Essays on Liquidity Frictions and Macroeconomic Dynamics

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Zusammenfassung


Zusammenfassung


Abstract

The essays collected in this dissertation explore macro-financial linkages arising from liquidity frictions. To this end, two distinct model frameworks are developed, which separately consider funding liquidity, i.e. the ease with which financial intermediaries can obtain short-term financing, and asset market liquidity, i.e. impediments to asset transactions. Both approaches highlight how liquidity conditions interact with macroeconomic dynamics. In particular, impairments in funding or asset market liquidity tighten financial constraints and trigger a flight to liquid assets, which serve as a hedge against future financing constraints. However, such liquidity hoarding is costly for the economy at large, since it crowds out funding for productive investment, thus amplifying and propagating initial shocks. By offering frameworks to study the nexus between liquidity frictions and macroeconomic dynamics, this dissertation attempts to further our understanding of the mechanisms that lie at the heart of financial crises.

Chapter 2 presents a model-based interpretation of the 2007-09 global financial crisis and the ensuing Great Recession, emphasizing a collateral crisis in the U.S. financial sector as its origin. To this end, banks operating subject to agency problems and funding liquidity risk in their intermediation activity are introduced into an otherwise standard Dynamic Stochastic General Equilibrium (DSGE) model. Balance sheet constraints force banks to trade off insurance against funding outflows with the scale of their loan portfolio. The amount of liquidity reserves held in the financial sector is, thus, determined endogenously. A financial crisis, simulated as an abrupt decline in the collateral value of bank assets, triggers a counter-cyclical flight to liquidity reserves, which strongly amplifies the initial shock and induces credit crunch dynamics. The model thus develops a new balance sheet channel of shock transmission which works through the composition of banks' asset portfolios.

While banks' exposure to funding liquidity frictions is at the centre of the mechanisms described in chapter 2, chapter 3 focuses on endogenous asset market liquidity by introducing search frictions into asset markets within a DSGE model. In this model, asset liquidity is tantamount to the ease of issuance and resaleability of private financial claims, which is driven by investors' participation in the asset market. This framework is able to generate
positive co-movement between asset saleability and asset prices. At the same
time, limited liquidity of private claims creates a role for liquid assets, such
as government bonds or fiat money, to ease funding constraints. When the
capacity of the asset market to channel funds to investors deteriorates due
to more severe frictions, the hedging value of liquid assets increases as in-
vestment falls. Thus, a version of the model calibrated for the U.S. economy
is able to match the liquidity hoarding observed during recessions, together
with the dynamics of key macroeconomic variables.

Chapter 4 extends the modelling framework developed in the previous
chapter to sovereign bond markets in order to study the transmission chan-
nels of stress from government debt markets to the real economy. Banks in
the euro area typically hold a large amount of government debt in their bond
portfolios, which are valued both for their low credit risk and high liquidity.
During the sovereign debt crisis, these characteristics of government debt
were severely impaired in stressed euro area countries. Chapter 4 presents
a standard DSGE model augmented with a banking sector and a market
for government debt characterized by search frictions with a view to disen-
tangling the interaction of sovereign credit and liquidity risk. A sovereign
solventch shock is modelled as a haircut on government bonds. As banks
react to this shock by rebalancing towards highly liquid short-term assets,
demand for government bonds collapses, which endogenously worsens their
market liquidity. Thus, a sovereign liquidity risk channel from government
bond markets to the real sector emerges. Endogenous government bond liq-
uidity negatively affects the funding conditions of the fiscal sector, tightens
financing constraints in the banking sector and lowers investment and out-
put. The model is able to match a number of stylised facts regarding the
behaviour of sovereign debt markets during the euro area sovereign debt
crisis, such as depressed turnover rates and rising bid-ask spreads.
## Acronyms

<table>
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<tr>
<td>DSGE</td>
<td>Dynamic Stochastic General Equilibrium</td>
</tr>
<tr>
<td>FTL</td>
<td>Flight To Liquidity</td>
</tr>
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<td>FTS</td>
<td>Flight To Safety</td>
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<tr>
<td>LTRO</td>
<td>Long-Term Refinancing Operation</td>
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<tr>
<td>MTS</td>
<td>Mercato Telematico dei titoli di Stato</td>
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<tr>
<td>OTC</td>
<td>Over-The-Counter</td>
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<td>TFP</td>
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Chapter 1

Introduction

1.1 Introductory Remarks

The essays collected in this dissertation explore macro-financial linkages arising from limited funding or asset market liquidity. Funding liquidity refers to the ease with which financial intermediaries can obtain funding, while market liquidity refers to the ease with which financial assets can be traded (either on markets, or via intermediaries). While being motivated by stylised facts that have emerged from the 2007-09 global financial crisis, this study of liquidity frictions harkens back to themes explored in the theory of banking as well as in early elaborations on the link between asset pricing and macroeconomic dynamics.

In the subsequent chapters, two distinct model frameworks are developed which consider funding frictions and impediments to asset transactions, respectively. Both approaches share a common theme in focussing on the feedback effects between liquidity conditions and macroeconomic dynamics. More specifically, macroeconomic - and in particular financial - shocks impair funding or asset market liquidity, such that financial constraints tighten endogenously. Both model frameworks embed a portfolio choice between liquid short-term assets and productive long-term investment. Deteriorating liquidity conditions trigger a Flight To Liquidity (FTL) as agents scramble for liquid assets to relieve their future financing constraints. However, liquidity hoarding is costly for the economy at large, since it crowds out funding for productive investment, thus amplifying and propagating initial shocks.

As the models presented in this thesis stand at the intersection of macroeconomics and finance, this chapter attempts to place them in the relevant theoretical literature of both fields. Empirical motivations are discussed in the introductory sections of the respective chapters.
1.1.1 Funding Liquidity and Macroeconomic Dynamics

The notion of funding liquidity frictions goes back to the theory of banks as inherently fragile entities. Banks create liquid claims on illiquid assets by issuing demand deposits. This process generally implies a maturity transformation between long-term assets and short-term liabilities. The resulting maturity mismatch exposes banks to the possibility of runs, as recognised in the seminal contribution of Diamond and Dybvig (1983). Bank runs typically trigger asset liquidations as banks struggle to obtain liquid funds to finance deposit outflows. Such liquidations are considered inefficient in the sense that recovery values of prematurely terminated investment or loan projects tend to be far below their continuation values. Similarly, secondary market prices for liquidated assets may not reflect the assets’ fundamental value even in the absence of asymmetric information problems. For instance, when many banks suffer funding outflows at the same time they may have to sell their assets at fire-sale prices. The anticipation of a fire-sale can, indeed, reinforce depositors’ incentives to run banks, thus triggering early asset liquidations and making the fire-sale expectation self-fulfilling (Allen and Gale, 2004). The mutually reinforcing nature of funding and market liquidity extends to securities markets with leveraged traders as shown by Brunnermeier and Pedersen (2008).

The Diamond-Dybvig style models of banking do not feature equity capital, i.e. the owners of banks do not suffer losses in case of bankruptcy. If, instead, banks have committed their own equity in funding long-term assets, the exposure to runs combined with the threat of costly asset liquidation introduces a motive to hedge against liquidity outflows. Such hedging can take the form of outright liquid asset holdings or insurance contracts that can be activated in case of deposit withdrawals. Both forms of hedging provide a buffer against funding liquidity risk. Holmström and Tirole (1998) develop a framework in which such liquidity buffers emerge endogenously from a trade-off between the initial size of investment projects and insurance against liquidity shocks. Although originally conceived to describe the creation of liquidity buffers by productive firms, this framework can be transferred to the banking sector, with liquidity shocks being interpreted as funding outflows and investment projects as bank loans. This model structure gives rise to an inverse relationship between the liquidation value (or market liquidity) of bank assets and the optimal size of liquidity buffers, i.e. the lower the liquidation value, the more banks hedge against funding liquidity shocks by flying to liquid assets. The analysis presented in chapter 2 studies the macroeconomic implications of this feedback mechanism by first adapting a version of the Holmström and Tirole (1998) model to the banking sector and then embedding it into a dynamic stochastic general equilibrium setting.
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1.1.2 Market Liquidity and Macroeconomic Dynamics

While theories of banking have entered the stage of dynamic micro-founded macroeconomic models fairly recently, asset pricing and portfolio choices have long been at the core of macroeconomic research. Tobin’s q-theory ascribes a central role to the ratio between the market price and the replacement cost of equity in linking asset markets, investment decisions and macroeconomic dynamics (Tobin, 1969). At the same time, Tobin emphasizes the coexistence of equity claims with other types of financial assets, such as money or bonds. In his view, these different types of financial assets are imperfect substitutes, which are held for distinct purposes. Money, in particular, is seen as catering to liquidity preferences of investors. Indeed, cash balances over and above what is necessary to satisfy transaction needs must reflect a desire to hedge against illiquidity of other financial claims with superior returns. ‘If cash is to have any part in the composition of investment balances, it must be because of expectations or fears of loss on other assets.’ (Tobin, 1958, p. 66) This idea harks back to Keynes’ speculative motive of cash balances (Keynes, 1936). In an economy, in which assets carry liquidity premia, a spectrum of asset returns emerges that reflects the respective functions of assets. By affecting this spectrum of returns, the relative supply of different assets can affect portfolio choices and macroeconomic dynamics; to the extent that open market operations by a central bank change the composition of assets held by the public, they may thus have real effects.

Drawing on these ideas, Kiyotaki and Moore (2012) develop a model of a monetary economy with assets of different liquidity. In this model, equity claims are exogenously assumed to be physically illiquid in the sense that they cannot be freely traded. Therefore, money emerges endogenously as a liquid store of value and hedge against funding constraints arising from limited market liquidity of equity claims. The spectrum of returns in this economy reflects these different asset characteristics. The irrelevance result of Wallace (1981) on the neutrality of a central bank’s asset portfolio no longer holds in such a setting. In fact, since the central bank controls the supply of money, open market operations change the composition of private agents’ portfolios and have real effects as suggested by Tobin. The Kiyotaki-Moore framework lends itself to the study of exogenous liquidity shocks as drivers of business cycles as well as the capacity of the central bank to attenuate such shocks through open-market operations.

However, as pointed out by Shi (2015), exogenous liquidity shocks imply counterfactual asset price dynamics: they act as negative supply shocks in the equity market and trigger persistent asset price booms which are at odds with the data. The framework proposed in chapter 3 addresses this short-coming by endogenously modelling limited asset liquidity as a consequence of search frictions. In this model, liquidity premia reflect primitive frictions, such that the spectrum of asset returns is fully endogenous. Em-
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Modeling asset market liquidity via search frictions follows the pioneering work of Duffie, Gârleanu, and Pedersen (2005), who apply search-theory to model trading frictions on Over-The-Counter (OTC) markets in a partial equilibrium framework. Building on this work, Lagos and Rocheteau (2009) show that asset demand in an OTC market depends not only on traders’ current asset valuations, but crucially also on their expected valuation over the holding period of an asset and the anticipation of future trading frictions. It is this sensitivity of asset demand to trading frictions which gives rise to a positive correlation between physical and price measures of asset market liquidity. Given inter-temporally optimising agents, this feature is also built into the model presented in chapter 3, i.e. demand for equity falls when trading is anticipated to be more costly in the future. In contrast, money becomes more attractive as a hedging device against future liquidity risks in these situations, such that the model generates a FTL.

The framework developed in chapter 3 provides a rationale for hedging portfolios, which are exposed to market liquidity risk, with liquid assets. As argued above, banks typically invest into illiquid assets. Rather than hedging the liquidity risk associated with these positions with large amounts of central bank reserves, they tend to hold sizeable portfolios of government bonds. In normal times, government bonds are highly liquid assets, which are also widely accepted as collateral in refinancing operations at small haircuts, and provide higher returns than central bank reserves. However, large bank-exposure to domestic government debt can give rise to a detrimental sovereign-bank nexus. In particular, a sovereign debt crisis severely impairs the liquidity of government bond markets. The funding constraints of banks with large exposure to government debt tighten as a result of a deterioration in the liquidity of government debt, which constrains their lending capacity. Stress on sovereign debt markets can, thus, propagate via banks to the real economy.

Such domestic costs of sovereign debt crises have been the subject of a host of recent research. For instance, Corsetti, Kuester, Meier, and Müller (2013) model a ‘sovereign risk channel’ through which increased risk premia on sovereign debt may feed into the refinancing costs of private firms, thereby constraining private sector borrowing and investment. While this study abstracts from financial intermediation, Gennaioli, Martin, and Rossi (2014) explicitly recognise the role of banks in propagating sovereign stress. In their set-up, banks use government bonds as liquidity reserves for financing future investment in the spirit of Holmström and Tirole (1998). Sovereign default is costly in that it dries up banks’ liquidity buffers, thereby reducing their intermediation capacity. The model presented in chapter 4 builds on these ideas by extending the search-theoretic framework for asset market liquidity to government bonds. In addition - and in contrast to the above-mentioned literature - it also increases the menu of available assets to include money. It thus provides an integrated framework to simultaneously think about a
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’sovereign liquidity risk channel’ and the real effects of the composition of public debt.

1.2 Overview and Main Results

Chapter 2 presents a model-based interpretation of the 2007-09 financial crisis and ensuing Great Recession as a collateral crisis in the financial sector. To this end, a DSGE model is augmented with a banking sector which is prone to FTL dynamics.

Banks are modelled as relationship-lenders, which extend loans to capital producers and provide monitoring services. Monitoring is assumed to be subject to moral hazard. Therefore, banks need to retain an equity stake in their loan projects (“skin in the game”) to satisfy incentive-compatibility and attract outside funding. Loan-origination is thus constrained by banks’ previously accumulated net worth. Idiosyncratic liquidity shocks hit banks in the form of deposit withdrawals before their loans mature. In order to keep loan projects afloat, banks need to refinance. However, they have limited access to outside funding at the refinancing stage due to the same frictions as during the initial loan origination. In anticipation of these constraints, banks endogenously choose to build liquidity reserves that can be tapped into once refinancing needs arise. Thus, a trade-off emerges between the initial loan scale and the amount of liquidity reserves that are set aside ex ante.

Since liquidity buffers are costly, banks do not fully insure against idiosyncratic liquidity risks, but choose an optimal amount of insurance that leaves them exposed to some residual risk of runs. Banks that are subject to deposit outflows in excess of their buffers are liquidated by outside investors. Banks’ equity is wiped out in the process and outside investors retain the collateral value of loan projects, which amounts to a fraction of the projects’ continuation value.

One key insight of this model is that the collateral value of loan projects is a crucial determinant of the liquidity reserves of banks. With lower collateral values, the liquidation of loan projects becomes more costly for outside investors. This weakens the threat of liquidation ex ante, such that banks can extract a higher share of the return to non-liquidated loan projects. Thus, when collateral values fall, banks respond by insuring more against liquidity risk at the expense of their loan scale.

Since the model does not distinguish between ordinary loan contracts and repurchase agreements, lower collateral values of loans are akin to higher haircuts in repurchase transactions. An abrupt exogenous decline in the collateral value of bank assets in the model thus mimics the ‘run on repo’ observed during the financial crisis in a stylised manner. By inducing a pronounced FTL and a concomitant credit crunch, the impact of numeri-
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cally small collateral shocks on investment and output is strongly amplified. The model can, thus, generate a recession that shares key features with the 2007-09 financial crisis and emphasizes a balance sheet channel of shock transmission that works through banks’ portfolio choices.

The collateral value, i.e. the market liquidity, of bank assets is exogenous in the above model. In contrast, the market liquidity of financial claims is modelled endogenously in the framework developed in chapter 3. This model thus lends itself to the study of interactions between fluctuations in market liquidity and macroeconomic dynamics. It sheds light on the cyclical behaviour of asset market liquidity, its impact on the funding constraints of firms and the propagation and amplification channels of shocks to the costs incurred in financial transactions.

The framework consists of an almost-standard real business cycle model augmented with a search market for financial asset. Financial assets are modelled as equity claims on investment projects undertaken by entrepreneurs. Sellers of such claims need to encounter buyers on a search market. As transactions can only occur when sellers are matched with buyers, a fraction of sellers will not be able to liquidate their assets offered for sale (both at issuance on primary markets and on secondary markets). Moreover, search market participation is costly for both sides. Both transaction costs and the physical impediments to asset transactions can give rise to illiquidity of financial claims. The transaction price of financial claims is determined in a bargaining process. This search market structure also lends itself to an interpretation of financial intermediation via banks, which provide a costly matching service for capital providers and seekers and determine the asset price so as to maximise the transaction surplus.

In this model set-up, market liquidity has both a quantity and a price dimension. The quantity dimension refers to the limited saleability of privately issued financial claims; the price dimension is captured by the sensitivity of the asset price and intermediation costs to the relative supply and demand conditions on the search market. The saleability of assets improves as demand increases relative to supply. Moreover, the search market structure gives rise to a non-trivial relationship between the asset price, asset saleability and intermediation costs, which is the main analytical result of the paper. In particular, the asset price correlates positively with asset saleability as long as the latter is low enough. Intuitively, when the saleability of assets is low, they provide little funding to entrepreneurs who seek liquid funds to finance new investment projects. Therefore, entrepreneurs are strongly financing-constrained and, thus, willing to accept a lower transaction price to ease their financing constraints somewhat with asset sales. Similarly, the asset price falls if the intermediation costs of buyers increase relative to those of sellers, as this shift in relative intermediation costs disproportionately deters demand and, thus, depresses the saleability of claims.

Since forward-looking entrepreneurs anticipate these funding constraints,
they wish to invest in a liquid asset to hedge against the possible deterioration of the market liquidity of privately issued financial assets. As a result of this hedging motive, an intrinsically worthless asset, such as money, is valued for its liquidity service and carries a liquidity premium. When market liquidity dries up, entrepreneurs rebalance their asset holdings towards liquid assets, such that the model can produce a FTL.

The model has several testable implications for the dynamic behaviour of asset market liquidity, investment and liquidity hoarding. Most notably, Total Factor Productivity (TFP) and shocks to the intermediation costs on the asset market have opposite effects on investors’ portfolio choices. Persistent TFP shocks reduce the current and future returns to investment projects and, thus, the incentive to hedge against future funding constraints due to asset market illiquidity. Intermediation cost shocks, on the other hand, do not affect the profitability of investment but rather impair the intermediation capacity of financial markets or banks. Investors thus value the hedging service provided by liquid assets more strongly and rebalance their portfolios toward them. As a result, demand on the asset market drops and asset price movements are amplified. Intermediation cost shocks thus allow the model to match the volatility of asset prices and liquidity hoarding and their co-movement with GDP in the data.

Finally, chapter 4 applies the theoretical apparatus developed in the previous chapter to understand interlinkages between sovereigns and banks arising from the special features of government bonds as highly liquid assets, which are widely held by financial institutions for refinancing purposes. This framework is employed to study the spillover of sovereign default risk via banks to the real sector and to identify feedback effects on the funding conditions of the fiscal sector.

In the model developed in this chapter, banks are assumed to extend loans to capital producers, which become illiquid after issuance. As banks commit resources in these long-term loan projects, they need to hold alternative saleable assets if they would like to take advantage of future profitable lending opportunities beyond their retained earnings. The model features two types of assets which can provide these liquidity services - short-term central bank deposits or money, which is assumed to be fully liquid, and long-term government bonds, whose liquidity is endogenously determined via a search market. The search-market structure can be thought of as capturing structural features of OTC government bond markets or refinancing operations collateralised by government debt. While long-term bonds are less liquid than money, they provide a higher return, such that banks invest in both types of assets in equilibrium.

In a scenario, which is reminiscent of the euro area sovereign debt crisis, an exogenous haircut on long-term government bonds is introduced to analyse the interaction between credit and liquidity risk in sovereign bond markets and their macroeconomic repercussions. A haircut impairs long-term
bonds’ hedging capacity against future financing constraints. As a result, investors exit the market and long-term bonds’ market liquidity deteriorates. Banks’ financing constraints tighten with the simultaneous surge in sovereign credit and liquidity risk exposure, thus amplifying the first-order wealth effects of the shocks. At the same time, banks rebalance their portfolios towards money as money’s liquidity service is more valuable in the sovereign default scenario. In sum, a sovereign liquidity risk channel from government bond markets to the real sector emerges.

The model also reveals a feedback loop between government finances and bond market conditions through general equilibrium effects. As demand for long-term bonds shrinks, so does their price, such that the funding costs of the government rise. Secondly, the FTL in the banking sector bids up the real value of debt, such that the government’s debt burden increases. If the government needs to cover its additional financing needs by issuing more debt, bond liquidity will further decline, which gives rise to a vicious circle between bond market stress and banks’ funding conditions. These rich macroeconomic interactions show that accounting for the economic role of government bonds and the microstructure of bond markets is key to understanding sovereign-bank interlinkages.
Chapter 2

Flight to Liquidity and the Great Recession


2.1 Introduction

The recent financial crisis and the ensuing Great Recession started with an abrupt surge in uncertainty about the value of assets on financial intermediaries’ balance sheets. As investors became worried about counter-party risk and the quality of collateral pools, financial institutions found it difficult to roll over short-term debt. In fact, several markets for short-term refinancing experienced runs: asset-backed commercial paper became illiquid in late 2007, followed by a freeze in the unsecured interbank-market after the fall of Lehman Brothers (Brunnermeier, 2009; Heider, Hoerova, and Holthausen, 2015). Similarly, average haircuts on collateral assets in repurchase agreements (repo) rose from 0 to 45% within the span of one year, effectively withdrawing $1.2 trillion in funding from the repo market (Gorton and Metrick, 2010, 2012; Duffie, 2010).

In response to the downward spiral of plummeting collateral values and rising funding liquidity risk, financial institutions took to hoarding liquid assets in order to reduce the maturity mismatch on their balance sheets. In the US, the flight to liquidity episode started in late 2007, manifested in a starkly rising share of liquid assets in total balance sheet size (see Figure 2.1). In fact, liquidity shares had been counter-cyclical since the early 2000s both in the traditional and in the shadow banking sector, with a contemporaneous cross-correlation of -0.46 and -0.40, respectively.¹

¹Liquid assets are defined as the sum of checkable deposits and currency, cash and reserves at the Federal Reserve, Treasury securities as well as agency- and securities backed...
However, the hoarding of liquid reserves locked up funds otherwise available for investment into riskier assets. This curtailed the lending capacity of the financial sector and eventually impaired non-financial firms’ access to external financing, thus propagating financial sector stress to the real economy.

Figure 2.1: Share of liquid assets of banks and market-based intermediaries

Notes: The liquidity share is computed as the sum of checkable deposits and currency, cash and reserves at the Federal Reserve, Treasury securities, agency and GSE-backed securities relative to total assets of the respective institutions. Source: US Flow of Funds (Federal Reserve)

In order to capture these events, this paper develops a model that features flight to liquidity in the banking sector as the key amplification and propagation mechanism of financial sector stress. Financial intermediaries operate subject to moral hazard problems in their monitoring activity and funding liquidity risk. They trade off the amount of liquidity reserves to hold as insurance against funding outflows with the amount of funds available for lending. Aggregate shocks to the collateral value of bank assets trigger a flight to liquidity in the sense of higher insurance against short-term funding risk. This amplifies the initial shock and induces credit crunch dynamics sharing key features with the Great Recession.

The model extends the canonical real business cycle model with Government-Sponsored-Enterprise (GSE). Of course, if these are truly liquid assets, they are expected to retain their value during a downturn, while prices of riskier assets would fall. Thus, the value of liquid assets relative to total balance sheet size would mechanically increase. However, the fact that liquidity buffers were not adjusted downwards suggests that a flight to liquidity occurred and banks’ willingness to lend declined.
2. Flight to Liquidity and the Great Recession

cial intermediation and liquidity risk at the bank level. In particular, banks intermediate funds between savers and capital producers and provide monitoring services for which they bear a cost. This private cost creates an agency problem between outside investors and banks: Banks need to retain an equity stake in their loans in order to receive a sufficient fraction of the return on lending that compensates them for their monitoring services.

As banks retain a fraction of the return on loans, the agency problem drives a wedge between the total return on loans and the amount that is pledgeable to outside investors. Building on Holmström and Tirole (1998), liquidity shocks arrive during the life-time of loan projects that require the input of additional resources. Here, such shocks are modelled as withdrawals of external funds, which are idiosyncratic at the bank level. Economically, they amount to rollover risk arising from a maturity mismatch between bank-assets and bank-funding. Limited pledgeability of loan returns constrains the funds that can be attracted to refinance these outflows, such that even projects with a higher continuation than liquidation value may have to be terminated.

Anticipating their financial constraints at the lending and at the refinancing stage, banks need to decide on how to optimally allocate their inside and outside funding between loans and liquidity reserves simultaneously. Given limited financial resources, earmarking funds as liquidity reserves decreases the scale of loans that banks can extend before liquidity shocks arrive. This trade-off implies that banks choose not to fully insure against liquidity risks. As a consequence, funding outflows cannot be entirely diversified despite being idiosyncratic. In particular, funding withdrawals in excess of liquidity reserves lead to the termination and inefficient liquidation of investment projects by the outside financiers.

Following evidence on rising haircuts in repo transactions for secured short-term finance, I introduce a shock to the liquidation - or collateral - value of bank assets as a novel type of aggregate risk. With lower collateral values, the liquidation of loan projects becomes more costly for outside investors. Banks can thus extract more resources after liquidity shocks have occurred, which makes liquidity reserves more compelling relative to the initial scale of loans.

The flight to liquidity unleashes a powerful amplification mechanism as higher liquidity reserves crowd out funds for initial bank lending (bank lending channel). These dynamics stand in sharp contrast to a frictionless economy where such crowding-out would not occur. Extending the model with nominal frictions, I also demonstrate how these interact with financial frictions to exacerbate the amplification from and recessionary impact of a liquidity crisis.

\[^2\]The liquidation value of a loan measures its resale value. This corresponds to the concept of market liquidity as defined in Brunnermeier, Eisenbach, and Sannikov (2012).
2. Flight to Liquidity and the Great Recession

The contribution of the paper is twofold: First, it introduces funding liquidity risk arising from a maturity mismatch between financial assets and external finance into a dynamic stochastic general equilibrium framework. Second, it explains how shocks operating directly on the balance sheets of financial intermediaries are amplified due to an endogenous increase in aggregate liquidity demand which emerges from the interaction between agency costs and funding liquidity risk. This adds to the literature on the balance sheet channel of shock transmission. However, amplification works through the endogenous composition of balance sheets, i.e. the choice between insurance against liquidity risk and lending scale, rather than fluctuations in the net worth of borrowers as in the financial accelerator literature. The model can explain a number of salient features observed during the recent financial crisis, such as the counter-cyclical flight to liquidity phenomenon as well as pro-cyclical lending and leverage.

2.1.1 Related Literature

This paper contributes to the growing body of literature on macro-financial linkages. It builds on two distinct strands of research. The first analyses financial frictions as the source of business cycle fluctuations. At the heart of this research is the balance sheet channel as surveyed by Bernanke and Gertler (1995), i.e. the amplification and propagation of business cycles due to a financial accelerator mechanism arising from the feedback between borrowing constraints and asset fire sales. Theoretical research in this area focuses on agency frictions between borrowers and lenders. Carlstrom and Fuerst (1997), Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999) and more recently Christiano, Motto, and Rostagno (2014) have embedded the costly-state-verification framework developed by Townsend (1979) in relation to financial contracts into business cycle models to study the dynamic impact of such agency costs. Other studies, such as Kiyotaki and Moore (1997), Gertler and Karadi (2011) and Gertler and Kiyotaki (2011), rely on limited or costly enforcement of financial contracts. Holmström and Tirole (1997) study an incentive model of financial intermediation where both firms and banks are capital constrained. The business cycle implications of this bank capital channel are analysed by Meh and Moran (2010), which is closely related to this paper.

However, the literature discussed so far does not accommodate the notion of endogenous liquidity demand. I introduce this feature following a second strand of literature initiated by Holmström and Tirole (1998). These authors develop a finite-horizon framework which motivates demand for corporate liquidity reserves with uncertain reinvestment needs during the lifetime of investment projects. Kato (2006) incorporates this structure into a dynamic general equilibrium setting to analyse the business cycle dynamics that result from liquidity risk at the corporate level. His model is able to replicate
firms’ counter-cyclical dependence on external finance and the hump-shaped response of output to shocks observed in the US. Covas and Fujita (2010) expand this analysis by adding regulatory capital requirements in the banking sector.

In Kurlat (2013) and Bigio (2013), liquid assets are needed to relax financial constraints in financing capital investment and working capital loans. Brunnermeier and Sannikov (2014) merge the financial accelerator framework of Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999) with the financial intermediation model of Holmström and Tirole (1997) to study asset fire sale episodes that endogenously decrease the market value of bank assets. These frameworks have in common that they focus on asset market liquidity, while funding liquidity risk is not considered. Similarly, they do not account for the flight to liquidity observed in the financial sector during the crisis.

The present paper merges the literature on the role of bank capital in the business cycle with the role of liquidity for relaxing financial constraints. However, I depart from previous research in a number of ways. First, liquidity risk is introduced at the bank level. Second, liquidity risk is modelled as funding risk. Third, the collateral value of liquidated investment projects for banks’ financiers is assumed to be non-zero.

Shocks related to financial markets are seen as potential drivers of the 2007-09 financial crisis. Gertler and Karadi (2011) model a shock to capital quality, which depresses the value of bank assets and triggers fire sales due to a leverage constraint imposed on banks. The resulting credit crunch drives the economic downturn. Meh and Moran (2010) investigate the business cycle consequences of a direct shock to bank capital, which in their model has limited recessionary impact. Other studies focus on shocks affecting the collateral value of financial assets. Kurlat (2013) and Bigio (2013) study the impact of information shocks that decrease the market liquidity of assets due to asymmetric information about asset quality, thus tightening the financing frictions in their respective models. In a model of endogenous information production on financial asset quality, Gorton and Ordoñez (2014) impose a shock on the perceived value of collateral assets whose true value is opaque. This leads to a breakdown of short-term debt markets relying on high-quality collateral. All of these scenarios have in common that the collateral value of assets held by financially constrained agents drops due to some exogenous disturbance.

Following these contributions, I introduce a shock to the liquidation value of bank assets as a new source of aggregate risk. This shock is intended to mimic the sudden drop in collateral values at the onset of the financial crisis described by Gorton and Metrick (2010, 2012). It is a parsimonious, reduced-form approach to capturing asset devaluations due to information, agency or search frictions. Moreover, it is more subtle than some of the financial-sector specific shocks introduced in the recent literature. For instance, unlike
Meh and Moran (2010) the shock does not operate directly on banks’ equity capital, but rather exploits their balance sheet constraints; unlike Gertler and Karadi (2011) it only assumes that the value of liquidated projects decreases, but not that of successful projects.

The remainder of this chapter is organized as follows. Section 2.2 develops the model. The baseline calibration of the model, the aggregate shock processes as well as simulation results are discussed in section 2.3. This section also offers insights into the role of nominal in addition to financial frictions. Section 2.4 concludes.

2.2 The Model

2.2.1 The Environment

Consider an economy populated by five types of agents: a continuum of agents with unit mass comprising a large fraction \( \eta^h \) of consumers (households) and a small fraction \( \eta^b = 1 - \eta^h \) of bankers; a mutual fund; and capital good and final good producers. Time is discrete and infinite (\( t = 0, 1, 2, ... \)).

There are two goods in the economy. Final or consumption goods are produced in a frictionless, competitive market. Capital goods are produced by entrepreneurs who convert final goods into capital goods subject to idiosyncratic risk of failure. Entrepreneurs are financed and monitored by banks, who suffer from agency problems towards their outside financiers and funding liquidity risk due to a maturity mismatch between their assets and liabilities. Banks receive funding in the form of deposits from a mutual fund, which intermediates the savings of the household sector. This fund also extends contingent credit lines that banks can draw on to partially buffer funding outflows. Each of these agents is now described in detail.

2.2.2 Capital Good Producers

Capital good producers manage investment projects in order to produce new capital goods. They have access to a stochastic constant returns to scale technology converting \( i_t \) units of consumption goods into \( R i_t \) units of capital, if successful, and zero otherwise.\(^3\) By exerting effort, entrepreneurs can ensure a high probability of success, \( \pi_H \). If they shirk, the success probability of investment projects is reduced to \( \pi_L < \pi_H \), but entrepreneurs earn a private benefit. This creates a moral hazard problem: in order to exert effort, capital producers have to be either compensated for foregoing private benefits or monitored.

\(^3\)The capital production technology is assumed to exhibit constant returns to scale in order to stay as close as possible to a frictionless real business cycle model. This avoids introducing distortions that would obscure the main mechanism analysed in this paper.
Any compensation for unmonitored entrepreneurs would reduce the return that an outside investor, who strives to induce effort, could extract from entrepreneurs. However, capital producers enter a close relationship with their lending banks when seeking external finance for their investment projects following Holmström and Tirole (1997). In this relationship, banks are assumed to have the capacity to detect shirking via some monitoring technology. Monitored entrepreneurs are prevented from shirking without having to be compensated. Hence, monitoring by banks eliminates the moral hazard problem in capital producers and ensures that entrepreneurs can channel all returns from investment projects to their lending banks.

2.2.3 Financial Sector

*Timing.* The intra-period timing assumptions are crucial for the set-up of the financial sector. Every period is divided into four subperiods (Figure 2.2). In the first subperiod, aggregate shocks occur and production of final goods takes place. Capital goods production extends over the last three subperiods: In subperiod 2, financial contracts are negotiated between banks and their outside investor (the mutual fund) and loans are extended to capital producers. Each bank extends a unique loan to an entrepreneur that is used to finance investment on a one-to-one basis, i.e. \( i_t = l_t \). \(^4\) After loans have been committed, liquidity shocks occur in subperiod 3 and loans need to be refinanced or liquidated. In the final subperiod, surviving loan projects have to be monitored by banks and new capital goods are produced by successful investment projects. Finally, all parties are paid according to their contracts.

*Liquidity shocks.* At the initial loan-financing stage in subperiod 2, banks receive outside funding in the form of intra-period deposits \( d_t \), which are collected from households and channelled to the banking sector by the mutual

\(^4\)Due to the lack of entrepreneurial capital, bank loans effectively amount to outside equity stakes in entrepreneurial projects.
fund. Liquidity shocks take the form of withdrawals of deposits that are idiosyncratic at the bank level and uncertain at the time of contracting. The funding outflows are proportional to the amount of external finance held by banks, i.e. a fraction \( \omega \) of deposits \( d_t \) is withdrawn, where the random variable \( \omega \in [0, U_H] \) is distributed according to the cumulative distribution function \( F(\omega) \) and density \( f(\omega) \). Withdrawals flow back to the household sector and are used for consumption purposes.\(^5\) Liquidity shocks may be thought of as rollover risk arising from a maturity mismatch between bank-assets and bank-funding. Assuming such idiosyncratic shocks at the bank-level serves as a short-cut for modelling heterogeneity in banks’ funding structures.

