Three essays on behavioral market design

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Zusammenfassung

Diese Dissertation umfasst drei eigenständige Essays über das Design von Märkten unter Berücksichtigung verhaltensökonomischer Aspekte. Der erste Essay analysiert die Ergebnisse eines Feldexperimentes zu dem Problem der Schulwahl unter der Anwendung des Top Trading Cycles Mechanismus. Insbesondere wird untersucht, wie sich zum einen die Erläuterung des verwendeten Zuteilungsmechanismus und zum anderen die Erklärung seiner Eigenschaften auf die Bereitschaft der Probanden, ihre wahren Präferenzen zu offenbaren, auswirkt. Der zweite Essay befasst sich mit der Zulassung zu Universitäten anhand von Aufnahmeprüfungen. Hierbei wird ein zentrales mit einem dezentralen Design sowohl theoretisch als auch experimentell verglichen. Der dritte Essay baut auf der im zweiten Essay entwickelten Theorie auf und vergleicht die Standardwettbewerbsform mit einem parallelen Wettbewerb anhand eines Feldexperiment mit ArbeitnehmerInnen einer Mikrokreditanstalt in Russland.

Schlüsselwörter: Schulwahl , Matching, Wettbewereb, Experimente, Feldexperiment

Abstract

This thesis consists of three self-contained essays on behavioral market design. The first essay presents the results of a field experiment which tests the effects of advice and mechanism disclosure on the truth-telling rates of participants in a school choice problem using the Top Trading Cycles mechanism. The second essay analyzes college admissions with entrance exams, comparing both theoretically and experimentally two alternative designs: centralized and decentralized. The third essay is built on the theory of the second one and compares standard and parallel contests in a field experiment with workers of a micro-credit company in Russia.

Keywords: school choice, matching, contest, experiments, field experiment

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Preface

This dissertation presents three essays on behavioral market design, which is a field that intersects two fields: market design and behavioral economics. Market design is a well-established and yet growing field in economics. Lately, it has spread to different subfields like matching theory. The market design perspectives have also been discovered for existing theories, like the theory of contests. From another perspective, behavioral economics became a broad field of economics, emphasizing the importance of behavior and systematic deviations from usual assumptions of rationality. While behavioral market design has already well-established itself in some subfields of market design, like auction theory, it has only recently become vivid in matching theory. Counting the fact that the market design field is very applied, experiments are essential for testing the predictions of the theory, and serve as a main bridge between theory and it successfully being into practice. However, the external validity of lab experiments is an important question to address, and the degree in which experimental findings can be generalized to practice varies from one question to another. There are some questions that are extremely sensitive to experimental demand effect like, for instance, the advice of an experimenter for the subjects, and some to the experimental design choices like, for instance, chosen versus real effort. To address these concerns, in this dissertation I present both lab and field experiments. I concentrate on behavioral implications of a matching mechanism and two contest designs.

One of the most important problems in matching theory is the school choice problem. Traditionally in most countries across the world students could only attend the district-school. In many countries, the schools are financed through the local taxes which has led to big differences in the quality of the schools. In the absence of school choice, children from families with a lower social-economic background were likely to be enrolled in bad quality schools, which led to higher ethical and economic segregation and lowered the prospect of the disadvantaged children. To address this problem school choice programs were introduced in many places around the world. In school choice programs parents rank schools as the best option, second best option, and so on, and the centralized authority uses a mechanism to match students to schools taking the preferences of the parents into account. However, depending on the mechanism at use, some parents might have to misrepresent their preferences in order to reach a satisfactory match, and yet again the families with a lower socio-economic background were disadvantaged. Thus, one of the most desirable properties of any allocation mechanism is strategy-proofness, i.e., a mechanism should not require strategic play by parents. Matching theory has developed strategy-proof mechanisms, but the theoretical properties do not guarantee success in practical implementation, mostly due to the complexity of the mechanisms. The bridge between the desirable theoretical properties and successful implementation can be reached through testing the mechanisms in the lab and in the field, to understand people's behavior, which is often not predicted by theory. In fact, experiments by Chen and Sönmez (2006) have been used as a tool to convinced policy makers to introduce changes to the current mechanism in use.

In the first chapter, which is a joint work with Pablo Guillen, we consider the school choice problem, when students have preferences over schools and schools have priorities over students. We use reformulation of the original problem in application to the topic allocation: students have preferences over topics and topics have priorities over students. We consider the Top Trading Cycles mechanism (TTC), which is Pareto efficient and strategy-proof, and put into practice in some school-districts in the US. Motivated by accumulating evidence of the misrepresentation of preferences both in the lab and in the practical implementation of the mechanism, i.e., not following the weakly-dominant strategy of truthful reporting, we test the effects of advice and disclosure of the details of the mechanism in a field experiment with students at the University of Sydney. First, we approached first-year economics students enrolled in an introductory microeconomics unit about which topic, among three, they would most like to write an essay on. Most students chose the same favorite topic. Then we used TTC to distribute students equally across the three options. We ran three treatments, varying the information the students received about the mechanism. In the first treatment students were given a description of the matching mechanism. In the second they received a description of the incentive properties of the mechanism (strategy-proofness) without details of the mechanism. Finally, in the third they were given both pieces of information. We find a significant and positive effect of describing the strategy-proofness on truth-telling rates. On the other hand, describing the matching mechanism has a significant and negative effect on truth-telling rates.

A related problem to the school choice problem is university admissions. If students are accepted to universities based on their grades then the admission mechanism can be modeled as a contest. The particular design of the admission process, like the level of centralization of admission procedure, has a drastic effect on the composition of the universities, the utility of students, and overall welfare. Once again, in contests a lot of behavioral deviations from predicted behavior have been observed in lab studies, and thus any theoretical conclusions should be tested in the lab or the field before being put into practice. Moreover, due to the complexity of equilibria predictions, sometimes alternative designs cannot be generally theoretically compared.

In the second chapter, which is a joint work with Isa Hafalir, Dorothea Kübler and Morimitsu Kurino, motivated by the observed variation between countries, we consider two alternatives of college admission designs: centralized, when students take one test and send ranked applications to many universities (for instance, like in Turkey); and decentralized, when students have to choose only one university and have to wait a year for the next admission round if they are not accepted (for instance, like in Japan). We study a college admissions problem in which colleges accept students by ranking students' efforts in entrance exams. Students' ability levels affect the cost of their efforts. We solve and compare the equilibria of "centralized college admissions" (CCA), where students apply to all colleges, and "decentralized college admissions" (DCA), where students only apply to one college. We show that lower ability students prefer DCA whereas higher ability students prefer CCA. Many predictions of the theory are supported by a lab experiment designed to test the theory, yet we find a number of differences that render DCA less attractive than CCA compared to the equilibrium benchmark: subjects overexert effort in both systems, but more so in the decentralized system. The latter leads to the fact that the benefits that the decentralized system can offer for students in some markets do not translate in the lab behavior, where the centralized system is better for subjects in all markets on average.

The third and last chapter of the dissertation extends the second chapter by testing the model of centralized and decentralized college admission in a field experiment. Motivated by the observation from the lab, that a decentralized system, or parallel contest, leads to significantly higher effort than the centralized system, or standard contest, I conduct a field experiment with 302 workers of a microcredit company in Russia to study the effects of the different designs of a contest for monetary prizes at the workplace. I consider a standard all-pay auction design with two and four prizes of different sizes and compare it to parallel contests with the same prizes, but where participants have to choose the prize prior to the start of the competition and then the winner is selected only among the players who chose the same prize. In all treatments the average number of new clients was higher than in the control group. Despite the theoretical predictions, the parallel contests lead to higher efforts for all players, but mainly by lower-ability players. Within the contests the treatment comparison leads to the predicted effects: the division of the prize in the standard contest led to a decrease in effort by high-ability workers in both contests, while it led to an increase in the efforts of low-ability workers in the standard contest only. As for the between-contest comparisons, the parallel contest led to higher efforts than standard contests for all abilities, irrespective of the theoretical predictions. Moreover in the parallel contests high ability workers chose the prize in line with equilibrium, while lower ability workers chose the higher prize too often. Thus, the motivated findings of the second chapter of this dissertation were qualitatively replicated in the field.

Chapter 1

How to Get Truthful Reporting in Matching Markets: a Field Experiment

with Pablo Guillen

1.1 Introduction

The use of matching mechanisms for school choice programs is one of the most relevant and successful real-world applications of game theory, see, for instance, Abdulkadiroğlu et al. (2005). In the programs parents are asked to rank the available public schools in the area. On the other side of the market students are ordered by a priority score. Submitted ranks and priorities are fed into a mathematical algorithm to produce a match of students to school seats. The use of strategy-proof matching mechanisms, in which participants have the right incentives to reveal their true ranking, is considered desirable. Indeed, if parents devote their energy to devise manipulation strategies they will have less time to discover the true quality of the schools available. Matching mechanisms are, however, complex and most likely difficult to understand for lay people. Whether theoretically strategy-proof mechanisms induce high truth-telling rates among participants is a critical empirical question for which answers are not readily available in the field. The true rankings of participants in matching markets are hard to elicit. That's why researchers turned to run matching experiments with induced valuations.

Chen and Sönmez (2006) pitched two theoretically strategy-proof mechanism, Top Trading Cycles (TTC) and Deferred Acceptance (DA) against the then most popular mechanism in use, the non-strategy-proof Boston (BOS). In Chen and Sönmez (2006) both TTC and DA do induce higher truth-telling rates (and efficiency) than BOS. More recent experiments also find remarkably high truth-telling rates for strategy-proof mechanisms, between 62% and 96% for TTC in particular.[1] All in all, experimental findings have been instrumental to convince school districts to adopt strategy-proof mechanisms, either DA or TTC.

However, the laboratory and real-life implementations of matching mechanisms often differ, critically, in terms of the information available to participants. That is, experimental subjects are generally given a very accurate, if not cumbersome, description of the mechanism together with a solved example. On the other hand, and although the details vary from one school choice program to another, it is generally quite difficult for participants in real life markets to obtain a description of the algorithm mechanics. Conversely, experimental subjects are typically not directly informed of the properties of the mechanism (strategy-proofness, stability, etc.), while participants in real-life markets are often told about strategy-proofness in one way or another. For instance, both the Boston Public Schools (BPS) system (Boston Public Schools, 2014) and the New Orleans Recovery District (Vanacore, 2012) websites do not contain algorithm descriptions (the last time we checked), but they both do inform participants that the best they can do is report their true preferences.

This paper focuses on the informational differences between matching markets implemented in the laboratory and the field. For that purpose we designed a controlled field experiment. To capture these differences, we run a TTC-based, in-class topic allocation task to compare three treatments that differ in the information given to participants: an only "mechanism description" (MD), only "properties description" (PD), and both "mechanisms and properties descriptions" (MPD). The main focus of the experiment is to assess which informational structure generates the highest truth-telling rate.

Our experiment took place at the University of Sydney, with first-year students of an introductory microeconomics course as participants.[2] The students had to write an essay on the structure of one of three markets: smartphones, TV sets or scanners. We simply elicited student's actual first preference by asking them to nominate their favorite topic (smartphone, TV set, scanner). The vast majority of students chose the smartphone. Then students were told in class that the topics had to be evenly allocated: one-third of the students to each topic. They were also told that to achieve this goal a matching mechanism would be used. Each one of the three sections of the course received the instructions for one of the three treatments. We find that describing the properties of TTC[3] leads to a significantly higher rate of truthful reporting. However, a description of the mechanism itself leads to a lower rate of truthful reporting.

[1] Calsamiglia et al. (2010): DA 57%-58%, TTC 62%-74% ; Pais and Pinter (2008): DA 67-82%, TTC 87%-96%; Pais et al. (2011): DA 58%-76%, TTC 62%-84%.

[2] It needs to be clear that this is a field experiment regardless of whether participants are university students. Indeed: 1) the experiment is conducted in its "natural" environment for the decision of interest, by real strategic agents; 2) we do not impose the preferences of the participants; 3) participants are not volunteers but students going over a classroom procedure.

[3] We use "properties description" and "advice" as interchangeable terms.

1.2 Literature review and motivation

After Abdulkadiroğlu and Sönmez (2003) proposed the use of the TTC mechanism to solve the school choice problem several experimental papers have examined the truth-telling rates in TTC, mostly by pitching it against competing mechanisms. In a path-finding experimental paper,Chen and Sönmez (2006) compared TTC (Pareto efficient and strategy-proof) against DA (non-Pareto efficient but strategy-proof) and the Boston mechanism (BOS) (non-strategy-proof mechanism) to show how both TTC and DA outperform BOS terms of truth-telling and efficiency. In light of their result, Chen and Sönmez (2006) advocated for the use of strategy-proof mechanisms and recommended educating the parents in order to increase truth-telling rates. Since then, many other experimental matching papers have reported fairly high truth-telling rates for TTC (62-95%). Given the desirable properties of TTC, the efficiency obtained and the relatively high truth-telling rates, there is a generally positive perception about the adequacy of the mechanism.

Nevertheless, an attentive reading of the experimental matching literature raises some doubts about the capacity of theoretical strategy-proof mechanism to generate high truth-telling rates. For instance, Pais and Pinter (2008) found that TTC outperforms DA and BOS with respect to the criterion of truthful preference revelation in all the informational settings tested. However, they also demonstrated that additional information leads to higher rates of preference misrepresentation in all three mechanisms. Klijn et al. (2013) compared BOS with DA, devoting special attention to individual behavior. In particular, they include a simple lottery to elicit risk aversion. Klijn et al. (2013) shows a positive correlation between risk aversion and the probability to play protective (out-of-equilibrium in any case) strategies under DA, thus showing that more risk-averse subjects are less likely to reveal their true preferences. Guillen and Hakimov (2014) found how truth-telling decreases dramatically when experimental subjects are informed about the strategies played by others. That is, there is a growing stream of the experimental matching literature eroding the idea of truth-telling driven by subjects understanding strategy-proofness from reading the instructions and working on the examples provided in the laboratory.

Interestingly, the school districts rarely provide any explanation of the matching mechanism equivalent to the experimental instructions used in the laboratory experiments and therefore they consciously refrain from trying to "educate" parents. For instance, the New Orleans Recovery District (NORD) adopted TTC for student assignment a few years ago (Hakimov and Kesten, 2014). In New Orleans, a sketch of TTC's mechanics was only made available to the public once through a poster published by the local newspaper. NORD does inform participants that the best they can do is report their true preferences (Vanacore, 2012). Following Chen and Sönmez (2006) recommendation to educate the initial DA implementation of Boston's BPS match offered a detailed explanation of DA together with seminars for interested parents. BPS also explained that truth-telling is the best strategy for parents in informational packages. Nowadays, the BPS website only includes a fade mention to strategy-proofness, but no procedural explanations. In summary, given the available information, it is unlikely that parents fully take the time and energy to seek out and understand the details involved in the procedure that assigns their children to schools.

The main interest of the current paper is, for practical purposes, to test the effect of strategy-proofness advice on truth-telling behavior. Note that explaining the properties of the mechanisms in the lab might help to overcome the gap with the field, but it could easily lead to methodological problems like demand effects and/or confusion. Nevertheless, a growing stream of the literature adds advice to matching experiments. For instance, Guillen and Hing (2014) provide experimental evidence that wrong third-party advice can easily mislead participants and result on very low truth-telling rates in the lab. Ding and Schotter (2013) find that chatting in between two DA matching markets does not increase truth-telling rates. Ding and Schotter (2015) find that after 20 rounds of intergenerational advice truth-telling decreases dramatically from above 70% to about just 45% .[4] Those three papers provide further indication of truth-telling not being driven by the transparency of the experimental instructions.[5] However, can correct advice induce participants to make the right choices? Braun et al. (2014) report some success in this direction: it includes correct advice in the experimental instructions which helps subjects to behave optimally. In contrast to Ding and Schotter (2015), Zhu (2015) shows that intergenerational advice might increase truth-telling rates in the simplest market of three agents and three objects in the lab, but only in cases when the preferences of subjects are uncorrelated. Is it possible to obtain positive results in the field? Some limited evidence indeed suggests that simple advice or simplified information might lead to improvement. For instance, Hastings and Weinstein (2008) show in their field experiments that the provision of simple information about school test scores to lower-income families increases the chance of higher-performing schools being listed in the choice lists for school choice program in the Charlotte-Mecklenburg Public School District, North Carolina. In similar vein, Bhargava and Manoli (2015) show that simplifying information and increasing the saliency of the benefit from information increases the take-up rate of earned income tax credits. Although the studies above do not provide direct advice, they try to help simplify the processing of information. Our study goes beyond that by providing advice about strategic behavior.

[4] Note that these truth-telling rates are observed for the only proposing side of DA with the possibility of submitting a full list. Truth-telling is a dominant strategy in this unilateral version of DA.

[5] An alternative explanation for the high truth telling rates in early matching experiments could be a demand effect stemming from induced preferences.

1.3 Experimental and procedures

We design a natural experiment to compare the behavior of students in a matching market under different information conditions. That is, we vary the explanation of the allocation procedure and the presence of advice across treatments.

1.3.1 Preliminaries

Students of an undergraduate introduction to economics class had to write a market structure essay in which they had to answer a series of questions to argue whether the market for a particular product approaches perfect competition, a monopoly or an oligopolistic structure. There were three possible products to write about (smartphone, TV set, and scanner) but other than for the product (or topic) the assignments were identical. More than 700 students were enrolled in the course which was taught across three sections. The essay mark was worth 15% of the final mark for the course.

The main challenge for the design of a field matching experiment is the elicitation of the true preferences of participants. We worked around this limitation in the following way: in Week 5 the lecturers announced that the students had to write an essay for which there were three available topics. All students were expected to submit their choice of topic into the online course management system. Thus, students were under impression that this was their final choice. We have little reason to believe that their submitted choices were not truthful. Students simply submitted their favorite topic to the system, most likely choosing their real top choice.[6]

Our method does not elicit the full preference list of students, but knowing the true top choice allows for a sufficiently rich analysis.

We tried to come up with topics for which student preferences are highly correlated. Our selection achieved the desired correlation in preferences (see Figure 1.1). As expected, a sizable majority of students reported the smartphone as their true top choice. Note that this design choice provides a straightforward interpretation of our results in terms of truth-telling, which is the focus of this paper. On the other hand, because the loss of one subject is most likely the gain of another, there would be only modest potential welfare increases achieved by universal truth-telling in our set-up. The lesson to be learned from our paper is, in any case, one about truth-telling and information. This lesson could be applied to markets with high potential for welfare increases from truth-telling.

[6] In previous years there was only one assigned topic and no topic choices. Thus, a possibility of learning from previous cohorts, or inferring any experiment-related knowledge, is excluded.

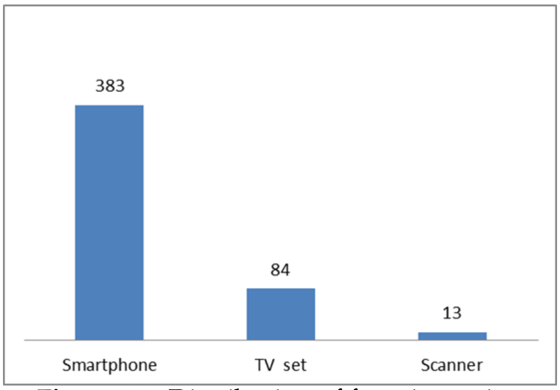


Figure 1.1. Distribution of favorite topics.

1.3.2 Procedures

The allocation of the topics to students was done through a direct reformulation of TTC for the school choice problem by Abdulkadiroğlu and Sönmez (2003).

Given preferences of students and priorities of schools, TTC works as follows:

"Step 1: Assign a counter for each school which keeps track of how many seats are still available at the school. Initially set the counters equal to the capacities of the schools. Each student points to her favorite school under her announced preferences. Each school points to the student who has the highest priority for the school. Since the number of students and schools are finite, there is at least one cycle. (A cycle is an ordered list of distinct schools and distinct students (s_1 ,i_1, s_2,..., s_k, i_k) where s_1 points to i_1, i_1 points to s_2....s_k points to i_k, i_k points to s_1.) Moreover, each school can be part of at most one cycle. Similarly, each student can be part of at most one cycle. Every student in a cycle is assigned a seat at the school she points to and is subsequently removed. The counter of each school in a cycle is reduced by one and if it is reduced to zero, the school is also removed. The counters of all the other schools stay put.

In general, at Step k: Each remaining student points to her favorite school among the remaining schools and each remaining school points to the student with the highest priority among the remaining students. There is at least one cycle. Every student in a cycle is assigned a seat at the school that she points to and is subsequently removed. The counter of each school in a cycle is reduced by one and if it is reduced to zero the school is also removed. The counters of all the other schools remain in place. The algorithm terminates when all students are assigned a seat. Note that there can be no more steps than the cardinality of the set of students."

In the case of the topic allocation task the modifications are straightforward. Each student has to be assigned to one of the three topics. Additionally, there are a maximum number of students who can be assigned to each of the topics, corresponding to the number of slots in schools in the original formulation. Each topic has priorities over all students.

The priorities of students for topics were generated as an analogue of the district school priority. Every student received a priority for one of the topics. The priority topic was written at the top of the instruction page and was called "Tentative topic." The allocation of tentative topics was random. The ties inside the same priority group as well as ties for non-priority students were broken randomly in the process of the topic allocation and the students were informed about it.

Let us consider a small example of topic allocation problem and its solution:

Imagine the preferences of six students are the following:

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
1st choice	Scanner	TV set	Smartph.	Smartph.	Smartph.	TV set
2nd choice	TV set	Scanner	TV set	TV set	TV set	Smartph.
3rd choice	Smartph.	Smartph.	Scanner	Scanner	Scanner	Scanner

The tentative topics are the following:

Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
Smartphone	Smartphone	TV set	TV set	Scanner	Scanner

Then the TTC works as follows:

First the ties in priority classes are broken randomly. Imagine the draw when Student 1 has higher priority than Student 2, Student 3 has higher priority than Student 4, and Student 5 has higher priority than Student 6.

1. First round:

• Student 1 points to Student 5, Student 5 points to Student 1.

• Student 3 points to Student 1.

There is one cycle: the beneficial trades are implemented between Student 1 and Student 5

2. Second round:

• Student 3 points to Student 2, Student 2 points to Student 3.

• Student 6 points to Student 3.

There is one cycle: the beneficial trades are implemented between Student 2 and Student 3

3. Third round: No more quotas are left for the topic Smartphone.

• Student 6 points to Student 4.

• Student 4 points to herself.

Thus, Student 4 receives TV set as a topic.

4. Last round: There is only Student 6 in the market and one quota for Scanner. She receives this topic.

Thus, the final assignments are:

Smartphone	TV set	Scanner
Student 3	Student 2	Student 1
Student 5	Student 4	Student 6

The three experimental treatments took place at the beginning of the corresponding Week 6 lecture for each of the three sections, exactly one week after the topics had been announced and just a couple of days after the deadline for reporting the choice through the class administration system. At the beginning of the class the lecturer announced that the distribution of submitted choices was skewed too much in favor of one topic (without mentioning which topic) and that there should be an approximately equal division of the topics among students. For that reason he announced that an allocation procedure would be implemented. Then the students had 10 to 15 minutes to read the instructions for the allocation mechanism and write down their preference order of the three topics.[7] We distributed the instruction and decision sheets. Students were asked to write their student ID at the top of the sheets.

1.3.3 Treatments

In all treatments students received the instruction and decision sheets including their tentative topic.

The mechanism description treatment (MD) In this treatment the instructions included an explanation of the TTC mechanism framed in the language of the topic allocation problem. We used a formulation similar to Chen and Sönmez (2006). The instructions for all treatments can be found in the online Appendix. The MD treatment is therefore very close to the typical laboratory setup.

The properties description treatment (PD) In this treatment, there is no explanation of the TTC mechanism, but the instruction sheet does include a description of the properties of the mechanism as follows:

"Each participant is first randomly assigned a tentative topic. Your tentative topic is .... (This assignment is random). You will be asked to submit Decision Sheet rankings, which are used to determine the final allocation. For these purposes we will use the Top Trading Cycles Mechanism.[8] This mechanism takes into account your preferences and the preferences of others in order to provide as many top choices as possible and it is strategy proof. Thus, no participant has an incentive to misrepresent her preferences, as no matter what other subjects do, she is always better off submitting true ranking lists."

The mechanism and properties description treatment (MPD) This treatment is the aggregation of the two previous treatments. Students received the instructions from MD with a typical TTC explanation and then, just like in PD, received the description of its properties at the end of the instructions.

1.3.4 Sessions

All the three sessions were run on April 18 and 19, 2013. We ran just one session per treatment, corresponding to one of the three sections. The MD treatment was run at the beginning of the 2pm to 4pm class on April 18. PD was run at the

[7] The instructions of all treatments, as well as the ranking list, had to fit one A4 sheet (double-sided for MD and MPD). Each participant had to read only one or two pages and submit her choice on the same paper sheet. For details check the instructions in the Appendix 1.A.

[8] We use the name of the mechanism to sound more scientific for the students, and also to be verifiable. We assume that none of the first-year students are familiar with the mechanism.

beginning of the 4pm to 6pm class on the same day. The MPD treatment was run the next day at the beginning of the 9am to 11am class. The order of the sessions and the relative short time frame allowed us to assume the minimum possibility of information transfer between students from different sections.[9]

Topics were allocated by inputting the submitted rank order lists to our custom-made TTC software and students were notified of their topic assignment on the Monday after the classes, April 22. Those students who did not show up to the class and thus did not submit their rankings were automatically allocated to the under-demanded topic.

A total of 505 students submitted their decision sheets with a rank list. We are able to use only 480 of them as 35 students who submitted a rank list in the classroom had failed to previously submit their favorite topic choice through the online system. As student attendance across sections was not uniform we ended up with 261 observations in MD, 106 in PD, and 113 in the MPD treatment.

1.3.5 Behavioral predictions

Strategy-proofness predicts that all students should report truthfully and should thus state their online favorite choice according to the online survey as the top choice in the rank list submitted in the classroom.

We believe that the complexity of the class submission task varies remarkably across tentative topics. The students whose tentative topic is their elicited favorite topic face a trivial decision which does not require much understanding of the mechanism properties. According to the data submitted online, the smartphone is clearly the most popular topic, thus, getting the smartphone as a tentative topic makes the decision trivial with a high probability.

Students whose tentative topic is the least preferred topic are in a nothing-to-lose situation. It is hard to find a behavioral justification to rank the scanner, the seemingly overall least favorite topic, first in this situation.[10]

The decisions of students who received the TV set as a tentative topic are the most interesting from a behavioral perspective. According to the online survey the TV set was the most likely second choice. These students may well be exposed to the kind of trade-off that often results in the so-called District School Bias (DSB, see Chen and Sönmez, 2006). That is, in the school choice context, ranking the pre-assigned school for which the applicant has a priority higher in the submitted preference list than it is in reality. DSB has been identified as being extremely

[9] The classes of MPD and PD treatments were in the same classroom one after another. There is short break between the end of the first class and the beginning of the second in which students rush to get to their next class. We did not observe any interaction between students of two sections.

[10] That could happen if the student actually likes the scanner best, which is quite unlikely given the survey. Note that students who got the scanner as their tentative choice might still lie about the way they rank the TV set vs the scanner. Our design does not allow for detecting these manipulation attempts. The situation is similar to the design in Guillen and Hakimov (2014) where the local district school was the least preferred school by design and therefore only 2% of subjects did not play the district school bias.

relevant in most subsequent matching experiments. In our context we will call this behavior tentative topic bias (TTB): if a student did not understand or trust the advice on strategy-proofness, she is likely to think that stating the true ranking list can lead to the loss of the priority for the second best topic and thus risk ending up with the least preferred topic.

Therefore we hypothesize that students with the TV set as their tentative topic are more likely to misreport their top choice when submitting their rank list.

We also hypothesize that the description of properties given to students in PD and MPD should increase the number of truthfully stated top choices by students. Note that in a field experiment such as ours advice comes from a reputable source, the lecturer, and therefore it has a better chance of succeeding than in previous laboratory experiments.

1.4 Results

Result 1: Across the three treatments, 13.5% of the experimental subjects misreported their top choice. Misrepresentation reached 18.8% in the MD treatment. When taking into account only non-trivial decisions, 20.3% of the subjects misreport. In that case misrepresentation reached 28.1% in the MD treatment.

Support. Table 1.1 shows the frequency and the corresponding percentage of the misrepresentations of the top choices by treatment. We also include both the results for the whole sample and for the non-trivial decisions in particular The exact Fisher test for the equality of proportions of the students who misrepresent their preferences provides the following p-values for one-sided tests for the full sample: p = 0.00 for MD versus PD; p = 0.01 for MD versus MPD treatment; p = 0.26 for PD versus MPD treatments. If only non-trivial decisions are considered, the exact Fisher test p-values for one-sided tests are as follows: p = 0.00 for MD versus PD; p = 0.00 for MD versus MPD treatment; p = 0.41 for PD versus MPD treatments.

Table 1.1. Misrepresentation rates by treatments

Treatment	Number of subjects	Number of top choice misrepres.	% of misrepres.	Number of subjects non-trivial	Number of top choice misrepres.	% of misrepres.
MD	261	49	18.80%	167	47	28.10%
PD	106	6	5.70%	63	6	9.50%
MPD	113	10	8.80%	74	9	12.10%
Total	480	65	13.50%	304	62	20.30%

The truth-telling rate for the MD treatment is the lowest among our treatments, but it is higher than in Chen and Sönmez (2006) (59% of misrepresentation in the random environment and 50% in the designed). We used Chen and Sönmez's (2006) formulation of TTC, but we cannot claim that in the natural environment subjects tend to report more truthfully than in the laboratory: the high truth-telling rate is driven by the students facing a trivial decision, which is ruled out by design in laboratory experiments. Excluding them, misreporting in MD reaches 28%, which is very much in line with Chen and Sönmez's results.

Result 2: The vast majority of subjects report a truthful top choice when they face a trivial decision.[11]

Support. Only 3 out of 176[12] students facing a trivial decision misreported their top choice in the experiment. The binomial probability test rejects the null hypothesis that the proportion of representation is higher than 5% (p=0.02) and thus we conclude that students reporting under a trivial decision situation is in line with our hypothesis. Trivial decisions are indeed trivial.

Next we look at the truth-telling rates by tentative topics.

Result 3: The proportion of misreported top choices is the highest among students with a TV set as a tentative topic, the second highest among students who have the scanner as a tentative topic and the lowest among students with the smartphone as a tentative topic. All those differences are significant at the 1% level.

Support. The last section of Table 1.2 (rows 13 to 16) presents the number of misreported choices and the proportion of truthful reporting for tentative topics. The exact Fisher test for the equality of proportions of the misreported top choices provides the following p-values for a one-sided test: p = 0.00 for smartphones versus TV set; p = 0.00 for smartphone versus scanner treatment; p = 0.00 for TV set versus scanner.

Thus, we find clear support for our hypothesis: students with the TV set as a tentative assignment are significantly more likely to misrepresent their top choices. Additionally, we are able to differentiate between misrepresentations of students in the form of TTB and other misrepresentations.

Result 4: TTB explains 78% of all the misrepresentations of top choices. TTB explains five out of five (100%) misrepresentations for the smartphone. TTB explains 41 out of 45 (91%) misrepresentations for the TV set. TTB explains only five out of 17 (29%) misrepresentations for the scanner.

Support. Column 4 of Table 1.2 presents the number of misrepresentations when the reported top choice is the tentative topic. In line with our hypothesis TTB most often occurs in the case of the TV set as the tentative topic, as students understand that they can guarantee themselves their most probable second choice by reporting the tentative topic as a top choice, thus escaping the worst option (scanner). The misrepresentations of preferences among students with the scanner as the tentative topic are harder to explain, but most likely they just skip the top choice, hoping that their chances of receiving the second choice are then higher. These

[11] Note that this result can be seen as a manipulation check for our top choice elicitation method.
[12] Two of these subjects were in the MD treatment and one in the MPD treatment.

Table 1.2. Summary of submitted choices

MD		N	Number of misrepres. of the top choice	Number of students affected by TTB	Proportion of truth
1	Smartphone	85	4	4	95.29%
2	TV set	93	31	30	66.67%
3	Scanner	83	14	5	83.13%
4	Total	261	49	39	81.23%
PD		N	Number of misrepres. of the top choice	Number of students affected by TTB	Proportion of truth
5	Smartphone	37	1	1	97.30%
6	TV set	40	3	3	92.50%
7	Scanner	29	2	0	93.10%
8	Total	106	6	4	94.34%
MPD		N	Number of misrepres. of the top choice	Number of students affected by TTB	Proportion of truth
9	Smartphone	35	0	0	100.00%
10	TV set	40	9	8	77.50%
11	Scanner	38	1	0	97.37%
12	Total	113	10	8	91.15%
All treatments		N	Number of misrepres. of the top choice	Number of students affected by TTB	Proportion of truth
13	Smartphone	157	5	5	96.82%
14	TV set	173	43	41	75.14%
15	Scanner	150	17	5	88.67%
16	Grand Total	480	65	51	86.46%

Notes: The table is grouped in 4 blocks by treatments. “N” in the second column represents the number of students with a given tentative topic.

Table 1.3. Summary of submitted choices by treatments

	Students with non- trivial decisions	Number of misreported top choices	p-value misreported top choices	p-value versus MD	p-value versus PD	p-value versus MPD
MD	73	30	41%	–	0.00 (0.00)	0.04 (0.07)
PD	30	3	10%	0.00 (0.00)	–	0.07 (0.12)
MPD	33	8	24%	0.04 (0.07)	0.07 (0.12)	–

*Notes:*Column 4-6 present the test for equality of proportion of truth-telling rates by treatments. One-sided p-values of the proportion test are presented, followed by one-sided p-values for the Fisher exact test for the equality of proportions in parenthesis.

students would not like to be assigned to their tentative topic and thus aim for the middle option. However, we cannot claim the latter with certainty as we know only the true top choice of the students.

As previously discussed, the most interesting group of students are those with the TV set as the tentative topic, as they are more likely exposed to TTB. To make a fair comparison, we consider only students for whom the decision is non-trivial, as otherwise the difference among the truth-telling rates could be driven by the unequal distribution of students with trivial situations across the treatments.

Result 5: The proportion of students who misreport when the TV set is their tentative topic and they face a non-trivial decision, is the highest in MD, the second highest in MPD, and the lowest in PD. All the differences are statistically significant.

Support. Column 4 of Table 1.3 reports the percentage of misreported top choices for students with TV set as a tentative topic among students with a non-trivial decision by treatments. The difference in the proportions of the misreported top choices between MD and PD treatments is significant at the 1% level; between MD and MPD treatments at the 5% level; between PD and MPD treatments a the 10% level (see columns 5–7 of Table 1.3 for the p-values of one-sided proportion tests).[13]

Next we use probit regressions to test jointly the effects of both properties and the mechanism description.

Result 6: TThe properties description increases the truthful reporting of the top choice. Conversely, describing the mechanism decreases the truthful reporting of the top choice. When including both variables at the same time, the properties description variable remains significant for the whole sample, while the mechanism description variable remains significant only when the sample is restricted to students with the tentative topic “TV set.”

Support.

[13] p-values differ for the Fisher exact test. The comparison of PD and MPD gives a p-value of 0.12. We still report the 10% significance of the result due to the high conservatism of the Fisher test.

