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Linus Mattauch[*], Ottmar Edenhofer[†], David Klenert[‡], Sophie Bénard[§]

Abstract

In developed economies, wealth inequality is high, while public capital is underprovided. Here, we study the impact of heterogeneity in saving behavior and income sources on the distributional effects of public investment. A capital tax is levied to finance productive public capital in an economy with two types of households: high income households who save dynastically and middle income households who save for retirement. We find that inequality is reduced the higher the capital tax rate is and that low tax rates are Pareto-improving. There is no clear-cut trade-off between efficiency and equality: middle income households' consumption is maximal at a capital tax rate that is higher than the rate which maximizes high income households' consumption.

JEL classification: E6, H23, H31, H40, H54

Keywords: Public capital, wealth disparity, inequality, household heterogeneity, Pasinetti Theorem, saving behavior

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1 Introduction

Capital taxation has for a long time not been favored by economists as a revenue-raising option, but has recently received much attention as a means of mitigating inequality in wealth (Piketty, 2014). "Capital is back" (Piketty and Zucman, 2014) as a factor to be taxed for financing public investment in industrialized countries: on the one hand, the capital-output ratio is increasing (Piketty, 2014) which in part explains great wealth disparities (Wolff, 2010). On the other hand, public investment may significantly enhance aggregate productivity (Romp and De Haan, 2007; Bom and Ligthart, 2014). Research on the distributional consequences of public investment has rarely analyzed the impact of distinct financing options. Further, realistic concepts of household heterogeneity in saving motives (Attanasio, 1994; Browning and Lusardi, 1996; Dynan et al., 2004), income sources (Wolff, 1998; Quadrini, 1997; Diaz-Gimenez et al., 2011) and time preference (Lawrance, 1991; Green et al., 1996) should crucially influence the outcome of different financing options for public investment, but previous analyses have neglected such heterogeneity.

This article introduces a model of two stylized types of households to study the effect of financing productive public investment by a capital tax. It answers the following question: is it possible to invest in public capital in a way that all households benefit while the financing mechanism mitigates wealth inequality? The answer is yes when there is significant underprovision of public capital and public investment is financed by a capital tax. We also prove that for each household type a different rate of capital taxation is optimal. These results hold for moderate to high values of the substitution elasticity between capital and labor.

Previous theoretical work on the provision of public capital has focused on homogenous households, not taking into account systematic differences in saving behavior and income. It has emphasized that a trade-off exists between investment into productive public capital and efficiency losses from taxes financing these investments. This trade-off determines an optimal tax rate (Barro, 1990). When household heterogeneity is taken into account to study the effects of public investment on *inequality*, all agents are typically assumed to have identical preferences, while the heterogeneity stems from different initial endowments. For example, Alesina and Rodrik (1994) show that when households differ by the division of their endowment in wealth and (inelastic) labor supply, households with higher labor to capital ratio prefer higher capital taxes. Their model implies that growth is always reduced when the preferred tax rate of all households but those who only hold capital is implemented. When public investment also affects the labor-leisure decision of households, Chatterjee and Turnovsky (2012) find that investing in public capital increases inequality in wealth (and welfare) in the long run, but also growth and average welfare, independent of the financing mechanism.

Here we introduce a two-class model of productive public capital and wealth inequality in which the heterogeneity stems from different saving motives, income sources and time preference rates. There are two different types of households: dynastically-saving high income households, whose only source of income is the interest from their capital stock and middle income households who live for two periods and save out of a life-cycle motive. A capital tax is levied by the government to finance productive public investment. The effect of other revenue sources, such as labor or consumption taxation, and the comparison with capital taxation is treated in our companion paper, Klenert et al. (2016), that extends the present analysis to less stylized functional forms. Our main point in the present article is to prove that capital taxation can both reduce inequality and be Pareto-improving.

Baranzini (1991) was the first to introduce two-class models in which there are capitalists, who leave bequests to their offspring, and workers who save over the life cycle only, thus providing earlier work by Kaldor (1955-1956) and Pasinetti (1962) with micro-foundations. Michl (2007, 2009) presents a version of a two-class model with a dynastically saving capitalist and workers living for two periods, on which our framework builds by modeling saving behavior across classes similarly.[1] We focus on financing public investment, whereas Michl (2009) instead employs a non-marginalist approach to production in order to assess different social security schemes.

In view of rising inequality, there is recently a renewed interest in understanding distributional conflict by means of two-class models: First, Stiglitz (2015a,b) considers a set-up similar to Michl (2009), but with exogenously given saving propensities, to analyze the role of rents for growth and distribution. Second, Dutt (2015) uses a two-class model in which the higher income group also receives wages as managers, in order to study the growth effects of an increasingly skewed distribution between the two classes. He finds that financialization and the increasing incomes of top managers lead to increased inequality and have adverse effects on growth. Third, Ryoo (2015) introduces financial assets and managerial pay into a two-class model and finds, in particular, that a declining growth rate can reduce inequality, in contrast to Piketty (2014). Fourth, Russo (2014) combines the division of the population into capitalists and workers with stochastic explanations for wealth inequality to show how a random process can amplify existing class distinctions.

Our model contains a version of the 'Pasinetti Theorem': In the steady-state the capitalists' propensity to save determines the interest rate (Pasinetti, 1962). We focus on the realistic case of the economy being populated by two classes, neglecting the "dual-Pasinetti equilibrium" of middle income households owning all capital (Samuelson and Modigliani, 1966), which is widely considered irrelevant (Pasinetti, 1966, 1974).[2] Given the empirical evidence summarized by Piketty (2014), one may rather wonder whether the 'anti-dual outcome', that high income households own all capital, is a real possibility (Taylor, 2014; Zamparelli, 2015).

To the best of our knowledge, Steedman (1972) was the first to examine how government activity influences the Pasinetti result, while La Marca (2005) more recently provided a synthesis of how government activity leads to the different possible equilibria. The case of the government providing productive public investment has, however, not been considered in the Post Keynesian strand of literature, but is otherwise standard (Barro, 1990; Futagami et al., 1993; Turnovsky, 1997). Recently, Tavani and Zamparelli (2015) study productive

[1] Michl and Foley (2004) and Michl (2009) label the two classes 'capitalists' and 'workers' respectively. We prefer to label these classes 'high income households' and 'middle income households' instead, because recent data (Wolff, 2010) suggests the existence of a third class, which hardly saves anything in some countries, see below. We believe that the term 'worker' is rather associated with such a third class.

[2] See Pasinetti (2012) for reflections from a contemporary perspective and implications for the current economic situation.

public capital also in a *classical* setting with a division between workers, who own no capital, and capital holders. They find that government action should differ according to the different growth model closures of elastic labor supply at a given wage share or fixed labor supply with adjusting real wage. Depending on the different closures, the government may be able to alleviate distributional conflicts by taxing profits to finance public investment and to provide transfers to workers.

