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# 1 **A strategy to determine DEM parameters for spherical and** 2 **non-spherical particles**

3 Frederik Elskamp<sup>1\*</sup>, Harald Kruggel-Emden<sup>1</sup>, Manuel Hennig<sup>2</sup>, Ulrich Teipel<sup>2</sup>

4 <sup>1</sup>Mechanical Process Engineering and Solids Processing, Technische Universität Berlin, Ernst-  
5 Reuter-Platz 1, D-10623 Berlin, Germany

6 <sup>2</sup>Department of Particle Technology, Technical University Nürnberg, Wassertorstrasse 10, D-  
7 90489 Nuremberg, Germany

8 \*Corresponding author. Tel.: +49-234-32-23492; Fax: +49-234-32-14227

9 E-mail address: frederik.elskamp@tu-berlin.de

## 10 **Abstract**

11 In Discrete Element Method (DEM) simulations the choice of appropriate contact parameters  
12 is significant to obtain reasonable results. Particularly, for the determination of DEM  
13 parameters for non-spherical particles a general straightforward procedure is not available.  
14 Therefore, in a first step of the investigation here, methods to obtain the friction and restitution  
15 coefficients experimentally for single particles (Polyoxymethylene (POM) spheres and quartz  
16 gravel) will be introduced. In the following, these predetermined DEM coefficients are used as  
17 initial values for the adjustment of bulk simulations to respective experiments. In the DEM  
18 simulations, the quartz gravel particles are represented by non-spherical particles  
19 approximated by clustered spheres. The best fit approximation of the non-spherical particles  
20 is performed automatically by a genetic algorithm. In order to optimize the sliding and rolling  
21 friction coefficients for DEM simulations, the static and dynamic angle of repose are determined  
22 from granular piles obtained by slump tests and rotating drum experiments, respectively.  
23 Additionally, a vibrating plate is used to obtain the dynamic bed height which is mainly  
24 influenced by the coefficient of restitution. The adjustment of the results of the bulk simulations  
25 to the experiments is conducted automatically by an optimization tool based on a genetic  
26 algorithm. The obtained contact parameters are later used to perform batch-screening DEM  
27 simulations and lead to accurate results. This underlines the applicability of the in parts  
28 automated strategy to obtain DEM parameters for particulate processes like screening.

29 **Keywords:** Discrete element method (DEM); Parameter determination; Automatic adjustment  
30 of DEM parameters; Arbitrary shaped particles; Screening

## 31 **1. Introduction**

32 The process step screening is important to separate bulk material in a wide range of industrial  
33 applications, where particles of non-spherical shape are classified according to desired size

34 class specifications [1,2]. In the interest of studying screening and its sub-processes in detail  
35 without performing extensive experimental tests the Discrete Element Method (DEM) dating  
36 back to Cundall and Strack [3] has been proved as a suitable tool [4–8]. To apply the DEM for  
37 complex processes like screening a proper calibration of DEM parameters and particle shape  
38 approximation has to be carried out. Several methods to calibrate DEM parameters have been  
39 proposed but particularly, for non-spherical particles general straightforward procedures with  
40 a high degree of automation are hardly available.

41 The first investigations published, addressing DEM parameter calibration, mostly concentrate  
42 on spherical particles without automated procedures. In one of them [9] Li et al. measured the  
43 coefficient of sliding friction in simple drag tests applying spheres and confirmed the DEM  
44 parameters by comparing quasi-two-dimensional hopper discharge and conical pile  
45 experiments with simulations. Based on this, Gryma and Wypych [10] as well as Chen et al.  
46 [11] applied particle clusters consisting of two spheres to measure the static angle of repose  
47 in a slump test to confirm and adjust the DEM parameters which had been determined in single  
48 particle tests before. Coetzee and Els [12] also applied clusters of two spheres and determined  
49 the DEM parameters in shear and compression tests before measuring the static angle of  
50 repose. The obtained values were later applied successfully to hopper discharge and bucket  
51 filling processes. Natsui et al. [13] determined the contact friction and used rolling friction to  
52 consider the irregular shape of coke in simulations where the angle of repose was measured.  
53 They stated that the choice of the Young's modulus is important to accelerate the calculation  
54 time in DEM simulations as long as the results are not influenced. Alonso-Marroquín et al. [14]  
55 determined the DEM parameters stiffness, the friction and restitution coefficients for polygonal  
56 wood particles with several single particle experiments, including sliding, triaxial and pendulum  
57 collision tests, respectively. They applied the obtained parameters to hopper discharge  
58 simulations and compared the results with respective experiments. Similar single particle tests  
59 are carried out by Barrios et al. [15] applying particle clusters consisting of four different sized  
60 spheres to approximate iron ore pellets. The validation of the DEM parameters by slump and  
61 tumbling tests clarified the significance of particle shape and the need of adjusting the DEM  
62 parameters after simple single particle tests. A review for DEM parameters and contact models  
63 for granular material has been done by Horabik and Molenda [16] who highlighted the  
64 importance of material and interaction properties for obtaining reliable information out of DEM  
65 simulations. Recently, Coetzee [17] investigated the influence of particle shape approximation  
66 with particle clusters created manually and by an automated optimization process with the  
67 multi-sphere method of up to eight spheres as main part of a DEM parameter calibration.  
68 Additionally, Dobrohotoff et al. [18] used spheropolygons to describe two-dimensional  
69 complex-shaped objects like pebbles, gravel and crushed shells where only a few iteration

70 steps were needed to get good results. The significant optimization of the shape was confirmed  
 71 by comparing anchor pull-out and hopper discharge simulations with respective experiments.  
 72 Höhner et al. [19,20] investigated the influence of the multi-sphere and the polyhedral method  
 73 on the mechanical behavior of particles during hopper discharge. Only minor differences were  
 74 obtained between both methods for complex shaped particles. However, the computational  
 75 time increased significantly if the polyhedral method was used for real particles like quartz  
 76 gravel. Accordingly, Li et al. [21] approximated real particles with the multi-sphere method  
 77 automatically by a greedy heuristic algorithm obtaining very accurate results. In order to  
 78 determine adequate DEM parameters automatically, Benvenuti et al. [22] developed a neural  
 79 network which can be trained by dedicated DEM simulations to predict granular bulk behavior  
 80 for a large number of DEM parameter sets.

81 Based on these previous investigations, the strategy to determine DEM parameters in this  
 82 investigation is as follows. First, DEM parameters and physical properties of single particles  
 83 are determined. Additionally, non-spherical particles are clustered with the multi-sphere  
 84 method automatically. Afterwards, the physical properties and the DEM parameters are  
 85 applied to bulk simulations where the DEM parameters are adjusted automatically with an  
 86 optimization tool to fit the results of corresponding experiments. To confirm the adjustment  
 87 procedure, initial and adjusted parameter values are compared against each other. The  
 88 adjusted DEM parameters are then applied to screening processes as example for a more  
 89 complex particulate process.