Table 1.4. Marginal effects of probit regressions for misreported top choices

Dummy for misreported top choice	(1)	(2)	(3)	(4)	(5)	(6)
Properties description	-.61***		-.51**	-.63***		-.35
	(.17)		(.20)	(.22)		(.26)
Trivial decision	-1.32***	-1.31***	-1.32***	-1.10***	-1.12***	-1.12***
	(.25)	(.25)	(.25)	(.36)	(.37)	(.37)
Mechanism description		.61***	.23		.94***	.69*
		(.23)	(.28)		(.33)	(.38)
Constant	-.59***	-1.33***	-.83***	-.26*	-1.30***	-.95**
	(.10)	(.21)	(.29)	(.15)	(.31)	(.41)
Observations	480	480	480	173	173	173
log(likelihood)	-161.98	-165.00	-161.62	-86.98	-86.14	-85.22

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Standard errors in parentheses

Table 1.4 presents probit regressions predicting the misrepresentation of the top choice by students under different specifications. We generate two dummy variables. "Properties description" equals 0 for the MD and 1 otherwise. "Mechanism description" equals 0 in PD and 1 otherwise.

Result 6 is the main result of the paper. We show that in our field experiment with student participants, who on average should be much better at understanding the mechanism than the general public, the explanation of the properties does matter for the successful practical implementation of a market. On the other hand, the explanation of the procedures of matching mechanism—the instructions—has a clear negative effect. We conjecture that this effect could be the result of participants being confused, and thus believing they understand more than they actually do. Such individuals could try to outsmart the mechanism even in the presence of advice.

1.5 Conclusions

We obtained overall high rates of truthful preference revelation in our field experiment. Nevertheless, this result is driven to a large extent by a substantial proportion of participants making a trivial decision. When the decision is non-trivial, that is, when the student is tentatively allocated her second best choice, truthful preference revelation is significantly lower and in line with previous laboratory experimentation. Furthermore, truth-telling in non-trivial decisions does not differ from truth-telling in trivial decisions when only advice about the properties of the mechanism is given. Conversely, truthful preference revelation is much lower in non-trivial decisions when only the mechanism description is provided. We obtain an intermediate result when both mechanism description and properties are provided.

We can therefore conclude that, in the context of our field experiment, providing a description of the mechanism, identical to the standard TTC experimental instructions, has a detrimental effect on truth-telling. That is, the standard experimental instructions are not transparent, strategy-proofness is hard to infer from them, and confused participants try to manipulate the mechanism. This is an important finding and thus school districts are right not to mention complicated details, at least for parents who do not request them. The good news is that providing advice about strategy-proofness (properties description) seems to work well, and school districts seem to be getting that bit right, too. This result stands in apparent contrast with previous research by Guillen and Hing (2014) and Ding and Schotter (2014), in which correct advice does not have a significant effect on truth-telling. We believe that the difference can be explained by the reputation of the source of advice. Indeed, Guillen and Hing (2014) use stylized advice from Internet sources and Ding and Schotter (2014) relies on advice from other participants. In our field experiment students obtain advice from their lecturer, a trustworthy source regarding classroom procedures.

Like our field experiment, real-life markets based on strategy-proof mechanisms both rely on advice about strategy-proofness and often avoid describing the mechanism in details. The result of our experiment gives strong support to this practice. Most likely, the key to success rests on the reputation of the source of advice. Distrust of the School Board, or more generally of the institution organizing the market and providing advice, may well end up with less efficient outcomes.

Appendix 1.A Experimental instructions and decision sheets.

MD treatment:

SID ....

Tentative topic: Smartphone

Please read instructions before submitting your preferences:

The first best topic ....

The second best topic ....

The third best topic ....

We will use the following procedure to allocate topics to students:

Each participant is first randomly assigned a tentative topic. Your tentative topic is Smartphone. You will be asked to submit Decision Sheet rankings, which are used to determine mutually beneficial exchanges between two or more participants. The order in which these exchanges are considered is determined by a fair lottery. This means each participant has an equal chance of being the first in line, the second in line, ... , as well as the last in line. The lottery will be run by computer and no one will know the outcome of it before making the decision.

The specific allocation process is explained below.

1. All participants are ordered in a queue based on the order in the lottery.

2. Next, the participant at the top of the queue applies for the topic of his top choice, based on her ranking list.

o If she applies for her tentative topic, she is assigned to the topic and this assignment is finalized. The participant and her assignment are removed from subsequent allocations. The process continues with the next participant in line.

o If she applies for a topic which is different from her tentative assignment, the procedure moves as follows:

o Say applicant Claudia's tentative topic is "topic A" and she is applying for "topic B." Then one of the students who is tentatively assigned topic B has to be chosen. In particular, among all these students we choose the student who is the first one in the queue. Then this student is moved to the top of the queue directly in front of the requester (Claudia).

3. Whenever the queue is modified, the process continues similarly: the participant at the top of the queue applies for the topic of his top choice, based on her ranking list.

o If she applies for her tentative topic, she is assigned to the topic and this assignment is finalized. The process continues with the next participant in line.

o If she applies for another topic, say "topic C," then we follow the procedure explained in the example with Claudia: the first participant in the queue who is tentatively assigned topic C is moved to the top of the queue directly in front of the requester.

4. A mutually-beneficial exchange is obtained when a cycle of applications are made in sequence, which benefits all affected participants, e.g., I apply for Stefan's tentative topic, Stefan applies for your tentative topic, and you apply for my tentative topic. In this case, the exchange is completed and the participants as well as their assignments are removed from subsequent allocations.

5. The process continues until all participants are assigned a topic.

PD treatment

SID ....

Tentative topic: Smartphone

Please read instructions before submitting your preferences:

The first best topic ....

The second best topic ....

The third best topic ....

We will use the following procedure to allocate topics to students:

Each participant is first randomly assigned a tentative topic. Your tentative topic is Smartphone (This assignment is random). You will be asked to submit Decision Sheet rankings, which are used to determine final allocation. For these purposes we will use the Top Trading Cycles Mechanism. This mechanism takes into account your preferences and the preferences of others in order to provide as many top choices as possible and it is strategy-proof. Thus, every participant has no incentive to misrepresent her preferences, as no matter what other subjects do she is always better off by submitting true ranking lists.

MPD treatment:

SID ....

Tentative topic: Smartphone

Please read instructions before submitting your preferences: The first best

topic ....

The second best topic ....

The third best topic ....

We will use the following procedure to allocate topics to students:

Each participant is first randomly assigned a tentative topic. Your tentative topic is Smartphone. You will be asked to submit Decision Sheet rankings, which are used to determine mutually beneficial exchanges between two or more participants. The order in which these exchanges are considered is determined by a fair lottery. This means each participant has an equal chance of being the first in line, the second in line, . . . , as well as the last in line. The lottery will be run by computer and no one will know the outcome of it before making the decision. The specific allocation process is explained below.

1. All participants are ordered in a queue based on the order in the lottery.

2. Next, the participant at the top of the queue applies for the topic of his top choice, based on her ranking list.

o If she applies for her tentative topic, she is assigned to the topic and this assignment is finalized. The participant and her assignment are removed from subsequent allocations. The process continues with the next participant in line.

o If she applies for a topic which is different from her tentative assignment, the procedure moves as follows:

o Say applicant Claudia's tentative topic is "topic A" and she is applying for "topic B." Then one of the students who is tentatively assigned topic B has to be chosen. In particular, among all these students we choose the student who is the first one in the queue. Then this student is moved to the top of the queue directly in front of the requester (Claudia).

3. Whenever the queue is modified, the process continues similarly: the participant at the top of the queue applies for the topic of his top choice, based on her ranking list.

o If she applies for her tentative topic, she is assigned to the topic and this assignment is finalized. The process continues with the next participant in line.

o If she applies for another topic, say "topic C", then we follow the procedure explained in the example with Claudia: the first participant in the queue who is tentatively assigned topic C is moved to the top of the queue directly in front of the requester.

4. A mutually-beneficial exchange is obtained when a cycle of applications are made in sequence, which benefits all affected participants, e.g., I apply for Stefan's tentative topic, Stefan applies for your tentative topic, and you apply for my tentative topic. In this case, the exchange is completed and the participants as well as their assignments are removed from subsequent allocations.

5. The process continues until all participants are assigned a topic.

This mechanism takes into account your preferences and preferences of others in order to provide as many top choices as possible and it is strategy proof. Thus every participant has no incentive to misrepresent her preferences, as no matter what other subjects do she is always better off by submitting true ranking lists.

Chapter 2

College Admissions with Entrance Exams: Centralized versus Decentralized

with Isa E. Hafalir, Dorothea Kübler, Morimitsu Kurino

2.1 Introduction

Throughout the world and every year, millions of prospective university students apply for admission to colleges or universities during their last year of high school. Admission mechanisms vary from country to country, yet in most countries there are government agencies or independent organizations that offer *standardized admission exams* to aid the *college admission* process. Students invest a lot of time and effort in doing well in these admission exams, and they are heterogeneous in terms of their ability to do so.

In some countries, the application and admission process is centralized. For instance, in Turkey university assignment is solely determined by a national examination called YGS/LYS. After learning their scores, students can then apply to a number of colleges. Applications are almost costless as all students need only to submit their rank-order of colleges to the central authority.[1] On the other hand, Japan has a centralized "National Center test," too, but all public universities, including the most prestigious universities, require the candidate to take another, institution-specific secondary exam which takes place on the same day. This effectively prevents the students from applying to more than one public university.[2]

[1] Greece, China, South Korea, and Taiwan have similar national exams that are the main criterion for the centralized mechanism of college admissions. In Hungary, the centralized admission mechanism is based on a score that combines grades from school with an entrance exam (Biro, 2012).

[2] There are actually two stages where the structure of each stage corresponds to our description and modeling of the decentralized mechanism in section 2.4. The difference between the stages is

The admissions mechanism in Japan is decentralized, in the sense that colleges decide on their admissions independent of each other. Institution-specific exams that prevent students from applying to all colleges have also been used and debated in the United Kingdom, notably between the University of Cambridge and the University of Oxford. Currently, the students cannot apply to both the University of Cambridge and the University of Oxford.[3] Moreover, till 1994 the college admission exams in South Korea were only offered on two dates each year, and students were allowed to apply for only one college per exam date (see Avery et al., 2014, for more details). In the Soviet Union, everyone had to submit the original of the school certificate together with the application to a college, and colleges had an institution-specific exam. Thus, college admissions were fully decentralized. Although most of the former Soviet republics and Russia have lately introduced centralized exams and a centralized college admissions process, some colleges, typically the best ones, still run their own entry exams and thus opt out of the centralized system.

In the United States, students take both centralized exams like the Scholastic Aptitude Test (SAT), and also complete college-specific requirements such as college admission essays. Students can apply to more than one college, but since the application process is costly, students typically send only a few applications (the majority being between two to six applications, see Chade et al., 2014). Hence, the United States college admissions mechanism falls inbetween the two extreme cases.

In this paper, we compare the institutional effects of different college admission mechanisms on the equilibrium efforts of students, student welfare, and sorting. To do this, we model college admissions with admission exams as contests (or all-pay auctions) in which the cost of effort represents the payment made by the students. We focus on two extreme cases: in the centralized model (as in the Turkish mechanism) students can freely apply to all colleges, whereas in the decentralized model (as in the Japanese mechanism for public colleges) students can only apply to one college. For simplicity, in our main model we consider two colleges that differ in quality and assume that students have homogeneous preferences for attending these colleges.[4]

More specifically, each of the n students gets a utility of v_1 by attending college 1 (which can accommodate q_1 students) and gets a utility of v_2 by attending college 2 (which can accommodate q_2 students). College 2 is the better and college

that the capacities in the first stage are much greater than those in the second stage. Those who do not get admitted to any college spend one year preparing for the next year's exam. Moreover, the Japanese high school admissions authorities have adopted similar mechanisms in local districts. Although the mechanism adopted varies across prefectures and is changing year by year, its basic structure is that each student chooses one among a specified set of public schools and then takes an entrance exam at his or her chosen school. The exams are held on the same day.

[3] We thank Aytek Erdil and Ken Binmore for discussions on college admission systems in the UK.

[4] In section 6, we discuss the case with three or more colleges.

1 is the worse of the two colleges. The students' utility from not being assigned to any college is normalized to 0. Hence, $0 < v_1 < v_2$. Following most of the literature on contests with incomplete information, we assume that an ability level in the interval $[0, 1]$, is drawn i.i.d. from the common distribution function, and the cost of exerting an effort e for a student with ability level a is given by $\frac{e}{a}$. Thus, given an effort level, the higher the ability the lower the cost of exerting effort.

In the centralized college admissions problem (CCA), all students rank college 2 over college 1. Hence, the students with the highest q_2 efforts get into college 2, students with the next highest q_1 efforts get into college 1, and students with the lowest $n - q_1 - q_2$ efforts are not assigned to any college. In the decentralized college admissions problem (DCA), students need to simultaneously choose which college to apply to and how much effort to exert. Then, for each college $i \in \{1, 2\}$, students with the highest q_i efforts *among the applicants to college i* get into college i.

It turns out that the equilibrium of CCA can be solved by standard techniques, such as those in Moldovanu et al. (2012). In this monotone equilibrium, higher ability students exert higher efforts, and therefore the students with the highest q_2 ability levels get admitted to the good college 2, and students with ability rankings between $q_2 + 1$ and $q_1 + q_2$ get admitted to the bad college 1 (Proposition 1).

Finding the equilibrium of DCA is not straightforward. It turns out that in equilibrium, there is a cutoff ability level that we denote by c. All higher ability students (with abilities in $(c, 1]$) apply to the good college, whereas lower ability students (with ability levels in $[0, c]$) use a *mixed strategy* when choosing between the good and the bad college. Students' effort functions are continuous and monotone in ability levels (Theorem 1). We also establish that the equilibrium we have found is the unique symmetric and monotone equilibrium.

Our paper therefore contributes to the all-pay contests literature. To the best of our knowledge, ours is the first paper to model and solve a game of competing contests with multiple prizes where the players have private information regarding their abilities and sort themselves into different contests.[5]

After solving for the equilibrium of CCA and DCA and proving their uniqueness, we compare the equilibria in terms of students' interim expected utilities. We show that students with lower abilities prefer DCA to CCA when the number of seats is smaller than the number of students (Proposition 2). The main intuition for this result is that students with very low abilities have almost no chance of getting a seat in CCA, whereas their probability of getting a seat in DCA is bounded away from zero due to the fewer number of applications than the capacity. Moreover, we show that students with higher abilities prefer CCA to DCA (Proposition 3).[6] The main intuition for this result is that high-ability students (i)

[5] There is a large literature on competing auctions and competing mechanisms, and competing contests with unit prizes and incomplete information are analyzed by DiPalantino and Vojnovic (2009). We discuss this literature in the next subsection.

[6] More specifically, we obtain a single crossing condition: if a student who applies to college 2 in

can only get a seat at the good college in DCA, whereas they can get seats at both the good and the bad college in CCA, and (ii) their equilibrium probability of getting a seat at the good college is the same across the two mechanisms.

We test the theory with the help of lab experiments. We implement five markets for the college admissions game that are designed to capture different levels of competition (in terms of the supply of seats, the demand ratio, and the quality difference between the two colleges). We compare the two college admission mechanisms and find that in some markets the comparisons of the students' ex-ante expected utilities, their effort levels, and the students' preferences regarding the two mechanisms given their ability are well organized by the theory. However, the experimental subjects exert a higher effort than predicted. The overexertion of effort is particularly pronounced in DCA, which makes it relatively less attractive for the applicants compared to CCA. We also find significant differences between the two mechanisms with respect to the sorting of students that are in part due to out-of-equilibrium choices of the experimental subjects.

2.1.1 Related literature

College admissions have been studied extensively in the economics literature. Following the seminal paper by Gale and Shapley (1962), the theory literature on two-sided matching mainly considers centralized college admissions and investigates stability, incentives, and the efficiency properties of various mechanisms, notably the deferred-acceptance and the top trading cycles algorithms. The student placement and school choice literature is motivated by the centralized mechanisms of public school admissions, rather than by the decentralized college admissions mechanism in the US. This literature was pioneered by Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003). We refer the reader to Sönmez and Ünver (2011) for a recent comprehensive survey regarding centralized college admission models in the two-sided matching literature. Recent work regarding centralized college admissions with entrance exams include Abizada and Chen (2015) and Tung (2009). Abizada and Chen (2015) model the entrance (eligibility) criterion in college admissions problems and extend models of Perach et al. (2007) and Perach and Rothblum (2010) by allowing the students to have the same scores from the central exam. On the other hand, by allowing students to submit their preferences after they receive the test results, Tung (2009) adjusts the multi-category serial dictatorship (MSD) analyzed by Balinski and Sönmez (1999) in order to make students better off.

One crucial difference between the modeling in our paper and the literature should be emphasized: In our paper student preferences affect college rankings over students through contests among students, while student preferences and college rankings are typically independent in the two-sided matching models and

DCA prefers CCA to DCA, then all higher ability students also have the same preference ranking.

school-choice models.

The analysis of decentralized college admissions in the literature is more recent. Chade et al. (2014) consider a model where two colleges receive noisy signals about the caliber of applicants. Students need to decide which colleges to apply to and application is costly. The two colleges choose admissions standards that act like market-clearing prices. The authors show that in equilibrium, college-student sorting may fail, and they also analyze the effects of affirmative action policies. In our model, the colleges are not strategic players as in Chade et al. (2014). Another important difference is that in our model the students do not only have to decide which colleges to apply to, but also how much effort to exert in order to do well in the entrance exams. Che and Koh (2015) study a model in which two colleges make admission decisions subject to aggregate uncertainty about student preferences and linear costs for any enrollment exceeding the capacity. They find that colleges' admission decisions become a tool for strategic yield management, and in equilibrium, colleges try to reduce their enrollment uncertainty by strategically targeting students. In their model, as in Chade et al. (2014), students' exam scores are costlessly obtained and given exogenously. Avery and Levin (2010), on the other hand, analyze a model of early admission at selective colleges where early admission programs give students an opportunity to signal their enthusiasm to the college they would like to attend. More recently, motivated by the South Korean college admission system that went through a policy change in 1994, Avery et al. (2014) compare the two (with and without early admissions) regimes and conclude that lower-ranked colleges may gain in competition with higher-ranked colleges by limiting the number of possible applications.

In another related paper, Hickman (2009) also models college admissions as a Bayesian game where heterogeneous students compete for seats at colleges. He presents a model in which there is a centralized allocation mechanism mapping each student's score into a seat at a college. Hickman (2009) is mostly interested in the effects of affirmative action policies and the solution concept used is "approximate equilibrium" in which the number of students is assumed to be large so that students approximately know their rankings within the realized sample of private costs.[7] Similarly, Olszewski and Siegel (2014) consider contests with many players and prizes and show that the equilibrium outcomes of such contests are approximated by the outcomes of an appropriately defined set of mechanisms. In contrast to Hickman (2009) and Olszewski and Siegel (2014), our results are also applicable when the number of agents is not large.

In another recent paper by Salgado-Torres (2013), students and colleges participate in a decentralized matching mechanism called Costly Signaling Mechanism (CSM) in which students first choose a costly observable score to signal their

[7] In a related paper, Morgan et al. (2012) study competition for promotion in *a continuum economy*. They show that a more meritocratic profession always succeeds in attracting the highest ability types, whereas a profession with superior promotion benefits attracts high types only under some assumptions.

abilities, then each college makes an offer to a student, and finally each student chooses one of the available offers. Salgado-Torres (2013) characterizes a symmetric equilibrium of CSM which is proven to be assertive and also performs some comparative statics analysis. CSM is decentralized just like the decentralized college admissions model developed in this paper. However, CSM cannot be used to model college admission mechanisms (such as the ones used in Japan) that require students to apply to *only one* college.

Our paper is also related to the all-pay auction and contests literature. Notably, Baye et al. (1996) and Siegel (2009) solve for all-pay auctions and contests with complete information. We refer the reader to the survey by Konrad (2009) about the vast literature on contests. Related to our decentralized mechanism, Amegashie and Wu (2006) and Konrad and Kovenock (2012) both model "competing contests" in a *complete information* setting. Amegashie and Wu (2006) study a model where one contest has a higher prize than the other. They show that sorting may fail in the sense that the top contestant may choose to participate in the contest with a lower prize. In contrast, Konrad and Kovenock (2012) study all-pay contests that are run simultaneously with multiple identical prizes. They characterize a set of pure strategy equilibria and a symmetric equilibrium that involves mixed strategies. In our decentralized college admissions model, the corresponding contest model is also a model of competing contests. The main difference in our model is that we consider incomplete information as students do not know each others' ability levels.

A series of papers by Moldovanu and Sela (and Shi) studies contests with incomplete information, but they do not consider competing contests in which the participation in contests is endogenously determined. In Moldovanu and Sela (2001), the contest designer's objective is to maximize expected effort. They show that when cost functions are linear or concave in effort, it is optimal to allocate the entire prize sum to a single first prize. Moldovanu and Sela (2006) compare the performance of dynamic sub-contests whose winners compete against each other with static contests. They show that with linear costs of effort, the expected total effort is maximized with a static contest, whereas the highest expected effort can be higher with contests with two divisions. Moldovanu et al. (2012) study optimal contest design where both awards and punishments can be used. Under some conditions, they show that punishing the bottom is more effective than rewarding the top.

There is a large literature on competing auctions and mechanisms; notable examples are Ellison et al. (2004), Biais et al. (2000), McAfee (1993), and more recently, Moldovanu et al. (2008), Virág (2010), and Ovadia (2014). Two papers that are most related to our papers are DiPalantino and Vojnovic (2009) and Buyukboyaci (2012). DiPalantino and Vojnovic (2009) consider multiple contests where each contest gives a single prize and show the existence of a symmetric monotone equilibrium using the revenue equivalence theorem. They are mostly interested in participation rates among different contests and establish that in the large system limit (i.e., as the population gets large) the number of players that participate in a given contest class is a Poisson random variable. Buyukboyaci (2012), on the other hand, theoretically and experimentally compares the performance of one contest with a single prize and two parallel contests each with a single prize. In her model agents can be either a high ability or a low ability type. Her main finding is that the designer's profit is higher in the parallel tournaments when the contestants' low and high ability levels are sufficiently differentiated.

This paper also contributes to the experimental literature on contests and all-pay auctions, summarized in a recent survey article by Dechenaux et al. (2014). Our setup in the centralized mechanism with heterogeneous agents, two non-identical prizes, and incomplete information is closely related to a number of existing studies by Barut et al. (2002), Noussair and Silver (2006), and Müller and Schotter (2010). These studies observe that agents overbid on average compared to the Nash prediction. Moreover, they find an interesting bifurcation, a term introduced by Müller and Schotter (2010), in that low types underbid and high types overbid. Regarding the optimal prize structure, it turns out that if players are heterogeneous, multiple prizes can be optimal to avoid the discouragement of weak players (see Müller and Schotter, 2010). Higher effort with multiple prizes than with a single prize was also found in a setting with homogeneous players by Harbring and Irlenbusch (2003).

We are not aware of any previous experimental work related to our decentralized admissions mechanism where agents simultaneously choose an effort level and decide whether to compete for the high or the low prize.

The paper also belongs to the experimental literature on two-sided matching mechanisms and school choice starting with Kagel and Roth (2000) and Chen and Sönmez (2006).[8] These studies as well as many follow-up papers in this strand of the literature focus on the rank-order lists submitted by students in the preference-revelation games, but do not study effort choice. Thus, the rankings of students by the schools are exogenously given in these studies unlike in our setup where the colleges' rankings are endogenous.

2.2 The Model

The college admissions problem with entrance exams, or simply the problem, is denoted by $(S, \mathcal{C}, (q_1, q_2), (v_1, v_2), F)$. There are two colleges – college 1 and college 2. We denote colleges by C. Each college $C \in \mathcal{C} := \{1, 2\}$ has a capacity q_C which represents the maximum number of students that can be admitted to college C, where $q_C \geq 1$.

There are n students. We denote the set of all students by S. Since we suppose

[8] A recent example of theory combined with experiments in the school choice literature is Chen and Kesten (2015).

homogeneous preferences of students, we assume that each student has the cardinal utility v_C from college $C \in \{1, 2\}$, where $v_2 > v_1 > 0$. Thus, we sometimes call college 2 the good college and college 1 the bad college. Each student's utility from not being assigned to any college is normalized to be 0. We assume that $q_1 + q_2 \leq n$.[9]

Each student $s \in S$ makes an effort e_s. Each student is assigned to one college or no seat in any college by the mechanisms which take the efforts into account while deciding on their admissions.[10] The students are heterogeneous in terms of their abilities, and the abilities are their private information. More specifically, for each $s \in S$, $a_s \in [0, 1]$ denotes student s's ability. Abilities are drawn identically and independently from the interval $[0, 1]$ according to a continuous distribution function F that is common knowledge. We assume that F has a continuous density $f = dF > 0$. For a student s with ability a_s, putting in an effort of e_s results in a disutility of $\frac{e_s}{a_s}$. Hence, the total utility of a student with ability a from making effort e is $v_C - e/a$ if she is assigned to college C, and $-e/a$ otherwise.

Before we move on to the analysis of the equilibrium of centralized and decentralized college admission mechanisms, we introduce some necessary notation.

2.2.1 Preliminary notation

First, for any continuous distribution T with density t, for $1 \leq k \leq m$, let $T_{k,m}$ denote the distribution of the $k^{th}-$(lowest) order statistics out of m independent random variables that are identically distributed according to T. That is,

$$T_{k,m}(a) \quad := \quad \sum_{j=k}^{m} \binom{m}{j} T(a)^j (1 - T(a))^{m-j}. \tag{2.1}$$

Moreover, let $t_{k,m}(\cdot)$ denote $T_{k,m}(\cdot)$'s density:

$$t_{k,m}(a) \quad := \quad \frac{d}{da} T_{k,m}(a) = \frac{m!}{(k-1)!\,(m-k)!} T(a)^{k-1} (1 - T(a))^{m-k} t(a). \tag{2.2}$$

For convenience, we let $T_{0,m}$ be a distribution with $T_{0,m}(a) = 1$ for all a, and $t_{0,m} \equiv dT_{0,m}/da = 0$.

Next, define the function $p_{j,k} : [0, 1] \to [0, 1]$ as follows: for all $j, k \in \{0, 1, \ldots, n\}$ and $x \in [0, 1]$,

$$p_{j,k}(x) := \binom{j+k}{j} x^j (1 - x)^k. \tag{2.3}$$

The function $p_{j,k}(x)$ is interpreted as the probability that when there are $(j + k)$

[9] Many college admissions, including ones in Turkey and Japan, are competitive in the sense that the total number of seats in colleges is smaller than the number of students who take the exams.

[10] In reality the performance in the entrance exams is only a noisy function of efforts. For simplicity, we assume that efforts completely determine the performance in the tests.

students, j students are selected for one event with probability x and k students are selected for another event with probability $(1-x)$. Suppose that $p_{0,0}(x) = 1$ for all x. Note that with this definition, we can write

$$T_{k,m}(a) \quad = \quad \sum_{j=k}^{m} p_{j,m-j}(T(a)). \tag{2.4}$$

2.3 The Centralized College Admissions Mechanism (CCA)

In the centralized college admissions game, each student $s \in S$ simultaneously makes an effort e_s. Students with the top q_2 efforts are assigned to college 2 and students with the efforts from the top (q_2+1) to (q_1+q_2) are assigned to college 1. The rest of the students are not assigned to any colleges.[11] We now solve for the symmetric Bayesian Nash equilibrium of this game. The following proposition is a special case of the all-pay auction equilibrium which has been studied by Moldovanu and Sela (2001) and Moldovanu et al. (2012).

Proposition 1. *In CCA, there is a unique symmetric equilibrium β^C such that for each $a \in [0,1]$, each student with ability a chooses an effort $\beta^C(a)$ according to*

$$\beta^C(a) = \int_0^a x\ \left\{ f_{n-q_2,n-1}(x)\, v_2 + \left(f_{n-q_1-q_2,n-1}(x) - f_{n-q_2,n-1}(x) \right) v_1 \right\} dx.$$

where $f_{k,m}(\cdot)$ for $k \geq 1$ is defined in Equation (2.2) *and $f_{0,m}(x)$ is defined to be* 0 *for all x.*

Proof. Suppose that β^C is a symmetric equilibrium effort function that is strictly increasing. Consider a student with ability a who chooses an effort as if her ability is a'. Her expected utility is

$$v_2\, F_{n-q_2,n-1}(a') + v_1\ \left(F_{n-q_1-q_2,n-1}(a') - F_{n-q_2,n-1}(a') \right) - \frac{\beta^C(a')}{a}.$$

The first-order condition at $a' = a$ is

$$v_2\, f_{n-q_2,n-1}(a) + v_1\ \left(f_{n-q_1-q_2,n-1}(a) - f_{n-q_2,n-1}(a) \right) - \frac{[\beta^C(a)]'}{a} = 0.$$

[11] In a setup with homogeneous student preferences, this game reflects how the Turkish college admission mechanism works. In the centralized test that the students take, since all students would put college 2 as their top choice and college 1 as their second top choice in their submitted preferences, the resulting assignment would be the same as the assignment described above. In a school choice context, this can be described as the following two-stage game. In the first stage, there is one contest where each student s simultaneously makes an effort e_s. The resulting effort profile $(e_s)_{s \in S}$ is used to construct a single priority profile $\succ$ such that a student with a higher effort has a higher priority. In the second stage, students participate in the centralized deferred acceptance mechanism where colleges use the common priority $\succ$.

Thus, by integration and as the boundary condition is $\beta^C(0) = 0$, we have

$$\beta^C(a) = \int_0^a x \,\{f_{n-q_2,n-1}(x)\, v_2 + \left(f_{n-q_1-q_2,n-1}(x) - f_{n-q_2,n-1}(x)\right) v_1\}\, dx.$$

The above strategy is the unique symmetric equilibrium candidate obtained via the "first-order approach" by requiring no benefit from local deviations. Standard arguments show that this is indeed an equilibrium by making sure that global deviations are not profitable (for instance, see section 2.3 of Krishna, 2002). ■

2.4 The Decentralized College Admissions Mechanism (DCA)

In the decentralized college admissions game, each student s chooses one college C_s and an effort e_s simultaneously. Given the college choices of students $(C_s)_{s\in S}$ and efforts $(e_s)_{s\in S}$, each college C admits students with the top q_C effort levels among its set of applicants ($\{s \in S \mid C_s = C\}$).[12]

For this game, we focus on "symmetric and monotone" Bayesian Nash equilibrium. More specifically, we consider the case in which (i) the students' strategies only depend on their ability levels and not their names, and (ii) when we consider the effort levels of students who are applying to a particular college, higher ability students choose higher efforts.

A natural equilibrium candidate is to have a cutoff $c \in (0, 1)$, students with abilities in $[0, c)$ to apply to college 1, and students with abilities in $[c, 1]$ to apply to college 2. It turns out that we cannot have an equilibrium of this kind. In such an equilibrium, (i) type c has to be indifferent between applying to college 1 or college 2, (ii) type c's effort is strictly positive in case of applying to college 1, and 0 when applying to college 2. Hence there is a discontinuity in the effort function. These two conditions together imply that a type $c + \epsilon$ student would benefit from mimicking type c. We show this in Proposition 4 in Appendix 2.B.1.

Therefore, some students have to use mixed strategies when choosing which college to apply to. Next, as we formally show in Proposition 5 in Appendix 2.B.1, we argue that when the students use mixed strategies in a symmetric and monotone equilibrium, they choose the same effort level when they apply to either of the colleges. This is surprising at first sight, yet it follows from a "revelation

[12] In a setup with homogeneous student preferences, this game reflects how the Japanese college admissions mechanism works: all public colleges hold their own tests and accept the top performers among the students *who take their tests*. In the school choice context, this can be described as the following two-stage game. In the first stage, students simultaneously choose which college to apply to, and without knowing how many other students have applied, they also choose their effort level. For each college $C \in \{1, 2\}$, the resulting effort profile $(e_s)_{\{s\in S \mid C_s = C\}}$ is used to construct one priority profile $\succ_C$ such that a student with a higher effort has a higher priority. In the second stage, students participate in two separate deferred acceptance mechanisms where each college C uses the priority $\succ_C$.

principle" argument: when students mix, they have to be indifferent between applying to either colleges, but since both games are Bayesian incentive compatible, expected utilities being the same implies expected payments or efforts being the same. In this equilibrium, lower ability students choose the same effort level independent of whether they are applying to college 1 or 2. Note that this is an equilibrium property, not a restriction on effort functions. In other words, students are allowed to choose different effort levels when they are applying to different colleges, yet they choose the same effort level in equilibrium.

In what follows, by considering a symmetric and monotone equilibrium we show that low-ability students use mixed strategies while the high-ability students are certain to apply to the better college. More specifically, $(\gamma(\cdot), \beta^D(\cdot); c)$ where $c \in (0,1)$ is a cutoff, $\gamma : [0,c] \to (0,1)$ is the mixed strategy that represents the probability of lower ability students applying to college 1, and $\beta^D : [0,1] \to R$ is the continuous and strictly increasing effort function. Each student with type $a \in [0,c]$ chooses college 1 with probability $\gamma(a)$ (hence chooses college 2 with probability $1-\gamma(a)$), and makes effort $\beta^D(a)$. Each student with type $a \in (c,1]$ chooses college 2 for sure, and makes effort $\beta^D(a)$.

We now move on to the derivation of symmetric and monotone Bayesian Nash equilibrium. Let a symmetric strategy profile $(\gamma(\cdot), \beta(\cdot); c)$ be given. For this strategy profile, the ex-ante probability that a student applies to college 1 is $\int_0^c \gamma(x) f(x) dx$, while the probability that a student applies to college 2 is $1 - \int_0^c \gamma(x) f(x) dx$. Let us define a function $\pi : [0,c] \to [0,1]$ that represents the ex-ante probability that a student has a type less than *a and* she applies to college 1:

$$\pi(a) := \int_0^a \gamma(x) f(x) dx. \tag{2.5}$$

With this definition, the ex-ante probability that a student applies to college 1 is $\pi(c)$, while the probability that a student applies to college 2 is $1-\pi(c)$. Moreover, $p_{m,k}(\pi(c))$ is the probability that m students apply to college 1 and k students apply to college 2 where $p_{m,k}(\cdot)$ is given in Equation (2.3) and $\pi(\cdot)$ is given in Equation (2.5).

Next, we define $G(\cdot) : [0,c] \to [0,1]$, where $G(a)$ is the probability that a type is less than or equal to a, conditional on the event that she applies to college 1. That is,

$$G(a) := \frac{\pi(a)}{\pi(c)}.$$

Moreover let $g(\cdot)$ denote $G(\cdot)$'s density. $G_{k,m}$ is the distribution of the $k^{th}-$order statistics out of m independent random variables that are identically distributed according to G as in equations (2.1) and (2.4). Also, $g_{k,m}(\cdot)$ denotes $G_{k,m}(\cdot)$'s density.

Similarly, let us define $H(\cdot) : [0,1] \to [0,1]$, where $H(a)$ is the probability that a type is less than or equal to a, conditional on the event that she applies to college 2. That is, for $a \in [0,1]$,

$$H(a) = \begin{cases} \frac{F(a)-\pi(a)}{1-\pi(c)} & \text{if } a \in [0,c], \\ \frac{F(a)-\pi(c)}{1-\pi(c)} & \text{if } a \in [c,1]. \end{cases}$$

Moreover, let $h(\cdot)$ denote $H(\cdot)$'s density. Note that h is continuous but is not differentiable at c. Let $H_{k,m}$ be the distribution of the $k^{th}-$order statistics out of m independent random variables distributed according to H as in equations (2.1) and (2.4). Also, $h_{k,m}(\cdot)$ denotes $H_{k,m}(\cdot)$'s density.

We are now ready to state the main result of this section, which characterizes the *unique* symmetric and monotone Bayesian Nash equilibrium of the decentralized college admissions mechanism. The sketch of the proof follows the Theorem, whereas the more technical part of the proof is relegated to Appendix 2.B.2.

