

Lukas Lentz, Huy The Nguyen, Utz von Wagner

# Energy Harvesting From Bistable Systems Under Random Excitation

**Journal article** | **Submitted manuscript (Preprint)**

This version is available at <https://doi.org/10.14279/depositonce-9837>



Lentz, L.; Nguyen, H. T.; von Wagner, U. (2017): Energy Harvesting From Bistable Systems Under Random Excitation. Machine Dynamics Research, Vol. 41, No. 1, pp. 5–16.

## Terms of Use

Copyright applies. A non-exclusive, non-transferable and limited right to use is granted. This document is intended solely for personal, non-commercial use.

**WISSEN IM ZENTRUM**  
**UNIVERSITÄTSBIBLIOTHEK**

Technische  
Universität  
Berlin

# Energy harvesting from bistable systems under random excitation

Lukas Lentz<sup>1a</sup>, Huy The Nguyen<sup>b</sup>, Utz von Wagner<sup>a</sup>

<sup>a</sup>Chair of Mechatronics and Machine Dynamics, Technische Universität Berlin

<sup>b</sup>Chair of Mechatronics and Maschine Dynamics, Technische Universität Berlin /  
Hanoi University of Mining and Geology

## Abstract

The transformation of otherwise unused vibrational energy into electric energy through the use of piezoelectric energy harvesting devices has been the subject of numerous investigations. The mechanical part of such a device is often constructed as a cantilever beam with applied piezo patches. If the harvester is designed as a linear resonator the power output relies strongly on the matching of the natural frequency of the beam and the frequency of the harvested vibration which restricts the applicability since most vibrations which are found in built environments are broad-banded or stochastic in nature. A possible approach to overcome this restriction is the use of permanent magnets to impose a nonlinear restoring force on the beam that leads to a broader operating range due to large amplitude motions over a large range of excitation frequencies.

In this paper such a system is considered introducing a refined modeling with a modal expansion that incorporates two modal functions and a refined modeling of the magnet beam interaction. The corresponding probability density function in case of random excitation is calculated by the solution of the corresponding Fokker-Planck equation and compared with results from Monte Carlo simulations. Finally some measurements of ambient excitations are discussed.

**Keywords:** energy harvesting, bistable oscillator, random excitation, Fokker-Planck equation, ambient excitation

## 1. Introduction

The generation of electrical energy from mechanical vibrations with ambient excitation by using a mechatronic system has found broad scientific consideration under the term “energy harvesting”. In this paper the authors will focus on piezo beam systems with base excitation. A broad review of such systems is e.g. given in

---

<sup>1</sup>Corresponding author adress: Einsteinufer 5, 10587 Berlin, Germany; Phone: +49 30 314-22922; Email: lukas.lentz@tu-berlin.de

[Priya 2007]. The simplest case would be that of a monofrequent (harmonic) excitation which allows for adapting the natural frequency of the harvesting system to the excitation frequency in order to obtain large displacement amplitudes and therefore intensive energy harvesting. Unfortunately the nature of most ambient vibrations will not be harmonic but broad band in frequency and include (in the modeling) random parts. Therefore broad band excitation of energy harvesting systems has been taken considerable interest in the last years, see e.g. [Tang et al. 2010] for an overview. The simplest case is the consideration of a linear system under white noise or colored noise (obtained by linear filters) excitation as considered e.g. in [Adhikari et al. 2009]. These problems can be investigated analytically as Gaussian excitation of a linear system produces also a Gaussian response. Nevertheless a refined modeling of either the excitation or the mechatronic system itself (or both of them) often results in a nonlinear model where this is not anymore possible. Also a possibility to adapt the harvesting systems to broad band excitation and to increase the level of harvested energy is the deliberate introduction of nonlinearities resulting in bi- or multistability, see e.g. [Harne 2013] for an overview. In this case frequent changes between the attractors of the multiple stable stationary solutions will result in a good energy harvesting result. If the restoring nonlinearity in a one degree of freedom system is described by a polynomial and excitation is by white noise, still an exact solution of the corresponding Fokker-Planck solution, i.e. an exact solution for the probability function (pdf) can be found. Systems of such a level are e.g. considered in [Gammaitoni et al. 2009], [Gammaitoni et al. 2010] or [Halvorsen 2008].

