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E V Shilko[1,2], A V Dimaki[1], V L Popov[2,3]
[1]Laboratory of computer-aided design, Institute of Strength Physics and Materials Science SB RAS, Tomsk 634055, Russia
[2]Tomsk State University, Tomsk 634050, Russia
[3]Berlin University of Technology, Berlin 10623, Germany

E-mail: shilko@ispms.tsc.ru

Abstract. In the paper, we developed a macroscopic discrete element model of permeable fluid-saturated materials with solid skeleton characterized by viscoelastic rheological properties. The Biot's linear model of poroelasticity was used as a mathematical basis for describing the mechanical interrelation between the solid skeleton and interstitial fluid. Using this model, we numerically studied the dependences of the effective Young's modulus and strength of fluid-saturated viscoelastic materials on the loading rate, sample size and the mechanical parameters, which determine the relaxation time of the solid-phase skeleton and the time scale of redistribution of fluid in the pore space. We revealed two dimensionless control parameters that determine the dynamic values of the effective mechanical characteristics of the samples under compression loading. We obtained the general relations that describe the above-mentioned dependences in terms of the two proposed control parameters. These relations have a logistic nature and are described by sigmoid functions. The importance of the proposed empirical expressions is determined by the possibility of their application for predicting the mechanical response of fluid-saturated materials of different nature (bone tissue, rocks, porous materials with polymeric skeleton, including elastomers, etc.) under dynamic loading.

1. Introduction

The regularities of the nonlinear behavior of viscoelastic materials and the related non-stationarity of mechanical characteristics (namely, their dependence on the loading rate) are the issues in a wide range of analytical and computational studies as these features determine the peculiarities of functioning of critical elements of various technical and natural structures including biological ones [1]-[6]. A broad subgroup of materials of this class comprises materials that contain viscous soft matter phases as structural constituents. Such kind of materials is characterized by a high contrast of local mechanical properties. Well-known examples are porous fluid-saturated materials, such as rocks, bone, and soft tissues, other natural and synthesized biomedical and engineering materials.

Interstitial fluid can change the behavior of material dramatically. In particular, the mechanical interaction of solid-phase and liquid-phase components can determine a strongly pronounced non-linear dependence of the mechanical behavior and effective mechanical characteristics of the material on the loading rate even in a quasistatic region in which the material of the solid-phase skeleton is insensitive to strain rate [7]-[11]. Moreover, at higher loading rates the material response is determined by the ratio of the time of stress relaxation in the skeleton and the characteristic time of redistribution of interstitial fluid in the pore space.

The above said determines the relevance of conducting theoretical studies of crucial aspects of nonlinear dynamic behavior (including fracture) of fluid-saturated viscoelastic materials under complex loading conditions. A useful tool for carrying out these studies is computer modeling using the discrete element method (DEM) [12][13]. A key advantage of this particle-based numerical method is the ability to explicitly model fracture processes (including multiple fractures) on various scales from micro- to macroscopic. This makes DEM attractive, in particular, for studying contact interaction, friction and wear in viscoelastic contact pairs of different nature, including articular joints. In spite of the broad application of this numerical method, its classical formalism does not allow modelling viscoelastic materials with taking into account mutual influence of the processes of stress relaxation in the deformed solid skeleton and fluid redistribution in the pore space due to change of pore volume.

In this work, we developed a DEM-based coupled model, which makes possible computer simulation of the mechanical behavior of porous fluid-saturated materials with a viscoelastic skeleton. With the use of the developed numerical model, we obtained the general expressions, which describe the dependences of uniaxial compressive strength and effective Young's modulus of viscoelastic fluid-saturated materials on the two dimensionless parameters related to characteristic times of relaxation processes in solid and fluid constituents.

2. Method

The model of mechanical behavior of viscoelastic materials was implemented within the formalism of homogeneously deformable discrete elements. This representative of the DEM group is called the method of movable cellular automata (MCA) [14]. It is based on the following assumptions.

A simulated body or consolidated fragment of the medium is represented by an ensemble of linked (chemically bonded) equiaxial discrete elements. Elements interact through contact faces. The initial value of the contact area is determined by the size of discrete elements and their packaging. This assumption is in many respects similar to the joint flat model of element-element interaction in the latest implementations of DEM [15].

