleaning apparatus to free the disc of wear particles was defect during this single measurement. Nevertheless, the faulty measurement was kept to incorporate all influences of the different velocity and amplitude sequences of all four data sets.

Regardless, the average values reproduce the expected characteristics. Without exci-
tation the friction decreases with increasing sliding velocity, while with excitation the slope is reversed and the friction is increasing with increasing \( \langle v \rangle \). In addition the friction is reduced. This effect is more prominent with increasing oscillation amplitudes. For all investigated amplitudes, a finite value of static friction can be assumed for vanishing sliding velocity. For the investigated velocity range, the friction coefficient converges against the finite value \( \mu_0 \) for the smaller amplitudes while this effect cannot be observed for the larger amplitudes.

Figure 4.11.: OOP experiments; stars represent average values of four measurements without an additional reduction gear box, circles illustrate average values of four measurements with additional reduction gear box
Coulomb’s Friction Law

Figure 4.12 compares the measurements with the solution of the equation of motion using Coulomb’s law to compute the friction force. To compare experimental data with theoretical values a relation between $\Delta l$ and $\Delta F_n$ has to be found. Since the amplitude of an oscillation of an initially indented sphere are measured, it is assumed that the recorded $\Delta l$ is the solely influencing parameter of the interchanging force $\Delta F_n$:

$$\Delta F_n = \lambda \cdot 2E^*a(F_n,0)\Delta l.$$  \hspace{1cm} (4.23)\n
Note that the contact radius used to compute the contact stiffness is assumed to be the one for the mean normal force $F_{n,0}$. Additionally, a fitting parameter $\lambda$ has been introduced. The contact stiffness of the probe $k_n = 2E^*a$ can be influenced easily by a changing test set-up. For instance, the sphere wears down and is flattened out. Besides, the pressure on the piezo elements has to be adjusted resulting in a rotation of the contacting sphere. Consequently a fitting of the contact stiffness within a remaining order of magnitude appears to be sensible.

Moreover, assuming the largest measured amplitude coincides with the case where no loss of contact appears, then for the largest measured amplitude

$$\frac{\Delta F_n}{F_{n,0}} = 1.$$  \hspace{1cm} (4.24)\n
Thus $\lambda$ can be computed using equation (4.23) and the relation between $F_n$ and $a$ of equation (2.17) to be $\lambda = 2.64$. As a mean friction coefficient the average value of $\mu_0,\text{slow}$ and $\mu_0,\text{fast}$, $\mu_0 = 0.4$ is used for the computations.

The results for the fitted data are shown in figure 4.12b. Compared to the experimental data in figure 4.12a the analogy is only of qualitative nature. It is well observable that the measured data and the theoretical ones converge against a finite static friction force for vanishing sliding velocity. Up until a certain velocity, for all oscillation amplitudes, a reduction in friction as well as a slope, increasing with increasing sliding velocity is visible. However, the critical velocity, for which no friction reduction is achieved is approximately three times smaller in theory than the ones measured. Concluding, the results underline the assumption that the simple Coulomb law is not suitable to describe the complicated contact mechanics of the stick-slip motion. Thus, comparing the results computing the friction force using the MDR with the experimental data is consequential.
Figure 4.12.: OOP experiments compared with results computing the friction force using Coulomb’s law

MDR

The same assumption about fitting the contact stiffness as in equation (4.23) has been made to compare the data gained by computing $F_r$ via the MDR with the experimental data. The result is shown in figure 4.13. In this graph the solid lines illustrate the theoretical data while the lines with marker show the experimental data. Both data sets cover the same velocity range. The values for the static friction force as well as the curve slopes are nicely predicted for the smallest and largest force ratio of $\Delta F_n/F_{n,0} = 0.25$.
and $\Delta F_n/F_{n,0} = 1$. The intermittent values can be interpreted as a very nice qualitative approximation.

Regardless and not very surprisingly, the MDR is capable of predicting the real contact mechanical behaviour of the stick-slip motion more accurately than the simple Coulomb law. It is however astonishing how well the MDR performs even for a simple contact form like a sphere. Not only is the macroscopic model a rough approximation but the contact mechanics of this case study are on a macroscopic scale compared to the stick-slip drives of chapter 3. It can thus be assumed that further influences on the macroscopic system as well as mechanism of the contact mechanics not yet considered, as e.g. wear, contact form and surface roughness will have a significant influence on the system’s behaviour. It is thus very pleasing how even the simplest model of macroscopic model in interplay with the MDR already captures the overall behaviour of the real measured data.