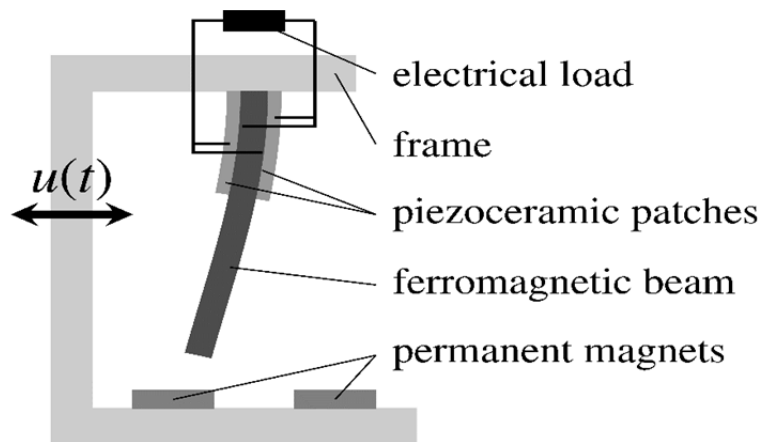


Figure 1: Bistable energy harvesting system with magnets as used in e.g. [Ertuk et al. 2009] or [Litak et al. 2010].

A typical bistable energy harvesting system as considered e.g. in [Ertuk et al. 2009] or [Litak et al. 2010] is sketched in Figure 1. A cantilever beam with base displacement excitation  $u$  becomes two stable stationary solutions by two applied magnets. The transfer from mechanical to electrical energy is performed by the piezoceramics applied on the beam. In [Litak et al. 2010] the base excitation is given by Gaussian white noise. A similar system is investigated in [Litak et al. 2012] under combined harmonic and white noise excitation. State of the art to perform such investigations is to use numerical integration in time, i.e. Monte Carlo simulation. As already mentioned an alternative way beside Monte Carlo simulations for the investigation of energy harvesting systems is the calculation of the pdf by solving the

corresponding Fokker-Planck equation. This equation is in most cases very hard to solve and only for a very limited class of problems analytical solutions of the Fokker-Planck equation are known. Typical approaches in literature for solving Fokker-Planck equations are using Finite Differences (e.g. [Bergman 1992], Finite Elements (e.g. [Bergman 2005], [Kumar et al. 2009]) or the path integral method (e.g. [Naess 2006]). Usually these methods are limited to the solution of problems with maximum dimensions between 4 and 6. A method capable for solving Fokker-Planck equations with higher dimensions is described in [Martens et al. 2012] which is based on earlier attempts e.g. [von Wagner 2000]. Herein the pdf is expanded in specially adapted orthogonal functions and the Galerkin method is applied. With this, problems up to dimension ten could be solved. In [Martens et al. 2013] this method is applied to a piezo beam energy harvester with random excitation, where the piezo beam system is discretized by just one modal ansatz function. In literature in almost all cases, the beam of such energy harvesters are discretized by just one ansatz function although using broad band excitation.

The present paper investigates energy harvesting systems as sketched in Figure 1 and intends to overcome the described limitations by using a spacial discretization by multiple ansatz functions. Additionally a refined modeling of the force between magnets and beam is introduced. The resulting Fokker-Planck equations of thereby refined models are solved and the results are compared with those from Monte Carlo simulations. Finally some results of measurements of ambient vibrations are presented which could be the base for future refined modeling of the excitation processes.

## 2. Modeling of the energy harvester

### 2.1 State of the art modeling

The energy harvesting system in Fig.1 is in general (e.g. [Litak et. al 2010] )modeled by discretizing the beam in space by one modal ansatz function resulting in the two coupled system equations

$$\ddot{x} + 2\xi\dot{x} - \alpha x + \beta x^3 - \chi v = f, \quad (1)$$

$$\dot{v} + \lambda v + \kappa \dot{x} = 0. \quad (2)$$

The first equation describes the mechanical domain, where  $x$  is the modal coordinate,  $\xi$  the modal damping ratio,  $\alpha$  the coefficient of the linear restoring forces,  $\beta$  the coefficient of the cubic restoring forces and  $f$  an external force which is proportional to the second time-derivative of the base excitation  $u$ . The second equation describes the electrical circuit, where  $v$  is the voltage across the electric load, which is in this case a simple resistance, and  $\lambda$  is proportional to the reciprocal of the product of the load resistance and the equivalent capacitance of the piezoceramic layers. The two equations are coupled via the dimensionless piezoceramic coupling coefficients  $\chi$  in the mechanical domain and  $\kappa$  in the electrical domain, see [Ertuk et al. 2009] for further reference. Following the approach in [Erturk et al. 2009] all variables including time are normalized and scaled to be dimensionless and to avoid ill-conditioned problems in the following investigations.

