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Ring Stiffened Cylindrical Shell Structures: State-of-the-Art Review

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Abstract: The cylindrical shell is a widely used structure in engineering practice, and its main form of failure is instability due to buckling. As a classical problem in the field of mechanics, the stability of cylindrical shells has been studied extensively. However, the large difference between the theoretically predicted results of the critical buckling load and the experimental results for the cylindrical shells subjected to uniform axial pressure has contributed to the continuous development of the shell stability theory. This paper briefly reviews the development of the shell stability theory, then presents an overview of the current status and trends of stability research on the stiffened cylindrical shell widely used in cylindrical shell structures in real engineering, and finally presents the difficulties and directions of future stability research on cylindrical shell structures in engineering applications.

Keywords: cylindrical shells; buckling; ring-stiffened; geometrical imperfection

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1. Introduction

Shell construction is one of the most efficient load-bearing structures, with the advantage of economizing on materials and reducing the weight of the structure. Cylindrical shells are used in many fields due to their special geometric configuration, including silos and tanks (see Figure 1) in civil engineering, where they are subjected to loads such as axial pressure, radial pressure, torsion, and combinations of these three types of loads. The dominant type of damage to cylindrical shells is buckling failure, which is generally sudden and often leads to the failure of the structure without visible symptoms or even catastrophic accidents, see Figure 2. Therefore, the buckling of cylindrical shells has long been a longstanding issue.

Typical cylindrical shell structures are made of (stainless) steel and achieve high economic efficiency through the optimal utilization of materials. Currently, the increasing cost of materials and the need for greater sustainability are prompting the manufacturers of cylindrical shells to rethink the previous concept and to constantly search for new types of shell structure construction in civil engineering. The application of orthotropic rotation shells, composed of thin shells with strong ring stiffeners, allows optimal matching to typical loads, such as wind. Ring stiffeners can contribute to a significant increase in resistance to wind and negative pressure. Research in the 1960s and 1970s showed that stiffeners also increased axial compression.

Recently, researchers [3] have re-evaluated published experiments on particularly thin-walled cylindrical shells. The results of parametric studies on closed-ring stiffened cylindrical shells confirm the high load-bearing capacity expected from published experiments. It has been proven that current standards are too conservative in some areas. In general, it is important to review the research history and development of ring-stiffened cylindrical shell structures and their application in steel structures for their further development and promotion, as well as for their wider application in practice.

(**a**) (**b**)

Figure 1. Cylindrical shells in Klaipėda (**a**) and Būtingė (**b**), Lithuania. [1].

Figure 2. Damaged silos after a storm, Marshalltown, Iowa, USA, 2020 [2].

2. Classic Shell Structures

Many researchers have conducted extensive research on the stability of cylindrical shells under axial compressions: a classic problem in mechanics. Due to the thin-walled nature of the silo structure, steel shells have become the central point of the silo structure. At the beginning of the twentieth century, researchers such as Lorenz [4] and Timoshenko [5] started to focus on the buckling of cylindrical shells and came up with theoretical methods. During the next 20 years, based on numerous experimental research, Flügge [6], Ballerstedt and Wagner [7], and Kanemitsu and Nojima [8] observed that the buckling load P_{lim} achieved in the laboratory was considerably lower than the elastic bifurcation load P_{Bif} calculated by the theoretical formulae, achieving less than one-quarter of the theoretical values. (Figure 3). In addition, the shell buckling loads measured by the tests were found to be significantly dispersed, even for identical or similar materials and geometric parameters. Further studies have shown that initial geometrical imperfections result in significant differences between the theoretical and experimental results. Defects in the shell also have a significant impact on the bearing capacity [9]. In practice, the maximum axial bearing capacity of cylindrical shells is almost impossible to calculate accurately on the basis of the shell theory, ignoring geometric imperfections.

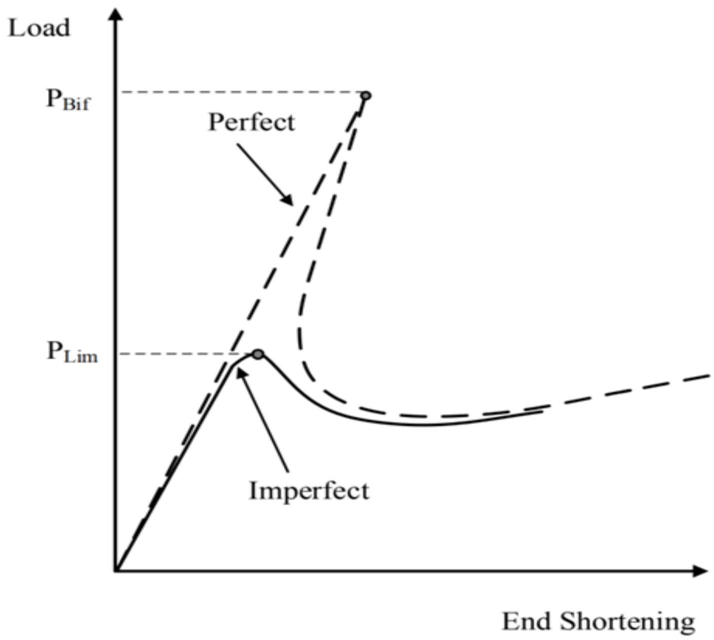

Figure 3. Equilibrium path of the cylindrical shell under axial pressure.

