7.2 Manufacturing strategy using new and reconditioned rotatable spare parts

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Abstract
The process of remanufacturing is attractive economically and environmentally for both manufacturers and consumers. It is important to properly use reconditioned parts in a production plan based on their availability and production costs. A mathematical model is derived to find the cost-optimal production strategy that incorporates reconditioned components in the manufacturing effort. New and reconditioned parts are used to carry out replacements upon failure under an unlimited free replacement warranty policy. Key production decisions, such as when remanufacturing should commence, how long the warranty period should be, and how many returned parts should be reconditioned are answered. The availability of reconditioned parts and their discounted costs are incorporated in the model. Interactions between these decisions and their impacts on the manufacturing system and the consumer are investigated. A case study on aircraft rotatable spare parts will be presented.

Keywords: Remanufacturing; End of Life; Reconditioning; Spare Parts; Unlimited Free Replacement Warranty

1 INTRODUCTION
The five options available at a product's end of life (EOL) are: disposal, recycling, repair for use by the same consumer, re-use by another consumer, and refurbishing/remanufacturing[1,2]. Remanufacturing is the process of restoring used products to like-new conditions by disassembly, cleaning, repairing and replacing parts, and reassembly[3]. In contrast, refurbishing entails minimal disassembly, and can be thought of as a lighter version of remanufacturing[4]. Both procedures enhance the environmental and economic value by reducing the consumption of virgin materials and energy. Moreover, the diminishing access to raw materials is forcing manufacturers to implement design methodologies incorporating global sustainability, using materials more efficiently, and participating in EOL product recovery[5]. In the refurbishing/remanufacturing EOL option, the returned product is disassembled, and used within the bill of materials (BOM) or a production plan. Potential benefits from remanufacturing are contingent on capacity utilization[6], and thus finding the ideal composition of new and reconditioned components is paramount in determining cost-effective production plans.

Economically, remanufacturing can be profitable, but it is heavily conditional on the product type and the industry. There is uncertainty in both the number of returned products and in the quality of the returns. Production plans for both new and remanufactured products were developed using a linear programming model and deterministic demand[7]. Mathematical models to deal with inventory control problems [8] and production planning[9,10] using reconditioned products have all been considered. A company must provide EOL services by supplying spare parts throughout the service period in order to remain competitive[11,12]; forcing manufacturers to carefully determine the warranty period to offer with their products. Warranty can be thought of as a contract where consumers will have their faulty product repaired/replaced at no cost or at reduced cost, before a specified time[13]. It indirectly conveys product quality to consumers through the terms and product reliability, which then dictate the associated costs[14-17]. Similarly, a key component in the resale of the remanufactured product is the connected warranty that must be offered with it. An expected warranty cost equation using second-hand components at the component level was presented [18]. A new strategy using reconditioned components for the replacements was proposed [19], however in real practice it is not always possible to have access to enough reconditioned components to honour the warranty. Since the supply of EOL or returned products is not steady, a manufacturer can be forced to use a combination or mixture of new and reconditioned components to carry-out replacements[20,21]. A mathematical model to determine the proportion of new and reconditioned components to be used, the age of the reconditioned components, the warranty length, and the profit margin in order to maximize the total profit was developed [22]. The role of EOL services in the context of the product’s entire lifecycle including the demand, production, inventory, and replacement upon failure during the EOL warranty period was investigated [23].

Kim and Park provided a robust production planning control model by incorporating manufacturing and EOL warranty[23]. This includes both the manufacture of the original products, as well as that of the spare parts to satisfy the warranty. The objective function consists of two major parts: the profit function for the production of new products, and the cost function to manufacture and supply spare parts as required. In the construction of the objective statement it is assumed that the company will produce and sell the product throughout its lifecycle and provide the customer with spare parts for the full length of the warranty period after the product has been discontinued. The major caveat here was that the customer could only get the spare part from the company once during the course of the entire product lifecycle – including the warranty period. This and the inability to handle reconditioned components were crucial shortcomings of this model. In this paper, the two-stage optimal control theory model proposed by [23] is extended to account for the collection and reuse of reconditioned parts in the manufacturing process (see Figure 1) along with an unlimited free replacement warranty (UFRW) policy offered on the products. The formulation of the system’s dynamics and numerical experiments will help understand the interactions between warranty length, production rate, refurbishing rate, and product reliability.
Part failures in the airline industry can have substantial cost implications due to the disruptions that they cause, stressing the importance to estimate their failure and have access to replacement parts. Spare parts demand is usually intermittent and can be classified as: slow moving demand, strictly intermittent demand, erratic demand, and lumpy demand [24]. Regattieri et al. focus on lumpy demand while performing a case study on Alitalia [24-26]. Rotatable parts require periodic replacement, and reconditioning these components can help mitigate the problems associated with their availability.

