Strategic Behaviour in Energy Markets

Advances in Complementarity and Multi-Stage Equilibrium Modelling

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Zusammenfassung

Diese Dissertation untersucht mehrere energiewirtschaftliche Fragestellungen, in denen Marktmacht eine besondere Rolle spielt; mittels spieltheoretischer Ansätze werden die strategischen Interaktionen mathematisch formuliert und anhand numerischer Methoden Gleichgewichte der Spiele identifiziert. Der Beitrag zum wissenschaftlichen Diskurs rund um den Themenbereich *Marktmacht in der Energiewirtschaft* liegt in der Entwicklung neuer Ansätze, strategisches Verhalten in partiellen Gleichgewichtsmodellen abzubilden, sowie in der Weiterentwicklung numerischer Methoden zu deren Lösung.

Der erste Teil dieser Arbeit befasst sich mit dem globalen Erdölmarkt und der Rolle der Organisation Erdöl-exportierender Länder (OPEC) im letzten Jahrzehnt. Ich formuliere ein *Stackelberg-Oligopol*, in dem die optimale Ausübung von Marktmacht durch die OPEC-Mitglieder davon abhängt, wie hoch die Kapazitätsauslastung der Nicht-OPEC-Produzenten ist. In einer numerischen Anwendung wird der tatsächliche Preisverlauf mit dem zweistufigen Modell besser abgebildet als mit Standard-Gleichgewichtsmodellen.

Das nächste Kapitel wendet sich dem internationalen Erdgasmarkt und – aus mathematischer Sicht – einem *mehrperiodigen Investitionsmodell* zu. Ich liefere einen Beweis, daß die Berücksichtigung von Investitionen in Erdgas-Marktgleichgewichtsmodellen ein konvexes Problem darstellt und daher mit Standard-Methoden gelöst werden kann.

Das folgende Kapitel entwickelt ein *partielles Energiesystem-Gleichgewichtsmodell*, in dem einzelne Produzenten Marktmacht über mehrere (fossile) Energieträger ausüben können. Dieser Ansatz erlaubt eine Verbindung von Energiesystemmodellen, die zur Analyse möglicher Entwicklungspfade des globalen Energiemixes verwendet werden, mit partiellen Sektormodellen für die Untersuchung von Marktmacht und detailliertem Infrastrukturausbau.

Der letzte Teil der Dissertation untersucht das Verhalten *national-strategischer Planer im Ausbau des (europäischen) Stromnetzes*: der Ausbau von Stromtrassen kann zu Wohlfahrtsverschiebungen zwischen verschiedenen Interessensgruppen (Konsumenten, Erzeuger, Netzbetreiber) sowie über Landesgrenzen führen. Durch das Fehlen effektiver Kompensationsmechanismen im europäischen Rahmen haben nationale Akteure unter Umständen Anreize, Netzausbau in ihrem Zuständigkeitsbereich zu reduzieren, um damit Renten zu ihren Interessensgruppen zu verschieben. Dies wird anhand eines stilisierten Beispiels illustriert.

Schlüsselwörter: Energiewirtschaft, Marktmacht, strategisches Verhalten, Investitionen, Infrastruktur, mehrstufiges Optimierungsmodell, allgemeines Nash-Gleichgewicht (GNE), Gleichgewichtsmodell unter Gleichgewichtsnebenbedingungen (EPEC)

Abstract

This dissertation combines three fields of economics: I take several topics from *energy economics*, use *game theory* as the framework to mathematically formulate strategic interaction between several players in these applications, and apply and further develop numerical methods from *Operations Research* to solve for equilibrium solutions of these games.

This work starts with a focus on the crude oil market and the role of the Organization of Petroleum Exporting Countries (OPEC) over the past decade. I propose a *Stackelberg oligopoly* to describe how market power exertion by OPEC members depends endogenously on the spare capacity of the competitive fringe (i.e., non-OPEC supply). In a numerical exercise, this two-stage model captures the crude oil price spike of 2008 better than standard, simultaneous-move equilibrium concepts widely used in applied work.

The following chapter turns from crude oil to *natural gas markets* and – mathematically – from a one-period quantity game to a multi-period investment model. I provide a proof that including production capacity investment decisions in large-scale partial-equilibrium models yields a convex problem, paving the way for an improvement of this widely used model class.

The next chapter develops a large-scale *energy system partial-equilibrium model*; it combines energy system models, which incorporate fuel substitution, and sector-specific partial-equilibrium models, which are used for market power analyses and consideration of infrastructure investments. The features of this model are illustrated using a large-scale data set and two scenarios: a reduction of shale gas potential in North America, and an ambitious EU policy initiative to reduce carbon dioxide emissions. Numerical results are discussed in terms of carbon leakage rates, trade flows, and fuel mix shifts.

The final chapter discusses investment in European *power transmission capacity*. Several zonal planners play a Nash game regarding their domestic network upgrades; each player anticipates the effect on the welfare allocation and seeks to shift rents to its constituents. The game is coordinated by a supra-national agency, which decides on cross-border investment; mathematically, this constitutes a three-level equilibrium model, which is reformulated using strong duality and a variant of a disjunctive-constraints approach. Using a stylized example, numerical results illustrate that the first-best network investment can not be reached when zonal planners act strategically and compensation mechanisms are not available.

Keywords: Energy economics, market power, multi-stage optimization problems, Generalized Nash equilibrium (GNE), equilibrium problems under equilibrium constraints (EPEC)

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And last, but certainly not least: thanks to *Sunshine* for moving to Berlin with me…

Rechtliche Erklärung

Hiermit versichere ich, daß ich die vorliegende Dissertation selbstständig und ohne unzulässige Hilfsmittel verfasst habe. Die verwendeten Quellen sind vollständig im Literaturverzeichnis angegeben. Die Arbeit wurde noch keiner Prüfungsbehörde in gleicher oder ähnlicher Form vorgelegt.

Daniel Huppmann

Berlin, 11. Juli 2014

Table of Contents

List of Tables

List of Figures

List of Abbreviations

CO_2 .... Carbon dioxide

AC .... Alternating-Current

bbl .... Barrel (crude oil)

CGE .... Computable General Equilibrium

CP .... Complementarity Problem

CQ .... Constraint Qualification

CV .... Conjectural Variations

DCLF .... Direct-Current Load Flow

DOE .... Department of Energy (US)

EIA .... Energy Information Administration (US)

EITE .... Energy-Intensive and Trade-Exposed

EMF .... Energy Modeling Forum

ENTSO-E .... European Network of Transmission System Operators for Electricity

EPEC .... Equilibrium Problem under Equilibrium Constraints

ESM .... Energy System Model

ETS .... Emission Trading System

EU .... European Union

GHG .... Greenhouse Gas (emission)

GNE .... Generalized Nash Equilibrium

IAM .... Integrated Assessment Model

IEA .... International Energy Agency

IEM .... Internal Energy Market (EU)

IPCC .... Intergovernmental Panel on Climate Change

ISO .... Independent System Operator

ITC .... Inter-TSO Compensation Mechanism

KKT .... Karush-Kuhn-Tucker (optimality condition)
LNG .... Liquefied Natural Gas
MCE .... Myopic Cournot Equilibrium
MCP (MiCP) .... Mixed Complementarity Problem
MIQCQP .... Mixed-Integer Quadratically Constrained Quadratic Problem
MISO .... Midwest ISO
MPCC .... Mathematical Program under Complementarity Constraints
MPEC .... Mathematical Program under Equilibrium Constraints
mtoe .... Million Tons of Oil Equivalent
NEMS .... National Energy Modeling System
NGL .... Natural Gas Liquids
NIMBY .... "not-in-my-backyard"
NO_x .... Nitrogen Oxides
NTC .... Net Transfer Capacity
OECD .... Organisation for Economic Co-operation and Development
OMR .... Oil Market Report
OPEC .... Organization of Petroleum Exporting Countries
OR .... Operations Research
PIES .... Project Independence Evaluation System
PJM .... Pennsylvania-Maryland-New Jersey Interconnection
PTDF .... Power Transmission Distribution Factor
SPC .... Sustainable Production Capacity
TSO .... Transmission System Operator
TYNDP .... Ten-Year Network Development Plan
US .... United States
VI .... Variational Inequality
WEO .... World Energy Outlook
WGM .... World Gas Model

Chapter 1

Introduction: Energy economics, game theory, and Operations Research

1.1. Modelling energy markets

When I joined DIW Berlin in the summer of 2008 as a student research assistant, my first task was the simulation of several scenarios using the World Gas Model (WGM), a large-scale multi-period partial-equilibrium model of the global natural gas market. One of the scenarios under investigation analysed whether sufficient capacity to import liquefied natural gas (LNG) could be built in the United States (US) in time to meet quickly rising demand. Local "not-in-my-backyard" (NIMBY) opposition threatened to prevent the construction of regasification terminals in California; in the scenario *Pretty Coast California*, LNG import capacity was built in Mexico instead and natural gas was then transported by pipeline to California (Huppmann et al., 2011). When defining the scenario assumptions for this article, shale gas production was already booming, but reliable data was not yet available and researchers were still unsure whether the bonanza would turn out to be a pipe dream.

As recent as a decade ago, the Energy Information Administration (EIA), a branch of the US government, expected significant imports of natural gas (cf. Richter, 2013); hence the concern about sufficient regasification capacity investment in our earlier analysis. Alas, due to the shale gas revolution made possible by hydraulic fracturing of wells, or *fracking*, the picture has changed dramatically. Regasification terminals built not even a decade ago are currently operating at a fraction of capacity. Several firms have applied for export licences, currently refurbishing regasification terminals to turn them into liquefaction plants. The EIA now expects the US to turn into a net exporter of LNG as soon as 2016 (EIA, 2014), and the International Energy Agency (IEA) announced a *golden age of gas* (IEA, 2011).

This anecdote illustrates the difficulty of modelling energy markets and using numerical approaches for scenario simulations and projections. Forecasts can quickly be overtaken by technological breakthroughs or shifts in the economics of one fuel with respect to another. The shale gas boom is only the most recent example of such a paradigm shift.

Nevertheless, not using such models is not a viable option either. National governments, supra-national organisations such as the IEA and the Intergovernmental Panel on Climate Change (IPCC), and the academic community at large need to provide projections and scenarios, if only to serve as a reference in national policy discussions and international negotiations. Numerical models are useful for evaluating and quantifying the impact of various policy measures, and to better understand how market participants interact.

It is this interaction of players that is the focus of my dissertation. I tackle the question of how to properly describe and mathematically formulate strategic behaviour and market power in the energy sector, and how to solve such models in numerical applications. This work uses examples from energy economics, analyses them through the prism of game theory, and applies methods from Operations Research (OR) to numerically solve the problems at hand.

When thinking about energy and market power, the Organisation of Petroleum Exporting Countries (OPEC) immediately comes to mind. It is frequently cited as a textbook example of a cartel (e.g., Alhajji and Huettner, 2000a); according to BP (2014a), its members controlled more than 70 % of global proved reserves and more than 40 % of production in 2012. Hence, this dissertation starts with a chapter on crude oil; I discuss (and mathematically describe) how OPEC market power has at first increased over the past decade due to the increasing energy demand in Asia, while the potential to maintain prices above marginal costs declined with the onset of the global recession.

The regular disputes between Russia and Ukraine warrant the conclusion that market power is also exerted in the natural gas market; this fuel has far higher shipping costs and requires capital-intensive transportation infrastructure. As a consequence, markets are less integrated than those for crude oil. Thus, the geographical aspects of strategic behaviour are of paramount importance when analysing natural gas markets. A number of large-scale models – such as the WGM mentioned above – have been developed over the past decade to describe this market. Several of these models treat production capacity in future periods as given, which may be driving scenario results. I provide a proof that including investment in production capacity as an endogenous decision variable yields a convex (and hence easily solvable) problem; this opens up an avenue to improve large-scale numerical energy market models that incorporate strategic behaviour by certain suppliers.

The subsequent chapter proposes a multi-fuel market equilibrium model, which bridges large-scale energy system models on the one hand, and sector-specific approaches on the other hand; the latter are able to capture market power exertion and detailed infrastructure considerations. Large-scale models of the global energy sector are an important tool when evaluating climate change mitigation efforts and policy measures to reduce greenhouse gas (GHG) emissions. Due to the high interdependence of energy markets, unilateral or regional initiatives such as the Emission Trading System (ETS) – introduced by the European Union (EU) a decade ago – may have global repercussions. Emission reductions in one region due to regulatory intervention may be offset in other regions; this effect is known as *carbon leakage*.

The model developed here has one particular feature which distinguishes it from other applications used in the arena of energy policy evaluation: suppliers of fossil fuels may be modelled as exerting market power across several fuels. For example, in the data set used in this chapter, Russia is aware that withholding natural gas supplies leads to price increases not only for this fuel, but also for crude oil it sells in the same markets (and vice versa). As a result, it is able to act strategically with respect to both fuels simultaneously.

The last chapter of this dissertation shifts the focus to the electricity sector; the particular characteristics of power flows in meshed networks give rise to a multitude of gaming possibilities that are difficult to solve analytically in real-world applications. There are numerous studies that treat generators as strategic players in the power market and develop numerical routines to model their strategic behaviour. However, there is little applied work related to the incentives of zonal planners in the context of the integration of European power markets. This work argues that compensation mechanisms for investment costs and welfare reallocations resulting from network upgrades are inadequate; it shows that the optimal transmission line expansion differs when incentives and strategic behaviour of zonal planners such as national governments are explicitly considered in an integrated European energy market.

The unifying thread through all these topics is the question of modelling market power and solving the resulting problems using numerical methods. The following section provides a brief introduction to the economic theory on non-cooperative strategic behaviour, as far as it is relevant for this dissertation; the subsequent section presents the mathematical framework to solve numerical optimization and equilibrium models. The last section summarizes the individual chapters of this dissertation in more detail and highlights their particular contribution to the literature.

1.2. Game theory

The topics and applications of this dissertation are taken from the field of energy economics, with a particular focus on strategic behaviour of market participants. *Game theory* offers a framework to mathematically formulate strategic interaction between several economic agents, where each player seeks to maximize its own payoff.

A simple equilibrium concept

The central concept here is that of a *Nash equilibrium*: given the decisions by all other players, no player has an incentive to deviate (i.e., depart from her equilibrium decision). Mathematically, this can be formulated in the following way:

Definition 1 (Nash equilibrium)**.** Assume player i's constrained optimization problem as follows:

$$\max_{x_i} f_i(x_i, x_{-i})$$
$$\text{s.t. } x_i \in K_i$$

Let $f_i(x_i, x_{-i})$ denote the objective function of player i, with her decision variable (vector) x_i, while x_{-i} is the decision variable (vector) of her rivals. Player i treats the latter as given parameters. Furthermore, her decision variable (vector) must be chosen from the set of feasible strategies K_i. The set of players is denoted by I.

A Nash equilibrium $(x_i^*)_{i\in I}$ is a vector such that for each player $i \in I$, the following inequality holds:

$$f_i(x_i^*, x_{-i}^*) \geq f_i(y_i, x_{-i}^*) \quad \forall\, y_i \in K_i$$

This can be interpreted as follows: given what all rivals do, no single player can improve her payoff by deviating from the equilibrium strategy; no unilateral profitable deviation exists.

In the standard Nash equilibrium, the interaction between players only takes place via their objective function $f_i(\cdot)$; each agent's feasible strategy space is not affected by her rivals' actions. Let me illustrate this with an example derived from energy economics, namely the global crude oil market, which will be the topic of Chapter 2. A supplier – for example Norway's *Statoil* – sets crude oil output so as to maximize its profits. Those depend on the global crude oil price, which is influenced by the crude oil production of other countries. Thus, the optimal quantity produced does depend on the output decision of all other suppliers – but the quantity that Statoil can produce, from a technical point of view, is only constrained by its own rigs and export capacities. These factors are not influenced by other suppliers, at least not in the short run.

As already described by Cournot (1838), a successive adaptation by each player to her rivals' decisions leads to an equilibrium; he coined the term *tâtonnement* process for such a convergence towards the equilibrium. The seminal work by Nash (1951), whose name is now widely associated with equilibria in non-cooperative games, was written more than a century later.

The central assumption of the Nash game of Definition 1 is that each player's set of feasible strategies is not affected by the decision taken by her rivals. However, there are plenty of real-world game theory applications where independence of the strategy space is not guaranteed. Thus, a broader concept is the *Generalized Nash equilibrium* (GNE). Here, the rivals' decision directly affects both a player's objective function and her feasible strategies.

Definition 2 (Generalized Nash equilibrium)**.** Assume player i's constrained optimization problem as follows:

$$\begin{aligned} \max_{x_i} \; & f_i(x_i, x_{-i}) \\ \text{s.t. } & x_i \in K_i(x_{-i}) \end{aligned}$$

The difference to the optimization problem introduced in Definition 1 is that the set of feasible strategies $K_i(x_{-i})$ now depends on the actions of the rivals, x_{-i}.

A Generalized Nash equilibrium $(x_i^*)_{i \in I}$ is a vector such that for each player, the following inequality holds:

$$f_i(x_i^*, x_{-i}^*) \geq f_i(y_i, x_{-i}^*) \quad \forall \, y_i \in K_i(x_{-i}^*)$$

This concept can be illustrated by another example from the crude oil market: in the US, there are several independent oil companies. Most of them use – to some extent – oilfield service companies to provide drilling rigs or other equipment. Thus, if all available rigs or drilling teams are operating, an oil company may not be able to increase its production level, even after it has obtained the required licenses and access to drilling sites. In this case, the decision by her rivals – how many rigs are already operating and whether there are idle rigs currently available – directly constrains the level of output by one firm.

Such games are far more prone to permitting a multitude of equilibria. In particular, the starting point for a tâtonnement process may have a significant impact on which equilibrium the players find themselves in. Returning to the example of several oil companies: one of the companies may already have signed a contract with an oilfield service company pertaining to a relatively expensive field; a rival now seeks an oilfield service company for a field with relatively cheaper production costs, but all available capacity is bound by that contract. In a very stylized world, one company would compensate the other, and the oilfield service company would move its rigs to the cheaper production field; all companies would benefit. However, in reality, the costs of negotiating a settlement for dissolving the contract may be too high, both in financial terms (i.e., the legal fees) and in non-monetary friction such as an unwillingness to cease operations for the benefit of a rival even at incurring own opportunity costs. Hence, a possibly arbitrary event – such as which company first signed the contract – can have a significant impact on the final outcome.

Of course, such a situation would only be a problem in the short run; new oilfield service companies will enter the market if there is a shortage of capacity, or contracts expire and thereby eliminate the inefficient allocation of resources. There are nevertheless numerous examples in energy economics where efficient short-term operation is hampered by such situations. I will return to this issue in the following section presenting the mathematical formulation and solution techniques for game-theoretic problems.

There is one important shortcoming of the equilibrium concepts introduced so far. Both the Nash equilibrium and the GNE sound like quite straightforward propositions and they are widely used in non-cooperative game theory. However, they are based on a substantial epistemological tenet: each player assumes that her rivals will not alter their decisions if she deviates.

Cournot vs. Bertrand, and the question of epistemology

In non-cooperative game theory and imperfect competition between firms, there are two workhorse models: *Cournot competition* refers to games in quantities; each firm announces the amount it is willing to sell, with the price being set such that the market clears. In particular, every player is aware of the impact that her own decision has on the market-clearing price. This is sometimes described as each firm acting as a monopolist with respect to the residual demand. In contrast, *Bertrand competition* is the term used when firms announce at which price they are willing to sell any quantity; consumers then decide how much to purchase at that price (Bertrand, 1883). Economists usually treat price games as equivalent to a perfectly competitive market on the following reasoning in a symmetric setting: by slightly under-cutting their rival with the offered price, a firm is able to capture the entire market. Therefore, the only Nash equilibrium is where each firm bids her marginal costs.

The main difference between the two approaches is not actually whether the decision variables of the players are quantities or prices: it revolves around the question which variable the Nash assumption is used on. In a Cournot quantity game, the player assumes that the rivals will not change their quantity sold when she increases her output; a higher total quantity sold in the market reduces the price consumers are willing to pay per unit. In a price game, each player assumes no change in prices offered by her rivals when she undercuts their offer. Assuming a price-elastic demand, this means that the total quantity sold increases. However, in the quantity game, the player can only capture a marginally larger market share by increasing her output. In the price game, she can capture the entire market – under the assumption that the rivals do not react to her deviation.

In reality, of course, no manager assumes that rival firms will not react to any change in her own decision; instead, she will have an expectation (or conjecture) of her rivals' reaction. The notion of *Conjectural Variations* (CV) was introduced to capture this effect and allow for intermediate market equilibria between the standard Cournot oligopoly and perfectly competitive markets (Figuières et al., 2004).

In the 1980s, a discussion (re-)started regarding the rationality (or consistency) of each players' conjectures regarding the rivals' reaction to her own marginal output changes. Bresnahan (1981) argued that under consistent conjectures, the "correct" Cournot equilibrium is equal to the Bertrand outcome under certain conditions. At the same time, Kreps and Scheinkman (1983) showed that in a two-stage game where firms first invest in capacity and then engage in Bertrand price competition, the Nash equilibrium is the one-stage Cournot equilibrium. After a decade of heated debate, the controversy subsided; I will return to this issue in more detail in Chapter 2.2.

Kimbrough et al. (2013) discuss the question of rationality of players' expectations more thoroughly and take the question of strategic behaviour to a meta-level: they ask which strategies a profit-maximizing firm would choose, knowing the equilibrium that would result for each choice of strategy. Kimbrough et al. relate this to a more complex notion of information or knowledge (or epistemology) on which any such decision must be based. They point out that the famous result of Allaz and Vila (1993), which states that forward markets are welfare-improving, is inconsistent because the information set of the agents shrinks over time; the agents "forget" from one stage of the game to the next.

Sequential games

So far, I have only discussed simultaneous-move games, where each player has the same knowledge (information) as her rivals; "move" in the game-theoretic framework means announcing one's decision. For most purposes, simultaneous-move games, where each player announces her decision – correctly anticipating the rivals' decisions – are equivalent to a repeated sequential game, where each player iteratively updates her decision until no player has an incentive to alter the chosen strategy any more; this would be the equilibrium to which a tâtonnement process converges.

Now I turn to multi-stage or *sequential games*, where one player has more information than the rest or is in position to make her decision at an earlier or later stage. The dominant player is usually called the *leader*, the others are the *followers*. Such a situation can be rationalized in several ways: either a player indeed has a larger information set than the rivals in a simultaneous-move game; or a player is able to move first in a sequential market setting, anticipating the optimal reaction of all rivals at the later stages of the market;[1] or

[1]In such settings, the term *first mover advantage* is often used. Strictly speaking, the advantage usu-

a player moves simultaneously with the rest, but is in a position to credibly commit to her decision beforehand; or a player is able to correctly anticipate the outcome and commit to not updating her decision in a tâtonnement process. These game structures are equivalent, only the setting and the rationalization differ explaining why one firm is dominant. They are usually called *Stackelberg game*, after the work by von Stackelberg (1934).

Of course, one can take multi-stage games further and introduce more than two stages or multiple players at each level, with distinct information sets. However, whether such intricate games can be solved for real-world applications depends on the actual game structure; an integrated framework is – to date – elusive. This dissertation is concerned with several examples of such games derived from energy economics and develops numerical solution techniques for the specific applications. The mathematical framework is introduced in the following section.

1.3. Operations Research

Deriving analytical solutions for games between several players quickly becomes intractable; here, the field of *Operations Research* (OR) comes into play and offers a toolbox to numerically solve large-scale applications. The methods used in this dissertation are taken from the field of complementarity problems (CP) and multi-stage optimization problems. Facchinei and Pang (2003) give a comprehensive treatise on CP and the related field of *Variational Inequalities* (VI). Gabriel et al. (2013) provide a more applied introduction, with many examples derived from energy economics.[2]

Both books also provide a detailed history of the development of OR methods over the past decades, where the difficulty of solving large-scale energy models provided important impetus for theoretical and algorithmic advances. One noteworthy stream of the literature is the *Project Independence Evaluation System* (PIES), research initiated by the US government at the time of the first oil crisis (Ahn and Hogan, 1982). This model evolved into the *National Energy Modeling System* (NEMS), which is used to this day by the *U.S. Department of Energy* (DOE) for projections and scenario simulations of the US energy markets (Gabriel et al., 2001).

ally comes not only from moving first, but more importantly from the knowledge required to correctly anticipate the followers' reactions.

[2] The mathematical notation presented in this section follows Facchinei and Pang (2003) for consistency; the definitions 3, 4, 5, and 7 are direct quotes.

Mathematical definitions

Let me now introduce several mathematical definitions, and then relate them to the game-theoretic equilibrium notions described earlier.

Definition 3 (Complementarity Problem)**.** Given a cone K and a mapping $F : K \to \mathbb{R}^n$, the *complementarity problem*, denoted $\mathrm{CP}(K, F)$, is to find a vector $x \in \mathbb{R}^n$ satisfying the following conditions:

$$K \ni x \quad \perp \quad F(x) \in K^*,$$

where the notation $\perp$ means "perpendicular" and K^* is the *dual cone* of K defined as:

$$K^* \equiv \{d \in \mathbb{R}^n : v^T d \geq 0 \quad \forall\, v \in K\};$$

that is, (the cone) K^* consists of all vectors that make a non-obtuse angle with every vector in K.

The *perpendicular operator* ($\perp$) indicates that $F(x)$ and the variable x are *complementary*; this means that for all $x \in K$ that solve the $\mathrm{CP}(K, F)$, it must hold that $x^T \cdot F(x) = 0$.

A related concept is the Variational Inequality (VI):

Definition 4 (Variational Inequality)**.** Given a subset K of the Euclidian n-dimensional space $\mathbb{R}^n$ and a mapping $F : K \to \mathbb{R}^n$, the *variational inequality*, denoted $\mathrm{VI}(F, K)$, is to find a vector $x \in K$ such that

$$(y - x)^T F(x) \geq 0, \qquad \forall\, y \in K. \tag{1.1}$$

The set of solutions to this problem is denoted $\mathrm{SOL}(K, F)$.

In the general case of the VI, the set K is closed and F is a continuous function (and differentiable, where appropriate). In a CP, the set K is a closed convex cone. In some cases (and many practical applications), the mapping $F(x)$ of a CP can be expressed as a collection of equalities and inequalities, and the set K as a product of either the real numbers or the positive orthant of the real numbers. This leads to the so-called *Mixed Complementarity Problem* (MCP):[3]

Definition 5 (Mixed Complementarity Problem)**.** Let G and H be two mappings from $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}_+$ into $\mathbb{R}^{n_1}$ and $\mathbb{R}^{n_2}$, respectively. The $\mathrm{MCP}(G, H)$ is to find a pair of vectors (u, v) belonging to $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ such that

$$\begin{aligned} G(u, v) = 0 \quad &, \quad u \text{ free} \\ 0 \leq v \quad &\perp \quad H(u, v) \geq 0. \end{aligned}$$

[3] Mixed Complementarity Problems are abbreviated as MiCP by Facchinei and Pang (2003); in this work, I follow the notation more commonly encountered in the literature.

An application of an MCP are the Karush-Kuhn-Tucker (KKT) optimality conditions, which are a standard tool in economics. The problem is derived from constrained optimization problems, such as profit maximization of economic agents under operational constraints.

Definition 6 (Karush-Kuhn-Tucker conditions)**.** Assume a constrained optimization problem as follows:

$$\begin{aligned} \max F(x,y) \\ \text{s.t. } K_1(x,y) = 0 \\ K_2(x,y) \leq 0 \\ x \geq 0 \end{aligned}$$

then the first order (KKT) conditions of this problem are:

$$\begin{aligned} 0 \geq \frac{\partial F}{\partial x} - \lambda \frac{\partial K_1}{\partial x} - \mu \frac{\partial K_2}{\partial x} \quad &\perp \quad x \geq 0 \\ 0 = \frac{\partial F}{\partial y} - \lambda \frac{\partial K_1}{\partial y} - \mu \frac{\partial K_2}{\partial y} \quad &, \quad y \text{ free} \\ 0 = K_1(x,y) \quad &, \quad \lambda \text{ free} \\ 0 \geq K_2(x,y) \quad &\perp \quad \mu \geq 0 \end{aligned}$$

In this formulation, λ and μ are the dual variables (or Lagrange multipliers) of the constraints $K_1(x,y)$ and $K_2(x,y)$. They can be interpreted as the improvement of the problem's objective value given a marginal relaxation of the associated constraint.

In economists' parlance, the term *complementary slackness* is frequently used; it states that if a constraint is not binding, the associated dual variable must be zero. In spite of the different wording, this is equivalent to the notion of complementarity as stated above: if the constraint is not binding, a marginal relaxation will not yield any improvement of the objective function.

In the MCP notation as defined above, the stationarity condition with respect to x, concatenated with constraints K_1 are equivalent to the constraints $G(\cdot)$ in Definition 5; the stationarity conditions of variable y combined with constraints K_2 are equivalent to the constraints $H(\cdot)$. The vector u is replaced by $(x,\lambda)^T$, the vector v by $(y,\mu)^T$. As above, the perpendicular operator ($\perp$) indicates that the equation $K_2(x,y)$ is complementary to the variable μ; $K_2(\bar{x},\bar{y}) \cdot \bar{\mu} = 0$ must hold in the optimum $(\bar{x},\bar{y},\bar{\mu},\bar{\lambda})$.

If for every optimal solution to the original problem, there exist multipliers such that the KKT conditions are satisfied, they are called *necessary*; if every point satisfying the KKT conditions is an optimum to the original problem, the KKT conditions are *sufficient.* There exist a number of (collections of) conditions under which either necessity or sufficiency (or both) hold; these are called *constraint qualifications* (CQ). One example for a simple CQ is the following: linearity of all constraints guarantees necessity of the KKT conditions; concavity of the objective function (to be maximized) yields sufficiency. This is the CQ applied in Chapter 4. The KKT system can be interpreted as a special case of a variational inequality, namely $\mathrm{VI}(K, \nabla F)$, where K is the set of all vectors (x, y) satisfying $K_1(x, y) = 0$ and $K_2(x, y) \leq 0$.

Equilibrium problems

The KKT conditions have a direct relation to the Nash equilibrium introduced in the previous section: if every player's optimization problem is such that a constraint qualification holds, then combining their respective KKT conditions and solving them simultaneously as one MCP yields a Nash equilibrium.

Another approach to finding the Nash equilibrium of such a game is to iteratively solve each player's optimization problem, holding each rivals optimal decision fixed, and then updating the player's decision in the rivals' optimization problems. This is continued until no player finds it optimal to alter her decision relative to the previous iteration; no profitable deviation exists. Such an approach is called *diagonalization*. It is the mathematical equivalent to Cournot's concept of the tâtonnement process; it is a variant of the *Gauss-Seidel method* for solving equation systems (Gabriel et al., 2013). Convergence of such an approach, as the existence of a solution to the MCP, is related to fixed-point properties of the underlying problem.

The problem is more intricate when one attempts to solve a GNE as an MCP; I will only discuss one particular case of a GNE at this point, namely when several players face so-called *shared constraints*. Such problems are encountered in Chapters 4 and 5. The problem arises from the fact that simply combining each player's KKT conditions leads to a non-square MCP; the shared constraint is identical for each player and can therefore be removed from the equation system, but the associated Lagrange multiplier may differ for each player. Mathematically speaking, the dimensions of the equation system $(G(u, v), H(u, v))$ and the complementary variable vector (u, v) do not match; the complementarity constraint $(u, v)^T \cdot (G(u, v), H(u, v)) = 0$ is not well-defined.

Harker (1991) proposes a simple solution: assume that all players value the shared resource equally; mathematically, this means assuming an identical dual variable of each player for the shared constraint. This assumption also has an economic interpretation: an implicit auction ensures that the player with the highest willingness-to-pay (the highest valuation) is given access or permission to use the resource. Alternatively, the dual variables may – again by assumption – be linearly related. Either way, "squareness" of the MCP is restored and standard solution approaches can be employed. In Chapter 4, I use this simplification; in Chapter 5, I propose an alternative approach that does not require a priori assumptions on the valuation of the shared constraint.

Bilevel problems

Let us now turn to the Stackelberg game, the simplest instance of a two-stage game; just like the standard Nash game has its mathematical counterpart in the MCP, the Stackelberg game can be written as a *Mathematical Program under Equilibrium Constraints* (MPEC).[4] This can be formalized as follows:

Definition 7 (Mathematical Program under Equilibrium Constraints)**.** Consider a constrained optimization problem:

$$\begin{aligned} \max\ & \theta(x,y) \\ \text{s.t. } & (x,y) \in Z \\ & y \in \mathrm{SOL}(K(x), F(\cdot,x)) \end{aligned}$$

Here, θ is a function $\mathbb{R}^{n+m} \to \mathbb{R}$ and Z is a given subset in $\mathbb{R}^{n+m}$. The variable (or vector) x is called the *design variable*, while y is the *state variable*. In addition to being an element of Z, the variable (or vector) y must be a solution to the parametric $\mathrm{VI}(K(x), F(\cdot,x))$.

The difficulty of solving MPECs arises from the fact that no suitable CQ holds for most relevant applications; hence, one usually has no guarantees whether a solution obtained numerically is indeed optimal.

The lower-level equilibrium is represented by the solution set of a VI; when the VI is replaced by a CP or MCP, strictly speaking, one faces a *Mathematical Program under Complementarity Constraints* (MPCC). The difference is subtle: in applied work, the equilibrium constraints are often replaced by complementarity constraints, as these are easier to handle. However, this risks eliminating potential equilibria from the lower level of the game when the VI and the CP are not equivalent. As a consequence, the MPCC is not necessarily an exact representation of the original MPEC in such instances.

Let me illustrate this problem using the two notions of *optimistic vs. pessimistic MPECs*: when solving bilevel problems such as Stackelberg games, the most commonly applied approach is to use first-order optimality conditions of the lower-level optimization problem and insert those as equilibrium constraints to the upper-level problem. The problem from Definition 7 can then be rewritten in the following way:

$$\begin{aligned} \max_{x,y}\ & \theta(x,y) \\ \text{s.t. } & (x,y) \in Z \\ & 0 \leq H(x,y) \perp y \geq 0 \ , \end{aligned}$$

where $0 \leq H(x,y) \perp y \geq 0$ is a representation of the problem $y \in \mathrm{SOL}(K(x), F(\cdot,x))$. However, when solving the bilevel problem in this way, an important assumption is implicitly taken: the decision variables of the lower-level player are then, mathematically speaking, decision variables of the upper-level player. The upper-level player only needs to make sure that the optimality constraints are satisfied. If the optimal response of the lower-

[4] In some publications, MPEC is understood to mean *Mathematical Problem under/with Equilibrium Constraints*. In this context, *problem* and *program* are equivalent.

level player is unique for any given decision of the upper-level player, this does not cause a problem. However, there may be cases where uniqueness of the lower-level best response is not guaranteed; the lower-level player may be indifferent between several options. In such cases, the upper-level player may then decide which of the options the lower-level player "chooses". This is commonly referred to as the *optimistic approach*. It is the best the upper-level player can do given the bilevel game.

In contrast, one may consider a case where the lower-level player wants to do what is worst for the upper-level player, given a situation where she is indifferent between several options; this is the *pessimistic approach*. Such a situation may be plausible in cases where the upper- and lower-level players are rivals, and the lower-level player wants to maximize her profits – but also has an incentive to reduce the profits of the rival, if it does not have a downside for her (by definition, own profits are not affected, as she is indifferent between two decisions). Mathematically, this leads to a *min-max-problem*, which is computationally difficult to solve. Therefore, most applied numerical research implicitly uses the optimistic approach, although this is not always clearly mentioned.

When seeking equilibria between several players, where each player solves an MPEC, one faces an *Equilibrium Problem under Equilibrium Constraints* (EPEC). This only exacerbates the difficulties posed by MPECs; to date, no canonical solution approach exists for this type of game. There is a host of literature identifying reformulations for specific problems with the aim of expressing EPEC-type models as a simpler mathematical problem: such a reformulation is proposed in Chapter 2 to reduce the EPEC representing a Stackelberg oligopoly to an MCP, which can then be solved using standard approaches.

When a reduction of the bilevel equilibrium problem to a simpler model is not easily possible, as is the case in the problem presented in Chapter 5 of this dissertation, two approaches are commonly used. First, similar to solving simultaneous-move games, diagonalization can work well in practice. However, similar to solving GNE, there may a multitude of equilibria, and it is not clear a priori which equilibria a diagonalization approach would identify.

The second approach is *enumeration*: each player's strategy space is discretrized, and the lower-level problem is then solved for each possible combination of strategies or actions of the upper-level game. The payoffs as well as the deviation incentives for each player are determined ex-post to identify stable Nash equilibria. The problem when implementing such an approach in practice comes from the difficulty of determining a suitable level of detail for the discretization, and a general issue of scalability to solve large-scale applied problems. I will discuss this in more detail in Chapter 5 and implement an advantageous method to identify global Nash equilibria in EPEC-type problems.