Theorem 1. *In DCA, there is a* unique *symmetric and monotone equilibrium* $(\gamma, \beta^D; c)$ *where a student with type* $a \in [0,c]$ *chooses college 1 with probability* $\gamma(a)$ *and makes effort* $\beta^D(a)$*; and a student with type* $a \in [c,1]$ *chooses college 2 for sure and makes effort* $\beta^D(a)$*. Specifically,*

$$\beta^D(a) = v_2 \int_0^a x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx.$$

The equilibrium cutoff c and the mixed strategies $\gamma(\cdot)$ *are determined by the following four requirements:*

(i) $\pi(c)$ *uniquely solves the following equation for x*

$$v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(x) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(x).$$

(ii) Given $\pi(c)$*, c uniquely solves the following equation for x*

$$\begin{aligned} v_1 = {} & v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) \\ & + v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\left(\frac{F(x)-\pi(c)}{1-\pi(c)}\right). \end{aligned}$$

(iii) Given $\pi(c)$ *and c, for each* $a \in [0,c)$*,* $\pi(a)$ *uniquely solves the following equation*

for $x(a)$

$$v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{F(a)-x(a)}{1-\pi(c)}\Big) = v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\Big(\frac{x(a)}{\pi(c)}\Big).$$

(iv) Finally, for each $a \in [0,c]$, $\gamma(a)$ *is given by*

$$\gamma(a) = \frac{\pi(c)B(a)}{(1-\pi(c))A(a)+\pi(c)B(a)} \in (0,1),$$

where

$$\begin{aligned} A(a) &:= v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))\, m\, p_{m-q_1,q_1-1}\Big(\frac{\pi(a)}{\pi(c)}\Big), \\ B(a) &:= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))\, m\, p_{m-q_2,q_2-1}\Big(\frac{F(a)-\pi(a)}{1-\pi(c)}\Big). \end{aligned}$$

Proof. Suppose that each student with type $a \in [0,1]$ follows a strictly increasing effort function β^D and a type $a \in [0,c]$ chooses college 1 with probability $\gamma(a) \in (0,1)$, and a type in $(c,1]$ chooses college 2 for sure.

We first show how to obtain the equilibrium cutoff c and the mixed strategy function γ. A necessary condition for this to be an equilibrium is that each type $a \in [0,c]$ has to be indifferent between applying to college 1 or 2. Thus, for all $a \in [0,c]$,

$$\begin{aligned} &v_1\left(\sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) + \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))G_{m-q_1+1,m}(a)\right) \\ &= v_2\left(\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))H_{m-q_2+1,m}(a)\right). \end{aligned} \tag{2.6}$$

The left-hand side is the expected utility of applying to college 1, while the right-hand side is the expected utility of applying to college 2. Note that $\sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c))$ and $\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c))$ are the probabilities that there are no more than (q_1-1) and (q_2-1) applicants in colleges 1 and 2, respectively. For $m \geq q_1$, $p_{m,n-m-1}(\pi(c))G_{m-q_1+1,m}(a)$ is the probability of getting a seat in college 1 with effort a when there are m other applicants in college 1. Similarly, for $m \geq q_2$, $p_{n-m-1,m}(\pi(c))H_{m-q_2+1,m}(a)$ is the probability of getting a seat in college 2 with effort a, when there are m other applicants in college 2.

Note that we have

$$G_{m-q_1+1,m}(a) = \sum_{j=m-q_1+1}^{m} p_{j,m-j}\left(\frac{\pi(a)}{\pi(c)}\right),$$

and

$$H_{m-q_2+1,m}(a) = \sum_{j=m-q_2+1}^{m} p_{j,m-j}\left(\frac{F(a)-\pi(a)}{1-\pi(c)}\right)$$

for all $a \in [0, c]$. The equation (2.6) at $a = 0$ and $a = c$ can hence be written as

$$v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)), \text{ and} \tag{2.7}$$

$$\begin{aligned} v_1 &= v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) \\ &+ v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\left(\frac{F(c)-\pi(c)}{1-\pi(c)}\right), \end{aligned} \tag{2.8}$$

respectively.

We show in Appendix 2.B.2 that there is a unique $\pi(c)$ that satisfies Equation (2.7), and that given $\pi(c)$, the only unknown c via $F(c)$ in Equation (2.8) is uniquely determined. Moreover, using (2.7), we can rewrite Equation (2.6) as

$$\begin{aligned} & v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\left(\frac{\pi(a)}{\pi(c)}\right) \\ &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\left(\frac{F(a)-\pi(a)}{1-\pi(c)}\right), \end{aligned} \tag{2.9}$$

for all $a \in [0, c]$. In Appendix 2.B, we show that given $\pi(c)$ and c, for each $a \in [0, c]$, there is a unique $\pi(a)$ that satisfies Equation (2.9) and, moreover, we show that we can get the mixed strategy function $\gamma(a)$ by differentiating Equation (2.9).

Finally we derive the unique symmetric effort function β^D by taking a "first-order approach" in terms of $G(\cdot)$ and $H(\cdot)$ which are determined by the equilibrium cutoff c and the mixed strategy function γ. Consider a student with type $a \in [0, c]$. A necessary condition for the strategy to be an equilibrium is that she does not want to mimic any other type a' in $[0, c]$. Her utility maximization problem is given by

$$\max_{a' \in [0,c]} v_2 \left(\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) H_{m-q_2+1,m}(a') \right) - \frac{\beta^D(a')}{a}.$$

where the indifference condition (2.6) is used to calculate the expected utility.[13] The first-order necessary condition requires the derivative of the objective function to be 0 at $a' = a$. Hence,

$$v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(a) - \frac{(\beta^D(a))'}{a} = 0.$$

Solving the differential equation with the boundary condition (which is $\beta^D(0) = 0$), we obtain

$$\beta^D(a) = v_2 \int_0^a x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx$$

for all $a \in [0, c]$

Next, consider a student with type $a \in [c, 1]$. A necessary condition is that she does not want to mimic any other type a' in $[c, 1]$. Her utility maximization problem is then

$$\max_{a' \in [c,1]} v_2 \left(\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) H_{m-q_2+1,m}(a') \right) - \frac{\beta^D(a')}{a}.$$

Note that although the objective function is the same for types in $[0, c]$ and $[c, 1]$, it is not differentiable at the cutoff c. The first-order necessary condition requires the derivative of the objective function to be 0 at $a' = a$. Hence,

$$v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(a) - \frac{(\beta^D(a))'}{a} = 0.$$

Solving the differential equation with the boundary condition of continuity (which is $\beta^D(c) = v_2 \int_0^c x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx$), we obtain

$$\beta^D(a) = v_2 \int_0^a x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx$$

for each $a \in [c, 1]$.

To complete the proof, we need to show that not only local deviations, but

[13] Equivalently, we can write the maximization problem as

$$\max_{a' \in [0,c]} v_1 \left(\sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) + \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) G_{m-q_1+1,m}(a) \right) - \frac{\beta^D(a')}{a},$$

With the same procedure, this gives the equivalent solution as

$$\beta^D(a) = v_1 \int_0^a x \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) g_{m-q_1+1,m}(x) dx$$

for each $a \in [0, c]$.

also global deviations cannot be profitable. In Appendix 2.B.3, we do that and hence show that the uniquely derived symmetric strategy $(\gamma, \beta^D; c)$ is indeed an equilibrium. ■

2.5 Comparisons

As illustrated in sections 2.3 and 2.4, the two mechanisms result in different equilibria. It is therefore natural to ask how the two equilibria compare in terms of interim student welfare. We denote by $EU^C(a)$ and $EU^D(a)$ the expected utility of a student with ability a under CCA and DCA, respectively.

Our first result concerns the preference of low-ability students.

Proposition 2. *Low-ability students prefer DCA to CCA if and only if $n > q_1 + q_2$.*

Proof. First, let us consider the case of $n > q_1 + q_2$. For this case it is not difficult to see that $EU^C(0) = 0$ (because the probability of being assigned to any college is zero), and $EU^D(0) > 0$ (because with a positive probability, type 0 will be assigned to a college). Since the utility functions are continuous, it follows that there exists an $\epsilon > 0$ such that for all $x \in [0, \epsilon]$, we have $EU^D(x) > EU^C(x)$.

Next, let us consider the case of $n = q_1 + q_2$. For this case, we have $EU^C(0) = v_1$. This is because with probability 1, type 0 will be assigned to college 1 by exerting 0 effort. Moreover, we have $EU^D(0) < v_1$. This is because type 0 should be indifferent between applying to college 1 and college 2, and in the case of applying to college 1, the probability of getting assigned to college 1 is strictly smaller than 1. Since the utility functions are continuous, it follows that there exists an $\epsilon > 0$ such that for all $x \in [0, \epsilon]$, we have $EU^C(x) > EU^D(x)$. ■ Intuitively, when the seats are over-demanded (i.e., when $n > q_1 + q_2$), very low-ability students have almost no chance of getting a seat in CCA, whereas their probability of getting a seat in DCA is bounded away from zero. Hence they prefer DCA.

Although this result merely shows that only students in the neighborhood of type 0 need to have these kinds of preferences, explicit equilibrium calculations for many examples (such as the markets we study in our experiments) result in a significant proportion of low-ability students preferring DCA. We provide a depiction of equilibrium effort levels and interim expected utilities for a specific example in Figure 2.1.

Moreover, we establish the reverse ranking for the high-ability students. That is, the high-ability students prefer CCA in the following single-crossing sense: if a student who applies to college 2 in DCA prefers CCA to DCA, then all higher ability students have the same preference ranking.

Proposition 3. *Let c be the equilibrium cutoff in DCA. We have (i) if $EU^C(a) \geq EU^D(a)$ for some $a > c$, then $EU^C(a') > EU^D(a')$ for all $a' > a$, and (ii) if $EU^C(a) < EU^D(a)$ for some $a > c$, then $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$.*

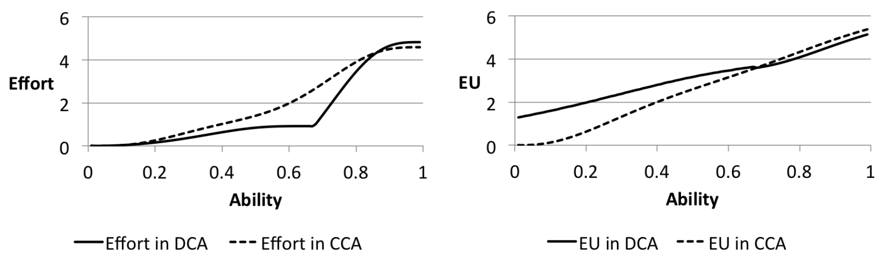


Figure 2.1. Efforts (left) and expected utility (right) under CCA and DCA

Note: The figures were created with the help of simulations for the following parameters: $n = 12$, $(q_1, q_2) = (5, 4)$, and $(v_1, v_2) = (5, 20)$. The equilibrium cutoff under DCA is calculated as $c = 0.675$.

Proof. Let us define

$$\begin{aligned}
K(a) &\equiv v_2 F_{n-q_2,n-1}(a), \\
L(a) &\equiv v_1 \left(F_{n-q_1-q_2,n-1}(a) - F_{n-q_2,n-1}(a)\right), \\
M(a) &\equiv K(a) + L(a), \\
N(a) &= v_2 \left(\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) H_{m-q_2+1,m}(a)\right).
\end{aligned}$$

Then we have

$$EU^C(a) = M(a) - \frac{\int_0^a M'(x)\, x dx}{a}.$$

By integration by parts, we obtain

$$EU^C(a) = \frac{\int_0^a M(x)\, dx}{a}.$$

Similarly,

$$EU^D(a) = N(a) - \frac{\int_0^a N'(x)\, x dx}{a},$$

and by integration by parts, we obtain

$$EU^D(a) = \frac{\int_0^a N(x)\, dx}{a}.$$

Note that, for $a > c$, we have

$$N(a) = K(a).$$

This is because students whose ability levels are greater than c apply to college 2 in DCA, and therefore a seat is granted to a student with ability level $a > c$ if and only if the number of students with ability levels greater than a is not greater than q_2. This is the same condition in CCA, which is given by the expression $K(a)$. (Also note that we have $N(a) \neq K(a)$ for $a < c$, in fact we have $N(a) > K(a)$, but this is irrelevant for what follows.)

Now, for any $a > c$, we obtain

$$\begin{aligned}\frac{d}{da}\left(aEU^C(a)\right) &= M(a) \\ &= K(a) + L(a)\end{aligned}$$

and

$$\begin{aligned}\frac{d}{da}\left(aEU^D(a)\right) &= N(a) \\ &= K(a).\end{aligned}$$

Since $L(a) > 0$, for any $a > c$, we have

$$\frac{d}{da}\left(aEU^C(a)\right) > \frac{d}{da}\left(aEU^D(a)\right),$$

or

$$EU^C(a) + a\frac{d}{da}EU^C(a) > EU^D(a) + a\frac{d}{da}EU^D(a).$$

This means that for any $a > c$, whenever $EU^C(a) = EU^D(a)$, we have $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$. Then we can conclude that once $EU^C(a)$ is higher than $EU^D(a)$, it cannot cut through $EU^D(a)$ from above to below and $EU^C(a)$ always stays above $EU^D(a)$. To see this suppose $EU^C(a) > EU^D(a)$ and $EU^C(a') < EU^D(a')$ for some $a' > a > c$, then (since both $EU^C(a)$ and $EU^D(a)$ are continuously differentiable) there exists $a'' \in (a, a')$ such that $EU^C(a'') = EU^D(a'')$ and $\frac{d}{da}EU^C(a'') < \frac{d}{da}EU^D(a'')$, a contradiction. Hence (i) is satisfied. Moreover, (ii) is obviously satisfied since whenever $EU^C(a) < EU^D(a)$, we have to have $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$. ■

Intuitively, since high-ability students (i) can only get a seat in the good college in DCA whereas they can get a seat in both the good and the bad college in CCA, and (ii) their equilibrium probability of getting a seat in the good college is the same across the two mechanisms, they prefer CCA.

One may also wonder whether there is a general ex-ante utility ranking of DCA and CCA. It turns out that examples where either DCA or CCA result in higher ex-ante utility (or social welfare) can be found. Specifically, markets 1 and 2 in our experimental sessions result in higher social welfare in CCA and DCA, respectively.

2.6 Extensions

In this section, we consider two extensions of the model. In the first, we allow for more than two colleges, again ranked in terms of quality. The second extension looks at a larger market in the following sense: as before, a setup is studied with two types of colleges resulting in utilities v_1 and v_2 and with capacities q_1 and q_2, but there are k colleges of each type and there are $k \times n$ students.

2.6.1 The case of ℓ colleges

Let us consider ℓ colleges, $1, ..., \ell$, where each college k has the capacity $q_k > 0$ and each student gets the utility of v_k from attending college k ($v_\ell > v_{\ell-1} > ... > v_2 > v_1 > 0$).

We conjecture that in the decentralized mechanism there will be a symmetric Bayesian Nash equilibrium $((\gamma_k)_{k=1}^{\ell}, \beta^D, (c_k)_{k=0}^{\ell})$:[14] (i) $c_0, \ldots, c_\ell$ are cutoffs such that $0 = c_0 < c_1 < \ldots < c_{\ell-1} < c_\ell = 1$; (ii) β^D is an effort function where each student with ability a makes an effort level of $\beta^D(a)$; (iii) $\gamma_1, \ldots, \gamma_\ell$ are mixed strategies such that for each $k \in \{1, \ldots, \ell - 1\}$, each student with ability $a \in [c_{k-1}, c_k)$ applies to college k with probability $\gamma_k(a)$ and college $k+1$ with probability $1 - \gamma_k(a)$. Moreover, each student with ability $a \in [c_{\ell-1}, 1]$ applies to college ℓ, equivalently, $\gamma_\ell(a) = 1$. The equilibrium effort levels can be identified as follows.

Let $k \in \{1, \ldots, \ell\}$ be given. Let $\pi^k(a)$ denote the ex-ante probability that a student has a type less than or equal to a and she applies to college k. Then, $\pi^1(a) = \int_0^a \gamma_1(x) dF(x)$. For $k \in \{2, \ldots, \ell\}$ and $a \in [c_{k-2}, c_k]$,

$$\pi^k(a) = \begin{cases} \int_{c_{k-2}}^{a} (1 - \gamma_{k-1}(x)) dF(x) & \text{if } a \le c_{k-1}, \\ \int_{c_{k-2}}^{c_{k-1}} (1 - \gamma_{k-1}(x)) dF(x) + \int_{c_{k-1}}^{a} \gamma_k(x) dF(x) & \text{if } a \ge c_{k-1}. \end{cases}$$

We define H^k to be the probability that a type is less than or equal to a, conditional on the event that she applies to college k:

$$H^k(a) = \frac{\pi^k(a)}{\pi^k(c_k)}.$$

In this equilibrium, each student with ability $a \in [c_{k-1}, c_k]$ exerts an effort of

$$\beta^D(a) = \beta^D(c_{k-1}) + \int_{c_{k-1}}^{a} x \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) h^k_{m-q_k+1,m}(x) dx$$

where $\beta^D(0) = 0$ and $h^k_{m-q_k+1,m}$ is the density of $H^k_{m-q_k+1,m}$. Similar to Theorem 1, it is possible to determine the formulation for cutoffs $c_1, \ldots, c_{\ell-1}$ and mixed strategies $\gamma_1, \ldots, \gamma_\ell$ using the indifference conditions (see Appendix C). This set of strategies can be shown to satisfy immunity for "local deviations," but prohibitively tedious arguments to check for immunity to global deviations (as we have done in Appendix B) prevent us from formally proving that it is indeed an

[14] As explained below, the strategies are not formally shown to be an equilibrium since we do not have a proof to show that global deviations are not profitable.

equilibrium.

By supposing an equilibrium of this kind, we can actually show that propositions 2 and 3 hold for ℓ colleges. Proposition 2 trivially holds, as students with the lowest ability levels get zero utility from CCA and strictly positive utility from DCA. We can also argue that Proposition 3 holds since the students with ability levels $a \in [c_{\ell-1}, 1]$ only apply to college ℓ. This can be observed by noting that a seat is granted to these students in college k if and only if the number of students with ability levels greater than a is no greater than q_ℓ, which is the same condition in CCA. Hence, even in this more general setup of ℓ colleges, we can argue that low-ability students prefer DCA whereas high-ability students prefer CCA.

2.6.2 The case of a k-replication

Consider an environment in which we have, (i) k type-1 colleges: $C_1^1, ..., C_1^k$ such that each of them has q_1 seats and gives a utility of v_1 to students, (ii) k type-2 colleges: $C_2^1, ..., C_2^k$ such that each of them has q_2 seats and gives a utility of v_1 to students, and (iii) $k \times n$ students. In other words, in this extension we consider a "k-replication" of our model.

With this extension, in CCA it is easy to see that there is a monotone equilibrium very similar to the original equilibrium. The students will list all type-2 colleges above all type-1 colleges (in an arbitrary fashion), students with the top $k \times q_2$ effort levels will get one of the type-2 colleges' seats, and students with the next top $k \times q_1$ effort levels will get one of the type-1 colleges' seats. In this equilibrium a student with type a will choose the effort

$$\beta^{C(k)}(a) = \int_0^a x\{f_{kn-kq_2,kn-1}(x)\, v_2 - \left(f_{kn-kq_2-kq_1,kn-1}(x) - f_{kn-kq_2,kn-1}(x)\right) v_1\} dx.$$

Moreover, we have that $\beta^{C(k)}(a)$ will be very close to $\beta^C(a)$ for all $k = 2, ..., \infty$. In fact, when F is uniform we have

$$\begin{aligned}\beta^{C(k)}(a) &= a\left(\frac{n-q_2}{n} v_2 - \left(\frac{n-q_1-q_2}{n} - \frac{n-q_2}{n}\right) v_1\right)\\ &= a\left(v_2 + \frac{q_1 v_1 - q_2 v_2}{n}\right)\end{aligned}$$

for all $k = 1, 2, ..., \infty$. Hence, for uniform distributions, any k-replica economy bidding function is the same as in the no-replica economy.

In DCA, on the other hand, one can observe that the equilibrium of the k-replica economy essentially remains the same as in the no-replica economy: the cutoff c and equilibrium effort functions will be the same. The only differences would be that (i) each student of ability lower than c will apply to each type-1 college with probability $\frac{\gamma(a)}{k}$ and each type-2 college with probability $\frac{1-\gamma(a)}{k}$, and

(ii) each student of ability higher than c will apply to each type-2 college with probability $\frac{1}{k}$.

Hence, if there are many students and many colleges (belonging to one of the two types), our predictions remain valid.

2.7 The Experiment

In this section, we present an experiment designed to test the results of the model and generate further insights into the performance of the centralized (CCA) and the decentralized college admissions mechanism (DCA). We compare the two mechanisms and study which of them leads to higher (interim and ex-ante) student welfare, higher efforts of the students, and how they affect the sorting of students by ability.

2.7.1 Design of the experiment

In the experiment, there are two colleges, college 1 (the bad college) and college 2 (the good college). There are 12 students who apply for positions, and these students differ with respect to their ability. Every student learns her ability a_s that is drawn from the uniform distribution over the interval from 1 to 100. Students choose an effort level e_s that determines their success in the application process. The cost of effort is determined by $100\frac{e_s}{a_s}$. (Note that we use the range of abilities from 1 to 100 instead of 0 to 1 in order to simplify the calculations for subjects. Accordingly, we scaled up the cost function by a constant of 100.)

In the centralized college admissions mechanism (CCA), all students simultaneously choose an effort level. Then the computer determines the matching by admitting the students with the highest effort levels to college 2 up to its capacity q_2 and the next best students, i.e., from rank $q_1 + 1$ to rank $q_1 + q_2$, to college 1. All other students are unassigned.

In the decentralized college admissions mechanism (DCA), the students simultaneously decide not only on their effort level but also on which college to apply to. The computer determines the matching by assigning the students with the highest effort among those who have applied to college C, up to its capacity q_C.

We implemented five different markets that differ with respect to the total number of open slots ($q_1 + q_2$), the number of slots at each college (q_1 and q_2) as well as the value of the colleges for the students (v_1 and v_2), see Table 2.1. This allows us to investigate behavior under very different market conditions. The parameters in each market were chosen so as to generate clear-cut predictions regarding the two main outcome variables, effort and the expected utility of each student.

Figure 2.2 shows the interim expected utility of students for each market. CCA dominates DCA with respect to the interim expected utility of students

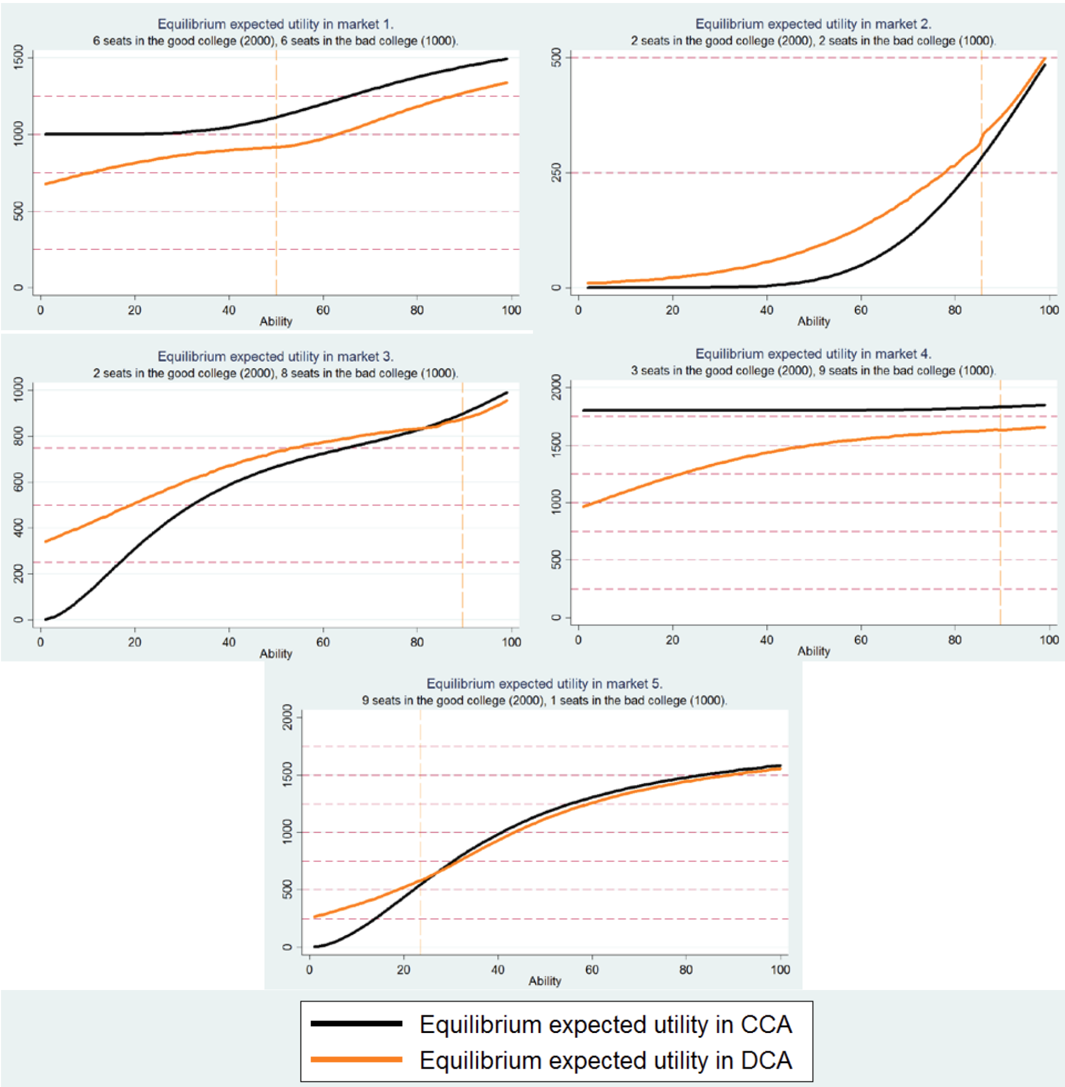


Figure 2.2. Equilibrium expected utility by ability

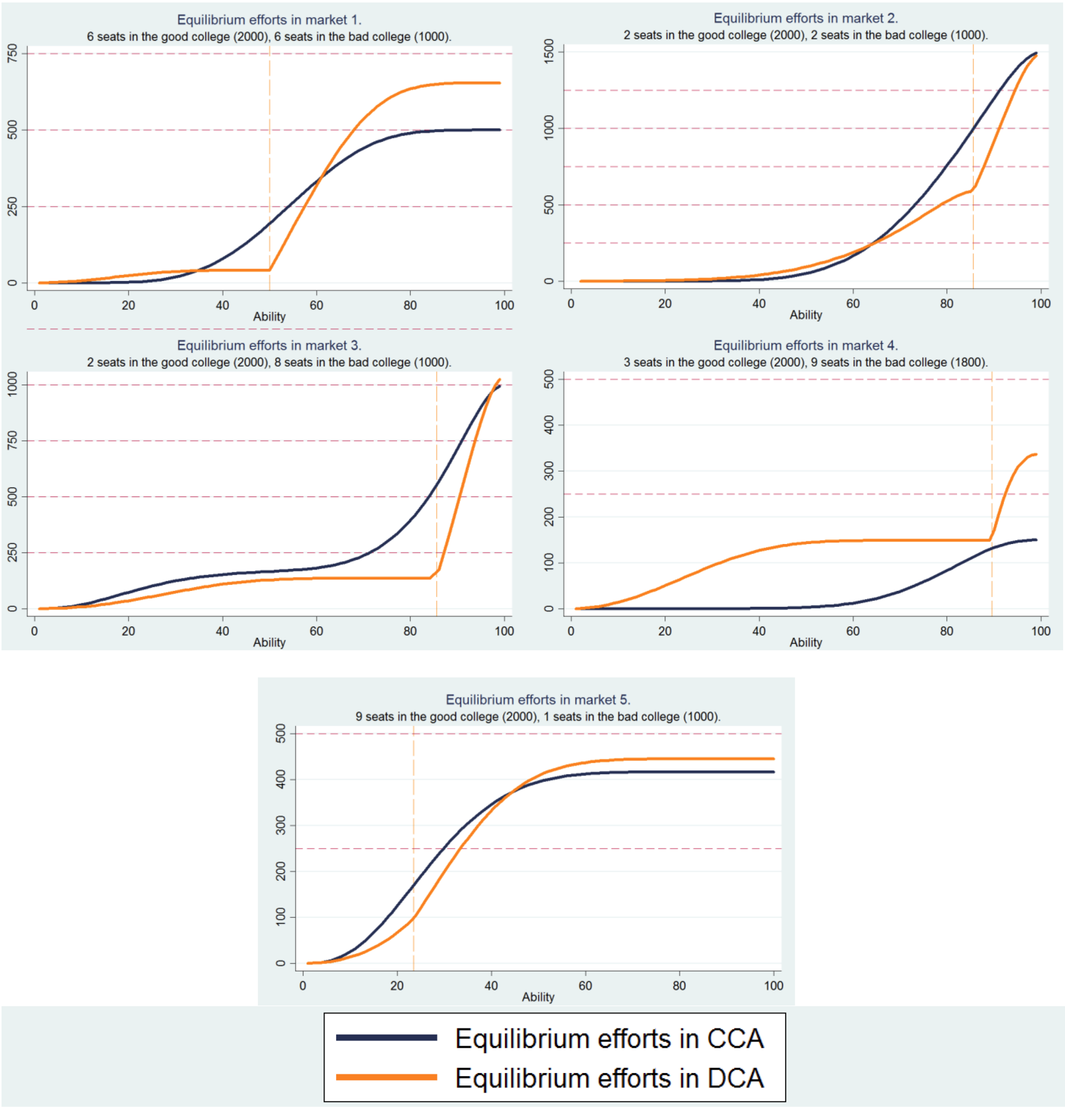


Figure 2.3. Equilibrium efforts by ability

Table 2.1. Overview of market characteristics

	Number of seats at [value of] college 2	college 1	Predicted utility higher	Predicted effort higher
Market 1	6 [2000]	6 [1000]	CCA	depends; DCA in expect.
Market 2	2 [2000]	2 [1000]	DCA	no diff. in expect.
Market 3	2 [2000]	8 [1000]	depends; DCA in expect.	CCA
Market 4	3 [2000]	9 [1800]	CCA	DCA
Market 5	9 [2000]	1 [1000]	no diff. in expect.	no diff. in expect.

Notes: In some markets, one of the two mechanisms dominates the other for all students. In other markets the ranking of the mechanisms depends on the students' ability in which case we compare the expected values."diff." stands for difference, "expect." stands for expectation.

(market 1) and the reverse (market 2). Figure 2.3 shows the equilibrium effort levels given abilities for each market. Effort is higher for all types in CCA in market 3 while the reverse holds for market 4. The fifth market is designed to make the two mechanisms as similar as possible.

In order to provide a valid comparison of the observed average effort and utility levels in the markets where there is no dominance relationship, i.e., the cells in Table 2.1 for which the predicted difference depends on the ability of the applicant, we compute the equilibrium effort and utility levels for the realizations of abilities in our experimental markets. We then take expected values given the realized abilities.

An important distinction for the theoretical predictions and for the intuition behind the predicted differences is whether the number of students is equal to the number of seats (markets 1 and 4) or whether there are more students than seats (markets 2, 3, and 5). As illustrated by Figure 2.3, in markets 1 and 4 with an equal number of seats and applicants, a positive effort in CCA is only exerted by those who can expect to get into the good college. In DCA, efforts are overall higher in these two markets because of the risk of miscoordination. In markets 2 and 3 in contrast, high-ability students tend to exert less effort in DCA than in CCA because the expected return is higher in CCA: in CCA one can obtain v_2, v_1 and 0, while in DCA only v_2 (or v_1) and 0 are achievable.

Note that our design aims at comparing the two mechanisms. We do not study the comparative statics of the equilibria of CCA and DCA by systematically varying one parameter. This would require a completely different design that is beyond the scope of this study.

We employed a between-subjects design. Students were randomly assigned either to the treatment with CCA or the treatment with DCA. In each treatment, subjects played 15 rounds with one market per round. Each of the five different markets was played three times by every participant, and abilities were drawn randomly for every round. These draws were independent, and each ability was equally likely. We employed the same randomly drawn ability profiles in both treatments in order to make them as comparable as possible. Markets were played in blocks: first, all five markets were played in a random order once, then all five markets were played in a random order for a second time, and then again randomly ordered for the last time. We chose this sequence of markets in order to ensure that the level of experience does not vary across markets. Participants faced a new situation in every round as they never played the same market with the same ability twice. They received feedback about their allocation and the points they earned after every round.

At the beginning of each round of the experiment, students received an endowment of 2,200 points. At the end of the experiment, one of the 15 rounds was randomly selected for payment. The points earned in this round plus the 2,200 endowment points were paid out in Euro with an exchange rate of 0.5 cents per point. The experiment lasted 90 minutes and the average earnings per subject were EUR 14.10.

The experiment was run at the experimental economics lab at the Technical University Berlin. We recruited student subjects from our pool with the help of ORSEE by Greiner (2004). The experiments were programmed in z-Tree, see Fischbacher (2007). For each of the two treatments, CCA and DCA, independent sessions were carried out. Each session consisted of 24 participants that were split into two matching groups of 12 for the entire session. In total, six sessions were conducted, that is, three sessions per treatment, with each session consisting of two independent matching groups of 12 participants. Thus, we end up with six fully independent matching groups and 72 participants per treatment.

At the beginning of the experiment, printed instructions were given to the participants (see Appendix 2.E). Participants were informed that the experiment was about the study of decision making, and that their payoff depended on their own decisions and the decisions of the other participants. The instructions were identical for all participants of a treatment, explaining in detail the experimental setting. Questions were answered in private. After reading the instructions, all individuals participated in a quiz to make sure that everybody understood the main features of the experiment.

2.7.2 Experimental results

We first present the aggregate results in order to compare the two mechanisms. In a second step, we study behavior in the two mechanisms separately to compare it to the point predictions and to shed light on the reasons for the aggregate findings. The significance level of all our results is 5%, unless otherwise stated.

Treatment comparisons: Aggregate results

We compare the two mechanisms with respect to three properties, summarized in results 1 to 3. The first comparison concerns the utility of students in the two mechanisms which is equal to the number of points earned, due to the assumption of risk neutrality. Second, we investigate whether one of the mechanisms induces higher effort levels than the other mechanism. And the third aspect is whether individuals of different abilities prefer different mechanisms.

Result 1 (Average utility): In markets 1 and 4 (where the number of seats equals the number of students), the average utility of students in CCA is significantly higher than in DCA, as predicted by the theory. In markets 2 and 3 (where there are less seats than applicants), the average utility of students in DCA is not significantly higher than in CCA, in contrast to the theoretical predictions. In market 5, there is no significant difference either in theory or in the data.

Support. Table 2.2 presents the average number of points or the average utility of the participants in the two mechanisms in all five markets. The third column provides the equilibrium prediction as to which mechanism, CCA or DCA, leads to a higher utility of the students. To generate this prediction, we compute the equilibrium utilities given the realized draws of abilities in the experiment for both mechanisms. And then we test for each market whether these equilibrium utilities are significantly different between the two mechanisms. Thus, the third column also displays the p-values for the two-sided Wilcoxon rank-sum test for the equality of distributions of equilibrium utilities. In markets 1 to 4, we expect that the utility of students in the two mechanisms is significantly different. The last column in the table provides the p-values for the two-sided Wilcoxon rank-sum test for the equality of distributions of the observed number of points earned in the two mechanisms.