In recent decades, the stability of the shell has been studied from different perspectives. For structural design purposes, Rotter [10] summarized the relationship between initial geometric imperfections and the cylindrical shell manufacturing process, updating the design approach based on Odland's concepts [11] in which cylindrical shells were divided into different levels depending on the quality of the manufactured components. Teng [12] gave an exhaustive review of the studies carried out without shell buckling until the end of the twentieth century. Schmidt [13], as a researcher, teacher, and structural engineer, focused on the stability design of steel shell structures from a Western European perspective. Knödel and Ummenhofer [14] evaluated the load-bearing capacity of thin-walled shell structures based on the different ways of modeling imperfections. Arbocz and Starnes [15] analyzed the structural reliability of the cylindrical shell buckle from a database of measured geometrical imperfections. Schenk and Schüeller [16] studied the influence of random properties of geometric imperfections on the buckling load of thin-walled cylindrical shells under axial compression using the Monte Carlo simulation approach and predicted the variation in the buckling load through experimental means.

Recently, Knödel et al. [17] identified the shortcomings of existing Eurocodes for shell buckling after systematic analysis, including inadequate design rules and a limited debate of imperfections. Using an artificial neural network (ANN), Tahir et al. [18] analyzed the experimental data from the literature and estimated the buckling load of the cylindrical shells. By comparison, it is concluded that the spacecraft design recommendations [19] issued by NASA and Eurocode for EN 1993-1-6 steel shell structures [20] were extremely conservative. Wagner et al. [21] proposed an improved knock-down factor method for the shell design in spacecraft using probability analysis with the generation of random variables. They discussed the influence of manufacturing imperfections on the buckling load. They also analyzed the experimental results of shell specimens under axial pressure using reliability theory and considering geometric imperfections and compared these results with the design standards. They found that although the design standards had improved considerably, they were still conservative compared to the experimental values [22]. In addition, recent contributions to the vibration behavior of shell structures can be mentioned [23–25].

As mentioned above, cylindrical shells have a wide field of applications in many areas of engineering. The types of loads and working environments to which the cylindrical shell structures are subjected and the manufacturing processes vary considerably from one application to another. This has led to a variety of forms for initial imperfections in cylindrical shells. Therefore, it is necessary to analyze the stability of cylindrical shells in engineering applications according to the type of load, the working environment condition, and the main types of geometrical imperfections. Currently, ring-stiffened cylindrical shells are the typical shell constructions in steel structures.

3. Stiffened Shell Structures

Stiffened cylindrical shells are a common structural form in engineering practice and are commonly used in civil engineering and aerospace structures. However, the references to the ring-stiffened cylindrical shells under axial compression are by far not as comprehensive as those of the axially pressure-loaded isotropic cylinder. The purpose of stiffening is to improve the stability of a cylindrical shell, which not only increases its critical buckling load for the same weight but also reduces the sensitivity of the buckling load of the cylindrical shell to the initial geometrical imperfections [26,27]. The design of the stiffened shells is a very real and serious issue in practice. In some aerospace structures, obtaining a lightweight structure with a high load-carrying capacity is a key design consideration, so the case of stiffened cylindrical shells is used frequently. Recently, some civil engineers have noticed this material-saving design solution and have applied it to some real constructions. However, the mechanical and buckling analysis of the stiffened shells is theoretically a complex problem because of the coupling of the mechanical behavior between the main shell structure and the stiffeners. Therefore, in the study of stiffened shells, it is necessary first to simplify the structure into a calculation model that is easy to process.