1.4. Outline and contributions of this dissertation

This dissertation uses examples from energy economics, motivated by game-theoretic considerations, to propose problem formulations and solution techniques based on the methods from Operations Research described above. The dissertation is based on four original research articles, which all deal with market power and strategic behaviour in energy markets. An overview of the chapters and their contribution to the literature of OR applications is given in Table 1.1 below; a list of my own contribution in each of them and where the research articles have been published is provided in Table 1.2 at the end of this chapter.

Chapter	Topic	Contribution to the literature	Model type
2	Crude oil	Reduces a two-stage multi-leader single-follower model to a one-stage equilibrium model using consistent conjectures	single-period MCP
3	Natural gas	Provides a proof that endogenously including investment in production capacity yields a convex problem	multi-period MCP
4	Energy system	Develops the first-of-its-kind large-scale energy system equilibrium model where market power is exerted across multiple fuels	multi-period MCP
5	Electricity	Develops a three-stage equilibrium model where zonal planners strategically upgrade the power network	EPEC solved as non-convex MIQCQP[5]

Table 1.1.: Overview of chapters: Topics and contribution to the literature

Chapter 2 – Strategic behaviour in the crude oil market

This chapter starts with a treatise on market power in the crude oil market. Decades after the first oil crisis, the actual role of the OPEC is still mired in (academic) controversy. I propose a *Stackelberg oligopoly* to describe the market structure: OPEC suppliers play a Cournot game against each other, taking each others' output decision as given in the Nash sense – but collectively, they take into account the reaction of the competitive fringe suppliers. OPEC exhibits two features that warrant the assumption that OPEC suppliers are Stackelberg leaders in the market: first, the OPEC quota serves as a credible commitment and signal towards the fringe suppliers. Second, the OPEC members have substantial knowledge of each rival's cost structure and available capacity, enabling them to correctly anticipate their reaction to OPEC output changes.

[5] Mixed-integer quadratically constrained quadratic problem (MIQCQP).

The example of the crude oil market allows to discuss limitations of representing non-competitive markets in a Nash-Cournot equilibrium framework. When combining strategic firms and a competitive fringe in a standard Nash-Cournot equilibrium model, as it is implemented in many energy models, the strategic firms fail to anticipate their impact on the competitive fringe's supply. They "over-exert market power", meaning that they reduce the output below the level which would be most profitable to them.

Due to the specific cost characteristics in oil (and natural gas) extraction, a logarithmic functional form is often used in applied numerical work (Golombek et al., 1995). When producing close to capacity, marginal costs increase sharply. Combining this cost function with the Stackelberg oligopoly model of several leaders and a competitive fringe as follower gives rise to an interesting effect: the optimal level of market power exertion increases in line with capacity utilization of the fringe suppliers. This is quite intuitive: the lower the spare capacity of the fringe, the lower is her ability to compensate for withholding of output by the OPEC member countries, hence a higher degree of market power exertion can be maintained.

In a numerical application using data from 2003–2012, the Stackelberg oligopoly model indeed yields a better match of results to the observed prices compared to other, standard models of imperfect competition. Optimal market power exertion by OPEC members gradually increases over time, culminating in the price spike of 2008. After the financial crisis and with the onset of a global recession, a reduction of demand for crude translates into higher levels of spare capacity of fringe suppliers and, as a consequence, less ability by OPEC to maintain prices above marginal-cost levels.

Chapter 3 – Investment in natural gas production capacity

The model for the crude oil market is a one-period game; here, I turn to multi-period partial-equilibrium models. The topic of this chapter is the global natural gas market and endogenous investment in production capacity. Numerous equilibrium models were proposed over the past decade to simulate scenarios of the future evolution of global natural gas markets. This research interest is due to its importance as a bridge fuel towards a low-carbon economy. Several of these models also use the logarithmic cost function which I already apply in Chapter 2. These models, in general, have a detailed depiction of transportation infrastructure, including the LNG value chain.

However, many of these large-scale models treat production capacity as exogenously given. Because the marginal costs depend strongly on the assumptions regarding production capacity in future periods at each node, this may be strongly driving the results in some scenarios. This chapter provides a proof that extending equilibrium problems to incorporate investment in production capacity endogenously yields a convex problem. As a consequence, it is no longer required to make a priori assumptions on production capacity in future periods; instead, the decision on production capacity can be treated as a variable in the supplier's optimization problem.

There is a subtle yet interesting additional effect arising from this extension: since investment today reduces marginal production costs tomorrow, ceteris paribus, the endogenous consideration of production capacity expansion can be interpreted as a *learning effect*. In models with linear or quadratic cost functions, investment is only optimal when the capacity constraint is reached. In contrast, due to the logarithmic cost function used here to account for the specific characteristics of natural gas extraction, it may be optimal to invest in additional capacity even if the constraint is never binding – but the reduction in future costs makes the investment profitable nevertheless.

Chapter 4 – A multi-fuel market equilibrium model

The next chapter makes use of the proof presented in Chapter 3 and includes endogenous investment in production capacity in a large-scale application; also, I depart from the single-fuel perspective to look at the interaction between different energy carriers. This chapter proposes a dynamic multi-fuel market equilibrium model. To the best of my knowledge, this is the first model that combines endogenous fuel substitution within demand sectors and in power generation, detailed infrastructure capacity constraints and investment, as well as strategic behaviour and market power aspects by suppliers across several fuels in a unified framework.

The model bridges the gap between *energy system models* (ESM) and models which focus on a specific fuel or sector. While the former are able to investigate fuel substitution and the optimal fuel mix in power generation, the latter are usually used for market structure analysis and detailed infrastructure considerations.

Most importantly, this model allows certain suppliers to exert market power across multiple fuels – for instance, in the data set compiled for this application, Russia is aware that withholding crude oil supply not only drives up the price of oil, but also has an impact on the revenue it receives for its natural gas exports (and vice versa). Nevertheless, the term "market power" is meant in a more simple way than in the model presented in Chapter 2. The multi-fuel market equilibrium model is subject to the criticism expressed there with regard to CV models; in light of the trade-off between sophistication and numerical tractability, it is the best we can do at the current time.

A data set based on the IEA's *World Energy Statistics* and the *World Energy Outlook* (WEO) is used to demonstrate the features of the model. A *Base Case* following the projections of the WEO's *New Policies* scenario is compared to two scenarios: in the first scenario, a reduction of North American natural gas availability relative to current projections leads to an even stronger increase of power generation from natural gas in the European Union relative to the Base Case. This is due to a shift in global fossil fuel trade. In the second scenario, a tightening of the EU ETS emission cap by 80 % in 2050 relative to 2010 combined with a stronger biofuel mandate leads to a total energy demand reduction of 25 % in 2050 and a strong electrification of the transportation sector. We observe carbon leakage rates from the unilateral mitigation effort of 70–90 %.

Chapter 5 – National-strategic investment in power grids

The last chapter of this dissertation turns the focus from the global energy system to a more regional and sector-specific application: the integration of the European electricity market and the efficient feed-in of renewables require substantial upgrade of the transmission grid. However, most applied studies aiming to determine the optimal network expansion abstract from the welfare shifts that may arise from changes in the network topology. These shifts can occur both between different stakeholder groups (i.e., generators, consumers, and transmission system operators, TSO) and across national boundaries.

To date, network investment is still mainly a national prerogative and there is no effective compensation mechanism to remunerate stakeholders for the welfare shifts from transmission line upgrades between different countries. In particular, due to specific characteristics of power flow in a meshed network, line upgrades in one area can directly influence the available capacity in other parts of the network. Thus, beneficiaries of investment could be located in different jurisdictions than those stakeholders bearing the costs. In these situations, zonal planners such as national governments, regulators, or TSO's in charge of a network area, have an incentive to hold back line upgrades or over-invest compared to the welfare-optimal expansion plan in order to induce a shift of rents towards stakeholders in their zone. This distortion of investment may impede the effective integration of the European power market and the efficient shift to a power system based on renewable energy sources.

In order to model the impact of strategic behaviour by zonal planners, we develop a three-stage equilibrium model: at the bottom stage, a competitive and well integrated market determines the optimal short-term dispatch of power plants and flows in the network; at the intermediate stage, zonal planners strategically decide on transmission investment on all lines within their jurisdiction. They specifically take into account how the network upgrades affect welfare of their constituents. The top stage of this game is the supra-national coordination agency; she seeks to maximize welfare in the entire region and decides on cross-border network upgrades, while being able to anticipate the strategic reaction by the zonal planners.

Our results based on a simple test case demonstrate that the national-strategic behaviour yields significant welfare losses compared to the system-optimal investment. A zonal planner has incentives to "under-invest" in domestic line upgrade in order for her constituents to obtain a share of the welfare gains, rather than seeing domestic welfare decreased by the system-optimal network upgrades.

Mathematically, this chapter is the most challenging work in this dissertation. As mentioned earlier, even solving a two-stage equilibrium (EPEC) model does not admit standard approaches, and existence or uniqueness of an equilibrium is in general not guaranteed. We extend an approach from the recent OR literature (Ruiz et al., 2012) to solve this three-stage model. Using an iterative algorithm, we are able to identify a multitude of equilibria. In the process, we also discuss a practical way to solve GNE-type problems without the need to make a priori assumptions on the relative valuation of shared constraints by the strategic players.

1.5. Concluding remarks and outlook for future research

Examples from energy economics are used in this dissertation to motivate more elaborate game-theoretic models than currently used in most applied research. Numerical solution approaches derived from the Operations Research literature are employed and further developed to identify equilibria between strategic players. Several avenues for future research are opened up through this work.

The notion that the crude oil market is a two-stage game, where some suppliers are in a dominant position relative to their rivals, is quite frequently discussed in the academic literature. However, few numerical models exist to describe this market. The Stackelberg oligopoly proposed in this work, where the optimal level of market power exertion depends on the spare capacity of fringe suppliers, yields a better match of observed crude oil prices over the past decade than the standard, simultaneous-move equilibrium concepts, which are widely used in applied energy market models. Nevertheless, the numerical results in terms of quantities are ambiguous and no market structure assumption yields a plausible fit. This points to more elaborate forms of collusion within OPEC; concepts discussed in the literature are the *bureaucratic cartel* (Smith, 2005) and the *Nash-Bargaining cartel* (Harrington et al., 2005). Including these more advanced collusion mechanisms in a numerical equilibrium model has the potential to provide a better understanding of past events in the crude oil market and allow a more informed outlook on future developments.

Another contribution of this dissertation tackles endogenous investment in production capacity in large-scale market equilibrium models employing a logarithmic cost function. This is used to capture the characteristics of crude oil and natural gas production. Including capacity investment endogenously in a mixed complementarity setting is already implemented in the multi-fuel partial-equilibrium model presented in Chapter 4, developed jointly by DIW Berlin and NTNU Trondheim. The WGM, a large-scale partial-equilibrium model developed under the leadership of the University of Maryland, is also being extended to include endogenous investment in production capacity at the time of writing.

The multi-fuel partial-equilibrium model is currently used within the *Energy Modeling Forum*,[6] Round 31 on "North American Natural Gas and Energy Markets in Transition" to contribute scenario simulations on potential pathways of the future US energy markets in a global context. Of course, the possibilities for further policy analysis and scenario simulation regarding the decarbonisation of the European energy sector are virtually endless; similarly, China – having recently become the largest emitter of carbon dioxide (CO_2) – should be given more focus in future research.

Methodologically, the model bridges the gap between energy system models and sector-specific partial-equilibrium models. The next logical step will be to also incorporate general equilibrium aspects, in order to better capture the interaction between the economy at large and the energy sector. When looking at questions such as the *green paradox* (cf. Sinn, 2008)

[6]The EMF (`http://emf.stanford.edu`) is an international forum organised by Stanford University, with the aim of sharing and facilitating discussions on energy policy and global climate issues among experts.

and strategic resource exploitation, such a hybrid approach will allow a more thorough analysis of the effectiveness of various policy instruments and emission reduction strategies in the light of strategic behaviour by fossil fuel suppliers.

The three-stage model developed in the last chapter of this dissertation opens up a number of avenues: the structure can be extended or modified to look at other instances of strategic interaction in the electricity sector. A companion paper (Zerrahn and Huppmann, 2014) uses a similar three-stage setup; in contrast to the work presented in this dissertation, however, the strategic players on the intermediate level are not zonal planners but power companies. By anticipating their impact on the system operator clearing the market, they may be able to deliberately congest the power network and thereby appropriating excess rents. The top-level player is the benevolent system planner, who expands the network with the aim of mitigating market power exertion by the strategic generators.

Returning to the model presented in this work – the strategic interaction between zonal planners – the next step in further developing this three-stage game would be to include investment in new power plants, in particular with respect to the location of additional renewable energy capacity. Furthermore, one may put more focus on the distribution of rents between the different stakeholder groups: so far, the zonal planner gives equal weight to consumer welfare, generator profits and congestion rents. However, one may consider the case that a national regulator gives a higher priority to consumer welfare, as long as generators and transmission system operators are able to recover their costs.

Chapter	Pre-publication and own contribution
2	Shifts in OPEC market power – A Stackelberg oligopoly with fringe
	This chapter is based on: *Discussion Papers of DIW Berlin 1313*, 2013b. It was presented at the Annual Conference 2013 (Düsseldorf) of the *Verein für Socialpolitik* (German Economic Association) and is published in the Conference Proceedings. The manuscript is submitted to the journal *Energy Economics* for publication.
	Single-author original research article.
3	Endogenous investment decisions in natural gas equilibrium models
	This chapter is based on: *Discussion Papers of DIW Berlin 1253*, 2012. A revised version was published as: *European Journal of Operational Research*, 231(2):503–506, 2013a.
	Single-author original research article.
4	Market power, fuel substitution, and infrastructure – A large-scale equilibrium model of global energy markets
	This chapter is based on: *Discussion Papers of DIW Berlin 1370*, 2014. The manuscript is submitted to the journal *Energy* for publication.
	This model was developed jointly with Ruud Egging. D. Huppmann had the lead role in the model development, the implementation in GAMS, data collection and coordination of the supporting student research assistants, scenario analysis, and writing of the manuscript.
5	National-strategic investment in European power transmission
	This chapter is based on: *Discussion Papers of DIW Berlin 1379*, 2014.
	This model was developed jointly with Jonas Egerer. D. Huppmann had the lead role in the model development, the implementation in GAMS, analysis of numerical results, and writing of the manuscript.

Table 1.2.: Overview of chapters: Pre-publication and own contribution

Chapter 2

Endogenous shifts in OPEC market power
A Stackelberg oligopoly with fringe

This chapter is based on:
Endogenous shifts in OPEC market power – A Stackelberg oligopoly with fringe.
Discussion Papers of DIW Berlin 1313, 2013b.
It was presented at the Annual Conference 2013 (Düsseldorf) of the *Verein für Socialpolitik* (German Economic Association) and is published in the Conference Proceedings.
The manuscript is submitted to the journal *Energy Economics* for publication.

2.1. Introduction: Market power in the crude oil sector

For the past four decades, oligopolistic behaviour in the crude oil market has been a recurring theme in the economic literature. Various theories were repeatedly tested to understand and explain the behaviour of the Organization of the Petroleum Exporting Countries (OPEC) or Saudi Arabia, its most prominent member. However, the aggregate of these studies is inconclusive at best and contradictory at worst, as summarized by Smith (2005), Alhajji and Huettner (2000a), and Griffin (1985).

The contribution of this chapter is threefold: first, I discuss how current large-scale equilibrium models over-simplify and, in a way, misinterpret market power exertion. Then, I propose a more elaborate approach than the standard Nash-Cournot oligopoly to model strategic behaviour when a competitive fringe is present – namely a two-stage game with several Stackelberg leaders that anticipate the reaction of the fringe. Third, this model is applied to the global crude oil market. By representing strategic behaviour using the two-stage game, the market power of OPEC members increases endogenously as the spare capacity of the fringe (i.e., the non-OPEC suppliers) goes down.

Thereby, this work ties into the discussion of the crude oil price increase over the past decade, culminating in the price spike of 2008. This phenomenon initiated a wide discussion in the academic literature regarding its causes, and whether it was rather driven by speculation or fundamentals of supply and demand. While Kaufmann and Ullman (2009) argue that speculation was an important factor for the price spike, Fattouh et al. (2013), Alquist and Gervais (2013), Hamilton (2009), Smith (2009) and Wirl (2008), amongst others, disagree and identify other, more important drivers: low demand elasticity, strong growth of newly industrialized countries, and insufficient production capacity expansion. The results of this work lend support to the latter view, but add increased market power of OPEC due to low spare capacity by other suppliers as an explanation.

Partial-equilibrium modelling of energy markets

OPEC first gained notoriety in the seventies and eighties. At that time, optimization and partial-equilibrium models were widely applied, both theoretically (e.g., Salant, 1976; Newbery, 1981) and numerically (e.g., Salant, 1982). These models usually combined a Hotelling-style exhaustible resources approach and Nash-Cournot or Stackelberg market power. Equilibrium models subsequently went out of fashion – for two reasons, I believe: first, the failure of the oil price to follow the path projected by a Hotelling-type model; and second, the debate regarding the consistency of Nash-Cournot equilibria. I will discuss both issues in more detail below.

With the liberalization of the electricity and natural gas markets in Europe, Nash-Cournot partial-equilibrium models were again widely used in large-scale numerical energy market applications. This was due to advances in algorithms and computation power, which allowed to drop many simplifications necessary in the early models. Recent applications for the natural gas market include the World Gas Model (Gabriel et al., 2012; Egging et al., 2010), GaMMES (Abada et al., 2013), and Gastale (Lise and Hobbs, 2008). There were also a number of models for electricity markets (e.g., Neuhoff et al., 2005; Bushnell, 2003), and – more recently – the global coal markets gained some attention (Trüby and Paulus, 2012; Haftendorn and Holz, 2010).

There are three recent numerical partial-equilibrium models for the crude oil market: Aune et al. (2010) present a dynamic equilibrium model in which both production and investment decisions of OPEC are strategic. They emphasize the requirement by financial markets that certain profitability measures are fulfilled so that investment can take place. Al-Qahtani et al. (2008), in contrast, focus on the role of Saudi Arabia; it is the only player that can behave strategically, while all other OPEC members charge an exogenously determined mark-up on top of marginal production costs.

Huppmann and Holz (2012) propose a spatial model for the crude oil market to compute prices and quantities produced and consumed, as well as trade flows, under different market structure assumptions over the time horizon 2005–2009. The approach includes arbitragers to account for liquid spot markets, which are an important characteristic of the global crude oil market. They find that a non-cooperative Nash-Cournot oligopoly by OPEC suppliers with Saudi Arabia as a Stackelberg leader and a competitive fringe best describes the crude oil market before the financial turmoil and the onset of a global recession in 2008; afterwards, the market was closer to the competitive benchmark.

Almoguera et al. (2011) approach the question of OPEC market power from the empirical side: rather than computing equilibria from fundamental cost and demand functions as it is done in the numerical work of Huppmann and Holz, they estimate the cost and mark-up parameters from a dataset ranging from 1974–2004. They also find evidence that OPEC is a non-cooperative oligopoly with a competitive fringe. However, their approach cannot capture two-stage market power in the Stackelberg sense, and they do not include the possibility of a shift in market power over time; instead, their approach only draws conclusions on the average behaviour over the entire period. These are the two issues I tackle in this work: the two-stage game aspect, where several Stackelberg leaders anticipate the optimal response of the competitive fringe given the credible commitment by the leaders; and the changing level of market power exertion depending on the spare capacity of the fringe.

I proceed as follows: first, I give an account of the debate concerning the consistency of a Nash-Cournot oligopoly and, more generally, conjectural variations. Then, I elaborate on the crude oil market and identify three characteristics that make it particularly interesting for the proposed Stackelberg oligopoly setting. Next, I formulate a simple bathtub model and derive conditions for equilibria in four different non-cooperative oligopoly market structures. Finally, I compute quarterly equilibria from 2003–2011 using these models and discuss how the market power of OPEC members changed before and after the financial crisis according to the proposed two-stage oligopoly setting.

2.2. Modelling market power in quantity games

It is quite natural to model fossil resource markets as a game in quantities. There are two standard cases: all players act perfectly competitive, i.e., they set price equal to marginal cost; and the Nash-Cournot equilibrium, where each player exerts market power, taking into account the reaction of the demand on its decision, while assuming that all rivals do not deviate from their quantity.[1]

Bowley (1924) and Frisch (1933) proposed *conjectural variations* (CV) as a way to elegantly model "intermediate" cases of imperfect competition or market power: instead of simply adding the mark-up warranted by a Cournot model on top of marginal costs in the price-setting of a supplier, a parameter is introduced to capture the expectation (or conjecture) of a supplier regarding the reaction (or variation of output) of the rivals. Setting this parameter accordingly allows to model a continuum of market power cases, ranging from the perfectly competitive market to the non-cooperative Nash-Cournot equilibrium to the cooperative cartel solution.[2]

However, an equilibrium computed from such an arbitrarily chosen, exogenous parameter is not based on any economic theory (cf. Figuières et al., 2004). Hence, a consistency problem arises: the conjecture of any agent need not be correct, i.e., it may not coincide with the actual reaction of the rival(s) (Laitner, 1980). In particular, in a Nash-Cournot equilibrium, each player follows the conjecture that all rivals will not react to any deviation. But in fact, when any player deviates from the equilibrium (by making a mistake, for instance), all rivals will also update their decision.

This observation initiated a stream of research which required the conjectures to be consistent or rational in equilibrium (e.g., Bresnahan, 1981; Perry, 1982). These extensions were subsequently criticized as well, since the rationality argument requires circular reasoning or information about the rival's cost function. To put it simply: in equilibrium, no agent can know how its rivals would respond to any deviation since no deviation ever actually occurs (cf. Makowski, 1987; Lindh, 1992).

Myopic strategic behaviour

Figuières et al. (2004) argue that, while an equilibrium based on CV may be in some sense arbitrary, it still offers a useful "shortcut" to capture more complex strategic interaction between players within a static framework. However, the applied partial-equilibrium models for natural gas and other energy markets mentioned in the introduction depart in one important way from the theoretical models: these application are usually interested in intermediate cases of non-cooperative oligopolistic behaviour, where some suppliers exert market power while others form a competitive fringe. This is commonly captured by assigning dif-

[1] One could also model the crude oil market as a game in *supply functions* (Klemperer and Meyer, 1989), but this would be mathematically challenging given the specific cost function, and distract from the main focus of this chapter. I therefore choose to remain in the realm of quantity games.

[2] A more extensive review of conjectural variations, including the mathematical formulation commonly used, is given in Haftendorn (2012) and Ruiz et al. (2010).

ferent conjectural variations to distinct suppliers, though these models have usually dropped the CV terminology and directly refer to "suppliers exerting Cournot market power" and "competitive or price-taking suppliers" (cf. Gabriel et al., 2013).

As shown by Ulph and Folie (1980), such an approach may yield rather counter-intuitive effects. They compare a competitive baseline to two models: first, one supplier acts as Cournot oligopolist vis-à-vis a competitive fringe, and treats the quantity supplied by the fringe as given; for the remainder of this discussion, I will refer to such a model as *Myopic Cournot Equilibrium* (MCE).[3] The authors show that under certain – not implausible – conditions, the myopic Nash-Cournot oligopolist in the MCE model earns lower profits than if he were to follow a competitive price rule (i.e., price equals marginal cost). This occurs because the myopic oligopolist does not consider that the fringe player will partly offset the quantity withheld.[4]

In the equilibrium of an MCE model, unilateral deviation would not improve the profits of the Nash-Cournot supplier(s); thus, the oligopolists fulfil the Nash equilibrium condition, while the competitive fringe follows the competitive price rule by assumption. But it is rather unsatisfactory to assume that certain suppliers pursue a strategy that leaves them worse off in equilibrium – and then claim that these players exert market power, as it is usually done in partial-equilibrium models that apply such an approach. Therefore, one must be rather careful in describing such a model as a representation of market power (cf. Ralph and Smeers, 2006).

A Stackelberg market is a two-stage game

In the second model studied by Ulph and Folie (1980), the supplier that exerts market power is a Stackelberg leader that takes into account the reaction of the fringe, rather than just the quantity it supplies. The model proposed in this work follows the intuition of this two-stage model: several Stackelberg leaders anticipate the response of the fringe in their optimization model.[5]

Mathematically, this yields a two-stage problem, and it can be treated formally as a Mathematical Problem under Equilibrium Constraints (MPEC), if there is one player in the upper-level problem, or Equilibrium Problem under Equilibrium Constraints (EPEC), if there are several players that interact non-cooperatively. EPECs have been proposed as the suitable approach to model electricity markets (Ralph and Smeers, 2006; Hu and Ralph, 2007), as well as more general hierarchical games (Kulkarni and Shanbhag, 2013).

A Stackelberg leader-follower model is discussed by Sherali et al. (1983); the followers are a number of Cournot players that take the quantity of the leader as given. The authors derive existence and uniqueness results under certain standard assumptions, and propose a

[3] The term "myopic" indicates that the oligopolistic supplier does not consider the reaction of the rivals.
[4] A numerical example of this effect is shown in Gabriel et al. (2013, p. 108).
[5] One alternative approach to including the first-order optimality conditions of the fringe in a Nash-Cournot oligopoly model is to subtract the quantity supplied by the fringe from the demand, and let the oligopoly face the residual demand curve. This is, however, problematic if the fringe supply cannot easily be computed (cf. Bushnell, 2003).

numerical solution algorithm. Lise and Kruseman (2008) propose an electricity market model where the level of market power exertion by the Stackelberg leader is directly modelled as a decision variable. They solve the model by discretizing the conjecture between competitive behaviour and standard Cournot market power exertion.

Chen et al. (2006) extend the Stackelberg game to two markets: a strategic firm is active both in the power market and the market for nitrogen oxide (NO_X) emission allowances, with several other firms active either as Cournot players or as competitive fringe. They apply the model to the Pennsylvania–New Jersey–Maryland (PJM) power market, where generators have to purchase emission certificates in order to produce electricity in conventional power plants. They find instances where the Stackelberg leader is able to increase its profits by withholding allowances in the NO_X market.

In contrast, Haftendorn (2012) analyses a model for a game in only one market (steam coal), but with several Stackelberg leaders vis-à-vis a competitive fringe. This model is applied to the Atlantic steam coal market, and the numerical results are then used to gauge whether market power was exerted in this market over the time period 2003–2006. Furthermore, the author also applies a Nash-Bargaining-type cartel (cf. Harrington et al., 2005). In this type of collusion, the objective of the cartel is not to maximize joint profits, but each player has to benefit above the level that it would earn in a Nash-Cournot market.

Sherali et al. (1983), Lise and Kruseman (2008), and Haftendorn (2012) use ways to elegantly circumvent the problems posed by the two-level nature of the game they study. Chen et al. (2006) propose an algorithm using several different solvers iteratively. The problem lies in the non-convexity of the feasible region when complementarity conditions are included in the lower level of the game. When tackling more general two-level problems directly, standard constraint qualifications fail for these models even in simple applications. As a consequence, establishing existence and uniqueness of a solution is non-trivial (cf., Gabriel et al., 2013), and numerical routines often fail to find the global optimum. In a way, this problem of violating constraint qualifications is reflected in the model I propose in this work, as I am not able to provide a succinct proof of uniqueness.

In this work, I propose a Stackelberg oligopoly model to properly capture market power exertion by OPEC members. They form a non-cooperative oligopoly amongst each other, but anticipate – in the Stackelberg sense – the response of the competitive fringe. This is accomplished by implicitly including the reaction function of the fringe in each oligopolist's profit maximization problem: the oligopolists have consistent conjectures regarding the fringe. But before turning to the model itself, I discuss several features and characteristics of the crude oil market that make it a particularly interesting and relevant application.

2.3. The crude oil market

The market structure in the crude oil sector in general and the role of OPEC, in particular, is still surrounded by controversy. As discussed in the introduction, the crude oil market underwent drastic upheaval in 2007–2008, and I believe that the proposed model sheds some light on this. In addition to these more general reasons, there are three aspects that have theoretical and practical import.

Three reasons why oil is interesting

A credible Stackelberg leader

The notion of a two-stage, Stackelberg game, in which one agent decides first taking the response of its rivals into account, is quite straightforward in theory. However, when applied to real world problems, one must argue carefully whether the two-stage setting is plausible – in game-theoretic terms: whether the commitment of the leader to maintain its decision is credible. Otherwise, the game would revert, following a tâtonnement process, to a Nash-Cournot equilibrium.

Almoguera et al. (2011) argue that OPEC can signal through its quota changes, both to other suppliers and to traders in the downstream market; this is the case even if the quota is not strictly adhered to by OPEC members (Dibooglu and AlGudhea, 2007). The quota allocations are only changed every couple of months, hence it is rational for other suppliers to assume that their short-term production decision will not affect the quota and hence OPEC output.

Instantaneous reactions & epistemology

The concept of consistent conjectural variations requires, in principle, that each player reacts instantaneously. However, instantaneous reactions are difficult to reconcile with most actual markets, due to rigidities and lack of information. This is not the case in the crude oil market. As I have argued, OPEC can credibly commit to a quota for an extended period of time; in contrast, crude oil is traded in very liquid markets at a high frequency, so the followers – not bound by a quota – feel the impact of their output decisions virtually immediately. This is, I believe, sufficiently close to instantaneous to warrant the use of consistent conjectural variations in this application.

There is one further aspect of both Stackelberg leadership and the use of consistent conjectural variations: the requirement that the leader knows the actual reaction of the rivals – and not just the equilibrium quantity as in a standard Nash-Cournot game. Again, the crude oil sector satisfies this requirement: the market for oil-related services – such as suppliers and operators of oilfield equipment, firms specializing in exploration, and business intelligence providers – is quite concentrated and the OPEC members, collectively, have substantial expertise. Hence, it is reasonable to assume that the OPEC members have a rather good understanding of their rivals' operations and cost structure, and can therefore predict their reactions to a price change.

Non-standard cost functions

In most theoretical and applied work on oligopoly theory, either linear or quadratic cost functions are used. This facilitates some proofs, but in combination with linear demand curves leads to a very strong simplification: the derivative of the optimality condition of each player, and hence the derivative of each player's reaction function, is constant. This translates to constant consistent conjectural variations.

When looking at crude oil production costs – and extractive industries in general – one notices that marginal production costs are quite flat for most of the feasible range, but then increase sharply when producing close to capacity. There are both engineering explanations, such as the need for additional equipment, increased wear-and-tear, more complex technology (water or CO_2 injection), as well as economic reasons: pumping oil too quickly leads to a deterioration of reservoir quality and even a decrease of recoverable resources. A production cost function that exhibits these characteristics will be formally introduced below. The important aspect, which is driving the model, is that the consistent conjecture is not constant any more.

The Hotelling rule isn't in the details

The above argument camouflages what many economists may consider a major omission in this work: theory postulates that when supplying a finite resource, its price must rise in lock-step with the rate of interest due to the consideration of inter-temporal arbitrage. This is known as "Hotelling rule" (Hotelling, 1931), and virtually all theoretical models use it in one way or another (e.g., Salant, 1976; Hoel, 1978; Newbery, 1981). Nevertheless, the real crude oil price fails to exhibit an exponential price increase over the long-term. Hart and Spiro (2011) and Livernois (2009) review extensions of the Hotelling rule to rationalize this phenomenon: these include technological progress, a backstop technology, increasing costs relative to remaining reserves, and uncertainty. The authors also cite empirical work that attempts to identify the scarcity rent as postulated by the Hotelling rule. They conclude – quite forcefully – that the Hotelling rule is of minor importance in today's crude oil market.

This work implicitly assumes that only short-term scarcity rent (i.e., insufficient production capacity and the resulting high marginal costs) is a major driver of oil prices, but that long-term scarcity rent (i.e., rents due to the exhaustibility of crude oil) are negligible. For simplicity, I therefore neglect all inter-temporal considerations other than what can be captured in the production cost function, as discussed above. The model presented in this work is – in each period – a one-shot quantity game comparing different behavioural assumptions. Capacity is fixed and exogenously given; I abstract from investment in new production capacity due to the significant lead-time. I will discuss possible extensions in the last section.

2.4. A bathtub model

A simple model is used to describe and compare several instances of non-cooperative supplier behaviour in the global crude oil market. As there are several suppliers of crude oil, but only one aggregated demand function and one global price for crude oil, such a model is usually called a bathtub model: several faucets, but only one drain. This simplification is frequently used in crude oil market analysis, in spite of quality differences and transport costs.

There is a set of suppliers that may form an oligopoly, denoted by S. In addition, there is one (aggregated) fringe supplier, f. For general notation relating to all suppliers, I use the indices i, j without stating the set, i.e., to be read as $i, j \in \{S \cup f\}$.

The profit maximization problem of a supplier i can be written as follows:

$$\max_{q_i \in \mathbb{R}_+} \quad p(Q)q_i - c_i(q_i) \tag{2.1}$$

Here, q_i is the quantity produced by that supplier, while Q is the total quantity supplied to the market. Price depends on total quantity supplied, given by an inverse demand function $p(\cdot)$, and production costs are denoted by $c_i(q_i)$; capacity constraints will be implicitly included in the cost function.

The first-order optimality condition (also called Karush-Kuhn-Tucker, or KKT, condition) of supplier i is then given by:

$$0 \geq p(\cdot) + p'q_i + p'\frac{\partial q_{-i}(q_i)}{\partial q_i}q_i - c_i'(q_i) \perp q_i \geq 0 \tag{2.2}$$

The production decision of a supplier i implicitly impacts the quantity produced by its rivals; hence, I write $q_{-i}(q_i)$ to express this effect. As is common in the conjectural variations literature, the term $r_i := \frac{\partial q_{-i}(q_i)}{\partial q_i}$ denotes the conjecture of player i regarding the aggregated reaction of the rivals. Note that this conjecture need not be correct. I distinguish three different conjectures: price-taking behaviour, where the supplier assumes to have no impact on the price; the Cournot conjecture, where the supplier believes to have no impact on the quantity supplied by its rivals, but considers the reaction of the price to its decision; and correct (i.e., consistent) conjectures, where the conjecture coincides with the actual reaction of (some of) the rivals.

For any solution q_i^* to the KKT condition (2.2) to be indeed a (local) maximum, rather than only a stationary point, one also has to check whether the profit function is concave (at that point).[6] This holds if the second derivative of the profit maximization problem is less than or equal than 0 (at that point). The condition is stated below; for simplicity, I assume that the second derivative of the price with respect to quantity is zero (i.e., a linear demand function). This will be formalized later.

$$p'\left(2 + 2\frac{\partial q_{-i}(q_i^*)}{\partial q_i} + \frac{\partial^2 q_{-i}(q_i^*)}{\partial q_i^2}\right) - c_i''(q_i^*) \leq 0 \tag{2.3}$$

This condition will be discussed in more detail after the specification of the different market power assumptions. Before I proceed, the functional form of the inverse demand and cost functions are specified by the following assumptions:

[6] In this work, quasiconcavity of the profit function cannot be guaranteed in general due to the consideration of the rivals' reaction in a suppliers optimization problem, hence multiple local maxima may exist; such a case will be illustrated in Example 8.

A1 The inverse demand function is linear, its slope is negative, and quantities from different suppliers are perfect substitutes, i.e., $p(Q) = a - b\,Q$, where Q is the total quantity supplied. Parameters a and b are strictly positive.

A2 The production cost function of each supplier i follows the form proposed by Golombek et al. (1995). It includes a logarithmic term depending on capacity utilization:

$$c_i(q_i) = (\alpha_i + \gamma_i)q_i + \beta_i q_i^2 + \gamma_i(\overline{q_i} - q_i)\ln\left(1 - \frac{q_i}{\overline{q_i}}\right) \tag{2.4a}$$

$$c_i'(q_i) = \alpha_i + 2\beta_i q_i - \gamma_i \ln\left(1 - \frac{q_i}{\overline{q_i}}\right) \tag{2.4b}$$

$$c_i''(q_i) = 2\beta_i + \gamma_i \frac{1}{\overline{q_i} - q_i} \tag{2.4c}$$

$$c_i'''(q_i) = \gamma_i \frac{1}{(\overline{q_i} - q_i)^2} \tag{2.4d}$$

The cost function parameters α_i, β_i and γ_i are strictly positive for each supplier i. The parameter $\overline{q_i}$ is the maximum production capacity.

Lemma 1. Under Assumption **A2**, the range of feasible production quantities is implicitly bounded from above by the capacity $\overline{q_i}$.

Lemma 2. Under Assumption **A2**, the cost function (2.4a) and its first, second and third derivative (2.4b–2.4d) are strictly positive, strictly monotone and strictly convex for any feasible production quantity $q_i \in (0, \overline{q_i})$.

Following Assumption **A1**, we can rewrite Equation (2.2):

$$0 \geq a - b\sum_j q_j - b\,(1 + r_i)\,q_i - c_i'(q_i) \perp q_i \geq 0 \tag{2.5}$$

Hence, the optimal output decision of supplier i is determined by the price in relation to production costs, adjusted by its (conjectured) impact on the price. This adjustment term can be interpreted as the mark-up that the supplier charges in addition to its marginal costs.

The oligopoly cases

I distinguish four cases of oligopoly: they differ in the assumptions of each supplier regarding the reaction of its rivals to any variation in his output. The first two cases are the "pure" oligopoly theories in the perfect competition and Cournot sense, respectively. The third case is the myopic oligopoly model with a competitive fringe, MCE, as discussed before. These three cases are well studied in theory and frequently applied in numerical equilibrium models.

