Supplementary information to "Generation of maximally entangled states and coherent control in quantum dot microlenses"

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1 Quantum State Tomography

In order to reconstruct the density matrix of the emitted two photon state, a full quantum state tomography with 16 cross correlation measurements was carried out by using a polarization resolved cross-correlation setup. Two quantum state tomographies were performed on the quantum dot microlenses QDM1 and QDM2, both of them emitting two maximally entangled photons via a biexciton-exciton radiative cascade. QDM1 exhibits an excitonic fine structure splitting of 16(1) µeV and QDM2 of 30(1) µeV. Fig. S1 shows the 16 measurements of QDM1 and QDM2 with projections in different photon polarization states, which are combinations of projections in the basis states horizontal (H), diagonal (D) and right-circular (R). Due to the excitonic fine structure splitting, the phase of the photonic state precesses, resulting in the oscillatory behavior of the cross correlations with RR, RD, DR and DD projections [1]. The two photon density matrix was calculated for every data point of the correlation measurements using a maximum likelihood technique [2] and the fidelity to the maximally entangled bell states ϕ^+ and ϕ^- was calculated from the density matrices. The values which resulted in the highest fidelity to the ϕ^+ state are marked in red in fig. S1 and the values with the highest fidelity to the ϕ^- state are marked in green. QDM1 and QDM2 reached a fidelity to the ϕ^+ state of 0.73(03) and 0.69(04). The fidelities to the ϕ^- state are 0.80(03) and 0.68(04). The density matrices and error matrices to these values are:

- QDM1 at $\tau = 0.020\,\mathrm{ns}$ with fidelity $F_{\mathrm{QDM1},\phi^+} = 0.73(03)$ to ϕ^+:

$$\begin{pmatrix} 0.4524+0.0000i & 0.0455-0.0471i & -0.0627+0.0212i & 0.2803-0.2134i \\ 0.0455+0.0471i & 0.0502+0.0000i & -0.0373+0.0232i & 0.0486-0.0756i \\ -0.0627-0.0212i & -0.0373-0.0232i & 0.0503+0.0000i & -0.1041+0.0746i \\ 0.2803+0.2134i & 0.0486+0.0756i & -0.1041-0.0746i & 0.4470+0.0000i \end{pmatrix}$$

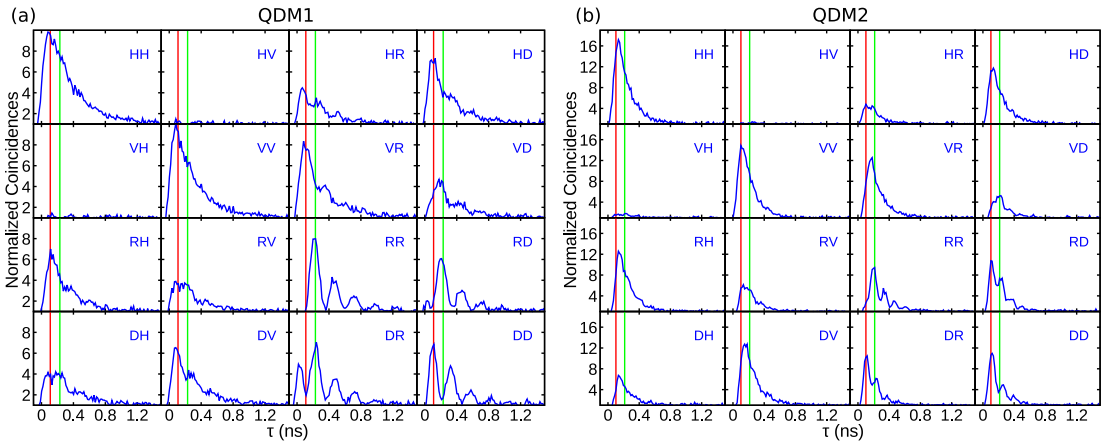

Figure S1: Cross correlation measurements of the emitted photons of (a) QDM1 and (b) QDM2 with projections in different polarization states. The biexciton photon triggered the measurement of the delay time τ and the exciton photon stopped it. The data points at the red line were used to reconstruct the density matrix which had the highest fidelity on the maximally entangled bell ϕ^+ state and the density matrix of the green line had the highest fidelity to the ϕ^- state.