3.1. Development and Progress of the Research in the Areas of Ring Stiffened Cylinder

The buckling of stiffened cylindrical shells is one of the main considerations in the design of stiffened shell structures. In the 1930s, Flügge [6] made the first findings on stiffened shells. One of the first findings was that the stiffener arrangement has an influence on the load-bearing behavior of the shell. Singer [28] presented a review of experimental and theoretical studies on the buckling of stiffened cylindrical shells. After that, Singer and Rosen et al. [29–31] investigated the influences of eccentric loads, geometrical imperfections, and boundary conditions on the buckling loads of stiffened cylindrical shells, respectively. An extensive compilation of critically selected tests was published in 1969 by Peterson in [32]. Since intensive research in the 1960s and 1970s, in the twentieth century, research almost came to a standstill, and only sporadic further research results were published that dealt with experiments around the topic of ring-strengthened cylinder shells under axial pressure. In this context, Grove and Didriksen [33] studied the buckling behavior of ring-stiffened cylindrical shells under axial pressure and transverse load at the upper edge of the specimens. Other publications, such as [34], highlighted the larger differences between welded test specimens and carefully machined cylinders. In the late 1980s and early 1990s, Seleim [35,36] investigated theoretically and experimentally the post-buckling of cylindrical ring-stiffened shells and analyzed the sensitivity of the shell buckling under external pressure. However, in many early papers, the parameters that determined the buckling behavior, such as geometrical imperfections associated with the ideal cylindrical shape and residual stresses, were often not adequately listed in experimental publications. The work by Baker [37] is one of the few publications that provide extensively documented measurements of the initial geometrical imperfections. They investigated the load-carrying capacity of eccentrically loaded cylindrical shells with variable ring stiffener spacing and then subsequently published the test results of the cylindrical shells with stiffeners. Das et al. [38] investigated the buckling behavior of stiffened cylindrical shells under combined loading based on reliability analysis. Li and Qiao [39] presented a post-buckling analysis of stiffened cylindrical shells under combined external and axial pressure loading.

Barlag [40] provides a comprehensive summary of the current research state and validates the cylindrical stiffened shell on the basis of various design codes. The results show that none of the current steel structure design codes specify cylinder loading with axial pressure and ring stiffness. A comparison of the test results using buckling reduction curves according to DIN 18800-4 shows that the provisions are significantly conservative with respect to the ring-stiffened cylindrical shell structures. Wirth [41] focused more marginally on stiffened ring shells when he performed tests on small-scale spiral-folded cylinder specimens under axial pressure and checked the suitability of Eurocode 3 for the design of the cylindrical shells. Jäger-Canás and Pasternak et al. [42] investigated the influence of ring-stiffeners on the bearing capacity of axially compressed cylindrical shells using a numerical approach. The potential material savings are suggested, and the beneficial influence of the ring stiffeners on the buckling load capacity is discussed. In his dissertation [3], Jäger-Canás proposed suggestions to address gaps in design standards. More recently, Li and Pasternak et al. [43–45] presented an investigation of cylindrical ring-stiffened shells based on axial buckling tests and numerical simulations. Several scaled-down welded cylindrical shell specimens with measured geometric imperfections by 3D laser scanning technologies were fabricated, tested, and analyzed based on efficient static analysis with accompanying eigenvalue analysis and high-fidelity numerical calculation (Figure 4).

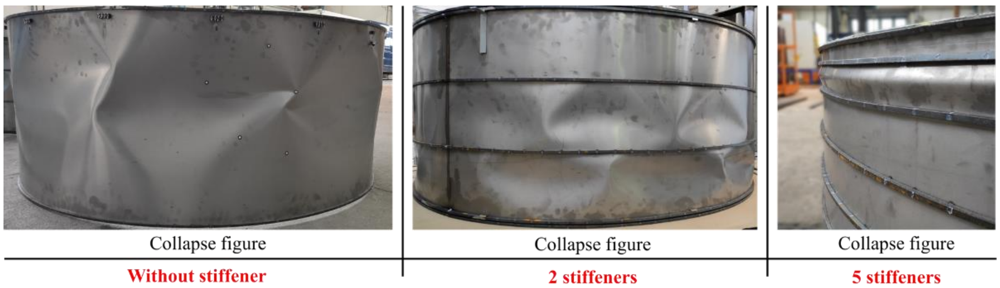

Figure 4. Collapse figure of ring-stiffened cylindrical shell structures with different stiffeners.