**First-best refinancing threshold.** Suppose that monitoring was costless for banks, such that the entire return on their loans could be pledged to outside investors. Initial loans could then be financed entirely with outside funding from the mutual fund in subperiod 2. In subperiod 3, banks need to refinance funding outflows \( \omega d_t \) in order to continue their loan project. In the absence of monitoring costs, the mutual fund would be willing to provide this additional funding up to the amount that makes it indifferent between the continuation of a loan project and its liquidation. The first-best refinancing or liquidity threshold after deposit withdrawals have materialised thus satisfies

\[
\underbrace{q_t \pi_H R_l t - \bar{\omega}_1 d_t}_{\text{continuation}} = \underbrace{q_t \xi^* l_t}_{\text{liquidation}}
\]

where \( \pi_H R_l t \) is the (expected) return to a successful, monitored loan project in terms of capital goods, \( q_t \) is the price of capital in terms of consumption goods and \( \xi^* \) is the exogenously determined fraction of the initial loan scale that the outside investor can salvage in case of liquidation. For any \( \bar{\omega}_1 > \bar{\omega}_1 \), loan projects have a lower continuation than liquidation value even in the absence of monitoring costs and the mutual fund would prefer liquidation. With monitoring costs, the first-best refinancing threshold cannot be implemented and liquidation will ensue following much smaller occurrences of the liquidity shock.\(^6\)

**Moral hazard.** By enforcing entrepreneurial effort through their monitoring activity, banks eliminate the agency friction in the capital production process. Monitoring services are assumed to involve private costs, such that the relationship between banks and their financier is affected by a moral hazard problem. In particular, monitoring banks incur costs in terms of fi-

\(^5\) An aggregate outflow of resources from the financial sector can be motivated by resource losses in financial intermediation arising from costs faced by outside investors. Institutional investors may, for instance, have to put up additional resources to investigate banks’ soundness or monitor their behaviour while funding them. Such costs would need to be covered by a premium on external funding arising while the relationship is ongoing.

\(^6\) For further details see Proposition 1.
nal goods which are proportional to project size, i.e. $\mu l_t$. They must earn a minimum return in order to cover this cost and thus cannot pledge the entire return on their loan to the mutual fund. Therefore, neither initial loans nor liquidity needs at the refinancing stage can be fully financed with outside funding. As a result, banks need to provide some inside funding in the form of bank capital $a_t$, which is accumulated through retained earnings.\(^7\)

**Key mechanism.** Anticipating their financial constraints both at the initial contracting and at the refinancing stage, banks need to decide on how to optimally allocate their available inside and outside funds between initial loans and liquidity reserves (plus expected monitoring costs) simultaneously. This portfolio allocation also needs to ensure a non-negative expected return for the fund (investor participation constraint) and induce monitoring efforts by banks (incentive compatibility).\(^8\) The financial contract that achieves these objectives is discussed in section 2.2.3. Given limited financial resources, earmarking funds as liquidity reserves reduces the scale of loans that banks can initially extend. This trade-off is the key mechanism of the model.

**Liquidity reserves.** The optimal amount of liquidity reserves that follows from the trade-off with banks’ loan scale is the central element spelled out by the financial contract between banks and their outside investor. Liquidity reserves take the form of uncommitted funds that banks leave with the mutual fund in subperiod 2, which entitle them to draw a credit line after having been hit by a funding outflow in subperiod 3. Banks choose the optimal liquidity buffer by capping their contingent credit line at some upper threshold $\bar{\omega}_t < \bar{\omega}_1$. This will allow them to obtain up to $\bar{\omega}_t d_t$ in additional funding from the mutual fund in case of liquidity shocks. Since these shocks are uncertain at the time of contracting, the corresponding reserve that is set aside in subperiod 2 amounts to the expected liquidity outflow given the optimal threshold, i.e. \(\left(\int_{0}^{\bar{\omega}_t} \omega dF(\omega)\right) d_t\).\(^9\)

Refinancing in excess of the liquidity threshold would violate banks’ incentive to engage in costly monitoring. Hence, projects with funding outflows $\omega > \bar{\omega}_t$ are liquidated by the outside investor. Given the distribution of the liquidity shock, the *ex ante* probability of an investment project to survive the funding shock is $\int_{0}^{\bar{\omega}} f(\omega) d\omega = F(\bar{\omega}) < F(\bar{\omega}_1)$.\(^10\) This shows that liquidity shocks - although idiosyncratic at the bank-level - cannot be fully

\(^7\)For details on the accumulation of bank capital see section 2.2.3.
\(^8\)I rule out equilibria in which effort is not induced for bankers.
\(^9\)This arrangement between a bank and the mutual fund is akin to an insurance contract with the liquidity buffer resembling the insurance premium. The financing of the actual credit lines that are granted by the mutual fund after liquidity shocks materialise is discussed in section 2.2.3.
\(^10\)Note that liquidation only occurs once liquidity buffers have been exhausted entirely. The liquidation value $\xi^*_t$ is hence independent of the funding characteristics of banks and the mutual fund has no resources other than unfinished loan projects to draw on.
diversified due to the agency problem between banks and the mutual fund. Accordingly, some investment projects with positive net present value, i.e. continuation values in excess of liquidation values, have to be terminated even when banks accumulate liquidity reserves.

**No partial liquidation.** Loan projects are assumed to be indivisible given banks’ role as relationship lenders. Banks acquire specific information about their debtors, which is costly, because it takes time to build. Partial liquidation of loan projects would disrupt this relationship and hence worsen both the monitoring incentives and ability of banks. Buyers of tranches of other intermediaries’ loan portfolios could at best substitute imperfectly for the monitoring services provided by the original lender. Thus, partial liquidation would aggravate the moral hazard problem between banks and depositors, increase further the wedge between total and pledgeable returns and thus make loan financing more difficult. For these underlying structural reasons, partial liquidation is ruled out.\(^{11}\)

**Intra-period Financial Contract**

*Constrained problem.* The optimal financial contract is a set \(\{l_t, d_t, R^b_t, R^h_t, \bar{\omega}_t\}\) which specifies the level of loans \(l_t\), the amount of deposits \(d_t\), the distribution of per unit project return \(R\) to banks, \(R^b_t\), and households, \(R^h_t\), as well as the threshold level of the liquidity shock, \(\bar{\omega}_t\), which banks can accommodate by tapping into their liquidity buffer. General equilibrium effects have an impact on the financial contract through the beginning-of-period relative price of capital \(q_t\), the previously accumulated capital of banks \(a_t\) and the stochastic liquidation value \(\xi^*\). At the time of contracting, these are, however, exogenous.

Since the contracting problem takes place within a period, time subscripts are omitted in the description of the optimal contract. Formally, the contract maximises banks’ expected return from loans to entrepreneurs subject to incentive compatibility, participation, and feasibility constraints:

\(^{11}\)Fixed monitoring costs would also be an obstacle to partial liquidation: Every time a loan tranche was sold, buyers would have to incur these fixed costs, which would amount to significant efficiency losses in financial intermediation and render partial liquidation unattractive. As discussed by Holmström and Tirole (1998), partial liquidation is also not an optimal policy in this setup. Agents prefer full continuation as long as they can accommodate liquidity shocks and full liquidation if shocks exceed their liquidity buffers.
The objective function accounts for the fact that the probability of successfully executing a project of scale \( l \) is \( F(\bar{\omega})\pi_H \), since the \textit{ex ante} probability of a non-excessive liquidity shock is \( F(\bar{\omega}) \), and the probability of yielding non-zero output is \( \pi_H \). As indicated by their incentive compatibility constraint (2.1), bankers need to be compensated with \( R^b \geq \mu q l (F(\bar{\omega}) - \pi_L) \) in order to monitor entrepreneurs. The share of loan returns that banks need to retain captures the severity of the moral hazard problem with respect to banks’ outside financiers and drives the crucial wedge between the full and the pledgeable return to loans as \( R - R^b = R^h \). Equation (2.2) is the participation constraint of the intermediating mutual fund. It requires that the expected return accruing to investors - composed of the expected return from successful projects as well as the liquidation value of unsuccessful ones - is sufficient to pay back the intra-period deposits lent to the financial sector at the beginning of the period. The balance sheet constraint (2.3) ensures that banks’ internal and external funds cover their expected expenses consisting of loans inclusive of monitoring costs related to surviving projects as well as the insurance set aside to accommodate anticipated funding outflows. Finally, (2.4) states that the returns accruing to individual agents add up to the total return from a successful project.

\textit{Unconstrained problem.} Since the objective function is linear in project scale \( l \), a finite solution for \( l \) can only exist when the balance sheet constraint binds. Moreover, banks maximise the amount of external resources by demanding the smallest feasible compensation for themselves and paying the smallest amount to depositors that ensures their participation. All constraints will hence bind at the optimum. Combining binding constraints (2.2) and (2.3) yields the loan scale as a function of bank capital and the liquidity cut-off:

\begin{align}
\max_{\{d, l, R^b, R^h, \omega\}} & \quad qF(\bar{\omega})\pi_H R^b l \\
\text{s.t.} & \quad qF(\bar{\omega})\pi_H R^b l - F(\bar{\omega})\mu l \geq qF(\bar{\omega})\pi_L R^b l \\
& \quad q \left[ F(\bar{\omega})\pi_H R^h + (1 - F(\bar{\omega}))\xi^* \right] l \geq d \quad (2.2) \\
& \quad d + a \geq (1 + F(\bar{\omega})\mu) l + \left( \int_{\bar{\omega}}^\infty \omega dF(\omega) \right) d \quad (2.3) \\
& \quad R = R^b + R^h \quad (2.4)
\end{align}

12The incentive compatibility constraint of banks tightens when monitoring costs rise or when the probability differential between monitored and non-monitored loans decreases. In both cases the expected payoff from monitoring over non-monitoring shrinks. Hence, the compensation for performing monitoring activities must rise, which reduces the amount of loan returns that is pledgeable to outside investors.
where \( H(\bar{\omega}) \equiv 1 + F(\bar{\omega})d - q \left( 1 - \int_{0}^{\bar{\omega}} \omega dF(\omega) \right) (F(\bar{\omega})\pi_{H}R^{h} + (1 - F(\bar{\omega})))\xi^* \).

Banks’ loan scale is thus linear in their capital with a leverage ratio of \( H(\bar{\omega})^{-1} \). Plugging the loan function back into the objective function yields the unconstrained problem with the liquidity threshold \( \bar{\omega} \) left as the only choice variable:

\[
\max_{\{\bar{\omega}\}} \frac{F(\bar{\omega})}{H(\bar{\omega})} q\pi_{H}R^{h}a
\]

Indifference threshold. Note that similar to the upper bound for the liquidity threshold \( \bar{\omega}_{1} \), there is a natural lower bound \( \bar{\omega}_{0} \). Suppose banks want to maximize the scale of their loan project. As bank capital is accumulated from retained earnings and therefore fixed, this is equivalent to attracting the largest possible amount of external financing, which consists of the sum of initial funding and the refinancing of liquidity shortfalls.\(^{13}\)

Intuitively, the outside investor, i.e. the mutual fund, is willing to provide external financing until the benefit of continuation of a loan project equals its liquidation value. Conveniently, maximising the total amount of external finance, i.e. the initial loan financing plus subsequent refinancing, can be reduced to the choice of the liquidity threshold. In particular, the loan scale is pinned down as a function of banks’ liquidity buffer by the participation and balance sheet constraints (2.2) and (2.3). Maximizing external funding thus amounts to choosing the threshold for liquidity reserves \( \bar{\omega}_{0} \) that maximizes the loan scale according to the funding constraint (2.5), i.e. \( \max_{\bar{\omega}} \frac{a}{H(\bar{\omega})} = \frac{a}{H(\bar{\omega}_{0})} \). To interpret this result, consider the corresponding first order condition, which can be expressed as\(^{14}\)

\[
\frac{q\pi_{H}R^{h}l}{\text{pledgable return}} - \left( 1 - \int_{0}^{\bar{\omega}_{0}} \omega dF(\omega) \right)^{-1} (\bar{\omega}_{0}d + \mu l) = \frac{q\xi^*l}{\text{liquidation value}}
\]

Indeed, this condition suggests that at \( \bar{\omega}_{0} \), the minimal continuation value of a loan project from the perspective of the outside investor equals its liquidation value. The minimal continuation value is the difference between the pledgeable return from the project and its maximum continuation cost.

\(^{13}\)Since capital is fixed at the time of contracting, maximizing external funding is tantamount to achieving the highest feasible leverage.

\(^{14}\)See appendix 2.B.1 for a derivation.
This cost consists of the highest amount $\bar{\omega} d$ that may be refinanced and monitoring costs $\mu l$. Both are scaled up by the fraction of deposits that is retained by the mutual fund needing to bear these costs.

Intuitively, for any liquidity threshold $\bar{\omega} < \bar{\omega}_0$, the fund always prefers continuation of the loan project because the return even after refinancing liquidity shocks up to $\bar{\omega} d$ exceeds the project’s liquidation value. The mutual fund would, in fact, be willing to expand its funding of liquidity shortfalls until being indifferent between continuation and liquidation of a project. Anticipating this incentive at the contracting stage, banks will never choose any buffer below the indifference threshold in order to take full advantage of external funding.

*Optimal liquidity threshold.* Since external resources are maximized at the indifference threshold for liquidity reserves, choosing liquidity reserves in excess of this threshold inevitably reduces the amount of total available external funding.\(^\text{15}\) Thus, for any $\bar{\omega}_0 < \bar{\omega} < \bar{\omega}_1$ a trade-off emerges between total external funding and liquidity reserves. Given a fixed amount of bank capital, lower external financing will also tighten the funding constraint (2.5) and reduce the loan scale.

Despite this trade-off, banks optimally choose liquidity reserves in excess of the indifference threshold. To see this, note that - besides the loan scale - banks’ expected return also depends on the survival probability of loan projects $F(\bar{\omega})$, which increases monotonically in the choice of the liquidity threshold $\bar{\omega}$. Therefore, banks will not seek to maximize their loan scale by setting $\bar{\omega} = \bar{\omega}_0$, but rather choose a liquidity buffer that lies between the indifference and the first-best threshold. These results are formally stated in the following proposition.

**Proposition 1:**

The liquidity threshold $\bar{\omega}$

(i) lies in the interval $(\bar{\omega}_0, \bar{\omega}_1)$, where the lower bound $\bar{\omega}_0$ designates the indifference threshold, which is implicitly determined by

$$\bar{\omega}_0 = \frac{\left(1 - \int_{0}^{\bar{\omega}_0} \omega dF(\omega)\right) (\pi_H R^h - \xi^*) - \frac{\mu}{q}}{F(\bar{\omega}_0)\pi_H R^h + (1 - F(\bar{\omega}_0))\xi^*} \quad \text{(2.7)}$$

(ii) the upper bound $\bar{\omega}_1$ is the first-best liquidity threshold, which is implicitly determined by

$$\bar{\omega}_1 = \frac{\pi_H R - \xi^*}{F(\bar{\omega}_1)\pi_H R + (1 - F(\bar{\omega}_1))\xi^*} \quad \text{(2.8)}$$

\(^{15}\)Analytically, this can be seen by considering the partial derivative of the leverage ratio, $\frac{\partial H(\bar{\omega})}{\partial \bar{\omega}}^{-1} \bigg|_{\bar{\omega} > \bar{\omega}_0} < 0$. See appendix 2.B.1 for a derivation.
(iii) and its optimal value is implicitly determined by the first-order condition of problem 2.6

\[ 1 = q \left[ 1 - \int_0^{\bar{\omega}} \omega dF(\omega) \right] \xi^* + \bar{\omega} F(\bar{\omega}) \left( F(\bar{\omega}) \pi_{HR} h + (1 - F(\bar{\omega})) \xi^* \right) \]

\[ \equiv Q(\bar{\omega}) \] (2.9)

where \( \xi^* = \xi + z\xi \).

**Proof.** See appendix 2.B.1.

**Comparative statics.** The tension between liquidity reserves and loan scale is the key mechanism that links financial sector outcomes with macroeconomic dynamics. The optimal liquidity buffer fluctuates with the exogenous or pre-determined factors entering the financial contract and, in turn, affects the amount of lending to entrepreneurs and capital production in the economy. To develop an intuition for the simulation results in section 2.3, consider the impact of changes in the stochastic liquidation value of loan projects \( \xi^* \) and the capital price \( q \) on optimal liquidity reserves. Under very mild parameter restrictions, the liquidity threshold correlates negatively with the liquidation value\(^{16} \)

\[ \frac{\partial \bar{\omega}}{\partial \xi^*} = -\frac{\partial Q/\partial \xi^*}{\partial Q/\partial \bar{\omega}} < 0 \] (2.10)

Intuitively, a lower liquidation value reduces the incentive of the outside investor to terminate a project after liquidity withdrawals have occurred, such that the indifference threshold \( \omega_0 \) increases. Thus, banks are able to extract more refinancing from the mutual fund *ex post* at any initial loan scale. In other words, the participation constraint of the outside investor becomes less sensitive to the liquidity threshold when the liquidation value shrinks. This shifts the *ex ante* trade-off between liquidity reserves and the loan scale in favour of a higher accumulation of reserves from the perspective of banks. Note that the trade-off does not disappear altogether, such that a higher liquidity threshold will still constrain the initial lending scale. However, with a lower liquidation value the contraction in the lending scale will be less severe for a given increase in the liquidity buffer.

The optimal liquidity threshold is also negatively correlated with the capital price, i.e.

\[ \frac{\partial \bar{\omega}}{\partial q} = -\frac{\partial Q/\partial q}{\partial Q/\partial \bar{\omega}} < 0 \] (2.11)

A lower asset price \( q \) decreases the market value of loan projects, and thus their marginal profitability. When the profitability of loan projects is low the

\(^{16}\)The partial derivatives of \( \bar{\omega} \) are derived in appendix 2.B.2.
opportunity cost of shifting resources to the liquidity buffer is lower. In other words, the balance-sheet trade-off between scale and reserves is weakened when the asset price falls. As a result, banks increase their liquidity reserves, which increases the survival probability of loans.

The Mutual Fund and Endogenous Liquidity Supply

The mutual fund intermediates households’ deposits at the beginning of subperiod 2 to provide external funding to banks. When loan contracts have been completed at the end of subperiod 4, it collects the pledgeable proceeds of loans from banks and channels them to depositors. The fund finances initial loans and also acts as a liquidity backstop for the banking sector by partially insuring liquidity risks. Although deposit withdrawals are idiosyncratic at the bank-level, the mutual fund cannot diversify them up to the first-best threshold $\bar{\omega}_1$, as discussed in section 2.2.3, because of the moral hazard problem faced by banks.

The insurance scheme allows banks to draw on a contingent credit line only up to a maximum amount $\bar{\omega}d < \bar{\omega}_1d$ after funding outflows, which is compatible with their incentive to provide monitoring services. However, banks do not set aside the full amount as a buffer with the mutual fund, but rather an amount equal to the expected liquidity shock $\left(\int_0^{\bar{\omega}} \omega dF(\omega)\right)d < \bar{\omega}d$. To understand how the mutual fund can refinance liquidity outflows up to the promised amount, note that liquidity shocks are independent across banks by assumption. Since there is a continuum of banks, the expectation of the refundable funding outflow from any bank is equal to the aggregate funds actually withdrawn from the financial sector ex post. Therefore, aggregate liquidity demand that the fund is asked to refinance in subperiod 3 is deterministic and given by

$$W = \left(\int_0^{\bar{\omega}} \omega dF(\omega)\right)D$$

where $D = \eta^b d$. In order to provide this amount, the mutual fund redistributes banks’ liquidity reserves from those with low outflows to those with high outflows. However, in order to ensure the participation of depositors, the fund can only satisfy the aggregate liquidity demand as long as it does not exceed the pledgeable returns to loans. The latter are equal to the market value of the banking sector in subperiod 3, which amounts to

$$V = q \left\{F(\bar{\omega})\pi_H R^b + (1 - F(\bar{\omega}))\xi^*\right\} L$$

where $L = \eta^b l$. This market value always exceeds banks’ liquidity need as the fraction of liquidity outflows is bounded by 1 from above, such that
2. Flight to Liquidity and the Great Recession

\[ V - W = \left[ 1 - \int_0^{\bar{\omega}} \omega \, dF(\omega) \right] D \geq 0. \]  

Hence, there is no aggregate shortage of valuable claims on the banking sector when liquidity shocks arrive, such that the insurance scheme up to the optimal liquidity threshold \( \bar{\omega} \) is feasible.

Since the aggregate refundable liquidity demand in the banking sector is deterministic, the fund can offer a riskless rate of return to depositors, which ensures risk neutrality of households with respect to deposits.

**Evolution of Bank Capital**

Each period, \( 1 - \tau^b \) bankers exit the financial sector and are replaced by a continuum of new bankers of the same mass. The share of bankers in the economy thus stays constant at \( n^b \). Bankers save the proceeds from their intermediation activity by accumulating capital \( k^b_t \). They derive income from renting their capital out to final goods producers and supplying one unit of labour inelastically to the same sector. After final goods production is completed, they earn the respective factor rents. Labour income provides small positive start-up funds even to assetless new bankers. Bank capital in subperiod 2 thus equals

\[ a_t = (q_t(1 - \delta) + r_t^a)k^b_t + w^b_t \]  

(2.12)

Each banker invests his entire capital into a loan project yielding \( R^b_t l_t \) if successful and zero otherwise. The proceeds can either be saved or consumed. The inter-temporal flow of funds of individual banks is

\[ c^b_t + q_t k^b_{t+1} = (1 + r^a_t) a_t = q_t F(\bar{\omega}_t) \pi_H R^b_t l_t \]

where \( 1 + r^a_t = \frac{q_t F(\bar{\omega}_t) \pi_H R^a_t}{H(\bar{\omega})} \) is the gross return on bank capital and the last line uses equation (2.5). Successful surviving bankers save the entire proceeds from their lending activity in capital goods. This is the optimal consumption-savings choice given bankers’ risk-neutrality and the high return on internal funds.\(^{19}\) Bankers’ whose projects yield no return lose all their capital and, accordingly, neither save nor consume. Exiting bankers consume their entire assets.

\(^{17}\)In particular, given a uniform distribution of the liquidity shock on the interval \([0, U_H]\), where \( U_H \in [0, 2] \):

\[ \bar{\omega} < U_H < \sqrt{2}U_H \implies \int_0^{\bar{\omega}} \omega \, dF(\omega) < 1 \implies V - W > 0 \]

\(^{18}\)Other arrangements that achieve the same risk-sharing outcome, such as banks directly holding a stake in the market portfolio of the banking sector, are discussed in Holmström and Tirole (2011).

\(^{19}\)The model calibration ensures that the marginal benefit always exceeds the marginal cost of saving for surviving bankers, i.e. \( (1 + r^a_{t+1}) (q_{t+1}(1 - \delta) + r_{t+1}) > q_t \).
The ad hoc assumption of a finite lifetime for bankers ensures the stationarity of aggregate bank capital. If bankers did not exit the economy to consume their assets they would eventually accumulate enough wealth to finance investments exclusively with internal funds.\(^{20}\)

### 2.2.4 Final Good Producers

Final good producers operate in a competitive, frictionless market. They use the aggregate capital stock \(K_t\) rented from households and bankers and aggregate labour supplied by households \(H^h_t\) and bankers \(H^b_t\) as inputs into production.

\[
Y_t = \exp(z_t) F(K_t, H^h_t, H^b_t) \tag{2.13}
\]

where \(\exp(z_t)\) is total factor productivity. Factors earn their marginal product, such that the interest rate on capital is \(r_t = \exp(z_t) F_K(K_t, H^h_t, H^b_t)\) and wages are given by \(w_i = \exp(z_t) F_{H_i}(K_t, H^h_t, H^b_t)\) for \(i \in \{b, h\}\).

### 2.2.5 Households

There exists a continuum of households of mass \(\eta^h\). Households are risk averse and maximise utility over consumption \(c^h_t\) and labour \(h^h_t\) subject to their individual budget constraints. At the beginning of each period, households lend previously accumulated capital \(k^h_t\) to final goods producers and supply labour to the same sector. Both factors are remunerated with their respective rents. Likewise, last period’s bonds pay a gross riskless return \(1 + r^b_t\). Capital depreciates at rate \(\delta\). Then households make their consumption-savings decision. In order to save, they have two options: purchasing one-period risk-free bonds or channelling funds to banks via the mutual fund. After banks have performed their intermediation activity and investment projects generate returns, \(q_t\) units of new capital goods are transferred to households for every unit of savings input. Choosing the amount of deposits is thus equivalent to choosing how much capital to hold in the future. Accordingly, the optimization problem takes the form

\[
\max \{c^h_t, k^h_{t+1}, b^h_{t+1}, h^h_t\} E_0 \sum_{t=0}^{\infty} \beta^t u(c^h_t, h^h_t) \\
\text{s.t. } c^h_t + q_t k^h_{t+1} + b^h_{t+1} = (1 + r^b_t)b^h_t + (q_t (1 - \delta) + r_t)k^h_t + w^h_t h^h_t \tag{2.14}
\]

\(^{20}\)This is a well-known property of macroeconomic models with financially constrained agents, shared, for instance, by Bernanke, Gertler, and Gilchrist (1999), Gertler and Karadi (2011), Gertler and Kiyotaki (2011) or Christiano, Motto, and Rostagno (2014).
The corresponding first order conditions for consumption, capital stock, bonds and labour supply read

\[ u_{c,t} = \lambda_t \]  
(2.15)

\[ \lambda_t = \beta E_t \left[ \frac{\lambda_{t+1} q_{t+1} (1 - \delta) + r_{t+1}}{q_t} \right] \]  
(2.16)

\[ \lambda_t = \beta E_t \left[ \lambda_{t+1} (1 + r_{t+1}^b) \right] \]  
(2.17)

\[ u_{h,t} = -\lambda_t w_t^h \]  
(2.18)

where (2.16) and (2.17) are the Euler equations with respect to capital and bonds, respectively.

2.2.6 Aggregation and Competitive Equilibrium

Due to linearities in the financing and production of capital goods, aggregation turns out to be straightforward. In particular, the production technology for new capital goods and monitoring costs are linear in loans. The distribution of bank capital, therefore, has no effect on aggregate loans \( L_t \) and investment \( I_t = L_t \), which are simply the sum of individual loans:

\[ L_t = \eta^b l_t \]

\[ = \eta^h a_t \]

\[ = \frac{A_t}{H(\bar{\omega}_t)} \]  
(2.19)

using the individual loan function (2.5).

The economy-wide equivalent to depositors’ participation constraint (2.2) pins down aggregate deposits.

\[ D_t = \eta^b d_t \]

\[ = q_t \left\{ F(\bar{\omega}_t) \pi_H R_t^b + (1 - F(\bar{\omega}_t))\xi^* \right\} L_t \]  
(2.20)

Aggregate stocks of capital holdings are the sum of individual stocks.

\[ K_t^b = \eta^b k_t^b, \quad K_t^h = \eta^h k_t^h \]  
(2.21)

The elasticity of labour supply differs across agents. Bankers individually supply one unit of labour inelastically, while households’ supply is elastic.

\[ H_t^b = \eta^b, \quad H_t^h = \eta^h h_t^h \]  
(2.22)

Aggregate bank capital is

\[ A_t = (q_t (1 - \delta) + r_t) K_t^b + H_t^b w_t^b \]  
(2.23)
2. Flight to Liquidity and the Great Recession

The average return on loans for bankers is $F(\bar{\omega}_t)\pi_H R_b^h L_t$. As discussed, surviving bankers invest all their proceeds into new capital goods. Since only a fraction $\tau^b$ survives, next period’s capital holdings by the banking sector will be

$$K_{t+1}^b = \tau^b F(\bar{\omega}_t)\pi_H R_b^h L_t \quad (2.24)$$

Exiting bankers consume their wealth and aggregate household consumption amounts to the sum of individual households’ consumption.

$$C_t^b = (1 - \tau^b)q_t F(\bar{\omega}_t)\pi_H R_b^h L_t \quad (2.25)$$

$$C_t^b = \eta_h c_t^b \quad (2.26)$$

The competitive equilibrium of the economy is a collection of (i) decision rules for $c_t^h, K_{t+1}^b, b_{t+1}^h, h_t^h$ that solve the maximization problem of households; (ii) decision rules for $K_t, H_t^b, H_t^h$ that solve the maximization problem of final good producers; (iii) decision rules for $l_t, d_t, R_t^b, R_t^h, \bar{\omega}_t$ associated with the financial contract that solves the maximization problem of banks; (iv) consumption $c_t^b$ and saving $k_{t+1}^b$ rules for bankers; (v) laws of motion for the exogenous processes $z_t, z_t^\xi$, and market clearing conditions for final goods, labour, capital goods, investment, loans and bonds:

$$C_t^h + q_t K_{t+1}^b + B_{t+1} = (1 + r_t^b)B_t + (q_t (1 - \delta) + r_t)K_t^b + w_t^h H_t^h \quad (2.27)$$

$$H_t = H_t^b + H_t^h \quad (2.28)$$

$$K_{t+1} = (1 - \delta)K_t + (F(\bar{\omega}_t)\pi_H R + (1 - F(\bar{\omega}_t))\xi) I_t \quad (2.29)$$

$$q_t L_t = q_t I_t \quad (2.30)$$

$$B_t = 0 \quad (2.31)$$

2.3 Quantitative Results

2.3.1 Calibration and Functional Forms

Period-utility - a function of consumption and hours worked - takes the following functional form:

$$u(c_t^h, h_t^h) = c_t^h \frac{1}{1 - \theta} + \nu \ln(1 - h_t^h) \quad (2.33)$$

The parameter $\theta$ governs the degree of relative risk aversion or the elasticity of intertemporal substitution of consumption. It is set to a standard value of 1.5 following Kato (2006). The weight on leisure, $\nu$, is chosen to match

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21 Appendix 2.A.1 lists the complete set of equilibrium conditions.
a fraction of working time of 30\%. Additionally, households\’ discount factor is set to a standard value of 0.99, which yields a riskless quarterly interest rate of 1\%.

Final goods are produced with a standard Cobb-Douglas technology

\[ F(K_t, H_t^h, H_t^b) = K_t^{\alpha_k} H_t^{\alpha_h} H_t^{\alpha_b} \]  

(2.34)

where \( \alpha_k + \alpha_h + \alpha_b = 1 \). This paper follows Meh and Moran (2010) in setting the capital share of output to 0.36 and the share of labour provided by bankers to a very small number \((5 \times 10^{-5})\), such that its effect on the dynamics is negligible.

Capital production is characterized by two parameters. A quarterly depreciation rate of capital of \( \delta = 0.025 \) is in line with many RBC studies of the US economy including King and Rebelo (1999), Kato (2006) and Covas and Fujita (2010). There is less precedent for the second parameter choice, \( R \), i.e. the return to investment in capital production. This parameter is calibrated such that the total return to investment with full buffering of liquidity shocks is one, i.e. \( \pi_H R = 1 \).\(^{22}\)

Financial intermediation and the associated frictions are characterized by the set of parameters \( \{\mu, \xi, \sigma^2(\omega), \tau^b, \pi_H, \pi_L\} \). The parameters \( \pi_H \) and \( \pi_L \) capture the idiosyncratic failure risk of entrepreneurs under effort and shirking. Following Meh and Moran (2010), \( \pi_H \) is set to 0.9903, which translates into a quarterly failure rate of entrepreneurs of 0.97\%, as in Carlstrom and Fuerst (1997), and \( \pi_L = 0.75 \).

The subset \( \{\mu, \xi, \sigma^2(\omega), \tau^b\} \) is jointly determined to match: (i) a bank-leverage ratio, defined as the ratio of debt to equity \( \Xi_t = \frac{D_t}{A_t} \), close to 13.44. This roughly corresponds to the average leverage ratio of the US financial sector composed of banks and market-based financial institutions over the past 30 years (Figure 2.6).\(^{23}\) (ii) A loss given default (LGD) on bank loans of roughly 40\% following Covas and Fujita (2010). In the model, the LGD corresponds to \( LGD_t = 1 - \frac{q^t \xi I_t}{A_t} \). (iii) The share of liquid assets in banks\’ balance sheets, \( \Omega_t = \frac{\int_{\bar{\omega}_t}^{\tilde{\omega}_t} \omega dF(\omega)D_t}{A_t + D_t} \). The sum of cash, central bank reserves as well as all government-backed assets relative to balance sheet size is used as the empirical counter-part. The evolution of this liquidity share for banks and market-based intermediaries was shown in Figure 2.1. While the ratio varied widely, between 13\% to 30\% for banks and 2\% to

\(^{22}\)In this case, the agency cost model collapses to the standard real business cycle model as consumption goods are converted into capital goods one-to-one \emph{ex post}.

\(^{23}\)For issuers of asset-backed securities (ABS), which make up an important fraction of the market-based intermediation sector as demonstrated in Figure 2.7, no data on leverage ratios was available. Since market-based intermediaries\’ leverage tends to exceed that of traditional banks, the average leverage of 13.44 computed for the financial sector without ABS issuers is likely downward-biased. Hence, I allow for a slightly higher leverage ratio in the model.
Table 2.1: Baseline calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household discount factor $\beta$</td>
<td>0.99</td>
<td>riskless interest rate: 1%</td>
</tr>
<tr>
<td>Relative risk aversion $\theta$</td>
<td>1.50</td>
<td>Kato (2006)</td>
</tr>
<tr>
<td>Utility weight on leisure $\nu$</td>
<td>2.67</td>
<td>working time: 30%</td>
</tr>
<tr>
<td><strong>Final goods production</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share of output $\alpha^h$</td>
<td>0.36</td>
<td>Meh and Moran (2010)</td>
</tr>
<tr>
<td>Labour share of output (households) $\alpha^h$</td>
<td>0.63995</td>
<td>Meh and Moran (2010)</td>
</tr>
<tr>
<td>Labour share of output (bankers) $\alpha^h$</td>
<td>0.00005</td>
<td>Meh and Moran (2010)</td>
</tr>
<tr>
<td><strong>Capital goods production</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depreciation rate of capital $\delta$</td>
<td>0.025</td>
<td>Kato (2006)</td>
</tr>
<tr>
<td>Return to investment $R$</td>
<td>1.0098</td>
<td>one-to-one transformation</td>
</tr>
<tr>
<td><strong>Financial intermediation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit-monitoring cost $\mu$</td>
<td>0.1308</td>
<td>investment-to-GDP: 0.15</td>
</tr>
<tr>
<td>Liquidation value to outsiders $\xi$</td>
<td>0.2400</td>
<td>loss-given-default: 0.4</td>
</tr>
<tr>
<td>Probability of success: effort $\pi_H$</td>
<td>0.9903</td>
<td>quarterly failure rate: 0.0097</td>
</tr>
<tr>
<td>Probability of success: shirking $\pi_L$</td>
<td>0.7500</td>
<td>Meh and Moran (2010)</td>
</tr>
<tr>
<td>Std. dev., idiosync. liquidity risk $\sigma_\omega$</td>
<td>0.3458</td>
<td>liquidity share: 0.21</td>
</tr>
<tr>
<td><strong>Population parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass of households $\eta^h$</td>
<td>0.97</td>
<td>Meh and Moran (2010)</td>
</tr>
<tr>
<td>Mass of bankers $\eta^b$</td>
<td>0.03</td>
<td>Meh and Moran (2010)</td>
</tr>
<tr>
<td>Share of surviving bankers $\tau^b$</td>
<td>0.26</td>
<td>bank-leverage ratio: 15</td>
</tr>
<tr>
<td><strong>Shock processes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence, productivity shock $\rho_a$</td>
<td>0.90</td>
<td>Kato (2006)</td>
</tr>
<tr>
<td>Std. dev., productivity shock $\sigma_a$</td>
<td>0.01</td>
<td>1%</td>
</tr>
<tr>
<td>Persistence, liquidity shock $\rho_\xi$</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>Std. dev., liquidity shock $\sigma_\xi$</td>
<td>0.048</td>
<td>20%</td>
</tr>
</tbody>
</table>

Notes: The model is calibrated for quarterly data.