For a refinement of the modeling and understanding of its limitations it is important to investigate the assumptions that it is based on. For refining the beam model with respect to a larger number of modal ansatz functions it is mandatory to separate the modeling of the magnetic effects from the modeling of the piezoceramic layers and the electrical circuit. The former was described quite early in [Moon 1979] where the system shown in Fig.1 without piezoceramic layers was analysed to investigate chaos in a deterministic mechanical system. The latter was described in [Erturk 2008] where a model for the system shown in Fig.1 without magnets was presented.

In [Moon 1979] the effect of the magnets on the ferromagnetic beam is modeled as a single load acting on the tip of the beam in the direction of displacement. The influence of the load on the beam displacement is assumed to be cubic one with a vanishing quadratic term. The further discretization is done using the first eigenfunction of the system without magnets, resulting in a model with one degree of freedom (dof).

Although the model for the beam with piezoceramic layers which is presented in [Erturk 2008] is in principle not restricted in the number of modal ansatz functions which are used for the discretization, it is in general restricted to one dof for the combination of both models.

## 2.2 Refined modeling

To overcome the limitation of the one-dof model the effect of the magnets is now interpreted as distributed load in the direction of the beam displacement. According to the approach in [Moon 1979] the influence of the load on the beam displacement is as well assumed to be cubic with a vanishing quadratic term. The describing set of equations for the case of a two-discretization then reads

$$\ddot{x}_1 + 2\xi_1\dot{x}_1 - \alpha_{11}x_1 - \alpha_{12}x_2 + \beta_{11}x_1^3 + \beta_{12}x_1^2x_2 + \beta_{13}x_1x_2^2 + \beta_{14}x_2^3 - \chi_1v = f_1, \quad (3)$$

$$\ddot{x}_2 + 2\xi_2\dot{x}_2 - \alpha_{21}x_1 - \alpha_{22}x_2 + \beta_{21}x_1^3 + \beta_{22}x_1^2x_2 + \beta_{23}x_1x_2^2 + \beta_{24}x_2^3 - \chi_2v = f_2, \quad (4)$$

$$\dot{v} + \lambda v + \kappa_1\dot{x}_1 + \kappa_2\dot{x}_2 = 0. \quad (5)$$

As it can be seen the equations of the mechanical domain are coupled in the linear and in the cubic terms. It shall be additionally mentioned, that the coefficients of the linear and cubic terms are not independent, but are calculated from an projection of the distributed load which resembles the magnetic effect on the eigenfunctions used in the discretization. The two external excitations  $f_1$  and  $f_2$  are as well not independent from each other.

## 3. Solution of the corresponding Fokker-Planck equation

If the base excitation of the energy harvester is modeled as white Gaussian noise the describing set of differential equations is given in the general form of the stochastic differential equations (SDE)

$$dX_t = f(X_t, t)dt + G(X_t, t)dW_t. \quad (6)$$

The probability density function (pdf)  $\mathbf{p}$  can be calculated as the solution of the corresponding Fokker-Planck equation (FPE) which reads

$$\sum_{i=1}^n \frac{\partial}{\partial x_i} (f_i(x) \mathbf{p}(x)) - \sum_{i,j=1}^n \frac{\partial^2}{\partial x_i \partial x_j} (b_{ij}(x) \mathbf{p}(x)) = 0. \quad (7)$$

Since the exact solutions of the FPEs for the given systems are not known, an approximate solution is calculated with an Galerkin-type scheme which is intensely discussed e.g. in [von Wagner 2000] and [Martens et al. 2012].

Herein, for the calculation of the approximate solution a starting solution  $\mathbf{p}_0$  is multiplied with a polynomial correction term  $\mathbf{k}$ , i.e.  $\mathbf{p} \approx \mathbf{k} \mathbf{p}_0$  where the polynomial is chosen from an ansatz-space which is orthonormal with respect to the weighted scalar product defined by  $\mathbf{p}_0$ . This ansatz is then inserted in the FPE yielding an error which is in the Galerkin sense supposed to vanish if projected on any of the members of the ansatz space. The unknown coefficients in the correction term  $\mathbf{k}$  are then computed to meet the afore mentioned conditions. As it is shown e.g. in [Martens et al. 2012] this method works very well even for small orders of  $\mathbf{k}$  if the starting solution  $\mathbf{p}_0$  is properly chosen. For the discussed energy harvesting system it is possible to use the exact solution of the system without piezoelements in the one-dof case.