4.4. In Plane Oscillations in Direction of Motion (IPI)

This kind of oscillation presents an asymmetric excitation with inherent friction reduction. Following [117] two models based on Coulomb’s friction law are presented, a dynamic model not considering the special geometry of the test stand and a quasi static theory adapted to the specifics of the test stand. Again, both theories will be compared to the experimental data. A modelling of the system with the MDR is not possible, since the contact points of the used probe are not rotationally symmetric.
4.4.1. Physical Modelling

**Model 1** The probe on a moving surface can be physically modelled as a mass $m$ with two contact points and changing length between these points, as depicted in figure 4.14. The movement of the mass is a result of the force $F$. The length between the contact points changes harmonically with

$$l(t) = l_0 + \Delta l \sin \omega t.$$  

The equation of motion can then be derived using Newtonian mechanics and Coulomb’s law according to the free body diagram in figure 4.15.

The friction force $F_r$ at each contact point is

$$F_r = \mu_0 \frac{F_n}{2} \text{sgn}(v_{rel}).$$  

Here, $\mu_0$ is the friction coefficient in absence of vibration, and $v_{rel}$ the velocity of the contact point with respect to the surface. The equation of motion yields

$$m \ddot{x} = F - \mu_0 \frac{F_n}{2} \text{sgn}(\dot{x} + \dot{l}(t)) - \mu_0 \frac{F_n}{2} \text{sgn}(\dot{x} - \dot{l}(t)),$$

with

$$\dot{l}(t) = \Delta l \omega \cos \omega t = 2\dot{v} \cos \omega t.$$  

Interpreting the force $F$ as the force required to drag the vibrating specimen, with a mean sliding velocity $\langle v \rangle$ the mean reduced friction coefficient yields

$$\langle \mu \rangle = \frac{F}{F_n}.$$  

which can be computed solving equation (4.27). This was conducted, computing \( \langle v \rangle \) averaged over one period for \( F \in [0, \mu_0 F_n] \) for different amplitudes. The normalized graph is depicted in figure 4.16.

![Figure 4.16.: Normalized friction reduction – IPI model 1](image)

Two characteristics are prominent: First, for very small amplitudes \( \Delta l \) the friction coefficient has a discontinuity at \( \langle v \rangle / \hat{v} = \frac{2}{\pi} \). At this point, the probe moves almost quasi-static like a caterpillar. One contact point rests, while the other one moves with the velocity \( \hat{v} \). For larger amplitudes, this effect cannot be observed. The velocity changes in the contact points are fast enough to prevent a movement of the contact points with the centre of mass of the probe.

**Model 2**  Another approach to compute the reduced friction force is given in \([109, 110]\) by averaging the alternating friction force over one oscillation period:

\[
\langle F_r \rangle = \frac{1}{T} \int_0^T F_r \, dt.
\]  

(4.30)

In case of the geometry given by the ultrasonic probe on the rotating pin-on-disc tribometer this equation has to be adapted according to the directions of the velocities at the contact points of the probe as indicated in figure 4.17.

Here, \( \langle v \rangle \) is the sliding velocity between rotating disc and probe. The friction force is directed opposite to the velocities of each contact point, which themselves are superpositions of the depicted velocities:

\[
\vec{v}_1 = -\langle v \rangle \sin \vartheta \hat{e}_x + \langle v \rangle \cos \vartheta + \hat{v} \cos \omega t \hat{e}_y
\]

and

\[
\vec{v}_2 = \langle v \rangle \sin \vartheta \hat{e}_x + \langle v \rangle \cos \vartheta - \hat{v} \cos \omega t \hat{e}_y.
\]

(4.31)  

(4.32)
Chapter 4. Active Control of Friction and Friction Reduction

The friction force then yields
\[ F_r = \mu_0 F_n \frac{\vec{v}_1}{|\vec{v}_1|} + \mu_0 F_n \frac{\vec{v}_2}{|\vec{v}_2|}, \] (4.33)

with
\[ |\vec{v}_1| = |\vec{v}_2| = \sqrt{\langle v \rangle^2 - 2 \langle v \rangle \dot{v} \cos \vartheta \cos \psi + \dot{v}^2 \cos^2 \psi}, \] (4.34)

where \( \psi = \omega t \). Consequently, the \( x \)-components of both summands of the friction force cancel out, while the \( y \)-components are identical if averaged over one oscillation period. Thus the mean friction force computes to
\[ \langle F_r \rangle = \frac{\mu_0 F_n}{2\pi} \int_0^{2\pi} \frac{\langle v \rangle \cos \vartheta - \dot{v} \cos \psi}{\sqrt{\langle v \rangle^2 - 2 \langle v \rangle \dot{v} \cos \vartheta \cos \psi + \dot{v}^2 \cos^2 \psi}} \, d\psi. \] (4.35)