23% for shadow banks, over the past three decades, the model targets the average empirical liquidity share of about 19%. (iv) An investment-to-GDP ratio $\Phi_t = \frac{I_{eff}^t}{Y_t}$ of 15%, where effective investment is defined as $I_{eff}^t \equiv [F(\bar{\omega}_t)\pi_H R_t + (1 - F(\bar{\omega}_t))\xi_t^*]I_t$.

Liquidity shocks are assumed to be distributed uniformly on the interval $[0, U_H]$, such that $\sigma^2(\omega) = \frac{U_H^2}{12}$. The assumption of a uniform distribution facilitates the analysis, but results do not depend on it. With the chosen calibration, we have $\bar{\omega}_0 = 0.68$, $\bar{\omega} = 0.73$ and $\bar{\omega}_1 = 1.04$. As the optimal liquidity threshold falls between the indifference threshold and the frictionless first-best refinancing threshold, the trade-off between the lending scale and the liquidity buffer is operative as discussed in section 2.2.3.

The full set of calibrated parameters including the remaining population parameters is listed in Table 2.1. Some key matched moments and their model-equivalents are summarized in Table 2.2.
Table 2.2: Selected targets: data vs. model

<table>
<thead>
<tr>
<th>Target</th>
<th>Concept</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss given default</td>
<td>LGD</td>
<td>39.8%</td>
<td>0.40</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>Ξ</td>
<td>13.44</td>
<td>15</td>
</tr>
<tr>
<td>Liquidity share</td>
<td>Ω</td>
<td>18.78%</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Notes: The average leverage ratio of the US financial industry is an asset-weighted average of the average leverage of bank- and market-based institutions. Due to lack of data for ABS issuers, this value is likely to be downward-biased. Data on loss given default derives from Araten et al. (2004), who report loan losses of a large US bank between 1982 and 1999. The empirical counterpart to the liquidity share is computed as the sum of checkable deposits and currency, cash and reserves at the Federal Reserve, Treasury securities and agency- and GSE-backed securities relative to total assets of the respective institutions. Sources: US Flow of Funds (Federal Reserve), Aratan et al. (2004)

2.3.2 Aggregate Shocks

I consider two types of aggregate risk in the economy. The first is a standard technology shock that follows the process $z_t = \rho z_{t-1} + \epsilon_t$.

The second source of aggregate risk is a collateral shock. I model this shock as a collapse in the liquidation value of bank loans to outside investors, i.e. $\xi_t = z^\xi_t$ where $z^\xi_t = \rho z^\xi_{t-1} + e^\xi_t$ and $e^\xi_t \sim N(0, 0.048)$. This shock to the liquidation value of loans is intended to capture the sudden decline in the collateral value of bank assets at the onset of the financial crisis. Gorton and Metrick (2010, 2012) investigate the development of the collateral value of bank assets during the Great Recession by analysing the repo market, a primary source of short-term refinancing among market-based financial intermediaries. The authors argue that haircuts on the underlying assets in repo transactions amount to a reduction in the collateral values of these assets. During the financial crisis, and particularly in the wake of the Lehman crash in September 2008, haircuts in repo contracts surged from close to zero to 45% on average. Even non-subprime-related assets suffered haircuts of up to 20%. In order to evaluate whether such a collateral shock may have contributed to the severe recession, the steady-state collateral value is hit with a (conservative) negative shock of 20%.

The following sections present the main findings regarding business cycle dynamics in the presence of idiosyncratic liquidity risk and a balance sheet channel of shock transmission working through the banking sector. The model is solved using a first-order approximation to the policy functions around the non-stochastic steady state.

Aggregate Technology Shock

The impulse response functions of key aggregate variables to a one-standard deviation technology shock are shown in Figure 2.3 along with impulse responses of the frictionless benchmark model without agency costs ($\mu = 0$), but identical technological constraints. In the absence of agency costs the
first-best refinancing threshold is constant at $\bar{\omega}_1$ and does not contribute to shock amplification and transmission. As a result, impulse responses in the benchmark model resemble those of a frictionless real business cycle model, while the agency-cost model exhibits slightly hump-shaped responses, particularly in output and investment.

**Figure 2.3:** Responses to a technology shock

Notes: Impulse responses to a negative one percent technology shock. The agency-cost model (solid lines) is contrasted with a frictionless benchmark model where $\mu = 0$ (dashed lines).

To understand the dynamics in the agency cost model, consider that lower productivity in the final goods sector reduces factor rents. Households react to lower expected rental income from holding capital by reducing their demand for new capital goods. This, in turn, puts downward pressure on the price of capital, $q_t$. The fall in the capital price reduces the profitability of the banking sector, and thus the returns on deposits as can be seen from households’ aggregate participation constraint (2.20). As a consequence, they provide less deposits to the banking sector.

At the same time, the low return on capital and the fall in the capital price reduce banks’ capital contemporaneously, as suggested by equation (2.23). In fact, the fall in bank capital is much more pronounced than in
the frictionless benchmark model due to the drop in the value of capital goods. With both external and internal funding eroding, banks are forced to shorten their balance sheets by curtailing their lending.

Further pressure on the volume of bank loans comes from the response of the liquidity buffer. As discussed in section 2.2.3, the liquidity threshold chosen by banks is negatively correlated with the relative price of capital, because the opportunity cost of holding liquidity reserves is lower when the capital price and the profitability of loans fall. Hence, $\bar{\omega}$ increases slightly. The share of liquid assets in banks’ balance sheets, $\Omega_t$, increases mainly on account of the balance sheet contraction (denominator effect). Overall, financial adjustments related to liquidity hoarding are rather short-lived.

Output drops immediately due to lower factor productivity. Its hump-shaped response derives from the sluggishness of bank capital. The latter continues to drop after the initial shock because of the decrease in the capital price. Lower bank capital propagates via more sluggish lending activity into capital formation and future output.

**Aggregate Collateral Shock and the Great Recession**

As argued in section 2.2.3, a negative shock to the collateral value of bank assets shifts the trade-off between loan scale and liquidity reserves in favour of the latter. The collateral shock thus unfolds its effects through the increase in banks’ liquidity buffers and the corresponding reduction in lending. These dynamics can be traced in the impulse responses shown in Figure 2.4.

In order to disentangle the effect of agency problems and liquidity hoarding from the impact of the collateral shock as such consider first the frictionless benchmark version of the model. In the absence of agency problems between investors and banks liquidity shocks are insured up to the first-best threshold $\bar{\omega}_1$, such that no projects whose continuation value exceeds the liquidation value are abandoned. Since the refinancing threshold is constant, the collateral shock only affects the economy through its direct impact on the participation constraint of the mutual fund. In particular, the fund reduces its overall funding to the banking sector in view of the lower expected liquidation value of loan projects. Given the pre-determined nature of banks’ capital at the time of contracting, banks are forced to react to the drop in external financing by curtailing lending. Recall the definition of effective investment as

$$I_t^{eff} = (F(\bar{\omega}_1)\pi_H R + (1 - F(\bar{\omega}_1))\xi^*_t) I_t$$

(2.35)

Investment falls both directly due to the impact of the lower liquidation value of failed investment projects, as well as indirectly through the reduction of the loan, and thus investment scale. The drop in effective investment reduces capital accumulation, which propagates the shock into the future.\(^\text{24}\)

\(^{24}\)The increase in the price of capital is closely linked to the investment dynamics: the
2. Flight to Liquidity and the Great Recession

Figure 2.4: Responses to a collateral shock

Notes: Impulse responses to a negative 20% percent collateral shock. The agency-cost model (solid lines) is contrasted with a frictionless benchmark model where \( \mu = 0 \) (dashed lines).

In the economy perturbed by agency conflicts, banks respond to a drop in the collateral value \( \xi_t^* \) by increasing their liquidity threshold \( \bar{\omega}_t \) as implied by the first-order condition (2.9). The higher liquidity threshold raises the survival probability of investment projects (marginal effect), which would by itself increase effective investment. However, given the trade-off between liquidity reserves and loan scale, higher liquidity reserves come at the cost of an amplified contraction in lending when compared with the frictionless economy (scale effect). The strong fall in effective investment suggests that the negative shock to the liquidation value in combination with the contraction in lending clearly dominates the effect of higher liquidity reserves on effective investment. Although the relative importance of the marginal ver-
sus the scale effect is an empirical question, intuitively the scale effect should be expected to dominate as it works through banks' leverage (see equation (2.19)), which is highly sensitive to changes in the liquidity threshold.\textsuperscript{25}

Since the collateral shock directly affects the choice between banks' lending scale and liquidity reserves, the response of the liquidity threshold $\bar{\omega}_t$ follows the shock’s AR(1) structure and is, thus, much more prolonged than in the case of a technology shock. The sustained increase in the liquidity share $\Omega_t$ reflects both the higher demand for liquidity reserves (numerator effect) and the contraction in banks’ balance sheets (denominator effect). Contrasting these results with the response of the frictionless benchmark economy, where the trade-off between liquidity and scale is absent, reveals liquidity hoarding by financial intermediaries as the key amplification mechanism of the initial collateral shock. Propagation works in much the same way as in the benchmark model. Depressed investment eats into banks' capital stock, forcing them to curtail lending in future periods as well. The sluggish response of bank capital thus translates into hump-shaped lending, investment and output. Interestingly, the model is able to replicate this hump-shaped response without recourse to adjustment costs, solely through balance sheet dynamics. As a second consequence of the sluggishness of bank capital relative to deposits, bank leverage becomes pro-cyclical. The model can thus rationalize both the scramble for liquidity and the strong deleveraging of financial intermediaries observed in the data during the Great Recession (Figures 2.1 and 2.6, respectively).\textsuperscript{26}

Although the initial aggregate shock is amplified through a balance sheet channel, the effects in this model are quite distinct from the financial accelerator framework. In that framework, fluctuations in borrower net worth affect the borrowing capacity of financially constrained agents. Negative shocks to borrowers’ net worth induce fire-sales which increase the initial losses and lead to further fire-sales. In the present model, the amplification mechanism works instead through the composition of the asset side of constrained borrowers’ balance sheets between liquidity buffers and loan scale. Borrowers’ net worth simply drops as a consequence of the negative impact of the credit crunch on investment and the capital stock, but is not the cause of the crunch. Hence, the model develops a novel type of shock transmission

\textsuperscript{25}Recall from equation (2.19) that the loan scale has a highly non-linear relationship with the liquidity threshold $\bar{\omega}$ through the leverage ratio $H(\bar{\omega})^{-1}$ and, therefore, reacts very sensitively to changes in $\bar{\omega}$ (see proof 2.B.1). In comparison, the survival probability of loan projects, i.e. the cumulative distribution function of $\bar{\omega}$, is much less sensitive to changes in the liquidity threshold by comparison (in fact, it is linear in the parameterization at hand), such that the leverage - or balance sheet - contraction dominates.

\textsuperscript{26}Note that the amplified drop in investment is tantamount to a larger negative supply shock on the capital goods market. Hence, the price reacts more strongly, increasing households’ net worth and consumption. This rise of consumption on impact prevents the economy from sliding into an even deeper recession. The initial jump in bank capital is also generated by the increase in the price of capital.
The key insight from this analysis is that even a modest drop in the collateral value of assets held in the financial sector triggers a flight to liquidity associated with output losses of 1.25%. During the financial crisis, average haircuts in repo contracts were more than twice as high as those modelled in this paper. The counter-cyclical flight to liquidity channel described here may thus have been an important amplification mechanism during the Great Recession.

2.3.3 Financial and Nominal Frictions

During the Great Recession, financial frictions are likely to have interacted with nominal rigidities. Christiano, Trabandt, and Walentin (2011), for instance, emphasize the Fisherian debt-deflation mechanism according to which deflationary pressures inflate the real value of nominal debt. At the same time, nominal frictions affect the consumption-savings decisions of households through their impact on the real interest rate. For instance, rising real interest rates are key for explaining the strong output losses experienced during the Great Recession in the model of Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011). A similar mechanism deepens the recession triggered by a collateral shock in the present model.

To add nominal rigidities in product markets an additional layer is introduced in the production process in the form of monopolistically competitive intermediate goods producers. Final goods are assembled from intermediate goods via a standard Dixit-Stiglitz aggregation technology with finite elasticity of substitution between different varieties of intermediate goods. Intermediary producers use their market power to price their goods at a mark-up over marginal costs. Moreover, they face price adjustment costs as in Rotemberg (1982), such that they do not adjust prices fully in response to variations in demand for their respective goods. Optimal price setting, thus, yields the familiar forward-looking New Keynesian Phillips curve.

Financing constraints induced by agency frictions make the present model sensitive to the distribution of wealth among agents. Therefore, the amplification mechanism presented here could be attenuated by transferring resources from unconstrained households to constrained banks. Such wealth transfers in the form of capital injections or public debt guarantees were the cornerstone of unconventional crisis policies adopted, for instance, by the US Government during the course of the financial crisis. They were ultimately aimed at restoring the capital and borrowing capacity of financial intermediaries. The present model lends itself to the study of such unconventional policies, which would reduce the recessionary impact of collateral shocks. However, since a fully-fledged welfare analysis is beyond the scope of the paper I do not pursue this avenue further.

For a detailed derivation see Appendix 2.C.
(\pi_t - 1) (\pi_t) = \frac{\varepsilon}{\chi} \left( m_{c,t} - \frac{\varepsilon - 1}{\varepsilon} \right) + E_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} \left( \frac{\pi_{t+1}}{\pi_t} - 1 \right) \left( \frac{\pi_{t+1}}{\pi_t} \right) \frac{Y_{t+1}}{Y_t} \right] (2.36)

Monetary policy is assumed to react to deviations of inflation and output from their respective non-stochastic steady states according to the following rule:

\[ i^b_t = (1 - \rho_i) i^b + \rho_i i^b_{t-1} + (1 - \rho_i) [\rho_\pi (\pi_t - \bar{\pi}) + \rho_y (Y_t - \bar{Y})] \] (2.37)

The elasticity of substitution between intermediate good varieties is calibrated to \( \varepsilon = 6 \) and the parameter governing price adjustment costs to \( \chi = 29 \). These choices are consistent with estimates of the slope coefficient of the log-linear Phillips curve as derived from the Calvo-Yun model (Galí and Gertler, 1999). The coefficients of the policy reaction function derive from those estimated in Clarida, Galí, and Gertler (2000), i.e. \( \rho_\pi = 1.5 \), \( \rho_i = 0.8 \), \( \rho_y = 0.1 \).

As the impulse responses in Figure 2.5 show, nominal frictions exacerbate the effect of a collateral shock on output significantly compared to the baseline model with flexible prices. The stronger decline in output, particularly in the first four quarters after the shock, results both from a further decline in investment as well as a more muted rise in consumption. As the increasing capital price raises households’ net worth after a collateral shock in the flexible-price baseline, they increase consumption. With price rigidities, however, the collateral shock allows inflation to drop below the steady state. Since the nominal interest rate lags economic dynamics, this causes the real interest rate to rise until the monetary authority reacts by cutting the nominal rate more aggressively. Ceteris paribus, higher real interest rates would tilt households towards saving rather than consuming. However, marginal costs drop with inflation such that wages are marked down further and hours worked fall strongly. Hence, households’ net worth declines leading to both lower savings and a reduced increase in consumption. In the absence of a substantial private demand boom the output decline is much stronger. The interaction of financial with nominal frictions is, hence, a key factor in explaining the severity of the recession. This observation underscores the importance of effective monetary policy and hints at the potential distortions introduced by the zero lower bound.

### 2.4 Conclusion

This paper studies the impact of idiosyncratic funding liquidity risk in the presence of financial frictions between banks and investors on business cycle fluctuations. A standard moral hazard problem induces a skin-in-the-game...
constraint which forces banks to retain an equity stake in their loans. The same moral hazard problem that puts a limit on bank loans at the initial lending stage also constrains the amount of funding outflows that can be refinanced during the lifetime of loans. Idiosyncratic liquidity shocks can thus not be fully diversified and may lead to the termination of highly productive loan projects. Anticipating this risk, banks reduce their initial lending scale in order to set aside resources as liquidity buffers. Hence, balance sheet constraints force banks to trade off insurance against idiosyncratic liquidity risk with initial loan scale.

A shock to the collateral value of bank assets is introduced as a novel source of aggregate risk, which directly operates on the participation constraint of banks’ outside investors. Banks react to such a shock by hoarding more liquidity at the expense of their lending scale. In the aggregate, this scale effect combined with the lower liquidation value of terminated loan projects dominates the higher survival probability of loans, such that net investment falls and economic activity contracts sharply. Decreases in bank

Figure 2.5: Responses to a collateral shock with nominal frictions

Notes: Impulse responses to a negative 20% collateral shock with (dashed) and without nominal frictions (solid).
capital propagate shocks through time and induce a hump-shaped response
of output. This credit crunch scenario shares key aspects with the Great Re-
cession, which was triggered by losses on financial assets resulting in a flight
to liquidity and a lending squeeze. Furthermore, the interaction of nominal
with financial frictions is shown to amplify the business cycle dynamics
stemming from the flight to liquidity channel.

The models identifies a new, quantitatively important type of amplifica-
tion mechanism working through endogenous portfolio choices of financial
intermediaries in the presence of idiosyncratic funding liquidity risk. This
paper thus contributes to the growing body of literature merging macroeco-
nomic models with financial frictions.

Appendix

2.A Equilibrium Conditions

2.A.1 Dynamic Equilibrium Conditions

Given the aggregate state variables \( \Gamma = (K_t, K^h_t, B_t; z_t, z^\xi_t) \), the competi-
tive equilibrium is a set of policy functions pinning down \( (K_{t+1}, K^h_{t+1}, K^b_{t+1},
B_{t+1}, C^b_t, C^h_t, I_t, A_t, H^b_t, H^h_t, R_t, R^b_t, R^h_t, q_t, r_t, r^b_t, w^b_t, w^h_t) \) together with the
exogenous laws of motion of \( (z_t, z^\xi_t) \). The solution to the dynamic program-
ing problem satisfies the following set of equilibrium conditions

1. Individual optimality

(a) Households

\[
1 = \beta E_t \left( \frac{C^h_{t+1}}{C^h_t} \right)^{-\theta} \frac{q_{t+1}(1-\delta) + r_{t+1}}{q_t} \tag{2.A.1}
\]

\[
1 = \beta E_t \left( \frac{C^b_{t+1}}{C^h_t} \right)^{-\theta} (1 + r^b_{t+1}) \tag{2.A.2}
\]

\[-\frac{\nu}{\eta^h - H^h_t} = -C^{h-\theta}_t w^h_t \tag{2.A.3}\]

(b) Final good producers

\[
r_t = \exp(z_t)\alpha^k K^\alpha^k H^\alpha^k H^{\alpha^k-1} \tag{2.A.4}
\]

\[
w^b_t = \exp(z_t)\alpha^b K^\alpha^b H^\alpha^b H^{\alpha^b-1} \tag{2.A.5}
\]

\[
w^h_t = \exp(z_t)\alpha^h K^\alpha^h H^\alpha^h H^{\alpha^h-1} \tag{2.A.6}
\]
2. Flight to Liquidity and the Great Recession

(c) Banks

\[ R_t^b = \mu \frac{\pi_H - \pi_L}{q_t} \]  \hspace{1cm} (2.A.7)

\[ R_t = R_t^b + R_t^h \]  \hspace{1cm} (2.A.8)

\[ L_t = \frac{A_t}{H(\bar{\omega}_t)} \]  \hspace{1cm} (2.A.9)

\[ 1 = Q(\bar{\omega}_t) \]  \hspace{1cm} (2.A.10)

\[ A_t = (q_t(1 - \delta) + r_t)K_t^b + H_t^b w_t \]  \hspace{1cm} (2.A.11)

\[ K_{t+1}^b = \tau^b F(\bar{\omega}_t) \pi_H R_t^b L_t \]  \hspace{1cm} (2.A.12)

\[ C_t^b = (1 - \tau^b)q_t F(\bar{\omega}_t) \pi_H R_t^b L_t \]  \hspace{1cm} (2.A.13)

\[ H_t^b = \eta^b \]  \hspace{1cm} (2.A.14)

where

\[ H(\bar{\omega}_t) = 1 + F(\bar{\omega}_t) \mu - q_t \left( 1 - \int_0^{\bar{\omega}_t} \omega dF(\omega) \right) \left( F(\bar{\omega}_t) \pi_H R_t^b + (1 - F(\bar{\omega}_t)) \xi_t^* \right) \]

\[ Q(\bar{\omega}_t) = q_t \left[ \left( 1 - \int_0^{\bar{\omega}_t} \omega dF(\omega) \right) \xi_t^* + \bar{\omega}_t F(\bar{\omega}_t) \left( F(\bar{\omega}_t) \pi_H R_t^b + (1 - F(\bar{\omega}_t)) \xi_t^* \right) \right] \]

\[ \xi_t^* = \xi + z_t^\xi \]

2. Market clearing conditions

(a) Goods

\[ C_t^b + q_t K_{t+1}^h + B_{t+1} = (1 + r_t^h)B_t + (q_t(1 - \delta) + r_t)K_t^b + w_t^b H_t^h \]  \hspace{1cm} (2.A.15)

(b) Capital

\[ K_t = K_t^b + K_t^h \]  \hspace{1cm} (2.A.16)

(c) Investment

\[ K_{t+1} = (1 - \delta)K_t + (F(\bar{\omega}_t) \pi_H R + (1 - F(\bar{\omega}_t)) \xi) I_t \]  \hspace{1cm} (2.A.17)

(d) Loans

\[ q_t L_t = q_t I_t \]  \hspace{1cm} (2.A.18)

(e) Bonds

\[ B_t = 0 \]  \hspace{1cm} (2.A.19)
3. Exogenous processes

\begin{align*}
  z_t &= \rho z_{t-1} + e_t \quad (2.20) \\
  z_t^\xi &= \rho^\xi z_{t-1}^\xi + e_t^\xi \quad (2.21)
\end{align*}

2.A.2 Steady State

Endogenous parameters \{\nu, R, \mu, \sigma^2(\omega), \tau^b, \xi\} are solved from the following calibration targets:

- \(\nu\): \(H^h = 0.3\)
- \(R\): \(\pi_H R = 1\)
- \(\{\mu, \sigma^2(\omega), \tau^b, \xi\}\):

\[
LGD = 1 - \frac{q\xi^*I}{I - A} \\
\Xi = \frac{D}{A} \\
\Omega = \int_0^{\bar{\omega}} \omega dF(\omega) \frac{D}{A + D} = \int_0^{\bar{\omega}} \omega dF(\omega) (1 + \Xi^{-1})^{-1} \\
\Phi = \left[ F(\bar{\omega})\pi_H R^h + (1 - F(\bar{\omega})) \xi \right] \frac{I}{Y}
\]

where \(D = q \left[ F(\bar{\omega})\pi_H R^h + (1 - F(\bar{\omega})) \xi \right] L \) and \(Y = K^a h^h \alpha^b \). Further, let \(\omega \sim U[0, U_H] \) such that \(\sigma^2(\omega) = \frac{U_H^2}{12} \). Given these targets, the steady state can be derived as follows:

\[
B = 0 \\
R = \frac{1}{\pi_H} \\
\tau^b = \frac{1}{\beta} - 1 \\
H^b = \eta^b
\]
To continue, guess \( \{q, \bar{\omega}, \mu, \xi\} \), then:

\[
F(\bar{\omega}) = 2 \left( 1 + \Xi^{-1} \right) \Omega \bar{\omega}^{-1}
\]

\[
r = \left( \frac{1}{\beta} - (1 - \delta) \right) q
\]

\[
R^b = \frac{\mu}{q(\pi_H - \pi_L)}
\]

\[
R^h = R - R^b
\]

\[
I = L
\]

\[
A \quad L = 1 - \frac{q \xi}{1 - \text{LGD}}
\]

\[
D = q \left[ F(\bar{\omega}) \pi_H R^h + (1 - F(\bar{\omega})) \xi \right] L
\]

\[
U_H = \bar{\omega}^2 \left( 1 + \frac{A}{D} \right)^{-1} \Omega^{-1}
\]

\[
K = \left( r^{-1} \alpha^h H^{h_{ab}} H^{b_{ab}} \right)^{\frac{1}{1 - \alpha^h}}
\]

\[
L = \frac{\delta}{F(\bar{\omega}) \pi_H R + (1 - F(\bar{\omega})) \xi} K
\]

\[
A = \left( 1 - \frac{q \xi}{1 - \text{LGD}} \right) L
\]

\[
w^b = \alpha^b K^{\alpha^b} H^{h_{ab}} H^{b_{ab}}
\]

\[
w^h = \alpha^h K^{\alpha^h} H^{h_{ab}} H^{b_{ab}}
\]

\[
K^b = (q(1 - \delta) + r)^{-1} \left( A - w^b H^b \right)
\]

\[
K^h = K - K^b
\]

\[
\tau^b = K^b \left( F(\bar{\omega}) \pi_H R^b L \right)^{-1}
\]

\[
C^b = (1 - \tau^b) q F(\bar{\omega}) \pi_H R^b L
\]

\[
C^h = (r - q \delta) K^h + w^h H^h
\]

\[
\nu = C^{h_\theta} w^h \left( \eta^h - H^h \right)
\]
To verify the guess, check the following equations for consistency:

\[ \Xi = q \left[ F(\bar{\omega}) \pi_H R^h + (1 - F(\bar{\omega})) \xi \right] \frac{L}{A} \]

\[ L = \frac{A}{H(\bar{\omega})} \]

\[ 1 = Q(\bar{\omega}) \]

\[ \Phi = \left[ F(\bar{\omega}) \pi_H R^h + (1 - F(\bar{\omega})) \xi \right] \frac{I}{Y} \]

### 2.B Proofs

#### 2.B.1 Proof of Proposition 1

(i) Note that

\[ \frac{\partial H(\bar{\omega})}{\partial \bar{\omega}} = f(\bar{\omega}_0) \left\{ \mu + q \left[ \bar{\omega}_0 \left( F(\bar{\omega}_0) \pi_H R^h + (1 - F(\bar{\omega}_0)) \xi^* \right) \right] \right\} \]

\[ - f(\bar{\omega}_0) \left\{ \left[ 1 - \int_{\bar{\omega}_0}^0 \omega \, dF(\omega) \right] \left( \pi_H R^h - \xi^* \right) \right\} \]

\[ = 0 \]

\[ \iff \bar{\omega}_0 = \frac{ \left( 1 - \int_{0}^{\bar{\omega}_0} \omega \, dF(\omega) \right) \left( \pi_H R^h - \xi^* \right) - \frac{\mu}{q} }{F(\bar{\omega}_0) \pi_H R^h + (1 - F(\bar{\omega}_0)) \xi^*} \]

Then, we have that

\[ \frac{\partial H(\bar{\omega})}{\partial \bar{\omega}} = \left\{ \begin{array}{ll} < 0 & : \bar{\omega} < \bar{\omega}_0 \\ = 0 & : \bar{\omega} = \bar{\omega}_0 \\ > 0 & : \bar{\omega} > \bar{\omega}_0 \end{array} \right. \]

i.e. \( H(\bar{\omega}) \) achieves a minimum at \( \bar{\omega}_0 \). Since the loan scale (2.5) is a function of \( H(\bar{\omega})^{-1} \), it is maximised at this point. Furthermore, \( F \) increases monotonically in \( \bar{\omega} \). Hence, bankers can always improve on \( \bar{\omega} < \bar{\omega}_0 \) by choosing \( \bar{\omega} = \bar{\omega}_0 \). The latter must, therefore, be the lower bound of the liquidity threshold.

To relate \( \bar{\omega}_0 \) to the interpretation as the indifference threshold mentioned in section 2.2.3, note that

\[ q \pi_H R^h l - \left( 1 - \int_{0}^{\bar{\omega}_0} \omega \, dF(\omega) \right)^{-1} (\bar{\omega}_0 d + \mu l) = q \xi^* l \]

\[ \iff \bar{\omega}_0 = \left[ \left( 1 - \int_{0}^{\bar{\omega}_0} \omega \, dF(\omega) \right) \left( \pi_H R^h - \xi^* \right) q - \mu \right] \frac{l}{d} \]

\[ \iff \bar{\omega}_0 = \frac{ \left( 1 - \int_{0}^{\bar{\omega}_0} \omega \, dF(\omega) \right) \left( \pi_H R^h - \xi^* \right) - \frac{\mu}{q} }{F(\bar{\omega}_0) \pi_H R^h + (1 - F(\bar{\omega}_0)) \xi^*} \]
where the last step uses the participation constraint of households, (2.2).

(ii) Completing the derivation of the first-best refinancing threshold in section 2.2.3 by again using households’ participation constraint (2.2) yields

\[
q \pi_H R l - \bar{\omega}_1 d = q \xi^* l \nonumber
\]

\[
\iff \bar{\omega}_1 = (\pi_H R - \xi^*) \frac{ql}{d} \nonumber
\]

\[
\iff \bar{\omega}_1 = \frac{\pi_H R - \xi^*}{F(\bar{\omega}_1)\pi_H R + (1 - F(\bar{\omega}_1))\xi^*} \nonumber
\]

(iii) Finally, (2.6) is convex in \( \bar{\omega} \), such that the necessary and sufficient condition for a maximum is given by the first order condition (2.9).\(^\text{29}\)

\[2. \text{B.2 Properties of the Optimal Liquidity Threshold}\]

First determine the partial derivatives of \( Q \) with respect to \( \bar{\omega}, q \), and \( \xi^* \):

\[
\frac{\partial Q}{\partial \bar{\omega}} = q \left\{ \bar{\omega} f(\bar{\omega}) \left[ 2F(\bar{\omega}) \left( \pi_H R^h - \xi^* \right) \right] + F(\bar{\omega}) \left( F(\bar{\omega}) \pi_H R^h + (1 - F(\bar{\omega})) \xi^* \right) \right\} \nonumber
\]

A sufficient condition for \( \frac{\partial Q}{\partial \bar{\omega}} > 0 \) is \( \pi_H R^h > \xi^* \), which will be satisfied in the calibration. Next,

\[
\frac{\partial Q}{\partial \xi^*} = 1 - \int_0^{\bar{\omega}} \omega dF(\omega) + \bar{\omega} F(\bar{\omega}) (1 - F(\bar{\omega})) > 0 \nonumber
\]

since the share of deposits that are not buffered by liquid reserves \( \int_0^{\bar{\omega}} \omega dF(\omega) \leq 1 \) by definition. Finally

\[
\frac{\partial Q}{\partial q} = \left( 1 - \int_0^{\bar{\omega}} \omega dF(\omega) \right) \xi^* + \bar{\omega} F(\bar{\omega}) \left( F(\bar{\omega}) \pi_H R^h + (1 - F(\bar{\omega})) \xi^* \right) > 0 \nonumber
\]

for the same reason. Now applying the implicit function theorem (IFT) we have that

\[
\frac{\partial \bar{\omega}}{\partial \xi} = -\frac{\partial Q/\partial \xi}{\partial Q/\partial \bar{\omega}} < 0 \nonumber
\]

and

\[
\frac{\partial \bar{\omega}}{\partial q} = -\frac{\partial Q/\partial q}{\partial Q/\partial \bar{\omega}} < 0 \nonumber
\]

\[\text{In the numerical exercise I will show that, indeed, } \bar{\omega}_0 < \bar{\omega} < \bar{\omega}_1.\]
2. C Derivation of the NK Phillips Curve

In the extended model, final good producers assemble intermediate goods via a Dixit-Stiglitz aggregator

\[ Y_t = \left[ \int_0^1 Y_{it}^{\epsilon - 1} \text{d}i \right]^{\frac{1}{\epsilon - 1}} \]

where \( \epsilon \) is the elasticity of substitution between different varieties of intermediate goods. From the optimization problem of final good producers demand for intermediate goods is given by

\[ Y_{it} = \left[ \frac{P_{it}}{P_t} \right]^{-\epsilon} Y_t \]

where \( P_t \) is the aggregate price level and \( P_{it} \) the price of variety \( i \). Demand from intermediate producers thus depends on the relative price of their product as well as the elasticity of substitution.