### 3.1 One-dof discretization

The one-dof discretization has already been solved and discussed in [Martens et al. 2013]. In preparation for the following discussion of the 2-dof discretization the results are shortly recapitulated. In Figure 2, the marginal probability density function with respect to the modal coordinate  $x_1$  and the modal velocity  $x_2$  are presented.

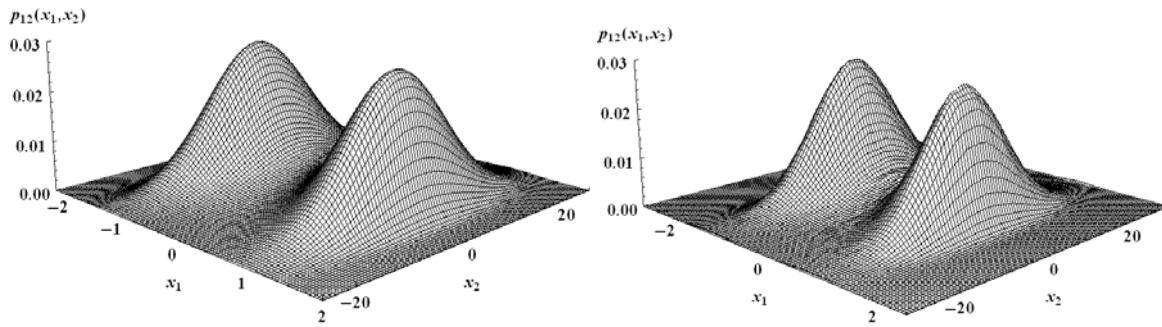


Figure 2: Marginal probability density function  $\mathbf{p}_{12}$  for the one-dof model as an approximate solution off the FPE (left) and from a Monte Carlo simulation (right).

The parameters used in the analysis correspond to an experimental set-up which is actually built up at the TU Berlin and will be used for experimental investigations of the energy harvesting system.

They read  $\xi = 8.4, \alpha = \beta = 443.8, \chi = 12.3, \lambda = 12555.0$  and  $\kappa_1 = -13.4$ . The intensity of the white noise excitation is  $\sigma = 50.5$ . The results on the left side are obtained as an approximate solution of the FPE and the results on the right side originate from a Monte Carlo simulation. As it can be seen, both results are in a very

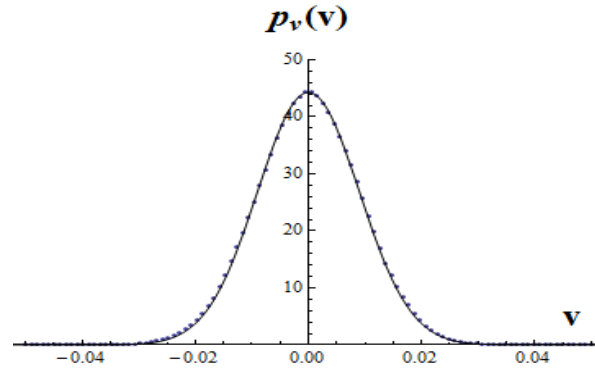


Figure 3: Marginal probability density function  $p_v$  for the one-dof discretization: result of solution of FPE (solid line) and Monte Carlo simulation (dots).

good agreement whereas the results on the left side are smoother due to their analytical nature. Both results show, that the system resides for the chosen parameters most probably in the vicinity of the two nontrivial equilibrium points.

As proposed in [Martens et al. 2013] the variance  $\sigma_v^2$  of the voltage  $v$  which is calculated from the marginal probability density function of the voltage is used for the assessment of the systems energy output. As it can be seen from Fig.3 the results from the approximate solution of FPE (solid line) and the results obtained from the Monte Carlo simulation (dots) are again in a very good agreement.