Table 2.2. Average utility

Market	Utility higher for all students (predicted)	Av. utility higher for realized types (predicted)	Av. utility in CCA (observed)	Av. utility in DCA (observed)	Observed utilities different in CCA and DCA
1	CCA	CCA, 0.00	1001	716	0.02
2	DCA	DCA, 0.02	-122	-169	0.75
3	N/A	DCA, 0.00	342	305	0.63
4	CCA	CCA, 0.00	1507	1014	0.00
5	N/A	N/A, 0.63	809	797	1.00

Notes: Columns 3 and 6 show the p-values of the Wilcoxon rank-sum test for equality of the distributions, based on averages of the six matching groups per treatment. "Av." stands for average.

In markets 1 and 4, the equilibrium predictions for the comparison of utilities of students are consistent with the experimental data, as the average utility in CCA is significantly higher in both markets. Thus, with an equal number of applicants and seats, CCA is preferable to DCA if the goal is to maximize the utility of the students. This is due to the potential miscoordination of applicants in DCA in these markets, inducing higher effort levels. However, we fail to observe the superiority of DCA in both markets where this is predicted, namely markets 2 and 3. The relationship is even reversed, with the average utility being higher in CCA than in DCA in both markets. Note also that the average utility is negative in the competitive market 2 (with only four seats for 12 students) such that, contrary to the prediction, the subjects earn less than the 2,200 points they are endowed with.

Result 2 (Average effort): In markets 1 and 4 (where the number of seats equals the number of students), the average effort level of students in DCA is significantly higher than in CCA. This is in line with the predictions. In market 3, the average effort levels of students in CCA are not significantly higher than in DCA, in contrast to the theoretical prediction. In markets 2 and 5, there is no significant difference in effort between the two mechanisms, as predicted.

Support. Table 2.3 presents the average effort levels of the participants by different mechanisms and markets. Analogously to Table 2.2, the third column displays the equilibrium prediction regarding which mechanism leads to significantly higher effort levels. For this prediction, we compute the equilibrium effort levels given the realization of abilities in the five markets. To generate this prediction, we compute the equilibrium utilities given the realized draws of abilities in our markets in the experiment. The column also indicates the p-values of the Wilcoxon rank-sum test regarding the difference between equilibrium efforts in CCA and DCA. We expect effort to differ significantly between the two mechanisms only in markets 3 and 4 (with a marginally significant difference in market 1). The last column provides the p-values for the two-sided Wilcoxon rank-sum test for the equality of distributions of the observed effort levels in the two mechanisms. The equilibrium predictions regarding the comparison of efforts in markets 1 and 4 are confirmed by the data because observed average effort is significantly higher in DCA. In market 3 average efforts are higher in CCA than in DCA as predicted, but the difference is not significant.

Table 2.3. Average effort

Market	Effort higher for all students (predicted)	Av. effort higher for realized types (predicted)	Av. effort in CCA (observed)	Av. effort in DCA (observed)	Observed efforts different in CCA and DCA
1	N/A	DCA, 0.06	276	362	0.04
2	N/A	N/A, 0.15	389	410	0.75
3	CCA	CCA, 0.00	397	354	0.42
4	DCA	DCA, 0.00	191	340	0.02
5	N/A	N/A, 0.75	400	395	1.00

Notes: Columns 3 and 6 show the p-values of the Wilcoxon rank-sum test for equality of the distributions, based on averages of the six matching groups per treatment. "Av." stands for average.

Taking results 1 and 2 together, we observe that in markets without a shortage of seats (market 1 and market 4) students are on average better off in CCA where they exert less effort. In market 5 the results are also in line with the theoretical predictions with almost identical effort and expected utility levels in both mechanisms. In the two remaining markets with a surplus of students over seats, markets 2 and 3, the results are not in line with the theory. Markets 2 and 3 should lead to a higher average utility of the students in DCA than in CCA, which is not observed in the lab. Therefore, the overall results suggest that with respect to the utility of students, CCA performs better than predicted relative to DCA.

Next we turn to the question of whether students of different abilities prefer different mechanisms by providing an experimental test of propositions 2 and 3. According to Proposition 2, low-ability students prefer DCA over CCA if there are more applicants than seats in the market, as in our markets 2, 3, and 5. Proposition 3 implies that if any student who is above the cutoff in DCA prefers CCA over DCA, then all students with a higher ability must also prefer CCA. (Remember that in markets 1 and 4, all students prefer CCA, and we therefore do not consider these markets here.)

Result 3 (Expected utility of low- and high-ability students): In markets 2 and 3 (with fewer seats than applicants), the average utilities of students with low abilities are higher in DCA, and the average utilities of students with high abilities are higher in CCA. However, significantly fewer students than predicted prefer DCA to CCA. There is no significant difference between the average utilities of students in DCA and CCA in market 5.

Support: In three of our markets - namely 2, 3, and 5 - low-ability students prefer DCA in equilibrium. We refer to the *predicted switching point* as the maximum ability at which students prefer DCA in equilibrium. The predicted switching points by markets are represented in Figure 2.4 by the intersection of the broken lines. For market 2, the switching point is 100, for market 3 it equals 81, and for market 5 it equals 26. Figure 2.4 also shows the *observed switching points* as the intersection of the solid lines in markets 2, 3, and 5. The figure reveals that in markets 2 and 3, the observed switching points are substantially lower than the predicted switching points. This suggests that fewer students than predicted prefer DCA to CCA in these markets.

To assess the statistical significance of these differences in switching points, we use bootstrapping. That is, we sample from the dataset with replacement to generate new samples and calculate the bootstrap confidence intervals of the observed switching points in markets 2 and 3.[15] Before turning to the bootstrap confidence intervals, we first use the bootstrap samples to assess the theoretical prediction of a unique switching point with ability types above the switching point preferring CCA in all markets. The vast majority of the bootstrap samples indeed produce a unique switching point in the predicted direction, i.e., lower-ability students prefer DCA while students with abilities above the switching point prefer CCA. In market 2, 77.5% of the bootstrap samples yield a unique bootstrapped

[15] Bootstrap confidence intervals are calculated by the percentile method (Efron, 1982). We perform block resampling to account for the dependence of observations within matching groups (see Davison and Hinkley, 1997). For each set of 50,000 bootstrap samples, we draw six random matching groups with replacement and calculate the bootstrap switching point for each market based on the polynomial smoothing of the observed utilities (we use lpoly in STATA with bandwidth 15 both for the bootstrap and for producing Figure 2.4) in the online appendix. We did not calculate bootstrap confidence intervals for market 5, because Figure 2.4 shows that there is no significant difference in the expected utility for high- and low-ability students in the two systems, as predicted.

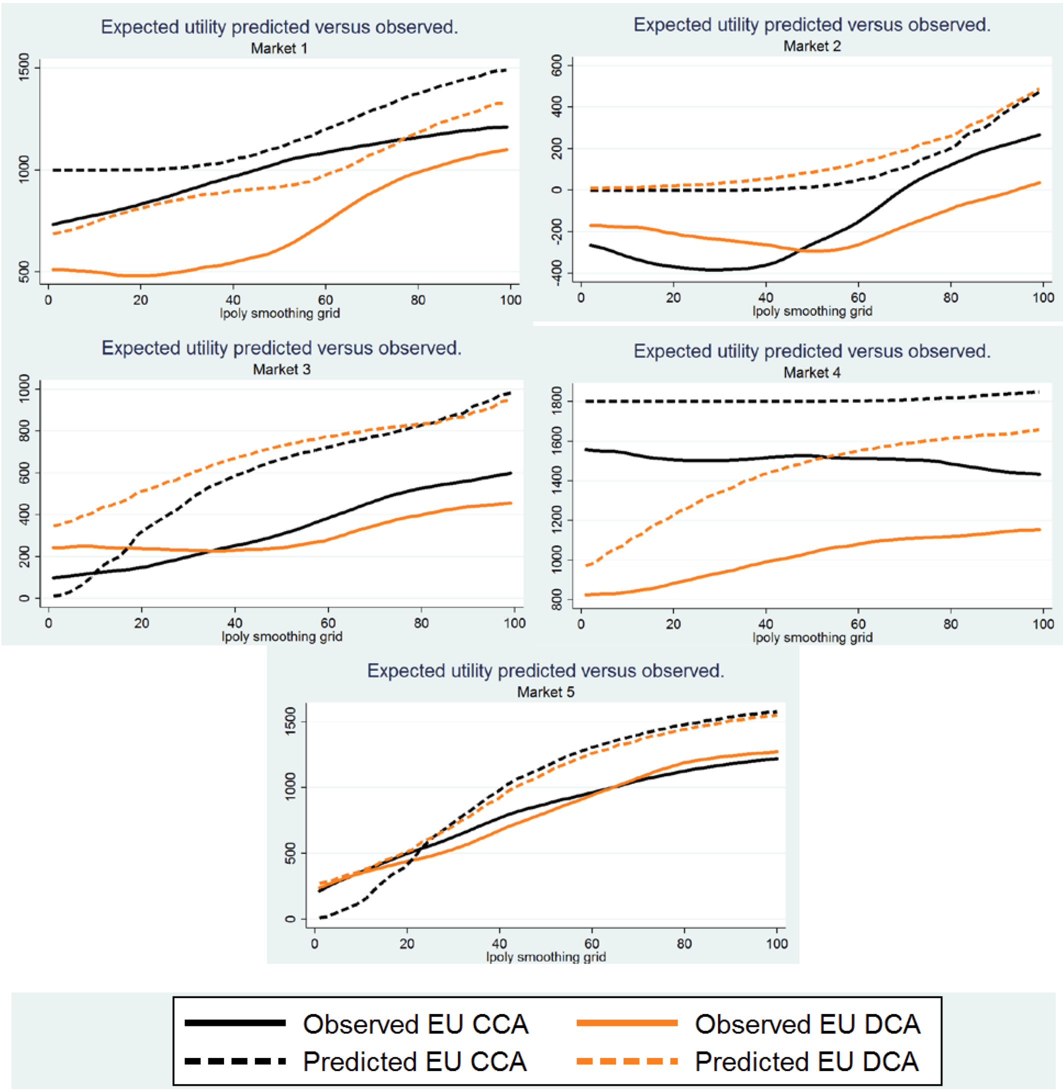


Figure 2.4. Predicted expected utilities and kernel regression of observed utilities by abilities.

switching point in the predicted direction.[16] In market 3, 80.1% of the bootstrap samples provide a bootstrapped switching point that is unique and in the predicted direction. Overall, the bootstrap samples confirm the single-crossing property of Proposition 3.[17]

Finally, we study the bootstrap distribution of the observed switching points where we restrict attention to bootstrap samples with a unique switching point in the predicted direction. The average switching point in market 2 is 48.4 with a 95% confidence interval of [30.5, 68.4], clearly indicating that the observed switching point is below the theoretical prediction of 100. In market 3, the average switching point is 37.6 with a 95% confidence interval of [9.2, 69.3], also indicating that the observed switching point is below the theoretical prediction of 81. Thus, we conclude that the observed switching points are significantly lower than the predicted switching points in markets 2 and 3, implying that students from a smaller range of abilities prefer DCA than the theory suggests.[18]

Point predictions for effort choices and utility

Next we test the point predictions regarding the utility and effort levels in CCA and DCA. This will help to gain a better understanding of the deviations from the predicted outcomes, in particular the relatively poor performance of DCA with respect to student utility.

Result 4 (Average utility and effort by markets): (i) The average utility is significantly lower than predicted across all markets and mechanisms. (ii) Average effort levels in the experiments are higher than the equilibrium efforts in all 10 markets. This overexertion of effort is significant in all five markets in DCA and in three out of five markets in CCA.

Support: Table 2.4 displays the equilibrium and observed averages for utility and effort levels by markets. Note that the average utility of subjects is significantly lower than predicted in all five markets and under both mechanisms. In addition, Figure 2.4 shows that average utility levels are always below the predicted level for all abilities. This is consistent with the fact that in all markets and mechanisms, average effort levels are higher than predicted, as can be taken from a comparison of columns (4) and (5) in Table 2.4.[19] In CCA the difference is significant for three

[16] In this market, a number of draws resulted in two switching points. This can be explained by the fact that in DCA two students with very low abilities of 2 and 6, respectively, took dominated effort choices by spending all their endowment, which resulted in a utility of -2200 points. In some bootstrap samples, these two observations shift the smoothed line of the expected utility in DCA below the line for CCA for the lowest ability types.

[17] See Table 2.7 in the online appendix for detailed results of bootstrapping and the number of switching points.

[18] See Figure 2.7 in the online appendix for a histogram of the bootstrapped switching points.

[19] Figure 2.6 in the online appendix depicts the observed efforts of individuals, the kernel regression estimation of efforts, and the equilibrium predictions for each of the markets and mechanisms. All 10 panels for the 10 markets show that the kernel of effort increases in ability, as predicted.

Table 2.4. Average utility and effort by markets

	Average equilibrium utility (1)	Average observed utility (2)	p-value obs.=pred. (3)	Average equilibrium efforts (4)	Average observed efforts (5)	p-value obs.=pred. (6)
CCA						
Market 1	1173	1001	0.04	230	276	0.19
Market 2	107	-122	0.01	364	389	0.54
Market 3	609	342	0.01	280	397	0.02
Market 4	1809	1507	0.01	35	191	0.01
Market 5	1011	809	0.02	305	400	0.05
DCA						
Market 1	975	715	0.01	262	362	0.02
Market 2	152	-169	0.00	309	410	0.00
Market 3	699	305	0.00	195	354	0.00
Market 4	1430	1014	0.00	125	340	0.00
Market 5	1019	797	0.00	307	395	0.00

Notes: Column (3) [(6)] shows the p-values for the significance of the constant when regressing the difference between (1) and (2) [(4) and (5)] on a constant, with standard errors clustered at the level of matching groups.

out of five markets (market 3, 4, and 5) while in DCA it is significant for all five markets. Thus, DCA leads to significant overexertion in more markets than CCA.

We also find that in spite of the negative results regarding the point predictions, the equilibrium effort levels have significant predictive power. This emerges from an OLS estimation of observed efforts based on clustered robust standard errors at the level of matching groups.[20] Furthermore, there is no significant difference with respect to the predictive power of the equilibrium efforts in the two mechanisms, as the interaction of the predicted effort and the dummy for CCA is not significant. Moreover, the regression confirms that there is on average less overexertion in CCA than in DCA, since the dummy for CCA is significant when controlling for equilibrium efforts.

Sorting of students

In a next step, we study how students sort across colleges with respect to their ability. In particular we ask whether the best students end up at the good college 2, the lower-ability students receive a seat at the bad college 1, and the students with the lowest ability are unassigned. In equilibrium, sorting by ability is always perfect in CCA while it is likely to be imperfect in DCA. Equilibrium miscoordination in DCA is due to the mixed strategy of low-ability students and the possibility that the number of students with realized abilities below and above the cutoff does not correspond to the number of seats in the two colleges. As a conse-

Moreover, the observed effort levels typically lie above the predicted values, except for high-ability students in a few markets.

[20] See Table 2.8 in the online appendix.

quence, miscoordination in DCA can lead to more unassigned students and less sorting by ability than in CCA.

Before investigating the average ability levels at the colleges, we study the choice of participants to apply to college 1 or college 2 in DCA. Recall that the symmetric Bayesian Nash equilibrium characterized in Theorem 1 has the property that students with an ability above the cutoff should always apply to the better college (college 2) whereas students with an ability below the cutoff should mix between the two colleges.

Result 5 (Choice of college in DCA): In DCA, students above the equilibrium ability cutoff choose the good college 2 more often than students below the cutoff. However, high-ability students apply to the good college significantly less often than predicted in all markets while low-ability students apply to the good college more often than predicted (significant in three markets).

Support: Table 2.5 displays the equilibrium cutoff ability for each market in column (1). In column (2) it provides the average equilibrium probability of choosing the good college 2 for students with abilities below the cutoff in the respective markets. This average is calculated given the actual realization of abilities in the experiment. It can be compared to the observed frequency of choosing the good college by these students in column (3) and the 95% confidence intervals with standard errors clustered at the level of matching groups in column (4). It emerges that subjects below the cutoff choose the good college 2 more often than predicted in all five markets. The difference is significant for markets 1, 3, and 5. Column (5) displays the proportion of subjects above the cutoff applying to college 2, followed by the 95% confidence interval with standard errors clustered at the level of matching groups in column (6). Note that in equilibrium these high-ability students should apply to college 2 with certainty, but we can reject this hypothesis in all five markets.[21] Finally, the last column of Table 2.5 presents the p-values for the Wilcoxon rank-sum test of equality of the distributions of the choice of college 2 below and above the market-specific equilibrium cutoff based on averages of six matching groups. In all markets except market 4, the differences are significant at the 1% significance level, and the difference is marginally significant for market 4. Further evidence of the predictive power of the model is provided by a probit regression of the observed choices of college 2. The coefficient for the equilibrium probability of choosing the good college is significant.[22] Thus we conclude that the choices of the subjects reflect the predicted equilibrium pattern, but that the point predictions fail.

[21] In markets 1, 2, and 5 the observed proportions are close to the equilibrium. In market 3 fewer high-ability students choose the good college, which may be due to the large bad college (eight seats) relative to the good college (two seats). In market 4, the relatively low proportion of high-ability students applying to the good college may be driven by the similarity of payoffs for both colleges (1,800 points versus 2,000 points).

[22] See Table 2.9 in the online appendix. The same table shows that there is no gender difference in the choice of the good college.

Table 2.5. Proportion of choices of good college 2

	Equ. ability cutoff	Equ. prop. of choices of college 2 below the cutoff	Obs. prop. of choices of college 2 below the cutoff mean	95% conf. int	Obs. prop. of choices of college 2 above the cutoff mean	95% conf. int	p-values for equality of prop. above and below the cutoff
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Market 1	50	13%	33%	[25%-44%]	85%	[75%-92%]	0.00
Market 2	85.5	43%	51%	[41%-61%]	92%	[77%-98%]	0.00
Market 3	85.5	15%	27%	[20%-36%]	68%	[49%-82%]	0.00
Market 4	89.5	16%	17%	[11%-27%]	42%	[21%-67%]	0.07
Market 5	23.5	51%	64%	[54%-72%]	91%	[84%-95%]	0.00

Notes: Column (7) displays the p-values of the Wilcoxon rank-sum test for equality of the distributions, based on averages of the six matching groups. Confidence intervals are estimated with standard errors clustered at the level of matching groups. "Equ" stands for equilibrium, "obs" for observed, "prop." for proportion.

In order to better understand why the point predictions fail, we investigate the application decision of students by ability. Figure 2.5 presents the choices of subjects in DCA by markets and ability quantiles, together with the equilibrium predictions. Students above the equilibrium cutoff in markets 1, 2, and 5 choose the good college 2 almost certainly, in line with the theory. The proportions of choices of students with low ability are also close to the equilibrium mixing probabilities. The biggest difference between the observed and the equilibrium proportions is due to students who are slightly above or below the cutoff. This finding is particularly evident in markets 1, 2, and 4. Remember that the equilibrium is characterized by a discontinuity regarding the probability of the choice of college 2: students with abilities just above the cutoff have a pure strategy of choosing college 2, while students just below the cutoff choose college 1 with almost 100% probability. Not surprisingly, the choices of universities by our subjects are smooth around the cutoff. In line with this, we also do not observe the predicted kink in the effort choices shown in Figure 2.6 in Appendix 2.C. These findings can be due to the fact that students with an ability level around the cutoff under- or overestimate the cutoff, which would result in the observed smoothing.

As a final step, we compare CCA and DCA with respect to the resulting average abilities of the students in each college. Panels A, B, and C of Table 2.6 present the equilibrium and observed average abilities of students assigned to the good and bad college and of unassigned students, respectively.[23] Panel D presents the equilibrium and the observed percentage of unfilled seats by markets.

Result 6 (Composition of colleges): (i) (Good college) There is no significant dif-

[23] The equilibrium assignment in CCA is straightforward to calculate given the ability draws. For DCA the choice of the college is random for students below the ability cutoff. We generate one realization of the choice of the college for all abilities below the cutoff, given the equilibrium probabilities. The resulting equilibrium allocation is determined and used for the calculation in this table.

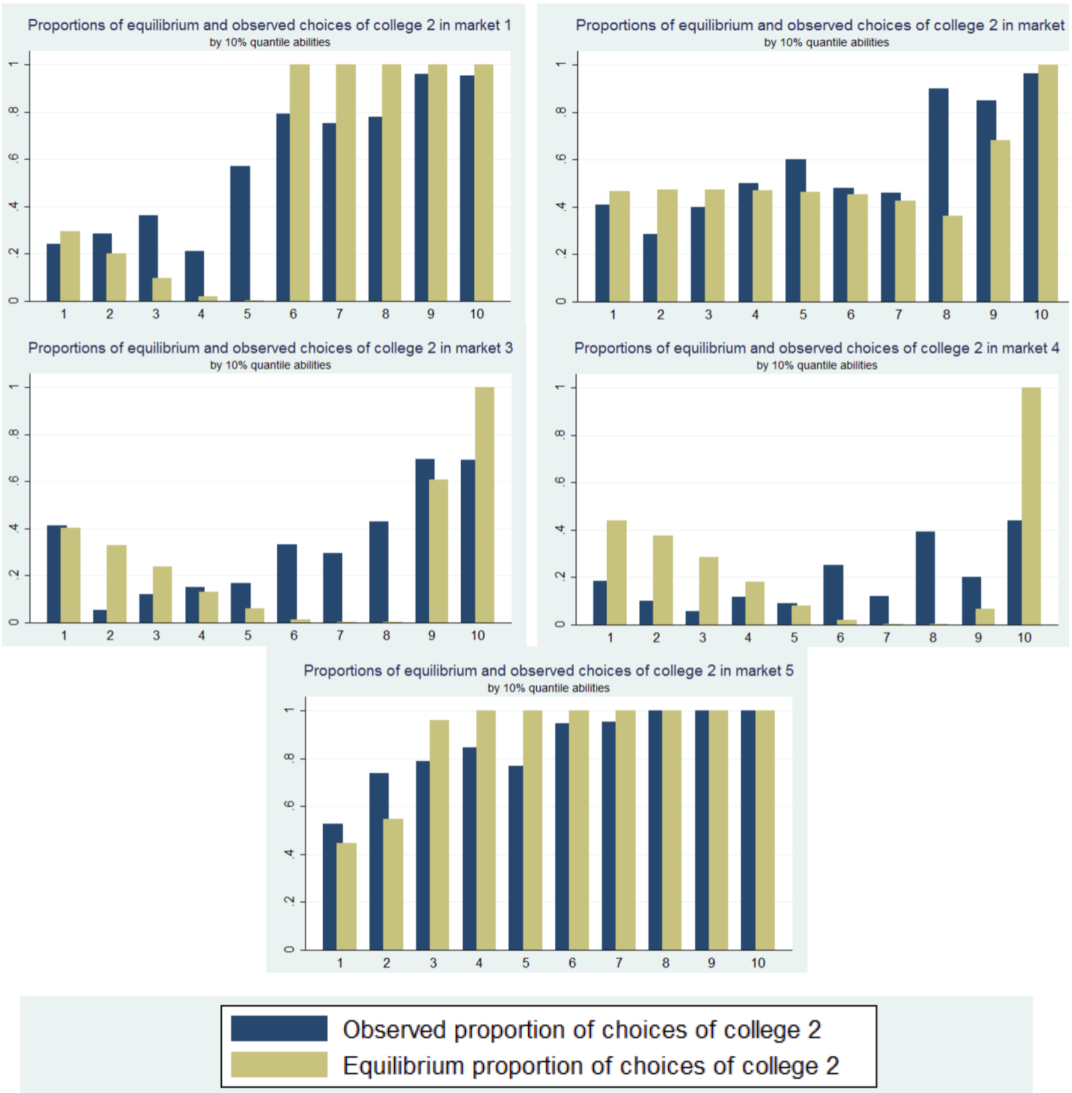


Figure 2.5. Choice of college in DCA

Table 2.6. Average abilities of students and unfilled seats by colleges

		Market				
		1	2	3	4	5
Panel A	Assigned to good college, equil.					
	CCA (1)	74.4	91.4	91.9	86.9	62.0
	DCA (2)	73.8	89.6	82.7	54.5	61.2
	CCA=DCA, equil., p-value (3)	*0.69*	*0.69*	*0.05*	*0.00*	*0.93*
	Assigned to good college, observed					
	CCA (4)	66.2	80.2	84.5	65.6	58.2
	DCA (5)	67.2	82.7	80.9	66.1	59.1
	CCA=DCA, observed, p-value (6)	*0.81*	*0.87*	*0.52*	*0.87*	*0.63*
	CCA observed=CCA equil., p-value (7)	*0.07*	*0.02*	*0.02*	*0.00*	*0.04*
	DCA observed=DCA equil., p-value (8)	*0.04*	*0.02*	*0.63*	*0.06*	*0.13*
Panel B	Assigned to bad college, equil.					
	CCA (1)	25.3	77.2	52.4	38.1	24.0
	DCA (2)	27.2	72.3	50.8	51.5	42.0
	CCA=DCA, equil., p-value (3)	*0.63*	*0.08*	*0.26*	*0.00*	*0.34*
	Assigned to bad college, observed					
	CCA (4)	33.5	75.3	52.4	45.2	40.9
	DCA (5)	31.0	43.7	45.5	49.6	34.5
	CCA=DCA, observed, p-value (6)	*0.42*	*0.01*	*0.04*	*0.15*	*0.52*
	CCA observed=CCA equil., p-value (7)	*0.00*	*0.70*	*1.0*	*0.00*	*0.05*
	DCA observed=DCA equil., p-value (8)	*0.40*	*0.00*	*0.05*	*0.29*	*0.04*
Panel C	Not assigned, equil.					
	CCA (1)		35.3	10.6		10.9
	DCA (2)	25.4	37.2	29.7	12.2	16.9
	CCA=DCA, equil., p-value (3)	*N/A*	*0.47*	*0.01*	*N/A*	*0.01*
	Not assigned, observed					
	CCA (4)	-	38.5	18.0	-	18.5
	DCA (5)	39.8	45.8	49.1	25.6	20.5
	CCA=DCA, observed, p-value (6)	*N/A*	*0.04*	*0.01*	*N/A*	*0.52*
	CCA observed=CCA equil., p-value (7)	*N/A*	*0.26*	*0.09*	*N/A*	*0.09*
	DCA observed=DCA equil., p-value (8)	*0.32*	*0.02*	*0.01*	*0.14*	*0.14*
Panel D	Percentage of unfilled seats, equil.					
	CCA (1)	0%	0%	0%	0%	0%
	DCA (2)	12.0%	1.4%	3.3%	5.1%	2.2%
	Percentage of unfilled seats, observed					
	CCA (3)	0%	0%	0%	0%	0%
	DCA (4)	10.2%	1.4%	7.8%	9.3%	2.8%

Notes: Rows (3) and (6) of panels A, B, and C display the p-values of the Wilcoxon rank-sum test for equality of the distributions, based on averages of the six matching groups. Rows (7) and (8) of panels A , B, and C display the p-values of t-test of equality of the averages of the six matching groups and the predicted constant value.

ference in the average ability of students in CCA and DCA. This is in line with the theory except for markets 3 and 4 where a significantly higher ability of students in CCA is predicted. (ii) (Bad college) Ability levels are not significantly different in markets 1 and 5, as predicted. The average ability of students in DCA is significantly lower than predicted and than in CCA in markets 2 and 3.

Support: We consider each market separately and mainly refer to rows (3) and (6) in panels A and B of Table 2.6. In markets 1 and 5, both the theory and the experimental data show no significant difference between ability levels in the good and bad college when comparing CCA with DCA.

In market 4 where the two colleges have almost the same value for the students, the average ability of students in the good college is predicted to be significantly lower in DCA, and conversely, the average ability is predicted to be higher in the bad college in DCA. We fail to observe this significant difference for both colleges because the average ability levels at both colleges are more similar than predicted under both mechanisms. Thus, there is no sorting advantage of CCA in market 4, other than predicted.

In markets 2 and 3, the observed abilities of students assigned to the bad college are significantly higher in CCA than in DCA (see row (6) of Panel B). In equilibrium the difference has the same sign but is much smaller and is not significant. Thus, in DCA low-ability students have a better chance than predicted of being admitted to the bad college in markets 2 and 3, at the cost of some high-ability students who remain unassigned (cf. rows (2) and (5) for markets 2 and 3 in Panel C). The reason for abilities being higher at the bad college in CCA than in DCA in these markets is due to a purely mechanical effect: in both mechanisms, students with abilities lower than predicted are able to get a seat in the bad college, due to imperfect sorting. But CCA allows high-ability students who are unable to get into the good college to obtain a seat in the bad college. This raises the average ability in the bad college compared to DCA where the students who are unsuccessful at the good college remain unassigned.

Table 2.6 also reports on the point predictions for each market separately, with test results in rows (7) and (8) of Panels A, B, and C. The point predictions are rejected in more than half of the cases, but we refrain from discussing them here in detail since our main focus is on the comparison of the two mechanisms.

2.7.3 Discussion

In this section we discuss possible explanations of the observed deviations from equilibrium behavior. Overbidding is a common finding in all-pay auction experiments (see Barut et al., 2002, and Noussair and Silver, 2006) and our results confirm this in the well-known context of a single contest with multiple prizes (CCA), but we also show it to hold in parallel contests (DCA). Our experiments allow us to compare the two mechanisms, and our main result is the relative unattractiveness of DCA relative to CCA, even in markets where it should be preferred by all students.

One candidate to explain the difference between predicted and observed utility levels in the two mechanisms is the number of unfilled seats in DCA. If students coordinate worse than predicted in equilibrium, the attractiveness of DCA is reduced relative to CCA. Table 2.6, Panel D presents the equilibrium and observed shares of unassigned seats by markets. The share of unfilled seats in DCA is somewhat higher than in equilibrium only in markets 3 and 4, and the difference is small. Thus, unfilled seats can at best partially explain the unattractiveness of DCA in our experiment relative to the equilibrium predictions.

Apart from welfare losses due to seats remaining unfilled, the aggregate welfare of students is affected by their choice of effort levels. Thus, overexertion of effort in DCA relative to CCA is a potential explanation. Inspecting the observed and predicted average effort levels in the two markets where DCA should be preferable for students (markets 2 and 3, see Table 2.4), it emerges that overbidding is more pronounced in DCA.[24] Moreover, it can be taken from Table 2.4 that efforts fail to be significantly higher in CCA than in DCA in market 3, in contrast to the prediction. Thus, the fact that DCA leads to more overbidding in markets 2 and 3 destroys its relative advantage for the students in these two markets.

To understand the structure of overbidding in DCA, we investigate whether the students condition their effort choice on the choice of the college in DCA.[25] Note that in equilibrium, students with abilities below the cutoff choose to apply to the good and the bad college with a certain probability, while they exert the same effort irrespective of the college choice. However, we observe that participants tend to exert higher effort when applying to the good as compared to the bad college.[26] We even observe differences in effort of high-ability students, depending on their choice of college although they should apply to the good college in equilibrium. Some of these students underbid, especially when applying to the bad college, but these are relatively rare instances. To sum up, relative overbidding in DCA goes along with students conditioning their effort choice on the choice of the college.

The level-k model and limited depth of reasoning provides a parsimonious explanation for overexertion of effort in DCA relative to CCA. Moreover, it can

[24] In market 2, average observed efforts and equilibrium efforts differ by (389-364)=25 points in CCA while the difference is 101 in DCA; similarly for market 3 with average overbidding of 117 in CCA and 159 in DCA.

[25] Table 2.10 in the online appendix presents the average overbidding in terms of cost of effort by markets for students above and below the equilibrium ability cutoff. One unit of cost of effort corresponds to 50 cents, thus the maximum effort is 11 units. We use the costs of effort instead of effort in order to control for the scale of the overbidding that depends on ability. Presenting this data in units of effort would not be informative as the same deviation of, say, 100 units of effort means a small deviation for a high-ability student but a very large deviation for a low-ability student.

[26] The difference is significant in two markets for low-ability students. Column 4 of Table 2.10 in the online appendix presents the p-values for the significance of the dummy variable for applying to the good college when regressing overbidding in money terms on the dummy and a constant for abilities below the theoretical cutoff in DCA (with standard errors clustered at the level of matching groups).

organize our other main findings. To see this, fix level-0 students as students who randomly choose the college to apply to in DCA. Thus, level-0 students do not sort according to ability. However, level-0 students apply to colleges in proportion to the number of seats at the college, i.e., they take into account the relative size of the two colleges. Furthermore, assume that level-0 students choose effort levels randomly from the interval of possible effort choices that is determined by their ability. This results in effort choices being proportional to ability.

Now consider level-1 students who play a best response to the belief that all other students are level-0, as described above. First, if the distribution of abilities is the same at the good and at the bad college and if the number of applicants is proportional to the number of seats, the good college is more attractive. This explains our observation that low-ability students apply to the good college more often than predicted. Regarding the effort level chosen by level-1 students, they will choose a different effort level at both colleges, namely a higher effort at the good college than at the bad college, because the competition is the same but the reward is higher at the good college. Again, this is reflected in our data.

Moreover, the level of effort is higher than in equilibrium if the randomization (or proportional rule) that level-1 players believe level-0 players to adopt yields higher than equilibrium efforts. This is due to the strategic complementarity of effort. The level-k model also predicts that the effort level of the highest-ability types may be lower than in equilibrium. This is the case if equilibrium efforts are higher than average random efforts, which is caused by competition for the good college being weaker than in equilibrium due to worse sorting. We observe this for markets with a strong competition for seats at the good college, namely markets 2 and 3.[27]

Finally, note that in the case of CCA, level-1 players overexert or underexert effort depending on whether the average random effort is smaller or greater than the equilibrium effort. For low enough abilities (i.e., not the very high types), the average random effort is always higher than the equilibrium effort, thus level-1 players overexert. In DCA overexertion is higher than in CCA for these types, as they exert similar efforts, but the predictions of efforts in DCA are much lower than in CCA for middle types around the cutoff. This also explains why we observe the biggest violations of equilibrium efforts for these middle types.

Another candidate explanation for the overexertion of effort especially in DCA is risk-aversion. Although we cannot provide a full analysis of this case due to technical difficulties, we would like to elaborate on the possible effects of risk aversion in our setup. Fibich et al. (2006) have shown that in a single contest, players with high values bid higher than they would have bid in the risk-neutral case (as compared to low-value bidders who bid less). The intuitive reason for this is that bidders who bid more have more to lose in case of not winning the

[27] To illustrate these points, the graphs of the random, predicted, and observed efforts (all smoothed) in market 2 can be found in the online appendix, Figure 2.8.

prize, due to concave utility functions. Let us use this intuition to compare the overexertion of effort in CCA versus DCA. In CCA, a high-ability student can get a high prize (v_2), a low prize (v_1), or no prize (0), whereas in DCA she would get either a high prize (v_2) or no prize (0). Therefore, in CCA just failing to win a high prize would still give this bidder a low prize, whereas this would result in no prize in DCA. In other words, this bidder has more to lose in a decentralized mechanism. Hence, we could expect that an overexertion of effort would be more pronounced in DCA than in CCA.

2.8 Conclusion

In this paper, we study college admissions exams which concern millions of students every year throughout the world. Our model abstracts from many aspects of real-world college admission games and focuses on the following two important aspects: (i) colleges accept students by considering student exam scores, (ii) students have differing abilities which are their private information, and the costs of getting ready for the exams are inversely related to ability levels. We focus on two extreme policies that capture practices in a number of countries. In the centralized model students can freely and without cost apply to all colleges, whereas in the decentralized mechanism students can only apply to one college. We consider a model that is as simple as possible by assuming two colleges and homogeneous student preferences over colleges in order to derive analytical results as Bayesian Nash solutions to the two mechanisms.[28]

The solution of the centralized admissions mechanism follows from standard techniques in the contest literature. The solution to the decentralized model, on the other hand, has interesting properties such as lower ability students using a mixed strategy when deciding which college to apply to. Our main theoretical result is that low- and high-ability students differ in terms of their preferences between the two mechanisms where high-ability students prefer the centralized mechanism and low-ability students the decentralized mechanism.