## 90 **2. Numerical method**

91 The DEM can be applied to systems with spherical and non-spherical shaped particles [23,24],  
 92 by tracking the translational and rotational motion of each particle. For this purpose the  
 93 Newton's and Euler's equations are integrated

$$m_i \frac{d^2 \vec{x}_i}{dt^2} = \vec{F}_i + m_i \vec{g}, \quad (1)$$

$$\hat{I}_i \frac{d\vec{\omega}_i}{dt} + \vec{\omega}_i \times (\hat{I}_i \vec{\omega}_i) = \Lambda_i^{-1} \vec{M}_i, \quad (2)$$

94 with particle mass  $m_i$ , particle acceleration  $d^2 \vec{x}_i / dt^2$ , contact force  $\vec{F}_i$ , gravitational force  $m_i \vec{g}$ ,  
 95 angular acceleration  $d\vec{\omega}_i / dt$ , angular velocity  $\vec{\omega}_i$ , external moments resulting out of contact  
 96 forces  $\vec{M}_i$ , the inertia tensor along the principal axis  $\hat{I}_i$  and the rotation matrix converting a  
 97 vector from the inertial into the body fixed frame  $\Lambda_i^{-1}$ . Explicit integration schemes (comp. e.g.  
 98 [25]) are used to solve both equations (eq. (1) and eq. (2)).

99 To model complex shaped particles in the DEM the multi-sphere method is used. Thereby, the  
 100 desired complex particle shape is represented by clustered spheres of arbitrary size [26] and  
 101 similar contact force laws as used for spherical particles are applied [27].

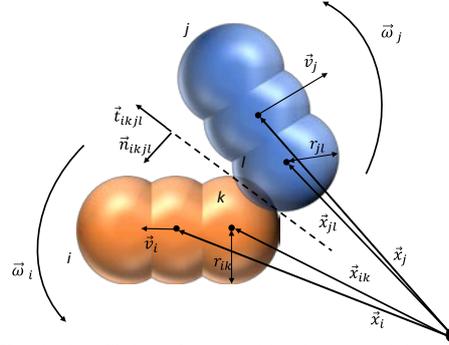


Fig. 1: A collision of two multi-sphere particles.

102  
103

104 In Fig. 1 a sketch of two complex shaped particles  $i$  and  $j$  is shown, where the spheres  $l$  and  $k$   
 105 collide. For further details on the contact scheme involving clustered spheres the works by  
 106 Kruggel-Emden and Kačianauskas and Kruggel-Emden et al. [28,29] are recommended.

107 The normal component of the contact forces is obtained from a linear spring damper model  
 108 which is exemplarily given for the contacting spheres  $k$  and  $l$  of particle  $i$  and  $j$  as

$$\vec{F}_{ikjl}^n = k^n \delta_{ikjl} \vec{n}_{ikjl} + \gamma^n \vec{v}_{ikjl}^n, \quad (3)$$

109 where  $k^n$  is the spring stiffness,  $\delta_{ikjl}$  the virtual overlap,  $\vec{n}_{ikjl}$  a normal vector,  $\gamma^n$  a damping  
 110 coefficient and  $\vec{v}_{ikjl}^n$  the normal velocity at the contact point [30]. The damping coefficient  $\gamma^n$  is  
 111 calculated as

$$\gamma^n = - (2 \ln(e^n) m_{eff}) / t^n \quad (4)$$

112 with the experimentally determined coefficient of restitution  $e^n$  (comp. section 3.5), the duration  
 113 of a collision

$$t^n = \pi / \left( \sqrt{\left( k_n / m_{eff} - (\gamma^n / (2m_{eff}))^2 \right)} \right) \quad (5)$$

114 and the effective mass  $m_{eff} = m_i m_j / (m_i + m_j)$ .

115 The tangential forces are calculated by applying a linear spring limited by the Coulomb  
 116 condition

$$\vec{F}_{ikjl}^t = \min(k^t |\vec{\xi}_{ikjl}|, \mu_c |\vec{F}_{ikjl}^n|) \vec{t}_{ikjl}, \quad (6)$$

117 where  $k^t$  is the tangential stiffness of a linear spring,  $\mu_c$  is the friction coefficient,  $\vec{\xi}_{ikjl}$  is the  
 118 relative tangential displacement and  $\vec{t}_{ikjl}$  is the tangential unit vector [31]. The tangential spring  
 119 stiffness  $k^t$  is obtained from

$$k^t = \kappa m_{eff} (\pi/t^n)^2, \quad (7)$$

120 where  $\kappa$  is given through the mechanical properties as

$$\kappa = ((1 - \nu_i)/G_i + (1 - \nu_j)/G_j) / ((1 - 0.5\nu_i)/G_i + (1 - 0.5\nu_j)/G_j), \quad (8)$$

121 where  $\nu$  is the Poisson's ratio and  $G = E/(2 + 2\nu)$  with Young's modulus  $E$  is the shear  
 122 modulus of the two interacting materials  $i$  and  $j$  [32].

### 123 3. Determination of DEM parameters and physical properties of single particles

124 In this section, the material and physical properties, the particle shapes, the sliding and rolling  
 125 friction coefficients and the coefficient of restitution are obtained by single particle  
 126 characterization. In case of the coefficients of friction and restitution the calibration at the  
 127 particle level is done to get a first approximation. These parameters can be used as initial  
 128 values for the parameter adjustment so that the calibration converges quickly; additionally they  
 129 are used as comparison for the values reached after the adjustment. If these values are already  
 130 accurate enough, the adjustment is very fast or is not necessary at all.

#### 131 3.1 Determination of material and physical properties

132 In a first step, material and physical properties like the size distribution, mass, volume and  
 133 density of the particles are determined and listed in Table 1.

134 **Table 1:** Mechanical particle and wall properties.

Mechanical particle property	Particle		Wall	
	POM	Gravel	Metal	Acryl
Diameter $d$ [mm]	5 / 7 / 10 $\pm$ 0.1	3.15-5.60	-	-
Mass $m$ [g]	0.0935 / 0.2459 / 0.7210 $\pm$ 0.02	0.0316-0.4440	-	-
Density $\rho$ [kg/m <sup>3</sup> ]	1.43E+03 / 1.37E+03 / 1.38E+03 $\pm$ 1.50E+03	2.76 E+03	7.85E+03	1.20E+03
Young's modulus $E$ [N/m <sup>2</sup> ]	2.84E+09	6.00E+10	2.08E+11	2.20E+09
Poisson's ratio $\nu$ [-]	0.35	0.25	0.30	0.37
Stiffness $k_{PP}^n / k_{PW}^n$ [N/m]	1.00E+05	1.00E+05	-	-

135

136 Polyoxymethylene (POM) spheres are applied in three different discrete size classes which  
 137 are equally distributed in each investigated case. For the quartz gravel the cumulative particle  
 138 size distribution as outlined in Fig. 2 which is obtained by an image analysis of a CAMSIZER®  
 139 is used. Note that the dots represent one size class in the DEM simulations.

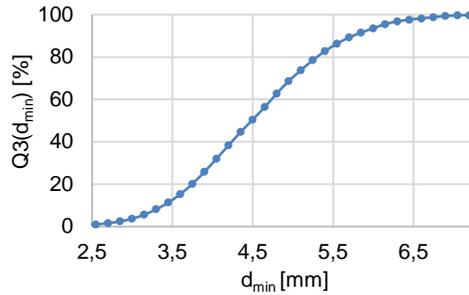


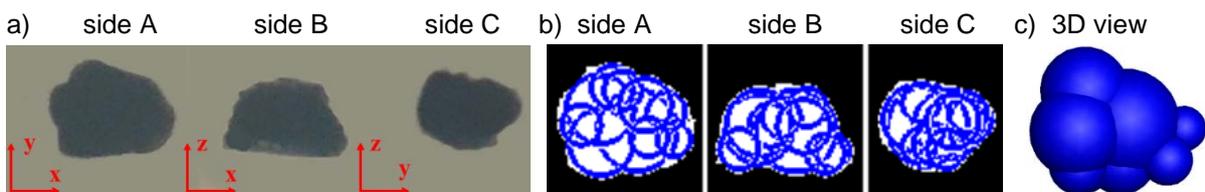
Fig. 2: Cumulative particle size distribution of quartz gravel.