3.2. Mechanics of Ring Stiffened Cylinder and Numerical Approaches

Due to the high stresses caused by axial pressure, bending, and circumferential pressure, ring stiffeners were often used in combination with thick plates. This resulted in either the stiffener or the shell failure first considering their isolation, but also the interaction of both components had to be taken into account. Traditionally, there are three main types of simplified models for stiffened shell structures in stability research. The first is to consider the stiffened shell structure by simplifying it to an equivalent orthotropic anisotropic structure [46]. This model is more suitable for densely stiffened structures with equivalent spacing, and it is difficult to determine a consistently satisfied equivalent value of orthogonal anisotropy for sparsely stiffened structures where the distance between the stiffeners is possibly greater than the buckling wavelength. In addition, this model is also difficult to simplify for nonequally spaced, nonuniformly stiffened shell structural types. The second method, called the beam-column method, is based on the consideration of stiffeners with their adjacent structural parts of the shell as a slat model [47]. This method is also not applicable to the cases of sparsely stiffened and non-uniform stiffeners. The third model is called the finite strip method [48]. The basic idea of this approach is to decompose the stiffened structure into the main structure and the stiffeners for separate investigations, which are related by displacement conditions and force equilibrium conditions at the junctions. This model is widely used in numerical methods, such as the finite-strip method and the finite-difference method. However, when applying the finite bar method for the stability analysis of stiffened shell and plate structures, it is still difficult to treat the issue of transverse stiffeners very effectively.

Over the past few decades, researchers have also conducted a large number of experimental studies for ring-stiffened cylindrical shells. According to the experimental results of Singer, either local shell buckling, global instability, or an axisymmetric plastic collapse regularly turns out to be the cause of failure. As early as 1967, Singer described in his experimental results that depending on the geometry, axisymmetric, or checkerboard, buckling patterns can show up as the failure pattern. Even with local failure, the ring stiffeners have a recognizable stiffening effect, and close ring stiffeners constrain the shell specimens to buckle in an axisymmetric shape. Global and local failure can interact and reduce the achievable ultimate buckling load. Although experiments for medium-length cylindrical shells show a high scattering of results, this cannot be transferred to the test results on short-length cylinders. One reason for this is the typically axisymmetric failure shape, which was classified by Singer as not imperfection sensitive due to the stable post-buckling behaviors. The second reason may be due to the greater influence of the torsional restraint at the edges of the shell specimen. One of the focuses of attention has also been the post-buckling shape. According to Singer's results, the chequerboard buckling pattern, which is typical of unstiffened cylindrical shells, could also be observed in the post-buckling area of widely stiffened cylinders with torsional weak ring stiffeners. However, on the other hand, if the stiffeners have high torsional stiffness, it is likely that a local, axisymmetric failure pattern will occur. Then, it is obvious that global failure is typical for thin-walled ring-stiffened cylinders with weak ring stiffeners. A thicker cylindrical shell specimen usually leads to an axisymmetric buckling or ring buckling in tightly as well as arbitrary ring-stiffened cylinders. Additionally, the internal stiffeners favor the formation of checkerboard buckling, whereby positive eccentricity reduces the buckling load compared to the centrally stiffened cylinders. However, with outside stiffeners, eccentricity has almost no influence, as the cylindrical shell tends to fail with an axisymmetric buckling shape.

Based on experimental observations, Singer [28–31] developed the "smeared stiffener theory", that is, the smeared consideration of the stiffeners in a calculation model (Figure 5). The basis of this simplification is the assumption that the stiffeners are so close together that no local buckling can occur, and thus, global failure becomes the design criteria. Wang and Hsu [49] investigated the applicability of the "smear stiffener theory" for stiffened cylinders under internal pressure, constant temperature variation, and axial compression, as well as a proposed improvement to consider the shear transfer between the stiffener and the shell. They examined the interaction of shear and normal forces using representative shell sections with shells, rings, and longitudinal stiffeners. Jaunky et al. [50] proposed a modified theory of applied stiffeners for the calculation of cross-stiffened shells by determining the bending stiffness and the coupling stiffness of the stiffening ribs and the shell to account for the interaction between the stiffeners and the shell. Kidane et al. [51] determined the global buckling loads of cross-stiffened shells using the "smear stiffener theory" and based on the energy method to determine the bifurcation loads for selected stiffener configurations and compared them to experiments on composite cylinders. Hao et al. [52] attempted to improve the prediction of buckling loads using the "smear stiffener theory" based on their research results. Using a hybrid optimization algorithm, they showed the potential savings of the stiffened shells in terms of weight. Wang et al. [53] proposed a numerically-based "smear stiffener theory". A representative section was considered, and a numerical implementation of asymptotic homogenization was employed. In this way, they were able to determine the equivalent stiffness coefficient very precisely and to use the Rayleigh–Ritz method to determine the critical load of the stiffened shell structures. However, due to the complexity of the stiffened cylindrical shell structure, the simplified analytical model of the stiffened cylindrical shell still needs to be improved in the future.

