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Truncation Error Estimates in Process Life Cycle Assessment Using Input-Output Analysis

Truncation error estimates in process lifecycle assessment using input-output analysis

Hauke Ward*, Leonie Wenz, Jan C Steckel, Jan C Minx

Keywords: truncation error estimate, process life-cycle assessment, input-output (IO) analysis, system boundary, service sectors

Abstract

Process life cycle assessment (PLCA) is widely used to quantify environmental flows associated with the manufacturing of products and other processes. As PLCA always depends on defining a system boundary, its application involves truncation errors. Different methods of estimating truncation errors are proposed in the literature; most of these are based on artificially constructed system complete counterfactuals. In this article we review the literature on truncation errors and their estimates and systematically explore factors that influence truncation error estimates. We classify estimation approaches, together with underlying factors influencing estimation results according to where in the estimation procedure they occur. By contrasting different PLCA truncation error modeling frameworks using the same underlying Input-Output (IO)-dataset and varying cut-off criteria we show that modeling choices can significantly influence estimates for PLCA truncation errors. In addition, we find that differences in IO- and process inventory databases, such as missing service sector activities, can significantly affect estimates of PLCA truncation errors. Our results expose the challenges related to explicit statements on the

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They also indicate that increasing the strictness of cut-off criteria in PLCA has only limited influence on the resulting truncation errors. We conclude that applying an additional input-output life cycle assessment (IOLCA) or a path exchange hybrid life cycle assessment (HLCA) to identify where significant contributions are located in upstream layers could significantly reduce PLCA truncation errors.
Quantifying environmental impacts of products and processes through the use of a life cycle assessment (LCA) has become standard procedure. Today it is widely applied in research and industry and encouraged by governments and NGOs (Guinée 2011). A range of LCA methodologies are available that utilize different types of data. These are LCA based process-level data (in the following referred to as process LCA or PLCA), LCAs which apply macro-economic input-output data (IOLCA) and methodologies which combine both datasets in so-called hybrid LCA (HLCA). The advantages and disadvantages of each different type of LCA are widely discussed in the literature (see Finnveden et al. (2009) and Rowley et al. (Peters et al. 2010)).

PLCA is one of the more frequently applied methodologies. This is possibly because of the high level of detail and availability of the underlying inventory data. These data are regularly distributed together with LCA software tools, making PLCA accessible and generally easy to use (Finnveden et al. 2009). PLCA refers to the iterative bottom-up approach demanded by the ISO norms investigating the environmental interventions related to a product or process by chasing upstream and downstream contributions (British Standards - ISO 14040 2006). Nonetheless, due to practicability, PLCA application requires a system boundary definition. According to the ISO norms, this boundary should be chosen such that the associated process chains are traced until, if possible, all inputs and outputs to the system are flows that have been directly drawn from the environment without human intervention (British Standards - ISO 14044 2006). This ideal condition is difficult to meet in practice, as the number of associated flows potentially grows exponentially and infinitely. Any system boundary definition not only determines the level of
50 detail that is applied in the analyses, but also the stages, processes, and inputs and outputs that
51 can be deleted if they are not expected to “significantly change the overall conclusion of the
52 study” (British Standards - ISO 14044 2006; British Standards - ISO 14040 2006, p. 8). This means
53 some process flows must be ignored by introducing (explicit) cut-off criteria (Suh et al. 2004),
54 leading to truncation errors. As the complete system is unknown, applying qualitative cut-off
55 criteria that only exclude relevant flows can be challenging (Suh et al. 2004). Put differently, unless
56 100% of the features of a system are already known, 95% cannot be calculated (Huang et al. 2009).
57
58 System complete alternative methodologies, based on input-output (IO) frameworks, have been
59 used to construct modeling counterfactuals and estimate the magnitude of truncation errors
60 occurring within a PLCA. In general, although modeling frameworks, as well as resulting estimates
61 vary (Junnila 2006; Lenzen 2000; Norris 2002; Rowley, Lundie, and Peters 2009), studies suggest
62 that truncation errors in PLCA can be significant in size. According to Lenzen (2000) they can be
63 in the order of 50% of total impacts. Sectors for which relevant impact shares are contained in
64 upstream production stages beyond the second or third layer, are especially prone to truncation
65 errors, as these layers are rarely contained in PLCA studies (Lenzen and Treloar 2003).
66 Approaches for estimating PLCA truncation errors, partially relying on IO data, are based on
67 specific modeling assumptions. They use different methodologies and underlying datasets differ,
68 inter alia, across sectoral resolution (Finnveden et al. 2009; Huang et al. 2009), sectors considered
69 (Majeau-Bettez, Hawkins, and Stromman 2011), and the point in time in which data have arisen
70 or impact categories considered (Finnveden et al. 2009; Suh et al. 2004). It has been shown that
71 depending on the product or process investigated, specific modeling characteristics can lead to
significant differences in results (Huang et al. 2009). Nevertheless, the scientific discourse lacks a systematic investigation into how far different modeling approaches influence PLCA truncation error estimates.