The normalized mean friction coefficient yields
\[ \frac{\langle \mu \rangle}{\mu_0} = \frac{\langle F_r \rangle}{\mu_0 F_n} \frac{1}{2\pi} \int_0^{2\pi} \frac{\langle v \rangle \cos \vartheta - \dot{v} \cos \psi}{\sqrt{\langle v \rangle^2 - 2 \langle v \rangle \dot{v} \cos \vartheta \cos \psi + \dot{v}^2 \cos^2 \psi}} \, d\psi. \] (4.36)

The results for \( \vartheta = 0^\circ \) and \( \vartheta = 15.25^\circ \) are depicted in figure 4.18, the latter one corresponds to the angle of the geometry used for the experiments. For the theory of a linear motion without rotating disc, \( \vartheta \) vanishes and the solution becomes the one of [109, 110]. For non-vanishing angles, the main difference between both cases is a mean friction coefficient which is only converging against the initial friction coefficient for very large sliding velocities. One inaccuracy of all inplane models may however be stated here. For decreasing velocities close to zero, the friction vanishes, resulting in a disappearing static friction force. This cannot be experimentally confirmed in general (see e.g. [98]).
4.4. Comparison of Theory and Experiments

Experimental Results

The experiments have been performed for a probe made of steel and a disc of polished steel. The relevant material and geometrical data can be found in table 4.2.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>probe’s mass $m_e$</td>
<td>0.5 g</td>
</tr>
<tr>
<td>unreduced friction coefficient $\mu_0$</td>
<td>0.4</td>
</tr>
<tr>
<td>unreduced friction coefficient for small velocities $\mu_{0,\text{slow}}$</td>
<td>0.37</td>
</tr>
<tr>
<td>unreduced friction coefficient for large velocities $\mu_{0,\text{fast}}$</td>
<td>0.42</td>
</tr>
<tr>
<td>mean normal force $F_{n,0}$</td>
<td>9 N</td>
</tr>
<tr>
<td>natural circular frequency $\omega$</td>
<td>$86 \cdot \pi$ kHz</td>
</tr>
<tr>
<td>angle $\vartheta$ (see figure 4.17)</td>
<td>15.25°</td>
</tr>
</tbody>
</table>

Table 4.2.: Properties of the test stand for oscillations in IPI direction

For two measurements the sequence of velocity and amplitudes were changed. The normalized mean values are presented in figure 4.18. The graphs for the absolute values and a graph with error-bounds are presented in appendix D. Different levels of friction have been measured for experiments with and without additional reduction gear box. As for the OOP vibration direction, a reason for this might be an overall smoother motion of the entire test stand due to additional damping brought into the system by the gear box. Nevertheless, for a better comparability, the mean values have been normalized with respect to the friction coefficients without vibration $\mu_{0,\text{slow}}$ and $\mu_{0,\text{fast}}$ for experiments without and with reduction gear box, respectively.
Figure 4.19.: IPI experiments; asterisks represent average values of four measurements without an additional reduction gear box, circles indicate average values of four measurements with additional reduction gear box.

In figure 4.19, circles are indicating average data points for measurements with reduction gear box and asterisks indicate those without. The measurements have been performed for various vibration amplitudes $\Delta l$. In figure 4.19 the amplitudes measured with respect to the velocity are shown. Only for certain amplitudes the exact same level could be measured for large and small velocities. Moreover steady amplitudes are
mostly achieved for measurements without reduction gear box. The derivations in the amplitudes between measurements with and without reduction gear box are most presumably a result of a change of the overall test stand properties. Measurements with and without gear box were taken several years apart. Naturally, the test stand was used in the meantime, resulting in irregularities of the measurements. The unsteady amplitude for the largest measured amplitude of about 0.6\( \mu \text{m} \) are due to the set-up of the test stand. In general, for all experiments, measuring a steady large amplitude for the entire velocity range proved to be difficult. However, in figure 4.19b, the normalized friction coefficients with respect to sliding velocity are depicted. The curves of slow and fast measurements coincide nicely except for the two smallest recorded amplitudes. This is in analogy to the experiments of the OOP direction. Regardless, the effect of friction reduction and reversed slope of the friction coefficient is observable. For the smallest measured amplitude of about 0.05 \( \mu \text{m} \) the friction coefficient is still a decreasing function with respect to \( \langle v \rangle \). For the other amplitudes this effect is again reversed. Consistently, the friction reduction increases with greater amplitude. While for the larger amplitudes the friction hints to have a finite value for vanishing sliding velocity the static friction vanishes for the two smallest measured amplitudes of 0.42 \( \mu \text{m} \) and 0.65 \( \mu \text{m} \). For large sliding velocities, it appears that the overall friction converges for all amplitudes to a constant level which is decreasing with increasing amplitude.