140  
141

142 The mass  $m_i$  of each POM particle size class is measured as average of 20 POM spheres and  
 143 the density  $\rho_i$  is determined with  $\rho_i = m/(1/(6\pi d^3))$ . The average volume  $V_i$  of the gravel is  
 144 obtained by water displacement of around 500 particles which are weighed before to obtain an  
 145 average density of  $\rho=2755\text{kg/m}^3$ . The normal spring stiffness is set to  $k^n=1.00\text{E}+05$  N/m for  
 146 both materials after carrying out test simulations of granular piles (comp. section 4.2) applying  
 147 values of  $k^n=1.00\text{E}+03$  N/m up to  $k^n=1.00\text{E}+06$  N/m and measuring no changes for values  
 148 larger than  $1.00\text{E}+05$  N/m. Additionally, the particle overlap is consistently below 0.5 % of the  
 149 particle diameter according to Cleary [33].

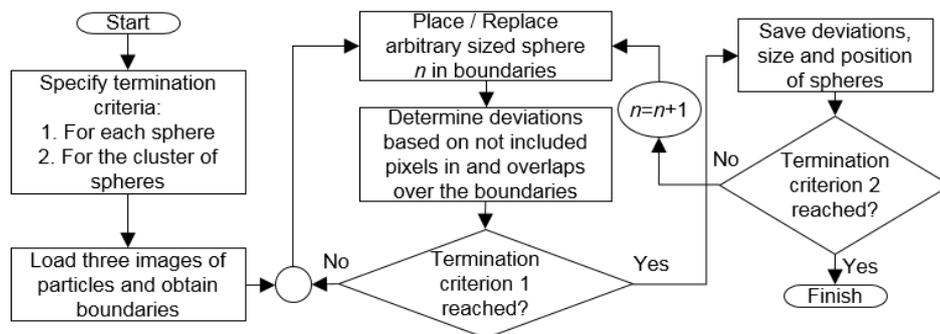
150 3.2 Shape approximation

151 As stated before, the POM particles are assumed to be ideal spheres of 5 mm, 7 mm and  
 152 10 mm which are applied in the DEM simulations accordingly. The gravel consists of non-  
 153 spherical particles with a sphericity between 0.35 and 0.87 (average: 0.75) and an aspect ratio  
 154 between 1.06 and 2.49 (average: 1.49) which is both obtained by an image analysis. To  
 155 represent such particles, various methods can be applied [17,18,21]. For this approximation a  
 156 genetic algorithm which is part of Matlab is used. This algorithm is very flexible and allows  
 157 adjusting many features such as population size, generation, mutation and crossover functions  
 158 as well as initial values (comp. [37]). It was already applied successfully in the work by Kruggel-  
 159 Emden et al. [34] for the adjustment of coefficients as part of multi-parameter models  
 160 describing reaction kinetics in the context of chemical looping where details on the algorithm  
 161 and possible settings can be found. Instead of modelling the gravel with a polyhedral shape,  
 162 the particles are approximated by clustered spheres like in Coetzee et al. [17] to save  
 163 computational time in the DEM simulation, which is important for an efficient adjustment  
 164 algorithm.



165 Fig. 3: (a) Cross sectional areas and (b) areas filled with circles to (c) approximate the shape of gravel particles (example).

166 Therefore, in a first step, a sample of gravel particles is filled into cast resin cubes and images  
 167 are taken from three sides giving the three cross sectional areas (Fig. 3a). With the help of an  
 168 optimization tool, arbitrary sized circles, which represent spheres, are placed one after the  
 169 other into these three areas to obtain the best fit for all areas while receiving penalty points for  
 170 not included pixels in or for overlaps over a projected zone (Fig. 3b). The number of applied  
 171 spheres determines the accuracy of the approximation but in contrary influences the simulation  
 172 time. One method to terminate the optimization is to specify the amount of clustered spheres  
 173 before the optimization, whereas another method is to automatically stop the tool if no further  
 174 improvement is detected when applying more spheres (decrease in deviation is lower than  
 175 defined). For the placing of each single sphere, the maximum number of iterations or a  
 176 specified change in deviations to the previous iteration can be defined. The whole algorithm is  
 177 presented in the flow chart in Fig. 4.



178 **Fig. 4:** Algorithm to best fit the representation of gravel particles with clustered spheres on the basis of cross-sectional areas.

179 One of the created gravel particles which is applied in the following simulations is shown in  
 180 Fig. 3c. It consists of 10 spheres which is sufficient enough to represent the gravel particles  
 181 (no further improvement by applying more spheres) and is still computationally efficient. It is  
 182 accomplished in less than 10 minutes requiring less than 100 generations where a population  
 183 consisting of 200 individuals was used. For the simulations, several different shapes for each  
 184 size class can be applied. A particle can be classified in one particle size class by measuring  
 185 its volume equivalent sphere diameter if the particle size distribution is based on this equivalent  
 186 diameter. Due to applying the particles for a screening process in the following, the  
 187 approximated particle is classified based on the minor axis diameter in this investigation (comp.  
 188 Fig. 2). The other particle size classes are created by scaling the approximated cluster with  
 189 respective scaling factors obtained through the ratio of the equivalent diameters of two size  
 190 classes.

### 191 3.3 Coefficient of sliding friction

192 To determine the sliding friction of the particles, samples of POM spheres of each size class  
 193 and gravel particles of each fraction are glued under a wooden plate and the force which is  
 194 necessary to drag these particles over a plate consisting of the material of one contact partner

195 is measured (comp. Fig. 5). Thereby, the evenness of the particle layer influences the quality  
 196 of the measurement. To stabilize the wooden plate a weight is placed on it. The value of the  
 197 weight is changed to test its influence resulting in deviations of below 1 %.

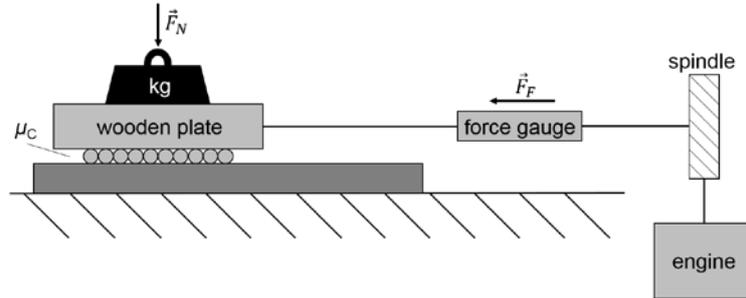


Fig. 5: Determination of sliding friction.

198  
 199 In this investigation, the classical friction theory is applied, which states that  $\mu_c = |\vec{F}_F|/|\vec{F}_N|$ ,  
 200 with the frictional force  $\vec{F}_F$  and the normal force  $\vec{F}_N$ . This is suitable for a point-point contact  
 201 when ignoring the effect of the contact area [9]. Due to having a very small contact area  
 202 between a particle and a plane wall like between two particles, a plate of the particle material  
 203 (POM spheres with a POM plate and quartz gravel with a gravel plate) is used as  
 204 approximation for the particle-particle friction.