### 3.2 Two-dof discretization

The analysis of the two-dof discretization is conducted in the same manner as the analysis of the one-dof problem. To validate the approximate results of the FPE a Monte Carlo simulation is carried out and is used as a reference. The parameters used in the analysis correspond as well to the experimental apparatus at the TU Berlin and read for the first equation  $\xi_1 = 8.4, \alpha_{11} = 443.8, \alpha_{12} = -3365.3, \beta_{11} = 468.3, \beta_{12} = 898.9, \beta_{13} = 860.0, \beta_{14} = 211.2, \chi_1 = 12.0$  for the second equation  $\xi_2 = 105.3, \alpha_{21} = -3004.1, \alpha_{22} = 267.5, \beta_{21} = -267.5, \beta_{22} = 767.2, \beta_{23} = -565.8, \beta_{24} = 232.7, \chi_2 = -39.3$  and for the third equation  $\lambda = 12555.0, \kappa_1 = -13.4, \kappa_2 = 49.1$ .

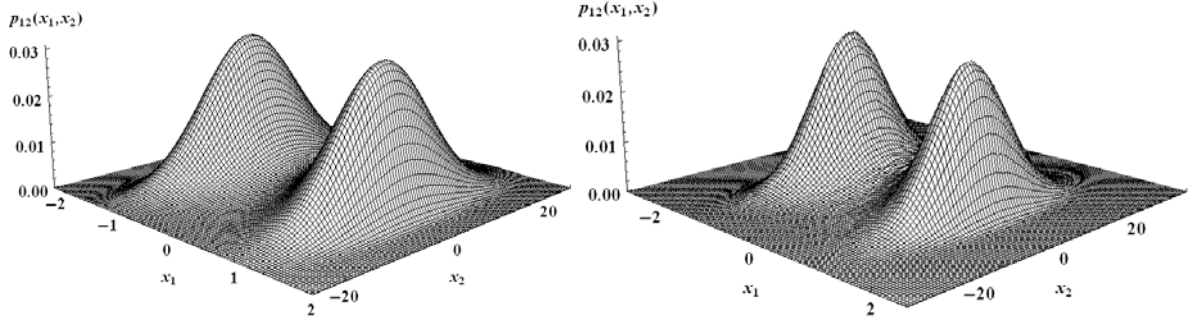


Figure 4: Marginal probability density function  $p_{12}$  for the two-dof discretization as an approximate solution of the FPE (left) and from a Monte Carlo simulation (right).

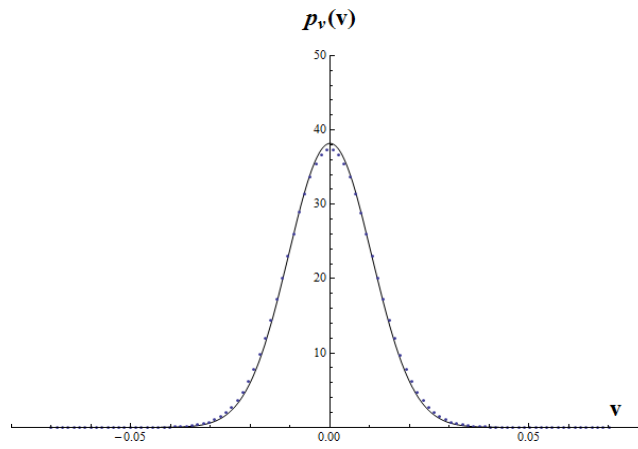


Figure 5: Marginal probability density function  $p_v$  for the two-dof discretization: solution of FPE (solid line) and Monte Carlo simulation (dots)

As it can be seen from Fig. 4 the results for the marginal probability density function of the modal coordinate  $x_1$  and the modal velocity  $x_2$  are again in a very good agreement.

Further it can be seen from Fig. 5 that the same is true for the marginal probability density function of the voltage  $p_v$ . However a comparison of Fig. 3 and Fig. 5 shows, that the calculated marginal probability density function of the voltage  $p_v$  shows a broader characteristic in the second case. This leads also to a difference in the variance  $\sigma_v^2$  and therefore deviations in the predicted energy output. This clearly demonstrates the influence of the refined model.

#### 4. Properties of ambient vibrations

So far, white noise has been used for the excitation of the examined systems. In this section more realistic excitation processes are considered with respect of future usage in the simulation of our energy harvesting system.



In [Kracht 2011], vibrations of oil paintings under different ambient sources, such as transport situations, visitors or traffic during exposition, were considered. The measurement results were clustered in two spectral densities, which are shown in Fig. 6. It can be seen that the stochastic excitations exhibit broad band property in these cases.