The errors of these values are shown in the following matrix and the norms of the errors are displayed in figure S2:

$$\begin{pmatrix} +0.0006+0.0000i & +0.0007+0.0000i & +0.0006+0.0012i & +0.0005+0.0003i \\ -0.0057-0.0000i & -0.0040-0.0030i & -0.0074-0.0059i & -0.0011-0.0026i \\ +0.0007+0.0000i & +0.0002+0.0000i & +0.0002+0.0001i & +0.0004+0.0004i \\ -0.0040-0.0030i & -0.0062-0.0000i & -0.0073-0.0030i & -0.0018-0.0024i \\ +0.0006+0.0012i & +0.0002+0.0001i & +0.0007+0.0000i & +0.0001+0.0013i \\ -0.0074-0.0059i & -0.0073-0.0030i & -0.0064-0.0000i & -0.0043-0.0031i \\ +0.0005+0.0003i & +0.0004+0.0004i & +0.0001+0.0013i & +0.0001+0.0000i \\ -0.0011-0.0026i & -0.0018-0.0024i & -0.0043-0.0031i & -0.0069-0.0000i \end{pmatrix}$$

- QDM1 at $\tau = 0.136\,\text{ns}$ with fidelity $F_{\text{QDM1},\phi^-} = 0.80(03)$ to ϕ^-:

$$\begin{pmatrix} 0.4680+0.0000i & 0.0201-0.0781i & 0.0059+0.0597i & -0.3440+0.1701i \\ 0.0201+0.0781i & 0.0763+0.0000i & 0.0163+0.0146i & -0.0507-0.0449i \\ 0.0059-0.0597i & 0.0163-0.0146i & 0.0335+0.0000i & 0.0451+0.0715i \\ -0.3440-0.1701i & -0.0507+0.0449i & 0.0451-0.0715i & 0.4222+0.0000i \end{pmatrix}$$

The errors of these values are shown in the following matrix and the norms of the errors are displayed in figure S2:

$$\begin{pmatrix} +0.0004+0.0000i & +0.0005+0.0005i & +0.0001+0.0008i & +0.0000+0.0006i \\ -0.0055-0.0000i & -0.0031-0.0061i & -0.0017-0.0019i & -0.0153-0.0145i \\ +0.0005+0.0005i & +0.0002+0.0000i & +0.0001+0.0001i & +0.0005+0.0007i \\ -0.0031-0.0061i & -0.0079-0.0000i & -0.0035-0.0023i & -0.0007-0.0010i \\ +0.0001+0.0008i & +0.0001+0.0001i & +0.0005+0.0000i & +0.0001+0.0014i \\ -0.0017-0.0019i & -0.0035-0.0023i & -0.0049-0.0000i & -0.0016-0.0022i \\ +0.0000+0.0006i & +0.0005+0.0007i & +0.0001+0.0014i & +0.0003+0.0000i \\ -0.0153-0.0145i & -0.0007-0.0010i & -0.0016-0.0022i & -0.0074-0.0000i \end{pmatrix}$$

- QDM2 bei $\tau = 0.024\,\text{ns}$ with fidelity $F_{\text{QP2},\phi^+} = 0.69(04)$ to ϕ^+:

$$\begin{pmatrix} 0.4602 + 0.0000i & 0.0981 + 0.0134i & -0.0947 + 0.0348i & 0.2404 - 0.2140i \\ 0.0981 - 0.0134i & 0.0420 + 0.0000i & -0.0133 - 0.0142i & 0.0401 + 0.0126i \\ -0.0947 - 0.0348i & -0.0133 + 0.0142i & 0.0540 + 0.0000i & -0.1469 + 0.0358i \\ 0.2404 + 0.2140i & 0.0401 - 0.0126i & -0.1469 - 0.0358i & 0.4438 + 0.0000i \end{pmatrix}$$