The fourth case is the addition to the literature by this work: a Nash-Cournot oligopoly, where each oligopolistic supplier has consistent conjectures regarding the reaction of the fringe. Furthermore, the consistent conjecture is not constant, but depends on the quantity supplied by the fringe due to the choice of cost function. As a consequence, the mark-up charged by oligopolistic suppliers in addition to marginal costs changes endogenously depending on the capacity utilization of the fringe.

Perfect competition

Each supplier assumes that its decision does not influence the market price, and he treats the price as a parameter. This is usually called "price-taking behaviour". Therefore, the first-order condition reduces to $0 \geq p - c_i'(q_i) \perp q_i \geq 0$. This can be mimicked in Equation 2.5 by setting $r_i = -1$; it can be interpreted as the player following the assumption that any own quantity change is fully compensated by the rivals, thus the total quantity on the market does not change.

Nash-Cournot oligopoly

Each supplier, including the fringe, assumes that its actions have no impact on the actions of his rivals: $r_i = 0$. Each supplier acts as a monopolist with respect to the residual demand curve given the quantity supplied by the rivals.

Nash-Cournot oligopoly with fringe

The suppliers that are members of the oligopoly (i.e., OPEC) act as Nash-Cournot players ($r_i = 0 \ \forall \ i \in S$); the fringe acts as price-taker ($r_f = -1$). This is the Myopic Cournot Equilibrium (MCE) discussed previously.

Lemma 3. Under Assumptions **A1** and **A2**, the KKT system (2.5) has a unique solution in each of the cases *Perfect competition*, *Nash-Cournot oligopoly* and *Nash-Cournot oligopoly with fringe*. The second-order derivative condition (2.3) holds everywhere on the feasible region.

Proof. The Jacobian matrix of the KKT system is symmetric and positive definite, hence existence and uniqueness is established (cf. Facchinei and Pang, 2003).

Realizing that r_i is constant by assumption and its partial derivative is thus 0, the second-order derivative condition (2.3) can be written as follows:

$$-b\,(2 + r_i) - c_i''(q_i) \leq 0$$

As $r_i \in [-1, 0]$ and $c_i''(q_i) > 0$ following Lemma 2, this condition holds trivially with strict inequality for all $q_i \in [0, \overline{q_i})$. The profit function of each supplier is thus strictly concave on the feasible region. □

In all of these cases, each supplier does not distinguish between its rivals; it has an aggregated conjecture regarding the total response. I now turn to an oligopoly that has a more elaborate approach to strategic behaviour.

Consistent conjecture oligopoly with fringe

The consistency requirement postulates that the conjecture of the player must be "correct", i.e., it must be equal to the actual reaction of the rivals to a variation in quantity. There is a conceptual difficulty in games with more than two players, as each rival's reaction in turn depends on the reaction of the other players. Kalashnikov et al. (2011), Ruiz et al. (2010), and Liu et al. (2007) all assume that all suppliers have consistent conjectures regarding all rivals (with the exception of a social welfare-maximizing player in the first article). Then, they derive closed-form expressions for this term under the assumption of quadratic cost functions and a linear demand curve.

In contrast, I use the idea of consistent conjectural variations to model a two-stage oligopoly: an oligopoly takes into consideration the reaction of a competitive fringe, but follows the Cournot conjecture amongst each other. Before formalizing this, I need to introduce some additional notation. Following Liu et al. (2007), the conjecture regarding the aggregated rivals' reaction can be separated:

$$r_i := \frac{\partial q_{-i}(q_i)}{\partial q_i} = \sum_{j \neq i} \frac{\partial q_j(q_i)}{\partial q_i} =: \sum_{j \neq i} r_{ij}$$

This term states that the aggregated reaction of the rivals can be separated into the sum of individual responses of each rival j, r_{ij}. Let $\rho_j(q_i)$ denote the actual reaction function of supplier j to the quantity supplied by supplier i – in contrast to $q_j(q_i)$ previously used, which denotes the conjecture of player i regarding the response of supplier j. Now, I can state the assumptions underlying the oligopoly with consistent conjectures regarding the fringe formally.

A3 The oligopoly suppliers $i \in S$ follow the Cournot conjecture amongst each other and have consistent conjectures regarding the fringe:

$$\begin{aligned} \frac{\partial q_j(q_i)}{\partial q_i} &= 0 && \forall\, j \in S \\ \frac{\partial q_f(q_i)}{\partial q_i} &= \frac{\partial \rho_f(q_i)}{\partial q_i} \end{aligned}$$

The fringe supplier f follows a competitive pricing rule: $r_f = -1$.

In addition, I need a restriction on the cost parameters of the fringe player to simplify the following notation.

A4 The marginal cost of the fringe supplier at zero production is strictly less than the price at maximum production of the oligopoly, which is the minimum possible price if the fringe player does not produce; mathematically:

$$c'_f(0) = \alpha_f < p\left(\sum_{i \in S} \overline{q_i}\right)$$

Lemma 4. Under Assumptions **A1**, **A2** and **A4**, the fringe supplier always produces a positive quantity q_f in equilibrium.

Proof. Assume that $q_f = 0$. Starting from Equation (2.5) and replacing $c'_f(0)$ by $p\left(\sum_{i\in S}\overline{q_i}\right)$ according to Assumption **A4** yields the following:

$$a - b\sum_{j\in S} q_j - c'_f(0) > a - b\sum_{j\in S} q_j - a + b\sum_{j\in S}\overline{q_j} = b\sum_{j\in S}(\overline{q_j} - q_j) > 0$$

This is a contradiction to the first-order condition of the fringe supplier. □

This lemma has an important interpretation: the oligopoly does not have sufficient production capacity to force the fringe supplier out of the market; hence, q_f will always be positive in equilibrium. Assumption **A4** and Lemma 4 allow us to omit the rather tedious case of establishing conjectures if a supplier is not producing, or of limit-pricing strategies by a monopolist (cf. Hoel, 1978).

Lemma 5. Under Assumptions **A1**, **A2**, **A3** and **A4**, each oligopoly supplier's conjectural variation equals the reaction of the fringe supplier, and it has the following functional form:

$$r_i = \sum_{j\neq i} r_{ij} = \frac{\partial q_f(q_i)}{\partial q_i} = -\frac{b}{b + c''_f(q_f)} \in (-1, 0) \quad \forall\, i \in S$$

Furthermore, r_i is continuous with respect to $q_f \in [0, \overline{q_f})$.

Proof. Following Assumption **A4** and Lemma 4, the first-order condition for the fringe supplier must hold with equality. Furthermore, this equality implicitly defines the fringe's output as a reaction to the output by firm i.

$$a - b\left(q_i + \sum_{j\in S\setminus\{i\}} q_j + q_f(q_i)\right) - c'_f\Big(q_f(q_i)\Big) = 0 \tag{2.6}$$

According to Assumption **A3**, each oligopoly supplier conjectures that the other oligopoly suppliers do not react to its output variation; hence, I write q_j rather than $q_j(q_i)\ \forall\, j \in S \setminus \{i\}$.

Taking the derivative of Equation (2.6) with respect to the output of an oligopoly supplier $q_i,\ i \in S$, and using the implicit function theorem yields the optimal response of the fringe to a variation in output by supplier i:

$$-b - b\frac{\partial q_f(q_i)}{\partial q_i} - c''_f\Big(q_f\Big)\frac{\partial q_f(q_i)}{\partial q_i} = 0$$

$$\Rightarrow r_{if} := \frac{\partial q_f(q_i)}{\partial q_i} = -\frac{b}{b + c''_f(q_f)} \tag{2.7}$$

Each oligopoly supplier conjectures that every rival apart from the fringe supplier does not react, hence its conjectural variation term reduces to the conjecture regarding the fringe. Noting that $c''_f(q_f) > 0\ \forall\, q_f \in [0, \overline{q_f})$ and continuous yields $r_i \in (-1, 0)$ and the continuity of r_i. □

Using Lemma 5, I can rewrite the system of suppliers' first-order conditions (2.5) as follows:

$$0 \le -a + b\sum_j q_j + b\left(1 - \frac{b}{b + c_f''(q_f)}\right) q_i + c_i'(q_i) \perp q_i \ge 0 \quad \forall\, i \in S \tag{2.8a}$$

$$0 \le -a + b\sum_j q_j + c_f'(q_f) \perp q_f \ge 0 \tag{2.8b}$$

Allow me to briefly discuss the term $r_i = r_{if} = {}^{-b}\!/_{b+c_f''(q_f)}$, the fringe's reaction, and relate it to earlier consistent conjectural variations literature. This term is similar to the examples discussed by Bresnahan (1981) and others if one uses quadratic or symmetric linear costs. In these cases, Assumption **A5** (see below) will hold trivially in the absence of capacity constraints. The equilibrium would not be as straightforward, however, in the case of asymmetric linear costs; in such a case, one would have to consider limit pricing or other more elaborate formulations.

Furthermore, due to the cost function used here, the fringe's reaction is not constant, and it is here that the proposed model departs from the previous literature – and it is here where two ambiguities arise, compared to the other oligopoly cases discussed before: first, uniqueness is not guaranteed, and the second-order derivative condition (Equation (2.3)) does not necessarily hold everywhere on the feasible region (i.e. the profit function may not be concave or not even quasi-concave). The second-order derivative condition is included in the theorem below; the following assumption provides a condition and test for uniqueness.

A5 Assume that the parameters satisfy the following inequality, where x and y are feasible production vectors:

$$\begin{aligned} &b\left(\sum_j (x_j - y_j)\right)^2 + \sum_j \left(c_j'(x_j) - c_j'(y_j)\right)(x_j - y_j) \\ &+\ b\sum_{i\in S}\left[\left(1 - \frac{b}{b + c_f''(x_f)}\right) x_i - \left(1 - \frac{b}{b + c_f''(y_f)}\right) y_i\right](x_i - y_i) > 0 \\ &\qquad \forall\, x, y \in \prod_j\, [0, \overline{q_j}), x \neq y \end{aligned}$$

Theorem 6. Under Assumptions **A1**, **A2**, **A3** and **A4**, a solution $\left((q_i^*)_{i\in S}, q_f^*\right)$ to the KKT system (2.8) always exists. It is indeed an equilibrium if it satisfies the second-order derivative condition:

$$-b\left(2 - 2\frac{b}{b + c_f''(q_f^*)} - \frac{b^2 c_f'''(q_f^*)}{(b + c_f''(q_f^*))^3} q_i^*\right) - c_i''(q_i^*) \le 0 \;\forall\, i \in S \tag{2.9}$$

Furthermore, if Assumption **A5** is satisfied, the solution is unique.

Proof. See Appendix A.

Before discussing the intuition and practical verification of Assumption **A5** for given parameters, I would like to refer to one other potential avenue to prove uniqueness: Sherali et al. (1983) present a model of one Stackelberg leader and a number of Cournot followers. They show that the total quantity supplied by the (lower-level) Nash-Cournot oligopoly is – as a function of the leader's quantity – unique, convex and decreasing under certain assumptions. If, in addition, the cost function of the leader is strictly convex, the leader's problem is a strictly concave maximization problem and has a unique solution.

However, the logarithmic cost function used in this work to represent the characteristics of extractive industries leads to a violation of the convexity of the lower-level's response. This would be true for any similar function with the desired properties for extractive industries, e.g., a piece-wise linear approximation of Equation (2.4b), as well as any model with capacity constraints in the lower level.

This leads to a problem when solving the model proposed in this chapter with parameters derived from actual data; it is difficult to show that Assumption **A5** holds and hence, that the solution obtained is unique. First, note that the Jacobian $J(\cdot)$ of the KKT system (2.8) is neither symmetric nor necessarily positive definite. Therefore, one cannot proceed as easily as in Lemma 3.

I therefore propose an alternative, numerical approach, namely requiring strong monotonicity of the equivalent Variational Inequality (VI, defined formally in Chapter 1.2, Definition 4, and discussed in the Appendix) – this is the interpretation of Assumption (**A5**). The first term is a square, hence positive. Because the marginal cost function is strictly monotone (cf. Lemma 2), the second term is a sum of strictly positive terms. The sign of the third term, however, is ambiguous. Furthermore, the entire term is not convex.

Nevertheless, whether Assumption (**A5**) holds for given parameters of a numerical application can be verified by solving the following optimization problem:

$$\begin{aligned}\min_{\substack{x,y\in K\\ x\neq y}} \; & b\left(\sum_j (x_j - y_j)\right)^2 + \sum_j \left(c'_j(x_j) - c'_j(y_j)\right)(x_j - y_j)\\ & + \; b\sum_{i\in S}\left[\left(1 - \frac{b}{b + c''_f(x_f)}\right)x_i - \left(1 - \frac{b}{b + c''_f(y_f)}\right)y_i\right](x_i - y_i) \qquad (2.10)\end{aligned}$$

The objective value is 0 for $x = y$. Hence, if the infimum of problem (2.10) is also equal to zero, the KKT system (2.8) has a unique solution. Solving this optimization problem is, in itself, a formidable challenge, because it is non-convex. One can try to solve it repeatedly using different starting points, but this, of course, does not guarantee finding the optimal solution. It would be more elegant to present a closed-form expression of sufficient assumptions for a unique equilibrium. However, I could not (yet) find a practical approach.

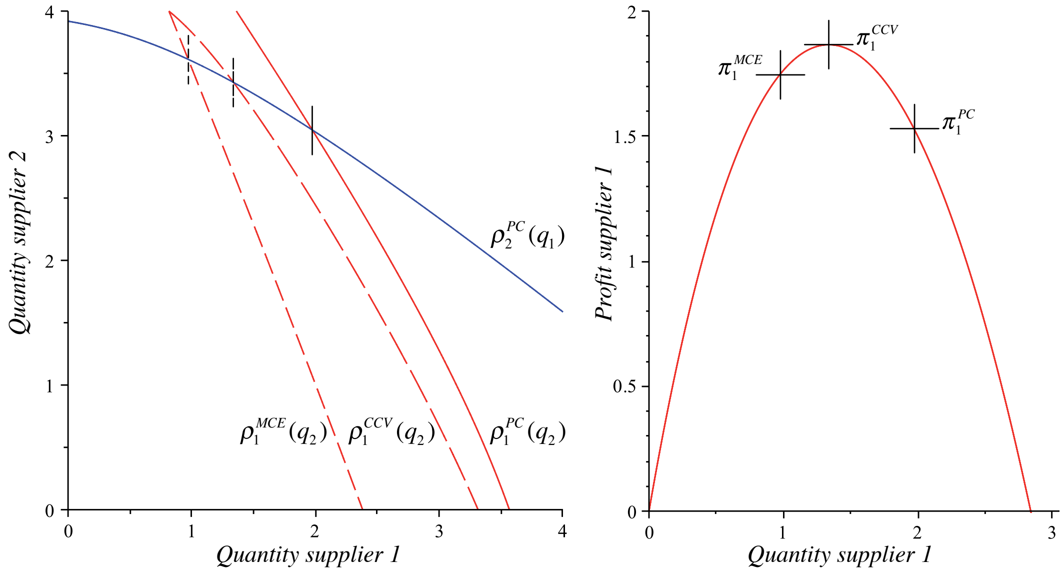

Figure 2.1.: Illustration to Example 7

2.5. A simple numerical example

In order to illustrate the differences between the oligopoly cases and the issue of non-uniqueness, I present two simple examples of two-player games. Supplier 2 acts as competitive fringe, while supplier 1 is an oligopolist exerting market power using the different conjectures discussed before: myopic Cournot behaviour (MCE), the Stackelberg leader-follower behaviour implemented using consistent conjectures regarding the fringe (CCV), and – as a benchmark – perfectly competitive behaviour (PC).

The first example serves to illustrate two points: the optimal exertion of market power "converges" to the Nash-Cournot solution when the fringe player reaches its capacity limit; and the profit earned when market power is exerted in the Stackelberg sense is always higher than under myopic Cournot behaviour.

Example 7. Assume two suppliers 1 and 2, facing the following cost curves and inverse demand function:

$$\begin{aligned} c_1(q_1) &= \ (1+1)q_1 + 0.2q_1^2 + 1(4-q_1)\ln\left(1 - \frac{q_1}{4}\right) \\ c_2(q_2) &= \ (1+0.6)q_2 + 0.1q_2^2 + 0.6(4-q_2)\ln\left(1 - \frac{q_2}{4}\right) \\ p(Q) &= 10 - 1.5Q \end{aligned}$$

The capacity limit of each supplier is 4 units.

The reaction functions are shown in the left-hand part of Figure 2.1: $\rho_i(q_j)$ is the reaction function of player i to the quantity supplied by player j, where the superscripts refer to the

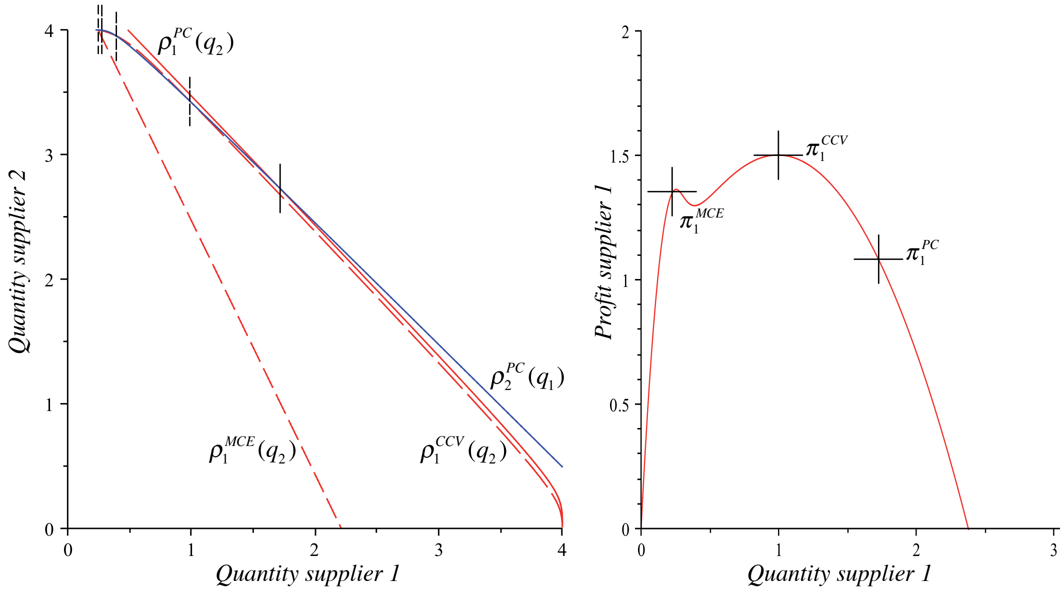

Figure 2.2.: Illustration to Example 8

market power case.[7] The equilibria are marked by vertical lines. This figure illustrates how the optimal response of the Stackelberg leader converges from the competitive case, if the fringe (supplier 2 in this example) is not constrained, to the Nash-Cournot reaction function when the fringe approaches its capacity limit.

The right-hand part of Figure 2.1 illustrates the profit of the oligopolist considering the reaction of the competitive fringe supplier. In this example, the profit generated under the myopic Cournot conjecture is indeed higher than the profit under competitive marginal-cost pricing. Many applied studies discussed in the introduction jump, from this observation, to the conclusion that MCE behaviour is equivalent to the optimal exertion of market power. This figure illustrates that this is not the case: instead, producing more than under the MCE conjecture yields higher payoff for the oligopolistic supplier, with the maximum attained at the consistent conjectural variations equilibrium.

The second example illustrates that there may exist several solutions at which the first- and second-order conditions are satisfied.

Example 8. The assumptions regarding the suppliers are identical to Example 7, but the inverse demand function is $p(Q) = 100 - 22Q$. As illustrated in Figure 2.2, the reaction functions $\rho_2^{PC}(q_1)$ and $\rho_1^{CCV}(q_2)$ now intersect multiple times. There exist two equilibria, and one point where the first-order conditions of both suppliers are satisfied, but the second-order derivative condition is violated. The latter point is actually a local profit minimum, as can be seen in the right-hand part of Figure 2.2. It is obvious that the profit curve of the

[7]The reaction functions are computed by solving the first-order condition given the quantity of the other player.

oligopolistic supplier is not quasi-concave.

It is straightforward that in this example, a Stackelberg leader that satisfies the epistemological qualifications (i.e., has sufficient knowledge to exert market power in the sense discussed here) would choose the equilibrium where it produces more, earning π_1^{CCV}. It is less clear, however, whether a numerical solver would find this equilibrium. If it were to terminate in the other local maximum, that would be unfortunate; if, however, it would terminate in the local minimum, this would be outright wrong, as this is not a Nash equilibrium. Indeed, when solving this problem in GAMS and setting the starting values for the PATH solver accordingly, all three intersections of the ρ_1^{CCV} and ρ_2^{PC} curves could be obtained as results, and the solver claimed optimality in all cases.

2.6. A crude oil application

Let's now turn to the actual question of this chapter: endogenous market power of OPEC suppliers over the past years. OPEC membership changed over the time period under investigation. To avoid shifts in the capacity share of OPEC, I assume the following countries to be OPEC members over the entire period: Algeria, Angola, Ecuador, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, Venezuela, and the United Arab Emirates.[8]

Data

In contrast to the model proposed in Huppmann and Holz (2012), I use quarterly data. The problem with any analysis of the crude oil market is the lack of available, reliable and consistent data, as pointed out by Smith (2005). Regarding quantities produced and consumed, I rely exclusively on IEA data, in particular the "Quarterly Statistics" (IEA, 2012, and earlier versions). Production data in the Quarterly Statistics are disaggregated by country and three oil types: crude, natural gas liquids (NGL), and non-conventional.[9] IEA frequently updates their published data, so I only use data from publications at least 6 quarters after the fact. The categories *Global biofuels production* and *processing gains*, which are not assigned to a country in the IEA reports, are treated as if produced by independent suppliers. This data is complemented with information from the monthly IEA Oil Market Reports (OMR).[10]

To derive production capacity, I use the following methodology: for each period, country and oil type (crude, NGL, and non-conventional), production is such that the average over the preceding and following four quarters is 95 % of capacity. If actual production is above this value due to a short-term spike, I assume that production is at 98 % of capacity in this period.[11]

[8] Angola and Ecuador (re-)joined in 2007; Indonesia suspended membership in 2009; OPEC website (`http://www.opec.org/opec_web/en/about_us/25.htm`, accessed Feb 14, 2013).

[9] Crude oil production in the "Neutral zone" are shared equally between Saudi Arabia and Kuwait, in line with IEA methodology.

[10] Oil market report website (`http://omrpublic.iea.org/`, accessed Feb 1, 2013).

[11] For Iraq (2003) and Lybia (2011), actual production is assumed to be at 98 % of capacity in each period

For OPEC countries, the IEA publishes a measure called *sustainable production capacity* (SPC) in the OMR. The definition of the sustainable production capacity is that the production level can be reached within 30 days and be sustained for 90 days. This fits nicely with the quarterly data that I use for this analysis. If available, I use this data rather than the capacity derived from the above methodology.[12]

This methodology guarantees three important aspects: first, I capture the trends in each country and oil type; second, in each period reference production is below capacity. Third, and most importantly, this approach yields aggregate capacity time series in line with those reported by the IEA and EIA, even though spare capacity in 2008 is most likely overestimated.

Production costs are divided into two parts: production/lifting costs, on the one hand, are derived from Aguilera et al. (2009), who estimate average production costs for a large number of fields worldwide. I assume that these costs increase by 5 % p.a.; unconventional oil is assumed to be twice as expensive as crude oil in every country. On the other hand, crude oil has to be shipped to market, and low-quality crude is traded at a discount. Shipping costs are a function of the oil price, therefore each country is assigned a scalar (ranging from 1–3 based on distance to markets and oil quality), and a linear trade cost term based on the actual crude oil price and multiplied with that scalar is added to the cost function.

To obtain the linear inverse demand curve, I use actual quarterly demand (from IEA, as above) and the global average crude oil price (obtained from Datastream, a Thomson Reuters information service) as reference demand points. The curve is then fitted assuming a demand elasticity at the reference of point of -0.10, as discussed by Hamilton (2009).

Results

This section presents the numerical results for the four market power cases: Perfect competition (*Competition*); Nash-Cournot oligopoly (all suppliers exert market power, *Nash-Cournot*);[13] Nash-Cournot oligopoly with fringe (OPEC members are Cournot players in the MCE sense, *Myopic Cournot*); and consistent conjecture oligopoly with fringe (OPEC members have consistent conjectures regarding the fringe, *Oligopoly*).

The equilibrium prices computed in each of the four market power cases and the reference crude oil price are shown in Figure 2.3; the order of results is intuitive: *Nash-Cournot* yields the highest prices, while *Competition* exhibits the lowest. *Myopic Cournot* is in between these two extremes, albeit it moves in relative lockstep to the first two cases. In contrast, the price according to the *Oligopoly* case fluctuates between the competitive and the myopic

to account for the war-related production stops. To consider hurricane-induced outages in the Gulf of Mexico, I assume that during the third and fourth quarter of 2005, total production capacity of the United States was reduced by 0.5 mbbl/d, and during the third quarter of 2008, it was reduced by 1 mbbl/d (cf. OMR Oct 10, 2008 and OMR Jan 17, 2006).

[12] There are instances where due to updates, actual production reported a year after the fact is higher than reported capacity in the quarter immediately after the fact. In this case, the estimate derived from actual production is used.

[13] For this case, the fringe was disaggregated by country; otherwise, one Cournot player would control around 60 % of capacity and this would not be a plausible benchmark.

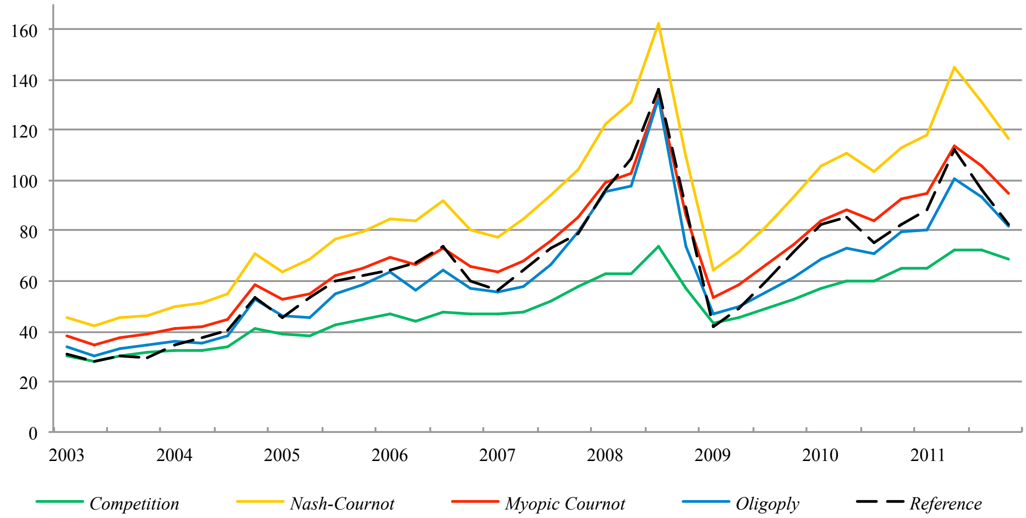

Figure 2.3.: Equilibrium price by market power case and reference (in US $/bbl)

Cournot case. In particular, it matches reasonably well the actual price path over the time period: close to competitive in 2003, converging to the myopic Cournot case until 2008, and a reversion to the competitive price benchmark after the onset of the financial crisis and the global recession.

The market power conjecture in the two-level, Stackelberg model is shown in Figure 2.4. The consistent conjecture of OPEC of its market power increases steadily until the third quarter of 2008, then drops drastically, and increases again over the time period 2010-2011.

The aggregate supply of OPEC is shown in Figure 2.5.[14] In any model with fixed endogenous conjectures, the share of a certain supplier to total supply is roughly constant; this is due to the characteristic that the mark-up charged by each Cournot player is a constant multiplied by this supplier's own production quantity. Hence, in a demand contraction, all suppliers reduce approximately by the same relative amount. This can be seen in the numerical results in the three constant conjecture cases.

When the market power conjecture is endogenous, however, there is a countervailing effect: demand is reduced, hence there is a downward pressure on supply; at the same time, the fringe has more spare capacity when it reduces its supply, thereby leading to a lower consistent conjecture. The supplier therefore reduces the mark-up on marginal costs that it demands, and this has an expansionary effect on its supply. This expansionary effect outweighs the contraction in the current application, as can be seen over the course of the year 2008 in the simulation results. This cannot be reconciled with the actual events in that time period. In general, no market power case fits the observed production levels; the Nash-Cournot case seems to match observed OPEC aggregate production, however, the aggregation masks that the individual OPEC suppliers are not as well matched as Figure 2.5 suggest.

[14]The OPEC production capacity reduction in 2003 is due to the war in Iraq, and the drop in 2011 is caused by the war in Lybia.

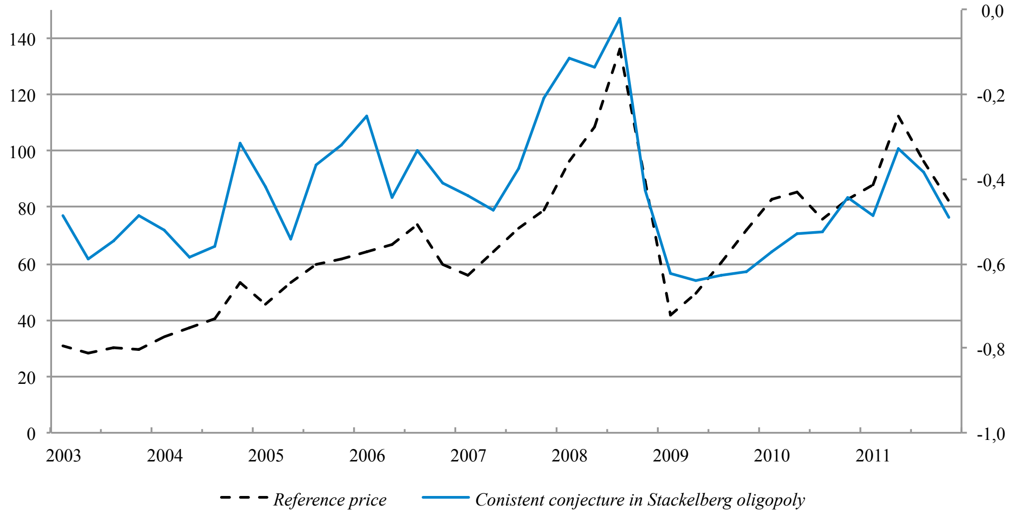

Figure 2.4.: Crude oil reference price (left axis, in US \$/bbl) and market power conjecture by OPEC oligopolists regarding the fringe (right axis; Cournot conjecture: 0; competitive behaviour: -1)

To provide a statistical measure of the goodness-of-fit of the different market power cases, I apply two empirical tests firstly, *Theil's inequality* coefficient U , which was also used by Trüby (2013). This is defined as follows:[15]

$$U = \frac{\sqrt{\sum_{k \in K} (X_k - A_k)^2}}{\sum_{k \in K} X_k^2 + \sum_{k \in K} A_k^2}$$

Here, X are the model results and A are the reference values. The coefficient is scaled and lies in the range $[0, 1]$, where 0 indicates a perfect fit. However, as we are looking at a time series, Theil's inequality may not be the accurate measure to compare and rank the results.

Hence, as an alternative test, I use the *dtw package* implemented in the statistics software **R** (Giorgino, 2009). The package computes the minimal distance between two time series, and it includes several practical features such as time warping and normalization.

Theil's inequality coefficient and the distance computed by the dtw package are presented in Table 2.1; for both metrics, a lower value indicates a better fit. Regarding the goodness-of-fit of the price, *Myopic Cournot* does slightly better than *Oligopoly* according to Theil's inequality, but the conclusion is reversed according to the distance measure. Concerning the quantity results, *Competition* yields the best fit, with *Oligopoly* a not-so-close runner-up. The case *Nash-Cournot*, which seems a very good fit in terms of aggregate OPEC production (Figure 2.5), is shown to yield a rather poor match to the reference quantities.

The statistical evaluation of the goodness-of-fit of each market power case is as ambiguous as the figures suggest. Of course, one could calibrate the input data and fine-tune underlying assumptions such that one oligopoly theory fits the data; but this is not the objective of this

[15]The definition is a quote form Trüby (2013), p. 151.

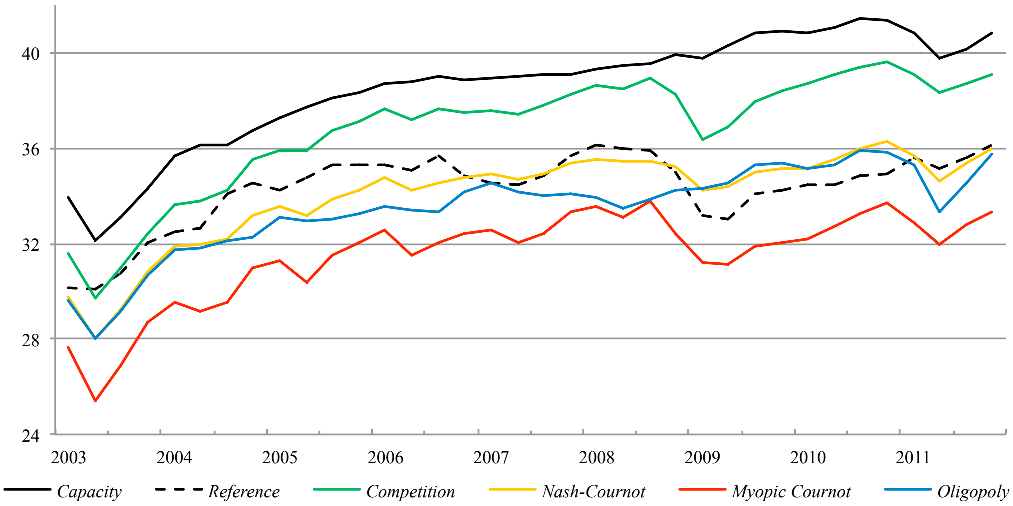

Figure 2.5.: Aggregate OPEC supply by market power case and reference supply (in mbbl/d)

	Competition	Nash-Cournot	Myopic Cournot	Oligopoly
Prices				
Theil's inequality	0.179	0.136	0.043	0.047
Distance (R/dtw)	417.83	527.16	235.35	202.38
Quantity				
Theil's inequality	0. 022	0.041	0.042	0.031
Distance (R/dtw)	112.08	225.55	311.80	181.06

Table 2.1.: Statistics of the goodness-of-fit between reference values and numerical results by market power case: low values indicate a good fit

work. I conclude that the endogenous shifts of market power exertion by OPEC suppliers may have played a role in the oil price shifts over the past decade; but other factors were certainly driving the market, too, which cannot be captured by this approach.

Numerical implementation, optimality, and uniqueness

The three oligopoly cases described above (Equations 2.5 and 2.8) as well as Problem (2.10) are implemented in GAMS using the PATH and CONOPT solvers, respectively. One shortcoming of the consistent-conjecture oligopoly model is that the uniqueness of equilibrium cannot be easily guaranteed, as discussed in Example 8 above. Assumption **A5** does indeed not hold for most periods in this application. In order to test numerically for multiple equilibria, each simulation was initialized from various randomly chosen starting values, and I always obtained the same equilibrium. This supports the notion that there exists only one equilibrium in the numerical application. The second-order derivative condition holds for all oligopolists at the equilibrium in each period.

2.7. Conclusions

This chapter argues that non-cooperative strategic behaviour – as it is frequently modelled in large-scale numerical equilibrium models – is used in a flawed way when combining dominant firms and a competitive fringe. The standard approach in applied models forces players to follow a strategy that may leave them worse off in equilibrium compared to simple price-taking behaviour, but the results are then nevertheless interpreted as these players "exerting market power". To remedy this inconsistency, and to properly model an oligopoly exerting market power considering the reaction of a competitive fringe, I propose a two-level model. Several oligopolists compete non-cooperatively following the Nash-Cournot assumption amongst each other, but take the reaction of the fringe into account – they anticipate the reaction of the fringe in the Stackelberg sense; hence the term *Stackelberg oligopoly* to describe this game. The optimal mark-up charged by the oligopolists is determined by including the consistent conjectural variation regarding the fringe in each oligopolist's profit maximization problem. As a consequence, the optimal level of quantity withholding – i.e., market power exertion – is endogenized in the model.

Representing the crude oil market and OPEC as a dominant-firms oligopoly in the Stackelberg sense is plausible for two reasons: the OPEC quota is a credible signalling and commitment device towards the fringe; and the liquid spot markets allow virtually instantaneous reactions between prices and output changes. In order to capture the specific characteristics of extractive industries, a logarithmic cost function is used; marginal costs increase sharply when producing close to capacity. As a result, the reaction of the fringe to a price change depends implicitly on its capacity utilization – the lower its spare capacity, the lower its reaction. Therefore, OPEC members can more easily exert market power when the fringe produces close to capacity, since their loss of market share from the fringe's response is small.