205 Results for the sliding friction coefficient between particles and the same material for the  
 206 particle-particle contacts and between particles and the wall materials (acryl, metal) for the  
 207 particle-wall contacts each averaged over 10 experiments are listed in Table 2. For two  
 208 different sized POM spheres the average of the respective values is applied.

209 **Table 2:** Coefficient of sliding friction.

Contact partner	POM 5 mm	POM 7 mm	POM 10 mm	Gravel
	$\mu_c$	$\mu_c$	$\mu_c$	$\mu_c$
Acryl (side walls)	$0.16 \pm 0.01$	$0.19 \pm 0.01$	$0.21 \pm 0.01$	$0.19 \pm 0.01$
Metal (screen wire)	$0.17 \pm 0.01$	$0.16 \pm 0.01$	$0.17 \pm 0.02$	$0.31 \pm 0.02$
POM / quartz gravel (particles)	$0.23 \pm 0.01$	$0.30 \pm 0.02$	$0.28 \pm 0.02$	$0.57 \pm 0.03$

210

### 211 3.4 Coefficient of rolling friction

212 For simulating spherical particles in the DEM (here for POM spheres) it is necessary to use a  
 213 model for rolling friction to oppose the rolling motion of the spheres with a decelerating moment  
 214  $\vec{M}_{roll}$ . Here, the model by Zhou et al. [35] is used which can be simplified for free rolling  
 215 spherical particles where the normal force  $\vec{F}_N$  is equal to the weight force. Therefore, the  
 216 coefficient of rolling friction can be calculated as

$$\mu_{roll} = |\vec{M}_{roll}|/|\vec{F}_N|. \quad (9)$$

217 To determine the coefficient of rolling friction experimentally, the rolling motion of a sphere on  
 218 a plane wall is recorded [36]. The moment  $\vec{M}_{roll}$  is then obtained as

$$|\vec{M}_{roll}| = \left( 1/2 m(v_1^2 - v_2^2) + 1/2 \theta(\omega_1^2 - \omega_2^2) \right) d/(2s) \quad (10)$$

219 with the velocities of the sphere at the beginning ( $v_1$ ) and at the end ( $v_2$ ) of the surface, the  
 220 sphere's mass moment of inertia  $\theta$ , its angular velocity at both time steps  $\omega_1$  and  $\omega_2$ , the  
 221 distance travelled by the sphere  $s$  and its half diameter  $d/2$ . The velocities of the sphere  $v_1$  and  
 222  $v_2$  as well as the distance travelled are measured by image analysis of records taken from  
 223 above with a high speed camera. It can be assumed that the spheres do not slip or bounce on  
 224 the applied plane surface and for the low velocities. Therefore, the angular velocity  $\omega$  can be  
 225 obtained through the translational velocity  $v$ .

226 Results for the coefficient of rolling friction between POM spheres and a POM plate for the  
 227 particle-particle contacts and between particles and the wall materials (acryl, metal) for the  
 228 particle-wall contacts each averaged over 10 experiments are listed in Table 3. For two  
 229 different sized POM spheres the average of the respective values is applied. The large  
 230 standard deviations indicate the need of a further adjustment of these initial values.

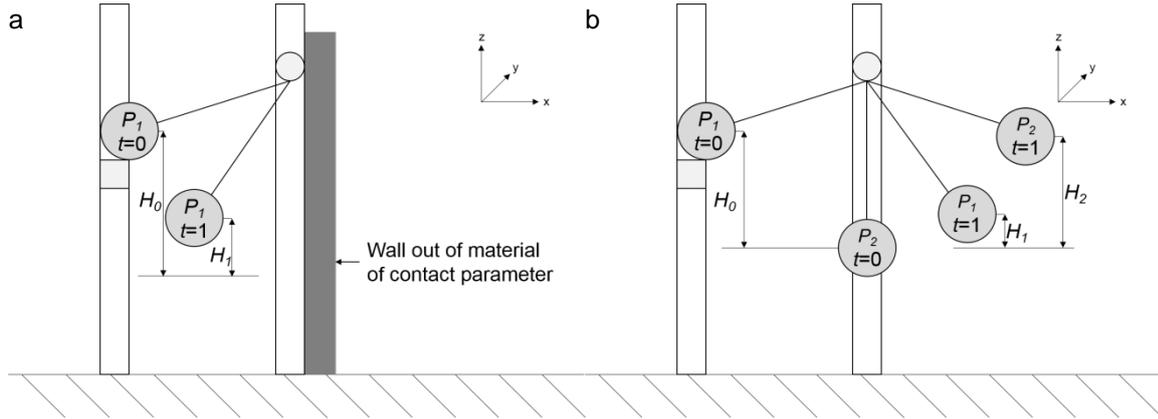
231 **Table 3:** Coefficient of rolling friction.

Contact partner	POM 5 mm	POM 7 mm	POM 10 mm
	$\mu_{roll} [10^{-5} \text{ m}]$	$\mu_{roll} [10^{-5} \text{ m}]$	$\mu_{roll} [10^{-5} \text{ m}]$
Acryl (side walls)	$5.44 \pm 1.1$	$7.76 \pm 2.7$	$6.41 \pm 3.3$
Metal (screen wire)	$7.22 \pm 2.2$	$8.13 \pm 3.6$	$7.07 \pm 3.7$
POM (particles)	$5.94 \pm 1.4$	$8.63 \pm 1.6$	$7.76 \pm 3.1$

232

### 233 3.5 Coefficient of restitution

234 In order to determine the coefficient of restitution, a particle which is on the end of a pendulum  
 235 is dropped so that it bounces against a wall (comp. Fig. 6a) or another particle which has a  
 236 velocity of  $v_2 = 0$  before the collision according to Alonso-Marroquín et al. [14] (comp. Fig. 6b).  
 237 For the particle wall collisions this experimental setup was chosen instead of a drop test to  
 238 have the same external effects in both experiments. Note that the procedure in Fig. 6b is only  
 239 applied for POM spheres, whereas for quartz gravel particles the setup in Fig. 6a is used with  
 240 a wall of the same material due to their arbitrary shapes, resulting in rotations or skewed  
 241 rebounds. Furthermore, it should be mentioned, that these experiments are more difficult if  
 242 smaller particles with a low mass compared to the mass of the thread are applied.



243 **Fig. 6:** Determination of the restitution coefficient for (a) particle-wall and (b) particle-particle contacts according to Alonso-  
 244 Marroquín et al. [14].

245 For the particle-wall contact, the velocities before ( $u_1$ ) and after the rebound ( $v_1$ ) are measured.  
 246 Alternatively, the heights before the particle drop ( $H_0$ ) and at the highest point after the rebound  
 247 ( $H_1$ ) could be measured. Thus, the particle-wall restitution coefficient is obtained by

$$e_{pW}^n = -v_1/u_1 = \sqrt{H_1/H_0}. \quad (11)$$

248 The restitution coefficient for a particle-particle contact is determined by

$$e_{pp}^n = -(v_1 - v_2)/u_1 = (\sqrt{H_2} - \sqrt{H_1})/\sqrt{H_0}, \quad (12)$$

249 where  $u_1$  and  $v_1$  are the velocities and  $H_0$  and  $H_1$  the heights of particle  $P_1$  before and after the  
 250 collision, respectively. Furthermore,  $v_2$  is the velocity and  $H_2$  the height of particle  $P_2$  after the  
 251 collision with particle  $P_1$ .