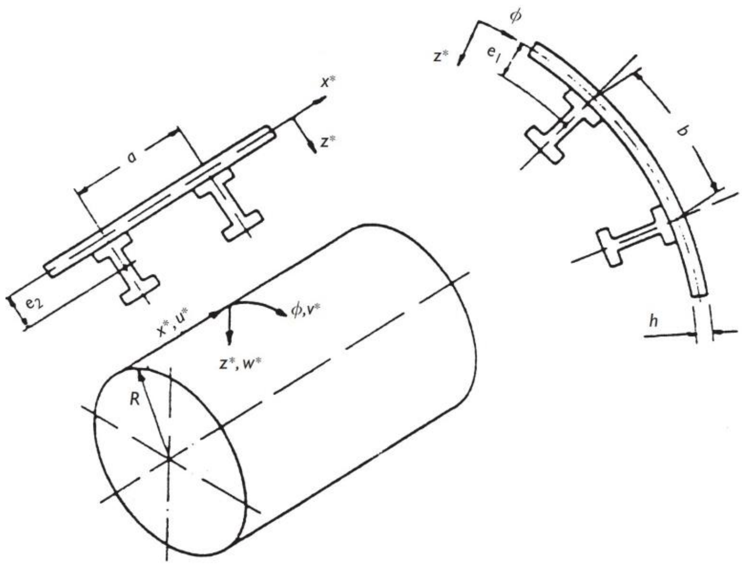

Figure 5. Schematic representation of the smeared stiffener theory from Singer [29].

With advances in computing and finite element theory, the numerical method can theoretically precisely analyze the complex nonlinear problems of shell stability by ignoring the geometrical complexity of the shell structures. On the basis of the recent efforts of researchers and engineers over a long period of time, the FEM program has become the most effective and popular tool for analyzing cylindrical shell buckling problems in recent years. A numerical method is an analytical tool aimed at researchers as well as engineers. They can use commercial finite element software, such as Abaqus® and Ansys® FE software, for structural design. However, some researchers are constantly striving for further improvements in numerical algorithms to solve specific problems. For analyzing the buckling of the stiffened cylindrical ring shell under axial compression, the time-dependent explicit dynamic method [54,55] has high robustness and is suitable for buckling and post-buckling analysis at the expense of time and computing efficiency [56]. Li and Pasternak et al. [43,57] attempted to determine whether the shell reached the bifurcation load using an iterative algorithm based on the accompanying eigenvalue method [58] in Abaqus®. This approach enables us to quickly obtain the ultimate load capacity of a stiffened cylindrical shell structure (Figure 6).

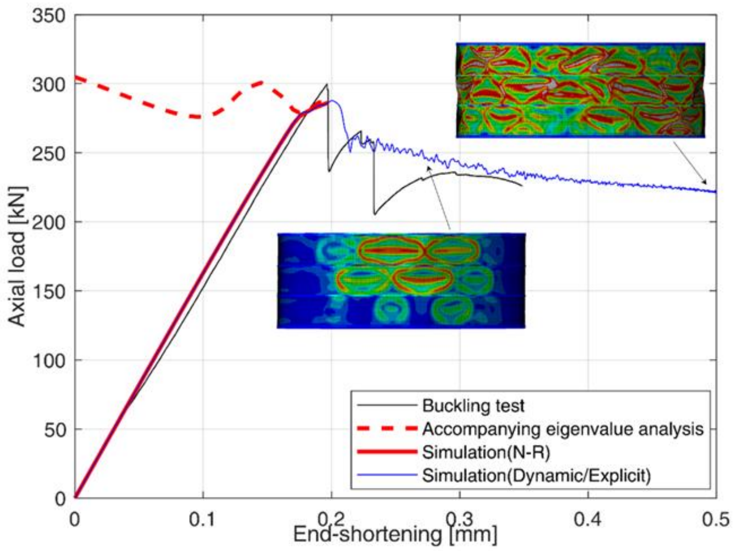

Figure 6. Load–displacement curves from finite element analysis and axial compression buckling test and collapse form with two ring-stiffeners.

3.3. Geometric Imperfection and Sensitivity of Ring Stiffened Cylindrical Shells

Some investigations show that the imperfection sensitivity of stiffened cylindrical shells is still clearly recognizable but probably much lower than that of unstiffened shells. Theoretically, the reduced sensitivity can be explained by the lack of clustering in the eigenvalues. For unstiffened cylinders, while many different possible buckling shapes can initiate failure at the identical load, ring stiffeners limit the possible failure modes. Moreover, more recent measurement studies [44,45] using 3D laser scanning technology have shown that the initial geometric imperfections of ring-stiffened cylindrical shells are related to the ring stiffener spacing. The results show that the sensitivity of the geometric imperfection is dependent on whether the ring spacing is in multiples of the half-wavelength of the axial buckling of the cylindrical shell. As research continues, the understanding of the stability of stiffened cylindrical shells is increasing among researchers and engineers, but due to the complexity of the structure of the stiffened cylindrical shell, the simplified analytical model for the stiffened cylindrical shell and the sensitivity to initial imperfections still need further improvement.