I compare the two-stage Stackelberg oligopoly model to the standard equilibrium models commonly used in large-scale numerical applications: perfect competition, a Nash-Cournot oligopoly, and a myopic Nash-Cournot oligopoly with a competitive, price-taking fringe. As the focus lies squarely on supplier behaviour, I cannot make a statement regarding the causes of the increased demand in 2008 – speculation or fundamentals. Nevertheless, according to the numerical results, the Stackelberg oligopoly approach can replicate quite well the price path over the past decade: starting from a competitive level in 2003, converging to a price level elevated above marginal costs until 2008 as warranted by an OPEC oligopoly with fringe, and then dropping drastically when demand contracted with the onset of the financial crisis and a global recession. This observation of a decline in market power is in line with the conclusions of Huppmann and Holz (2012), though the two-stage model is able to explain the shift in market power endogenously through the high level of spare capacity following the price collapse in the fall of 2008 and the global recession. This high level of spare capacity reduced the optimal mark-up charged by OPEC suppliers and the Stackelberg oligopoly equilibrium was closer to the competitive benchmark.

Obviously, the lack of reliable data on the crude oil market makes any application on this sector particularly difficult. Furthermore, the assumption of straightforward profit maximization by each supplier ignores a wide array of other potential objectives: targeting a certain level of revenue (Alhajji and Huettner, 2000b); preventing high oil prices to discourage substitution efforts; and the complex negotiations within OPEC, where quota allocations are based on (stated) reserves. This may explain why the quantity results are ambiguous and do not strongly favour one market structure as an explanation. In general, numerical equilibrium models are quite sensitive to underlying assumptions. Nevertheless, the aim of this chapter is to offer a better approach to model market power exertion when a fringe is present, and to determine whether endogenous market power may be a factor in explaining the crude oil price path over the past decade. I claim that in this respect, the model and the numerical application succeed.

Three avenues for future research are opened up through this work. First, the assumption of a one-shot Nash-Cournot oligopoly among OPEC members should be replaced by a richer model of collusion. This may follow a "bureaucratic cartel" (Smith, 2005) or a "Nash-bargaining cartel" (Harrington et al., 2005). The former considers the rigidities and dynamics of intra-OPEC negotiation; the latter includes cartel-stability considerations, such that each cartel member must have an incentive to remain in the cartel, rather than simply maximizing total revenue of the entire group. This is particularly relevant since OPEC does not have a formal compensation mechanism. Furthermore, inter-temporal optimization by crude oil suppliers should be considered; not necessarily in a Hotelling-type model, but by including endogenous investment in new production capacity as a strategic decision. This should ideally be implemented in a game-theoretic approach using closed-loop equilibria (cf. Murphy and Smeers, 2005).

Second, endogeneity of the conjectural variations and hence the mark-up on top of marginal costs (i.e., market power exertion) should be implemented in long-term numerical equilibrium models. These are widely used to analyse scenarios and investment requirements, most prominently in natural gas (e.g., Gabriel et al., 2012; Egging et al., 2010; Lise and Hobbs, 2008). However, such models are usually calibrated to reflect a certain situation in the base year, and the assumptions regarding the conjectural variations of players are then assumed to remain fixed for the entire simulation horizon. A model where market power exertion depends on the rivals' spare capacity may be a significant extension of such models.

Third, the methodology of using the reaction of the followers in an equilibrium model to properly capture the dominant-firm aspect can be applied to other sectors: equilibrium problems under equilibrium constraints (EPEC) are now widely proposed as appropriate to model hierarchical markets in general (Kulkarni and Shanbhag, 2013), and the electricity market in particular (Ralph and Smeers, 2006). In the power market, supply curves steepen when generation is close to capacity (cf. Hortaçsu and Puller, 2008; Chen et al., 2006), and capacity constraints are an important factor in this sector, leading to kinks in the reaction functions and thus theoretical as well as algorithmic problems. I believe that a consistent conjecture Stackelberg formulation may be a natural way to circumvent the multiplicity of equilibria in EPECs and offer a way to compute numerical solutions more easily.

Chapter 3

Endogenous investment decisions in natural gas equilibrium models with logarithmic cost functions

This chapter is based on:
Endogenous investment decisions in natural gas equilibrium models with logarithmic cost functions. *Discussion Papers of DIW Berlin 1253*, 2012.
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Endogenous production capacity investment in natural gas market equilibrium models. *European Journal of Operational Research*, 231(2):503–506, 2013a.

3.1. Introduction: Modelling global natural gas markets

The global natural gas market has changed significantly over the last two decades. Liberalisation of natural gas markets in Europe has in part led to the gradual replacement of long-term contracts by short-term spot markets. Stringent carbon dioxide (CO_2) emission constraints are intended to induce a shift from coal and oil to comparatively clean natural gas (EC, 2011). Unconventional reserves are a game-changer in North America and maybe other regions, with the OECD speaking of a "*golden age of gas*" (IEA, 2011). Last but not least, the question of supply security and European dependence on a small number of suppliers for a substantial share of its imports arises frequently (cf. Lévêque et al., 2010).

These factors have led to a considerable interest in modelling the future development of natural gas markets. A number of equilibrium models have been developed to provide numerical analysis of different scenarios regarding supply and demand patterns, environmental regulation and infrastructure investment options. These efforts were made possible by theoretical and algorithmic advances in solving such problems (Mathiesen, 1985; Harker and Pang, 1990; Ferris and Munson, 2000; Facchinei and Pang, 2003). One early example of applied work can be found in Mathiesen et al. (1987).

Two large-scale natural gas equilibrium models developed in the past decade stand out in particular: the *GASTALE* model, developed by ECN (Lise and Hobbs, 2008), and the *World Gas Model (WGM)*, joint work by the University of Maryland and DIW Berlin (Egging et al., 2010).[1] These models share a number of characteristics: they are spatial partial-equilibrium models with a detailed geographic disaggregation, allowing for analysis and comparison of different pipeline and LNG export/import options; they consider seasonality within a year and explicitly model storage to shift natural gas between low- and high-demand seasons; they are multi-year models and endogenously determine optimal investment in infrastructure; and they allow for oligopolistic behaviour by (a subset of) suppliers, i.e., Cournot competition. Both models also apply a logarithmic cost function, as first proposed by Golombek et al. (1995), in order to capture the specific characteristics of natural gas production: sharply increasing costs when producing close to full capacity.

However, neither of the models allows for endogenous investment in production capacity; instead, the production capacity in future periods is defined exogenously. Given that production capacity is a significant determinant of results and that these models simulate price and quantity trajectories for several decades into the future, this omission is certainly a major drawback. It is owed, in all likelihood, to the rather complicated functional form when including investment decision variables in the logarithmic cost function. This chapter provides a proof that this extension yields a convex problem, which is a prerequisite for solving this problem as an equilibrium model.

Let me also mention two more recent natural gas models: the *GaMMES* model was developed by EDF and IFPEN (Abada et al., 2013). In contrast to the models presented

[1] Both models were published in different versions and used extensively for scenario simulations; only one recent publication for each model is cited here.

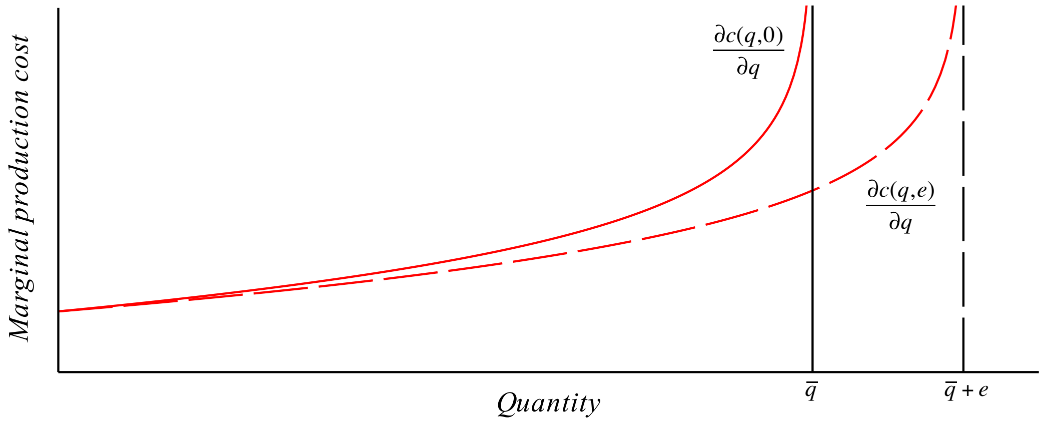

Figure 3.1.: Illustration of the marginal cost function (production capacity $\overline{q}$) without investment (—) and with additional investment e (– –)

above, it distinguishes between spot market sales and long-term contracts. It also assumes a slightly different formulation of the logarithmic cost function: production costs are not increasing relative to capacity utilization, as in the other models, but relative to remaining reserves. This is an interesting approach, but differs from what is discussed in this work. EWI Cologne is currently developing the *COLUMBUS* model (Hecking and Panke, 2012). It is – at this stage – formulated as a linear complementarity problem and does not use a logarithmic cost function.

All these models are formulated as Mixed Complementarity Problem (MCP). The optimization problems of different players subject to engineering and other constraints are solved simultaneously by deriving their respective Karush-Kuhn-Tucker (KKT) conditions, combined with market clearing constraints. The MCP framework is convenient for this type of exercise, as it allows to include Cournot market power for certain suppliers, in contrast to welfare maximization or cost minimization problems. In addition, these models can easily be extended to include stochasticity (e.g., Gabriel et al., 2009) or two-level problems such as Stackelberg competition (e.g., Siddiqui and Gabriel, 2013).

3.2. Mathematical formulation

Assume a supplier with decision variables q_y (production quantity) and e_y (production capacity expansion/investment). The periods are denoted by $y \in \{1, \dots, \overline{y}\}$. In order to keep the notation concise, y denotes both a period as well as its position in the set. Hence, $\overline{y}$ stands for both the last period as well as the number of periods in the set. Following this logic, I use $y' < y$ for "all periods y' prior to period y" in sums and indices, and $y' > y$ for the inverse statement. The price at which the produced quantity is sold is denoted by p_y, and the initial production capacity is $\overline{q}$.

Production costs $c_y(\cdot)$ are determined by a logarithmic cost function as introduced by

Golombek et al. (1995) related to capacity utilization (see Equation (3.3a) below). This function is illustrated in Figure 3.1: Marginal production costs increase sharply when operating close to capacity. Hence, if capacity is expanded, marginal production costs for the same quantity decrease. In line with the literature, capacity investment costs are assumed to be linear.

The parameters of the cost function are denoted by greek letters and may vary by period: $\alpha_y, \beta_y, \gamma_y$ are the parameters for the production cost function; κ_y is the (linear) unit production capacity investment cost. All cost parameters are positive. Discounting of future profits may be implicitly included in the price and cost parameters. For now, I abstract from Cournot market power and other considerations such as reserve horizon or maximum investment constraints. These extensions are briefly discussed below.

The profit maximization problem of the supplier can then be written as follows, converted to a minimization problem:

$$\min_{q,e} f(q,e) = \sum_{y} -p_y q_y + c_y(q_y, e_1, \ldots, e_{y-1}) + \kappa_y e_y \qquad (3.1)$$
$$\text{s.t.} \quad q, e \in \mathbb{R}_+^{\overline{y}}$$

This yields the following Karush-Kuhn-Tucker conditions:

$$0 \leq -p_y + \frac{\partial c_y(\cdot)}{\partial q_y} \perp q_y \geq 0 \qquad (3.2a)$$

$$0 \leq \sum_{y'>y} \frac{\partial c_{y'}(\cdot)}{\partial e_y} + \kappa_y \perp e_y \geq 0 \qquad (3.2b)$$

It is straightforward to see that there will never be investment in the last period; the KKT condition reduces to $0 + \kappa_y \geq 0$, implying $e_y = 0$ if $\kappa_y > 0$. This variable and the associated equation can thus be omitted from further consideration.

The production cost function and its partial derivatives are listed below. In order to make the notation more concise, the sum of previous investments, $\sum_{y'<y} e_{y'}$, is replaced by $e(y)$ for the remainder of this work.

$$c_y(\cdot) = (\alpha_y + \gamma_y) q_y + \beta q_y^2$$
$$+ \gamma_y \left(\overline{q} + e(y) - q_y\right) \ln\left(1 - \frac{q_y}{\overline{q} + e(y)}\right) \qquad (3.3a)$$

$$\frac{\partial c_y(\cdot)}{\partial q_y} = \alpha_y + 2\beta_y q_y - \gamma_y \ln\left(1 - \frac{q_y}{\overline{q} + e(y)}\right) \qquad (3.3b)$$

$$\frac{\partial c_y(\cdot)}{\partial e_{\hat{y}}} = \gamma_y \ln\left(1 - \frac{q_y}{\overline{q} + e(y)}\right) + \gamma_y \frac{q_y}{\overline{q} + e(y)} \qquad \text{for } \hat{y} < y \qquad (3.3c)$$

$$\frac{\partial^2 c_y(\cdot)}{\partial q_y^2} = 2\beta_y + \gamma_y \frac{1}{\overline{q} + e(y) - q_y} \tag{3.3d}$$

$$\frac{\partial^2 c_y(\cdot)}{\partial e_{\hat{y}} \partial e_{\tilde{y}}} = \gamma_y \frac{q_y^2}{\left(\overline{q} + e(y) - q_y\right)\left(\overline{q} + e(y)\right)^2} \qquad \text{for } \hat{y} < y \wedge \tilde{y} < y \tag{3.3e}$$

$$\frac{\partial^2 c_y(\cdot)}{\partial q_y \partial e_{\hat{y}}} = -\gamma_y \frac{q_y}{\left(\overline{q} + e(y) - q_y\right)\left(\overline{q} + e(y)\right)} \qquad \text{for } \hat{y} < y \tag{3.3f}$$

Given this cost function and assuming $\gamma_y > 0\ \forall y$, marginal production costs tend to infinity when the produced quantity tends to initial capacity plus expansions in previous periods. Hence, production quantity q_y is implicitly bounded by capacity. Mathematically speaking, for any $p_y > 0$, there exists a quantity q_y with $\frac{\partial c_y(\cdot)}{\partial q_y} \geq p_y$ and $q_y < \overline{q} + e(y)$. Hence, an explicit production capacity condition is not required as a constraint in the optimization problem (3.1).

Marginal production costs $\frac{\partial c_y(\cdot)}{\partial q_y}$ are non-negative, while the effect of an expansion on costs $\frac{\partial c_y(\cdot)}{\partial e_y}$ is non-positive.[2] A capacity expansion today reduces total cost in subsequent periods periods for any fixed quantity (cf. Figure 3.1). In contrast, in a model with linear or quadratic production cost functions and endogenous capacity expansion, only the dual (or shadow price) to the capacity constraint in future periods determines the investment; hence, only the possibility of producing a higher quantity drives capacity expansion decisions.

Theorem 9. Any solution to the KKT system (3.2) is a global optimum of the supplier profit maximization problem (3.1).

Proof. Sufficiency of the KKT conditions can be established by showing convexity of the objective function $f(q,e)$, which is equivalent to its Hessian matrix being positive semidefinite for any feasible vector (q,e). The matrix has the following form:

$$H(f(q,e)) = \begin{pmatrix} f_{qq} & f_{qe} \\ (f_{qe})^T & f_{ee} \end{pmatrix} = \tag{3.4}$$

$$= \begin{pmatrix} \left(\frac{\partial^2 c_i(\cdot)}{\partial q_i^2}\right)_{\substack{ij, i=j \\ i\in\{1,..,\overline{y}\} \\ j\in\{1,..,\overline{y}\}}} & \left(\frac{\partial^2 c_i(\cdot)}{\partial q_i \partial e_j}\right)_{\substack{ij, i>j \\ i\in\{1,..,\overline{y}\} \\ j\in\{1,..,\overline{y}-1\}}} \\ \left(\frac{\partial^2 c_j(\cdot)}{\partial e_i \partial q_j}\right)_{\substack{ij, i<j \\ i\in\{1,..,\overline{y}-1\} \\ j\in\{1,..,\overline{y}\}}} & \left(\sum_{k>\max\{i,j\}} \frac{\partial^2 c_k(\cdot)}{\partial e_i \partial e_j}\right)_{\substack{ij \\ i\in\{1,..,\overline{y}-1\} \\ j\in\{1,..,\overline{y}-1\}}} \end{pmatrix} \tag{3.5}$$

This matrix is symmetric and all its diagonal entries are positive, but it is not necessarily diagonally dominant; hence, we cannot apply a standard result to show positive semidefiniteness. Nevertheless, the partial derivatives of the cost function have a certain structure which can be exploited to show $x^T M x \geq 0$ for all $x \in \mathbb{R}^n$.

Define $x \in \mathbb{R}^{2\overline{y}-1}$ as follows: $x = \left[(a_y),(b_y)\right]^T$. This is to exploit the different interpretation

[2]That the marginal effect of expansion on future marginal production costs is non-positive can be shown as follows: capacity utilization $\frac{q_y}{\overline{q}+e(y)} \in [0,1]$ for any feasible quantity q_y. Since $\ln(1-x)+x \leq 0\ \forall x \in [0,1]$ (cf. Equation 3.3c), $\frac{\partial c_y(\cdot)}{\partial e_y}$ is non-positive.

of the production vs. expansion variables.

$$
\begin{bmatrix} (a_y)_{y\in\{1,..,\overline{y}\}} \\ (b_y)_{y\in\{1,..,\overline{y}-1\}} \end{bmatrix}^T H(f(q,e)) \begin{bmatrix} (a_y)_{y\in\{1,..,\overline{y}\}} \\ (b_y)_{y\in\{1,..,\overline{y}-1\}} \end{bmatrix} = \tag{3.6}
$$

$$
= \begin{bmatrix} \left(a_y \frac{\partial^2 c_y(\cdot)}{\partial q_y^2} + \sum_{\hat{y}<y} b_{\hat{y}} \frac{\partial^2 c_y(\cdot)}{\partial e_{\hat{y}} \partial q_y} \right)_{y\in\{1,..,\overline{y}\}} \\ \left(\sum_{\hat{y}>y} a_{\hat{y}} \frac{\partial^2 c_{\hat{y}}(\cdot)}{\partial q_{\hat{y}} \partial e_y} + \sum_{\hat{y}} b_{\hat{y}} \sum_{k>\max\{y,\hat{y}\}} \frac{\partial^2 c_k(\cdot)}{\partial e_y \partial e_{\hat{y}}} \right)_{y\in\{1,..,\overline{y}-1\}} \end{bmatrix}^T \begin{bmatrix} (a_y)_{y\in\{1,..,\overline{y}\}} \\ (b_y)_{y\in\{1,..,\overline{y}-1\}} \end{bmatrix} = \tag{3.7}
$$

$$
\begin{aligned}
&= \sum_y a_y^2 \frac{\partial^2 c_y(\cdot)}{\partial q_y^2} + \sum_y a_y \sum_{\hat{y}<y} b_{\hat{y}} \frac{\partial^2 c_y(\cdot)}{\partial e_{\hat{y}} \partial q_y} \\
&+ \sum_y b_y \sum_{\hat{y}>y} a_{\hat{y}} \frac{\partial^2 c_{\hat{y}}(\cdot)}{\partial q_{\hat{y}} \partial e_y} + 2 \sum_y b_y \sum_{\hat{y}<y} b_{\hat{y}} \sum_{k>y} \frac{\partial^2 c_k(\cdot)}{\partial e_y \partial e_{\hat{y}}} + \sum_y b_y^2 \sum_{k>y} \frac{\partial^2 c_k(\cdot)}{\partial e_y^2}
\end{aligned} \tag{3.8}
$$

The reduced index set of b_y is implicitly covered in the summations. Plugging in the partial derivatives stated above and rearranging terms yields the following:

$$
\begin{aligned}
&\sum_y a_y^2 \left(2\beta_y + \gamma_y \frac{1}{\overline{q}+e(y)-q_y} \right) - 2 \sum_y a_y \sum_{\hat{y}<y} b_{\hat{y}} \left(\gamma_y \frac{q_y}{(\overline{q}+e(y)-q_y)(\overline{q}+e(y))} \right) \\
&+2 \sum_y b_y \sum_{\hat{y}<y} b_{\hat{y}} \sum_{\tilde{y}>y} \left(\gamma_{\tilde{y}} \frac{q_{\tilde{y}}^2}{(\overline{q}+e(\tilde{y})-q_{\tilde{y}})(\overline{q}+e(\tilde{y}))^2} \right) \\
&+ \sum_y b_y^2 \sum_{\tilde{y}>y} \left(\gamma_{\tilde{y}} \frac{q_{\tilde{y}}^2}{(\overline{q}+e(\tilde{y})-q_{\tilde{y}})(\overline{q}+e(\tilde{y}))^2} \right) =
\end{aligned} \tag{3.9}
$$

$$
\begin{aligned}
&= 2 \sum_y a_y^2 \beta_y \\
&+ \sum_y \frac{\gamma_y}{\overline{q}+e(y)-q_y} \underbrace{\left(a_y^2 - 2 \left(a_y \sum_{\hat{y}<y} b_{\hat{y}} \right) \frac{q_y}{\overline{q}+e(y)} + \sum_{\hat{y}<y} \left(2b_{\hat{y}} \sum_{\tilde{y}<\hat{y}} b_{\tilde{y}} + b_{\hat{y}}^2 \right) \frac{q_y^2}{(\overline{q}+e(y))^2} \right)}_{\circledast(y)}
\end{aligned} \tag{3.10}
$$

For positive semidefiniteness of the Hessian and hence convexity of the optimization problem, we require that Equation (3.10) is non-negative for every feasible vector (q,e). For the first part, $\sum_y a_y^2 \beta_y$, that is straightforward for any vector (a). The term $\frac{\gamma_y}{\overline{q}+e(y)-q_y}$ is non-negative in every period y, as otherwise the implicit production capacity constraint would be violated. For the last part, note that this term can actually be written as

$$
\circledast(y) = \left(a_y - \left(\sum_{\hat{y}<y} b_{\hat{y}} \right) \frac{q_y}{\overline{q}+e(y)} \right)^2 ,
$$

hence it is always non-negative. Equation (3.10) is thus non-negative for any positive cost parameter vectors $\alpha, \beta, \gamma, \kappa$ and any vector $x \in \mathbb{R}^{2\overline{y}-1}$. $\square$

The proof allows one additional interesting observation: the upper left part of the Hessian matrix is positive definite for strictly positive cost parameters β_y, indicating that the quantity produced in equilibrium is unique in these cases. However, depending on the parameters, the part of the matrix pertaining to the investment decision may be either positive definite or positive semidefinite, depending on the parameters. This can be interpreted that there may be cases when the timing of the investment does not matter.

The following three corollaries show that the supplier's optimization problem with endogenous expansion decision variables can be easily included in the framework of the models discussed in the introduction: the assumption of price-taking behaviour (i.e., exogenous price p_y) may be relaxed and replaced by Cournot market power; engineering and other constraints may be considered in the optimization problem; and several suppliers may compete non-cooperatively, possibly in a richer equilibrium model with other types of players and price-sensitive demand.

Corollary 10. The exogenous price parameter p_y may be replaced by an inverse demand function $P_y(\cdot)$, if it is linear and negatively sloped. Then Theorem 9 remains valid.

Proof. The Hessian matrix of this extended problem $\tilde{f}(q, e)$ is as follows:

$$H(\tilde{f}(q, e)) = H(f(q, e)) + \operatorname{diag}\Big(\big(-2P'_y(\cdot)\big)_{y\in\{1,\dots,\overline{y}\}}, \big(\,0\,\big)_{y\in\{1,\dots,\overline{y}-1\}}\Big)$$

Since $P'_y(\cdot) < 0$, the second part of the extended Hessian is positive semidefinite. The sum of two positive semidefinite matrices is again positive semidefinite. □

Corollary 11. Constraints of the form $g(q, e) \leq 0, h(q, e) = 0$ may be added to the supplier's profit maximization problem (3.1) if $g(q, e)$ is convex and $h(q, e)$ is affine. Then Theorem 9 remains valid.

Proof. The KKT conditions of an optimization problem of the form

$$\min_x f(x) \text{ s.t. } g(x) \leq 0, h(x) = 0,$$

with $f(x), g(x)$ convex and $h(x)$ affine are both necessary and sufficient. □

Corollary 12. Equilibria between several suppliers with endogenous investment decisions and other players can be solved simultaneously as a MCP by taking the respective KKT conditions of each player, combined with appropriate market-clearing conditions, given that the other players' optimization problems are of a form such that their KKT conditions are sufficient for optimality.

Proof. Cf. Facchinei and Pang (2003). □

3.3. Conclusions

This chapter provides a brief overview of current large-scale natural gas equilibrium models and points out their omission of endogenous investment decisions in production capacity, instead relying on exogenously determined capacity increases for future periods. I propose a mathematical formulation to incorporate capacity investments into the state-of-the-literature equilibrium models with logarithmic cost functions and show that this formulation is indeed a convex problem.

Nevertheless, implementing this formulation in a large-scale equilibrium model will increase the number of non-linear terms in the KKT conditions. While any solution obtained will indeed be an equilibrium, the logarithmic terms may lead to numerical problems and increased computation time. Therefore, the proposed formulation shall be integrated into a large-scale equilibrium model to test the numerical properties of this approach. The endogenous consideration of production capacity expansions may significantly improve the scope and validity of scenario simulations of the natural gas as well as other energy and resource markets. Such a large-scale application is presented in the following chapter.

Chapter 4

Market power, fuel substitution, and infrastructure – A large-scale equilibrium model of global energy markets

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4.1. Introduction: Energy system modelling

The global energy system is constantly changing, driven by technological advances and economic shifts as well as regulatory interventions. The shale gas boom in the United States, for example, drastically shifted the economics between the different fossil fuels. Other trends are a result of governmental regulation, such as the establishment of an Emission Trading System (ETS) by the European Union, or biofuel mandates in North America and Europe. Many of these regulations are motivated by potential threats of global warming and climate change (cf. IPCC, 2013), and intend to curb greenhouse gas (GHG) emissions – most importantly carbon dioxide (CO_2) – or reduce local air pollution. Other measures are motivated by public pressure, for instance the nuclear phase-out in several OECD countries following the Fukushima incident. Another driver of energy policy interventions are concerns regarding security of supply and import dependency, which have been raised in Europe in particular by the recurring natural gas transit disputes between Russia and Ukraine (Lévêque et al., 2010).

When combined, these trends may create paradoxical effects. For example, the shale gas boom in North America led to an increase of coal exports from the US to Europe, and Germany saw an increase in the use of coal and lignite in recent years (AGEB, 2014). This occurred despite the EU ETS, as well as ambitious national policy goals to reduce CO_2 emissions and substantial renewable energy feed-in. At the same time, European policy makers express concerns about a loss of competitiveness with North America due to low energy prices overseas (IEA, 2013) – while European utilities consider mothballing natural gas power plants, because they are not able to compete with subsidized renewables and coal at current low CO_2 permit prices.

There is one further intricate aspect with regard to energy and emissions: carbon leakage. Unilateral or regional emission reduction may shift fossil fuel consumption to other regions and thus have limited benefit to the global climate. These effects can work either directly through reduced world prices for fossil fuels, increasing consumption in other regions; or it may work indirectly via the goods market channel, where production of consumer goods is shifted to regions with lower environmental standards, and the goods are then exported to the countries with more stringent standards. Energy-intensive and trade-exposed (EITE) industries such as steel or pulp-and-paper are particularly vulnerable in this regard, further fuelling the fear of reduced competitiveness and industry relocation.

These examples illustrate the complex and integrated nature of global energy markets and climate policy. This interdependence poses various challenges to companies, governments and supra-national entities when considering long-term trends. To gauge the economic impacts of technology-related shifts as well as the effects of regulation and policy measures on the global energy system, policy makers and academics rely on large-scale numerical models of the energy sector.

There is an inherent trade-off in energy modelling: a broad research scope requires substantial aggregation, which necessarily omits many details; on the other hand, many relevant questions with regard to energy, in particular infrastructure investment, can only be tackled adequately while accounting for operational or seasonal detail. Depending on the research question posed, models therefore set different priorities and vary with respect to spatial disaggregation, the time horizon under consideration, and the level of detail with which fuels and technologies (e.g., in power generation) are modelled. They also treat different variables as endogenous (i.e., determined by the model) or exogenous (i.e., taken as a given parameter from some external source or assumption).

The energy market modelling approaches can be broadly classified into four categories, albeit the distinction is not always clear-cut and there is some overlap. *Integrated assessment models* (IAM) such as ETSAP-TIAM (Loulou and Labriet, 2008) and MIT-EPPA (Jacoby et al., 2006) typically have a global and long-term scope and explicitly capture the interaction between the economy, the energy sector, and climate. Several *computable general equilibrium* (CGE) models specifically include emissions and climate aspects, for example PHOENIX (Wing et al., 2011) or GTAP-E (McDougall and Golub, 2007).

Energy system models (ESM) abstract from other sectors of the economy and focus only on the energy sector; this allows for an even more detailed analysis. ESM are usually based on an explicit optimization or equilibrium model.[1] Examples include the *PRIMES* model (EC, 2011) and the many applications based on TIMES-MARKAL.[2] In the US, the *National Energy Modeling System* (NEMS) is used by the *Department of Energy* (DOE) for projections of the national production and consumption of energy (Gabriel et al., 2001).

Lastly, *sector models* only cover one particular fuel (e.g., natural gas, Egging et al., 2010) or sector (e.g., power generation, Leuthold et al., 2012); this focus allows for the inclusion of a high level of detail with regard to market characteristics, infrastructure constraints (e.g., flow of electricity in a network), or variability over time. Some models in this area of research focus on market structure and strategic behaviour by certain dominant players, which we discuss in more detail below.

The model proposed in this chapter combines the advantages of partial-equilibrium modelling (strategic behaviour and a high level of infrastructure detail) with the broad scope of energy system modelling. In particular, fuel substitution is included endogenously in the final demand sectors and in power generation. Furthermore, we make provisions for taxes and emission quota on multiple emissions and pollutants at various levels (nodal, regional, global), and we include constraints on the fuel mix in transformation and final demand to represent governmental regulation. This enables us to conduct detailed analyses of the impact of various energy and climate policies on global fossil fuel markets and the integration of renewable energy.

Compared to Egging and Huppmann (2012), this work extends the framework in the following respects: i. it is a multi-period model allowing for endogenous investments in and

[1] Some ESM are "simulation" models, which are not based on any optimization rationale. Rather, there are certain if-then-assumptions or rules underlying the model. To simulate a scenario, the future energy demand or fuel mix is extrapolated based on these rules. Some hybrid models employ a combination of optimization and simulation approaches. For the sake of conciseness, we do not address simulation approaches in detail, but focus on optimization or partial-equilibrium-based ESM.

[2] See `http://www.iea-etsap.org` for more information.

depreciation of all infrastructure types; ii. it includes seasonality, storage, and load variation; iii. it allows for endogenous fuel substitution in the final demand sectors; iv. the data set is more detailed in terms of geographical coverage, demand sectors, and with respect to the fuels.

The remainder of this chapter is organized as follows: the next section details how different model classes tackle fuel substitution, infrastructure, and market power. Section 4.3 provides the mathematical formulation of the model; Section 4.4 gives a brief overview of the current data set and shows two scenarios to illustrate the types of analysis that can be performed with the model: a pessimistic scenario regarding the future of shale gas in North America, and a scenario where the EU unilaterally reduces its CO_2 emissions by 80 % until 2050. Section 4.5 concludes and proposes a number of potential avenues for further research and model development.

4.2. Three modelling aspects of particular interest

Numerical models for assessing potential developments of the global energy landscape have been used for decades. We refer to Hirth (2015) and Connolly et al. (2010) for a detailed classification and a comparison of models currently used for policy analyses. Instead of providing an extensive overview, we focus on three key aspects, and how they are covered in state-of-the-art models. These aspects are: fuel substitution within demand sectors as well as in power generation; infrastructure for production, transportation, storage, and transformation of different energy carriers; and finally, the explicit consideration of strategic behaviour by certain suppliers, i.e., Nash-Cournot market power exertion. This allows us to highlight how the proposed model departs from and extends the current state-of-the-art in energy modelling.

Endogenous fuel substitution

There are, in principle, two approaches for incorporating fuel substitution: a *top-down* formulation follows the computable general equilibrium methodology (CGE), using elasticities of substitution. This approach is advantageous because the energy sector can be embedded in the broader economy, thereby allowing for well-founded welfare analyses specifically including the interdependence between economic activity and energy prices. However, due to the large aggregation necessary for such models, many details are lost. In addition, a drawback of using elasticities is that, if a fuel is not used at all in the base year (or only to a small extent relative to total energy consumption), such models exhibit a high degree of inertia in the fuel mix and are not capable of showing large future penetration rates of these fuels even when economic considerations would warrant that. This is a significant disadvantage when modelling potentially “game-changing” technologies.

In contrast, energy system models (ESM) usually start from a *bottom-up* assessment of the energy sector such as production/generation and investments costs. They are based on optimization or partial-equilibrium techniques and often follow a (linear) least-cost approach where pre-specified demand levels (often determined by another, CGE-type model) must be met at lowest total cost. Alternatively, the objective may be welfare maximization, where a decreasing willingness-to-pay for energy (or inverse demand function) is assumed. This approach may have the opposite drawbacks of CGE-type models, namely "bang-bang"-results: if a small shift in relative costs makes one fuel cheaper than the one currently used, the entire energy demand by the respective sector shifts from one fuel to the other.[3] This is obviously unrealistic, so these models need to specify (assumptions on) limits or costs of fuel substitution.

Some models only investigate fuel substitution within the electricity sector; this can be problematic due to the implicit assumption that parameters outside the scope of the analysis do not change even in drastic scenarios. For example, studying the effects of a nuclear phase-out while keeping the price of fossil fuels fixed obviously neglects the interdependence between gas and coal prices and their use in power generation to replace nuclear power (cf. Knopf et al., 2014).

Supply chain infrastructure

In typical CGE models, a long-term equilibrium perspective combined with yearly averages ensures that each sector attracts "just enough" capital to execute all activities. Hence, such models cannot account for explicit capacity constraints. This, in combination with taking yearly averages, makes such an approach unsuitable for analysing infrastructure bottlenecks in detail or addressing the many short-term operational considerations important in energy markets.

Energy system models, in comparison, can easily capture "hard" infrastructure capacity constraints, and can be extended to include seasonal or hourly variation and stochasticity. In addition, many multi-period models allow for endogenous investments, and explicitly account for the lead-time of capacity expansions, which is neglected by the "just enough" capital approach in CGE models.

The simplifications regarding the economy made in sector models allow more sector-specific peculiarities to be tackled, such as loop flows in an electricity network. This enables a correct analysis where bottlenecks occur in the transmission networks. The problem with such an approach is one of scaling; extending such a detailed model to a realistic hourly disaggregation will quickly reach the limit of computational tractability.

The model REMIND-R (Leimbach et al., 2010) and the approach proposed by Böhringer and Rutherford (2009, 2008) combine the advantages of CGE-type models to capture the economy at large and the detail allowed by sector models. In these hybrid models, the energy sector is represented with the technological detail common for partial-equilibrium models, and the rest of the economy is modelled according to the top-down approach typical for CGE models. They can thereby capture the feedback loops between the market for various fuels and the economy at large, which are typically overlooked by partial-equilibrium models. At

[3] Mathematically speaking, the solution may jump from one corner of the feasible space to another corner (or vertex); this is a common characteristic in particular of linear optimization problems.

the same time, their approach allows for completely inactive energy technologies to develop over time (thereby addressing the *inertia* issue discussed above) and for consideration of hard capacity limits, which standard CGE models cannot address. However, REMIND-R does not incorporate the third aspect relevant to energy markets, which we consider important – market power.

Strategic behaviour and market power

Energy system models usually apply a least-cost or welfare maximization approach, and therefore, they cannot incorporate strategic behaviour. Rather, market structure analysis is frequently conducted using a sector-specific modelling approach: for example, Trüby (2013) and Haftendorn and Holz (2010) test several market power assumptions for the global coal market in a partial-equilibrium framework; Huppmann and Holz (2012) conduct a similar analysis for crude oil, with a specific emphasis on the role of OPEC members. For natural gas, a large number of models have been developed over the last years; several of these compute market equilibria for the next decades taking into account Nash-Cournot behaviour by certain suppliers (e.g., Abada et al., 2013; Egging et al., 2010; Lise and Hobbs, 2008). The generalization of Nash-Cournot market power are conjectural variations (CV): each supplier has an expectation (or conjecture) regarding how its rivals will react to a variation in its own output. This allows to model "intermediate" cases of market power exertion. As discussed in Chapter 2, the assumption of Nash-Cournot behaviour (or any fixed CV parameter) to represent strategic interaction is in itself a strong simplification, but it may still be a useful approximation.