The errors of these values are shown in the following matrix and the norms of the errors are displayed in figure S2:

$$\begin{pmatrix} +0.0047 + 0.0000i & +0.0042 + 0.0072i & +0.0053 + 0.0026i & +0.0112 + 0.0106i \\ -0.0142 - 0.0000i & -0.0036 - 0.0028i & -0.0038 - 0.0143i & -0.0010 - 0.0174i \\ +0.0042 + 0.0072i & +0.0058 + 0.0000i & +0.0012 + 0.0026i & +0.0018 + 0.0054i \\ -0.0036 - 0.0028i & -0.0011 - 0.0000i & -0.0052 - 0.0040i & -0.0065 - 0.0001i \\ +0.0053 + 0.0026i & +0.0012 + 0.0026i & +0.0009 + 0.0000i & +0.0030 + 0.0040i \\ -0.0038 - 0.0143i & -0.0052 - 0.0040i & -0.0008 - 0.0000i & -0.0022 - 0.0051i \\ +0.0112 + 0.0106i & +0.0018 + 0.0054i & +0.0030 + 0.0040i & +0.0096 + 0.0000i \\ -0.0010 - 0.0174i & -0.0065 - 0.0001i & -0.0022 - 0.0051i & -0.0144 - 0.0000i \end{pmatrix}$$

- QDM2 at $\tau = 0.124\,\text{ns}$ with fidelity $F_{\text{QP2},\phi^-} = 0.68(04)$ to ϕ^-:

$$\begin{pmatrix} 0.4296 + 0.0000i & 0.0582 + 0.0615i & -0.0466 - 0.0942i & -0.2322 + 0.1385i \\ 0.0582 - 0.0615i & 0.0468 + 0.0000i & -0.0385 - 0.0027i & 0.0733 + 0.0649i \\ -0.0466 + 0.0942i & -0.0385 + 0.0027i & 0.0564 + 0.0000i & -0.0463 - 0.1128i \\ -0.2322 - 0.1385i & 0.0733 - 0.0649i & -0.0463 + 0.1128i & 0.4673 + 0.0000i \end{pmatrix}$$

The errors of these values are shown in the following matrix and the norms of the errors are displayed in figure S2:

$$\begin{pmatrix} +0.0033 + 0.0000i & +0.0006 + 0.0008i & +0.0015 + 0.0040i & +0.0049 + 0.0028i \\ -0.0017 - 0.0000i & -0.0003 - 0.0004i & -0.0012 - 0.0029i & -0.0034 - 0.0013i \\ +0.0006 + 0.0008i & +0.0007 + 0.0000i & +0.0004 + 0.0011i & +0.0009 + 0.0000i \\ -0.0003 - 0.0004i & -0.0016 - 0.0000i & -0.0008 - 0.0015i & -0.0015 - 0.0006i \\ +0.0015 + 0.0040i & +0.0004 + 0.0011i & +0.0012 + 0.0000i & +0.0004 + 0.0034i \\ -0.0012 - 0.0029i & -0.0008 - 0.0015i & -0.0018 - 0.0000i & -0.0007 - 0.0026i \\ +0.0049 + 0.0028i & +0.0009 + 0.0000i & +0.0004 + 0.0034i & +0.0052 + 0.0000i \\ -0.0034 - 0.0013i & -0.0015 - 0.0006i & -0.0007 - 0.0026i & -0.0016 - 0.0000i \end{pmatrix}$$

As the phase of the two photon state precesses between the ϕ^+ and ϕ^- bell state, the fidelity oscillates between zero and one with the delay time. The highest fidelities of the first three periods are shown in table S1.

2 Deconvolution of the correlation measurements

Due to the time resolution of 100 ps of the detectors, the phases of the photon states are getting mixed, which results in a lower fidelity and negativity. To examine the effect of the time resolution to the fidelity and negativity, the cross correlation data was deconvoluted.