252 Results for the coefficient of restitution between particles and the same material (each size of  
 253 POM sphere with itself and the other sizes and quartz gravel with a gravel plate) for the particle-  
 254 particle contact and between particles and the wall materials (acryl, metal) for the particle-wall  
 255 contacts are listed in Table 4.

256 **Table 4:** Coefficient of restitution.

Contact partner		POM 5 mm	POM 7 mm	POM 10 mm	Contact partner	Gravel
Acryl (side walls)	$e_{pW}^n$	0.88 ± 0.01	0.76 ± 0.01	0.81 ± 0.05	Acryl (side walls)	$e_{pW}^n$ 0.81 ± 0.01
Metal (screen wire)	$e_{pW}^n$	0.80 ± 0.01	0.83 ± 0.01	0.74 ± 0.04	Metal (screen wire)	$e_{pW}^n$ 0.48 ± 0.02
POM (5 mm particle)	$e_{pp}^n$	0.84 ± 0.02	0.81 ± 0.04	0.85 ± 0.03	Quartz gravel plate (gravel particle)	$e_{pp}^n$ 0.77 ± 0.01
POM (7 mm particle)	$e_{pp}^n$	0.81 ± 0.04	0.82 ± 0.02	0.86 ± 0.03		
POM (10 mm particle)	$e_{pp}^n$	0.85 ± 0.03	0.86 ± 0.03	0.87 ± 0.03		

257

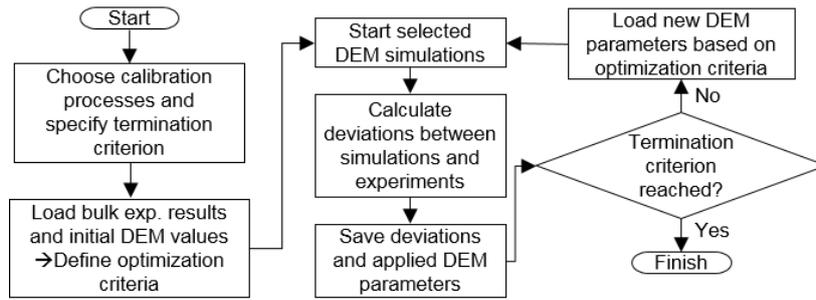
#### 258 4. Adjustment of DEM parameters of particles as bulk material

259 After the calibration at the particle level in section 3 the parameters have to be adjusted based  
 260 on the bulk behavior due to two reasons. Firstly, it could be possible, that due to particle size

261 or shape it is not possible to perform an accurate calibration at the single particle level where  
262 assumptions have to be made. A second aspect is that parameter sets obtained at the single  
263 particle level and at the bulk level differ from each other. This is a result of inaccuracies related  
264 to the single particle measurements for which it is compensated for in the bulk calibration or of  
265 the models (e.g. insufficient approximation of shape or of contact models). In a first step before  
266 the adjustment, three different bulk experiments with several settings are conducted. They are  
267 later compared with respective simulations to test the DEM parameters obtained by the single  
268 particle experiments before adjusting the DEM parameters to minimize the differences  
269 between the results of simulations and experiments. In the bulk experiments the static and  
270 dynamic angles of repose which are mainly influenced by the friction coefficients between  
271 particles (static) and particles and walls (dynamic) are measured. Furthermore, the bed height  
272 on a vibrating plate is determined, where the restitution coefficient is the crucial parameter.  
273 Note that in all investigations with POM spheres the same mass for each fraction and for gravel  
274 the particle size distribution from Fig. 2 is applied. In all investigations the appliances are filled  
275 according to a defined filling degree or level. Due to different bulk densities in case of POM  
276 spheres, the resulting average particle mass varies slightly dependent on the applied particle  
277 size classes.

#### 278 4.1 Adjustment of DEM parameters by a genetic algorithm

279 For obtaining the best fit between simulations and experiments for all investigated bulk  
280 experiments, an optimization tool based on a genetic algorithm [37] like the one used for the  
281 shape approximation in section 3.2 is used. The whole adjustment procedure for one particle  
282 distribution and an arbitrary number of bulk calibration processes is outlined in Fig. 7. Therein,  
283 the termination criterion can be a defined number of generations or a specified change in  
284 deviations between two generations. The algorithm is fed with the results shown in Fig. 9,  
285 Table 7 and Table 9 and the initial DEM parameters listed in Table 1, Table 2 and Table 4. The  
286 DEM parameters are changed in defined physical boundaries, supporting a quick convergence  
287 of the optimization. As DEM simulations are dependent on input parameters in a complex way  
288 the generations required for convergence are varying. It depends on the quality of the initial  
289 values, their difference to the values after optimization and if the initial values lead to consistent  
290 results. At the most 100 generations were enough to get good results which were not further  
291 improved in consecutive generations; often 50 generations were sufficient. Due to calculating  
292 up to 100 generations for each adjustment, one main criterion is the time needed for one  
293 simulation. Thus, the dimensions of the bulk tests are chosen as small as possible while still  
294 obtaining suitable results.



295 **Fig. 7:** Algorithm to adjust the DEM parameters for the best fit between simulations and experiments.

296 In case of the gravel particles the procedure is performed one time for all size classes due to  
 297 the simplification of using the same friction and restitution coefficients for each particle. In case  
 298 of the POM spheres with three discrete size classes, one possible method would be to directly  
 299 fit the DEM parameters for all size classes. However, in this investigation, all monodisperse  
 300 cases are fitted before the bidisperse cases to reduce the amount of adjustable parameters in  
 301 one adjustment procedure and thereby, to enhance the quality of the optimization. The  
 302 obtained parameters are then applied for the case with three different particle size classes.

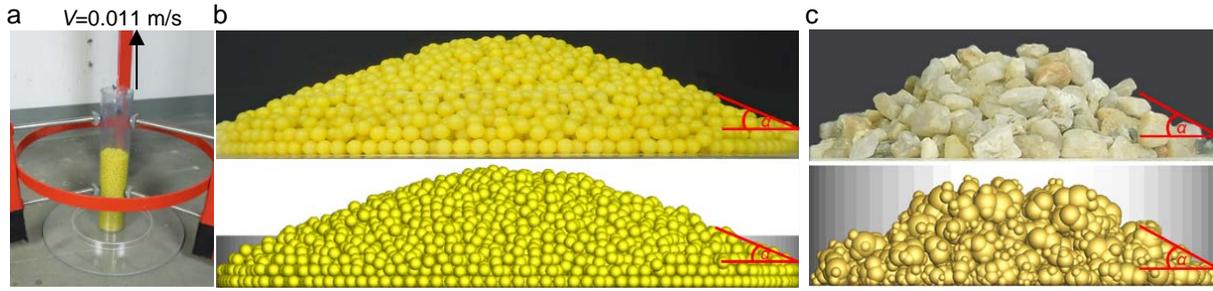
### 303 4.2 Static angle of repose

304 Static angle of repose measurements were conducted in a slump test by releasing POM  
 305 spheres and gravel particles contained in a hollow acrylic cylinder onto an acrylic and a steel  
 306 surface (Fig. 8a). To prevent excessive spreading of spheres, an acrylic ring as boundary is  
 307 used for both materials. The static angle of repose  $\alpha_{sta}$  is measured after reaching a steady  
 308 state. The experimental and simulative properties (Fig. 8b,c) are listed in Table 5.

309 **Table 5:** Experimental and simulative properties for measuring the static angle of repose.