CGE-type models can – in principle – incorporate imperfect competition; however, to our knowledge, most large-scale applications used in the policy evaluation arena assume perfectly competitive behaviour by all players. Böhringer et al. (2014) are the exception; in their article, they compare several assumptions concerning OPEC market power in a CGE framework and discuss the implications on carbon leakage. However, they only treat oil as a homogeneously traded fuel, and thus (to some extent) omit carbon leakage via the other fossil fuels.[4]

Another application of a CGE-type model with market power is presented by Golombek et al. (2013): they use the market power version of the LIBEMOD model (Aune et al., 2008) to investigate how different energy market liberalization trajectories affect prices and social welfare in Western Europe.[5] They model market power through incorporating price mark-ups, representing various market imperfections, to the supply cost curves. The values of these mark-ups are based on data observations and determined in the model calibration phase. As such, the origins and magnitudes of the various market imperfections are neither

[4]Böhringer et al. (2014) assume that there exists a global integrated market only for crude oil; they use Armington elasticities for other fossil fuels, which implicitly assumes that coal or gas from different regions are not perfect substitutes.

[5]Strictly speaking, the LIBEMOD model is a hybrid between a CGE model and a bottom-up energy system model. It is a partial-equilibrium model for the energy sectors using mathematical expressions that are commonly used in CGE models, in particular nested constant elasticity of substitution (CES) functions.

assessed nor quantified. Merely the aggregate effect in each agent's behaviour is accounted for. The market liberalization in a specific market can then be represented by removing the mark-ups for the agents involved in that market.

The approach used by Golombek et al. follows the reasoning of Smeers (2008), who argues that just adding a mark-up in addition to marginal costs is an easier way to include non-competitive behaviour than conjectural variations, in terms of mathematical formulation and computational burden. We disagree; in spite of some theoretical shortcomings, using a CV parameter allows to incorporate a relation between the level of market power exertion and the market share of the respective supplier in various markets. As a consequence, a supplier exerting market power will seek to diversify its sales. Standard optimization energy models, in contrast, implicitly minimize transportation costs. For this reason, one will not observe trade flows of the same fuels in opposite directions in such models – but such trade flows do occur in the real world. Assuming Cournot behaviour or a CV by (some) suppliers results in trade flows that are more diverse, and thus allow for better matching of actual sales and trade patterns; just increasing marginal costs cannot capture such an effect. Using CV assumptions therefore offers a significant practical advantage when calibrating a model to replicate real-world observations.

We discuss the solution approach and some mathematical and game-theoretic considerations in more detail in Section 4.3.8, after presenting the structure and the mathematical formulation of the model.

Combining fuel substitution, infrastructure, and market power

The model presented here follows the bottom-up approach and allows for linear substitution between fuels in the final demand sectors. Nevertheless, to avoid the problem of bang-bang-results, we introduce fuel-specific end use costs increasing in the amount of each specific fuel used in a sector. This represents "stickiness" of fuel usage due to existing capital stock, the fact that fuels are used for various purposes within a sector, as well as the costs associated with installing additional equipment.

Regarding infrastructure, we allow for the level of technological detail common in bottom-up energy system models. In particular, we consider endogenous investment in production capacity and transport, transformation, and storage infrastructure. The model covers multiple periods (i.e., years) and includes seasonality within each year. It is therefore able to address both short-term operational concerns as well as long-term trends.

Because the model is formulated as a partial-equilibrium model derived from individual players' profit maximization problems (mathematically, a mixed complementarity problem, MCP), it allows for the inclusion of Nash-Cournot market power of several suppliers using a CV approach (Gabriel et al., 2013; Facchinei and Pang, 2003). To our knowledge, this is the first large-scale energy system model that is spatially disaggregated and explicitly models "cross-fuel" market power – a supplier is able to consider how its sales of one fuel affect not only the price of that fuel, but also the price of other fuels it is selling in the same market.

4.3. A multi-fuel market equilibrium model

The model represents the entire supply chain of multiple fossil fuels and renewable energy sources in a spatially disaggregated framework. It captures game-theoretic considerations (i.e., supplier market power), infrastructure capacity constraints, endogenous fuel substitution by consumers in various end use sectors, multiple governmental regulatory constraints, and greenhouse gas or other pollutants' emission caps in an integrated mathematical framework.

There are three levels of the supply chain: first, fuels are produced (upstream activities); then, they are transformed, stored and/or transported to other nodes by the service providers (midstream); and finally, they are sold to the demand sectors (downstream). We solve for a Nash equilibrium of profit-maximizing players using a deterministic open-loop approach (i.e., assuming perfect foresight and full information). The players are suppliers and infrastructure operators, each aiming to maximize profits. Final demand sectors seek to optimize their welfare from consuming energy. When formulating the model, we took great care to develop a flexible and generic framework; the following examples should be understood as motivation for the model formulation and functionality.

The *suppliers* produce various energy carriers (fuel) and use infrastructure services purchased from *service providers* to transport fuels, to transform them into other types of energy, and to store fuels across seasons. They ultimately sell the fuels to *final demand sectors* which endogenously substitute between different fuels depending on relative costs, efficiencies, as well as regulatory and technical constraints. The suppliers may be modelled as Nash-Cournot players, exerting market power vis-à-vis consumers, or as price-takers (i.e., perfectly competitive). Suppliers may control production capacity at several nodes and of several fuels.

The *transformation technology operators* can be parametrized with multiple input fuels and several output fuels; hence, this can represent crude oil refineries, different types of power generation plants, and combined power-and-heat plants. This setup also provides flexibility to model different technologies for the same input-output combination, for example single- vs. combined-cycle gas turbines to generate electricity from natural gas.

The *arc operators* can represent pipelines, tanker ships, the LNG supply chain components, and power transmission lines (albeit no consideration is given to alternating-current power flow characteristics). The *storage operators* allow the supplier to shift fuels between different seasons within a year; one may think of natural gas storage (summer-winter) or pump-hydro electricity storage (night-day). All service providers are assumed to act in a perfectly competitive manner; whenever capacity is scarce, an implicit auction between the suppliers using the infrastructure allocates capacity according to the highest willingness-to-pay.

The *emission permit auctioneer* allocates permits and collects taxes on emissions of greenhouse gases or other pollutants. Both the taxes and the emission quota can be set on a nodal, regional and/or global level, and are additive. The model is thereby able to replicate a situation where, for example, Norway is part of the EU ETS, but levies an additional

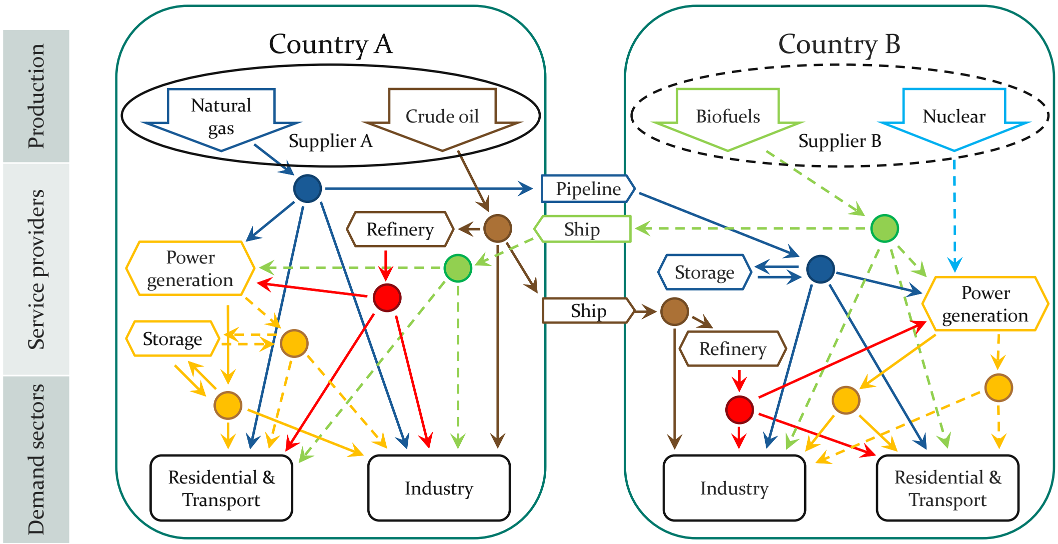


Figure 4.1.: Illustration of the supply chain in a two-node, six-fuel setup; ◯ represents the nodal mass balance of each supplier regarding a fuel at the node.

methane emissions tax within its jurisdiction. The constraints can also be set such that only one sector is faced with a tax or a quota, so that the model is able to replicate sector-specific carbon prices (IEA, 2013).

The suppliers and the service providers face infrastructure constraints, i.e., limited production, transportation or transformation capacity, or maximum injection (extraction) rates into (from) storage. The players invest endogenously in additional capacity if this is economically viable.

Figure 4.1 illustrates how the supply chain is modelled in a simplified two-node, four-fuel setup: In *Country A*, *Supplier A* produces both natural gas and crude oil, which it can either sell domestically or export via ship (oil) or pipeline (natural gas). There are two demand sectors, *Industry* and *Residential & Transport*. Only the industry sector can use crude oil directly; natural gas, oil products (crude oil processed by a refinery), biofuels, and electricity can be used by both sectors. Nuclear power, "produced" by *supplier B* in *country B*, is modelled as an input to the power sector. For illustrative purposes, the power sector is very stylized here: it converts any input fuel except crude oil into electricity. There exists a natural gas storage facility in country B and an electricity storage plant in country A (e.g., a pump-storage hydro reservoir); the latter can be used by both suppliers, since both can convert some of their fuels into electric power.

In this example, there is one supplier per node to keep the illustration simple, but one could easily include a supplier active at several nodes, or multiple suppliers producing identical fuels at the same node, each supplier with an individual capacity and cost function. We assume that each supplier remains the owner of the fuel it produces, even after transformation and storage. This is necessary to properly account for market power exertion. We discuss this in more detail below.

4.3.1. General notation

First, we introduce some general notation before presenting the optimization problem of each player in detail. Table 4.1 lists the most important sets and mappings used in the model. The remaining notation is introduced in the explanation of each player's problem. A complete list of sets, parameters and variables is provided in Appendix B.1. In general, $q^{\circledast}$ are decision variables of the supplier, where $^{\circledast}$ is to be replaced by a letter representing the activity in the supply chain. Accordingly, $f^{\circledast}$ are decision variables of the service providers, and $z^{\circledast}$ are the respective infrastructure investment variables.

The relative duration of season h is given by dur_h; the sum of all season durations (over a year) must equal to 1. The discount factor of future revenues and costs is df_y. We use $y' < y$ as shorthand for "all years y' prior to year y", and $y' > y$ for the inverse statement.

4.3.2. The supplier

The supplier maximizes its profits earned by producing and selling fuels while considering costs for production, transport, transformation and storage. Losses during each step of the

$y \in Y$	... years
$h \in H$	... hours/days/seasons
$v \in V$	... loading (injection/extraction) cycles of storage
$s \in S$	... suppliers
$n, k \in N$	... nodes
$d \in D$	... demand sectors
$a \in A$	... arcs
$c \in C$	... transformation technology (e.g., oil refineries, power plants)
$o \in O$	... storage operators/technology
$e, f \in E$	... energy carriers/fuels
$r \in R$	... regions
$g \in G$	... emission types (greenhouse gases)
$n, k \in N_r$	... node-to-region mapping
$r \in R_n$	... region-to-node mapping (any node can be part of several regions)
$a \in A^+_{ne}$	... subset of arcs ending at node n transporting fuel e
$a \in A^-_{ne}$	... subset of arcs starting at node n transporting fuel e
$e \in E^A_a$	... fuel(s) transported via arc a (singleton)
$n^{A+}(a)$	... end node of arc a (singleton)
$n^{A-}(a)$	... start node of arc a (singleton)
$f \in E^{C+}_c$	... subset of output fuel(s) f obtained from transformation technology c
$e \in E^{C-}_c$	... subset of input fuel(s) e for transformation technology c
$(e, f) \in E^C_c$	... input/output fuel mapping of transformation technology c
$o \in O^E_e$	... subset of technologies storing fuel e
$h \in H^V_{vo}$	... mapping between loading cycle and hour/day/season

Table 4.1.: Selected sets and mappings

supply chain are considered by the supplier in the nodal mass balance constraint. Final demand emission costs as well as fuel-specific end use costs are included in the final demand price, which the supplier receives per unit of fuel. The supplier's profit optimization problem is given by the following; variables in parentheses denote the duals of the constraints.

$$\begin{aligned}
\max_{\substack{q^P,q^A,q^C\\ q^{O-},q^{O+},q^D\\ z^P}} \sum_{\substack{y\in Y,h\in H\\ n\in N,e\in E}} df_y dur_h \Bigg(& \sum_{d\in D} \left[cour^S_{ysnd}\Pi^D_{yhnde}(\cdot) + (1-cour^S_{ysnd})p^D_{yhnde}\right] q^D_{yhsnde} \\
& - cost^P_{yhsne}(\cdot) - \sum_{a\in A^+_{ne}} p^A_{yha}q^A_{yhsa} - \sum_{c\in C} p^C_{yhnce}q^C_{yhsnce} \\
& - \sum_{o\in O^E_e} \left(p^{O-}_{yhno}q^{O-}_{yhsno} + p^{O+}_{yhno}q^{O+}_{yhsno} \right) \\
& - \sum_{g\in G} p^G_{yng} ems^P_{ysneg} q^P_{yhsne} - inv^P_{ysne} z^P_{ysne} \Bigg) && (4.1\text{a})
\end{aligned}$$

$$\text{s.t. } q^P_{yhsne} \le avl^P_{yhsne}\left(cap^P_{ysne} + \sum_{y'<y} dep^P_{y'ysne} z^P_{y'sne} \right) \quad (\alpha^P_{yhsne}) \qquad (4.1\text{b})$$

$$\sum_{h\in H^V_{vo}} dur_h q^{O+}_{yhsno} = \sum_{h\in H^V_{vo}} dur_h (1-loss^{O-}_o) q^{O-}_{yhsno} \quad (\alpha^O_{yvsno}) \qquad (4.1\text{c})$$

$$\begin{aligned}
& (1-loss^P_{sne})q^P_{yhsne} - \sum_{d\in D} q^D_{yhsnde} \\
& + \sum_{c\in C, f\in E^{C-}_c} transf^C_{yncfe} q^C_{yhsncf} - \sum_{c\in C} q^C_{yhsnce} \\
& + \sum_{a\in A^+_{ne}} (1-loss^A_a) q^A_{yhsa} - \sum_{a\in A^-_{ne}} q^A_{yhsa} \\
& + \sum_{o\in O^E_e} \left(q^{O+}_{yhsno} - q^{O-}_{yhsno} \right) = 0 \quad (\phi_{yhsne}) && (4.1\text{d})
\end{aligned}$$

$$z^P_{ysne} \le exp^P_{ysne} \quad (\zeta^P_{ysne}) \qquad (4.1\text{e})$$

$$\sum_{y\in Y,h\in H} dur_h q^P_{yhsne} \le hor^P_{sne} \quad (\gamma^P_{sne}) \qquad (4.1\text{f})$$

The linear combination of price and inverse demand function is multiplied by the quantity sold to final demand q^D to yield the revenue in the supplier's objective function (Equation 4.1a). As discussed in Section 4.2, the supplier may either act competitively (i.e., price-taking behaviour) or as a Cournot player. In the competitive case, the parameter $cour^S$ is set to 0, and the supplier takes the final demand price p^D as given in its optimization. In contrast, if the supplier acts as a pure Cournot player ($cour^S = 1$), it is aware of the inverse demand function $\Pi^D(\cdot)$, and hence the impact of its own sales on the market price.[6] In order

[6] The term $cour^S$ may lie in the range $[0, 1]$. The standard conjectural variations literature uses a slightly different notation, so the CV values applied there are in the range $[-1, 0]$. The interpretation, however,

to properly account for market power, all fuels remain the property (and hence under the control) of the supplier throughout the entire supply chain; therefore, the supplier is aware not only of the impact of its sales of one fuel on the price for that specific fuel ("direct" market power), but also of its impact on the price of other fuels ("cross-fuel" market power).

The supplier faces production costs $cost^P(\cdot)$, specified below. It also has to pay the market-determined price p^A for each unit transported via arcs, q^A; price p^C for each unit of fuel put into a transformation unit, q^C; and the prices p^{O-} and p^{O+} for each unit of fuel injected into storage q^{O-} and extracted from storage q^{O+}, respectively. The prices charged by the service operators are derived from a market-clearing constraint; they include operating costs, emission charges incurred by the service providers, and congestion rent. Only the price for emissions p^G during production q^P have to be paid by the supplier directly, where ems^P gives the emission intensity. The investment costs in additional production capacity are given by inv^P, and z^P denotes the capacity expansion (i.e., investment) variable of the supplier.

The supplier must satisfy a production capacity constraint (Equation 4.1b) at each node where he is active, where avl^P is the availability factor of capacity, cap^P is the existing capacity (with depreciation included for future periods) and dep^P is the depreciation factor of newly-built capacity.

Equation (4.1c) is a mass-balance constraint over each loading cycle; the amount extracted from storage must equal the amount injected, after losses ($loss^{O-}$). The nodal mass balance constraint (4.1d) states that production, sales to final demand, transformation into other fuels (where $transf^C$ is the transformation rate), imports and exports, and injection into and extraction from storage must be balanced at each node, for each fuel, and in each time period.

The parameter exp^P sets the maximum production capacity expansion level (Equation 4.1e); the reserve horizon hor^P gives the maximum cumulative production over the model horizon by the supplier (Equation 4.1f) of a specific fuel at a node.

Last, let us specify the production cost function and its partial derivatives (Equations 4.2a–4.2c): it follows the functional form proposed by Golombek et al. (1995). The logarithmic part allows to add two interesting effects to be included, compared to a simpler linear or quadratic cost function. First, marginal costs increase sharply when producing close to capacity; this is realistic for natural gas or crude oil production. Second, since costs depend on total capacity, an investment reduces production costs for all future periods, ceteris paribus, as illustrated in Chapter 3. For conciseness, $\widehat{cap}^P_{yhsne}$ defines the available capacity including prior expansions as defined in Equation 4.1b. The parameters lin^P, qud^P, and

is identical: a value at the lower end of the range indicates competitive behaviour, while a high value represents standard Nash-Cournot behaviour. A similar approach is applied by several models using a CV representation if market power, as discussed in the previous section.

gol^P specify the shape of the cost function.

$$\begin{aligned}
cost^P_{yhsne}(\cdot) &= (lin^P_{ysne} + gol^P_{ysne})q^P_{yhsne} + qud^P_{ysne}(q^P_{yhsne})^2 \\
&\quad + gol^P_{ysne}\left(\widehat{cap}^P_{yhsne} - q^P_{yhsne}\right)\ln\left(1 - \frac{q^P_{yhsne}}{\widehat{cap}^P_{yhsne}}\right) && (4.2a) \\
\frac{\partial\, cost^P_{yhsne}(\cdot)}{\partial\, q^P_{yhsne}} &= lin^P_{ysne} + 2qud^P_{ysne}q^P_{yhsne} - gol^P_{ysne}\ln\left(1 - \frac{q^P_{yhsne}}{\widehat{cap}^P_{yhsne}}\right) && (4.2b) \\
\frac{\partial\, cost^P_{yhsne}(\cdot)}{\partial\, z^P_{\hat{y}sne}} &= gol^P_{ysne}avl^P_{yhsne}dep^P_{\hat{y}ysne}\left(\ln\left(1 - \frac{q^P_{yhsne}}{\widehat{cap}^P_{yhsne}}\right) + \frac{q^P_{yhsne}}{\widehat{cap}^P_{yhsne}}\right) \qquad \text{if } \hat{y} < y && (4.2c)
\end{aligned}$$

$$\text{where } \widehat{cap}^P_{yhsne} = avl^P_{yhsne}\left(cap^P_{ysne} + \sum_{y'<y} dep^P_{y'ysne}z^P_{y'sne}\right)$$

4.3.3. The arc operator

The arc operator maximizes its profits from transporting fuel f^A considering operation costs trf^A and emission costs.[7] Each arc is fuel-specific; losses are accounted for in the supplier's mass balance constraint (Constraint 4.1d). The operator allocates congested capacity to suppliers, such that the market clearing constraint (Equation 4.4) is satisfied. The dual variable to this constraint is the price p^A for using the arc. Capacity investment z^A is undertaken if the shadow price of future congestion (τ^A) is greater than investment costs inv^A.

$$\max_{f^A, z^A} \sum_{y\in Y, h\in H} df_y dur_h \left((p^A_{yha} - trf^A_{ya})f^A_{yha} - \sum_{g\in G} p^G_{yng}ems^A_{yag}f^A_{yha} - inv^A_{ya}z^A_{ya}\right) \tag{4.3a}$$

$$\text{s.t.} \quad f^A_{yha} \le cap^A_{ya} + \sum_{y'<y} dep^A_{y'ya}z^A_{y'a} \quad (\tau^A_{yha}) \tag{4.3b}$$

$$z^A_{ya} \le exp^A_{ya} \quad (\zeta^A_{ya}) \tag{4.3c}$$

Market clearing

$$\sum_{s\in S} q^A_{yhsa} = f^A_{yha} \quad (p^A_{yha}) \tag{4.4}$$

4.3.4. The transformation technology operator

The transformation technology operator transforms fuels into other energy carriers. The input into the unit is denoted by f^C; the share of output fuels produced $transf^C$ is fixed, but

[7] All emissions are accounted for at the starting node of the arc.

depends on the input fuel. There is one unit per technology at each node, which represents total (aggregate) capacity; the capacity constraint (Equation 4.5b) is stated in terms of output quantities. In order to capture technical or regulatory constraints, Equation (4.5c) allows us to impose that a certain minimum share shr^C of total output must be produced from a certain input fuel.

$$\max_{f^C,z^C} \sum_{\substack{y\in Y,h\in H\\ e\in E_c^{C-}}} df_y dur_h \left((p^C_{yhnce} - trf^C_{ync}) f^C_{yhnce} - \sum_{g\in G} p^G_{yng} ems^C_{yceg} f^C_{yhnce} - inv^C_{ync} z^C_{ync} \right) \tag{4.5a}$$

$$\text{s.t.} \quad \sum_{(e,f)\in E_c^C} transf^C_{yncef} f^C_{yhnce} \leq cap^C_{ync} + \sum_{y'<y} dep^C_{y'ync} z^C_{y'nc} \quad (\tau^C_{yhnc}) \tag{4.5b}$$

$$shr^C_{ynce} \sum_{(e',f)\in E_c^C} transf^C_{ynce'f} f^C_{yhnce'} \leq \sum_{f\in E_c^{C+}} transf^C_{yncef} f^C_{yhnce} \quad (\beta^C_{yhnce}) \tag{4.5c}$$

$$z^C_{ync} \leq exp^C_{ync} \quad (\zeta^C_{ync}) \tag{4.5d}$$

Market clearing

$$\sum_{s\in S} q^C_{yhsnce} = f^C_{yhnce} \quad (p^C_{yhnce}) \tag{4.6}$$

4.3.5. The storage operator

The storage operator allows suppliers to transfer fuels between different hours/days/seasons. We assume one or several loading (f^{O-})/unloading (f^{O+}) cycles for each technology, and all costs, losses and emissions are accounted for during injection. The capacity constraint (i.e., maximum quantity stored, Equation 4.7b) is the summation over all energy injected during one loading cycle. In addition, there is a capacity constraint on injection (Equation 4.7c) and extraction (Equation 4.7d), and the respective investment variables may also be constrained (Equations 4.7e–4.7g).

$$\begin{aligned}\max_{\substack{f^{O-},f^{O+}\\ z^O,z^{O-},z^{O+}}} \sum_{y\in Y,h\in H} df_y dur_h \Bigg(&(p^{O-}_{yhno} - trf^{O-}_{yno}) f^{O-}_{yhno} + p^{O+}_{yhno} f^{O+}_{yhno} - \sum_{g\in G} p^G_{yng} ems^{O-}_{yog} f^{O-}_{yhno} \\ &- inv^O_{yno} z^O_{yno} - inv^{O-}_{yno} z^{O-}_{yno} - inv^{O+}_{yno} z^{O+}_{yno} \Bigg)\end{aligned} \tag{4.7a}$$

$$\text{s.t.} \sum_{h \in H^V_{vo}} dur_h f^{O-}_{yhno} \leq cap^O_{yno} + \sum_{y'<y} dep^O_{y'yno} z^O_{y'no} \quad (\tau^O_{yvno}) \tag{4.7b}$$

$$f^{O-}_{yhno} \leq cap^{O-}_{yno} + \sum_{y'<y} dep^{O-}_{y'yno} z^{O-}_{y'no} \quad (\kappa^{O-}_{yhno}) \tag{4.7c}$$

$$f^{O+}_{yhno} \leq cap^{O+}_{yno} + \sum_{y'<y} dep^{O+}_{y'yno} z^{O+}_{y'no} \quad (\kappa^{O+}_{yhno}) \tag{4.7d}$$

$$z^O_{yno} \leq exp^O_{yno} \quad (\zeta^O_{yno}) \tag{4.7e}$$

$$z^{O-}_{yno} \leq exp^{O-}_{yno} \quad (\zeta^{O-}_{yno}) \tag{4.7f}$$

$$z^{O+}_{yno} \leq exp^{O+}_{yno} \quad (\zeta^{O+}_{yno}) \tag{4.7g}$$

Market clearing

$$\sum_{s \in S} q^{O-}_{yhsno} = f^{O-}_{yhno} \quad (p^{O-}_{yhno}) \tag{4.8a}$$

$$\sum_{s \in S} q^{O+}_{yhsno} = f^{O+}_{yhno} \quad (p^{O+}_{yhno}) \tag{4.8b}$$

Quantities must be extracted within the same loading cycle in which they were injected, but we do not model the storage level explicitly. This and the consideration that only amounts injected (after losses) can be extracted is included in the supplier's optimization problem (Equation 4.1c).

4.3.6. The emission permit auctioneer

The emission permit auctioneer allocates constrained emission quota. This mechanism represents an implicit auction of permits. It is possible to have multiple constraints on global ($quota^{glob}$), regional ($quota^{reg}$) or nodal ($quota^{nod}$) levels. In addition, a tax can be introduced for any emission type, again on a global, regional and/or nodal level (tax^{glob}, tax^{reg}, tax^{nod}); the equilibrium price at any node cannot be lower than the respective tax(es), so this can be interpreted as a floor price.[8] If a node belongs to more than one region, the taxes and shadow prices (dual variables to Equations 4.9b–4.9d) from both regions apply.

[8] Mathematically speaking, the dual to a constraint and a tax (i.e., mark-up) are equivalent, since each supplier and service provider takes the emission price as given. However, one may want to investigate scenarios with specific carbon prices, in which setting a parameter is a more straightforward approach; hence, we include both options to provide more flexibility when defining scenarios.

The optimization problem of the emission permit auctioneer reads as follows:

$$\max_{f^G} \sum_{\substack{y\in Y, n\in N\\ g\in G}} df_y \left(p^G_{yng} - tax^{glob}_{yg} - \sum_{r\in R_n} tax^{reg}_{yrg} - tax^{nod}_{yng} \right) f^G_{yng} \tag{4.9a}$$

$$\text{s.t.} \quad \sum_{n\in N} f^G_{yng} \leq quota^{glob}_{yg} \quad (\mu^{glob}_{yg}) \tag{4.9b}$$

$$\sum_{n\in N_r} f^G_{yng} \leq quota^{reg}_{yrg} \quad (\mu^{reg}_{yrg}) \tag{4.9c}$$

$$f^G_{yng} \leq quota^{nod}_{yng} \quad (\mu^{nod}_{yng}) \tag{4.9d}$$

Market clearing

$$\sum_{h\in H, e\in E} dur_h \Bigg(\sum_{s\in S} ems^P_{ysneg} q^P_{yhsne} + \sum_{s\in S, d\in D} ems^D_{ydeg} q^D_{yhsnde} + \sum_{a\in A^+_{ne}} ems^A_{yag} f^A_{yha} + \sum_{c\in C} ems^C_{yceg} f^C_{yhnce} + \sum_{o\in O} ems^{O-}_{yog} f^{O-}_{yhno} \Bigg) = f^G_{yng} \quad (p^G_{yng}) \tag{4.10}$$

4.3.7. Final demand

The final demand sector aims to maximize its utility from using energy services, where each fuel provides a certain level of service (eff^D). When taking the derivative of the utility maximization problem with respect to the amount of a particular fuel, one obtains the inverse demand curve for this fuel (Equation 4.11); this is the demand function Π^D included in the supplier's objective function (4.1a):

$$p^D_{yhnde} = eff^D_{ynde} \left[int^D_{yhnd} - slp^D_{yhnd} \left(\sum_{s\in S, f\in E} eff^D_{yndf} q^D_{yhsndf} \right) \right] - eucc^D_{yhnde} - eucl^D_{yhnde} \left(\sum_{s\in S} q^D_{yhsnde} \right) - \sum_{g\in G} p^G_{yng} ems^D_{ydeg} \tag{4.11}$$

Final demand price of a fuel p^D equals the following: the first part of the equation is a composite sector price index in terms of energy services, weighted by the efficiency of the particular fuel in supplying the service. Parameters int^D and slp^D give the form of the sector's energy service demand function.

The second part concerns the fuel-specific end use costs. We use a linear function with parameters $eucc^D$ and $eucl^D$ to capture stickiness of usage of a fuel. The third part of the inverse demand curve represents the emission costs for consuming the fuel.

In addition to emission constraints and end use costs, consumers may face restrictions in their energy consumption due to governmental regulation, technical constraints or a limited capital stock. The first constraint type (denoted L) concerns the fuel mix of demand for a certain sector: a minimum share shr^L of total consumption of demand sector(s) $d \in D^L$ satisfied from fuels $e \in \widehat{E}^L$ must be supplied using fuel(s) $e \in E^L$.

$$shr^L_{ynl} \sum_{\substack{s \in S, d \in D^L_l \\ e \in \widehat{E}^L_l}} \mathit{eff}^D_{ynde} q^D_{yhsnde} \leq \sum_{\substack{s \in S, d \in D^L_l \\ e \in E^L_l}} \mathit{eff}^D_{ynde} q^D_{yhsnde} \quad (\beta^L_{yhnl}) \tag{4.12}$$

The second constraint type (denoted M) concerns the "origin" or transformation of fuels. A minimum share shr^M of consumption of fuel(s) $e \in E^M$ must be supplied from transformation technology(s) $c \in C^M$ at that node n.

$$shr^M_{ynm} \sum_{\substack{s \in S, d \in D \\ f \in E^M_m}} q^D_{yhsndf} \leq \sum_{\substack{s \in S, (e,f) \in E^C_c \\ c \in C^M_m}} \mathit{transf}^C_{yncef} q^C_{yhsnce} \quad (\beta^M_{yhnm}) \tag{4.13}$$

4.3.8. Solution approach

We formulate the problem as a Mixed Complementarity Problem (MCP, cf. Facchinei and Pang, 2003) by deriving the first-order (Karush-Kuhn-Tucker, or KKT) optimality condition of each player's optimization problem and solving them simultaneously. This can be interpreted as a Nash equilibrium solution of the market, in which some players act as price-takers (i.e., perfectly competitive), while others may exert Cournot market power vis-à-vis final demand. We use an exogenous conjectural variations parameter ($cour^S$) to incorporate this behaviour and allow intermediate cases. Each player's optimization problem is convex and all constraints are linear, so the KKT conditions are necessary and sufficient for optimality. Nevertheless, since not all optimization problems are strictly convex, we cannot guarantee uniqueness of the equilibrium. This is an inherent problem common to all large-scale energy models that are (partly) linear.

The KKT conditions are stated in Appendix B.2. The equilibrium price p^D is replaced by the inverse demand curve to reduce the number of variables and equations. Hence, there is no equivalent to Equation 4.11 in the KKT conditions.

Since the demand sectors do not have specific decision variables, the constraints (4.12) and (4.13) are included in the profit maximization problem of the suppliers. We therefore have a Generalized Nash Equilibrium (GNE), because the suppliers face so-called "shared constraints": the constraints are identical, but in principle, each supplier may value the constraint differently. Mathematically, this is equivalent to assigning a different Lagrange multiplier to it. Hence, the MCP would no longer be a square system. We follow the approach by Harker (1991) and assume that each supplier values a shared constraint identically. Several recent models of generalized Nash games in energy markets use a similar approach (e.g., Abada et al., 2013; Oggioni et al., 2012).

A challenging task of building a large-scale bottom-up equilibrium model is the calibration to match observed values in the base year and projections from a reference source reasonably well. While data on production and transformation costs as well as other technology aspects (e.g., transformation efficiency, losses) are available, making defensible assumptions concerning the demand side is more difficult (e.g., willingness-to-pay for energy services, possibility for fuel substitution). Furthermore, one has to avoid the aforementioned problem of "bang-bang"-results. To this end, we introduce the concept of fuel-specific end use costs in the final demand curve. We also implement an iterative semi-automated calibration algorithm; the end use costs and this algorithm are discussed in more detail below.

For the reference data set (explained further in the next section), the resulting model contains approximately 150,000 variables and equations. It is solved in GAMS using the PATH solver (Ferris and Munson, 2000). Due to the logarithmic function used, solving it without initializing from a well-chosen starting point does not converge. We implemented several subroutines to facilitate obtaining a valid starting point.

4.3.9. End use costs and model calibration

As discussed previously, linear fuel substitution may yield "bang-bang"-results. Given that a fuel is usually used for many different purposes even within a sector, a certain measure of "stickiness" of fuel usage is plausible. Furthermore, fuel-switching often requires a change of equipment, which entails investment costs. One may also consider start-up costs as a rationale for sticky fuel-switching in a seasonal model. Overall, we expect substitution to occur gradually, purpose-by-purpose. However, this is difficult to capture while keeping the model linear, which is an important consideration for the sake of efficient numerical computation.

We therefore introduce the concept of fuel-specific end use costs to mimic this gradual substitution; they are represented by an affine function. Parameters are chosen such that, in equilibrium, each demand sector chooses exactly the reference demand quantity of each fuel in its consumption mix.

An illustration of end use costs

The end use costs are depicted, in a stylized example, in Figure 4.2. There are two fuels (1 and 2) used in the sector; reference consumption values are given by q^{D1} and q^{D2}. For simplicity, we assume that both fuels have the same *efficiency*, i.e., the same level of *energy service* provided per unit of fuel.

The prices on the y-axis (p^{S1} and p^{S2}) can be interpreted as supply price, i.e., how much does it cost to deliver (supply) the reference quantity of the respective fuel to that node. The consumer has to pay the supply costs plus the end use costs, depicted by the intercept and slope of the end use cost function. The slope of the end use cost curve of fuel 1 is flatter, which means that consumption of this fuel is more elastic. The slope is determined from assumptions on the relative elasticity of substitution between the fuels in the sector.

When comparing the intersection of the demand cost curves (supply cost plus linear end use costs) with the reference demand quantity for both fuels, one notices that the "price" to

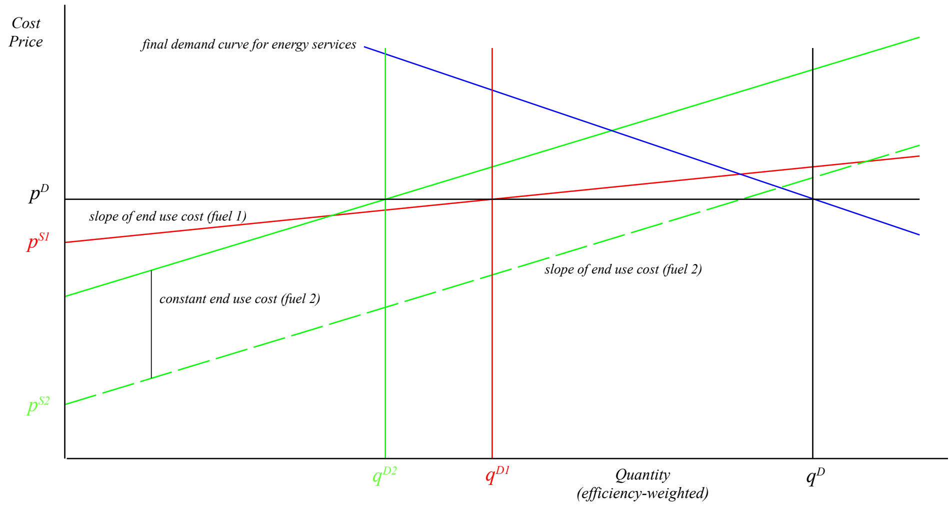


Figure 4.2.: Illustration of end use costs in a sector which can consume two fuels

be paid by the consumer for fuel 2 is much lower than the price for fuel 1; hence, this would not be a plausible equilibrium for the consumer to choose less of fuel 2 than of fuel 1. We therefore shift the demand cost curve upwards by adding a constant end use cost term; this could be, for example, interpreted as distribution costs to deliver the fuel.

Now, the intersection of the demand cost curve and the reference demand quantity for both fuels are equal; hence, it is indeed optimal for the consumer to use exactly the reference quantities, paying the final demand price p^D for a unit of either.

Summing up the reference demand quantities, we obtain the total quantity (energy service) consumed (q^D). Now, we can use the standard approach in partial equilibrium energy sector models, fitting a linear inverse demand curve through the reference point (q^D, p^D). The slope of the inverse demand curve at that point is derived from the elasticity of the entire sector.

An iterative calibration algorithm

As stated before, calibrating a large-scale model is a challenge. We use the concept of the end use costs and supply prices to implement a semi-automatic calibration algorithm.