**Comparison to Analytical Models**

In figure 4.20, the experimental data are compared to the introduced models. The theoretical graphs have been normalized using a mean measured friction coefficient \( \mu_0 = 0.4 \). The analytical graphs for model 1 are depicted in figure 4.20b. It is important to mention that for the simulation an equivalent mass of \( m_e = 0.5 \text{g} \) has been used as a fitting parameter. While for this mass the velocity interval for the model can be matched with the one of the experiments, it is a rather unrealistic mass, considering the real mass of the unmounted probe is 52 g. The only theoretical curve similar to the experiments is the one for the largest measured amplitude. Otherwise, the curves can qualitatively cover an increasing slope of the friction coefficient with respect to sliding velocity. The vanishing friction coefficient for vanishing sliding velocity does not represent the finite static friction indicated by the measurements.

For the second model, the values won by equation (4.36) are depicted in figure 4.20c. No fitting parameters have been used. The overall tendencies are in better accordance with the experimental data. Especially the form of the curve converging against \( \mu_0 \) for large sliding velocities represents the trend of the experimental data. However, regarding the vanishing static friction in the model and the fact that all curves converge against an average friction coefficient \( \mu_0 \) it can be concluded that both simple models based on Coulomb’s law can only give a first description of the overall friction controlling mechanisms. For a profound and more realistic description of reality, more sophisticated friction laws or numerical methods to describe the contact interaction must be implemented.
(a) IPI experiments, normalized friction coefficient with respect to the sliding velocity

(b) IPI data of model 1 using Coulomb’s law to compute the friction force

(c) IPI data of model 2 using Coulomb’s law to compute the friction force

Figure 4.20.: IPI experiments compared with numerical data won by model 1 and model 2. For model 1, the mass of the probe was fitted to be $m_e = 0.5 \text{ g}$
4.5. In Plane Oscillations Perpendicular to the Direction of Motion (IPP)

Regarding figure 4.1, this vibration direction is not included in the definition of an asymmetric or symmetric excitation from the introduction. However, for the form of the specific geometry of the test stand, the perpendicular motion of the contact point results in an oscillating tangential force and can thus be understood as an asymmetric excitation. Again, a quasi-static model will be introduced and compared to the experimental data as similarly done in [117].

4.5.1. Physical Modelling

As has been shown in the previous section, the friction reduction effect of vibrations in the IPI direction is due to the change in direction of the overall velocity. The same applies for the IPP direction. Once more, a quasi-static consideration of the sliding probe appears reasonable due to no overall acceleration of the probe. The overall direction of the friction force remains in sliding direction, as indicated in figure 4.21 illustrating the velocity and force distribution at the contact points of the ultrasonic probe. The concept presented here follows the model introduced by Littmann et al. in [109].

The velocities of the contact points are given by

\[ \vec{v}_{1,2} = \langle v \rangle \hat{e}_y \pm \hat{v} \cos \omega t \hat{e}_x. \]  \hspace{1cm} (4.37)

Using in addition equation (4.33) to compute the overall macroscopic friction force yields

\[ F_r = \mu_0 F_n \frac{\langle v \rangle}{\sqrt{\langle v \rangle^2 + \hat{v}^2 \cos^2 \omega t}}. \]  \hspace{1cm} (4.38)
The remaining frictional force is only directed in the direction of motion, since the horizontal velocity components are identical and cancel out. The mean friction force is again the average force over one period of the vibration:

\[
\langle F_r \rangle = \frac{\mu_0 F_n}{2\pi} \int_0^{2\pi} \frac{d\psi}{\sqrt{1 + \left(\frac{\hat{v}}{\langle \hat{v} \rangle}\right)^2 \cos^2 \psi}}.
\] (4.39)

With \( \zeta = \frac{\langle \hat{v} \rangle}{\hat{v}} \) this can be expressed as a complete elliptic integral of first kind

\[
Q(\zeta) = \int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1 - \zeta^2 \sin^2 \psi}}.
\] (4.40)

where \( 0 \leq \zeta \leq 1 \). The normalized mean friction coefficient yields

\[
\frac{\langle \mu \rangle}{\mu_0} = \frac{2}{\pi \sqrt{1 + \frac{1}{\zeta^2}}} Q \left( \frac{1}{1 + \zeta^2} \right).
\] (4.41)

This result is presented in figure 4.22. As for the IPI models a vanishing friction coefficient for small sliding velocities is predicted. For large sliding velocities the mean friction converges against the value for sliding without ultrasonic oscillations.