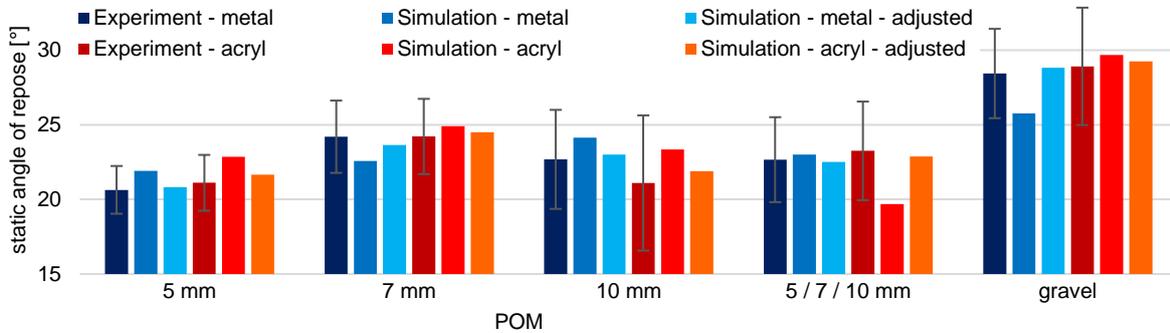
Properties	POM	Gravel
Bed height [m]	0.065	0.040
Cylinder velocity [m/s]	0.011	0.011
Cylinder diameter [m]	0.070	0.035
Ring diameter [m]	0.125	0.072
Particle mass [kg]	0.195-0.221	0.060

310  
 311 For the simulations, DEM parameters obtained in section 3 are applied in the first step of the  
 312 adjustment. Therein, the angle of repose is mainly influenced by the rolling (in case of spheres)  
 313 and sliding friction coefficient between particles. This simulation is time-determining for the  
 314 whole adjustment procedure, due to needing the longest time for reaching the steady state.  
 315 Therefore, it should be calculated with a larger number of processors  $n_{proc}$  as used for the other  
 316 simulations ( $n_{proc}/2$  for the tumbling and approx.  $n_{proc}/4$  for the vibrating plate simulation). Note,  
 317 that the DEM code used is parallelized using domain decomposition.



318 **Fig. 8:** (a) Experimental set-up to measure the static angle of repose and (b) resulting piles of 5mm POM spheres as well as (c)  
 319 piles of gravel in the experiments (top) and the simulations (bottom).

320 Most of the results for POM spheres and for gravel particles applying the initial DEM  
 321 parameters reveal a good agreement between simulations and experiments with deviations  
 322 below the standard deviations (comp. Fig. 9). In contrast, the simulations with 7 mm and 10 mm  
 323 POM spheres (not shown in Fig. 9) and with 3 size classes applying an acryl surface form out  
 324 too flat static angles of repose.



325 **Fig. 9:** Experimental results of measuring the static angle of repose  $\alpha_{sta}$  and comparison to simulations applying DEM parameters  
 326 determined in section 3 as well as after the adjustment.

327 By adjusting the DEM parameters (comp. Table 10), the deviations are reduced for all cases  
 328 but particularly for the polydisperse simulations where 7 mm and 10 mm spheres are in contact  
 329 (averagely from 9.59 % to 2.15 %).

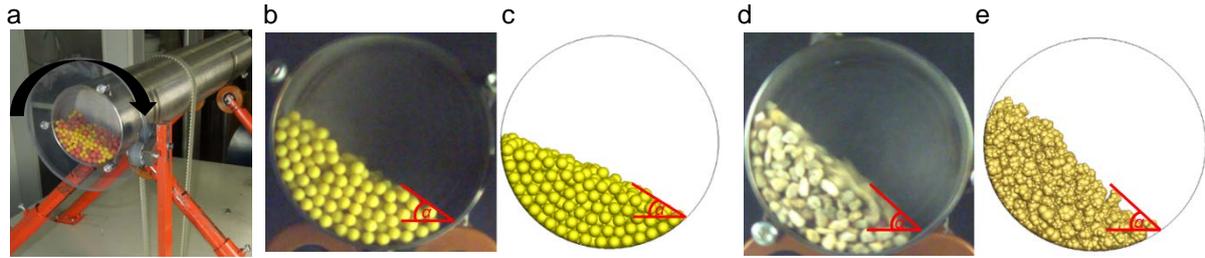
### 330 4.3 Dynamic angle of repose

331 For determining the dynamic angle of repose  $\alpha_{dyn}$ , tumbling tests with a hollow acrylic and a  
 332 metal cylinder (Fig. 10a) are conducted for POM spheres (Fig. 10b,c) and gravel particles (Fig.  
 333 10d,e) with three different velocities and 30 % filling (comp. Table 6). The dynamic angle of  
 334 repose was measured at 10 different points in time after a transient period of one second.

335 **Table 6:** Experimental and simulative properties for measuring the dynamic angle of repose.

Properties	POM	Gravel
Cylinder velocity [rpm]	10 / 15 / 20	
Cylinder diameter [m]	0.036	
Cylinder depth [m]	0.250	
Filling degree [%]	30	
Particle mass [kg]	0.061-0.069	0.130

336



337 **Fig. 10:** (a) Experimental set-up to measure the dynamic angle of repose as well as resulting piles of (b,c) 5mm POM spheres  
 338 and (d,e) gravel in (b,d) the experiments and (c,e) the simulations.

339 The results, which are mainly influenced by the friction coefficients between particles and walls  
 340 are presented in Table 7. All investigations for POM spheres and for gravel with the initial DEM  
 341 parameters for an acrylic cylinder obtained in section 3 reveal a much lower dynamic angle of  
 342 repose in the simulations than in the experiments (average deviations of 26.39 %). The  
 343 simulations with the metallic cylinder also reveal lower angles which are however closer to the  
 344 experimental ones (average deviations of 18 %). The low initial friction coefficient between  
 345 particles and acrylic walls leads to slip which is prevented to some extent by the larger  
 346 coefficient in case of metallic walls.

347 **Table 7:** Experimental results of measuring the dynamic angle of repose  $\alpha_{dyn}$  and comparison to simulations applying DEM  
 348 parameters determined in section 3 as well as after the adjustment.

Static angle of repose $\alpha_{dyn}$ [°]	POM				gravel
	5 mm	7 mm	10 mm	5 / 7 / 10 mm	3.15 - 5.6 mm
Experiment - metal - 10 rpm	28.26 ± 2.15	31.73 ± 4.06	23.26 ± 2.74	32.03 ± 3.76	40.81 ± 1.98
Simulation - metal - 10 rpm	25.39	23.92	20.65	23.76	39.49
Simulation - metal - 10 rpm - adjusted	29.79	32.92	23.15	30.64	41.49
Experiment - acryl - 10 rpm	33.40 ± 2.14	38.81 ± 1.34	32.25 ± 3.37	33.91 ± 2.61	42.82 ± 1.68
Simulation - acryl - 10 rpm	24.01	27.87	22.81	25.68	28.97
Simulation - acryl - 10 rpm - adjusted	33.01	37.87	29.81	32.25	42.54
Experiment - metal - 15 rpm	29.80 ± 1.73	32.16 ± 4.32	24.70 ± 4.74	31.31 ± 3.51	40.99 ± 1.28
Simulation - metal - 15 rpm	25.06	24.58	22.8	24.93	38.96
Simulation - metal - 15 rpm - adjusted	29.49	33.91	25.49	31.17	41.16
Experiment - acryl - 15 rpm	32.82 ± 1.31	35.75 ± 3.88	32.59 ± 3.52	34.85 ± 2.53	42.95 ± 1.73
Simulation - acryl - 15 rpm	23.5	25.6	22.6	26.82	29.23
Simulation - acryl - 15 rpm - adjusted	32.5	35.35	29.68	33.58	43.89
Experiment - metal - 20 rpm	30.33 ± 2.31	29.87 ± 2.89	26.42 ± 2.55	32.58 ± 2.38	40.39 ± 1.61
Simulation - metal - 20 rpm	24.62	21.7	21.28	25.02	41.19
Simulation - metal - 20 rpm - adjusted	29.06	30.7	24.78	32.39	41.59
Experiment - acryl - 20 rpm	33.93 ± 2.77	35.29 ± 2.56	33.72 ± 2.71	34.29 ± 2.33	42.57 ± 1.87
Simulation - acryl - 20 rpm	23.3	25.38	24.99	25.8	29.44
Simulation - acryl - 20 rpm - adjusted	32.3	34.38	31.99	32.56	43.13