When describing the kinematics of an element, its shape is approximated by an equivalent sphere. This approximation is the most widely used in DEM and allows one to consider the forces of central and tangential interaction of elements as formally independent. This makes possible to use the simplified equations of element dynamics (Newton-Euler equations of motion).

Discrete elements are treated as deformable. Strains and stresses are assumed to be uniformly distributed in the volume of the element (averaged stresses and strains). Within the framework of this approximation, the values of averaged stresses in the volume of the element are calculated as the superposition of forces applied to different parts of the surface of the element. In other words, averaged stresses are expressed in terms of the element-element interaction forces [16]:

$$\overline{\sigma}^{i}_{\alpha\beta} = \frac{R_i S^0_{ij}}{\Omega^0_i} \sum_{j=1}^{N_i} \left[\sigma_{ij} \left(\vec{n}_{ij} \right)_\beta + \tau_{ij} \left(\vec{t}_{ij} \right)_\beta \right] \tag{1}$$

where i is element number, $\overline{\sigma}^{i}_{\alpha\beta}$ is component $\alpha\beta$ of averaged stress tensor, $\alpha,\beta = x,y,z$ (XYZ is the laboratory coordinate system), Ω^0_i is the initial volume of element, S^0_{ij} is initial value of an area of the contact face of the element i with the neighbour j, R_i is radius of equivalent sphere (semi-size of the element i), σ_{ij} and τ_{ij} are specific values of central and tangential forces of interaction of the elements i and j, $\left(\vec{n}_{ij}\right)_\alpha$ and $\left(\vec{t}_{ij}\right)_\alpha$ are the projections of unit normal and unit tangent vectors onto the α-axis, N_i is the number of interacting neighbors.

Invariants of the averaged stress tensor $\overline{\sigma}^{i}_{\alpha\beta}$ are used to calculate element-element central interaction force σ_{ij} and criterion of an interelement bond break (criterion of local fracture). The components of the averaged strain tensor $\overline{\varepsilon}^{i}_{\alpha\beta}$ are calculated in increments using the specified equation

of state of the simulated material and the calculated increments of averaged stresses [16].

In the present work, we performed the numerical implementation of a particular three-parameter model of viscoelastic materials. This well-known model is sometimes called the standard model of elastomers (figure 1). It is a superposition of Kelvin element (spring with elastic modulus E_K) and Maxwell element (spring with elastic modulus E_M, connected in series with viscosity damper η_M).

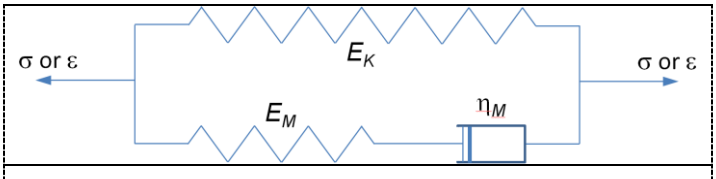

Figure 1. Schematic representation of a one-dimensional standard model of elastomers.

The 3D constitutive relations between stresses and strains in the considered model of a viscoelastic material are formulated in hypoelastic form:

$$\begin{cases} \dot{\sigma}_{\alpha\beta} = \left(2G_{\mathrm{K}} + 2G_{\mathrm{M}}\right)\dot{\varepsilon}_{\alpha\beta} + \left[1 - \dfrac{\left(2G_{\mathrm{K}} + 2G_{\mathrm{M}}\right)}{\left(K_{\mathrm{K}} + K_{\mathrm{M}}\right)}\right]\delta_{\alpha\beta}\dot{\sigma}_{\mathrm{mean}} - \dfrac{2G_{\mathrm{M}}}{\eta_{\mathrm{M}}}\left(\sigma_{\alpha\beta} - \sigma^{\mathrm{K}}_{\alpha\beta}\right) + \dfrac{2G_{\mathrm{M}}}{\eta_{\mathrm{M}}}\dfrac{K_{\mathrm{M}}}{\left(K_{\mathrm{K}} + K_{\mathrm{M}}\right)}\delta_{\alpha\beta}\sigma_{\mathrm{mean}} \\ \dot{\sigma}^{\mathrm{K}}_{\alpha\beta} = 2G_{\mathrm{K}}\dot{\varepsilon}_{\alpha\beta} + \dfrac{\left(K_{\mathrm{K}} - 2G_{\mathrm{K}}\right)}{\left(K_{\mathrm{K}} + K_{\mathrm{M}}\right)}\delta_{\alpha\beta}\dot{\sigma}_{\mathrm{mean}} \end{cases}, \quad (2)$$