In this article we initially provide a comprehensive review of the relevant literature. Based on this, we develop a classification scheme of existing truncation error estimation frameworks and a typology of factors influencing PLCA truncation error estimates. We then investigate in detail the impact of some of the most relevant factors discussed in the literature. We also implement different scenarios, or modeling set-ups. For example we vary cut-off criteria and modeling frameworks, and investigate the influence on PLCA truncation error estimates of service sectors ignored in process inventory databases (Majeau-Bettez, Hawkins, and Stromman 2011). We implement all scenarios using a single IO database for the USA covering more than 400 sectors. For simplicity we only focus on investigating embodied CO₂ emissions.

Our results show that PLCA truncation error estimates crucially depend on modeling specifications, challenging explicit statements on the magnitudes of truncation errors made in the literature. Note that our results do not examine the quality of PLCA, IOLCA and HLCA techniques as assessment tools. We are primarily concerned with how modeling choices influence truncation error estimates. An investigation of the overall quality of LCA analyses needs a different framework and an understanding of the influence of multiple factors identified in this article. In addition a precise estimate of the overall associated impact is needed, which is difficult to achieve.
Truncation errors in PLCA and their estimates

This section formally defines truncation errors in LCAs and reviews the literature on PLCA truncation errors and their estimates. It aims to identify and structure factors that are of relevance for these estimates.

Truncation error

We define a truncation error as the proportion of impact (investigated value) not covered by the system boundaries of the LCA. Truncation errors can occur when flows are knowingly ignored, that is, when their contributions and their upstream flow contributions are – often mistakenly – assumed not to affect the overall impact. They can also occur inadvertently when relevant data for the study are (unknowingly) missing and hence flows are disregarded.

More formally, we define \( MI \) as the measured impact, by which we mean the impact as given by a (P)LCA study of a process or product. \( TI \) denotes the corresponding total (unknown) associated impact and \( EI \) the estimated total associated impact, as for instance derived from a system complete alternative approach. The related truncation error corresponds to the proportion of the impact that is missing in the assessment as follows:

\[
TE = 1 - \frac{MI}{TI} \tag{1}
\]

The corresponding truncation error estimate (TEE) can be expressed as

\[
TEE = 1 - \frac{MI}{EI} \tag{2}
\]

The quality of the estimate consequently depends on how well \( TI \) is approximated by \( EI \). Nevertheless, a precise number cannot be given, as \( TI \) cannot be completely known.
Factors influencing PLCA truncation errors and their estimates

System Boundary

The system boundary determines cut-off conditions for flows and associated impacts. It therefore directly influences $M_I$. The same holds when the result of a PLCA analysis is approximated within an IO or hybrid framework, see for instance Norris (2002) and Lenzen (2000). A critical point is that the system boundary has to be drawn at the beginning of an LCA study, which is prior to data collection, without knowing the total system and often lacking a scientific basis. This leaves a lot of room for individual interpretation of a “significant contribution” (Suh et al. 2004). ii Suh et al. (2004) note that an accumulation of small, but disregarded flows could become relevant. Also, small mass or energy content, often used as proxies for the relevance of an impact, do not necessarily correspond to small impacts (Suh et al. 2004).

Cut-off criteria

No consensus on cut-off methodology across truncation error modeling literature exists; different modeling approaches exist in parallel, indicating that there is no distinct truncation procedure for flows in PLCA, that influences results (see table 1). Considering a specific share of the total footprint or accounting for all flows above a specific anticipated contribution share (British Standard Institute 2011) is difficult in the absence of complete system knowledge (Huang et al. 2009).
On the (system-complete) counterfactual side, multiple, different cut-off criteria have been utilized, leading to different results. These cut-off criteria are further explained at the end of this chapter.

Data

Missing or incomplete data is another important issue related to the estimation of \( MI \) and \( TI \). The choice of data inevitably (and unintentionally) influences truncation errors; only flows included in a database can be considered. For instance some regions are not represented in process databases (PE International 2015), which have also been criticized for ignoring specific service sectors and capital goods (Suh et al. 2004; Junnila 2006). More evidence is given by Majeau-Bettez et al. (2011) who identify explicit sectors contained in an IO dataset which are omitted from process databases, such as government defense, non-defense government and finance services. Additionally, data contained in dissimilar process databases differ from each other (Finnveden et al. 2009; Zhang, Gibbemeyer, and Bakshi 2014). Hence the choice of data has an influence on PLCA results and consequently also on truncation error estimates (as indicated by Huang et al. (2009) by using different IO datasets).