This paper is structured as follows: In Section 2, the model is introduced, the notation is defined, and the mathematical model is developed. Section 3 is dedicated to the discussion of the results obtained using the model on a rotatable spare part from a Canadian carrier.

2 THE OPTIMAL CONTROL THEORY MODEL

An optimal control theory model of an MRP-based production plan using new and reconditioned components in the context of remanufacturing has been developed to determine the optimal production lifecycle length and warranty period (Figure 1).

![Figure 1: Model Stages](image)

The company produces and sells a product until $T$, the end of Stage 1B, when it is discontinued. Concurrently, new spare parts are fabricated and are used to repair the products that fail during the warranty coverage period. In Stage 1A, all spare parts are created using new components. The start of Stage 1B sees the introduction of reconditioned components harvested from failed products to be used in the assembly of spare parts. The next stage represents the warranty obligations of the firm after the product is discontinued. Here, there is a point $(\gamma)$, where the cumulative number of product failures, $F(t)$, is for the first time equivalent to the cumulative production of new and reconditioned spare parts, $Q(t)$. All the accrued spare parts are completely consumed within Stage 2A, and subsequently spare parts are produced as required in Stage 2B. These four periods are listed below:

- **Stage 1A ($0 \leq t \leq \gamma T$):** The company manufactures the product and also manufactures new spare parts using solely new components;
- **Stage 1B ($\gamma T \leq t \leq T$):** Spare parts using both new and reconditioned components are produced;
- **Stage 2A ($T \leq t \leq \tau$):** Production of the product is discontinued, but production of reconditioned spare parts continues, and new spare parts are produced as needed;
- **Stage 2B ($\tau \leq t \leq T + w$):** Production of reconditioned spare parts continues, and new spare parts produced as needed.

With the incorporation of reconditioned components in this model, the following additional decisions must be considered:

1. When should the reconditioned components be introduced into the manufacturing production plan?
2. How many reconditioned parts are available?
3. What is the economic benefit?

The introduction of reconditioned components is a key factor in this investigation, where the time of commencement is defined as a fraction $(\gamma)$ of the product’s lifecycle $(T)$ with $0 \leq \gamma \leq 1$. A value of $\gamma = 0$ means reconditioned components are utilized at time $0$, effectively eliminating Stage 1A; whereas $\gamma = 1$ implies that reconditioned components are introduced in the 2nd Stage. The two considerations to be made with regards to availability of reconditioned parts are: the total amount of failed products $F(t)$, and how much of it is usable. $\eta$ represents the proportion of failed products that will be in a state of degradation such that their key components can technically and economically be tested, removed, and reconditioned. Lastly, there is a cost to use reconditioned components, so $k_r$ will model the production cost of remanufacturing as a function of $\gamma$. Furthermore, all pertinent terms relating to spare parts will be split into their new $(\alpha)$ and reconditioned $(\beta)$ constituents. The following notation is adopted.

- **$W$** Warranty period
- **$T$** Lifecycle of the current product
- **$\gamma$** Time at which cumulative production of spare parts equals the cumulative number of failure
- **$\gamma T$** Time at which remanufacturing begins as a fraction of $T$, $0 \leq \gamma \leq 1$
- **$\rho(t)$** Sale price of the current product at $t$
- **$d(t)$** Instantaneous sales units of the current product at $t$
- **$D(t)$** Cumulative sales units of the current product at $t$
- **$F(t)$** Cumulative parts failures/losses at $t$
- **$q(t)$** Spare parts produced at $t$
- **$q_r(t)$** New spare parts produced at $t$
- **$q_{\beta}(t)$** Reconditioned spare parts produced at $t$
- **$Q(t)$** Cumulative production of spare parts at $t$
- **$Q_{\beta}(t)$** Cumulative production of spare parts at $t$ using reconditioned components
- **$\alpha$** Failure rate
- **$\eta$** Proportion of failed products that can be reconditioned
- **$h$** Unit inventory cost of the spare part
- **$d_1$** Potential market size when price and warranty is zero
- **$d_2$** Price coefficient
- **$d_3$** Warranty coefficient
- **$c_p$** Unit production cost of new products
- **$c_r$** Unit cost to replace spare parts
- **$k_i$** Cost to manufacture a spare part at $i, i=1,2$
- **$k_{\Omega}$** Remanufacturing parameter