where upper dot denotes time derivative of the variable, $\delta_{\alpha\beta}$ is Kronecker delta, total stresses $\sigma_{\alpha\beta}$ are a superposition of contributions of Kelvin and Maxwell elements: $\sigma_{\alpha\beta} = \sigma^{\mathrm{K}}_{\alpha\beta} + \sigma^{\mathrm{M}}_{\alpha\beta}$, σ_{mean} is mean stress. Note that volumetric stresses are associated with volumetric strains by a linear-elastic law (only shear stresses relax): $\dot{\sigma}_{\mathrm{mean}} = \left(K_{\mathrm{K}} + K_{\mathrm{M}}\right)\dot{\varepsilon}_{\mathrm{mean}} = K\dot{\varepsilon}_{\mathrm{mean}}$.

The authors previously showed that the relation for the force of central interaction of elements is formulated based on constitutive equation of the material for the diagonal components of the stress tensor, while the force of tangential interaction is formulated on the basis of similar equations for non-diagonal stresses [16]. When implementing the considered model of viscoelastic materials, the expressions for specific values of central and tangential forces of mechanical response of the element *i* to mechanical action from the neighbouring element *j* are written as follows:

$$\begin{cases} \Delta\sigma_{ij} = 2G_i\Delta\varepsilon_{ij} + D_i\Delta\bar{\sigma}^i_{\mathrm{mean}} - \dfrac{\Delta t}{T^i_{\mathrm{M}}}\left(\sigma_{ij} - \sigma^{\mathrm{K}}_{ij}\right) + \dfrac{\Delta t}{T^i_{\mathrm{M}}}N^i_{\mathrm{M}}\bar{\sigma}^i_{\mathrm{mean}} \\ \Delta\sigma^{\mathrm{K}}_{ij} = 2G^i_{\mathrm{K}}\Delta\varepsilon_{ij} + M_i\Delta\bar{\sigma}^i_{\mathrm{mean}} \\ \Delta\tau_{ij} = 2G_i\Delta\gamma_{ij} - \dfrac{\Delta t}{T^i_{\mathrm{M}}}\left(\tau_{ij} - \tau^{\mathrm{K}}_{ij}\right) \\ \Delta\tau^{\mathrm{K}}_{ij} = 2G^i_{\mathrm{K}}\Delta\gamma_{ij} \end{cases}, \quad (3)$$

where the symbol Δ means increment of the corresponding variable during time step Δt of the numerical scheme of integration of motion equations, total values of interaction forces are the sums of "Kelvin" and "Maxwell" contributions ($\sigma_{ij} = \sigma^{\mathrm{K}}_{ij} + \sigma^{\mathrm{M}}_{ij}$ and $\tau_{ij} = \tau^{\mathrm{K}}_{ij} + \tau^{\mathrm{M}}_{ij}$), $\Delta\varepsilon_{ij}$ and $\Delta\gamma_{ij}$ are increments of normal and shear strains of the element *i* in *i-j* pair, $2G_i = 2G^i_{\mathrm{K}} + 2G^i_{\mathrm{M}}$ is total shear modulus of the material of the element *i*, $K_i = K^i_{\mathrm{K}} + K^i_{\mathrm{M}}$ is total bulk modulus, $D_i = 1 - 2G_i/K_i$, $T^i_{\mathrm{M}} = \eta^i_{\mathrm{M}}/2G^i_{\mathrm{M}}$ is relaxation time of Maxwell element, $N^i_{\mathrm{M}} = K^i_{\mathrm{M}}/\left(K^i_{\mathrm{K}} + K^i_{\mathrm{M}}\right)$, $M_i = \left(K^i_{\mathrm{K}} - 2G^i_{\mathrm{K}}\right)/\left(K^i_{\mathrm{K}} + K^i_{\mathrm{M}}\right)$.