On the counterfactual side, all identified approaches are based (at least partially) on IO data, which differ from process inventory databases (see Introduction). IO and PLCA data show differences in the level of sectoral aggregation (Majeau-Bettez, Hawkins, and Stromman 2011; Suh et al. 2004; Junnila 2006; Rowley, Lundie, and Peters 2009; Lenzen 2000) and data contained (Majeau-Bettez, Hawkins, and Stromman 2011). Input-output data uses a monetary accounting system, whereas some PLCA studies account for physical flows (Bruckner et al. 2015). In contrast
to process inventory data, IO tables typically assume proportionality between monetary and underlying physical flows (Rowley, Lundie, and Peters 2009) and do not consider the gate-to-grave component (Lenzen 2000). Consequently, they only consider impacts related to production.

Many approaches to the estimation of truncation errors are based on IO datasets for a single region (Rowley, Lundie, and Peters 2009; Lenzen 2000). These yield a higher sectoral resolution than multi-regional IO datasets (Tukker and Dietzenbacher 2013), thus influencing the results (Su et al. 2010). On the downside, they do not consider differences in inter-regional production technologies.

As a wide range of IO data exists - constructed in different ways - specific characteristics regarding underlying sectors, regions and impact categories may differ considerably (Tukker and Dietzenbacher 2013). Sectoral aggregation schemes and underlying countries may also vary (Bruckner et al. 2015). Using a specific IO database therefore, has an impact on truncation error estimates (as MI and the approximated TI change), indicated by differences in IOLCA results when using multiple datasets (Ward et al. 2016; Steen-Olsen et al. 2015; Alexeeva-Talebi et al. 2012; Huang et al. 2009).

The measured impact is also influenced and potentially falsified by approximating missing data. For example grains have been approximated by wheat due to unavailable data (Peters et al. 2010). In such a case, the resulting truncation error depends on the similarity of the substitute to the missing data. Data can also be supplied by applying matrix inversion techniques to partially compensate for the proportion of impact that has been cut off, or by adding an IO correction term
Both of these influence total measured impacts and can lead to overestimates (Suh and Huppes 2005; Rowley, Lundie, and Peters 2009).

**Sectors investigated**

Truncation error estimates also depend on the sector being investigated, as the related impacts vary in their distribution across different layers (Lenzen and Treloar 2003). For instance, it has been shown that for gas and oil production more than 80% of the carbon footprint associated with production is contained within the final production step and first upstream layer, whereas in the publishing sector, more than 50% of the carbon footprint is connected to higher layers (Huang et al. 2009).

**Impact investigated**

A variety of different impacts has been investigated. For instance energy footprints (Treloar 1997), emissions footprints (Peters et al. 2010), material footprints (Wiedmann et al. 2013), land use footprints (Bruckner et al. 2015), water footprints (Lenzen et al. 2013) and bad labor, which also considers child labor (Simas et al. 2014) have been assessed. Each impact category has its own characteristic distribution of where relevant impacts are located. Impacts can be difficult or easy to cover by (P)LCA studies depending on this distribution, the supply chain length and structure. A potential truncation error is also dependent on the quality of data coverage. For an impact category, whose impact is insufficiently reported, the occurrence of truncation errors cannot be prevented.
Network properties

Other sources of truncation errors that have not yet been discussed in the literature might also be relevant. In particular, differences in network properties of IO data and process databases need to be investigated. Some literature, however, hints of such differences (Mongelli, Suh, and Huppes 2005; Norris 2002). These papers cite differences in the average numbers of network links and differences across other network properties. Typically, for IOLCA, first order upstream flows exceed 300 in number (Norris 2002); it can be expected that this number is much smaller for process inventory databases. A smaller number of direct upstream links implies that more activities are associated with higher process tiers. These characteristics can influence results when investigating (environmental) impacts, using either inventory data or IO data and applying similar cut-off criteria. This is because the second or third layer are rarely contained in PLCA studies (Lenzen and Treloar 2003).

Process databases are updated each time a new process is modeled. Consequently, their link density increases over time and converges towards the real density. In contrast, IO data is already system complete and it is likely that this conceptual difference influences corresponding results. To investigate the influence of incomplete link density (and also estimate their real density) the identified power law for self-organized networks (SON) (both, IO tables and complete process inventory database are in principle SON) could be utilized (Laurienti et al. 2011).

Reference Systems

In order to estimate the magnitude of a PLCA truncation error, an estimate of \( T' I \) is needed as well as an estimate of \( M I \). Several approaches have been proposed using system complete data (Suh
Estimation frameworks have so far (partially) relied on IO data, in which two estimate classes can be identified (see table 1).

Firstly there are approaches that compare PLCA results with results from system complete alternatives (HLCA or IOLCA) to conclude on computed truncation errors (“between system”) (Rowley, Lundie, and Peters 2009). For instance, the IOLCA approach has been used to estimate PLCA truncation errors for energy embodied in basic iron and steel products (estimates are in the order of 50%) (Lenzen and Dey 2000). By applying two different types of hybrid analyses, a process-based hybrid analysis and an IO-based hybrid analysis, PLCA truncation errors of the life-cycle energy embodied in passive houses were estimated to be 69% and 77%, respectively (Crawford and Stephan 2013).