In this article, we prove the following results: first, the higher the capital tax rate, the higher the share of capital owned by the middle income households, thus mitigating inequality in wealth. This result is due to the different saving motives of the two household types and hence the Pasinetti property of our model. Second, there is not one optimal level of capital taxation, but each type of household desires a different level. Middle income households are more favorable to the investment in public capital and desire a higher capital tax rate than high income households. The reason is that middle income households' savings depend only on their labor income. These first two results are reversed for low values of the substitution elasticity between capital and labor. Third, a Pareto-improving capital tax rate exists nevertheless: it can be proved that if public capital is underprovided, both classes are made better off as long as public capital is not unreasonably productive. The results together imply that there is no clear trade-off between equity and efficiency.

The division of households into two types with respect to their saving behavior is highlighted by recent data on wealth ownership. For example Wolff (2010) finds for the U.S. that the wealthiest 5 % of the population own roughly 62 % of total wealth, the next 55 % of the population own close to the remaining 38 %, while the rest of the population owns virtually no wealth. What distinguishes these cohorts? An analysis of the literature yields three major characteristics. First, the saving motive: The wealthiest cohort has been found to save dynastically, while the middle part of the wealth distribution is known to save in a life-cycle fashion (Attanasio, 1994; Browning and Lusardi, 1996; Dynan et al., 2004). Second, the income source: The wealthiest individuals are more likely to be self-employed entrepreneurs and to receive a higher share of capital income (Wolff, 1998; Quadrini, 1997; Diaz-Gimenez et al., 2011; Wolff and Zacharias).[3] Third, lower income households have higher time preference rates, as shown by Lawrance (1991) and Green et al. (1996). These empirical findings suggest dividing households into three groups with distinctly different saving motives: The wealthiest income cohorts are mainly self-employed and save dynastically, which suggests that they should be modeled as infinitely-lived households who only receive capital income. The cohorts in the middle of the wealth distribution save mainly for their retirement and are thus best represented as overlapping generation agents that have income from both labor and capital. The poorest households do not save. They are excluded from our analysis as the benefits they may derive from public investment are unlikely to affect their saving behavior.[4]

[3]According to Wolff (1998) 72 percent of the richest 1% were self-employed entrepreneurs for data on U.S. households for 1995.

[4]Their welfare may still be affected through an effect on wages, but this is not the focus of the present article. Our analysis of Pareto-improving policies seems valid even when including this cohort because the wage effect is likely to be positive, see the discussion of our results in Section 3.

Finally, evidence indicates that public investment is indeed suboptimally low in OECD countries. Building on Aschauer (1989) and Gramlich (1994), Bom and Ligthart (2014) recently conducted a meta-analysis: the output elasticity of public capital is estimated to be between 0.08 and 0.19, with short-run effects and a broad definition of public capital giving the lower bound. The higher estimate is reached for the long run and particularly for transport infrastructure and utilities at the regional level. It follows that public capital is underprovided because its corresponding marginal productivity is (significantly) higher than that of private capital.

The remainder of this article is structured as follows: Section 2 sets out the model. Section 3 first presents its steady-state properties and then contains the proofs of the main results. Further subsections subsequently elucidate the role of the elasticity of substitution between labor and capital as well as the robustness of our modeling assumptions about public capital, partially by means of numerical solutions. Section 4 summarizes the results and considers possible extensions.

2 Model

We model a one-good economy in which the government can finance public capital that enhances productivity. The population consists of two classes, high income households and middle income households. The representative middle income household provides labor when its members are young and they save for retirement; the household leaves no bequests. Middle income households are modeled as members of overlapping generations. The representative high income household holds capital and interest is its sole source of income. It has a dynastic saving motive and is thus modeled as an infinitely-lived agent. Both types of agents derive utility from consumption only. Factor markets clear and on the capital market, the supply consists of both high income households' investment and middle income households' savings. There are decreasing returns to scale in private and public capital, but constant returns to scale in private capital and labor. We first describe the households' behavior before detailing the role of the firms and the government.

High income household The high income household owns a capital stock K_t^c and maximizes intertemporal utility given by

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho_c)^t} \ln(C_t^c), \tag{1}$$

with consumption C_t^c and time preference rate ρ_c. Its budget constraint is

$$K_{t+1}^c - K_t^c = (1-\tau) r_t K_t^c - C_t^c, \tag{2}$$

where r_t is the interest rate and τ is the capital tax.

The initial capital stock is given as $K_1^c = K_0^c$. The high income household respects a transversality condition: $\lim_{t\to\infty} \left(K_t^c \prod_{s=1}^{t-1} \frac{1}{1+r_s} \right) \geq 0$.

Solving the maximization problem yields an Euler equation for this household:

$$\frac{C_{t+1}^c}{C_t^c} = \frac{1+(1-\tau) r_{t+1}}{1+\rho_c}. \tag{3}$$

Middle income household The middle income household lives for two periods, a 'young' (y) and an 'old' (o) stage. It maximizes its lifetime utility, where utility from consumption in the second period is discounted by the time preference rate ρ_w:

$$\ln(C_t^y) + \frac{1}{1+\rho_w}\ln(C_{t+1}^o). \tag{4}$$

In the first period, the middle income household rents its fixed labor L to the producing firm, which in turn pays a wage rate w_t. Labor income can either be consumed or saved for the old age:

$$w_t L = S_t + C_t^y. \tag{5}$$

In the second period the middle income household consumes its savings and the interest on them:

$$C_{t+1}^o = (1+(1-\tau)r_{t+1})S_t. \tag{6}$$

Solving the optimization problem subject to the budget constraints leads to an Euler equation for this household:

$$\frac{C_{t+1}^o}{C_t^y} = \frac{1+(1-\tau)\cdot r_{t+1}}{1+\rho_w}. \tag{7}$$

From Equations (5-7) explicit expressions for saving and consumption in the two periods can be derived:

$$S_t = \frac{1}{2+\rho_w} w_t L \tag{8}$$

$$C_t^y = \left(\frac{1+\rho_w}{2+\rho_w}\right) w_t L \tag{9}$$

$$C_{t+1}^o = \left(\frac{1+(1-\tau)r_{t+1}}{2+\rho_w}\right) w_t L. \tag{10}$$

The savings rate is constant, as is standard in discrete OLG models when the utility function is logarithmic. The same is true for the consumption of the young agent, while C_{t+1}^o is still dependent on the interest rate r. Moreover, combining Equations (8-10) implies that C_t^y and C_{t+1}^o depend linearly on savings S.