In view of the necessity of fulfilling the third Newton's law ($\sigma_{ij}=\sigma_{ji}$ and $\tau_{ij}=\tau_{ji}$), the increments of the reaction forces of the elements i and j are calculated based on the solution of the following system of equations:

$$\begin{cases} \Delta\sigma_{ij} = \Delta\sigma_{ji} \\ R_i\Delta\varepsilon_{ij} + R_j\Delta\varepsilon_{ji} = \Delta r_{ij} \\ \Delta\tau_{ij} = \Delta\tau_{ji} \\ R_i\Delta\gamma_{ij} + R_j\Delta\gamma_{ji} = \Delta l_{ij}^{\mathrm{sh}} \end{cases}, \tag{4}$$

where Δr_{ij} is the change in the distance between the centers of the automata for a time step Δt, $\Delta l_{ij}^{\mathrm{sh}}$ is the value of the relative shear displacement of the interacting automata i and j [16][17]. The system of equations (4) is solved for the increments of strains. This allows calculation of the increments of the specific interaction forces. When solving the system (4), the increments of mean stress and the values of specific forces in the right-hand sides of relations (3) are taken from the previous time step or are evaluated and further refined within the predictor-corrector scheme.

Discrete elements that model porous fluid-saturated material are described as porous and permeable. Pore space of such an element can be saturated with liquid. The characteristics of the pore space are taken into account implicitly through specified integral parameters, namely, the porosity ϕ, permeability k, the ratio of the macroscopic value of bulk modulus K to the bulk modulus of the walls of porous skeleton K_s. The mechanical influence of the pore fluid on the stresses and strains in the solid skeleton of an element is taken into account on the basis of the linear Biot's model of poroelasticity [18][19]. Within this model, the mechanical response of a "dry" discrete element is assumed to be viscoelastic and is described on the basis of the above-shown relations. The mechanical effect of the pore fluid on the element behavior is described in terms of the local pore pressure P_{pore} (fluid pore pressure in the volume of the element). In the Biot model, the pore pressure only affects the diagonal components of the stress tensor. Therefore, it is necessary to modify only the relations for the central interaction forces in (3):

$$\begin{cases} \Delta\sigma_{ij} = 2G_i\left(\Delta\varepsilon_{ij} - \dfrac{a_i\Delta P^i_{\mathrm{pore}}}{K_i}\right) + D_i\Delta\bar{\sigma}^i_{\mathrm{mean}} - \dfrac{\Delta t}{T^i_{\mathrm{M}}}\left(\sigma_{ij} - \sigma^{\mathrm{K}}_{ij}\right) + \dfrac{\Delta t}{T^i_{\mathrm{M}}}N^i_{\mathrm{M}}\bar{\sigma}^i_{\mathrm{mean}} \\ \Delta\sigma^{\mathrm{K}}_{ij} = 2G^i_{\mathrm{K}}\left(\Delta\varepsilon_{ij} - \dfrac{a_i\Delta P^i_{\mathrm{pore}}}{K_i}\right) + M_i\Delta\bar{\sigma}^i_{\mathrm{mean}} \end{cases}, \tag{5}$$

where $a=1-K/K_s$ is coefficient of poroelasticity.

Interstitial fluid is assumed to be linearly compressible. The value of fluid pore pressure in the volume of a discrete element is calculated on the basis of relationships of Biot's poroelasticity model with the use of the current value of pore volume [19][20]. The pore space of discrete elements is assumed to be interconnected and provides the possibility of redistribution (filtration) of interstitial fluid between the interacting elements. A pore pressure gradient is considered as the "driving force" of filtration. The redistribution of fluid between elements is carried out by numerical solution of the classical equation of transfer of fluid density [21]. This equation is solved by the finite volume method on an ensemble of discrete elements.

Local fracture is modeled by the method of discrete elements as the loss of cohesion/adhesion in a pair of elements (crack formation on the contact surface) [15]-[17]. In the developed model, the Drucker-Prager two-parameter criterion (a generalization of the von Mises criterion for a wide class of materials of different nature, including brittle solids) is used as a criterion for breaking a bond in a pair of automata i-j:

$$0.5\left(\lambda_{ij}+1\right)\sigma^{ij}_{\mathrm{eq}} + 1.5\left(\lambda_{ij}-1\right)\left(\sigma^{ij}_{\mathrm{mean}} + b_{ij}P^{ij}_{\mathrm{pore}}\right) = \sigma^{ij}_{\mathrm{c}}, \tag{6}$$