4. Construction Examples of Ring-Stiffened Shells

In practical constructions, the ring-stiffened tanks are typically used as agricultural and industrial storage tanks (e.g., by the Lipp company, Tannhausen, Germany), in the biogas industry, as shown in Figure 7, or as oil storage tanks in Figure 8. This type of very thin-walled tank or silo is usually assembled from steel plates and ring stiffeners. Various options for ring-stiffened shells are given in [42]. The most complex construction method must be applied to conventional construction welded tanks. Following the assembly of preformed steel plates, ring stiffeners must be welded on all around. In addition to high internal stresses, large pre-buckling is also observed in the meridian and circumferential directions (Figure 8). The construction process can be used in a very wide range since both thick and thin sheets with high strength can be used and combined as desired. From small pressure vessels to large tanks for refineries, many construction projects have been completed so far. For spiral-folded silos, as shown in Figure 9, the horizontal joints of steel shells are usually folded to form a ring stiffener. For the cylindrical welded shell structures in Figure 10, the application of the ring stiffeners is usually limited to a minimum distance, as the welding process is very time-consuming. In addition, external welds reduce the dimensional accuracy of the cylinder shape and introduce residual stresses. In the process used by some German companies such as Lipp to manufacture welding cylinders, the tanks are spirally wrapped from the steel plate and are automatically welded to the lower edge of the following section, as shown in Figure 10. During the wrapping process, the steel plate strip, whose thickness is between 2 and 6 mm, is folded at the upper edge. Therefore, a high-quality welded cylinder with ring stiffeners can be formed, and a high degree of geometric fidelity and constant weld quality can be achieved. However, residual stresses and manufacturing eccentricities that can have a negative impact remain unpreventable. Moreover, some manufacturers use bolted tanks, including stiffeners, as shown in Figure 11. The process, which is very complex in terms of assembly technology, is impressed with its high adaptability to the expected loads and enables a wide range of applications. This construction is particularly typical for bins used in agriculture. The ring stiffeners form edge profiles, which serve not only to stabilize the bin but also as a horizontal butt plate between two wall panels. They are also used as an intermediate support for the trapezoidal sheet cladding and are, therefore, a necessary construction element for the manufacture of the tank. As a result, the additional expense in terms of costs arises only from the production of the stiffeners, not from their assembly. All the procedures presented have a wide range of materials that can be used. Ordinary structural steels (suitable for cold forming) are used, as are pure stainless steels and galvanized steels, either plated or enameled with stainless steel.

Figure 7. Tank for biogas industry (Stallkamp company, Dinklage, Germany).

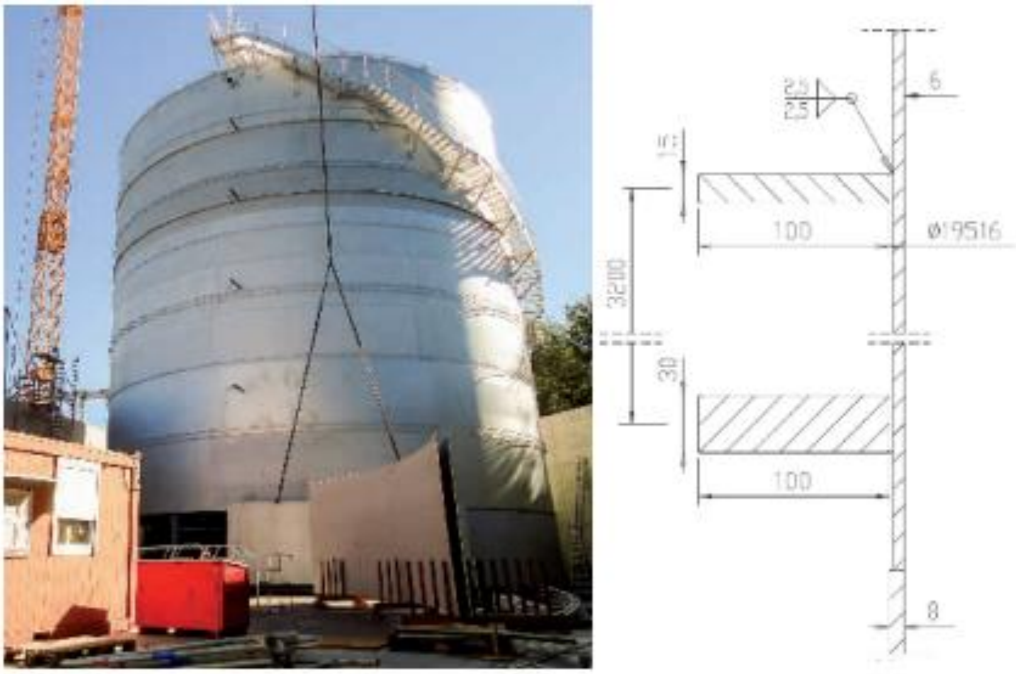

Figure 8. Tank for strong lye. Left: view. Right: cross section (Kremsmüller company, Steinhaus bei Wels, Austria).