We employ experiments to test the theory and to develop insights into the functioning of centralized and decentralized mechanisms that take into account behavioral aspects. Overall, many predictions of the theory are supported by the data, despite a few important differences. We find that in our markets with an equal number of seats and applicants, the centralized mechanism is better for all applicants, as predicted by the theory. Again in line with the theory we observe that in the markets with an overdemand for seats, low-ability students prefer a decentralized admissions mechanism whereas high-ability students prefer a centralized mechanism. However, in these markets the predicted superiority of the decentralized mechanism for the students is weaker than predicted. Thus, only a smaller group of (low-ability) students than predicted profits from the

[28] We also discuss the extension to more colleges in section 6.

decentralized system. This can be ascribed to one robust and stark difference between theory and observed behavior, namely overexertion of effort, which is more pronounced in the decentralized mechanism. Moreover, the decentralized mechanism leads to less sorting by ability and to more high-ability students being unassigned, both compared to the centralized mechanism and compared to the equilibrium prediction. Overall, our findings resonate with a number of countries having moved from a decentralized to a more centralized procedure in the past years, e.g., Russia and other former Soviet states as well as South Korea.

For the evaluation of the two mechanisms from a welfare perspective, it matters whether the effort spent preparing for the exam has no benefits beyond improving the performance in the exam or whether this effort is useful. If effort is purely a cost, then welfare can be measured by the mean utility of the students. In all our markets, the centralized mechanism outperforms the decentralized mechanism with respect to this criterion. However, if the effort exerted by the students increases their productivity, then the decentralized mechanism becomes relatively more attractive, where efforts are weakly higher than in the centralized mechanism across markets.

Appendix 2.A Preliminaries

The following lemmata are useful for the results given in the rest of the Appendix.

Lemma 1. *Let l, m be given integers. Then,*

$$\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{j,m-j}(x)\Big) &= -m\, p_{l,m-l-1}(x) \quad \textit{when } 0 \le l < m, \\
\frac{d}{dx}\Big(\sum_{j=l}^{m} p_{j,m-j}(x)\Big) &= m\, p_{l-1,m-l}(x) \quad \textit{when } 0 < l \le m,
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{m-j,j}(x)\Big) &= m\, p_{m-l-1,l}(x) \quad \text{when } 0 \le l < m, \\
\frac{d}{dx}\Big(\sum_{j=l}^{m} p_{m-j,j}(x)\Big) &= -m\, p_{m-l,l-1}(x) \quad \text{when } 0 < l \le m.
\end{aligned}$$

Proof. We use the following equation:

$$\begin{aligned}
\binom{m}{j-1}(m-j+1) &= \frac{m!}{(j-1)!(m-j+1)!}(m-j+1) \\
&= \frac{m!}{(j-1)!(m-j)!} = \binom{m}{j} j. \qquad (2.10)
\end{aligned}$$

The first formula: Suppose $0 = l$. Then, $\sum_{j=0}^{l} p_{j,m-j}(x) = p_{0,m}(x) = (1-x)^m$. Its derivative is $-m(1-x)^{m-1} = -m\, p_{0,m-1}(x)$. Thus the formula holds. Consider another case where $0 < l$. Then we have

$$\begin{aligned}
&\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{j,m-j}(x)\Big) \\
&\qquad = \frac{d}{dx}\Big(\sum_{j=0}^{l}\binom{m}{j}x^j(1-x)^{m-j}\Big) \\
&\qquad = \sum_{j=1}^{l}\binom{m}{j}\,j\,x^{j-1}(1-x)^{m-j} - \sum_{j=0}^{l}\binom{m}{j}(m-j)x^j(1-x)^{m-j-1} \\
&\qquad = \sum_{j=1}^{l}\binom{m}{j}\,j\,x^{j-1}(1-x)^{m-j} \\
&\qquad - \sum_{j=1}^{l+1}\binom{m}{j-1}(m-j+1)x^{j-1}(1-x)^{m-j} \\
&\qquad = \sum_{j=1}^{l}\binom{m}{j}\,j\,x^{j-1}(1-x)^{m-j} - \sum_{j=1}^{l+1}\binom{m}{j}\,j\,x^{j-1}(1-x)^{m-j} \\
&\qquad\quad \text{(by (2.10))}
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{j,m-j}(x)\Big) &= -\binom{m}{l+1}(l+1)\,x^l(1-x)^{m-l-1} \\
&= -\frac{m!}{l!(m-l-1)!}x^l(1-x)^{m-l-1} \\
&= -m\frac{(m-1)!}{l!(m-l-1)!}x^l(1-x)^{m-l-1} = -m\,p_{l,m-l-1}(x).
\end{aligned}$$

The second formula: Suppose $l = m$. Then, $\sum_{j=l}^{m} p_{j,m-j}(x) = p_{m,0}(x) = x^m$. Its derivative is $mx^{m-1} = mp_{m-1,0}(x)$. Thus the formula holds. Consider another case where $l < m$. Then we have

$$
\begin{aligned}
&\frac{d}{dx}\Big(\sum_{j=l}^{m} p_{j,m-j}(x)\Big) \\
&\qquad = \frac{d}{dx}\Big(\sum_{j=l}^{m}\binom{m}{j}x^j(1-x)^{m-j}\Big) \\
&\qquad = \sum_{j=l}^{m}\binom{m}{j}\, j\, x^{j-1}(1-x)^{m-j} \\
&\qquad - \sum_{j=l}^{m-1}\binom{m}{j}(m-j)x^j(1-x)^{m-j-1} \\
&\qquad = \sum_{j=l}^{m}\binom{m}{j}\, j\, x^{j-1}(1-x)^{m-j} \\
&\qquad - \sum_{j=l+1}^{m}\binom{m}{j-1}(m-j+1)x^{j-1}(1-x)^{m-j} \\
&\qquad = \sum_{j=l}^{m}\binom{m}{j}\, j\, x^{j-1}(1-x)^{m-j} \\
&\qquad - \sum_{j=l+1}^{m}\binom{m}{j}\, j\, x^{j-1}(1-x)^{m-j} \quad \text{(by (2.10))}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{j,m-j}(x)\Big) &= \binom{m}{l}\, l\, x^{l-1}(1-x)^{m-l} = \frac{m!}{(l-1)!(m-l)!}x^{l-1}(1-x)^{m-l} \\
&= m\frac{(m-1)!}{(l-1)!(m-l)!}\, x^{l-1}(1-x)^{m-l} = m\, p_{l-1,m-l}(x).
\end{aligned}
$$

The third formula: By the second formula, we have

$$
\frac{d}{dx}\Big(\sum_{j=0}^{l} p_{m-j,j}(x)\Big) = \frac{d}{dx}\Big(\sum_{j=m-l}^{m} p_{j,m-j}(x)\Big) = m\, p_{m-l-1,l}(x).
$$

The fourth formula: By the first formula, we have

$$
\frac{d}{dx}\Big(\sum_{j=l}^{m} p_{m-j,j}(x)\Big) = \frac{d}{dx}\Big(\sum_{j=0}^{m-l} p_{j,m-j}(x)\Big) = m\, p_{m-l,l-1}(x).
$$

■

Appendix 2.B On Equilibria of Decentralized College Admissions

2.B.1 On properties of monotone and symmetric equilibrium of decentralized college admissions

We focus on symmetric and monotone equilibrium. More specifically, each student will use the same probability mixing function $\gamma(a)$, and the same effort function $\beta_i(a)$ while applying to college $i \in \{1, 2\}$. Moreover, for all values β_i is defined (i.e., for all types a which apply to college i with positive probability) $\beta_i(a)$ is increasing in a and $\gamma(a)$ is integrable (continuous except for a zero measure set). We define

$$\pi(a) = \int_0^a \gamma(x) f(x)\, dx.$$

We then define the conditional distributions

$$H^1(a) = \frac{\pi(a)}{\pi(1)} \text{ and } H^2(a) = \frac{F(a) - \pi(a)}{1 - \pi(1)}$$

We then define the probability of being in the top q_i among applicants to college i by $K_i(a)$ and its corresponding density by $k_i(a)$. That is, we have

$$\begin{aligned}
K_1(a) &= \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(1)) + \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(1)) H^1_{m-q_1+1,m}(a), \\
K_2(a) &= \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(1)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(1)) H^2_{m-q_2+1,m}(a).
\end{aligned}$$

We first prove that there cannot be a pure strategy equilibrium (i.e., $\gamma(a) \in \{0, 1\}$ for all $a \in [0, 1]$).

Proposition 4. *There cannot be a pure-strategy symmetric and monotone equilibrium.*

Proof. By method of contradiction, suppose that there is one. It is easy to see that the measure of both $\{a \in [0, 1] : \gamma(a) = 0\}$ and $\{a \in [0, 1] : \gamma(a) = 1\}$ are strictly positive. Then there exist c, d with $0 < c < d \leq 1$ such that all $a \in [0, c]$ applies to college i and all $a \in (c, d]$ applies to college j. Then it is easy to see that (i) $\beta_i(c) > 0$ and $\lim_{a \downarrow c} \beta_j(a) = 0$, and (ii) $v_i K_i(c) - \frac{\beta_i(c)}{c} = v_j K_j(c)$. Now, we argue that any $a \in \left(c, \beta_j^{-1}(\beta_i(c))\right)$ would be strictly better off by "mimicking" type c. To see this, consider

$$f(a) \equiv v_i K_i(c) - \frac{\beta_i(c)}{a} - \left(v_j K_j(a) - \frac{\beta_j(a)}{a}\right)$$

which represents the gain from mimicking type c. We obtain $f(a) > 0$ for all

$a \in \left(c, \beta_j^{-1}\left(\beta_i\left(c\right)\right)\right)$ by noting that $f\left(c\right) = 0$ and

$$f'\left(a\right) = \frac{\beta_i\left(c\right)}{a^2} - \frac{\beta_j\left(a\right)}{a^2} > 0$$

by the envelope theorem. ■

We then show that in any mixed strategy equilibrium, $\gamma\left(a\right) \in \left(0, 1\right)$ implies that $\beta_1\left(a\right) = \beta_2\left(a\right)$.

Proposition 5. *If $\gamma\left(a\right) \in \left(0, 1\right)$, then $\beta_1\left(a\right) = \beta_2\left(a\right)$.*

Proof. Let $a \in [0, 1]$ such that $\gamma(a) \in (0, 1)$. There is an interval $I = [\underline{a}, \overline{a}]$ such that $a \in I$ and for all $b \in I$, $\gamma\left(b\right) \in \left(0, 1\right)$. Then, for all $b \in I$, since type b is indifferent applying to colleges 1 and 2, we have

$$EU_1\left(b\right) \equiv v_1 K_1\left(b\right) - \frac{\beta_1\left(b\right)}{b} = v_2 K_2\left(b\right) - \frac{\beta_2\left(b\right)}{b} \equiv EU_2\left(b\right).$$

Since the first-order conditions imply

$$\beta_i\left(b\right) = v_i \int_{\underline{a}}^{b} x k_i\left(x\right) dx + D_i = v_i \left(K_i\left(b\right) b - K_i(\underline{a})\underline{a} - \int_{\underline{a}}^{b} K_i\left(x\right) dx\right) + D_i,$$

where D_i is a constant. Thus

$$\begin{aligned} EU_i(b) &= v_i K_i(b) - v_i K_i(b) + v_i K_i(\underline{a})\frac{\underline{a}}{b} + v_i \frac{\int_{\underline{a}}^{b} K_i(x) dx}{b} - \frac{D_i}{b} \\ &= v_i K_i(\underline{a})\frac{\underline{a}}{b} + v_i \frac{\int_{\underline{a}}^{b} K_i(x) dx}{b} - \frac{D_i}{b}. \end{aligned}$$

Then, as $EU_1(\underline{a}) = EU_2(\underline{a})$, we have

$$v_1 K_1(\underline{a}) - \frac{D_1}{\underline{a}} = v_2 K_2(\underline{a}) - \frac{D_2}{\underline{a}}. \tag{2.11}$$

Moreover, for all $b \in I$, as $EU_1(b) = EU_2(b)$,

$$\begin{aligned} & v_1 K_1(\underline{a})\frac{\underline{a}}{b} + v_1 \frac{\int_{\underline{a}}^{b} K_1(x) dx}{b} - \frac{D_1}{b} = v_2 K_2(\underline{a})\frac{\underline{a}}{b} + v_2 \frac{\int_{\underline{a}}^{b} K_2(x) dx}{b} - \frac{D_2}{b} \\ \Rightarrow\ & v_1 K_1(\underline{a}) + v_1 \frac{\int_{\underline{a}}^{b} K_1(x) dx}{\underline{a}} - \frac{D_1}{\underline{a}} = v_2 K_2(\underline{a}) + v_2 \frac{\int_{\underline{a}}^{b} K_2(x) dx}{\underline{a}} - \frac{D_2}{\underline{a}} \\ \Rightarrow\ & v_1 \frac{\int_{\underline{a}}^{b} K_1(x) dx}{\underline{a}} = v_2 \frac{\int_{\underline{a}}^{b} K_2(x) dx}{\underline{a}} \quad (\because (2.11)) \\ \Rightarrow\ & v_1 \int_{\underline{a}}^{b} K_1(x) dx = v_2 \int_{\underline{a}}^{b} K_2(x) dx. \end{aligned} \tag{2.12}$$

Thus, we have

$$\text{for all } b \in I,\ v_1 K_1(b) = v_2 K_2(b). \tag{2.13}$$

Therefore, using the equalities (2.11), (2.12), and (2.13), we can conclude that for all $b \in I$, $\beta_1(b) = \beta_2(b)$. Hence,

$$\beta_1(a) = \beta_2(a).$$

■ Hence, it is without loss of generality that we focus on the equilibria in the main body: when students mix between applying to colleges, they choose the same effort level while applying to either college.

2.B.2 Derivation of the symmetric equilibrium

We show how to obtain the function $\gamma : [0, c] \to (0, 1)$ and the cutoff c from Equation (2.6).

Step 1: We show that there is a unique value $\pi(c)$ that satisfies Equation (2.7). Define a function $\varphi_1 : [0, 1] \to R$: for each $x \in [0, 1]$,

$$\varphi_1(x) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(x) - v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(x).$$

Differentiate φ_1 at each $x \in (0, 1)$: using Lemma 1, we have

$$\varphi_1'(x) \quad = \quad v_2(n-1)\, p_{(n-1)-(q_2-1)-1,q_2-1}(x) + v_1(n-1)\, p_{q_1-1,(n-1)-(q_1-1)-1}(x) > 0.$$

Thus, φ_1 is strictly increasing. Moreover, $\varphi_1(0) = -v_1 < 0$ and $\varphi_2(1) = v_2 > 0$. Thus, since φ_1 is a continuous function on $[0, 1]$, there is a unique $x^* \in (0, 1)$ such that $\varphi_1(x^*) = 0$. Thus, since $\varphi_1(\pi(c)) = 0$ by (2.7), there is a unique $\pi(c) \in (0, 1)$ that satisfies Equation (2.7).

Step 2: Given a unique $\pi(c)$, we now show that there is a unique cutoff $c \in (0, 1)$. In Equation (2.8), since $\pi(c)$ is known by Step 1, the the only unknown is c via $F(c)$. Define a function $\varphi_2 : [\pi(c), 1] \to R$ as follows: for each $x \in [\pi(c), 1]$,

$$\begin{aligned}\varphi_2(x) \quad &= \quad v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) \\ &+ \quad v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{x-\pi(c)}{1-\pi(c)}\Big) - v_1.\end{aligned}$$

Differentiate φ_2 at each point $x \in (\pi(c), 1)$: using Lemma 1, we have

$$\varphi_2'(x) \;=\; v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))\Big(\frac{1}{1-\pi(c)}\Big)\, m\, p_{m-q_2,q_2-1}\Big(\frac{x-\pi(c)}{1-\pi(c)}\Big) > 0.$$

Thus, φ is strictly increasing. Moreover, $\varphi_2(1) = v_2 - v_1 > 0$ and

$$\begin{aligned}
\varphi_2(\pi(c)) &= v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) \\
&+ v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}(0) - v_1 \\
&= v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) \\
&- v_1 \quad (\because p_{j,m-j}(0) = 0 \text{ for } j \geq m - q_2 + 1 \geq 1) \\
&= v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) - v_1 \quad (\because (2.7)) \\
&< 0.
\end{aligned}$$

Therefore, there is a unique $x^* \in (\pi(c), 1)$ such that $\varphi_2(x^*) = 0$. Since $\varphi_2(F(c)) = 0$, $x^* = F(c)$. Thus, since F is strictly increasing, there is a unique cutoff $c \in (F^{-1}(\pi(c)), 1)$ such that $c = F^{-1}(x^*)$.

Step 3: From steps 1 and 2, $\pi(c)$ and c are uniquely determined. We now show that for each $a \in [0, c)$, there is a unique $\pi(a) \in (0, 1)$ that satisfies (2.9). Fix $a \in [0, c)$. Define a function $\varphi_3 : [0, F(a)] \to R$:

$$\begin{aligned}
\varphi_3(x) = v_2 &\sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{F(a)-x}{1-\pi(c)}\Big) \\
-v_1 &\sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\Big(\frac{x}{\pi(c)}\Big).
\end{aligned}$$

Let us differentiate φ_3 at each $x \in (0, F(a))$ by using Lemma 1:

$$\begin{aligned}
\varphi_3'(x) \;=\; & v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))\Big(-\frac{1}{1-\pi(c)}\Big)\, m\, p_{m-q_2,q_2-1}\Big(\frac{F(a)-x}{1-\pi(c)}\Big) \\
& -v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))\Big(\frac{1}{\pi(c)}\Big)\, m\, p_{m-q_1,q_1-1}\Big(\frac{x}{\pi(c)}\Big) < 0.
\end{aligned}$$

Thus, φ is strictly decreasing. Moreover,

$$\begin{aligned}
\varphi_3(0) &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{F(a)}{1-\pi(c)}\Big) \\
&\quad - v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}(0) \\
&= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}\Big(\frac{F(a)}{1-\pi(c)}\Big) \quad (\because p_{j,m-j}(0)=0) \\
&> 0.
\end{aligned}$$

and

$$\begin{aligned}
\varphi_3(F(a)) &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^{m} p_{j,m-j}(0) \\
&\quad - v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\Big(\frac{F(a)}{\pi(c)}\Big) \\
&= -v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^{m} p_{j,m-j}\Big(\frac{F(a)}{\pi(c)}\Big) \quad (\because p_{j,m-j}(0)=0) \\
&< 0.
\end{aligned}$$

Thus, there is a unique $x^* \in (0, F(a))$ such that $\varphi_3(x^*) = 0$. Since $\varphi_3(\pi(a)) = 0$, $x^* = \pi(a)$. Hence, there is a unique $\pi(a) \in (0,1)$ that satisfies Equation (2.9).

Step 4: Finally, we derive $\gamma(a)$ for each $a \in (0, c)$. Recall that in (2.9), $\pi(a) = \int_0^a \gamma(x) f(x) dx$ and $\pi(c)$ and $\pi(a)$ are known by previous steps. Differentiate (2.9) with respect to a by using Lemma 1:

$$\begin{aligned}
& v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \Big(\frac{\gamma(a) f(a)}{\pi(c)}\Big)\, m\, p_{m-q_1,q_1-1}\Big(\frac{\pi(a)}{\pi(c)}\Big) \\
&= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \Big(\frac{f(a) - \gamma(a) f(a)}{1-\pi(c)}\Big)\, m\, p_{m-q_2,q_2-1}\Big(\frac{F(a)-\pi(a)}{1-\pi(c)}\Big).
\end{aligned} \tag{2.14}$$

Let us define the following functions:

$$\begin{aligned}
A(a) &:= v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c))\, m\, p_{m-q_1,q_1-1}\Big(\frac{\pi(a)}{\pi(c)}\Big) > 0, \\
B(a) &:= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c))\, m\, p_{m-q_2,q_2-1}\Big(\frac{F(a)-\pi(a)}{1-\pi(c)}\Big) > 0.
\end{aligned}$$

Then, we can write (2.14) as

$$\frac{\gamma(a)f(a)}{\pi(c)}A(a) = \frac{f(a)(1-\gamma(a))}{1-\pi(c)}B(a). \tag{2.15}$$

Solving for $\gamma(a)$ in (2.15), we obtain

$$\gamma(a) = \frac{\pi(c)B(a)}{(1-\pi(c))A(a) + \pi(c)B(a)} \in (0,1).$$

By construction, function γ we have derived satisfies Equation (2.9).

2.B.3 Verification: the candidate is an equilibrium

In this appendix, we check for global deviations and confirm that the unique symmetric equilibrium candidate we have derived in Theorem 1 is indeed an equilibrium. As a preliminary notation and analysis, let us calculate the probability, denoted by $P[1,b|c,\gamma,\beta^D]$, that a student who makes effort $e = \beta^D(b)$ and applies to college 1 ends up getting a seat in college 1:

$$P[1,b|\gamma,\beta^D] = \begin{cases} \sum_{m=0}^{q_1-1} \hat{p}_{m,n-m-1}(c) + \sum_{m=q_1}^{n-1} \hat{p}_{m,n-m-1}(c)G_{m-q_1+1,m}(b) & \text{if } b \in [0,c] \\ 1 & \text{if } b \geq c. \end{cases}$$

Obviously, if the student chooses an effort more than $\beta(c)$, he will definitely get a seat in college 1. Otherwise, the first line represents the sums of the probability of events in which e is one of the highest q_1 efforts among the students who apply to college 1.

Similarly, let us calculate the probability, denoted by $P[2,b|\beta,\gamma]$, that a student who makes effort $e = \beta(b)$ and applies to college 2 ends up getting a seat in college 2.

$$P[2,b|\gamma,\beta^D] = \begin{cases} \sum_{m=0}^{q_2-1} \hat{p}_{n-m-1,m}(c) + \sum_{m=q_2}^{n-1} \hat{p}_{n-m-1,m}(c)H_{m-q_2+1,m}(b) & \text{if } b \in [0,1] \\ 1 & \text{if } b \geq 1. \end{cases}$$

Obviously, if the student chooses an effort greater than $\beta(1)$, he will definitely get a seat in college 2.[29] Otherwise, the first line represents the sums of probability of events in which e is one of the highest q_2 efforts among the students who apply to college 2.

Next, denote by $U(r,b|\gamma,\beta^D,a)$ (or $U(r,b|a)$ for short) the expected utility of type a who chooses college 1 with probability r and makes effort $e = \beta^D(b)$ when all of the other students follow the strategy (γ,β^D). We have,

$$U(r,b|a) := rP[1,b|\gamma,\beta^D]v_1 + (1-r)P[2,b|\gamma,\beta^D]v_2 - \frac{e}{a}.$$

[29] Of course, there is no type b with $b > 1$, if a student chooses an effort e strictly greater than $\beta^D(1)$, we represent him as mimicking a type $b > 1$.

We need to show that for each $a \in [0,1]$, each $r \in [0,1]$ and each $b \geq 0$, $\hat{U}(a) \equiv U(\gamma(a), a|a) \geq U(r, b|a)$. Fix $a \in [0,1]$. It is sufficient to show that $\hat{U}(a) \geq U(0, b|a)$ and $\hat{U}(a) \geq U(1, b|a)$, as these two conditions together implies required "no global deviation" condition. Below, we show that for any $a \in [0,1]$, and for $b \geq 0$, both $\hat{U}(a) \geq U(0, b|a)$ and $\hat{U}(a) \geq U(1, b|a)$ hold. We consider two cases, one for lower-ability students ($a \in [0, c]$), one for higher-ability students ($a \in [c, 1]$). As sub-cases, we analyze b to be in the same region (b is low for a low, and b is high for a high), different region (a high, b low; and a low, b high), and b being over 1. The no-deviation results for the same region is standard, whereas deviations across regions need to be carefully analyzed.

Case 1: Type $a \in [0, c]$

Case 1-1: $b \in [0, c]$. Then, by our derivation, we have $U(0, b|a) = U(1, b|a)$ and also $\hat{U}(a) \geq U(1, b|a)$ can be shown via standard arguments (for instance, see section 3.2.1 and Proposition 2.2 in Krishna, 2002). Hence, we can conclude that $\hat{U}(a) \geq U(1, e|a) = U(0, e|a)$.

Case 1-2: $b \in (c, 1]$. We first show $\hat{U}(a) \geq U(1, b|a)$.

$$\begin{aligned}
\hat{U}(a) &\geq U(1, c|a) = v_1 - \frac{\beta^D(c)}{a} \\
&\geq v_1 - \frac{\beta^D(b)}{a} \quad (\because \beta^D(c) \leq \beta^D(b)). \\
&= U(1, b|a).
\end{aligned}$$

Next, we show $\hat{U}(a) \geq U(0, b|a)$.

$$\begin{aligned}
\hat{U}(a) &\geq U(\gamma(c), c|a) = P[2, c|\gamma, \beta^D]v_2 - \frac{\beta^D(c)}{a} \\
&= \left(P[2, \beta^D(c)|\gamma, \beta^D]v_2 - \frac{\beta^D(c)}{c}\right) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(0, c|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&\geq U(0, b|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} = P[2, b|\gamma, \beta^D]v_2 - \frac{\beta^D(b)}{c} + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= \left(P[2, b|\gamma, \beta^D] - \frac{\beta^D(b)}{a}\right) + \frac{\beta^D(b)}{a} - \frac{\beta^D(b)}{c} + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(0, b|a) + \left(\beta^D(b) - \beta^D(c)\right)\left(\frac{1}{a} - \frac{1}{c}\right) \\
&\geq U(0, b|a) \quad (\because \beta^D(b) \geq \beta^D(c),\ a < c).
\end{aligned}$$

Case 1-3: $b > 1$ (or $e > \beta^D(1)$).

$$\begin{aligned}
\hat{U}(a) &\geq U(\gamma(c), c|a) = v_1 - \frac{\beta^D(c)}{a} \\
&> v_1 - \frac{e}{a} \quad (\because \beta^D(c) \leq \beta^D(1) < e) \\
&= U(1, b|a).
\end{aligned}$$

Moreover,

$$\begin{aligned}
\hat{U}(a) &\geq U(0, 1|a) \quad \text{(by Case 1-2)} \\
&= v_2 - \frac{\beta^D(1)}{a} \\
&> v_2 - \frac{e}{a} \quad (\because e > \beta^D(1)) \\
&= U(0, b|a).
\end{aligned}$$

Case 2: Type $a \in [c, 1]$

Case 2-1: $b \in [0, c]$. We first show $\hat{U}(a) \geq U(1, b|a)$.

$$\begin{aligned}
\hat{U}(a) &\geq U(0, c|a) = v_2 P[2, c|\gamma, \beta^D] - \frac{\beta^D(c)}{a} \\
&= U(\gamma(c), c|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&\geq U(\gamma(b), b|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(\gamma(b), b|a) + \frac{\beta^D(b)}{a} - \frac{\beta^D(b)}{c} + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(1, b|a) + (\beta^D(c) - \beta^D(b)) \left(\frac{1}{c} - \frac{1}{a} \right) \quad (\because U(\gamma(b), b|a) = U(1, b|a)) \\
&\geq U(1, b|a) \quad (\because \beta^D(c) - \beta^D(b) \geq 0,\ c < a).
\end{aligned}$$

To obtain $\hat{U}(a) \geq U(0, b|a)$, note that in the above inequalities, if we use $U(\gamma(b), b|a) = U(0, b|a)$ in the fourth line, we obtained the desired inequality.
Case 2-2: $b \in (c, 1]$. First, by our derivation, $\hat{U}(a) \geq U(0, e|\gamma, \beta^D, a)$ can be shown via standard arguments (for instance, see section 3.2.1 and Proposition 2.2 in Krishna, 2002). Next, we show $\hat{U}(a) \geq U(1, b|a)$.

$$\begin{aligned}
\hat{U}(a) &\geq U(0, c|a) = v_2 P[2, c|\gamma, \beta^D] - \frac{\beta^D(c)}{a} \\
&= v_1 - \frac{\beta^D(c)}{a} \quad (\because v_2 P[2, c|\gamma, \beta^D] = v_1) \\
&\geq v_1 - \frac{\beta^D(b)}{a} = U(1, b|a) \quad (\because \beta^D(c) \leq \beta^D(b)).
\end{aligned}$$

Case 2-3: $b > 1$ (or $e > \beta^D(1)$)

$$\begin{aligned}
\hat{U}(a) &\geq U(\gamma(c), c|a) = U(1, c|a) = v_1 - \frac{\beta^D(c)}{a} \\
&\geq v_1 - \frac{e}{a} \quad (\because e > \beta^D(1) > \beta^D(c)) \\
&\geq U(1, b|a).
\end{aligned}$$

and

$$\begin{aligned}
\hat{U}(a) &\geq U(0, 1|a) = v_2 - \frac{\beta^D(1)}{a} \\
&\geq v_2 - \frac{e}{a} \quad (\because e > \beta^D(1)) \\
&= U(0, b|a).
\end{aligned}$$

Appendix 2.C Additional tables and figures (online)

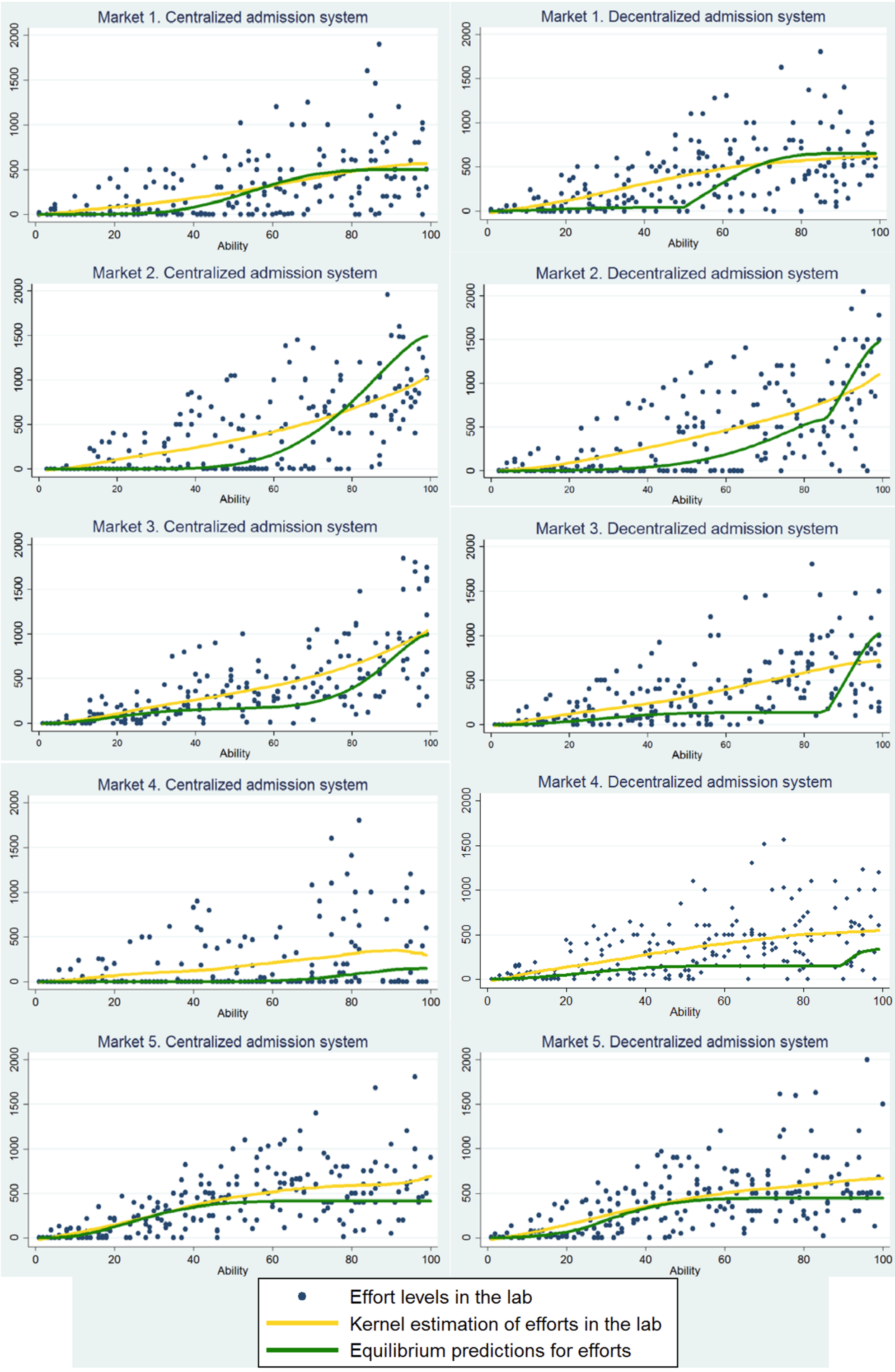


Figure 2.6. Individual efforts by ability

Table 2.7. Number of switching points in the 50,000 bootstrapped samples, by markets

	Market	
	2	3
Unique switching point in the predicted direction	77.5%	80.1%
Two switching points	17.3%	4.5%
Three or more switching points	0.8%	4.6%
No switching points	4.2%	6.9%
Unique switching point in the opposite direction	0.2%	3.9%

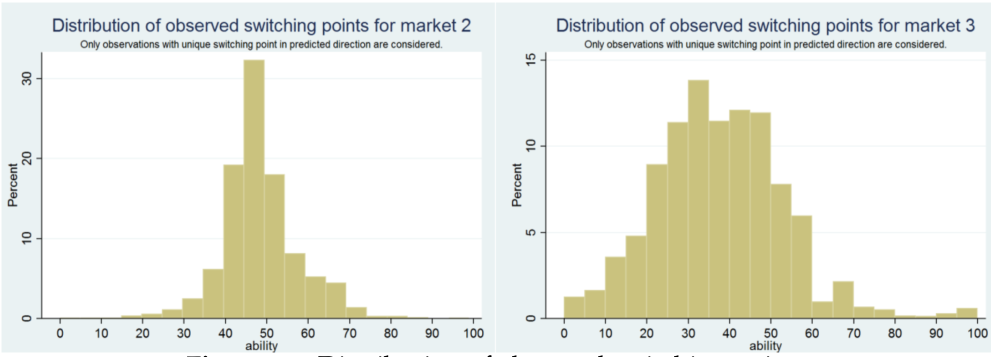


Figure 2.7. Distribution of observed switching points

Table 2.8. Observed effort choices and equilibrium predictions

	(1) Efforts	(2) Efforts
Equilibrium effort	.739***	.326***
	(.047)	(.083)
Dummy for CCA	-48.099	-61.923*
	(30.206)	(28.710)
Equilibrium effort in CCA	.0162	.0790
	(.084)	(.085)
Ability		5.210***
		(.627)
Constant	195.187***	29.430
	(17.909)	(20.487)
Observations	2160	2160
No. of individuals		
Overall-R^2		
R^2	.307	.379
F-test	140.829	186.695

Notes: OLS estimation of effort levels based on clustered robust standard errors at the level of matching groups. Equilibrium effort in CCA is an interaction of the CCA dummy and equilibrium effort. *** denotes statistical significance at the 1%-level, ** at the 5%-level, and * at the 10%-level. Standard errors in parentheses.