Secondly there are approaches that investigate PLCA truncation errors solely within the alternative framework (“within system”). In this way, PLCA application is simulated within an IO framework. The results are then compared to total impacts, which are calculated by IOLCA for the same database (Lenzen 2000; Treloar 1997; Norris 2002).

“Within-system” approaches can be sub-classified further. Firstly, a finite layer matrix approach, which we will refer to as “matrix layer approach”, was proposed by Lenzen (2000). This uses a power series calculation, where each series element corresponds to a complete layer of upstream flows. For this approach it is crucial to assume that the (applied) truncation of flows in PLCA corresponds to the exclusion of all flows beyond a specific matrix layer $k$ ($k \in \{0,1,2,3\}$ is often
 Nonetheless, it is questionable whether PLCA flow cut-offs correspond to the practice of matrix layer approaches (Suh et al. 2004). Considering the ISO norms, a judgment on single flows is more appropriate, resulting in flows being cut off in different layers (British Standards - ISO 14040 2006; British Standards - ISO 14044 2006).

Secondly, path analyses (which we will refer to as “path approaches”) have been used to estimate PLCA truncation errors, see for instance Treloar (1997) and Norris (2002). In this way, single entries from IO tables are used to construct a branching and exponentially growing network of upstream supply flows. In this approach, single flows, that is branches, are traced and investigated. Different variations of this approach exist. For instance, flows can be ranked according to their environmental impact, or a specific number of top contributing flows can be considered (path approach 2.i) (Treloar 1997). Another variant initially ranks all flows and then considers all elements above a specific threshold (path approach 2.iii) (Treloar 1997). Norris (2002) considers a specific share of total contribution (90%, 95%, 99%) in each layer to select flows with sufficient contribution (path approach 2.ii)).

All these approaches postulate that the total impact is known whereas, in practice, a PLCA applicant has no information on the total impact. Hence, by applying PLCA alone, the entirety of flows cannot be ranked, or the relevant flows located. In this paper we will implement a slightly modified path approach, which in our view better simulates PLCA application considering the ISO norms. This works as follows: if a branch is judged to be significant, its impact is considered and all its direct upstream branches are further investigated; if it is insignificant, it is excluded from the analysis, together with all its upstream flows.

An overview of how different estimation frameworks utilize data is given in table 1.
Table 1: Schematic illustration of different approaches to estimate PLCA truncation errors with their corresponding data requirements.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Path approach 1</th>
<th>Path approach 2</th>
<th>Matrix Layer approach</th>
<th>IOLCA</th>
<th>Hybrid LCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within or between System</td>
<td>Within</td>
<td>Within</td>
<td>Within</td>
<td>Between</td>
<td>Between</td>
</tr>
<tr>
<td>Data used</td>
<td>IO data</td>
<td>IO data</td>
<td>IO data</td>
<td>IO data</td>
<td>IO + process data</td>
</tr>
<tr>
<td>Characteristics</td>
<td>Upstream paths are iteratively traced</td>
<td>Paths are ranked according to their contribution. Different possibilities: i) a specific number of paths are considered, ii) a specific contribution in each layer is considered, iii) all paths above a specific threshold are considered</td>
<td>Complete layers are considered</td>
<td>IOLCA is performed. Results are compared to PLCA results.</td>
<td>PLOCA and IO are combined. Results are compared to PLCA results. Different approaches exist.</td>
</tr>
</tbody>
</table>

Varying cut-off criteria within single approaches

Threshold schemata differ across path- and matrix-layer approaches, potentially impacting truncation error estimates. For the latter the maximal layer considered varies. For path approaches, absolute and relative thresholds may vary, as well as the specific number of flows that are considered. Generally, other (more) realistic cut-off procedures are conceivable for approximating PLCA within IOLCA. For instance, it is likely that real world LCA applicants add a random component when cutting off flows because of an individual judgment on the relevance of connected impacts (Suh et al. 2004) (please see the Supplementary Information (SI) for a modeling approach).
Multiple frameworks using different underlying datasets have been applied (see table 1) to estimate PLCA truncation errors (see table 2, which gives examples of different modeling approaches). The findings in the literature suggest a significant variance in magnitudes of truncation error estimates, across modeling frameworks (see table 2) and across sectors (Lenzen 2000; Lee and Ma 2013). HLCA frameworks have also been applied; these are separately discussed in the next section. Even though a substantial variation in results can be identified, an examination of how underlying modeling specifications, and the factors identified above, can influence truncation error estimates, using a single reference dataset, is missing. An overview of factors influencing truncation errors and the corresponding estimates is given in figure 1. Please note that the grey arrows indicate how different datasets are being used to provide an estimate. Our goal is to show that changing modeling specifications can cause a variation in truncation error estimates, using a single underlying dataset.