where $\lambda_{ij} = \sigma_c^{ij} / \sigma_t^{ij}$ is the ratio of uniaxial compression strength (σ_c^{ij}) and uniaxial tension strength (σ_t^{ij}) of the pair *i-j* in a "dry state", σ_{mean}^{ij} and σ_{eq}^{ij} are invariants of the stress tensor on the contact face of the pair (stress tensor $\sigma_{\alpha\beta}^{ij}$ is calculated by linear interpolation of the values of the components of tensors $\bar{\sigma}_{\alpha\beta}^{i}$ and $\bar{\sigma}_{\alpha\beta}^{j}$), $0 \le b_{ij} \le 1$ is the dimensionless coefficient of the influence of pore pressure on critical stress (typical values is b=1), P_{pore}^{ij} is pore pressure on the contact face (it is determined by linear interpolation of the pore pressures P_{pore}^{i} and P_{pore}^{j} in interacting elements).

The developed discrete-element model of viscoelastic materials was verified by performing a numerical analysis of the frequency dependence of the steady-state values of the real and imaginary Young's moduli of dry samples subjected to cyclic alternating loading. Comparison of simulation results with analytical estimates [22] confirmed the correctness of the developed model and the possibility of modeling with high precision the mechanical behavior of viscoelastic materials described by Kelvin and Maxwell models as well as by the standard model of elastomers.

3. Results and discussion

The developed model was applied to study the dynamic mechanical characteristics of porous fluid-saturated materials under uniaxial compression at a constant speed. We considered a model isotropic viscoelastic material with mechanical characteristics of the same order of magnitude as the compact bone tissue: E_K=1 GPa, $E=E_K+E_M$=11 GPa (E_K/E=0.1 what is a lower estimate for this class of materials), ν=0.3, η_M=10 MPa·s (lower-bound estimate for bone tissue), ρ=2500 kg/m^3, $\sigma_c \approx$20 MPa and $\sigma_t \approx$10 MPa (λ=2 what does not go beyond the bounds of the literature data interval for compact bone tissue), ϕ_0=10% (the lower limit of the interval of porosities of compact bone tissue), $a \approx$0.67 (poroelastic constant), b=1. We considered water as a reference fluid. Using mentioned values as a reference, we studied and analyzed the dependences of the dynamic compressive strength and the effective Young's modulus of viscoelastic fluid-saturated materials on the strain rate, the relaxation time of the solid skeleton and the characteristic time of redistribution of fluid in the pore space. In our calculations, the material parameters and the strain rate varied within wide limits: the permeability of the material varied within 4 orders of magnitude, the viscosity of the fluid varied within 2 orders of magnitude, the sample size changed within the order of magnitude and the strain rate varied within 3 orders of magnitude.

We simulated uniaxial compression of 3D cylindrical samples along the axis of the cylinder. The initial pore pressure of interstitial fluid was assumed to be atmospheric. Fluid could flow out freely from the compressed sample through the side surface.

Analysis of the simulation results showed that under compression, the values of the mechanical characteristics of fluid-saturated samples including the elastic modulus and strength are determined by the balance of two competing processes [23,24]:

- deformation of the solid skeleton, providing compression of the pore space and a corresponding increase in the pore pressure of the interstitial fluid;
- the outflow of interstitial fluid through the side surfaces, which leads to the inverse effect of lowering pore pressure.

We revealed two key control parameters that determine the specific dynamic value of the mechanical characteristics of fluid-saturated viscoelastic materials.

The first one is the pore fluid related dimensionless parameter. It is formulated by the analogy with dimensionless Darcy number:

$$Da = \frac{T_{Darcy}}{T_{load}} \sim \frac{\eta_{fl} L^2}{k \Delta P} \dot{\varepsilon}_{def}, \tag{7}$$

where T_{Darcy} is the characteristic time of fluid filtration (Darcy time scale), $T_{load} \sim 1/\dot{\varepsilon}_{def}$ is time scale of

deformation of the sample, $\dot{\varepsilon}_{\text{def}}$ is applied strain rate, η_{fl} is dynamic viscosity of pore fluid, L is the length of the segment with the pore pressure difference ΔP at the ends (radius of deformed cylinder in the considered case). The parameter Da characterizes the ratio of the timescales of deformation of the skeleton (and the fluid-filled pore space) and filtration of the pore fluid.