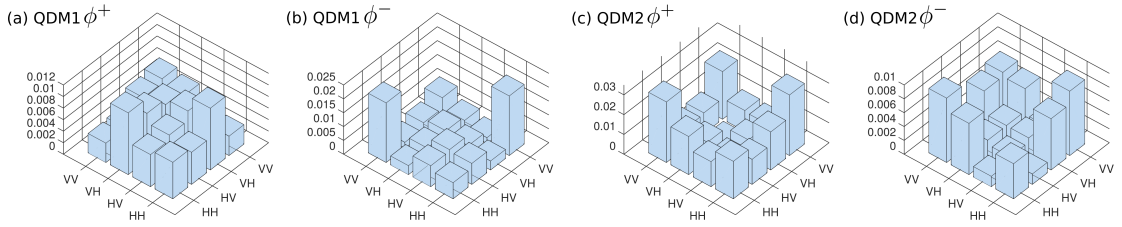

Figure S2: Norms of the errors of the density matrices with a high fidelity to the maximally entangled ϕ^+ and ϕ^- Bell states. (a) and (b) show the values of QDM1 and (c) and (d) of QDM2.

Table S1: Fidelities of the two photon states to the maximally entangled bell states ϕ^+ und ϕ^-. The fidelities of the first three periods of the photon state phase oscillation are listed.

	QP1		QP2	
	fidelity to ϕ^+	fidelity to ϕ^-	fidelity to ϕ^+	fidelity to ϕ^-
first maximum	0.73(03)	0.80(03)	0.69(04)	0.68(04)
second maximum	0.68(07)	0.77(09)	0.52(09)	0.61(08)
third maximum	0.57(07)	0.65(07)	0.45(07)	0.54(09)

The theoretical progress of the cross correlation data is described by [1]

$$p(\tau,\theta_1,\theta_2,\phi_1,\phi_2) = \frac{e^{-\frac{\tau}{\tau_R}}}{2\tau_R}\left|\cos\frac{\theta_1-\theta_2}{2}\cos\left(\frac{\phi_1+\phi_2}{2}+\frac{\pi\tau}{T_P}\right) + i\cos\frac{\theta_1+\theta_2}{2}\sin\left(\frac{\phi_1+\phi_2}{2}+\frac{\pi\tau}{T_P}\right)\right|^2, \tag{S1}$$

where τ_R is the exciton radiative lifetime and T_P is the period of the phase precession of the photonic state. The formula depends on the delay time τ and the two angles θ and ϕ, which describe the polarization of one of the two photons. Equation S1 was convoluted with a normal distribution with a standard deviation of $\sigma = 0.04\,\text{ns}$ to simulate the time resolution of the detectors of 0.1 ns. The obtained function was then fitted to the cross correlation measurements with the fixed parameters σ, T_P, τ_R and fixed angles θ and ϕ, while two values, an offset and the amplitude, were used as fit parameters. For QDM1 the exciton radiative lifetime is $\tau_R = 0.29\,\text{ns}$ with a period of the phase precession of $T_P = 0.25\,\text{ns}$ and for QDM2 $\tau_R = 0.15\,\text{ns}$ with $T_P = 0.15\,\text{ns}$. The new density matrices were then calculated from the received parameters. The density matrices and error matrices are:

- QDM1 at $\tau = 0.020\,\text{ns}$ with fidelity $F_{\text{QDM1},\phi^+} = 0.87$ to ϕ^+:

$$\begin{pmatrix} 0.4715 + 0.0000i & 0.0156 - 0.0163i & -0.0179 + 0.0242i & 0.4103 - 0.0158i \\ 0.0156 + 0.0163i & 0.0258 + 0.0000i & -0.0116 - 0.0025i & -0.0025 - 0.0355i \\ -0.0179 - 0.0242i & -0.0116 + 0.0025i & 0.0081 + 0.0000i & -0.0055 - 0.0088i \\ 0.4103 + 0.0158i & -0.0025 + 0.0355i & -0.0055 + 0.0088i & 0.4945 + 0.0000i \end{pmatrix}$$

