J. Non-Equilib. Thermodyn.
2007 · Vol. 32 · pp. 181–185

Scientific Correspondence

Comment on the Article "Relativistic Non-Equilibrium Thermodynamics Revisited"

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Abstract

There are two problematic items in García-Colín and Sandoval-Villalbazo's approach to "relativistic non-equilibrium thermodynamics" (L.S. García-Colín and A. Sandoval-Villalbazo, J. Non-Equilib. Thermodyn. 31, 2006, pp. 11–22). The paper does not follow the fundamentals of relativity theory; according to them, the energy-momentum tensor (EMT) has to include all energies of the considered system. Secondly, strange thermodynamic consequences result by using the presuppositions made by the authors. The paper is critically discussed and some shortcomings are elucidated.

1. The citations

García-Colín and Sandoval-Villalbazo argue that Eckart's construction of the EMT which includes the internal energy and the heat flux density "goes against the tenets of the theory of general relativity; the stress energy tensor only includes all forms of mechanical energy." The authors pretend that this had been pointed out already by Tolman and Weyl [1, 2]. However, the truth is that these relativists maintained just the contrary. This fact shall be discussed in more detail.

Weyl founded his view in the English translation of the fourth edition of his book *Raum*, *Zeit*, *Materie* [2] and, in more detail, in the fifth German edition [3]. In this note, we follow the line of Weyl's arguments. This is done because these arguments show very clearly that the theory of relativity demands to interpret the EMT as a description of the total energy including mechanical and thermal energy.

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We restrict ourselves, like Weyl, to the theory of special relativity, but formally all relations are of the shape that they are also valid in general relativity.

The starting point of the authors is to reject Eckart's EMT, which includes the terms $\varrho u^i u^k$ and $2u^{(i}q^{k)}$. Here, ϱ is the total energy density, u^i is the four-velocity, and q^k the heat flux density. The authors assume that ϱ has to be interpreted as "mass density" and that the EMT must not contain the heat flux density. Therefore, they feel entitled

i) to consider

$$T^{ik}_{;i} = 0, \quad T^{ik} \equiv \varrho u^i u^k + t^{ik}, \quad t^{ik} u_k = 0, \quad t^{ik} u_i = 0, \tag{1}$$

as a balance equation of "purely mechanical stress-energy" and

ii) to introduce a further energy balance which is interpreted as "the first law of relativistic thermodynamics", thus representing the balance of total energy:

$$J^k_{;k} = 0, \quad J^k := u^i T^k_i + n\varepsilon u^k + q^k, \quad q^k u_k = 0. \tag{2}$$

By multiplying $(1)_1$ with the projector $h^s_k := \delta^s_k - (1/c^2)u_k u^s$ and inserting $(1)_2$, we obtain a balance for ϱu^i:

$$T^{ik}_{;i} = 0, \rightarrow [\varrho u^i]_{;i} u^k + \varrho \dot{u}^k + t^{ik}_{;i} = 0, \quad ^{\cdot} := {}_{;k} u^k, \tag{3}$$

and presupposing Eckart's balance of particles, this results in

$$N^k_{;k} := [\varrho u^k]_{;k} = 0 \rightarrow \varrho \dot{u}^k + t^{ik}_{;i} = 0. \tag{4}$$

The additional balance (2) is similarly built as the balance of internal energy in non-relativistic thermodynamics: It includes the heat flux density q^k, the flux of the "internal energy density" $n\varepsilon u^k$, and according to Eq. (1),

$$u^i T^k_i = c^2 \varrho u^k, \tag{5}$$

also the flux of the "mechanical energy density" $c^2 \varrho u^k$, which does not appear as a flux, but as a production in the non-relativistic balance of internal energy. Thus, the mechanical power does not appear in the authors' approach, neither in the balance (3) nor in Eq. (2).

Of course, this approach is only acceptable if at least two conditions are satisfied: First, the assertion that ϱ can really solely be ascribed to mechanical forms of energy, and second that the balance (2) does not conflict with the balance equation (1). Now both of these conditions are investigated.

For detecting the meaning of ϱ, we consider a material whose EMT is especially given by

$$^{0}T^{ik} = \varrho u^{i} u^{k}. \tag{6}$$

Considering the 4-component of the equation of motion $(4)_2$, we obtain by introducing the proper time τ:

$$\varrho \frac{du^4}{d\tau} = 0, \rightarrow \frac{d}{d\tau}\left(\frac{\varrho c}{\sqrt{1-(v/c)^2}}\right) - \frac{d\varrho}{d\tau}\frac{c}{\sqrt{1-(v/c)^2}} = 0. \tag{7}$$