![Figure 4.22.: Normalized friction reduction – IPP](image_url)

### 4.5.2. Comparison of Theory and Experiments

#### Experimental Results

The experiments for vibrations in IPP direction have been conducted for small and large velocities with the same steel probe and the same polished steel plate as used for
the experiments in IPI vibration direction. The material properties are recapitulated in table 4.3.

Figure 4.23.: IPP experiments; stars represent average values of four measurements without an additional reduction gear box, circles depict average values of four measurements with additional reduction gear box.
unreduced friction coefficient \( \mu_0 \) | 0.37
unreduced friction coefficient for small velocities \( \mu_{0,\text{slow}} \) | 0.44
unreduced friction coefficient for large velocities \( \mu_{0,\text{fast}} \) | 0.3
mean normal force \( F_{n,0} \) | 4 N
natural circular frequency \( \omega \) | \( 86 \cdot \pi \) kHz

Table 4.3.: Properties of the test stand for oscillations in IPP direction

For this direction measurements for each amplitude and velocity range have been performed four times with interchanging order of velocities and amplitudes. Again, while the average normalized values are shown in figure 4.23, the single measurements along with the error-bounds can be found in appendix E.

The measurements with and without gear box are depicted by circles and asterisks, respectively in figure 4.24. In figure 4.24a the supposedly steady amplitude with respect to sliding velocity is shown. Observably it proved troublesome to measure steady amplitudes for the experiments with included gear box. This may be the reason why for these measurements no positive slope of the friction coefficient with respect to \( \langle v \rangle \) can be observed. Nevertheless a small effect of friction reduction can be observed.

The experimental data with and without gear box do not coincide at any point, except for the curve measured in both cases without oscillation. Due to the obvious difficulties of regulating the amplitude for the small velocity measurements, it appears that no sinusoidal vibration was achieved due to a defective test stand. Consequently, the measurements with gearbox cannot be taken as trustworthy.

The data gained without gear box show a steady amplitude except for one outlier. Again, the effects already identified for OOP and IPI directions can be observed: a changing slope and reduction of the overall friction. Both effects increase with larger oscillation amplitudes. For all investigated amplitudes there appears to exist a finite static friction force and a friction force against which the curves converge for large sliding velocities.

Comparison to the Analytical Model

The measurements taken without gear box are compared to the analytical curves in figure 4.24. To illustrate the qualitative prediction accuracy, figure 4.24b shows two sets of oscillation amplitudes. The real oscillation amplitudes are plotted as solid lines. An effect of friction reduction is visible but does not match the measured velocity range. For the tenfold larger amplitudes, indicated by the dashed lines, a better accordance with the measurements can be observed. However, it is immanent that the simple model based on Coulomb’s law can only explain the major effect of the oscillation and cannot draw a quantitative picture of the effect. Again, working with numerical contact mechanics or with more sophisticated friction laws might allow to close this gap.

\[ ^1 \text{Indeed, a faulty cable connecting oscillating probe with the amplifier was exchanged shortly after the measurements were taken} \]
For all investigated oscillation directions two effects can be observed: an apparent reduction of friction is achieved for small sliding velocities. While the friction coefficient is a monotonic decreasing function with respect to sliding velocity it becomes a monotonically increasing function under the influence of vibrations. Both effects highly depend on the applied oscillation amplitude, the larger the amplitude, the more prominent the effect becomes. Furthermore, comparing the three investigated oscillation directions, the

Figure 4.24.: IPP experiments compared with data won by the analytical model

4.6. Conclusions
vibration show the largest effect for the OOP vibration direction. The IPP vibration direction occurs to be least influenced by the vibrations.

The effect of friction reduction might be used to decrease friction in a specific contact. It will however be difficult to efficiently save energy in a system. Energy savings might only be possible for the OOP vibration direction. For both in plane directions the energy input will always be larger than the amount of energy saved.

The technically interesting effect of reversing the slope of the friction coefficient with respect to sliding velocity may be used to induce effective damping into the system. This effect is known to omit frictional instabilities and consequently suppress effects like braking and cornering squeal.