Intermediate producers use capital and labour as inputs into their production function

\[ Y_{it} = \exp(z_t) F(K_{it}, H_{h}^b, H_{b}^b) \]

Their optimization problem can be broken down into two separate steps: a cost minimization step in the production of a given quantity of intermediate goods and a price setting step. To minimize costs, intermediate producers solve

\[ \min_{\{K_{it}, H_{h}^b, H_{b}^b\}} r_t K_{it} + w_t^h H_{h}^b + w_t^b H_{b}^b - mc_{it}[\exp(z) F(K_{it}, H_{h}^b, H_{b}^b) - Y_{it}] \]

where the Lagrange multiplier represents the marginal cost of the firm. Taking the first order conditions and imposing symmetry across firms yields

\[ r_t = mc_{it} \exp(z_t) F_K(K_{it}, H_{h}^b, H_{b}^b) \] (2.C.1)

\[ w_t^h = mc_{it} \exp(z_t) F_{H_h}(K_{it}, H_{h}^b, H_{b}^b) \] (2.C.2)

\[ w_t^b = mc_{it} \exp(z_t) F_{H_b}(K_{it}, H_{h}^b, H_{b}^b) \] (2.C.3)

In a second step, intermediate producers set their optimal relative price given quadratic price adjustment costs subject to their individual demand schedule

\[ \max_{\{P_{is}\}} \mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{st} \left\{ \left[ \frac{P_{is}}{P_s} - mc_s \right] Y_{is} - \frac{\chi}{2} \left[ \frac{P_{is}}{\bar{\pi} P_{is} - 1} - 1 \right]^2 Y_s \right\} \]

subject to \( Y_{is} = \left[ \frac{P_{is}}{P_s} \right]^{-\epsilon} Y_s \) and where \( \lambda_{st} = \frac{\partial \lambda_s}{\partial t} \) is households’ stochastic discount factor between periods \( s \) and \( t \). In a symmetric equilibrium all
intermediate good producers set the same price, such that type subscript \( i \) can be dropped and \( P_{it} = P_t, \forall i \in [0, 1] \). After some manipulations, the first order condition yields the forward-looking New Keynesian Phillips curve

\[
\left( \frac{\pi_t}{\pi} - 1 \right) \left( \frac{\pi_t}{\pi} \right) = \frac{\epsilon}{\chi} \left( m_{c,t} - \frac{\epsilon - 1}{\epsilon} \right) + E_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} \left( \frac{\pi_{t+1}}{\pi}_{-1} \right) \left( \frac{\pi_{t+1}}{\pi}_{-1} \right) \frac{Y_{t+1}}{Y_t} \right]
\]

(2.C.4)

The second sector directly affected by the introduction of nominal rigidities is the household sector. Households now choose the level of nominal rather than real bonds

\[
\max \left\{ c^h_t, h^h_{t+1}, b^h_{t+1}, h^h_t \right\} \ E_0 \sum_{t=0}^{\infty} \beta^t u(c^h_t, h^h_t)
\]

s.t.

\[
c^h_t + q_t^h k^h_{t+1} + \frac{b_{t+1}^h}{P_t} = (1 + \frac{b_t^h}{P_t}) \frac{b_t^h}{P_t} + (q_t(1 - \delta) + r_t) h^h_t + w_t^h h^h_t
\]

(2.C.5)

and the first order condition for \( b^h_{t+1} \) accordingly becomes

\[
\lambda_t = \beta E_t \left[ \frac{1}{\pi_{t+1}} \right]
\]

(2.C.6)
2. Flight to Liquidity and the Great Recession

2.D Graphs

Figure 2.6: Leverage ratios of banks and market-based intermediaries

Notes: US-chartered commercial banks, savings institutions and credit unions are identified as traditional banks. The shadow banking sector comprises securities and broker dealers, issuers of asset-backed securities, finance companies and government-sponsored enterprises. This follows the classification in Adrian and Shin (2009). The leverage ratio is defined as the ratio of debt to equity. Source: US Flow of Funds (Federal Reserve)

Figure 2.7: Asset-to-GDP ratios of banks and market-based intermediaries

Source: US Flow of Funds (Federal Reserve)
Chapter 3

Search-Based Endogenous Illiquidity and the Macroeconomy

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3.1 Introduction

Illiquidity of privately issued financial assets arises from impediments to their issuance and subsequent transactions. Empirical evidence points to procyclical variation in the market liquidity of a wide range of financial assets.\(^1\) The view that asset liquidity dries up during recessions has been further reinforced by the 2007-2009 financial crisis, when illiquidity problems were most pronounced for commercial paper and asset-backed securities.\(^2\)

\(^1\text{Studies by Huberman and Halka (2001), Chordia, Roll, and Subrahmanyam (2001), Chordia, Sarkar, and Subrahmanyam (2005) and Naes, Skjeltorp, and Oedegaard (2011) assert that market liquidity is procyclical and highly correlated across asset classes such as bonds and stocks in the US.}\)

\(^2\text{Dick-Nielsen, Feldhütter, and Lando (2012) identify a structural break in the market liquidity of corporate bonds at the onset of the sub-prime crisis. The liquidity component of spreads of all but AAA rated bonds increased and turnover rates declined, making refinancing on the market more difficult. Commercial paper, which is largely traded on a search market with dealers as match-makers, experienced pronounced illiquidity reported by Anderson and Gascon (2009). In addition, money market mutual funds, the main investors in the commercial paper market, shifted to highly liquid and secure government securities. Finally, Gorton and Metrick (2012) show that the repo market has registered strongly increasing haircuts during the crisis.}\)
Illiquid primary or secondary equity and debt markets reduce firms’ ability to finance investment, which creates a role for liquid assets, such as fiat money or government bonds. These liquid assets provide insurance against funding constraints as they can be readily used for financing purposes at any time. When funding constraints tighten in recessions, firms tend to re-balance their portfolios towards such liquid assets— a phenomenon referred to as ‘Flight To Liquidity’. Variations in asset liquidity and the idea of liquidity hoarding as a hedging device against funding constraints goes back to Keynes (1936) and Tobin (1969). Nevertheless, the link between asset liquidity and aggregate fluctuations is often ignored in state-of-the-art dynamic general equilibrium models.

We propose a framework in which endogenous variation in asset liquidity interacts with macroeconomic conditions. To this end, we incorporate a search market for financial assets into an almost-standard real business cycle model. Search frictions give rise to asset illiquidity both on primary markets (issuance of new assets) and secondary markets (liquidation of existing assets). Asset liquidity has both a physical and a price dimension: the physical dimension, to which we refer to as saleability, is measured by the endogenous fraction of new or existing assets offered for sale that are successfully traded. The price dimension is captured by the sensitivity of the transaction price and intermediation costs to relative supply on the asset market. The search market structure in our model can be interpreted as a stand-in for financial intermediation via either markets or banks, both of which involve a costly matching process between capital providers and seekers.

The model shows how a drop in investor participation in the search market simultaneously reduces asset saleability, pushes down asset prices and tightens funding constraints, which further dampens real investment and production. The central contribution of this paper is to demonstrate that (i) endogenizing asset liquidity is essential to generate positive correlation between asset saleability and asset prices; and (ii) shocks to the cost of financial intermediation can be an important source of flight to liquidity and business cycles.

Consider an economy where privately issued financial claims are backed by cash flow from physical capital, which is rented to final goods producers and owned by households. There is a continuum of households whose members are temporarily separated during periods. Some become workers, others entrepreneurs. Only the latter have access to investment opportunities for capital goods creation. All household members are endowed with a portfolio of liquid assets (money)4 and private claims, which we interpret as

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3 In fact, U.S. nonfinancial firms only fund 35% of fixed investment through financial markets, of which 76% through debt and equity issuance and 24% through portfolio liquidations (Ajello, 2012).

4 For the sake of simplicity, all government-issued assets are considered as money. The framework could easily be extended to general interest bearing liquid assets as illustrated.
3. Search-Based Endogenous Illiquidity and the Macroeconomy

a catch-all for privately issued assets such as corporate bonds and equity.

To finance investment, entrepreneurs exploit all available modes of funding: they issue new financial claims to their investment projects and liquidate their existing asset portfolio. Private financial claims (both new and old) are only partially liquid, because they are traded on a search market. Participation in the search market is costly for both buyers and sellers as they are matched by an intermediary for a fee. Moreover, the intermediary determines the transaction price by maximizing the total surplus of a match, similar to the bargaining process in the labour search literature (Diamond, 1982; Mortensen and Pissarides, 1994; Shimer, 2005). It can be shown that the transaction price correlates positively with asset saleability under mild parameter restrictions. Since the physical and the price dimension of asset liquidity are, thus, interconnected, the concepts of asset saleability and asset liquidity are to some degree interchangeable in our set-up.

As the funding ability of private claims is limited by their partial liquidity and the intermediation costs incurred by buyers and sellers, entrepreneurs are financing constrained and cannot fund the first-best level of investment. Money, on the other hand, is readily available for financing purposes at any time. Since households value this hedging value against the illiquidity of private financial claims, money commands a liquidity premium.

Market- and bank-based financial intermediation share the essential feature of matching savers and borrowers, such that the framework presented in this paper admits both interpretations of the intermediation process. On the one hand, the search and matching framework echoes features of over-the-counter (OTC) markets, in which a large fraction of corporate bonds, asset-backed securities, and private equity is traded. Participation costs in these markets arise from information acquisition as well as brokerage and settlement services from dealers and market makers. On the other hand, the framework can be seen as a reduced-form approach towards modelling the costly matching process between savers (investors) and the corporate sector through financial intermediaries.

Two types of exogenous shocks are considered: an aggregate productivity shock and a symmetric shock to the participation costs of buyers and sellers, which we interpret as an ‘intermediation cost shock’. The latter captures any generic disruption in the financial sector that increases the cost of providing intermediation services. Both shock scenarios are used to illustrate the effect of endogenous asset liquidity on macroeconomic dynamics.

Negative aggregate productivity (TFP) shocks decrease the return to capital, make investment into capital goods less attractive, and, hence crowd

\[5\] For simplicity, this fee is modelled as a dead-weight cost, which drives a wedge between the transaction price and the effective purchase and sale prices of financial claims.

out investors from the search market. Negative intermediation cost shocks, on the other hand, make investment into liquid assets more attractive to hedge future investment. This reduces the incentive for investors to post costly buy orders on the search market. In either case, the fall in demand on the asset market exceeds that of supply (under some regularity conditions), such that sellers have a lower chance of encountering a buyer. Hence, the sales rate - or saleability - of financial claims drops. Because lower saleability implies that entrepreneurs need to retain a larger equity stake in new investment projects, their financing constraints tighten and the option of breaking off negotiations becomes less valuable. Entrepreneurs are thus willing to accept a lower transaction price. In the aggregate, deteriorating asset saleability and prices restrict the funding available to entrepreneurs and, thereby, reduce real investment.

While both shocks generate procyclical asset liquidity and prices, only intermediation cost shocks induce a pronounced flight to liquidity. In the case of persistent negative TFP shocks, investors have a weaker incentive to hedge against future investment, because of lower current and future returns to capital. Adverse intermediation cost shocks, however, do not deteriorate the quality of investment itself either today or tomorrow. Investors thus value the hedging service from liquid assets more strongly and rebalance towards liquid assets. Because of the portfolio rebalancing, asset price movements are more pronounced. Intermediation cost shocks thus allow the model to match the volatility of asset prices and liquidity hoarding and their co-movement with GDP in the data.

To the best of the authors’ knowledge, this paper is the first attempt to incorporate endogenous asset liquidity in a dynamic macroeconomic model in a tractable way and to explore the feedback effects between asset liquidity and the real economy.\(^7\) Kiyotaki and Moore (2012) (henceforth, KM) demonstrate how exogenous asset market liquidity interacts with aggregate fluctuations, in a model in which firms can only sell an exogenous fraction of private claims to finance new investment. However, as pointed out by Shi (2015), exogenous liquidity fluctuations lead to counter-factual asset price dynamics: a negative shock to asset saleability reduces the supply of financial assets, while demand remains relatively stable since the quality of investment projects is unaffected by liquidity shocks. The negative supply shock induces a persistent asset price boom that is at odds with the data. This counter-factual finding highlights the need to model asset liquidity endogenously, as is done in this paper.

\(^7\)A recent study by Yang (2013) also considers endogenous asset liquidity. The difference is that we model liquid and illiquid assets together and the corresponding portfolio choice simultaneously.
3. Search-Based Endogenous Illiquidity and the Macroeconomy

3.1.1 Related Literature

Following KM, we model liquidity differences between private claims and government-issued assets. The irrelevance result of Wallace (1981) on the neutrality of central banks’ portfolios no longer holds in such a setting. In fact, open market operations that change the composition of liquid and illiquid assets in agents’ portfolios have real effects. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) analyze such ‘unconventional policy’ after an exogenous fall in liquidity in an extended KM model with a ‘zero lower bound’.8

The search literature provides a natural theory of endogenous liquidity as in Lagos and Rocheteau (2009) and has been applied to a wide range of markets such as OTC markets for asset-backed securities, corporate bonds, federal funds, private equity and housing amongst others.9 This literature shows that search frictions can explain substantial variation in a wide range of measures of asset market liquidity (e.g., bid-ask spreads and trading delays). Further, work by Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), and Petrosky-Nadeau and Wasmer (2013) has emphasized the role of search and matching frictions in credit markets and their impact on aggregate dynamics.10 Nevertheless, the joint behaviour of asset prices and asset saleability is generally not explored in a general equilibrium setting, such that mutual feedback effects are not considered.

An alternative approach to endogenizing liquidity uses information frictions, such as adverse selection models in Eisfeldt (2004) and Guerrieri and Shimer (2012). While endogenizing asset liquidity, these studies do not consider the feedback effects of fluctuations in liquidity on production and employment. A notable exception is Kurlat (2013), who extends KM with endogenous resaleability through adverse selection but neglecting the role of liquid assets. In Eisfeldt and Rampini (2009) firms need to accumulate liquid funds in order to finance investment opportunities. While the supply of liquid assets affects investment, secondary markets for asset sales are shut off as an alternative means of financing. In contrast to these contributions, we jointly model endogenous liquidity on primary and secondary asset markets, the role of liquid assets as the lubricant of investment financing, and feedback effects between asset liquidity and business cycles.11

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8 More generally, Kara and Sin (2013) show that market liquidity frictions induce a trade-off between output and inflation stabilization off the ZLB that can be attenuated by quantitative easing measures.

9 See e.g., Duffie, Gärleanu, and Pedersen (2005, 2007); Ashcraft and Duffie (2007); Feldhutter (2011); Wheaton (1990); Ungerer (2012).

10 Further, Kurmann and Petrosky-Nadeau (2007) study search frictions associated with physical capital in a macroeconomic setting. As shown in Beaubrun-Diant and Tripier (2013), search frictions also help explain salient business cycle features of bank lending relationships.

11 In this sense, we thus compliment the studies of cyclical capital reallocation, such as
The present framework also differs along important dimensions from search-theoretic models of money, such as Lagos and Wright (2005) and Rocheteau and Wright (2005). In this literature, money has a transaction function in anonymous search markets. Recent extensions include privately created liquid assets such as claims to capital (Lagos and Rocheteau, 2008) or bank-deposits (Williamson, 2012) as media of exchange. The model presented in this paper rather emphasizes the role of financial assets - both public and private - as stores of value, i.e. money and equity claims are used for financing purposes. Moreover, this approach is able to generate endogenous variation in asset liquidity and the associated premia, because private claims are subject to search frictions themselves, rather than serving to overcome such frictions on other markets. These differences notwithstanding, a common tenet is that liquid assets play an important role in economic transactions by relaxing deep financial frictions.

By studying intermediation cost shocks which affect asset market liquidity, we also complement the literature on financial shocks. Recent contributions by Jermann and Quadrini (2012), Christiano, Motto, and Rostagno (2014), and Jaccard (2013) identify financial shocks as an important source of business cycle fluctuations. This paper shows how such shocks may be endogenously amplified within financial markets.

### 3.2 The Model Environment

The model presented in this paper is a variant of a standard real business cycle (RBC) model. Time is discrete and infinite \( t = 0, 1, 2, \ldots \). The economy has three sectors: final goods producers, households (with entrepreneurs and workers) and financial intermediaries. Final goods producers generate output by renting capital and labour from households.\(^\text{12}\) There are search frictions afflicting the purchase and sale of financial assets issued by previous and current entrepreneurs. Financial intermediaries facilitate asset transactions between sellers and buyers by implementing a costly matching process. Liquid government-issued assets, on the other hand, can be traded on a frictionless spot market. To abstract from government policies, we model liquid assets as non-interest bearing money. We focus on an equilibrium in which this intrinsically worthless asset is valued for its liquidity service and accepted by all market participants.\(^\text{13}\)

\(^{12}\)They rebate profits back to households. In equilibrium, profits are zero because of perfect competition.

\(^{13}\)The derivation with interest-bearing government bonds and taxation is available upon request.
3.2.1 Final Good Producers

Competitive firms rent aggregate capital stock $K_t$ and hire aggregate labour $N_t$ from households to produce output (general consumption goods) according to

$$Y_t = e^{z_{a,t}} F(K_t, N_t),$$

where $F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}$, $\alpha \in (0, 1)$ and $z_{a,t}$ measures exogenous aggregate productivity. The profit-maximizing rental rate and wage rate are thus

$$r_t = e^{z_{a,t}} F_K(K_t, N_t), \quad w_t = e^{z_{a,t}} F_N(K_t, N_t). \quad \text{(3.2.1)}$$

3.2.2 Households

Households comprise of entrepreneurs and workers. Workers earn wages by supplying labour. Entrepreneurs do not work, but only they have investment opportunities. They issue new claims and/or sell existing claims to finance new investment, to the extent possible. Claims other than money need to be issued or resold through intermediaries with a matching technology. These claims are illiquid in the sense that for every unit of capital put on sale only a fraction $\phi_{t,t} \in [0, 1]$ (to be determined endogenously) is sold. Financial frictions are thus represented by the fact that entrepreneurs have to retain a fraction $(1 - \phi_{t,t})$ of claims to new investment.

A Representative Household

At the beginning of $t$, the aggregate productivity and the unit cost of trading private claims are realized. A representative household specifies policy rules for its members, who receive equal shares of assets accumulated from previous periods. Then, they receive a shock that determines their type, which is idiosyncratic across members and through time. With probability $\chi$, a member becomes an investing entrepreneur (called type $i$); with probability $(1 - \chi)$ a non-investing worker (called type $n$). By the law of large numbers, each household thus consists of a fraction $\chi$ of entrepreneurs and a fraction $(1 - \chi)$ of workers. Both groups are temporarily separated during each period and there is no consumption insurance between them.

In the middle of $t$, final goods producers rent capital and labour from households to produce consumption goods and the payoffs from private claims are thus realized. At the same time, household members trade liquid assets on a competitive market in exchange for consumption goods. Equity claims on capital goods are offered for sale by entrepreneurs, while workers post buy orders. Financial intermediaries match potential buyers and sellers and intermediate a transaction price. Entrepreneurs then invest in physical capital, after which workers and entrepreneurs consume.
At the end of $t$, members come together again to share their accumulated assets. All members hence enter the next period again with an equal share of their household’s assets.  

Preferences. The household objective is to maximize

$$E_t \sum_{s=0}^{\infty} \beta^{t+s} [U(c_{i,t+s}, c_{n,t+s}) - (1 - \chi)h(n_{t+s})],$$

(3.2.2)

where $\beta \in (0, 1)$ is the discount factor, $E_t(c_{i,t}, c_{n,t}) = \chi u(c_{i,t}) + (1 - \chi)u(c_{n,t})$ is the total utility derived from consumption by entrepreneurs ($c_{i,t}$) and workers ($c_{n,t}$). $E_t(.)$ is a standard strictly increasing and concave utility function and $h(.)$ captures the dis-utility derived from labour supply $n_t$. $E_t$ is the expectation operator conditional on information at time $t$.

Balance Sheet. Physical capital ($K_t$), earning a return $r_t$, is owned by households and rented to final goods producers. There is a claim to the future return of every unit of capital, which household members can either retain or offer for sale to outside investors on the search market. We normalize equity by the capital stock, such that both depreciate at the same rate (denoted by $\delta$). Equity claims can be sold in a successful match at unit price $q_t$, which is determined by the intermediary as explained in section 3.2.3.

In addition, households can invest into nominal, liquid assets (money). Hence, at the onset of period $t$, households own a portfolio of liquid assets, equity claims on other households’ return on capital and own physical capital. These assets are financed by net worth plus equity claims issued against their own physical capital. This financing structure gives rise to the following beginning-of-period balance sheet in real terms.

<table>
<thead>
<tr>
<th>Table 3.1: Household’s balance sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>liquid assets</td>
</tr>
<tr>
<td>$B_t/P_t$</td>
</tr>
<tr>
<td>other’s equity</td>
</tr>
<tr>
<td>$q_tS_t^O$</td>
</tr>
<tr>
<td>capital stock</td>
</tr>
<tr>
<td>$q_tK_t$</td>
</tr>
<tr>
<td>equity issued</td>
</tr>
<tr>
<td>$q_tS_t$</td>
</tr>
<tr>
<td>net worth</td>
</tr>
<tr>
<td>$q_tS_t + B_t/P_t$</td>
</tr>
</tbody>
</table>

All existing claims to capital need to be traded on the search market at price $q_t$ for refinancing purposes. Similarly, the fraction of the capital stock owned by a given household on which no outside equity claims have been written yet would need to be offered on the search market if the household wanted to raise additional funds. It is, therefore, also valued at $q_t$, such that

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14 The representative household with temporarily separated agents has been introduced in Lucas (1990) and applied to the KM framework in Shi (2015) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011).
we only need to keep track of net equity, defined as
\[ S_t = S_t^O + (K_t - S_t^I). \]

**Individual Members**

Let \( s_{jt} \) and \( b_{jt} \) be net equity and money for a typical household member \( j \). Then, the net equity evolves according to
\[ s_{j,t+1} = (1 - \delta) s_{j,t} + i_{j,t} - m_{j,t}, \]
where \( i_{j,t} \) is investment into capital goods and \( m_{j,t} \) corresponds to asset sales. Let \( c_{jt} \) and \( n_{jt} \) denote consumption and labour supply, respectively.

**Workers’ flow-of-funds.** The household delegates equity purchases on the search market to workers because they do not have investment opportunities \((i_{n,t} = 0)\). Therefore, workers \( j = n \) post asset positions \( e_{n,t} \) to acquire new or old equity at unit cost \( \kappa_n \). On the search market, each posted position is filled with a probability \( \phi_{n,t} \in [0, 1] \) and an individual buyer expects to purchase an amount \( m_{n,t} = -\phi_{n,t} e_{n,t} \). Notice that a worker’s flow-of-funds constraint reads
\[ c_{n,t} + (\kappa_n + q_t \phi_{n,t}) e_{n,t} + \frac{b_{n,t+1}}{P_t} = w_t n_{n,t} + r_t s_{n,t} + \frac{b_{n,t}}{P_t}, \]
where labour income and the return on equity and money are used to finance consumption, search costs and the new accumulation of equity claims and money. To simplify, we define the effective purchase price per unit of equity as
\[ q_{n,t} \equiv q_t + \frac{\kappa_n}{\phi_{n,t}}, \]
where \( q \) captures the transaction price and \( \frac{\kappa_n}{\phi_{n,t}} \) represents search costs per transaction (scaled by the probability of encountering a seller \( \phi_n \)). By using (3.2.3) and \( m_{n,t} = -\phi_{n,t} e_{n,t} \), the flow-of-funds constraint (3.2.4) becomes
\[ c_{n,t} + q_{n,t} s_{n,t+1} + \frac{b_{n,t+1}}{P_t} = w_t n_{n,t} + r_t s_{n,t} + (1 - \delta) q_{n,t} s_{n,t} + \frac{b_{n,t}}{P_t}. \]

**Entrepreneurs’ flow-of-funds.** Entrepreneurs \( j = i \) decide which fraction \( e_{i,t} \in [0, 1] \) of their assets to put up for sale at unit cost \( \kappa_i \) in order to finance new investment \((i_{i,t} > 0)\). These assets include existing equity claims on other households’ capital stock and their own unissued capital stock (in total \( s_{i,t} \)), plus claims on new investment, \( i_{i,t} \). Then, the amount of private financial claims that are up for sale is bounded from above by an entrepreneur’s existing equity holdings and the volume of new investment, \((1 - \delta) s_{i,t} + i_{i,t}\). Offers are matched with a buyer with probability \( \phi_{i,t} \in [0, 1] \). Therefore, an
individual entrepreneur expects to sell $m_{i,t} = \phi_{i,t}e_{i,t}[(1 - \delta)s_{i,t} + i_{i,t}]$. Notice that the returns on equity and money are used to finance consumption, search costs and the accumulation of equity (with new investment taken into account) and money. The flow-of-funds constraint can, thus, be written as

$$c_{i,t} + i_{i,t} + \frac{b_{i,t+1}}{P_t} = r_t s_{i,t} + \frac{b_{i,t}}{P_t} + (q_t\phi_{i,t} - \kappa_i) e_{i,t} [(1 - \delta)s_{i,t} + i_{i,t}], \quad (3.2.7)$$

We define the effective selling price of a unit of financial asset as

$$q_{i,t} \equiv q_t - \frac{\kappa_i}{\phi_{i,t}}. \quad (3.2.8)$$

When $\kappa_i > 0$, the effective selling price is below the transaction price. Hence, entrepreneurs not only face constraints regarding the quantity of equity that can be issued and resold, they also have to sell at a discount due to the intermediation cost $\kappa_i\phi_{i,t}$ when liquidating financial claims. These factors have important implications for the tightness of financing constraints and households’ portfolio choice. Using (3.2.3) and $m_{i,t} = \phi_{i,t}e_{i,t}[(1 - \delta)s_{i,t} + i_{i,t}]$, investment can be substituted out from (3.2.7), such that the flow-of-funds constraint becomes

$$c_{i,t} + q_{r,t}s_{i,t+1} + \frac{b_{i,t+1}}{P_t} = \phi_{i,t}e_{i,t} + (1 - \phi_{i,t}e_{i,t})q_{r,t} (1 - \delta)s_{i,t}$$

$$+ r_t s_{i,t} + \frac{b_{i,t}}{P_t}, \quad (3.2.9)$$

where

$$q_{r,t} = \frac{1 - \phi_{i,t}e_{i,t}}{1 - \phi_{i,t}e_{i,t}}. \quad (3.2.10)$$

The left-hand side of (3.2.9) captures entrepreneurs’ spending on consumptions and accumulation of equity and money, while the right-hand side represents entrepreneurial (total) net-worth including rental income from capital claims, the value of existing equity claims and the real value of money. Note that a fraction $\phi_{i,t}e_{i,t}$ is saleable and, hence, valued at $q_{i,t}$, while a fraction $(1 - \phi_{i,t}e_{i,t})$ is retained and valued at $q_{r,t}$, which is the effective replacement cost of existing assets. To see this, notice that entrepreneurs can sell a fraction $e_{i}\phi_{i,t}$ of their financial assets at price $q_{i,t}$. For every unit of new investment, they will accordingly need to make a “down-payment” $(1 - \phi_{i,t}e_{i,t}q_{i,t})$ and retain a fraction $(1 - \phi_{i,t}e_{i,t})$ as inside equity. With this interpretation, if entrepreneurs replace existing assets by new assets issued against investment, $q_{r,t}$ is indeed the effective replacement cost.\(^{15}\)

\(^{15}\)Further, $q_r$ captures the effect of search costs on equity accumulation: higher search costs decrease the effective sales price, which increases the down-payment that in turn depresses equity accumulation. Therefore, the entrepreneurs’ ability to leverage will be lower if search costs are higher.
Notice that (3.2.9) involves gross investment. New investment can be backed out from \( s_{i,t+1} = (1 - \phi_i e_{i,t}) (s_{i,t} + i_t) \) and (3.2.9). Formally, 

\[
\bar{i}_{i,t} = \frac{\left( r_t + \phi_i e_{i,t} q_{i,t} (1 - \delta) \right) s_{i,t} + b_{i,t} }{1 - \phi_i e_{i,t} q_{i,t} } - c_{i,t},
\]

which says that entrepreneurs’ liquid net-worth net of consumption can be levered at \((1 - e_i \phi_i q_i)^{-1}\) to invest in new capital.

**A Household’s Problem**

*Aggregation.* Recall that \( j \in \{i, n\} \) indicates workers and entrepreneurs, respectively. We define aggregate type-specific variables as \( X_{i,t} \equiv \chi x_{i,t} \) and \( X_{n,t} \equiv (1 - \chi) x_{n,t} \). Household-wide variables correspond to the aggregation of workers’ and entrepreneurs’ quantities, i.e., \( X_t = X_{n,t} + X_{i,t} \). For example, aggregate consumption is the sum of consumption of workers and entrepreneurs, i.e., \( C_t = C_{n,t} + C_{i,t} \).

For simplicity, we switch to recursive notation, i.e., let \( x \) and \( x' \) denote \( x_t \) and \( x_{t+1} \). Since previously accumulated assets are homogeneously divided across all household members, we have \( S_i = \chi S \), \( S_n = (1 - \chi) S \), \( B_i = \chi B \) and \( B_n = (1 - \chi) B \). Because entrepreneurs do not work, we also have that \( N = N_n \). Given these simplifications, the individual budget constraints (3.2.6) and (3.2.9) aggregate to

\[
C_n + q_n S'_n + \frac{B'_n}{P} = wN + [r + q_n (1 - \delta)] (1 - \chi) S + (1 - \chi) \frac{B}{P}, \tag{3.2.12}
\]

\[
C_i + q_i S'_i + \frac{B'_i}{P} = [r + \phi_i e_i q_i + (1 - \phi_i e_i) q_r] (1 - \delta) \chi S + \chi \frac{B}{P}, \tag{3.2.13}
\]

Note that total investment can be aggregated from (3.2.11) to

\[
I = \chi \left[ (r + \phi_i e_i q_i (1 - \delta)) K + \frac{B}{P} \right] - C_i. \tag{3.2.14}
\]

*Notice that we implicitly impose that \( S'_n \geq (1 - \delta)(1 - \chi)S \) such that workers in a household are always buyers. Such condition is satisfied in our later numerical analysis because we focus on shocks that will not push workers to sell assets to smooth consumption. Aggregation takes into account type-specific transactions on the search market and evolutions of equity.

*Once we proceed to the equilibrium definition, \( \Gamma \equiv (K, B; z_a, z_{\kappa}) \) where \( K \) is the total capital stock, \( B \) is the total amount of money circulated, \( z_a \) is total factor productivity in final goods production and \( z_{\kappa} \) is an intermediation cost shock in the search market. The exogenous stochastic processes for \( z_a \) and \( z_{\kappa} \) are specified in the numerical examples in section 3.4.*
Since at the end of the period workers and entrepreneurs reunite to share their stocks of equity and money, we have

\[ S' = S'_n + S'_i, \quad B' = B'_n + B'_i. \]  

(3.2.15)

Then, the value satisfies the following Bellman equation,

**Problem 1:**

\[
J(S, B; \Gamma) = \max_{\{e, N, C, S'_i, B'_n\}_{k \in \mathbb{N}}} \chi u \left( \frac{C_i}{\chi} \right) + (1 - \chi) \left[ u \left( \frac{C_n}{1 - \chi} \right) - h(N - \chi) \right] \\
+ \beta E \mathbb{E} \left[ J(S', B'_i; \Gamma') \right]
\]

s.t. (3.2.12), (3.2.13) and (3.2.15).

### 3.2.3 Search, Matching and the Asset Price

**Search and Matching.** Matching between buyers and sellers of private claims is handled by zero-profit intermediaries owned by households, which provide specialized services to find counterparts. The implicit assumption is that individual search for trading partners is extremely costly for buyers and sellers alike. The - comparatively small - costs of matching counterparts through intermediaries motivate the remaining search costs. Note that we do not distinguish financial institutions (e.g., banks) and dealers in financial markets in our model. They are both captured by the financial sector with a costly matching technology, which intermediates the asset price.\(^{18}\)

Buyers post total asset positions \(E_n = \phi_n^{-1} [S'_n - (1 - \delta)S_n]\) that are to be filled. Sellers put their new and old assets on sale, offering \(E_i = e_i [(1 - \delta)S + I]\). After \(E_n\) and \(E_i\) are determined, the number of aggregate matches \(M\) is determined by intermediaries’ matching technology

\[ M(E_n, E_i) = \xi E_n^{1-\eta} E_i^\eta, \]

where \(\eta \in (0, 1)\) is the elasticity of matches with respect to assets on sale and \(\xi\) measures the matching efficiency.

Defining \(\theta\) as the ratio of vacant asset positions \(E_n\) to assets on sale \(E_i\), we have

\[ \theta \equiv \frac{E_n}{E_i}, \quad \phi_n \equiv \frac{M}{E_n} = \xi^{1-\eta}, \quad \phi_i \equiv \frac{M}{E_i} = \xi^{1-\eta}, \]

(3.2.16)

\(^{18}\) To the extent that intermediaries resemble banks, the matching fee could be interpreted as comprising screening and monitoring costs associated with successful matches. For a detailed discussion of these two types of agents and their impact on macroeconomic dynamics refer to De Fiore and Uhlig (2011)
where $\phi_n$ captures the probability of a buyer meeting a seller for each unit of asset positions posted and $\phi_i$ the probability of a seller meeting a buyer for each unit of assets put on sale. Recall that $\phi_i$ also represents the fraction of financial assets that can be sold ex post in a given period. Therefore, we refer to $\phi_i$ as asset saleability.

Notice that $\theta$ expresses the search market tightness from a buyer’s perspective. A larger $\theta$ indicates that buyers have difficulty in finding appropriate investment opportunities on the search market, such that $E_i$ are relatively small compared to $E_n$. Lastly, noticing that $\phi_n^{-1} \phi_i = \theta$, we can link the relationship between $\phi_n$ and $\phi_i$ as

$$\phi_n = \xi^{-1} \phi_i ^{\frac{n}{n-1}}. \quad (3.2.17)$$

The Asset Price. Once a unit of offered assets is matched to a vacant asset position, intermediaries offer a price $q$ to both parties. Since intermediaries make zero profits and are owned by the households, they seek to maximize the total surplus by bargaining on behalf of each side of the trade. Notice that the amount of matched assets $m_{j,t}$ is predetermined at the point of bargaining. Therefore, buyers and sellers interact at the margin $m_{j,t}$, i.e., the match surplus for both buyers and sellers is the respective marginal value of an additional transaction.

Denote by $J^n$ and $J^i$ the value of individual workers and entrepreneurs from the point of view of the household. In consumption goods unit, a buyer’s surplus amounts to

$$-J^m_n = -q + \beta \mathbb{E}_\Gamma \left[ \frac{J_S(S', B'; \Gamma')}{w'(c_n)} \right]$$

where $m$ as a subscript indicates the marginal value of a successful match. Intuitively, if the deal is successful, the buyer sacrifices $q$ today but gains the household’s value of one more unit of assets tomorrow (normalized by the marginal utility of a worker’s consumption). Similarly, the sellers’ surplus is the marginal value to the household of an additional match for entrepreneurs, i.e.

$$J^i_m = q - \frac{1}{e_i \phi_i} + \beta \left( \frac{1}{e_i \phi_i} - 1 \right) \mathbb{E}_\Gamma \left[ \frac{J_S(S', B'; \Gamma')}{w'(c_i)} \right].$$

In other words, the seller receives a gain of $(q - e_i^{-1} \phi_i^{-1})$ in the current period plus a continuation value from a successful match. The contemporary surplus reflects that entrepreneurs earn the bargaining price $q$, but spend $e_i^{-1} \phi_i^{-1}$ resources per additional match on new investment projects. The evolution of entrepreneurs’ equity position can be expressed as the difference

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Note that search market participation costs are already sunk at the bargaining stage. However, search costs are not ignored since households take them into account when determining optimal asset posting decisions by workers and entrepreneurs.
between offered and sold assets (i.e., \( s'_i = (e^{-1}\phi^{-1}_i - 1) m_i \)). Entrepreneurs retain a fraction \( (e^{-1}\phi^{-1}_i - 1) \) for each unit of successful matches as inside equity, which is brought back to the household. Therefore, the continuation value of a match consists of the marginal value of future assets to the household multiplied by this factor (and normalized by the marginal utility of an entrepreneur’s consumption).