Table 2 Examples of different approaches used in literature to estimate PLCA truncation errors.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Type of approach</th>
<th>IO data used</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Matrix Layer approach for layers k={0,1,2,3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Path approach 2.ii); for each sector, input shares of (90%, 95%, 99%) are considered</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Path approach 2.iii) with absolute threshold value (0.00001 GJ/ $ 100 Aus)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Path approach 2.iii) with absolute threshold value (0.00001 GJ/ $ 100 Aus)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Path approach 2.iii) with absolute threshold value (0.00001 GJ/ $ 100 Aus)</td>
</tr>
</tbody>
</table>
### Estimated magnitude

| Even when $k=3$, truncation error of up to 50% can occur | Truncation errors of 23% and 35% result from shares of 90% and 95% | A truncation error of 7.5% results (Australian building sector considered only) | Truncation error (evaluated by HLCA) of 3% to 55% for PLCA, depending on sector and impact category |

### Further insights

| Truncation error magnitude varies across sectors | A high degree of system completeness requires a huge volume of flows | It is important to consider energy embodied in processes more than four stages upstream | Some IOLCA impacts are significantly larger than HLCA and PLCA impacts |

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**Sources influencing magnitudes of PLCA TEs**

- Matrix inversion applied
- Data approximated
- Data considered (regional & sectoral)
- Cut-off criteria applied
- System boundary definition applied
- Impact content misestimated
- Refining of system boundary
- Impact category considered
- Relevance approximated

**Systems used for estimating TEs**

- IOLCA
- HLCA
- Regions considered
- Differences between datasets
- Differences in link density
- PLCA cut-off criteria approximated
- Aggregation of sectors
- Sectors investigated
- Difference in reference years
- Use- & End-of-Life phase not considered
- Assumption of proportionality of physical and monetary flows

**Sources influencing IO based TEEs**

- Additional factors

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**Figure 1:** Schematic illustration of factors influencing truncation errors (TEs) and truncation error estimates (TEEs).

The left side (red) shows factors with direct influence on (unknown) truncation errors in PLCA and PLCA results. The right side (blue) represents factors influencing (system-complete) truncation error estimates. The grey arrows show how estimation approaches utilize different underlying datasets. The origin of an arrow indicates how the total impact has been approximated (either IOLCA or HLCA, which combines PLCA and IOLCA). The head of the arrow indicates how measured impact has been calculated (either using PLCA or an IO based counterfactual).

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**Excursus Hybrid LCA for estimating truncation errors**

Both PLCA and IOLCA have short-comings. PLCA analysis misses the impact associated with higher layers (Lenzen and Treloar 2002; Crawford and Stephan 2013) that can significantly change the
conclusion of (comparative) studies (Lenzen and Treloar 2003). System complete IOLCA lacks the extensive detail of process inventory databases, as well a gate-to-grave component (Suh and Huppes 2005; Lenzen 2000). Hence, literature judges the application of HLCA, combining the advantages of both approaches, to be superior (Wiedmann et al. 2011; Suh and Huppes 2005; Lenzen and Crawford 2009; Huang et al. 2009; Crawford and Stephan 2013; Rowley, Lundie, and Peters 2009; Guinée 2011). Different approaches for HLCA exist, inter-alia tiered-HLCA (Suh and Huppes 2005), the path exchange method (Lenzen and Crawford 2009) and the IO-based HLCA (Joshi 1999).

It has been shown that, depending on the type of hybrid LCA that is applied to the activity investigated, both IOLCA and PLCA can miss relevant impact shares if HLCA is assumed to be precise (Rowley, Lundie, and Peters 2009; Wiedmann et al. 2011; Crawford and Stephan 2013). It has also been revealed that IOLCA can overestimate specific impacts (Rowley, Lundie, and Peters 2009); depending on the environmental indicator and sector considered, and assuming that HLCA results can serve as a reference, estimates for truncation errors for PLCA are in the range of 2% to 77% (Rowley, Lundie, and Peters 2009; Wiedmann et al. 2011; Crawford and Stephan 2013). Corresponding estimates for IOLCA are in the range of -51% to 96% (Rowley, Lundie, and Peters 2009; Crawford and Stephan 2013).

Nevertheless, as one cannot compare HLCA to a better reference system (as it is partially consists of PLCA whose complete impact remains unknown), it is difficult to judge the quality of the estimate. In addition, if two HLCA approaches are applied, as in Wiedmann et al. (2011) and Crawford and Stephan (2013), it is impossible to judge which one is closer to the real unknown impact.
Although it is highly likely that HLCA produces better results, we do not use it for our simulations, as a qualitative reference, to which results can be compared, is lacking. In addition, implementing HLCA within an IO framework, where the total impact is known, would result in an IOLCA.