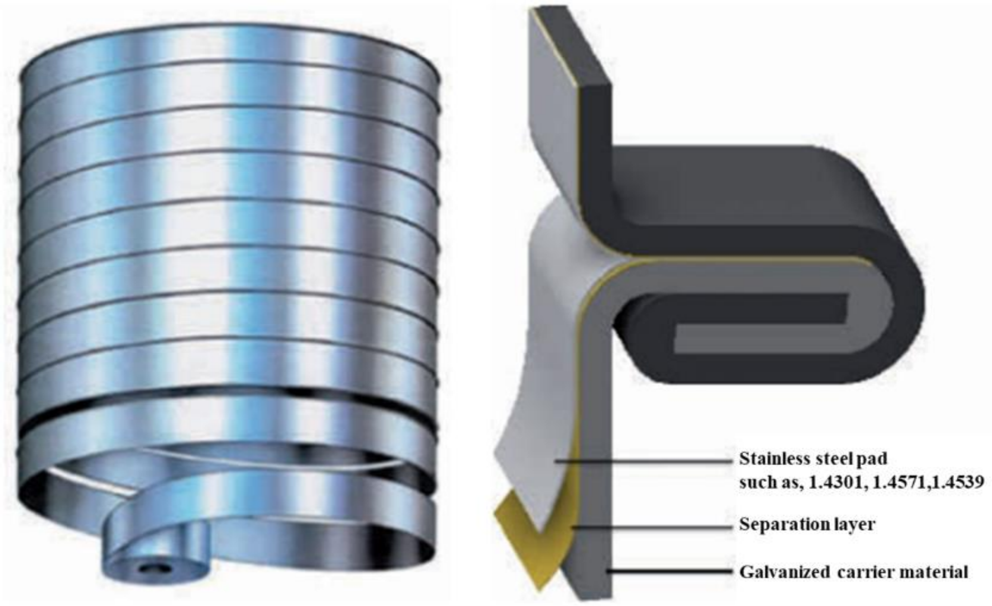

Figure 9. Double-fold tank system. Left: view. Right: cross section (Lipp company, Tannhausen, Germany).