Note that all members within the groups of buyers and sellers are homogeneous, such that the type-specific valuations are identical in all matched pairs. We consider the case in which the transaction price \( q \) is determined by surplus division between buyers and sellers. That is, intermediaries set a price \( q \) to maximize

\[
\max_q \{ (J^1_m)^\omega (-J^m_n)^{1-\omega} \}
\]

where \( \omega \in (0, 1) \) is the fraction of the surplus that goes to sellers. Notice that this set-up is similar to bilateral (generalized) Nash bargaining between buyers and sellers over the match surplus. In the bilateral bargaining case, \( \omega \) is the bargaining power of sellers. In this sense, our price setting is similar to the wage determining process in Morten O. Ravn (2008) and Ebell (2011), where individual workers come to bargain on behalf of their respective households.

### 3.2.4 Recursive Competitive Equilibrium

We close the model by defining the recursive competitive equilibrium.

**Definition 1:**

The recursive competitive equilibrium is a mapping \( K \rightarrow K' \), with associated consumption, investment, labour and portfolio choices \( \{C_n, C_i, N, e, I, S'_n, S'_i, B'_n, B'_i\} \), asset liquidity \( \{\phi_i, \phi_n\} \) and a collection of prices \( \{P, q_n, q_i, q_r, w, r\} \), given exogenous evolutions of aggregate productivity \( z_a \) and search costs \( (\kappa_n, \kappa_i) \), such that

1. final goods producers’ optimality conditions in (3.2.1) hold;
2. \( S_n = (1 - \chi)S, S_i = \chi S, B_n = (1 - \chi)B \) and \( B_i = \chi B \). Given prices, the policy functions solve the representative household’s problem (Problem 1), subject to the households’ budget constraints (3.2.12) and (3.2.13) and aggregate investment in (3.2.14);
3. market clearing conditions hold, i.e.,
   - (a) the capital market clears: \( K' = (1 - \delta) K + I \) and \( K = S \);
   - (b) the search market clears: (3.2.17) holds and \( q \) solves (3.2.18), with the effective prices defined in (3.2.5), (3.2.8) and (3.2.10).
   - (c) the market for liquid assets clears: \( B' = B \);
To verify that Walras’ Law is satisfied, notice that the investment equation and the household budget constraint resemble the entrepreneurs’ and workers’ budget constraints (3.2.12) and (3.2.13). These two constraints imply the aggregate resource constraint

\[ C + I + \kappa_n E_n + \kappa_i E_i = e_i^{\gamma} K^\alpha N^{1-\alpha}, \]  

(3.2.19)

where \( E_i \) and \( E_n \) are again the total number of assets on sale and asset positions to be filled.

### 3.3 Equilibrium Characterization

We mainly focus on the interesting equilibrium with positive participation costs on both sides, i.e., \( \kappa_n > 0 \) and \( \kappa_i > 0 \). The limiting case when costs become zero is discussed in the end.

Note that the cost of search market participation may large enough for the households to be forced to finance investment entirely from internal funds, i.e. retained earning, such that the search market is inactive. In contrast, we restrict our attention to an economy in which the search market is active. That is, the replacement costs \( q_r \leq 1 \). Using the definition of \( q_r \), this implies that effective selling price \( q_i \geq 1 \). Then, the effective purchase price \( q_n \) is strictly greater than 1, since \( \kappa_n > 0 \) and \( \kappa_i > 0 \). Thus, \( q_n > q > q_i \geq 1 \geq q_r \).\(^{20}\) Compared to workers, who value equity at price \( q_n \), the price of equity is strictly lower from the perspective of entrepreneurs. Therefore, the household will prompt entrepreneurs to spend whatever net worth they are not consuming on creating new equity. Entrepreneurs thus sell as many existing equity claims as possible and do not invest into money, i.e., \( e_i = 1 \) and \( B'_i = 0 \).

To ensure \( q_r \leq 1 \), we restrict exogenous parameters. To see this, first define

\[ \gamma \equiv \frac{\omega \kappa_n}{1 - \omega \kappa_i} \]

and let the steady state value of \( \kappa_n, \kappa_i \), and \( \gamma \) be \( \bar{\kappa}_n, \bar{\kappa}_i \), and \( \bar{\gamma} \). The parameter restriction is then formally expressed in the following lemma.

---

\(^{20}\) As shown in Corollary 1, in a frictionless economy with costless search market participation the capital price approaches \( q_t = 1 \). In this case, the internal equals the external cost of creating capital goods, such that capital production yields zero profits and financial constraints cease to exist. Empirically, the capital price captures Tobin’s \( q \), which ranges between 1.1 and 1.21 in the U.S. economy, i.e. well above 1. For this empirically relevant case, capital production is profitable, which reflects financial constraints of firms. During recessions, \( q_t \) typically falls and erodes firms’ net worth, which tightens financing constraints further. This tightening derives from the fact that firms are leveraged, such that the contraction in their funding base due to the negative shock to net worth is strongly amplified.
Lemma 1:
Suppose $\kappa_n > 0$ and $\kappa_i > 0$. If the following condition is satisfied

$$\frac{\beta^{-1} - (1 - \chi)}{\chi} \geq \frac{\kappa_n (\beta^{-1} - (1 - \chi) \chi)^\eta}{\xi^\gamma} + \frac{\kappa_i (\beta^{-1} - (1 - \chi) \chi)^{\eta-1}}{\xi^\gamma} + 1,$$

(A1)

then $q_n > q > q_i \geq 1 \geq q_r$ in the neighborhood around steady state.

Proof. See the Appendix 3.B.1.

As an illustration, if we further restrict $\bar{\kappa}_n = \bar{\kappa}_i = \kappa$, the above restriction implies an upper bound for the search costs $\kappa$. (A1) then directly implies that costs of participation should not be too large.

3.3.1 The Households’ Optimality Conditions

To reduce the number of prices, we define the ratio of the effective purchase price and the effective replacement cost:

$$\rho \equiv \frac{q_n}{q_r}.$$  

(3.3.1)

By using the types’ budget constraints (3.2.12) and (3.2.13) to substitute out $C_i$ and $C_n$ in Problem 1, and using $e = 1$ and $B'_i = 0$, we know that a household’s optimal choice can be reduced to the set $\{N, S'_i, S'_n, B'_n\}$.21

Then, the first-order condition for labour is

$$u'(c_n) w = \mu.$$  

(3.3.2)

The first-order conditions for $S'_i$ and $S'_n$ are

$$u'(c_i) q_r = \beta E_\Gamma [J_S(S', B'; \Gamma')] , \quad u'(c_n) q_n = \beta E_\Gamma [J_S(S', B'; \Gamma')] ,$$

from which we learn that

$$u'(c_i) = \rho u'(c_n).$$  

(3.3.3)

$\rho$ is inversely related to risk-sharing among workers and entrepreneurs. When $\rho = 1$, search frictions disappear and entrepreneurs are not financing constrained (see Corollary 1). In this case, (3.3.4) naturally implies $c_i = c_n$, i.e., perfect consumption risk-sharing among household members. In an economy where the search market structure imposes financing frictions, we have $\rho > 1$. Therefore, $c_i < c_n$ and the risk-sharing capacity of the household decreases in $\rho$. Finally, the optimality condition for money holdings $B'_n$ is

$$u'(c_n) \frac{1}{P} = \beta E_\Gamma [J_B(S', B'; \Gamma')] .$$

21As in a portfolio choice problem, the corresponding first-order conditions are also sufficient due to the concavity of the objective function.
3. Search-Based Endogenous Illiquidity and the Macroeconomy

We derive asset pricing formulae for equity and money. Using the envelope condition and noticing that
\[ \phi_i q_i + (1 - \phi_i) q_r = 1, \]
we obtain
\[ J_S = u'(c_i) \chi [r + 1 - \delta] + u'(c_n) (1 - \chi) [r + q_n (1 - \delta)] \]
\[ = u'(c_n) [(\chi \rho + 1 - \chi) r + (1 - \delta) (\chi \rho + (1 - \chi) q_n)], \]
together with the first-order condition for equity
\[ S_n = \frac{\beta u'(C_n') (\chi \rho' + (1 - \chi)) r' + (1 - \delta) (\chi \rho' + (1 - \chi) q_n')}{q_n} = 1, \]
(3.3.5)
where the second term in the expectations operator captures the internal return on equity from the perspective of the household. Similarly, we can derive another asset pricing formula for money by applying the envelope condition again
\[ E \left[ \frac{\beta u'(C_n') (\chi \rho' + (1 - \chi)) r' + (1 - \delta) (\chi \rho' + (1 - \chi) q_n')}{q_n} \right] = 1, \]
(3.3.6)
where the second term in the expectations operator is the internal return on money from the perspective of the household, and inflation is defined as
\[ \pi' \equiv \frac{P'}{P}. \]

In the steady state, condition (3.3.6) implies that
\[ [\chi \rho + 1 - \chi] \pi^{-1} = \beta^{-1}. \]
If money is valued, \( \bar{\pi} = 1 \) and
\[ \bar{\rho} = \chi^{-1} [\beta^{-1} - (1 - \chi)] > 1. \]

As a result, the real interest rate on money \( \pi^{-1} \) will be lower than the rate of time preference \( \beta^{-1}. \) This fact shows that money provides a liquidity service and, accordingly, carries a liquidity premium. This is easiest to be seen in the steady state.

**Proposition 2:**

*Suppose (A1) holds. Then, money provides a liquidity service in a neighborhood around the steady state. The steady state liquidity premium amounts to*

\[ \Delta B \equiv [\chi \rho + (1 - \chi) - 1] \frac{1}{\pi} = \frac{(\bar{\rho} - 1) \chi}{\bar{\pi}} = \beta^{-1} - 1 > 0. \]

---

22 One could interpret \( \phi_i q_i + (1 - \phi_i) q_r = 1 \) in the following way. The fully resaleable fraction of existing equity is worth \( \phi_i q_i, \) while the non-resaleable fraction of existing equity is \( 1 - \phi_i q_r, \) which corresponds to the net-worth put up by the entrepreneurs.

23 Though we focus on fiat money, such that \( P' = P \) in the steady state (and \( \pi^{-1} = 1 < \beta^{-1} \)), one can easily imagine an economy where the government steps in and may run inflation or deflation.
When $\rho > 1$, liquidity frictions matter and entrepreneurs are financing constrained. An additional unit of money then relaxes entrepreneurs’ constraints by increasing their net-worth, which allows them to leverage their investment or, equivalently, their future equity position. We will show in Corollary 1 that in the limiting case ($\rho \to 1$) equity can be sold without frictions and money loses its liquidity value. The asset pricing formulae then collapse to a single standard Euler equation as in a RBC model. However, if the search market is not frictionless, the liquidity premium is non-zero and may vary substantially over the business cycle. The liquid asset share in households’ portfolio composition will then fluctuate correspondingly.

### 3.3.2 The Bargained Asset Price

The asset price is set to maximize the total surplus of buyers and sellers. Assuming an interior solution, the sufficient and necessary first-order condition yields

$$\frac{\omega}{u'(c_i) (q - \phi_i^{-1}) + (\phi_i^{-1} - 1) \beta E_T J_S (S', B'; \Gamma')} = 1 - \frac{1}{u'(c_n)q + \beta E_T J_S (S', B'; \Gamma')}.$$  

By using the household’s optimality condition for asset holdings, $u'(c_n)q_n = \beta E_T J_S (S', B'; \Gamma')$, and the risk-sharing condition, $u'(c_i) = \rho u'(c_n)$, we can derive an analytical solution for the asset price, stated in the following proposition:

**Lemma 2:**

Suppose (A1) holds. The asset price solution simplifies to

$$\rho = \frac{\omega}{1 - \omega} \frac{\kappa_n}{\kappa_i} \theta_i,$$  

which can be solved for $q$ as a function of saleability $\phi_i$:

$$q = \frac{\gamma (1 + \frac{\xi}{\phi_i}) \phi_i - \kappa_n}{\xi^{\frac{1}{1-\eta}} \phi_i^{\frac{1}{1-\eta}} \left[ 1 + \left( \gamma (\xi^{-1} \phi_i)^{\frac{1}{1-\eta}} - 1 \right) \phi_i \right]}.$$  

**Proof.** See Appendix 3.B.2.

Proposition 2 is the main result of our model linking the equity price with intermediation costs and asset saleability. If the parameter restriction (A1)
is satisfied, i.e. if intermediation costs are not prohibitively high, then the price of equity exceeds the cost of capital (which is one). Therefore, creating capital by issuing equity claims is profitable and entrepreneurs would like to issue additional claims. If intermediation costs were removed, they would do so until the price of equity would equal the cost of capital, thereby erasing any profits. However, financial frictions due to non-zero intermediation costs endogenously limit the saleability of financial claims, which prevents entrepreneurs from issuing a sufficient amount of claims, such that the price of equity permanently stays above the cost of capital.

In order to understand the relationship between the asset price and financial frictions, it is instructive to first consider a comparative static analysis before moving to the discussion of equilibrium dynamics in the next section. First, consider how the asset price responds to small increases in intermediation costs, which capture a deterioration of the intermediation capacity of the search market. On the one hand, \((3.3.7)\) implies that a higher \(\kappa_n\) decreases the asset price since participation in the search market becomes more costly for the demand side, such that buyers participate less; on the other hand, an increase of \(\kappa_i\) in \((3.3.7)\) pushes the asset price up as it increases the costs of posting assets for sale. The net effect of higher intermediation costs on the equity price, therefore, depends on the parameters.

For ease of exposition, we restrict our attention to the case in which the ratio of participation costs of buyers and sellers stays constant.

**Proposition 3:** Suppose \((A1)\) holds and the ratio of \(\bar{\kappa}_n/\bar{\kappa}_i = g\) is fixed. If the following condition is satisfied

\[
\xi \left[\frac{\beta^{-1} - (1 - \chi)}{\chi}\right]^{1 - \eta} < (1 - \omega)^{\eta} \omega^{1 - \eta} g^{1 - \eta},
\]

then the steady state asset price \(q\) drops when \(g\) increases.

**Proof.** See the Appendix 3.B.3. □

The higher the level of \(g\), i.e. the cost of posting buy orders relative to the cost of posting sale orders, the more likely it is that the above condition is satisfied. In other words, if the sensitivity of the demand side to search market participation increases relative to the sensitivity of the supply side, the bargained asset price will fall. The fact that the endogenous response of the demand relative to the supply side to variation in intermediation costs determines the sign of asset price changes underpins a key insight gleaned from the equilibrium dynamics presented in the next section.

Endogenizing asset liquidity further gives rise to a non-trivial relationship between \(q\) and saleability \(\phi_i\). In particular, if \(\phi_i\) is sufficiently small, the asset price can be shown to correlate positively with \(\phi_i\).
Proposition 4: 
$q$ correlates positively with asset saleability $\phi_i$ (i.e. $\frac{\partial q}{\partial \phi_i} > 0$) and negatively with the purchase rate $\phi_n$ (i.e. $\frac{\partial q}{\partial \phi_n} < 0$), if

$$\phi_i < \left[ \frac{\eta}{1 - \eta} + \left(1 + \frac{\kappa_i}{\omega}\right) \right] \left[ \frac{\eta}{1 - \eta} + 2\gamma(\xi^{-1}\phi_i)^{\frac{1}{\xi}} - 1 \right]^{-1}.$$  \hspace{1cm} (A3)

When $\eta = 0.5$, the above sufficient condition simplifies to $\phi_i < \sqrt{\left(1 + \frac{\kappa_i}{\omega}\right)\gamma^{-1}\xi}$. 


Intuitively, the drop in saleability implies that a larger share of investment needs to be financed out of entrepreneurs’ own funds. On the one hand, this tightens the contemporaneous financing constraints of entrepreneurs. The threat point for entrepreneurs of breaking off negotiations over an additional asset sale and self-financing at the margin becomes less attractive. Entrepreneurs are thus more willing to accept a lower bargaining price. On the other hand, retaining a larger fraction of equity stakes also implies that entrepreneurs return more assets to the household, which relaxes the funding constraints of future generations of entrepreneurs. This effect supports the threat point, such that entrepreneurs ask for a higher transaction price in a successful match. Thus, a trade-off emerges between current and future funding constraints.

Proposition 4 shows that the contemporaneous effect dominates as long as the sales rate is small enough, because current financial constraints bind strongly. If financial frictions are sufficiently tight, entrepreneurs will have to accept a lower price when the demand side is less willing to participate. Our model can thus generate simultaneous decreases in asset saleability and the asset price through the simultaneous reaction of supply and demand.

Remark 1: An exogenous drop in asset saleability, such as in KM and Shi (2015), acts like a negative supply shock on the asset market: the decline in saleability translates into a tighter financing constraint for entrepreneurs and less supply of financial claims on the asset market. However, the productivity of capital is not affected by the shock such that asset demand does not fall much. The dominating supply contraction triggers an asset prices boom - a counter-factual phenomenon in recessions. Although this effect is still present in our framework, there are competing forces from the demand side that outweigh the supply contraction under the condition expressed in Proposition 4.

Remark 2: Frictionless limiting case. When there are no search costs for either buyers or sellers, i.e. $\kappa_n = 0$ and $\kappa_i = 0$, the search market price will go to $q = 1$, i.e. it will equal the cost of capital. In this case, money loses its liquidity premium and the economy collapses to the RBC framework.
Households’ Euler equations then simplify to the standard ones in a RBC framework since $\rho = 1$. In summary,

**Corollary 1:**

When $\kappa_i \to 0$ and $\kappa_n \to 0$, $q_n \to q_i \to q \to 1$ and $\rho \to 1$.

**Proof.** See Appendix 3.B.5.

To back out asset saleability $\phi_i$ when $\kappa_i \to 0$ and $\kappa_n \to 0$, one can first solve for consumption and investment in the corresponding RBC model, because of $q = 1$ and perfect consumption risk-sharing. Then, $\phi_i$ can be backed out from aggregate investment given by (3.2.14).

### 3.4 Numerical Examples

While we discussed some analytical results in previous discussion, this section uses numerical tools to illustrate system dynamics after exogenous shocks. We consider a standard AR(1) process for aggregate productivity, i.e.,

$$z'_a = \rho_a z_a + \epsilon'_a,$$

with i.i.d. $\epsilon'_a \sim N(0, \sigma^2_a)$. We further introduce a symmetric shock to the cost of financial intermediation, which in our asset search framework corresponds to an increase in the participation costs. We let

$$\kappa_i = e^{z_{\kappa_i}} \bar{\kappa}_i, \quad \kappa_n = e^{z_{\kappa_n}} \bar{\kappa}_n,$$

where $z_{\kappa}$ follows an AR(1) process

$$z'_\kappa = \rho_{\kappa} z_\kappa + \epsilon'_{\kappa},$$

with i.i.d. $\epsilon'_{\kappa} \sim N(0, \sigma^2_{\kappa})$. Rather than affecting the production frontier of the economy, this shock simply impairs the capacity of the financial sector to intermediate funds between workers and entrepreneurs. Both in a market and a banking context, such an increase in intermediation costs may, for example, be triggered by rising uncertainty about counterparty risk. In contrast to productivity shocks, the resource loss associated with intermediation cost shocks is small, such that they unfold their effects primarily through the endogenous responses of asset saleability and prices.

Using the model as a laboratory, we study the different equilibrium responses to aggregate productivity shocks and intermediation cost shocks. In particular, when $\kappa_i = 0$ but $\kappa_n > 0$, we have a similar result, except that households always spend a fixed fraction of resources to purchase equity. When $\kappa_n = 0$ but $\kappa_i > 0$, the household will not participate in the search market. This is because when $\kappa_n = 0$, the bargain price collapses to $q = 1$, which implies that $q_i < 1$ (given $\kappa_i > 0$). That is, (A1) is violated when $\kappa_n = 0$ and $\kappa_i > 0$. 

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26When $\kappa_i = 0$ but $\kappa_n > 0$, we have a similar result, except that households always spend a fixed fraction of resources to purchase equity. When $\kappa_n = 0$ but $\kappa_i > 0$, the household will not participate in the search market. This is because when $\kappa_n = 0$, the bargain price collapses to $q = 1$, which implies that $q_i < 1$ (given $\kappa_i > 0$). That is, (A1) is violated when $\kappa_n = 0$ and $\kappa_i > 0$. 

particular, we compare the model’s predictions for the dynamics of standard macroeconomic variables, the asset price and the liquidity share with the corresponding time series for the U.S. economy. The empirical counterpart for the liquidity share is defined as the total amount of nominal liquid assets $PB$ circulated within the U.S. (essentially money and government bonds, see Appendix 3.C) over total assets $PB + PqK$ (where $PqK$ is the value of capital within the U.S., see Appendix 3.C). Fluctuations in the liquidity share indicate changes in private investors’ willingness to hold liquid assets. For asset price, we use the Wilshire 5000 price full cap index from 1971Q1-2013Q4. Because we do not model government policies, we identify GDP as the sum of real private consumption and real private fixed investment between 1971Q1 and 2013Q4.

We use the HP filter (with a smoothing coefficient of 1600) to de-trend all time series. In the data, the liquidity share correlates negatively with GDP (-0.58), while asset prices correlate positively with GDP (0.50). These two correlations suggest that asset prices fall in recessions, which tends to be associated with portfolio rebalancing towards liquid assets.

### 3.4.1 Calibration

We calibrate the steady state of the model to several long-run U.S. statistics. We choose a conventional CRRA utility function for consumption, $u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$, and a linear dis-utility of labour, $h(n) = \mu n$.\textsuperscript{27} Parameters $\beta$, $\sigma$, and $\delta$ are chosen exogenously and are similar to a standard calibration. $\alpha$ and $\mu$ are set to target the investment-to-GDP ratio and working hours (Table 3.2). Note that the parameter $\chi$ can be interpreted as the fraction of firms which adjust capital in a period. According to Doms and Dunne (1998), the annual fraction is 0.20 which translates to $\chi = 0.054$ at quarterly frequency (similar to Shi (2015)).

There are five parameters related to the search market $\{\xi, \bar{\kappa}_n, \bar{\kappa}_i, \eta, \omega\}$. $\xi$ and $\eta$ are not independent due to the constant returns to scale matching technology on the search market. Without loss of generality, $\eta$ can, therefore, be fixed at 0.5, while $\xi$ is calibrated. This leaves four independent parameters to be calibrated to match four targets: Tobin’s $q$ ranges from 1.1 to 1.21 in the U.S. economy according to Compustat data. This paper targets a value of $q = 1.15$. The liquidity share is around 10% on average, which is used to calibrate $\omega$. Steady state saleability $\phi_i$ is calibrated at 0.28, which corresponds the ratio of funds raised in the market to fixed investment in the U.S. flow-of-funds data.\textsuperscript{28} Finally, the total cost of intermediation is estimated at 10% of total investment, in line with the findings in the cost

\textsuperscript{27}This dis-utility function facilitates the steady state solution. The main results are robust to a more complicated specification (available upon request).

\textsuperscript{28}Nezafat and Slavik (2011) use the US flow-of-funds data for non-financial firms to estimate the stochastic process of $\phi_i$. 
of initial public offerings (IPO) (e.g., Chen, Ritter, McCormick, Naranjo, Schultz, Thomas, and Wallman (2000)). Notice that with these parameters, assumptions (A1), (A2), and (A3) are satisfied and one should expect the asset price and asset saleability to positively co-move.

We set the persistence and the size of shocks to target the volatility (0.02) and first order correlation (0.89) of GDP’s cyclical components. By using only productivity shocks, we have

$$\rho_a = 0.88, \sigma_a = 0.008.$$  

By using only shocks to intermediation costs, the exercise gives

$$\rho_\kappa = 0.81, \sigma_\kappa = 0.69.$$  

We use these parameters to show two numerical simulations in the following. By design, these two shocks will generate very similar aggregate output dynamics. The focus will be the different paths of other variables.

### 3.4.2 Equilibrium Responses to Shocks

**Adverse aggregate productivity shocks.** Suppose a shock hits the economy at time 0 (see the dynamics of $z$ in Figure 3.1). This shock depresses the rental rate of capital and its value to the household. Search for investment into entrepreneurs is less attractive and the amount of purchase orders from workers drops. The demand-driven fall is reflected in the sharp drop in asset saleability $\phi_i$. This endogenous decline of asset liquidity amplifies the initial shock in two ways: (1) it reduces the quantity of assets that entrepreneurs are able to sell; (2) the bargaining price, i.e. private assets’ resale value falls - though only modestly - in line with our analytical result in Proposition 4. Both effects constrain entrepreneurs and thus tighten their financing
constraints. As a result, investment falls. Note that consumption also falls because of fewer resources.

In principle, money’s liquidity service becomes more valuable to households when private claims’ liquidity declines. However, in the case of a persistent TFP shock, lower expected returns to capital make future investment less attractive. This effect works against the incentive to hedge against asset illiquidity for future investment. Which effect dominates depends on the calibration and is thus an empirical question. In our calibration, the decline in profitability of investment projects is sufficient for the liquidity share to drop. This fall in demand for liquid assets is reflected in the decrease of their price $1/P$, which leads to a surge in inflation $\pi = P/P_{-1}$ on impact. To the extent that total factor productivity reverts back to the steady state while asset liquidity is still subdued, hedging becomes more attractive which explains the relatively fast recovery of the liquidity share, $B/(B + PqK)$.

*Intermediation shocks.* Suppose a shock hits the economy at time 0 (see
the dynamics $\kappa$ in Figure 3.1). The output dynamics in this scenario are by construction similar to those of the productivity shock.

Note that higher search costs bind resources. Both the substitution and income effects induce households to adjust their portfolios. Realizing that search market participation is more costly now and later, households seek to reduce their exposure to private financial claims. On the supply side, financing-constrained entrepreneurs would still like to sell as many assets as possible in order to take full advantage of profitable investment opportunities. Thus, asset demand on the search market shrinks relative to supply, which reduces the likelihood for sellers to be matched with buyers, such that asset saleability falls.

Since the sharp drop in asset liquidity tightens entrepreneurs’ financing constraints substantially, the threat point of abandoning the bargaining process with a potential buyer worsens. Entrepreneurs as sellers are willing to accept a lower price. The bargaining price thus falls strongly and amplifies the initial shock by depressing entrepreneurs’ net worth further. This effect is mirrored in a significant decline of investment activity, the impact response of which is about six times stronger than that of output.

As saving via the bond market becomes more expensive, workers reduce their labour supply and consume slightly more for three quarters after the initial shock. As a result, consumption increases initially, while output falls immediately. However, lower investment into the capital stock soon reduces the marginal product of labour and the wage rate. As labour income falls, consumption persistently drops below the steady state from the fourth quarter.

While the intermediation cost shock depresses the demand for and liquidity of private assets, it substantially increases the hedging value of money. To see this, note that future investment remains profitable since the productivity of capital is not affected by the shock. To take advantage of future investment opportunities, households seek to hedge against the persistent illiquidity of private claims by expanding their liquidity holdings. Therefore, the liquidity share rises.

### 3.4.3 Discussion

The equilibrium dynamics suggest two key results. (1) In order to reconcile declining asset saleability with falling asset prices, liquidity must be an endogenous phenomenon. In other words, it must be a consequence, rather than a cause of economic disturbances. (2) Both standard productivity and intermediation cost shocks affect the hedging value of liquid assets. However, only the latter unambiguously implies a negative co-movement between the liquidity share and aggregate output.
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Business Cycle Statistics

Some key business cycle statistics of the model in comparison to the data are reported in Table 3.3, where only aggregate productivity shocks are considered. Our main targets are consumption, investment, asset prices, and the liquidity share. As in a RBC model, consumption and investment volatility, the correlation of macroeconomic variables with GDP, and first-order autocorrelations are roughly in line with the data. However, the liquidity share and the asset price move too little in the model. Besides, the model-implied negative correlation between the liquidity share and GDP falls short of the data, while the asset price co-moves too strongly with GDP.

Table 3.3: Business cycle statistics with only aggregate productivity shocks

<table>
<thead>
<tr>
<th>Variable</th>
<th>Relative volatility $\frac{x}{y}$</th>
<th>Correlation $\rho(x, y)$</th>
<th>1st auto-correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Output</td>
<td>0.02</td>
<td>0.02</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.44</td>
<td>0.67</td>
<td>0.88</td>
</tr>
<tr>
<td>Investment</td>
<td>3.45</td>
<td>2.43</td>
<td>0.96</td>
</tr>
<tr>
<td>Liquidity Share</td>
<td>3.44</td>
<td>0.87</td>
<td>-0.58</td>
</tr>
<tr>
<td>Asset Price</td>
<td>5.23</td>
<td>0.79</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note: The volatility of output ($y$) is reported in absolute terms.

Table 3.4 shows the relevant statistics when there are only intermediation shocks. Unlike the economy with productivity shocks only, the volatility of the liquidity share and the asset price are much higher. The volatility of investment and the asset price are much closer to the data, while the liquidity share fluctuates more than in the data. The model successfully generates countercyclical movements in the liquidity share, mimicking the liquidity hoarding typically observed in recessions. However, the correlation between the liquidity share and output is exaggerated. This fact suggests that some recessions are still best explained by aggregate productivity shocks.

Table 3.4: Business cycle statistics with only intermediation shocks

<table>
<thead>
<tr>
<th>Variable</th>
<th>Relative volatility $\frac{x}{y}$</th>
<th>Correlation $\rho(x, y)$</th>
<th>1st auto-correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Output</td>
<td>0.02</td>
<td>0.02</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.44</td>
<td>0.79</td>
<td>0.88</td>
</tr>
<tr>
<td>Investment</td>
<td>3.45</td>
<td>4.61</td>
<td>0.96</td>
</tr>
<tr>
<td>Liquidity Share</td>
<td>3.44</td>
<td>7.85</td>
<td>-0.58</td>
</tr>
<tr>
<td>Asset Price</td>
<td>5.23</td>
<td>4.36</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note: The volatility of output ($y$) is reported in absolute terms.
Shocks with Different Persistence

To illustrate the impact of the shocks’ persistence on the system’s dynamics, we set $\rho_a$ or $\rho_c$ to a higher (0.90) and a lower number (0.80). The different persistence may affect macroeconomic variables either directly (through production) or indirectly (through investment).

Qualitatively, the differences are small (Figures 3.2 and 3.3). Still, different persistences do change the speed and magnitude of the adjustment of macroeconomic variables, the liquidity share, and the asset price in the two experiments. These different dynamics are due to the fact that the hedging value of liquid assets against the illiquidity of equity changes in the persistence of shocks.

For example, if low aggregate productivity shocks are perceived to be more persistent in the future, the hedging value of liquid assets will be depressed for longer. As a result, the liquidity share drops to a slightly lower value than in the baseline and takes longer time to revert back to the steady state. In contrast, when intermediation shocks are perceived to be

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**Figure 3.2:** Impulse responses (in percentage deviations from steady state values) after aggregate productivity shocks with different persistence ($\rho_a = 0.90, \rho_a = 0.80$, and the baseline).
more persistent, liquid assets will be valued for a longer period, such that the liquidity share increases more on impact and recovers more slowly. This exercise also shows that the liquidity share is always above the steady state value regardless of the persistence of intermediation cost shocks.

**Non-symmetric Shocks**

The intermediation cost shock discussed so far symmetrically affects both the demand and the supply side of the bond market. However, as mentioned in section 3.3.2, higher search market participation costs of buyers and sellers push the steady state equity price in opposite directions. We would, therefore, expect different equilibrium responses to one-sided intermediation cost shocks and discuss both in turn.

Suppose we fix the parameters of the cost shock process, but restrict its effect to the demand side, i.e. \( \kappa_i = \bar{\kappa}_i \), \( \kappa_n = e^{\gamma \kappa} \bar{\kappa}_n \). The system dynamics after such a demand-side cost shock turn out to be similar to the baseline impulse responses after a symmetric shock, albeit with a slightly smaller
magnitude (Figure 3.4). In contrast, a shock affecting the supply side only (i.e. \( \kappa_i = e^{z_1} \hat{\kappa}_i, \quad \kappa_n = \hat{\kappa}_n \)) leads to much subdued responses of macroeconomic variables and asset saleability, and altogether different responses of the asset price and the liquidity share. In fact, the asset price increases, which contributes to a much more contained response of the liquidity share.

Importantly, the equilibrium response to an increase in sellers’ participation costs is very similar to the dynamics generated by exogenous liquidity shocks in Kiyotaki and Moore (2012) and Shi (2015). In their studies, an exogenous and persistent reduction of \( \phi_i \) depresses the supply of assets on financial markets relative to demand and thus leads to an asset price boom. In our endogenous asset liquidity framework, one-sided supply cost shocks lead to a fall of asset demand relative to asset supply. This can be seen from the contraction of asset saleability \( \phi_i \), which is a function of the ratio of asset demand to asset supply. However, sellers’ sensitivity to cost shocks is contained by their need to resort to equity issuance and resale in order to relax their financing constraint. Therefore, if buyers are not directly affected by the intermediation cost shock, their demand does not fall sufficiently relative to asset supply to prevent an asset price boom (see Proposition 3).

The endogenous liquidity framework thus demonstrates that financial shocks need to strongly affect the demand side in order to overturn this anomaly in the reaction of asset prices. Moreover, more persistent demand side shocks are more severe on impact. This is because buyers who perceive equity markets to be illiquid for an extended period, anticipate that holding additional equity claims may constrain their own funding ability in the future and thus become even less inclined to buy them.

While Shi (2015) suggests that aggregate productivity shocks are necessary to overturn the asset price anomaly generated by exogenous liquidity shocks, our simulations show that pure financial shocks are sufficient, provided that liquidity is modelled endogenously. The financial shocks considered here are different from productivity shocks, since they affect investment via financing constraints rather than directly reducing the production frontier of the economy. To see this, recall the goods market clearing condition (3.2.19)

\[
C + I + \kappa_n E_n + \kappa_i E_i = Y.
\]

Aggregate productivity shocks affect the right-hand side, while intermediation cost shocks affect the investment-related costs \( \kappa_n E_n + \kappa_i E_i \) on the left-hand side. Such cost shocks may thus be interpreted as a particular form of investment-specific technology shocks (see, e.g., Fisher (2006) and Justiniano, Primiceri, and Tambalotti (2010)), whose impact is amplified by their effect on endogenous market participation.
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**Figure 3.4:** Impulse responses (in percentage deviations from steady state values) after symmetric and one-sided intermediation cost shocks.