The errors of these values are shown in the following matrix and the norms of the errors are displayed in figure S3:

$$\begin{pmatrix} +0.0062 + 0.0000i & +0.0036 + 0.0057i & +0.0005 + 0.0018i & +0.0072 + 0.0036i \\ -0.0007 - 0.0000i & -0.0004 - 0.0010i & -0.0008 - 0.0009i & -0.0038 - 0.0008i \\ +0.0036 + 0.0057i & +0.0078 + 0.0000i & +0.0042 + 0.0010i & +0.0012 + 0.0023i \\ -0.0004 - 0.0010i & -0.0012 - 0.0000i & -0.0008 - 0.0002i & -0.0003 - 0.0011i \\ +0.0005 + 0.0018i & +0.0042 + 0.0010i & +0.0025 + 0.0000i & +0.0020 + 0.0013i \\ -0.0008 - 0.0009i & -0.0008 - 0.0002i & -0.0007 - 0.0000i & -0.0000 - 0.0001i \\ +0.0072 + 0.0036i & +0.0012 + 0.0023i & +0.0020 + 0.0013i & +0.0041 + 0.0000i \\ -0.0038 - 0.0008i & -0.0003 - 0.0011i & -0.0000 - 0.0001i & -0.0013 - 0.0000i \end{pmatrix}$$

- QDM1 at $\tau = 0.136\,\text{ns}$ with fidelity $F_{\text{QDM1},\phi^-} = 0.97$ to ϕ^-:

$$\begin{pmatrix} 0.4880 + 0.0000i & 0.0078 - 0.0060i & -0.0113 + 0.0168i & -0.4760 - 0.0158i \\ 0.0078 + 0.0060i & 0.0023 + 0.0000i & -0.0009 - 0.0002i & -0.0101 - 0.0092i \\ -0.0113 - 0.0168i & -0.0009 + 0.0002i & 0.0017 + 0.0000i & 0.0070 + 0.0164i \\ -0.4760 + 0.0158i & -0.0101 + 0.0092i & 0.0070 - 0.0164i & 0.5080 + 0.0000i \end{pmatrix}$$

The errors of these values are shown in the following matrix and the norms of the errors are displayed in figure S3:

$$\begin{pmatrix} +0.00007 + 0.00000i & +0.00025 + 0.00011i & +0.00011 + 0.00009i & +0.00012 + 0.00012i \\ -0.00008 - 0.00000i & -0.00023 - 0.00010i & -0.00010 - 0.00005i & -0.00008 - 0.00006i \\ +0.00025 + 0.00011i & +0.00015 + 0.00000i & +0.00002 + 0.00000i & +0.00005 + 0.00002i \\ -0.00023 - 0.00010i & -0.00012 - 0.00000i & -0.00002 - 0.00000i & -0.00005 - 0.00000i \\ +0.00011 + 0.00009i & +0.00002 + 0.00000i & +0.00005 + 0.00000i & +0.00001 + 0.00008i \\ -0.00010 - 0.00005i & -0.00002 - 0.00000i & -0.00005 - 0.00000i & -0.00002 - 0.00004i \\ +0.00012 + 0.00012i & +0.00005 + 0.00002i & +0.00001 + 0.00008i & +0.00016 + 0.00000i \\ -0.00008 - 0.00006i & -0.00005 - 0.00000i & -0.00002 - 0.00004i & -0.00016 - 0.00000i \end{pmatrix}$$

- QDM2 bei $\tau = 0.024\,\text{ns}$ with fidelity $F_{\text{QP2},\phi^+} = 0.93$ to ϕ^+:

$$\begin{pmatrix} 0.5141 + 0.0000i & 0.0250 - 0.0004i & 0.0306 - 0.0086i & 0.4657 + 0.1096i \\ 0.0250 + 0.0004i & 0.0165 + 0.0000i & 0.0050 - 0.0004i & 0.0178 - 0.0066i \\ 0.0306 + 0.0086i & 0.0050 + 0.0004i & 0.0028 + 0.0000i & 0.0250 + 0.0120i \\ 0.4657 - 0.1096i & 0.0178 + 0.0066i & 0.0250 - 0.0120i & 0.4666 + 0.0000i \end{pmatrix}$$