<heading level 2> Analysis framework

This section describes the framework used to investigate the influence of the most relevant modeling configurations of PLCA truncation error estimates in literature. We implement different estimation frameworks, vary threshold rules and disregard specific service sectors in scenarios. Finally, we contrast their results. When doing so, we do not aim to provide exact estimates; this is impossible as the total impact in the process inventory system remains unknown. Our goal is to assess whether there is relevant influence by modeling specifications. We avoid estimation errors linked to the comparison of two different systems, that is “between system”, by implementing scenarios and evaluating truncation error estimates within a single IO system (within system).

<heading level 3> Data

In order to implement the different modeling frameworks we use the single region Open IO database for the US (Applied Sustainability Center, University of Arkansas. Sylvatica 2010), representing the US economy in 2002. The database has a resolution of 430 sectors and also provides data on different greenhouse gas releases. For demonstration purposes, only CO₂ emissions are considered in this article. As we use a single region IO table, exports and imports are not accounted for separately. It is assumed that other countries produce commodities with the same sectoral coefficients.
Although IO databases with higher resolution exists, for instance the national Australian IO with 1284 sectors (available at (IELab 2017)), we use the US database, as it fits our purposes perfectly. This is because it has been used by Majeau-Bettez et al. (2011), who precisely identified and named the sectors that are excluded from inventory databases. Hence, it allows us to easily exclude such sectors. Taking a different dataset, with varying sectors, would require us to approximate the sectors identified by Majeau-Bettez et al. (2011), potentially introducing additional errors.

**IO notations**

Standardized IO data consist of an inter-industry flow matrix \( Z \in \mathbb{R}^{m \times m} \) and a final demand vector \( Y \in \mathbb{R}^m \). Entries \( Z_{uv} \) of \( Z \) reflect the total monetary value (in USD) of flows from sector \( u \) to sector \( v \) with \( u, v \in M = \{1, ..., m\} \); where \( m \) denotes the number of all sectors. Analogously, \( Y_u \) represents the sum of all monetary flows from sector \( u \) into final demand.

By \( O \in \mathbb{R}^m \), we denote the total output vector, with entries \( O_u = \sum_v Z_{uv} + Y_u \), giving the total output of sector \( u \). \( A \in \mathbb{R}^{m \times m} \) denotes the technology matrix, consisting of entries \( A_{uv} = Z_{uv} / O_v \), that describe the amount of each input \( u \) (in USD) that is required by sector \( v \) in order to produce one unit of output (in USD).

The Leontief inverse is calculated as \( L = (I - A)^{-1} \), where \( I \) denotes the unity matrix. It accounts for all pre-products that have been used at some stage during the production process. Further,

\[ \sum_{l=0} A^l \rightarrow (I - A)^{-1} \text{ for } l \rightarrow \infty \] holds, where each term of the power series refers to a complete upstream production tier.
Additionally, we use data on released CO₂ emissions. We let \( F \in \mathbb{R}^m \) denote the vector whose \( m \) entries \( F^u \) denote total emissions released by sector \( u \). Dividing \( F \) by total sectoral outputs \( O \) results in vector \( f \in \mathbb{R}^m \) whose entries \( f^u \) reflect CO₂ emissions associated with one USD of output of sector \( u \).

\[
\begin{align*}
\text{IO – Matrix layer approach} \\
\text{When applying the matrix layer approach to estimate truncation errors, it has been assumed that the PLCA application can be approximated by a power series (Lenzen 2000), considering all elements up until a specific layer } k \in \mathbb{N}. \text{ The truncation error estimate of the matrix layer approach (TEE\_MLA) for sector } u \text{ results as:}
\end{align*}
\]

\[
TEE_{\text{MLA}}^u = \frac{(f(I-A)^{-1})^u - (f(\sum_{l=0}^{k} A^l)^u)}{(f(I-A)^{-1})^u} \quad \forall u \in M. \quad (3)
\]

In order to adjust the strictness of the cut-off criterion, the number of the maximal layer \( k \) can be varied.

\[
\text{Exclusion of sectors using the matrix approach} \\
\text{To account for sectors frequently disregarded in PLCA analyses using the matrix approach we define } S \subset M \text{ to be the set of sectors ignored in the analysis (Majeau-Bettez, Hawkins, and Stromman 2011). Please refer to the Supporting Information (SI) for further details. } A \text{ is modified such that entries referring to } S \text{ are set to zero, resulting in matrix } A^*.\text{ The truncation error estimates then result in:}
\end{align*}
\]

\[
TEE_i = \frac{(f(I-A)^{-1})^u - (f(I-A^*)^{-1})^u}{(f(I-A)^{-1})^u}. \quad (4)
\]
Although, literature indicates that specific capital goods are also ignored in process inventory databases (Suh et al. 2004; Junnila 2006), we decline to implement a corresponding scenario, as a precise list of sectors is missing. Hence, investigating their impact is left to future research.

