Preloading Effects on Dynamic Sand Behavior by Resonant Column Tests

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Vorsitzender:	Prof. DrIng. Y. Petryna
Berichter:	Prof. DrIng. S. Savidis
Berichter:	Prof. DrIng. C. Vrettos

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То

My Wife, Wei Wei for her support and understanding

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ABSTRACT

Dynamic behavior of Berlin sand and the preloading effects on shear modulus and damping properties of sand were investigated by resonant column tests in this study, in addition, a new reliable calibration method for the Stokoe resonant column apparatus is also presented. The influences of confining pressure, void ratio, water content, sampling method, stress history, confinement duration and others on dynamic properties of Berlin sand were examined by resonant column tests. An empirical equation was proposed to predict the small-strain shear modulus, and two empirical models were proposed to simulate the nonlinear modulus and damping properties of Berlin sand, a brief comparison of small-strain shear modulus by resonant column and bender element tests is addressed as well.

In this study, the author initially introduced the *preloading concept* to investigate vibration history effects on dynamic sand behavior, which is quite different from the prestraining concept conventionally employed in previous investigations. For the preloading concept, the previous vibration applied to specimen is employed by the non-resonant vibration mode of stress-controlled shear by resonant column apparatus. With this concept the number of loading cycles can be enlarged to a range from one to any desired number. By contrast the prestraining concept cannot investigate low number of cycles due to the necessary several hundreds of cycles to determine the resonant frequency. In addition, the use of the preloading concept can also ensure constant preloading stress during the previbration is applied to the tested specimen at the same vibration frequency and input drive voltage. The prestraining concept may introduce less precision of the calculation of prestraining amplitude if the set vibration frequency and input drive voltage are not adjusted during previbration. That is due to variation of the resonant vibration frequency and other parameters, if the stiffness of tested specimen varies with number of cycles, and therefore the set vibration frequency is not the resonant frequency of the vibration system any more.

The effects of many factors which may influence the preloading effects on the dynamic behavior of sand were fully explored in this study. One of the most important findings is that the shear modulus or stiffness of sand decreases with number of cycles if it does not exceed a threshold number and increases when the number of cycles exceeds this threshold. A theoretical interpretation of the reduction of stiffness of sand subjected to preloading was proposed herein.

Key Words: Berlin sand, shear modulus, damping ratio, resonant column test, number of cycles, preloading frequency, unloading, reloading, water content

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CHAPTER 1 INTRODUCTION

1.1 Background

In nature, there exist all kinds of vibrations action on the ground, such as earthquake, traffic loads, water wave, storm, vibration machinery, wind power, construction operations, and so forth. In practical geotechnical engineering, many problems are focus on the response of dynamic properties of subsoil subjected to these vibrations. Shear modulus and damping properties are required for analyzing and understanding the response of subsoil subjected to dynamic loads.

The prediction of settlements or transforms of earth constructions under cyclic loading under drained condition for saturated soils as well as unsaturated soils has been attracted by researches and civil engineers in practice in the last decades. Some high speed transport system (e.g. express way, high speed rail way, airstrip) have developed, which transmits dynamic loading to subsoil and whose serviceability is extremely sensitive to the differential settlements. In addition, other examples may be subjected to steady-state vibration are construction and operation of a facility, wind power plant, bulwark, and pile penetration, which may induce magnitude of vibration exceeding elastic range.

The settlements of subsoil in these cases are very much related to the stiffness of soil, there is of importance to well understand the dynamic shear modulus of soil subjected to such a long term dynamic loading for a better knowledge of work capacity of soil. A few investigations had focused on the influence of previbration history on dynamic properties of sand using the previbration shearing strain amplitude and number of cycles as the controlled conditionings. However, there seems no study using the preloading stress amplitude as conditioning to investigate this effect. In addition, the effect of previbration history on dynamic sand properties is far from consistent; especially the interpretation of the increase in shear modulus and variation in damping ratio after samples subjected to a given number of loading cycles. In addition, in previous investigations the number of loading cycles is normally greater than 1,000 cycles due to the prestraining was applied by resonant vibration mode, which resulting a lack of low number of cycles effects.

Berlin has been one of Europe's biggest construction sites since 1990s; reconstruction project of inner city traffic lines (VZB-Project) takes a primary role among these projects. Berlin sand is the dominating soil which extensively distributes in most areas of Berlin; a lot of investigations have been conducted by carrying out conventional tests using triaxial, and cyclic triaxial devices in laboratory; and many field tests have also been made in site, extensive data have been collected. Unfortunately, however, few studies have been carried out to the dynamic properties of Berlin sand by resonant column method. As known, the resonant column apparatus is one of the most reliable

equipment to determine the shear modulus of soil at very small strain level, and as well as at the shearing strain not exceeding the threshold volume strain, normally 0.01% for most soils.

In addition, a few numbers of literatures present the detailed drive system calibration of a Stokoe fixed-free resonant column apparatus, especially tests on higher stiffness specimen like frozen soil. There are efforts (Kumar and Clayton 2007; Clayton, Priest et al. 2009) focused on this aspect, especially Clayton et al. (2009) made a detailed investigation on the factors influence calibration results. They suggested using the equivalent mass polar moment of inertia of drive system according to the determined resonant frequency. Using the equivalent mass polar moment of drive system can derive a relative accurate shear modulus; however, this may result in an inconvenience in calculation of shear wave for a given specimen under different confining pressure.

1.2 Objectives

Based the background and problems mentioned in previous paragraphs, this study aimed at proposing a simple and reliable calibration method for a fixed-free resonant column apparatus, comprehensively investigating the dynamic properties of Berlin sand, and preloading effects on dynamic sand properties, the followings give a brief.

- (1) By preparation of eight aluminum calibration bars with various torsional stiffness to achieve a large range of resonant frequency, the calibration tests were carried out to determine the mass polar moment if inertia of the drive system of resonant column apparatus. A novel simple and reliable method for determination of mass polar moment of inertia of drive system was proposed based on testing results. The influence of torsional stiffness of calibration bar on tested mass polar moment of inertia of drive system will be examined and compared with conclusions drawn in literature. In addition, the calibration factors for torsional shear test mode added by the used resonant column apparatus were determined. One factor was used to calculate the actual torsional stress during test; the other was used to estimate the preloading stress amplitude replacing prestraining to study the preloading effects.
- (2) To examine factors influence small-strain shear modulus, shear modulus reduction curve, and damping curve of Berlin sand, and propose an empirical equation to estimate the small-strain shear modulus for the case of no tested data at hand. Built up two empirical models for predicting shear modulus and damping ratio at a given shearing strain amplitude under various confining pressure for dry Berlin sand. In addition, some bender element tests were performed on the same specimen before the resonant column tests were conducted. The relationship between the values of small-strain shear modulus measured by resonant column and bender element methods were correlated for Berlin sand.
- (3) To investigate the preloading effects on small-strain shear modulus, modulus reduction relationship with shearing as well as the damping increase curve. Besides effect of number of cycles, effects of confining pressure, density of soil, frequency of preloading, preloading ratio, unloading and reloading of confining pressure, water content, sample preparation method, and duration of confinement

on the preloading effects on dynamic properties of sands were detailedly investigated. Established an empirical relationship between void ratio and accumulated axial strain of specimen during testing to consider the densification effect by remeasuring the dimensions of tested specimens after tests. Interpreted the variation of dynamic properties of sands after preloading was applied.

1.3 Organization

This dissertation is organized in seven chapters. Factors influence dynamic properties of sands (shear modulus and damping ratio) are reviewed based on existing literature in Chapter 2. The existing empirical equations for predicting small-strain shear modulus of cohesionless soils are summarized in this chapter.

In Chapter 3, the configurations of the used resonant column apparatus and bender element test, the fundamental of resonant column test, torsional shear test, definitions of shear modulus and damping ratio, calibration of mass polar moment of inertia of drive system and torque factor in torsional shear test mode, are described in detail.

In Chapter 4, the basic properties of the used sands, sample preparation method (raining and tamping methods), dimension measurement method and its precision, deviation of small-strain shear modulus induced by the deviation in dimension measured, apparatus installation, and testing procedures are presented herein.

In Chapter 5, dynamic properties of Berlin sand are presented. Some influence factors, an empirical equation for predicting small-strain shear modulus, two empirical models for evaluating nonlinear dynamic shear modulus and damping ratio, and comparison between resonant column and bender element testing results, are addressed.

In Chapter 6, initially an empirical equation for correlating the accumulated axial strain to void ratio during is developed, then some factors influence small-strain shear modulus are analyzed, in succession, factors affecting nonlinear shear modulus and damping properties are addressed. Theoretical interpretation of preloading effects on dynamic properties of sand are presented in the end of this chapter.

In Chapter 7, some important findings are drawn from this study, and a few worthy suggestions are pointed out for further investigation.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

In engineering practice, soil properties can be divided into static and dynamic soil properties according to the loading type to which soil are subjected. Soil demonstrates two different engineering properties under static load and dynamic load. As well known, static soil properties are rather complicated according to the knowledge of current researches, there is not one perfect model that can exactly characterize these various properties. However, dynamic soil properties seem more complicated due to the sources of loading with more various characteristics, regularity and irregularity.

Hardin and Drnevich (1972; 1972) pointed out that the critical parameters for many dynamic soil properties were shear modulus and damping ratio. Soil dynamics primarily focuses on small-strain shear modulus, G_{max} , reduction of shear modulus with strain amplitude, damping ratio, and variation of damping with strain amplitude. To well understand the nature of dynamic soil properties, the influence factors should be completely investigated. A comprehensive general stress-strain relation for soil was extremely complicated simply because of the large number of parameters that affect the behavior of soils (Hardin and Drnevich 1972). Dynamic soil properties are affected by various factors such as strain amplitude, confining pressure, void ratio, overconsolidation ratio, loading frequency, temperature, anisotropic stress, and so forth. Hardin and Black (1968) proposed a function to describe factors influence shear modulus as follows:

$$G = F(\sigma_0, e, H, S, \tau_0, C, A, f, T, \theta, K)$$

$$(2.1)$$

in which, G is shear modulus, σ_0 is mean principal effective stress (or isotropic confining pressure), e is void ratio, H is ambient stress history, S is degree of saturation, τ_0 is deviatoric component of ambient stress, C is grain characteristics such as grain shape, gradation and mineralogy, A is strain amplitude of vibration or loading, f is frequency of vibration or loading, T is secondary effects that are functions of time and magnitude of load increment, θ is soil structure, and K is temperature.

Equation 2.1 does not necessarily imply independence between factors. For instance, effective confining stress and void ratio are often observed to affect each other. Hardin and Drnevich(1972) classified the importance of these factors on shear modulus and damping ratio into three groups: very important, less important, and relatively unimportant, see Table 2.1.

The shear modulus keeps at a highest value as long as the shearing strain is less than one certain value because of the linearity of the curve in nature. The modulus is well known as the small-strain shear modulus or the maximum shear modulus, whose value is the slope of the linear part of the curve. With an increase in strain amplitude beyond a threshold level, this curve demonstrates an apparent nonlinearity in nature. And the shear modulus related to these strain is known as the secant shear modulus.

As the response of soil deposit under cyclic loading at different strain amplitude ranges varies so significantly, this chapter will review these effects in detail through two aspects, namely small-strain and nonlinear dynamic soil properties, based on the existing literatures. As mentioned above, the dynamic soil properties vary with the strain amplitude. To understand the small-strain and nonlinear dynamic soil properties, the cyclic threshold strains should be exactly defined. On the basis of a synthesis of previous studies on various types of soils, Vucetic (1994) systematically discussed two types of cyclic threshold shearing strains. Figure 2.1 shows the variation of normalized modulus and damping with an increase in shearing strain amplitude as well as the zones of cyclic shearing strain. As shown in Figure 2.1, the cyclic threshold shear stain is classified as the linear threshold cyclic shearing strain, symbolized with γ_{μ} , and the volumetric cyclic shearing strain, symbolized with γ_{μ} .

	Importance				
Daramatar	Modulus		Damping		
r arameter	Clean	Cohesive	Clean	Cohesive	
	sands	soils	sands	soils	
Strain amplitude	V^{a}	V	V	V	
Effective mean principal stress	V	V	V	V	
Void ratio	V	V	V	V	
Number of cycles of loading	R ^b	R	V	V	
Degree of Saturation	R	V	L°	U^d	
Overconsolidation ratio	R	L	R	L	
Frequency of loading (above 0.1	R	R	R	L	
Other time effects (thixotropy)	R	L	R	L	
Grain characteristics, size, shape, gradation, mineralogy	R	R	R	R	
Soil structure	R	R	R	R	
Volume change due to shearing strain (for strain less than 0.5%)	U	R	U	R	

Table 2. 1Parameters affecting shear modulus and damping Ratio (after Hardin and
Drnevich 1972)

^a V means very important, ^b R means relatively unimportant except as it may affect another parameter, ^c L means less important, and ^d U means relative importance is not clearly known at that time.



Figure 2.1 Normalized modulus and damping curves with different zones of cyclic shearing strain amplitude for soil (slightly modified from Vucetic 1994)

Volumetric cyclic threshold shearing strain γ_{ν}

For any soil cyclic shearing strain amplitude below volumetric cyclic threshold shearing strain amplitude, permanent microstructural change of soil essentially does not occur; residual cyclic pore-water pressure essentially does not develop if the soil is fully saturated and cyclically sheared in undrained conditions; and the permanent volume change is negligible if the soil is dry, partially saturated, or fully saturated in drained conditions. If the shearing strain amplitude exceeds this threshold value, the microstructure changes irreversibly; soil stiffness changes permanently; a permanent pore-water pressure builds up in fully saturated cyclically shear loading in drained conditions, and for dry soil, partially saturated soil, or fully saturated soils with drainage allowed, a permanent volume change accumulates. Vucetic (1994) determined the value of γ_{ν} at which the ratio of the modulus to maximum modulus is proximately 0.65 in his study. He also summarized the range of γ_{ν} as shown in Figure 2.2 considering the soil plasticity index varying from 0 to 200 and with various overconsolidation ratios.

Linear cyclic threshold shearing strain γ_{tl}

As shearing strain amplitude below the linear cyclic threshold shearing strain, which less than the volumetric cyclic threshold shearing strain, soil behaves essentially as a perfect linearly elastic material. The shear modulus at strains less than γ_{d} is the initial tangent shear modulus or the maximum shear modulus. The γ_{d} was defined as the strain at which the ratio of the modulus to maximum modulus is 0.99 by Vucetic (1994).

As the shearing strain amplitude within the range of the linear cyclic threshold shearing strain and volumetric cyclic threshold shearing strain ($\gamma_d < \gamma < \gamma_w$), soil behaves slightly elastoplastic material. In this range, the permanent microstructural change can be considered as negligible.

These two threshold shearing strains generally increase as an increase in the plasticity index (PI) of soil due to the increase of the size of soil particles (or the clay mineral composition). The correlation of these two threshold strains with the PI can be described as Figure 2.3. Normally the linear cyclic threshold shearing strain and the volumetric cyclic threshold shearing strain vary from 4×10^{-6} to 4×10^{-5} , and 10^{-4} to 10^{-3} , respectively.



Figure 2.2 Relationship between volumetric cyclic threshold shearing strain and modulus reduction and damping curves (Vucetic and Dobry 1991; Vucetic 1994)



Figure 2.3 Variation of cyclic threshold shearing strain with plasticity index from cyclic triaxial tests (from Vucetic 1994)

2.2 Factors Influence Small-strain Shear Modulus

The importance of small-strain shear modulus for many repeated load or dynamic problem was referred to as the same level as that of shear strength for stability analysis (Hardin and Black 1968). Lefebvre, et al. (1994) pointed out small-strain shear modulus is a basic characterization of soil deformability and plays a key role in dynamic response analyses. G_{max} is widely regarded as one of the most essential parameters for earthquake engineering, traffic engineering, vibration machine foundation, vibration isolation measures, and analysis of dynamic soil structure interactions (Savidis, Vrettos et al. 1993; Kalteziotis, savidis et al. 1994; Park 1998; Savidis and Vrettos 1998; Vrettos and Savidis 1999). Dobry, et al.(1980) employed the stiffness method for predicting the liquefaction potential of saturated sand under the condition of knowing of the shear modulus at small strains for soil layers to determine the threshold ground acceleration. This threshold acceleration is that which is sufficient to initiate the development of excess pore pressure in the layer. Chen and Lee (1994) found that liquefaction resistance increases linearly with for different densities and confining pressures.

2.2.1 Void Ratio

Void ratio is a very important factor influence small-strain shear modulus. Hardin and Richart (1963) evaluated the shear wave velocity of granular soils, and drew the conclusion that shear wave velocity decreased linearly with void ratio (from 0.37 to 1.40), independent of grain size, gradation, and relative density. Richart et al.(1970) compiled data and illustrated the influence of void ratio on the shear wave velocities (V_s) of clean sands with void ratio from 0.37 to 1.26, and reported similar conclusions.

Hardin and his colleagurs (1968; 1969; 1972) pointed out that the shear modulus of soils decreased with void ratio by using resonant column technique and proposed the following Equations to express the effects of void ratio on G_{max} ,

$$F(e) = \frac{(2.17 - e)^2}{(1 + e)}$$
(2.2)

for round-grained sands (e<0.80), and

$$F(e) = \frac{(2.973 - e)^2}{(1 + e)}$$
(2.3)

for angular-grained sands and clayey soil.

Hardin and Drnevich (1968) also limited the validity of the F(e) for soils with void ratio less than 2.0 to account for the effects of void ratio on G_{max} . To consider a further large range of void ratio of clayey soil, Marcuson and Wahls (1972) proposed another equation to replace Equation 2.3 as

$$F(e) = \frac{(4.4 - e)^2}{(1 + e)}$$
(2.4)

Considering the weakness of Equation 2.3 in evaluation clayey soils with void ratio higher than 2.0, Hardin (1978) proposed the following expression to describe the effects of void ratio,

$$F(e) = \frac{1}{(0.3 + 0.7e^2)}$$
(2.5)

While Kokusho et al. (1982) gave a higher influence function of void ratio on the small-strain shear modulus for clayey soils as follows,

$$F(e) = \frac{(7.32 - e)^2}{(1 + e)}$$
(2.6)

Later, Seed et al. (1986) drew a similar conclusion that the shear modulus of well-graded gravels increased with an increase in the relative density by the evaluation of the shear modulus coefficient, K_2 , of a simplified equation earlier proposed to estimate the small-strain shear modulus for sands.

The void ratio effects on small-strain shear modulus and the conclusion that the small-strain shear modulus decreased with an increase in void ratio for soils have been studied and confirmed by several investigators since 1990s (Qian, Gray et al. 1991; Kagawa 1992; Qian, Gray et al. 1993; Guha 1995; Baig, Picornell et al. 1997; Lo Presti, Jamiolkowski et al. 1997; Simonini and Cola 2000; Fam, Cascante et al. 2002; Kallioglou, Tika et al. 2008). Some empirical equations for sand are summarized in Table 2.2.

2.2.2 Confining Pressure

The effects of confining pressure on small-strain shear modulus were studied by extensive investigators in the past few decades. The effects of confining pressure are admittedly assumed as one of the two very important factors (another is void ratio)

which significantly influence the maximum shear modulus of sandy and clayey soils (Hardin and Drnevich 1972). Confining pressure (or mean principal effective stress) together with void ratio are recognized as the very important parameters which influence the small-strain shear modulus of soil by many investigators (Hardin and Drnevich 1972; Marcuson and Wahls 1972; Afifi and Richart 1973; Yu and Richart 1984; Chien and Oh 2002; Kallioglou, Tika et al. 2008; Mayoral, Rorno et al. 2008, and others). From the very beginning, investigators have managed to build up this relation to evaluate the small-strain shear modulus in the case of without knowledge of G_{max} at hand.

As foregoing, a correlation of small-strain shear modulus with void ratio and confining pressure for sandy soils was initially developed by Hardin and Richart (1963) by measured the modulus of round-grained and angular-grained Ottawa sands as with resonant column method at the strain level less than 10⁻⁴. Hardin and Music(1965) confirmed the variation of shear wave velocity with confining pressure by carrying out tests on dry sands with developing a new apparatus at that time. Drnevich et al. (1966) reported that the small-strain shear modulus was a function of void ratio and confining pressure by determining the small-strain shear modulus of the C-190 Ottawa sand with resonant column apparatus by tests on both solid and hollow cylindrical specimens.

The effects of confining pressure on shear modulus are presented in Figure 2.7. Figure 2.7 illustrates the small-strain shear modulus of Ottawa 20-30 sands almost linearly increases with an increase in mean confining pressure.



Figure 2.4 Relation between shear modulus with mean confining pressure (Alarcon-Guzman, Chameau et al. 1989)

Normally, the small-strain shear modulus increase with confining pressure in a power ranges from 0.4 to 0.7, a various equations for predicting the small-strain shear modulus of many particular sands were continually reported in literatures (Seed and Idriss 1970; Hardin and Drnevich 1972; Afifi and Richart 1973; Iwasaki and Tatsuoka 1977; Hardin 1978; Kokusho 1980; Chung, Yokel et al. 1984; Seed, Wong et al. 1986; Saxena and Reddy 1989; Chien and Oh 2002; Sawangsuriya, Bosscher et al. 2006). Although these proposed equations are quite different in some parameters, they could be classified into three general formulas in general as follows,

$$G_{\max} = SF(e)\sigma_0^n \tag{2.7}$$

$$G_{\max} = SF(e)P_a^{(1-n)}\sigma_0^n$$
(2.8)

$$G_{\max} = 1,000(K_2)_{\max}\sigma_0^{\ n}$$
(2.9)

in which, G_{max} is small-strain shear modulus, S is soil stiffness coefficient depends upon soil type, F(e) is void ratio function, Pa is atmospheric pressure (reference pressure), expressed as the same system of units of σ_0 , $\sigma_0 = (\sigma_1 + \sigma_2 + \sigma_3)/3$ is mean principal effective stress or isotropic confining pressure, $(K_2)_{\text{max}}$ is maximum soil modulus coefficient, for sands, $30 < (K_2)_{\text{max}} < 75$, and for gravels, $80 < (K_2)_{\text{max}} < 180$ (Seed, Wong et al. 1986). These empirical equations are summarized in Table 2.2

 Table 2.2
 Reported Relations to Estimate Small-strain shear modulus for Sandy

 Soils Based on Laboratory Tests

	D 4		a . •	Valid Units	
General formula	neral formula Reference Equation		Strain	$\overline{\sigma}_0$	$G_{\rm max}$
$G_{\max} = SF(e)\sigma_0^n$ Hardin Richar	Hardin and Richart (1963) ^a	$G_{\text{max}} = 326 \frac{(2.17 - e)^2}{(1 + e)} \overline{\sigma_0}^{0.5}$ for round-grained sands ($e < 0.80$)	<10-4	kg/cm ²	kg/cm ²
		$G_{\max} = 700 \frac{(2.97 - e)^2}{(1 + e)} \overline{\sigma_0}^{0.5}$			
		for angular-grained sands			
	Afifi and Richart (1973) ^a	$G_{\max} = 2630 \frac{(2.17 - e)^2}{(1 + e)} \overline{\sigma_0}^{0.5},$ for round-grained sands (0.30 < e < 0.80), and $G_{\max} = 1230 \frac{(2.97 - e)^2}{(1 + e)} \overline{\sigma_0}^{0.5},$ for angular-grained sands $(0.60 < e < 1.30)$	<10 ⁻⁵	psi	psi
	Hardin (1965) ^a	$G_{\max} = \frac{(32.17 - 14.8e)^2}{(1 + e)} \overline{\sigma}_0^{0.5},$ ($\overline{\sigma}_0 > 2000 \text{psf}$) $G_{\max} = \frac{(22.52 - 10.6e)^2}{(1 + e)} \overline{\sigma}_0^{0.6},$ ($\overline{\sigma}_0 < 2000 \text{ psf}$)	2.5×10 ⁻⁵	Psf	Psi

				Valid	Units
	Drnevich and Richart (1970) ^a	$G_{\max} = \frac{(32.17 - 14.8e)^2}{(1+e)} \overline{\sigma}_0^{0.5}$	<10-5	psf	psi
	Iwasaki and Tatsuoka (1977) ^b	$G_{\max} = 900 \frac{(2.17 - e)^2}{(1 + e)} \overline{\sigma_0}^{0.4}$	10 ⁻⁶	kg/cm ²	kg/cm ²
		$G_{\max} = 850 \frac{(2.17 - e)^2}{(1 + e)} \overline{\sigma_0}^{0.44}$	10-5		
		$G_{\max} = 700 \frac{(2.17 - e)^2}{(1 + e)} \overline{\sigma}_0^{0.5}$	10-4		
	Hardin and Drnevich (1972) ^b	$G_{\max} = 1230 \frac{(2.97 - e)^2}{(1 + e)} \overline{\sigma}_0^{0.5}$	<2.5×10 ⁻⁵	psi	psi
	Dmevich (1978) ^c	$G_{\max} = 1230 \frac{(2.97 - e)^2}{(1 + e)} \overline{\sigma}_0^{0.5}$	<10-5	psi	psi
$G_{\max} = SF(e) \times P_a^{(1-n)} \sigma_0^n$	Hardin (1978) ^b	$G_{\rm max} = \frac{625}{(0.3 + 0.7e^2)} P_a^{0.5} \overline{\sigma_0}^{0.5}$	10 ⁻⁵	any units	same as $\overline{\sigma}_0$
	Kokusho (1980) ^d	$G_{\max} = 840 \frac{(2.17 - e)^2}{(1 + e)} \overline{\sigma_0}^{0.5}$	10 ⁻⁶	kg/cm ²	kg/cm ²
	Chien and Oh (2002) ^e	$G_{\max} = 87.296 \frac{(2.17 - e)^2}{(1 + e)} \sigma_0^{0.553}$	<10-5	MPa	MPa
	Sawangsuriya et al. (2006) ^a Roundness=0.85	$G_{\max} = 6900 \frac{(2.17 - e)^2}{(1 + e)} \overline{\sigma}_0^{0.5}$	5×10 ⁻⁵	kPa	kPa
	Chung et al. (1984) ^c	$G_{\max} = \frac{523}{(0.3 + 0.7e^2)} P_a^{0.52} \overline{\sigma}_0^{0.48}$	10 ⁻⁵	any units	$same_{as \overline{\sigma}_0}$
	Saxena and Reddy (1989) ^c	$G_{\max} = \frac{428.2}{(0.3 + 0.7e^2)} P_a^{0.426} \overline{\sigma}_0^{0.574}$	<10-5	any units	same $as \overline{\sigma}_0$
$G_{\max} = 1,000 \times (K_2)_{\max} \sigma_0^n$	Seed and Idriss (1970), Seed et al. (1986) ^f	$G_{\rm max} = 1,000(K_2)_{\rm max} \overline{\sigma}_0^{0.5}$	<10-4	psf	psf

^a Ottawa Sand, ^b Clean Sand, ^c Monterey No.0 sand, ^d Toyoura and Gifu Sand, ^e Reclaimed soils (sand) ^f Sand and Gravel;

2.2.3 Stress Ratio

The effects of anisotropic stress condition or initial static state of stress on small-strain shear modulus for soils have been investigated by a few investigators in the past five decades (Hardin and Richart 1963; Lawrence 1965; Hardin and Black 1966; Hardin and Black 1968; Hardin and Drnevich 1972; Schmertmann 1978; Roesler 1979; Tatsuoka, Iwasaki et al. 1979; Knox, Stokoe II et al. 1982; Yu and Richart 1984; Ahlvin 1985; Schmertmann 1985; Tatsuoka 1985; Yu and Richart 1985; Taya, Hatanaka et al. 1999; Chien and Oh 2002; Hoque and Tatsuoka 2004; Yuan, Sun et al. 2005; Wang and Siu 2006; Vassallo, Mancuso et al. 2007). The effects of stress condition (or stress ratio) on small-strain shear modulus are far from the last word. There are some different arguments regarding this aspect from these investigations during the past decades.

Minor Influence

Lawrence (1965) measured the ultrasonic wave velocity pulses in triaxial samples built by three type soils (a coarse, round-grained sand, and two types of saturated clay) subjected to anisotropic stress condition and concluded that the level of anisotropic stress has a minor influence on shear wave velocity. Hardin and Black (1966) used resonant column technique to measured the shear modulus of dry sands subjected to various static states of stress and reported that the shear modulus was independent of the deviatoric component $(\sigma_1 - \sigma_3)$ of the initial static state of stress. This conclusion was found to be suitable also for clays (Hardin and Black 1968; Hardin and Black 1969). Schmertmann (1978) reported that no effects of stress ratio from his tests results were found by measuring the shear wave velocity propagation in a larger chamber dry sand sample 1.2 m in diameter and 1.2 m in height with burying four accelerometers in the sample. The scope of the testing included the relative densities of sand with 30% and 80%, and stress ratio σ_1 / σ_3 as 1 and 3 under the mean principal stress with 34.5 kPa, 69kPa, and 138 kPa; and concluded that the shear wave velocity, and therefore shear modulus did not change significantly with the direction of travel in an anisotropic stress state. Schmertmann and Woods (1980), further argued this findings as discussions of (Roesler 1979) and (Yu and Richart 1984). Schmertmann (1985) further proved their findings by comparison their data in (Schmertmann and Woods 1980) with the prediction value obtained from the two modified equations suggested by Yu and Richart (1984). Sully and Campanella (1995) reported the shear wave velocity ratio of anisotropic stress state to isotropic stress state is relatively independent of stress ratio based on the results obtained from the in situ velocity data measured by crosshole and downhole shear wave velocity technique. A similar relation was also reported by Taya et al. (1999) that the influence of stress ratio on small-strain shear modulus is negligible as the stress ratio (σ_r / σ_a) within 0.5 to 1.5 for sandy or gravel soils.

Recently, based on the measurement of the small-strain Young's moduli of several large prismatic sand specimens in which mounted four pairs lateral and one pair of vertical local displacement transducers, Hoque and Tatsuoka (2004) reported small-strain stiffness of sand under triaxial shearing was independent of stress ratio as the ratio of vertical principal stress to horizontal principal stress was less than 3.0. However, an increase in stress ratio greater than 3.0 would reduce the Young's modulus remarkably due to damage of the initial fabric caused by large increment of plastic straining (Figure 2.13). The similar conclusion was also reported by Kuwano and Jardine (2002) by using three pairs of bender elements installed on the specimens, and by Teachavorasinskun (2006) by tests on clayey soils. They did, however, not conducted these tests under a constant mean principal stress but a variable confining pressure situations, therefore, the effects of stress ratio on the small stiffness of sand only demonstrated by the damage of initial fabric caused by over plastic straining.

Significant Influence

Tatsuoka et al.(1979) reported that under a constant mean principal stress, the effects of stress ratio on shear modulus were more significant in triaxial extension case than in triaxial compression case, namely the shear modulus decreased with an increase in stress ratio both triaxial compression and triaxial extension cases. Additionally, shear

modulus could be considered almost independent of the stress ratio on as less than 4.0 for shear strain variation from 5×10^{-5} to 3×10^{-3} , in triaxial compression cases.

Roseler (1979) pointed out that anisotropic stress state induced the shear modulus to be a vector not a scalar quantity, and then the shear wave velocity depended upon the direction of wave propagation and polarization, was independent of the third direction. He proposed that the shear wave velocity was proportional to the principal stresses as follows:

$$V_s \sim \sigma_a^{0.149} \sigma_p^{0.107} \sigma_s^{0}$$
 (2.10)

and resulting small-strain shear modulus as

$$G_{\max} \sim \sigma_a^{0.298} \sigma_p^{0.214} \sigma_s^0 \tag{2.11}$$

in which, σ_a is the principal stress along the direction of shear wave propagation, σ_p is the principal stress in the direction of the soil grain vibration or the applied dynamic shear stress; and σ_s is the third principal stress.

Knox et al.(1982) reported that the small-strain shear modulus depended equally on the principal stress in the direction of the shear wave propagation and in the direction of particles motion, and was independent of the principal stress in the out-of-plane direction. The relationship between small-strain shear modulus with the three principal stresses was proposed by Knox et al.(1982) as follows:

$$G_{\max,1} = S_1 \sigma_a^{0.18} \sigma_p^{0.18} \sigma_s^{0.12}$$
(2.12)

$$G_{\max,A} = S_2 \sigma_a^{0.22} \sigma_p^{0.22} \sigma_s^{0}$$
(2.13)

in which, $G_{\max,I}$ is the shear modulus in isotropic plane, $G_{\max,A}$ is the shear modulus in anisotropic plane for biaxial loading, and C_1 , C_2 are the simple coefficients.

Yu and Richart (1984) investigated the effects of stress ratio on the shear modulus by resonant column tests on three clean dry sands solid cylindrical samples under biaxial loading conditions with both compression and extension cases. Yu and Richart (1984) concluded that the effects of stress ratio on shear modulus did exist, increasing the stress ratio decreased the shear modulus, up to a maximum of 20-30%. They suggested two modified empirical equations for Hardin's and Roseler's equations considering the effects of stress ratio as follows, respectively:

$$G_{\max} = SF(e)P_a^{0.5} \left(\frac{\sigma_a + \sigma_p}{2}\right)^{0.5} (1 - 0.3K_n^{1.5})$$
(2.14)

$$G_{\max} = SF(e)P_a^{0.49}\sigma_a^{0.26}\sigma_p^{0.25}(1-0.18K_n^2)$$
(2.15)

$$K_{n} = \frac{K_{13} - 1}{\left(K_{13}\right)_{\max} - 1} = \frac{\sigma_{1} / \sigma_{3} - 1}{\left(\sigma_{1} / \sigma_{3}\right)_{\max} - 1}$$
(2.16)

in which, K_{13} is stress ratio, expressed as σ_1 / σ_3 in compression case, and σ_3 / σ_1 in extension case, K_n is normalized stress ratio; $(K_{13})_{max}$ is the maximum stress ratio possible, or the failure criterion of sands, and P_a is atmospheric pressure in the same unit system as principal stress, other notations are the same as previous.

In contrast, some recent investigations indicate that the shear modulus of soils increase with an increase in stress ratio (Chien and Oh 2002; Yuan, Sun et al. 2005). Chien and Oh(2002) performed resonant column tests on hydraulic claimed soil (sand fill) obtained from the offshore area at Yunlin on the west coast of Taiwan. They compared the tested small-strain shear modulus with the predicted modulus obtained from Hardin equation built in their paper, and presented that the small-strain shear modulus remarkably increased as the increase of stress ratio from 1.0 to 3.0, and the increment may be up to 32% at the stress ratio of 3.0 under the confining pressure of 196 kPa, see Figure 2.5.

Yuan et al. (2005) reported that the practical influence of stress ratio on the small-strain shear modulus may much higher than that predicted from Hardin equation (Hardin and Richart 1963) by conducted some resonant column tests on Harbin sand and Fujian sand in China, and concluded that the increase of shear modulus caused by the increase of stress ratio may increase up to 60% when the stress ratio went up to 3.0. Unfortunately, however, they did not compare the tested results under the condition that kept the mean principal stress level constant, but those values calculated from Hardin equation based on using the confining pressure as the confining pressure (σ_3 or σ_r), which will magnify the effects of the stress ratio on shear modulus because the increase of stress ratio will result in an increase of mean principal stress.