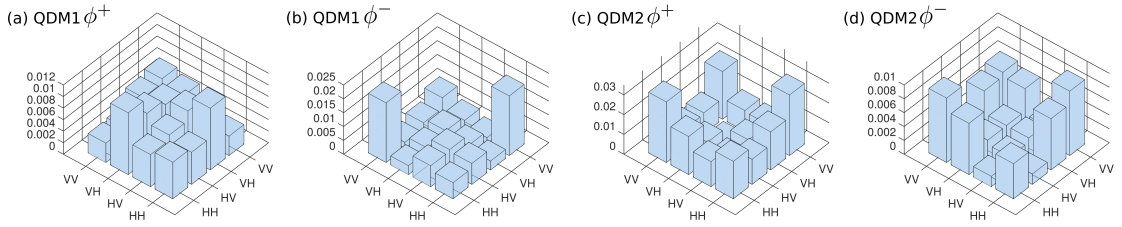

Figure S3: Norms of the errors of the deconvoluted density matrices with a high fidelity to the maximally entangled ϕ^+ and ϕ^- Bell states. (a) and (b) show the values of QDM1 and (c) and (d) of QDM2.

The errors of these values are shown in the following matrix and the norms of the errors are displayed in figure S3:

$$\begin{pmatrix} +0.0051+0.0000i & +0.0195+0.0051i & +0.0008+0.0006i & +0.0046+0.0099i \\ -0.0036-0.0000i & -0.0208-0.0053i & -0.0023-0.0012i & -0.0097-0.0205i \\ +0.0195+0.0051i & +0.0104+0.0000i & +0.0033+0.0014i & +0.0039+0.0022i \\ -0.0208-0.0053i & -0.0052-0.0000i & -0.0042-0.0013i & -0.0003-0.0021i \\ +0.0008+0.0006i & +0.0033+0.0014i & +0.0007+0.0000i & +0.0017+0.0025i \\ -0.0023-0.0012i & -0.0042-0.0013i & -0.0009-0.0000i & -0.0009-0.0015i \\ +0.0046+0.0099i & +0.0039+0.0022i & +0.0017+0.0025i & +0.0046+0.0000i \\ -0.0097-0.0205i & -0.0003-0.0021i & -0.0009-0.0015i & -0.0097-0.0000i \end{pmatrix}$$

- QDM2 at $\tau = 0.124\,\text{ns}$ with fidelity $F_{\text{QP2},\phi^-} = 0.93$ to ϕ^-:

$$\begin{pmatrix} 0.5028+0.0000i & 0.0070-0.0111i & -0.0054+0.0013i & -0.4800+0.0117i \\ 0.0070+0.0111i & 0.0033+0.0000i & -0.0008+0.0000i & -0.0134-0.0039i \\ -0.0054-0.0013i & -0.0008-0.0000i & 0.0002+0.0000i & 0.0066-0.0001i \\ -0.4800-0.0117i & -0.0134+0.0039i & 0.0066+0.0001i & 0.4937+0.0000i \end{pmatrix}$$

The errors of these values are shown in the following matrix and the norms of the errors are displayed in figure S3:

$$\begin{pmatrix} +0.00031+0.00000i & +0.00121+0.00100i & +0.00030+0.00043i & +0.00016+0.00629i \\ -0.00263-0.00000i & -0.00291-0.00063i & -0.00128-0.00022i & -0.00541-0.00695i \\ +0.00121+0.00100i & +0.00007+0.00000i & +0.00015+0.00004i & +0.00003+0.00111i \\ -0.00291-0.00063i & -0.00110-0.00000i & -0.00005-0.00012i & -0.00005-0.00027i \\ +0.00030+0.00043i & +0.00015+0.00004i & +0.00013+0.00000i & +0.00009+0.00009i \\ -0.00128-0.00022i & -0.00005-0.00012i & -0.00000-0.00000i & -0.00111-0.00009i \\ +0.00016+0.00629i & +0.00003+0.00111i & +0.00009+0.00009i & +0.00011+0.00000i \\ -0.00541-0.00695i & -0.00005-0.00027i & -0.00111-0.00009i & -0.00374-0.00000i \end{pmatrix}$$

Figure S4 shows the norms of the density matrices, received from convoluted and deconvoluted cross correlation data.

