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Multi-chimera states in the Leaky Integrate-and-Fire model

N.D. Tsigkri-DeSmedt[1], J. Hizanidis[1,2], P. Hövel[3,4], and A. Provata[1*]

[1] Institute of Nanoscience and Nanotechnology, National Center for Scientific Research "Demokritos" Athens, Greece
a.provata@inn.demokritos.gr; n.tsigkri@inn.demokritos.gr
[2] Crete Center for Quantum Complexity and Nanotechnology, Department of Physics, University of Crete Heraklion, Crete, Greece.
hizanidis@physics.uoc.gr
[3] Institut für Theoretische Physik, Technische Universität Berlin, Berlin, Germany
[4] Bernstein Center for Computational Neuroscience Berlin, Humboldt-Universität zu Berlin, Berlin, Germany
phoevel@physik.tu-berlin.de

Abstract
We study the dynamics of identical leaky integrate-and-fire neurons with symmetric non-local coupling. Upon varying control parameters (coupling strength, coupling range, refractory period) we investigate the system's behaviour and highlight the formation of chimera states. We show that the introduction of a refractory period enlarges the parameter region where chimera states appear and affects the chimera multiplicity.

Keywords: Chimera state, synchronisation, leaky integrate-and-fire, neuron models, refractory period

1 Introduction

The study of the dynamics and in particular collective behaviour of coupled oscillators has received great interest from scientists in different fields varying from chemical and mechanical systems to neuroscience and beyond [19]. A very interesting and unexpected synchronisation phenomenon that was first observed in identical coupled oscillators is the so-called *chimera state*. This is a dynamical scenario in which part of the oscillators are synchronised, while simultaneously others are not synchronised. These states were first observed in 2002 by Kuramoto and Battogtokh [9], while the term "chimera" was coined later, in 2004, by Abrams and Strogatz [3]. Potential applications of chimera states include the unihemispheric sleep that appears in dolphins and some birds, which sleep with one eye open meaning that half of the brain is synchronised and half is not synchronised, power grids and social systems [18]. On one hand, this surprising phenomenon has been observed numerically in various neuron models such as leaky integrate-and-fire, Kuramoto phase oscillators, Hindmarsh-Rose, FitzHugh-Nagumo, and

*Corresponding Author

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SNIPER/SNIC model [2, 1, 4, 7, 9, 15, 16, 23]. On the other hand, experimental verifications [6, 10, 13, 21, 24] do not include examples from neuroscience so far. This gives rise to an even greater interest to study chimera states as it may lead to a better understanding of information processing in neuron networks.

In this study we examine the effect of different control parameters on the appearance of chimera states for Leaky Integrate-and-Fire (LIF) neuronal oscillators that are arranged in a 1-dimensional regular ring topology. We compare the behaviour of coupled LIF units with and without refractory period and we find that in both cases chimera states appear. We show that when the refractory period is introduced the chimera states are enhanced and their multiplicity increase.

In the next section we introduce the single and coupled LIF models. Subsections 2.2 and 2.3 describe the coupled LIF model with and without a refractory period, respectively. In Sec. 3 we show the development of chimera states in a network of coupled LIF neurons. We demonstrate the differences in the form of chimera states between a network of coupled LIF neurons with and without a refractory period. Finally, the main conclusions are recapitulated in Sec. 4.

2 The leaky integrate-and-fire model

2.1 The single neuron model

The LIF model is a simple model for spiking neurons [5] which was introduced in 1907 by Louis Lapicque. It describes the dynamical evolution of the membrane potential of a single neuron. Figure 1 depicts the spiking behaviour of the membrane potential of a single LIF neuron in time.

The membrane potential $u(t)$ evolves according to the following equation

$$\dot{u}(t) = -u(t) + \mu \tag{1}$$

with a reset condition

$$\forall\, u(t) = u_{\rm th} \;\Rightarrow\; \lim_{\varepsilon\to 0} u(t+\epsilon) = u_{\rm rest} \tag{2}$$

where $u_{\rm th}$ is the threshold of the potential and $\mu > u_{\rm th}$ denotes a constant. In LIF, whenever the membrane potential reaches the threshold $u(t) = u_{\rm th}$, a spike is fired and the membrane potential is instantaneously reset to the rest state $u_{\rm rest}$. In this study the potential in the rest state is set equal to zero, $u_{\rm rest} = 0$.