349  
 350 After adjusting the DEM parameters (comp. Table 10) and particularly by increasing the friction  
 351 coefficient between particles and walls, the deviations of the dynamic angle of repose are all  
 352 minimized significantly and below the standard deviation (average deviations of 3.18 %).

353 4.4 Bed heights on a vibrating plate

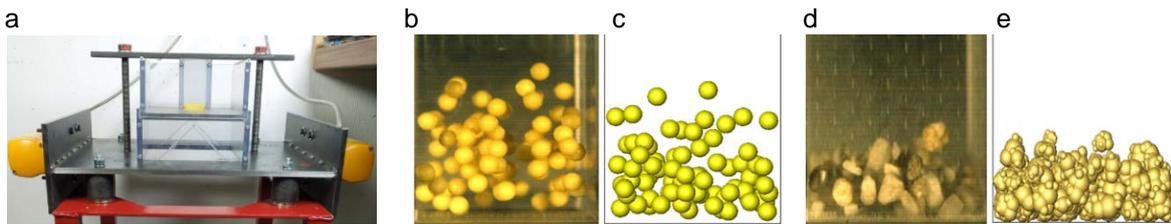
354 In order to obtain the dynamic bed height, tests on a vibrating metal plate with three different  
 355 degrees of filling are conducted (comp. Fig. 11; here with one particle layer of 5 mm spheres  
 356 or 5 % filling) for POM spheres (Fig. 11b,c) and gravel particles (Fig. 11d,e). The average  
 357 maximum bed height (referred to as “Top”) and the average distance between the lowest  
 358 particles and the bottom plate (referred to as “Bottom”) at different points in time are measured  
 359 using the properties listed in Table 8. Therein, the vibration parameters are obtained by an  
 360 accelerometer which measures an amplitude of around  $1.2 \pm 0.04$  mm in z-direction and only  
 361 minor amplitudes in the horizontal (x- / y-stroke < 0.1mm) at a frequency of 54 Hz. Note that  
 362 the bed height is analyzed after one second when a continuous motion of the plate is ensured.

363 **Table 8:** Experimental and simulative properties for measuring the dynamic bed height.

Properties	POM	Gravel
Length, width and height [m]	0.045 x 0.045 x 0.100	
Amplitude [mm]	1.2 ± 0.04	
Frequency [Hz]	54	
Stroke angle [°]	~90	
Filling degree [%]	1 Layer (5-10) / 20 / 50	
Particle mass [kg]	0.007-0.013 / 0.029- 0.033 / 0.076-0.083	0.013 / 0.065 / 0.162

364

365 The experimental results presented in Table 9, which are mainly influenced by the coefficient  
 366 of restitution, are compared to simulations applying the same properties (comp. Table 8). They  
 367 are initially obtained with the DEM parameters determined in section 3 and thereafter, with the  
 368 best fit of the adjustment.



369 **Fig. 11:** (a) Experimental set-up to measure the bed height on a vibrating plate as well as resulting particle beds for a filling of one  
 370 layer of (b,c) 5 mm POM spheres and (d,e) gravel in (b,d) the experiments and (c,e) the simulations.

371 Applying the initial DEM parameters, the results reveal some good agreements between  
 372 simulations and experiments but also deviations up to 17 % between the “Top” values in some  
 373 cases. The “Bottom” values are mostly lower in the simulations than in the experiments but the  
 374 absolute deviations are less than one respective particle in all cases. After the adjustment, the  
 375 deviations of the “Top” values are minimized significantly (all below 10 %), whereas the  
 376 “Bottom” values in the simulations are only slightly adjusted and still reveal a few deviations.

377

378  
379

**Table 9:** Experimental results of measuring the dynamic bed height on a vibrating plate and comparison to simulations applying DEM parameters determined in section 3 as well as after the adjustment.

Average bed height [mm]	POM				gravel
	5 mm	7 mm	10 mm	5 / 7 / 10 mm	3.15 - 5.6 mm
Experiment - 1 Layer - Top	41.40	35.70	35.50	49.90	20.70
Simulation - 1 Layer - Top	39.80	42.10	38.86	43.64	21.54
Simulation - 1 Layer - Top - adjusted	41.20	37.31	36.87	49.64	20.88
Experiment - 1 Layer - Bottom	3.81	3.99	6.26	8.07	2.74
Simulation - 1 Layer - Bottom	2.69	4.09	6.83	3.58	1.14
Simulation - 1 Layer - Bottom - adjusted	4.20	4.05	6.45	4.62	1.97
Experiment - 20 % - Top	35.90	37.10	46.00	40.90	29.00
Simulation - 20 % - Top	30.85	36.55	46.46	34.12	26.40
Simulation - 20 % - Top - adjusted	35.77	37.29	46.44	40.38	27.50
Experiment - 20 % - Bottom	1.39	1.82	4.07	3.46	2.44
Simulation - 20 % - Bottom	0.74	1.24	2.50	0.87	0.80
Simulation - 20 % - Bottom - adjusted	1.35	1.34	2.69	1.21	0.95
Experiment - 50 % - Top	56.20	54.10	58.60	59.30	58.22
Simulation - 50 % - Top	53.15	53.69	54.41	54.53	59.97
Simulation - 50 % - Top - adjusted	55.90	54.4	57.80	57.24	59.00
Experiment - 50 % - Bottom	1.17	1.29	1.18	0.98	2.42
Simulation - 50 % - Bottom	0.81	0.92	0.73	0.64	1.20
Simulation - 50 % - Bottom - adjusted	1.90	2.10	2.50	1.93	1.23

380

### 381 5. Application of the adjusted DEM parameters for batch-screening

382 The DEM parameters after the adjustment of section 4 are listed in Table 10. Particularly, the  
 383 sliding friction coefficients between particles and walls have to be increased by an average  
 384 factor of 2.68 and 1.96 for acryl and metal, respectively. Additionally, the sliding friction  
 385 between particles is increased by an average factor of 1.27 whereas the rolling friction and the  
 386 restitution coefficients are adjusted in both directions. All these parameters are applied for  
 387 batch-screening of well mixed POM spheres with three different size classes and gravel  
 388 particles with the particle size distribution outlined in Fig. 2. The particle and experimental  
 389 properties as outlined in Table 1 and Table 11 are used in the simulations.