3 Correction of the polarization parameters

At the bottom of fig. S5 (a) the negativity of QDM2 is shown. It can be seen, that the negativity drops to 0.02 at $t_X - t_{XX} = 0.25\,\text{ns}$, which is an error in the measurement in the R basis, as will be explained in the following.

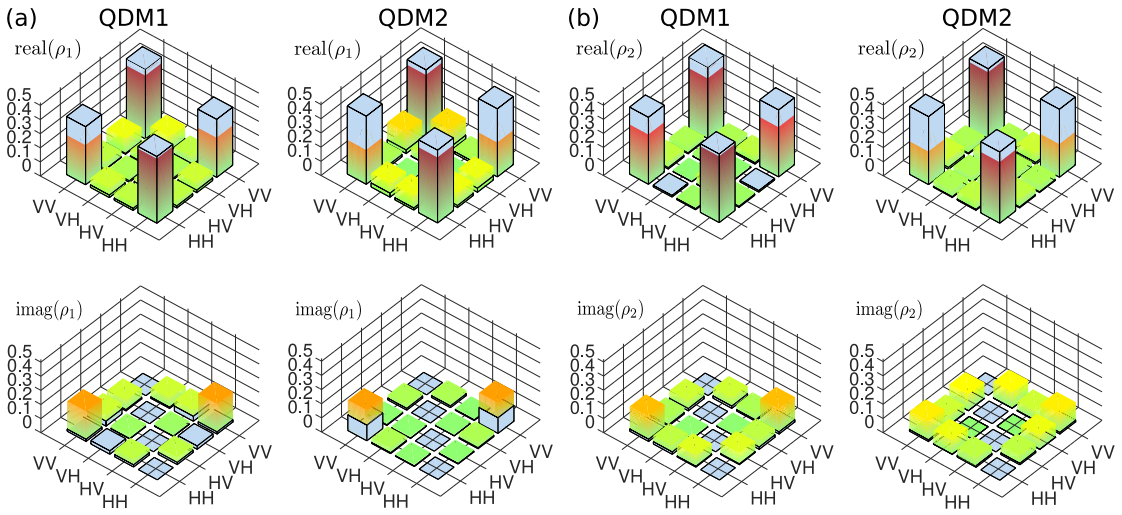

Figure S4: Norms of the density matrices, received from (a) convoluted and (b) deconvoluted cross correlation data.