Figure 2.5 Increment of small-strain shear modulus with Stress Ratio (after Chien and Oh 2002)
2.2.4 Grain Characteristics

Hardin and Richart (1963) determined the compression and shear wave velocities in specimens of Ottawa sand, crushed quartz sand, and crushed quartz silt by using resonant column method in laboratory, and indicated that the grain size, grain shape, and grading of sands retained on the No. 120 sieve did not affected shear modulus, and interpreted that grain size affects the shear wave velocity in sand only entered their influence on void ratio. Generally, the smaller grain sizes the larger void ratios; thus, small-grained materials have a lower shear wave velocity, resulting in lower shear modulus.

Iwasaki and Tatsuoka (1977) proposed a modified empirical expression to describe the shear modulus of various clean sands as follows:

$$G = A(\gamma)B\frac{(2.17 - e)^2}{1 + e}\sigma_0^{m(\gamma)}$$
(2.17)

in which, $A(\gamma)$ and $m(\gamma)$ are functions of low shear strain amplitude, $A(\gamma)=900$ and $m(\gamma)=0.4$ for $\gamma=10^{-6}$, $A(\gamma)=850$ and $m(\gamma)=0.44$ for $\gamma=10^{-5}$, $A(\gamma)=700$ and $m(\gamma)=0.5$ for $\gamma=10^{-4}$, respectively; and *B* is a parameter which is dependent on the variety of normally consolidated clean sands, and independent of γ , *e*, and σ_0 .

Iwasaki and Tatsuoka (1977) presented the relationship between the value of *B* and mean grain size D_{50} in Figure 2.6, and reported that the grain sizes had no effect on shear modulus for clean sands with uniform coefficient less than 1.8 and the D_{50} within the range of 0.16 mm to 3.2 mm.



Figure 2.6 Relation between parameter B and mean grain size D₅₀ for clean sands (after Iwasaki and Tatsuoka 1977)

Similarly, Iwasaki and Tatsuoka (1977) used uniform coefficient (C_u) and the content of fine particles less than 0.074 mm in diameter as the indices of grain size characteristics to develop the relationship between B-value and uniform coefficient (C_u) and fine particle content for various sands, and concluded that shear modulus of normally consolidated sands were significantly influenced by grain size distribution characteristics; in detail, various sands with well-graded (higher C_u) and having higher fine particle content had smaller shear modulus than clean sands under the condition of the same mean confining pressure, void ratio, and shear strain amplitude.

2.2.5 Degree of Saturation

Hardin and Richart (1963), and Lawrence (1965) reported that the degree of saturation had only small effect on small-strain shear modulus for sand at low pressures. In a very excellent paper, Hardin and Drnevich (1972) classified degree of saturation as the very important parameter for cohesive soils but unimportant parameter for cohesionless soils. Pore pressures may build up in saturated cohesionless soils but are accounted for by applying effective stress theory. For the difficulty in determining the effective stress in partially saturated cohesive soils, degree of saturation has been used as a parameter for such soils. Hardin and Drnevich (1972) reported that the small-strain shear modulus of a silty clay with the liquid limit of 48% and plasticity limit of 28% decreased to a half with an increase in degree of saturation from 70% to 100% under mean principal stress of about 390 kPa.

Wu (1983) and Wu et al. (1984) reported the small-strain shear modulus rapidly increased from the value at dry condition to a peak value corresponding to an optimum degree of saturation, thereafter slowly decreased as the increase of degree of saturation as shown in Figure 2.7.



Figure 2.7 Variation of small-strain shear modulus with degree of saturation for Glacier Way silt (after Wu, Gray et al. 1984)

The normalized small-strain shear modulus, $G_{\max(wet)} / G_{\max(dry)}$, based on data in Figure 2.7 varies with degree of saturation is illustrated in Figure 2.8. The value of $G_{\max(wet)} / G_{\max(dry)}$ increased up to 2.0 at the lowest confining pressure of 3.6 psi, this value decreased with an increase in confining pressure. In addition, the $G_{\max(wet)} / G_{\max(dry)}$ did not monotonically decrease from the peak value to 1, but continuously decreased to some values which were less than 1.0 as the degree of saturation is within the range of about 70% to 100% as shown in Figure 2.8. They accounted for this phenomenon by the method of sample constructing on the pedestal of resonant column apparatus at pre-specified moisture contents. At high degree of saturation, some air bubbles were closed in the soil, as high confining pressure applied, as the volume decreases, at the same time the air bubbles are compressed resulting in an increase in air pressure, and caused resultant effective pressure less than external applied confining pressure.

Qian et al. (1993) further investigated the degree of saturation on small-strain shear modulus by using resonant column method, and reported that grain shape affected both small-strain shear modulus ratio and optimum degree of saturation, these two parameters for angular-grained cohesionless soils were higher than for subround-grained cohesionless soils. The small-strain shear modulus ratio linearly decreased with void ratio for both angular-grained sand and subround-grained sands, and the decreasing slope was not affected by the confining pressure and grain size distribution, and only depended on the grain shape.



Figure 2.8 Variation of Normalized small-strain shear modulus with degree of saturation for Glacier Way silt (after Wu, Gray et al. 1984)

2.2.6 Frequency of Loading

It is very important to assess the shear modulus and hysteretic damping of soils when predicting or back analyzing the response of ground or soil structures subjected to various types of cyclic loading. The frequency of transient loadings from wave, seismic, traffic, and machine loadings may range from 0.01 to 100 Hz (Shibuya, Mitachi et al. 1995).

Several investigations had revealed that loading frequency or strain rate had only small or no influence on small-strain shear modulus for cohesionless soils. Hardin and Richart (1963) found that frequency of loading had no obviously effect on shear wave velocity propagation in granular soils for the frequencies in the range from 200 to 2500 Hz. Hardin (1965) measured the static (frequency of loading lower than 0.1 Hz) small-strain shear modulus of solid specimens of dry sand, by comparing with dynamic shear modulus previously published, concluded that rate of loading had no effect on small-strain shear modulus for sands. He addressed that the difference between the static and dynamic values were caused by the failure of the devices used for static test for measuring the accurate small deformation for conventional dynamic vibration tests.

Hardin and Black (1966) founded small-strain stiffness for sands was independent of rate of loading. Iwasaki et al. (1978) compared the values of shear modulus obtained from torsional shear and resonant column tests at the shear strain amplitudes were 10^{-4} or less for dry and saturated Toyoura sands, and reported that frequency of loading had little effect on shear modulus even the frequencies of loading of resonant column tests were about 500 to 1000 times higher than those of torsional shear tests. Bolton and Wilson (1989) investigated the soil properties of a dry sand at medium to large strain amplitude with a torsional shear device at the frequency of 0.001 Hz and with resonant column apparatus at 45 to 95 Hz, and test results showed no appreciable difference between the values for torsional shear tests and for resonant column tests.

Thus, It could be said that the frequency of loading or strain rate has little effect on the shear modulus at small shear amplitude for sands (Lo Presti, Jamiolkowski et al. 1997; Tatsuoka, de Magistris et al. 1998; Tatsuoka, Modoni et al. 1999; Matesic and Vucetic 2003).

2.2.7 Duration of Confinement

The dynamic properties of soil may also vary with time elapse under certain steady confining pressure; this time-dependency has attracted some investigators (Hardin and Black 1968; Afifi and Woods 1971; Hardin and Drnevich 1972; Marcuson and Wahls 1972; Afifi and Richart 1973; Anderson and Stokoe 1978; Marcuson and Wahls 1978; Amini 1995; Tatsuoka, de Magistris et al. 1998; Amini 1999 and others) since 1960s.

Hardin and Black (1968) reported that a secondary increase of vibration shear modulus with time did exist at constant confining pressure, not accounted for the change of void ratio. They presented that the stiffness built-up was sensitive to particle disturbance and can be partially or totally destroyed by the change of effective confining stress, and addressed that this time-dependency may be quite important for soil in situ, this conclusion was further confirmed in Hardin and Drnevich (1972).

Normally in the laboratory, the time-dependent behavior for shear modulus could be separated into primary and second behaviors. The secondary period is characterized by shear modulus increases with time following the primary period. In general, it has been shown for cohesive soils that the shear wave velocities determined in laboratory after one day of sample consolidation are much lower than those obtain from in situ. However, the difference values decrease as the increase in the duration of sample consolidation at a certain confining pressure. Therefore, it is essential to consider an appropriate increase in velocity caused by the secondary time effects when the in situ shear wave velocities are predicted on the basis of laboratory tests.

Two significant features of the secondary property of the shear modulus for soils have been identified in literature (Marcuson and Wahls 1972; Anderson and Stokoe 1978; Athanasopoulos 1993; Amini 1995; Tatsuoka, de Magistris et al. 1998; Amini 1999), involving the rate and magnitude of increase. The rate of secondary increase in shear modulus or shear wave velocity is a function of the logarithm of time. The magnitude of secondary increase, as defined by change in modulus per logarithmic cycle of time, was found to vary with soil type.

Some investigations (Hardin and Scott 1966; Hardin and Black 1968; Humphries and Wahls 1968), Afifi and Woods (1971) initiated the quantitative study of the rate, extent or characteristics of the stiffness build-up with time which lasted up to 803 days for dry soils. Three types of soils including air-dry sand, air-dry silt, and air-dry kaolinite clay, were employed to study using resonant column device. On the basis of tested data, they concluded that:

- (1) The shear modulus of air-dry sands and silts increased with time up to 430 days, the shear modulus of air-dry kaolinite clays might reach a peak or level off near a time of about 200 days, and the percentage increase in shear modulus increase with decreasing particle size (2%-5% for sands, 5%-12% for air-dry silt and kaolinite clays).
- (2) The shear modulus of an air-dry soil under a constant confining pressure for a period of 2 years could be predicted with 10%-15% from a 48-hr test (3,000 min).
- (3) The increases of shear modulus was accounted for the decrease of void ratio with time were 0%, 0%-5%, and 10%-15% for air-dry sand, silt, and kaolinite clay, respectively.
- (4) A step increase of confining pressure of 10 psi (68.95 kPa) did not significantly affect the shear modulus accumulated with time in air-dry sands and silts, such increase, however, might be destroyed up to 40% of shear modulus accumulated with time in air-dry kaolinite clays, and this loss stiffness might accumulate with a period of time. Figure 2.9 and 2.10 are the typical variation of shear modulus with time from Afifi and Woods (1971) for dry sand and kaolinite clay, respectively.

It is interesting to mention that the sudden drop of low-amplitude shear modulus after the increment of confining pressure, this decrease is temporary and the modulus regained its initial value after a certain time, which was also reported by Hardin and Black (1968). This phenomenon is quite different from the drop when a temporary release of confinement. Hardin and Black (1968) interpreted that this decrease of modulus was due to the destruction of the stiffness built-up during secondary compression under the previous pressure increment because this stiffness build-up was sensitive to particle disturbance and can be partially or totally destroyed by changes in effective stress. It was also found that the drop occurred even when only primary consolidation was allowed under the previous pressure increment (Afifi and Woods 1971; Anderson and Woods 1976). Athanasopoulos and Richart (1983) assumed that the fast rate deformation developing immediately after the application of a confining pressure increment are responsible for the immediate decrease of the modulus for cohesive soils.



Figure 2.9 Variation of shear modulus and vertical strain with time for dry sand at constant confining pressure (after Afifi and Woods 1971)



Figure 2.10 Variation in Shear Modulus and Vertical Strain with Time for Dry Sand at Constant Confining Pressure (after Afifi and Woods 1971)

2.2.8 Prestraining

The effects of number of loading cycles have been extensively investigated in the past decades. Drnevich et al. (1966; 1967) investigated the large amplitude vibration effect

on shear modulus of dry C-190 Ottawa sand by measurement of shear modulus of hollow and solid specimens using resonant column apparatus. The variation of small-strain shear modulus with number of large amplitude was reported as shown in Figure 2.11. The small-strain shear modulus was hardly influenced by the number of loading cycles for the case that the prestraining is 1.6×10^{-4} ; and significantly increased with number of cycles for the case that the prestraining is 6×10^{-4} ; this increment may be up to 100% after 1,000,000 cycles. Drnevich and Richart (1970) presented that the shear modulus of both dense and loose dry sands increased after application a large number of cycles of loading with shear strain amplitudes larger between 10^{-5} and 10^{-4} . The small-strain shear modulus increased up to 300% after the application of 22 millions cycles of vibrations at strain amplitude of 6×10^{-4} . They addressed that this stiffness could not be accounted for the densification of samples, related to the large number of the high frequency vibrations.



Figure 2.11 Variation of small-strain shear modulus with number of cycles at various cyclic shear strain amplitude for hollow dry sand specimen (Drnevich, Hall et al. 1967)

Anderson and Richart (1976) measured the shear wave velocity in hollow saturated clay specimen after some desired number of cycle loading (500-100,000) applied by resonant column apparatus. They stated that the low amplitude of shear modulus was significantly affected by number of loading cycles at strain amplitudes larger than 10⁻⁴, and not effect was detected when the strain amplitude less than this threshold value. They normalized the shear modulus after large number of loading cycles at various prestraining amplitudes to the shear modus before cyclic loading to evaluate the number of cycle loading on low amplitude shear moduli; and reported that the small-strain shear modulus decreased as an increase in number of loading cycles at

various prestaining amplitudes exceeding the threshold strain 0.01%. Based on tests on dry sand by torsional shear technique, Sherif and Ishibashi (1976) reported that the shear modulus increased up to 28% as number of loading cycles equal to 25 and remained nearly constant thereafter, for sands.

Shen et al.(1985) measured the small-strain shear modulus of both dense sands and loose sands by resonant column technique after a given number of cyclic loading cycles applied by free vibration generated by the resonant column apparatus at the shear strain amplitude at about 1.4×10^{-4} with the frequency of 30 Hz, and stated that after application of the first 500 cycles of vibration, the small-strain shear modulus increased by more than 25% for loose sand and 20% for dense sand, respectively, and the shear modulus measured at the end of 20,000 cycles, could increase by up to 35% for dense sand and 80% for loose sand, respectively. Lo Presti et al. (1997) reported that the shear modulus of sands was not influenced by the number of cycles of loading at small shear strain amplitude.



Figure 2.12 Development of small-strain shear modulus with number of cycles at various prestraining amplitude for fine sand (after Wichtmann and Triantafyllidis 2004)

Alarcon-Guzman et al.(1989) investigated the prestraining effect on small-strain shear modulus for dry coarse sand by application of a large number of cycles with the amplitude of 1.3×10^{-4} in the resonant column mode of the apparatus. The increase of small-strain shear modulus after the prestraining was observed only 5%. They stated that the increase was caused by the reduction of void ratio due to prestraining, thus small-strain shear modulus was insensitive to strain history. Recently, Wichtmann and Triantafyllidis (2004) reported that the small-strain shear modulus for hollow fine sand specimens was affected by the number of loading cycles from 10^4 to 5×10^6 at the cyclic strain amplitudes of 0.5×10^{-4} , 1×10^{-4} , and 2×10^{-4} ; by contraries, the small-strain shear modulus was not influenced by the number of cycles as the prestraining less than 0.5×10^{-4} , as the prestraining amplitude exceeded 0.5×10^{-4} the small-strain shear modulus slightly decreased with number of cycles at these cyclic

strain amplitudes for hollow medium sand specimens as shown in Figures 2.13.



Figure 2.13 Development of small-strain shear modulus with number of cycles at various prestraining amplitude for medium sand (after Wichtmann and Triantafyllidis 2004)

2.3 Factors Influence Nonlinear Dynamic Soil Properties

As the same importance as small-strain dynamic properties of soil, the nonlinear dynamic soil properties of soil play an essential role in analyzing the dynamic behavior of ground motion during strong earthquakes, such as Sichuan Earthquake (Richter magnitude scale, M = 8.0), which recently took place at 14:28 on May 12, 2008 in Sichuan province, China, killed more than 70,000, injured around 300,000 people, and caused millions of residents homeless. The shear strains induced in surface deposits during such earthquakes motions may be estimated around 10^{-2} % to 1% (Iwasaki and Tatsuoka 1977). It is very necessary to investigate the strain-dependent dynamic characteristics of soils at shear strain within the range from 10^{-4} to 10^{-2} in laboratory.

As known, the shear modulus decreases and damping ratio increases as shearing strain amplitude increases exceeding the threshold level defined in Section 2.2, these properties is regarded as the nonlinear dynamic properties. These properties are usually described by the modulus reduction and damping curves, which are of importance for the response analysis of several dynamic problems especially for high strain cases such as strong ground motion caused by horizontal force due to strong earthquake.

2.3.1 Confining Pressure

Confining pressure is recognized as one of the most important factors influence

small-strain shear modulus, its influence on nonlinear shear modulus properties of soil. Figure 2.14 illustrates the influence of confining pressure on shear modulus of Berlin sand with the relative densities around 63%. It is seen that the shear modulus versus shearing strain amplitude reduction curve plots higher with increasing confining pressure from 50 kPa to 400 kPa. Usually the shear modulus is normalized by the small-strain shear modulus to analyze the nonlinear shear modulus properties versus shearing strain amplitude. Figure 2.15 is the typical normalized shear modulus versus shearing strain amplitude obtained from sand. The normalized shear modulus reduction curve is shifted to right higher position as increasing confining pressure.



Figure 2.14 Variation of shear modulus with shearing strain amplitude for Berlin sand under various confining pressures

Several investigators have synthesized this work and proposed nonlinear generic curves for use in earthquake analyses (Seed, Wong et al. 1986; Vucetic and Dobry 1991; Darendeli 2001; Zhang, Andrus et al. 2005). The threshold shear strain, at which G/G_{max} starts to decrease, is greater at high effective confining pressures than at low effective confining pressures(Vucetic 1994). Modulus reduction behavior is more influenced by effective confining pressure, particularly for soils of low plasticity (Iwasaki, Tatsuoka et al. 1978; Kokusho 1980; Ishibashi and Zhang 1993; Zhang, Andrus et al. 2005). Hardin and Drnevich (1972) classified the effect of confining pressure as an very important parameter on the damping properties for both sands and cohesive soils, the damping ratios for both sands and cohesive soils obviously decreased with the square root of confining pressure at a given strain amplitude. Figure 2.16 illustrates the variation of damping ratio with shear strain amplitude under various confining pressure for sands. This figure shows that the damping ratio decreases with confining pressure when the shear strain amplitudes are greater than a given value, say 0.01%; in other words, the damping curve shifts to the right lower location as the confining pressure increases.



Figure 2.15 Effect of confining pressure on normalized shear modulus reduction curve for sand (after Ishibashi 1992)



Figure 2.16 Effect of confining pressure on damping curve for silty sand (after Darendeli 2001)

2.3.2 Frequency of Loading

Hardin and Drnevich (1972; 1972) reported the shear moduli of sands were not sensitive to loading frequency. Iwasaki et al. (1978) presented that the values of shear modulus obtained from torsional shear and resonant column tests at the shear strain amplitudes of 10^{-4} for dry and saturated Toyoura sands in some figures, from those

figures, it can be seen that frequency of loading has little effect on shear modulus even the frequencies of loading of resonant column tests were about 500 to 1000 times higher than of torsional shear tests. Bolton and Wilson (1989) investigated the soil properties of a dry sand at medium to large strain amplitude with a torsional shear device at the frequency of 0.001 Hz and with resonant column apparatus at 45 to 95 Hz, and test results showed no appreciable difference between the values for torsional shear tests and for resonant column tests. Alarcon-Guzman et al.(1989) also reported that well agreement of the shear modulus of Ottawa sand were obtained from resonant column and torsional shear methods at the shear strain amplitude of 0.01% despite of the great disparity in the loading frequencies.

Shibuya et al. (1995) used torsional shear method to investigate strain rate effect on shear modulus and damping of normally consolidated clay with relatively low loading frequencies ranging from 0.005 to 0.1 Hz. Test data indicated that the shear modulus was essentially not affected by the loading frequency within the range between 0.005 and 0.1 Hz with the shear strain amplitudes between 1.5×10^{-5} to 2×10^{-4} . The damping ratio, however, significantly influenced by the loading frequency with the identical frequency and shear strain ranges; i.e. the damping ratio decreased with increasing loading frequencies from 0.005 to 0.1 Hz, especially in the range of 0.01 to 0.1 Hz.

Lo Presti et al. (1997) compared the data obtained from torsional shear and resonant coloumn tests, and reported that loading frequency had no influence on shear modulus and the normalized shear modulus reduction relationship of sands, however, the damping ratios obtained from resonant column tests were greater than from torsional shear tests at small shear strain amplitude, and at higher strains, the opposite was true.

2.3.3 Stress Ratio

Kuribayashi et al. (1975) pointed out that stress ratio had no influence on shear modulus reduction curves was observed because the same decrease rate of shear modulus at shear strain amplitude of 0.005% and 0.01% was demonstrated, but the damping ratio increased with stress ratio. Similar results were presented by Tatsuoka et al.(1979) for shear modulus, i.e., the shear modulus ratio versus shear strain amplitude curves was not affected by the stress ratios ranging from 1.5 to 5.0 for Toyoura sand. Additionally, they pointed out that shear modulus could be considered almost independent of the stress ratio on less than 4.0 for shear strain amplitude ranging from 0.005% to 0.3%, in triaxial compression cases. There was no obviously dependent relationship between damping ratio and stress ratio due to the scatter of the data, however, a trend of illustrating that damping ratio increased slightly with increasing stress ratio in both compression and extension cases for shear strain amplitude less than 0.05% was observed.

Hoque and Tatsuoka (2004) reported small-strain stiffness of sand under triaxial shearing was independent of stress ratio as the ratio of vertical principal stress to horizontal principal stress was less than 3.0. However, an increase in stress ratio greater than 3.0 would reduce the Young's modulus remarkably due to damage of the initial fabric caused by large increment of plastic straining.

2.3.4 Number of loading cycles

Silver and Seed (1971) measured the shear modulus and damping ratio of dry sands at the 1st, 10th, and 300th cycles of loading, and indicated that, in general, the shear modulus slightly increased and damping ratio significantly decreased with increasing number of loading cycles, the primary increase in shear modulus and decrease in damping ratio occurred within the first 10 cycles of loading.

Hardin and Drnevich (1972) presented the variation of the shear modulus and damping ratio for clean dry sands versus shear strain amplitude varying from around 10^{-6} to more than 10^{-3} as shown in Figures 2.17 and 2.18. As shown in these figures, the shear modulus reduction curves trend to slightly locate higher as if the number of cycles increases, which is similar to the conclusion drown by Silver and Seed (1971) however, this effect on damping ratio is quite clear, the damping ratio clearly decreases as an increase in the number of loading cycles, especially when the shear strain amplitudes vary from 0.02% to 0.06%.

As for the effect of the number of loading cycles on shear modulus for dry sands, Sherif and Ishibashi (1976) reported that shear modulus could increase up to 28% at N=25 and remained essentially constant thereafter at the shear strain amplitude of 0.03%. Ray and Woods (1988) also reported similar results that the shear modulus could increase up to 20% at N=200 for a given strain level for sands, and deceased for silt; and the damping ratio decreased for both sands and silts, especially for silty soils might decrease to 50% of the initial values at N=200.



Figure 2.17 Effects of number of cycles on the location of shear modulus curves for clean dry sand (after Hardin and Drnevich 1972)



Figure 2.18 Effects of number of cycles on the location of damping curves for clean dry sand (after Hardin and Drnevich 1972)

Lo Presti et al. (1997) showed that the shear modulus increased and damping ratio decreased with increasing number of cycles for both virgin and stepwise specimens; and the increase in shear modulus for virgin specimens was larger than for multistage specimens, which well agrees with the findings of Silver and Park (1975), and Silver and Seed (1971), it reached a stable value as the number larger than 5 or 10 cycles. Unlike shear modulus, the damping ratio was strongly influenced by the number of cycles in both tests, which might continuously decease even after hundreds of cycles. Li and Cai (1999) reported that the shear modulus moderately increased the damping ratio might strongly decrease as the increase of number of cycles at the shear strain amplitude of 0.025% as shown in Figure 2.19.



Figure 2.19 Effects of number of cycles on shear modulus and damping ratio for dry sand (Li and Cai 1999)

2.3.5 Prestraining

Prestraining means that the tested specimen is initially subjected to a given number of loading cycles of at a given strain amplitude before its dynamic parameters are tested. Drnevich and Richart (1970) conducted resonant column tests on hollow, cylindrical Ottawa sand specimens, test results indicated that prestraining at shear strain amplitudes of 0.06% can double the dynamic shear modulus and damping as shown in Figure 2.20 for shear modulus and Figure 2.21 for damping ratio. Density changes can not account for this no changes occur for prestraining at shearing strains less than 0.001%. The effects were dependent on confining pressure, initial void ratio, prestraining amplitude, and number of loading cycles. Millions of cycles may be necessary to produce these changes. They reported that the variation of density or void ratio could not account for the total change in these parameters imparted to dry sands by vibratory loading. This is important in determining the dynamic response of foundations and the compaction improvement of sand soils. As shown in Figure 2.20, the normalized shear modulus reduction curves plot for specimens subjected to prestraining decreased faster compared with the shear modulus reduction curve for the virgin specimen. Figure 2.21 shows that the damping curves for specimens subjected to prestraining are always higher than for virgin specimen at all confining pressures, with the confining pressure increases the rate of increase in damping ratio decreases. They explained that, the wearing process at particle contact may be responsible for the prestraining effects on the increase in shear modulus increased and damping ratio. Prestraining generated abrasive action and caused the original minute asperities to wear, flattening these asperities, increasing contact areas, and forming additional contacts. These changes can occur without considerably changing the porosity.



Figure 2.20 Effect of number of cycles on shear modulus reduction curves (after Drnevich and Richart 1970)

Tatsuoka et al. (1979) measured the shear modulus and damping ratio of hollow cylindrical specimen of saturated Toyoura sand, and presented that the shear stress history have only slight influence on the location of the shear modulus ratio versus logarithmic shearing strain amplitude. Unlike the findings of Drnevich and Richart (1970), the shear strain history slightly decreased damping ratio of sand. And they concluded that shear strain history at large prestraining amplitude had less influence on both shear modulus reduction and damping curves.



Figure 2.21 Effect of prestraining on damping ratio of Ottawa sand at various confining pressures (data from Drnevich and Richart 1970))

Li and Yang (1998) used a multilayer multipath closed-loop control scheme called the energy-injecting virtual-mass (EIVM) resonant column system, whose idea was originally proposed by Li in 1982 and extend and proposed by Li et al. (1998), to study the influence of vibration history on dynamic properties of dry sand. Specimens with four combinations of relative density and confining pressure were prestrained at various given prestraining amplitudes with increasing number of loading cycles. To the writer's knowledge, it seems that Li and Yang (1998) initially attempted to measured and plotted the shear modulus and damping ratio versus shear strain amplitude beyond the applied prestraining amplitude. They reported that there existed a signature of vibration history in both shear modulus and damping curves if the prestraining amplitude exceeds 0.01%. Figure 2.22 is representative modulus and

damping curves with and without vibration history found by Li and Yang (1998). This figure showed that after 1.2×10^6 cycles of loading at γ =0.0254%, the shear modulus after vibration keeps slightly higher than their counterparts before vibration if the increasing shear strain amplitude was lower than the defined prestraining amplitude; whereas, there is no difference between the damping curves of before and after vibration when the increasing shear strain amplitude is lower than the elastic shear strain threshold, 0.01%. A significant plateau is developed as the shear strain amplitude approaches the defined prestraining amplitude in both curves for modulus and damping versus shear strain amplitude, especially for damping curve; Li and Yang (1998) reported that the signature of vibration history for damping curve was influenced by the number of loading cycles, a vibration signature was identified if the number of cycles was greater than 900 at prestraining shear strain amplitude γ =0.0667%. In addition, they also reported that the signature of vibration history was not affected by the initial relative density and confining pressure applied to the sand.



Figure 2.22 Modulus and damping curves before and after 1,200,000 cycles of vibration (after Li and Yang 1998)

Wichtmann and Triantafyllidis (2004) further investigated the influence of vibration history on the shear modulus and damping properties of sand by conducting series of resonant column tests on both solid and hollow cylindrical specimens. They further confirmed the signature of vibration history for dry sand which originally reported by Li and Yang (1998). Figure 2.23-1 and2.23-2 is the typical shear modulus and damping curves before and after 3,000,000 cycles of loading with the prestraining amplitude of 0.01% for dry sand with Dr = 64% under confining pressure of 200 kPa. It is interesting to note that, as illustrated in Figure 2.23, the modulus reduction curve of the first increase of shear strain amplitude plotted at the highest location in Figure 2.23-1 and the corresponding damping curves for the first increase of shear strain amplitude at the lowest location in Figure 2.23-2, furthermore, the modulus and damping curves for the first increase of shear strain amplitude are much higher and lower than those of re-increase and reduction of shear strain amplitude, respectively. And the modulus reduction and damping curves of all the re-increase and reduction curves in the case of the re-increase of shear

strain amplitude are always plotted higher than in the case of the reduction of shear strain amplitude, and the curves in the case of re-increase of shear strain amplitude keep lower than in the case of reduction shear strain amplitude, for damping curves. It is notable that, contrarily, unlike previous findings (Tatsuoka, Iwasaki et al. 1979; Li and Yang 1998), test results presented by Wichtmann and Triantafyllidis (2004) showed that the shear modulus decreased and damping increased if the specimens are subjected to shear stress or strain history. Further work was made using the areas of the development plateau to quantitative analyze the influence of vibration history on the shear modulus and damping properties of dry sands.



Figure 2.23 Modulus and damping curves (1) shear modulus, and (2) damping ratio, for fine dry sand before and after 3×10^6 cycles of loading (after Wichtmann and Triantafyllidis 2004)

2.4 Summary

The factors influence shear modulus and damping ratio of sands are addressed in this chapter, these factors including void ratio, confining pressure, stress ratio, grain characteristics, degree of saturation, frequency of loading, number of loading cycles, and previbration history. Among these factors, void ratio and confining pressure are the primary parameters affecting dynamic properties of sand.

CHAPTER 3 TESTING EQUIPMENT AND CALIBRATION

3.1 Introduction

The resonant column method has been used to analyze the dynamic soil behavior since the 1930's when it was originally developed by Japanese engineers (Ishimoto and Iida 1936; 1937); one of earlier type of resonant column device in the United States was used to determine the torsional shear velocities of rock specimens(Birch and Bancroft 1938). In the 1960's, the resonant column device was popularly employed in the study of the dynamic response of soil in geotechnical engineering by many researchers such as Hall and Richart (1963), Hardin and Richart (1963), and Drnevich, et al(1967). These achievements have included the application of anisotropic stresses(Hardin and Black 1966), modifications to apparatus to allow hollow specimens (application of constant shearing strain(Drnevich 1967)), ability to test at large strains(Anderson and Stokoe 1978) and testing at high confining stresses (Hardin, Drnevich et al. 1994). Modifications have also been made to extend the tests undertaken to allow torsional shear(Kim and Stokoe 1994). Drnevich has contributed to this subject area extensively and has helped standardize the test procedure(Dmevich 1978; Dmevich, Hardin et al. 1978) so that the assumptions made in the mathematical model are valid during tests. More recent advancements were achieved high strain amplitudes in combined cyclic torsional shear and resonant column apparatus developed by Professor Stokoe and his graduate students at the University of Texas at Austin, which is well known as the Stokoe torsional shear/resonant column device (TS/RC), and has been continually refined in the last four decades. At present, the RC testing method is regarded as one of the most reliable, efficient, and pragmatic laboratory test methods for testing shear modulus and material damping ratio of soils and other materials (ASTM-D4015-92).

During the last two decades, the bender element method has been employed as a swift measure procedure to determine the maximum shear modulus of soils (Dyvik and Madshus 1985; Thomann and Hryciw 1990; Souto, Hartikainen et al. 1994; Viggiani and A tkinson 1997; Arulnathan, Boulanger et al. 1998; Zeng and Ni 1998; Blewett, Blewett et al. 2000; Clayton, Theron et al. 2004; Lee and Santamarina 2005; Zeng and Grolewski 2005). In this study, the bender elements used to measure the wave velocity transmitting in cylinder specimen were mounted in the top cap and the pedestal in the resonant column device.

In this chapter, the resonant column apparatus and bender elements testing system employed in study are described, as well as the calibration methods are proposed in this chapter.

3.2 Detail of Resonant Column Apparatus

The resonant column apparatus employed in this study was developed by GDS Instruments Ltd. (England). It is a Stokoe fixed-free type resonant column device. The specimen is fixed to the pedestal at bottom end, to the drive plate through the top cap at other end. This system has four testing modules; they are resonant column test, torsional shear test, triaxial compression test, and bender element test. These tests can be performed on the same specimen; therefore, testing results from different testing modes are readily compared in case of the discrepancy in individual specimen.

The used testing system are composed of testing unit (or testing chamber), axial loader, axial loader controller, control computer, back pressure system, cell pressure controller, resonant column controller, data acquisition box, temperature controlling system, as well as bender element system. Their arrangement is presented in Figures 3.1 and 3.2, and the section of testing cell in Figure 3.3. In this study, samples were tested under unsaturated state and isotropic confining conditions; therefore, axial load component, back pressure component, and fridge component were removed. A majority of tests were carried out under resonant column technique, as well as bender element method.



Figure 3.1 Arrangement of the testing components of resonant column apparatus and bender element testing instrument

3.2.1 Procedure of Resonant Column Test

Figure 3.2 presents the arrangement of the combing resonant column and bender element apparatus. After the installation of specimen is completed, switch on all components the testing program may concern. When a resonant column test is run, the control computer sends an order signal to the resonant column controller to generate a desired voltage signal (sine wave signal with a given amplitude and frequency), after that the signal is transferred to the power amplifier to magnify, then sent back to the resonant column controller, and split into four even parts and sent to four pairs of coils in the testing unit, then the specimen is vibrated by the drive plate by the torque generated by the electromagnetic system. As vibration, the charge signal measured by the accelerometer is sent to charge amplifier, then sent to resonant column controller and sent back to the computer and the relative vibration amplitude versus time curve is displayed on the screen, and the vibration frequency and its corresponding mean value of vibration amplitudes for the stack are recorded. After that the system repeats this procedure at the subsequent higher frequency. As the sweep is finished, the amplitude frequency response curve is given, and X-coordinate of the peak point on



Figure 3. 2 Arrangement of main components of resonant column apparatus and bender element testing instrument

the curve is the resonant frequency corresponding to the input signal. The Y-coordinate of the peak point is the amplitude of the specimen when the resonance occurs. The calculation of shear modulus and damping ratio will be described in detail later this chapter.

3.2.2 Drive system

The drive system is the core part of resonant column testing unit. It is composed of a flat aluminum four-armed drive plate, with a cubic permanent magnet encircled by a pair of drive coils at each end, and the top cap and four screws connecting the drive plate to the top cap, and two drainage tubes for saturated specimen test, see Figure 3.3. The masses of the components which added to the top of tested specimen are 352 g for the top cap, 15.35 g for these screws, 777 g for the drive plate, and the total additional axial stress on the specimen is 5.7 kPa considering the diameter of specimen being 5 cm. The core part of drive system is normally referred to the drive plate and the drive coils. The magnets are securely fixed to the four ends of the spider. Each pair of drive coils is housed by a cubic polymeric box. The four cubic boxes are rigidly mounted on a flat aluminum ring plate, which is used to hold the coil boxes and to provide a platform for connecting the drive system to the sport cylinder. Figure 3.4 presents the photographs of top and side views of the drive system.