**IO – Path approach**

The path approach, whose foundation has been described by Treloar (1997) and Norris (2002), investigates the environmental impact of a process/product by tracing single flows in a typically exponentially growing set of paths. This set of paths is often referred to as process tree in literature, as upstream flows branch increasingly. Each element (single path), is described by the equation \( \omega_{op \ldots qu} = A_o^{p} \cdot \ldots \cdot A_q^{u} \cdot f^{u} \), which refers to associated CO\(_2\) emissions in sector \( u \) of one unit of output in sector \( o \) with a corresponding production path \( u \rightarrow q \rightarrow \ldots \rightarrow p \rightarrow o \), \( u, q, \ldots, p, o \in M \) (emissions have been released in sector \( u \)). We modify the approaches in the literature to get a new procedure of tracing branches that is similar to tracing flows in PLCA. Cut-off criteria are defined accordingly, that is flows which are likely to have an insignificant contribution are deleted. Whether a flow with emissions \( \omega_{op \ldots qu} \) has a sufficiently significant contribution to the study is judged on the basis of a threshold \( t \). If \( \omega_{op \ldots qu} > t \) holds, all of \( \omega_{op \ldots qu} \) first order upstream flows are added to the set of flows that need to be investigated. Otherwise the flow is ignored, together with all of its upstream flows. This procedure is repeated until no flows remain to be investigated.

**Defining scenarios**

We use a modified IOLCA approach, see equation (4), to investigate how omitting service sectors influences truncation error estimates. With regard to varying threshold rules, we recall that
approaches in the literature estimating truncation errors consider different cut-off criteria. When using the matrix layer approach, flows up to a specific layer are considered (Lenzen 2000). In the case of a path approach, (ordered) flows have so far been cut off using an absolute threshold (Treloar 1997) or a specific share of impacts within each layer (Norris 2002). To investigate the influence of the choice of modeling frameworks, we first implement relative thresholds for the path approach (each relative threshold corresponds to a specific absolute threshold by transformation), where flows below a specific share of contribution are disregarded. Secondly we use the matrix layer approach with varying maximal layer. A third path approach, using stochastic thresholding, is implemented in the SI.

<heading level 1> Results & Discussion

This section presents and discusses results of the scenarios introduced above. It then continues with a discussion of the influence of various cut-off criteria and modeling frameworks on truncation error estimates. Finally it provides implications for further research.

<heading level 2> Different cut-off criteria and modeling frameworks

In this subsection, different modeling frameworks are applied and compared, see figure 2. The matrix layer approach with varying maximal layers $k \in \{0,1,2,3\}$, and the process approach with changing relative thresholds $t \in \{2\%,1\%,0.5\%\}$ for each sector are implemented. Service sectors are included as they would impact all modeling approaches equally and their exclusion would not provide any additional relevant information.
Figure 2: Mean truncation error estimate (for a sector group) when applying different cut-off criteria. Left: Matrix layer approach with increasing maximum layers ($k \in \{0,1,2,3\}$). Right: Increasing strictness of (relative) threshold ($t \in \{2\%, 1\%, 0.5\%\}$).

We find that for the path approach, mean truncation error estimates are between 28\% and 76\% for different sector groups. Increasing the strictness of the threshold level from 2\% to 0.5\%, leads
to reductions in mean truncation error estimates of less than ten percentage points for all sector groups. In comparison, results for the matrix layer approach show that significant reductions in mean truncation error estimates for aggregated sectors occur with increasing strictness of cut-off criteria. Mean truncation error estimates of 33% to 75% arise for all flows up to the first layer (that is $k = 1$). In contrast, truncation error estimates of 7% to 28% result when $k = 3$. The smallest truncation error estimations for both modeling frameworks (matrix layer and path approach) are observable for agricultural and transportation sectors. The largest truncation errors are identified for textile and electronic manufacturing sectors. These results are consistent with the literature (Lenzen 2000). They reveal that the matrix layer approach is sensitive to an increase in the strictness of the cut-off criterion, whereas the path approach is not. This is because each approach incorporates flows differently. The matrix layer approach considers whole flow layers, independent of the size of individual flows, whereas the path approach considers single flows. The results become more distinct when investigating the distribution of truncation error estimates (figure 3).

The distribution of truncation error estimates in the path approach, slowly shifts to the left when the relative threshold is reduced (figure 3). In contrast, when increasing the strictness of cut-off criteria for the matrix layer approach, the entire distribution quickly shifts to the left.
Figure 3: Relative distribution of truncation error estimates derived from the implemented path approach (left) and relative distribution of truncation error estimates derived from the matrix layer approach (right).

The results imply that there is a tremendous decline in the contribution of single flows within higher flow layers. More importantly, the outcomes indicate that truncation errors of traditional PLCA applications, which iteratively trace flows, are barely reduced when the strictness of the cut-off criterion is increased. As a further reduction of the relative threshold imposes computational difficulties, we cannot give a threshold \( t \) that is sufficiently small to reduce truncation errors below a specific level.