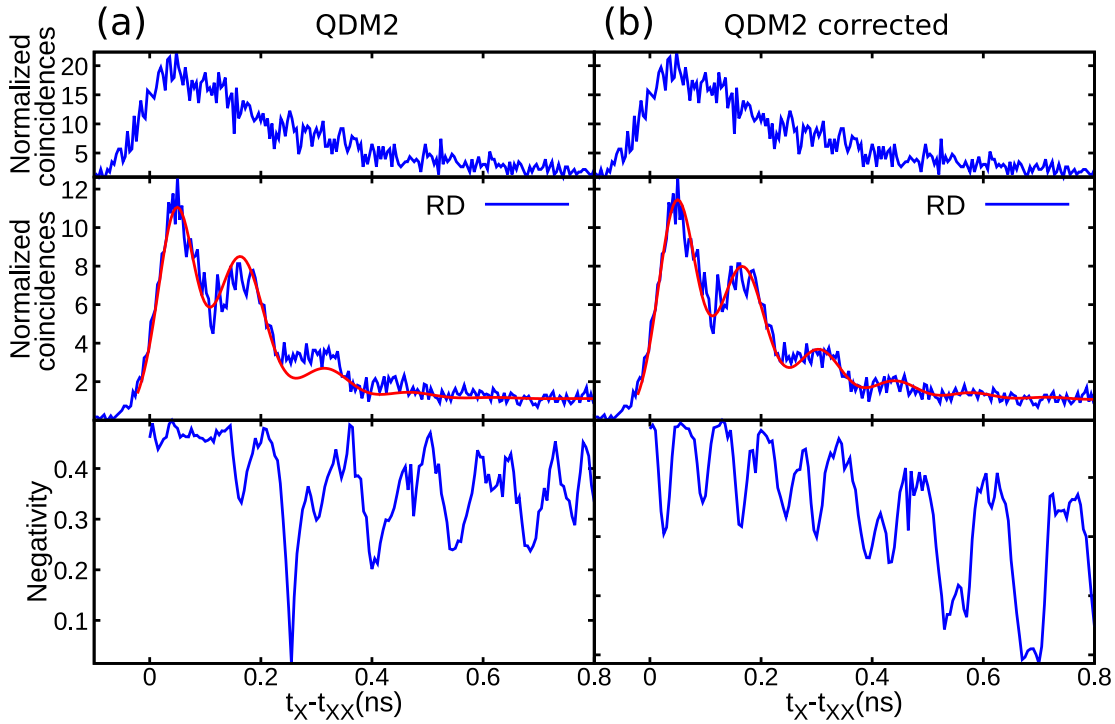

Figure S5: Fit of the cross correlations and negativity of QDM2 (a) without correction and (b) with correction of the R-polarization in the RD basis. Top: cross correlations without projections for comparison. Middle: fit of the cross correlations in RD basis. Bottom: negativity of the quantum state without (a) and with (b) correction.

In the middle of Fig. S5 (a) the cross correlation measurements in the RD basis are shown. Here the measurements were fitted with the same parameters mentioned in section 2. As one can see, the fit does not match well with the measurements, in particular at $t_X - t_{XX} = 0.25\,\text{ns}$, where the drop of the negativity occurs. No initial condition for the fit without changing the angles θ and ϕ in equation S1 was found. Fig. S5 (b) shows the fit to the cross correlation measurement with the angle ϕ_1 for the R polarization as free parameter. This is the only way to match the progress of the data. It is likely, that during the measurements the photon state was not projected correctly in the RD basis. The resulting negativity with corrected R polarization can be seen at the bottom of fig. S5 (b). The RD measurement is the only measurement where the R polarization was corrected in the fit. Every other measurement of QDM2 had a fixed value for ϕ_1.

4 Determination of the extraction efficiency

To determine the extraction efficiency, we excited the QDs with a two-photon resonant pulsed Ti Sapphire laser ($f = 77MHz$) at the power corresponding to the first maximum of the Rabi oscillations. We collected the luminescence using a microscope objective with a NA of 0.4 and spectrally filtered the X emission using the monochromator. Under these conditions we observed a total count-rate of 78($\pm$4) kHz (56($\pm$3) kHz) at the superconduction nanowire single-photon detector (SNSPD) for QD1 (for QD2). For the single-photon flux into the first lens the setup efficiency was derived using a tunable laser which was focused onto a gold mirror mounted in the cryostat and tuned to the wavelength of the X emission line. The laser was attenuated using neutral density filters in front of the monochromator to achieve SNSPD count-rates comparable to those observed for the QD emission. Taking into account the laser power, the reflection of the gold mirror, the transmission of the cryostat window, the attenuation of the density filters, and the maximal count-rates on the SNSPD we determined a setup efficiency of Setup=1.2%. From the detected count-rate, the setup efficiency η and the laser repetition rate f, we finally deduce an extraction efficiency of for the X emission. $\eta_{QD1} = (8.4 \pm 0.1)\%$ ($\eta_{QD2} = (6 \pm 0.1)\%$).

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