Figure 3.3 Sketch map of the configuration of the resonant column testing unit

This drive system of this apparatus can provide two kinds of excitation to the specimen, one is torsional excitation, and the other is flexural excitation. For achievement of torsional excitation, the four pair of drive coils is connected in series so that a net torque is applied to the specimen (Figure 3.5(a)). To apply flexural vibrations, the coils are automatically switched (controlled by testing program) so that only two magnets are used to produce a horizontal force to the specimen inducing flexural excitation (Figure 3.5(b)). This allows the same coil and magnet arrangement to be used in both flexural and torsional vibration.



a) Top view

b) Side view

Figure 3.4 Photographs for drive system of resonant column apparatus



Figure 3. 5 Sketch map for drive excitation modes for drive system

3.2.3 Rotation Monitoring

Torsional motion monitoring system is consisted of an accelerometer rigidly mounted on the drive plate and an associated counter balance mounted on the opposite side of the four-armed drive plate, and a proximeter, mounted on the top plate, measuring the rotation of specimen when the torsional shear test is performed, see Figures 3.3 and 3.6. As known, the accelerometer may generate a high impedance charge signal proportional to the imposed acceleration, which requires conditioning to a low impedance voltage suitable for measurement. This is achieved by a charge amplifier. Then the charge signal is sent from the charge amplifier to resonant column controller, finally is sent to data acquisition card in computer terminal, see Figure 3.2.

The proximeter is mounted on the top plate for monitoring the distance between the front face of proximeter and the target mental plate as shown in Figure 3.3. Normally, the proximeter is mounted at a location 1.5 mm far from the target mental plate.



Figure 3.6 Accelerometer and counter balance on the drive system (top view)

3.2.4 Confining Chamber and Cell Pressure

Figure 3.7 is the photographs of the hollow cylinder and base plate. In the bottom plate, there are many ports, which are used to house pore pressure cables, back pressure cables, accelerometer cable, current input cables of drive coils, and others. The confining chamber is consisted of a bottom metal plate and a up hollow cylinder with one closed top and one open bottom end; the top end is enforced by a metal plate and the bottom end is reinforced by a metal ring, the metal ring and plated are connected with six pairs of metal rods used to strengthen the stiffness of the up hollow cylinder. The confining chamber is proofed to normally work under the confining pressure of 1.7MPa, and the maximum can be 2.5MPa, noted by the manufacturer.

The air cell pressure is applied to specimen by the computer controlled pressure controller from the top of the confining chamber; a secondary cell pressure sensor is mounted on the air tube close to the confining chamber. The air is supplied by the laboratory house air system. The used cell pressure controller can supply the maximum pressure up to 1000 kPa.



Figure 3.7 Up and bottom parts of confining chamber used in this study

3.3 Resonant Column Test

3.3.1 Shear Modulus

Shear modulus, G, is an extremely significant parameter in the analysis on the dynamic response of soil properties; it can be obtained by the following equations:

$$G = \rho V_s^2 \tag{3.1}$$

$$\rho = \frac{m}{V} \tag{3.2}$$

$$V = \frac{\pi \times d^2 \times H}{4} \tag{3.3}$$

in which, $\rho = \text{mass}$ density of specimen, $V_s = \text{shear}$ wave velocity propagation in the specimen, m = mass of specimen, d = diameter of specimen, H = height or length of specimen.

The mass, diameter, and height of the tested specimen can be readily determined by the balance and appropriate calipers, respectively. The shear wave velocity, however, cannot be directly measured with the resonant column method. Based on the theory of one dimension wave propagation in a fixed-free solid cylinder rod, shear wave velocity, V_s can be expressed as following equation:

$$V_s = \frac{2\pi f_n H}{\beta} \tag{3.4}$$

in which, f_n = natural frequency of system (Hz).

The frequency equation of motion of a fix-free resonant column specimen subjected to harmonic torque at the free end can be expressed as

$$\frac{I}{I_0} = \beta \tan \beta \tag{3.5}$$

in which, $I = \text{mass polar moment of inertia of specimen}; I_0 = \text{mass polar moment of inertia of resonant column driving system.}$

The mass polar moment of inertia of the tested specimen considering its shape as a cylinder, can be obtained as

$$I = \frac{md^2}{8} \tag{3.6}$$

The mass polar moment of inertia of drive system is the summation of the mass polar moment of each component consisted of the excitation head connected to the top end of the specimen, it can be obtained by

$$I_0 = \sum_{i=1}^{n} I_i \tag{3.7}$$

in which, i = the i^{th} component; n = the number of all components; $I_i =$ the mass polar moment of inertia of the i^{th} component connected to the top of specimen.

Actually, it is very difficult to calculate the mass polar moment of inertia of excitation head from Equation 3.7 due to the irregular shape of these components. Normally, therefore, the value of I_0 be obtained by the calibration of excitation head with a known properties of standard specimen. As the ratio of the mass polar moment of inertia of specimen to that of the excitation head obtained, the value of β can be readily calculated from Equation 3.5. Consequently, the shear wave velocity of specimen can be easily calculated by Equation 3.4.

To get shear wave velocity, the natural frequency of vibration system (specimen and all components added to the top of specimen), f_n , must be determined. In practice, the natural frequency can be replaced by the resonant frequency of vibration system, f_r , which is obtained from resonant column test. In theory, using the f_r , to substitute the f_n in Equation 3.4 is exact under the condition that no damping occurs in the testing material. The relationship between the f_r and the f_n can be expressed as

$$f_r = f_n \sqrt{1 - 2D^2}$$
(3.8)

in which D = Damping ratio of sample material;

As shown in Equation 3.8, the distance between resonant frequency and natural frequency increases with an increase in material damping ratio. As the damping ratio decreases to zero, resonant frequency equals to natural frequency; there is no a zero damping material in the universe, thus the resonant frequency of material is always less than its natural frequency. Yet, normally, material damping ratio of most soils is less than 20%. Substituting the damping ratio as 20% into Equation 3.8, yields to

$$f_r = 0.9592 f_n \tag{3.9}$$

The distance between resonant frequency and natural frequency equals to 4.08%, hence, it is reasonable to replace natural frequency with resonant frequency in determination of shear wave velocity from Equation 3.4.

Substituting Equation 3.4 with replacing f_n with f_r into Equation 3.1 yields

$$G = \rho \frac{4\pi^2 f_r^2 L^2}{\beta^2}$$
(3.10)

Reviewing on Equation 3.10, all parameters except the resonant frequency can be calculated or directly measured, hence in order to obtain the shear modulus of a sample; the resonant frequency should be determined by resonant column test. The resonant frequency of excitation head can be read from the frequency response curve obtained from resonant test, see Figure 3.8.

3.3.2 Damping Ratio

Nothing can freely vibrate forever, thus, energy dissipation always exists. This inherent dynamic property, attenuation or energy losses, is very important to analyze the dynamic response of ground amplification by earthquake motion (Vucetic and Dobry 1991). Total material damping consist of many component such as geometric spreading, apparent attenuation, and material losses, and so on; the detailed damping mechanisms were discussed in Wang (2001).

Another important function of resonant column apparatus is to determine material damping ratio for sample. Damping ratio measurement can be classified into two different techniques, namely, one is the bandwidth method using the frequency response curve obtained from resonant column test (Figure 3.8); the other is the logarithmic decrement method from decaying curve of free vibration of specimen. In this study, the latter was employed in calculation of the damping ratio from resonant column test.



Figure 3.8 Typical frequency response curve obtained from resonant column test

The decaying curve of free vibration of specimen is acquired by the vibration amplitude record of the accelerometer mounted on the resonant column drive plate. After a resonant frequency determination test at specific signal amplitude is complete, a sinusoidal signal whose amplitude and frequency are the same characteristics as those during the system resonance occurs is applied to the specimen; as the vibration becomes steady, then the power (or signal) is switched off and the resulting decaying vibrations are recorded from which the logarithmic decrement is calculated. Figure 3.9 shows a typical decaying curve obtained from such resonant column test.

The logarithmic decrement (δ) of the decaying curve is calculated by the following equation (Richart, Hall et al. 1970)

$$\delta = \frac{1}{n} \ln \left(\frac{A_1}{A_{n+1}} \right) \tag{3.11}$$

in which, A_1 = the amplitude of the first cycle vibration after applied power turned off; A_{n+1} = the amplitude of the (n+1)th cycle vibration after applied power turned off; and n = the number cycles of between two peak points in the recorded time.

The material damping ratio (D) can then be calculated from Equation 3.12 using the logarithmic decrement

$$D = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} \tag{3.12}$$



Figure 3.9 Typical free vibration decaying curve for measurement of material damping ratio from resonant column test

3.3.3 Shearing Strain

Torsional shearing strain is calculated from the twist angle (θ) of the tested specimen. When the top of specimen is subjected to a given torque (*T*), resulting in a given torsional displacement, and the resulting torsional strain (γ) within this specimen cylinder depends on the distance between this point and the axis of specimen cylinder, as well as the height of the horizontal section of this specimen cylinder which this point locates. As for the fixed-free vibration mode, the twist angle (θ) of a given height horizontal section depends on its height, it varies from zero at the bottom to the maximum value (θ_{max}) at the top of this test specimen cylinder. In theory, therefore, the zero torsional strain of those points of the test specimen cylinder distribute within the bottom section and the axis of this specimen. This concept is schematically depicted as shown in Figure 3.10.

The torsional strain of specimen column can be determined by the following equation

$$\gamma = \frac{r \times \theta}{h} \tag{3.13}$$

in which, θ and h are the twist angle and height of the section which the calculated point locates, and r is the distance between the calculated point and the axis of the specimen.

Normally, the top section of tested specimen is used to evaluate the properties of this material. As for the top section, the torsional shearing strain (γ) can be calculated by

$$\gamma = \frac{r \times \theta_{\max}}{H} \tag{3.14}$$

$$\theta_{\max} = \frac{x}{R} = \frac{x_A}{l_A}$$
(3.15)

in which, x = length of the arc which a given point at the edge of specimen during vibration; R =radius of tested cylindrical specimen; x_A = displacement of accelerometer mounted on the drive plate; and l_A = offset of accelerometer from the axis of the tested specimen.



Figure 3. 10 Diagram of the concept for torsional strain in a fixed-free cylinder specimen

Strictly, the displacement of accelerometer is less than the exact length of the arc through which the accelerometer vibrates during testing. In practice, however, it is sufficiently exact to substitute the length of this arc with the displacement of the accelerometer thanks to the torsional strain is small during resonant column testing. Therefore, the maximum rotation of the specimen column can be measured by the accelerometer mounted on the drive plate using Equation 3.15.

To calculate the angle of twist of the specimen cylinder, firstly the voltage output of accelerometer must be converted from the peak output of \pm -G into m/s². According to GDS instruments Ltd.(2003), the conversion is performed by

$$a = 0.981 V_{out} \tag{3.16}$$

in which, V_{out} = voltage output from charge amplifier.

The displacement of the accelerometer, x_A , is calculated by the following Equations

$$x_A = a \,/\,\omega^2 \tag{3.17}$$

$$\omega = 2\pi f_r \tag{3.18}$$

in which, f_r = the resonant frequency of vibration system.

Hence the maximum displacement of the accelerometer is

$$x_{A} = \frac{a}{4\pi^{2} f_{r}^{2}} = \frac{0.981 V_{out}}{39.4784 f_{r}^{2}} = \frac{0.024849 V_{out}}{f_{r}^{2}}$$
(3.19)

Submitting Equation 3.19 and the offset of the accelerometer (l_A) from the axis of specimen cylinder into Equation 3.15) yields

$$\theta_{\max} = \frac{x_A}{l_A} = \frac{0.024849 V_{out}}{l_A f_r^{22}}$$
(3.20)

Then submitting Equation 3.20 into Equation 3.14 yields the torsional strain (γ) within the top section of specimen cylinder as

$$\gamma = \frac{0.024849 r V_{out}}{H l_A f_r^2}$$
(3.21)

For the resonant column apparatus used in this study, the offset of the accelerometer from the center of rotation is 0.04325 m, therefore Equation 3.21 is rewritten as

$$\gamma = \frac{0.5745 r V_{out}}{H f_r^2} \tag{3.22}$$

Equation 3.22 is used to calculate a discretionary point within the top section of a tested specimen. Considering the specimen geometry, the average torsional shearing strain of at the top section of solid cylindrical specimen is assumed equal to that of point whose distance to the axis is 0.8 time of the radius of the specimen, it is given as (ASTM-D4015-92 2000)

$$\gamma = \frac{0.8 \times 0.5745 RV_{out}}{Hf_r^2} = \frac{0.4596 RV_{out}}{Hf_r^2}$$
(3.23)

The use of Equation 3.23 is more than the calculation of torsional shearing strain when the drive system vibrates at resonant frequency; it can be employed to approximately evaluate the applied load of the test using the rearrangement of Equation 3.23 as

$$V_{out} = \frac{\gamma H f_r^2}{0.4596R}$$
(3.24)

3.4 Torsional Shear Test

An additional test mode of the used resonant column apparatus in this study is to determine the shear modulus of specimen by running a torsional shear test. The applied torque or force is applied by the some drive system. A torque is calculated by the rotation of a known shear modulus calibration bar specimen, which is measured by a proximeter mounted at the top plated as shown in Figure 3.3. With achievement of the torque, the shear stress is obtained based on mass polar moment of inertia of specimen, and the radius of calculated point. And the shearing strain of specimen is obtained by the readings of proximeter. The derivation of the torque and shear stress in torsional shear test mode will present later in this chapter. The shearing strain of this test mode is calculated by Equation 3.14. However, the torsional shear test is not the primary tests used in this study; the necessity of presentation herein is to quantify the preloading stress amplitude which was used as an important controlled parameter in this study.

The theory of the application of cyclic torsional shear test to measure the shear modulus and material damping of soil sample is based on the stress-strain relationship hysteresis loop generated by the torque applied at the top of the specimen and the resulting displacement monitored by a proximeter mounted on the top plate (Figure 3.3); the voltage on drive coils is monitored, and the applied torque is calculated. The calculation of shearing strain is similar to the concept depicted in Figure 3.10.

The secant shear modulus for each loading cycle is calculated by the evaluation of the slope of the line which connected the two ends of the stress-strain hysteresis loop as described in Figure 3.11.

Material damping ratio is defined by calculating the ratio of the area within the hysteresis loop (A_L) to the maximum potential energy stored in each cycle of vibration as represented by the triangular area (A_T) . The area A_T is calculated using the end point of the hysteresis loop as illustrated in Figure 3.11. Material damping ratio is computered by

$$D = \frac{A_L}{4\pi A_T} = \frac{A_L}{2\pi G\gamma^2}$$
(3.25)



Figure 3. 11 Concept of shear modulus and material damping ratio in torsional shear test

3.5 Bender Element Test

Bender element method is a swift and simple technique to determine the small-strain shear modulus of soil by measuring the wave velocity transmitting through sample at small strain. This method is initially developed and regarded as the most useful measurement to determine the wave velocity of high-porosity laboratory sediments by Shirley and Hampton(1978).

The used bender element is manufacture as an insert that can be mounted in top cap or pedestal by GDS instrument Ltd as shown in Figure 3.12. The insert for the pedestal is made from stainless steel. The insert for the top cap is manufactured from titanium. This reduces the weight of the insert by half, minimizing the axial load on the sample caused by the top cap. The inserts are mounted in a modified top cap and pedestal.

The bender element is made from piezoelectric ceramic bimorphs. Two sheets are bonded together with a flexible shim in-between them as shown in Figures 3.13. An excitation voltage is applied to generate a displacement in the source transducer, resulting in a wave propagating through the specimen. This wave poses a displacement in the receiver, which induces a voltage that can be measured by an automatic acquisition testing system (Figure 3.14). The used bender element is measured to be 2.5 mm in the length out from the top cap and pedestal (L_c), 11.0 mm in width, 1.35 mm in thickness, and the total length is unknown.



Figure 3. 12 An example of bender element insert made by GDS used in this study

The principle of shear wave propagation through a specimen is illustrated in Figure 3.14-1. In this testing mode, the bender element embedded in top cap service as the source transmitter, and that mounted in pedestal works as the receiver. The shear wave source element is polarized in the same direction, and the two sheets of the receiver in opposite direction, when excitation voltage is applied to the bender elements, resulting in one piezoelectric ceramic bimorph contracts and the other stretches simultaneity, which induces a displacement of the free end of the bender element and a motion in sample, as shown in Figure 3.14-1. Hence the generating shear wave to the receiver results in a motion in receiver bender element, and generating a voltage. A source voltage reverses inducing the bender element motioning in opposite direction.



Figure 3. 13 Configuration of bender element (left: dimension, right: inside configuration) (after Yamashita, Fujiwara et al. 2009)



Figure 3. 14 Transmission of shear and compression wave in bender elements test

In compression wave velocity testing mode, as shown in Figure 3.14-2, the bender element embedded in the pedestal services as an excitation source and the other in top cap as a receiver. When an excitation voltage is applied both sheets extend; when the excitation voltage reverses these sheets contract. As extension and contraction of bender elements, a compression wave propagates in sample. The arrival of compression wave at the bender element in the top cap induces a motion generating a voltage.

With time distance (ΔT) between the excitation moment and receiving moment, wave velocity is calculated by

$$V = \frac{L}{\Delta T} \tag{3.26}$$

in which, V = velocity of shear (compression) wave propagation in specimen depends on the testing mode; and L = distance between the source and receiver element tips.

The input signal used in bender element test in this study is a sine wave with the frequency of 10 Hz. Based on a large number of investigations, the start to start method is the most popular method which is used to determine the time arrival of wave propagation in a sample. The detailed description of identification of time arrival could be found in literatures (Lee and Santamarina 2005; Yamashita, Fujiwara et al. 2009). A brief of this method is illustrated in Figure 3.15. Generally, the section of received wave between Point A and Point C is caused by the near field effect, and Point C is used to identify the arrival of the received wave.



Figure 3. 15 Example of shear wave signal within near field: (A) first deflection, (B) first bump maximum, (C) zero after first bump, and (D) major first peak

3.6 Calibration of Drive System for Resonant Column Test

Testing instrument calibration is extremely of importance to achieve reliable testing results. As known, the instrument components may vary with using age, especially those electronic components, the permanent magnets, as well as the bender of drive plate arms after a great number of oscillations at very high level. In this section, drive system of resonant column testing mode and torque calibrations are addressed in detail. The calibration certificates of cell pressure controller, secondary cell pressure transducer, linear variable differential transducer (LVDT), and proximeter for torsional shear test, are given in Appendix.

Note that the drive system herein is referred to the components added on the top of specimen during resonant column test; it includes the top cap, drive plate, and the screws using to connect these two components.

From the theory of wave propagation, the velocity of the shear wave propagation is computed using measured value of resonant frequency, specimen dimension, and the mass polar moment of inertia of drive system (ASTM-D4015-92 2000). Due to the complicated design of drive system, the mass polar moment of inertia of drive system normally is determined from a number of separate on calibration bars (Dmevich 1978; ASTM-D4015-92 2000; GDSRCA-Manual 2003; Kumar and Clayton 2007; Clayton, Priest et al. 2009).

In this study, drive system calibration was performed with a large range of resonant frequency from 23.5 Hz to 235.8 Hz on different torsional stiffness calibration bars
inspired by the investigation on dynamic properties on frozen soil by resonant column test. In general, it is assumed that during interpretation of resonant column test data that the measured resonant frequency of the specimen results from torsional single-degree of freedom oscillation, due only to distortion of the specimen. However, there are many causes, in theory, why this might not be the case. These causes were well reported by (Dmevich 1978; Kumar and Clayton 2007; Clayton, Priest et al. 2009). In this section, a brief is listed as follows.

- (1) The existence of other patterns of distortion. In theory, the resonant column data are analyzed on the basis of single-freedom horizontal oscillation, however, other specimen oscillations might occur, such as flexure, axial shortening, and so forth, interference of these oscillations may lead to underestimate the resonant frequency.
- (2) Compliance in test apparatus or calibration system. Additional distortion may arise, say, from the bender of drive components; as a result the measured frequency might be significantly reduced, particularly testing on stiffer material. These compliances may be induced from design of calibration bar(Clayton, Priest et al. 2009), slippage between stiffer specimen and top cap and pedestal (Dmevich 1978), drive system and platen (Clayton, Priest et al. 2009), and support system (Dmevich 1978).
- (3) Introduction of additional degree freedom due to significant compliance within drive and slippage between them (Dmevich 1978). All these may induce the system to resonance at a number of frequencies, even under torsional excitation alone, the fundamental frequency of the whole system may be changed (Clayton, Priest et al. 2009).

3.6.1 Theory Background

Usually, experimental procedure is used to determine the mass polar moment of inertial of the drive system; this may involve submitting a calibration bar as the soil specimen and measure the resonant frequency of the system. Combining Equations 3.4 and 3.5 and rearranging yields

$$\frac{I_s}{I_0} = \frac{\omega_n L}{V_s} \tan \frac{\omega_n L}{V_s}$$
(3.27)

And the system is considered as a torsional pendulum with a single degree of freedom, where the calibration is regarded as s torsional spring and the drive system is considered as the pendulum mass, the circular frequency of motion for this system can be expressed as

$$\omega_n = \sqrt{\frac{K_T}{I_0 + I_s / 3}} \tag{3.28}$$

in which, K_T is the torsional stiffness of the calibration bar, I_s is the mass polar moment of inertia of calibration bar.

The resonant frequency of system is remeasured after additional masses are added on the drive plate; and Equation 3.28 can be rewritten as

$$I_{\rm am} = \frac{K_T}{\omega_n^2} - (I_0 + I_{\rm s}/3)$$
(3.29)

in which, I_{am} is the mass polar moment of the added masses, which can be calculated from the mass and dimension of the standard shape added masses. It is clear that Equation 3.28 take a form of a standard linear equation. Through plotting the value of I_{am} as a function of $1/\omega_n^2$ for each test, $(I_0 + I_s/3)$ is identified by the negative Y-axis intercept, and the torsional stiffness by the slope of the straight line. The mass polar moment of the calibration bar is assumed negligible in interpretation of Equations 3. 28 and 3.29 by Clayton et al. (2009), they reported that the maximum error for the biggest bar (28.1 mm in diameter) they used is less than 1% when the small-strain shear modulus measured is compared with its corresponding value calculated by the more rigorous approach. The used biggest bar used in this study has a diameter of 26 mm, which is smaller than that used in Clayton, Priest et al. (2009). However, for minimizing this error, the I_s of central stem of each calibration bar is considered in this study.

3.6.2 Calibration Bars and Weights

Eight calibration bars and three calibration weights were prepared to for the purpose of the determination of the mass polar moment of inertia of drive system; their photographs are shown in Figure 3.16 Each bar has a central column to connect a large and thin circular plate at two ends, enabling the bar can be bolted to the pedestal and drive head. The central stem has a diameter varying from 7.5 mm to 26 mm, and two circular plates have the same dimensions with 50 mm in diameter and 5 mm in thickness. Within each plate, six small screw holes with 4.5 mm in diameter were opened for tightening the bar to the top cap and pedestal, and a cylindrical pit with 20.5 mm in diameter and 3 mm in depth was made for accommodating the bender element embedded in the top cap and pedestal, the design of calibration bar is illustrated in Figure 3.17. All the bars are made of aluminum. Three copper calibration weights have a shape in rectangular block with 20 mm in width, 100 mm in length, and 8 mm in height, and have 132 g in mass. The calibration weight was tightly fixed on the drive plate by two screws as shown in Figure 3.18.

The torsional stiffness, $K_{\rm T}$, of calibration bar is not only measured from resonant column test, but calculated by the following equation (Higdon, Ohlsen et al. 1985):

$$\frac{1}{K_{\rm T}} = \frac{H_1}{J_{\rm pl}G_1} + \frac{H_2}{J_{\rm p2}G_2} + \dots + \frac{H_i}{J_{\rm pi}G_i} + \dots + \frac{H_n}{J_{\rm pn}G_n}$$
(3.30)

in which, H_i is the height of center stem of calibration bar, the J_{pi} is the polar moment of inertia of section of round bar, and G_i is the shear modulus of calibration bar. For the design calibration bar as shown in Figure 3.16, the central stem is used to calculate the value of K_T , which is simply dependent on the height, the polar moment of inertial of section, and the shear modulus, the number, n, is 1 herein.



Figure 3. 16 Design of the calibration bar used in resonant column test



Figure 3. 17 Photographs of calibration bars and weights used in this study



Figure 3. 18 Set up of resonant column calibration test

The masses, dimensions, mass polar moment of inertias, and the torsional stiffness based on Equation 3.30 of the central stems of calibration bars are listed in Table 3.1. The mass polar moment of inertias of calibration weights and other unimportant components on calibration bar are summarized in Table 3.2.

Bar Number	Height H (mm)	Diameter D _b (mm)	Mass [†] m (kg)	Mass Polar Moment of Inertia, I_s (10^{-3} kg m^2)	Calculated Torsional Stiffness, K_{T}^{\ddagger} (kN m/rad)
1	100	7.5	0.01177	8.27E-05	0.082
2	100	10	0.02092	2.61E-04	0.260
3	100	12.5	0.03269	6.38E-04	0.635
4	100	15	0.04707	1.32E-03	1.317
5	100	17.5	0.06406	2.45E-03	2.440
6	100	20	0.08367	4.18E-03	4.163
7	100	23	0.11066	7.32E-03	7.280
8	100	26	0.14141	1.19E-02	11.889

Table 3.1 Dimension and mass polar moment of inertia of the central rod calibration bars

 † The density of aluminum was measured as 2663.42 kg/m³. ‡ Calculated from Equation 3.30 by assuming 26.5 GPa as the shear modulus of aluminum.

Component on Calibration Bar	Ι	Number	Total I
	(10^{-3} kg m^2)	Number	(10^{-3} kg m^2)
One Calibration Weight	0. 1144	1	0. 1144
Screw Tightening One Weight	3.85×10 ⁻⁴	2	7.7×10 ⁻⁴
One Calibration Weight and Screws			0.1152
Two Calibration Weights	0. 1144	2	0. 2288
Screw Tightening Two Weights	5.25×10 ⁻⁴	2	1.05×10^{-3}
Two Calibration Weight and Screws			0.2299
Three Calibration Weights	0. 1144	3	0.3432
Screw Tightening Three Weights	6.29×10 ⁻⁴	2	1.26×10^{-3}
Three Calibration Weight and Screws			0.3445
Screw Tightening Calibration Bar	6.14×10 ⁻⁴	3	1.84×10^{-3}
BE Pit in Upper plate	-1.48×10^{-4}	1	-1.48×10 ⁻⁴
Screw Hole in Upper plate	-5.90×10 ⁻⁵	6	-3.54×10 ⁻⁴
Upper plate no Considering Holes	8.17×10 ⁻³	1	8.17×10 ⁻³
Upper plate Considering Holes			7.75×10 ⁻³
Upper plate Including Tightening Screws			9.51×10 ⁻³
Screw Tightening Filter Stone on Top Cap	2.79×10 ⁻⁴	3	8.37×10 ⁻⁴

Table 3.2 Dimension and mass polar moment of inertia of calibration weights

Note: The mass polar moment of inertias of screw and holes in these components are calculated on the basis of assuming them as mass points using $I = mr^2$, where *m* is the mass, and *r* is the distance of mass point to the O-O axis. The *I* of weight is calculated using $I = m(W^2 + L^2)/12$, where *W* is the width, and *L* is the length.

3.6.3 Testing Results

The resonant column tests were carried out by the increment of frequency of 0.1 Hz, which is usually used in soil properties determination in resonant column test. The tested results of calibration bars are presented in Table 3.3 and Figures 3.19 and 3.20.

Figure 3.21 illustrates the comparison of the tested and calculated torsional stiffness for three sets of data presented in Tables 3.4 and 3.5. It is seen that the calculated the data points locate at the 45 degree line when stiffness of calibration bar is relatively small, as the stiffness increase the data point starts deviate from the straight line, which implies that the compliance within the drive system increases as the system vibrates at higher frequency. The data points of this study starts to deviate earlier than those of two previous investigations, which is accounted for the case that top cap was mounted in this study but were not in used in the previous investigations, as a results more compliance might occur in this study. It should be noted that the calculated torsional stiffness is generally greater than its corresponding tested value in this study; however, as for the data reported in previous investigations show an opposite regularity, namely the calculated torsional stiffness is greater than the tested value, which is possibly contributed to the fact that the calibration discs were directly mounded on the top of the calibration bar in these two investigations (Kumar and Clayton 2007; Clayton, Priest et al. 2009) resulting in an increase in stiffness of the tested bar to a value greater than the real value. Considering the calculation of torsional stiffness is based on an assumption that there is no compliance between the top and bottom plates of the calibration bar (Clayton, Priest et al. 2009), then the calculated values should be theoretically greater than those obtained by tests due to some compliance of bar or other components actually occurs during vibrating. In this regard, the data of this study seems better than those in previous studies.

Dor	No Weight		With One weight		With Tw	vo Weights	With Three Weights	
Dal	$f_{\rm r}({\rm Hz})$	γ (%)	$f_{\rm r}({\rm Hz})$	γ (%)	$f_{\rm r}({\rm Hz})$	γ (%)	$f_{\rm r}({\rm Hz})$	γ (%)
1	23.2	0.00105	22.8	0.00105	22.5	0.00110	22.2	0.00108
2	40.2	0.00067	39.6	0.00067	39.1	0.00065	38.6	0.00061
3	61.5	0.00048	60.6	0.00048	59.8	0.00048	59	0.00049
4	87.6	0.00031	86.3	0.00032	85.2	0.00035	84.0	0.00033
5	118.5	0.00035	116.9	0.00036	115.3	0.00037	113.8	0.00038
6	151.1	0.00055	148.1	0.00055	146.1	0.00057	144.3	0.00059
7	195.8	0.00074	193.4	0.00075	191.0	0.00076	188.8	0.00077
8	235.8	0.00055	233.3	0.00059	230.6	0.00057	228.0	0.00058

Table 3.3 Tested results of all calibration bars used in this study



Figure 3. 19 Results of calibration of a Stokoe resonant column apparatus with different calibration bars (Bars 1-3): I_{am} plotted against $1/\omega_n^2$



Results of calibration of a Stokoe resonant column apparatus with different calibration bars (Bars 4-8): I_{am} plotted against $1/\omega_n^2$ Figure 3. 20

Table 3.4	Tested and calculated torsional stiffness of all calibration bars used in this
	study

Dor	D _b	$f_{\rm r}$	K_{T}	$K_{ ext{T}}^{C\ \ddagger}$	I_0	I_0^{C1}	I_0^{C2}
Dal	(mm)	(Hz)	(kN m/rad)	(kN m/rad)	(10^{-3} kg m^2)	(10^{-3} kg m^2)	(10^{-3} kg m^2)
1	7.5	23.2	0.081	0.082	3.796	3.864	3.791
2	10	40.2	0.260	0.260	4.079	4.068	3.991
3	12.5	61.5	0.595	0.635	3.971	4.244	4.163
4	15	87.6	1.227	1.317	4.044	4.338	4.255
5	17.5	118.5	2.281	2.440	4.104	4.391	4.308
6	20	151.1	3.756	4.163	4.214	4.607	4.520
7	23	195.8	6.945	7.280	4.579	4.798	4.707
8	26	235.8	10.917	11.889	4.960	5.403	5.300

^{*}Based on Equation 3.29 using $I_{am} = 9.51 \times 10^{-6}$ kg m² as the mass polar moment of inertia of upper plate of the calibration bar, seen in Table 3.2. ^{C1} and ^{C2} calculated based on Equation 3.29 and Equation 3.27, respectively.

Investi- gations	Bar	D _b (mm)	f _r (Hz)	K _T (kN m/rad)	$K_{\rm T}^C$ † (kN m/rad)	I_0 (10 ⁻³ kg m ²)	I_0^{C1} (10 ⁻³ kg m ²)	I_0^{C2} (10 ⁻³ kg m ²)
	1	12	50.0	0.44	0.422			kg III)
Clayton	1	15	59.8	0.44	0.432	2.99	2.968	2.910
et al	2	18	111.7	1.55	1.590	2.98	3.132	3.069
(2009)	3	23	175.2	4.23	4.238	3.37	3.402	3.331
	4	28.1	244.3	10.48	9.441	4.32	3.912	3.826
Kumar	A1	13	61.03	0.42	0.41	2.734	2.692	2.704
and	A2	18.03	114.64	1.47	1.52	2.828	2.832	2.839
Clayton	A3	22.94	178.46	4.14	4.00	3.093	3.081	3.076
(2007)	A4	28.1	247.27	9.35	8.99	3.812	3.619	3.619

Table 3. 5Tested and calculated results from other investigators (data after Kumar
and Clayton 2007; Clayton, Priest et al. 2009)

[†] To calculate the $K_{\rm T}$, Clayton et al. (2009) considered only the central stem of calibration bar based on G=27 GPa, however, Kumar and Clayton (2007) considered the whole dimension not just the central stem of the calibration bar based on $V_{\rm s}=3086$ m/s, considering the density of aluminum is 2700 kg/m³, accordingly the G=26.2 GPa is obtained. The $I_{\rm am}$ of the upper plate of the calibration bar is taken as 9.6×10^{-5} kg m² according to Clayton et al. (2009).



Figure 3. 21 Comparison of the tested and calculated torsional stiffness of calibration bar by a Stokoe resonant column apparatus

3.6.4 Mass Polar Moment of Inertia of Drive System

Figure 3.22 presents the variation of the tested mass polar moment of inertia of drive system with the tested torsional stiffness of calibration bar; both results of previous investigations are also illustrated in this figure for comparison. It can be seen that, in general, the mass polar moment of inertia of drive system increases with torsional stiffness except for the data of the 10 mm calibration bar. This relationship is in agreement with those of reported by Kumar and Clayton (2007), and Clayton et al. (2009).

Figure 3.23 plots the tested and calculated values of mass polar moment of inertia of drive system against the tested resonant frequency. Both tested and calculated mass polar moments of inertia of the drive system increase with the tested resonant frequency. It can be seen that the value of the calculated of mass polar moment of inertia of drive system is greater than its corresponding tested value for a given tested frequency; the values calculated by Equation 3.29 are plotted highest among these values. This implies that in real soil specimen test, the mass polar moment of inertial of drive system should be increased to get an accurate shear modulus when the system vibrates at higher frequency.



Figure 3. 22 Variation of the tested mass polar moment of inertia of drive system with the tested torsional stiffness of calibration bar



Figure 3. 23 Variation of the mass polar moment of inertia of drive system (tested and calculated) with the tested resonant frequency of calibration bar

3.6.5 Discussion

Conventionally, the tested mass polar moment of drive system may be approximately obtained from a test on a softer torsional stiffness calibration bar due to the compliance in the drive system is usually negligible. The average value method is suggested by GDS instrument Ltd, to achieve the mass polar moment of inertia of drive system, several softer stiffness known shear modulus calibration bars are used to determine the I_0 , and takes the mean value of these I_0 as the mass polar moment of inertia of inertia of drive system (GDSRCA-Manual 2003).