0 *Initialize:* Assume a supply price for each node/fuel/sector combination

1 Derive the end use cost parameters and the inverse demand function based on the supply prices

2 Solve the model

3 Compare model results (quantities consumed) with demand reference values; if model results are too high, decrease the supply price of this fuel; otherwise, increase the

supply price

4 Repeat from step 1; end after n iterations

At first glance, it may seem counter-intuitive to decrease the fuel supply price when this fuel is used too much relative to the reference quantity, so let us elaborate on the rationale: if this fuel is not the most expensive one (including end use costs), a change in this fuel's supply price will not influence the final demand price p^D, set by the most expensive fuel. Instead, the constant end use cost term will be increased, making the fuel relatively more expensive in the next model run. In contrast, if a fuel is already the most expensive, reducing its supply price will also reduce the final demand price. This will also reduce all other fuels' constant end use cost parameter, making all other fuels relatively cheaper. Either way, the share of this fuel in the sector's total consumption will decrease in the next model run, which should get model results closer to reference values.

Obviously, this algorithm only seeks to match the demand reference quantities. Therefore, after a sufficient number of iterations, we compare production, transformation, and trade flow results to the reference values, and adapt relative costs and other parameters accordingly (and manually). Then, we repeat steps 1–4. In practical application, this algorithm works quite well.

4.4. Some illustrative results

The main contribution of this chapter is the mathematical formulation of a multi-fuel equilibrium model with market power considerations. In the following section, we illustrate that this model can be used to solve large-scale applications of the global energy system. We provide a brief overview of the underlying data set and two scenarios, to demonstrate the advantages of our approach compared to both energy system models and fuel-specific equilibrium models.

The Base Case

The base year of the data set is 2010; the model proceeds in 10-year steps until 2050. Data on quantities produced, transformed and consumed in the base year are compiled from the *Extended World Energy Balances* of the International Energy Agency (IEA).[9] Reference demand values for future years are derived from the IEA's *World Energy Outlook* (WEO, IEA, 2013), *New Policies* scenario; however, since our assumptions on demand sector and fuel disaggregation partly differ from the WEO, one must be careful when comparing our results with the IEA's publications. For EU members countries, we compared the IEA projections to the *Energy Roadmap 2050* (EC, 2011) and the European Commission's *Reference Scenario* (EC, 2013) to obtain projections on a more disaggregated level.

[9] IEA (2013), "Extended world energy balances", *IEA World Energy Statistics and Balances* (database). doi: 10.1787/data-00513-en

Production capacities and costs, both for the base year of 2010 as well as the development over the next decades, are taken from our research group's sector models (Egging, 2013; Schröder et al., 2013a; Huppmann and Holz, 2012; Leuthold et al., 2012; Haftendorn and Holz, 2010). Cost estimates and the potential of different renewable energy sources are taken from the respective IEA publications; for Europe, we use data based on the *Reshaping* project.[10] During calibration, we compiled reference values for future production and the fuel mix in power generation from the WEO 2013, various publications by the US Energy Information Administration (EIA), and BP (2014b); however, since we derive and use our own cost estimates for production and transformation, model results may deviate with respect to production and fuel mix in power generation.[11]

The fuels included are: crude oil, (refined) oil products, natural gas, coal, lignite, electricity, biofuels, renewables, and (as input for power generation only) nuclear and hydro. We intend to use this data set mainly for studies with a European focus, so Europe is represented by 15 nodes, of which eleven are EU member states (or aggregates thereof). The rest of the world is aggregated into thirteen nodes by continent or major regions. Noting that capacity constraints in global LNG trade are due to the liquefaction and regasification infrastructure (rather than along the shipping routes), we add 34 nodes to represent LNG terminals. There are two seasons per year, summer and winter, with natural gas storage facilities to shift this fuel from the low demand to high demand months.

The Russian production of crude oil and natural gas is assumed to be controlled by one strategic supplier. In the Middle East, the production of crude oil and natural gas is divided between two suppliers: a strategic supplier representing the OPEC members in the region (93 % of crude oil and 87 % of natural gas production in the Middle East region in 2010), and a competitive supplier controlling the remaining capacity. The reserves in the Middle East are separated by OPEC membership as well. We assume a CV value of 0.5 for the strategic suppliers (cf. Huppmann, 2013b, and Chapter 5 of this dissertation), except for their respective domestic markets where they act competitively.

A shale disappointment – A reduction of US natural gas supply

The strongly increasing production of shale gas in the US over the past years has dramatically changed expectations regarding the future energy landscape. Only a few years ago, substantial investment went into regasification terminals in North America to import liquefied natural gas (LNG). Now, the IEA and the Energy Information Administration (EIA) project that North America will be a net exporter of natural gas in the near future (EIA, 2014).

Richter (2013) discusses how current projections of US shale gas production may be substantially over-optimistic. He uses a single-fuel partial-equilibrium model to analyse shifts in trade flows as well as production and consumption patterns if the shale gas potential

[10] See http://www.reshaping-res-policy.eu for a list of publications.

[11] The numerical results presented below are based on the data set as of March 17, 2014. We intend to continue improving the underlying data and assumptions, in particular with respect to renewable energy sources, and to incorporate updated projections from IEA, EIA, BP, and other publications.

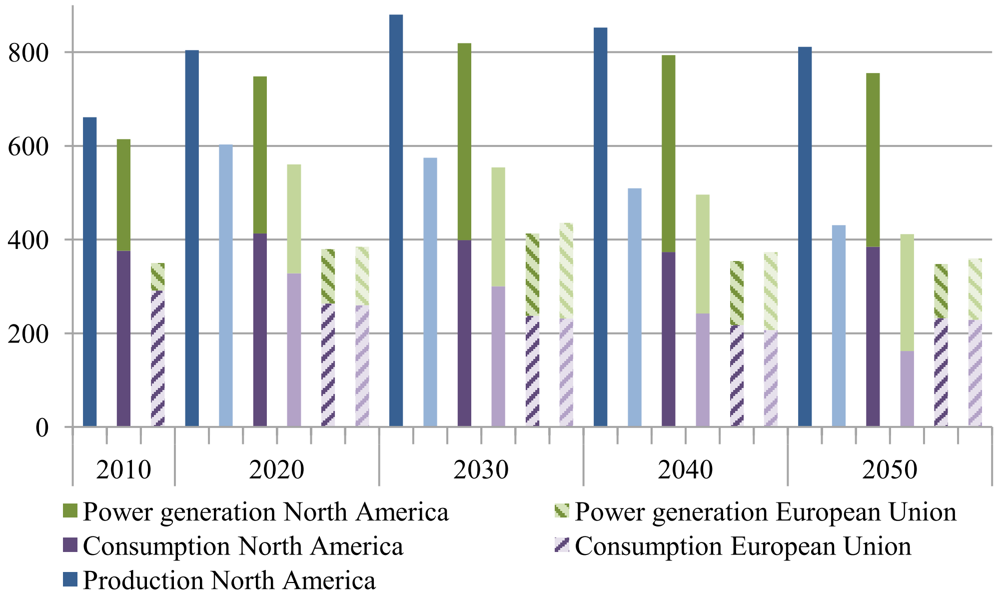


Figure 4.3.: Scenario *A shale disappointment*: Natural gas production and consumption in North America and the EU, in mtoe/y (Base Case: dark; scenario results: light colours)

does not meet current expectations. We replicate his work, but our model allows to analyse endogenous fuel substitution due to reduced availability of shale gas compared to our Base Case derived from the WEO and other sources.

Production and consumption of natural gas in North America and the European Union is presented in Figur 4.3: instead of increasing natural gas production in North America, as projected by the EIA and others, output decreases steadily until 2050 in this scenario.[12] Direct consumption and use in power generation in North America decline accordingly.

The interesting effect from this scenario can be seen in the use of natural gas in European power generation: it increases relative to the Base Case, while in a single-fuel model as used by Richter, use of natural gas would decline globally across all uses. In a multi-fuel model, this can be explained by a shift in global fossil fuel trade: North America makes up for reduced availability of domestic natural gas by increasing their imports of crude oil and coal; it even starts to import natural gas by 2030 from South America, as predicted by the EIA as recent as a decade ago (cf. Richter, 2013). The EU in turn compensates for higher world prices of coal and oil by importing more natural gas by pipeline from Russia and the Caspian region.

[12]The scenario assumptions are set such that the North American production follows the trajectory of US production in the "Low Shale" scenario presented by Richter (2013).

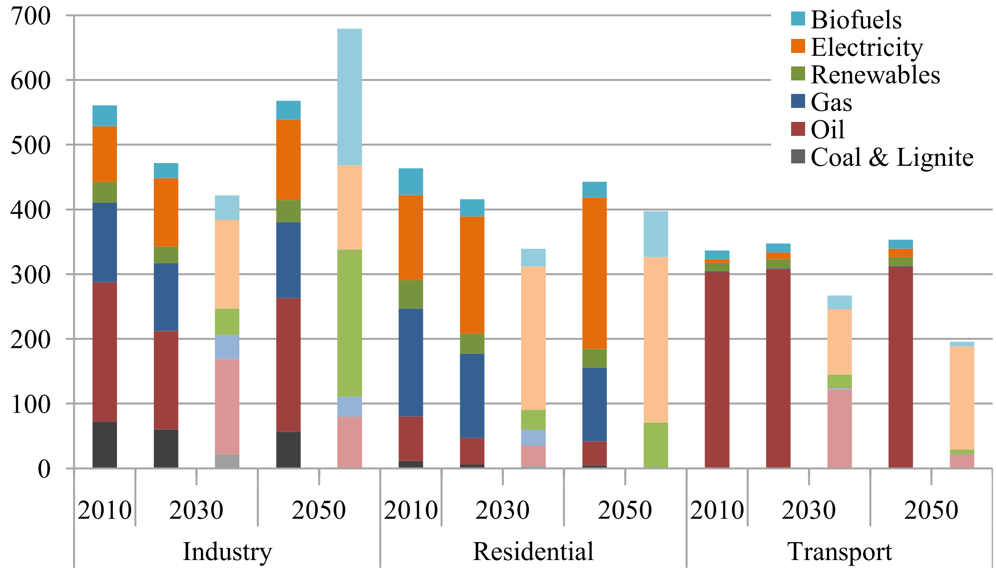


Figure 4.4.: Scenario *Truly Renewable*: Energy consumption by sector in the EU, in mtoe/y (Base Case: dark; scenario results: light colours)

Truly Renewable – Strong emission reduction policies in Europe[13]

The European Commission recently announced its intention to reduce greenhouse gas emissions by 40 % until 2030 relative to 1990 levels, amongst other objectives (EC, 2014); a reduction by 80-95 % until 2050 is regularly discussed (EC, 2011), even though no legally binding decisions have been made. We simulate a scenario where all emissions from the energy sector are reduced by 40 % until 2030 and 80 % until 2050.[14] We assume that all energy consumed is covered by this mandate, in contrast to the current setup of the ETS. In addition, we introduce an increasing biofuel mandate, reaching 25 % of all oil products consumed in the transportation sector by 2050.

The energy mix in the demand sectors is depicted in Figure 4.4: the sector most affected is transportation, where the combination of a biofuel mandate and the emissions quota induces a strong shift to electrification. Crude oil, natural gas, and coal in industrial consumption are mostly replaced by renewables and biofuels; because renewable energy sources cannot equivalently replace fossil fuels in all uses, aggregate consumption of secondary energy increases in the energy sector in 2050 relative to the Base Case. Coal is phased out almost

[13]This scenario is not included in the DIW Discussion Paper version of this chapter. It was added following the publication of the *DIW Wochenbericht* 13 (2014): "Atomkraft ohne Zukunftsaussichten", March 26, 2014. In contrast to the scenario *Ambitious EU* in the Discussion Paper, this scenario assumes that no nuclear or CCS power plants will be constructed due to high implicit costs, the unresolved question of storage, and low public acceptance.

[14]Since we do not include non-energy sector emissions (e.g., industrial process emissions), we abstract from those. The difference between the WEO reference emission values presented in Figure 4.6 and model results is mostly due to this omission, as well as the aggregation to keep our data set sufficiently small to be numerically tractable. To avoid distortion due to the model aggregation and calibration, we set the reduction relative to model results of emissions in 2010.

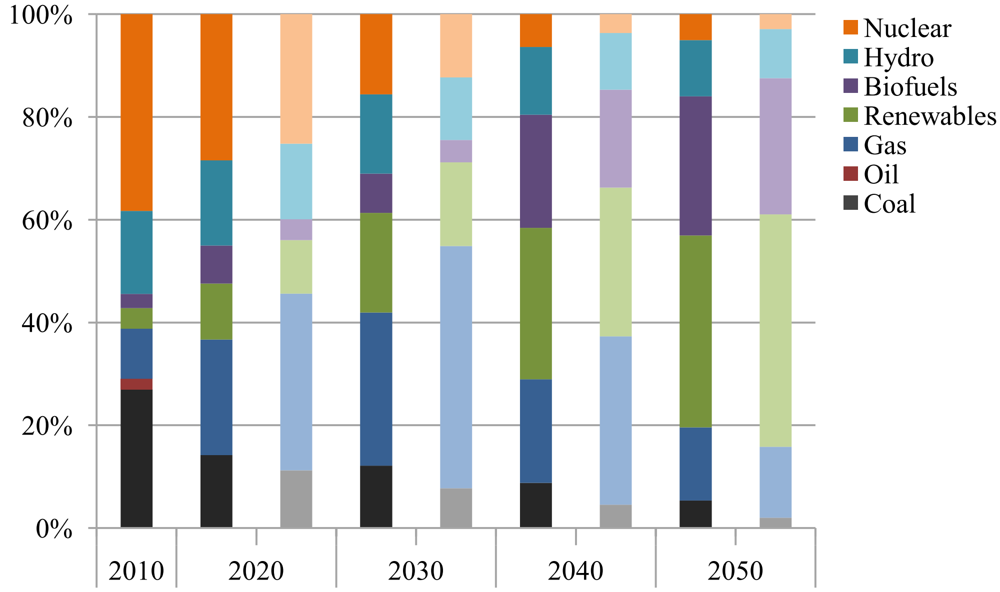


Figure 4.5.: Scenario *Truly Renewable*: Relative fuel mix in power generation in the EU (Base Case: dark; scenario results: light colours)

completely by 2050 in the scenario both in direct use and in power generation, as shown in Figure 4.5.

In the scenario, the CO_2 emission permit price in the EU ETS increases to 51 USD/t CO_2 in 2030 and more than 200 USD in 2050, compared to the Base Case emission costs of 40 USD in 2030 (according to the WEO) and 60 USD in 2050 (own assumption). Aggregate energy consumption is reduced by 25 % in 2050 relative to the Base Case.

Last, let us turn to emission reductions and carbon leakage, which is illustrated in Figure 4.6: Emissions from coal and oil are reduced more strongly than those from natural gas, because it is shifted to the power generation and serves as a bridge fuel. Leakage rates are in the range of 70–90 %. At first, most of the leakage occurs in the European periphery (the Balkans and Ukraine): power generators use their comparative cost advantage of not being covered by the EU ETS and export electricity to Eastern EU member states, in spite of relatively high losses of power transmission. After 2030, the major part of the leakage shifts to Asia, where China and other countries increase their consumption of fossil fuels due to reduced prices on the world market.

4.5. Conclusions

Understanding the global energy system and quantifying the impacts of technological and economic changes as well as the effects of policy measures requires large-scale numerical models. A multitude of such models exist and are widely used for policy evaluation and scenario simulations. They either follow a *top-down* methodology, which allows embedding

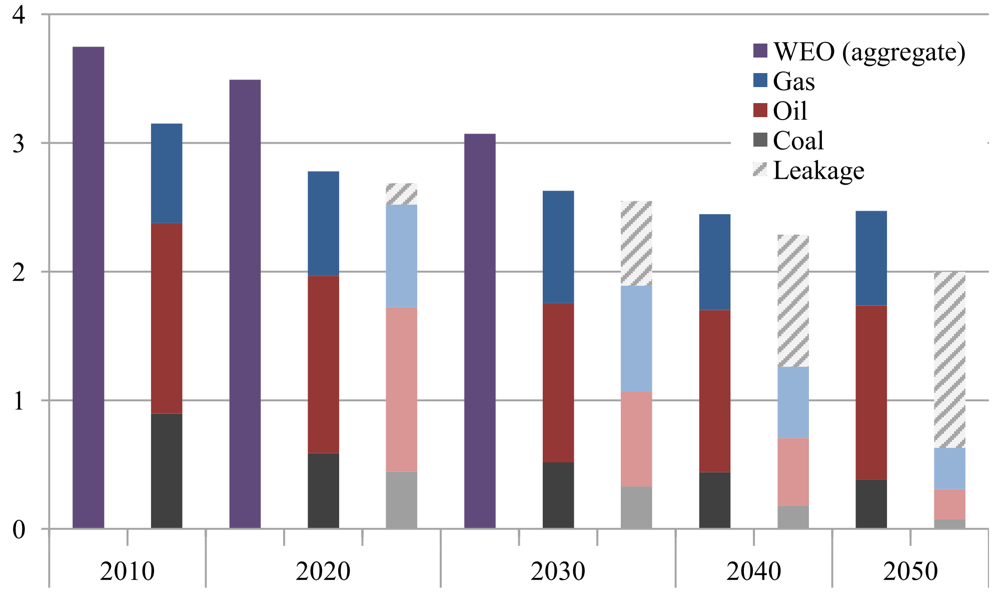


Figure 4.6.: Scenario *Truly Renewable*: CO_2 emissions, model results by fossil fuel, and WEO projections, in Gt/y (Base Case: dark; scenario results: light colours)

the energy sector in the economy at large, or a *bottom-up* approach, which allows for a detailed analysis of the energy system and infrastructure constraints. Market structure analysis is usually done in a sector- or fuel-specific modelling framework.

We present a first-of-its-kind large-scale model that combines endogenous fuel substitution, detailed infrastructure constraints, and strategic behaviour by suppliers in a unified framework. Most importantly, in the model proposed here, suppliers can exert market power across several fuels; strategic firms are aware how their sales of one fuel impact not only the price of that fuel ("direct" market power), but also the revenue they earn from selling other fuels in the same market. We believe that this is the first large-scale market equilibrium model that captures such an effect.

The model covers multiple time periods and allows for endogenous investment in production and mid-stream infrastructure capacity. We include seasonality and storage, and are thus able to incorporate short-term operational concerns, which are highly relevant for energy market analysis.

We illustrate the model functionality using a data set compiled from the IEA's Extended World Energy Balances and other sources. The first scenario analyses the impacts of strongly reduced shale gas availability in North America, in contrast to current optimistic projections of LNG exports from the US in the next years. The decreasing output of North American natural gas is compensated in the scenario with increased imports of crude oil and coal. This induces a shift in global trade patterns, and Europe increases its use of natural gas in power generation due to higher world market prices of coal and oil.

In the second scenario, we assume a tightening of the EU ETS emission cap by 80 % in

2050 combined with a stronger biofuel mandate. This leadsto a strong electrification of the transportation sector. We observe significant carbon leakage ratios from the unilateral mitigation effort in the order of 70–90 %. In the first decades, the leakage mostly takes place in the European periphery (the Balkans and Ukraine), which benefits from being adjacent to the ETS jurisdiction and exports electricity to Eastern European countries. As more renewable generation capacity comes online after 2030, the leakage shifts to the Asian region, where China and other countries soak up the global fossil fuel supplies not exported to Europe.

Both scenarios illustrate fuel substitution effects which can hardly be captured using single-fuel sector models. At the same time, the market power aspects ignored by ESM, IAM (and most CGE models) affect supply, trade and infrastructure developments and fuel prices.

We plan to extend this modelling framework in two directions: on the one hand, we will use the model for detailed policy analysis, going beyond the two illustrative scenarios presented here. This will require regular updates of the underlying data set and continuous improvement regarding implicit assumptions, in particular with respect to renewable energy sources. Furthermore, we have not yet discussed the impact of market power, which is included in the calibration of our Base Case and the scenarios. Egging et al. (2009) analyse the impact of a counterfactual cartel of "GasPEC" member countries in the natural gas market; the model presented here allows us to replicate their analysis for market power exertion by a joint OPEC-GasPEC cartel in both crude oil and natural gas.

On the other hand, we will extend the model mathematically: one avenue is to include stochasticity, which was recently implemented in other large-scale sector models (e.g., Egging, 2013; Gabriel and Fuller, 2010). Short-term stochasticity could be interpreted as operational or seasonal uncertainty. Long-term stochasticity may represent the ambiguity of resource availability or the "threat" of a global climate change mitigation agreement. Uncertain prospects over an extended time horizon are especially relevant when one considers long-lived infrastructure investment.

Another interesting question to be tackled in the context of energy markets, strategic behaviour, and emissions is the issue of carbon leakage: mitigation efforts in one region may just displace GHG emissions to other regions. This may happen either directly via the channel of fossil fuel prices, as exemplified in the second illustrative scenario. But it may also occur indirectly via the goods channel, including relocation of firms to regions with less stringent climate policies. To properly capture both channels simultaneously requires combining a detailed bottom-up energy system model with a CGE-type module to reflect the impact of energy prices on the economy (Böhringer and Rutherford, 2008). This would allow to more accurately quantify the extent of carbon leakage due to unilateral or regional climate policies with particular regard to strategic behaviour by fossil fuel suppliers.

Chapter 5

National-strategic investment in European power transmission

This chapter is based on:
National-strategic investment in European power transmission.
Discussion Papers of DIW Berlin 1379, 2014.
Joint work with Jonas Egerer.

5.1. Introduction: Investment in power transmission

The creation of a well-integrated *Internal Energy Market* (IEM, EC, 2012) and the decarbonisation of the European electricity sector require both a paradigm shift within the generation portfolio as well as a significant expansion of the (cross-border) power transmission system. The switch to renewable power generation is fundamental to meeting European climate goals (*EU Energy Road Map 2050,* EC, 2011); however, wind and solar are inherently volatile and the most effective sites are often not located close to cities and industrial regions. As a consequence, the transmission system must be strengthened to ensure the effective and efficient integration of these energy sources. Due to the national scope of planning electricity systems for most of the 20$^{\text{th}}$ century, additional cross-border transmission capacity is required in particular to achieve these goals.

There are several recent studies that aim to determine the optimal network investment plans to support such a decarbonisation of the electricity sector (e.g., Egerer et al., 2013a; Tröster et al., 2011). These models take a long-term view (i.e., until the year 2050), and they typically use a pan-European welfare maximization approach, where all investment decisions are taken by a benevolent central planner. In contrast, the *European Ten-Year Network Development Plan* (TYNDP, ENTSO-E, 2013) only covers investments over the following decade. In contrast to the other studies mentioned, the TYNDP is not derived from an explicit welfare-maximization rationale. Instead, it lists transmission expansion projects as planned by the individual transmission system operators (TSO) and agreed within the *European Network of Transmission System Operators for Electricity* (ENTSO-E), the TSO's trade association, following a public stakeholder process. The TYNDP also specifies a list of prioritized "projects of common interest". However, all these studies neglect that transmission investments may have a strong impact on welfare distribution both across national borders and among different stakeholders: generators, consumers, and TSOs.

Network expansion is still a national prerogative in Europe, both regarding planning and financing. Grid investments are highly capital-intensive and, once built, constitute a "lock-in" regarding the grid topology for many decades to come. Funding for grid upgrades is usually guaranteed by the national regulator, who allows the TSO to levy network usage fees to recover expansion costs. If the income earned by the TSO from usage tariffs and congestion rents exceeds the approved funding base, the surplus must be used to either maintain and guarantee availability of the existing grid, invest in transmission capacity upgrades, or to lower usage fees (EC, 2009).

Due to the nature of electricity flows in an integrated network, beneficiaries of transmission investment may be located in a different jurisdiction than those bearing the costs. National governments, regulators and/or the TSOs may then be reluctant to invest if the benefits accrue elsewhere, unless an appropriate compensation mechanism is in place. Furthermore, any analysis of the power market is particularly complicated due to the specific characteristics of electricity transmission: if one line between two zones is expanded, the changes in power flow patterns may adversely affect other TSOs due to an increase of loop flows or a reduction of effectively available transmission capacity along other lines.

Egerer et al. (2013b) discuss the welfare implications for different topologies of the North and Baltic Seas offshore connectors. They show a significant re-allocation of rents, both across jurisdictions, as well as between the different stakeholder groups. This indicates that the cost allocation of network investment is of paramount importance (cf. Buijs et al., 2010). In principle, the beneficiaries of any network upgrade should bear the costs. With this in mind, the EU has introduced the *Inter-TSO compensation mechanism* (ITC, EC, 2010), intended to remunerate TSOs for transit flows and grid upgrades of regional importance.

Theoretically, an appropriate allocation of benefits and costs through side payments would result in a grand coalition and yield a welfare-optimal expansion on the European level. There is a string of scientific literature examining various allocation methods based on cooperative game theory (Nylund, 2013; Gately, 1974). However, quantifying re-allocation of welfare might prove impossible due to issues of measurement; for instance, one may debate whether the no-investment welfare allocation should serve as a reference (i.e., the status quo), or a hypothetical case of unilateral investment. We believe that this difficulty is one of the reasons why the ITC is still mainly used for compensation of transit flows (where a reference can be more easily established and the monetary transfers involved are fairly small), rather than for compensation of actual network upgrades (cf. Buijs et al., 2010). The current annual budget dispensed through the ITC is around EUR 100 million (Rüster et al., 2012, p. 25ff); the investment volume planned according to the TYNDP is more than EUR 100 billion (ENTSO-E, 2013) over the next decade.[1]

So far, we have argued specifically with the European situation in mind; nevertheless, Buijs et al. (2010) point out that while the institutional setting in the US power markets differs from Europe in many aspects, the same underlying conundrum is present in renewables support and network upgrades between different states as well as between regional system operators such as the Pennsylvania-Maryland-New Jersey Interconnection (PJM) and the Midwest ISO (MISO).

Following this train of thought, we ask whether zonal planners would alter their transmission capacity investment if they were only concerned with national welfare, rather than the welfare of the entire region. We use the term "national-strategic" to differentiate our work from other studies that treat generators as strategic players (e.g., Zerrahn and Huppmann, 2014; Pozo et al., 2013; Schröder et al., 2013b; Neuhoff et al., 2005), as well as from the literature that treats one TSO responsible for the entire network as the strategic player (e.g., Léautier and Thelen, 2009; Rosellón and Weigt, 2011). In contrast to these articles, we assume that the strategic players in the game are zonal planners such as national governments, regulators, or TSOs in charge of a certain network area: they anticipate the effects of network expansion within their jurisdiction on the welfare allocation. They may thus have an incentive to withhold line upgrades or over-invest compared to the welfare-optimal expansion plan in order to induce a shift of rents towards stakeholders in their zone. This

[1] Of course, only a fraction of the investment costs need to be financed through a pan-European fund; most line upgrades are viable from a national perspective alone. Nevertheless, the discrepancy is significant.

distortion of investment may impede the effective integration of the European power market and the efficient shift to a power system based on renewable energy sources.

The next section will briefly present the theory of network formation and discuss two previous models similar to our research question. Section 5.3 describes the three-stage model depicting the national-strategic investment game, while Section 5.4 provides the mathematical formulation. This part also extensively discusses the problems of solving multi-level games and illustrates how our approach deals with them. Section 5.5 provides a numerical example of the three-stage model to illustrate the impact of strategic network planning; Section 5.6 concludes and suggests potential avenues for further research.

5.2. Network formation and strategic zonal planners

The question of how networks are formed when some players act strategically and have to agree whether or not to build a link between them is not new in the economics and game theory literature. However, to date no canonical approach exists to solve such games (cf. Bloch and Jackson, 2007, 2006). To further complicate the research question which we tackle in this work, the size of the link in electricity networks (i.e, capacity of the power line) is of paramount importance for resulting flows and nodal prices. We therefore opt for the most straightforward game structure: zonal planners strategically decide only on the upgrades of lines within their jurisdiction. Cross-border lines are decided by a supra-national planner. This allows us to sidestep, for the time being, the question of how to properly model negotiation on links between zones, while we are still able to focus on the impact of strategic zonal planners. We formulate a methodology to identify Nash equilibria and quantify the welfare "loss" due to strategic behaviour on the network planning level.[2]

A similar research question was tackled by Daxhelet and Smeers (2007), albeit their focus lies on network usage tariffs: the authors compare several proposals discussed within the *Florence Regulatory Forum* and discuss the different implementations of supra-national coordination. They propose a two-stage model in which national regulators set network usage tariffs for generators (G-component) and consumers (or load, hence L-component) in their jurisdiction, aiming to maximize welfare (defined as consumer surplus, generator profits and congestion rent) in their zone. In their model, the regulators play a Nash game, which forms the upper level of the model, while the spot market forms the lower level and takes the network usage tariffs in each zone as given. Mathematically, each regulator faces a mathematical program under equilibrium constraints (MPEC); the equilibrium constraints represent the lower-level optimization problem. Since several regulators interact simultaneously, the entire problem is an equilibrium problem under equilibrium constraints (EPEC), in particular a multi-leader-single-follower game (cf. Kulkarni and Shanbhag, 2013). The network in this work is treated as exogenously given.

In contrast, Buijs and Belmans (2011) focus on network upgrades; they compare the

[2]To be precise, it is not a welfare loss, but a failure to reap all welfare gains from network investment and a more efficient and integrated spot market.

supra-national welfare-optimal investment to two other approaches: first, a setup where the planner responsible for one zone (i.e., a government/regulator) decides on investment in the entire network seeking to maximize domestic welfare comprising consumer surplus, generator profits and congestion rents less investment costs. Similar to the work by Daxhelet and Smeers, the zonal planner takes into account the impact of her decisions on the spot market, which forms the lower level; it is agnostic with respect to the welfare effects in other zones.

As a second model, Buijs and Belmans introduce a variation of the supra-national planner model, which they refer to as *Pareto planner*; they add explicit constraints that the aggregate welfare in each zone must not be reduced due to investment relative to the status quo. Both the zonal planner and the Pareto planner solve an MPEC; however, Buijs and Belmans do not actually compute Nash equilibria between the zonal planners. Their results only serve as upper bounds: how much could a zone benefit at most if all transmission investment was carried out in the most beneficial way for this particular zone. Similarly, their Pareto planner approach exhibits an inconsistency: it is only constrained relative to the status quo. This may not be adequate if a zonal planner can unilaterally improve domestic welfare, for instance by expanding a line which is entirely within her jurisdiction.

Buijs and Belmans (2011) use genetic algorithms to solve each MPEC; it is not straightforward how this could be generalized to obtain a Nash equilibrium between the zonal planners. Two approaches to solve such EPEC-type problems are most commonly used in the literature: *enumeration* discretizes the strategy space for each strategic player; the lower-level optimization problem is then solved for each possible strategy of the upper-level game, and the pay-offs as well as deviation incentives for each player are determined ex-post. Such an approach is used by Egerer and Nylund (2014). The problem when implementing this approach in practice comes from the difficulty of determining a suitable discretization, and a general issue of scalability to solve large-scale problems.

In contrast to enumeration, *diagonalization* describes an iterative algorithm: each MPEC is solved consecutively, holding the decisions variables of the rivals fixed. This approach is sometimes referred to as a variant of the "Gauss-Seidel" algorithm. While it does work in certain applications, convergence to a stable Nash equilibrium is not guaranteed. In particular, several Nash equilibria may exist, and it is not clear whether diagonalization would identify all solutions; more importantly, one can usually make no informed statement whether other equilibria may exist; in particular, a Pareto improvement may be possible (i.e., a new equilibrium where at least one player is strictly better off, and no player has a worse payoff than in the previously found equilibrium). We discuss this in more detail below. In the following section, we present a closed-form expression of the game between zonal planners, as well as an iterative algorithm to determine a number of Nash equilibria ranked according to a welfare metric.

Stage	Player	Decision Variable
I	*Supra-national planner* seeks to maximize welfare in the entire region less investment cost	cross-border network investment
II	Nash game between *Zonal planners* seeks to maximize zonal welfare less domestic investment cost	domestic network investment (within her jurisdiction)
III	*Competitive and integrated spot market* (represented by ISO)	generation, demand, power flow, nodal prices

Figure 5.1.: Illustration of the three-stage game structure

5.3. A three-stage equilibrium problem

We propose a three-stage model to describe the Nash game between zonal planners; Figure 5.1 illustrates the stages, players and decision variables of the game. We solve this game by backward induction; therefore, we present the structure from the bottom stage to the top-level player. We first only describe the model stages and players; the mathematical formulation is presented in the subsequent section.

Stage III: A competitive spot market

The third (bottom) level of the game is the competitive and integrated spot market, where generators, consumers, and TSOs of each zone take the network capacity as given for determining short-term dispatch and demand. The characteristics of the power network are taken into account using a loss-free direct-current load flow (DCLF) representation of Kirchhoff's laws, as is the current standard in this stream of modelling (e.g., Leuthold et al., 2012).[3] This approach explicitly accounts for the externalities occurring in meshed electricity systems.[4]

Mathematically, the problem of a competitive market between generators and demand is equivalent to an Independent System Operator (ISO) optimizing welfare of the entire region (with the network taken as given), so we will use the latter notion in the mathematical derivation. Most importantly, in our subsequent formulation, congestion rents earned by TSOs are based on actual flows rather than financial transmission rights to properly account for these externalities.

[3] The term DCLF is actually a misnomer, as it is not based on the assumption of direct-current electricity flows; instead, it is a linearization of the non-convex alternating-current power flow model.

[4] The current European market design for trading power across zone borders is based on auctions of transmission capacity, which are capped by net transfer capacity (NTC). Because power does not actually flow in such a way, the stated NTC values include security margins to ensure network feasibility in spite of loop flows. Oggioni and Smeers (2013) and Oggioni et al. (2012) discuss the difficulty of combining zonal pricing and trade based on net transfer capacities. The negative externalities arising out of NTC-based trade increase in lock-step with market integration and cross-border trade; European TSOs are currently looking into flow-based approaches to replace the NTC trading framework.

Stage II: The zonal planners

The second (intermediate) stage is the Nash game between the zonal planners, each deciding on network upgrades in her jurisdiction. Every zonal planner aims to maximize welfare in her zone; congestion rents earned by the zonal TSO are explicitly included in the objective function. Rents from cross-border transmission and the expansion costs for these lines are shared equally between the adjacent zones.

The decision on cross-border line upgrades is not taken by the zonal planners, but by the player at the first stage of the game. This is the simplest setup for our illustrative model; we will return to potential modifications and more elaborate formulations of bargaining in the concluding section, when we discuss further research.

Stage I: The supra-national coordination agency

The first (top) level of our model is a supra-national coordination agency; she can represent, for example, the Agency for the Cooperation of Energy Regulators (ACER). While this player cannot influence the decision by the zonal planners on their domestic line upgrades, she can decide on the expansion of cross-border lines; because she anticipates the strategic reaction of zonal planners, she can set the cross-border line investment in such a ways as to induce them to end up in a second-best Nash equilibrium.[5]

Realizing that there may exist multiple equilibria in the game between the zonal planners (stage II), the coordination agency can therefore also be interpreted as an equilibrium selection mechanism – it coordinates among the zonal planners to make sure a "good" equilibrium is attained. The coordination agency has a central role in the mathematical implementation and the solution approach, which we will discuss in the next section.

5.4. Mathematical formulation

We now turn to the mathematical formulation; Table 5.1 lists selected important notation used in this chapter; the remaining sets, variables and parameters are introduced where necessary.

Our approach follows the methodology developed by Ruiz et al. (2012); we will discuss their approach and the extensions we introduce below, when we discuss the difficulties posed by multi-stage games.[6]

[5] The first-best solution would be the supra-national network investment plan without strategic considerations by the zonal planners.

[6] A similar three-stage model is developed by Zerrahn and Huppmann (2014); the authors use analogous reformulations to the lowest-level equilibrium problem, derive Karush-Kuhn-Tucker (KKT) conditions of the intermediate strategic players and use the top-level player as an equilibrium selection mechanism. In their work, however, the intermediate (strategic) players are generators rather than zonal planners, seeking to maximize profits by strategically withholding generation capacity or intentionally congesting the network. The top-level player is the network planner facing a trade-off between investment costs and the welfare-enhancing of additional line capacity; the welfare gains can be distinguished between efficiency gains (less congestion) and a higher degree of competition between strategic generators.