The implemented scenarios, using a single database, show that large differences in resulting truncation error estimates can be observed. This depends on the cut-off criterion, underlying framework, and strictness of thresholds chosen. Thus modeling specifications have a significant influence on estimating the PLCA truncation error.
Disregarded sectors in PCLA

Our results show that ignoring service sectors not covered by life-cycle inventory databases, as identified by Majeau-Bettez et al. (2011), causes median truncation error estimates of 3-13% depending on the sector group being analyzed; results vary across different sectors. The largest truncation error estimates are located in manufacturing sectors (see figure 4), where median truncation error estimates exceed 10%. For a few specific sectors, such as fishing, electronic computer manufacturing, computer storage, broadcast and wireless communications equipment or analytical laboratory instruments, truncation error estimates can even exceed 20%. The results therefore indicate that disregarding specific sectors can be relevant. The smallest median truncation error estimates occur in agricultural sectors, indicating that a high proportion of emissions are associated with tier zero activities. As the most important direct emissions in the agricultural sectors are greenhouse gases other than CO₂ (Peters et al. 2010), analyzing different impact categories could even augment the outcome by further reducing the estimate.
Figure 4: Influence on PLCA truncation error estimates through the disregard of service-related sectors that are typically omitted in process databases. Red bars correspond to median truncation error estimates of the corresponding sector group and blue boxes span from 1st to 3rd quartile. A detailed overview of aggregated sector groups and disregarded service sectors can be found in the Supporting Information.

Impact and future research

Conventional PLCA arguably suffers from truncation errors (Suh et al. 2004; Lenzen 2000). Consequently total impacts assessed by PLCA (for example environmental impacts) associated with specific products and processes remain unknown. Reviewing the literature on truncation errors and their estimates, we find that the latter are influenced at three different levels: first when estimating the impact itself, either through the PLCA or by approximating the PLCA estimate; second when estimating the system complete counterfactual; and third when
(possibly) concluding on two different systems, which is comparing IO or HLCA results with PLCA results.

Our results show that large differences in estimates occur when investigating factors of these different levels. Estimates crucially depend on the chosen modeling framework and the applied cut-off criterion, even for a single database. Our results challenge explicit results and statements on the size of PLCA truncation errors given in the literature, as the influence of the modeling configurations has not been considered. In this respect the identified factors and the investigated scenarios indicate that estimating PLCA truncation errors correctly is not possible; there are too many interacting model factors at different levels that cannot simultaneously be accounted for. Our results hence stress the necessity to carefully consider the influence of the modeling framework on results in future assessments.

The findings are important for PLCA applicants in multiple dimensions. Firstly, they suggest that not considering specific service sectors in process inventory databases can lead to relevant error-prone results. Secondly, our results imply that the procedures for artificially curing the system incompleteness of PLCA, for example by including an IO correction term as in HLCA, cannot be precisely evaluated as knowledge of the complete truncation error would be necessary.

Thirdly, the developed and implemented new variant of the path approach gives a (rough) indication of the magnitude of PLCA truncation errors. We find that mean truncation error estimates are likely to be in the range of 30% to 80%. These depend on the sector group
investigated. Nevertheless, we cannot quantify the influence on results that is introduced by our modeling assumptions, for instance by using IO data.

Fourthly, our results show that path approach truncation error estimations are relatively stable across different threshold levels. This indicates that truncation errors associated with pure PLCA can barely be reduced by increasing the strictness of the cut-off criterion. A feasible solution to reduce truncation errors in a targeted manner might be the application of a preceding IOLCA analysis (as an early-warning system) that orders the flows contributions as suggested by Treloar (1997). It could indicate where relevant contribution shares are hidden in the process tree. HLCA methodologies are widely judged to more precisely account for total (environmental) impacts. The application of HLCA, such as the path exchange method (Lenzen and Crawford 2009), could prevent severe truncation errors, while retaining the detail of PLCA.

Clearly our analysis alone could not consider all factors potentially influencing truncation errors estimates that have been identified in the literature and in this article. It remains unclear how they bias the results obtained. Hence, our results do not incorporate the full complexity necessary to adequately quantify PLCA truncation errors, which seems to be difficult to achieve.

In the scenarios presented in the article we have evaluated the influence of some factors that are characteristic for different PLCA truncation error estimates in the literature. Future investigations need to assess the influence of other factors identified (for example differences in regional data considered or network properties). More research is also required to clarify how the different factors interact (for example, how do different network properties and varying cut-off criteria jointly influence truncation error estimates). Finally, the most relevant factors biasing PLCA
truncation error estimates have to be identified in order to help reduce unknown truncation errors within PLCA applications.

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Please note that we chose a database representing the US economy in 2002. Although, these data are non-actual, the derived results are valid and representative for modeling approaches. This database was been chosen because it coincides with the one used by Majeu-Bettez et al. (2011).
In the latest version of (British Standards - ISO 14044 2006) it is stated that "The initial system boundary shall be revised, as appropriate, in accordance with the cut-off criteria established in the definition of the scope."