Figure 3.24 illustrates the shear modulus and reduction of shear modulus of each calibration bar based on Equation 3.27 using the measured value of I_0 obtained from the smallest bar as well as the mean value of I_0 for all bars. As shown in this figure, the shear modulus rapidly reduces with the resonant frequency (or torsional stiffness) for these three sets of data, the speed of reduction of the data in this study is faster than those by Clayton et al. (2009) and Kumar and Clayton (2007) due to the more components on the upper plate in the calibration test in this study. The reduction of shear modulus starts from 5% at 40.2 Hz to 29% at 235.8 Hz in this study. When the average I_0 is used to analysis shear modulus, it may overestimate the shear modulus at lower frequency and underestimate at higher frequency. It may overestimate the shear modulus to 10.9% at the frequency of 23.2 Hz and underestimate the shear modulus to

21.5% at 235.8 Hz. It is concluded that both the smallest I_0 and average I_0 methods may cause very great deviation of shear modulus from the reference value, which is not acceptable in practice.



Figure 3. 24 Variation of shear modulus with the tested resonant frequency based on the smallest and average mass polar moment of inertia of drive system

Figure 3.25 presents the variation of the measured shear wave velocity of aluminum with tested resonant frequency. Equation 3.27 is used to interpret the shear wave velocity based on the tested values of mass polar moment of inertia of drive system and corresponding resonant frequency. Data of previous investigations are also plotted in this figure for comparison. As shown in this figure, all the wave velocities (except for the value at 244.4 Hz for Clayton et al. (2009)) locate around the V_s =3086 m/s suggested by De Billy (1980) within the range reported by Lambe and Whitman (1979). The shear wave velocity obtained from previous investigations suddenly jumps up when the frequency reaches maximum, which seems somewhat mysterious according to the fact that the shear wave velocity should be smallest when the torsional stiffness of specimen is greatest in theory. Furthermore, actually, these four calibration bars and resonant column apparatus used in their studies should be the same in these two investigations due to both tests were carried out on the same calibration bars in Southampton University by Clayton. In this regard, the testing results presented in this study show a better result. As shown in this figure, the measured shear wave velocities are always lower than the reference velocity of 3124.19 m/s considering the measured density of 2663.42 kg/m³ and referent the shear modulus of 26.5 GPa. Even though these tested mass polar moments of inertia of drive system are individually used to interpret shear modulus, some deviation of shear modulus from reference value can not be avoided. It is necessary to propose a correction method to eliminate the influence of torsional stiffness on shear modulus.



Figure 3. 25 Shear wave velocity with the tested resonant frequency of calibration bar based on the tested I_0 from resonant column test

3.6.6 Correction Procedure

To correct the tested shear modulus Clayton et al. (2009) suggested using a series of calculated values of I_0 instead of the I_0 determined from the smallest calibration bar. By this method, the I_0 should be accordingly adjusted with measured resonant frequency, as a result the value of I_s/I_0 varies with resonant frequency, consequently induces inconvenience in calculation of shear velocity, as shown in Equation 3.27. Using a constant I_0 during test may simplify Equations 3.4 and 3.5 1 into a single variable ($\omega_n=2\pi f_r$) function. In theory, for a given drive system, there should be a unique mass polar moment of inertia; therefore, the application of a constant I_0 has more significance. The mass polar moments of inertia of drive system (including the I_0 of upper plate of calibration bar) calculated by Equation 3.27 are plotted against the tested frequency in Figure 3.26. As shown in this figure, a four order polynomial function may fit testing data,

$$I_0^{\rm C} = af_{\rm r}^4 + bf_{\rm r}^3 + cf_{\rm r}^2 + df_{\rm r} + (I_0 + I_{\rm up})$$
(3.31)

in which, I_0^C is the calculated mass polar moment of inertia of drive system for each calibration test, a, b, c, and d are the fitting constants, they are listed in Table 3.6 for the data in this study, I_0 is the mass polar moment of inertia of drive system, and I_{up} is the mass polar moment of inertia of drive system, and I_{up} is the mass polar moment of the upper plate of calibration bar. The I_0 is obtained by subtracting the intercept of the I_0^C versus resonant frequency curve to the I_{up} .



Figure 3. 26 Determination of mass polar moment of inertia of drive system of a fixed-free resonant column apparatus

Table 3. 6Fitting parameters and mass polar moment of inertia of drive system for
Equation 3.31

a	b	с	d	$I_0 + I_{up}$ (10 ⁻³ kg m ²)	(10^{-3} kg m^2)	
9.949×10 ⁻¹¹	3.273×10 ⁻⁷	-1.241×10 ⁻⁴	0.01754	3.4655	3.4560	

Using the determined I_0 for all tested frequency without correction may underestimate the shear modulus, especially at high frequency. To accurately interpret shear modulus the tested frequency must be corrected to an appropriate value. The correction is carried out by developing a relationship between the calculated frequency by Equation 3.27 with the I_0 determined by Equation 3.31 with knowledge of V_s and the tested frequency as shown in Figure 3.27. The relationship between the calculated and tested resonant frequency is given as

$$f_{\rm r}^{\rm C} = lf_{\rm r}^{4} + mf_{\rm r}^{3} + nf_{\rm r}^{2} + of_{\rm r} + p \qquad (3.32)$$

in which, the l, m, n, o, and p are fitting constants, these values and correlative coefficients are listed in Table 3.7.



Figure 3. 27 Correction of resonant frequency of the used calibration bars based on Equation 3.27 for the fixed-free resonant column apparatus used in this study

 Table 3.7
 Fitting parameters and correlative coefficients for Equation 3.32

1	m	n	n o		r ²	
5.5085×10 ⁻⁸	-2.0972×10 ⁻⁵	0.00305	0.95817	0.82978	0.99996	

Figure 3.28 presents the shear moduli of calibration bars obtained from Equation 3.27 after frequency correction by Equation 3.32 using $I_0=3.456\times10-3$ kg m². It is seen that shear modulus after correction well agrees with the reference value of shear modulus of 26.5 GPa for aluminum, the relative deviation of shear modulus plots within a small range from -1.29% to 1.37% for the frequency range from 23.2 Hz to 235.8 Hz with, which is acceptable in practice.

The application of the method proposed in this study, the following steps may concern,

- (1) Prepare several calibration bars whose shear wave velocity is known with a large range of torsional stiffness which may cover a large range of resonant frequency in tests.
- (2) Conduct resonant column tests on these calibration bars to obtain the resonant frequencies without using extra calibration weights.
- (3) Use Equation 3.27 to calculate the mass polar moment of inertia of drive system corresponding to each calibration bar by using the measured frequency.
- (4) Plot the calculated mass polar moment of inertia of drive system against the

measured resonant frequency in a diagram like Figure 3. 26

- (5) Find an appropriate function to fit the data like Equation 3.31, and then take the intercept of the fitting curve as I_0+I_{up} , and then the mass polar moment of inertia of drive system is obtained.
- (6) Calculate the frequency by Equation 3.27 using the I_0 obtained in Step 5 for each calibration bar.
- (7) Plot the calculated frequency in Step 6 against the tested frequency in a diagram like Figure 3.27, and find a suitable function like Equation 3.32 to fit the data.
- (8) Analyze shear modulus of specimen based on specimen density and shear wave velocity calculating by Equation 3.27 using the resonant frequency corrected by Equation 3.32.



Figure 3. 28 Shear modulus of aluminum before and after correction in this study

3.7 Calibration of Torque for Torsional Shear Test

3.7.1 Theory Background

The torque action on the top of specimen is applied by the same drive system as resonant test mode for the used apparatus. For such a resonant column combining torsional test, in torsional shear test mode, the test is stress controlled not strain controlled; the torque can not be directly controlled as that applied by a standard torsional shear device. Usually the torque is obtained by calibration test on a known shear modulus round bar, say aluminum bar. The torque is related to the angle of twist for a solid cylindrical bar as (Shigley and Mishcke 1990)

$$T = \frac{GJ\theta}{H} \tag{3.33}$$

in which, T is the torque action at the top of the specimen, G is shear modulus of

tested specimen, J is the polar moment of inertia of section of specimen, θ is angle of twist, and H is the height. For a solid round rod, the shear stress is zero at the center and maximum at surface, at the free end as for the fixed-free model. The distribution of stress at the top section is proportional to the radius ρ as

$$\tau = \frac{T\rho}{J} \tag{3.34}$$

$$\tau_{\max} = \frac{TR}{J} \tag{3.35}$$

in which *R* is the radius of specimen, an average torsional stress is taken as $0.8\tau_{max}$ according to ASTM-D4015-92 (2000).

Note that: some assumptions are adopted in the analysis as follows,

- (1) The bar is acted upon by a pure torque, and the sections under consideration are remote from the point of application of the load and from a change in diameter.
- (2) Adjacent cross sections originally plane and parallel remain plane and parallel after twisting, and any radial line remains straight.
- (3) The material obeys Hook's law.

For the used apparatus, the twist angle (θ) is measured by a proximeter mounted on the top plate as shown in Figure 3.3. During a calibration test, the torque generated by the electromagnetic system and displacement of target mental plate from the front surface of the proximeter are both output in voltages. 0.2860 mm/V was taken as the calibration factor of the proximeter used in this study, which is slightly lower than 0.2920 mm/V calibrated by GDS Ltd according to the author's measured calibration results. Consequently, the torque related to the corresponding displacement of the mental target is calculated by Equation 3.33. Then correlate the calculated torque to the output voltage in drive coils, and the torque calibration factor is obtained. The used calibration bars were Bar 1 to 6, whose dimensions and relative physical properties are already listed in Table 3.1.

3.7.2 Effect of Torsional Stiffness

Herein two kinds of calibration factor are introduced, F1 is the factor obtained by dividing the torque to the output voltage in drive coils, which is used to input in the testing program; and F2 is the factor obtained by dividing the torque to the input voltage in the testing program, which is used to estimate the shear stress based on Equation 3.35.

Figure 3.29 illustrates the relationship between torque calibration factor and torsional stiffness of calibration bar under input voltage of 2 volts at the shear frequency of 1 Hz. It is seen that the calibration factors increase with an increasing in torsional stiffness of calibration bar, especially for the F2. Inputting a medium F1 is in the testing program will lead to an overestimation of stress for a specimen which has lower stiffness than that of the calibration bar corresponding to F1, and an underestimation of stress for a specimen which has higher stiffness than that of the

calibration bar. This can be interpreted by the decrease of electromagnetic force acting on permanent magnets when they offset from the center of drive coils during test. Figure 3.30 illustrates the relationship between the calibration factor and the offset of the metal target plate from the proximeter, which qualitatively stands for the offset of magnet from the center of the coils. As shown in this figure, the calibration factors decrease with an increase in the offset of target plate from the proximeter.



Figure 3. 29 Influence of torsional stiffness of calibration bar on the torque factor



Figure 3. 30 Variation of torque calibration factor with offset of proximeter from target mental plate

3.7.3 Effect of Input Voltage

0.5 V, 1 V, 1.5 V, and 2 V were employed to analysis the influence on torque factor in torque calibration test. In analysis, data of Bars 1-5 are considered herein, and data of Bar 6 are not included due to less precision of measurement by the proximeter because the displacement of target plat is too small when the input voltage is less than 2 V.

Figure 3.31 presents the variation of calibration factor with the input voltage. As shown in this figure, in general, the tested values of F1decrease with increasing input voltage, however, the values of F2 do not show a clear tendency as F1 due to data scatter. The reduction is possibly accounted for the offset of the proximeter from the target plate as addressed above. Figure 3.32 presents the variation of relative deviation of torque factor with input voltage for five calibration bars. In analysis, the mean torque factor of each bar is used as the reference factor. The relative deviation of torque factor slightly increases with input voltage, but the relative deviation is relative low. In general, the relative deviation just varies from -1.733% to 1% under input voltage increasing from 0 V to 2 V, and the maximum range of deviation is still within the range from -2% to 2%. One can conclude that influence of input voltage on testing results of shear modulus is negligible in practice.



Figure 3. 31 Variation of torque calibration factor with offset of proximeter from target mental plate



Figure 3. 32 Relative deviation of torque factor versus input voltage in torsional shear test

3.7.4 Proposed Procedure

Considering the fact that the input voltage has less influence on the torque factor during test, the influence of torsional stiffness is the unique factor considered in the proposed method herein. To minimize the influence of input voltage on torque factor, the average value of torque factors obtained from all input voltage for each bar is plotted against torsional stiffness in Figure 3. 33.

As shown in Figure 3.33, the relationship between torque factor and torsional stiffness can be predicted by an exponential function as

$$F = \mathbf{a}K_T^{\ n} + \mathbf{b} \tag{3.36}$$

in which, a, b, and *n* are fitting parameters, $K_{\rm T}$ is the torsional stiffness of tested specimen.

Figure 3. 34 compares the relative deviation of torque factor with torsional stiffness of calibration bar based on the mean torque factor (a factor obtained from a medium bar is suggested by GDS instrument Ltd (GDSRCA-Manual 2003)) with the correction method (Equation 3. 36) proposed herein. As shown in this figure, using mean torque factor may overestimate torque factor resulting the shear modulus of softer specimen and underestimate that of stiffer specimen. The deviation of torque factor and resulting shear modulus may varies from -19% to 14% for the tested calibration bars. The proposed method may significantly reduce the deviation to nearly zero; the

maximum deviation is only 2%.

The proposed method for torque calibration is summarized as follows,

- (1) Prepare several round bars with knowledge of shear modulus; calculate the torsional stiffness of calibration bar by Equation 3.30.
- (2) Measure the torque factor by testing on these bars.
- (3) Plot torque factor against torsional stiffness in a diagram like Figure 3.33, and find an empirical relationship between these two parameters as Equation 3.36.
- (4) In a torsional shear test on soil specimen, the torsional stiffness of specimen can be calculated by Equation 3.30, and the shear modulus of tested specimen may be roughly estimated by resonant column test.

Though the proposed method may well estimate shear modulus of calibration bar, when it is applied in soil test, special care should be taken due to the torsional stiffness decreases with shearing strain amplitude. Using the shear modulus by resonant column test at lower strain amplitude may be somewhat overestimated torsional stiffness of specimen at higher strain level; as a result shear modulus by torsional shear test will be overestimated at higher strain shear amplitude, especially for stiffer specimen.



Figure 3. 33 Diagram of determination of torque factor for torsional shear test



Figure 3. 34 Comparison of the relative deviation of torque factor versus torsional stiffness by the mean value and proposed methods

3.8 Summary

The used apparatus and relative fundamentals are described in chapter. Two simple and reliable methods are presented for determination of the mass polar moment of inertia of drive system and the torque factor for torsional shear test. The procedure for resonant column test can minimize the deviation of shear modulus to around 1% compared to the maximum deviation of conventional method, which may reach 28.3% when the mass polar moment of inertia from the smallest calibration bar and 20.35% from the mean value of data for all calibration bars at the frequency of 235.8 Hz. The method for calibration of torque factor may also reduce the deviation of shear modulus from by the mean factor method from 19% to 1%.

CHAPTER 4 MATERIALS AND TESTING PROCEDURES

4.1 Introduction

The geotechnical properties of soils in site are influenced by a lot of factors, such as grain characters, plasticity, Water Contents, void ratio, stress state, and so forth. The testing boundary conditions simulating that of soil in the field are also of importance. Therefore, it is essential to well know the basic physical properties of testing soils and design an appropriate testing program to simulate in-situ conditions during tests. In the following sections, the characters, testing procedures, as well as the sample preparation will be described in detail.

4.2 Properties of Testing Sands

Four different source sands, Berlin sand, Cuxhaven fine sand, Cuxhaven medium sand, and Braunschweig coarse sand with gravel, were used in this study. Considering the distribution and importance of Berlin sand, most tests were carried out on this sand, and some accessorial tests were performed to confirm the findings or comparison purpose. In addition, Berlin sand is the primary soil material used in the Geotechnical Engineering Institute of the Technical University of Berlin for other research projects for many years. Soil particles of the used sands are round or secondary round in shape, Figures 4.1-4.4 are the microscopes of the material used in this study. The primary coarse mineral of used sands is quartz, but Berlin sand includes a small content of feldspar, as well as a little amount of mica. The basic properties of these soils are listed in Table 4.1, and the grain size characteristic curves are shown in Figure 4.5.

Sands	d ₁₀	d ₃₀	d ₅₀	d ₆₀	Cu	Cc	emin	emax	ρ_s	φ _c
	(mm)	(mm)	(mm)	(mm)	- u	č		ших	(g/cm ³)	(°)
BS	0.229	0.390	0.620	0.777	3.39	0.86	0.368	0.708	2.65	32.64
CFS	0.094	0.103	0.112	0.116	1.23	0.96	0.569	0.980	2.61	33.20
CMS	0.142	0.263	0.369	0.437	3.08	1.12	0.409	0.753	2.66	32.69
BRCS	0.253	0.502	1.000	1.459	5.76	0.68	0.349	0.690	2.67	

Table 4. 1 Basic physical and mechanical properties of sands used in this study

BS=Berlin sand, CFS= Cuxhaven fine sand, CMS= Cuxhaven medium sand, BRCS= Braunschweig coarse sand with gravel



Figure 4.1 Micrographs of Berlin sand (Left: 2000 µm, Right: 1000 µm)



Figure 4. 2 Micrographs of Cuxhaven fine sand (Left: 1000 µm, Right: 200 µm)



Figure 4.3 Micrographs of Cuxhaven medium sand (Left: 1000 µm, Right: 500 µm)



Figure 4. 4 Micrographs of Braunschweig coarse sand with gravel (Left: 2000 μm, Right: 2000 μm)



Figure 4. 5 Grain size distribution of the used sands in this study according to German Industry Standard (DIN 18123 (1996))

4.3 Specimen Preparation

Two methods were adopted to prepare specimens in this study, one is raining technique, and the other is tamping method. Both methods make specimens inside the triaxial chamber. Cylindrical specimens of 5 cm in diameter and 10 cm in height were prepared no matter which technique was taken. In general, the raining technique was used to prepare dry specimens with the relative densities lower than 50%, and the tamping technique was used to make wet specimens and those specimens having the relative densities larger than 50%. In the following sections, these two methods will be addressed in detail.

4.3.1 Tamping Method

Tamping method has an advantage of raining method in controlling the density of specimen even though the operator is not very much familiar with it. Tools used in this method are shown in Figure 4.6. They are five cups to contain soil, one split specimen mold with two tubes connecting to a vacuum, callipers, funnel, scoop, screwdrivers, feather brush, mold clamp, scissors, mold extension, membrane, two strips of filter paper, three O-rings, spirit bubble, and tamping hammer.

For getting a uniform density, the under-compaction effect induced by the energy from tamping next layers during specimen preparation. The under-compaction of each layer can be calculated by the following equation, and the relative parameters are illustrated in Figure 4.7.

$$\mathbf{u}_n = \frac{\mathbf{u}_1}{m-1} \times (m-n) \tag{4.1}$$

$$h_n = \frac{H}{m} \times ((n-1) + (1 + u_n / 100))$$
(4.2)

$$h_n^* = h_{rel} - h_n \tag{4.3}$$

in which, u_n is the under-compaction of the nth layer, u_1 is the under-compaction of the first layer, giving by experience (3% is taken for the sands used in this study), *m* and *n* are the total number and number of layer, respectively, h_n is the height between the top of the nth layer and top plane of filter stone, *H* is the height between the top of the split mold and top plane of the filter stone (102.35 mm is the length of the used mold), h_n^* is the controlling length of tamping hammer for the nth layer tamping, and h_{rel} is the height between the top of extension and the top of bottom filter stone.



Figure 4.6 Tools and accessories used for specimen preparation in this study



Figure 4. 7 Sketch map of specimen preparation by tamping method and relative controlled parameters

Specimens were tamping in five layers; the total mass required can be readily calculated based on the knowledge of the potential volume of specimen and sand specific gravity, maximum and minimum void ratios, and the desired relative density. Five parts of sand with identical mass were stored in five glass cups as shown in Figure 4.6. The progress of specimen preparation is shown in Figure 4.8. The steps of specimen preparation are given as follows.

- (1) Weigh five parts of sand with identical mass, and store in five cups;
- (2) Cut the membrane in an appropriate length, and place it on the base pedestal, then entangle two O-rings to seal the membrane to the pedestal with the help of the specimen mold;
- (3) Place the split mold on the pedestal, and fix it with clamp, then entangle one O-ring to the mold head, connect the vacuum to the mold and drain tube and close the drain valves.
- (4) Insert two trips of filter paper, and out entangle the membrane to mold head, and then apply -30 kPa by the vacuum to ensure the membrane is well stick to the mold wall; thereafter mount the mold extension;
- (5) Place the funnel with the extensional tube stays on the filter stone inside the mold, then pluviate dry sand into the funnel, then slowly fall sand by lifting the funnel with tube mouth gently contacting sand surface. For wet sample preparation, sand is immediately fallen into the mold after being mixed with given amount of water to avoid too much water evaporation.



Figure 4.8 Steps of specimen preparation by tamping method used in this study

- (6) Adjust the length of tamping hammer according to Equation 4.3, and then use the feather brush to gently flatten the sand surface, thereafter, tamp the sand surface to the desired elevation. Repeat this step to complete sand tamping.
- (7) Carefully mount the top cap to the specimen after the mold extension is removed, then open the drain values to apply -30 kPa to hold the specimen, then remove the negative pressure which is used to hold the membrane stick to the mold wall.
- (8) Up push the membrane and entangle the O-ring to seal the membrane to the top cap, then dismount the split mold and the preparation steps are complete (see Figure 4.8-8).

4.3.2 Raining Method

Steps of raining method are similar to those of tamping method addressed above except missing Step 6 and a little bit difference in Step 5. Normally, this method can just be suitable for dry specimen preparation; the desired density is obtained by controlling the opening and falling height from funnel mouth to the sand surface. During the falling, the height between the mouth and sand surface is kept stable to get a uniform specimen. At the end of Step 5, the top of specimen is flattened by a scraper. The mass of specimen can be measured by the distance of sand mass weighed before and after sample preparation.

4.3.3 Dimension Measurement

After specimen is prepared, the next step followed is dimension measurement. Normally, five elevations of specimen with heights of rough 1 cm, 3 cm, 5 cm, 7 cm and 9 cm from the top of bottom filter stone are measured, and the mean value is taken as the diameter; three locations distributing in 120 degrees interval on the top view of the circle are measured, and the mean value is taken as the height of specimen, measurement were conducted as shown in Figure 4.9. All performances of measurement were carried out by the callipers with the precision of 0.01 mm.



Figure 4.9 Dimension measurement by callipers (Left: Diameter, Right: Height)

4.3.4 Deviation in Dimension Measurement

The precision of dimension measurement is an important and difficult subject in geotechnical investigation both in the laboratory and in situ. The deviation in measurement may be caused by precision of measuring instruments, measuring methods, and operators, and so forth. Among these factors, the deviation intruded by operators may be the primary factor, and it can not be eliminated even the operator is very skilled.

The core investigation of this study involves preloading effects on dynamic properties of sand, the dimension of specimen may vary a lot during the preloading is applying. The accumulated axial strain at the end of preloading on medium dens specimen may be up to over 3% after a large number of preloading cycles under large vibration amplitude. No considering the change of dimension may introduce some errors in results analysis; therefore, it is very of importance to know the deviation in specimen dimension caused by measuring. Herein, the deviation caused by operator is addressed by taking three density specimens as example. In addition, the deviation in small-strain shear modulus introduced by the measuring deviation in specimen dimensions is also discussed in the end of this section.

Precision Evaluation

In this section, three relative densities of 30%, 60%, and 90% of Berlin sand are here analyzed for the evaluation of the precision of dimension measurement. Standard cylindrical specimens prepared in this study may have a dimension of 4.900 cm in diameter and 10.150 cm in height for the Dr=30% specimen, 4.970 cm in diameter and 10.230 cm in height for the Dr=60% specimen, 5.000 cm in diameter and 10.240 cm in height for the Dr=90% specimen based on a large number of dimension measurement data. The basic physical properties are shown in Table 4.1. Based on previous experience of measurement, the range of measuring deviation in diameter may vary from -0.02 cm to 0.02 cm with the increment of 0.005 cm, and that of height may vary from -0.01 cm to 0.01 cm with the increment of 0.005 cm.

Figure 4.10 illustrates the variation of relative density with deviation in specimen dimension for three different reference densities specimen. It is shown in this that, the measured relative density may range from 25.7% to 34.3% for the Dr=30% specimen, from 56% to 64% for the Dr=60% specimen, and from 86.3% to 93.7% for the Dr=90% specimen, respectively, under the same range of deviation in specimen dimension. The total range of deviation in relative density is 8.6% for the Dr=30% specimen, 8% for the Dr=60% specimen, and 7.4% for the Dr=90% specimen, respectively. The total deviation of relative density seems decreasing with the relative density of specimen, which may be partially influenced by the choosing dimension of the reference specimen.

It should be noted that the deviation in relative density of relative dense specimen normally is lower than that of relative looser specimen; the cause of this deviation is due to higher precision of dimension measurement of relative dense specimen. As known, the fact that the looser specimen is easier compressed than that of denser one during the measurement is carried out by calipers, results in the reading being much more deviation even the pressure applied by hand can be well controlled.



Figure 4. 10 Variation of relative density with deviation in diameter at various deviations in height for various density specimens

Deviation in Small-strain Shear Modulus

Deviation in small-strain shear modulus analyzed herein is based on the deviation of specimen dimension measurement presented above. Several confining pressures are used to analyze this effect, they are 15 kPa, 25 kPa, 50 kPa, 100 kPa, 200 kPa, and 400 kPa, and 800 kPa. The empirical equation (Equation 5.4) of small-strain shear modulus proposed for Berlin sand is adopted for this purpose, the deriving process will be presented in Chapter 5 later. The maximum positive and negative deviations of dimension are adopted to analyze their influence on small-strain shear modulus. They are 0.02 cm in diameter and 0.01 in height for positive value, and 0.02 cm in diameter and 0.01 in height for negative value, respectively.

Figure 4.11 illustrates the variation in small-strain shear modulus with isotropic confining pressure due to the maximum positive and negative deviations for three reference density specimens. Figure 4.12 presents the deviation of small-strain shear modulus of these three reference specimens with confining pressure due to the

deviation of measuring dimension. It is seen that in this figure, the deviation of small-strain shear modulus increase with confining pressure for all specimens, this deviation increases with the density of specimen as well. The deviation under the confining pressure of 800 kPa ranges from -23.3 MPa to 24.5 MPa for the 30% specimen, from -16.5 MPa to 17.3 MPa for the 60% specimen, and from -11.7 MPa to 13.3 MPa for the 30% specimen for the range of dimension deviation from -0.02 cm to 0.02 cm in diameter and from -0.01 cm to 0.01 cm in height, respectively. The relative deviation in small-strain shear modulus under any confining pressure level is relative small, it ranges from -3.7% to 3.9% for the Dr=30% specimen, from -3.8% to 4.0% for the Dr=60% specimen, and from -4.0% to 4.2% for the Dr=90% specimen, respectively.

Actually, the measuring relative density of specimen normally deviates from the expected density by -2% to 3% based on a large number of measuring data, which implies that the actual deviation of relative density may less. The deviation range of relative density falls into the range of the deviation range which has been discussed above.

Based on the discussion foregoing, one can conclude that the influence of deviation in dimension measuring on small-strain shear modulus is relatively less; therefore the precision of dimension measurement can be acceptable.



Figure 4. 11 Variation of small-strain shear modulus due to the deviation of measuring dimension for various reference specimens



Figure 4. 12 Deviation in small-strain shear modulus due to the deviation of measuring dimension for three reference specimens

4.4 Testing System Installation

After specimen is prepared as shown in Figure 4.8-8, the installation of drive system is followed as the following steps.

- (1) Rigidly mount the hollow support cylinder on the base plate by six screws (Figure 4.13-1 and 4.13-2).
- (2) Place the drive system on the support cylinder and gently adjust the drive plate in place (Figure 4.13-3).
- (3) Connect the accelerometer cable to the accelerometer mounted on the drive plate (Figure 4.13-4).
- (4) Place four screw shoots to the bottom of leveling screws in case that the penetration of the screws into the top of support cylinder occurs when the drive system is tighten to support cylinder (Figure 4.13-5).
- (5) Carefully screw in four screws to tightly connect the drive plate to the top cap (Figure 4.13-6-4.13-8).
- (6) Level and center the drive coils to ensure the permanent magnets staying at the centers of drive coils for conventional resonant column test, and slightly higher of the centers of coils to have sufficient place for the sediment of drive plate (Figure 4.14-1 and 4.14-2).
- (7) Screw the fixing screws to ensure the drive system rigidly connected to support cylinder tightened on the base plate (Figure 4.14-3).
- (8) Connect the coil cables to their corresponding connection ports (Figure 4.14-4 and 4.15-1). Now make a trial resonant column test at minimum small strain level to test if the test is successful, if not, find the problem.



Figure 4. 13 Steps of drive system installation of resonant column apparatus (1)

- (9) Mount the LVDT on the top of drive plate and connect the LVDT cable to its connection port (Figure 4.15-2 and 4.15-3). Then go to computer to read the LVDT reading to test if the connection is correct or not. If correct, fix the top plate to drive coil boxes (Figure 4.15-4).
- (10) Mount the proximeter to the top plate and adjust the proximeter an appropriate location, normally 1.4 mm is taken as the reasonal distance between the proximeter and metal target plate considering the accurate measurement scope of the used proximeter is within the range from 0.2 mm to 3.0 mm (Figure 4.15-5).
- (11) Lower the up part of confining chamber in place and fix it to the base plate by six dig screws (Figure 4.15-6).

Note that, if the torsional shear test is not conducted, Steps 10 and 11 are not included for the system set up. In addition, the bender elements are permanent mounted in the base pedestal and the top cap; therefore, the installation is automatically completed after the drive system installation is complete.



Figure 4. 14 Steps of drive system installation of resonant column apparatus (2)



Figure 4. 15 Steps of drive system installation of resonant column apparatus (3)

4.5 Testing Procedures

4.5.1 Confining Pressure

Dynamic properties of Berlin sand (Chapter 5)

The confining pressure mentioned in this study is isotropic pressure; its range varied from 10 kPa to 800 kPa. For stage tests, the pressure was applied starting from low stress level to the maximum of 800 kPa for small-strain shear modulus and damping

ratio tests. The nonlinear dynamic properties were measured under the maximum confining pressure for each specimen, these maximum pressures were 50 kPa, 100 kPa, 200 kPa, 400 kPa, and 800 kPa, the test for each sample was stopped after the high strain amplitude dynamic shear modulus and damping properties had been complete.

Preloading Effects on Dynamic sand Properties (Chapter 6)

Three levels of confining pressure, under which the preloading was applied to specimens, were adopted in this study. They are 50 kPa, 100, and 200 kPa, based on the fact that long term vibrations normally occur at a relative shallow depth under ground, which involving traffic engineering, wind power plant, offshore engineering, machine foundation, and so forth. In addition, the confining pressure may increase from 50 kPa to 400 kPa after the preloading under 50kPa was completed to investigate reloading effect on the small-strain shear modulus, and decrease from 200 kPa to 10 kPa after the preloading under 200kPa was completed to investigate unloading effect on the small-strain shear modulus.

4.5.2 Specimen Density

The densities of specimens range from 18% to 90.6% were prepared for the investigation of dynamic properties of Berlin sand, and the empirical equation for predicting small-strain shear modulus was proposed based on this range of density. The nonlinear dynamic properties were primarily tested on four groups of the densities of 32.5%-34.4%, 48.1%-49.6%, 62.2%-63.7%, and 77.5%-78.2%.

Specimens of medium dense density around 40% and dense density around 75% were prepared to study the preloading frequency effects and preloading ratio on dynamic properties of sand. In addition, two specimens with the Dr=88% and 96.4% were prepared for the preloading ratio effect as well. Five specimens of the relative densities around 40% and one specimen of 90.4% relative density and the 96.4% density specimen prepared for the preloading ratio effect were reloaded to study the reloading effect on small-strain dynamic properties. Nine medium density (Dr=40%) and six high density (Dr=90%) Berlin sand specimens and three other type medium dense sand specimens were prepared to investigate the unloading effect after preloading.

Seven Berlin sand specimens with the relative density ranging from 42.2% to 55.9% and one high dense specimen with relative density of 85.5% are prepared to study the water content effect on dynamic properties of Berlin sand. **4.5.3 Water Content**

Four medium dense specimens with the relative density ranging from 51.3% to 55.9% were prepared with the water content ranging from 0% to 9% in the in increment of 3%, they were initially tested at small strain amplitude under the confining pressure increasing from 15 kPa 200 kPa to investigate the water content influence on small-strain dynamic properties, thereafter the preloading stress amplitude of 40 kPa was applied to the specimens under the confining pressure of 200 kPa up to 100,000 cycles of vibration except for the Dr=51.3% specimen with 6% water content
specimen which was subjected to 800,000 cycles of preloading. The 6% and 9% water content specimens were made the nonlinear dynamic properties tests after the vibration rested for 10 minutes. And the 0% and 3% water content specimens were conducted small-strain shear modulus tests at unloading path of confining pressure from 200 kPa to 15 kpa after the pressure was unloaded the desired level for 10 minutes, then the confining pressure was reloaded to 200 kPa to make the nonlinear dynamic properties tests after the pressure was stable for 10 minutes.

In addition, two medium dense specimens with relative density of 55.5% and 55.8% were prepared with water content of 3% to study the water content effects under the confining pressure of 50 kPa and 100 kPa. The Dr=55.8% and Dr=55.5% specimens were vibrated with the preloading stress of 10 kPa and 20 kPa to 100,000 cycles under the confining pressure of 50 kPa and 100 kPa, respectively. After the vibration was complete, the Dr=55.8% specimen was made the small-strain dynamic properties tests under high confining pressure by increasing the pressure up to 200 kPa, meanwhile made the nonlinear dynamic properties tests as well. As for the Dr=55.5% specimen, after the preloading was complete, the confining pressure was unloaded to study the effect of water content on small-strain shear modulus at unloading path.

4.5.4 Shearing Strain Amplitude

As reported in previous investigations, strain has an extreme influence on dynamic properties of soil; therefore tests should be carried out at definite strain amplitude. For small-strain tests, the shearing strain amplitude was controlled bellow 0.001% for any test in this study. As for the nonlinear dynamic properties tests, the shearing strain amplitude was increased from possible low level to possible high level, in this study, normally the shearing strain amplitude is range from 0.0001% to 0.09%.

4.5.5 Number of Preloading Cycles

In previous investigations, most researchers applied the prestraining on specimens by resonant vibration mode; the starting previous vibration number normally was beyond several hundred cycles, the starting number of cycles was 1,000 for Drnevich (1967), , 10,000 for Wichtmann and Triantafyllidis (2004), the absence of low number data is attributed to the resonant vibration mode by which around hundreds of vibrating cycles are necessary to find the system vibration resonant frequency. Therefore the influence of low number of cycles on small-strain shear modulus could not be studied when applied the previbration by resonant column mode. In this study, the influence of number of cycles can be studied from the first cycle to very large cycles as expected by preloading concept (stress-controlled) replacing prestraining concept (strain-controlled). Thus the absent information of previous works could be well supplemented in this study. The number of preloading cycles applied for each specimen is presented on its corresponding curve.

