

# The legacy of Coulomb and generalized laws of friction

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The simple formulation of “Coulomb’s law of friction” which can be found in most textbooks—the force of friction is proportional to the normal force and does not depend on the contact area and velocity—has little to do with the real work of Coulomb. On the contrary, Coulomb found that Amontons’ law, as well as the independence of the coefficient of friction on velocity, normal force, contact area and roughness are only a first, very rough approximation. He differentiated between material couples (e.g. metal-metal), where Amontons’ law is a good approximation, and other (wood on metal or wood on wood), where there are significant deviations from Amontons’ law. In all cases, however, the dependencies are relatively weak. In contemporary language, we would say they are of logarithmic character: the geometric and loading parameters have to be changed by several orders of magnitude to achieve a change in the coefficient of friction by a factor of two. Coulomb also gives simple two-term relations, which empirically summarize these experimental findings.

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## 1 Biographical note on Coulomb’s works on friction

Charles Augustin de Coulomb (1736-1806) is one of the central personalities in the history of Tribology. Following the first fundamental experimental work on dry friction by Amontons [1], he expanded our knowledge about external friction and laid solid foundation for research in this area up to our days. It is interesting to look at the biography of Coulomb to try to understand the roots of his influential work. Most of his work on friction Coulomb carried out in the years between 1773 and 1780.

The first scientific work of Coulomb was his memoir of 1773 “On an application of the rules of maximum and minimum to some static problems, relevant to architecture” [2] (for English translation see [3]). The research results presented in the memoir were obtained by Coulomb in Martinique, where he served in 1764-1772 and supervised the construction of Fort Bourbon. Coulomb recalled the time spent on Martinique: “For eight years I (almost always alone) was responsible for the construction of Fort Bourbon and for the work of 1,200 people, and at that time I often found myself in a situation where I found out to what extent theories based on hypotheses or experiments conducted in miniature in the cabinet de physique, turn out to be unsatisfactory benchmarks in practice. I have dedicated my life to all types of research that can be used in the work undertaken by officers of the engineering troops” [4].

In 1774, Coulomb was transferred to La Hougue, near the large port of Cherbourg (1774 - 1777) [4] and later (1777-1778) to Besancon, where he had a lot of free time. At this time, Coulomb presented a series of works for the Military Ministry.

In 1779, Coulomb was transferred to Marseille and in 1779 - 1780 to Rochefort, where he performed a series of experiments devoted to the study of external friction, the results of which were presented in the memoir “Theory of Simple Machines”, the fundamental work in the theory of friction [5].

1781 brought to an end the career of Coulomb as an engineer. From that time, he worked on physical problems (in particular made his famous discovery of the law of interaction of electrically charged bodies).

As shown in [6], this “sudden” change of his field of research can be logically explained by his previous works. Through the preceding studies of torsion of wires Coulomb had created a very sensitive device for measuring forces. He then only “reinterpreted” his set-up for studying the elastic properties of wires as a device for measuring forces. After this act of “exaptation”, Coulomb started measuring various forces, including electrical ones. A very famous portrait of Coulomb, reproduced in Fig. 1, shows Coulomb with the torsional balance. This portrait can be considered as a symbol of his life as it depicts the object unifying both his studies in mechanics of materials and electricity.

## 2 Coulomb and generalized laws of friction

Based on a wealth of experimental data, Coulomb confirmed the Amontons’ law, but at the same time showed its limitations. He systematically studied the dependencies of the coefficient of friction on normal force,  $F_N$ , sliding velocity,  $v$ , size of the contact,  $L$ , as well as atmospheric conditions and showed that the friction phenomenon is too complicated to be described by one single equation. However, in some limited ranges of external parameters he managed to describe friction with two-term-equations, of which the first term was a constant coefficient of friction  $\mu_0$  and the second described a relatively weak (often logarithmic) dependence on the parameter in question. Schematically, his approximations can be roughly reformulated in the

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**Fig. 1:** Portrait of Charles Augustin de Coulomb with his torsion balance (by Hippolyte Lecomte)

form

$$\mu = \mu_0 + a \ln F_N + b \ln v + c \ln L \dots \quad (1)$$

The existence of such a dependence means that the stationary coefficient of friction can be written as

$$\mu = \mu_0 + \ln(F_N^a v^b L^c) \quad (2)$$

thus depending only on one single variable of the form  $F_N^a v^b L^c$ .

Some of dependencies studied by Coulomb have been studied in subsequent years, particularly the velocity dependence [7]. The explicit dependence on time (kinetics of friction) was studied since the works of Dieterich [8][7]. The dependence of the coefficient of friction on the normal force is an area, which only begins to develop [9]. The dependence of the dry force of friction on the size of the system has not been studied systematically yet.

Coulomb has left us a work program that the tribological community still has not yet finished. Formulating effective theoretical and empirical procedures for constructing “generalized laws of friction” of the form 3, including the dependencies on the normal force and the shape remain hot topics in modern tribology.

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