Note that in the process of deformation of a fluid-saturated sample, the pore pressure difference ΔP continuously changes from zero to some maximum value (at the time of fracture beginning). Therefore, when analyzing the results of the calculations, we used some constant value that characterizes the scale of the pore pressure difference in the sample. It is the difference between the Terzaghi estimate of the maximum pore pressure in the compressed material $P_{\text{pore}}^{\max} = \sigma_{\text{c}}\phi_0$ and the atmospheric pressure at the lateral sides of the sample $P_{\text{pore}}^{\min} = 10^5\,Pa$: $\Delta P = P_{\text{pore}}^{\max} - P_{\text{pore}}^{\min}$.

The second control parameter is the dimensionless parameter, which characterizes the ratio of the time scale of deformation to the stress relaxation time in the viscoelastic skeleton:

$$B = \frac{T_{\text{M}}}{T_{\text{def}}^{\text{elast}}} = T_{\text{M}}\left(\varepsilon_{\text{def}}^{\text{elast}} / \dot{\varepsilon}_{\text{def}}\right)^{-1} = T_{\text{M}}\dot{\varepsilon}_{\text{def}}\left(\sigma_{\text{c}} / \left(E_{\text{K}} + E_{\text{M}}\right)\right)^{-1}, \tag{8}$$

where $T_{\text{M}} = \eta_{\text{M}} / 2G_{\text{M}}$ is a characteristic time of stress relaxation in the skeleton, $T_{\text{def}}^{\text{elast}} = \varepsilon_{\text{def}}^{\text{elast}} / \dot{\varepsilon}_{\text{def}} = \left(\sigma_{\text{c}} / \left(E_{\text{K}} + E_{\text{M}}\right)\right)\left(\dot{\varepsilon}_{\text{def}}\right)^{-1}$ is the time of deformation of the sample of the virtual linear elastic material with the value of strength equal to the considered viscoelastic material and Young's modulus $E = E_{\text{K}} + E_{\text{M}}$. Note that parameter B is an indicator of the balance of two competing processes: linear elastic growth of stress deviators during deformation and relaxation of deviator stresses.

The determining effect of these two parameters can be vividly illustrated by two extreme cases: a fluid-filled viscoelastic material with an infinitely large relaxation time of the skeleton $T_M \to \infty$ (Kelvin material with $E = E_{\text{K}} + E_{\text{M}}$) and a "dry" material ($P_{\text{pore}}$=0) with a finite relaxation time of the solid skeleton.

The simulation results showed that the dynamic values of Young's modulus and the uniaxial compression strength (UCT) of fluid-saturated Kelvin material samples non-linearly depend on the strain rate $\dot{\varepsilon}_{def}$, the radius of the sample L, the dynamic viscosity of pore fluid η_{fl} and the permeability of solid skeleton k. In particular, Young's modulus of a fluid-saturated sample is minimal (equal to E) at infinitely small strain rates and tends to the maximum value (Young's modulus of undrained sample E_{u} [19]) at large ones. In turn, UCT of the sample at extremely small strain rate approaches the maximum value (strength of dry sample σ_{c}) and non-linearly decreases to some minimum value at large ones. The key result is the established ability to build a single "gauge" dependence applicable to samples of porous materials of various widths, characterized by different strength and permeability of solid skeleton, different pore fluid viscosities and deformed at different rates. An argument of such a "master curve" is the dimensionless Darcy number Da (figure 2):

$$\begin{cases} E = E_{\max} + \dfrac{E_{\min} - E_{\max}}{1 + \left(Da / Da_0\right)} \\ \sigma_{\text{UCT}} = \sigma_{\text{UCT,min}} + \dfrac{\sigma_{\text{UCT,max}} - \sigma_{\text{UCT,min}}}{1 + \left(Da / Da_0\right)^2} \end{cases}, \tag{9}$$

where $E_{\min}$ and $\sigma_{\text{UCT,max}}$ correspond to $Da \to 0$ (parameters for "dry" sample), $E_{\max}$ and $\sigma_{\text{UCT,min}}$ correspond to $Da \to \infty$ (undrained sample), Da_0 is normalizing constant. Both master curves (for Young's modulus and UCT) have a logistic (sigmoidal) form. Note that sigmoid dependence is typical

for the functions determined by the balance of competing processes.