Following Weyl [2], we consider the following example: Two point particles of the masses m^1 and m^2 are performing a mass-constant motion for which according to Eq. (7),

$$\frac{m^{\alpha} c}{\sqrt{1-(v_{\alpha}/c)^2}} = const_{\alpha}, \quad \alpha = 1, 2, \tag{8}$$

is valid. Both the particles shall have the same masses $m^1 = m^2 := m$, and opposite velocities of the amount of $v_1 = -v_2 := v$. Consequently, we obtain for the system consisting of these two point masses:

$$\frac{2mc}{\sqrt{1-(v/c)^2}} = const_1 + const_2 := const, \tag{9}$$

during their mass-constant motion. Now, the two particles undergo an inelastic collision in such a way, that after the collision both the particles are unified in one which is now resting, $v = 0$, and thus performs a mass-constant motion after the inelastic collision. For this particle, Eq. (9) results in

$$2mc = konst \leq const, \tag{10}$$

$$c\Delta m := const - konst = 2mc\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right) \geq 0, \tag{11}$$

a relation which explicitly ($c \doteq 1$) can be found in Weyl's book [4].

Weyl now concludes correctly [3]: In this process heat is generated. Since the mass of a body is only determined by its internal state, this heating, independently of its mechanism, is always related to the same mass change Δm. Thus, $c^2\Delta m$ represents an energy measure for each thermal change of state. Already in [2] he writes: "Inertial mass varies with the contained energy. If a body is heated, its inertial mass increases; if it is cooled, it decreases."

To repeat, the mass defect Δm which changes with the internal state corresponds to the mass density ϱ in Eq. (1). Therefore, to assume that ϱ is of purely mechanical nature contradicts the inertia of energy underlying the basis of the theory of relativity. Obviously, the authors' basic assumption is not correct.

To complete the discussion of this item, it should be mentioned that also Tolman [1], to whom the authors referred, too, had the above-described view at ϱ in Eq. (1). Looking for the relativistic equivalent of the first law of classical thermodynamics, he stressed: "In view of the fact that the classical law has also to be regarded as expressing the conservation of total energy, one must conclude that the relativistic balance equation (1) for momentum and energy appearing inevitably in the theory of relativity has to be considered as the relativistic equivalent". According to Tolman, one has still to introduce a distinction between flow of heat and performance of work with respect to the relativistic second law of thermodynamics.

2. Discussion of the balances

In almost all textbooks on relativity theory one can find the expression of the four-momentum of a mass point:

$$p^i := m_0 u^i = \frac{m_0}{\sqrt{1-(v/c)^2}}(v^1, v^2, v^3, c) = (p^1, p^2, p^3, E/c). \tag{12}$$

Here the energy E is as a 4-component of a Lorentz vector coupled to the space components p^α, which are the 3-momenta. Consequently, there is no pure conservation law of the energy, because energy is always coupled to momementum in relativity theory. The authors do not take this coupling into consideration, because they do not introduce a momentum density in Eq. (2). An equation such as Eq. (2) does not exist in relativistic thermodynamics. Thus, the concept that "the stress energy tensor only includes all forms of mechanical energy" is erroneous because of two different reasons:

1. The energy in the EMT is the total energy, and
2. According to the laws of the theory of relativity, energy balance and momentum balance are coupled to each other in another way as described by the authors.

For completing this reply, we want to sketch the correct procedure. Starting out with

$$T^{ik}_{;i} = 0, \quad T^{ik} := \varrho u^i u^k + t^{ik} + q^i u^k + u^i p^k, \tag{13}$$

$$t^{ik} u_k = 0, \quad t^{ik} u_i = 0, \quad q^i u_i = 0, \quad p^k u_k = 0, \tag{14}$$

we obtain from Eq. (5) by differentiation and by Eq. $(13)_1$

$$T^k_{j;k} u^j = 0 \to [c^2 \varrho u^k + c^2 q^k]_{;k} - u_{j;k}(t^{kj} + q^k u^j + u^k p^j) = 0, \tag{15}$$

and by the analog procedure (see the special case in Eq. (3))

$$T^k_{j;k} h^{js} = 0 \to [p^s u^k + t^{ks}]_{;k} + [(\varrho u^k + q^k) u^s]_{;k} = 0. \tag{16}$$

Here the energy flux is

$${}^e J^k := \varrho u^k + q^k, \tag{17}$$

and the flux of momentum

$${}^m J^{ks} := p^s u^k + t^{ks}. \tag{18}$$

Because of the coupling between energy and momentum, the divergences of both of the fluxes do not vanish on one's own. The question of how to define balances of the internal and the mechanical energy is beyond the scope of this reply.

References

[1] Tolman, R.C., Relativity, Thermodynamics and Cosmology, Chap. IX, Dover, New York, 1987.
[2] Weyl, H., Space-Time-Matter, Chap. IV, Dover, New York, 1950.
[3] Weyl, H., Raum, Zeit, Materie, Springer, Berlin, 1988. (This is an unchanged reprint of the 5th edition from 1923.)

Paper received: 2006-08-13
Paper accepted: 2006-12-13