2.2 Non-locally coupled LIF neurons

Neurons "fire" electrical signals as a result of receiving inputs from other neurons. This observation sets the need of studying a network of coupled neurons. We study a network of N LIF neurons that are arranged in a regular ring topology with non-local connections, that is, each element is coupled to R nearest neighbours on either side, as schematically depicted in Fig. 2. The dynamic evolution in time of this system is determined by

$$\dot{u}_i(t) \quad = -u_i(t) + \mu + \tfrac{\sigma}{2R} \sum_{j=i-R}^{i+R} [u_i(t) - u_j(t)] \tag{3}$$

with the same reset mechanism for each element as described in Sec. 2.1. Here, σ is the coupling strength and R denotes the coupling range. The index i has to be taken modulo N. The network nodes are considered identical, that is, they have the same system and coupling parameters.

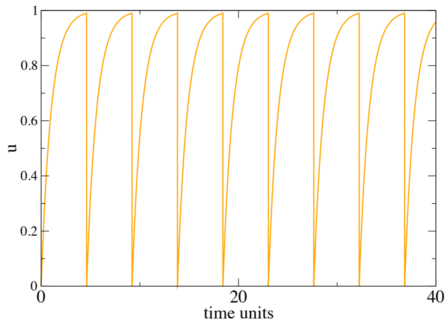

Figure 1: (Colour online) Dynamic evolution of a single neuron in time according to Eq. (1). Parameters: $\mu = 1$, $u_{\mathrm{th}} = 0.99$, $u_{\mathrm{rest}} = 0$.

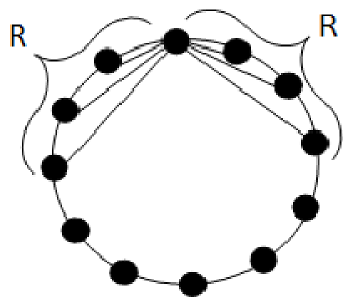

Figure 2: Topology the considered one-dimensional ring network.

The study of a system of coupled oscillators [22, 11, 17] involves the identification of parameter regions where synchronisation occurs. In the next sections, we investigate the effect of the coupling strength σ and the coupling range R on synchronisation phenomena with special focus on chimera states.

2.3 Coupled neurons with a refractory period

In many neuron models, the neuron stays in its rest state for a certain period of time after firing. In order to take this into account, we consider a refractory period p_r [8]. The refractory period is a time interval, during which a neuron remains at rest after firing and is not able to trigger an additional spike.

The dynamics of the refractory LIF model is described by the equations of the coupled LIF neurons system Eq. (3), except that after firing each neuron remains at the rest state for time p_r. Figure 3 depicts the spiking behaviour of the membrane potential of a single LIF neuron with a refractory period $p_r = 1$.

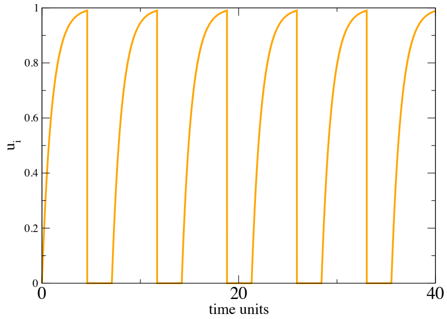

Figure 3: (Colour online) Dynamic evolution of a LIF neuron with a refractory period of $p_r = 1$. Other parameters as in Fig. 1.

3 Chimera states in coupled leaky integrate-and-fire neurons

We investigate the appearance of chimera states of a network of LIF neurons with and without refractory period. In references [18, 20, 12, 25, 14] chimera states appear in the LIF system, for different realisations of the model and of the coupling geometry. In reference [14] the authors have shown the existence of chimera states in coupled LIF systems with delay dynamics. In this study, we show that the presence of a refractory period favours their appearance, while at the same time has an effect on their multiplicity. The refractory period is different from delayed self-feedback in the sense that the former introduces a dead (resting) time after firing while in the latter each neuron receives input not only by its neighbours but also by its past states.