Production The firm produces output according to a Cobb-Douglas production function. Public capital P_t enhances productivity of both factors:

$$F(K_t, L) = A_t K_t^{\alpha} L^{1-\alpha} \tag{11}$$

with $A_t = P_t^{\beta}$, and $0 < \alpha, \beta < 1$. So β denotes the efficiency factor of public capital P_t. Throughout, we assume $\alpha + \beta < 1$ to exclude the case of long-run or explosive growth.

K_t denotes the sum of the individual capital stocks

$$K_t = K_t^c + S_{t-1}. \tag{12}$$

Profit maximization yields the standard rates of return for capital and labor (with δ_K denoting depreciation of private capital):

$$r_t + \delta_K = \frac{\partial F(K_t, L)}{\partial K_t} = \alpha \frac{F(K_t, L)}{K_t} \tag{13}$$

$$w_t = \frac{\partial F(K_t, L)}{\partial L} = (1-\alpha)\frac{F(K_t, L)}{L}. \tag{14}$$

Government The sole function of the government in this model is the provision of public capital. It finances its investments by the capital tax, thus influencing the interest rate. Hence the government's activity is summarized as the change in the stock of public capital (with δ_P denoting its depreciation):

$$P_{t+1} = P_M + (1-\delta_P)P_t + \tau r_t K_t. \tag{15}$$

For the following analytical results, we assume that for non-governmental provision of public capital P_M,

$$P_M = 0. \tag{16}$$

One could object to such a stylized role for public capital in our model that the finding of Pareto-improving public investment relies on very high marginal returns to public capital if public investment approaches zero. We address this concern by confirming numerically (in Subsection 3.3) that even if a minimal provision of public good $P_M > 0$ is present without government intervention our qualitative results do not change.

3 Results

In this section we show that inequality in wealth is mitigated by a capital tax levied for public investment in our model. We also characterize the optimal tax rate for each household type: the middle income household is more favorable to capital taxation and higher public investment than the high income household. We point out that low capital tax rates lead to a Pareto improvement, even for the case in which the economy is functioning without any public investment. Figures 1 and 2 below illustrate the main findings. First, we characterize the steady-state and the validity range of the model. We then prove the results just stated for the case of a Cobb-Douglas production function. Further, the model is calibrated and analyzed numerically to determine the optimal tax rates, which cannot be calculated explicitly and to examine the role of potential non-publicly provided existing infrastructure. The numerical analysis also ensures that all critical values for the tax rate are within the validity limit of the two-class model for a wide range of parameters. Finally, we generalize the analysis to a wider range of elasticities of substitution between capital and labor.

3.1 Steady state and validity range

In our model, a version of the Pasinetti Theorem holds. In a model with two types of households, in which one household only receives income from capital – the "capitalist" –, the Pasinetti Theorem (Pasinetti, 1962) states that the capitalist will determine the steady state interest rate independently of the saving rate of the other household type – the "worker" – or the production technology. This is true unless the worker's saving propensity is so high that the capitalist class ceases to exist (Samuelson and Modigliani, 1966).

In the steady-state of our model we find a similar duality although the saving behavior of our household types is derived from their intertemporal preferences: Either (i) the high income household (corresponding to the "capitalist") determines the steady-state interest rate *or* (ii) its capital stock and consumption is zero and the economy is populated only by the middle income household (corresponding to the "worker"). Which regime holds in the steady-state of our model depends on its parameters. In the following analysis, we are exclusively concerned with (the applicable) case (i) (Pasinetti, 1966, 2012). We study the effect of the capital tax on the wealth distribution and call the tax rate at which the high income household's share of capital approaches zero, the limit of case (i), the model's validity limit.

The (unique, non-trivial) steady-state is saddle-point stable and the economy converges to it on a stable path because the high income household's behavior determines the overall dynamics.[5] The Pasinetti-type behavior of the model in the steady-state can be explained as follows: The high income household's saving behavior determines the interest rate because reducing or increasing its investment is its only means of obtaining its desired long-term distribution of consumption to capital, as is also true in a standard Ramsey model. Any attempt of the middle income household to obtain a different interest rate would thus be balanced by the high income household adjusting its saving rate. Thus the middle income household accepts the interest rate as given. However, its propensity to save (which is independent of the interest rate) influences the amount the high income household saves, who is bound to own the share of total capital net of what the middle income household saves. Thus the high income household determines the interest rate and with it the total capital stock, but the middle income household determines the capital share owned by the high income household.

Steady-state values of variables are denoted by a tilde. We first assume that the high income household's consumption is strictly positive, $\widetilde{C^c} > 0$ and then derive the validity range of this assumption.

It follows from the high income household's Euler Equation (3) that

$$\widetilde{r} = \frac{\rho_c}{(1-\tau)} \tag{17}$$

and from its budget constraint (2) that

$$\widetilde{C^c} = \rho_c \widetilde{K^c}. \tag{18}$$

The steady-state level of public capital is given by:

$$\widetilde{P} = \frac{1}{\delta_P} \tau \widetilde{r} \widetilde{K}. \tag{19}$$

[5]A heuristic argument for saddle-point stability is as follows: The dynamics of the model are captured by four Equations for the variables K^c, C^c, P and K, namely Equations (2), (3), (15) and substituting Equation (8) into Equation (12). If it were the case that $K^c = K$, then the model would be a neoclassical growth model with public capital in discrete time. The dynamics of public capital is such that the required stability properties carry over from the neoclassical growth model, where C^c is a "jump variable". What does Equation (12) add to the dynamics of the case $K^c = K$? The only modification is that in Equations (2) and (3) the interest rate is lower than if K^c was the only capital input (the revenue in Equation (8) stays a constant fraction of total output). This implies that there are no qualitative differences in the dynamics, only the steady-state value of K^c is smaller than the Keynes-Ramsey level of capital K by exactly S. This can be shown by transforming the original system by dividing all variables through Y, and noting that $\frac{K}{Y} = \frac{K^c}{Y} - \frac{1-\alpha}{1+\rho_c}$, which reduces the transformed system to three dimensions.

For later reference only, we note that in the steady-state an explicit expression for $\widetilde{K}$ can be obtained. From Equation (13) a steady-state relationship for the production factors $\widetilde{P}$ and $\widetilde{K}$ can be derived:

$$\alpha \widetilde{P}^{\beta} \widetilde{K}^{\alpha-1} L^{1-\alpha} = \frac{\rho_c}{1-\tau} + \delta_k. \tag{20}$$

Rearranging and inserting Equation (19) into Equation (20) gives an explicit expression for $\widetilde{K}$:

$$\widetilde{K}^{(1-\left(\frac{\beta}{1-\alpha}\right))} = L\left(\tau\frac{\rho_c}{\delta_P(1-\tau)}\right)^{\left(\frac{\beta}{1-\alpha}\right)}\left(\frac{\frac{\rho_c}{(1-\tau)}+\delta_k}{\alpha}\right)^{\left(-\frac{1}{1-\alpha}\right)}. \tag{21}$$

The equation shows that K as a function of τ is inverted U-shaped (recalling from above that we exclude the case $\beta > 1-\alpha$). This is because of two counteracting effects: as τ increases, K initially increases due to the productivity-enhancing effect of public capital (given by the first term of the product). However, it eventually decreases because of the distortionary effect of capital taxation that discourages capital accumulation (represented by the second term above).