The objective function is composed of 3 main parts: the profit function (sales revenue minus holding, production, and repair costs) at the first stage, the cost function at the second stage (holding, production, and repair costs) and a constant term of $mD(T) - nD(T)^{T}$ representing a lump sum profit (LSP) based on
the market share (installed base – cumulative) held by the company. The demand increases with a longer warranty period and a lower price, however as they increase the net profit decreases. Additionally, due to economies of scale, it is cheaper to manufacture a spare part in Stage 1, but this is countered by inventory holding costs (h). All these trade-offs are included in Eq. (1):  

\[
Z = \int_{0}^{T} \left[ d(t)\rho(t) + c_p + h\left[ Q_1(t) + Q_2(t) - F(t) \right] \right] dt + \int_{T}^{T+w} \left[ -h\left[ Q_1(t) + Q_2(t) - F(t) \right] \right] dt + nT - nD(T)^2 
\]

Subject to:

\[
Q(t) \geq F(t) 
\]

\[
Q(0) = 0, D(0) = 0, F(0) = 0 
\]

\[
t, w, T, \gamma, \rho(t), d(t), D(t), F(t), q(t), Q(t) \geq 0
\]

First, \( k_o \), the remanufacturing parameter is defined as:

\[
k_o = \hat{\alpha}(1 - \gamma) + \hat{\delta}_2
\]

where \( \hat{\delta}_1 \) and \( \hat{\delta}_2 \) are parameters to represent the slope and intercept respectively. The instantaneous demand is then:

\[
d(t) = \begin{cases} 
\frac{d_1 + d_3 w}{2} + c_p + hT at, & \text{if } t \leq T \\
0, & \text{if } t > T 
\end{cases}
\]

Using a two-stage optimal control model similar to [23], the Hamiltonian, the necessary conditions for optimality and the optimal solution functions are derived for each stage. Due to the limited number of pages allowed for this article, only the optimal functions are presented. All mathematical derivations are available from the authors. The first term introduced is the sale price of the product, which is modelled as:

\[
\rho(t) = \begin{cases} 
\frac{1}{2} d_1 + d_3 w - \frac{d_2}{2} c_p + hT at, & \text{if } t \leq T 
\end{cases}
\]

The cumulative sales units of the product are:

\[
D(t) = \begin{cases} 
\frac{d_1 + d_3 w}{2} + c_p T - d_2 - \frac{X_3 T + h a T^3}{12}, & \text{if } 0 \leq t \leq T \\
\frac{d_1 + d_3 w}{4} T + c_p T - \frac{X_3 T + h a T^3}{6}, & \text{if } T \leq t \leq T + w 
\end{cases}
\]

With the UFRW policy as long as a product fails during the original warranty period \( w \), it is replaced with another product free of charge. The instantaneous failure rate is a proportion of the total volume of products under warranty coverage:

\[
f(t) = \alpha[D(t) - V(t)]
\]

\( V(t) \) is the volume of products no longer covered:

\[
V(t) = \begin{cases} 
0, & \text{if } t < w \\
D(t - w), & \text{if } t \geq w
\end{cases}
\]

Therefore:

\[
f(t) = \begin{cases} 
\alpha D(t), & \text{if } 0 \leq t < w \\
\alpha D(T) - D(t - w), & \text{if } w \leq t < T \\
\alpha D(T) - D(t - w), & \text{if } t \geq T
\end{cases}
\]

Finally, \( F(t) = \int_0^t f(x)dx \).

The cumulative production units of the spare parts are:

\[
Q(t) = \begin{cases} 
\frac{hT^2}{2k_1} + \frac{X_1 T}{k_1}, & \text{if } 0 \leq t \leq T \\
\frac{hT^2}{2k_2} + \frac{X_1 T}{k_2} + \frac{1}{k_1} \left\{ \frac{hT^2}{2} + X_1 T \right\}, & \text{if } T \leq t \leq T + w
\end{cases}
\]

Similarly the instantaneous production amount is:

\[
q(t) = \dot{Q}(t)
\]