3.1 Without refractory period

In the following, starting from random initial conditions u_i, $\{i = 1, ..., N\}$ distributed over the interval [0,1], we investigate the appearance of chimera states for a finite network of $N = 1000$ neurons by varying the coupling strength σ and the coupling range R. See Fig. 4. We observe that for $R = 100$ chimera states do not appear for very small values of the coupling strength $\sigma \leq 0.5$ nor for $\sigma \geq 0.6$. They are found for intermediate values of the coupling strength, such as $\sigma = 0.565$, as shown in Fig. 4(e). Notice that the chimera states observed in this case are transient and disappear for longer times. See red curves in Fig. 4.

The appearance of chimera states at a certain value of the coupling strength also depends on the value of the coupling range R. More specifically, we show that as we increase the coupling range R, chimera states appear for a larger value of σ, as shown in Fig. 5. Thus the range of the values of the coupling strength that favour the appearance of chimera states, shifts following the change of the coupling range.

Chimera states are highly dependent on initial conditions. Figure 6 shows the temporal evolution of chimera states starting from two different random initial conditions in columns (a) and (b), respectively. All other parameters are the same. We find that when the system starts from an initial state (a) it reaches complete synchronisation, while when it starts from initial state (b) a chimera state is formed as shown in the plots.

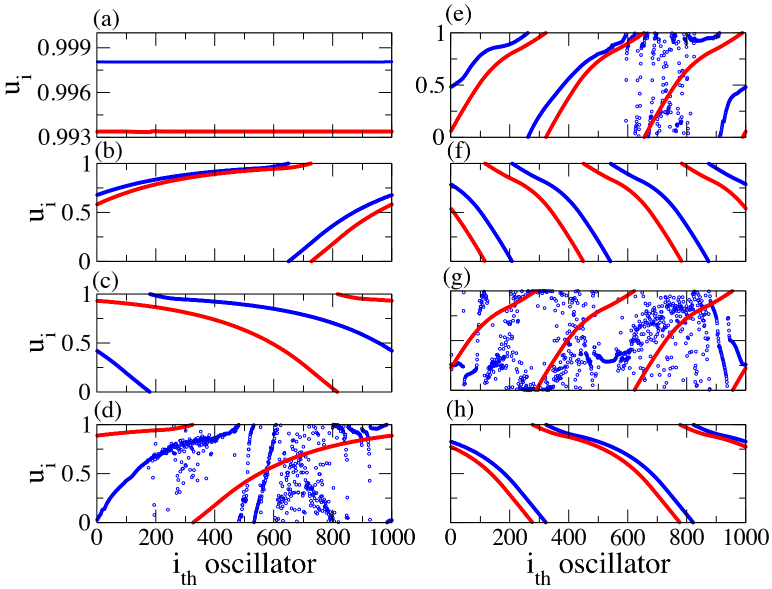

Figure 4: (Colour online) Snapshot of the membrane potential u_i for different values of the coupling strength: (a) $\sigma = 0.4$, (b) $\sigma = 0.52$, (c) $\sigma = 0.54$, (d) $\sigma = 0.56$, (e) $\sigma = 0.565$, (f) $\sigma = 0.57$, (e) $\sigma = 58$, (g) $\sigma = 0.6$. The blue line corresponds to $t = 1000$ time units and the red line to $t = 9000$ time units. Other parameters: $N = 1000$, $u_{\rm th} = 0.98$, $R = 100$ and $\mu = 0.99$.