4.5.6 Preloading Stress and Frequency

In previous investigations, the prestraining level was generally adopted as the

controlling vibration parameter. Actually, however, the shearing strain amplitude calculated in resonant column tests is a function of resonant frequency (f_r) and the voltage output (V_{out}) from charge amplifier corresponding to this resonant frequency. In resonant column test, even if the same the voltage input (V_{in}) is applied, the V_{out} and f_r corresponding to this V_{in} may change with the stiffness (shear modulus) of specimen, therefore the shearing strain amplitude may vary due to the change in these parameters during the test. Hence, one should alter the input voltage after each stack of vibration to achieve constant shearing strain amplitude, as well as the vibration frequency. However, it is impossible to keep changing these parameters if a large number of cycles are applied to specimen. Therefore, in previous researches, the shearing strain amplitude was approximately kept by high strain modulus test by changing the input voltage value and frequency (Drnevich and Richart 1970; Wichtmann and Triantafyllidis 2004). Thus the prestraining is not at a constant level using this concept as the change of the stiffness.

In practice, the dynamic response is not strain controlled but stress controlled problem, therefore using dynamic preloading standing for prestraining has more practical significance than, furthermore, using preloading can also overcome the disadvantage of the deviation due to the usage of prestraining to study the effect of vibration history on dynamic properties of sand. The calculation of preloading stress is based on the torsional shear test calibration method described in Chapter 3.

The preloading stress amplitudes used in this study were 5 kPa, 10 kPa, 15 kPa, and 20 kPa for the case of the confining pressure is equal to 50 kPa to investigate the preloading ratio effects; 20 kPa for the case of the confining pressure is equal to 100 kPa to investigate the preloading frequency effects; and 40 kPa for the case that specimens were vibrated under the confining pressure of 200 kPa.

4.5.7 Pressure Release Effects

Pressure release effect was accidentally inspired by one trial preloading test on on medium sand under the confining pressure of 100 kPa. This test was carried out by applying preloading stress amplitude of 20 kPa at the frequencies of 10 Hz. Figure 4.13 illustrate the variation of small-strain shear modulus with number of preloading cycles including the pressure release effect on small-strain shear modulus. The specimen was prepared by raining sand method; there was a flaw in the shape of the specimen that the size of two ends were much bigger than the middle due to the air leakage when the vacuum was applied to suck the membrane to stick on the sample mold. As the small-strain shear modulus was complete after 20,000 cycles of preloading, the preloading was applied again; unfortunately, however, as the vibration continuously further ran for around 1,000 cycle, one magnet turned to lean on the coils of drive system thanks to uneven sediment of specimen and partial slippage between the top cap and specimen, which resulted in the preloading had to be stopped to adjust the coils the right place. The same problem occurred again after 15,000 cycles of vibration was complete, and confining pressure had to be released and applied the vacuum to hold the specimen during the confining pressure was removed. For these to stage, the vacuum was applied to 30 kPa as the level during the preparation of specimen. Additionally, to further confirm the pressure release effect on dynamic shear modulus, confining pressure was intentionally unloaded to 30 kPa for

20 minutes after applying 200,000 cycles of preloading, then reloaded the pressure to 100 kPa again to determine the small-strain shear modulus after pressure release. Thereafter the preloading vibration was applied to 1,000,000 cycles.

As shown in Figure 4.13, the small-strain shear modulus after pressure release jumps much higher its corresponding value before pressure release. As the vibration continues the small-strain shear modulus rapidly reduce, if one connects the those data points without pressure release the $G_{max}(Nc)$ curve becomes near smooth sunken in shape.

To systemically investigate this effect, two dense specimens with similar density were subjected to 100,000 cycles of preloading with stress amplitude of 40 kPa at the frequency of 20 Hz under 200 kPa. The confining pressure was gradually unloaded to a lower pressure and held for 10 minutes, then reloaded to 200 kPa to make the small-strain shear modulus test, repeated this procedure to the minimum pressure of 25 kPa.



Figure 4. 16 Development of small-strain shear modulus with number of cycles after confining pressure release for Berlin sand under the confining pressure of 100 kPa.

4.5.8 Accumulated Axial Strain

The accumulated axial strain was monitored by a linear variable differential transducer (LVDT) mounted at top center of drive plate for each test in this study. Based on the LVDT data, the variation of void ratio during testing was obtained. The calculation process and discussion will be presented in Chapter 6 later. In addition, the accumulated axial displacement can also used to recalculate the distance between two bender elements mounted at the top cap and bottom pedestal when the bender element test is performed.

For getting the LVDT data, the initial reading was set zero after the cell chamber was fixed before the confining pressure was applied. The negative readings mean specimen sediment and positive readings indicate extension during tests.

4.5.9 Bender Element Test

For the purpose of comparison of the small-strain shear modulus obtained from resonant column tests, some bender element tests were made on the same specimen. Normally, the bender element test was carried out before the resonant column test was performed to avoid the influence of vibration. The testing results presented in this study are the data obtained from a sinusoidal signal with excitation frequency of 10 Hz. The LVDT was used to monitor the variation of the height of specimen, and the height of specimen could be changed all the time during testing. Note that the tested velocity was shear wave velocity, and the compression wave velocity was not determined in this study.

4.6 Summary

The properties of testing four used sands are presented in detail in the beginning parts, the tamping and raining method of specimen preparation are detailedly descried, influence of deviation in specimen dimension on the relative density of sample is presented, within the deviation range from -0.02 cm to 0.02 cm for diameter and from -0.01 cm to 0.01 cm for height. Meanwhile the influence of deviation in dimension on small-strain shear modulus of Berlin sand are analyzed, and finally testing procedures are addressed in detail in this chapter.

CHAPTER 5 DYNAMIC PROPERTIES OF BERLIN SAND

5.1 Introduction

Berlin sand extensively distributes in Berlin area, which have been employed in many practical projects, in which many geotechnical problem are observed especially dynamic problems which are related to traffic foundations, and tunneling. It is of importance to well understand the dynamic properties of Berlin sand not only for its basic dynamic behaviors but also giving evidence to analyze the dynamic preloading effects on dynamic behaviors of sand in Chapter 6.

5.2 Small-strain Shear Modulus

By definition, the small-strain shear modulus, or maximum shear modulus, G_{max} , is assumed to be constant with increasing shearing strain amplitude when the strain level is lower than a cyclic threshold, which is called elastic or linear cyclic threshold shearing strain, γ_a . Berlin sand is nonplastic material, whose γ_a is assumed to be 4×10^{-6} according to large numbers of testing data by resonant column test. Drnevich and Richart (1970) reported that sand could be vibrated for many cycles at the shearing strain amplitude less than 1×10^{-4} without appreciable change in density and shear modulus. For the purpose of practical application, Hardin and Drnevich (1972) suggested 2.5×10^{-5} as the linear shearing strain threshold to determine the G_{max} . Some researchers (Hardin and Richart 1963; Isenhower and Stokoe 1981; Shen, Li et al. 1985; Qian, Gray et al. 1993) has reported that when the shearing strain amplitude less than 1×10^{-5} , shearing strain influence on shear modulus of various soils is negligible. As mentioned in Chapter 2, the γ_a was defined as the strain at which the ratio of the modulus to the G_{max} is 0.99, it ranges from 4×10^{-6} to 4×10^{-5} with an increase in plasticity index of soils (Vucetic 1994).

Figure 5.1 shows the reduction tendency of shear modulus with increasing shearing strain amplitude for Berlin sand. It is seen that the reduction of G_{max} is less than 1% when the shearing stain is less than 6×10^{-6} for confining pressure ranging from 50 kPa to 800 kPa. The shear modulus measured at the strain of 1×10^{-5} is 0.97 of that measured at 6×10^{-6} under confining pressure of 50 kPa, which means that using the strain of 1×10^{-5} to determine the G_{max} is not reliable when the confining pressure is relatively low. Therefore, the G_{max} should be determined at the shearing strain amplitude below 6×10^{-6} for Berlin sand if possible. However, for some very low confining pressure tests, it is very difficult to reach this small strain due to the limit of instrument to read a clear signal at very low vibrating amplitude thanks to the softer sample. Consequently, the small-strain shear moduli measured in this study is actually measured at strain larger than 6×10^{-6} but lower than 1×10^{-5} for low pressure tests.



Figure 5.1 Normalized modulus curves at various confining pressures for Berlin sand

5.2.1 Effect of Void Ratio

Small-strain shear moduli of Berlin sand with various densities determined under four levels of confining pressure are plotted on Figure 5.2. As shown in this figure, the G_{max} decreases with increasing void ratio, which is in agreement with previous results in literature. It seems that the decrease of modulus with void ratio is faster for higher confining pressure than for lower pressure. This phenomenon is possibly caused by the influence of confining pressure. To eliminate the influence of confining pressure, the G_{max} is normalized by square root of confining pressures ($\sigma_0^{0.5}$), which was suggested by most investigators. The analyzing results are plotted in Figure 5.3. The variation of shear modulus with void ratio can be approximately expressed by the following empirical equation:

$$G_{\max} / \sigma_0^{0.5} = S / (0.3 + 0.7e)^{b}$$
(5.1)

$$F(e) = \frac{1}{(0.3 + 0.7e)^{b}}$$
(5.2)

in which, G_{max} and σ_0 are small-strain shear modulus and isotropic confining pressure, respectively, have the unit in kilo pascal (kPa); *S*=4285 is stiffness coefficient, and b=2.96 is fitting parameter.

For the purpose of comparison, previous empirical equations were also employed to fit these data; unfortunately, however, these equations fail to predict these data, this failure exhibits not only in large maximum deviation but also low correction coefficient as depicted in Figure 5.3.



Figure 5.2 Variation of Small-strain shear modulus with void ratio under different confining pressures



Figure 5.3 Relationship between small-strain shear modulus normalized by square root of confining pressure and void ratio of Berlin sand

5.2.2 Effect of Confining Pressure

Confining pressure and void ratio are acknowledged as two of the most important factors which affect the stiffness of particulate materials. It has been extensively confirmed that shear modulus increases with an increase in confining pressure.

Figure 5.4 plots the relationship between small-strain shear modulus and isotropic confining pressure both in logarithmic scale for various densities of Berlin sand. It is shown that the relationship could be well fitted by exponential function. As Figure 5.4 shown, it seems that the pressure effect exponent, n, slightly decreases from 0.515 to 0.477 with an increase in relative density from 33.7% to 90.6%. As we all know, the density of sand may increase with increasing pressure which it is subjected to, the magnitude of increase is greater for lower density sample than for higher density sample. However, the densities of samples are assumed constant under all stress level during tests. As a result, shear modulus shows a slightly higher pressure dependency for lower density sand.

In despite of the slight variation of the pressure exponent, n, the variation of small-strain shear modulus with confining pressure well follows linear relation on double logarithmic graph, which well agrees with existing results.



Figure 5.4 Relationship between small-strain shear modulus and confining pressure of Berlin sand

Measured small-strain shear moduli are normalized void ratio function Equation 5.2, for comparison, they were also normalized by the following void ratio functions: $F(e)=1/(0.3+0.7e^2)$, $F(e)=(2.17-e)^2/(1+e)$, and $F(e)=1/(0.3+0.7e^{2.96})$, respectively. Fitting results are illustrated in Figure 5.5.

As illustrated in this figure, it can be clearly seen that the normalized modulus linearly increases with confining pressure for all these functions. Previous functions have less

capacity to fit the relationship between the normalized shear modulus and confining pressure. It should be noted that relationship between confining pressure and shear modulus normalized by Equation 5.2 is well fitted by $G_{\text{max}}/F(e)=S101.3^{(1-n)}(\sigma_0)^n$. This pressure power is 0.504, which is quite close to 0.5, which was employed to derive the void ratio function in this study.



Isotropic Confining Pressure, σ_0 (kPa)

Figure 5.5 Influence of isotropic confining pressure on the small-strain shear modulus normalized by various void ratio functions

5.2.3 Empirical Equation

5.2.3.1 Equation proposal

In Sections 5.2.2 and 5.2.3, the effects of confining pressure and void ratio on G_{max} are discussed separately; in fact, however, both factors jointly influence the G_{max} , it is difficult to purely analyze one effect by separating this effect from other effect. In this section, data obtained from 43 specimens with the void ratios vary from 0.38 to 0.608. The tested confining pressures vary from 10 kPa to 800 kPa. The data analysis code *EasyPlot4* was used to perform the three D curve fitting.

Considering the void ratio function proposed in Section 5.2.1, the following expression is used to fit all data,

$$G_{\max}(e,\sigma_0) = \frac{S}{(0.3+0.7e)^b} p_a^{(1-n)} \sigma_0^n$$
(5.3)

in which, p_a is atmospheric pressure, and the standard value of 101.3 kPa is adopted in this study, *b* and *n* are the powers of void ratio and confining pressure, respectively, the others are the same as previous notations. The fitting results are presented in Figure 5.6.



Figure 5. 6 Variation of small-strain shear modulus with void ratio and confining pressure for Berlin sand

To examine the validity of the proposed empirical equation, some classic empirical equations are also used to fitting these data. The relative fitting parameters are listed in Table 5.1 On the basis of the comparison of the maximum deviation of these fitting empirical equations, it can be seen that the first two expressions with similar fitting parameters can best fit the whole data points; especially the second expression not only has the pressure power of 0.5, which is the most popular value suggested in literature, but the minimum deviation. Therefore, Equation 5.4 is recommended to predict the small-strain shear modulus of dry sands.

$$G_{\max}(e,\sigma_0) = \frac{S}{(0.3+0.7e)^{2.67}} p_a^{(1-n)} \sigma_0^n$$
(5.4)

The meanings of these notations are the same as Equation 5.3, for Berlin sand the pressure power, n=0.5, S=488, G_{max} and σ_0 have the same unit in kPa.

No.	Expressions	S	b	n	Mean Dev (MPa)	Max dev. (MPa)
1	$G_{\max}(e, p) = S/(0.3 + 0.7e)^b p_a^{(1-n)} \sigma_0^n$	490	2.68	0.494	3.56	24.6
2	$G_{\max}(e,p) = S/(0.3+0.7e)^b p_a^{0.5} \sigma_0^{0.5}$	488	2.67	0.500	2.66	23.8
3	$G_{\max}(e, p) = S/(0.3 + 0.7e)^2 p_a^{(1-n)} \sigma_0^n$	663	2	0.499	6.05	37.4
4	$G_{\max}(e, p) = S/(0.3 + 0.7e^2) p_a^{(1-n)} \sigma_0^n$	751		0.504	7.54	55.8
5	$G_{\max}(e,p) = S(2.17-e)^2 / (1+e) p_a^{(1-n)} \sigma_0^n$	851		0.500	6.95	43.5
6	$G_{\max}(e,p) = S(2.97-e)^2 / (1+e) p_a^{(1-n)} \sigma_0^n$	388		0.504	7.51	59.9

Table 5. 1 Laboratory parameters of Berlin sand for various G_{max} predicting equations

Note: $p_a = 101.3$ kPa is atmospheric pressure

5.2.3.2 Application to other sands

Four other type sands were tested by resonant column apparatus to examine the validity of the applicability of Equation 5.4. Equations N0.3 to No.5 in Table 5.1 are employed to compare with Equation 5.4. The following general expression stands for these equations,

$$G_{\max}(e,\sigma_0) = SF(e) p_a^{(1-n)} \sigma_0^n$$
(5.5)

As shown in Table 5.2 and Figure 5.7, all these empirical equations can very well fit test results with identical pressure power, correlative coefficient, and maximum deviation but different stiffness coefficients. This well agreement confirms the validity of proposed equation to other sources of soils.

F(e)	Parameters	Sands					
		BRFS	BRCS	CFS	CMS		
	S	574	405	653	519		
1	n	0.484	0.603	0.490	0.476		
$(0.3+0.7e)^{2.67}$	r ²	1	0.999	0.999	1		
	Max dev.(kPa)	727	1770	3705	2008		
	S	667	571	748	710		
1	n	0.484	0.603	0.490	0.476		
$(0.3+0.7e^2)$	r ²	1	0.999	0.999	1		
	Max dev.(kPa)	727	1770	3705	2008		
	S	654	523	736	658		
1	n	0.484	0.603	0.490	0.476		
$(0.3+0.7e)^2$	r ²	1	0.999	0.999	1		
	Max dev.(kPa)	727	1770	3705	2008		
	S	833	658	944	826		
$(2.17 - e)^2$	n	0.484	0.603	0.490	0.476		
(1+e)	r ²	1	0.999	0.999	1		
	Max dev.(kPa)	727	1770	3705	2008		

Table	5.	2	Laboratory	fitting	parameters	of	four	sands	for	various	G_{\max}	predicting
			equations									



Figure 5. 7 Comparison of Equation 5.4 with previous equations for four different sands

5.2.4 Effect of stress history

It is recognized that small-strain shear moduli of soils increase with overconsolidation ratio (OCR), especially for clayey soils; Hardin and Richart (1963), Hardin and Black (1966) reported that the OCR had only small or no effect on the small-strain shear wave for sand, others also reported the OCR had negligible effect on shear modulus of sand. In theory, as soil is loaded to higher stress, the relative movement between particles occurs resulting in soil having higher density, resulting in an increase in G_{max} . To examine this effect on sand, three different densities Berlin sand were tested under confining pressure varying from 10 kPa to 800 kPa.

On basis of previous knowledge, OCR has the positive increase effect on shear modulus of soils, the relative increase of G_{max} , I_G , is expressed as

$$I_{G} = \frac{G_{\max}(\text{Unloading}) - G_{\max}(\text{Loading})}{G_{\max}(\text{Loading})} \times 100\%$$
(5.5)

in which, I_G is relative increase of G_{max} , has the unite of %; G_{max} (Loading) and G_{max} (Unloading) are the G_{max} determined at loading and unloading paths of confining pressure, respectively.

The testing results of three density samples are illustrated in Figure 5.8. Unlike previous observation, the small-strain shear moduli obtained at loading path slightly keep higher than their counterparts at unloading path for these samples but the scatter

value obtained under 150 kPa for the Dr=64.2% sample. As indicated in Figures 5.8 and 5.9, the magnitudes of the distance in G_{max} between loading and unloading paths are observed larger for the Dr=88.2% and Dr=39.7% samples which subjected to the maximum confining pressure of 800 kPa than for the Dr=39.7% sample experienced the maximum confining pressure of 200 kPa. The decrease in G_{max} is dependent on the maximum confining pressure which the sample has experienced. The maximum reduction of G_{max} may be up to 8% for these two samples which have experienced to 800 kPa.



Figure 5.8 Comparison of Small-strain shear modulus at loading and unloading paths for three different densities of Berlin sand

For further investigate the influence of stress history, the overconsolidation ratio (OCR) is used to analyze this finding. The OCR is defined as follows:

$$OCR = \frac{\sigma_0(\max)}{\sigma_0}$$
(5.6)

in which, $\sigma_0(\max)$ is the maximum confining pressure which the sample experienced herein. The variation of I_G with OCR is illustrated in Figure 5.10

As shown in Figures 5.9 and 5.10, in general, the reduction degree of G_{max} with OCR is dependent upon the density of the specimen, which decreases with increasing the density. This decrease in G_{max} is thanks to the reorientation of soil particle in vertical direction induced by increasing confining pressure. As the density increases, the reorientation is not easy to occur, and further discussion on the change of microstructure will be addressed in Chapter 6 later. As the OCR continuously increases, the G_{max} starts to increases, which is accounted for the reduction of void ratio after high confining pressure.



Figure 5.9 Relative increase of G_{max} versus isotropic confining pressure



Figure 5. 10 Relative increase of $G_{max}(I_G)$ versus overconsolidation ratio (OCR)

5.2.5 Effect of Confinement Duration

Figure 5.11 illustrates the development of small-strain shear modulus with increasing confinement duration under confining pressure of 200 kPa for the relative density of 64.3% of dry Berlin sand. It can be seen that the G_{max} rapidly increases within the first 20 minutes especially within the first 10 minutes, thereafter slowly increases as time lapses, which indicates that the primary consolidation is complete after 20 minutes. As a whole, the increase in G_{max} with time is small even if the sample rests under the constant pressure for 200 minutes; the increase in G_{max} at the 10th and 200th minutes

merely reaches 1% and 3% compared to that at the 3th minute. Therefore, to measure the G_{max} obtained at the 10th minute is acceptable in this study for shortening testing period. Normally influence of time on the shear modulus of sand is unimportant compared to those of confining pressure and void ratio.



Figure 5. 11 Variation of small-strain shear modulus with confinement duration under confining pressure of 200 kPa for the Dr=64.2% Berlin sand

5.2.6 Effect of Water Content

Four similar densities Berlin sand samples were tested to investigate the influence of water content on small-strain shear modulus and damping ratio (will presented in Section 5.4.3), water contents of 0%, 3%, 6%, and 9% were used to prepare the specimens. The test results are presented in Figure 5.12. To eliminate the influence of void ratio, the G_{max} is also normalized by $F(e) = 1/(0.3 + 0.7e)^{2.67}$, the results are presented in Figures 5.12 and 5.13 that moisture has certain influence on the G_{max} of sand; the G_{max} of the w=3% sample is slightly higher than that of the identical dry sand. Small-strain shear modulus increases with increasing water content as it is lower than 6% beyond which it starts to decrease.

Figure 5.14 illustrates the percentage increase of $G_{max}/F(e)$ normalized by that of dry sample with water content based on the data depicted in Figure 5.13. The small-strain shear moduli determined under confining pressures of 50 kPa, 75 kPa, 100 kPa, 150 kPa, and 200 kPa were used to demonstrate this influence. For the purpose of analysis, the following equations are introduced:

$$N_G^e = \frac{G_{\max}}{F(e)} \tag{5.6}$$

$$I_{G}^{w} = \frac{N_{G}^{e}(\text{wet}) - N_{G}^{e}(\text{dry})}{N_{G}^{e}(\text{dry})} \times 100\%$$
(5.7)

in which, N_G^e is small-strain shear modulus normalized by $F(e)=1/(0.3+0.7e)^{2.67}$; I_G^w is the relative increase of G_{max} due to water content in percentage (%); N_G^e (wet) and N_G^e (dry) are G_{max} normalized by void ratio function for wet and dry samples, respectively, both have the unit in MPa;



Figure 5. 12 Variation of Small-strain shear modulus with confining pressure at various water contents for Berlin sand



Figure 5. 13 Influence of water content on normalized small-strain shear modulus against confining pressure

As addressed previously, Figure 5.14 clearly shows the increase in shear modulus with water content as it reaches the water content of 6%; thereafter the increase in shear modulus starts to decrease as water content continuously increases. It is very interesting to note that the increase magnitude of G_{max} is dependent on the confining pressure, namely, the I_G^w decreases with confining pressure as the water content is lower than 6%. The increase in shear modulus with increasing water content may reach the maximum 15.2% at water content of 6% under confining pressure of 50 kPa. One can conclude that there exists an optimum water content below which the shear modulus increases and beyond which decrease with increasing water content. This influence is affected by confining pressure, namely increasing confining pressure may decrease this influence.



Figure 5. 14 Variation of increase of normalized small-strain shear modulus with water content under various confining pressures

5.2.7 Effect of Sampling Technique

Raining sand and dry tamping methods were employed to build tested samples. It is very difficult to achieve the samples with absolutely identical density; therefore, to investigate this influence on dynamic properties of sand, the feasible way is to carry out the analysis by comparing the normalized shear modulus. Testing results of three different samples are presented in Figure 5.15, the small-strain shear moduli are normalized by the proposed void ratio function in this study.

As shown in this figure that samples prepared by tamping method have a higher stiffness than that of specimen prepared by raining method. The distance between these two values increases with an increase in sample density. The G_{max} of the Dr=77.3% sample by tamping method plots highest among these samples, which indicates a fact that particles are highly orientated due to large number of blows by tamping hammer, results in a larger number of contacts in vertical direction than samples prepared by raining method, consequently demonstrates higher stiffness.



Figure 5. 15 Sampling technique effect on small-strain shear modulus

5.3 Nonlinearity in Shear Modulus

5.3.1 Effect of Confining Pressure

Figures 5.16-5.19 present the degradation of shear modulus with shearing strain amplitude for four group samples with different densities under various confining pressures varying from 50 kPa to 800 kPa. It can be seen in these figures that, at the same shearing strain level, the shear modulus increases with increasing confining pressure. The distance in shear modulus under various confining pressures decreases with increasing shearing strain amplitude, which implies that shearing strain amplitude may eliminate the influence of confining pressure on shear modulus at very high strain level.

It should be noted that the shear modulus of lower density samples starts to increase with strain as the shearing strain amplitude exceeding a given strain threshold at rear part of curve, especially for those curves obtained from low confining pressure as shown in Figure 5.16. This increase with shearing strain amplitude is attributed to an increase in density after a given number (normally 1,000 cycles induced by the resonant column frequency finding) of high strain vibrations, especially for low density sand. As the density increases, this tendency vanishes, but the degradation of shear modulus with strain slows down as shown in Figure 5.16, the increase in shear modulus with increasing strain also disappears due to less reduction in void ratio under the 400 kPa curve. The threshold strain for increasing shear modulus increase as the confining pressure increase, under high confining pressure, the threshold strain is out of the range of testing strain.

Considering the influences of void ratio and other influences (such as deviation in measurement of dimension of specimen) on the shear modulus reduction curves, it is

much more convenient to normalize shear modulus by small-strain shear modulus for analyzing the nonlinear shear modulus behavior of soil. Figures 5.20-5.23 present the development of the normalized shear modulus and accumulated axial strain with shearing strain amplitude. For the purpose of analyzing the axial strain due to the high strain vibration, the accumulated axial strains presented in these figures are obtained by subtracting the total strain to the initial axial strain which is induced by the confining pressure. As shown in Figures 5.20-5.23, the normalized shear modulus reduction curve shifts to right higher position as confining pressure increases, which indicates that the degradation in shear modulus under higher confining pressure is less than under lower confining pressure at the same strain level.



Figure 5. 16 Shear modulus versus shearing strain amplitude for Berlin sand $(Dr=32.5\% \sim 34.4\%)$



Figure 5. 17 Shear modulus versus shearing strain amplitude for Berlin sand (Dr=48.1% ~ 49.6%)



Figure 5. 18 Shear modulus versus shearing strain amplitude for Berlin sand $(Dr=62.2\% \sim 63.7\%)$



Figure 5. 19 Shear modulus versus shearing strain amplitude for Berlin sand (Dr=77.5% ~ 78.0%)



Figure 5. 20 Normalized shear modulus and accumulated vertical strain versus shearing strain amplitude (Dr=33.1% ~ 34.2.0%)



Figure 5. 21 Normalized shear modulus and accumulated vertical strain versus shearing strain amplitude (Dr=48.1% ~ 49.6%)



Figure 5. 22 Normalized shear modulus and accumulated vertical strain versus shearing strain amplitude ($Dr=62.2\% \sim 63.7\%$)



Figure 5. 23 Normalized shear modulus and accumulated vertical strain versus shearing strain amplitude (Dr=77.5% ~ 78.0%)

Table 5.3 lists the linear and volumetric threshold strains for all samples. As listed in this table, these threshold strains are dependent on confining pressure; the density of sand shows no significant influence on the magnitude of linear threshold strain but on volumetric threshold strain. In general, the volumetric threshold strain increase with soil density. The linear threshold strain of Berlin sand is within the range from 0.00065% to 0.0014% under confining pressure from 50 kPa to 800 kPa.

Number	Polotivo donaitu	Confining	Linear threshold	Volumetric threshold
Number	Relative density	pressure (kPa)	strain [†]	strain [‡]
1	33.1%	50	0.0007%	0.0044%
2	33.2%	100	0.0009%	0.0051%
3	34.4%	200	0.0011%	0.0045%
4	32.5%	400	0.0012%	0.0056%
5	48.3%	50	0.0008%	0.0055%
6	48.1%	100	0.00075%	0.0061%
7	49.6%	200	0.00095%	0.0043%
8	48.1%	400	0.0011%	0.0042%
9	62.2%	50	0.0007%	0.0063%
10	63.7%	100	0.00065%	0.0042%
11	62.6%	200	0.0011%	0.0057%
12	62.6%	400	0.0014%	0.008%
13	78.0%	50	0.0007%	0.0038%
14	77.5%	100	0.0008%	0.0033%
15	77.7%	200	0.0085%	0.0038%
16	77.5%	400	0.0011%	0.0085%
17	78.8%	800	0.0014%	0.0148%

 Table 5.3
 Summarization of linear threshold and volumetric strains of Berlin sand

[†] Linear threshold strain is the strain at which G/G_{max} is 0.99

[‡] Volumetric threshold strain is the strain at which \mathcal{E}_{a}^{Acc} is 0.01% for resonant column test

5.3.2 Effect of Void Ratio

Data in Figures 5.16-5.19 are replotted at the same confining pressure with various densities and depicted in Figure 5. 24-5.27 except for the Dr=78.8% sample. The shear modulus analogically reduces with the shearing strain amplitude under each level of confining pressures except for that of the Dr=33.1% sample under confining pressure of 50 kPa, which increase with shearing strain amplitude exceeding 0.034% due to greater reduction of void ratio. Turnup tails are observed for the lowest density samples when the confining pressure is lower than 400 kPa. The influence of void ratio on shear modulus is affected by the shearing strain amplitude; the increase in shear modulus due to reduction of void ratio is diminished by the shearing strain amplitude similar to that of confining pressure. The normalized shear modulus reduction curves are plotted on the same diagram to analyze the effect of density. Figures 5.28-5.32 show that density has no significant influence on the position of the normalized shear modulus reduction curve under the same level of all confining pressures. Figure 5.31 illustrated the normalized shear modulus reduction curves for the samples with relative density ranging from 32.5% to 87.7% under confining pressure of 400 kPa. It is seen in this figure that all curves plot in a very narrow range, which further confirms the fact that void ratio has no significant influence on the reduction of shear modulus with shearing strain amplitude. The observation is in agreement with the conclusion reported in literature.



Figure 5. 24 Effect of density on shear modulus versus shearing strain amplitude reduction curves under confining pressure of 50 kPa



Figure 5. 25 Effect of density on shear modulus versus shearing strain amplitude reduction curves under confining pressure of 100 kPa



Figure 5. 26 Effect of density on shear modulus versus shearing strain amplitude reduction curves under confining pressure of 200 kPa



Figure 5. 27 Effect of density on shear modulus versus shearing strain amplitude reduction curves under confining pressure of 400 kPa



Figure 5. 28 Effect of density on normalized shear modulus versus shearing strain amplitude reduction curves under confining pressure of 50 kPa



Figure 5. 29 Effect of density on normalized shear modulus versus shearing strain amplitude reduction curves under confining pressure of 100 kPa



Figure 5. 30 Effect of density on normalized shear modulus versus shearing strain amplitude reduction curves under confining pressure of 200 kPa



Figure 5. 31 Effect of density on normalized shear modulus versus shearing strain amplitude reduction curves under confining pressure of 400 kPa



Figure 5. 32 Effect of density on normalized shear modulus versus shearing strain amplitude reduction curves under confining pressure of 800 kPa

5.3.3 Empirical Modeling

5.3.3.1 Proposed Equation

Figure 5.33 presents five typical normalized shear modulus reduction curves for Berlin sand under the confining pressure ranges from 50 kPa to 800 kPa. Thanks to the last three data points in the 50 kPa and 100 kPa curves do not well follow the general reduction tendency due to large reduction in void ratio during test, therefore, these data points are not included in the fitting. The fitting is carried out by the following modified hyperbolic equation:

$$\frac{G}{G_{\max}} = \frac{1}{a + b\gamma^m}$$
(5.8)

in which, a is experimental constant, b and m are fitting variables, both are dependent on confining pressure, and γ is the shearing strain amplitude.

The initial fitting parameters and correlative coefficient are listed in Table 5.4. As listed in this table, the parameter *a* seems no significant dependency on confining pressure, for the purpose of easy application of Equation 5.8, the mean value of parameter $a=0.9602\approx0.96$ is taken to perform the second time fitting on these data. The second fitting curves are illustrated in Figure 5.34 and their fitting parameters are summarized in Table 5.5. One can see that these fitting curves may well predict the normalized shear modulus at any given shearing strain amplitude.



Figure 5. 33 Typical normalized shear modulus reduction curves under various confining pressures for Berlin sand



Figure 5. 34 Fitting curves for typical normalized shear modulus versus shearing strain amplitude reduction curves under various confining pressure for Berlin sand

Table 5. 4Fitting parameters of typical normalized shear modulus versus shearing
strain amplitude reduction curves under various confining pressure for
Berlin sand

$\sigma_{_0}$ (kPa)	а	b	m	Dev Max	r ²
50	0.952	1065	0.831	0.0138	0.998
100	0.963	369	0.741	0.0115	0.998
200	0.948	139	0.657	0.0175	0.995
400	0.964	106	0.658	0.0098	0.998
800	0.974	98.8	0.695	0.0102	0.997

Table 5. 5 Second fitting parameters of typical normalized shear modulus versusshearing strain amplitude reduction curves under various confiningpressure for Berlin sand

$\sigma_{_0}$ (kPa)	а	b	m	Dev Max	r ²
50	0.96	1293	0.854	0.0145	0.997
100	0.96	338	0.730	0.0103	0.998
200	0.96	154	0.674	0.0227	0.994
400	0.96	98.3	0.648	0.0110	0.998
800	0.96	64.6	0.638	0.0162	0.995

As shown in Table 5.5, both parameters b and parameter m decrease with confining pressure, they rapidly decrease when the confining pressure is less than 200 kPa, hereafter slow down, particularly parameter b, the relationship between these parameters and confining pressure is illustrated in Figure 5.35.

For practical application of Equation 5.8, both parameters are correlated to the confining pressure by the following equations,

$$b = 110 + 1.29 \text{E7} \frac{1}{\sigma_0^k} \tag{5.9-1}$$

for 50 kPa $\leq \sigma_0 \leq$ 200 kPa , and

$$b = 13 + 6567 \frac{1}{\sigma_0^k} \tag{5.9-2}$$

for 200 kPa $\leq \sigma_0 \leq 800$ kPa

$$m = 0.619 + 11.6 \frac{1}{\sigma_0} \tag{5.10}$$

in which, k is experimental constant, 2.38 and 0.725 are taken for Equation 5.9-1 and Equation 5.9-2, respectively.



Figure 5. 35 Relationship between parameters b and m and confining pressure

5.3.3.2 Comparison

The equation suggested by Stokoe et al.(1999) is employed to compare with Equation 5.8, the equation is given as follows,

$$\frac{G}{G_{\text{max}}} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_r}\right)^k}$$
(5.11)

in which, γ_r is reference strain, and k is second curve-fitting variable called the curvature parameter. The reference strain corresponds to the shear strain amplitude when G/G_{max} is equal to 0.5.

Stokoe et al. (1995) suggested the following expression to determine γ_r ,

$$\gamma_r = \gamma_{r1} \left(\frac{\sigma_0}{P_a}\right)^j \tag{5.12}$$

in which, γ_{r1} is reference strain at mean effective principal stress of 100 kPa; P_a is reference stress of 100 kPa; and *j* is stress correction exponent, which can be determined by combining Equations 5.11 and 5.12, and conducting regression on data for specimens tests at multiple confining pressure using the resulting equation.

According to this standard, the reference strain for the data in Figure 5.36 is summarized in Table 5.6. In this study, there is no need to determine the *j* value due to the γ_r can be directly determined from the normalized shear modulus reduction curves illustrated in Figure 5.36.