Validity limit The above equations (17–21) are only valid if both agents have positive capital and consumption (see the discussion at the beginning of this section). This is ensured as long as the middle income household's savings are smaller than the total capital, that is

$$\frac{\widetilde{S}}{\widetilde{K}} = \frac{L\widetilde{w}}{(2+\rho_w)\widetilde{K}} = \frac{(1-\alpha)}{\alpha(2+\rho_w)}\left(\frac{\rho_c}{1-\tau}+\delta_k\right) < 1, \tag{22}$$

where Equations (8) , (14) and (20) were used to obtain an expression in terms of parameters only. We use the ratio $\frac{\widetilde{S}}{\widetilde{K}}$, the share of capital held by the high income household, as an indicator for wealth inequality below.

From Equation (22), one can derive that there exists a constant $\tau_{lim} < 1$ for which the steady-state characterization of the agents' behavior is valid in exactly the interval $(0; \tau_{lim})$. The expression of τ_{lim} is:

$$\tau_{lim} = 1 - \rho_c\left(\frac{1-\alpha}{\alpha(2+\rho_w) - \delta_k(1-\alpha)}\right). \tag{23}$$

For the remainder of the analysis, we assume that all critical values of τ are within the interval on which the analysis is valid. We check numerically in Section 3.3 that this assumption holds for a wide range of parameters, including those that best represent developed economies in a stylized way.

We next describe the impact of the capital tax rate on the steady-state behavior.

3.2 The effects of policy

The three main results regarding the role of fiscal policy in our model are stated as three propositions below:

- a capital tax levied for public investment decreases inequality in wealth (Proposition 1),

- middle income households prefer a higher capital tax rate than high income households (Proposition 2) and

- there exists a Pareto-improving range of capital tax rates (Proposition 3).

Below, inequality and optimality for the two household types are exclusively discussed in terms of their *wealth*: This is sufficient as the *consumption* of the high income household and the old and young middle income household are linear functions of their wealth. For the case of the high income household this is due to Equation (18), for middle income household's consumption it is an immediate consequence of Equations (8–10).

The economic intuition behind the three propositions can be developed as follows: The first result is a consequence of the Pasinetti Theorem. The middle income household's savings are proportional to its wage income, which is proportional to total output. So the share of the middle income household's savings to total capital – the indicator for inequality – depends linearly on the ratio of total output to total capital. By the properties of the neoclassical production function, the output-capital ratio depends positively on the marginal productivity of capital. However, the marginal productivity of capital increases for higher capital taxes, even independently of how the tax revenue is used. This follows from the Pasinetti-behavior of the model – as the interest rate is fixed by the high income household's behavior to be an increasing function of the tax rate (see Equation (17)).

The second result is derived from the fact that total capital depends in a convex way on the capital tax. The relationship is determined by the counteracting effects of the distortion through the capital tax and the beneficial effect of spending it on public investment that also impacts the productivity of private capital positively.[6] The maximal wealth of the middle income household then occurs for a higher capital tax value because his savings depend *only* on his labor income. His labor income depends monotonically on accumulated total private capital, but also on public capital, and the impact of the latter is always positive. The maximal wealth of the high income household occurs for lower capital values as a consequence of the first result: The share of total capital belonging to the middle income household increases faster than total capital the higher the capital tax. Hence the maximal share of the high income household must be reached for lower values of the capital tax than the maximal total capital.

The third result is due to the fact that the marginal productivity of public capital is higher than the efficiency loss from distortionary capital taxation, if little public capital is provided. Section 3.3 provides a numerical analysis of the magnitude of this effect: The possibility that some public capital may exist even if there is no government intervention is considered there, but it is found that the level of non-government financed public capital would need to be very high to rule out the possibility of a Pareto improvement.

Figures 1 and 2 illustrate these results.

Proposition 1. *Capital taxation (used for public investment) decreases inequality in wealth:* $\frac{\mathrm{d}(\frac{\tilde{S}}{\tilde{K}})}{\mathrm{d}\tau} > 0$ *for* $0 < \tau < 1$.

[6]This is the same trade-off between efficiency-enhancing public investment and distortionary capital taxation studied by Barro (1990) for a single infinitely-lived agent.

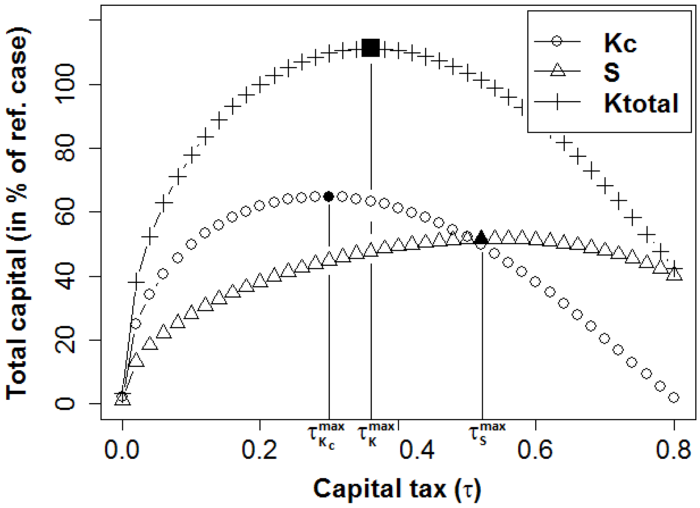

Figure 1: The graph shows the size of the different private capital stocks for capital taxes between 0 and τ_{lim} = 81%. Each capital stock has its maximum for a different value of the capital tax τ. The high income household prefers a lower tax rate than the middle income household, with the capital tax rate that maximizes total capital in between (i.e. $\tau^{max}_{K_c} < \tau^{max}_{K} < \tau^{max}_{S}$). Units of capital are normalized to 100% at the reference case of $\tau = 21\%$.

Proof. From Equation (22),

$$\frac{\widetilde{S}}{\widetilde{K}} = \frac{(1-\alpha)}{\alpha(2+\rho_w)}\Big(\frac{\rho_c}{1-\tau} + \delta_k\Big), \tag{24}$$

which is increasing in τ. □

Proposition 2. *Middle income households prefer a higher capital tax rate than high income households: for some threshold rate* τ^*, $\frac{\mathrm{d}\widetilde{S}(\tau)}{\mathrm{d}\tau} > 0$ *for* $0 < \tau < \tau^* + \varepsilon$ *and* $\frac{\mathrm{d}\widetilde{K}^C(\tau)}{\mathrm{d}\tau} < 0$ *for* $1 > \tau > \tau^* - \varepsilon$ *for some* $\varepsilon > 0$.