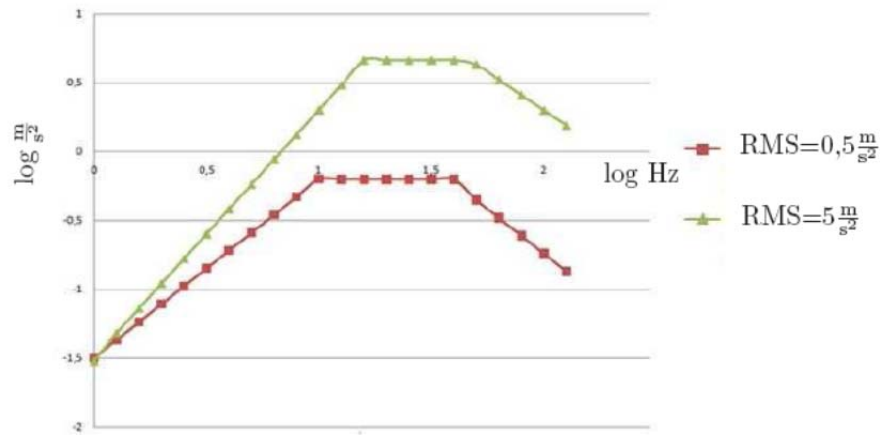


Figure 6: Spectral densities clustered from excitation data [Kracht 2011].

In order to obtain some additional measurement results, experiments with a bicycle were performed as described in the following.

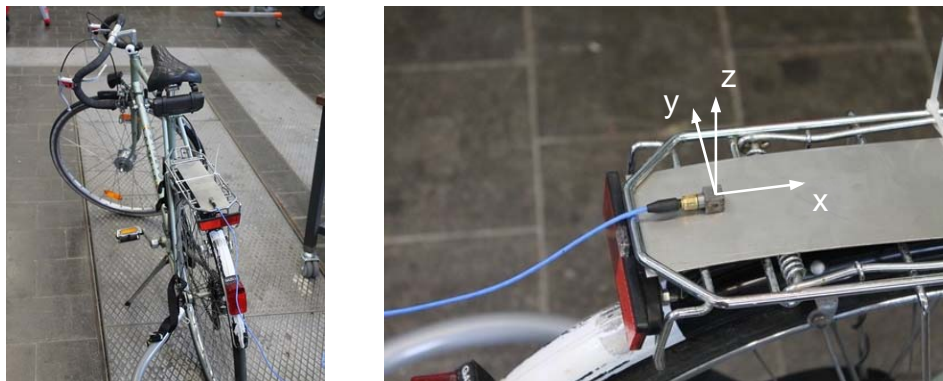


Figure 7: Experimental setup for measurement of ambient vibrations.

Two photos of the experimental setup are shown in Fig.7. A triaxial accelerometer was attached on a sheet of iron, which is fixed to the pannier rack of a bicycle. Acceleration signals were collected by an USB data acquisition device and recorded in a measuring computer. Accelerometer data were sampled at a frequency of 2560 Hz. Both the data acquisition device and the computer worked with 220 V power source, which is supplied by a 12 V car battery via a power inverter. All these equipments were put in a bike trailer and carried together with the test bicycle. The experiments were performed with a variety of road surfaces, e.g. bicycle lane, asphalt street, cobbled street and sand path. Figure 8 (a) shows the frequency spectrum of the vertical acceleration at one point on the pannier rack of a bicycle that was ridden normally on a road with asphalt surface. The corresponding pdf is given in Fig. 9 (a). The results show, that there are two distinct frequencies peaks in the spectrum while the pdf has a non-Gaussian character. Compared to this, measurements performed

in planes in [Kracht 2011] show a somewhat different characteristic. One set of these measurements performed by Kracht were evaluated with respect to frequency spectrum and pdf as shown in Fig. 8 (b) and Fig. 9 (b).