Table 5. 6	Reference strain and fitting parameters for Equation 5.11 under variou	us
	confining pressures	

$\sigma_{_0}$ (kPa)	k	γ_r	Dev Max	r^2
50	0.976	2.37×10 ⁻⁴	0.0268	0.988
100	0.834	3.62×10 ⁻⁴	0.0196	0.991
200	0.766	6.0×10 ⁻⁴	0.0262	0.995
400	0.735	9.00×10 ⁻⁴	0.0277	0.991
800	0.844	1.15×10 ⁻³	0.0323	0.987

The fitting curves predicted by Equation 5.11 are depicted in Figure 5.36 combining with those by Equation 5.8. The corresponding curvature parameter and correlative coefficients are listed in Table 5.6. As shown in Table 5.6 and Figure 5.36, Equation 5.11 may fit the tested data, but are slightly higher and scatter compared to Equation 5.8.



Shearing Strain Amplitude, y (-)

Figure 5. 36 Comparison of the fitting capacity of Equations 5.8 and Equation 5.11 for Berlin sand

Comparison of Equations 5.8 and Equation 5.11 gives following advantages of Equation 5.8,

(1) Equation 5.8 can predict the variation of normalized shear modulus with shearing strain amplitude at any confining pressure without knowledge of the reference

strain, γ_r . However, the reference strain should be required before the Equations 5.11 is used.

- (2) The k of Equation 5.11 has no dependency on confining pressure; it can only be obtained by fitting the existing data.
- (3) The curve fitting parameters in Equation 5.8 can easily determined by Equations 5.9 and 5.10; they have a good dependence of confining pressure.
- (4) The only disadvantage of Equation 5.8 is based on Berlin sand, if it is application on other sands more investigations are necessary.

5.4 Damping Properties

Damping ratio is another important parameter in the analysis on dynamic problems. Small-strain damping ratio, D_{\min} , is rather difficult to accurately determine due to many factors such as equipment damping, environmental noise, back electromagnetic force (Back-EMF), therefore, D_{\min} measured by resonant column method is somewhat higher scatter, but these effects become unimportant when tests are made at high strain amplitude.

5.4.1 Effect of Void Ratio

Small-strain damping ratios presented herein were determined at the shearing strain amplitude lower than 1×10^5 under all confining pressures; they are illustrated in Figures 5.37-5.41. It can be seen in these figures that, the density or void ratio has no influence on the small-strain damping ratio in spite of confining pressure under which tests are made. The damping ratio of Berlin sand slightly increases with shearing strain amplitude when it does not exceed the strain of 1×10^5 , the rate of increase decreases with confining pressure. Testing scatter decreases with increasing confining pressure, this is contributed to better coupling between tested sample and top cap under higher confining pressure.



Shearing Strain Amplitude, y (-)

Figure 5. 37 Variation of damping ratio with shearing strain amplitude on small-strain test under confining pressure of 50 kPa

Figures 5.42-5.46 indicate the variation of damping ratio with shearing strain amplitude under various confining pressures. As shown in these figures, void ratio or density demonstrates no significant influence on damping ratio at any shearing strain amplitude under all confining pressures. Data scatter significantly decreases with increasing confining pressure as well, especially after confining pressure increases to beyond 100 kPa. In general, one can draw a conclusion that damping ratio slightly increases with decreasing void ratio.



Figure 5. 38 Variation of damping ratio with shearing strain amplitude on small-strain test under confining pressure of 100 kPa



Figure 5. 39 Variation of damping ratio with shearing strain amplitude on small-strain test under confining pressure of 200 kPa



Figure 5. 40 Variation of damping ratio with shearing strain amplitude on small-strain test under confining pressure of 400 kPa



Figure 5. 41 Variation of damping ratio with shearing strain amplitude on small-strain test under confining pressure of 800 kPa



Figure 5. 42 Variation of damping ratio with shearing strain amplitude under confining pressure of 50 kPa



Figure 5. 43 Variation of damping ratio with shearing strain amplitude under confining pressure of 100 kPa


Figure 5. 44 Variation of damping ratio with shearing strain amplitude under confining pressure of 200 kPa



Figure 5. 45 Variation of damping ratio with shearing strain amplitude under confining pressure of 400 kPa



Figure 5. 46 Variation of damping ratio with shearing strain amplitude under confining pressure of 800 kPa

5.4.2 Effect of Confining Pressure

The fitting curves of damping ratio illustrated in Figures 5.42 to 5.46 are replotted on Figure 5.47 to project the effect of confining pressure. As shown in this figure, the damping curve shifts to a right lower with increasing confining pressure, namely damping ratio decreases with increasing confining pressure when shearing strain amplitude is the same.



Figure 5. 47 Effect of confining pressure on damping ratio for Berlin sand

The tested data can be well fitted by the following empirical equation,

$$D = (a\gamma^n + b) + D_{\min}$$
(5.13)

in which, *D* is damping ratio, *a* and *b* are laboratory variables, D_{\min} is the small-strain damping ratio (take the damping ratio corresponding to 0.0004%, they vary from 0.33%~0.39%, for convenience 0.036% is taken) for each confining pressure, *n* is the power of shear strain amplitude, *a*, *b*, and *n* are dependent on confining pressure, they can be determined by Equation 5.14-5.16, the relationship between them and confining pressure is presented in Figures 5.48 and 5.49.

$$a = 4.63 \times 10^{-8} \sigma_0^{4.17} + 335 \tag{5.14-1}$$

for 50 kPa $\leq \sigma_0 \leq$ 200 kPa , and

$$a = 1.69 \times 10^{-3} \sigma_0^{1.82} + 487 \tag{5.14-2}$$

for 200 kPa $\leq \sigma_0 \leq$ 800 kPa. $n = 1.17 \times 10^{-4} \sigma_0^{-1.35} + 0.454$ (5.15-1) for 50 kPa $\leq \sigma_0 \leq$ 200 kPa, and

$$n = 3.04 \times 10^{-3} \sigma_0^{0.647} + 0.507 \tag{5.15-2}$$

for 200 kPa $\leq \sigma_0 \leq 800$ kPa.

$$b = -1/(-44.481 + 33.777\sigma_0^{0.0688})$$
(5.16)



Figure 5. 48 Diagram for determination of parameters *a* and *n* of Equation 5.13



Figure 5. 49 Diagram for determination of parameters *b* of Equation 5.13

5.4.3 Effect of Water Content

One dry and three wet samples with various water contents were tested under confining pressures of 100 kPa and 200 kPa to determine the small-strain damping ratio, the small-strain shear moduli of these samples, which have been presented in Section 5.2.6. Testing results are illustrated in Figures 5.50 and 5.51.



Figure 5. 50 Water content effect on small-strain damping ratio under 100 kPa



Figure 5. 51 Water content effect on small-strain damping ratio under 200 kPa

As shown in Figures 5.50 and 5.51, there is no distinguishable distance between the damping ratios of these samples. The influence of water content on the small-strain damping is negligible based on the range of water content used in this study. The influence of water content on dynamic properties of soil is another study subject; it is out of range to go into detail in this study.

5.5 Comparison of G_{max} by RC and BE Tests

Bender element technique has been broadly applied to determine small-strain shear wave velocity of soil in practice thanks to the simple, fast performance of application. To further go insight into the dynamic properties of Berlin sand, several dry samples with the density ranging from 18% to 82.5% were tested under confining pressure varying from 10 kPa to 800 kPa. Source signal used is sinusoidal wave with the frequency of 10 kHz and amplitude of 10 V. Bender element tests were performed before resonant column tests were carried out. Testing results are presented in Figures 5.52-5.55, based on data in Figures 5.52 and 5.53, the following conclusions are drawn.

- (1) The G_{max} linearly increases with confining pressure on the double logarithmic diagram, curves tested by resonant column method (RC) have significantly higher slopes than those by bender element method (BE). In another word, the G_{max} obtained by RC test increases with confining pressure slightly faster than by BE test.
- (2) As illustrated in Figure 5.52, there is a threshold pressure at which the $G_{\text{max}}^{\text{RC}}$ by RC is equal to $G_{\text{max}}^{\text{BE}}$ by BE, below which $G_{\text{max}}^{\text{RC}}$ is lower and beyond which is higher than $G_{\text{max}}^{\text{BE}}$, and the distance between $G_{\text{max}}^{\text{RC}}$ and $G_{\text{max}}^{\text{BE}}$ increases with confining pressure. Based on these data, the threshold pressure is normally lower than 20 kPa

The V_s tested by RC method marked as V_s^{RC} , and by BE method marked as V_s^{BE} . As shown in Figures 5.54 and 5.55, both G_{max} and V_s by RC test are higher than their counterparts by BE test if the G_{max} and V_s are higher than a threshold value. It is very clear that G_{max}^{RC} is a function of G_{max}^{BE} , and similarly the V_s^{RC} is also a function of V_s^{BE} . Their relationships can be expressed by the following linear equations:

$$G_{\max}^{\text{RC}} = 1.38G_{\max}^{\text{BE}} - 17.6 \tag{5.17}$$

$$Vs^{\rm RC} = 1.24Vs^{\rm BE} - 36.3 \tag{5.18}$$

As mentioned above, the threshold values of G_{max} and V_{s} can be obtained by assuming $G_{\text{max}}^{\text{RC}} = G_{\text{max}}^{\text{BE}}$ and $Vs^{\text{RC}} = Vs^{\text{BE}}$ in Equations 5.17 and 5.18, for Berlin sand, the threshold values are $G_{\text{max}}^{\text{t}} = 46.3 \text{ MPa}$, $Vs^{\text{t}} = 151.25 \text{ m/s}$.



Figure 5. 52 Comparison of small-strain shear modulus by RC and BE tests under various confining pressures (Dr=18%-51.3%)



Figure 5. 53 Comparison of small-strain shear modulus by RC and BE tests under various confining pressures (Dr=60.1%-82.5%)



Figure 5. 54 Comparison of small-strain shear modulus by RC and BE methods for a large number of data of dry Berlin sand

To comprehensively investigate capacity of the bender element technique, a three D fitting method is adopted to fit the testing data. The fitting surface of the BE testing results is illustrated in Figure 5.56. The fitting surface is given by

$$G_{\max}(e,\sigma_0) = \frac{393}{(0.3+0.7e)^{2.67}} p_a^{0.542} \sigma_0^{0.458}$$
(5.19)

The S is 393 for Equation 5.19, which is smaller than 488 in Equation 5.4, the n=0.458 is less than 0.5 in Equation 5.4. The smaller S and n in Equation 5.19 induce

a lower value in G_{max} when the confining pressure exceeds 20 kPa. This is possibly attributed to the fact that the G_{max} measured by resonant column test is normally determined at relatively higher shearing strain amplitude under low confining pressure under which the minimum tested small-strain actually exceeds the elastic strain range. In addition, the coupling between tested sample and top cap and bottom platen is another factor inducing lower value in shear modulus. Therefore, this effect should be considered when resonant column test is carried out under low confining pressure based on the information given above.



Figure 5. 55 Comparison of small-strain shear wave velocity by RC and BE methods for a large number of data of dry Berlin sand



Figure 5. 56 Comparison of empirical equations for G_{max} by BE and RC tests for Berlin sand

5.6 Summary

In this chapter, the dynamic properties (G and D) of Berlin sand are fully addressed based on resonant column tests on 43 dry and 3 three wet sand samples. Many factors including void ratio, confining pressure, stress history, duration of confinement, water content, and sample preparation method, were affected on dynamic properties of Berlin sand. An empirical equation is proposed to predict the small-strain shear modulus based on the void ratio and confining pressure. Tow equations are developed to model nonlinear dynamic sand properties at any give shearing strain amplitude under various confining pressures. Bender element tests are compared to resonant column tests, the relationship between the G_{max} measured by the resonant column and bender element tests was developed as well.

CHAPTER 6 PRELOADING EFFECTS ON DYNAMIC PROPERTIES OF SAND

6.1 Introduction

Along with the fast development of modern society, more and more traffic infrastructures (such as highroad and railroad), wind power plants, and offshore engineering, machine engineering have been booming after the second war. The dynamic problems regarding these engineering projects have become more and more important to rationally design and analyze. Berlin sand is the largest distribution and most application in practice in Europe, the shear modulus and damping ratio of Berlin sand are the two key parameters in these dynamic problems. The preloading effects on dynamic properties of sand become more and more important; therefore, it is essential to well know the development of shear modulus and damping properties. In the following sections, the preloading effects are fully presented.

6.2 Void Ratio during Testing

The volumetric strain of saturated specimen in triaxial chamber can be readily determined by measuring the water volume change in measuring cylinder or back pressure controller; and the axial strain is normally measured by the linear vertical differential transducer (LVDT) mounted on the testing apparatus, with knowledge of volumetric and axial strains, the radial strain is obtained accordingly.

Unfortunately, however, it is nearly impossible to directly and accurately measure the volume change of dry specimen during a test due to a lack of water inside the sample. The void ratio or density of soil is one of the most important parameters influencing the stiffness of soil. As the variation of confining pressure and application of preloading on the specimen during test, the density changes accordingly. Normally, the density of sample increases with increasing confining pressure and the number of preloading cycle as the strain exceeds the volume change threshold strain in resonant column tests. Shear dilatation is reckoned no occurring in resonant column tests due to the shearing strain amplitude is far below the destruct strain. Therefore, it is essential to investigate the variation of void ratio during tests for the purpose of well knowing the preloading effect on dynamic properties of sand.

In general, the change of the height of specimen is conventionally the unique dimension parameter which could be monitored during a test on dry sample; therefore, it is indispensable to build an approximate relationship between the void ratio and

axial strain. For this purpose, dimensions of 39 sand samples were measured at the end of tests by applying a negative pressure of 30 kPa to support the sample avoiding collapse. The initial void ratios of these samples measured after tests vary from 0.38 to 0.578 for Berlin sand, 0.574 for Braunschweig coarse sand and 0.828 for Cuxhaven fine sand.

6.2.1 Empirical Expression

The testing data of these 39 samples are presented in Figures 6.1 to 6.3. All three sets of data can be well fitted by a linear function. The empirical equation of the relationships between volumetric strain, ε_v , radial strain, ε_r , and relative change in void ratio, $\varepsilon_e = (e_f - e_0)/e_0$ (e_f is the void ratio at the end of test, and e_0 is the initial void ratio), and accumulated axial strain, ε_a^{Acc} , can be expressed by the following general formula,

$$\varepsilon = a \varepsilon_a^{Acc}$$
 (6.1)

in which, ε is the general symbol for strain of volume, radius, and void ratio, and a is the incremental coefficient of volumetric strain, radial strain, and relative change in void ratio, a is equal to 4.05 for volumetric strain, 1.57 for radial strain, and 11.3 for relative change in void ratio, respectively. All these strains have the same unit in percentage.

For the purpose of examination of the proposed relationship of Equation 6.1, the relationship between volumetric strain and axial strain and radial strain is correlated as

$$\varepsilon_{\rm v} = a\varepsilon_{\rm a}^{\rm Acc} + b\varepsilon_{\rm r} \tag{6.2}$$

in which, a=0.992, b=1.95 according to the fitting surface illustrated in Figure 6.4.

The measured data and fitting equation are compared to the typical relationship equation (Wood 1990)

$$\varepsilon_{\rm v} = \varepsilon_{\rm a}^{\rm Acc} + 2\varepsilon_{\rm r} \tag{6.3}$$

Comparison of Equation 6.2 with Equation 6.3 shows that the parameters a=0.992, and b=1.95 for Equation 6.2 are slightly lower than 1 and 2 for Equation 6.3. The distance between these two equations is possibly attributed to the different Water Contents and data scatter in the measurement of dimensions. This similar relationship of dry sand confirms the validity of the proposed Equation 6.1. Therefore, the following equation is proposed to predict the void ratio of sample during the tests.

$$e = e_0 (1 - 11.3\varepsilon_a^{\text{Acc}} / 100) \tag{6.4}$$

in which, e_0 is initial void ratio, and e is void ratio corresponding to the accumulated axial strain, ε_a^{Acc} , during the test.



Figure 6.1 Relationship between volumetric strain and accumulated axial strain



Figure 6.2 Relationship between Radial strain and accumulated axial strain



Figure 6. 3 Relationship between relative change in void ratio and accumulated axial strain



Figure 6.4 Relationship between volumetric strain and accumulated axial strain and radial strain

The relative deviation of void ratio calculated by Equation 6.4 from the measured void ratio at the end of test is depicted in Figures 6.5 to 6.7. The relative deviation is expressed by

$$\delta_e = \frac{e_{\rm cal} - e_{\rm mea}}{e_{\rm mea}} \times 100 \tag{6.5}$$

in which, δ_e is relative deviation in percentage, e_{mea} and e_{cal} are the void ratio measured at the end of test and the void ratio calculated by Equation 6.5, respectively.

Figures 6.5 to 6.7 show that the relative deviation of the void ratio calculated by Equation 6.4 is relatively small, the maximum deviation does not exceed 2.5%. These errors demonstrate no dependency on the final axial strain, initial void ratio and relative density. Evidences presented above indicate that it is advisable to use Equation 6.4 to predict the void ratio of sample during test.



Figure 6. 5 Relative deviation of radial strain by Equation 6.5 versus accumulated axial strain



Figure 6. 6 Relative deviation of void ratio by Equation 6.5 versus initial relative density



Figure 6.7 Relative deviation of void ratio by Equation 6.5 versus initial void ratio

6.2.2 Variation of Void Ratio with Number of Cycles

Figure 6.8 presents the variation of void ratio with number of preloading cycles for three types of sands. It can be seen that the void ratios of all samples decrease with

increasing number of cycles, the rate of reduction decreases with increasing density. For instance, the reduction of void ratio of the 90.4% relative density Berlin sand is very small even if the number of cycles reaches 10^5 . In addition, the grain characters has an influence on this reduction, which can be shown by the comparison of the curves of the 36.9 relative density Cuxhaven fine sand, the 43.7% relative density Berlin sand, and the 41.7% relative density Braunschweig coarse sand, it is accounted for the uniform coefficients of these two sands (the values of C_u these sands are 1.23, 3.39, and 5.76, respectively).

The reduction of void ratio is not only affected by the density of soil but the prestraining applied to the sample. Taking Berlin sand for an example, both samples are preloaded under the same condition of preloading of 40 kPa and frequency of 20 Hz under 200 kPa, the prestraining on the 43.7% relative density sample is higher than that of 90.4% sample because of the higher stiffness of the dense sample. This effect will be addressed in the next coming section in this chapter.



Figure 6. 8 Reduction of void ratio with number of preloading cycles for various sands

6.3 Small-strain Shear Modulus Correction

As known, the void ratio of sample decreases with increasing confining pressure, however the reduction of void ratio due to an increase in confining pressure normally is minor and not taken into account during resonant column test, therefore the initial void ratio is reckoned as a representative of tested sample. In this section, the variation of void ratio is taken into account based on the proposed empirical equation in Section 6.2.1 (Equation 6.4). The analysis is based on monitoring the axial sediment of the height of soil sample by the linear variable differential transducer (LVDT) placed on the top of tested specimen.

For the purposed of comparison, the small-strain shear modulus measured by

assuming the dimension constant in the whole test, is marked $G_{\text{max}}^{\text{Uncor}}$, the G_{max} measured based on accurately considering dimensions (both the height and diameter) change (the change of height of specimen can be readily measured by LVDT, and the change of diameter could be obtained from Equation 6.1), is marked $G_{\text{max}}^{\text{Real}}$, and the G_{max} corrected considering void ratio variation by Equation 6.4, is marked $G_{\text{max}}^{\text{Cor}}$. These small-strain shear moduli are defined by,

$$G_{\max}^{\text{Uncor}} = \rho_d^0 (V_s^0)^2 \tag{6.5}$$

$$G_{\max}^{\text{Cor}} = \rho_d^1 (V_s^0)^2$$
 (6.6)

$$G_{\max}^{\text{Real}} = \rho_d^2 (V_s)^2 \tag{6.7}$$

$$\rho_d^0 = m \,/\, V_0 \tag{6.8}$$

$$\rho_d^1 = \frac{\rho_s}{1+e} \tag{6.9}$$

$$\rho_d^2 = m / V \tag{6.10}$$

$$V_0 = \frac{\pi H_0(d_0)^2}{4} \tag{6.11}$$

$$V = \frac{\pi H_0 (1 - \varepsilon_a^{\text{Acc}} / 100) (d_0 (1 - 1.5657 \varepsilon_a^{\text{Acc}} / 100))^2}{4}$$
(6.12)

in which, ρ_d^0 is initial dry density of specimen, ρ_d^1 and ρ_d^2 are dry density considering the dimension variation, ρ_s is specific gravity of soil, *m* is the mass of specimen, H_0 and d_0 are initial height and diameter of specimen, V_s^0 is shear wave velocity based on the measured system resonant frequency and initial dimensions, and V_s is the shear wave velocity based on the measured system resonant frequency and frequency and dimensions after correction, *e* is the void ratio after correction based on Equation 6.4.

Data of two Berlin sand samples are contrastively analyzed herein for this purpose, one sample with the initial relative density of 70.9% experienced multi-stage test under the confining pressure varying from 50 kPa to 800 kPa at the shearing strain amplitude around 6×10^{-6} , and the other sample with the relative density of 43.9% was vibrated by the shear stress of 20 kPa at the frequency of 20 Hz up to one million cycles under the confining pressure of 100 kPa.

Figures 6.9-6.11 illustrate the analysis results of the Dr=70.9% Berlin sand. As shown in Figure 6.9, $G_{\text{max}}^{\text{Real}}$ and $G_{\text{max}}^{\text{Cor}}$ are visibly higher than $G_{\text{max}}^{\text{Uncor}}$ as the confining pressure exceeds 400 kPa, and the corresponding accumulated axial strain exceeds

0.15%. The deviation of G_{\max}^{Cor} from G_{\max}^{Real} is relatively small even if the confining pressure reaches 800 kPa with the corresponding accumulated axial strain of 0.35%. The magnitude relationship among these small-strain shear moduli is $G_{\max}^{\text{Real}} > G_{\max}^{\text{Cor}} > G_{\max}^{\text{Uncor}}$. The distance between G_{\max}^{Cor} , and G_{\max}^{Uncor} and G_{\max}^{Real} are presented in Figures 6.10 and 6.11. Both the deviation and relative deviation of G_{\max} increase with an increase axial strain due to increasing confining pressure. The deviation of G_{\max}^{Real} from G_{\max}^{Real} keeps lower than its counterpart of G_{\max}^{Uncor} from G_{\max}^{Real} , the maximum relative deviation of G_{\max}^{Uncor} from G_{\max}^{Real} is around 1.7%. Therefore, it is acceptable to take G_{\max}^{Uncor} as G_{\max}^{Real} in conventional resonant column tests. In other words, the error resulting from assuming dimensions constant in the whole test is negligible in practice.

Figures 6.12-6.14 illustrate the analysis results of the Dr=43.9% Berlin sand, which were subjected to one million cycles of vibration with the shear stress amplitude of 20 kPa at the frequency of 20 Hz under the confining pressure 100 kPa. Similar to the Dr=70.9% Berlin sand, as illustrated in Figure 6.12, the $G_{\max}^{\text{Real}}(Nc)$ curve locates at the highest position, the $G_{\max}^{\text{Cor}}(Nc)$ curve at the second, and the $G_{\max}^{\text{Uncor}}(Nc)$ curve at the lowest position. The distances between G_{\max}^{Real} and G_{\max}^{Cor} , and G_{\max}^{Uncor} increase with increasing the accumulated axial strain due to the increase of number of preloading cycles, especially that between G_{\max}^{Real} and G_{\max}^{Uncor} .



Figure 6. 9 Various type small-strain shear modulus and accumulated axial strain under various confining pressures



Figure 6. 10 Deviation of small-strain shear modulus under various confining pressures



Figure 6. 11 Relative deviation small-strain shear modulus under various confining pressures

Figures 6.13 and 6.14 show the deviations of G_{max} between $G_{\text{max}}^{\text{Real}}$ and $G_{\text{max}}^{\text{Uncor}}$, and $G_{\text{max}}^{\text{Real}}$ and $G_{\text{max}}^{\text{Cor}}$. The distance between $G_{\text{max}}^{\text{Real}}$ and $G_{\text{max}}^{\text{Uncor}}$ is much greater than that between $G_{\text{max}}^{\text{Real}}$ and $G_{\text{max}}^{\text{Cor}}$, both deviations increase with increasing number of cycles

due to the increasing accumulated axial strain, the deviation of $G_{\text{max}}^{\text{Uncor}}$ from $G_{\text{max}}^{\text{Real}}$ may be up to 11 MPa after one million cycles, however, that of $G_{\text{max}}^{\text{Cor}}$ from $G_{\text{max}}^{\text{Real}}$ is merely 2.3 MPa. As shown in Figure 6.14, the relative deviation of $G_{\text{max}}^{\text{Uncor}}$ from $G_{\text{max}}^{\text{Real}}$ may be up to 7.5%, however, that of G_{max} from $G_{\text{max}}^{\text{Cor}}$ is only 1.7% as the accumulated axial strain increases to 1.5% when the number of cycles reaches one million. Actually, for some tests, the accumulated axial strains may exceed 2.5%, which may induce the relative deviation over 10%.

As mentioned above, the enormous deviation G_{\max}^{Real} and G_{\max}^{Uncor} is attributed to use the initial dimensions to calculate the shear modulus during the whole test and no considering the densification of sample during test. Comparison shows that the deviation between G_{\max}^{Real} and G_{\max}^{Cor} is relatively small even if the accumulated axial strain increases up to 1.5% after one million cycles of vibration. The shear wave velocity obtained by resonant column test is dependent upon the mass polar moment of inertial of specimen (diameter of specimen) and specimen height. In practice, however, the dimensions (Diameter and height) of specimen and polar inertial moment are reckoned constant during conventional resonant column test. It is a very time consuming work to modify these parameters even if the dimensions of specimen can be approximately obtained from Equation 6.1 based on the accumulated axial strain measured by LVDT. Therefore, it is rational to assume G_{\max}^{Cor} as G_{\max}^{Real} when the influence of dimension variation during test is taken into account.



Figure 6. 12 Development of small-strain shear modulus and accumulated axial strain with number of cycles (Dr=43.9%)



Figure 6. 13 Deviation of small-strain shear modulus with number of cycles (Dr=43.9%)



Figure 6. 14 Relative deviation of small-strain shear modulus with number of cycles (Dr=43.9%)

6.4 Small-strain Shear Modulus

Note that, the small-strain shear moduli mentioned in the following sections are referred to G_{\max}^{Cor} but marked as G_{\max} . In this section, the small-strain shear

modulus determined before the preloading, is marked as $G_{max}(0)$ and plotted on the half logarithmic diagram at the horizontal coordinate of 10^{-1} due to 0 can not be plotted in logarithmic scale.

6.4.1 Effect of Number of Cycles

Figures 6.15 and 6.16 illustrate the variation of small-strain shear modulus and accumulated axial strain with number of cycles for four different source sands tested under the confining pressure of 200 kPa. The preloading stress amplitude of 40 kPa was applied at the frequency of 20 Hz. As shown in these two figures, the $G_{max}(Nc)$ of Berlin sand decreases with number of preloading cycles when it less than twenty thousand cycles thereafter nearly keeps stable. As for the Braunschweig coarse sand, the $G_{max}(Nc)$ greatly reduces after one cycle of preloading then slightly decreases with increasing number of cycles up to 100,000 except the data point obtained after 20,000 cycles. The variation of $G_{max}(Nc)$ of Cuxhaven medium sand shows a similar tendency to that of Braunschweig coarse sand when the number of cycles is lower than 2,000, thereafter nearly keeps stable up to 20,000, then start to increase as number of cycles continues. Unlike the three foregoing sands, the $G_{max}(Nc)$ of the Cuxhaven fine sand slightly degrades after the first three cycles of vibration, then slightly increases with an increase in the number of cycles as shown in Figure 6.16.



Figure 6.15 Development of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles of various sands



Figure 6. 16 Development of normalized G_{max} with number of preloading cycles for various sands

6.4.2 Effect of Preloading Frequency

Eight Berlin sand samples with medium density and four samples with high density were preloaded by the same torsional shear stress of 20 kPa under the confining pressure of 100 kPa at the vibrating frequency varying from 0.1 Hz to 40 Hz. Testing results are presented in Figures 6.17-6.20.

As mentioned above, Figures 6.17 and 6.18 indicate that the small-strain shear modulus decreases with number of cycles when it is lower than a given number thereafter slightly increases for these samples except for the sample excited at 40 Hz. The data presented in Figure 6.17 are scatter due to the tested error and individual diversity in sample. Figure 6.18 presents the variation of G_{max} with number of cycles, N_c , by the ratio of G_{max} after preloading, $G_{\text{max}}(N_c)$, to the $G_{\text{max}}(0)$ curve is dependent on the applied vibrating frequency, it plots higher as the frequency increase. And the corresponding accumulated axial strain curve plots higher accordingly.

The maximum reduction of G_{max} with Nc was found at the sample excited at 0.1 Hz, in this case the G_{max} may decrease to a value greater than 20% of the $G_{\text{max}}(0)$. In general, this maximum reduction decreases with increasing the applied preloading frequency, especially when the frequency increases beyond 30 Hz as shown in Figure 6.18.

Unlike the cases of lower vibrating frequency, in the case of 40 Hz, the reduction in G_{max} with Nc was not observed partially thanks to the lack of data at lower number of cycles, but a significant increase of G_{max} is seen as indicated in Figures 6.17 and 6.18. The rate of the increase in G_{max} with Nc may be up to 155% after 1,000,000 cycles at

which the accumulated axial strain accordingly develops to 2.3%. The increase of G_{max} is accounted for great densification of sample and possibly wear process occur during vibration. The greatest reduction of void ratio was caused by the corresponding high strain amplitude because the vibrating frequency of 40 Hz is possibly close to the resonant frequency of the system. As illustrated in Figure 3.8, the vibrating amplitude decreases with the distance between the resonant frequency and vibrating frequency. It should be noted that the increase relationship of 40 Hz case is similar to previous investigations, in which the applied vibrating frequencies were normally around the resonant frequency and the starting number of cycles is normally larger than 1000.

As for the dense Berlin sand, the testing scatter is relatively lower, it can been clearly seen that the influence of preloading frequency on the is much more obvious when the frequency is greater than 20 Hz, as shown in Figures 6.19 and 6.20. It There is no significant influence of preloading on the $G_{max}(Nc)/G_{max}(0)$ curve when it is lower than 20 Hz due to the frequency is far away from the system resonant frequency (normally is 80 Hz). Therefore, it can be concluded that the decrease in small-strain shear modulus with number of cycles is affected by the preloading frequency. Normally, the decrease degree in small-strain shear modulus increases with increasing frequency when the number of cycles is less than a given number for a given sample.



Figure 6. 17 Development of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles at various preloading frequencies for medium dense Berlin sand



Figure 6. 18 Development of normalized G_{max} with number of cycles at various preloading frequencies for medium dense Berlin sand



Figure 6. 19 Development of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles at various preloading frequencies for dense Berlin sand



Figure 6. 20 Development of normalized G_{max} with number of cycles at various preloading frequencies for high dense Berlin sand

6.4.3 Effect of Preloading Ratio

Three medium dense and four dense Berlin sand samples were tested under the confining pressure of 50 kPa at the preloading frequency of 20 Hz. The preloading stresses of 5 kPa, 10 kPa, and 15 kPa for the medium dense samples, and 5 kPa, 10 kPa , and 20 kPa for the dense samples, were applied, respectively. The preloading ratio is defined by the ratio of preloading stress (τ_{pre}) to confining pressure (σ_0), τ_{pre}/σ_0 ; then the τ_{pre}/σ_0 of 0.1, 0.2, 0.3 and 0.4 are obtained respectively. Testing results are presented in Figures 6.21-6.24.

Figures 6.21 and 6.22 illustrate the results for the medium dense samples. As shown in figures, the rate of reduction in shear modulus increases with increasing preloading ratio for the first ten cycles thereafter decreases to cause the $G_{max}(Nc)/G_{max}(0)$ plots higher with increasing preloading ratio. The small-strain shear moduli of the samples preloaded by the preloading ratios of 0.1 and 0.2 continue decreasing with number of cycles to around the number of 10,000, beyond which start to increase. Unlike the cases of the preloading ratio of 0.1 and 0.3, the G_{max} in the case of 0.3 starts to increase to larger than the G_{max} measured before preloading. As for the sample subjected to the preloading ratio of 0.1; the G_{max} may reduce to 78% after 20,000 cycles. The accumulated axial strain indicates that the densification of sample account for the increase of small-strain shear modulus. For the case of $\tau_{pre}/\sigma_0=0.1$, the axial strain is very small even if the sample was subjected to 700,000 cycles. The decrease of G_{max} is related to some other factors, which will be expatiated in the end of this chapter.

As for the cases of the dense samples, the influence of preloading ratio on G_{max} is not so much significant. The small-strain shear modulus of the sample which was subjected to preloading ratio of 0.1 decreases very slowly with number of cycles. As for the sample subjected to preloading ratio of 0.4, the G_{max} fast decreases with number of cycles at the first 5 cycles, then keeps nearly stable from 5 to 10,000 cycles, thereafter fast increases in the half logarithmic diagram. As mentioned previously, as the preloading ratio increases, resulting in the corresponding prestraining increase, consequently the reduction of void ratio of sample becomes faster. The maximum magnitude of decrease in shear modulus was observed for the sample subjected to lower preloading ratio generally higher than the sample subjected to higher preloading ratio. The preloading ratio effect on small-strain shear modulus is attributed to the prestraining amplitude induced by preloading stress amplitude.



Figure 6. 21 Development of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles at various preloading ratios for medium dense Berlin sand



Figure 6. 22 Development of normalized G_{max} with number of cycles at various preloading ratios for medium dense Berlin sand



Figure 6. 23 Development of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles at various preloading ratios for dense Berlin sand (Dr=75.1%-76.1%)



Figure 6. 24 Development of normalized G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles at various preloading ratios for dense Berlin sand (Dr=75.1%-76.1%)

6.4.4 Effect of Void Ratio

Three dense samples were tested under the confining pressure of 50 kPa with the preloading stress of 20 kPa at the frequency of 20 Hz, and one medium dense sample was also tested under the same conditions but the preloading stress of 5 kPa. Testing results are presented in Figures 6.25 and 6.26.



Figure 6. 25 Development of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles for Berlin sand with various void ratios

As shown in Figure 6.26, the $G_{max}(Nc)/G_{max}(0)$ curve plots higher as the void ratio decreases. It can be seen that the decrease in small-strain shear modulus of the Dr=75.4% sample with number of cycles is more significant than the two higher dense samples of 88.4% and 96.4%. Normally, the small-strain shear modulus rapidly decreases before 5 cycle of preloading, thereafter keeps nearly stable until the number of cycles reaches 1,000, then start to increase for the Dr=96.4% sample, its maximum reduction is just 5%, and the increase in shear modulus after 1.2 million cycles of preloading is also just around 5%. For the Dr=75.4% sample, the small-strain shear modulus after 5 cycles of preloading may decrease to 90% of the modulus without preloading, thereafter keeps nearly stable until 50,000 cycles, after that starts to increase. The G_{max} of the Dr=43.7% sample decreases to the maximum at the fifth cycles and gradually increases with number of cycles up to 10, 000 cycles, and beyond which rapidly increase in the semilogarithmic diagram and reaches 55% after one million cycles. The fact that the higher density the lower maximum reduction in G_{max} is attributed to the particle reorientation during preloading, this aspect will discussed in the end of this chapter.