Proof. Let τ^* be the value of τ that maximizes $\widetilde{K}$ as a function of τ on $(0, 1)$: it can be calculated that for $\tau^* = \frac{\beta(\rho_c+\delta_k)}{\beta\delta_k+\rho_c}$, $\frac{\mathrm{d}\widetilde{K}(\tau^*)}{\mathrm{d}\tau} = 0$ and shown that $\frac{\mathrm{d}\widetilde{K}(\tau)}{\mathrm{d}\tau} > 0$ for $\tau < \tau^*$ and $\frac{\mathrm{d}\widetilde{K}(\tau)}{\mathrm{d}\tau} < 0$ for $\tau > \tau^*$ (see Supplementary Material, available from the authors upon request).

By combining Equations (8), (14) and (19), we obtain:

$$\widetilde{S} = \underbrace{\frac{1-\alpha}{2+\rho_w} L^{(1-\alpha)} \big(\frac{\rho_c}{\delta_P}\big)^{\beta}}_{=\vartheta} \left(\frac{\tau}{(1-\tau)}\right)^{\beta} \widetilde{K}^{\alpha+\beta}.$$

Thus:

$$\frac{\partial\widetilde{S}}{\partial\tau} = \vartheta\left(\frac{\tau}{1-\tau}\right)^{\beta}\left[\frac{\beta}{\tau(1-\tau)} + \frac{\alpha+\beta}{\widetilde{K}}\frac{\partial\widetilde{K}}{\partial\tau}\right]\widetilde{K}^{\alpha+\beta}. \tag{25}$$

Hence $\frac{\mathrm{d}S(\tau)}{\mathrm{d}\tau} > 0$ for $0 < \tau < \tau^*$. Also $\frac{\mathrm{d}S(\tau)}{\mathrm{d}\tau}$ is continuous and strictly positive at τ^*, thus positive on $[\tau^*, \tau^* + \varepsilon]$ for some ε.

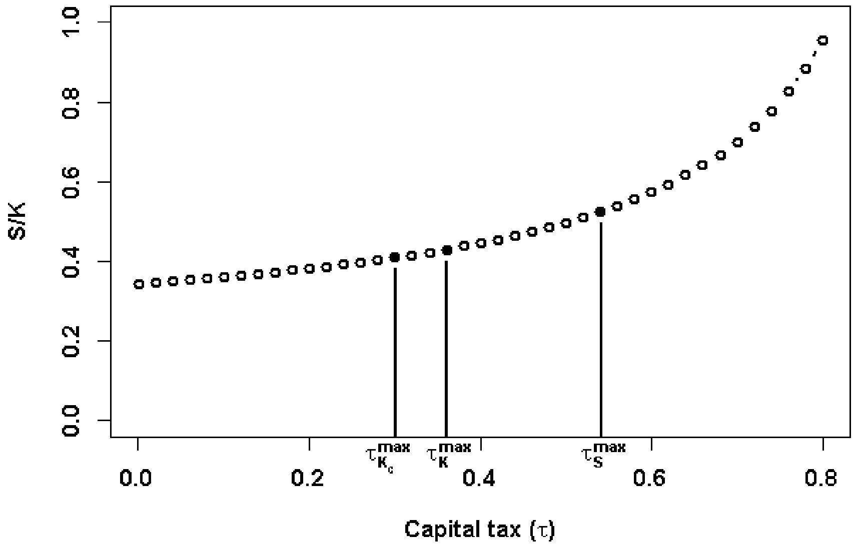

Figure 2: This graph illustrates Proposition 1: the middle income household's share of total capital increases within the validity range of capital taxes between 0 and $\tau_{lim} = 81\%$.

For a similar argument for $\widetilde{K}^C$, use that

$$\widetilde{K}^C = \widetilde{K}(1 - \frac{\widetilde{S}}{\widetilde{K}}).$$

For $1 > \tau > \tau^*$, $\widetilde{K}^C$ is thus the product of two positive decreasing functions and hence $\frac{\mathrm{d}\widetilde{K}^C(\tau)}{\mathrm{d}\tau} < 0$ there. At τ^*, $\frac{\mathrm{d}\widetilde{K}^C(\tau)}{\mathrm{d}\tau}$ is strictly negative and continuous, hence it is also negative on $[\tau^* - \varepsilon, \tau^*]$. □

Proposition 3. *If* $2\beta < 1 - \alpha$, *there exists a Pareto-improving range of capital tax rates:* $\frac{\mathrm{d}\widetilde{S}(\tau)}{\mathrm{d}\tau} > 0$, $\frac{\mathrm{d}\widetilde{K}^C(\tau)}{\mathrm{d}\tau} > 0$ *for small* $\tau > 0$.

Proof. From Equation (25), $\frac{\mathrm{d}\widetilde{S}(\tau)}{\mathrm{d}\tau} > 0$ is positive, if $\frac{\mathrm{d}\widetilde{K}(\tau)}{\mathrm{d}\tau} > 0$ is, which is true for all $0 < \tau < \tau^*$.

For $\widetilde{K}^C = \widetilde{K}(1 - \frac{\widetilde{S}}{\widetilde{K}})$, it is sufficient to prove that $\lim_{\tau\to 0} \frac{\mathrm{d}\widetilde{K}(\tau)}{\mathrm{d}\tau} > 0$, because

$$\frac{\mathrm{d}\widetilde{K}^C(\tau)}{\mathrm{d}\tau} = \frac{\mathrm{d}\widetilde{K}(\tau)}{\mathrm{d}\tau}(1 - \frac{\widetilde{S}}{\widetilde{K}}) + \widetilde{K}\frac{\mathrm{d}}{\mathrm{d}\tau}(1 - \frac{S}{K}) \tag{26}$$

and the second summand goes to zero by Equation (24) as well as noting $\lim_{\tau\to 0} \widetilde{K}(\tau)$. It can be shown that

$$\lim_{\tau\to 0} \frac{\mathrm{d}\widetilde{K}(\tau)}{\mathrm{d}\tau} = \begin{cases} \infty & \text{if } 2\beta < 1 - \alpha, \\ 0 & \text{if } 2\beta > 1 - \alpha. \end{cases} \tag{27}$$

(See the Supplementary Material for the derivation of this claim). □

In the following subsection we verify numerically that the proposition also holds for $\beta > \frac{1}{2}(1-\alpha)$. Empirically, with current estimates of $\beta = 0.2$ and $\alpha = 0.38$ (see page 14 below), the parameter restriction needed here to obtain the analytical result is rather harmless.