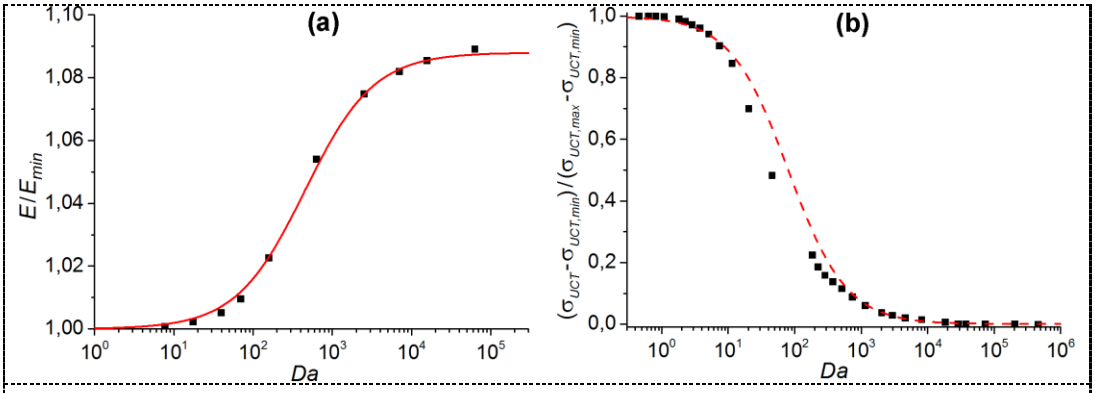

Figure 2. Dependences of Young's modulus (a) and uniaxial compression strength (b) of fluid-saturated samples of Kelvin material on the dimensionless Darcy number. Dots show numerically obtained values, lines are approximating sigmoids.

A similar study was carried out for "dry" samples of viscoelastic materials with finite relaxation times T_M. Simulation results showed that dependences of the value of Young's modulus on strain rate, dynamic viscosity η_M and compression strength σ_c of the skeleton and Young's moduli E_K and E_M can be brought together in the following sigmoid master curve (figure 3):

$$E = \left(E_K + E_M\right) - \frac{E_M}{1+B^2} \tag{10}$$

with introduced dimensionless parameter B as an argument. Here E is an effective (dynamic) secant modulus of the sample. It is derived from numerical experiments as a ratio of the strength of the sample to ultimate strain (at the moment of fracture). Note that UCT of the samples is a constant value (σ_c) until the range of high strain rates.

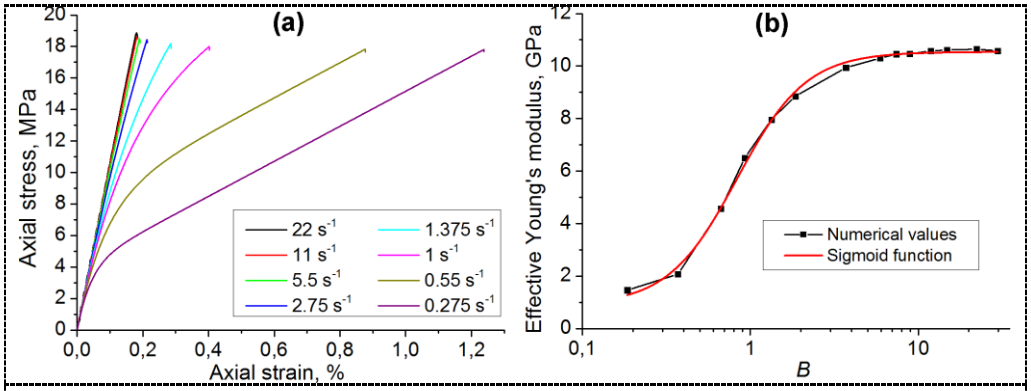

Figure 3. Typical diagrams of uniaxial compression of a "dry" sample of a model viscoelastic material at different strain rates (a) and the dependence of the secant elastic modulus of the sample on the dimensionless timescale parameter B (b). Dots show numerically obtained values, lines are approximating sigmoids.