Table 2.9. Choice of the good college 2 in DCA

	(1)	(2)	(3)
Equil. probability of choosing the good college	1.684***	1.464***	1.465***
	(.106)	(.118)	(.113)
Ability		.009***	.009***
		(.002)	(.002)
Female dummy			-.016
			(.114)
Constant	-.793***	-1.144***	-1.139***
	(.079)	(.110)	(.115)
Observations	1080	1080	1080
log(likelihood)	-615.461	-596.561	-596.543

Notes: Probit estimation of dummy for the choice of the good college based on clustered robust standard errors at the subject level. *** denotes statistical significance at the 1%-level, ** at the 5%-level, and * at the 10%-level. Standard errors in parentheses

Table 2.10. Average overbidding in money terms, given the choice of the college in DCA.

		Ability below cutoff				Ability above cutoff			
		CCA (1)	DCA Bad college (2)	DCA Good college (3)	P-value Good= Bad (4)	CCA (5)	DCA Bad college (6)	DCA Good college (7)	P-value Good= Bad (8)
M. 1	N	106	70	36		110	16	92	
	Overbid.	3.2	3.4	7.7	0.02	0.4	-0.9	1.8	0.10
M. 2	N	177	86	91		39	3	36	
	Overbid.	3.2	2.5	4.9	0.10	-4.1	-8.6	-0.8	0.02
M. 3	N	179	130	49		37	12	25	
	Overbid.	2.5	3.5	5.8	0.24	0.7	-1.3	0.3	0.26
M. 4	N	190	157	33		26	15	11	
	Overbid.	3.1	4.0	5.8	0.06	2.2	1.4	6.8	0.10
M. 5	N	44	16	28		172	15	157	
	Overbid.	2.3	-0.5	5.4	0.02	1.6	0.4	1.8	0.35

Notes: Columns (4) and (8) show the p-values for the significance of the dummy variable for applying to the good college when regressing overbidding in money terms on the dummy and a constant for abilities below and above the theoretical cutoff in DCA, respectively, with standard errors clustered at the level of matching groups."M." stands for market, "overbid." stands for overbidding.

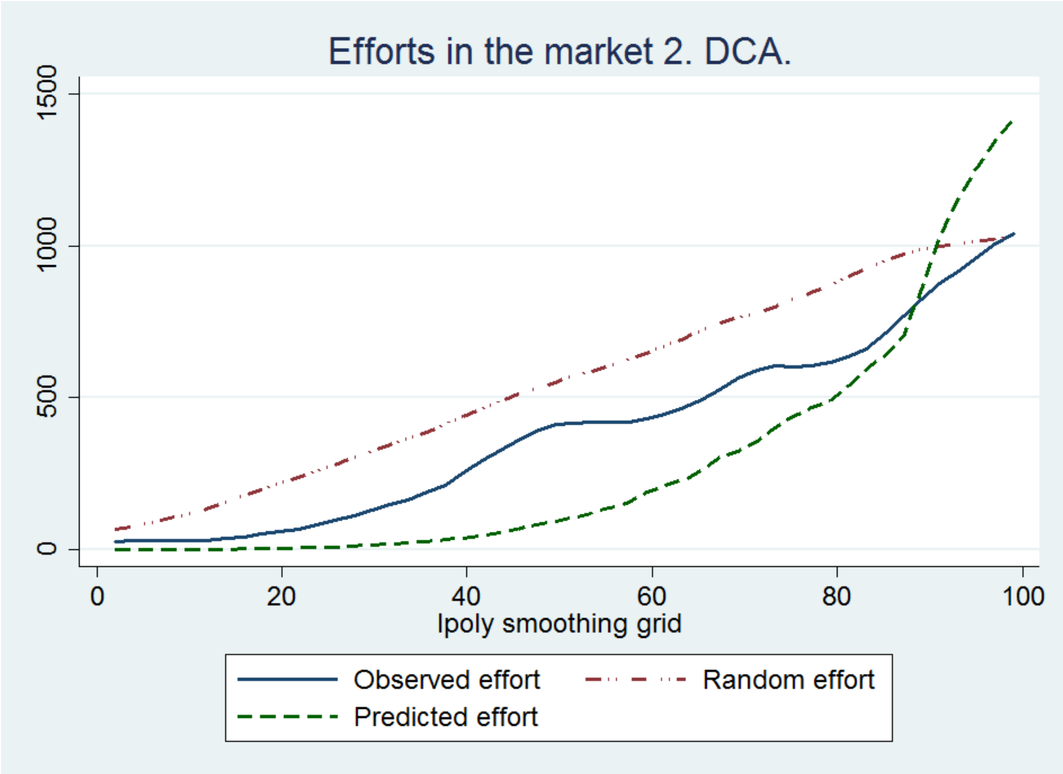


Figure 2.8. Equilibrium, average random and average observed efforts in market 2

Appendix 2.D Equilibrium derivation for ℓ colleges

We show how to derive cutoffs, mixed strategies, and cost functions provided there exists an equilibrium as specified in section 2.6.1. The basic procedure follows the one in Theorem 1.

We first show how to obtain the equilibrium cutoffs $c_1, \ldots, c_{\ell-1}$ and the mixed strategy function $\gamma_1, \ldots, \gamma_{\ell-1}$. Let $k \in \{1, \ldots, \ell-1\}$. A necessary condition for this to be an equilibrium is that each type $a \in [c_{k-1}, c_k]$ has to be indifferent between applying to college 1 and college 2. Thus, for all $a \in [c_{k-1}, c_k]$,

$$
\begin{aligned}
&v_k \left(\sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) + \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) H^k_{m-q_k+1,m}(a) \right) \\
&= v_{k+1} \left(\sum_{m=0}^{q_{k+1}-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \right. \\
&\left. + \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) H^{k+1}_{m-q_{k+1}+1,m}(a) \right). \qquad (2.16)
\end{aligned}
$$

Step 1: Find $\pi^1(c_1), \ldots, \pi^l(c_\ell)$. Equation (2.16) can be written as

$$v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi^1(c_1)) = v_2 \sum_{m=0}^{q_2-1} p_{m,n-m-1}(\pi^2(c_2)),$$
$$v_{k-2} = v_k \sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) \quad \text{for } k \in \{3, ..., \ell\}, \tag{2.17}$$

where the first equation is Equation (2.16) at $a = 0$ under $k = 1$, which says that a type $a = 0$ is indifferent between college 1 and 2; the second equation follows from Equation (2.16) at $a = c_k$ under $k-1$ and k, which says that a type $a = c_{k-2}$ is indifferent between colleges $k-2$ and k. Therefore, $\pi^1(c_1), \ldots, \pi^\ell(c_\ell)$ can be obtained by solving Equation (2.17).

Step 2: Given $\pi^1(c_1), \ldots, \pi^\ell(c_\ell)$, find cutoffs $c_1, \ldots, c_{\ell-1}$. We first show the following claim that shows how to obtain $\pi^k(c_{k-1})$ from $\pi^1(c_1), \ldots, \pi^\ell(c_\ell)$.

Proof. For $k = 2$: Note that $\pi^1(c_1) = \int_0^{c_1} \gamma_1(x)dF(x)$. Thus $\pi^2(c_1) := \int_0^{c_1}(1 - \gamma_1(x))dF(x) = F(c_1) - \pi^1(c_1)$. Suppose that the claim is true up to $k-1$ where $k \geq 3$. Then $\pi^{k-1}(c_{k-1}) := \pi^{k-1}(c_{k-2}) + \int_{c_{k-2}}^{c_{k-1}} \gamma_{k-1}(x)dF(x)$. Thus $\int_{c_{k-2}}^{c_{k-1}} \gamma_{k-1}(x)dF(x) = \pi^{k-1}(c_{k-1}) - \pi^{k-1}(c_{k-2})$. Hence, by the induction hypothesis, we have

$$\begin{aligned}
\pi^k(c_{k-1}) : &= \int_{c_{k-2}}^{c_{k-1}} (1 - \gamma_{k-1}(x))dF(x) \\
&= F(c_{k-1}) - F(c_{k-2}) - \int_{c_{k-2}}^{c_{k-1}} \gamma_{k-1}(x)dF(x) \\
&= F(c_{k-1}) - F(c_{k-2}) - \pi^{k-1}(c_{k-1}) + \pi^{k-1}(c_{k-2}) \\
&= F(c_{k-1}) - F(c_{k-2}) - \pi^{k-1}(c_{k-1}) + (F(c_{k-2}) - \sum_{j=1}^{k-2} \pi^j(c_j)) \\
&= F(c_{k-1}) - \sum_{j=1}^{k-1} \pi^j(c_j).
\end{aligned}$$

■ Now Equation (2.16) at $a = c_k$ can be rewritten as, for each $k \in \{1, \ldots, \ell - 1\}$,

$$\begin{aligned}
v_k &= v_{k+1} \sum_{m=0}^{q_{k+1}-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \\
&\quad + v_{k+1} \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1}))L
\end{aligned} \tag{2.18}$$

where L=$\sum_{j=m-q_{k+1}+1}^{m} p_{j,m-j}\left(\frac{F(c_k)-(\pi^1(c_1)+\ldots+\pi^k(c_k))}{\pi^{k+1}(c_{k+1})}\right)$

where we use induction claim and

$$H^{k+1}_{m-q_{k+1}+1,m}(c_k) = \sum_{j=m-q_{k+1}+1}^{m} p_{j,m-j}\left(\frac{\pi^{k+1}(c_k)}{\pi^{k+1}(c_{k+1})}\right).$$

Hence, given $\pi^1(c_1), \dots, \pi^\ell(c_\ell)$, we can find c_k by solving Equation (2.18).
Step 3: Given $\pi^1(c_1), \dots, \pi^\ell(c_\ell)$ and $c_1, \dots, c_{\ell-1}$, for each $k \in \{1, \dots, \ell-1\}$ and each $a \in [c_{k-1}, c_k]$, there is a unique $\pi^k(a)$ that satisfies Equation (2.19). Moreover, we can get the mixed strategy function $\gamma^k(a)$ by differentiating Equation (2.19).

Equation (2.16) at $a \in [c_{k-1}, c_k]$ can be rewritten as, for each $k \in \{1, \dots, \ell-1\}$,

$$\begin{aligned} &v_k \textstyle\sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) + v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) \sum_{j=m-q_k+1}^{m} p_{j,m-j}\left(\frac{\pi^k(a)}{\pi^k(c_k)}\right) \\ &= v_{k+1} \textstyle\sum_{m=0}^{q_{k+1}-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \\ &+ v_{k+1} \textstyle\sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \sum_{j=m-q_{k+1}+1}^{m} p_{j,m-j}\left(\frac{F(a)-F(c_{k-1})-\pi^k(a)+\pi^k(c_{k-1})}{\pi^{k+1}(c_{k+1})}\right). \end{aligned} \tag{2.19}$$

where we used the following equation: for each $a \in [c_{k-1}, c_k]$, since $\pi^k(a) := \pi^k(c_{k-1}) + \int_{c_{k-1}}^{a} \gamma_k(x)dF(x)$,

$$\begin{aligned} \pi^{k+1}(a) &:= \int_{c_{k-1}}^{a} (1-\gamma_k(x))dF(x) \\ &= F(a) - F(c_{k-1}) - \pi^k(a) + \pi^k(c_{k-1}). \end{aligned}$$

Differentiate Equation (2.19) with respect to a by using Lemma 1:

$$\begin{aligned} &v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) \frac{\gamma_k(a)f(a)}{\pi^k(c_k)} m p_{m-q_k,q_k-1}\Big(\frac{\pi^k(a)}{\pi^k(c_k)}\Big) \\ &= v_{k+1} \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \frac{f(a)-\gamma_k(a)f(a)}{\pi^{k+1}(c_{k+1})} m p_{m-q_{k+1},q_{k+1}-1}\Big(\frac{\pi^{k+1}(a)}{\pi^{k+1}(c_{k+1})}\Big). \end{aligned} \tag{2.20}$$

Let us define the following functions:

$$\begin{aligned} A^k(a) &= v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) m p_{m-q_k,q_k-1}\Big(\frac{\pi^k(a)}{\pi^k(c_k)}\Big) > 0 \\ B^k(a) &= v_{k+1} \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) m p_{m-q_{k+1},q_{k+1}-1}\Big(\frac{\pi^{k+1}(a)}{\pi^{k+1}(c_{k+1})}\Big) > 0. \end{aligned}$$

Then we can write (2.20) as

$$\frac{\gamma_k(a)f(a)}{\pi^k(c_k)}A^k(a) = \frac{f(a)(1-\gamma_k(a))}{\pi^{k+1}(c_{k+1})}B^k(a). \tag{2.21}$$

Solving for $\gamma_k(a)$ in (2.21), we obtain

$$\gamma_k(a) = \frac{\pi^k(c_k)B^k(a)}{\pi^{k+1}(c_{k+1})A^k(a) + \pi^k(c_k)B^k(a)}.$$

Step 4: We find the effort function β^D. Consider a student with type $a \in [c_{k-1}, c_k]$. A necessary condition is that she does not want to mimic any other type a' in $[c_{k-1}, c_k]$. Her utility maximization problem is

$$\max_{a' \in [c_{k-1}, c_k]} v_k \left(\sum_{m=0}^{q_{k-1}} p_{m,n-m-1}(\pi^k(c_k)) + \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) H^k_{m-q_k+1,m}(a') \right) - \frac{\beta^D(a')}{a}.$$

The first-order necessary condition requires the derivative of the objective function to be 0 at $a' = a$. Hence

$$v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) h_{m-q_k+1,m}(a) - \frac{(\beta^D(a))'}{a} = 0.$$

Solving the differential equation with the boundary condition at $\beta^D(c_{k-1})$, we obtain

$$\beta^D(a) = \beta^D(c_{k-1}) + v_k \int_{c_{k-1}}^{a} x \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) h^k_{m-q_k+1,m}(x)dx$$

for all $a \in [c_{k-1}, c_k]$.

Appendix 2.E Instructions of the experiment (for online publication)

Welcome! This is an experiment about decision making. You and the other participants in the experiment will participate in a situation where you have to make a number of choices. In this situation, you can earn money that will be paid out to you in cash at the end of the experiment. How much you will earn depends on the decisions that you and the other participants in the experiment make.

During the experiment you are not allowed to use any electronic devices or to communicate with other participants. Please use exclusively the programs and functions that are intended to be used in the experiment.

These instructions describe the situation in which you have to make a decision. The instructions are identical for all participants in the experiment. It is important that you read the instructions carefully so that you understand the decision-making problem well. If something is unclear to you while reading, or if you have other questions, please let us know by raising your hand. We will then answer your questions individually.

Please do not, under any circumstances, ask your question(s) aloud. You are not permitted to give information of any kind to the other participants. You are also not permitted to speak to other participants at any time throughout the experiment. Whenever you have a question, please raise your hand and we will come to you and answer it. If you break these rules, we may have to terminate the experiment.

Once everyone has read the instructions and there are no further questions, we will conduct a short quiz where each of you will complete some tasks on your own. We will walk around, look over your answers, and solve any remaining comprehension problems. The only purpose of the quiz is to ensure that you thoroughly understand the crucial details of the decision-making problem.

Your anonymity and the anonymity of the other participants will be guaranteed throughout the entire experiment. You will neither learn about the identity of the other participants, nor will they learn about your identity.

General description

This experiment is about students who try to enter the university. The 24 participants in the room are grouped into two groups of 12 persons each. These 12 participants represent students competing for university seats. The experiment consists of 15 independent decisions (15 rounds), which represent different student admission processes. At the end of each round every student will receive at most one seat in one of the universities or will remain unassigned.

There are two universities that differ in quality. We refer to the best university as University 1. Admission to the best university (University 1) yields a payoff of 2,000 points for the students. Admission to University 2 yields a smaller payoff for the students, which can vary across the rounds. Each university has a certain number of seats to be filled, a factor which can also be different for each of the rounds.

Instructions for CCA

The allocation procedure is implemented in the following way:

At the beginning of the each round, every student learns her ability. The ability of each student is drawn uniformly from the interval from 0 to 100. Thus every student has an equal chance of being assigned every level of ability from the interval. You will learn your own ability but not the ability of the other 11 students competing with you for the seats. The ability is drawn independently for all participants in every round.

Admission to universities is centralized and is based on the amount of effort that each student puts into a final exam. In the experiment you can choose a level of effort. This effort is costly. The price of effort depends on your ability. The higher the ability the easier (cheaper) the effort. The higher the ability the easier (cheaper) the effort. The price of one unit of effort is determined as: 100 divided by the ability, 100/ability. On your screen you will see your ability for the round and the corresponding price of one unit of effort. You will have to decide on the amount of the effort that you choose.

In each of the rounds you can use the calculator which will be on your screen. You can use it to find out what possible payoffs a particular effort in points can yield. To gain a better understanding of the experiment you can insert different values. This will help you with your decision.

In the beginning of each round, every participant receives 2,200 points that can either be used to exert effort or kept.

After each student has decided how much effort to buy, these effort levels are sent to the centralized clearing house which then determines the assignments to universities. The students who have chosen the highest effort levels are assigned to University 1 up to the capacity of this university. They receive 2,000 points. The students with the next higher levels of effort are assigned to University 2 up to its capacity and receive the corresponding amount of points. All other students who have applied remain unassigned and will receive no points. Participants that have chosen the same amount of effort will be ranked according to a random draw.

Each participant receives a payoff that is determined as the sum of the non-invested endowment and the payoff f rom university admission. Thus:

Payoff $=$ Endowment $-$ price of effort$*$units of effort $+$ payoff from assignment

Note that your ability, the ability of the other participants, and the number of seats at University 1 and University 2 vary in every round.

Every point corresponds to 0.5 cents. Only one of the rounds will be relevant for you actual payoff. This round will be selected randomly by the computer at the end of the experiment.

Example

Let us consider an example with three hypothetical persons: Julia, Peter, and Simon.

Imagine the following round: University 1 has four seats, and University 2 has five seats. The admission to University 1 yields 2,000 points and the admission to University 2 yields 1,000 points.

Julia has an ability of 25. Thus the cost of one unit of effort is 100/25 = 4 points for her. Her endowment is 2,200 points, which means that she can buy a maximum of 2,200/4 = 550 units of effort. Let us imagine that Julia decided to buy 400 units of effort. Thus she has to pay 400*4 = 1,600 points and keeps 600 points of her endowment.

Peter has an ability of 50. Thus the cost of effort for him is 100/50 = 2 points for one unit of effort. His endowment is 2,200 points. Thus he can buy a maximum of 2,200/2 = 1100 units of effort. Let us assume that Peter chose 600 units of effort. Thus he has to pay 600*2 = 1,200 points.

Simon has an ability of 80. Thus the cost of one unit of effort is 100/80 = 1.25 points for one unit of effort. His endowment is 2,200 points. Thus he can buy a maximum of 2,200/1.25 = 1760 units of effort. Let us imagine that Simon decides to buy 500 units of effort. Thus he has to pay 500*1.25 = 625 points.

Imagine that the following effort levels were chosen by the other 9 participants: 10, 70, 200, 250, 420, 450, 550, 700, 1,200.

Thus, the four students with the highest effort levels are assigned to University 1 and receive a payoff of 2,000 points. These are the students with effort levels 1,200, 700, 600 (Peter), and 550. Of the remaining eight students, five students with the highest levels of efforts are assigned to University 2 and receive a payoff of 1,000 points. These are the students with the efforts levels 500 (Simon), 450, 420, 400 (Julia) and 250.

The students with effort levels 10, 70, and 200 remain unassigned.

Thus, the payoff for Julia is $2,200 - 1,600 + 1,000 = 1,600$, for Peter $2,200 - 1,200 + 2,000 = 3,000$ and for Simon $2,200 - 625 + 1,000 = 2,575$.

Instructions for DCA

The allocation procedure is implemented as follows:

At the beginning of the each round, every student learns her ability. The ability of each student is drawn uniformly from the interval from 0 to 100. Thus every student has an equal chance of being assigned every level of ability from the interval. You will learn your own ability but not the ability of the other 11 students competing with you for the seats. The ability is drawn independently for all participants in every round.

The admission to universities is decentralized. Students first decide which university they want to apply to. Thus, you have to choose one university you want to apply to. After the decision is made, you will compete only with students who have decided to apply to the same university. The assignment of seats at each university is based on the amount of the effort that each student puts into a final test. In the experiment you can choose a level of effort. This effort is costly. The price of effort depends on your ability. The higher the ability the easier (cheaper) is the effort. The price of one unit of effort is determined as: 100 divided by the ability, 100/ability. On your screen you will see your ability for the round and the corresponding price of one unit of effort. You will have to decide on the amount of the effort that you choose.

In each of the rounds you can use the calculator which will be on your screen.

You can use it to find out what possible payoffs a particular effort in points can yield. To gain a better understanding for the experiment you can insert different values. This will help you with your decision.

In the beginning of each round, every participant receives 2,200 points that can be used to exert effort or kept.

After each student decides how much effort to buy, these efforts are used to determine the assignments to universities. Among the students who apply to University 1, the students with the highest effort levels are assigned to this university up to its capacity and receive 2,000 points. All other students who applied to University 1 remain unassigned. Among those students who apply to University 2, the students with the highest effort levels are assigned a seat up to the capacity of University 2. They receive the corresponding amount of points. All other students who have applied to University 2 remain unassigned. Participants that have chosen the same amount of effort will be ranked according to a random draw.

Each participant receives a payoff that is determined as the sum of the non-invested endowment and the payoff from university admission. Thus:

Payoff $=$ Endowment $-$ price of effort$*$units of effort $+$ payoff from assignment

Note that your ability, the ability of the other participants, and the number of seats at University 1 and University 2 vary in every round.

Every point corresponds to 0.5 cents. Only one of the rounds will be relevant for you actual payoff. This round will be selected randomly by the computer at the end of the experiment.

Example

Let us consider an example with three hypothetical persons: Julia, Peter, and Simon.

Imagine the following round: University 1 has four seats, and University 2 has five seats.

Julia has an ability of 25 and decides to apply to University 2. Thus the cost of one unit of effort is 100/25 = 4 points for her. Her endowment is 2,200 points, which means that she can buy a maximum of 2,200/4 = 550 units of effort. Let us imagine that Julia decided to buy 400 units of effort. Thus she has to pay 400*4 = 1,600 points and keeps 600 points of her endowment.

Peter has an ability of 50. He applies to University 1. Thus the cost of effort for him is 100/50 = 2 points for one unit of effort. His endowment is 2,200 points. Thus he can buy a maximum of 2,200/2 = 1100 units of effort. Let us assume that Peter chose 600 units of effort. Thus he has to pay 600*2 = 1,200 points.

Simon has an ability of 80. He applies to University 2. Thus the cost of one unit of effort is $100/80 = 1.25$ points for one unit of effort. His endowment is 2,200 points. Thus he can buy a maximum of $2,200/1.25 = 1,760$ units of effort.

Let us imagine that Simon decides to buy 500 units of effort. Thus he has to pay $500 * 1.25 = 625$ points.

Imagine that there are an additional four students who decide to apply to University 2 (competing with Julia and Simon), and five students who decide to apply to University 1 (competing with Peter). The following efforts were bought by the four participants who apply to University 2, together with Julia: $10, 70, 450,$ 550.

Thus, there are 6 contenders for 5 seats. All students, but one with the effort of 10, receive a seat at University 2 and thus a payoff of 1,000 points.

The following efforts were bought by the five other participants who apply to University 1, together with Peter: 200, 250, 420, 700, 1,200.

Thus, there are 6 contenders for 4 seats. The four students with the highest efforts are assigned to University 1, including Peter, and all receive 2,000 points.

The students with effort levels 200 and 250 remain unassigned.

Thus, the payoff for Julia is , $2200 - 1,600 + 1,000 = 1,600$, for Simon $2,200 - 625 + 1,000 = 2575$ and for Peter $2,200 - 1,200 + 2,000 = 3000$.

Chapter 3

Contests at the workplace with and without prize selection: Testing theory in a field experiment

3.1 Introduction

Millions of firms around the world introduce contests for their employees for monetary and non-monetary prizes. The term "contest" can describe a lot of different economic environments, from sport competitions to elections and college admissions. However, finding the optimal contest design is still a complex question. Its complexity stems from the fact that predictions of existing contest models are sensitive to assumptions about the number of players, number and sizes of prizes, participants' heterogeneity in terms of abilities, and other parameters. Moreover, the different alternatives of contests, which vary in the way the winners are selected, are modeled in the literature. Comparison between different models, or contest designs, is even more difficult to make and is sensitive to the parameters of the models.

One of the most common ways to determine winners in contests is through an all-pay auction design, where all participants are ranked according to their level of effort and the participant with the highest effort receives the best prize, the participant with the second-highest effort receives the second prize, and so on. These types of contests are quite common.[1] The examples of these types of contests are broad, and include all kinds of sport championships, competition for leadership positions and grants, centralized college admissions, and so forth. They are fairly common for firms, too: best employee of the month, best salesmen of the month,

[1] These contests are also modeled as rank-order tournaments. In the rank-order tournaments the level of efforts is not observable and the outcome, which is a noisy measure of efforts, is used to determine the winners of the contest. In what follows we assume full observability of efforts or absence of randomness in rank-order tournaments. Thus, two models coincide.

etc. In the following we refer to these contests as “standard” contests.

Recent theoretical and experimental evidence on the standard contest design shows some undesirable features of standard contests: in case of heterogeneity among players the efforts may be too heterogeneous, and lower ability players will not actively participate in the contest (Müller and Schotter, 2010). The authors call this behavioral regularity “bifurcation.” A similar observation was found by De Paola et al. (2012) in a field experiment, when only higher ability students increased their productivity in response to a contest introduced for obtaining better grades in exams. Other undesirable features of standard contests include the observed great variance of efforts even given the same ability (Nalbantian and Schotter, 1997, and Eriksson et al., 2009) and possibilities of sabotage (Harbring and Irlenbusch, 2011).

Some of the problems might be mitigated by varying the parameters of the contests, like introducing multiple prizes and varying the number of participants. The equilibrium of all-pay auction contests with multiple prizes under incomplete information was characterized by Moldovanu and Sela (2001). The optimal allocation of a fixed amount of prize money depends on the convexity of the cost function, keeping other parameters fixed. A convex enough cost function leads to higher effort under multiple prizes than under a single prize (Moldovanu and Sela, 2001). This theoretical result was supported by experiments by Müller and Schotter (2010).[2]

Lately, another variation of the contest has been discussed and analyzed in the literature, when participants must choose the prize they compete for and the effort simultaneously. The winners are then determined for each prize only from the subset of players who chose to compete for the same prize. In what follows we refer to them as “parallel” contests. Examples of such contests are decentralized college admission, parallel tournaments of the same professional association (like in tennis, WTA and ATP), some TV shows, like the popular singing competition “The Voice,” where everyone chooses their coach and competes for the final only against those who chose the same coach. The equilibrium of parallel contests with incomplete information is characterized by Hafalir et al. (2014). The authors also show that in the lab players exert more effort in parallel contests than in standard all-pay auction contests, even when theory predicts the reverse relation. Buyukboyaci (2012) finds similar results in a simpler environment with just two players: the total effort exerted in the lab in parallel contests is higher when players have heterogeneous abilities. Importantly, in parallel contests there is not much support for a bifurcation of efforts in the lab. These findings motivate the current field experiment. Can the positive effect of parallel contests on effort

[2] Lately Cason et al. (2010) show benefits of the contest similar to the lottery contest modeled by Tullock in Buchanan et al. (1980). Authors show that a contest with proportional prize devision attracts a higher participation of lower ability participants that standard contest. We do not consider these types of contests in the paper. For a recent review of experimental evidence on both standard and lottery contest check Dechenaux et al. (2015).

provision in the lab be replicated in a field setting?

We conduct an experiment with 302 workers at a microcredit company in Russia to compare the effects of standard and parallel contests for monetary prizes at the workplace. For one month the workers competed for different prizes with nine other random workers from the company without knowing their identities. We use a 2x2 design, varying the number and the size of the prizes and the contest design. In the first two treatments we used an all-pay auction contest. We varied the number of prizes: either two prizes, one of 20,000 rubles and one of 10,000 rubles, or four prizes, two of 10,000 and two of 5,000 rubles.[3] In the other two treatments we used parallel contests. Before the start of the contest participants had to choose the prize for which they wanted to compete, and then the winner was determined as the best performer among those who had chosen the same prize. We used the same variation of prizes in these treatments. The fifth group of credit specialists was in the control group, where no contest was introduced. The effort was defined as the number of new clients that were attracted by the worker in the course of the month and the winners were determined by the highest efforts.

Regarding the results, the average effort levels in all contest treatments are significantly higher than the average efforts in the control group. The highest average efforts were reached in the parallel contest's treatments. In line with the lab findings of Hafalir et al. (2014) we observe higher efforts in the parallel contests than in standard contests. We check the efforts relation of participants with different abilities and find that efforts are higher in the parallel contests for all abilities, even when the theoretical predictions go in the opposite direction. For within-model comparison, we find some support for the effects of the prize division in both contest designs: higher ability players exert more effort in treatments with a small number of high prizes in both contests, while lower ability types exert higher efforts in treatments with a higher number of smaller prizes in standard contests. In parallel contests, the high-ability types chose the prize in line with equilibrium, while lower ability types chose the high prize too often. In line with equilibrium, we find no difference in efforts depending on the choice of the prizes, controlling for ability. The efforts predicted by Moldovanu and Sela (2001) and Hafalir et al. (2014) have strong predictive power for the realized efforts by ability types: they fully explain the treatment difference between control and standard contest treatments, but an additional positive difference remains between the control group and the parallel contests treatments. This observation is in line with the lab findings that motivated this experiment where parallel contests lead to higher effort than the theory currently suggests.

[3] The exchange rate in February 2015 was around 70 rubles per 1 EUR. The average monthly salary of a credit specialist was 35,000 rubles.

3.2 Experimental design

3.2.1 Preliminaries

The experiment was conducted in February 2015 at the offices of Mol.Bulak microfinance company in Russia. Mol.Bulak focuses on microloans to the labor migrants that come to Russia for work, mostly as cheap labor, from the countries of Central Asia: Kyrgyzstan, Tadjikistan, and Uzbekistan.[4] The company had 45 offices all over the Russia from Vladivostok to Kaliningrad with 302 credit specialists in total in February 2015. The company's management decides on the range of available loan options and the terms at which they can be taken up. There were in total eight available combinations of the length of the loan term and the sum of loan for new clients during the period of the experiment. The company's credit specialists are responsible for finding clients, assessing their creditworthiness, and supervising regular repayments of the loans. Usually, the company's clients are in direct contact with a credit specialist, who is typically the only contact person between the company and the clients. All the company's credit specialists participated in the experiment.

The payment scheme for all credit specialists throughout the firm and all its branches in Russia was as follows before and during the experiment: a flat fixed salary plus a progressive bonus, depending on productivity. However, there is a threshold of productivity under which the bonus is 0. Productivity at the company is defined as a combination of the number of clients and the quality of the returns (low portfolio at risk value). Thus, the design of the compensation scheme led to the fact that a credit specialist below the threshold for bonuses could have a low motivation for exerting effort, as the bonus seemed to be too far away. Moreover, due to frequent changes in compensation schemes in the past, some workers could not believe that the long-term effort of reaching the threshold would pay off, as new changes could be introduced. These considerations led the management to believe that a short-term intervention in the form of contests with the possibility to earn money could have a strong effect on the efforts of the specialists.

For one month the credit specialists competed for different prizes with nine other random workers from the company without knowing their identities. For the goals of the contest their productivity was defined as the number of new clients which they attracted in February. The specificity of the business allows

[4] Mol.Bulak is a unique microfinance company that offers loans without collateral to migrants in Russia. The target group of migrants is a very risky group, as they do not have a permanent registration address, often switch jobs, and have a high probability of leaving Russia for the country of origin or moving to another region of Russia. They do not have any credit history records, except the one inside of the company. However, the migrants often need money for registration in Russia, payment for housing or sending cash back to their families in the home-country who are most of the time dependent on their transfers. These migrants do not have access to credit in Russian credit institutions. The efficient rate of credit was around 75% and the percent of returned credits around 90%.

Table 3.1. Treatments

		Within -contests variation	
		2 Prizes (rubles) 20,000; 10,000	4 Prizes (rubles) 2 of 10,000; 2 of 5,000
Between-model variation	Standard	**Standard2**	**Standard4**
	Parallel	**Parallel2**	**Parallel4**

us to assume that the number of new clients is a direct result of the effort exerted by the credit specialist. Moreover, the management was convinced that most of the workers did not work at the maximum of their aptitude to attract new clients. Another important motivation for choosing this specific performance indicator as a main input of contest is that all the company's credit specialists had the same probability of winning, as it did not depend on previous productivity. We chose not to inform the credit specialists of the name of their competitors, and guaranteed that they had been matched randomly by demonstration of the assignment procedure. By doing so we tried to minimize the possibility of discouragement effects which are typically observed in contests in the lab. Finally, the lack of information about competitors allowed us to exclude all potential sabotage possibilities.

3.2.2 Treatments

We use a 2x2 design, varying the number and the size of prizes (within-model) and the contest architecture (between models). Table 3.1 presents the summary of the parameters of the treatments.

In the first two treatments we use the well-known all-pay auction contest by Moldovanu and Sela (2001). We vary the number of prizes: either two prizes, one of 20,000 and one of 10,000 rubles, or four prizes, two of 10,000 and two of 5,000 rubles. Thus, in the first treatment, which we call "Standard2", all groups of 10 workers compete for two prizes. The credit specialist with the highest number of new clients in the month of February received the prize of 20,000 rubles, and the credit specialist with the second-highest number of new clients received 10,000 rubles. In the second treatment there are four prizes in total: two of the credit specialists with the highest number of the new clients received 10,000 rubles, and the credit specialists with the third and fourth-highest number of new clients received 5,000 rubles each. We call this treatment "Standard4." In the other two treatments contestants could choose the prize they would compete for prior to the start of the competition, thus separating themselves from the contestants who had chosen the other prize. The winner was determined as the best performer among those who had chosen the same prize. We use the same prize variations as in standard treatments. Thus, in the third treatment, which we call "Parallel2," every credit specialist had to choose a prize of either 20,000 or 10,000 rubles, and the credit specialist with the highest number of new clients in February, among those who had chosen the same prize, received the corresponding prize. In the fourth treatment, there were two prizes of 10,000 rubles and two prizes of 5,000 rubles, and the credit specialists had to choose the prize (10,000 or 5,000), and the two credit specialists with the highest number of clients, among those who had chosen the same prize, received the corresponding prize. We call this treatment "Parallel4." The fifth group of credit specialists was the control group, where no contest was introduced. Note that the explained bonus incentives were still present in all treatments and the control group, thus the incentives of contests are introduced additionally to the existing incentives in the company.

The treatment randomization was done at the office level, so every credit specialist of an office was in the same treatment condition. According to the management of the company there is no interaction between the credit specialists of the different offices even if there are several offices in the same city (which was only the case for Moscow and Saint-Petersburg). The first criterion for a balanced randomization was the office size with respect to the number of specialists. According to the management, there is a big difference in the environment and manner of interaction between large and small offices. We ranked all offices by the number of employees and randomly assigned a treatment number between 1 and 5 starting from the five largest offices, then to the five offices of rank six to 10, and so on. Thus, offices with the same number were assigned to the same treatment. The resulting assignment led to an unequal distribution of the number of credit specialists between treatments, with a maximum of 64 credit specialists per treatment and a minimum of 57. As we aimed to have 60 credit specialists in each treatment with a contest, we reallocated small offices from the treatments with too many workers to treatments with less than 60 workers. This was possible due to the presence of three offices with two credit specialists and three offices with one credit specialist.

The procedure of the randomization at the first level left few possibilities for additional adjustments. We checked for the average number of clients per credit specialist by treatments as it can be a proxy for the ability of the credit specialists. This parameter was lower in the Standard4 treatment than in other treatments, though in a non-significant way due to high variation. In order to exclude a possible imbalance in ability, we manually switched the assignment of one successful office with five credit specialists from treatment Parallel2 to treatment Standard4, while moving a less successful office with five employees from Standard4 to Parallel2. This switch finalized the treatment assignments.