However, the investigated models based on Coulomb’s laws can only qualitatively describe the effects of friction reduction due to oscillations. Especially an explanation for a finite static friction force for in plane oscillations is lacking. The implementation of the MDR to describe the effects of frictional behaviour is very promising. Even with the most fundamental assumed model of a sphere on half space contact, the overall quantities and qualities of the OOP vibration direction can be explained.

Consequently, to investigate a practical application of the influence on vibrations on sliding friction, it would be beneficial to further explore the applicability of the MDR. The test stand could be remodelled such that the probes for the in plane vibration directions are of a rotationally symmetric form and could be modelled using the MDR. Furthermore different material pairings should be investigated, as well as the influence of the normal force on the effect of friction reduction or inversion of the slope. Only if a comprehensive investigation is performed, sufficient insight can be won for what kind of systems the positive effects of a vibrating contact are economically marketable.
5. Results and Perspectives

In this work oscillating tangential contacts have been investigated. Two classes of contacts were examined: An asymmetric excitation in form of a sawtooth tangential load resulted in a directed motion of the stick-slip drive RaMoNa and active control of friction was studied, regarding a pin-on-disc tribometer. For both classes extensive experimental investigations along with qualitative theoretical models and more sophisticated numerical models have been conducted.

Stick-Slip Drives  The force-generating axis of the nano robot RaMoNa has been examined for different scales of contacting ruby hemispheres. The dynamic modelling of the contact, combining macroscopic Newtonian mechanics with a microscopic dynamic description of the tangential contact presents a new approach. The method allows the implementation of the tangential contact geometry and its dynamic behaviour without the drawback of an exceeding computation time. The results of the computations are qualitatively excellent and quantitatively very good for the smaller range of inspected actuation amplitudes. A main achievement of this work is the correspondence of numerical and experimental data without using a single fitting parameter.

In addition, the numerical results match the experiments for all three investigated contact scales. This implies a universality of the method not given for any single state friction law. The agreement between numerically and experimentally observed 0-amplitude was exceptionally good. It has also been shown that the 0-amplitude is a mean value of the contact’s presliding distances $u_{x_{\text{max}}}$. Thus this work confirms the assumptions of previous publications on the origin of presliding in nano devices. Different model-influencing characteristics, including surface roughness and contact geometry, have been considered but show no significant amelioration of the numerical results. Furthermore, a simple dynamic model based on the ideas of the MDR has been introduced. This model allows an approximative derivation of the 0-amplitude, the description of the vibration damping after the back-step of the runner and most notably the development of a new actuation method to enhance the drive’s performance.

Since it has been shown that the MDR and hence the classical tangential contact theory is suitable to describe dry friction in nano devices, a next aim should be to even further test the method’s capabilities. Two ideas come to mind: First a program to describe the entire locomotion platform of RaMoNa or any other desired drive may be implemented. Second, for further investigations of the correlation between 0-amplitude and presliding distance $u_{x_{\text{max}}}$ the 0-amplitude for a simplified test stand could be directly measured and compared with numerical and analytical results.
**Friction Reduction and Active Control of Friction**

The influence of ultrasonic vibration on friction was systematically studied performing experiments on a pin-on-disc tribometer. For a steel-steel friction pairing multiple actuation amplitudes for three vibration directions have been inquired. It has been experimentally shown that for sliding friction the dependence of friction with respect to the sliding velocity can be altered if oscillations are applied. While in general the friction decreases with increasing sliding velocity, if vibrations are applied, this effect is reversed and the friction force becomes a monotonically increasing function with respect to sliding velocity. In addition, for small sliding velocities the amount of friction is highly reducible. Both influences appear to be most prominent for vibrations in the direction normal to the contact plane.

Analytically all oscillation directions have been described using the simple Coulomb’s law. As was expected these considerations only provide a rough qualitative explanation of the measured effects. For the rotational symmetric probe used for the experiments with oscillations normal to the contact plane, a MDR model has been implemented. Even though the normal contact stiffness for this model is fitted, the results for the simple contact geometry of a sphere agrees well with the measured data.

To further investigate if the MDR can describe the macroscopic contacts of the tribometer, multiple routes may be followed. For instance, the influence of wear, roughness and velocity dependent friction coefficient should be investigated for the existing numerical program. In particular, wear was observable during the experiments and appears to have an influence on the behaviour of the test stand. It would therefore be interesting to pair the implementational work of this thesis with studies conducted to describe fretting wear by the MDR [121].

Furthermore, the probes used for the measurements of vibrations in the in plane directions should be remodelled. If they had the shape of rotational symmetric bodies, the MDR could be applied to numerically describe the remaining oscillation directions.