Figure 6. 26 Development of normalized G_{max} with number of cycles for Berlin sand with various void ratios

6.4.5 Effect of Confining Pressure

Three medium dense and two dense Berlin sand samples were tested under the confining pressures of 50 kPa, 100 kPa, and 200 kPa. The applied preloading ratio is equal to 0.2, and the preloading frequency is 20 Hz. The testing results of medium dense samples are illustrated in Figures 6.27 and 6.28.



Figure 6. 27 Development of G_{max} with number of cycles under various confining pressures for medium dense Berlin sand



Figure 6. 28 Development of normalized G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles under various confining pressures for medium dense Berlin sand

As shown in these figures, there seems no significant difference between the $G_{\max}(\text{Nc})/G_{\max}(0)$ curves of 100 kPa and 200 kPa for the first 10,000 cycles. The curve of 50 kPa keeps slightly higher than the others after the number of cycles exceeds 5 cycles. One can conclude that the influence of confining pressure on the development of G_{\max} with number of cycles is less if the confining pressure exceeds 100 kPa for medium sand.

As for the dense samples, Figures 6.29 and 30 show that the $G_{max}(Nc)/G_{max}(0)$ curves of 100 kPa is higher than the curve of 50 kPa. The maximum decrease of shear modulus may reach 15% when the number of cycles reaches 50,000; thereafter, the small-strain shear modulus starts to increase for the case of 50 kPa confining pressure. Confining pressure demonstrates a different influence on the variation of G_{max} for different density of sample.



Figure 6. 29 Development of G_{max} with number of cycles under various confining pressures for dense Berlin sand



Figure 6. 30 Development of normalized G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles for dense Berlin sand under various confining pressures

6.4.6 Effect of Reloading

The concept "*reloading*" herein means that the confining pressure increases to a higher level after the expected number of cycles is complete. The prepared samples were initially tested under the confining pressure of 50 kPa at the small strain

amplitude below 1×10^{-5} to determine the G_{max} , thereafter the previbration was applied and the G_{max} was measured at a series of given number of cycles, as the preloading was complete, the G_{max} was measured after the preloading had been stopped for 10 minutes under the confining pressure of 50 kPa. After that the confining pressure was increased to a series of higher pressure level and the G_{max} was determined after each target higher confining pressure was stable for 10 minutes.

Note that, in these diagrams, the solid triangle down marks stand for the measured value of G_{max} , marked $G_{max}^{Mea.}$, the solid triangle up marks stand for the predicted value, marked $G_{max}^{Cal.}$ and the solid circle marks stand for the value measured before preloading.

Figures 6.31-6.37 illustrate the variation of small-strain shear modulus, obtained by measuring and predicting by Equation 5.4 based on the void ratio corrected by Equation 6.4 after preloading, with confining pressure after preloading. Testing results show that the values of measured G_{max} were observed to plot higher than their corresponding predicted values, both values obtained after preloading plot higher than the G_{max} predicted curve by Equation 5.4 under all higher confining pressures based on the initial void ratio of the sample before preloading.



Figure 6. 31 Reloading effect on G_{max} after 1,000 cycles of preloading (Dr=38.8%)



Reloading effect on G_{max} after 10,000 cycles of preloading (Dr=45.6%) Figure 6. 32



Figure 6.33 Reloading effect on G_{max} after 100,000 cycles of preloading

(Dr=42.4%)

Figures 6.34 and 6.35 illustrate the increase of small-strain shear modulus with confining pressure for two samples which have been subjected to 700,000 and 1,000,000 cycles of the preloading stress of 5 kPa and 15 kPa at the same vibrating frequency under the confining pressure of 50 kPa, respectively. It is seen in Figures 6.33-6.35 that the preloading stress has significant influence on the shear modulus confining pressure relationship after preloading. The distance between the measured and predicted G_{max} increases with the applied stress amplitude, this magnitude of distance in Figure 6.34 is much smaller than that in Figure 6.33 even if the number of cycles for the case in Figure 6.34 is seven times of that in Figure 6.33. The distance between these two values is smallest among all the cases presented herein.



Isotropic Confining Pressure, σ_0 (kPa)

Figure 6. 34 Reloading effect on G_{max} after 700,000 cycles of preloading (Dr=41.3%)



Isotropic Confining Pressure, σ_0 (kPa)

Figure 6. 35 Reloading effect on G_{max} after 1,000,000 cycles of preloading (Dr=43.7%)



Isotropic Confining Pressure, σ_0 (kPa)

Figure 6. 36 Reloading effect on G_{max} after 1,100,000 cycles of preloading (Dr=76.1%)



Isotropic Confining Pressure, σ_0 (kPa)

Figure 6. 37 Reloading effect on G_{max} after 1,200,000 cycles of preloading (Dr=96.4%)

As shown in Figures 6.31-6.33, the distance between measured and predicted G_{max} under all confining pressures higher than 50 kPa under which the preloading was applied increase with number of cycles. Comparison of Figure 6.33 with Figure 6.35 shows that the influence of number of cycles on this distance becomes stable when the number of cycles exceeds 100,000 even if the preloading stress in the case of Figure
6.35 is 15 kPa. The wear process of soil particles accounts for this phenomenon, which will be addressed later in Section 6.6.

Comparison of Figure 6.35 with Figure 6.36 indicates that the distance between measured and predicted G_{max} decreases with decreasing void ratio of sample, which is accounted for the higher resulting prestraining amplitude for lower density sample than that for higher density sample because of higher stiffness of the denser sample when they are subjected to the same preloading stress.

Table 6.1 summarizes the fitting parameters and correlative coefficient of Equation 5.4 for the measured small-strain shear moduli for seven samples after preloading. The relative characteristic indices are also listed in this table. Fitting results show that the Gmax measured under reloading path after preloading can still be well fitted by the general formula of Equation 5.4. It is seen in this table that the influence powers, n, of confining pressures on G_{max} are higher than 0.5 for virgin samples except for fitting curves for the 43.7% and 96.4%, which means that the G_{max} has a more pressure dependency after preloading. For the cases of 43.7% and 96.4%, the G_{max} show a lower pressure dependency with the values of n less than 0.5 due to a rather higher soil stiffness coefficient because of the high density after preloading. The measured G_{max} is normalized by the calculated G_{max} based on the void ratio predicted by Equation 6.4.

The application of $G_{\max}^{\text{Mea.}}(Nc)/G_{\max}^{\text{Cal.}}(Nc)$ eliminates the influence of soil densification and indicates structure alternation of sand under the confining pressure of 50 kPa after preloading. The magnitude relationship of the $G_{\max}^{\text{Mea.}}(Nc)/G_{\max}^{\text{Cal.}}(Nc)$ approximately agrees with that of the stiffness coefficient as shown in Table 6.1. The $G_{\max}^{\text{Mea.}}(Nc)/G_{\max}^{\text{Cal.}}(Nc)$ implies the stable state of soil structure, generally the lower $G_{\max}^{\text{Mea.}}(Nc)/G_{\max}^{\text{Cal.}}(Nc)$ the less stable of soil structure, there the G_{\max} shows higher pressure sensitivity after preloading. The $G_{\max}^{\text{Mea.}}(Nc)/G_{\max}^{\text{Cal.}}(Nc)$ curves of all samples are presented in Figure 6.38 to show the reloading influence on soil structure of sample after preloading.

No.	Nc	Dr (Initial)	Dr [†] (Final)	$\frac{G_{\max}^{\text{Mea.}}(Nc)}{G_{\max}^{\text{Mea.}}(0)}$	$rac{G_{ m max}^{ m Mea.}(Nc)}{G_{ m max}^{ m Cal.}(Nc)}$	S	n	r ²
1	700,000	41.3%	45.8%	0.82	0.79	0.464	0.576	0.998
2	1,000	38.8%	47.0%	0.86	0.80	0.461	0.554	0.998
3	10,000	45.6%	58.8%	0.91	0.80	0.456	0.579	0.997
4	100,000	42.4%	64.5%	0.96	0.78	0.472	0.585	0.996
5	1,000,000	43.7%	88.1%	1.55	1.00	0.575	0.487	0.994
6	1,200,000	96.4%	99.1%	1.06	1.03	0.582	0.420	0.998
7	1,100,000	76.1%	83.3%	0.89	0.84	0.504	0.586	0.996

Table 6. 1Fitting parameters and correlative coefficient of Equation 5.4 for G_{max} of
various samples after preloading

[†] Calculated based on the void ratio calculated by Equation 6.4 after preloading under the confining pressure of 50 kPa.



Isotropic Confining Pressure, σ₀ (kPa)

Figure 6. 38 Influence of reloading on G_{max} after preloading under the confining pressure of 50 kPa (Reloading path)

Figure 6.38 presents the influence of reloading on the G_{max} depicted in Figures 6.31-6.37. As shown in Figure 6.38, the values of $G_{\text{max}}^{\text{Mea.}}(Nc)/G_{\text{max}}^{\text{Cal.}}(Nc)$ for all cases are apparently affected by the increase of confining pressure after preloading. As for all cases (1-7), $G_{\text{max}}^{\text{Mea.}}(Nc)/G_{\text{max}}^{\text{Cal.}}(Nc)$ rapidly increase with increasing confining pressure when it increases not exceeding 75 kPa, thereafter the G_{max} ratios slowly increase with pressure when it is not exceeding100 kPa for case 7 and 200 kPa for Cases 1-4, and then the values of $G_{\text{max}}^{\text{Mea.}}(Nc)/G_{\text{max}}^{\text{Cal.}}(Nc)$ keep stable. It should be noted that the $G_{\text{max}}^{\text{Mea.}}(Nc)/G_{\text{max}}^{\text{Cal.}}(Nc)$ curve plots higher with the number of preloading cycles for Cases 2-4 as the confining pressure starts to increase. The shape of curve 1 is similar to Curves 2-4 but plots lowest among these curves. In these cases, the wear process of inter-particles, which is related to the number of cycles and preloading stress amplitude, plays an important role on the influence of confining pressure on the value of $G_{\text{max}}^{\text{Mea.}}(Nc)/G_{\text{max}}^{\text{Cal.}}(Nc)$, which will be discussed in Section 6.6. Case 7 demonstrates a similar regularity to the four foregoing cases.

Unlike the five cases mentioned above, Cases 5 and 6 indicate a different variation of $G_{\text{max}}^{\text{Mea.}}(Nc)/G_{\text{max}}^{\text{Cal.}}(Nc)$ with confining pressure after preloading. As shown in Figure 6.38, the $G_{\text{max}}^{\text{Mea.}}(Nc)/G_{\text{max}}^{\text{Cal.}}(Nc)$ increases up to around 120% when the pressure increases to 100 kPa for the Dr=96.4% sample and to 123% when the pressure increases to 150 kPa for the Dr=43.7% sample, thereafter starts to decrease as confining pressure increases but it still keeps higher than 100% for both cases. The decrease of $G_{\text{max}}^{\text{Mea.}}(Nc)/G_{\text{max}}^{\text{Cal.}}(Nc)$ is faster for Case 6 than Case 5. The decrease of $G_{\text{max}}^{\text{Mea.}}(Nc)/G_{\text{max}}^{\text{Cal.}}(Nc)$ is possibly accounted for the final density of sample after preloading.

In conclusion, reloading confining pressure transforms soil structure into a stabler state for the sample subjected to preloading under lower confining pressure, which induces the small-strain shear modulus greater than its counterpart of the sample without experiencing vibration history. The maximum increase of G_{max} may be up to 20% when confining pressure increases to a range from 100 kPa to 200 kPa, thereafter G_{max} keeps stable or decrease but still greater than that obtained before preloading for the used samples. The greater reduction of G_{max} under preloading confining pressure the greater increase of G_{max} under confining pressure the preloading confining pressure at reloading path.

6.4.7 Effect of Unloading

The concept "unloading" herein means that the confining pressure is unloaded to a lower level after the desired number of preloading cycles has been complete. Multistage tests were initially performed at strain amplitude below 1×10^{-5} under a series of confining pressures which were increased from a very low level to 100 kPa or 200 kPa, under which dynamic preloading is applied. After the small-strain shear modulus has been measured, preloading was applied to the sample until a desired number of cycles was reached, then the G_{max} was determined after the vibrating was stopped for 10 minutes. After that the confining pressure was unloaded to a lower level to measure the G_{max} after the confining pressure was stable for 10 minutes. Testing results are presented in Figures 6.39-6.51.

As shown in these figures, the small-strain shear moduli measured under each confining pressure at unloading path after preloading are much lower than those determined at loading path before preloading except for the measured G_{max} under 200 kPa for the Dr=43.7% sample due to tremendous reduction of void ratio (see Figure 6.41). The small-strain shear moduli calculated by Equation 5.4 based on the change of void ratio corrected by Equation 6.4 are presented in these figures marked as the empty triangle up marks. The calculated small-strain shear moduli plot at the highest positions compared to those measured values before and after preloading. The distances between calculated G_{max} after preloading and measured G_{max} before preloading are dependent on the densification of sample after the preloading.

As shown in Figures 6.39 to 6.41 and Figure 6.47, the reduction of shear modulus increases with increasing number of preloading cycles, however, when the number of cycles reaches 1,000,000 the reduction becomes less than that of 100,000 cycles but larger than that of 10,000 cycles under the same preloading coditions. This tendency demonstrates that the microstructure of sample develops to an unstablest state when the number of preloading cycles reaches 100,000 cycles; more cycles of vibration cause the structure change into a stabler state than that after 100,000 cycles. As shown in Figures 6.42 and 6.47, the Dr=47.6 % sample was subjected to 2,000 cycles of vibration with the preloading stress amplitude of 20 kPa at the frequency of 0.1 Hz under the confining pressure of 100 kPa. The distance between the measured G_{max} before preloading and the calculated G_{max} after preloading is relatively small due to small reduction of void ratio thanks to the low vibrating frequency and low number of cycles applied. The $G_{\text{max}}^{\text{Mea}}(Nc)/G_{\text{max}}^{\text{Cal}}(Nc)$ curve in this case plots much higher than those obtained from similar density samples under 200 kPa as shown in Figure 6.47.



Figure 6. 39 Unloading effect on G_{max} after 10,000 cycles of preloading (Dr=43.4%)



Figure 6. 40 Unloading effect on G_{max} after 100,000 cycles of preloading (Dr=43.5%)

Figure 6.43 illustrates the results of the Dr=90.4% sample, which was subjected to 100,000 cycles of vibration with the same preloading amplitude and frequency as the first three medium dense samples, the distance between the measured and calculated

 G_{max} after preloading is relative smaller compared to those medium samples illustrated in Figures 6.39-6.41 due to low reduction of void ratio after preloading. The relative reduction in shear modulus can also be clearly seen in Figure 6.47, the $G_{\text{max}}^{\text{Mea.}} / G_{\text{max}}^{\text{Cal.}}$ plots higher than those medium dense samples due to less change of mircostructure as mentioned previously.

Figures 6.44-6.46 present the testing results of three Berlin sand samples of the similar densities varying from 74.9% to 77.5%. These samples were subjected to the same preloading amplitude but different frequencies and number of cycles under the same confining pressure of 100 kPa. It is very clear that the vibration frequency has strongly influence on the reduction of shear modulus, which increases with increasing frequency of preloading as shown in Figure 6.47. The influence of frequency is regarded to be the prestraining action on the soil structure when the same preloading stress amplitude is applied on the sample.

It is very interesting to note that the $G_{\max}^{\text{Mea.}} / G_{\max}^{\text{Cal.}}$ decreases with decreasing confining pressure as the confining pressure reaches 50 kPa or 35 kPa, the maximum reduction of $G_{\max}^{\text{Mea.}} / G_{\max}^{\text{Cal.}}$ may reach 52% for these medium samples, then starts to increase as the confining pressure continues decreasing for some cases, the reason that accounts for this increase is in question. Sample density and preloading frequency have significant influence on the influence of unloading on the $G_{\max}^{\text{Mea.}} / G_{\max}^{\text{Cal.}}$.



Figure 6. 41 Unloading effect on G_{max} after 1,000,000 cycles of preloading (Dr=43.7%)



Figure 6. 42 Unloading effect on G_{max} after 2,000 cycles of preloading (Dr=47.6%)



Figure 6. 43 Unloading effect on G_{max} after 100,000 cycles of preloading (Dr=90.4%)



Figure 6. 44 Unloading effect on G_{max} after 20,000 cycles of preloading (Dr=77.5%)



Figure 6. 45 Unloading effect on G_{max} after 200,000 cycles of preloading (Dr=74.9%)



Figure 6. 46 Unloading effect on G_{max} after 100,000 cycles of preloading (Dr=77.2%)



Figure 6. 47 Influence of unloading on G_{max} after preloading under the confining pressure of 100 kPa and 200 kPa (unloading path)

For the purpose of investigation on the influence of sand type on the unloading effect on small-strain shear modulus after preloading, three more different sand samples with medium density were prepared to test under the same preloading conditions (preloading amplitude of 40 kPa, frequency of 20 kPa, number of cycles of 100,000, and confining pressure of 200 kPa). Similarly, the G_{max} was also determined at loading path with confining pressure increasing from 15 kPa to 200 kPa. For analysis, the measured G_{max} of these three sands were initially fitted by Equation 5.4 to determine the stiffness coefficients, *S*, and pressure influence power, *n*, for these sands; and the obtained equation was used to predict the G_{max} base on void ratio after preloading. Figures 6.48 to 6.50 show the testing results of Cuxhaven fine and medium sands, and Braunschweig coarse sand with gravel, respectively. As shown in Figure 6.48, it is clear that the distance between the measured G_{max} before and calculated G_{max} after preloading is very small even though the relative density of the sample is merely 36.9% due to the densification of sample is relative low. The measured G_{max} at unloading path after preloading is just slightly lower than the predicted value under each confining pressure.



Figure 6. 48 Unloading effect on G_{max} after 100,000 cycles of preloading for Cuxhaven fine sand (Dr=36.9%)

Unlike Cuxhaven fine sand, as shown in Figures 6.49 and 6.50, the distances among these G_{max} of Cuxhaven medium sand and Braunschweig coarse sand are much larger than those illustrated in Figure 6.48, the variation of G_{max} is similar to that of Berlin sand. Figure 6.51 presents the testing results of these three different type sands together with Berlin sand with similar densities, which were subjected to the same preloading conditions. It is clear that soil type has very significant influence on the unloading effect on the $G_{\text{max}}^{\text{Mea.}} / G_{\text{max}}^{\text{Cal.}}$ after preloading. The $G_{\text{max}}^{\text{Mea.}} / G_{\text{max}}^{\text{Cal.}}$ of Cuxhaven fine sand is slightest affected by the unloading of confining pressure, it decreases from 102% under 200 kPa to 77.6% under 25 kPa after preloading. As shown in Figure 6.51, the $G_{\text{max}}^{\text{Mea.}} / G_{\text{max}}^{\text{Cal.}}$ curve of Braunschweig coarse sand with gravel, plots lowest compared to others, the $G_{\text{max}}^{\text{Mea.}} / G_{\text{max}}^{\text{Cal.}}$ decreases from 65.6% under 200 kPa to 40.9% as the confining pressure was unloaded to 15 kPa. As for the Cuxhaven medium sand and Berlin sand, the $G_{\text{max}}^{\text{Mea.}} / G_{\text{max}}^{\text{Cal.}}$ curves locate at middle positions. Therefore, it can be concluded that the decrease of G_{max} after preloading is dependent on the mean grain size of soils, namely increasing grain size results in an increase in the reduction of G_{max} at unloading path of confining pressure.



Figure 6. 49 Unloading effect on G_{max} after 100,000 cycles of preloading for Cuxhaven medium sand (Dr=52%)



Figure 6. 50 Unloading effect on G_{max} after 100,000 cycles of preloading for Braunschweig coarse sand (Dr=41.7%)



Figure 6. 51 Influence of unloading on G_{max} after preloading under the confining pressure of 200 kPa for various sands (unloading path)

It is worthy to note that, as shown in Figures 6.48-4.50, unlike Equation 5.4 can well fit the G_{max} at reloading path after preloading, Equation 5.4 fails to fit the G_{max} measured at unloading paths after preloading for all the samples, but a linear equation can well do this work, see Figures 6.39-6.46 and Figures 6.48-6.50.Unfortunately, it is not clear what makes the change of the relationship between G_{max} and confining pressure after preloading, further investigation and theoretical interpretation should be carried out to better understand this phenomenon.

6.4.8 Effect of Water Content

6.4.8.1 on Number of Cycles

Figures 6.52 and 6.53 illustrate the variation of small-strain shear modulus with number of cycles for two similar medium dense Berlin sand samples with water contents of 0% and 3%, which were subjected to the preloading stress amplitude of 20 kPa at the frequency of 20 Hz under the confining pressure of 100 kPa.

Figures 6.52 and 6.53 show that the small-strain shear moduli of both samples decrease with increasing number of cycles before it reaches 5,000 cycles thereafter start to increase. As shown in Figure 6.53, the $G_{\text{max}}^{\text{Mea.}}(\text{Nc})/G_{\text{max}}^{\text{Mea.}}(0)$ curve of the w=3% sample plots lower than that of the dry sample in the range of the number of cycles applied in these two tests. The G_{max} of the dry sample merely decreases to 87% of that obtain before preloading when the number of cycles reaches 5,000, however, the reduction of G_{max} of the w=3% sample may reach 68% of that obtain before preloading after the same number of cycles of 5,000 was reached. In addition, the accumulated axial strain of the dry sample keeps higher than that of the w=3% sample

within the range of number of cycles applied due to the capillary effect of the existence of water.

Besides two samples tested under the confining pressure of 100 kPa, four similar medium dense samples with different water contents were tested under the confining pressure of 200 kPa with preloading stress amplitude of 40 kPa at the frequency of 20 Hz, the testing results are presented in Figures 6.54 and 6.55. The water contents employed in these tests vary from 0% to 9% at the increment of 3%.

Similar to Figure 6.53, Figure 6.55 shows that the small strain shear moduli decrease with increasing number of preloading cycles as it reaches a given cycles thereafter starts to increase. The $G_{\max}^{\text{Mea.}}(\text{Nc})/G_{\max}(0)$ curve plots lower as the water content increases from 0% to 6%, the reduction of small-strain shear modulus may up to 32%, beyond 6% the $G_{\max}^{\text{Mea.}}(\text{Nc})/G_{\max}(0)$ curve shifts to a higher position but lower than the curves of 0% and 3%. This finding is opposite to the influence of water content on G_{\max} obtained before preloading as addressed in Chapter 5. Comparison of the $G_{\max}^{\text{Mea.}}(\text{Nc})/G_{\max}(0)$ curve of the Dr=42.2% sample with that of the Dr=55.9% sample shows void ratio and the preloading confining pressure might have no influence on the water content effect on the variation of G_{\max} with number of cycles for medium dense sands.

Therefore, one can conclude that Water Content has significant influence on the effect of number of cycles on small-strain shear modulus of sand. The reduction of shear modulus with number increase with an increase in water content before it reaches the optimum value of 6%, beyond which the influence becomes less for the used Berlin sand in this study.



Figure 6. 52 Effect of water content on the variation of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles under confining pressure of 100 kPa



Figure 6. 53 Effect of water content on the variation of normalized G_{max} with number of cycles under confining pressure of 100 kPa



Figure 6. 54 Effect of water content on the variation of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles under confining pressure of 200 kPa



Figure 6. 55 Effect of water content on the variation of normalized G_{max} with number of cycles under confining pressure of 200 kPa

6.4.8.2 on Effect of Unloading

The Dr=55.5% dry sample and the Dr=55.9% wet sample with the water content of 3% were used to analyze the influence of Water Content on the G_{max} determined at unloading path. The water content influence on the development of G_{max} with number of cycles can be seen in Figures 6.54 and 6.55. The small-strain shear modulus tests were initially performed under the confining pressure increasing from 15 kPa to 200 kPa, and then the preloading was applied and G_{max} was determined at a given interval number of cycles until 100,000 cycles of vibration was reached; thereafter determined the G_{max} after the sample had rested under the preloading confining pressure of 200 kPa for 10 minutes. Then the G_{max} test was repeated after the confining pressure was released to a series of lower levels for 10 minutes at the unloading path of the pressure from 200 kPa to 25 kPa. Another Dr=55.5% sample with the water content of 3% was also tested through a testing program similar to that of these two foregoing samples but the preloading confining pressure is 100 kPa, the detailed preloading conditions are presented on Figure 6.58.

Figures 6.56-6.58 present the values of three different type small-strain shear moduli $(G_{\max}^{\text{Mea.}}(Nc), G_{\max}^{\text{Cal.}}(Nc))$, and $G_{\max}(0)$ for the dry sample and wet samples, respectively. Figure 6.56 shows an influence of unloading effect on G_{\max} of the dry sample similar to those described in Section 6.47. As for the two w=3% wet samples with the same density, the measured G_{\max} obtained before preloading keeps higher than the measured G_{\max} after preloading under the same confining pressure at unloading path, and the calculated G_{\max} based on the void ratio corrected after preloading locates at the highest position. As mentioned previously, the G_{\max} measured after preloading linearly decreases with decreasing confining pressure at the unloading path for the dry samples as shown in Figure 6.56. It is very interesting to note that, unlike the variation pf G_{\max} with confining pressure in a linearity way at unloading path after preloading for dry sample, the relationship between G_{max} and confining pressure for the wet samples can not be expressed by a linear function at unloading path of confining pressure after preloading, but well expressed by an exponential function as illustrated in Figures 6.57 and 6.58. In addition, confining pressure under which preloading is applied shows no influence on this relationship between G_{max} and confining pressure observed at unloading path. Unfortunately, however, the theoretical interpretation of the change of relationship between G_{max} and confining pressure at unloading path for the wet samples after preloading.

Figure 6.59 presents the variation of the ratio of the measured G_{max} to calculated G_{max} for these two samples which were subjected to the same preloading conditions under the confining pressure of 200 kPa. It can be seen in this figure that the $G_{\text{max}}^{\text{Mea.}} / G_{\text{max}}^{\text{Cal.}}$ curve of the w=3% sample locates lower than that of the dry sample, the maximum reduction of G_{max} may increase 67% as the confining pressure is unloaded to 50 kPa which is much greater than 55% of the dry sample under the confining pressure of 25 kPa. This finding further confirms that Water Content has influence on the unloading effect on the reduction of G_{max} after preloading.



Figure 6. 56 Effect of water content on unloading effect on G_{max} after 100,000 cycles of preloading under confining pressure of 200 kPa (w=0%, Dr=55.5%)



Figure 6. 57 Effect of water content on unloading effect on Gmax after 100,000 cycles of preloading under confining pressure of 200 kPa (w=3%, Dr=55.9%)



Figure 6. 58 Effect of water content on unloading effect on Gmax after 100,000 cycles of preloading under confining pressure of 100 kPa (w=3%, Dr=55.5%)



Figure 6. 59 Influence of water content on unloading effect on normalized G_{max} after preloading of 100,000 cycles under confining pressure of 200 kPa

6.4.9 Miscellaneous Effects

Sample Preparation Method

Two dense samples were prepared to investigate the influence of sample preparation method on the preloading effect on shear modulus of sand. One sample was prepared by the raining technique, which has the relative density of 65.7%; the other was built by the tamping method, which has the relative density of 66.7%. These two samples were tested under the same testing program, multistage small-strain shear modulus tests were carried out under the confining pressure varying from 15 kPa to 200 kPa, thereafter the preloading was applied and G_{max} was determined at a given interval of number of cycles until it reached 100,000 cycles.

Figure 6.60 presents both measured and calculated G_{max} predicted by Equation 5.4 based on the variation of void ratio during vibrating versus number of cycles for these two samples. Figure 6.61 presents $G_{\text{Max}}^{\text{Mea.}}(Nc)/G_{\text{Max}}(0)$, $G_{\text{Max}}^{\text{Mea.}}(Nc)/G_{\text{Max}}^{\text{Cal.}}(Nc)$, and accumulated axial strain with number of cycles. Figures 6.62 and 6.63 illustrate the unloading effect on the G_{max} after preloading for these samples prepared by tamping and raining methods. As illustrated in these figures, sampling technique has no significant influence on the variation of G_{max} with number of cycles for the samples prepared by raining and tamping methods, as well as the reduction of G_{max} at unloading path after the sample were subjected to 100,000 cycles of preloading under the confining pressure of 200 kPa.



Figure 6. 60 Sampling method effect on the development of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles for dense sample



Figure 6. 61 Sampling method effect on the development of normalized G_{max} with number of cycles for dense sample



Figure 6. 62 Sampling method effect on unloading effect on G_{max} after preloading under confining pressure of 200 kPa



Figure 6. 63 Sampling method effect on unloading effect on normalized G_{max} after preloading under confining pressure of 200 kPa

Pressure Release

The term "pressure release" means that the confining pressure is unloaded to a lower pressure than the preloading confining pressure after preloading is applied, then it is reloaded to the preloading confining pressure. Data presented herein were obtained from two samples mentioned above. After 100,000 cycles of vibration were completely applied, the sample rested at 200 kPa for 10 minutes and made the G_{max} test, then unloaded confining pressure to 150 kPa and made G_{max} test after pressure reached the target confining pressure for 10 minutes, in succession reloaded confining pressure to 200 kPa and made the G_{max} test after the pressure was stable for 10 minutes. The confining pressure was released from 200 to 100 kPa, 75 kPa, 50 kPa, and 25 kPa in decreasing order as the procedure described for the 150 kPa.

Figure 6.64 indicates the effect of pressure release on the small-strain shear modulus of two samples subject to the preloading conditions as described in above paragraphs. In this figure, the G_{max} measured under the confining pressure of 200 kPa without pressure release after preloading is marked as G_{max}^{200} , and those measured after pressure release are marked as $G_{\text{max}}^{\text{PR}}$ in general. It is seen that the $G_{\text{max}}^{\text{PR}}$ increases with decreasing the target release pressure. The increase of $G_{\text{max}}^{\text{PR}}$ may be up to 22% after the pressure was released to 25 kPa. In addition, there is no significant influence of sample preparation technique on the effect of pressure release on the small-strain shear modulus after preloading.



Figure 6. 64 Influence of pressure release on G_{max} under confining pressure of 200 kPa after preloading

Vibration Mode and Time

One dry sample was prepared with the relative density of 61.3% by raining technique; which was subjected to multistage small-strain tests under the confining pressure varying from 15 kPa to 200 kPa. The sample was preloaded to 100,000 cycles by application of the flexural vibration generated by the two magnets of the drive system of the resonant column apparatus with the input voltage of 1.90 V at the frequency of 20 Hz under the confining pressure of 200 kPa, unfortunately the stress amplitude of the vibration is unkown due to a shortage of calculation technique. After the

preloading, the sample was consolidated under 200 kPa for 900 minutes to study the effect of time on small-strain shear modulus of the sample. Thereafter the confining pressure was released by the procedure described in the previous paragraph to investigate the influence of vibration mode on the G_{max} after preloading.

For the purpose of comparison, data of the Dr=65.7% sample were replotted in Figures 6.65 and 6.66 together with those of the Dr=61.3% sample preloading by flexural vibration. As illustrated in these two figures, the qualitative variation of G_{max} with number of cycles for the Dr=61.3% sample preloaded by flexural vibration similar to that for the Dr=65.7% sample preloaded by torsional vibration. The $G_{\text{max}}^{\text{Mea.}}(\text{Nc})/G_{\text{max}}(0)$ and $G_{\text{max}}^{\text{Mea.}}(\text{Nc})/G_{\text{max}}(Nc)$ curves of the Dr=61.3% sample are slightly and significantly higher than those of the 65.7% sample, respectively. The accumulated axial strain during vibrating of the sample subjected to flexural vibration at the same number of cycles. The distance between these two vibrating modes is accounted for the fact that the flexure is generated by the electromagnetic power induced by a pair of magnets and two pairs of coils as illustrated in Figure 3.5; consequently, the resulting shear stress is smaller than that by torsional vibration.

Figures 6.67 and 6.68 illustrate the influence of the different types of vibration on the unloading effect on the small-strain shear modulus after 100,000 cycles of vibration. As shown in these figures, the tendency of the decease in G_{max} with confining pressure for the sample vibrated by flexural vibrating mode is similar to by torsional vibrating mode. Similarly, the reduction degree of G_{max} obtained at unloading path after preloading is slightly smaller for flexural vibration mode than for torsional vibration mode due to a much unstabler structure caused by the torsional vibration compared to that by the flexural vibration thanks to a higher preloading stress amplitude induced by the electromagnetic force generating by the four pairs of coils of drive system.

Figure 6.69 presents the increase of small-strain shear modulus with confinement of duration under confining pressure of 200 kPa for the Dr=61.3% sample after preloading together with that of the Dr=64.2% sample without preloading illustrated in Figure 5.11. The testing history of the Dr=61.3% sample has already been described in previous paragraph in this section, and that of the Dr=64.2% sample can be seen in Section 5.2.5 in Chapter 5. In this figure, the G_{max} is normalized by the G_{max} measured after the sample rested for three minutes after the preloading was stopped. It can be seen in this figure that both small-strain shear moduli of virgin sample and the sample subjected to vibration history slightly increase with time, the increase of G_{max} after preloading is slightly greater than that of the virgin sample. The $G_{\text{max}}(T)/G_{\text{max}}(3)$ might increase to 1.07 after the sample rested for 900 minutes after preloading, which indicates that time effect on the G_{max} of sample subjected to preloading history is also unimportant as for virgin sample.



Figure 6. 65 Vibration mode effect on the development of G_{max} and $\varepsilon_a^{\text{Acc}}$ with number of cycles under confining pressure of 200 kPa



Figure 6. 66 Comparison of the reduction of G_{max} with number of cycles by torsional and flexural vibrations under confining pressure of 200 kPa



Figure 6. 67 Effect of vibration mode on unloading effect on G_{max} after 100,000 cycles of preloading



Figure 6. 68 Effect of preloading vibration mode on the unloading effect on relative reduction of G_{max} after preloading



Figure 6. 69 Effect of time on small-strain shear modulus of samples without preloading and after preloading under confining pressure of 200 kPa

6.5 Nonlinear Dynamic Properties

6.5.1 Effect of Preloading Frequency

Three medium dense samples were measured the shear modulus and damping ratio at shearing strain varying from small-strain to high strain after they were preloaded under the confining pressure of 100 kPa with the torsional shear stress of 20 kPa at the frequency of 5 Hz, 20 Hz, and 40 Hz, respectively, the total number of cycles of each test is presented in Figures 6.70-6.72.

Figure 6.70 presents the variations of shear modulus and damping ratio with shearing strain amplitude under confining pressure of 100 kPa for the Dr=41.4% sample, which had been subjected to 1,000,000 cycles of preloading at the frequency of 40 Hz under the confining pressure of 100 kPa before the nonlinear properties were determined. The Dr=43.9% sample experienced the preloading conditions similar to those for the Dr=41.4% sample was also preloaded to 200,000 cycles under the confining pressure of 100 kPa with identical preloading stress amplitude at the frequency of 5 Hz. For the purpose of comparison, shear modulus and damping ratio at various strains are predicted by Equations 5.8 and 5.13, respectively, and collectively depicted in these figures.