3.5 Conclusion

This paper presents a macroeconomic model which introduces endogenous asset liquidity via search frictions on asset markets. Endogenous fluctuation of asset liquidity may be triggered by shocks that affect asset demand and supply on the search market either directly (intermediation cost shocks), or indirectly (productivity shocks). By tightening entrepreneurs’ financing constraints, they feed into investment, consumption and output. The model is able to capture both a physical and a price dimension of asset liquidity. In particular, we show that asset prices can co-move with asset saleability. The *endogenous* nature of asset liquidity is key to match this positive correlation, as *exogenous* liquidity shocks would act as negative supply shocks on the asset market and lead to asset price booms in recessions.

We also show that the liquidity service provided by intrinsically worthless government-issued assets, such as money, is higher when financing constraints bind tightly. As a result, shocks to the cost of financial intermediation increase the hedging value of liquid assets, enabling our model to replicate the ‘flight to liquidity’ or countercyclical share of liquid assets (over
total assets) observed in U.S. data.

Our search framework can be interpreted as a model of market-based financial intermediations. It can also be seen as a short-cut to modelling bank-based financial intermediation: financial intermediaries help channel funds from investors to suitable creditors in need of outside funding, which resembles a matching process. Adding further texture by explicitly accounting for intermediaries’ balance sheets would open interesting interactions between liquidity cycles and financial sector leverage and maturity transformation.

Regarding government interventions, the framework developed in this paper suggests that, as in KM, open market operations in the form of asset purchase programs can have real effects by easing liquidity frictions. However, government demand may crowd out private demand due to congestion externalities in an endogenous liquidity framework. Therefore, future research could focus on the optimal design of conventional and unconventional monetary as well as fiscal policy measures in the presence of illiquid asset markets.

Appendix

3.A Equilibrium Conditions

3.A.1 Recursive Competitive Equilibrium

Notice that \( C = C_n + C_i \), such that \(^{29}\)

\[
C_n = \rho_n C, \quad C_i = \rho_i C,
\]

where

\[
\rho_n \equiv \frac{1 - \chi}{1 - \chi + \rho^{-1/\sigma} \chi}, \quad \rho_i \equiv \frac{\chi}{\rho^{1/\sigma} (1 - \chi) + \chi}.
\]

We use \( \rho_n \) and \( \rho_i \) in the subsequent analysis. We change the recursive equilibrium slightly by using \( \rho = \frac{q_n}{q_r} \) (instead of using \( q_r \) and \( q_i \)), defining real liquidity \( L = \frac{B}{P-1} \), and adding aggregate output \( Y \). Given the aggregate state variables \( \Gamma = (K; z_a, z_\kappa) \), we solve the equilibrium system

\[
(K', L, C, I, N, Y, \rho, \rho_i, \rho_n, \phi_n, \phi_i, \eta, q, q_r, w, \pi)
\]

together with the exogenous laws of motion of \( (z_a, z_\kappa) \), i.e., \( z'_a = \rho_a z_a + \epsilon'_a \) and \( z'_\kappa = \rho_\kappa z_\kappa + \epsilon'_\kappa \). To solve for these 16 endogenous variables, we use the following 16 equations:

\(^{29}\)Using the utility function \( u(c_j) = \frac{c_j^{1-\sigma} - 1}{1-\sigma} \) in (3.3.4) and noting that \( C = C_n + C_i \) yields \( C_n = \rho_n C \) and \( C_i = \rho_i C \), where \( \rho_n \equiv \frac{1}{1-\chi + \rho^{-1/\sigma} \chi} \) and \( \rho_i \equiv \frac{\chi}{\rho^{1/\sigma} (1 - \chi) + \chi} \).
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1. The representative household’s optimality conditions:

\[ w = \mu \left( \frac{\rho_n C}{1 - \chi} \right)^{\sigma} \]  
(3.A.1)

\[ \rho_n = \frac{1 - \chi}{1 + \rho^{-1/\sigma} \chi} \]  
(3.A.2)

\[ \rho_i = \frac{\chi}{\rho^{1/\sigma} (1 - \chi) + \chi} \]  
(3.A.3)

\[ 1 = \beta \mathbb{E}_{\Gamma} \left[ \left( \frac{\rho_n C'}{\rho_n C} \right)^{-\gamma} \left[ \chi \rho' + 1 - \chi \right] \frac{1}{\pi} \right] \]  
(3.A.4)

\[ 1 = \beta \mathbb{E}_{\Gamma} \left[ \left( \frac{\rho_n C'}{\rho_n C} \right)^{-\gamma} (\chi \rho' + 1 - \chi) r' + (1 - \delta) (\chi \rho' + (1 - \chi) q_n') \right] \]  
(3.A.5)

\[ I = \chi \left[ (r + \left( \phi_i q_n - \left[ \frac{(1 - \omega)}{\omega} + 1 \right] \kappa_i \right) (1 - \delta)) K + \frac{L}{\pi} \right] - \rho_i C \]  
(3.A.6)

2. Final goods producers:

\[ r = e^{\gamma_n} F_K (K, N) \]  
(3.A.7)

\[ w = e^{\gamma_n} F_N (K, N) \]  
(3.A.8)

\[ Y = e^{\gamma_n} F (K, N) \]  
(3.A.9)

3. Market clearing:

(a) Consumption goods:

\[ (\rho_n + \rho \rho_i) C + q_n K' + L' = w N + \left[ \chi \rho + (1 - \chi) \right] \frac{L}{\pi} \]  

\[ + \left[ (\chi \rho + (1 - \chi)) r \right] \]  

\[ + (1 - \delta) (\chi \rho + (1 - \chi) q_n) \]  

\[ K \]  

(b) Capital:

\[ K' = (1 - \delta) K + I \]  
(3.A.10)

(c) Search market (note: \( \gamma \equiv \frac{\omega}{1 - \omega} \kappa_i \), \( \kappa_i = e^{\gamma_n} \bar{\kappa}_i \), \( \kappa_n = e^{\gamma_n} \bar{\kappa}_n \)): 
\[ \phi_i = \xi (\gamma^{-1} \rho)^{1-\eta} \]  
\[ \phi_n = \xi \frac{1}{\gamma} \phi_i^{\frac{\omega}{\gamma-\omega}} \]  
\[ q_n = \rho \frac{1 + \kappa_i + \rho \frac{(1-\omega)\kappa_i}{\omega}}{1 + (\rho - 1) \phi_i} \]  
\[ q = q_n - \frac{\kappa_n}{\phi_n} \]

\text{(3.A.11) \hspace{1cm} (3.A.12) \hspace{1cm} (3.A.13) \hspace{1cm} (3.A.14)}

(d) Liquid assets (note: \( L' = \frac{B'}{\pi}, \pi' = \frac{P'}{P} \)):
\[ L' = \frac{L}{\pi}. \]  
\text{(3.A.15)}

3.A.2 Steady State
In the deterministic steady state, any variable \( X = X' \). With a slight abuse of notation, the steady state of \( X \) is denoted as \( X \) itself in this section. First notice that \( z_a = 0, z_n = 0 \) such that \( \kappa_i = \bar{\kappa}_i, \kappa_n = \bar{\kappa}_n \).

Now all prices can be solved for analytically. Market clearing for liquid assets implies \( \pi = 1 \). Next, use (3.A.4) to obtain
\[ \rho = \chi^{-1} [\beta^{-1} - (1 - \chi)] , \quad \rho_n \equiv \frac{1 - \chi}{1 - \chi + \rho^{-1/\sigma} \chi}, \quad \rho_i \equiv \frac{\chi}{\rho^{1/\sigma} (1 - \chi) + \chi} \]

This directly implies
\[ \phi_i = \xi (\gamma^{-1} \rho)^{1-\eta}, \quad q_n = \rho \frac{1 + \kappa_i + \rho \frac{(1-\omega)\kappa_i}{\omega}}{1 + (\rho - 1) \phi_i} \]

From (3.A.5), (3.A.7) and (3.A.8) we have
\[ r = \frac{q_n}{\beta} - (1 - \delta) (\chi \rho + (1 - \chi) q_n) \chi \rho + 1 - \chi, \quad w = (1 - \alpha) \left( \frac{r}{\alpha} \right)^{1-\alpha}, \quad \text{and} \]
\[ C = \left( \frac{w}{\mu} \right)^{1/\sigma} \frac{1 - \chi}{\rho_n}. \]

Now, labour supply \( N \) can be expressed as a function of \( K \)
\[ N = \left( \frac{r}{\alpha} \right)^{1-\alpha} K. \]

Investment \( I = \delta K \) and real liquidity can be rewritten as a function of \( K \) using (3.A.6)
\[ L = \chi^{-1} \{ \rho_i C + [\delta - \chi r - \phi_i q_i (\delta + \chi (1 - \delta))] K \} . \]
Since \( N \) and \( L \) are both linear in \( K \), \( K \) can be derived from the household’s budget constraint
\[
K = \frac{(\rho_n + \rho_i) C}{(1 - \alpha)r + A_K + (\rho - 1) [\delta - \chi r - \phi_i q [\delta + \chi (1 - \delta)]]},
\]
where \( A_K = (\chi \rho + 1 - \chi) r + (1 - \delta) (\chi \rho + (1 - \chi) q_n) - q_n \).

### 3.B Proofs

#### 3.B.1 Lemma 1

Using a guess-and-verify strategy, suppose that \( q_i \geq 1 \). Then the search market for private claims is active and we seek the parameter restriction that yields \( q_i \geq 1 \). Using asset price derived in Lemma 2, \( q = \frac{\rho(1 + \frac{\kappa}{\omega}) - \frac{\kappa n}{\phi} - \frac{q n}{\phi} - (\rho - 1) \kappa_i}{1 + (\rho - 1) \phi_i} \), the selling price \( q_i = q - \frac{\kappa_i}{\phi_i} \) becomes
\[
q_i = \frac{\rho (1 + \frac{\kappa_i}{\omega}) - \frac{\kappa n}{\phi} - \frac{q n}{\phi}}{1 + (\rho - 1) \phi_i} - (\rho - 1) \kappa_i.
\]
Therefore, \( q_i \geq 1 \) is equivalent to
\[
\rho \left( 1 + \frac{\kappa_i}{\omega} \right) - (\rho - 1)(\kappa_i + \phi_i) \geq 1 + \frac{\kappa n}{\phi} + \frac{\kappa_i}{\phi}.
\]
or
\[
\rho \left( 1 + \frac{\kappa_i}{\omega} - \kappa_i - \phi_i \right) + \kappa_i + \phi_i \geq 1 + \frac{\kappa n}{\phi} + \frac{\kappa_i}{\phi}.
\]
Using again the asset price solution \( \rho = \frac{\omega - \frac{\kappa n}{\phi} - \frac{q n}{\phi}}{1 - \phi} \), one can simplify the above inequality to
\[
(1 - \phi_i)(\rho - 1 - \frac{\kappa n}{\phi} - \frac{\kappa_i}{\phi}) \geq 0.
\]
Since \( \phi_i \in [0, 1] \), we have
\[
\rho \geq 1 + \frac{\kappa n}{\phi} + \frac{\kappa_i}{\phi} = 1 + \frac{\kappa n}{\xi (\gamma^{-1} \rho)^{-\eta}} + \frac{\kappa_i}{\xi (\gamma^{-1} \rho)^{1-\eta}},
\]
where the last equality uses the fact that \( \phi_i = \xi \theta^{1-\eta} \) and \( \rho = \gamma \theta \). Finally, notice that from the steady state derivation, \( \rho \) can be expressed as \( \rho = \chi^{-1} [\beta^{-1} - (1 - \chi)] \), which delivers the condition stated in the lemma.

#### 3.B.2 Lemma 2

First simplify the first-order condition associated with the bargaining solution to
\[
\frac{\omega}{\rho (q - \frac{1}{\phi}) + \frac{1 - \phi_i}{\phi} q n} = 1 - \omega - q_n - q.
\]
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by using \( u'(c_n) q_n = \beta \mathbb{E}_t J_S (S', B'; \Gamma') \) and \( u'(c_i) = \rho u'(c_n) \). Then

\[
\frac{\kappa_n}{\phi_n} = (1 - \omega) \left[ \rho \left( q - \frac{1}{\phi_i} \right) + \frac{1 - \phi_i q_i (1 - \phi_i) q_n}{1 - \phi_i q_i} \right],
\]

which can be further simplified to

\[
\frac{\kappa_n}{\phi_n} = (1 - \omega) \rho (q - q_i),
\]

by realizing that \( \rho \equiv \frac{q_n}{q_r} = \frac{(1 - \phi_i) q_n}{\phi_i q_i} \). Solving the above equation for \( \rho \) yields

\[
\rho = \frac{\omega}{1 - \omega} \frac{\kappa_n}{\phi_n} \phi_i = \gamma \theta,
\]

which is (3.3.7).

One can further express \( q \) in terms of \( \rho \) or \( \phi_i \). Using the above expression along with the definition of \( \rho \)

\[
\rho \equiv \frac{q_n}{q_r} = \frac{(1 - \phi_i) q_n}{1 - \phi_i q_i} = \frac{(1 - \phi_i) \left( q + \frac{\kappa_n}{\phi_n} \right)}{1 - \phi_i q + \kappa_i},
\]

\( q \) can be express as

\[
q = \frac{\rho (1 + \kappa_i) - (1 - \phi_i) \frac{\kappa_n}{\phi_n} \phi_i}{1 + (\rho - 1) \phi_i} = \frac{\rho (1 + \frac{\kappa_n}{\phi_n}) - \frac{\kappa_n}{\phi_n}}{1 + (\rho - 1) \phi_i},
\]

where the last line uses (3.3.7) again. Realizing that \( \phi_n = \xi (\gamma^{-1} \rho)^{-\eta} \) and \( \phi_i = \xi (\gamma^{-1} \rho)^{1-\eta} \), \( q \) can be rewritten as a function of \( \rho \)

\[
q = \frac{\rho^{1-\eta} (1 + \frac{\kappa_n}{\phi_i}) - \kappa_n \gamma^{-\eta} \xi^{-1}}{\rho^{-\eta} \left[ 1 + (\rho - 1) \xi (\gamma^{-1} \rho)^{1-\eta} \right]},
\]

or, equivalently as a function of \( \phi_i \)

\[
q = \frac{\gamma^{1-\eta} \xi^{-1} \phi_i (1 + \frac{\kappa_n}{\phi_i}) - \kappa_n \gamma^{-\eta} \xi^{-1}}{\gamma^{-\eta} (\xi^{-1} \phi_i)^{\eta-1} \left[ 1 + \left( \gamma (\xi^{-1} \phi_i)^{1-\eta} - 1 \right) \phi_i \right]}
\]

\[
= \frac{\gamma \phi_i (1 + \frac{\kappa_n}{\phi_i}) - \kappa_n}{\xi^{\eta-1} \phi_i^{\eta-1} \left[ 1 + \left( \gamma (\xi^{-1} \phi_i)^{1-\eta} - 1 \right) \phi_i \right]}.
\]
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3.B.3 Proposition 3

Notice that in steady state, $\rho$, $\phi_i$, and $\phi_n$ are functions of parameters that are independent of search costs $\kappa_n$ and $\kappa_i$, when $\kappa_n/\kappa_i = g$ is fixed (such that $\gamma = \frac{\omega}{1-\omega} \frac{\kappa_n}{\kappa_i}$ is also fixed). Therefore, $\frac{\partial q}{\partial \kappa_n} > 0$ is equivalent to

$$\frac{\gamma}{\omega g} \phi_i - 1 < 0.$$

Using the definition for $\gamma$ and $g$ yields

$$\phi_i < 1 - \omega.$$  

Since in the steady state $\rho = \chi^{-1} (\beta^{-1} - (1 - \chi))$ and $\phi_i = \xi \left( \gamma^{-1} \rho \right)^{1-\eta}$, the above inequality is equivalent to

$$\xi \left[ \chi^{-1} (\beta^{-1} - (1 - \chi)) \right]^{1-\eta} < (1 - \omega) \eta \omega^{1-\eta}.$$

3.B.4 Proposition 4

By differentiating the asset price from (3.3.8) with respect to $\phi_i$, one gets

$$\frac{\partial q}{\partial \phi_i} \left[ \xi^{\frac{1}{1-\eta}} \phi_i^{\frac{\eta}{1-\eta}} \left[ 1 + \left( \frac{\kappa_i}{\omega} \right) \phi_i \right] \right] = \gamma \left( 1 + \frac{\kappa_i}{\omega} \right) - q \frac{\partial}{\partial \phi_i} \left[ \xi^{\frac{1}{1-\eta}} \phi_i^{\frac{\eta}{1-\eta}} \left[ 1 + \left( \gamma \left( \gamma^{-1} \phi_i \right)^{1-\eta} - 1 \right) \phi_i \right] \right], \quad (3.B.1)$$

where

$$\frac{\partial}{\partial \phi_i} \left[ \xi^{\frac{1}{1-\eta}} \phi_i^{\frac{\eta}{1-\eta}} \left[ 1 + \left( \gamma \left( \gamma^{-1} \phi_i \right)^{1-\eta} - 1 \right) \phi_i \right] \right] = \rho^{-1} \gamma \left[ \phi_i \left( 2 \rho - \frac{1 - 2\eta}{1 - \eta} \right) - \frac{\eta}{1 - \eta} \right].$$

Note that $2\rho - \frac{1 - 2\eta}{1 - \eta} = \frac{\eta}{1 - \eta} + 2\rho - 1$. A necessary and sufficient condition for $\frac{\partial q}{\partial \phi_i} > 0$ is for the RHS of (3.B.1) to be non-negative. This is the case, whenever

$$\phi_i < \left[ \frac{\eta}{1 - \eta} + (1 + \frac{\kappa_i}{\omega}) \frac{\rho}{q} \right] \left[ \frac{\eta}{1 - \eta} + 2\rho - 1 \right]^{-1}.$$

This condition requires $\phi_i$ to be small enough for the asset price and asset liquidity to correlate positively. Replacing $\rho$ and notice that $\rho/q < 1$, a sufficient condition is

$$\phi_i < \left[ \frac{\eta}{1 - \eta} + (1 + \frac{\kappa_i}{\omega}) \right] \left[ \frac{\eta}{1 - \eta} + 2\gamma \left( \gamma^{-1} \phi_i \right)^2 - 1 \right]^{-1}.$$
When $\eta = 0.5$, $\frac{\eta}{\eta-1} = 1$ and the sufficient condition becomes $\phi_i < \sqrt{\frac{\xi^2(1+\frac{2}{4})}{1}}$.

Note that $\frac{\partial q}{\partial \phi} > 0$ implies $\frac{\partial q}{\partial \phi_n} < 0$, because $\frac{\partial q}{\partial \phi} = \frac{\partial q}{\partial \phi_i} \frac{\partial \phi_i}{\partial \phi_n}$ and

$$\frac{\partial \phi_i}{\partial \phi_n} = \frac{\eta - 1}{\eta} - \frac{1}{\eta} \frac{\xi^2}{\phi_n} < 0.$$ 

Hence, the same parameter restriction that ensures $\frac{\partial q}{\partial \phi_i} > 0$ also ensures $\frac{\partial q}{\partial \phi_n} < 0$.

3.B.5 Corollary 1

Notice that the surplus from entrepreneurs are

$$J_{m}^u = u'(c_i) \left( q - \frac{1}{\phi_i} \right) + \frac{1-\phi_i}{\phi_i} \beta \mathbb{E}_{\Gamma} \left[ J_S \left( S', B'; \Gamma' \right) \right]$$

$$= u'(c_n) \left[ \rho \left( q - \frac{1}{\phi_i} \right) + \frac{(1-\phi_i)q_n}{\phi_i} \rho \right].$$

Since $\frac{(1-\phi_i)q_n}{\phi_i} = \frac{1-\phi_i q_n}{\phi_i}$, $\frac{(1-\phi_i)q_n}{\phi_i} = \frac{1-\phi_i q_n}{\phi_i} \rho$, the term in the bracket is equal to $\rho(q - q_i)$. When $\kappa_i \to 0$, the surplus of the entrepreneurs goes to zero because $q_i \to q$. In this case, intermediation will set price $q = 1$ in order to maximize workers’ surplus. Further, $\kappa_n \to 0$ implies that $q_n \to q \to q_i \to 1$ and $\rho \to 1$.

3.C Data

The measure of liquid assets $B_t$ consists of all liabilities of the federal government circulated inside the US economy, obtained from U.S. flow-of-funds data. In particular, we use Treasury securities, net of saving bonds (for financing World War II), net of holdings by the monetary authority and the rest of the world, plus reserves and vault cash of depository institutions with the monetary authority, plus checkable deposits and currency net of the monetary authority’s liabilities due to the rest of the world and due to the federal government. This measure is similar to the one in Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), but is cleaned from liquid assets held by agents outside the US.

Following again Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), $PqK$ measure the value of capital in the economy. We use the balance sheet of households, the non-corporate and the corporate sectors to obtain the market value of aggregate capital. On households’ side, we add real estate, equipment and software of non-profit organizations, and consumer durables. As for the non-corporate sector, we add real estate, equipment and software and inventories. As for the corporate sector, we obtain the market value of the capital stock by summing the market value of equity and liabilities net
of financial assets. Finally, we subtract from the market value of capital the government credit market instruments, TARP, and trade receivables.
Chapter 4

Government Bond Liquidity and Sovereign-Bank Interlinkages

This chapter is based on joint work with Wei Cui (UCL) and Cristina Checherita-Westphal (ECB). It has been published as RADDE, S., W. CUI and C. CHECHERITA-WESTPHAL (2015): “Government Bond Liquidity and Sovereign-Bank Interlinkages,” SFB 649 Discussion Paper, 2015-032.

4.1 Introduction

The euro area sovereign debt crisis exposed a powerful sovereign-bank nexus. Concerns about the sustainability of public debt sent government bond yields soaring. Credit risk premia surged and the liquidity of sovereign debt markets was severely impaired as captured by sharply rising bid-ask spreads and strong declines in turnover volumes. At the same time, the euro area interbank market became deeply fragmented and bank funding costs rose, such that lending conditions for non-financial firms and households deteriorated. This further stifled the already depressed economic activity with adverse consequences for fiscal revenues and public debt sustainability, thereby reinforcing the debt crisis.

This paper aims to understand interlinkages between the government and the banking sector arising from the features of government bonds as highly liquid assets, which are held in financial sector for refinancing purposes. It asks how a deterioration of the market liquidity of government bonds, triggered for instance by a shock to a sovereign’s credit quality, impairs the funding conditions of banks and spills over to the macroeconomy. To answer this question, this paper proposes a general equilibrium model in which (long-term) government bonds are traded on a search market, which is intended to capture both outright transactions in government bonds as well
as refinancing operations collateralised by government debt. This stylised set-up allows us to study the endogenous propagation - and amplification - of stress from government bond markets via banks to the real sector and to identify feedback effects on the funding conditions of the fiscal sector.

Our approach complements the growing literature that models the spill-over of sovereign credit risk to the macroeconomy, e.g. Bi (2012); Bi and Traum (2012b); Bi and Leeper (2013); Broner, Erce, Martin, and Ventura (2014); Corsetti, Kuester, Meier, and Müller (2013). This literature typically argues that sovereign funding strains spill over into private credit markets by adversely affecting nonfinancial firms’ funding costs. However, it does not shed light on the precise transmission channels through which stress on sovereign debt markets propagates to the real economy. By explicitly modelling the economic function of government bonds for the financial sector as a hedging instrument against market liquidity risks in normal times, a sovereign liquidity risk channel of shock transmission and amplification can be identified that operates in times of stress.

The search market set-up for government bond transactions is designed to capture the structural features of limit order inter-dealer markets on which large parts of euro area government bonds are traded (Pelizzon, Subrahmanyam, Tomio, and Uno, 2013; Cheung, Jong, and Rindi, 2005). In these markets, bond portfolios are only partially resalable and market liquidity fluctuates over time as shown by both price and volume measures. During the euro area sovereign debt crisis, for instance, bid-ask spreads in transactions of government bonds of stressed countries soared (see Figure 4.1) and market depth in terms of turnover volumes declined substantially. Following Cui and Radde (2014), these features can be generated endogenously by explicitly modelling impediments to transactions in government bonds due to search and matching frictions. Lower turnover on the market for government bonds is analytically shown to result in widening bid-ask spreads and higher liquidity risk premia, which increase the financing costs of the government.

In a version of the model calibrated to the Italian economy, an exogenous haircut on long-term government bonds is introduced as a laboratory experiment to analyse the interaction between credit and liquidity risk in sovereign bond markets and their macroeconomic implications. The impulse responses to the sovereign solvency shock reveal that its first-order wealth effects are amplified by tightening financing constraints in the banking sector due to an endogenous deterioration of government bond saleability and prices. This impairment of government bonds’ market liquidity results from a reduction in demand for such assets as their value as a hedge against future financing constraints falls. Instead, the model predicts a flight to more liquid assets such as central bank deposits or money, which squares well with the FTL dynamics observed during the sovereign debt crisis (García and Gimeno, 2014).
The reaction of banks to the solvency shock has direct and indirect implications for the fiscal position of the government. The fall in the market price of government bonds in response to lower demand immediately increases the government’s funding costs as the value of bond issuance declines. More indirectly, the concomitant flight to liquid assets bids up the relative price of long-term bonds in real terms, such that the government’s debt burden rises. If the government covers its additional financing needs by issuing more debt, bond liquidity from the perspective of private investors deteriorates further as the additional public supply crowds out private sellers in the market. These rich macroeconomic interactions show that accounting for the economic role of government bonds and the microstructure of bond markets is key to understanding sovereign-bank interlinkages.

4.1.1 Related Literature

There is a vast amount of empirical studies suggesting that liquidity risk is a crucial driver of sovereign yield spreads during crises, while a growing body of both theoretical and empirical research attests to the existence of sovereign-bank interlinkages, whose degree depends, inter alia, on banks’ government bond holdings. This literature has guided the choice of key features of the model presented in this paper.

The role of liquidity in pricing government bond risk. Since the onset of the euro area crisis, a large body of research has focused on the topic of sovereign risk pricing, i.e. government bond yield spreads, with considerable attention given to the fiscal-financial nexus. The literature generally distinguishes three factors influencing the premia asked by investors to hold a
4. Government Bond Liquidity and Sovereign-Bank Interlinkages

euro area government bond versus a benchmark bond (for instance, the German bund). First, sovereign bond spreads comprise a credit risk premium that accounts for a country’s creditworthiness as reflected by its fiscal and macroeconomic fundamentals. Second, spreads reflect a liquidity risk premium that compensates investors for impediments to trading government bonds, which depends on the size, depth and structure of the government bond market. Third, government bond spreads may be influenced by international risk aversion, i.e. investor sentiment towards this asset class across different sovereigns.

These three categories of factors are not orthogonal; interdependencies exist and are usually found to strengthen in times of crisis. For instance, credit risk may influence both market liquidity and investor risks aversion, while the liquidity factor may be essential in explaining why investors demand similar premia for countries with very different fundamentals.

There is by now compelling empirical evidence that, while not being a dominant pricing factor during normal times, liquidity risk becomes acute during crisis episodes. The euro area sovereign debt crisis stands out in this respect. Many studies covering the global financial crisis starting in 2008 find that liquidity risk was statistically significant in explaining sovereign yield spreads in the euro area, although its impact was smaller compared to credit risk or international risk aversion (Attinasi, Checherita, and Nickel, 2009; Barrios, Iversen, Lewandowska, and Setzer, 2009; Haugh, Ollivaud, and Turner, 2009; Sgherri and Zoli, 2009). A common international risk factor is generally found to have played an important role in explaining spreads on account of a broad-based Flight To Safety (FTS). This view is corroborated by García and Gimeno (2014), who find strong co-movement of liquidity premia across euro area sovereign debt markets during the global financial crisis. The euro area sovereign debt crisis, in contrast, is characterised by an asymmetric response of liquidity spreads in stressed and non-stressed countries due to portfolio reallocations, which is interpreted as a strong indication for a FTL. Extracting a common euro area liquidity shock from the KfW-Bund spread, Monfort and Renne (2013) disentangle credit and liquidity risk premia in European sovereign bond markets. Their study shows that the common liquidity factor is an important driver of the dynamics of sovereign yield spreads during liquidity-related crises. These findings are corroborated by studies using liquidity measures directly gleaned from tick-by-tick trade and quote data from individual broker-dealers for European sovereign bonds.\footnote{The data is obtained from the Mercato Telematico dei Titoli di Stato (MTS) Global Market bond trading system. The MTS market is the largest interdealer trading system for Euro-zone government bonds, which is largely based on electronic transactions. See Cheung, Jong, and Rindi (2005).}

1 For instance, Darbha and Dufour (2013) show that liquidity was an important pricing factor for sovereign bond spreads during the global financial crisis and the early stages of the euro area sovereign debt
crisis for bonds rated AA and lower, but not for AAA rated bonds. Moreover, the authors show that the bid-ask spread is the single-best predictor of bond yields among the trade- and quote-based liquidity measures. Exploiting similar data, Pelizzon, Subrahmanyam, Tomio, and Uno (2013) find evidence of a dynamic relationship between market liquidity (using alternative quote- and trade-based liquidity measures) and credit risk (as measured by CDS spreads) during the sovereign debt crisis. Their analysis suggests that market liquidity reacted to contemporaneous and lagged changes in credit risk during this period. However, this relationship is found to be non-linear and it changes in the presence of heightened credit risk (e.g. spreads above 500bps). In the latter cases, evidence suggests a contemporaneous and stronger interaction between credit and liquidity risk.\(^2\) Finally, Dewachter, Iania, Lyrio, and de Sola Perea (2015) suggest that euro area sovereign bond spreads are largely explained by fundamental shocks, but that non-fundamental shocks have become more dominant since the onset of sovereign debt crisis.

Studies focusing on longer pre-crisis periods generally find that liquidity is not a significant driver of sovereign bond spreads during ‘normal’ times, while gaining relevance during episodes of uncertainty. Schuknecht, von Hagen, and Wolswijk (2009, 2010); Pagano and von Thadden (2004); Jankowitsch, Mosenbacher, and Pichler (2006) all find that over long horizons liquidity differences play at most a minor role in explaining sovereign bond spreads in the euro area. Pasquariello and Vega (2007) present similar evidence for the US Treasury bond market, where swings in order flow are generally linked to macroeconomic, i.e. fundamental, news. On the other hand, Gomez-Puig (2006) finds that liquidity risk is the most important factor explaining sovereign bond spreads after the introduction of the euro. Studying the pre-crisis period, Beber, Brandt, and Kavajecz (2009) find that while the bulk of sovereign yield spreads is explained by differences in credit quality, liquidity plays a larger role during times of heightened market uncertainty and especially for low credit risk countries. In particular, the study shows that large order inflows are strongly determined by search for liquid assets. Moreover, in periods of large flows, liquidity explains a larger fraction of sovereign yield spreads, particularly at the long end of the yield curve. The study concludes that credit quality matters for bond valuation but that, in times of market stress, investors chase liquidity. These findings are in line with Goyenko, Subrahmanyam, and Ukhov (2010), who study liquidity premia of on-the-run and off-the-run US Treasury bonds. Their results suggest that liquidity differences between these two groups widen during recessions, attesting to a flight to liquidity.

\(^2\)Interestingly, the analysis also finds that the causality reversed after the introduction of the ECB’s Long-Term Refinancing Operation (LTRO) programme, with a reduction in liquidity risk Granger causing an improvement in credit risk.

\textit{Sovereign-bank interlinkages.} Although still limited, the literature pro-
4. Government Bond Liquidity and Sovereign-Bank Interlinkages

provides theoretical and empirical support for important fiscal-financial inter-
linkages, whose severity depends on several factors, most importantly: (i) the amount of domestic sovereign bonds held by domestic banks, depend-
ing, inter alia, on the role of government debt for bank liquidity management;
(ii) the size, capital structure, and initial financial condition of the banking
sector; (iii) the initial fiscal condition of the sovereign; and (vi) the extent
to which banking crises cut through to the real sector, thereby adversely
affecting tax revenues.

As for the role of sovereign debt for the liquidity management in the
banking sector, Gennaioli, Martin, and Rossi (2014) point to the importance
of government bonds as collateral in the secured interbank market. One of
the hypotheses they test concerning banks’ holdings of government bonds
and their effects during sovereign defaults reflects the liquidity view, that is,
banks hold sizeable amounts of government bonds on a regular basis to store
liquidity and to post them as collateral in borrowing arrangements. They
find that high pre-crisis exposure to government bonds negatively affects
banks’ lending capacity after sovereign defaults.

Several papers analyse the impact of bank rescue packages and financial
spillovers in the euro area. Attinasi, Checherita, and Nickel (2009) use dy-
namic panel regression techniques and show that the announcement of bank
bailout measures triggered a transfer of risk from the financial to the gov-
ernment sector. The risk for sovereign-bank spillovers is especially severe in
post-bank bailout periods. To the extent that the domestic banking sector
holds large amounts of government bonds, yield- and CDS spreads in both
the banking and sovereign sectors co-move as a result of increased sovereign
credit risk. Acharya, Drechsler, and Schnabl (2011) provide an empirical
study and a model to document and capture such a two-way feedback be-
tween banks and governments. Focusing on different compositions of Euro
area countries, the studies of Ejsing and Lenke (2011), Mody and Sandri
(2012) and Stanga (2011) show a similar pattern.

In terms of theoretical literature, Bi (2012); Bi and Traum (2012b); Bi and Leeper (2013); Bocola (2014); Corsetti, Kuester, Meier, and Müller
(2013) amongst others provide DSGE models to study the impact of strained
government finances on macroeconomic stability and the transmission of fis-
cal stimulus or austerity policies. They analyse a sovereign risk channel
according to which higher risk premia on sovereign debt feed into the refi-
nancing costs of private firms, thereby constraining private sector borrowing
and investment. Rather than providing a rationale for the financial sector
holdings of sovereign bonds, this strand of literature focusses on the macroe-
onomic implications arising from fiscal limits, i.e. the point at which ad-
justments to the primary balance can no longer accommodate the sovereign
debt burden and a haircut becomes inevitable.

Although we have a similar propagation of sovereign risk premia to pri-
ivate sector funding costs in mind, we focus on endogenous fluctuations in
the liquidity service provided by sovereign bonds to the banking sector in response to a sovereign solvency shock as the key transmission mechanism.