Sets	$n \in N$	... nodes
	$s \in S$	... power plant unit, generation technology
	$r \in R$	... regulatory zone (price, TSO)
	$l \in L$	... power lines
Variables	d_n	... demand
	g_{ns}	... generation
	p_n	... locational marginal price
	e_l	... line expansion/investment
Parameters	$\overline{g_{ns}}$	... generation capacity
	c^G_{ns}	... linear generation costs
	c^T_l	... linear transmission investment costs
	c^R_r	... fixed costs/guaranteed return for TSO
	B_{nk}, H_{lk}	... line/node impedance matrices

Table 5.1.: Selected notation

The Independent System Operator

The ISO optimizes the short-term dispatch of power plants, deciding on the generation level g_{ns} at each node n and of technology s, demand d_n, and the voltage angle δ_n.[7] Consumer welfare (utility from consuming electricity) is given by the function $(a_n - \frac{1}{2}b_n d_n)d_n$, while c^G_{ns} are the per-unit (i.e., marginal) generation costs. From the point of view of the ISO, the line expansion e_l is an exogenous parameter.

$$\max_{g,d,\delta} \sum_{n \in N} \left[\left(a_n - \tfrac{1}{2}b_n d_n\right) d_n - \sum_s c^G_{ns} g_{ns} \right] \tag{5.1a}$$

$$\text{s.t.} \sum_s g_{ns} - \sum_k B_{nk}\delta_k - d_n = 0 \quad (p_n) \tag{5.1b}$$

$$\overline{f}_l + e_l - \sum_k H_{lk}\delta_k \geq 0 \quad (\overline{\mu}_l) \tag{5.1c}$$

$$\overline{f}_l + e_l + \sum_k H_{lk}\delta_k \geq 0 \quad (\underline{\mu}_l) \tag{5.1d}$$

$$\overline{g}_{ns} - g_{ns} \geq 0 \quad (\beta_{ns}) \tag{5.1e}$$

$$\delta_{\hat{n}} = 0 \quad (\gamma) \tag{5.1f}$$

$$g_{ns} \geq 0 \quad (\psi_{ns}) \tag{5.1g}$$

$$d_n \geq 0 \quad (\phi_n) \tag{5.1h}$$

[7] This approach using voltage angles is equivalent to another method frequently used when formulating DCLF models, namely using power transmission distribution factors (PTDF).

Constraint (5.1b) is the energy balance constraint; the dual variable to this is the price of energy at each node. Constraints (5.1c–5.1d) are the thermal line capacity limits (in positive and negative power flow direction). The maximum available generation by node and technology is given by Constraint (5.1e). Using voltage angles to determine power flows requires the definition of a *slack bus* ($\hat{n}$), at which the voltage angle is 0 by assumption (Constraint 5.1f). The dual variables for all constraints are given in parentheses.

The ISO problem forms the lower stage to the optimization problem of the zonal planners, the subsequent game stage II; hence, the standard approach is to insert the ISO problem to the upper-level problem as equilibrium constraints, specified below:[8]

$$0 = c^G_{ns} - p_n + \beta_{ns} - \psi_{ns} \quad , \quad g_{ns} \text{ (free)} \tag{5.2a}$$

$$0 = -a_n + b_n d_n + p_n - \phi_n \quad , \quad d_n \text{ (free)} \tag{5.2b}$$

$$0 = \sum_k B_{kn} p_k + \sum_l H_{ln} \left(\overline{\mu}_l - \underline{\mu}_l\right) - \begin{Bmatrix} \gamma & \text{if } n = \hat{n} \\ 0 & \text{else} \end{Bmatrix} \quad , \quad \delta_n \text{ (free)} \tag{5.2c}$$

$$0 = \sum_s g_{ns} - \sum_k B_{nk} \delta_k - d_n \quad , \quad p_n \text{ (free)} \tag{5.2d}$$

$$0 \leq \overline{f}_l + e_l - \sum_k H_{lk} \delta_k \quad \perp \quad \overline{\mu}_l \geq 0 \tag{5.2e}$$

$$0 \leq \overline{f}_l + e_l + \sum_k H_{lk} \delta_k \quad \perp \quad \underline{\mu}_l \geq 0 \tag{5.2f}$$

$$0 \leq \overline{g}_{ns} - g_{ns} \quad \perp \quad \beta_{ns} \geq 0 \tag{5.2g}$$

$$0 = \delta_{\hat{n}} \quad , \quad \gamma \text{ (free)} \tag{5.2h}$$

$$0 \leq g_{ns} \quad \perp \quad \psi_{ns} \geq 0 \tag{5.2i}$$

$$0 \leq d_n \quad \perp \quad \phi_n \geq 0 \tag{5.2j}$$

Gabriel and Leuthold (2010), as one example, formulate such a model by taking first-order (KKT) conditions of the ISO problem (5.1), and replace the complementarity (or slackness) constraints using disjunctive constraints (Fortuny-Amat and McCarl, 1981). This yields a mixed-integer problem; however, deriving stationarity conditions for the zonal planners requires to take the derivative of the objective function. When binary variables are included due to the disjunctive constraints formulation, this is not possible; hence, this method is not suitable for our approach. Another possibility to replace complementarity conditions was proposed by Siddiqui and Gabriel (2013), using Schur's decomposition and so-called SOS1-type variables. However, implementing this approach in a mathematically exact way also requires binary variables, and thus suffers from the same caveat.

Therefore, we apply the aforementioned methodology by Ruiz et al. (2012) as an alternative: we use strong duality to replace the optimization problem by a set of constraints, which can be included in the next stage's optimization problems without the need for binary variables.

[8]The equilibrium model (5.2) could be simplified by combining equations (5.2a,5.2b) and equations (5.2i,5.2j); however, we will require the more extensive version in the later reformulations.

All constraints of Problem (5.1) are linear and the objective function (5.1a) is concave; therefore, strong duality holds (for any non-trivial set of parameters that admits a feasible solution). The Lagrangian dual problem then reads as follows:

$$\min_{p,\overline{\mu},\underline{\mu},\beta,\gamma,\phi} \tfrac{1}{2} \sum_{n\in N} \frac{1}{b_n} \left((a_n)^2 + (p_n)^2 + (\phi_n)^2 - 2a_n p_n + 2a_n\phi_n - 2p_n\phi_n \right)$$
$$+ \sum_{l\in L} (\overline{\mu}_l + \underline{\mu}_l)(\overline{f}_l + e_l) + \sum_{n\in N, s\in S} \beta_{ns}\overline{g}_{ns} \tag{5.3a}$$

$$\text{s.t.} \qquad c^G_{ns} - p_n + \beta_{ns} - \psi_{ns} = 0 \quad (g_{ns}) \tag{5.3b}$$

$$-a_n + b_n d_n + p_n - \phi_n = 0 \quad (d_n) \tag{5.3c}$$

$$\sum_k B_{kn} p_k + \sum_l H_{ln} \left(\overline{\mu}_l - \underline{\mu}_l \right) - \begin{Bmatrix} \gamma & \text{if } n = \hat{n} \\ 0 & \text{else} \end{Bmatrix} = 0 \quad (\delta_n) \tag{5.3d}$$

Strong duality holds, hence the objective values at the optimal solution of Problems (5.1) and (5.3) are identical. This can be stated formally in the following constraint:

$$-\tfrac{1}{2} \sum_{n\in N} \frac{1}{b_n} \left((a_n)^2 + (p_n)^2 + (\phi_n)^2 - 2a_n p_n + 2a_n\phi_n - 2p_n\phi_n \right)$$
$$- \sum_{l\in L} (\overline{\mu}_l + \underline{\mu}_l)((\overline{f}_l + e_l) - \sum_{n\in N, s\in S} \beta_{ns}\overline{g}_{ns}$$
$$+ \sum_{n\in N} \left[\left(a_n - \tfrac{1}{2} b_n d_n \right) d_n - \sum_s c^G_{ns} g_{ns} \right] \geq 0 \tag{5.4}$$

Strong duality entails that any vector $(g, d, \delta, p, \overline{\mu}, \underline{\mu}, \beta, \gamma, \phi)$ satisfying the primal constraints (5.1b–5.1h), the dual constraints (5.3b–5.3d) as well as the strong-duality constraint (5.4) is an optimal solution to both the primal and the dual problem. We can now use this set of linear and quadratic constraints to replace the ISO optimality conditions in the optimization problems of the players at the higher stages of the game, without having to deal with complementarity constraints or binary variables due to using disjunctive constraints.[9]

Normally when using strong duality, Constraint (5.4) would have to hold with equality; we write it as an inequality such that the quadratic constraint remains convex (for a given network, i.e., fixed e_l). Since the objective value of Problem (5.1) for any feasible solution is always smaller than the objective value of Problem (5.3) for any feasible solution due to weak duality, Constraint (5.4) can never hold with a strict inequality for a vector that is primal and dual feasible.

[9] The constraints to the dual problem (5.3b–5.3d) are also the stationarity (KKT) conditions, if one would solve this problem as an equilibrium problem (or mathematically, a mixed complementarity problem, MCP), as given by Equations (5.2). The solution vector of the strong duality problem would also solve the MCP, and vice versa.

The zonal planner

Each zonal planner r aims to maximize the sum of generator profits, consumer surplus and congestion rent earned by the zonal TSO at all nodes within her zone $N_r \subset N$ by deciding on network upgrades for all lines e_l within her jurisdiction; these are the lines in the set $L_r \subset L$. For cross-border lines, the investment level is taken as given by the zonal planner; both investment costs and congestions rents are shared equally between the two adjacent zonal TSOs by assumption. The parameter shr_{lr} denotes the share of costs and rents of line l to be attributed to the TSO of zone r; it is 1 for domestic lines, 0.5 for cross-border lines, and 0 otherwise.

Mathematically, the optimization problem of the zonal planners reads as follows:

$$\max_{e_l \in L_r} \sum_{n \in N_r} \left[\left(a_n - \tfrac{1}{2} b_n d_{nt} - p_n\right) d_n + \sum_s \left(p_n g_{ns} - c^G_{ns} g_{ns}\right) \right] - \sum_{l \in L_r} \left[shr_{lr} \left(\sum_k H_{lk} \delta_k\right) \left(\sum_k I_{lk} p_k\right) - \sum_{l \in L_r} c^T_l e_l \right] \tag{5.5a}$$

$$\text{s.t. } c^G_{ns} - p_n + \beta_{ns} - \psi_{ns} = 0 \quad (\eta^R_{nsr}) \tag{5.5b}$$

$$-a_n + b_n d_n + p_n - \phi_n = 0 \quad (\rho^R_{nr}) \tag{5.5c}$$

$$\sum_k B_{kn} p_k + \sum_l H_{ln} \left(\overline{\mu}_l - \underline{\mu}_l\right) - \begin{Bmatrix} \gamma & \text{if } n = \hat{n} \\ 0 & \text{else} \end{Bmatrix} = 0 \quad (\nu^R_{nr}) \tag{5.5d}$$

$$\sum_s g_{ns} - \sum_k B_{nk} \delta_k - d_n = 0 \quad (\lambda^R_{nr}) \tag{5.5e}$$

$$\overline{f}_l + e_l - \sum_k H_{lk} \delta_k \geq 0 \quad (\overline{\mu}^R_{lr}) \tag{5.5f}$$

$$\overline{f}_l + e_l + \sum_k H_{lk} \delta_k \geq 0 \quad (\underline{\mu}^R_{lr}) \tag{5.5g}$$

$$\overline{g}_{ns} - g_{ns} \geq 0 \quad (\beta^R_{nsr}) \tag{5.5h}$$

$$\delta_{\hat{n}} = 0 \quad (\gamma^R_r) \tag{5.5i}$$

$$g_{ns} \geq 0 \quad (\psi^R_{nsr}) \tag{5.5j}$$

$$d_n \geq 0 \quad (\phi^R_{nr}) \tag{5.5k}$$

$$-\tfrac{1}{2} \sum_{n \in N} \frac{1}{b_n} \left((a_n)^2 + (p_n)^2 + (\phi_n)^2 - 2a_n p_n + 2a_n \phi_n - 2p_n \phi_n \right) - \sum_{l \in L} (\overline{\mu}_l + \underline{\mu}_l)((\overline{f}_l + e_l) - \sum_{n \in N, s \in S} \beta_{ns} \overline{g}_{ns} + \sum_{n \in N} \left[\left(a_n - \tfrac{1}{2} b_n d_n\right) d_n - \sum_s c^G_{ns} g_{ns} \right] \geq 0 \quad (\kappa^R_r) \tag{5.5l}$$

Constraints 5.5b–5.5l are the primal and dual constraints from the ISO (lower-level) problem and the strong-duality constraint; they ensure optimality of the ISO lower-level problem given the network investment decision variables.

The Nash game between zonal planners

The optimization problem faced by one zonal planner (Problem 5.5) is similar to the model presented by Buijs and Belmans (2011); since the lower-level is an equilibrium problem, each zonal planner faces an MPEC. Searching for Nash equilibria between several MPECs, we obtain an EPEC. The constraints faced by each zonal planner are identical; nevertheless, the valuation that the zonal planner accords to them (i.e., the dual variables marked with an R superscript) may differ across the planners. Therefore, the problem at hand is a *Generalized Nash game* (GNE). Harker (1991) proposes to assign identical duals to all shared constraints by assumption; this facilitates the solution of such problems considerably, since the problem then reduces to a *Nash equilibrium* (NE), which can be solved as a Variational Inequality (VI) or Mixed Complementarity Problem (MCP) using standard methods. This assumption can be interpreted as an implicit auction of the scarce resource or constraining factor in some cases; there exists a coordination mechanism between the players affected by the shared constraint. However, since one of the main tenets of our work is that such a coordination mechanism does not exist, we do not use this approach here. Instead, we will present a reformulation based on disjunctive constraints below to circumvent this problem without having to make a-priori assumptions.

We proceed by taking first-order (KKT) conditions of the zonal planner's MPEC, stated in the Appendix; however, both the objective function (5.5a) and the strong-duality constraint (5.5l) are non-convex due to the congestion rents in the objective function (5.5a)and the line expansion in the strong-duality constraint (5.5l). As a consequence, the KKT conditions are neither necessary nor sufficient for an equilibrium, and there may be a multitude of KKT points. Let us first discuss the caveat that the KKT conditions are not necessary: this means that there may exist equilibria of the game between zonal planners that our approach does not capture. Mathematically, such equilibria are related to relaxed definitions of the Nash equilibrium, such as a *Nash Bouligand stationary point* (or Nash B-stationary point, Kulkarni and Shanbhag, 2013). In our defense, it is not clear whether different approaches such as enumeration or diagonalization would be able to identify all equilibria in applied problems.

The caveat that KKT conditions are not sufficient for an equilibrium is, in contrast, easy to overcome. We treat each KKT point as a candidate equilibrium and check for each zonal planner whether a profitable deviation exists, keeping all line capacities not within the jurisdiction of the respective zonal planner fixed. The algorithm is explained in more detail below, when we summarize the methodological advances of our approach. If no profitable deviation exists for any zonal planner, we keep the KKT point as an equilibrium; otherwise, we discard it. Our approach therefore only admits global Nash equilibria, in contrast to other approaches that only focus on deviation incentives in the neighbourhood of equilibrium candidates (*local Nash equilibrium*, Hu and Ralph, 2007).

In this way, we can use the KKT conditions as constraints to describe strong global Nash equilibria; the decision which equilibrium to choose is taken by the top-level player in this game, the supra-national planner.

The supra-national planner

The supra-national planner decides on cross-border network investment, anticipating the optimal strategic reaction of the zonal planners. She seeks to maximize welfare in the entire region, less the required investment costs:

$$\max_{g,d,\delta} \sum_{n \in N} \left[\left(a_n - \tfrac{1}{2} b_n d_n\right) d_n - \sum_s c^G_{ns} g_{ns} \right] - \sum_{l \in L} c^T_l e_l \tag{5.6a}$$

$$\text{s.t. equilibrium between the zonal regulators (KKT conditions C.1)}$$
$$\text{and optimal dispatch by the ISO}$$

As already mentioned above, there may exist multiple Nash equilibria. We therefore implement an iterative algorithm seeking to identify a range of possible outcomes. This allows us to give a broader interpretation to the top-level player than Ruiz et al. (2012) have done: rather than just presenting one Nash equilibrium, we can compare several equilibria. We will return to this issue when discussing the numerical results.

Methodological advances

Our model extends the approach used by Ruiz et al. (2012) in three important aspects: firstly, they assume a piece-wise constant willingness-to-pay function of consumers, while we use a linear inverse demand curve. This allows to identify marginal benefits of network expansion. Secondly, Ruiz et al. hold the dual variable associated with lower-level complementarity (κ^R in our formulation) fixed and test for different (exogenous) values.

The third contribution of our work relative to Ruiz et al. lies in the interpretation of the top level player: they use this player merely as an equilibrium selection mechanism, but do not discuss whether multiple Nash equilibria exist. We develop an iterative algorithm to identify alternative equilibria, and thus provide more room for interpretation of the three-stage nature of the model.

Reformulating the Generalized Nash game using disjunctive constraints

One general methodological novelty (not discussed by Ruiz et al.) concerns the nature of the Generalized Nash Equilibrium (GNE) between the players at the intermediate model stage. In particular, as we mentioned earlier, we decide against the common simplification when solving GNE models of assuming identical multipliers for shared constraints (Harker, 1991) or a fixed, exogenously specified ratio between the multipliers (Oggioni et al., 2012). Instead, we replace the complementarity constraints of the zonal planners' and the ISO's KKT conditions by disjunctive constraints. This allows us to circumvent the problem of a non-square MCP model: we illustrate the approach using the example of the nodal energy balance faced by the ISO (constraint 5.1e).

$$\overline{g}_{ns} - g_{ns} \geq 0$$

For optimality, either this constraint is not binding or the associated dual variable for the ISO (β_{ns}) equals 0. However, each zonal planner must also consider this constraint in her own optimization problem (Constraint 5.5h), with the associated dual variable (β^R_{nsr}). The entire problem then reads as follows:

$$\begin{aligned} 0 \leq \overline{g}_{ns} - g_{ns} \quad &\perp \quad \beta_{ns} \\ 0 \leq \overline{g}_{ns} - g_{ns} \quad &\perp \quad \beta^R_{nsr} \quad \forall\, r \in R \end{aligned} \tag{5.7a}$$

It is not possible to solve such a model with shared constraints as a Mixed Complementarity Problem (MCP), because it is essentially a non-square system (more variables than unique equations). However, we can replace the entire system using disjunctive constraints. We introduce one large scalar K and a binary variable r_{ns} for each constraint.

$$\begin{aligned} \overline{g}_{ns} - g_{ns} &\geq 0 \\ \overline{g}_{ns} - g_{ns} &\leq K\, r_{ns} \\ \beta_{ns} + \textstyle\sum_r \beta^R_{nsr} &\leq K\,(1 - r_{ns}) \end{aligned} \tag{5.7b}$$

This reformulation allows to include shared constraints without the requirement to make assumptions on the ratio of the dual variables of the shared constraints between the different players. For shared equality constraints, the complementarity condition can be neglected altogether. We reformulate all inequality constraints of the zonal planners' KKT conditions (stated in Appendix C) in this way. After this reformulation, we can also eliminate the strong-duality constraint (5.5l): the primal and dual feasibility of the ISO problem is already included in the zonal planners' constraints, and complementarity of the ISO's variables is enforced by the disjunctive constraints reformulation. After this simplification, the resulting problem is only bilinear in the dual variable to the strong duality constraint (κ^R_r).

Optimistic vs. pessimistic bilevel games

There is one general concern of modelling bilevel games that we still need to address: when solving two-stage games, the most commonly applied approach uses first-order optimality condition of the lower-level optimization problems and inserts those as constraints to the upper-level optimization problem. An important assumption is implicitly taken here: the decision variables of the lower-level player are then, mathematically speaking, decision variables of the upper-level player, respecting optimality constraints. If the optimal response of the lower-level player is unique for any given decision of the upper-level player, this need not be a concern. However, there may be cases where uniqueness of the lower-level best response is not given; the lower-level player is indifferent between several options.

In such cases, the upper-level player may then decide which of the options the lower-level player "chooses". This is commonly referred to as the *optimistic approach*; it is the best the upper-level player can do given the bilevel game.

In contrast, one may consider a case where the lower-level player wants to do what is worst for the upper-level player, given a situation where she is indifferent between several options; this is the *pessimistic approach.* Such a situation may be plausible in cases where the upper- and lower-level players are rivals, and the lower-level player wants to maximize her profits – but also has an incentive to reduce the profits of the rival, if it does not have a downside for her (by definition, own profits are not affected, as she is indifferent between two decisions). Mathematically, this leads to a *min-max-problem*, which is computationally difficult to solve.

In the three-stage model (Problem 5.6), we implicitly use the optimistic approach, where optimistic is to be seen from the supra-national planner's point of view. We believe that this makes sense: the supra-national planner and the ISO's objective are well aligned. For the zonal planners, we assume that they are only concerned with national welfare – but we do not presume that they are malevolent towards other states/zones.

For the deviation-incentive checking algorithm (explained in the following section), we solve the optimization model of the zonal planners (Problem 5.5); here, we also use the optimistic approach (now optimistic from the point of view of the respective zonal planner). Again, this makes sense, because our question when solving this model is to check whether the zonal planner has a deviation incentive from the equilibrium candidate solution identified by the three-stage model. If we do no find a deviation incentive under the optimistic assumption, we can be confident that indeed there is none.[10]

An iterative solution algorithm to identify multiple equilibria

Let us now turn to the iterative algorithm to identify multiple equilibria, which we already referred to earlier. The problem of our three-stage model is as follows: there may exist multiple equilibria in the national-strategic game, and there may be KKT points which are not stable in the Nash sense, because a zonal planner finds a profitable deviation – in this case, the KKT conditions would hold because it is a local welfare minimum or saddle point in one or more variables of a zonal planner. This section describes an iterative algorithm to identify multiple equilibria, and to check for deviation incentives:

1 Solve the national-strategic model (Problem 5.6). The optimal line expansion is denoted as $e_l^*(i)$, where i is the iteration counter; let the set I collect all iterations of this algorithm.

2 Loop over zonal planners: Solve the zonal MPEC problem (Problem 5.5), with all line investments but the domestic ones fixed to the level determined in step 1, $e_l = e_l^*(i)\ \forall\ l \notin L_r$. If at least one zonal planner has a profitable deviation, discard the KKT point as an equilibrium candidate.

[10] We are aware that this assumption may exclude equilibria in the game between zonal planners, which are not deviation-proof under the optimistic assumption, but are incentive-compatible if the ISO can credibly commit to "retaliate" by following the pessimistic strategy. However, identifying whether such a case may even exist is beyond the scope of this work.

3 If the KKT point identified is an equilibrium candidate: check whether a previously found equilibrium dominates the current candidate solution. Definition: a solution A dominates another solution B if each zonal planner is (weakly) better off in solution A than in B. If no dominating previous solution is found, keep this KKT point as an equilibrium.

4 Add constraints to the national-strategic model (Problem 5.6) such that the solution $e_l^*(i)$ is excluded from the feasible region. To this end, we introduce auxiliary binary variables $z_l^+(i), z_l^-(i)$; the parameter $\overline{e_l}$ is a suitable large upper bound on the line expansion:

$$\left.\begin{array}{l} e_l > z_l^+(i)\, e_l^*(i) \\ e_l < \overline{e_l} - z_l^-(i)\,(\overline{e_l} - e_l^*(i)) \\ \sum\limits_l z_l^+(i) + z_l^-(i) \geq 1 \end{array}\right\} \quad \forall\, i \in I \tag{5.8}$$

The logic is straightforward: at least one of the auxiliary binary variables must equal 1 for each previous iteration; hence, these constraints ensure the next KKT point found in Problem (5.6) differs with regard to at least one line expansion variable from all previously found KKT points. This must hold irrespective of whether a point was a stable equilibrium or not.[11]

... Repeat at step 1; stop after n iterations

There is one important methodological and conceptual benefit of this iterative solution to the more "hands-on" approach of solving such a game by diagonalization and attempting to find different equilibria by using various starting points: using diagonalization, one can make no informed statement whether "better" equilibria exist, in the sense that there is a dominating equilibrium (Definition in Step 3 of the iterative algorithm). In contrast, using our iterative algorithm, the first KKT point will be the candidate equilibrium with the highest level of aggregate welfare – irrespective of whether this point is an equilibrium. The second iteration will provide the KKT point with the second-highest overall welfare, and so forth. As we stated earlier, there may exist equilibria which are not KKT points – our approach cannot find these, so the above statement does not apply to such equilibria. However, our approach is at least an improvement to the diagonalization approach.[12]

Solution strategy and two benchmark models

The three-stage model is, after the disjunctive constraints reformulation, a quadratic program with mixed-integer non-convex constraints. We implement the problem in GAMS and use

[11]Otherwise, the iterative algorithm would get stuck at this point. In the GAMS implementation, we use an ϵ distance to replace the strict by a weak inequality.

[12]For further research, we intend to attempt reformulations such that the KKT conditions of the zonal planners are necessary; in this case, our iterative approach would indeed identify all equilibria, ranked by aggregate welfare. Of course, our statements only hold under the assumption that the BARON solver identifies the correct global optimum of the non-convex problem.

the BARON solver (Tawarmalani and Sahinidis, 2005); this solver aims to solve non-convex problems to global optimality.

In order to identify and quantify the impact of strategic zonal planners, we compute two benchmark cases: first, a model without any investment at all, which is exactly the ISO dispatch problem (Problem 5.1). As a second benchmark, we compute the optimal supra-national investment problem without strategic consideration by zonal planners. This can be computed by combining the objective function of the supra-national planner (5.6a) with the operational constraints of the ISO problem (Constraints 5.1b–5.1h), allowing investments on all lines. Both benchmark models are convex quadratic optimization problems (QP) and are numerically solved using the CPLEX solver.

5.5. An illustrative example

We apply this model to a four-node, two-country example, which is motivated by the current situation between Germany and Poland. We assume two countries with two nodes each; the south of country A has a large industrial base, while the north of country A has only low demand for electricity. Country B has intermediate demand at both nodes. The energy system of both countries used to be based on autarky of each zone, with limited transmission capacity between them.

Two shifts caused a significant imbalance in the power network: large installation of generation capacity with low variable costs in the north of Country A (e.g., wind power in the coastal region), and decommissioning of base-load generation capacity in the south of Countries A and B (e.g., the nuclear phase-out in Germany, less power generation from coal due to emission reduction efforts). We abstract from investment in power generation; we assume that the shifts in the generation portfolio have already taken place, but not at the locations where it would be optimal from the point of view of the entire system. This can be interpreted as the effects caused by fixed feed-in tariffs or other subsidies for renewables; in some cases, renewable support does not give sufficient consideration to locational aspects. The decommissioning of nuclear power plants is also a political decision rather than based on the short-run economics of power markets.

Figure 5.2 depicts the network, the installed generation capacity by type, the reference demand at each node, the initial transmission capacity, and the equilibrium prices in the absence of network expansion. All quantities, flows and capacities presented here are given in Gigawatthours (GWh)[13], all prices and costs are given in Euro per Megawatthour (MWh). We assume three types of power plants: "renewables" with zero marginal generation costs, "base" with 40 €/MWh and "peak" with 70 €/MWh. The parameters for the inverse demand function are computed using a price of 70 €/MWh and an elasticity of -0.25. The costs for network expansion are assumed to be 2 €/MWh.[14] The impedance of all lines is assumed

[13]Capacities are actually in Gigwatt (GW) rather than GWh (energy). By assuming one representative hour, the two are equivalent.

[14]This value is derived from the following assumptions: a double-circuit 380 kV line costs 1.4 million €/km; the distance between two nodes is 400 km; the line is used at full capacity in half of the hours every year;

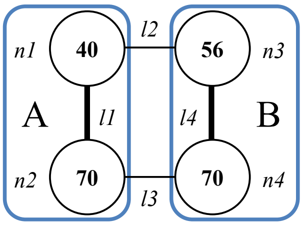
Benchmark: *No investment*

Node *n1*		
Generation		
Renewables	20	(20)
Base	22.21	(40)
Load	33.21	(30)

Line *l1*	
Capacity	10

Node *n2*		
Generation		
Base	30	(30)
Peak	11	(40)
Load	50	(50)

Line *l2*	
Capacity	1



Line *l3*	
Capacity	.5

Node *n3*		
Generation		
Base	30	(30)
Load	21	(20)

Line *l4*	
Capacity	10

Node *n4*		
Generation		
Peak	9	(25)
Load	20	(20)

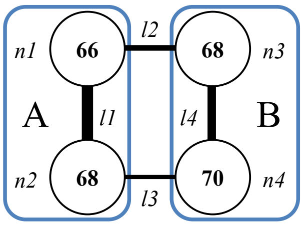
Benchmark: *Welfare-optimal investment*

Node *n1*		
Generation		
Renewable	20	(20)
Base	40	(40)
Load	30.43	(30)

Line *l1*	
Capacity	10 + **12.34**

Node *n2*		
Generation		
Base	30	(30)
Peak	0	(40)
Load	50.36	(50)

Line *l2*	
Capacity	1 + **6.23**



Line *l3*	
Capacity	.5 + **.98**

Node *n3*		
Generation		
Base	30	(30)
Load	20.14	(20)

Line *l4*	
Capacity	10 + **7.09**

Node *n4*		
Generation		
Peak	0.93	(25)
Load	20	(20)

Figure 5.2.: Benchmark cases in the illustrative two-zone, four-node network; generation capacity and reference load in parentheses; investment in bold; nodal prices given in the circles in the figure

to be identical.

Without substantial upgrades of the transmission grid, we observe a large price differential between the two nodes of Country A. However, in order to transmit power to the node with the highest willingness-to-pay, upgrades must also be made in the northern cross-border line and the line within Country B due to the particular power flow characteristics in an alternating-current (AC) power grid.

We first compute two benchmark cases: *no investment* and (supra-national) *welfare-optimal investment*. In the initial (current) network, peak-load generators in both southern nodes are the price-setting plants – there is idle capacity at node $n1$, but it cannot be transported to the high-price nodes due to the constraints of the network. With welfare-optimal investment in transmission capacity, prices in node $n2$ slightly decrease, as more base-load

the lifetime of the line is 40 years; and the interest rate is 4 % p.a.

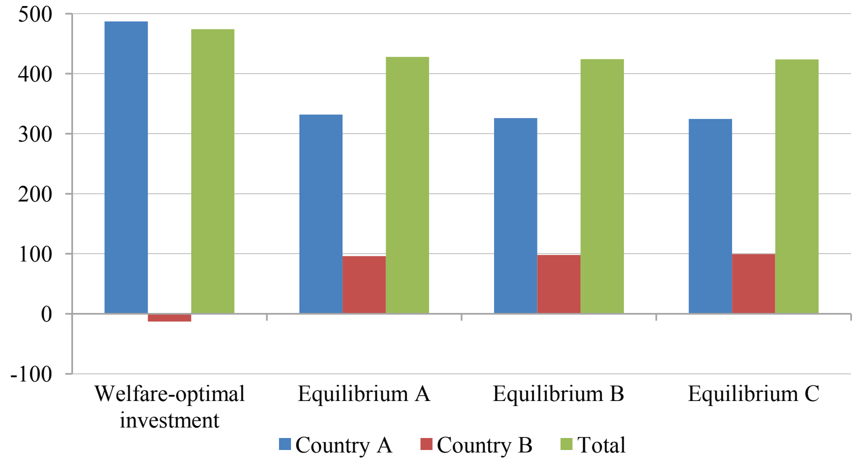


Figure 5.3.: Welfare gains relative to the *No investment* benchmark in 1000 €

generation can be shipped to that node. Prices in the northern nodes increase in line with a general price convergence – the resulting price differential between the nodes is exactly equal to the costs of building an additional marginal unit of line capacity.

Because the plant in node $n4$ is still the price-setting plant in the system even with welfare-optimal line expansions, prices converge towards the high price at that node; this may seem counter-intuitive at first given that network investment is generally expected to exhibit a price-decreasing effect. Nevertheless, welfare is increased due to the network expansion, because peak-load (i.e., high variable cost) generation can be replaced by the base-load plant at node $n1$ which was idle in the no-investment benchmark case.

However, when analysing the welfare shifts due to the network expansion as illustrated in Figure 5.3 and Table 5.2, one notices that all welfare gains accrue in *Country A*, while *Country B* finds its welfare decreased relative to the status quo. This begs the question whether a zonal planner would not find it in her best interest to restrict the network expansion within her jurisdiction, since explicit transfers between the zones is not possible in the current setting.

To investigate this effect, we compute equilibria of the three-stage game; we use the iterative algorithm presented in Section 5.4 to identify 46 KKT points of the game.[15] Of these 46 candidate solutions, 5 are not Nash equilibria (i.e., at least one of the zonal planner identified a profitable deviation). Another 35 KKT points are Nash equilibria in the sense that no zonal planner has a unilateral deviation incentive, but the equilibria are not *Pareto efficient* (also referred to as Pareto optimal). A Pareto improvement exists; the solutions are dominated by other equilibria.[16] Three iterations experienced numerical difficulties and

[15] Due to the "holes" cut into the feasible region by the iterative algorithm to identify other KKT points, the solver requires more time after each iteration. The 47$^{\text{th}}$ iteration was aborted after 24 hours of computation time.

[16] A dominated equilibrium is one where another equilibrium was found which is a welfare improvement for all zonal planners. Thus, agreeing to move to the dominating equilibrium is in every player's best interest;

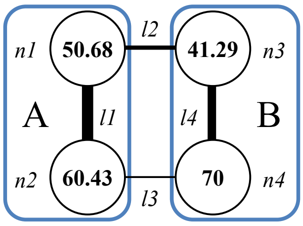
Equilibrium *A* – Iteration 1

Node *n1*		
Generation		
Renewables	20	(20)
Base	40	(40)
Load	32.05	(30)

Line *l1*	
Capacity	10 + **11.39**

Node *n2*		
Generation		
Base	30	(30)
Peak	0	(40)
Load	51.71	(50)

Line *l2*	
Capacity	1 + **5.56**



Line *l3*	
Capacity	.5 + **0.37**

Node *n3*		
Generation		
Base	30	(30)
Load	22.05	(20)

Line *l4*	
Capacity	10 + **4.51**

Node *n4*		
Generation		
Peak	5.81	(25)
Load	20	(20)

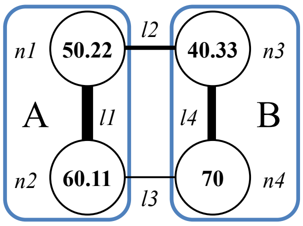
Equilibrium *B* – Iteration 23

Node *n1*		
Generation		
Renewable	20	(20)
Base	40	(40)
Load	32.12	(30)

Line *l1*	
Capacity	10 + **11.35**

Node *n2*		
Generation		
Base	30	(30)
Peak	0	(40)
Load	51.77	(50)

Line *l2*	
Capacity	1 + **5.53**



Line *l3*	
Capacity	.5 + **.47**

Node *n3*		
Generation		
Base	30	(30)
Load	22.12	(20)

Line *l4*	
Capacity	10 + **4.41**

Node *n4*		
Generation		
Peak	6	(25)
Load	20	(20)

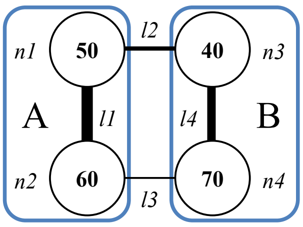
Equilibrium *C* – Iteration 27

Node *n1*		
Generation		
Renewable	20	(20)
Base	40	(40)
Load	32.14	(30)

Line *l1*	
Capacity	10 + **11.34**

Node *n2*		
Generation		
Base	30	(30)
Peak	0	(40)
Load	51.79	(50)

Line *l2*	
Capacity	1 + **5.52**



Line *l3*	
Capacity	.5 + **0**

Node *n3*		
Generation		
Base	30	(30)
Load	22.14	(20)

Line *l4*	
Capacity	10 + **4.38**

Node *n4*		
Generation		
Peak	6.07	(25)
Load	20	(20)

Figure 5.4.: Three-stage game equilibria in the illustrative two-zone, four-node network; generation capacity and reference load in parentheses; investment in bold; nodal prices given in the circles in the figure

where therefore excluded. This leaves three Nash equilibria found at iterations 1, 23 and 27; they are presented in Figure 5.4.

In general, network expansion is lower than in the welfare-optimal benchmark. The difference is most pronounced on line $l4$, where expansion is lower by one third relative to the benchmark. This makes sense intuitively: the zonal planner of country B specifically "withholds" line expansion; this has the effect to reduce prices at node $n3$ and thereby shifts rents from generators at node $n1$ to the consumers at node $n3$. Equilibrium C is the only solution where not all lines are fully congested; this is line $l3$, which is consequently not expanded.

In the three national-strategic equilibria, aggregate welfare is 10 % lower compared to the welfare-optimal benchmark (see Table 5.2). At the same time, by strategically reducing line expansion on the domestic line, the zonal planner of country B is able to appropriate more than 20 % of the welfare gains, rather than seeing her constituents worse off due to the network expansion.

As a last remark, let us compare the three equilibria: it is clear that Equilibrium A is the network expansion with the highest aggregate welfare. However, one may argue that Equilibrium C is the more "equitable" outcome from the point of view of the supra-national planner, as both zones benefit more evenly from the expansion. In any case, the difference between the three equilibria is relatively small; we are not sure whether this is due to the small scale of the sample network, an artefact arising from the numerical aspects of the non-convex problem, or an inherent property of such a game led by the supra-national planner.

5.6. Conclusions

In a meshed power network, line expansions may lead to significant re-allocations of profits and rents across different stakeholder groups and national boundaries. Zonal (or national) interests and the lack of an efficient compensation mechanism may prevent reaping all potential welfare gains from network investments, because the beneficiaries of network expansion are located in different jurisdiction than those stakeholders bearing the costs. Zonal planners, such as governments, regulators, or national TSOs, may therefore have incentives to over-invest or intentionally withhold power line upgrades in their jurisdiction to induce a shift of rents towards their constituents. This may impede the efficient integration of the European energy market and the transformation towards a low-carbon power sector.