\( q(t) \) is the average amount of reconditioned spare parts are:

\[
Q(t) = \begin{cases} 
0, & \text{if } 0 \leq t \leq \gamma T \\
\gamma F(t), & \text{if } \gamma T \leq t \leq T + w
\end{cases}
\]

Finally, the production amount and cumulative production units of new spare parts are:

\[
q_\alpha(t) = q(t) - q_\alpha(t)
\]

\[
Q_\alpha(t) = \int_0^t q_\alpha(t)dt
\]

The integration constants resulting from the differential equations in the mathematical model are:

\[
X_1 = k_2 f(\tau) - hT
\]

\[
X_3 = \frac{d_1 + d_3 w}{2} + c_p + \frac{h a T^2}{3} - \frac{m}{n T}
\]
Theorem 1: Given \( T \), the optimal price is always increasing.

Proof: The derivative of Eq. (7), arrives at:

\[
\dot{p}(t) = \frac{d}{dT} \left( \frac{d_1 + d_2 w}{2d_2} + c_p h_T e^{-\frac{1}{2} \alpha t} - \frac{3}{4} h_\alpha t^3 \right)
\]

(20)

Since \( h, \alpha, \) and \( T \) are all positive quantities of holding costs, failure rate, and product lifecycle, it can be seen that the price continues to increase until \( t=T \) at which point, \( \dot{p}(T) = 0 \).

When products are sold late in the lifecycle, the probability that they will fail after \( T \) increases. The reserve fund has to increase to compensate for the higher production costs of spare parts in Stage 2; increasing the price of the product.

3 RESULTS FROM A CASE STUDY

The model developed above, is applied to a case study from a large Canadian regional airline. The chosen rotatable spare part is the EHSI/EADI Display (Electronic Horizontal Situation Indicator / Electronic Attitude Directional Indicator). The data obtained from the company has been partially anonymized, however proportions are preserved. This model examines four key factors in the remanufacturing process: the length of the product’s lifecycle \( T \), the warranty period \( w \), when to commence remanufacturing \( \delta_1 \), and what proportion should be reconditioned components \( \delta_2 \).

Using the parametric values shown in Table 1 arrives at an optimal solution, but also some interesting intermediary results.

Table 1: Parameters Used in Numerical Computations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.05</td>
</tr>
<tr>
<td>( h )</td>
<td>1</td>
</tr>
<tr>
<td>( c_p )</td>
<td>10</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>5</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>4</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>8</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>10</td>
</tr>
<tr>
<td>( m )</td>
<td>0.2</td>
</tr>
<tr>
<td>( n )</td>
<td>100</td>
</tr>
<tr>
<td>( n )</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The total profit in the objective function is maximized by having \( w \) as small as possible, however it is constrained by Eq. (2), which enforces that there is a greater cumulative production of spare parts at any given time to replace the cumulative failed ones. This in turn stipulates that there is a smallest allowable value of \( w \) for an associated value of \( T \). These optimal pairs are shown in Figure 2. It can be seen that the value of \( w \) rises much more rapidly as the lifecycle duration increases with a noticeable change in the slope of the curve at \( T=16 \). The final feasible solution occurs at \( T=20.7 \), here \( w = T \).

As \( T \) and \( w \) are increased the probability of failure occurring during the warranty period is also increased, augmenting the cost to repair and replace. Since the bulk of the failures occur towards the end of the warranty period, an escalation of costs is seen in the 2nd Stage (Figure 3), where at \( T=16 \) the described behaviour occurs.

Figure 3: Profitability per Stage

Figure 4 presents the correlation between \( \delta_1, \delta_2 \) and the optimal values of \( \eta \) and \( \gamma \). By fixing \( \delta_2=0 \), increasing \( \delta_1 \) results in a decrease of \( \eta \) and an increase in \( \gamma \). The value of \( \eta \) can be approximated as \( (\delta_1+1)^{\delta_1} \).

For each given set of remanufacturing cost parameters, there is a clear optimal value of the four variables (Figure 5):

Table 2 summarizes the results and trends discussed this far.
The optimal values for the no remanufacturing (baseline) scenario are: \( T^* = 8.3, \ w^* = 3.83, \ \tau^* = 9.55, \ P^* = 4796.25 \).