The employment of the friction controlling mechanism in practical applications should be a future aim. Achieving this, more systematic studies of the oscillation influences have to be performed. For example the influence of varying normal load is of great interest if friction is to be controlled in a broad range of real life applications. This work has been commenced at the department in [90] concerning the static coefficient of friction. In addition, for sliding friction examinations, more material pairings found in technical devices have to be experimentally studied.

Beside the systematic experimental investigations of this work, a general result lies in the description of friction as dynamically changing tangential contacts using the MDR. As of yet microscopic, mesoscopic, and macroscopic friction has been described by more or less sophisticated friction laws. While this might be sufficient for some applications, for those it is not, another approach is needed. The method derived in this thesis to combine a macroscopic dynamic model with a microscopic contact model based on the classical tangential contact theory proved to be very promising and time efficient. It consequently is preferable to very time consuming FEM or BEM models. Furthermore, the MDRs wide application range including parameters as e.g. roughness, viscousness, and wear, along with its simplicity should result in an implementation of an interface with standard FEM software as a logical next step.
Concluding, this work shows a promising method investigating any kind of dynamically changing contacts. The interplay of experimental work along with a qualitative model of the system allows a deep understanding of the mechanisms at play. In addition a non-intuitive numerical, quantitative description of the contact mechanics with the means of the MDR allows for a precise physical description of the measured effects.
Appendices
A. Identity for the proof of $u_{x,\text{max}}$

The tangential displacement and the indentation depth have been shown to be $u_{x,\text{max}} = \mu \frac{E}{G} d$ for randomly rough surfaces of different material properties in chapter 2.2.2. To completely show this correlation the following identity has to be computed:

\[
1 + \frac{\gamma^2}{2} = E^* \left( \frac{1}{E^*} + \frac{\nu_1}{4G_1} + \frac{\nu_2}{4G_2} \right) \quad (A.1)
\]

\[
= E^* \left( \frac{2 - 2\nu_1}{4G_1} + \frac{2 - 2\nu_2}{4G_1} + \frac{\nu_1}{4G_1} + \frac{\nu_2}{4G_2} \right) \quad (A.2)
\]

\[
= \frac{E^*}{G^*}. \quad (A.3)
\]
B. All Measurements and Simulations of the Generated Force

In the following, all experimental data along with the numerical simulations of the force-generating device will be depicted. The simulation data shown are the ones using a simple Hertz contact between a sphere and the runner.

B.1. Measurements and Simulation for a Sphere

B.1.1. Radius $R = 1$ mm

![Graph showing generated force $F_{gen}$ as a function of the actuation amplitude for different preloads $F_p$.](image)

Figure B.1.: Generated force $F_{gen}$ as a function of the actuation amplitude, experimental data for $R = 1$ mm and different preloads $F_p$. 
Appendix B. All Measurements and Simulations of the Generated Force

Figure B.2.: Generated force $F_{gen}$ as a function of the actuation amplitude, data for $R = 1$ mm and different preloads $F_p$

Figure B.3.: Generated force $F_{gen}$ as a function of the actuation amplitude, data for $R = 1$ mm and different preloads $F_p$
B.1. Measurements and Simulation for a Sphere

B.1.2. Radius $R = 0.5 \text{ mm}$

Figure B.5.: Generated force $F_{\text{gen}}$ as a function of the actuation amplitude, experimental data for $R = 0.5 \text{ mm}$ and different preloads $F_p$
Figure B.6.: Generated force $F_{gen}$ as a function of the actuation amplitude, data for $R = 0.5\, \text{mm}$ and different preloads $F_p$. 

Figure B.7.: Generated force $F_{gen}$ as a function of the actuation amplitude, data for $R = 0.5\, \text{mm}$ and different preloads $F_p$. 
B.1. Measurements and Simulation for a Sphere

B.1.3. Radius $R = 0.25 \text{ mm}$

Figure B.8.: Generated force $F_{\text{gen}}$ as a function of the actuation amplitude, data for $R = 0.5 \text{ mm}$ and different preloads $F_p$

Figure B.9.: Generated force $F_{\text{gen}}$ as a function of the actuation amplitude, experimental data for $R = 0.25 \text{ mm}$ and different preloads $F_p$
Appendix B. All Measurements and Simulations of the Generated Force

Figure B.10.: Generated force $F_{gen}$ as a function of the actuation amplitude, data for $R = 0.25 \text{ mm}$ and different preloads $F_p$

Figure B.11.: Generated force $F_{gen}$ as a function of the actuation amplitude, data for $R = 0.25 \text{ mm}$ and different preloads $F_p$
B.2. Measurements and Simulation with Changing Friction Coefficient

Figure B.12.: Generated force $F_{gen}$ as a function of the actuation amplitude, numerical data for $R = 0.5$ mm for a runner with and without changing friction coefficient.