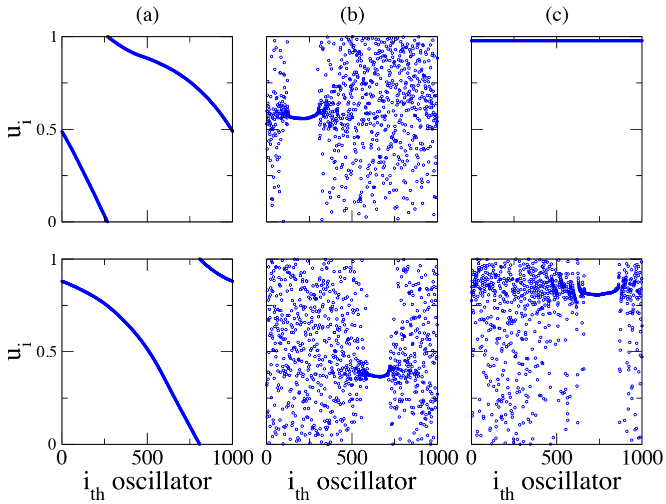

Figure 5: (Colour online) Snapshots of the membrane potential u_i for different values of the coupling parameters R and σ. The upper panel corresponds to $\sigma = 0.565$, while the lower one to $\sigma = 0.7$. The coupling range is (a) $R = 200$, (b) $R = 300$ and (c) $R = 400$. Other parameters as in Fig 4.

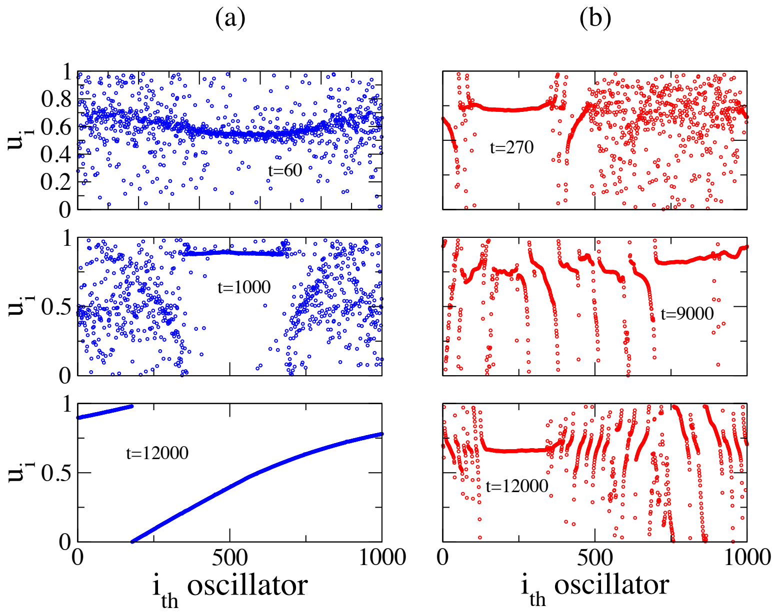

Figure 6: (Colour online) Snapshots of the membrane potential u_i for different time units and for different initial conditions in panels (a) and (b). Parameters: $N = 1000$, $u_{\mathrm{th}} = 0.98$, $\sigma = 0.565$, $R = 350$ and $\mu = 1$.

3.2 With refractory period

The study of the network of coupled LIF neurons shows that this network displays the phenomenon of chimera states which are mostly transients. We now examine the effect of the refractory period in their spatial form and temporal evolution. Using values of the coupling strength σ and the coupling range R for which we observed chimera states in the original coupled LIF system, we now consider the influence of a non-zero refractory period p_r.

In Fig. 7 we demonstrate that the refractory period enhances the appearance of chimera states. When varying the refractory period it is natural to use a time scale for comparison that is intrinsic to the system. As this reference, we use the period T of the oscillations of the coupled LIF unit.

In Fig. 8 we compare the system of the $N = 1000$ oscillators behaviour with and without a refractory period. For $p_r = 0$, as show in Fig. 8(a), the chimera state has one coherent and one incoherent region. On the contrary, in Fig. 8(b) the chimera state that is formed in the system with $p_r = 0.5T$ has four incoherent and four coherent regions.