3.3 Calibration and robustness with respect to modeling public capital

In the derivation of the three propositions above, we assumed that there is no public capital when there are no taxes levied to finance it ($P_M = 0$). This implies a very high, potentially implausible, marginal productivity of public capital when very little of it is provided. To relax this assumption (needed for analytical tractability above), we present results from a numerical simulation as a robustness check, in which it is assumed that some public capital is provided even without government intervention. We also determine the tax values which cannot be calculated analytically, such as $\tau_{k_c}^{max}$ and τ_s^{max} and list the corresponding values for the distribution of capital between the agents $\frac{S}{K}$. We finally determine the range of each input parameter, within which the validity condition (23) from Section 3.1 holds: outside the validity range, the model is not meaningful because only one class would exist. Table 2 gives the broad range of parameters in which the model is within the validity limit.

The simulation yields that the results of Propositions 1–3 also hold for low to moderate base levels of the public capital stock and illustrates the dependency of optimal tax rates on different public capital productivities β. The results are summed up in Table 1. The corresponding figures show the trajectories of capital K, K^C and S and consumption C (Figure 1) and of capital ownership $\frac{S}{K}$ (Figure 2) for tax rates between 0 and $\tau_{lim} = 0.81$. Values of the stocks displayed are steady-state values without indicating this by a tilde in this subsection.

Parameter	$\tau_{k_c}^{max}$	τ_k^{max}	τ_s^{max}	$\frac{S}{K}\lvert\tau_{k_c}^{max}$	$\frac{S}{K}\lvert\tau_k^{max}$	$\frac{S}{K}\lvert\tau_S^{max}$
$\beta^1 = 0$	0	0	0	0.34	0.34	0.34
$\beta^2 = 0.1$	0.16	0.2	0.37	0.37	0.38	0.43
$\beta^3 = 0.2$	0.29	0.36	0.54	0.41	0.43	0.53
$\beta^4 = 0.3$	0.41	0.49	0.64	0.45	0.49	0.62
$\beta^5 = 0.4$	0.52	0.60	0.70	0.51	0.57	0.71
$\beta^6 = 0.5$	0.63	0.69	0.75	0.60	0.68	0.80
$P_M^1 = 0$	0.29	0.36	0.54	0.41	0.43	0.53
$P_M^2 = 4$	0.14	0.24	0.48	0.37	0.39	0.48
$P_M^3 = 8$	0	0.12	0.44	0.34	0.36	0.46
$P_M^4 = 12$	0	0	0.40	0.34	0.34	0.45

Table 1: Numerical results for varied public capital parameters β and P_M. The table displays the capital tax values which maximize the different capital stocks and the ratio of middle income household's savings to total capital for these tax rates. The highlighted rows correspond to the standard calibration, see Table (2) for the other parameter values. Units of P_0 can be converted to units of the private capital stock K as follows: $P_M = 4$ corresponds to $P_M \approx 21\%$ of the steady-state value of K in the baseline calibration.

The non-highlighted rows in Table (1) show the results for varied public capital productivity (β), and varied initial public capital stock (P_M), displaying the change in the numerical values for τ and $\frac{S}{K}$. The main results remain true up to a base level of public capital of $P_M < 8$ (corresponding to $\approx 21\%$ of the steady-state value of K in the baseline calibration). For values $P_M \geq 8$, no Pareto improving policy is possible anymore as further public investment is of no value to the high income household, who prefers a tax rate of 0. For the case of totally unproductive public capital ($\beta = 0$) each agent prefers a tax rate of 0.

A more extensive sensitivity analysis of all parameters of the model shows that for the ranges given in Table (2), the results obtained with the standard calibration are robust. In particular all capital stocks as functions of the capital tax rate reach their maximum within the model's validity limit for a large parameter range.

We calibrated the model so that for a capital tax of 21 %, which is the average capital tax rate in OECD countries between the years 1970 and 2000 (Carey and Rabesona, 2002), the distribution of wealth is as in Wolff (2010): in the U.S. in 2007, 62 % of net worth are held by the top 5 % of the population and almost 38 % of net worth by the next 55 % (while the bottom 40 % hardly possess any net worth). In accordance with findings on significant differences in intertemporal behavior of different income cohorts, the time preference rate of high income households is chosen lower than that of middle income households (Lawrance, 1991; Green et al., 1996).

The capital's share of income α in the production function was chosen to be 0.38. This is in accordance with observations by the OECD, that in 26 OECD countries with reliable data available, the labor share of income was dropping from 66.1 % to 61.7 % from 1990 to 2009 (OECD, 2012). The productivity of public capital β, which is varied above to highlight the robustness of the result, has been estimated to be between 0.08 and 0.19 (Bom and Ligthart, 2014), downwardly correcting higher estimates from earlier studies (Aschauer, 1989; Gramlich, 1994). Labor L, the total working hours, is a fixed factor in our model. Its value scales all variables.[7] We normalize labor $L = 100$ and measure the other variables in this unit to obtain values in a convenient range. Time is measured in steps of 30 years, as middle income households are assumed to live for two periods.

3.4 The role of the elasticity of substitution between capital and labor

The above results are proved for the case of a Cobb-Douglas production function (see Equation 11), implying an elasticity of substitution of capital and labor $\sigma = 1$. In this subsection we relax this assumption to show that the first two results of the article depend on the value of σ. Its empirical value has recently been debated: For instance, a meta-study finds that "the weight of the evidence suggests a value of σ in the range of 0.40 to 0.60" (Chirinko, 2008) with 26 out of 31 studies finding an elasticity below 1 (see also Rognlie, 2014). This would imply that a Cobb-Douglas production function is not a good approximation to reality. In contrast and more recently, Piketty and Saez (2014) and Piketty and

[7]This can be seen from Equations (8), (9) and (10) for the overlapping generations household, from Equation (18) and (21) for the infinitely-lived household and Equation (19) for public capital.

Parameter	Range	Standard value	Corresponding annual value
ρ_c	$0.2-1.6$	0.56	1.5%
ρ_w	$3.0-8.0$	3.98	5.5%
δ_k	$0.3-1.7$	0.7	4%
δ_P	$0.3-3.1$	0.7	4%
β	$0.0-0.5$	0.2	–
P_M	$0.0-5.0$	0.0	–

Table 2: For values inside the range given in column two, the results of the model are economically meaningful: that is, the functions $K(\tau)$ and $S(\tau)$ reach their maximum within the validity range ($0 < \tau < \tau^{lim}$). In the third and fourth column the standard values used in the simulation and the corresponding yearly values are given.

Zucman (2015) argue, based on the data on changes in the capital-output ratio collected in Piketty and Zucman (2014), that the elasticity must be higher than 1. It is thus vital to understand how the above results change if an elasticity unequal to 1 is considered.