Figure B.13.: Generated force $F_{gen}$ as a function of the actuation amplitude, numerical data for $R = 0.5$ mm for runner with changing friction coefficient and experimental data.
Appendix B. All Measurements and Simulations of the Generated Force

Figure B.14.: Generated force $F_{\text{gen}}$ as a function of the actuation amplitude, numerical data for $R = 0.25$ mm for a runner with and without changing friction coefficient.

Figure B.15.: Generated force $F_{\text{gen}}$ as a function of the actuation amplitude, numerical data for $R = 0.25$ mm for a runner with changing friction coefficient and experimental data.
C. All Measurements of the OOP Oscillations

The experiments for the OOP vibration direction have been performed four times with changing order of sliding velocities or amplitudes. All performed measurements are shown in figure C.1, C.2, C.3, C.4. The average values with error bound for an average friction coefficient of $\mu_{0,\text{slow}} = 0.44$ and $\mu_{0,\text{fast}} = 0.33$ are shown in figure C.5.

Figure C.1.: OOP experiments, friction coefficient with respect to time, measurement 1, stars represent measurements without an additional reduction gear box, circles indicate measurements with additional reduction gear box.
Figure C.2.: OOP experiments, friction coefficient with respect to time, measurement 2, stars are measurements without an additional reduction gear box, circles are measurements with additional reduction gear box.

Figure C.3.: OOP experiments, friction coefficient with respect to time, measurement 3, stars represent measurements without an additional reduction gear box, circles indicate measurements with additional reduction gear box.
Figure C.4.: OOP experiments, friction coefficient with respect to time, measurement 4, stars represent measurements without an additional reduction gear box, circles indicate measurements with additional reduction gear box.

Figure C.5.: OOP experiments, normalized friction coefficient with respect to time with error margins, dotted lines represent measurements without an additional reduction gear box, solid lines indicate measurements with an additional reduction gear box.
D. All Measurements of the IPI
Oscillations

The experiments for the IPI vibration direction have been performed two times with
changing order of sliding velocities or amplitudes. All performed measurements are
shown in figure D.1 and D.2. The average values with error bound for an averaged
friction coefficient of $\mu_{0,\text{slow}} = 0.37$ and $\mu_{0,\text{fast}} = 0.42$ are shown in figure D.3.

![Graph](image)

Figure D.1.: IPI experiments, friction coefficient with respect to time, measurement 1, stars
represent measurements without an additional reduction gear box, circles indicate measurements with additional reduction gear box.
Figure D.2.: IPI experiments, friction coefficient with respect to time, measurement 2, stars represent measurements without an additional reduction gear box, circles indicate measurements with additional reduction gear box.

Figure D.3.: IPI experiments, normalized friction coefficient with respect to time with error margins, dotted lines represent measurements without an additional reduction gear box, solid lines indicate measurements with an additional reduction gear box.
E. All Measurements of the IPP Oscillations

The experiments for the IPP vibration direction have been performed two times with changing order of sliding velocities or amplitudes. All performed measurements are shown in figure E.1 and E.2. The average values with error bound for an averaged friction coefficient of $\mu_{0,\text{slow}} = 0.44$ and $\mu_{0,\text{fast}} = 0.3$ are shown in figure E.5.

Figure E.1.: IPP experiments, friction coefficient with respect to time, measurement 1, stars represent measurements without an additional reduction gear box, circles indicate measurements with additional reduction gear box.
Appendix E. All Measurements of the IPP Oscillations

Figure E.2.: IPP experiments, friction coefficient with respect to time, measurement 2, stars represent measurements without an additional reduction gear box, circles indicate measurements with additional reduction gear box.

Figure E.3.: IPP experiments, friction coefficient with respect to time, measurement 3, stars represent measurements without an additional reduction gear box, circles indicate measurements with additional reduction gear box.
Figure E.4.: IPP experiments, friction coefficient with respect to time, measurement 4, stars represent measurements without an additional reduction gear box, circles indicate measurements with additional reduction gear box.

Figure E.5.: IPP experiments, normalized friction coefficient with respect to time with error margins, dotted lines represent measurements without an additional reduction gear box, solid lines indicate measurements with additional reduction gear box.
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