One can see the generality of the relations for the effective elastic moduli of viscoelastic material (9) and a fluid-saturated linear elastic material (8). In both cases, relaxation processes strongly influence the deformation process by reducing the magnitude of local stresses. In the first case, they are concerned with the motion of defects and/or reconfiguration of the system of molecules in the

viscoelastic skeleton. In the second case, the stress relaxation mechanism is transport/redistribution of the pore fluid (outflow from the sample). The revealed generality suggests that the effective mechanical properties of fluid-saturated viscoelastic materials should also have a sigmoid dependence on the parameter, which is a combination of the dimensionless parameters *Da* and *B*.

The established co-directionality of the effects of the redistribution of pore fluid and stress relaxation in solid skeleton on the effective mechanical characteristics of viscoelastic permeable materials allowed us to obtain a generalized relation for the dynamic strength of fluid-saturated viscoelastic materials. Such a master curve has a logistic character, as in the limiting cases described above (figure 4):

$$\sigma_{\text{UCT}} = \sigma_{\text{UCT,min}} + \frac{\sigma_{\text{UCT,max}} - \sigma_{\text{UCT,min}}}{1 + \left(\dfrac{Da}{Da_0} B^{1/2} \right)^2}. \qquad (11)$$

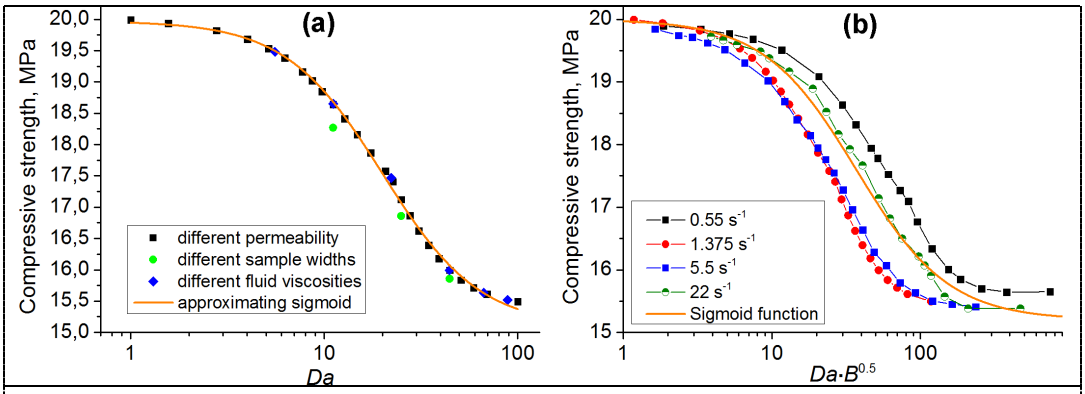

Figure 4. An example of the calibration dependence of the uniaxial compressive strength of the fluid-saturated sample of a model viscoelastic material on Darcy number at $\dot{\varepsilon}_{\text{def}}$ =1.375 s^{-1} (a) and strength dependences for various strain rates $\dot{\varepsilon}_{\text{def}}$ (b). Dots show numerically obtained values, lines are approximating sigmoids.

Similar to the first relation (8), the dependence of the secant Young's modulus of samples of a viscoelastic fluid-saturated sample on a combination $Da \cdot B$ also has a sigmoidal form.

4. Conclusions

The mechanical behavior of porous fluid-saturated materials with a viscoelastic skeleton is determined by the combined effect of two key factors: fluid redistribution in a deformed pore volume and stress relaxation in the walls of the porous skeleton. Both factors provide not only effective relaxation of local stresses but also an extremely non-linear (logistic) character of the dependence of the dynamic mechanical characteristics of such contrast materials on the loading parameters and mechanical characteristics of the constituents (fluid and solid phases).

The key result of the study is the established possibility of constructing generalized dependencies (master curves) of effective mechanical characteristics on the strain rate, relaxation times of the skeleton and redistribution of fluid in the pore space, as well as the size factor. In the paper, this was illustrated by the example of constructing such dependences for effective elastic moduli and dynamic compression strength. It is important to note that the argument of the obtained master curves is a dimensionless combination of dimensional parameters. This allows these master curves to be used to evaluate and predict the mechanical response of fluid-saturated materials of various nature (including bone tissue, polymer-based porous materials, primarily elastomers, and rocks) under dynamic loading. The “reference” mechanical characteristics of the solid-phase and liquid-phase components of the

contrast material can be used as input parameters for such an assessment.

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