We develop a three-stage model to represent the Generalized Nash game between zonal planners, each taking into account how line capacity upgrades impacts the outcome in the competitive spot market. The game is led by a supra-national planner, who decides on cross-border line expansion. We make use of strong duality to replace the equilibrium constraints of the lowest-level player, the ISO managing the competitive spot market, and take first-order conditions of the zonal planners' optimization problem. Adapting a disjunctive constraints reformulation, we circumvent the common problem when solving Generalized Nash games, and do not need to make *a priori* assumptions on the relative valuations of shared constraints.

a joint deviation incentive for all players without transfers/compensation exists.

	Consumer surplus			Generator profit			Congestion rent			Investment costs			Welfare		
	A	B	Total	A	B	Total	A	B	Total	A	B	Total	A	B	Total
Benchmark cases:															
No investment	12148.2	5887	18035.2	1700	480	2180	308	148	456				14156.2	6515	20671.2
Welfare-optimal investment	11421.2	5640.1	17061.4	3200	840	4040	53.9	43.4	97.3	31.9	21.4	53.3	14643.2	6502.1	21145.4
National-strategic model:															
Equilibrium A (Iteration 1)	12280.3	6203.5	18483.8	2064.8	38.9	2103.6	171.8	383.6	555.3	28.7	15	43.7	14488.1	6611	21099.1
Equilibrium B (Iteration 23)	12317.4	6224.7	18542.1	2016.8	10	2026.8	176.8	393.2	570	28.7	14.8	43.5	14482.3	6613.1	21095.4
Equilibrium C (Iteration 27)	12330.4	6232.1	18562.5	2000	0	2000	178.6	396.4	575	28.2	14.3	42.5	14480.7	6614.3	21095

Table 5.2.: Welfare results of the sample network, in 1000 €; summations may not match due to rounding errors

Finally, we implement an iterative algorithm to determine multiple equilibria.

Our results based on a simple test case demonstrate that the national-strategic behaviour yields significant welfare loss compared to the system-optimal investment. A zonal planner has incentives to "under-invest" in its domestic line in order for its constituents to obtain a share of the welfare gains, rather than seeing its domestic welfare decreased by the system-optimal network upgrades.

For further research and future model extensions, we intend to develop the three-stage approach in several directions: firstly, we want to relax the current simplification that all cross-border network investment is decided by the supra-national planner. Instead, those line upgrades should be subject to a bargaining process between the adjacent zonal planners. This will require the explicit modelling of a compensation mechanism, in the spirit of the ITC and the notion of "projects of common interest". It will be interesting to see whether an explicit transfer scheme can alleviate the failure to reach the first-best equilibrium.

As a second line of further research, we intend to focus on the welfare allocation between the different stakeholder groups, and the shifts between them due to network upgrades. One may imagine a situation where the regulator has a preference for the welfare of consumers, rather than the equal weight of consumer welfare, generator profits, and TSO rents, which we assumed so far. Furthermore, in such a setting, an explicit funding constraint of the TSO has to be considered; network usage tariffs will be used to finance additional transmission capacity. Thereby, we will be able to explicitly include the trade-off between the benefits of additional transmission capacity and the consumer welfare loss due to the tariffs. This is similar to the work by Daxhelet and Smeers (2007), but extends it for network investment.

Appendix A

Mathematical Appendix to Chapter 2: Proof of Theorem 6

This proof consists of three parts: first, I derive the second-order derivative condition stated in the Theorem. Then, I show existence of a solution to the KKT system via the equivalent Variational Inequality. Last, uniqueness of the solution is shown if Assumption **A5** holds.

Equation (2.3) states:

$$p'\left(2 + 2\frac{\partial q_{-i}(q_i^*)}{\partial q_i} + \frac{\partial^2 q_{-i}(q_i^*)}{\partial q_i^2}\right) - c_i''(q_i^*) \leq 0$$

Continuing the proof of Lemma 5, it follows that:

$$\begin{aligned} -b - b\frac{\partial q_f(q_i)}{\partial q_i} - c_f''\Big(q_f\Big)\frac{\partial q_f(q_i)}{\partial q_i} &= 0 \\ -b\frac{\partial^2 q_f(q_i)}{\partial q_i^2} - c_f'''\Big(q_f\Big)\left(\frac{\partial q_f(q_i)}{\partial q_i}\right)^2 - c_f''\Big(q_f\Big)\frac{\partial^2 q_f(q_i)}{\partial q_i^2} &= 0 \\ \Rightarrow \frac{\partial^2 q_f(q_i)}{\partial q_i^2} &= -\frac{b^2 c_f'''(q_f)}{\Big(b + c_f''(q_f)\Big)^3} \end{aligned} \tag{A.1}$$

Inserting this term into Equation (2.3), in combination with the assumptions, yields the stated second-order derivative condition for the oligopoly suppliers. The profit maximization function of the fringe supplier f is strictly concave following the reasoning of Lemma 3.

Existence of a solution is shown by looking at the equivalent Variational Inequality (VI) to the KKT system (2.8); equivalence is given since the each player's objective function is continuous and the feasible space is convex.
This is to find a vector $q^* = \Big[(q_i^*)_{i\in S}, q_f^*\Big]^T \in K$ such that:

$$\begin{aligned} &F(q^*)(q - q^*) = \\ &= \begin{bmatrix} \left(-a+b\sum\limits_j q_j^* + b\left(1-\frac{b}{b+c_f''(q_f^*)}\right)q_i^* + c_i'(q_i^*)\right)_{i\in S} \\ -a+b\sum\limits_j q_j^* + c_f'(q_f^*) \end{bmatrix}^T \begin{pmatrix} (q_i - q_i^*)_{i\in S} \\ q_f - q_f^* \end{pmatrix} \geq \ 0 \quad \forall\, q \in K \end{aligned} \tag{A.2}$$

The set K is the Cartesian product of each suppliers' feasible quantity decisions (cf. Lemma 1); however, the marginal cost function $c_j'(q_j)$ and hence F is not defined at $q_j = \overline{q_j}$, and therefore, F is not continuous at the limit. To circumvent this problem, a bound is introduced on the produced quantity. Choose $\widetilde{q_j}$ such that:

$$a < c_j'(\widetilde{q_j}) \text{ and } \widetilde{q_j} < \overline{q_j}.$$

Such a bound obviously exists for every supplier. Producing a quantity greater than $\widetilde{q_j}$ would violate the complementarity condition; hence, I can safely restrict the supplier's feasible region to $q_j \in [0, \widetilde{q_j}]$. Furthermore, $c_j'(q_j)$ and $c_j''(q_j)$ are continuous on that range.

Let n denote the number of oligopoly suppliers. Now, I can formally define the feasible region of VI (A.2):

$$K = \prod_j [0, \widetilde{q_j}] \subset \mathbb{R}_+^{n+1}$$

Because K is closed, convex and compact, and F is continuous on K, the solution set to the VI is non-empty (cf. Facchinei and Pang, 2003, Corollary 2.2.5).

Uniqueness of the solution can be shown through strict monotonicity of F on K, defined as:

$$(F(x) - F(y))^T (x - y) > 0 \quad \forall\, x, y \in K,\ x \neq y$$

Here, x and y vector elements of the feasible region K.

$$\left[\begin{pmatrix} \left(b \sum_j (x_j - y_j) + b\left(1 - \frac{b}{b + c_f''(x_f)}\right) x_i + c_i'(x_i) - b\left(1 - \frac{b}{b + c_f''(y_f)}\right) y_i - c_i'(y_i) \right)_{i \in S} \\ b \sum_j (x_j - y_j) + c_f'(x_f) - c_f'(y_f) \end{pmatrix} \right]^T \begin{pmatrix} (x_i - y_i)_{i \in S} \\ x_f - y_f \end{pmatrix} =$$

$$= b \left(\sum_j (x_j - y_j) \right)^2 + \sum_j \left(c_j'(x_j) - c_j'(y_j) \right) (x_j - y_j)$$
$$+ b \sum_{i \in S} \left[\left(1 - \frac{b}{b + c_f''(x_f)} \right) x_i - \left(1 - \frac{b}{b + c_f''(y_f)} \right) y_i \right] (x_i - y_i) > 0$$

This condition is stated in Assumption **A5**. K is closed and convex, and F is strictly monotone under this assumption, so there exists at most one solution (cf. Facchinei and Pang, 2003, Theorem 2.3.3).

Combining the results of existence and (at most) uniqueness yields that the solution to the VI is indeed unique. If it satisfies the second-order derivative condition, it is the unique equilibrium of the Stackelberg oligopoly problem with a competitive fringe. □

Appendix B

Mathematical Appendix to Chapter 4

B.1. Nomenclature

Sets and mappings

Sets

$y \in Y$	... years (#1)
$h \in H$	... hours/days/seasons
$v \in V$	... loading (injection/extraction) cycles of storage
$s \in S$	... suppliers
$n, k \in N$	... nodes
$d \in D$	... demand sectors
$l \in L$	... sector fuel mix constraints
$m \in M$	... transformation mix constraints
$a \in A$	... arcs (#2)
$c \in C$	... transformation technology (e.g., oil refineries, power plants)
$o \in O$	... storage operators/technology (#2)
$e, f \in E$	... energy carriers/fuels
$r \in R$	... regions
$g \in G$	... emission types (greenhouse gases)

Mappings

$n, k \in N_r$	... node-to-region mapping
$r \in R_n$	... region-to-node mapping (any node can be part of several regions)
$a \in A^+_{ne}$	... subset of arcs ending at node n transporting fuel e
$a \in A^-_{ne}$	... subset of arcs starting at node n transporting fuel e
$e \in E^A_a$	... fuel(s) transported via arc a (singleton)

$n^{A+}(a)$	... end node of arc a (singleton)
$n^{A-}(a)$	... start node of arc a (singleton)
$f \in E^{C+}_c$	... subset of output fuel(s) f from transformation technology c
$e \in E^{C-}_c$	... subset of input fuel(s) e for transformation technology c
$(e, f) \in E^{C}_c$	... input/output fuel mapping of transformation technology c
$e^{O}(o)$	... fuel stored by technology o (singleton)
$o \in O^{E}_e$	... subset of technologies storing fuel e
$h \in H^{V}_{vo}$	... mapping between loading cycle and hour/day/season
$v^{H}(h, o)$	... loading cycle of hour/day/season (singleton)
$e \in E^{L}_l$	... fuel(s) that satisfies fuel mix constraint l
$e \in \widehat{E}^{L}_l$	... fuel(s) that are included in fuel mix constraint l
$e \in E^{M}_m$	... fuel(s) that satisfies transformation mix constraint m
$d \in D^{L}_l$	... demand sector(s) to which fuel mix constraint l applies
$c \in C^{M}_m$	... transformation technologies that satisfy transformation mix constraint m

Parameters and functions

General Parameters

df_y	... discount factor
dur_h	... relative duration of hour/day/season h (with $\sum_h dur_h = 1$, #3)

Supplier

$cour^{S}_{ysnd}$	... CV market power parameter of supplier s at node n regarding sector d
$cost^{P}_{ysne}(\cdot)$	... production cost function faced by supplier s at node n for fuel e
lin^{P}_{ysne}	... linear term of the production cost function ($lin^{P} \geq 0$)
qud^{P}_{ysne}	... quadratic term of the production cost function ($qud^{P} \geq 0$)
gol^{P}_{ysne}	... logarithmic (Golombek) term of the production cost function ($gol^{P} \geq 0$)
cap^{P}_{ysne}	... gross production capacity (#4)
avl^{P}_{yhsne}	... availability factor of production capacity
exp^{P}_{ysne}	... production capacity expansion limit
inv^{P}_{ysne}	... production capacity expansion (per-unit) costs
$dep^{P}_{yy'sne}$	... production capacity expansion depreciation factor (#6)
hor^{P}_{sne}	... production horizon (reserves)
$loss^{P}_{sne}$	... loss rate during production of fuel e at node n
ems^{P}_{ysneg}	... emission of type g during production of fuel e at node n by supplier s

Arc operator

trf^{A}_{ya} ... tariff for using arc a

cap^{A}_{ya} ... gross capacity of arc a (#4)

exp^{A}_{ya} ... arc capacity expansion limit

inv^{A}_{ya} ... arc capacity expansion (per-unit) costs

$dep^{A}_{yy'a}$ ... arc capacity expansion depreciation (#6)

$loss^{A}_{a}$ ... loss rate during transportation through arc a

ems^{A}_{yag} ... emission of type g during transportation through arc a

Transformation technology operator

trf^{C}_{ync} ... tariff for using transformation technology c for input fuel e

cap^{C}_{ync} ... capacity of transformation technology c (as measured in input fuel, #4)

exp^{C}_{ync} ... transformation technology capacity expansion limit

inv^{C}_{ync} ... transformation technology expansion (per-unit) costs

$dep^{C}_{yy'nc}$ ... transformation capacity expansion depreciation (#6)

$transf^{C}_{yncef}$ ... transformation rate by technology c at node n from input e to output f

shr^{C}_{ynce} ... minimum share of fuel e by transformation technology c

ems^{C}_{yceg} ... emission of type g during transformation of (input) fuel e

Storage technology operator

trf^{O-}_{yno} ... tariff for injecting into storage technology o

cap^{O}_{yno} ... capacity for fuel stored in storage technology o over loading cycle (#4)

exp^{O}_{yno} ... yearly storage capacity expansion limit

inv^{O}_{yno} ... yearly storage capacity expansion (per-unit) costs

$dep^{O}_{yy'no}$ ... yearly storage capacity expansion depreciation (#6)

cap^{O-}_{yno} ... capacity for fuel injection into storage

exp^{O-}_{yno} ... storage injection capacity expansion limit

inv^{O-}_{yno} ... storage injection capacity expansion (per-unit) costs

$dep^{O-}_{yy'no}$ ... storage injection capacity expansion depreciation (#6)

cap^{O+}_{yno} ... capacity for fuel extraction rate from storage technology o (#4)

exp^{O+}_{yno} ... storage extraction capacity expansion limit

inv^{O+}_{yno} ... storage extraction capacity expansion (per-unit) costs

$dep^{O+}_{yy'no}$ ... storage extraction capacity expansion depreciation (#6)

$loss^{O-}_{o}$ ... loss rate of storage technology o (accounted at injection)

ems^{O-}_{yog} ... emission of type g of storage technology o (accounted at injection)

Emissions auctioneer

$quota^{glob}_{yg}$	... quota for global emissions
$quota^{reg}_{yrg}$	... quota for regional emissions
$quota^{nod}_{yng}$	... quota for nodal emissions
tax^{glob}_{yg}	... tax for emission g (global)
tax^{reg}_{yrg}	... tax for emission g in region r
tax^{nod}_{yng}	... tax for emission g at node n

Demand

$\Pi^{D}_{yhnde}(\cdot)$	... inverse demand curve of sector d
int^{D}_{yhnd}	... intercept of inverse demand curve for fuel e at node n
slp^{D}_{yhnd}	... slope of inverse demand curve for fuel e at node n
eff^{D}_{ynde}	... efficiency of demand satisfaction of sector d by fuel e at node n
$eucc^{D}_{yhnde}$	... constant end use cost parameter
$eucl^{D}_{yhnde}$	... linear end use cost parameter
ems^{D}_{ydeg}	... emission of type g during consumption of fuel e at node n
shr^{L}_{ynl}	... minimum share of sector fuel mix constraint l (in energy services)
shr^{M}_{ynm}	... minimum share of generation mix constraint m (in gross energy)

Variables

q^{P}_{yhsne}	... quantity produced
q^{A}_{yhsa}	... quantity transported through arc a
q^{C}_{yhsnce}	... quantity put into transformation technology c
q^{O-}_{yhsno}	... quantity injected into storage o
q^{O+}_{yhsno}	... quantity extracted from storage o
q^{D}_{yhsnde}	... quantity sold to final demand sector d
z^{P}_{ysne}	... expansion of production capacity (#5)
α^{P}_{yhsne}	... dual for production capacity constraint
α^{O}_{yvsno}	... dual for injection/extraction constraint
γ^{P}_{sne}	... dual for production horizon constraint
ζ^{P}_{ysne}	... dual for production capacity expansion limit
ϕ_{yhsne}	... dual for mass-balance constraint
f^{A}_{yha}	... quantity transported by the arc operator
z^{A}_{ya}	... expansion of arc capacity (#5)

τ^A_{yha} ... dual for arc capacity constraint
ζ^A_{ya} ... dual to arc capacity expansion limit
p^A_{yha} ... market-clearing price of arc capacity
f^C_{yhnce} ... quantity of fuel e put into transformation technology c
z^C_{ync} ... expansion of transformation technology capacity (#5)
τ^C_{yhnc} ... dual for capacity constraint of transformation technology c
β^C_{yhnce} ... dual for minimum input share of fuel e to transformation technology c
ζ^C_{ync} ... dual to transformation technology capacity expansion limit
p^C_{yhnce} ... market-clearing price of transformation technology capacity
f^{O-}_{yhno} ... quantity injected into storage
f^{O+}_{yhno} ... quantity extracted from storage
z^O_{yno} ... expansion of yearly storage capacity(#5)
z^{O-}_{yno} ... expansion of injection capacity (#5)
z^{O+}_{yno} ... expansion of extraction capacity (#5)
ζ^O_{yno} ... dual to yearly storage capacity expansion limit
ζ^{O-}_{yno} ... dual to injection capacity expansion limit
ζ^{O+}_{yno} ... dual to extraction capacity expansion limit
p^{O-}_{yhno} ... market-clearing price for injection into storage
p^{O+}_{yhno} ... market-clearing price for extraction from storage
τ^O_{yvno} ... dual for capacity constraint of storage technology in loading cycle v
κ^{O-}_{yhno} ... dual for injection capacity constraint of storage technology
κ^{O+}_{yhno} ... dual for extraction capacity constraint of storage technology
f^G_{yng} ... quantity of emissions of type g at node n
μ^{glob}_{yg} ... dual for emission constraints of type g (global)
μ^{reg}_{yrg} ... dual for emission constraints of type g in region r
μ^{nod}_{yng} ... dual for emission constraints of type g at node n
p^G_{yng} ... market-clearing price for emission of type g
p^D_{yhnde} ... final demand price by fuel
β^L_{yhnl} ... dual for fuel mix constraint l
β^M_{yhnm} ... dual for generation mix constraint m

Comments

#1 The notation $y' < y$ indicates "all years y' prior to year y", and $y' > y$ the reverse.

#2 Arcs and storage technologies can be used for exactly one fuel. This saves indices, but may be a problem later (e.g., capacity constraints of oil products pipelines).

#3 We want to maintain flexibility of the model, and there is no general "natural" time unit for a multi-fuel model (such as mcm/d for natural gas or kWh for power). All variables have to be interpreted as "full-year equivalent" (i.e., the quantities produced, transported, transformed, or consumed if that time slice lasted one year).

#4 Initial capacity in the base year net of depreciation or planned shut-down; capacities are given in gross terms (before losses).

#5 Investment in additional capacity is available from the following period at the earliest.

#6 The first subscript (y) refers to the time period at which the expansion was undertaken; the second subscript refers to the "current" period. The parameter specifies how much of an expansion is still available in the current period. (Note that $dep^{\circledast}_{yy'\ldots} = 0$ if $y \geq y'$, since any expansion can only be available after the investment period.)

B.2. The Karush-Kuhn-Tucker conditions

We write $f(x) \geq 0 \perp x \geq 0$ to indicate the complementarity constraint $f(x) \cdot x = 0$. This allows to have the KKT conditions below to be written exactly as they are coded in GAMS.

$$
\begin{aligned}
df_y dur_h \left[\frac{\partial\ cost^P_{yhsne}(\cdot)}{\partial\ q^P_{yhsne}} + \sum_{g \in G} p^G_{yng} ems^P_{ysneg} \right] & \\
+ \alpha^P_{yhsne} - (1 - loss^P_{sne}) \phi_{yhsne} + \gamma^P_{sne} \geq 0 \quad \perp \quad q^P_{yhsne} \geq 0 & \qquad \text{(B.1)}
\end{aligned}
$$

$$
df_y dur_h p^A_{yha} + \phi_{yhsn^{A-}(a)e} - (1 - loss^A_a) \phi_{yhsn^{A+}(a)e} \geq 0 \quad \perp \quad q^A_{yhsa} \geq 0 \qquad \text{(B.2)}
$$

$$
\begin{aligned}
df_y dur_h p^C_{yhnce} + \phi_{yhsne} - \sum_{f \in E^{C+}_c} transf^C_{yncef} \phi_{yhsnf} & \\
- \sum_{\substack{m \in M, f \in E^{C+}_c \\ \text{if } c \in C^M_m}} transf^C_{yncef} \beta^M_{yhnm} \geq 0 \quad \perp \quad q^C_{yhsnec} \geq 0 & \qquad \text{(B.3)}
\end{aligned}
$$

$$
df_y dur_h p^{O-}_{yhno} - dur_h (1 - loss^{O-}_o) \alpha^O_{yvH(ho)sno} + \phi_{yhsne} \geq 0 \quad \perp \quad q^{O-}_{yhsno} \geq 0 \qquad \text{(B.4)}
$$

$$
df_y dur_h p^{O+}_{yhno} + dur_h \alpha^O_{yvH(ho)sno} - \phi_{yhsne} \geq 0 \quad \perp \quad q^{O+}_{yhsno} \geq 0 \qquad \text{(B.5)}
$$

$$
\begin{aligned}
df_y dur_h \Bigg[\Bigg(-int^D_{yhnd} + slp^D_{yhnd} \Bigg(\sum_{s' \in S, f \in E} eff^D_{yndf} q^D_{yhs'ndf} \Bigg) \Bigg) eff^D_{ynde} & \\
+ eucc^D_{yhnde} + eucl^D_{yhnde} \sum_{s' \in S} q^D_{yhs'nde} + \sum_{g \in G} p^G_{yng} ems^D_{ydeg} & \\
+ cour^S_{ysnd} \Bigg(eff^D_{ynde} slp^D_{yhnd} \sum_{f \in E} eff^D_{yndf} q^D_{yhsndf} + eucl^D_{yhnde} q^D_{yhsnde} \Bigg) \Bigg] & \\
+ \phi_{yhsne} + \sum_{\substack{l \in L \\ \text{if } d \in D^L_l}} \left\{ \begin{array}{ll} (shr^L_{ynl} - 1) eff^D_{ynde} \beta^L_{yhnl} & \text{if } e \in E^L_l \\ shr^L_{ynl} eff^D_{ynde} \beta^L_{yhnl} & \text{if } e \in \widehat{E}^L_l, e \notin E^L_l \end{array} \right\} & \\
+ \sum_{\substack{m \in M \\ \text{if } e \in E^M_m}} shr^M_{ynm} \beta^M_{yhnm} \geq 0 \quad \perp \quad q^D_{yhsnde} \geq 0 & \qquad \text{(B.6)}
\end{aligned}
$$

$$
\begin{aligned}
df_y inv^P_{ysne} + \sum_{y' > y, h \in H} \Bigg[df_{y'} dur_h \frac{\partial\ cost^P_{y'hsne}(\cdot)}{\partial\ z^P_{ysne}} & \\
- avl^P_{y'hsne} dep^P_{yy'sne} \alpha^P_{y'hsne} \Bigg] + \zeta^P_{ysne} \geq 0 \quad \perp \quad z^P_{ysne} \geq 0 & \qquad \text{(B.7)}
\end{aligned}
$$

$$
avl^P_{yhsne} \Bigg(cap^P_{ysne} + \sum_{y' < y} dep^P_{y'ysne} z^P_{y'sne} \Bigg) - q^P_{yhsne} \geq 0 \quad \perp \quad \alpha^P_{yhsne} \geq 0 \qquad \text{(B.8)}
$$

$$
\sum_{h \in H^V_{vo}} dur_h \left((1 - loss^{O-}_o) q^{O-}_{yhsno} - q^{O+}_{yhsno} \right) = 0 \quad \perp \quad \alpha^O_{yvsno} \text{ (free)} \qquad \text{(B.9)}
$$

$$(1-loss^P_{sne})q^P_{yhsne} - \sum_{d\in D} q^D_{yhsnde} + \sum_{c\in C, f\in E^{C-}_c} transf^C_{yncfe} q^C_{yhsncf}$$
$$- \sum_{c\in C} q^C_{yhsnce} + \sum_{a\in A^+_{ne}} (1-loss^A_a)q^A_{yhsa} - \sum_{a\in A^-_{ne}} q^A_{yhsa}$$
$$+ \sum_{o\in O^E_e} \left(q^{O+}_{yhsno} - q^{O-}_{yhsno}\right) = 0 \quad , \quad \phi_{yhsne} \text{ (free)} \tag{B.10}$$

$$exp^P_{ysne} - z^P_{ysne} \geq 0 \quad \perp \quad \zeta^P_{ysne} \geq 0 \tag{B.11}$$

$$hor^P_{sne} - \sum_{y\in Y, h\in H} dur_h q^P_{yhsne} \geq 0 \quad \perp \quad \gamma^P_{sne} \geq 0 \tag{B.12}$$

$$-shr^L_{ynl} \sum_{\substack{s\in S, d\in D^L_l \\ e\in \widehat{E}^L_l}} eff^D_{ynde} q^D_{yhsnde} + \sum_{\substack{s\in S, d\in D^L_l \\ e\in E^L_l}} eff^D_{ynde} q^D_{yhsnde} \geq 0 \quad \perp \quad \beta^L_{yhnl} \geq 0 \tag{B.13}$$

$$-shr^M_{ynm} \sum_{\substack{s\in S, d\in D \\ f\in E^M_m}} q^D_{yhsndf} + \sum_{\substack{s\in S, (e,f)\in E^C_c \\ c\in C^M_m}} transf^C_{yncef} q^C_{yhsnce} \geq 0 \quad \perp \quad \beta^M_{yhnm} \geq 0 \tag{B.14}$$

$$df_y dur_h \left(-p^A_{yha} + trf^A_{ya} + \sum_{g\in G} p^G_{ynA-(a)g} ems^A_{yag}\right) + \tau^A_{yha} \geq 0 \quad \perp \quad f^A_{yha} \geq 0 \tag{B.15}$$

$$df_y inv^A_{ya} - \sum_{y'>y, h\in H} dep^A_{yy'a} \tau^A_{y'ha} + \zeta^A_{ya} \geq 0 \quad \perp \quad z^A_{ya} \geq 0 \tag{B.16}$$

$$cap^A_{ya} + \sum_{y'<y} dep^A_{y'ya} z^A_{y'a} - f^A_{yha} \geq 0 \quad \perp \quad \tau^A_{yha} \geq 0 \tag{B.17}$$

$$exp^A_{ya} - z^A_{ya} \geq 0 \quad \perp \quad \zeta^A_{ya} \geq 0 \tag{B.18}$$

$$df_y dur_h \left(-p^C_{yhnce} + trf^C_{ync} + \sum_{g\in G} p^G_{yng} ems^C_{yceg}\right)$$
$$+ \sum_{f\in E^{C+}_c} transf^C_{yncef} \tau^C_{yhnc}$$
$$+ \sum_{(e',f)\in E^C_c} shr^C_{ynce'} transf^C_{yncef} \beta^C_{yhnce'}$$
$$- \sum_{f\in E^{C+}_c} transf^C_{yncef} \beta^C_{yhnce} \geq 0 \quad \perp \quad f^C_{yhnce} \geq 0 \tag{B.19}$$

$$df_y inv^C_{ync} - \sum_{y'>y, h\in H} dep^C_{yy'nc} \tau^C_{y'hnc} + \zeta^C_{ync} \geq 0 \quad \perp \quad z^C_{ync} \geq 0 \tag{B.20}$$

$$cap^C_{ync} + \sum_{y'<y} dep^C_{y'ync} z^C_{y'nc} - \sum_{(e,f)\in E^C_c} transf^C_{yncef} f^C_{yhnce} \geq 0 \quad \perp \quad \tau^C_{yhnc} \geq 0 \tag{B.21}$$

$$-shr^C_{ynce} \sum_{(e',f)\in E^C_c} transf^C_{ynce'f} f^C_{yhnce'}$$
$$+ \sum_{f\in E^{C+}_c} transf^C_{yncef} f^C_{yhnce} \geq 0 \quad \perp \quad \beta^C_{yhnce} \geq 0 \tag{B.22}$$

$$exp^C_{ync} - z^C_{ync} \geq 0 \quad \perp \quad \zeta^C_{ync} \geq 0 \tag{B.23}$$

$$df_y dur_h \left(-p^{O-}_{yhno} + trf^{O-}_{yno} + \sum_{g\in G} p^G_{yng} ems^{O-}_{yog}\right)$$
$$+ dur_h \tau^O_{yv^H(h,o)no} + \kappa^{O-}_{yhno} \geq 0 \quad \perp \quad f^{O-}_{h,n,o} \geq 0 \tag{B.24}$$

$$-df_y dur_h p^{O+}_{yhno} + \kappa^{O+}_{yhno} \geq 0 \quad \perp \quad f^{O+}_{yhno} \geq 0 \tag{B.25}$$

$$df_y inv^O_{yno} - \sum_{y'>y,v\in V} dep^O_{yy'no}\tau^O_{y'vno} + \zeta^O_{yno} \geq 0 \quad \perp \quad z^O_{yno} \geq 0 \tag{B.26}$$

$$df_y inv^{O-}_{yno} - \sum_{y'>y,h\in H} dep^{O-}_{yy'no}\kappa^{O-}_{y'hno} + \zeta^{O-}_{yno} \geq 0 \quad \perp \quad z^{O-}_{yno} \geq 0 \tag{B.27}$$

$$df_y inv^{O+}_{yno} - \sum_{y'>y,h\in H} dep^{O+}_{yy'no}\kappa^{O+}_{y'hno} + \zeta^{O+}_{yno} \geq 0 \quad \perp \quad z^{O+}_{yno} \geq 0 \tag{B.28}$$

$$cap^O_{yno} + \sum_{y'<y} dep^O_{y'yno} z^O_{y'no} - \sum_{h\in H^V_{v,o}} dur_h f^{O-}_{yhno} \geq 0 \quad \perp \quad \tau^O_{yvno} \geq 0 \tag{B.29}$$

$$cap^{O-}_{yno} + \sum_{y'<y} dep^{O-}_{y'yno} z^{O-}_{y'no} - f^{O-}_{yhno} \geq 0 \quad \perp \quad \kappa^{O-}_{yhno} \geq 0 \tag{B.30}$$

$$cap^{O+}_{yno} + \sum_{y'<y} dep^{O+}_{y'yno} z^{O+}_{y'no} - f^{O+}_{yhno} \geq 0 \quad \perp \quad \kappa^{O+}_{yhno} \geq 0 \tag{B.31}$$

$$exp^O_{yno} - z^O_{yno} \geq 0 \quad \perp \quad \zeta^O_{yno} \geq 0 \tag{B.32}$$

$$exp^{O-}_{yno} - z^{O-}_{yno} \geq 0 \quad \perp \quad \zeta^{O-}_{yno} \geq 0 \tag{B.33}$$

$$exp^{O+}_{yno} - z^{O+}_{yno} \geq 0 \quad \perp \quad \zeta^{O+}_{yno} \geq 0 \tag{B.34}$$

$$\begin{aligned} df_y \left(-p^G_{yng} + tax^{glob}_{yg} + \sum_{r\in R_n} tax^{reg}_{yrg} + tax^{nod}_{yng} \right) & \\ + \mu^{glob}_{yg} + \sum_{r\in R_n} \mu^{reg}_{yrg} + \mu^{nod}_{yng} \geq 0 \quad \perp \quad f^G_{yng} \geq 0 & \end{aligned} \tag{B.35}$$

$$quota^{glob}_{yg} - \sum_{n\in N} f^G_{yng} \geq 0 \quad \perp \quad \mu^{glob}_{yg} \geq 0 \tag{B.36}$$

$$quota^{reg}_{yrg} - \sum_{n\in N_r} f^G_{yng} \geq 0 \quad \perp \quad \mu^{reg}_{yrg} \geq 0 \tag{B.37}$$

$$quota^{nod}_{yng} - f^G_{yng} \geq 0 \quad \perp \quad \mu^{nod}_{yng} \geq 0 \tag{B.38}$$

$$f^A_{yha} - \sum_{s\in S} q^A_{yhsa} = 0 \quad , \quad p^A_{yha} \text{ (free)} \tag{B.39}$$

$$f^C_{yhnce} - \sum_{s\in S} q^C_{yhsnce} = 0 \quad , \quad p^C_{yhnce} \text{ (free)} \tag{B.40}$$

$$f^{O-}_{yhno} - \sum_{s\in S} q^{O-}_{yhsno} = 0 \quad , \quad p^{O-}_{yhno} \text{ (free)} \tag{B.41}$$

$$f^{O+}_{yhno} - \sum_{s\in S} q^{O+}_{yhsno} = 0 \quad , \quad p^{O+}_{yhno} \text{ (free)} \tag{B.42}$$

$$\begin{aligned} f^G_{yng} - \sum_{h\in H,e\in E} dur_h \Bigg(\sum_{s\in S} ems^P_{ysneg} q^P_{yhsne} + \sum_{s\in S,d\in D} ems^D_{ydeg} q^D_{yhsnde} & \\ + \sum_{a\in A^+_{ne}} ems^A_{yag} f^A_{yha} + \sum_{c\in C} ems^C_{yceg} f^C_{yhnce} + \sum_{o\in O} ems^{O-}_{yog} f^{O-}_{yhno} \Bigg) = 0 \quad , \quad p^G_{yng} \text{ (free)} & \end{aligned} \tag{B.43}$$

Appendix C

Mathematical Appendix to Chapter 5

The KKT conditions of the national regulator

$$0 \le c_l^T - (\overline{\mu}_{lr}^R + \underline{\mu}_{lr}^R) + \kappa_r^R(\overline{\mu}_l + \underline{\mu}_l) \quad \perp \quad e_l \ge 0 \quad \text{if } l \in L_r \tag{C.1a}$$

$$\begin{aligned}0 = \Big[-p_n + c_{ns}^G \Big]_{\text{if } n \in N_r} \\ -\lambda_{nr}^R + \beta_{nsr}^R - \psi_{rns}^R + \kappa_r^R c_{ns}^G \quad \perp \quad g_{ns} \text{ (free)}\end{aligned} \tag{C.1b}$$

$$\begin{aligned}0 = \Big[-a_n + b_n d_n + p_n \Big]_{\text{if } n \in N_r} \\ -\rho_{nr}^R b_n + \lambda_{nr}^R - \phi_{nr}^R - \kappa_r^R(a_n - b_n d_n) \quad \perp \quad d_n \text{ (free)}\end{aligned} \tag{C.1c}$$

$$\begin{aligned}0 = \Bigg[\sum_{l \in L_r} shr_{lr} H_{ln} \sum_{k \in N} I_{lk} p_k \Bigg]_{\text{if } n \in N_r} \\ + \sum_{k \in N} B_{kn} \lambda_{kr}^R + \sum_{l \in L} H_{ln}(\overline{\mu}_{lr}^R - \underline{\mu}_{lr}^R) - \begin{Bmatrix} \gamma_r^R & \text{if } n = \hat{n} \\ 0 & \text{else} \end{Bmatrix} \quad \perp \quad \delta_n \text{ (free)}\end{aligned} \tag{C.1d}$$

$$\begin{aligned}0 = \Bigg[d_n - \sum_{s \in S} g_{ns} + \sum_{l \in L_r} shr_{lr} I_{ln} \sum_{k \in N} H_{lk} \delta_k \Bigg]_{\text{if } n \in N_r} \\ + \sum_{s \in S} \eta_{nsr}^R - \rho_{nr}^R - \sum_{k \in N} \nu_{kr}^R B_{nk} + \kappa_r^R \frac{1}{b_n}(p_n - a_n - \phi_n) \quad \perp \quad p_n \text{ (free)}\end{aligned} \tag{C.1e}$$

$$0 \le -\sum_{n \in N} \nu_{nr}^R H_{ln} + \kappa_r^R(\overline{f}_l + e_l) \quad \perp \quad \overline{\mu}_l \ge 0 \tag{C.1f}$$

$$0 \le \sum_{n \in N} \nu_{nr}^R H_{ln} + \kappa_r^R(\overline{f}_l + e_l) \quad \perp \quad \underline{\mu}_l \ge 0 \tag{C.1g}$$

$$0 \le -\eta_{nsr}^R + \kappa_r^R \overline{g}_{ns} \quad \perp \quad \beta_{ns} \ge 0 \tag{C.1h}$$

$$0 = \nu_{\hat{n}r}^R \quad \perp \quad \gamma \text{ (free)} \tag{C.1i}$$

$$0 \le \eta_{nsr}^R \quad \perp \quad \psi_{ns} \ge 0 \tag{C.1j}$$

$$0 \le \rho_{nr}^R + \kappa_r^R \frac{1}{b_n}(\phi_n + a_n - p_n) \quad \perp \quad \phi_n \ge 0 \tag{C.1k}$$

Equations (5.5b–(5.5l) in complementarity form

As explained in Section 5.4, the strong duality constraint is replaced by the ISO's equilibrium constraints (Equations 5.2). All complementarity conditions are replaced by the disjunctive constraints reformulation. The constraints are bilinear (and non-convex) in the dual variable to the strong-duality constraint (κ_r^R).

Index

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