In the following, we elaborate on the effect of the refractory period. As shown in Fig. 9 the chimera multiplicity changes as p_r varies from $0.1T$ to T. Notice that the chimera states appear only for intermediate values of the refractory period and that the number of coherent and incoherent regions for fixed values of the coupling strength and coupling range remains constant. A potential interpretation of this behaviour is that the neurons by sections slightly differ in phase. Intuitively, the small but substantial values of the refractory period facilitate the grouping because the condition $u_i(t) = 0$ forces neighbouring elements to synchronise locally in the rest state. The grouping of neurons in sections, influences the neurons on the boundaries

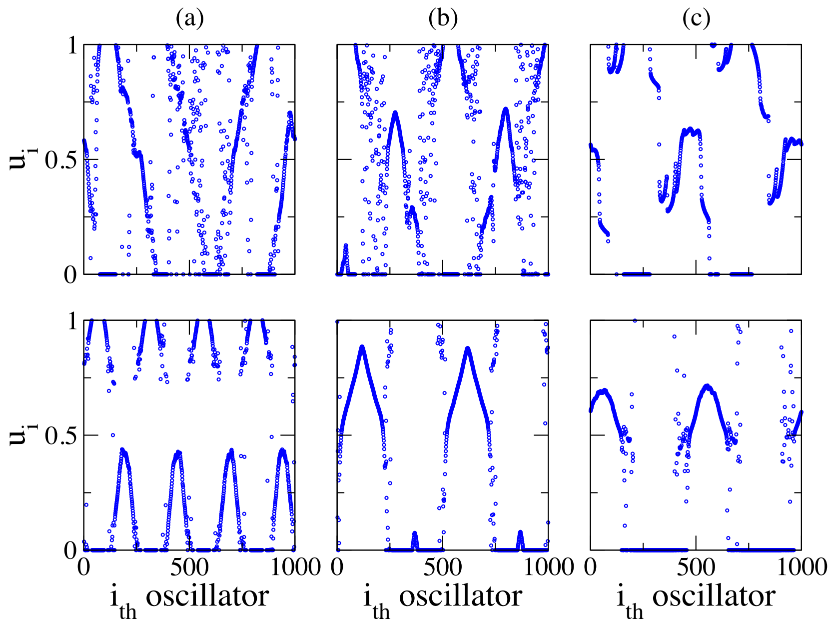

Figure 7: (Colour online) Snapshots of the membrane potential u_i for different values of the coupling parameters R and p_r. The upper panel corresponds to $p_r = 500$ time units and the lower panel corresponds to $p_r = 1000$. The coupling range is (a) $R = 200$, (b) $R = 300$ and (c) $R = 400$. Other parameters are $N = 1000$, $u_{\rm th} = 0.98$, $\sigma = 0.565$, $\mu = 0.99$, $t = 9000$ time units.

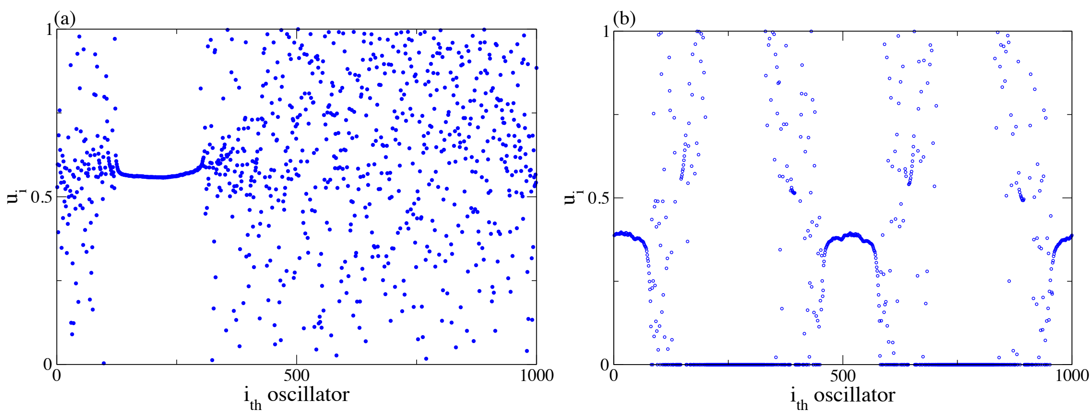

Figure 8: (Colour online) Snapshots of the membrane potential u_i: panel (a) depicts the formation of chimera states in a network without a refractory period and panel (b) shows the formation of chimera states in a network with $p_r = 0.5T$. Other parameters are: $N = 1000$, $\sigma = 0.565$, $u_{\rm th} = 0.98$, $R = 300$ and $\mu = 0.99$.

between sections, which destabilise and become asynchronous.