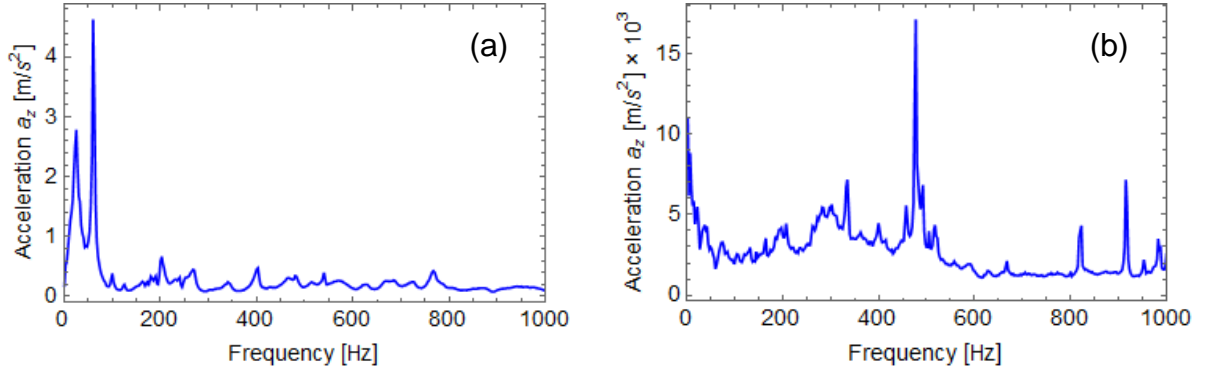


Figure 8: Acceleration spectra of ambient vibrations: Vibrations on the pannier rack of a bicycle (a) and in a plane (b).

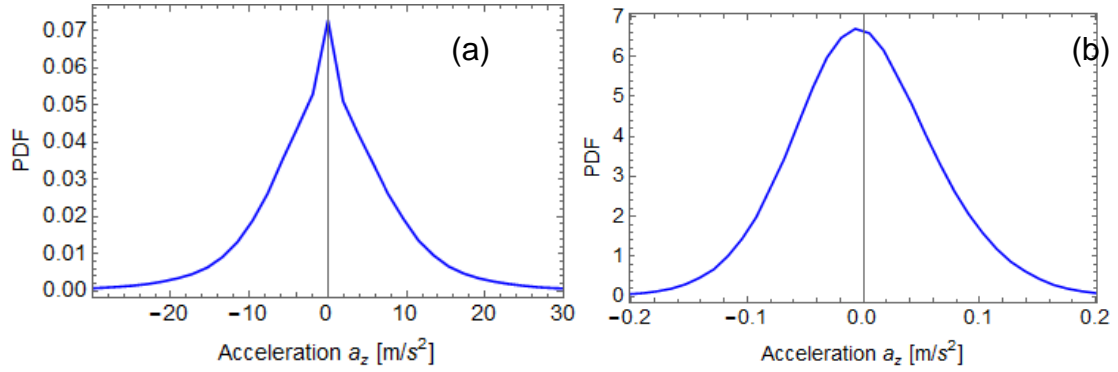


Figure 9: Pdf of ambient vibrations: Vibrations on the pannier rack of a bicycle (a) and in a plane (b).

These results from the seat of a plane show a broad band characteristic with one distinct frequency peak in the spectrum (Fig. 8 (b)) and a Gaussian-like pdf (Fig. 9 (b)).

To include such ambient excitation characteristics in the analysis described in section 3, corresponding linear or nonlinear filters are necessary to obtain the respective characteristics. A nonlinear filter offering wide possibilities in shaping desired pdfs is given in [Wedig 2010] which produces also amplitude limited noise. This will be part of intended future work.

## 5. Conclusions

The transformation of otherwise unused vibrational energy into electric energy is studied in the present paper with respect to a piezo beam system with base excitation. To increase the energy output, a bistability is introduced by using magnets and random excitation by Gaussian white noise is considered. State of the art of modeling such systems is to discretize the beam vibrations using one modal ansatz function (despite the broad band excitation) and solving the corresponding system of

stochastic differential equations by Monte Carlo simulation. In this paper, the modeling is refined by using two instead of just one modal ansatz function for the discretization of the beam and the pdf of the system is calculated by using a Galerkin method based solution of the Fokker-Planck equation. The results are compared with those obtained by Monte Carlo simulations and show a good coincidence. When the results from the refined model are compared to those of a state of the art model, it becomes obvious that there are some differences in the marginal pdf of the voltage  $v$ . This difference shows the influence of the type of modeling used for the prediction of the energy output.

For replacing in future steps the white noise excitation by a more realistic one, measurements of ambient excitations were also considered. Next intended steps are the usage of additional mode(s) in the discretization of the beam and the usage of more sophisticated excitation processes, based on measurements of ambient accelerations.