390 **Table 10:** DEM parameters after the adjustment for POM spheres and quartz gravel particles. For gravel the rolling friction is  
 391 neglected.

Contact partner 1	Contact partner 2	$\mu_c$	$\mu_{roll}$	$e^n$
5 mm POM	Acryl (side walls) / metal (screen wire)	0.6649 / 0.4074	4.30E-05 / 6.99E-05	0.8145 / 0.8412
7 mm POM		0.4415 / 0.3561	8.66E-05 / 6.01E-05	0.7531 / 0.8013
10 mm POM		0.5330 / 0.2816	5.16E-05 / 4.91E-05	0.8195 / 0.8055
Gravel particle		0.3710 / 0.5461	- / -	0.5334 / 0.3966
5 mm POM	5 mm POM / 7 mm POM / 10 mm POM	0.2395 / 0.3237 / 0.2935	3.83E-05 / 4.00E-05 / 3.30E-05	0.8676 / 0.8686 / 0.7701
7 mm POM	7 mm POM / 10 mm POM	0.4882 / 0.4673	3.89E-05 / 8.92E-05	0.7303 / 0.8209
10 mm POM	10 mm POM	0.4229	3.844E-05	0.7653
Gravel particle	Gravel particle	0.5381	-	0.8676

392

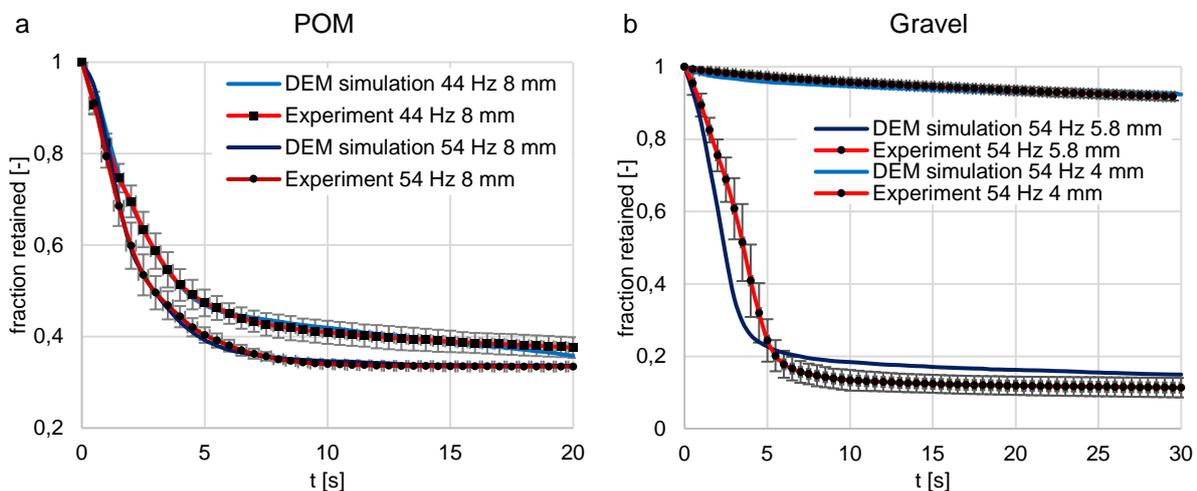
393 The vibration parameters are obtained like in section 4.4, but here the transient period of the  
 394 screen motion before  $t=0.61$  s is also measured and applied to the simulations as well as the  
 395 removal of the retaining plate below the apertures which lasts  $t=0.1 \pm 0.02$  s.

396 **Table 11:** Experimental properties for batch-screening experiments.

Properties	POM / gravel
Floor area [m]	0.185 x 0.185
Aperture size [mm]	$8.00 \pm 0.02$ / $5.80 \pm 0.01$ and $4.00 \pm 0.01$
Wire Diameter [mm]	$1.5 \pm 0.01$
Scree wire profile [-]	rectangular
Particle mass [kg]	1
Amplitude [mm]	$1.20 \pm 0.04$
Frequency [Hz]	44 and 54
Stroke angle [°]	$\sim 90$

397

398 Investigations for POM spheres are performed for two different frequencies and the results  
 399 averaged over a sufficient number of experiments (standard deviation represented by the  
 400 vertical error bars) and simulations reveal a good agreement with only a few deviations  
 401 between simulations and experiments (comp. Fig. 12a). The horizontal error bars represent  
 402 the response time of the balance. The average deviations are for 44 Hz 0.0164 and for 54 Hz  
 403 0.0083. In case of quartz gravel, a frequency of 54 Hz and two different mesh sizes are applied  
 404 resulting in a fast particle depletion for the aperture size of 5.8 mm and in a low passage rate  
 405 for the aperture size of 4 mm.



406 **Fig. 12:** Fraction retained on the screen over time applying (a) POM spheres for two different frequencies with a mesh size of  
 407 8 mm and (b) gravel for one frequency with mesh sizes of 5.8 mm and 4 mm all obtained by experimental investigations (results  
 408 are averaged over 10 experiments) and DEM simulations, respectively.

409 The results for the latter also reveal low deviations (average: 0.0074). In contrast, the results  
 410 with an aperture size of 5.8 mm reveal larger deviations (average: 0.0554) particularly in the  
 411 first five seconds, where the particles in the simulations pass faster than in the experiment.  
 412 Thereafter, the passage rate is reduced and too many particles remain on the screen. Reasons  
 413 for the deviations can be the rounded shape of the particles increasing the stratification and

414 passing possibility, the scaling in the particle size classes based on pixels of images or the low  
415 number of different shapes per size class used in the simulations. To overcome these  
416 discrepancies will be focused on in further investigations.

## 417 **6. Conclusions**

418 DEM parameters for POM spheres of three different size classes and quartz gravel particles  
419 with a realistic size distribution have first been determined by single particle tests and later  
420 applied to simulations representing three small bulk experiments. The static and dynamic angle  
421 of repose as well as the dynamic bed height are measured and particularly, the measurement  
422 of the dynamic angle of repose reveals large deviations between simulations and experiments.  
423 Motivated to reduce these discrepancies, the DEM parameters are automatically adjusted with  
424 an optimization tool, whereby the deviations between simulations and experiments are  
425 minimized significantly. The deviations averaged over all bulk tests are reduced from 9.59 %  
426 to 2.15 % for the static angle of repose, from 22.19 % to 3.18 % for the dynamic angle of  
427 repose and from 7.25 % to 4.15 % for the dynamic bed height (only the “Top” value). The  
428 adjusted DEM parameters are then applied to batch-screening simulations and reveal a good  
429 agreement with respective experiments in case of POM spheres and for quartz gravel with a  
430 low passage rate. In contrast, in case of the quartz gravel particles applying a large aperture  
431 size, the simulation overpredicts the passage rate in the first five seconds but more particles  
432 remain on the screen afterwards which indicates that the screening simulation needs further  
433 improvement. Investigations with another less rounded particle shape are currently ongoing  
434 and more different shapes will be applied for each size class. Nevertheless, it could be shown  
435 that the results of single particle tests can only be used as initial values for DEM parameters  
436 which have to be further adjusted to minimize deviations between simulations and bulk  
437 experiments. After an appropriate automatic adjustment of the DEM parameters, the results of  
438 bulk experiments applying spherical and non-spherical particles and the results of more  
439 complex processes applying spheres are represented well. In further studies, the batch-  
440 screening will be integrated in the adjustment process to enhance the accuracy.

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