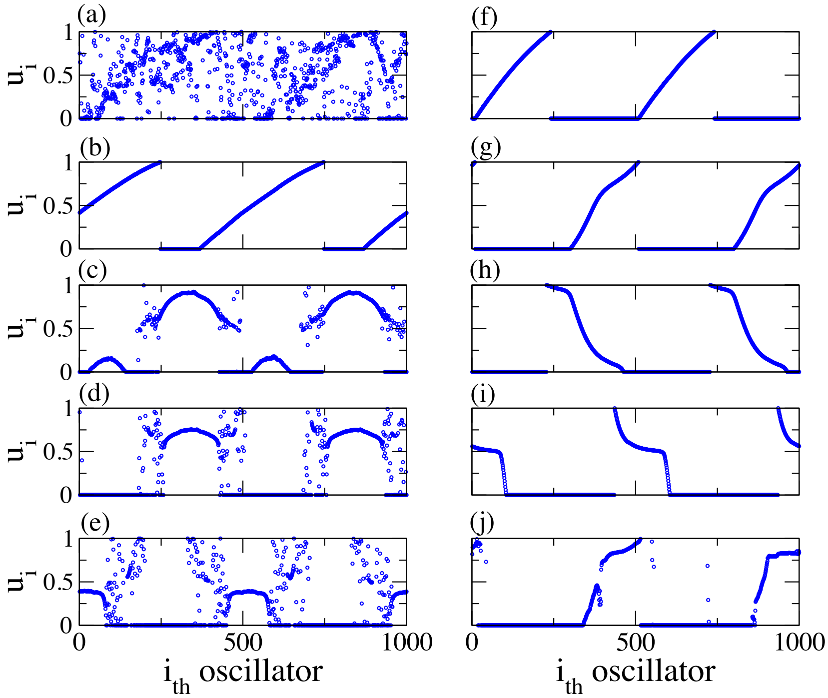

Figure 9: (Colour online) Snapshots of the membrane potential u_i in space for different values of the refractory period p_r, (a) $p_r = 0.1T$, (b) $p_r = 0.2T$, (c) $p_r = 0.3T$, (d) $p_r = 0.4T$, (e) $p_r = 0.5T$, (f) $p_r = 0.6T$, (g) $p_r = 0.7T$. (h) $p_r = 0.8T$, (i) $p_r = 0.9T$, (j) $p_r = T$. Other parameters are: $N = 1000$, $u_{\rm th} = 0.98$, $R = 300$ and $\mu = 0.99$.

4 Conclusions

Chimera states on a non-locally coupled network of LIF neurons highly depend on the combination of coupling strength, coupling range and refractory period p_r. The analysis of a network of $N = 1000$ neurons has shown that chimera states appear for intermediate values of the coupling strength. Furthermore, the emergence of chimera states also depends on the value of the coupling range. More specifically, we have observed that as the coupling range increases, the range of coupling strengths that favour the appearance of chimera states shifts to higher values. Additionally, we have noticed that a crucial control parameter for the occurrence of chimera states is the refractory period, a resting period between two consecutive excitations of a neuron. We have shown that the refractory period helps the chimera states survive for longer periods, while at the same time is responsible for the formation of multiple coherent and incoherent regions. The number of coherent and incoherent regions for fixed values of the coupling strength and the coupling range, does not depend on the value of the refractory period.

Our results represent only a first approach to the study of the effect of control parameters in a LIF network and to the phenomenon of partial synchronisation (more specifically chimera states). Future work should address quantitative investigation of the parameter regions which favour chimera states and could include additional parameters related to the experimentally measured time-scales of biological neurons.

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