The introduction of the treatments was done as follows: on Friday, January 30, every head of the office[5] in all treatments other than the control announced to the credit specialists that contests would be held for monetary prizes. The com-

[5] In each of the company's offices there is an office chief who is the direct superior of the credit specialists. The heads of the office received the rules by email with the request to go through them in public on Monday, February 2, and were asked directly by the head of the regions whether the rules were clear.

petition result would be determined by the number of new clients in the month of February and that everyone would compete with nine other random credit specialists from all the company's offices, and that they would not know each other's identities. The prizes and exact rules were not declared, but the managers announced that the CEO of the company would provide further instructions by email. It is important to note that emails from the CEO are a common way of communication in the company. Every credit specialist receives a smartphone after one month of work with the installed internal email application. The push notifications of new emails are set, so that every time the credit specialist receives an email it pops up on her smartphone homescreen. Additionally, every day they receive an email with a list of credits with delayed payments and the number of credits issued during the month. The CEO received the list of emails addresses of the credit specialists grouped by treatments and had to send four different emails to these groups with hidden emails of the recipients on the afternoon of January 30. The process was supervised by me in person.

The emails for each of the treatments are presented in Appendix 3.B.

Additionally, all the office managers received the same email as credit specialists and were instructed to announce the start of the contest orally at the traditional office meeting on Monday, February 2. In case of any questions they could clarify them with the head of regions – their direct superiors. The two heads of regions (Asia and Europe) were given instructions by me personally about the contest and all treatments, and also had the list of offices with their assignment to each treatment. It was done to insure that any possible questions about the contests rules would be answered correctly.

3.2.3 Theoretical predictions

In this section we derive the equilibrium predictions for each of the treatments. Our model is the all-pay auction with incomplete information on ability and heterogeneous players in terms of ability. In particular, we adopt the models of Molodovanu and Sela (2001) for the standard contest, and the model of Hafalir et al. (2014) for the parallel contest, to derive the predictions for the setup of the experiment.

The model

Ten risk-neutral players compete for two types of monetary prizes: Monetary Prize 1 and Monetary Prize 2. We denote the set of players by I and a generic player by i. We denote the set of prizes by M. The monetary prizes are available in different quantities, denoted by q_1 and q_2 respectively. Each player has a cardinal utility of m_1 for Monetary Prize 1 and m_2 for Monetary Prize 2. We assume that Monetary Prize 2 is more desirable than Monetary Prize 1 to every player, that is, $m_2 > m_1$. Each player i exerts the effort e_i. Players are heterogeneous in terms of their abilities. We take the interval $(0,1)$ as the ability space. The abilities are private information and are drawn identically and independently from the interval $(0,1)$ according to a continuous distribution function F. The distribution of abilities is common knowledge. A player i with ability a_i who exerts effort e_i bears a cost of effort $\frac{1}{a_i} \cdot e_i^{\alpha}$, $\alpha \geq 1$. The player's performance is determined by her effort e_i.

In the standard contest each player $i \in I$ exerts e_i. Given an effort profile $(e_i)_{i \in I}$, players with the top q_2 efforts receive the prize 2, and players with the efforts from top $(q_2 + 1)$ to $(q_2 + q_1)$ receive the prize 1.

Proposition 6. *In the standard contest, there is a unique symmetric equilibrium such that for each $a \in (0,1)$, each player with ability a chooses effort $e(a)$ where the effort function e is a closed-form solution. See details in Appendix 3.A*

In the parallel contests, each player $i \in I$ simultaneously chooses one of the prizes, M_i and an effort e_i. Given the prize choices of students $(M_i)_{i \in I}$ and efforts $(e_i)_{i \in I}$, each prize M is awarded to players with the top q_M effort levels among the set of players who chose the prize ($\{i \in I \mid M_i = M\}$). The unique symmetric Bayesian equilibrium of this game is characterized in Hafalir et al. (2014), and the following Proposition is a special case of it, with a modification allowing for convexity of the cost function.

Proposition 7. *In a parallel contest, there is a* unique *symmetric equilibrium $(\gamma, e; c)$ where a player with type $a \in (0, c]$ chooses the monetary prize 1 (smaller prize) with probability $\gamma(a)$ and makes effort $e(a)$; and a player with type $a \in [c, 1)$ chooses prize 2 (higher prize) for sure and makes effort $e(a)$. The closed form solutions and details are in Appendix 3.A.*

Treatment comparisons

Hypothesis 1 (Incentives): The average effort in all contest treatments should be higher than in the control group. Introducing contests incentives should lead to an increase in effort, as we assume that before the introduction of the incentives the credit specialists did not work with maximum motivation.

The equilibrium comparisons of the treatments is not possible without assuming a shape of the cost functions and the distribution of the abilities. We use a simulation approach to develop hypotheses for the experiment. Using Proposition 6 and Proposition 7 , the equilibrium effort level for each ability is calculated in each treatment given assumptions on two parameters: α and F. We use a large variation of parameters that cover the most realistic cases.

For parameter α we use the values of {1,1.2,1.4}. Taking as extreme the linear shape of the cost function, we increase α, thus increasing the convexity of costs.

As for the distribution of the ability F, the theory restricts it to be bounded between $(0,1)$. Three alternative distributions are used in the simulations to derive predictions for the treatments: uniform (or beta distribution with parameters

$\alpha = 1, \beta = 1$), beta distribution with parameters $\alpha = 2, \beta = 2$, and beta distribution with parameters $\alpha = 3, \beta = 3$. All the distributions have a mean of 0.5 and are symmetric but vary in terms of thickness of the tails. Section 3.B.1 of the appendix presents the figures with the simulated efforts comparing the standard and the parallel contests and the figures of simulated efforts comparing the contests with two and four prizes within each contest design.

We use the robust findings from the simulations to develop the following hypotheses that are not sensitive to the convexity of the cost function and the distribution of efforts.

Hypothesis 2 (Between-model comparison): The average effort of the intermediate ability types should be higher in the standard treatments than in parallel treatments with the same number of prizes.

Note that in equilibrium players with abilities around the theoretical cutoff c of the parallel contests always exert less effort in the parallel contest than in the standard contests. See figures 3.7 and 3.8 of the appendix. The intuition comes from the fact that players just above the ability cutoff win the good prize only if there are fewer higher ability players than number of good prizes and, unlike in standard contests, they do not have any chance of getting the smaller prize in the parallel contests, thus they exert less effort than the same ability types in the standard contests. For the abilities just below the cutoff, they most likely receive the smaller prize if they choose it in the parallel contests, and they do not face any competition from higher ability players in the equilibrium as the latter choose the high prize with certainty. Thus, players with abilities below the cutoff also exert less effort in the parallel contests than in the standard contests.

Hypothesis 3 (Within-model comparison): For high-ability types, efforts are higher in contests with two prizes. For low-ability types efforts are higher in contests with four prizes, but only in the standard contests.

Figure 3.9 in the appendix shows the relation of efforts in standard contests for players of different abilities. High-ability players always exert higher effort in two-prize treatments than in four-prize treatments, while the lower ability players exert higher efforts in four-prize treatments. The intuition is straightforward: in case of two prizes high-ability players have a chance to win a higher prize and exert more effort, while in case of four prizes more players have a high chance of a prize, and thus a larger part of the support of abilities exert a positive effort.

Figure 3.10 in the appendix shows the relation of efforts in parallel contests for players of different abilities. As in the standard contest treatments, high-ability players exert higher efforts in the case of two prizes. As for lower types, the relation is unlike in the standard contests: though the efforts of the lower ability players are higher in case of four prizes, the level of efforts are almost the same in the two parallel treatments. The intuition for it is that due to sorting in Parallel2, a larger support of abilities below the cutoff exert positive effort than one prize would suggest in the case of standard contests (as around 50% of the players choose the high prize). This moves the interval of the abilities, when four prizes could lead to higher effort, to the left where the cost of efforts is already much higher, and thus the difference in efforts is much smaller.

Hypothesis 4 (The choice of prizes in parallel contests): High-ability players choose the high prize with certainty. Lower ability players randomize the choice of prizes.

The hypothesis is a property of the equilibrium from Proposition 7. Figures 3.7 and 3.8 of the appendix show that the cutoff is sensitive to the distribution of abilities and is independent of the convexity of the cost function.

Hypothesis 5 (Effort choice in parallel contests): Irrespective of the prize choice, contenders exert the same effort, given ability.

Again, the hypothesis is a property of the equilibrium from Proposition 7, and we test it with our data.

Table 3.2. Descriptive statistics of the credit specialists characteristics by treatments

Treatment		Standard2	Parallel2	Standard4	Parallel4	Control
Portfolio,	Mean	3,565	3,550	3,418	3,515	3,451
1000 RUB	Sd	1,876	2,047	2,202	1,769	1,866
Number	Mean	165.9	170.6	167.1	166.1	166.6
of clients	Sd	78.1	107.2	103.3	80.7	86.9
PAR7	Mean	0.1	0.08	0.08	0.1	0.09
	Sd	0.09	0.06	0.07	0.06	0.08
Experience, month	Mean	15.8	18.2	15.3	19.5	15.7
	Sd	11.2	14.4	12.1	10.4	9.8
Age, years	Mean	34.5	32.9	34.2	32.6	34.7
	Sd	8.4	8.1	8.0	7.6	7.6
Nr of new clients	Mean	14.8	15.7	16.0	14.9	14.9
in January	Sd	9.3	8.8	8.2	7.7	7.0
Bonus dummy	Mean	0.07	0.07	0.08	0.05	0.07
	Sd	0.25	0.26	0.28	0.22	0.25

Notes: Portfolio shows the total sum of the balance of current loans issued by credit specialists. PAR7 is a coefficient of portfolio at risk with a delay of seven days or more. Bonus dummy is equal to 1 if a credit specialist received a bonus in December 2014 and 0 otherwise.

3.3 Results

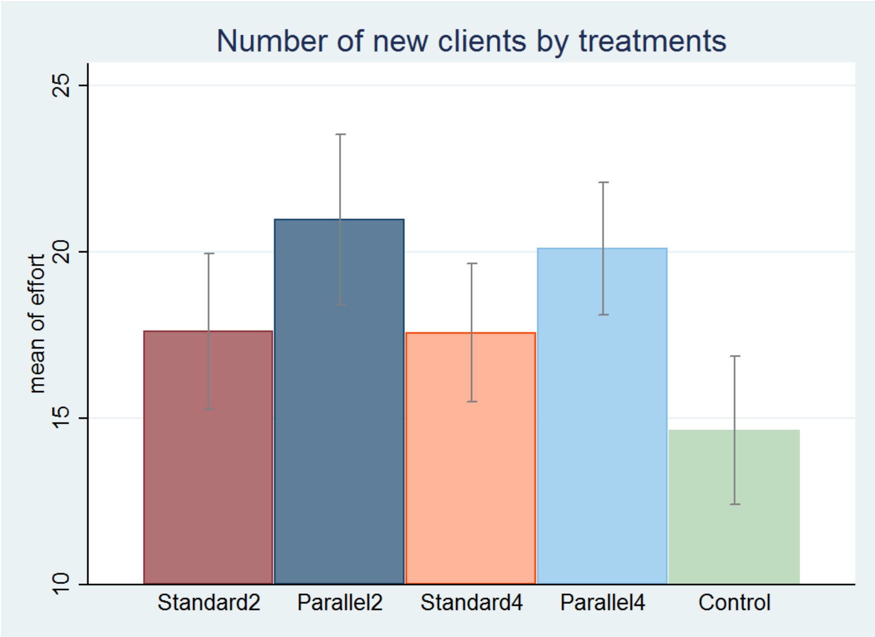
3.3.1 Randomization. Descriptive statistics

Table 3.2 presents the summary of characteristics of credit specialists by treatments. We present the data only for a sample of 286 credit specialists. The number of specialists in each treatment is smaller than was originally assigned. This is due to a number of employees being fired and specialists taking more than seven days vacation or being ill for more than seven days. Out of the 302 credit specialists on January 25 when the treatment allocation was carried out, only 293 were still employed in the company. In addition, seven were ill or took more than seven days vacations and are not used in the analysis. Thus, we use 286 observations.

There is no significant difference in the size of portfolio, number of new clients, age, number of clients in January, and the proportion of credit specialists who received a bonus between treatments. However, there are differences between the specialists in different treatments in PAR7. Credit specialists in Parallel2 and Standard4 have, on average, lower PAR7 than credit specialists in Parallel4 (the difference is significant at a 10% significance level according to a two-sided Mann-Whitney test). Nevertheless we do not consider it to be a problem in treatment assignments as the relation of the high PAR7 to the potential to grow is unclear. On one hand, with a high portfolio at risk most of the time the specialists devote their resources to reclaiming some of the delayed payments. However, the specialists only have limited control over this and at some stage of the process most of the work of reclaiming the delayed payments will be in the hands of the company's credit control department. In the latter case, a credit specialist might feel motivated to find new clients to increase the overall credit balance and may thus decrease the PAR7 coefficient. We control for the difference in PAR7 in the subsequent analysis. The credit specialists in the Parallel4 treatment have, on average, more experience than in Standard2, Standard4, and the Control treatments (Mann-Whitney test two-sided p-values for corresponding pairwise comparisons are: 0.03, 0.01, 0.06). All other pairwise differences are not significant. Once again, as in the case of PAR7, it is unclear how experience can affect the reaction to the treatment, and thus we control for it in the analysis of the results. In the following, all significant results are reported at a 5% level and all p-values are two-sided, if not otherwise stated.

3.3.2 Treatment comparison

Result 1 (Incentive effect): All contests lead to a significantly higher average effort of credit specialists than in the control group (only 10'% significance in Standard2). The efforts in Parallel2 are higher than in Standard2 and Standard4 (both 10% significance) and efforts in Parallel4 is higher than in Standard2 (10% significance) and Standard4. **Support.**



Notes: Confidence intervals are for 5% significance level.
Figure 3.1. Number of new clients by treatments.

Figure 3.1 shows the average number of new clients in the month of February by treatments. Credit specialists of the Parallel2 treatment attracted the highest number of new clients – 20.96 on average. In the control group the average number of new clients was only 14.93. For comparison, in January the average number of new clients among all the company's credit specialists was 15.4 per specialist. Wilcoxon rank-sum test for equality of the distribution of efforts between the control and Standard2 leads p-value 0.07, between the control and Parallel2 p-value 0.00, between the control and Standard4 p-value 0.05, and between the control and Parallel4 p-value 0.00. Thus we observe positive effect of the contests in all treatments. Moreover, comparison of Parallel2 to Standard2 leads p-value 0.07, Parallel2 to Standard4 0.07, Parallel4 to Standard2 0.09, and Parallel4 to Standard4 0.05. Thus, even in the raw data we see benefits of the parallel contests relative to the standard contest to the firm. Note that we observe result one in the raw data using a non-parametric test. Additionally we run regressions in order to control for the individual characteristics of the credit specialists by treatments. Columns 1 and 2 of Table 3.3 show the coefficients of linear regressions of the number of new clients on the dummy for each of the treatments. The number of new clients in Parallel2 and Parallel4 treatments is significantly higher than in Control. The number of new clients in Standard2 and Standard4 are 10% significantly higher than in Control. The significance is robust to controlling for the individual characteristics. Thus, the incentives of the experiment led to an increase in efforts, as expected in hypothesis 1. The difference to some other studies that failed to find the effect may be due to the relatively high incentives in the current experiment and the absence of a performance-based compensation scheme for the majority of the participants and in the control group. Note that this is the main result of the paper and is totally free from any assumptions on the parameters like ability distribution and the shape of the cost function.

Table 3.3. OLS regressions of efforts in contests relative to the control

	(1)	(2)	(3)
Standard2	2.96*	2.74*	2.20*
	(1.58)	(1.41)	(1.18)
Parallel2	6.33***	6.66***	5.44***
	(1.59)	(1.43)	(1.20)
Standard4	2.93*	2.86**	2.99**
	(1.57)	(1.40)	(1.17)
Parallel4	5.45***	6.33***	5.78***
	(1.57)	(1.42)	(1.19)
Individual controls		(yes)	(yes)
Ability estimate			26.12***
			(2.40)
Constant	14.64***	7.71***	1.93
	(1.11)	(2.40)	(2.07)
Observations	286	286	286
R^2	.0652	.4636	.5265
F-test	4.90	19.66	23.26

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Standard errors in parentheses
Individual controls include all variables from Table 3.2

In order to provide further insights on the experimental results and understand the sources of the treatments effects some assumptions have to be made. To test hypotheses 2 to 5, the abilities of the credit specialists have to be estimated. One of the disadvantages of field experiments relative to lab experiments is the difficulty in disentangling efforts and abilities. To address this issue, two additional data points were collected for all the credit specialists. The first includes the number of new clients by all credit specialists participating in the current study for the previous 11 months (March 2014 to January 2015) or since the start of their employment at the company. Assuming that every credit specialist was fully motivated and worked to the maximum of his or her ability at least once during their period of employment, we can reconstruct the abilities from the maximum monthly effort for the previous 12 months or for the period of employment in the company.

The estimation of the α parameter of the cost function is not feasible given the data available. Given the specificity of the business and the task of the credit specialists, we assume that the costs are close to linear: finding a second client requires the same number of steps or even fewer than finding the first. Fatigue may be present but it is unlikely to generate high values of the parameter α. The hypotheses under test are not sensitive to the parameter α, so we assume a low level of convexity, or α=1.2 for the purpose of deriving theoretical predictions

	Coef.	Std. Err.	Z	P¿z
alpha	2.53	0.20	12.43	0.00
beta	2.34	0.18	12.53	0.00

Table 3.4. Estimation of parameters alpha and beta for Beta distribution, based on the distribution of observed abilities

and estimating abilities. Thus, we can calculate the ability of each credit specialist from the maximum exerted effort by using the following equation: $maxcost = \frac{e_i^{\alpha}}{a_i} = \frac{median^{\alpha}}{0.5}$

Thus, due to an assumption of the symmetric distribution of abilities, we assign an ability equal to 0.5 to credit specialists with the median maximum monthly effort. The median maximum monthly effort is 21. Using the equation above we calculate the abilities of each credit specialist. This procedure, however, does not guarantee that abilities will lie in the interval from 0 to 1. In fact, four credit specialists received an ability higher than 1. In this case they are assigned an ability equal to 0.99.[6]

The second data we collected is used for a robustness check. The heads of the regions, who are the direct superiors of the credit specialists, were asked the following question: "If the prize was 1 million rubles per each client, how many clients could each credit specialist find?" We use their expert estimate as the proxy for ability. By deriving the ability from these estimates in a similar way as shown above, the correlation of ability estimates from maximum effort and expert estimates is 0.70.

Given the estimated ability of each credit specialist, the parameters of distribution are estimated based on the actual distribution of abilities by fitting a two-parameter beta distribution to the distribution of estimated abilities by a conventional parameterization with shape parameters $\alpha > 0$ and $\beta > 0$ (Forbes et al., 2011). The results of the estimation are presented in Table 3.4. Figure 3.2 presents the fit of the estimated Beta distribution to the observed distribution of abilities.

Thus, for each treatment the theoretical prediction of effort is calculated assuming parameter $\alpha = 1.2$ and Beta(2.53, 2.34) distribution of abilities. Figure 3.3 shows the predicted efforts for each of the treatments, assuming zero effort in the control group, as only contest incentives are considered. It allows us to determine areas of abilities for which we expect a particular effort relation between the treatments, thus specifying hypotheses 2 to 5. It is useful to note that given the parameters of the model, the cutoff in the Parallel2 treatment is 0.715 and in the Parallel4 treatment 0.815.

Result 2 (Efforts by ability types): The effort level in the parallel contests is higher than in the standard contests with the same number of prizes for all abil-

[6] We assign ability 0.99 and not 1, as in what follows we assume Beta-type of distribution, and estimate its parameters. As Beta distributions exclude values 1, we substitute extreme high values of abilities by 0.99.

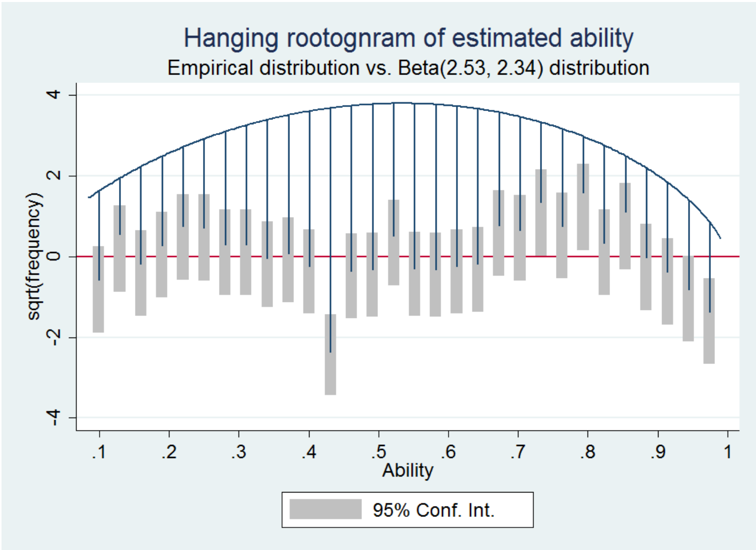


Figure 3.2. Empirical versus theoretical distribution of abilities.

Notes: Solid line above presents the density of Beta(2.53, 2.34) distribution. Each vertical gray rectangle shows the 5% confidence interval in a square root scale of the difference between the observed and the theoretical frequency of data in each of the ability intervals on the horizontal axis.

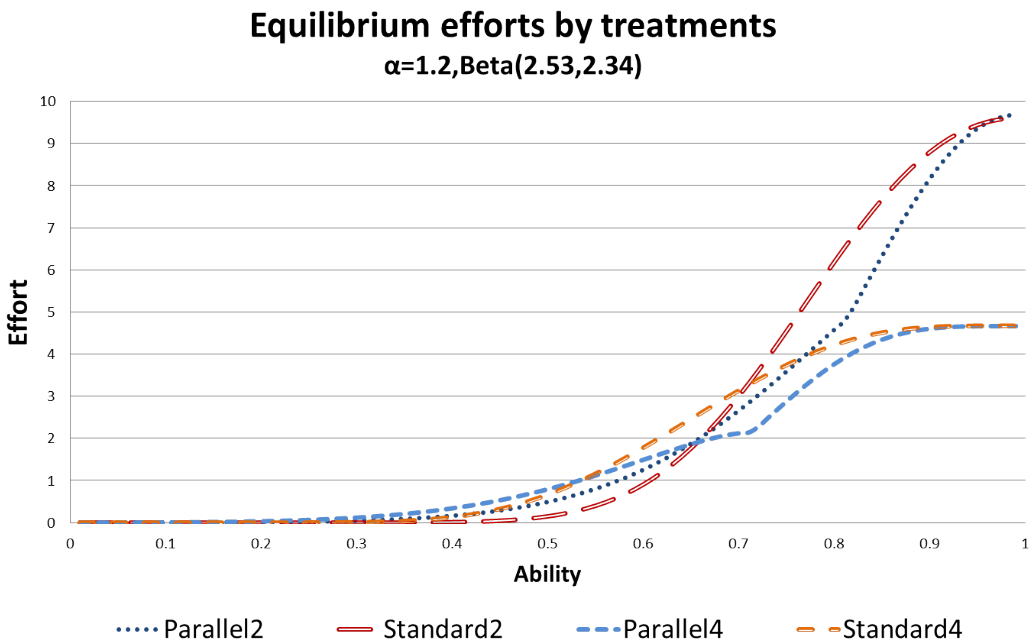


Figure 3.3. Equilibrium efforts by treatments.

ities, irrespective of the theoretical predictions. The difference is significant for abilities from 0.40 to 0.59 in the case of two prizes, and for abilities below 0.47 in the case of four prizes.

Support. First, figure 3.4 shows the smoothed efforts by abilities grouped by treatments with the same number of prizes. Note that in equilibrium players with abilities around the cutoff always exert less effort in the parallel contest (hypothesis 2). In contrast to the predictions, efforts in the parallel contest are higher for all abilities, thus we observe no support for hypothesis 2. In order to identify the significance of the difference in efforts between treatments for different abilities, we first create dummy variables for each 10% quantile of the abilities distribution. For the intervals of the 10% quantiles we use the quantiles of Beta (2.53, 2.34) distribution. Table 3.5 presents the results of the linear regression of efforts in Standard2 and Parallel2 contests (columns 1 to 3) and Standard4 and Parallel4 contests (columns 4 to 6).

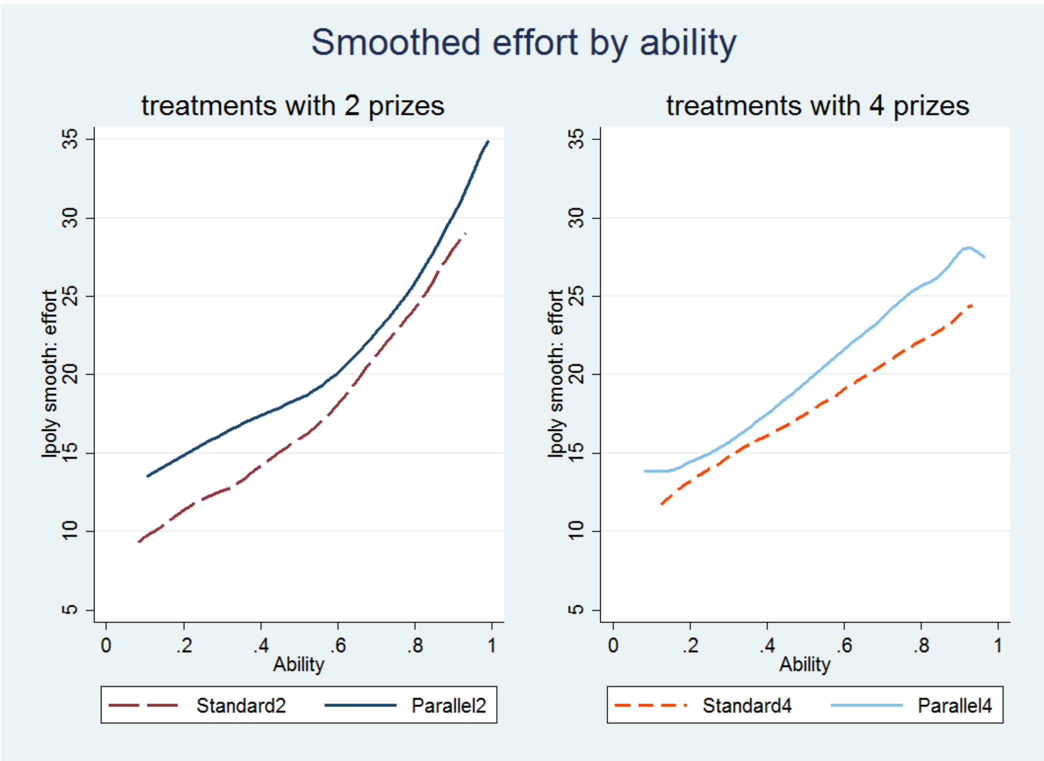


Figure 3.4. Smoothed effort by ability

Columns 1 and 4 show that there is a significant treatment difference in the average efforts controlling for the ability of all participants (10% significance in case of four-prize contests). However, we are more interested in differences in efforts by ability groups because theoretical predictions vary across the different ability groups. For the contests with two prizes, efforts in Standard2 are significantly lower than efforts in Parallel2 for abilities of the third, fourth, fifth, and sixth 10% quantiles of abilities. For the contests with four prizes, efforts in Standard4 are

Table 3.5. Efforts in contests

	(1) effort 2 prizes	(2) effort 2 prizes	(3) effort 2 prizes	(4) effort 4 prizes	(5) effort 4 prizes	(6) effort 4 prizes
Ability	38.51***	38.66***	38.04***	25.19***	23.97***	25.84***
	(2.68)	(3.63)	(4.01)	(3.09)	(4.47)	(5.01)
Standard	-2.59**			-2.24*		
	(1.02)			(1.15)		
quantile 1-2 in Standard dummy		-.58	-.06		-2.62	-4.78**
		(2.41)	(2.44)		(2.34)	(2.26)
quantile 3-4 in Standard dummy		-3.47**	-3.48**		-3.66*	-4.10**
		(1.60)	(1.66)		(1.92)	(1.77)
quantile 5-6 in Standard dummy		-3.52**	-3.36**		-.87	-.68
		(1.63)	(1.69)		(1.92)	(1.87)
quantile 7-8 in Standard dummy		-.89	-.75		-2.02	-2.86
		(1.91)	(1.98)		(2.14)	(2.00)
quantile 9-10 in Standard dummy		-2.92	-3.12		-1.77	-4.80*
		(2.22)	(2.37)		(2.75)	(2.63)
Constant	.23	.15	.24	7.35***	7.97***	11.00***
	(1.62)	(2.09)	(3.11)	(1.76)	(2.40)	(3.09)
Individual controls	no	no	yes	no	no	yes
Observations	112	112	112	116	116	116
R^2	.67	.67	.70	.39	.39	.54
F-test	108.54	36.26	17.39	35.73	11.84	9.05

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Standard errors in parentheses
Individual controls include all variables from Table 3.2

significantly lower than efforts in Parallel4 for credit specialists of abilities in the first, second, third and fourth 10% quantiles of abilities. Note that there is also a 10% significance of the differences in efforts in Standard4 and Parallel4 for the ninth and 10th quantiles of abilities.

Result 2 shows that between-model predictions for the relation of effort do not find much support in the data, thus we reject hypothesis 2. The biggest deviation comes from the players with abilities for which the standard contests should lead to higher efforts. Instead, we observe higher efforts in parallel contests, and this observation is in line with the results of the lab experiment of Hafalir et al. (2014). We also observe that the significance of the difference comes from the lower quantiles of abilities. Participants with these abilities almost certainly do not win a prize and barely exert any effort in the equilibrium of the standard contests. This is in line with the intuition that the lower ability players in the parallel contests might feel motivated by the belief that higher ability players are sorted out from competition and will choose the bigger prize. This observation is also in line with Buyukboyaci (2012).

3.3.3 Within-model comparison

Result 3 (Efforts by ability types): In the standard contests, the level of effort is significantly higher in the case of two prizes than in the case of four prizes for

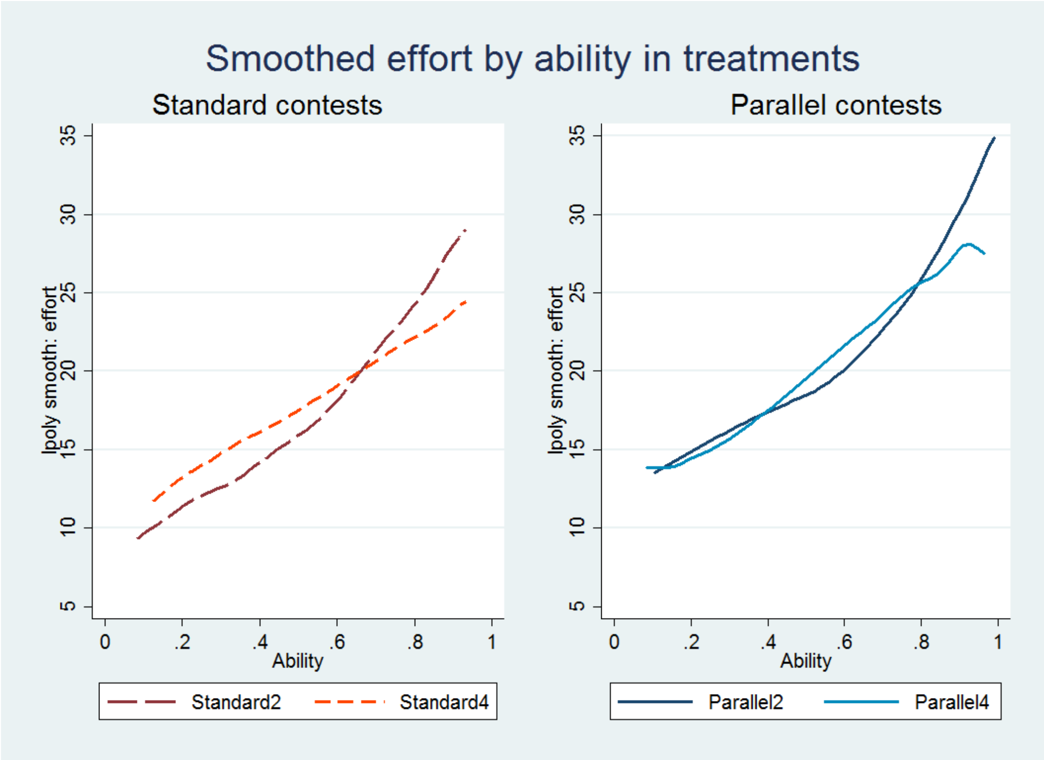


Figure 3.5. Smoothed effort by ability

high-ability players. The reverse is true for low-ability players and the difference is 10% significant. In the parallel contests, the level of effort is 10% significantly higher in the case of two prizes than in the case of four prizes for high-ability players. There is no significant difference in the efforts of the lower ability players.

Support. First, figure 3.5 shows the smoothed efforts by abilities grouped by treatments with the same contest design. The standard contest treatments are depicted on the left side. The smoothed lines show the relation in line with the theory: lower ability credit specialists exert, on average, higher efforts in four-prize treatment while higher ability credit specialists exert, on average, higher effort in two-prize treatment. The smoothed lines cross at an ability equal 0.66, while the theoretical crossing point is 0.70 (see figure 3.3). In order to check the significance of the differences of efforts we regress the efforts in the standard treatments on the ability and the interaction of ability and the dummy variables for ability being below and above the crossing point of smoothed lines (0.66). Columns 1 to 3 of Table 3.6 presents the results of the OLS estimation. Controlling for individual characteristics, the average effort in Standard2 is significantly higher than effort in Standard4 for abilities above 0.66. The average effort in Standard4 is 10% significantly lower than in Standard2 for abilities below 0.66. Thus, we find clear support for the predictions of the effect of the splitting of the prizes in standard contests.

Table 3.6. Efforts in contests relative to the control group

	(1) effort Standard	(2) effort Standard	(3) effort Standard	(4) effort Parallel	(5) effort Parallel	(6) effort Parallel
Ability	31.74***	25.90***	25.70***	32.05***	27.02***	23.88***
	(2.87)	(3.28)	(3.83)	(3.06)	(3.61)	(4.78)
Dummy for 2 prizes	-.73			-.17		
	(1.09)			(1.15)		
Dummy below in 2 prizes		-2.23*	-2.04*		-1.00	-1.52
		(1.14)	(1.15)		(1.17)	(1.16)
Dummy above in 2 prizes		4.47**	3.90*		5.81**	4.95*
		(1.90)	(1.98)		(2.66)	(2.89)
Constant	1.88	4.76***	5.77*	3.88**	6.43***	8.71***
	(1.61)	(1.78)	(2.92)	(1.74)	(1.99)	(3.04)
Individual controls	no	no	yes	no	no	yes
Observations	115	115	115	113	113	113
R^2	.52	.55	.60	.50	.54	.60
F-test	61.31	22.09	11.43	55.17	20.99	11.29

Dummy below in 2 prizes is 1 if ability is below the crossing point in 2 prizes treatment
Dummy above in 2 prizes is 1 if ability is above the crossing point in 2 prizes treatment
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Standard errors in parentheses
Individual controls include all variables from Table 3.2

The right side of Figure 3.5 shows smoothed efforts by abilities in the parallel treatments. Unlike the theoretical predictions, the observed smoothed lines cross three times. In theory they cross at ability 0.65. In order to formally test hypothesis 3 we take the last crossing point, at ability 0.72, and test the significance of the differences of efforts in treatments in the same way as in standard treatments. Columns 4 to 6 of Table 3.6 present the results of the OLS regression of efforts on the dummy variable for two prizes and the interaction of this dummy variable with dummy variables for being above and below 0.72 in the parallel treatments. Controlling for individual characteristics, the average effort in Parallel2 is 10% significantly higher than effort in Parallel4 for abilities above 0.72. There is no significant difference in efforts for abilities below 0.72. Note that in equilibrium we expect almost the same level of efforts in the parallel contests for lower ability players (hypothesis 3), thus we find support for the predictions of the effect of splitting the prizes in the parallel contests.