The shear moduli measured after preloading illustrated in Figures 6.70 and 6.71, are greater than the measure and predicted values before preloading. The increase in shear

modulus is dependent on the frequency at which the previbration was applied. It can been seen that the increase in shear moduli for the sample vibrated at the frequency of 40 Hz are much higher than those measured and predicted before preloading, see Figure 6.70. However, the modulus increment is relatively lower for that of the Dr=43.9% sample vibrated at the frequency of 20 Hz compared to the Dr=41.4% sample as shown in Figure 6.71. The more increase in shear modulus of the sample vibrated at higher frequency is accounted for the higher prestraining due to higher frequency. For the Dr=44.3% sample which was vibrated at the frequency of 5 Hz, the shear modulus was measured lower than both measured and predicted shear moduli before preloading, see Figure 6.72. The decrease in shear modulus is because of the number of preloading cycles was insufficient large to increase the shear modulus higher than before preloading.



Figure 6. 70 Shear modulus and accumulated axial strain versus shearing strain before and after preloading at 40 Hz (Dr=41.4%)

The development of the accumulated axial strain with shearing strain amplitude is also depicted in Figures 6.70-6.72. As shown in these figures, the ε_a^{Acc} nearly keeps stable with increasing shearing strain amplitude increases if the shearing strain is lower than its corresponding threshold strain, thereafter slightly increases. The rate of increase in ε_a^{Acc} with shearing strain decreases with the preloading frequency due to greater densification of the sample when it was preloading at higher frequency. For instance, the ε_a^{Acc} of the sample after preloading at 5 Hz is smallest compared to the others, and the densification resulting from preloading is lowest among these samples.

Figure 6.73 indicates the normalized shear modulus and damping ratio versus shearing strain amplitude after preloading. As shown in this figure, there seems no significant influence of preloading frequency on the normalized shear modulus for the samples preloaded at the frequency of 20 Hz and 40 Hz. The normalized shear modulus reduction curve of the sample subjected to 200,000 cycles at the frequency

of 5 Hz plots highest among these curves. The rate of increase in normalized shear modulus with shearing strain amplitude increases with decreasing preloading frequency as illustrated in Figure 6.73.



Figure 6. 71 Shear modulus and accumulated axial strain versus shearing strain before and after preloading at 20 Hz (Dr=43.9%)



Figure 6. 72 Shear modulus and accumulated axial strain versus shearing strain before and after preloading at 5 Hz (Dr=44.3%)



Figure 6. 73 Normalized shear modulus and damping ratio with shearing strain after preloading under confining pressure of 100 kPa

It is clear that damping ratios of these samples are well in agreement with the predicted curve by Equation 5.13 under the confining pressure of 100 kPa for virgin samples, and they were measured significantly greater than the predicted curve as the shearing strain ranges from 0.001% to 0.04%. The preloading frequency shows no significant influence on the damping ratio curve.

6.5.2 Effect of Number of Cycles

Five medium dense samples are here used to investigate the influence of number of preloading cycles on the nonlinear dynamic properties of sand after preloading. Four samples were initially previbrated by the preloading stress amplitude of 10 kPa at the frequency of 20 Hz under the confining pressure of 50 kPa with the number of cycles varying from 1,000 to 1,000,000 cycles, thereafter they were tested by increasing the shearing strain amplitude from possible small to large level, one sample was tested without vibration history for the purpose of comparison.

Figure 6.74 illustrates the normalized modulus and damping versus shearing strain amplitude curves for these five samples, the normalized modulus and damping curves predicted by Equations 5.8 and 5.13 for virgin specimen are also plotted for analysis. As shown in this figure, the number of cycles has significant influence on the positions of these curves. Both normalized modulus and damping curves of the virgin specimen follow the predicted tendencies very well; which further confirms the precision of the predicted equations proposed in Chapter 5. The normalized modulus curve plots at the highest position for the sample subject to 1,000 cycles of preloading, when the number of cycles is lower than 100,000 it shifts to a left lower position if the number of cycles increases but still higher than the virgin curve. As the number of cycles increases to 1,000,000, the normalized modulus curve shifts to a position

totally lower than the virgin curve.

Unlike the tendency of normalized shear modulus curve with number of cycles, the damping curve contrarily shifts left higher with increasing number of cycles especially the damping curve obtained after 1,000,000 cycles of preloading shifts to a position which is much higher than the curve for the virgin sample. This phenomenon further confirms the fact that damping ratio increases with reducing normalized shear modulus at the same shearing strain amplitude reported by previous investigations.



Figure 6. 74 Influence of number of cycles on nonlinear dynamic properties of medium dense sand under the confining pressure of 50 kPa

6.5.3 Effect of Reloading

Figure 6.75 presents the nonlinear dynamic properties of four medium dense Berlin sand samples (Dr=40%-48.3%) which had been previbrated to 10,000 cycles with the preloading stress amplitude of 10 kPa at frequency of 20 Hz under confining pressure of 50 kPa. One sample was determined the nonlinear dynamic properties under 50 kPa, the others were tested under the confining pressure of 100 kPa, 200 kPa, and 400 kPa after preloading under confining pressure of 50 kPa, respectively.

As shown in this figure that both normalized shear modulus and damping ratio measured under higher confining pressure are influenced by the preloading applied under the confining pressure of 50 kPa. The normalized shear modulus curve of sample after preloading determined under the confining pressure greater than 50 kPa keeps higher than its counterpart for the virgin sample under the same confining pressure if the confining pressure was increased not exceeding 400 kPa. It can be seen that the distance between the normalized modulus curve of the sample subjected to preloading under 50 kPa and that of virgin sample is decreased by increasing confining pressure. By contraries, the damping curves locate at lower positions than their counterparts of virgin samples, as the confining pressure increases to 400 kPa,

the damping ratios well agree with the curves of the virgin sample obtained under confining pressure of 400 kPa. It can be concluded that increasing the confining pressure may reduce the influence of preloading under lower confining pressure.



Figure 6. 75 Reloading effect on nonlinear dynamic properties after preloading under confining pressure of 50 kPa for medium dense sand

6.5.4 Effect of Unloading

Two medium sand samples (44.4% and 44.8%) were preloaded under confining pressure of 200 kPa with preloading stress amplitude of 40 kPa at frequency of 20 Hz to 10,000 cycles. The confining pressures were decreased to 100 kPa and 50 kPa; then the shear modulus and damping ratio were tested with the shearing strain amplitude ranging from a possible low to high strain level. Besides, three high dense samples (89.7% to 90.5%) were subjected to the same preloading conditions as previous two medium samples but 100,000 cycles of vibration. One sample was directly measured the shear modulus and damping ratio under the preloading confining pressure (200 kPa), and the others were tested after the confining pressure was unloaded to 100 kPa and 50 kPa, respectively. For comparison, three similar dense density samples were prepared to test the nonlinear properties under confining pressures of 50 kPa, 100 kPa, and 200 kPa without vibration history, respectively. Two medium dense samples with the relative densities of 46.7% and 42.1% were preloaded under identical preloading conditions from 100 and 1,000,000 cycles and unloaded the confining pressure to 100 kPa to show the influence of number of cycles on the unloading effect on nonlinear dynamic properties.

As shown in Figure 6.76, the normalized shear modulus reduction curves of both medium dense samples plot much higher and flatter than those predicted by Equation

5.8 for virgin samples as shearing strain amplitude exceeds 0.001%. For the case that the confining pressure was unloaded to 50 kPa, the modulus reduction curve plots higher than the 50 kPa predicted curve and even higher than the 100 kPa curve, and for the case that pressure was unloaded to 100 kPa the curve locates higher than the 100 kPa predicted curve even higher than the 200 kPa curve. Accordingly the damping curve for the case that confining pressure was unloaded to 50 kPa plots lower than their corresponding curve of virgin sample; the curve for the case that confining pressure was unloaded to 100 kPa plots much lower the predicted damping curve, and agrees with the 200 kPa predicted damping curve.

As shown in Figure 6.77 for dense samples, unlike that in Figure 6.76, confining pressure shows no significant influence on the location of the normalized modulus reduction curves measured after these samples were subjected to the same preloading conditions. These curves are much flatter than those measured from virgin samples. The normalized modulus starts to up deviate from its counterpart as shearing strain amplitude exceeds the strain of 0.0032% for the 50 kPa curve; and 0.0046% for the 100 kPa curve, the magnitude of deviation increases with an increase in shearing strain amplitude. Unlike those measured under 50 kPa and 100 kPa, the normalized shear modulus measured under confining pressure of 200 kPa after preloading was observed obviously lower than that of virgin sample when the shearing strain is lower than 0.017% beyond which it becomes greater.



Figure 6. 76 Unloading effect on nonlinear dynamic properties after preloading under confining pressure of 200 kPa for medium dense sand



Figure 6. 77 Unloading effect on nonlinear dynamic properties after preloading under confining pressure of 200 kPa for dense sand

Damping ratios measured under the confining pressures of 50 kPa after preloading are much lower than those of virgin sample as shearing strain greater than 0.005%. The damping ratio of the 100 curve was found to the virgin damping curve of 200 kPa. And damping ratios of the 200 kPa curve are measured larger than those of its corresponding of virgin sample when the shearing strain is lower than 0.022% beyond which they are nearly the same.

Figure 6.78 indicates that these normalized modulus reduction curves after various numbers of preloading cycles all plot much higher than the 100 kPa curve of virgin sample even over the 400 kPa curve of virgin sample as shearing strain exceeds 0.02%. The modulus reduction curves of 100 and 10,000 cycles of preloading slightly higher than that of 1,000,000 cycles as the shearing strain amplitude is lower than 0.013%, beyond which the curve of 10,000 cycles plots always highest. The damping curves also plot lower than the 100 kPa curve of virgin specimen and well agree with the 200 kPa curve of virgin sample as the shearing stress lower than 0.013% for the 100-cycle curve. The 1,000,000-cycle damping curve has a good agreement with the 200 kPa damping curves. Number of preloading cycles under higher confining pressure has relatively small influence on the nonlinear dynamic properties measured under the confining pressure lower than the preloading confining pressure.



Figure 6. 78 Nonlinear dynamic properties under confining pressure of 100 kPa after various numbers of cycles under confining pressure of 200 kPa for medium dense sand

6.5.5 Effect of Water Content

Four samples were prepared by tamping technique with relative densities varying from 51.3% to 55.9%, their water contents varies from 0% to 9%. All the samples were previbrated with the preloading stress of 40 kPa at 20 Hz under the confining pressure of 200 kPa, all samples except the w=6% sample were preloaded to 100,000 cycles of vibration before the nonlinear shear modulus and damping ratio were determined. The w=6% sample was subjected to 800,000 cycles before the nonlinear dynamic properties were tested.

As shown in Figure 6.79, the normalized modulus curve of the dry sample plots clearly higher than the predicted curve and the curves of wet samples. As the water content increases the curve starts to shift left lower and reach to the lowest position when the water content reaches the water content of 6%, as the water content continuously increases to 9% the curve shifts right higher but lower than the w=3% curve. It should be noted that the number of cycles of the w=6% sample is much higher than others, which might contribute some influence on the position of the curve. Accordingly, the damping curves shift in the opposite direction to that of normalized modulus reduction curves. Water content does not change the qualitative relationship between normalized shear modulus and damping ratio.



Figure 6. 79 Influence of water content on normalized shear modulus and damping ratio after preloading under the confining pressure of 200 kPa for medium dense sample (Dr=51.3%-55.9%)

6.5.6 Effect of Prestraining

A medium dense sample (Dr=44.1%) was prepared to apply the vibration as previous investigations. This sample was initially tested from a possible low to the shearing strain amplitude of around 0.026% under the confining pressure of 200 kPa, shear modulus and damping determined at the first increase of shearing strain is regarded as no prestraining. And then directly decreased the shearing strain to the possible low level to make shear modulus and damping once more, in this stage, the sample is assumed to be vibrated with 1,000 cycle of prestraining amplitude of 0.026% according the testing program settings. After the sample was subjected 2.3 million cycles of prestraining at 0.026%, the tests was carried out from a possible low strain to 0.069%; and then decreased the strain to a possible low level repeated the tested as described for the application of the 0.026% prestraining for that of the prestraining amplitude of 0.069% to 10,000 cycles. Testing results are presented in Figures 6.80 and 6.81.

The variation of shear modulus and damping ratio with number of cycles is similar to the cases that samples vibrated by the preloading concept (stress-controlled shear). Shear modulus decreases with number of cycles when it does not exceed 100,000 cycles for the prestraining amplitude of 0.026%, the distance between shear modulus before and after prestraining decreases with increasing shearing strain amplitude and approaches smallest at the prestraining amplitude. After 2.3 million cycles of prestraining of 0.026%, shear modulus was measured higher when the strain lower than 0.0003% beyond which lower than those obtained after 10,000 cycles

prestraining.

The shear moduli rapidly increase to approach those before prestraining when the shearing strains are lower than 0.0062% after 1,000 cycles of prestraining of 0.069%. Further cycles of prestraining seem have less increasing effect on the shear modulus. All normalized modulus curves obtained after prestraining but the curve after 2.3 million cycles plot higher than the first increasing strain curve. Number of cycles seems no significant influence on the normalized shear modulus if it is lower than 100,000. The normalized shear modulus curve plots much lower and damping curve plots much higher than their corresponding virgin curves after 2.3 million cycles of prestraining, which is similar to those of the sample subjected to 1 million cycle of preloading as illustrated in Figure 6.74. Possibly large number of cycles is accounted for the greater deviations of normalized shear modulus and damping ratio from those obtained before prestraining. In addition, after 1,000 and 10,000 cycle of prestraining at 0.069% the normalized shear modulus curve slightly plots higher than those prestraining at 0.26%.



Figure 6. 80 Shear modulus and accumulated axial strain versus shearing strain amplitude after various numbers of cycles of prestraining



Figure 6. 81 Normalized shear modulus and damping ratio versus shearing strain amplitude after various numbers of cycles of prestraining

6.6 Theoretical Interpretation

Previous investigations indicate that shear modulus of dry sand increases with number of loading cycles if the prestraining amplitude exceeds the threshold volumetric strain. Some researches reported the increase of shear modulus could not be accounted for the densification of sand (Drnevich and Richart 1970), but the wear process of soil particles, they pointed out that the large amplitude shearing strains occurring within the specimen were sufficient large to make the relative motion between particles occur, but the large amplitude shearing strains were not large enough to cause large the particles reorientation (densification or dilation) to occur, the prestraining applied abrasive action and caused the nature of these points of contact to wear, and generated additional contacts, hence the actual contact area increased; however, others (Alarcon-Guzman, Chameau et al. 1989) indicated that the influenced of prestraining on small-strain shear modulus was small, the increase is due to the densification after prestraining. Wichtmann and Triantafyllidis (2004) interpreted that two possible causes were accounted for the increase of shear modulus of sand, one was the change of the shape of particle contacts from conus-sphere to sphere-sphere contact shape after prestraining; the other was the reduction of stress fluctuation due to the application of prestraining. Reversely, Cundall and Strack (1979), Cundall et al.

(1982), Chen and Ishibashi (1990), and Chen and Hung (1991) studied the variation of fabric of granular material by numerical simulation, and indicated that the number of contacts might loss with the development of anisotropy of even in the case that the material was contracting.

In torsional resonant column test, it typically is recognized that the wave travels through isotropic media, consequently has the same wave speed in all directions. Actually, the wave is vertically propagated from the top to bottom of tested sample along the axis of the sample. As mentioned in Chapter 2, the shear wave velocity is much more sensitive to the stress in the direction in which the wave propagating. In nature, the increase of velocity is accounted for the increase of contact number or area in the plane which is perpendicular to the wave propagating direction. The structural anisotropy effect on natural soil stiffness has already been investigated by previous investigations by various means.

Saada, et al. (1978) demonstrated the importance of the clay fabric and its influence on the dynamic behavior. They concluded that, when determining the shear moduli of clay, the relative arrangement of its particles should not be ignored. It was shown that substantial difference exists among the moduli obtained from vertical and horizontal specimen. Macari and Ko (1994) presented that the small-strain shear moduli of remolded silt in vertical direction (deposit direction) kept greater than those in horizontal direction. This phenomenon can be accounted for the orientation of soil particle resulting in the loss contact area in the direction in which particle aligning.

Berlin sand, Cuxhaven medium sand, and Braunschweig coarse sand are round and elliptical in shape, and Cuxhaven fine sand has somewhat lower roundness than others, and higher sphericity, as shown in Figures 4.1-4.4. Medium dense sample shows a homogenous performance in structure, orientation of soil particles is hardly formed, as shown in Figure 6.82-left. When the sample is subjected to sufficient large shearing strain amplitude exceeding the threshold volumetric strain, the motion or slippage between particles occurs, resulting in the volume of sample contracting under isotropic confinement condition, which usually occurs in resonant column test at high strain. As vibration goes on, the soil particles gradually align in vertical direction (Figure 6.82-right), which induces a reduction of number of contacts in vertical direction and an increase in radial direction. This change can be confirmed by the relationship between the vertical and radial strain as presented in Figure 6.2, in which the radial strain is 1.57 times great of the vertical strain. The orientation of soil structure is dependent on the sphericity of individual soil particle, the lower sphericity the higher orientation occurs after vibration. In nature, soil particles of sands are almost ellipsoid in shape, applying a shear to soil causes structure orientation, especially for granular material like sand.

In the author's opinion, during the process of vibrating in torsional shear or simple shear mode, the wear process of soil particles, densification, the reorientation of soil particles, and loss of net number of contact occur simultaneity. The former two processes increase the number or area of contacts; contrarily the latter two ones reduces particle contact area in the direction in which wave propagation, and increase the contact area in horizontal direction in resonant column test. The reorientation of particles and contact number loss easier occur in looser sample than in denser sample, and the wear process easier occurs in dense sample. Shear modulus is a function of
the area of contact in the direction in which the shear wave propagates. The area of contact is dependent on three foregoing actions, as for a given sample, there exists a critical state below which decrease effect has an overwhelming action on changing contact area than increasing effect of densification and wear process in the direction that the shear wave propagates, and beyond which the wear process and or densification play an more significant role than particle reorientation and contact number loss effect. The wear process and densification are both dependent on the shearing strain, namely the degree of wear process and densification increases with the shearing strain after same number of vibrating cycles, besides the degree of wear process also increase with a decrease in particle roundness due to abrasion of sharp points during motion between particles.



Figure 6. 82 Sketch map of development of the microstructure of sand before and after preloading by torsional shear: (1) wear process; (2) reorientation of soil particles

As indicated in Figure 6.16, Cuxhaven fine sand demonstrates a significantly different variation tendency of small-strain shear modulus with number of loading cycles, the observed reduction in G_{max} is very small; however, other sands show much greater reduction in shear modulus compared to the fine sand. It seems that the magnitude of reduction in G_{max} increases with an increase in grain size. Inspired by this finding, two different diameter uniform glass beads (2.5 g/cm³ was taken as the specific gravity); one has 0.06 mm in diameter the other has 0.5 mm, and a sieved Berlin sand which were stay on the 0.125 mm opening sieve but passing the 0.25 mm sieve were tested for this purpose, 0.188 mm is regarded as the mean grain size. All the samples are prepared by raining technique by keeping the opening of tube of funnel slightly contacting material surface during falling to have a relatively low density. These samples were subjected to the same preloading stress amplitude of 30 kPa at the frequency of 20 Hz under confining pressure of 200 kPa. The small-strain shear moduli were measured at given interval of number of cycles after the vibration had been stopped for three minutes. Figure 6.83 presents the testing results of these three materials combing with the data of Cuxhaven fine sand illustrated in Figure 6.16. Note that the accumulated axial strains in Figure 6.83 were obtained by subtracting the total axial strains to those induced by the confining pressure before vibration.

As shown in Figure 6.83, the sieved Berlin sand (fine sand) shows a similar development tendency of G_{max} with number of cycles as that of Cuxhaven fine sand, the slightly increase in Cuxhaven find is contributed to the reduction of void ratio demonstrated by the greater accumulate axial strain. As for the case of 0.06 mm glass beads, the shear modulus decreases to 88% when the number of cycles reaches 5, thereafter starts to increase and reaches 150% after 1,000,000 cycles of vibration, which is accounted for the reduction of void ratio as well wear process as mentioned above. As for the case of 0.5 mm glass bead, the shear modulus continuously decreases with increasing number of cycles and may reduce to approximate 82% when it is lower than 20,000, thereafter recovers with number of cycles and increases to 97.4% after the number reaches 1,3000,000 cycles. The change of void ratio of sample is very small as exhibited by the accumulated axial strain curve as shown in Figure 6.83. The decrease in shear modulus is accounted for the loss of net contacts after large amplitude preloading (Cundall and Strack 1979; Cundall, Drescher et al. 1982; Chen and Ishibashi 1990; Chen and Hung 1991). The increase in shear modulus after 100,000 cycles is attributed to the abrasive action between particles and no change of net contacts. Comparison the sieved Berlin sand curve and the 0.5 mm glass bead curve shows the influence of grain size the development of shear modulus with number of cycles.

The measured G_{max} is normalized the calculated G_{max} considering reduction of void ratio based on Equation 6.4 after preloading to present the vibration amplitude influence on the effect of wear process and particle reorientation on shear modulus. Analytical results are presented in Figure 6.84.

As shown in Figure 6.84, the reduction speed of ratio of G_{max} with number of cycles decreases with increasing preloading frequency, the maximum reduction of shear modulus for each curve similarly decreases with increasing preloading frequency, which implies that shearing strain plays an important role in the reduction action of shear modulus. As known, the shearing strain amplitude increases with vibrating frequency if the frequency is lower than the resonant frequency with a given shear stress amplitude.

When the sample is sheared at relatively lower strain level, the abrasive action between particles is small and the reorientation action plays an overwhelming role in change of shear modulus, consequently shear modulus demonstrates to decrease with number of cycle. As the number of cycles increases the reorientation of particles continues develops and reaches a critical state when the number achieves a relatively larger threshold value then soil stiffness reaches the minimum value. Thereafter, the shear modulus is not affected by reorientation action but the wear process continues with number of cycles, and the shear modulus demonstrates to increase with number of cycles. If the sample is sheared at relative higher strain amplitude, the wear process plays a larger role in increasing shear modulus; hence the maximum reduction in shear modulus is lower and reached earlier, especially for the case of 40 Hz as shown in this figure.



Figure 6. 83 Grain size influence on the development of small-strain shear modulus with number of cycles for various granular materials



Figure 6. 84 Influence of preloading frequency on G_{max} considering variation of void ratio with number of cycles under confining pressure of 100 kPa for medium dense Berlin sand

Increasing confining pressure may significantly increase the contact area of sample subjected to preloading under lower confining pressure, which may increase the contact area to an extent that is larger than the area before preloading, consequently shear modulus demonstrates to be higher than that no preloading at the same higher confining pressure. However, unloading the confining pressure to a lower level may further decrease the contact area, therefore the shear modulus exhibits lower than both moduli measured before and after preloading.

The normalized shear modulus curve higher and damping curve plots lower if the soil structure is at an unstabler state, which is observed in the reloading and unloading effect on the nonlinear dynamic properties of sample after preloading. This can be explained by the fact that after preloading the shear modulus of soil has already decreased a lot further shear can only decrease a small magnitude of shear modulus. As for the damping ratio, as the soil structure is in unstable state and the friction between particles is less than in stable state, which resulting in less energy loss during damping test.

6.7 Summary

An empirical equation was proposed to consider the variation of void ratio during vibrating is developed based on the measured dimensions before and after test. A simple G_{max} correction method is also generated to improve the calculation precision of G_{max} if great change of dimensions of specimen occurs during tests. The factors affecting preloading effect on small-strain and nonlinear shear modulus and damping properties are detailedly analyzed. The nonlinear dynamic properties of sand after preloading are presented and discussed. Reorientation of soil particle during preloading is assumed to be the cause of the reduction in shear modulus even if the void ratio decreases after preloading.

CHAPTER 7 CONCLUSIONS AND OUTLOOK

7.1 Conclusions

7.1.1 Calibration

A simple and reliable calibration method for determination of the mass polar moment of inertia of drive system (I_0) of the Stokoe resonant column apparatus was proposed based on the testing results of eight various torsional stiffness aluminum calibration bars. Study shows that the tested I_0 increases with an increase in torsional stiffness of calibration bar, which is accounted for the compliance among the components on the top of specimen. To achieve the I_0 , the equivalent I_0 for each resonant frequency of corresponding calibration bar was back compute based on the tested resonant frequency by assuming the shear modulus of aluminum as 26.5 GPa. The equivalent values of I_0 were plotted against the tested resonant frequencies; the data were fitted by a polynomial with the correlative coefficient larger than 0.999; finally the intercept of the polynomial was taken as the mass polar moment of initial of the drive system. The proposed method may decrease the deviation of the tested shear modulus from the average I_0 method, which is suggested by GDS instrument Ltd, from 20% to around 1% at the frequency of 235.8 Hz. For the torsional shear test, the influence of torsional stiffness and input voltage on calibration factor was analyzed, torque factor slightly decrease with the input voltage, but apparently increases with torsional stiffness of calibration bar, which is accounted for the offset of permanent magnets from the center of drive coils.

7.1.2 Dynamic Properties of Berlin Sand

Small-strain Shear Modulus

- (1) Small-strain shear modulus significantly increases with confining pressure and distinctly decreases with void ratio, this relationship can be well expressed by Equation 5.4. It should be noted that Equation 5.4 shows the best fitting results compared to those popular equations listed in Table 5.1 with the lowest mean and maximum deviation.
- (2) Unlike clayey soil, a slight influence of overconsolidation ratio on small-strain shear modulus was observed on Berlin sand, the values of G_{max} obtained at loading path were measured slightly higher than those at unloading path. Shear modulus increase with an increase in confinement of duration, this increase is relatively unimportant compared to confining pressure and void ratio.
- (3) Water Content has a visible influence on shear modulus of Berlin sand. For a given density specimen, there exists optimum water content below which the G_{max}

initially increases with increasing water content and beyond which decreases with water content. The rate of increase in G_{max} with water content decreases with increasing confining pressure.

(4) Small-strain shear moduli were measured slightly higher in specimens prepared by the tamping method than those by raining method, which is accounted for the orientation of soil particles for samples prepared by tamping method especially for relatively higher density samples.

Nonlinear Dynamic Properties

Confining pressure has a significant influence on the shear modulus reduction and damping curves. As confining pressure increases the modulus reduction curve shifts to at a higher position and the damping curve shifts to a lower position. These relationships can be well modeled by Equations 5.8 and 5.13 for Berlin sand. Void ratio shows no significant influence on both modulus reduction and damping curves. Water content also was observed no affecting on the damping curve.

Comparison of RC and BE Tests

Comparison of resonant column and bender element tests indicates that small-strain shear moduli measured by resonant column tests were determined greater than by bender element tests when the stiffness of specimen exceeds a threshold value. The distance in G_{max} between RC and BE tests increases with increasing confining pressure, and the relationship between these two values can be correlated by Equations 5.17 or 5.18.

7.1.3 Effects of Preloading on Dynamic Properties of Sand

Small-strain Shear Modulus Correction

A simple and feasible small-strain shear modulus correcting method considering the change of dimensions and density of specimen due to the variation of confining pressure and preshear contracting was propose at the beginning of Chapter 6. In general, the deviation of G_{max} without correction is relatively low due to the pressure contracting; however, it may be very large up to around 8% for a medium dense sample when the accumulated axial strain approaches 1.5% as the preloading stress amplitude is relative high after large number of cycles. Actually, if the number of cycles reaches very large number, the deviation may increase up more than 10%. The proposed method may minimize the deviation of shear modulus to lower than 2%.

Small-strain Shear Modulus

- (1) Small-strain shear modulus initially decreases with number of preloading cycles if the number does not exceed a threshold value, thereafter, it starts to increase with number of cycles. The reduction in G_{max} may reach over 20% for looser samples.
- (2) Preloading frequency has a significant influence on the variation of G_{max} with number of cycles. The $G_{\text{max}}(\text{Nc})/G_{\text{max}}(0)$ curve plots higher with an increase in preloading frequency, which is accounted for the higher densification and strong

wear process of interparticles due to higher prestraining amplitude resulting from higher preloading frequency. The influence of preloading frequency decreases with increasing density of sample.

- (3) Preloading ratio was observed to have influence on the variation of small-strain shear modulus with number of preloading cycles. The magnitude of reduction of G_{max} with number of cycles increases with decreasing preloading ratio if the ratio induces the prestraining amplitude exceeds the threshold volumetric strain.
- (4) Density has an influence on the variation of small-strain shear modulus with number of cycles. The maximum magnitude of the reduction in G_{max} decreases with increasing density, as the density of sample increases the capacity of particle reorientation decreases resulting in the less reduction in G_{max} .
- (5) After preloading increasing the confining pressure (Reloading) may increase the small-strain shear modulus under the same confining pressure compared to that of specimen without preloading. The increase in G_{max} may be up to 20% under a concern confining pressure. The greater reduction of G_{max} under the preloading confining pressure the greater increase of G_{max} under the confining pressure which is larger than the preloading confining pressure at reloading path.
- (6) By contraries, after preloading decreasing the confining pressure (Unloading) may further decrease the small-strain shear modulus under the same confining pressure compared to that of specimen without preloading. The decrease in G_{max} may be up to 40.9% under the confining pressure of 15 kPa for Braunschweig coarse sand. The greater reduction of G_{max} under preloading confining pressure the greater reduction of G_{max} under confining pressure which is less than the preloading confining pressure at unloading path.
- (7) Water Content has significant influence on the development of G_{max} with number of cycles and the unloading effect on G_{max} after preloading. The degree of reduction in G_{max} with an increase in water content when it does not exceed a threshold content beyond which the reduction starts to decrease at a given number of cycles. The magnitude of reduction in G_{max} at unloading path after preloading for the wet sample was observed greater than that for dry sample. The relationship between the confining pressure and G_{max} measured at unloading path after preloading can not predicted by linear function but exponential function.
- (8) Sample preparation method and vibration mode were observed to have no influence on the preloading effects on small-strain shear modulus of sand; the increase of G_{max} with time after preloading is unimportant as that for virgin sample. Releasing the confining pressure to a lower level then reloading to the preloading confining pressure may increase the G_{max} after preloading, the lower target pressure the confining pressure has been released to the greater of increase of G_{max} is.

Nonlinear Dynamic Properties

(1) The preloading frequency was observed to has no significant influence on the normalize shear modulus and damping ratio. The damping ratio measured after

preloading was found to be higher than that of virgin sample if the shearing strain amplitude exceeds the elastic threshold shearing strain amplitude if the number of cycles increases to a number larger than 100,000.

- (2) Number of cycles has significant influence on the normalized shear modulus and damping ratio of sample subjected to vibration history. The normalized shear modulus decreases and damping ratio increases with number of cycles. The normalized shear modulus reduction curves of those medium dense samples subject to preloading stress amplitude of 10 kPa at frequency of 20 Hz under confining pressure of 50 kPa the number of cycle lower than 100,000 all plot higher than that of the virgin sample, however as the number of cycles reaches 1,000,000 the curve plots lower than this curve of virgin sample. And the damping ratio shows a contrary tendency to that of the normalized shear modulus.
- (3) The normalized shear modulus reduction and damping measured after preloading are dependent on the variation of small-strain shear modulus after preloading, if the G_{max} after preloading is larger than the G_{max} before preloading the modulus reduction curve after preloading plots lower than the virgin sample curve under identical confining pressure, accordingly the damping curve plots higher; contrariwise, the modulus reduction curve after preloading plots higher and the damping curve plots lower. Unloading confining pressure may further urge the normalized shear modulus reduction curve to plot higher and damping ratio curve lower. Number of cycles under higher confining pressure has no significant influence on the normalized shear modulus and damping properties if the confining pressure is unloaded to the same lower confining pressure for medium dense sand. Confining pressure has less influence on the normalized shear modulus after preloading under higher confining pressure at unloading path, but has significant influence on damping ratio for dense sample.
- (4) Water content plays an important role in the variation of normalized shear modulus and damping ratio with shearing strain amplitude after preloading. The normalized shear modulus decreases and damping ratio increases with increasing water content if the water content does not exceed the optimum value, over which the normalized modulus starts to increase and damping ratio decreases.
- (5) Prestraining and preloading have similar influence on dynamic properties of sand. In practice, the concept of preloading may have more practical significance than prestraining.
- (6) Preloading effects on shear modulus and damping ratio of sand may be accounted for the jointed effects of wear process and reorientation of interparticles during vibration.

7.2 Outlook

Although a lot of work has been performed to investigate the dynamic sand properties and dynamic preloading effects on shear modulus and damping properties of sand, some more investigations are necessary to be carried out in the future. They are given as follows:

- (1) As addressed in Chapter 3, the torque factor for torsional shear test of the Stokoe resonant column apparatus varies with the sample stiffness. Using a mean value of calibration factor may overestimate the shear modulus of softer sample and underestimate that of stiffer sample. A reliable correction method is necessary proposed to analyze the shear modulus of various stiffness samples.
- (2) Water content was found having significant influence on small-strain shear modulus of virgin sample and on preloading effects on dynamic properties of Berlin sand. More levels of water content, particularly fully saturated, are needed to prepare for further study on the influence of water content on shear modulus and damping ratio, especially water contents on nonlinear dynamic sand properties.
- (3) Comparison of resonant column and bender element tests shows that the small-strain shear modulus measured by resonant column test is greater than that by bender element test. This finding is disagreement with some results presented in literature. More comparison tests are need to carried out on various source soil including clayey and sandy soils.
- (4) Although Equation 6.4 may approximately predict the variation of void ratio of dry Berlin sand based on the axial strain measured by LVDT by measuring the dimensions of sample before and after test, which may include some extent measuring deviation. To accurately monitor the change of diameter of sample during test a high precision method should be used if possible, such as the application of proximeter.
- (5) The cause of the increase effect of pressure release on shear modulus after preloading is possibly caused the alternation of microstructure of soil due to the fluctuation of confining pressure; further work is needed to be done to better understanding.
- (6) Flexural vibrating is similar to the dynamic shear existing in the wind power plant engineering. However, the flexural stress is unable to estimate in this study, an estimating method is essential to find for further investigation on the dynamic properties of sample preloaded by this vibration mode.
- (7) Grain size characteristic was found to have influence on preloading effects on dynamic sand properties; more source sands with different grain properties are worthy to investigate for better understanding the nature of preloading effects.

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APPENDIX

CALIBRATION CERTIFICATE

Transducer: LVDT Sensor Transducer Serial Number: 3 Sensitivity: 0.00105 kPa/mV Range: 0-18 mm



Certificate Date: 13-Jul-2009

Calibrated by: Lidong Bai

Signature:

CALIBRATION CERTIFICATE

Transducer: Cell Pressure Sensor Transducer Serial Number: 7 Sensitivity: 0.102138 kPa/mV Range: 0.77-840 kPa



Certificate Date: 13-Jul-2009

Calibrated by: Lidong Bai Signature:

CALIBRATION CERTIFICATE

Transducer: Cell Pressure Secondary Sensor Transducer Serial Number: 8 Sensitivity: 17.07716 kPa/mV Range: 18.5-988 kPa



Certificate Date: 13-Jul-2009

Calibrated by: Lidong Bai

Signature: Rozt 100

CALIBRATION CERTIFICATE

Serial Number:

Sensitivity	0.0002920 mm/mV	
Range	2 mm	
Range	2 11111	
Supply	-24 Volts DC	
Linearity	, ,	



Calibration Date: 06-Apr-09 Signature:

Transducer: Proximeter (Berlin RCA) Transducer