Table 2: Results Summary \( (\delta=0) \)

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( T^* )</th>
<th>( w^* )</th>
<th>( \tau^* )</th>
<th>( \eta^* )</th>
<th>( \gamma^* )</th>
<th>( P^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>8.3</td>
<td>3.83</td>
<td>9.55</td>
<td>-</td>
<td>-</td>
<td>4796.25</td>
</tr>
<tr>
<td>1.0</td>
<td>8.7</td>
<td>4.11</td>
<td>10.02</td>
<td>48%</td>
<td>8%</td>
<td>4837.67</td>
</tr>
<tr>
<td>1.5</td>
<td>8.7</td>
<td>4.11</td>
<td>10.02</td>
<td>39%</td>
<td>9%</td>
<td>4829.15</td>
</tr>
<tr>
<td>2.0</td>
<td>8.7</td>
<td>4.11</td>
<td>10.02</td>
<td>33%</td>
<td>11%</td>
<td>4823.43</td>
</tr>
<tr>
<td>3.0</td>
<td>8.6</td>
<td>4.05</td>
<td>9.91</td>
<td>25%</td>
<td>12%</td>
<td>4816.30</td>
</tr>
<tr>
<td>4.0</td>
<td>8.3</td>
<td>3.83</td>
<td>9.55</td>
<td>20%</td>
<td>13%</td>
<td>4812.83</td>
</tr>
<tr>
<td>5.0</td>
<td>8.3</td>
<td>3.83</td>
<td>9.55</td>
<td>17%</td>
<td>14%</td>
<td>4810.14</td>
</tr>
<tr>
<td>8.0</td>
<td>8.3</td>
<td>3.83</td>
<td>9.55</td>
<td>12%</td>
<td>16%</td>
<td>4805.60</td>
</tr>
<tr>
<td>10.0</td>
<td>8.3</td>
<td>3.83</td>
<td>9.55</td>
<td>10%</td>
<td>17%</td>
<td>4803.92</td>
</tr>
</tbody>
</table>

The value of \( \delta_1 \) not only has an effect on \( \eta \) and \( \gamma \) as previously stated, but it also dictates the optimal values of \( T \) and \( w \). \( T \) is slightly larger when \( \delta_1 \) is small, but decreases to the baseline solution as \( \delta_1 \) increases. Figure 6 depicts the dynamics of the variables for the optimal solution of \( \delta_1 = 1, \ \delta_2 = 0 \).

Reassessing the prior outcomes, the trends observed will be discussed, and the results rationalized. While it is clear that the optimal values of \( T, w, \) and \( \gamma \) should neither be 0 or their maximum allowable value (\( T=w=20.7, \ \gamma=1 \)), the intermediate optimal values of \( 0<\gamma<1 \) is an interesting quandary. There are a few reasons why these limitations exist, and they revolve around the fact that the time-line starts at the point of production of the new products. As there are not any legacy products in the market, there is not any reconditioned material available at time 0. Analogously, because products typically tend not to fail so rapidly, it takes some time for them to become accessible. In contrast, the model requires production to take place in the early stages to service failed products and to take advantage of economies of scale.

Finally, the extra holding cost from the premature manufacture of excess reconditioned parts comes into play. So, it may be possible to have too much of a good thing. The appropriate selection of \( \delta_1, \delta_2 \) will help determine both the time to start remanufacturing \( \gamma \) and the amount of reconditioned products \( \eta \) to use when setting the production plan. It appears that the determination of \( w \) and \( T \) are independent of this parameter.
4 CONCLUSIONS AND FURTHER RESEARCH

In this paper, a mathematical model for the production of spare parts with new and reconditioned components has been developed. The demand for the product was modelled to be proportional to the length of the warranty period to translate consumers’ preference and perception of better reliability through longer warranty. The model obtained was solved numerically and yielded valid decision parameters that were discussed and explained. It demonstrated how an appropriate warranty model and associated production decisions can make reconditioned products attractive from both economic and environmental perspectives. The key observation in this model is that with the introduction of a declining cost parameter ($k_0$), optimal, non-zero values are established for the time to start reconditioning and the amount of reconditioned components to use. Without a declining $k_0$, the sooner reconditioning starts, the lower the overall cost, but there is an upper limit to the quantity of reconditioned components to be used in the remanufacturing effort. In all examined cases remanufacturing results in higher profits. Extensions being investigated include using the reconditioned components in the primary production in addition to the spare parts; multiple quality grades of reconditioned components; and solving the system in a stochastic version of the problem. The current model assumed constant failure rate which is true for electronics as in the case study. For other types of components, the failure rate can be non-constant. Therefore, another extension will be to derive a model for time dependent failure rates.

5 REFERENCES