## Acknowledgement

This work has been funded by Deutsche Forschungsgemeinschaft (DFG) by grant WA 1427/23-1

## References

**Adhikari, S., Friswell, M.I., Inman, D.J.**, 2009, Piezoelectric energy harvesting from broadband random vibrations, *Smart Materials and Structures*, 18, 115005, doi: 10.1088/0964-1726/18/11/115005.

**Bergman, L. A.**, 1992, On the Numerical Solution of the Fokker-Planck Equation for Nonlinear Stochastic Systems, *Nonlinear Dynamics*, 4, 357-372.

**Bergman, L. A.**, 2005, Application of multi-scale finite element methods to the solution of the Fokker-Planck equation, *Computer Methods in applied mechanics and engineering*, 194, 1513-1526.

**Erturk, A., Inman, D. J.**, 2008, A distributed parameter electromechanical model for cantilevered piezoelectric energy harvesters, *Journal of Vibration and Acoustics*, Vol. 130, No.4, 041002, doi: 10.1115/1.2890402.

**Erturk, A., Hoffmann, J., Inman, D.J.**, 2009, A piezomagnetoelastic structure for broadband vibration energy harvesting, *Applied Physics, Lett.* 94, 11-14.

**Gammaitoni, L., Neri, I., Vocca, H.**, 2009, Nonlinear oscillators for vibration energy harvesting. *Applied Physics, Letters* 94, 164102, doi: 10.1063/1.3120279.

**Gammaitoni, L., Neri, I., Vocca, H.**, 2010, The benefits of noise and nonlinearity: Extracting energy from random vibrations, *Chemical Physics*, 375, 435-438.

**Halvorsen, E.**, 2008, Energy Harvesters Driven by Broadband Random Vibrations, *Journal of Microelectromechanical Systems*, Vol. 17, No.5, 1061-1070.

**Harne R. L., Wang K. W.**, 2013, A review of the recent research on vibration energy harvesting via bistable systems, *Smart Materials and Structures*, 22, 023001, doi: 10.1088/0964-1726/22/2/023001.

**Kracht, K.**, 2011, *Die Untersuchung des Schwingungsverhaltens von Ölgemälden in Abhängigkeit der Alterung*, Dissertation TU Berlin. Shaker Verlag, Aachen.

**Kumar, P., Chakravorty, S., Singla, P., Junkins, J.**, 2009, The Partition of Unity Finite Element Approach with HP-Refinement for the Stationary Fokker-Planck Equation, *Journal of Sound and Vibration*, 327, 144-162.

**Litak, G., Friswell, M. I., Adhikari, S.**, 2010, Magnetopiezoelectric energy harvesting driven by random excitations, *Applied Physics Letters*, 96, 214103, doi: 10.1063/1.3436553.

**Litak, G., Kitio Kwuimy, C. A., Borowiec, M., Nataraj, C.**, 2012, Performance of a piezoelectric energy harvester driven by air flow, *Applied Physics Letters*, 100, 024103, doi:10.1063/1.3676272.

**Martens, W., von Wagner, U., Mehrmann, V.**, 2012, Calculation of high-dimensional probability density functions of stochastically excited nonlinear mechanical systems, *Nonlinear Dynamics*, 67, 2089-2099.

**Martens, W., von Wagner, U., Litak, G.**, 2013, Stationary response of nonlinear magneto-piezoelectric energy harvester systems under stochastic excitation, *European Physical Journal – Special Topics*, 222, 1665-1673.

**Moon, F., Holmes, P.**, 1979, A magnetoelastic strange attractor, *Journal of Sound and Vibration*, 65, 275-296

**Naess, A.**, 2006, Response probability density functions of strongly non-linear systems by the path integration method, *International Journal of Non-Linear Mechanics*, 41, 693-705.

**Priya, S.**, 2007, Advances in energy harvesting using low profile piezoelectric transducers, *Journal of electroceramics*, 19, 165-182.

**Tang, L., Yang, Y., Soh, C.K.**, 2010, Toward Broadband Vibration-based Energy Harvesting, *Journal of Intelligent Material Systems and Structures*, Vol. 21, 1867-1897.

**Von Wagner, U., Wedig, W. V.**, 2000, On the calculation of stationary solutions of multi-dimensional Fokker-Planck equations by orthogonal functions, *Nonlinear Dynamics*, 21, 289–306.

**Wedig, W. V.**, 2010, Nonlinear Models of Cars Riding on Bounded Road Realizations. Proceedings of the 1st Joint International Conference on Multibody System Dynamics, Lappeenranta, Finland, 744-752.