Overall, in spite of the lack of strong significance, the relation of average efforts by ability in the case of two and four prizes gives support for hypothesis 3. Thus, unlike the between-contest comparisons, the results of the experiment support the equilibrium predictions within contests.

Next, the results for parallel contests are analyzed in more detail. The equilibrium of the parallel contests model makes prediction about the choices of prizes and the effort, given the choice, which resulted in hypotheses 4 and 5.

Result 4 (Choice of prizes in parallel contests): There is no significant difference

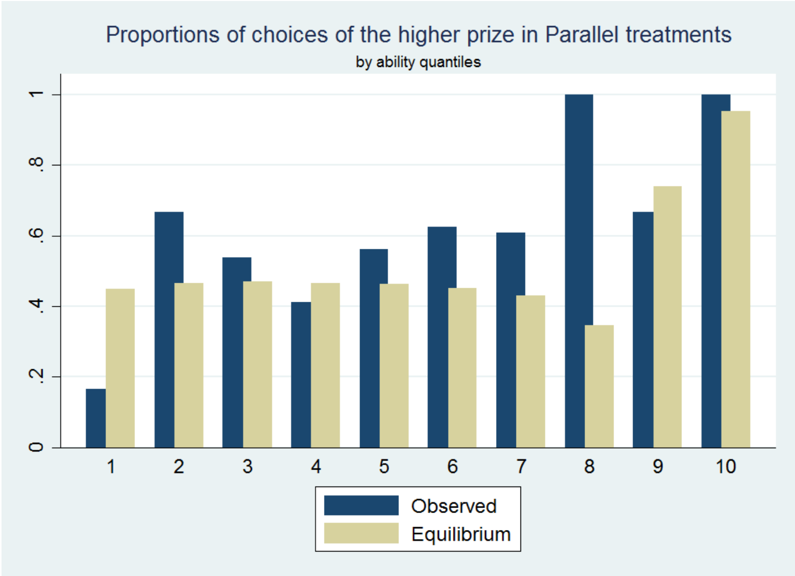


Figure 3.6. Choice of the higher prize, by ability quantiles.

between the observed and theoretical probability of choosing the high prize for specialists with abilities above the cutoff. Specialists with abilities below the cutoff choose the high prize significantly more often than the equilibrium suggests.

Support. Figure 3.6 presents the average proportion of credit specialists that choose the high prize by ability quantiles in both parallel treatments. For abilities above the theoretical cutoffs, the Wilcoxon matched-pairs signed-ranks test for equality of the observed and equilibrium probability of choosing the best colleges fails to reject the difference (two sided p-value is 0.32).

For the abilities below the cutoff, there is a significant difference between the predicted probability of choice and the observed average probability (Wilcoxon matched-pairs signed-ranks p¡0.01). The main difference comes from the credit specialist just below the cutoff, thus, middle-ability players choose the high prize significantly more often than the equilibrium suggests. This is consistent with possible overconfidence in own abilities or over-optimism about the draw of abilities of the competitors. It is worth noticing that this observation is in line with the results of Hafalir et al.'s (2014) lab experiment. Their subjects also choose the good college more often than suggested by equilibrium.

One important feature of the equilibrium in parallel contests is the independence of the effort from the choice of the prize. The following result tests this pattern in the data.

Result 5 (Choice of effort given prize choice in Parallel contests): Controlling for ability, there is no significant difference in effort, depending on the choice of prize in Parallel2 and Parallel4 treatments.

Support. Table 3.7 presents the results of the estimation of the regressions of effort levels in Parallel2 and Parallel4 depending on the choice of the prize. The variable choice is a dummy variable that equals one if the high prize is chosen. Row 2 of the table shows the coefficient of the choice dummy under different model specifications. The coefficients in Parallel2 treatments are negative and not significant. The coefficients in Parallel4 treatment are positive and not significant. Thus, we conclude that there is no evidence that credit specialists condition their effort on the actual choice of prize, which is in line with the equilibrium prediction. Thus, we cannot reject hypothesis 5.

Table 3.7. Linear regressions of efforts in contests depending on the choice of the prize.

Effort	Paral2	Paral2	Paral4	Paral4
Ability	39.55***	40.23***	22.75***	16.32**
	(4.28)	(4.60)	(4.71)	(6.72)
Choice	-.51	-1.29	2.08	1.60
	(1.78)	(1.82)	(1.69)	(1.60)
Constant	.02	5.37	7.43***	12.15***
	(2.24)	(4.07)	(2.49)	(4.14)
Controls	no	yes	no	yes
Observations	55	55	58	58
R^2	.66	.72	.35	.54

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ Standard errors in parentheses
Choice is a dummy equal to 1 if the high prize is chosen
Individual controls include all variables from Table 3.2

Finally, we test whether the theoretical predictions for the effort level have explanatory power for the observed efforts.

Result 6 (Point predictions and realized efforts): The theoretical predictions have significant predictive power for the realized efforts. Moreover, controlling for the predictions, the parallel treatments lead to significantly higher efforts than the standard treatments.

Support. Table 3.8 shows the results of the OLS regressions of the efforts on the predicted effort, controlling for abilities and the individual controls. Predicted effort has a significant effect on the observed efforts in all specifications. The inclusion of predicted effort explains the treatment difference of standard contests treatments relative to the control group, while dummy variables for the parallel treatments still remain significant. Thus, we can interpret this as an overexertion of effort relative to the equilibrium predictions in the parallel contests. This observation is in line with previous observations from the lab. One possible explanation is the limited level of reasoning. If level-0 players randomly choose the prize, ignoring ability, and then exert some random effort which is proportional to their ability, then level-1 players will choose the high prize more often than in the equilibrium, as they expect fewer high-ability players to choose it. This is in line with result 4. As for the higher efforts relative to predictions, if the random effort is higher than the equilibrium efforts, which is always the case for the lower ability players, the best response to it would imply higher than equilibrium efforts, and thus overexertion.

Table 3.8. Linear regression of efforts of all credit specialists on the predicted effort.

	effort	effort
Ability	17.51***	17.25***
	(2.78)	(3.25)
Predicted effort	1.50***	1.16***
	(.28)	(.29)
Standard2		.72
		(1.21)
Parallel2		3.82***
		(1.24)
Standard4		1.50
		(1.20)
Parallel4		4.54***
		(1.20)
Constant	7.53***	5.55**
	(1.28)	(2.22)
Controls	no	yes
Observations	286	286
R^2	.45	.53

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Standard errors in parentheses
Individual controls include all variables from Table 3.2

Concluding the analysis of individual efforts, in spite of the fact that we observe support for the equilibrium predictions of the parallel contests in the way the prizes are chosen and regarding the independence of efforts from the choice of the prize, it seems that the equilibrium predictions cannot explain the observed higher efforts in the parallel contests relative to standard counterparts. Thus, the field experiment supports the lab findings of Hafalir et al. (2014) and Buyukboyaci (2012) that in case of heterogeneous players parallel contests are attractive for designers aiming to maximize the total effort.

3.3.4 Profitability of the contests

In this section we evaluate the results of the introduction of the contests with regard to their profitability to the firm. One important question to clarify is the effect of the contests' incentives on the quality of issued credits. As credit specialists are the ones to make the final decision on the creditworthiness of a client, one can expect that, in a bid to win the contest, a credit specialist might be less strict in the assessment of the risk of the client. The quality for each credit specialist is determined as a coefficient of the portfolio at risk of the credits issued to the new clients in February. This consideration is an important step in evaluating the overall profitability of the contest for the firm.

To address this question, the data on the PAR7 coefficient of the new clients issued in February 2015 were collected for each credit specialist on December 1, 2015. Note that the average term of credits issued for new clients was approximately 6.5 months. There are also some credits with the term of 12 months, thus the data on the PAR7 of these credits are not final. However, in most cases any problems with credit returns appear at the beginning of the term, and almost never in the last two months of the long-term credits. Note that some credit specialists no longer work in the company but the credits are still registered to their names, thus we do not lose this data. Table 3.9 presents the marginal effects of the probit regressions of the average PAR7 coefficients of credits issued in February to new clients by each credit specialist on the effort and the treatments dummy.

Table 3.9. Marginal effect of probit model for PAR7 of issued credits

	(1) PAR7	(2) PAR7	(3) PAR7
Standard2 (d)	.03	.02	.03
	(.08)	(.08)	(.08)
Parallel2 (d)	.11	.10	.11
	(.07)	(.08)	(.08)
Standard4 (d)	.08	.07	.08
	(.07)	(.08)	(.08)
Parallel4 (d)	.09	.07	.08
	(.08)	(.08)	(.08)
Effort	.00	.01*	.00
	(.00)	(.00)	(.00)
Ability			.14
			(.21)
Controls	no	yes	yes
Observations	286	286	286
log(likelihood)	-166.88	-164.06	-163.83

(d) for discrete change of dummy variable from 0 to 1
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Standard errors in parentheses
Individual controls include all variables from Table 3.2

The effort of the credit specialists has no significant effect on the quality of the issued credit, neither do contests, on average, relative to the control group. Thus, in an estimation of the profitability of the contests we assume that additional efforts in the contest treatments have the same average quality as the credits in the control group.

Given the budget of 3,000 rubles per participant we estimate the average cost of new clients in each treatment, taking an estimation of the treatment effects from column 3 of Table 3.3. This leads to the following estimated costs of one new client: 1,364 rubles in Standard2, 556 rubles in Parallel2, 1,003 rubles in Standard4, and 519 rubles in Parallel4. In all treatments the cost of new credit is lower than the expected profit of an average credit.[7] However, the company has an alternative source of attracting new clients, called "the agents' scheme", when non-employed people who attract a client receive 400 rubles for each attracted client and give a 200-ruble discount to the client. Thus, the benchmark to compare the

[7] Minimum expected (given the average repayment rate) profit of a credit to new clients was 2,160 rubles, without cost of capital. That was a credit for three months of 10,000 rubles.

effectiveness of the contest for the company is 600 rubles (insisted upon by the management of the company). Both parallel contests led to lower costs. The results of the parallel treatments were accepted as successful for the company by the CEO, with a plan to repeat the contests with parallel design. While the calculation of the benefits of the contest and its profitability is simplified, it provides a first idea about the magnitude of the effect for the company.

3.4 Conclusions

The theoretical contest literature has developed various alternatives of contest designs, each of which offers different advantages. Motivated by Hafalir et al.'s (2014) recent model and the evidence from lab experiments that the parallel design leads to higher effort than the standard design, even when theory predicts the opposite, we conducted a field experiment to test this contest variation against the standard all-pay auction design.

Our main finding is that, as in the lab, the parallel contests lead to the significantly higher efforts of participants than a standard all-pay auction design. In line with the theory, the main benefit comes from lower ability players, who exert higher efforts in parallel contests due to being able to separate themselves from the high-ability competitors in the standard contests. In contrast to the theory, we do not observe a relative under-exertion of efforts in parallel contests for the players around the cutoff-ability. The predictions of equilibrium efforts explain the treatment effect of standard contests, but in parallel contests there is an additional positive effect, which is not explained by the theory. This might be explained by playing against level-0 players, when every credit specialist assumes a random choice of prizes by other credit specialists, ignoring the sorting.

As for the effect of splitting the prizes, the effect is in line with the theoretical prediction: high-ability specialists prefer a small number of high prizes in all contests, while the lower ability specialists prefer a higher number of smaller prizes in standard contests.

In the parallel contests we find evidence that credit specialists choose the higher prize too often relative to equilibrium. The biggest difference comes from the credit specialists just below the ability cutoff, which might be partially explained by the overestimation of one's own relative ability. This finding is also in line with the lab findings of Hafalir et al. (2014) and Buyukboyaci (2012). We find no support for specialists conditioning their effort on the choice of the prize, which is in line with the equilibrium prediction.

Finally, we check the effects of the short-term contest incentives on the quality of the issued credits, and find no significant effect. Altogether, this leads to estimated costs of new clients for the company that are lower than accepted by the management in parallel contests but not in standard contests. The experiment shows that parallel contests might be a good and easy change from standard contests for a designer who aims to maximize the effort. However, long-term effects of the parallel design might differ and should be further studied.

Appendix 3.A Theoretical predictions

Proposition 6 *In the standard contest, there is a unique symmetric equilibrium such that for each $a \in [0,1]$, each student with ability a chooses an effort e(a) according to*

$$e(a) = \left[\int_0^a x\left\{f_{10-q_2,9}(x)\,m_2 + \left(f_{10-q_1-q_2,9}(x) - f_{10-q_2,9}(x)\right)m_1\right\}dx.\right]^{\frac{1}{\alpha}} \tag{3.1}$$

where $f_{k,m}(\cdot)$ for $k \geq 1$ is a density of the k^{th}- (lowest) order statistic out of m independent random variables that are identically distributed according to F.

This is a special case of the all-pay auction studied by Moldovanu and Sela (2001). We use Hafalir et al.'s (2014) formulation. One modification from Proposition 6 of Hafalir et al. (2014) is to allow for the cost function to be convex. We used their proof with a modified function for the costs of effort to derive the equilibrium effort formula (3.1).

Proof. Suppose that $e(a)$ is a symmetric equilibrium effort function that is strictly increasing. Consider a student with ability a who chooses an effort as if her ability is a'. Her expected utility is

$$m_2\,F_{10-q_2,9}(a') + m_1\,\left(F_{10-q_2-q_1,11}(a') - F_{10-q_2,9}(a')\right) - \frac{1}{a}\cdot e(a')^{\alpha}.$$

The first-order condition at $a' = a$ is

$$m_2\,f_{10-q_2,9}(a) + m_1\,\left(f_{10-q_2-q_1,9}(a) - f_{10-q_2,9}(a)\right) - \frac{1}{a}\cdot\alpha\cdot e(a)^{\alpha-1}\cdot e'(a) = 0.$$

$$\Rightarrow \alpha\cdot e(a)^{\alpha-1}\cdot e'(a) = a\cdot m_2\,f_{10-q_2,9}(a) + a\cdot m_1\,\left(f_{10-q_2-q_1,9}(a) - f_{10-q_2,9}(a)\right).$$

Thus, by integration and as the boundary condition is $e(0) = 0$, we have

$$[e(a)]^{\alpha} = \left[\int_0^a x\left\{f_{10-q_2,9}(x)\,m_2 + \left(f_{10-q_1-q_2,9}(x) - f_{10-q_2,9}(x)\right)m_1\right\}dx.\right].$$

■

In the parallel contests, each player $i \in I$ simultaneously chooses one of the prizes, M_i and an effort e_i. Given the prize choices of students $(M_i)_{i\in I}$ and efforts $(e_i)_{i\in I}$, each Prize M is awarded to players with the top q_M effort levels among the set of players who chose the prize ($\{i \in I \mid M_i = M\}$). The unique symmetric

Bayesian equilibrium of this game is characterized in Hafalir et al. (2014), and the following Proposition is a special case of it, with a modification allowing for convexity of the cost function.

Proposition 7 *In a parallel contest, there is a* unique *symmetric equilibrium* $(\gamma, e; c)$ *where a player with type* $a \in (0, c]$ *chooses the monetary prize 1 (smaller prize) with probability* $\gamma(a)$ *and makes effort* $e(a)$*; and a player with type* $a \in [c, 1)$ *chooses prize 2 (higher prize) for sure and makes effort* $e(a)$*. Specifically,*

$$e(a) = \left[m_2 \int_0^a x \sum_{j=q_2}^{9} p_{9-j,j}(\pi(c)) h_{j-q_2+1,j}(x) dx \right]^{\frac{1}{\alpha}}.$$

where $\pi(a)$ *is the ex-ante probability that a player has a type less than a, and is given by:*

$$\pi(a) := \int_0^a \gamma(x) f(x) dx.$$

where $p_{j,k}(\pi(c))$*is the probability that out of* $j + k$ *students, j players choose Monetary Prize 1 and k players choose Monetary Prize 2, and is given by:*

$$p_{j,k}(x) := \binom{j+k}{j} x^j (1-x)^k.$$

where $h_{j,k}$ *is the density of* $H(a)$ *– the probability that a type is less or equal to a, conditional on the event that she chooses Monetary Prize 2, and it is given by:*

$$H(a) = \begin{cases} \frac{F(a)-\pi(a)}{1-\pi(c)} & \text{if } a \in (0, c], \\ \frac{F(a)-\pi(c)}{1-\pi(c)} & \text{if } a \in [c, 1]. \end{cases}$$

The equilibrium cutoff c and the mixed strategies $\gamma(\cdot)$ *are determined by the following four requirements:*

(i) $\pi(c)$ *uniquely solves the following equation for x*

$$m_1 \sum_{j=0}^{q_1-1} p_{j,9-j}(x) = m_2 \sum_{m=0}^{q_2-1} p_{9-j,j}(x).$$

(ii) *Given* $\pi(c)$*, c uniquely solves the following equation for x*

$$m_1 = m_2 \sum_{j=0}^{q_2-1} p_{9-j,j}(\pi(c)) + m_2 \sum_{j=q_2}^{9} p_{9-j,j}(\pi(c)) \sum_{l=j-q_2+1}^{j} p_{l,j-l} \left(\frac{F(x) - \pi(c)}{1 - \pi(c)} \right).$$

(iii) *Given* $\pi(c)$ *and c, for each* $a \in [0, c)$*,* $\pi(a)$ *uniquely solves the following equa-*

tion for $x(a)$

$$m_2 \sum_{j=q_2}^{9} p_{9-j,j}(\pi(c)) \sum_{l=j-q_2+1}^{j} p_{l,j-l}\left(\frac{F(a)-x(a)}{1-\pi(c)}\right)$$
$$= m_1 \sum_{j=q_1}^{9} p_{j,9-j}(\pi(c)) \sum_{l=j-q_1+1}^{j} p_{l,m-l}\left(\frac{x(a)}{\pi(c)}\right).$$

(iv) *Finally, for each* $a \in [0, c]$, $\gamma(a)$ *is given by*

$$\gamma(a) = \frac{\pi(c)B(a)}{(1-\pi(c))A(a) + \pi(c)B(a)} \in (0,1),$$

where

$$A(a) := m_1 \sum_{j=q_1}^{9} p_{j,9-j}(\pi(c))\, j\, p_{j-q_1,q_1-1}\left(\frac{\pi(a)}{\pi(c)}\right),$$
$$B(a) := m_2 \sum_{m=q_2}^{9} p_{9-j,j}(\pi(c))\, j\, p_{j-q_2,q_2-1}\left(\frac{F(a)-\pi(a)}{1-\pi(c)}\right).$$

Proof. The proof of Proposition 7 is given in Hafalir et al. (2014). The only modification is made in the step of derivation of the symmetric effort function, where the linear cost function is substituted with $\frac{1}{a_i} \cdot e_i^{\alpha}$. Thus, similar mechanics are performed as in the case of the standard contest. ■

Appendix 3.B Letters to credit specialists in each of the treatments

Standard2

In order to increase productivity we are introducing a competition for monetary prizes. You will compete in attracting NEW clients. Every one of you is competing with nine other credit specialists who were randomly selected from all our branches in Russia. There is only a small probability that you will be competing against someone from your own office. Your task is to attract the maximum number of new clients in February.

In every group of 10, the winner (the credit specialist who attracts the highest number of new clients) will receive the prize of 20,000 RUB. The second place will receive a prize of 10,000 RUB. After the competition is over, you will receive the names of your direct competitors and the number of clients they attracted.[8] In case of attracting the same number of clients, the credit specialist who attracted the last client earlier has priority.

For instance, a computer randomly determines the following credit specialists to be in one competing group: Asel from Vladivostok, Bahodyr from Moscow,

[8] We have done this to guarantee the transparency of results

Djazgul from Novokuznetsk, Mirbek from Kaliningrad, Tahmina from Saint-Petersburg, Nasiba from Tver, Sergei from Ufa, Kunduz from Murmansk, Elena from Irkutsk and Jyldyz from Kazan. None of these credit specialists knows who was picked to be in the same group with him or her. Imagine they attract the following number of NEW clients in February:

Credit specialist	Number of new clients in February
Tahmina from Saint-Petersburg	41
Asel from Vladivostok	35
Sergei from Ufa	35
Jyldyz from Kazan	29
Nasiba from Tver	24
Bahodyr from Moscow	21
Djazgul from Novokuznetsk	12
Kunduz from Murmansk	12
Mirbek from Kaliningrad	9
Elena from Irkutsk	8

Tahmina attracted the highest number of new clients and won 20,000 RUB. Asel and Ulan attracted the same number of new clients, but the last client of Asel took a credit on February 27, while the last client of Sergei took a credit on February 28. That is why Asel won 10,000 RUB.

Each one of you has a big chance to win a prize! Everything is in your hands! Good luck!

Parallel2:

In order to increase productivity we are introducing a competition for monetary prizes. You will compete in attracting NEW clients. Every one of you is competing with nine other credit specialists who were randomly selected from all our branches in Russia. There is only a small probability that you will be competing against someone from your own office. Your task is to attract the maximum number of new clients in February.

In every group of 10 credit specialists, there are two prizes to compete for 20,000 RUB and 10,000 RUB. Each one of you has to choose the prize for which you want to compete before February 1. Send an sms with your name and surname, and the prize you choose (10 or 20) to the number +79267608072. You will only be competing with those credit specialists from your group of 10 who choose the same prize. The credit specialist with the highest number of new clients among those who chose 20,000 RUB will receive 20,000 RUB. The credit specialist with the highest number of new clients who chose 10,000 RUB will receive 10,000 RUB. It may be that everyone in the group of credit specialists will choose the same prize, then the second prize will not be awarded to anyone. If one of the prizes is chosen just by one credit specialist out of 10, then he or she will receive it in any case. However, you will not know how many credit specialists of your group have chosen the same prize until the end of the competition. After the competition is over, you will receive the names of your direct competitors, their choice of prizes, and the number of clients they attracted. In case of attracting the same number of clients, the credit specialist who attracted the last client earlier has priority.

For instance, a computer randomly determines the following credit specialists to be in one competing group: Asel from Vladivostok, Bahodyr from Moscow, Djazgul from Novokuznetsk, Mirbek from Kaliningrad, Tahmina from Saint-Petersburg, Nasiba from Tver, Sergei from Ufa, Kunduz from Murmansk, Elena from Irkutsk and Jyldyz from Kazan. None of these picked credit specialists knows who was picked to be in the same group with him or her. Asel, Tahmina, Bahodyr, Sergei, Djazgul, and Mirbek chose 20,000 RUB before the start of the competition. Jyldyz, Nasiba, Kunduz, and Elena chose 10,000 RUB. Imagine they attract the following number of NEW clients in February:

Credit specialists who chose 20,000 RUB	
Credit specialist	Number of new clients in February
Tahmina from Saint-Petersburg	41
Asel from Vladivostok	35
Sergei from Ufa	35
Bahodyr from Moscow	21
Djazgul from Novokuznetsk	12
Mirbek from Kaliningrad	9

Credit specialists who chose 10,000 RUB	
Credit specialist	Number of new clients in February
Jyldyz from Kazan	29
Nasiba from Tver	24
Kunduz from Murmansk	12
Elena from Irkutsk	8

Tahmina attracted the highest number of new clients among those who chose 20,000 RUB, thus she won 20,000 RUB. Jyldyz attracted the highest number of new clients among those who chose 10,000 RUB, thus she won 10,000 RUB.

Each one of you has a big chance to win a prize! Everything is in your hands! Good luck!

Standard4:

In order to increase productivity we are introducing a competition for monetary prizes. You will compete in attracting NEW clients. Every one of you is competing with nine other credit specialists who were randomly selected from all our branches in Russia. There is only a small probability that you will be competing against someone from your own office. Your task is to attract the maximum number of new clients in February.

In every group of 10, the credit specialist who attract the first highest and the second highest number of new clients will receive the prize of 10,000 RUB. The third and the fourth places will receive a prize of 5,000 RUB. After the competition is over, you will receive the names of your direct competitors and the number of clients they attracted. In case of attracting the same number of clients, the credit specialist who attracted the last client earlier has priority.

For instance, a computer randomly determines the following credit specialists to be in one competing group: Asel from Vladivostok, Bahodyr from Moscow, Djazgul from Novokuznetsk, Mirbek from Kaliningrad, Tahmina from Saint-Petersburg, Nasiba from Tver, Sergei from Ufa, Kunduz from Murmansk, Elena from Irkutsk and Jyldyz from Kazan. None of these picked credit specialists knows who was picked to be in the same group with him or her. Imagine they attract the following number of NEW clients in February:

Credit specialist	Number of new clients in February
Tahmina from Saint-Petersburg	41
Asel from Vladivostok	35
Sergei from Ufa	35
Jyldyz from Kazan	29
Nasiba from Tver	24
Bahodyr from Moscow	21
Djazgul from Novokuznetsk	12
Kunduz from Murmansk	12
Mirbek from Kaliningrad	9
Elena from Irkutsk	8

Tahmina attracted the highest number of new clients and won 10,000 RUB. Asel and Ulan attracted the same number of new clients, but Asel's last client took a credit on February 27, while Sergei's last client a took credit on February 28. That is why Asel won 10,000 RUB, while Sergei won 5,000 RUB. Jyldyz took fourth place and won 5,000 RUB too.

Each one of you has a big chance to win a prize! Everything is in your hands! Good luck!

Parallel4:

In order to increase productivity we are introducing a competition for monetary prizes. You will compete in attracting NEW clients. Every one of you is competing with nine other credit specialists who were randomly selected from all our branches in Russia. There is only a small probability that you will be competing against someone from your own office. Your task is to attract the maximum number of new clients in February.

In every group of 10 credit specialists, there are two prizes to compete for 10,000 RUB and 5,000 RUB. Every one of you has to choose the prize for which you want to compete before February 1. Send an sms with your name and surname, and the prize you choose (10 or 5) to the number +79267608072. You will compete only with those credit specialists from your group of 10 who chose the same prize. Two credit specialists with the two highest numbers of new clients among those who chose 10,000 RUB will receive 10,000 RUB. Two credit specialists with two highest number of new clients who chose 5,000 RUB will receive 5,000 RUB. It may be that everyone in the group of credit specialists chooses the same prize, then the second prize will not be awarded to anyone. If one of the prizes is chosen just by one or two credit specialists out of 10, then they will receive it in any case. However, you will not know how many credit specialists of your group have chosen the same prize until the end of the competition. After the competition is over, you will receive the names of your direct competitors, their choice of prizes, and the number of clients they attracted. In case of attracting the same number of clients, the credit specialist who attracted the last client earlier has priority.

For instance, a computer randomly determines the following credit specialists to be in one competing group: Asel from Vladivostok, Bahodyr from Moscow, Djazgul from Novokuznetsk, Mirbek from Kaliningrad, Tahmina from Saint-Petersburg, Nasiba from Tver, Sergei from Ufa, Kunduz from Murmansk, Elena from Irkutsk and Jyldyz from Kazan. None of these picked credit specialists knows who was picked to be in the same group with him or her. Asel, Tahmina, Bahodyr, Sergei, Djazgul, and Mirbek chose 10,000 RUB before the start of the competition. Jyldyz, Nasiba, Kunduz, and Elena chose 5,000 RUB. Imagine they attract the following number of NEW clients in February:

Credit specialists who chose 10,000 RUB	
Credit specialist	Number of new clients in February
Tahmina from Saint-Petersburg	41
Asel from Vladivostok	35
Sergei from Ufa	35
Bahodyr from Moscow	21
Djazgul from Novokuznetsk	12
Mirbek from Kaliningrad	9

Credit specialists who chose 5,000 RUB	
Credit specialist	Number of new clients in February
Jyldyz from Kazan	29
Nasiba from Tver	24
Kunduz from Murmansk	12
Elena from Irkutsk	8

Tahmina attracted the highest number of new clients among those who chose 10,000 RUB, thus she won 10,000 RUB. Asel and Ulan attracted the same number of new clients and both chose 10,000 RUB, but Asel's last client took credit on February 27, while Sergei's last client took credit on February 28, that is why Asel

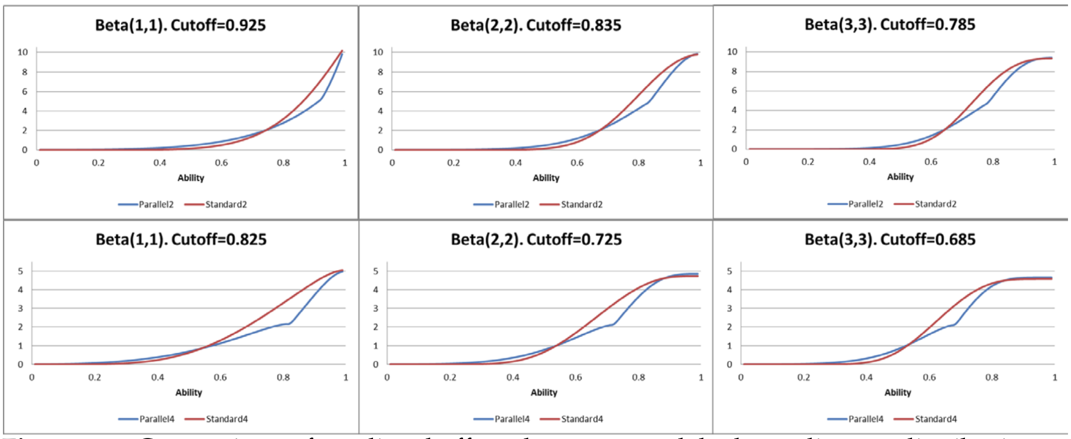


Figure 3.7. Comparison of predicted efforts between models depending on distribution of abilities

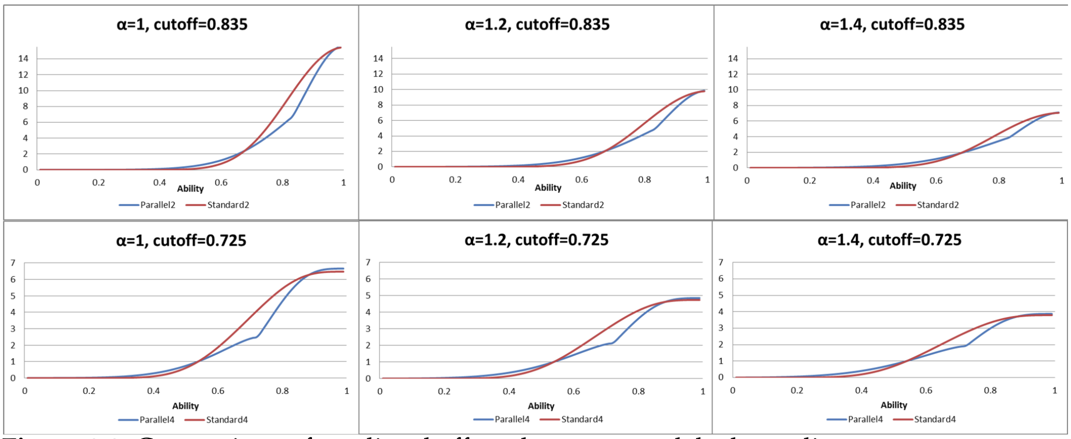


Figure 3.8. Comparison of predicted efforts between models depending on parameter α of the cost function

won 10,000 RUB. Jyldyz attracted the highest number of new clients among those who chose 5,000 RUB, thus she won 5,000 RUB, as well as Nasiba who took the second place among those who chose the 5,000 RUB.

Each one of you has a big chance to win a prize! Everything is in your hands! Good luck!

3.B.1 Simulations of effort functions by treatments given different cost functions and ability distributions

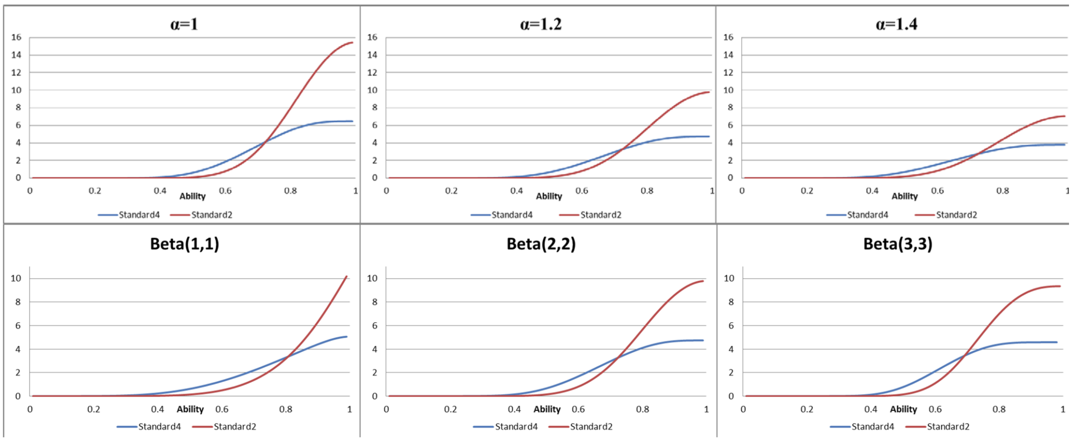


Figure 3.9. Comparison of predicted efforts of standard contests depending on parameter α of the cost function and distribution of abilities

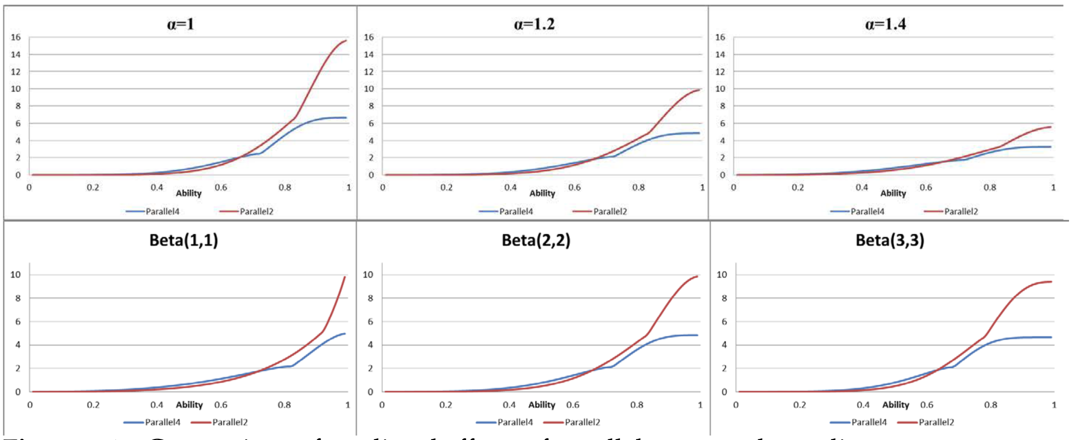


Figure 3.10. Comparison of predicted efforts of parallel contests depending on parameter α of the cost function and distribution of abilities

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Rechtliche Erklärung

Ich erkläre, dass ich die vorliegende Arbeit selbstständig und nur unter Verwendung der angegebenen Literatur und Hilfsmittel angefertigt habe.

Diese Dissertation ist aus Forschungsarbeiten mit den zu Beginn des jeweiligen Kapitels genannten Personen entstanden.

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Berlin, den 1. March 2016 Rustamdjan Hakimov