We show below that for elasticities significantly lower than 1, there is an additional effect that determines the level of inequality when public capital is financed by a capital tax: wealth inequality can also be rising because, for fairly inelastic production factors, a capital tax harms wages more than capital. Below, we first prove that wealth inequality could also *increase* for low elasticities and high capital tax rates or for all capital tax rates and very low elasticities. We subsequently determine numerically for which values this happens: under our standard parametrization (see Subsection 3.3) we find that: (i) inequality is unambiguously decreasing for all values of $\sigma > 0.82$, (ii) it is declining for low tax values and then rising for high tax values for $0.61 < \sigma \leq 0.82$. and (iii) inequality is rising for $\sigma \leq 0.61$.

To obtain these results, let production be given by

$$F(K_t, L) = \left(\alpha K^\gamma + (1-\alpha)(AL)^\gamma\right)^{\frac{1}{\gamma}} \tag{28}$$

with $A = P^\beta, P_M = 0, \gamma < 1, \gamma \neq 0$.

This implies that the elasticity of substitution between capital and labor σ is given by $\sigma = \frac{1}{1-\gamma}$. For this particular CES-production function, an explicit expression for the steady-state capital share of middle income households $\frac{\widetilde{S}}{\widetilde{K}}$ can still be derived (derivation in the Supplementary Material):

$$\frac{\widetilde{S}}{\widetilde{K}} = \frac{(1-\alpha)}{\alpha(2+\rho_w)}\Big(\frac{\rho_c}{1-\tau} + \delta_k\Big) \times \Big(\frac{1}{(1-\alpha)}\big((\frac{1}{\alpha}(\frac{\rho_c}{1-\tau} + \delta_k))^{\frac{\gamma}{(1-\gamma)}} - \alpha\big)\Big). \tag{29}$$

It can further be shown that the function $\frac{\widetilde{S}}{\widetilde{K}}(\tau)$ has a maximum at:

$$\tau_z = 1 - \frac{\rho_c}{\alpha(\alpha(1-\gamma))^{\frac{1-\gamma}{\gamma}} - \delta_K}. \tag{30}$$

For the relevant parameter range, the derivative is positive for $\tau < \tau_z$ and negative for $\tau > \tau_z$ (see Supplementary Material): $\frac{\widetilde{S}}{\widetilde{K}}(\tau)$ has exactly one maximum at τ_z, which is in the economically relevant range of $\tau \in (0,1)$ for specific values of γ only.

This finding gives rise to different cases: If $\tau_z < 0$, or $\tau_z > 1$, wealth inequality will increase or decrease monotonically, respectively. Further, the maximum of the function $\frac{\widetilde{S}}{\widetilde{K}}(\tau)$ matters only for the economic outcome if it is within the validity limit of the model, which is now dependent on γ, as it is determined by $\frac{\widetilde{S}}{\widetilde{K}}(\gamma) < 1$. If the maximum occurs within this range, inequality will decrease for tax values lower than this maximum and increase for tax values higher than it. If instead the maximum occurs for $\tau_z \in (0,1)$ but yields $\frac{\widetilde{S}}{\widetilde{K}}(\tau_z) > 1$, inequality is decreasing for all (meaningful) tax rates. We verify numerically below that all these cases occur and that they monotonically depend on γ.

We next determine the specific cases for γ for which the limiting cases (that is, $\frac{\widetilde{S}}{\widetilde{K}}(\tau_z)$=0 and $\frac{\widetilde{S}}{\widetilde{K}}(\tau_z)$=1) occur. First, set $\tau = 0$ in Equation (30) and solve for γ. This yields:

$$(\frac{1}{\alpha}(\rho_c + \delta_K)) = (\alpha(1-\gamma_0))^{\frac{1-\gamma_0}{\gamma_0}} \tag{31}$$

Second, to find out for which γ, $\frac{\widetilde{S}}{\widetilde{K}}(\tau_z) = 1$, it can be verified that

$$\frac{\widetilde{S}}{\widetilde{K}}(\tau_z) = -\frac{\alpha\gamma_1}{(2+\rho_w)}\big(\alpha(1-\gamma_1)\big)^{\frac{1-\gamma_1}{\gamma_1}}. \tag{32}$$

Setting this equal to 1 leads to:

$$(-\frac{2+\rho_w}{\alpha\gamma_1})^{\frac{\gamma}{1-\gamma_1}} = \alpha(1-\gamma_1). \tag{33}$$

Equations (31) and (33) are not generally solvable analytically for γ. We instead solve these equations numerically: solutions for γ_0 and γ_1 and the corresponding elasticities σ_0 and σ_1, depending on α for robustness, are given in Table 3. Further, Figure 3 illustrates the behavior of $\frac{\widetilde{S}}{\widetilde{K}}(\tau)$ for key values of the elasticity. Table 4 then presents how the preferred tax rates of the two cohorts depend on it. It illustrates that public investment is still Pareto-improving for any elasticity, but that middle income households may prefer lower capital taxes for low elasticities.

α	γ_0	σ_0	γ_1	σ_1
0.2	-1	0.5	- 0.42	0.7
0.3	-0.78	0.56	-0.29	0.78
0.38	-0.645	0.61	-0.22	0.82
0.5	-0.48	0.68	-0.15	0.87
0.6	-0.37	0.73	-0.1	0.91

Table 3: Threshold values of the elasticity given for various values of α. At σ_0 the highest share of middle income households' capital $\widetilde{S}/\widetilde{K}$ occurs for $\tau = 0$ and thus wealth inequality is continuously increasing. At σ_1, $(\widetilde{S}/\widetilde{K})(\tau_z) = 1$ and thus wealth inequality is continuously decreasing over the range of all meaningful tax rates. The highlighted row is the standard calibration.