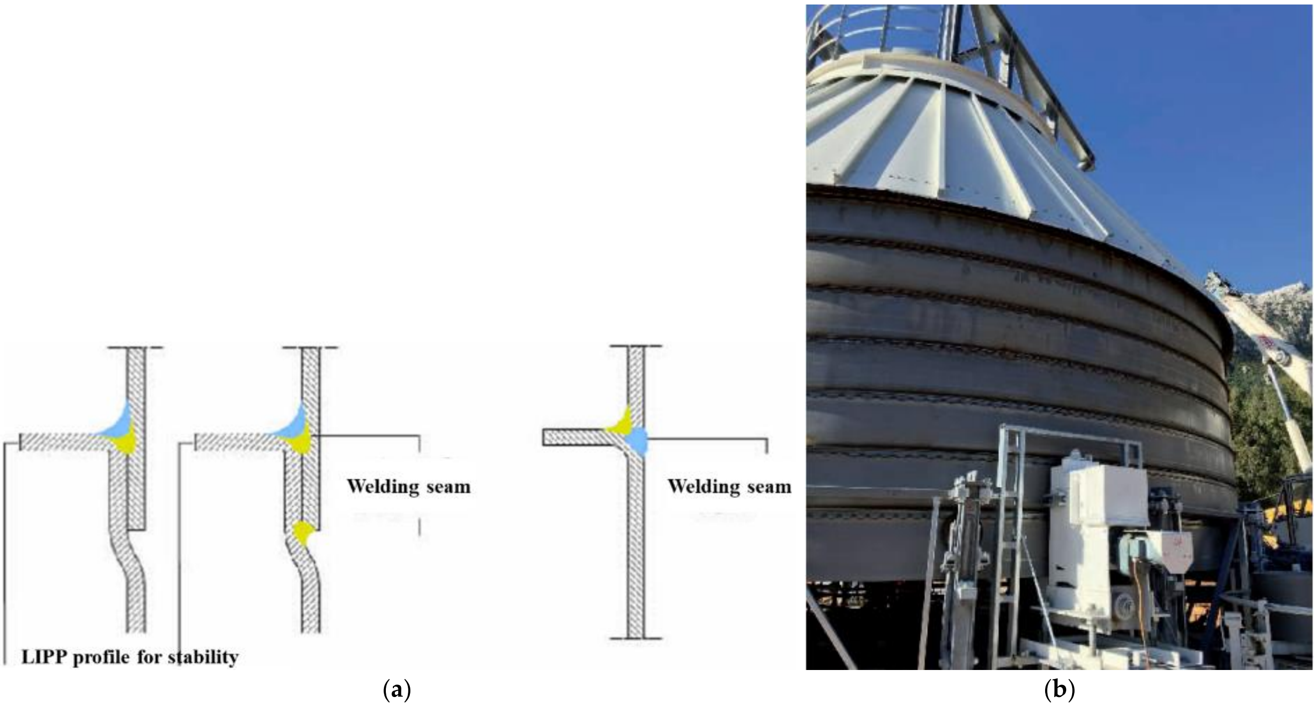

Figure 10. Welded tank with stiffeners: cross section of the tank wall (**a**), welded tank in construction stage (**b**) (Lipp company, Tannhausen, Germany).

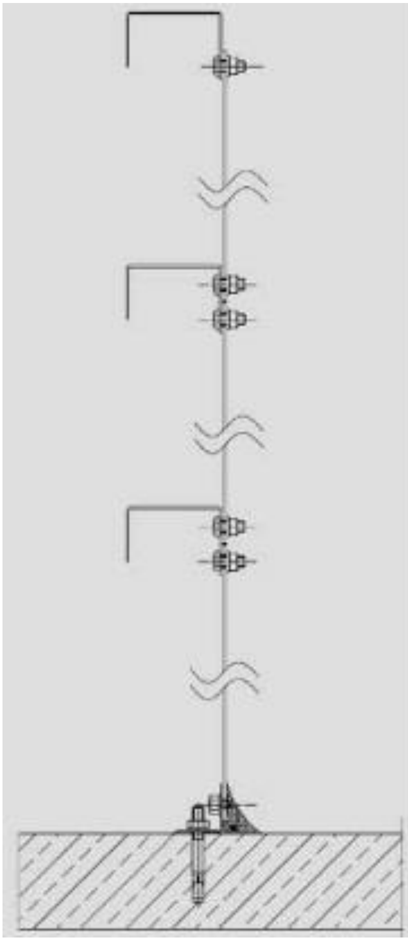

Figure 11. Construction principle of bolted tanks.

5. Conclusions

The buckling issue of cylindrical and stiffened shells, especially axially pressed cylinders, has contributed to the continuous development of the stability theory of shell structures. To explain the dispersion of axially compressed cylindrical shell experimental results and the large differences between them and the predictions of the classical linear buckling theory, the stability theory of shells has gradually evolved from the initial linear theory to a relatively well-developed nonlinear buckling and post-buckling theory. Nowadays, theoretical explanations for the sensitivity of geometrical imperfections and the post-buckling properties of the stiffened and unstiffened shell structure have been presented. With the continuous development of the shell buckling theory, the stability of the cylindrical shell with and without stiffeners has been solved from a general theoretical point of view. However, there are still the following issues in cylindrical shell structures, especially ring-stiffened shells, that need to be solved urgently:

(1) The strong dispersion of the ultimate load of the experimental results and the large differences between these results and the theoretical calculations are mainly due to the high sensitivity of the initial geometrical imperfections, whereas there are always unavoidable and unmeasurable differences between the practical shell structures and the theoretical model. Hence, an approach that can reasonably quantify geometric imperfections needs to be proposed.

(2) For stiffened cylindrical shell structures used in steel structures, the simplified analytical models employed for stability analysis still need to be improved, for example, for the ring-stiffened cylindrical shell. The application of the finite strip method and the smeared stiffener theory for buckling analysis is still not well addressed.

(3) Since ring-stiffened shells are already insensitive to geometric imperfections, a transfer of the method of unstiffened cylinders for ring-stiffened shell structures does not seem to be justified, and numerical simulations are needed to develop a design method for circumferentially variable meridional loads that are economical and safe.
(4) The investigations that have been carried out do not cover the circumferentially unevenly distributed axial load in ring-stiffened cylindrical shells. It may cause the local buckling of the ring-stiffened cylindrical shells.

Throughout the history of the research into cylindrical shells with and/or without ring stiffeners for more than a century, shell buckling has been established as a complex but also crucial task for structural safety. Accompanying analytical methods and the computer analysis of shell buckling problems has been a very important area of computational mechanics. Computer analysis is actually a new research paradigm for shell buckling. For further investigation of stiffened cylindrical shell structures, computer analysis is an essential research tool because "in fact, computer analysis is nowadays on the same footing as laboratory experiments, and computer packages play a role comparable to that of testing machines" [59].

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