4.2 The Model

The model environment closely follows Cui and Radde (2014) in extending a standard real business cycle (RBC) model with endogenous asset market liquidity. Throughout the model, the liquidity of financial assets is captured by the ease of issuance and resaleability and the price impact of trading these assets. There are three types of assets: equity stakes in capital producers are risky and only liquid at issuance. Long-term government bonds are traded on a search market, such that their degree of liquidity is endogenously determined. Finally, the government sector also issues a risk-less and fully liquid one-period asset, which can be traded on a spot market and may be thought of as central bank deposits or short-term bonds.

Time is discrete and infinite ($t = 0, 1, 2, \ldots$). The economy comprises three sectors: households - split into populations of bankers with lending opportunities and workers -, capital goods producers and final goods producers. Final goods producers rent capital and labour to generate the numeraire (consumption) good. Capital goods producers undertake new investment under perfect competition and finance by selling state-contingent securities to lending banks. Banks intermediate the savings of the household sector and finance private securities. In addition to these agents, there is a government sector, which comprises a monetary and a fiscal authority.

4.2.1 Capital and Final Goods Producers

*Capital goods.* There is a measure one continuum of capital producers, some of which face investment opportunities. They enlarge their capital stock by converting final goods one-to-one into capital goods. In order to finance this investment, producers issue state-contingent securities to financial intermediaries at price $q$. Each security represents a state-contingent claim to the flow of returns from one unit of capital. Due to perfect competition, intermediate goods producers earn zero-profits in every state of the world and the value of a security equals the holding banks’ valuation. To keep the model simple, the capital structure of capital producers does not distinguish between equity and debt. Hence, private financial claims issued by capital producers are also referred to as loans.

*Final goods.* A measure one continuum of producers assembles final goods ($Y^t$) in a perfectly competitive environment. They operate a constant returns to scale technology using capital ($K^t$) and labour ($L$) as inputs

$$Y^t = e^{z_{a,t}} K^t H^{1-\alpha}_t$$
with $\alpha \in (0, 1)$ and $z_{a,t}$ capturing total factor productivity. The profit-maximising labour demand is

$$w_t = (1 - \alpha) \frac{Y_t}{H_t} \quad (4.2.1)$$

and the return to capital amounts to

$$r_t = \frac{Y_t - w_t H_t}{K_t} = \alpha \frac{Y_t}{K_t} \quad (4.2.2)$$

### 4.2.2 Households

*Household sector structure and financial frictions.* The economy features a representative household comprising a unit measure of members. Household members are either workers or bankers. Workers earn wages by supplying labour. Bankers do not work, but provide intermediation services by channeling part of the household’s savings to capital producers in the form of loans. In addition to loans, the household sector owns a portfolio of long-term government bonds and liquid assets, which are held by its members. During each period, there is perfect consumption insurance within each group of household members.

As discussed in section 4.4, this paper focuses on an equilibrium in which lending to capital producers is profitable as the market price of loans exceeds banks’ cost of creating new loans. In order to take full advantage of such profitable lending opportunities, banks will want to maximise funding by liquidating their government bond portfolios as well as selling stakes in loans on the market. However, asset sales are restricted by two key frictions: i) only a fraction $\theta$ of newly issued loans can be sold in the market, while existing loans are entirely illiquid, i.e. there is no market for these claims; ii) search frictions afflict the trading of long-term bonds. In particular, for every unit of long-term bonds offered for sale only a fraction $\phi_i$ which will be endogenously determined - can be liquidated. As a result of these frictions, lending banks need to retain a fraction $(1 - \theta)$ of new loans as well as a fraction $(1 - \phi_i)$ of long-term bonds, such that they become financially constrained.

*Timing.* The timing of events is shown in Figure 4.2. At the beginning of each period $t$, aggregate shocks materialise. The representative household specifies policy rules for all its members regarding consumption and dividends, labour supply, (costly) selling and buying offers for long-term bonds, liquid asset positions and loan sales.

Then, an idiosyncratic type shock splits workers from banks. More specifically, a member becomes a banker with probability $\chi$. By the law of large numbers, a share $\chi$ of household members thus become intermediaries, while the remaining fraction $1 - \chi$ are workers. Assets are distributed unequally
across household members with bankers retaining a fraction $\eta$.\(^3\)

**Figure 4.2: Intra-period timing**

After types have been determined, final goods producers employ labour from households and rent capital to produce consumption goods. The rental income on capital accrues to banks as their payoff on loans to capital producers. At this stage, banks redeem liquid assets in exchange for consumption goods. Then, each individual bank meets a counterpart for trading long-term government bonds in the search market and bargains over the price (in terms of consumption goods). The counterparts are workers, to whom households delegate saving. Banks subsequently extend new loans to capital producers and issue as many claims to these on the market as possible. Once asset portfolios have been determined, banks consume their dividends.

At the end of period $t$, household members of both types meet again and share their newly accumulated assets.\(^4\)

**Maximisation Problem**

*Preferences.* The household objective is to maximize

$$
\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} [U(c_{i,t+s}, c_{n,t+s}) - (1 - \chi)h(h_{t+s})],
$$

where $\beta \in (0, 1)$ is the discount factor, $U(c_{i,t}, c_{n,t}) = \chi u(c_{i,t}) + (1 - \chi)u(c_{n,t})$ is the total utility derived from consumption by banks ($c_{i,t}$) and workers ($c_{n,t}$). $u(.)$ is a standard strictly increasing and concave utility function, and $h(.)$

---

\(^3\)Both parameters are used to separately calibrate the ratio of banks’ reserve accumulation to household consumption as well as the bank equity to household net worth ratio. For details on the calibration see section 4.4.1.

\(^4\)The idea of capturing intra-period heterogeneity by a representative family with temporarily separated agents goes back to Lucas (1990).
captures the dis-utility derived from labour supply \( n_t \). \( \mathbb{E}_t \) is the expectation operator conditional on information at time \( t \).

**Aggregation.** As all household members of the same type are homogeneous, one can aggregate individual member’s type-specific variables for the sub-sectors of banks \( i \) (investing) and workers \( n \) (non-investing). In particular, aggregate variables in each sub-sector are defined as \( X_{i,t} \equiv \chi x_{i,t} \) and \( X_{n,t} \equiv (1 - \chi) x_{n,t} \). As banks are allocated a fraction \( \eta \) of the household’s assets, while workers are endowed with the remaining fraction \( (1 - \eta) \), one has \( X_{i,t} = \eta X_t, X_{n,t} = (1 - \eta) X_t \) for \( X_t \in \{ S_t, B_t, D_t \} \), where \( S_t, B_t, D_t \) denote the representative household’s claims on capital producers (loans), long-term government bonds and net liquid assets.

For simplicity, now switch to recursive notation such that \( x \) and \( x' \) denote \( x_t \) and \( x_{t+1} \). First consider the evolution of both types’ loan and long-term bond portfolios. Lending banks purchase claims on the proceeds of new capital goods \( I_i \) from capital producers, which are akin to loans. A fraction \( \theta \) of these can be resold to workers. Loans accumulated in previous periods, on the other hand, are illiquid and need to be retained.\(^5\) Given capital depreciation at rate \( \delta \), the loan portfolios of banks and workers thus evolve according to

\[
S'_{i} = (1 - \delta) \eta S_t + (1 - \theta) I_t, \quad S'_n \geq (1 - \delta) (1 - \eta) S_t \tag{4.2.4}
\]

Lending banks can obtain further funding for new loans by liquidating their bond portfolios. While short-term government bonds mature each period, long-term bonds are modelled as perpetuities with coupon payments decaying at rate \( \lambda \) following Woodford (2001). The parameter \( \lambda \) can also be interpreted as controlling the average maturity of the long-term bond portfolio. This specification thus offers a parsimonious way of capturing a portfolio of bonds with diverse maturities.\(^6\) In addition, a haircut \( \Delta \) imposed on bondholders in a partial default by the government is introduced following Bi (2012); Bi and Traum (2012a) and Bocola (2014).\(^7\) \( \Delta \) is treated as an exogenous mean-zero stochastic process.\(^8\)

Long-term bonds need to be sold on a search market as explained in detail in section 4.2.3. For each unit \( E_i \) of previously accumulated long-term bonds \( (1 - \Delta) \lambda \eta B \) offered on this market, only a fraction \( \phi_i \) can actually be sold to a counterparty. Therefore, banks need to retain a fraction \( (1 - \phi_i E_i) \) of their beginning-of-period long-term bond position. Workers, on the other

\(^5\) As claims on capital producers are normalised to the size of the capital stock, both depreciate at the same rate.

\(^6\) It also nests the special cases of a consol paying \( \frac{1}{P} \) perpetually \( (\lambda = 1) \) and a one-period bond \( (\lambda = 0) \).

\(^7\) While these papers introduce a non-linearity by distinguishing a default- from a non-default regime, this paper models shocks to \( \delta \) as a continuous process in order to maintain tractability.

\(^8\) See section 4.4.
hand, may choose to post $E_n$ orders for long-term bond purchases, a fraction $\phi_n$ of which will be matched. Long-term bond positions of both types at the end of subperiod 2 are, thus,

$$B'_i = (1 - \phi_i E_i) (1 - \Delta) \lambda \eta B, \quad B'_n = (1 - \Delta) \lambda (1 - \eta) B + \phi_n E_n \quad (4.2.5)$$

Given these asset evolutions, workers’ and banks’ aggregate balance sheet constraints can be expressed as

$$C^b_n + q S'_n + Q_n \frac{B'_n}{P} + \frac{D'_n}{P} = N_n \quad (4.2.6)$$

$$C^b_i + q_r S'_i + Q_i \frac{B'_i}{P} + \frac{D'_i}{P} = N_i \quad (4.2.7)$$

Let $j \in \{i, n\}$ denote banks and workers. Both types use their net worth $N_j$ to finance consumption and dividend payments $C_j$, respectively, and asset portfolios at the end of subperiod 2 consisting of loans $S'_j$, long-term bonds $B'_j$ and liquid assets $D'_j$. Note that the valuation of loans and long-term bonds differs across types. Workers value their loans at the market price $q$. Lending banks, on the other hand, can originate and partially sell new loans, which is reflected in the valuation $q_r$, defined as

$$q_r \equiv \frac{1 - \theta q}{1 - \theta} \quad (4.2.8)$$

This price captures the effective replacement cost of loans for banks. Recall that lending banks can sell a fraction $\theta$ of new loans at price $q$ to workers. Hence, they need to finance an amount $(1 - \theta q) / (1 - \theta)$ of every unit of new loans, of which they retain a fraction $1 - \theta$; in other words, they need $(1 - \theta q) / (1 - \theta)$ numeraire goods to acquire one unit of future loans.

On the search market for long-term bonds, buyers and sellers bargain for a transaction price $q^b$. As search market participation is assumed to be costly for both sides, the effective long-term bond prices also reflect these type-specific search costs. Search costs are intended to capture dealer fees and commissions incurred through direct trading on government bond markets or more general costs of participating in collateralised interbank loan markets. In particular, assume that buyers face nominal costs $\kappa^n$ in proportion to posted orders $E_q$. As only a fraction $\phi_n$ of orders is matched, the effective cost per unit of accumulated long-term bonds is $\frac{\kappa^n}{\phi_n t}$, such that buyers effectively value long-term bonds at unit price

$$Q_{n,t} \equiv q^b + \frac{\kappa^n}{\phi_{n,t}}. \quad (4.2.9)$$

---

9These are obtained from individual banks’ type-specific constraints, which are derived in Appendix 4.A.1.
Similarly, sellers face nominal costs $\kappa^i$ in proportion to the amount of assets posted for sale $E_i (1 - \Delta) \lambda B$. Since only a fraction $\phi_i$ of posted assets are sold, sellers bear an effective cost $\frac{\kappa^i}{\phi_{i,t}}$ per unit of long-term bonds, such that they effectively value long-term bonds at unit price

$$Q_{i,t} \equiv q^b_t - \frac{\kappa^i}{\phi_{i,t}}.$$  \hspace{1cm} (4.2.10)

Household members’ current net worth derives from the return on previously accumulated assets and labour income for workers, i.e.

$$N_n = (1 - \eta) \left[ (r + (1 - \delta) q) S + (1 + (1 - \Delta) \lambda Q_n) \frac{B}{P} + R \frac{D}{P} \right] + w H$$

$$N_i = \eta \left[ (r + (1 - \delta) q_r) S + (1 + (1 - \Delta) \lambda Q_i) \frac{B}{P} + R \frac{D}{P} \right]$$  \hspace{1cm} (4.2.11, 4.2.12)

where loans and bonds are valued at the type-specific prices discussed above.

Maximisation problem. Let $V (S, B, D; \Gamma)$ denote the value function of the representative household with loans $S$, long-term bonds $B$ and net liquid assets $D$, given a vector of aggregate state variables $\Gamma$ whose evolution is taken as given by the household. Since banks and workers reunite at the end of every period to share their assets and liabilities, one has $S' = S'_i + S'_n$, $B' = B'_i + B'_n$, $D = D_i + D_n$. Finally, let $E \Gamma$ denote expectations taken in period $t$ given the aggregate state variables $\Gamma$. The Bellman equation associated with the household’s optimisation problem then reads

Problem 2:

$$V (S, B, D; \Gamma) = \max_{\{H, C_k, S'_k, B'_k, D_k\}_{k=1,n}} \chi u \left( \frac{C_k}{\chi} \right)$$

$$+ (1 - \chi) \left[ u \left( \frac{C_n}{1 - \chi} \right) - h \left( \frac{H}{1 - \chi} \right) \right]$$

$$+ \beta E \Gamma \left[ V (S', B', D'; \Gamma') \right]$$  \hspace{1cm} (4.2.13)

s.t. (4.2.6), (4.2.7).

4.2.3 Search and Matching in the Bond Market

Search and matching. Matching between buyers and sellers of long-term government bonds takes place in a decentralized market. Lending banks seek to sell their long-term bonds to workers in order to free up resources for new loans. Recall that banks offer a fraction $E_i$ of their beginning-of-period portfolio of long-term bonds for sale, while workers post buy orders
4. Government Bond Liquidity and Sovereign-Bank Interlinkages

\[ E_n = \phi_n^{-1} [B'_n - (1 - \Delta) \lambda (1 - \eta) B] \] as per equation (4.2.5). Buy and sell orders are randomly matched, with the number of aggregate matches \( M \) being determined by the matching function

\[ M \equiv \xi [E_i (1 - \Delta) \lambda \eta B] E_n^{1 - \gamma} \] (4.2.14)

where \( \gamma \in (0, 1) \) is the elasticity of matches w.r.t posted assets. Defining \( \theta \) as the ratio of buy to sell orders, one has

\[ \theta \equiv \frac{E_n}{E_i (1 - \Delta) \lambda \eta B}, \quad \phi_n \equiv \frac{M}{E_n} = \xi \theta^{-\gamma}, \quad \phi_i \equiv \frac{M}{E_i (1 - \Delta) \lambda \eta B} = \xi \theta^{1 - \gamma}, \] (4.2.15)

where \( \phi_i \) captures the endogenously determined probability of a sell order being matched by a buy order, and \( \phi_n \) the probability of a buy order being matched by a sell order. Note that \( \theta \) reflects search market tightness from a buyer’s perspective. A larger \( \theta \) indicates that buyers find it harder to find an appropriate counterpart on the search market, such that buy orders are large compared to sell orders. Finally, noticing that \( \phi_n^{-1} \phi_i = \theta \), the relationship between the matching probabilities can be linked as

\[ \phi_n = \xi^{\frac{1}{1 - \gamma}} \phi_i^{\frac{\gamma}{1 - \gamma}} \] (4.2.16)

**Bond price.** Once their positions have been matched, banks and workers bargain over how to split the surplus of a transaction of long-term bonds. Importantly, the amount of matched assets is predetermined at the point of bargaining. Therefore, buyers and sellers interact at the margin, i.e. the match surplus for both buyers and sellers is the respective marginal value of an additional transaction.

Let \( V^i \) and \( V^n \) denote banks’ and workers’ value from the perspective of the household. The marginal value of a match for either type is then given by the envelope condition on the respective type’s value with respect to a successful match, i.e. \( V^i_m \) for sellers and \( V^n_m \) for buyers. An individual buyer’s surplus in terms of consumption goods then amounts to\(^{10}\)

\[ -V^n_m = -u' (c_n) (1 - \beta) \frac{q^b}{P} + \beta \left[ E \left[ V_B \right] - \beta \frac{q^b}{P} \frac{1}{q} E \left[ V_S \right] \right] \]

Intuitively, a buyer pays \( \frac{q^b}{P} \) for the matched government bonds, which decreases consumption by a fraction \( 1 - \beta \) of that amount given logarithmic utility. The consumption sacrificed today for the purchase is valued at marginal utility \( u' (c_n) \). At the same time, the purchase increases future bond

\(^{10}\)Individual’ values are derived in appendix 4.B.1. Note that search costs are sunk at the time of bargaining. However, they are not ignored, since they are taken into account by the representative household when deciding on bond market participation.
holdings while reducing available resources for additional future lending. In particular, for every unit of government bonds purchased at \( \frac{q_b}{P} \) today, the household forgoes the opportunity to spend a fraction \( \beta \) of it’s member’s net worth on future loans at current price \( q \).

The seller’s surplus is the marginal value to the household of an additional match for banks, which is given by

\[
V_i^m = u'(c_i) (1 - \beta) \frac{q_b}{P} + \beta \left( \left( c_i^1 \phi_i \right)^{-1} - 1 \right) \mathbb{E}_\Gamma [V_B] + \beta \frac{q_b}{P} \frac{1}{q_r} \mathbb{E}_\Gamma [V_S].
\]

A seller receives additional resources \( \frac{q_b}{P} \) per match, increasing his current consumption by \( 1 - \beta \) valued at marginal utility \( u'(c_i) \). The continuation value captures the effect of the sale on the future asset portfolio of the lending bank. On the one hand, banks retain a fraction \( \left( c_i^1 \phi_i \right)^{-1} - 1 \) for each unit of matches as bonds are only partially saleable. These assets are returned to the household at the end of the period and, thus, increase future long-term bond holdings \( B' \). Therefore, the continuation value of a match comprises the marginal value of future long-term bonds to the representative household multiplied by this factor. Moreover, banks invest a fraction \( \beta \) of the additional resources \( \frac{q_b}{P} \) gained in a match on new loans at internal cost \( q_r \), thus increasing the household’s future loan portfolio \( S' \).

Due to the homogeneity of members within the groups of buyers and sellers, the type-specific valuations are identical in all matched pairs. The transaction price \( q_b \) is determined via (generalized) Nash bargaining between buyers and sellers over the total match surplus, i.e. agents bargain over \( q_b \) to solve

\[
\max_{q_b} \left\{ \omega \ln \left( V_i^m \right) + (1 - \omega) \ln \left( -V_i^m \right) \right\}
\]

where \( \omega \) is the fraction of the surplus going to sellers. In the case of bilateral bargaining, \( \omega \) also represents the bargaining power of sellers.

### 4.2.4 The Government

*The fiscal authority.* Government expenditures consist of (fixed) consumption, coupon payments on long-term bonds and interest payments on liquid assets. To finance its budget, the fiscal authority collects taxes, issues long-term bonds \( B_g \) and receives transfers from the monetary authority corresponding to the new issuance of liquid assets \( D' - D \). The consolidated budget constraint of the government thus reads

\[
\bar{G} + \frac{B}{P} + R \frac{D}{P} = T + \frac{D'}{P} + Q \frac{B_g}{P}
\]

where

\[
B' = (1 - \Delta') \lambda B + B_g
\]
and assuming that the government never purchases long-term bonds, i.e. \( B_g \geq 0 \). Further assume that the fiscal authority takes the bargained bond price as given rather than entering the bargaining process itself. However, by changing the supply relative to the demand of long-term bonds, new issuance affects the matching probability of sellers \( \phi_i \). This, in turn, affects the bargaining price through general equilibrium effects as shown in 4.3.2.\(^{11}\)

The government levies lump-sum taxes, which are modelled by a simple rule that ensures stationarity of government indebtedness, in particular

\[
T_l = \psi T_1 + \psi T_2 \left( \frac{\Psi}{\bar{\Psi}} - 1 \right) \tag{4.2.20}
\]

where \( \Psi = \frac{(1-\Delta) \lambda P Y}{P Y} \) is the ratio of government debt at face value to GDP, \( \psi T_1 \) is the steady state level of taxes and \( \psi T_2 \) captures the elasticity of lump-sum taxes to deviations of the debt-to-GDP ratio from its steady state. \( \psi T_2 > 0 \) ensures that taxes increase in response to rising government indebtedness.

**The monetary authority.** Conventional monetary policy involves setting the nominal interest rate according to a Taylor-type feedback rule

\[
R' = \psi R \left( \frac{\pi}{\bar{\pi}} \right) \psi \tag{4.2.21}
\]

where \( \psi R \) calibrates the steady state gross riskless interest rate and \( \psi \) determines the reaction of the central bank to deviations of inflation from its steady state. Inflation is defined as \( \pi' \equiv \frac{\pi'}{\bar{\pi}} \).

Moreover, the central bank controls the supply of liquid assets. Thus, the fiscal authority cannot decide on the contribution of liquid asset issuance to the consolidated government budget (4.2.19).\(^{12}\)

\[
D' = D + \psi D \left( 1 - \frac{\phi_i}{\bar{\phi}_i} \right) P \tag{4.2.22}
\]

Unconventional monetary policy can be captured by an expansion of the stock of liquid assets in response to a tightening of liquidity conditions in the long-term bond market with \( \psi D > 0 \).

\(^{11}\)With government issuance of long-term bonds the amount of assets for sale on the search market is

\[
B_M \equiv E_i (1-\Delta) \lambda \eta B + \phi_i^{-1} B_g = \phi_i^{-1} \left[ B' - B'_i + (1-\Delta) \lambda (\eta - 1) B \right]
\]

Scaling the targeted government issuance by the inverse of the probability of finding a buyer ensures that the government actually issues the targeted amount.

\(^{12}\)This modelling choice follows Gertler and Karadi (2013). It is a short-cut to modelling liquid assets as money or an international safe-haven asset, neither of which is controlled by the domestic fiscal authority.
4. Government Bond Liquidity and Sovereign-Bank Interlinkages

4.2.5 Competitive Equilibrium

The recursive competitive equilibrium is characterised by a set of equations determining the consumption, labour and portfolio choices \((C_i, C_n, H, K', S', S'_n, B'_i, B'_n, D'_i, D'_n)\); search market features \((\phi_i, \phi_n)\) and prices \((q, q_r, q^b, Q_i, Q_n, r, P, \pi)\) given exogenous stochastic processes for \((z'_a, \Delta')\) and policy rules for \((T_l, D', R')\). This set of equations satisfies

1. individual optimality: given prices and search market characteristics, the policy functions satisfy the optimality conditions of final goods producers (4.2.1)-(4.2.2) and solve the representative household’s problem

2. Investment \(I\) is determined by (4.2.7).

3. market clearing conditions for

(a) securities/loans and capital: \(S' = (1 - \delta) K + I\) and \(K' = S'\); 
(b) the search market: (4.2.16) holds, the search market price \(q^b\) solves (4.2.17), while the effective prices are defined in (4.2.9) and (4.2.10); 
(c) long-term bonds: the government budget constraint (4.2.18) is satisfied with outstanding long-term bonds evolving according to (4.2.19); 
(d) short-term bonds: (4.2.22) holds; 
(e) final goods: the workers’ budget constraint (4.2.6) is satisfied; 

3. policy rules: for lump-sum taxes (4.2.20) and the riskless interest rate (4.2.21) with inflation defined as \(\pi' \equiv \frac{P'}{P}\).

4.3 Equilibrium Characterisation

This paper focuses on an equilibrium in which the search market is active, i.e. in which banks are financially constrained and want to trade long-term government bonds to finance a larger loan portfolio. For this to be the case, the market price of loans must exceed one, i.e. \(q > 1\). Using the definition of \(q_r\) in equation (4.2.8), implies a replacement cost strictly below one, such that one has \(q > 1 > q_r\). In other words, creating loans at the replacement cost and selling them at the market price is a profitable business. Therefore, the representative household will prompt lending banks to spend whatever net worth they are not consuming on issuing new loans. Accordingly, banks quote as many long-term bonds for sale as feasible, i.e. \(E_i = 1\) (or \(B'_i = (1 - \phi_i) (1 - \delta^b) \eta B\)). They also go short on liquid assets as much as possible, i.e. \(D'_i = 0\).

4.3.1 The Household’s Optimality Conditions

By using the type-specific budget constraints (4.2.6) and (4.2.7) to substitute out consumption \(C_n\) and dividends \(C_i\) in Problem 2, and using \(E_i = 1\)
and $D'_i = 0$ the representative household’s choice set can be reduced to $\{H, S'_i, S'_n, B'_n, D'_n\}$.

The household’s optimal labour supply satisfies

$$u'(c_n)w = \mu \quad (4.3.1)$$

The first-order conditions for household members’ loan holdings $S_i$ and $S_n$ read

$$u'(c_i)q_r = \beta \mathbb{E}_t \left[V_S \left(S', B', D'; \Gamma'\right)\right], \quad u'(c_n)q = \beta \mathbb{E}_t \left[V_S \left(S', B', D'; \Gamma'\right)\right]$$

which implies

$$u'(c_i) = \rho u'(c_n) \quad (4.3.2)$$

where $\rho \equiv \frac{q_r}{q}$ is inversely related to risk sharing between lending and non-lending banks.\textsuperscript{13} As long as lending banks are financially constrained, such that loan origination is profitable, one has $\rho > 1$, and, therefore, $c_i < c_n$. Finally, the first-order conditions for bonds and liquid assets are

$$u'(c_n) \frac{Q_n}{P} = \beta \mathbb{E}_t \left[V_B \left(S', B', D'; \Gamma'\right)\right], \quad u'(c_n) \frac{1}{P} = \beta \mathbb{E}_t \left[V_D \left(S', B', D'; \Gamma'\right)\right].$$

Using the appropriate envelope conditions, one can derive the corresponding asset pricing formulae for each asset class

$$\mathbb{E}_t \left[\Lambda' \frac{\zeta'_1}{q}\right] = 1, \quad \mathbb{E}_t \left[\Lambda' \frac{\zeta'_2}{\pi Q_n}\right] = 1, \quad \mathbb{E}_t \left[\Lambda' \frac{\zeta'_3}{\pi}\right] = 1 \quad (4.3.3)$$

where banks’ stochastic discount factor between two successive periods is $\Lambda' \equiv \frac{\hat{\beta}u'(c')}{u'(c')}$, inflation is defined as $\pi' \equiv \frac{P'}{P}$ and

$$\zeta'_1 \equiv [\eta \rho' + (1 - \eta)] r' + (1 - \delta) q'$$

$$\zeta'_2 \equiv [\eta \rho' + (1 - \eta)] + (1 - \Delta') \lambda [\eta \rho' \rho'_i Q'_i + (1 - \eta) Q'_n]$$

$$\zeta'_3 \equiv [\eta \rho' + (1 - \eta)] R'.$$

### 4.3.2 The Bargained Bond Price

The sufficient and necessary first-order-condition to the bargaining problem (4.2.17) that maximises the total surplus of buyers and sellers yields

$$\frac{\omega}{V_m} = \frac{1 - \omega}{-V_m}.$$ 

This condition can be further consolidated to obtain an analytical expression for the bargaining price summarized in the following proposition.

\textsuperscript{13}Note that aggregate have been replaced with individual consumption levels using $c_i = \frac{c_i}{\chi}$ and $c_n = \frac{c_n}{\chi}$.
Proposition 5:
The search market bargaining price

1. is given by

\[ q^b = \frac{\omega + \phi_i - 1}{(1 - \omega) [1 + \phi_i (\rho - 1)]} \kappa^n \]  \hspace{1cm} (4.3.4)

2. correlates positively with bond saleability (i.e. \( \frac{\partial q^b}{\partial \phi_i} > 0 \)), and negatively with the purchase rate (i.e. \( \frac{\partial q^b}{\partial \phi_n} < 0 \)), if

\[ \phi_i \left[ \phi_i - \frac{1}{3} \left( \frac{1}{\gamma (\rho - 1)} + (1 - \omega) (\gamma^{-1} - 2) \right) \right] + \frac{11 - \omega}{3 \rho - 1} < 0. \]

When \( \gamma = 0.5 \), the above sufficient condition simplifies to

\[ -\left( \frac{1}{9} - (\rho - 1) (1 - \omega) \right)^{\frac{1}{2}} < (\rho - 1) \phi_i - \frac{1}{3} < \left( \frac{1}{9} - (\rho - 1) (1 - \omega) \right)^{\frac{1}{2}}. \]

(4.3.5)

With \( \omega \approx 1 \) and \( \rho \in (1, 1.5) \), the bounds approximately collapse to \( 0 \lesssim \phi_i \lesssim \frac{4}{3} \), such that the upper bound will never be binding, since by construction \( \phi_i \in [0, 1] \).


Proposition 5.1 links the bond price with demand-side search market participation costs. It immediately implies that higher search costs crowd out demand for long-term bonds and erode the surplus that can be garnered from a match, which leads to a fall in the bond price. Search costs thus capture the intermediation capacity of bond markets in the present model.\(^{14}\)

The search market features of the bond market further give rise to a non-trivial relationship between the bond price and endogenous bond saleability. When bond saleability drops, banks need to finance a larger share of loans out of yields on own funds rather than asset liquidations. This tightens their contemporaneous financing constraints. Therefore, banks’ threat point of breaking off negotiations over the marginal bond sale and self-financing becomes less attractive. They are thus willing to accept a lower bargaining price. On the other hand, with lower bond saleability lending banks retain a larger fraction of bonds, which will be returned to the representative household. This relaxes the funding constraints of future generations of banks.

\(^{14}(4.3.4)\) implies that when \( \kappa^n \to 0 \), the long-term bond prices goes to zero. Without search costs, long-term bonds lose their liquidity premium and thus yield less than short-term bonds (or deposits) as long as \( R \geq 1 \). Therefore, investors would prefer short- to long-term bonds and the market for long-term bonds would collapse with the price going to zero.
with lending opportunities, which is valued by the household and, hence, increases the bargaining price. Proposition 5.2 shows that the contemporaneous effect dominates in this trade-off between current and future funding constraints as long as the sales rate is small enough, because current financial constraints bind strongly in this case.\footnote{In fact, the contemporaneous effect dominates for all plausible calibrations.}

This feedback between bond saleability and the bond price is triggered whenever bond market participation becomes less attractive for buyers, such that bond saleability drops. The model can thus generate simultaneous decreases in bond saleability and the bond price through the simultaneous reaction of supply and demand.

\section{4.4 Numerical Results}

This section presents the results of the numerical solution of the model in order to illustrate macroeconomic dynamics in response to exogenous shocks. The model is solved using a first-order approximation around the non-stochastic steady state.

\textit{Shock processes.} Total factor productivity is assumed to evolve according to an AR(1) process, i.e.

\begin{equation}
    z'_a = \rho a z_a + \varepsilon'_a
\end{equation}

where $0 < \rho a < 1$ and $\varepsilon'_a$ is a normally distributed random variable with mean zero and standard deviation $\sigma a$. In the baseline calibration, $\rho a = 0.9$ and $\sigma a = 0.01$.

In addition, an exogenous haircut on long-term bonds is introduced to capture a partial sovereign default. In expectation, this sovereign solvency shock is zero. In particular, $\Delta = z_{\Delta}$ where $z_{\Delta}$ follows an AR(1) process

\begin{equation}
    z'_\Delta = \rho \Delta z_\Delta + \varepsilon'_\Delta
\end{equation}

where $0 < \rho \Delta < 1$ and $\varepsilon'_\Delta$ is a normally distributed random variable with mean zero and standard deviation $\sigma \Delta$. In the baseline calibration, $\rho \Delta = 0.8$ and $\sigma \Delta = 0.01$. While this shock destroys resources, it does not affect the production frontier of the economy. This is a key property to understanding the endogenous response of bond market liquidity and the fight to liquidity in the banking sector.

\textit{Utility.} To facilitate the analysis, log-utility is assumed without loss of generality for both workers and banks, i.e. $u (c_{jt}) = \ln (c_{jt})$ for $j \in \{i, n\}$.

\subsection{4.4.1 Calibration}

The model is calibrated to match long-run characteristics of the Italian economy. An overview of all parameters and calibration targets is shown in
Table 4.1. All data sources and definitions of targets are detailed in Appendix 4.C.

Preferences and Production Technology. Household preferences are parametrized exogenously with standard values for the discount factor $\beta$ and the coefficient capturing relative risk aversion $\sigma$. The utility weight of leisure targets a steady state working time of 30%. The depreciation rate of capital $\delta$ and the capital share of output $\alpha$ are chosen such that the model replicates the long-run (post 2001) averages of the capital-output and the investment-output ratio, which amount to 300% and 20.3%, respectively. Taken together, these two targets imply an investment-capital ratio of about 7%.

The parameters that govern the mass of bankers and their share of total financial assets are less common. The former is chosen to match a target for banks’ dividends $C_i$ as a share of households’ consumption $C_n$. Since the model does not allow for inside equity accumulation and all retained earnings are essentially consumed by bankers, this share is measured as the accumulation of capital and reserves in the Italian banking sector relative to final consumption expenditure by the private sector, which was on average 1.54% since 2001. This target yields a population share of bankers of $\chi = 1.63\%$ in the model economy. In addition, the fact that banks are strongly leveraged compared to households is accounted for by obtaining the banking sectors’ share of all financial assets as $\eta = 4.76\%$, i.e. about three times bankers’ population share. In other words, compared to households, banks finance a much larger amount of assets relative to their population share. Since both sectors’ asset shares are proportional to their respective net worth in the model, banks’ asset share is calibrated by targeting the model net worth ratio with the average ratio of bank equity to household net worth observed between 1997 and 2006. During the said period, this ratio was very stable at around 4.8%.

Liquidity Frictions. Frictions associated with the sale and purchase of financial assets are parameterized by the vector $\{\gamma, \xi, \bar{\kappa}_n, \bar{\kappa}_i, \omega, \theta\}$. Since the supply sensitivity of matching $\gamma$ and matching efficiency $\xi$ never occur independently of each other, one can exogenously set $\gamma = 0.5$ without loss of generality and determine matching efficiency endogenously as a function of some steady-state target. The remaining parameters related to search frictions $\{\xi, \bar{\kappa}_n, \bar{\kappa}_i, \omega\}$ are jointly chosen to match the following targets: i) average turnover rates from the perspective of sellers and buyers of Italian government bonds; ii) the long-run average of bid-ask spreads for 10-year Italian government bonds; iii) the ratio of liquid to total assets in Italian banks’ balance sheets.