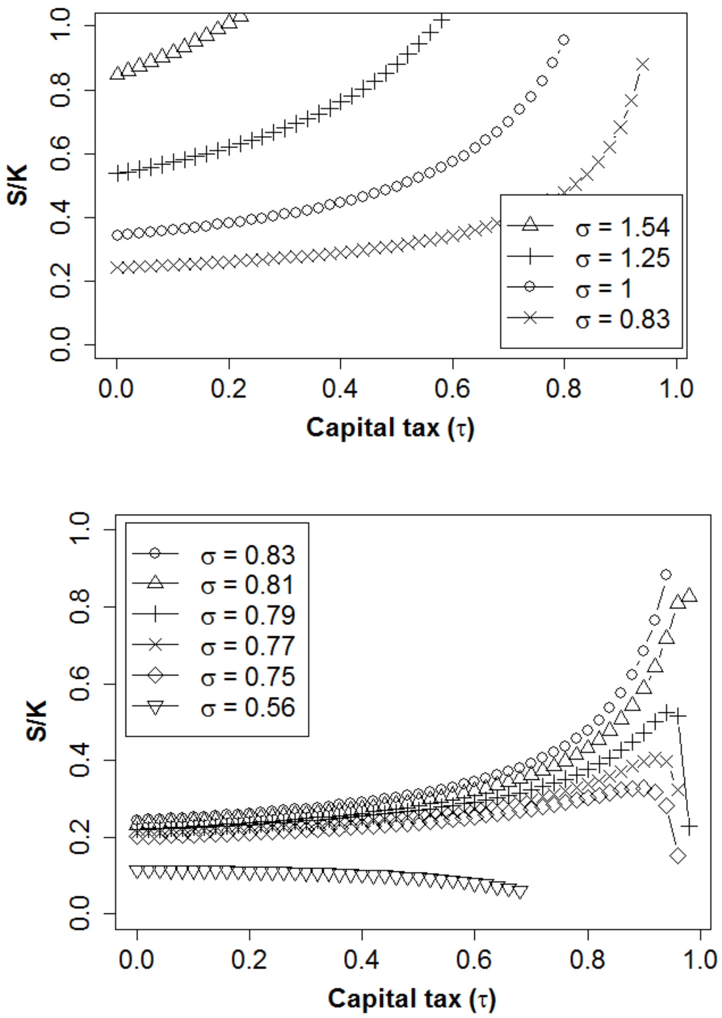

Figure 3: Wealth inequality as a function of the capital tax rate for various elasticties. The upper panel shows selected cases of high elasticities that reproduce the behavior of the Cobb-Douglas function, the lower panel shows cases of low elasticities that illustrate the deviating cases. The middle income households' share of total capital increases monotonically only with high elasticities and decreases monotonically with low elasticites. For middle cases, it increases for low tax rates, but then decreases for high tax rates.

The numerical results on the role of the elasticity of substitution can thus be summarized as follows: Propositions 1 and 2 above are reversed for elasticities of substitution between capital and labor significantly lower than 1, while Proposition 3 still holds. That is, while financing public investment by capital taxation is still Pareto-improving, inequality may be rising and the middle income households may be the class which prefers comparatively lower tax rates. There are critical values of the elasticity of substitution σ at which the effects are reversed: For $\sigma > 0.82$ all of the results obtained for the Cobb-Douglas case $\sigma = 1$ still hold. For $\sigma \leq 0.61$, inequality is rising for all tax rates and middle income households prefer a lower capital tax rate than high income households. In between these values, inequality will first decrease for lower tax

γ	σ	$\tau_{k_c}^{max}$	τ_k^{max}	τ_s^{max}	$\frac{S}{K}\vert\tau_{k_c}^{max}$	$\frac{S}{K}\vert\tau_k^{max}$	$\frac{S}{K}\vert\tau_S^{max}$
0.3	1.43	0.08	0.22	N/A	0.76	0.86	N/A
0	1	0.29	0.36	0.54	0.41	0.43	0.53
-0.2	0.83	0.22	0.24	0.34	0.26	0.27	0.28
-0.3	0.77	0.22	0.24	0.32	0.22	0.23	0.23
-0.5	0.67	0.24	0.24	0.26	0.156	0.156	0.166
-0.8	0.56	0.22	0.22	0.20	0.110	0.110	0.111
-3	0.25	0.08	0.08	0.04	0.009	0.009	0.012

Table 4: The table displays the capital tax values which maximize the different capital stocks and the ratio of middle income household's savings to total capital for these tax rates. It extends Table 1 to varied elasticities between capital and labor. The highlighted row corresponds to the original case of a Cobb-Douglas production function. In the first row, middle income households own all capital for $\tau = 0.35$, which is lower than the value at which $\widetilde{S}$ reaches its maximum.

rates and then increase for higher values. Also in this range, there is a reversal of the class which prefers the higher tax rate. To sum up, in the set-up of the present article, the traditional conclusion that capital taxation is bad for middle and lower classes even if it is spent productively thus holds for low elasticities, despite the Pasinetti property of the model.

4 Conclusion and outlook

This paper shows that under stylized assumptions about heterogeneous saving behavior of households there is no simple equity-efficiency trade-off. We assume that the heterogeneity in saving behavior can be captured in two types of households: High income households save dynastically and their only source of income is capital interest. Middle income households save a portion of their wages for retirement.[8] Under this assumption about households we prove that public investment financed by capital taxation decreases inequality in wealth for any capital tax rate. Middle income households are in favor of a higher capital tax rate than high income households. These results are reversed if the elasticity of substitution between labor and capital is significantly lower than 1. However, low capital tax rates constitute a Pareto improvement over the unregulated outcome for any elasticity. Further, the results establish that for the assumed type of heterogeneity and a high substitution elasticity, balancing the goals of equity and efficiency is not a single trade-off, but is rather characterized by three stages. While the higher the tax, the more equal the wealth distribution, there are three distinct stages regarding efficiency: (i) Low capital taxes (up to 29 % in our model) increase consumption for both classes, there is no trade-off; (ii) higher capital taxes (up to 54 %) still increase *aggregate* output, but decrease consumption of high income households[9]; (iii) all even higher capital taxes decrease both household types' consumption.

[8]Depending on the economy the model represents, low income households can be either assumed to behave similarly or it can be assumed that they do not save and are irrelevant to the present analysis of wealth inequality.

[9]One might expect yet another stage: some range in which there is an increase in the income of the middle income households, but a decrease in aggregate output – which would represent the conventional view on equity and efficiency as conflicting goals. Such a stage does not exist

There are two ways in which the analysis of this article could be extended: First, we only characterized potential policy interventions by their effect on inequality and consumption of the two types of households, eschewing the question which outcome is *socially optimal*. While the question of social optimality in overlapping generation models has been widely discussed (Calvo and Obstfeld, 1988; Heijdra, 2009), we do not know of any treatment of the role of a social planner in models with heterogeneous agents in which some households evolve as overlapping generations and some are infinitely-lived. Several reasonable normative viewpoints are conceivable in such a context. Defending one particular of them will need to answer the following question: With two household types having different time preference rates, does the time preference rate of the social planner only apply to the birth date of subsequent overlapping generations or should the utility of one or both household types also be discounted by this rate?

Second, the model employed in the present analysis relies on a set of very specific assumptions, introduced for isolating the effect of heterogeneous saving behavior and tractability. The results of this article also hold for more general production and utility functions (resulting in non-constant savings rate) and can be extended to other forms of generating fiscal revenue: labor taxation under non-fixed labor supply and consumption taxation. Our companion article confirms numerically that the results of this study hold under these more general assumptions (Klenert et al., 2016).

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in this model because there is no direct transfer to middle income households: When output decreases, both their capital and labor income also decrease.

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