Bond saleability $\phi_i$ and purchase rate $\phi_n$ are set to match average turnover

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16Starting with the financial crisis, this ratio embarks on an increasing path due to both nominator and denominator effects, which are not intended to be capture here.
rates of Italian government bonds on the MTS interdealer trading system gleaned from Pelizzon, Subrahmanyam, Tomio, and Uno (2013). Exploiting both transaction and quote data reported in this study, average turnover rates are calculated as the ratio of the total *traded* quantity of bonds to the total *quoted* quantity on either side of the market during the entire sample period.\(^\text{17}\) The bid-ask spread, normalised by the mid-point between bid and ask prices, is set to the post 2001 average of 10 basis points observed for 10-year Italian Government Benchmark bonds reported by Reuters. We target the bid-ask spread of 10-year benchmark bonds since these capture long-term refinancing conditions. The extent to which liquidity frictions affect financing conditions and the real economy crucially depends on banks’ dependence on liquid assets. In order to quantify the importance of liquid assets in the Italian banking sector, the ratio of Italian banks’ holdings of liquid assets to their total assets is targeted. According to the World Bank’s Global Financial Development database, this ratio amounted on average to 24.5% post 2001.

Finally, the fraction of loans that is liquid at issuance is calibrated to match Tobin’s Q estimated in Hall and Oriani (2006) at 1.1 based on firm-level panel data for the Italian economy.

*Government.* As discussed in section 4.2.2, modelling long-term government bonds as perpetuities with decay rate \(\lambda\) may, equivalently, be interpreted as a portfolio of government bonds with average maturity \(\frac{1}{1-\lambda}\). Broner, Erce, Martin, and Ventura (2014) show that the Italian sovereign’s maturity structure was stable since 2001 with an average maturity of seven years. The government consumption-output ratio is set to its post 2001 average of 19% to calibrate \(\bar{G}\). The parameters of the tax policy rule, \(\psi_{T1}\) and \(\psi_{T2}\), are chosen exogenously to ensure stationarity of the government budget constraint. The quarterly steady-state riskfree interest rate is set to 1.005, implying an annual gross nominal rate of 1.02. Interest rate policy is shut off in the baseline calibration by setting \(\psi_{x} = 0\), i.e. the central bank only controls the supply of liquid assets.

### 4.4.2 Equilibrium Responses to Shocks

*Adverse aggregate productivity shocks.* The impact of a negative productivity shock on real variables shown in Figure 4.3 is similar to that in a real business cycle model. A decrease in total factor productivity lowers the return to capital and, hence, the demand for capital goods. This is reflected in the falling capital price and a sharp reduction in investment activity. Output drops on impact on account of lower factor productivity and returns only sluggishly due to the depressed capital stock.

\(^{17}\)Since the reported quote statistics are average across both sides of the market, one cannot distinguish between turnover rates for sellers and buyers. For this reason, turnover rates are assumed to be symmetric for both sides.
The investment dynamics are reinforced by the portfolio choices of workers and bankers. As the negative productivity shock persistently decreases the return to capital, it also makes loan-financing less profitable in the future. As a result, the hedging value of long-term bonds and liquid assets drops and the demand for both assets falls. On the bond market, less buy offers result in lower bond liquidity from the sellers’ perspective, which is reflected in their lower turnover rate \( \phi_i \). On the market for liquid assets, the fall in demand has indirect fiscal implications that interact with demand and supply conditions on the bond market. Notice that the price of liquid assets is one over the price of the numeraire good, i.e. \( \frac{1}{P} \). This price of liquid assets decreases with weaker demand. As a result, the real value of government debt \( \frac{B}{P} \) falls, such that the fiscal authority can reduce its issuance of long-term bonds. The drop in government supply on the bond market pushes up the bargaining price through general equilibrium effects.

The decrease in the real value of government bonds is compensated on impact by the fall in GDP, such that the debt-to-GDP ratio rises temporarily. However, the debt-to-GDP ratio eventually contracts as bond issuance falls. Lump-sum taxes follow the path of the debt-to-GDP ratio by assumption.

As the demand for liquid assets shrinks, so does banks’ liquidity ratio. Over time, it gets pushed back up, however, by a denominator effect as the value of total assets shrinks with the decrease of the capital price. The bid-ask price declines as buyers’ turnover rate increases, such that the demand-side search costs per match fall.

Finally, the decline in lending and investment is propagated through a sustained drop in banks’ net worth.

### Table 4.1: Baseline calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences and Production Technology</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Household discount factor</td>
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<td>Exogenous</td>
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<td>Relative risk aversion</td>
<td>( \sigma )</td>
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<td>Utility weight on leisure</td>
<td>( \mu )</td>
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<td>Working time: 30%</td>
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<td>Mass of bankers</td>
<td>( \chi )</td>
<td>0.0163</td>
<td>Reserve accumulation-Consumption ratio: 0.01</td>
</tr>
<tr>
<td>Asset share of bankers</td>
<td>( \eta )</td>
<td>0.0476</td>
<td>Bank equity-Household net worth ratio: 0.05</td>
</tr>
<tr>
<td>Depreciation rate of capital</td>
<td>( \delta )</td>
<td>0.0067</td>
<td>Investment-Capital ratio: 0.07</td>
</tr>
<tr>
<td>Capital share of output</td>
<td>( \alpha )</td>
<td>0.2521</td>
<td>Capital-Output ratio: 3.00</td>
</tr>
<tr>
<td><strong>Liquidity Frictions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supply sensitivity of matching</td>
<td>( \gamma )</td>
<td>0.5</td>
<td>Exogenous</td>
</tr>
<tr>
<td>Matching efficiency</td>
<td>( \xi )</td>
<td>0.4000</td>
<td>Bond market turnover, sellers: 40%</td>
</tr>
<tr>
<td>Buyer search costs</td>
<td>( \kappa_b )</td>
<td>0.0035</td>
<td>Bond market turnover, buyers: 40%</td>
</tr>
<tr>
<td>Seller search costs</td>
<td>( \kappa_s )</td>
<td>0.0021</td>
<td>Bid-ask spread: 10 bps</td>
</tr>
<tr>
<td>Bargaining weight of sellers</td>
<td>( \omega )</td>
<td>0.9998</td>
<td>Banks’ liquidity-ratio: 0.25</td>
</tr>
<tr>
<td>Liquid issuance of loans</td>
<td>( \theta )</td>
<td>0.0562</td>
<td>Tobin’s Q: 1.1</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay rate of coupon payments</td>
<td>( \lambda )</td>
<td>0.0643</td>
<td>Average maturity of bonds: 7 years</td>
</tr>
<tr>
<td>Government consumption</td>
<td>( \bar{G} )</td>
<td>0.0825</td>
<td>Government consumption-Output ratio: 0.19</td>
</tr>
<tr>
<td>Lump-sum tax, steady state</td>
<td>( \psi_{11} )</td>
<td>0.1474</td>
<td>Exogenous</td>
</tr>
<tr>
<td>Lump-sum tax, elasticity</td>
<td>( \psi_{12} )</td>
<td>0.1300</td>
<td>Exogenous</td>
</tr>
<tr>
<td>Steady state gross nominal interest rate</td>
<td>( \psi_{N} )</td>
<td>1.0050</td>
<td>Annual gross riskless rate: 1.02</td>
</tr>
<tr>
<td>Taylor coefficient on inflation</td>
<td>( \psi_{e} )</td>
<td>0.00</td>
<td>Exogenous: money (=0) or deposits (&gt;0)</td>
</tr>
</tbody>
</table>

Notes: The model is calibrated to quarterly frequency.
Figure 4.3: Impulse responses after a negative one standard deviation productivity shock.

Aggregate Sovereign Solvency Shocks. We introduce an exogenous haircut on long-term government bonds as a laboratory experiment to analyse the interaction between credit and liquidity risk in sovereign bond markets. The impulse responses are shown in Figure 4.4. The sovereign solvency shock destroys financial wealth of bond holders while providing debt-relief to the sovereign. These first-order effects of the shock are amplified by tightening financing constraints in the banking sector due to endogenously deteriorating bond liquidity. Again, these effects are best understood by considering the portfolio choices of banks and savers.

Being persistent in nature, the haircut reduces both the current and future value of long-term bonds holdings. As a result, the hedging value of long-term bonds against future financing constraints for banks erodes, such that households reduce their demand for long-term bonds. Declining buy orders on the search market for bonds depress bond saleability $\phi_s$ as the market becomes more congested from the point of view of bond-selling banks. Conversely, buyers encounter sellers more easily, such that the purchase rate $\phi_n$ increases. Due to this asymmetry in the response of bond liquidity for buyers and sellers the intermediation capacity of the bond market declines, i.e. its capacity to channel liquid funds where financing constraints are felt most ostensibly. As their financing constraints tighten due to the
falling saleability of bonds, banks are willing to accept a lower transaction price on the bond market, such that \( q_b \) strongly declines (see Proposition 5.2). The price decline tightens financing constraints further, giving rise to a downward spiral.

In contrast to the negative productivity shock, the solvency shock does not impair the productivity of capital goods, such that investment projects remain profitable. Anticipating continued illiquidity in the bond market, households would like to hedge against future financing constraints by shoring up their liquidity buffers. Hence, savers fly to liquid assets, which is reflected in the strong and persistent increase in the liquidity ratio. The strong fall in bond saleability also increases search costs per unit of bonds offered for sale, such that the bid-ask spread soars. These model predictions for the behaviour of variables capturing liquidity frictions contrast starkly with the productivity shock dynamics.

As for real variables, investment declines on impact as tighter financing constraints in the banking sector weigh on loan supply. Again, the decline in lending and investment is propagated through a sustained drop in banks’ net worth, which depresses capital accumulation going forward. Output also contracts on impact as households reduce their demand in response to the negative wealth shock associated with the haircut on bonds and lower bond prices.

The solvency shock also has fiscal implications beyond the first-order debt relief. The flight to liquidity triggered by the endogenous decline in bond saleability bids up the price of liquid assets \( \frac{1}{P} \), which also inflates the real debt-burden \( \frac{B}{P} \). The falling bond price \( q_b \) exerts further upward pressure the government’s financing costs. The government compensates these additional costs by strongly increasing long-term bond issuance.

Note that the increase in the price of liquid assets \( \frac{1}{P} \) corresponds to a fall in the price level \( P \), i.e. lower inflation. The drop in inflation combined with higher issuance initially increase the real value of bonds outstanding despite the debt relief provided by the haircut. Coincidentally, the sharp increase in bond issuance contributes to the deterioration of bond saleability as the additional government supply crowds out private sellers.

4.4.3 Robustness

Having argued that the anticipation of future financing constraints due to prolonged liquidity frictions in the bond market is key for the dynamics observed after sovereign solvency shocks, one can now illustrate the impact of variation in shock persistence on aggregate dynamics. Varying degrees of persistence may effect macroeconomic variables both directly via the negative wealth effect associated with exogenous haircuts, and indirectly via the impact of financing constraints on investment.

Impulse responses are shown in Figure 4.5 for the baseline calibration
with high persistence ($\rho_\Delta = 0.8$), medium ($\rho_\Delta = 0.6$) and low persistence ($\rho_\Delta = 0.2$). Although the responses are qualitatively similar, this comparison confirms that the persistence of solvency shocks is, indeed, crucial for both the magnitude of macroeconomic responses on impact and the speed of adjustment. In particular, these results suggest a non-linear relationship between shock persistence and liquidity frictions. As solvency shocks are expected to last longer, bond liquidity is anticipated to be low both today and in the future. The protracted illiquidity of bonds tightens financing constraints of banks, such that the hedging value of liquid assets increases. Banks react by rebalancing their portfolios towards liquid assets, thereby increasing their liquidity ratios.

The increasing demand for liquid assets implies a more pronounced rise in their price $1/P$, which is mirrored in more subdued inflation. The higher is inflation, the longer the eventual decrease in real bond holdings - due to weaker demand for nominal bonds - is delayed.
4. Government Bond Liquidity and Sovereign-Bank Interlinkages

Figure 4.5: Impulse responses after a one standard deviation sovereign solvency shock (increase of haircut) with different degrees of persistence.

4.5 Conclusion

This paper analyses the interaction of sovereign credit and liquidity risk and the impact of endogenous fluctuations in government bond liquidity on financial intermediation, bank lending, investment and government finances. A dynamic general equilibrium model is proposed, which endogenises government bond liquidity through search and matching frictions. The model is able to match a number of stylised facts regarding the behaviour of sovereign debt markets during the euro area sovereign debt crisis, such as depressed turnover rates and rising bid-ask spreads. In the model, lower bond market liquidity constrains the funding capacity and ultimately the profitability of the banking sector, leading to subdued lending, slower capital accumulation and declining economic activity. As bond prices fall together with bond liquidity, the revenue of bond issuance accruing to the public sector falls as well. Thus, the model demonstrates how endogenous declines in government bond liquidity reinforce the sovereign-bank nexus.
Appendix

4.A Model Characteristics

4.A.1 Individuals’ Constraints

Preferences and Flow-of-Funds. The representative household maximises the present value of dividends

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} [U(c_{i,t+s}, c_{n,t+s}) - (1 - \chi)h(s_{t+s})],$$

(4.A.1)

where $\beta \in (0, 1)$ is the discount factor, $U(c_{i,t}, c_{n,t}) = \chi u(c_{i,t}) + (1 - \chi)u(c_{n,t})$ is the total utility derived from consumption by banks ($c_{i,t}$) and workers ($c_{n,t}$) and $h(.)$ captures the dis-utility derived from labour supply $n_t$. Suppose that $u(c_{j,t}) = \ln(c_{j,t})$ for $j \in \{i, n\}$.

The flow-of-funds constraint of a typical household member $x$ then reads:

$$c_{j,t} + i_{j,t} + q_t \left[s_{j,t+1} - i_{j,t} - (1 - \delta) s_{j,t}\right] + \frac{d_{j,t+1}}{P_t} + \kappa^i e_{j,t} \frac{1}{P_t}$$

$$+ \kappa^i e_{j,t} (1 - \Delta) \lambda b_{j,t} = r_t s_{j,t} + \frac{b_{j,t}}{P_t} + R_t \frac{d_{j,t}}{P_t} + q_t b_{j,t} + w_t h_{j,t}$$

(4.A.2)

Expenditures consist of i) dividend payments $c_{j,t}$, ii) newly issued loans $i_{j,t}$, iii) issuance or purchases of claims on the marketable fraction of these new loans $[s_{j,t+1} - i_{j,t} - (1 - \delta) s_{j,t}]$ at price $q_t$, iv) adjustments to the portfolio of liquid assets $\frac{d_{j,t+1}}{P_t}$ and v) search costs associated with purchases and sales of nominal long-term bonds. We assume that buyers face nominal costs $\kappa^i$ in proportion to posted orders $e_{j,t}^i$. Similarly, sellers face nominal costs $\kappa^n$ in proportion to the amount of assets posted for sale $e_{j,t}^i (1 - \Delta) \lambda b_{j,t}$.

Income derives from returns to the stock of loans (equity stakes in capital producers) $r_t s_{j,t}$, the real return on long-term bonds $\frac{b_{j,t}}{P_t}$, the real return on liquid assets $R_t \frac{d_{j,t}}{P_t}$, and revenue from sales of long-term bonds $\frac{m_{i,t}}{P_t}$ at bargained price $q_t^b$.

Bonds and loans evolve according to

$$b_{j,t+1} = (1 - \Delta) \lambda b_{j,t} - m_{j,t}$$

$$s_{j,t+1} \geq (1 - \delta) s_{j,t} + (1 - \theta) i_{j,t}$$

As existing loans are entirely illiquid, only a fraction $\theta$ of newly issued loans can be sold in the market.

Workers. An individual worker $j = n$ supplies labour $h_{n,t} > 0$, but does not have new lending opportunities, such that

$$i_{n,t} = 0, \quad e_{n,t}^n \geq 0, \quad e_{n,t}^i = 0$$
Purchases of long-term bonds amount to \( m_{n,t} = -\phi_{n,t} e_{n,t}^n \), such that the evolutions of bond- and loan-holdings become

\[
\begin{align*}
  b_{n,t+1} &= (1 - \Delta) \lambda b_{n,t} + \phi_{n,t} e_{n,t}^n \\
  s_{n,t+1} &\geq (1 - \delta) s_{n,t}
\end{align*}
\]

Using these, the flow-of-funds constraint simplifies to the balance sheet constraint

\[
c_{n,t} + q_t s_{n,t+1} + \left( q_t^b + \frac{\kappa^n}{\phi_{n,t}} \right) \frac{b_{n,t+1}}{P_t} + \frac{d_{n,t+1}}{P_t} = n_{n,t}
\]

where net worth is defined as

\[
n_{n,t} = (r_t + (1 - \delta) q_t) s_{n,t} + \left[ 1 + (1 - \Delta) \lambda \left( q_t^b + \frac{\kappa^n}{\phi_{n,t}} \right) \right] \frac{b_{n,t}}{P_t} + R_t \frac{d_{n,t}}{P_t} w_t h_{n,t}
\]

**Banks.** An individual bank \( j = i \) has new lending opportunities, but does not supply labour \( h_{i,t} = 0 \), such that \( e_{i,t}^n = 0 \), \( e_{i,t}^i \geq 0 \)

Sales of long-term bonds amount to \( m_{i,t} = \phi_{i,t} e_{i,t}^i (1 - \Delta) \lambda b_{i,t} \), such that the flow-of-funds constraint can be simplified to

\[
c_{i,t} + (1 - \theta q_t) i_{j,t} + \frac{d_{j,t+1}}{P_t} = r_t s_{j,t} + b_{j,t} + \left( q_t^b - \frac{\kappa^i}{\phi_{i,t}} \right) m_{j,t} + R_t \frac{d_{j,t}}{P_t} + \left[ 1 + (1 - \Delta) \lambda \left( q_t^b - \frac{\kappa^i}{\phi_{i,t}} \right) \phi_{i,t} e_{i,t}^i \right] \frac{b_{j,t}}{P_t} + R_t \frac{d_{j,t}}{P_t}
\]

The evolutions of bond- and loan-holdings can be expressed as

\[
\begin{align*}
  b_{i,t+1} &= (1 - \phi_{i,t} e_{i,t}^i) (1 - \Delta) \lambda b_{i,t} \\
  s_{i,t+1} &= (1 - \delta) s_{i,t} + (1 - \theta) i_{i,t}
\end{align*}
\]

By replacing investment and matches, the flow-of-funds constraint simplifies to the balance sheet constraint

\[
c_{i,t} + \frac{1 - \theta q_t}{1 - \theta} s_{i,t+1} + \left( q_t^b - \frac{\kappa^i}{\phi_{i,t}} \right) \frac{b_{i,t+1}}{P_t} + \frac{d_{i,t+1}}{P_t} = n_{i,t}
\]

where net worth is defined as

\[
n_{i,t} = \left( r_t + (1 - \delta) \frac{1 - \theta q_t}{1 - \theta} \right) s_{i,t} + \left[ 1 + (1 - \Delta) \lambda \left( q_t^b - \frac{\kappa^i}{\phi_{i,t}} \right) \right] \frac{b_{i,t}}{P_t} + R_t \frac{d_{i,t}}{P_t}
\]

(4.A.6)
4. Government Bond Liquidity and Sovereign-Bank Interlinkages

4.A.2 Dynamic Equilibrium Conditions

For computational convenience, redefine all nominal variables in real terms:

\[
\frac{B'}{P} = B', \quad \frac{D'}{P} = D'
\]

Given the aggregate state variables \((S, B, D, R, z_a, z_\xi)\), we are then solving for

\[
(C_i, C_n, H, S', K', I, B', B'_i, D', T, \phi_i, \phi_n, q, q_r, \rho, q^p_i, Q_i, Q_n, r, w, \pi)
\]

together with exogenous stochastic processes for \((z'_a, z'_\xi, z'_\delta)\) and policy rules for \((T, D', R')\) from the following set of dynamic equations

1. Individual optimality

   (a) Banks

   \[
   u'(\frac{C_n}{1-\chi})w = \mu \tag{4.A.7}
   \]

   \[
   u'(\frac{C_n}{1-\chi})q = \beta E \left[u'(\frac{C_n'}{1-\chi})\zeta_1'\right] \tag{4.A.8}
   \]

   \[
   u'(\frac{C_n}{1-\chi})Q_n = \beta E \left[u'(\frac{C_n'}{1-\chi})\zeta_2'\right] \tag{4.A.9}
   \]

   \[
   u'(\frac{C_i}{1-\chi}) = \rho u'(\frac{C_n}{1-\chi}) \tag{4.A.10}
   \]

\[
B'_i = (1-\phi_i)(1-\Delta)\lambda\eta\frac{B}{\pi} \tag{4.A.12}
\]

\[
(1-\theta q)I + C_i = \eta \left[rS + (1+\phi_i(1-\Delta))\lambda Q_i\frac{B}{\pi} + R\frac{D}{\pi} - T_i\right] \tag{4.A.13}
\]

\[
q_r \equiv \frac{1-\theta q}{1-\theta} \tag{4.A.14}
\]

\[
\rho \equiv \frac{q}{q_r} \tag{4.A.15}
\]

where

\[
\zeta_1' = \left[\eta p' + (1-\eta)\right]r' + (1-\delta)q'
\]

\[
\zeta_2' = \left[\eta p' + (1-\eta)\right] + (1-\Delta')\lambda\left[\eta p'\phi_i'Q_i' + (1-\eta)Q_n'ight]
\]

\[
\zeta_3' = \left[\eta p' + (1-\eta)\right]R'
\]
(b) Final goods producers

\[ r = \alpha e^{sa} \left( \frac{K}{H} \right)^{\alpha - 1} \]  
\[ w = (1 - \alpha) e^{sa} \left( \frac{K}{H} \right)^{\alpha} \]  
(4.A.16)

(4.A.17)

2. Government policy rules

(a) Fiscal policy

\[ T = \psi_{T1} + \psi_{T2} \left( \frac{\Psi}{\Psi} - 1 \right) \]  
(4.A.18)

where \( \Psi = \frac{(1-\Delta)\lambda B}{1} \).

(b) Monetary policy

\[ D' = D\pi + \psi_{D} \left( 1 - \frac{\phi_{i}}{\phi_{i}} \right) \]  
(4.A.19)

\[ R' = \max \left\{ \psi_{R} \left( \frac{\pi}{\pi} \right) \psi_{s}, 1 \right\} \]  
(4.A.20)

3. Market clearing conditions

(a) Goods

\[ C_{n} + C_{i} + qI + Q_{n}B' + D' + T = \]
\[ = rS + [1 + (1 - \Delta) \lambda ((1 - \eta) Q_{n} + \eta \phi_{i} Q_{i})] \frac{B}{\pi} + R \frac{D}{\pi} + wH \]  
(4.A.21)

(b) Bonds (inflation)

\[ \bar{G} + (1 + (1 - \Delta) \lambda Q_{i}) \frac{B}{\pi} + R \frac{D}{\pi} = T + D' + Q_{i} B' \]  
(4.A.22)

(c) Securities/credit

\[ S' = (1 - \delta) K + I \]  
(4.A.23)

(d) Capital

\[ K' = S' \]  
(4.A.24)
4. Government Bond Liquidity and Sovereign-Bank Interlinkages

(e) Matches

$$\phi_i = (\xi e^{z_i})^\frac{1}{\gamma} \frac{1}{1 + \phi_i - 1} \kappa^n$$ (4.A.25)

$$q^b = \frac{\omega + \phi_i - 1}{(1 - \omega)(1 + \phi_i (\rho - 1))} \phi_n$$ (4.A.26)

$$Q_n \equiv q^b + \frac{\kappa^n}{\phi_n}$$ (4.A.27)

$$Q_i \equiv q^b - \frac{\kappa^i}{\phi_i}$$ (4.A.28)

4. Exogenous processes

$$z'_a = \rho_z z_a + \varepsilon'_a$$ (4.A.29)

$$z'_\Delta = \rho_\Delta z_\Delta - \varepsilon'_\Delta$$ (4.A.30)

4.B Proofs

4.B.1 Proof of Proposition 5

Proof of Proposition 5.1. Workers’ value satisfies

$$V^n (m_n) = \int_{j \in i} u (c_j) \; dj + \int_{j \in n} u (c_j) \; dj + \beta \mathbb{E} \left[ V (S', B', D'; \Gamma) \right]$$

s.t. $c_n + q s'_n + \frac{d_n'}{P} + \kappa^n e_n \frac{1}{P} = n_n$

$$n_n = (r + (1 - \delta) q) s_n + \frac{b_n}{P} + R \frac{d_n}{P} + q s_m + wh_n$$

$$b'_n = (1 - \Delta) \lambda b_n - m_n$$

$$S' = \int s'_j \; dj, \quad B' = \int b'_n \; dj$$

The value function consists of current utility as well as the discounted future value of the household, since workers join the representative household at the end of a period and type-shocks are idiosyncratic over members and time. Note that with logarithmic utility, agents optimally consume a fraction $\beta$ of their net worth, i.e. the consumption-savings choice is given by

$$c_n = (1 - \beta) n_n, \quad q s'_n + \frac{d'_n}{P} + \kappa^n e_n \frac{1}{P} = \beta n_n$$

Then, the marginal or excess value of an additional match can be expressed as

$$-V^n_m = -u' (c_n) \frac{\partial c_n}{\partial n_m} \frac{\partial n_m}{\partial m_n} - \beta \left[ \mathbb{E}_{\Gamma} [V_B] \frac{\partial B'}{\partial b_n'} \frac{\partial b_n'}{\partial m_n} + \mathbb{E}_{\Gamma} [V_S] \frac{\partial S'}{\partial s'_n} \frac{\partial s'_n}{\partial m_n} \frac{\partial m_n}{\partial m_n} \right]$$

$$= -u' (c_n) (1 - \beta) \frac{q^b}{P} + \beta \left[ \mathbb{E}_{\Gamma} [V_B] - \mathbb{E}_{\Gamma} [V_S] \beta \frac{q^b}{P} \right]$$
Note that using the definition of matches from the perspective of banks, $m_i = e_i^i \phi_i (1 - \delta) b_i$, the evolution of bonds can be rewritten in terms of matches rather than the beginning-of-period stock of bonds as $b_j' = (1 - \delta) b_j - m_j = \left((e_i^j \phi_i)^{-1} - 1\right) m_i$. Then, lending banks’ value satisfies

$$V^i (m_i) = \int_{t \in I} u (c_j) \, dj + \int_{j \in \mathbb{N}} u (c_j) \, dj + \beta \mathbb{E} \left[ V (S', B', D', \Gamma) \right]$$

s.t. $c_i + q_r s_i' + \frac{d_i^j}{P} + \kappa_i e_i^i (1 - \delta) \frac{b_i}{P} = n_i$

$$n_i = (r + (1 - \delta) q_r) s_i + \frac{b_i}{P} + R \frac{d_i^j}{P} + q^b m_i$$

$$b_i' = \left((e_i^j \phi_i)^{-1} - 1\right) m_i$$

$$S' = \int s_j' \, dj, \quad B' = \int b_n' \, dj$$

The consumption-savings choice is, again, given by

$$c_i = (1 - \beta) n_i, \quad q s_i' + \frac{d_i^j}{P} + \kappa_i e_i^i \frac{1}{P} = \beta n_i$$

The marginal or excess value of an additional match from the perspective of a selling bank is

$$V^i_m = u' (c_i) \frac{\partial c_i}{\partial m_i} \frac{\partial m_i}{\partial m_i} + \beta \mathbb{E} \left[ V (S') \frac{\partial S'}{\partial s_i'} \frac{\partial s_i'}{\partial m_i} + \mathbb{E} \left[ V (B') \frac{\partial B'}{\partial b_i'} \frac{\partial b_i'}{\partial m_i} \right] \right]$$

$$= u' (c_i) (1 - \beta) \frac{q^b}{P} + \beta \mathbb{E} \left[ V (B) \left( (e_i^j \phi_i)^{-1} - 1 \right) + \mathbb{E} \left[ V (S) \beta \frac{q^b}{q_r} \frac{1}{P} \right] \right]$$

Note that $E_i = 1$ implies $e_i^1 = 1$ due to homogeneity. Using the first-order-conditions for loans, $u'(c_n) q = \beta \mathbb{E} \mathbb{E} [V_S]$ and $u'(c_i) = \rho u'(c_n)$, and bonds, $u'(c_n) \frac{q^b}{P} = \beta \mathbb{E} \mathbb{E} [V_B]$, we can then simplify the valuations of workers and banks and plug them into the bargaining solution $(1 - \omega) V^i_m = \omega (-V^m_m)$ to obtain

$$(1 - \omega) \left[ \rho q^b + \frac{\phi_i}{\phi_i} Q_n \right] = \omega \left[ Q_n - (1 - \beta) q^b \right]$$

By applying the definitions of the effective bond-price from buyers’ perspective $Q_n = q^b + \frac{n_n}{\phi_i}$, this expression can be rewritten as (4.3.4).

**Proof of Proposition 5.2.** By substituting $\phi_n = \frac{\gamma}{\frac{\gamma}{\phi_i} + 1}$ in the analytical expression for the bargained bond price in 4.3.4 and differentiating with respect to $\phi_i$, we get

$$\frac{\partial q^b}{\partial \phi_i} (1 - \omega) \xi^{\frac{1}{\gamma - 1}} \left\{ [1 + \phi_i (\rho - 1)] \phi_i^{\gamma - 1} \right\}$$

$$= \kappa_n - q^b (1 - \omega) \xi^{\frac{1}{\gamma - 1}} \phi_i^{\gamma - 1} \left[ \frac{1 - 2 \gamma}{1 - \gamma} \phi_i (\rho - 1) - \frac{\gamma}{1 - \gamma} \phi_i^{\gamma - 1} \right]$$

(4.B.1)
A necessary and sufficient condition for \( \frac{\partial q}{\partial \phi_i} > 0 \) is for the RHS of (4.B.1) to be non-negative. This is the case, whenever
\[
\phi_i \left( \frac{1}{3} \left( \frac{1}{\gamma (\rho - 1)} + (1 - \omega) \left( \gamma^{-1} - 2 \right) \right) + \frac{1}{3} \frac{1 - \omega}{\rho - 1} \right) < 0.
\]

In our calibration, we set \( \gamma = 0.5 \) without loss of generality, since \( \gamma \) is not independent of \( \xi \) in the model. Then, above sufficient condition reduces to
\[
\phi_i \left( \frac{1}{3} \left( \frac{1}{\rho - 1} + (1 - \omega) \left( 2 \right) \right) + \frac{1}{3} \frac{1 - \omega}{\rho - 1} \right) < 0.
\]

It follows that the necessary condition for the bargaining price to correlate negatively with saleability is \( \phi_i < \frac{2}{3} \frac{1}{\rho - 1} \). The simplified sufficient condition can be further solved to obtain a lower and an upper bound on \( \phi_i \), between which the sufficient condition will be satisfied. In particular, we have that \( \frac{\partial q}{\partial \phi_i} > 0 \) whenever
\[
- \left( \frac{1}{9} - (\rho - 1)(1 - \omega) \right) \frac{1}{3} < \phi_i - \frac{1}{3} < \left( \frac{1}{9} - (\rho - 1)(1 - \omega) \right) \frac{1}{3}.
\]

Suppose that \( \omega \approx 1 \), then the bounds approximately collapse to \( 0 \leq \phi_i \leq \frac{2}{3} \frac{1}{\rho - 1} \). Suppose further that \( \rho \in (1,1.5] \), such that the minimum value for the upper bound is \( \frac{4}{3} \). Then, the upper bound will never be binding, since by construction \( \phi_i \in [0,1] \).

Note that \( \frac{\partial q}{\partial \phi_i} > 0 \) implies \( \frac{\partial q}{\partial \phi_n} < 0 \), because \( \frac{\partial q}{\partial \phi_n} = \frac{\partial q}{\partial \phi_i} \frac{\partial \phi_i}{\partial \phi_n} \) and
\[
\frac{\partial \phi_i}{\partial \phi_n} = \frac{\gamma - 1}{\gamma} \frac{1}{\xi \phi_n} < 0.
\]

Hence, the same parameter restriction that ensures \( \frac{\partial q}{\partial \phi_i} > 0 \) also ensures \( \frac{\partial q}{\partial \phi_n} < 0 \). \( \Box \)

4.C Data

Investment- and Government consumption-output ratios

Series: fixed capital formation, final consumption expenditure (general government), gross domestic product (all at market prices). Source: Eurostat, National Accounts (ESA2010). All series in EUR, quarterly frequency, neither seasonally nor working day adjusted. Ratios are computed as four-quarter moving averages.

Capital-output ratio

Series: Net capital stock per unit of gross domestic product at constant market prices. Source: European Commission, AMECO. Annual frequency.
Reserve accumulation-consumption ratio
Series: a) Final consumption expenditure (total less general government); b) capital and reserves (MFIs), net, flows. Sources: a) Eurostat, National Accounts (ESA2010); b) ECB, Balance Sheet Items Statistics. All series in EUR, annual frequency.

Bank equity-household net worth ratio
Series: a) Financial net worth (households); b) Capital and reserves (MFIs excluding ECB reporting sector), net. Sources: a) Eurostat, Annual Sector Accounts (ESA2010); b) ECB, Balance Sheet Items Statistics. All series in EUR, annual frequency, outstanding amounts at the end of the period (stocks).

Banks’ liquidity ratio
Series: a) Bank deposits to output: the total value of demand, time and saving deposits at domestic deposit money banks as a share of GDP. Deposit money banks comprise commercial banks and other financial institutions that accept transferable deposits, such as demand deposits. b) Deposit money banks’ assets to output: total assets held by deposit money banks as a share of GDP. Assets include claims on domestic real nonfinancial sector which includes central, state and local governments, nonfinancial public enterprises and private sector. Deposit money banks comprise commercial banks and other financial institutions that accept transferable deposits, such as demand deposits. c) Liquid assets to deposits and short term funding: The ratio of the value of liquid assets (easily converted to cash) to short-term funding plus total deposits. Liquid assets include cash and due from banks, trading securities and at fair value through income, loans and advances to banks, reverse repos and cash collaterals. Deposits and short term funding includes total customer deposits (current, savings and term) and short term borrowing (money market instruments, CDs and other deposits). Source: World Bank, Global Financial Development database. All series in percent, annual frequency. Banks’ liquidity ratio is computed as liquid assets to deposits and short term funding divided by bank deposits to output divided by deposit money banks’ assets to output.

Bid-ask spread
Series: 10-year Government Benchmark bond, bid and ask prices, end-of-day. Source: Reuters and ECB. Percentage p.a., daily frequency. The bid-ask spread is computed as the difference between the bid and ask price on a given day normalised by the mid-point said bid and ask price in basis points.
Bibliography


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